U.S. Investment 1901-2005: Incumbents, Entrants, and Q^*

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Abstract

(PRELIMINARY) Our paper shows that investment by new firms responds to Tobin's Q much more elastically than does investment by incumbent firms. To explain this fact we build a model in which the investment-supply curve of incumbent firms is highly elastic and positively related to Q. However, when variation in Q is caused by shifts in this supply curve, the equilibrium relation between Q and investment that it traces out is negative. That alone causes a negative equilibrium relation between Q and investment. At high levels of Q, however, the investment of incumbents is further reduced, or crowded out, by the positive response of the investment by entering firms to the rise in Q

1 Introduction

Investment in new firms appears to be significantly more elastic with respect to Tobin's Q than investment of established firms. We document this fact with aggregate measures. We show, however, that aggregate investment is negatively related to Q when variation in the latter is caused by shifts in the supply curve for new capital. A rise in Q has the side effect of drawing in more new firms, and this tends to lower incumbent investment even more through a crowding-out effect.

Most economic models imply that investment should respond positively to movements in Tobin's Q. Yet, measured investment of firms shows little response to movements in measured Tobin's Q. So little, in fact, that one needs puzzlingly high capital-adjustment costs to explain the pattern. The puzzle is there in aggregate data and at the firm level.

No such puzzle exists for investment in new firms, however. Venture capitalists invest almost exclusively in young start-up firms. Venture investment as we know it today did not really get off the ground until the 1980s. Figure 1 shows that such

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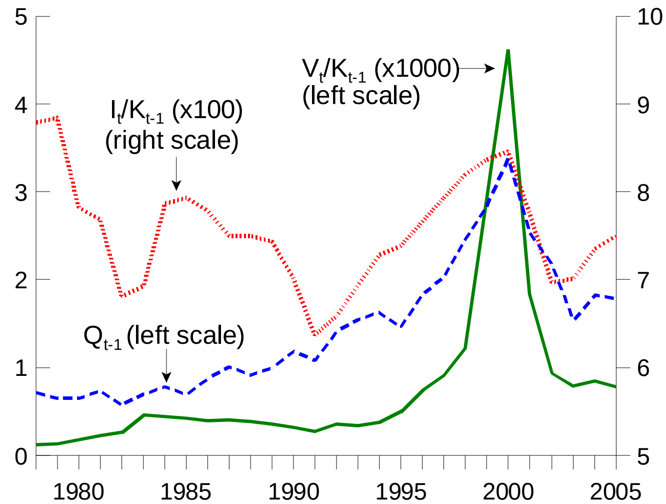

Figure 1: Venture investment, aggregate investment, and Q 1978-2005

investment responds elastically to Q, quite in contrast to aggregate investment which bears little relation to Q. The correlations with Q are 0.844 and 0.186, respectively.[1]

The same is true for IPOs; while IPOs are an imperfect and delayed measure of investment in new firms, they are better than any other century-long time series that is available.[23] They are shown along with Q and aggregate investment in Figure 2.[4]

[1]In Figure 1, data on venture capital investment are from the "Venture Xpert Database" of Thompson Venture Economics, Inc, and represent flows over each calendar year from 1978 through 2005. K_t is measured as the year-end stock of private fixed assets from the detailed fixed assets tables of the Bureau of Economic Analysis (2006, Table 6.1, line 1). I_t is gross private fixed investment from the National Income and Product Accounts. For Tobin's Q, we use fourth quarter observations underlying Hall (2001) for 1978-1999, and ratio splice estimates from Abel (????) to Hall's series for 1999 to 2005.

[2]The incorporations data and establishments data are dominated by dry cleaners, corner stores and such, and therefore not much to do with the model.

[3]Of course it is reasonable that investment would not respond to Q in the region where $Q < 1$. If adverse shocks cause the value of capital in place to fall, and if firms cannot reduce the stock of their capital, the value of their capital can fall below its replacement cost and nothing will happen to investment. In a representative firm model, however, this hypothesis (Sargent 1980) faces the problem that aggregate investment is always positive. When firms can differ, the Qs of some firms would fall below one, while for others – and hence for the economy at large – investment would remain positive. One problem with this is that the capital-weighted fraction of plants with zero investment never rises above 6 percent (Kashyap and Gourio 2007, Figure 1), and so there seems little chance that such a model can ever realistically deliver an aggregate Q below unity.

[4]In Figure 2, K_t is the end-of-year stock of private fixed assets from the BEA (2006, Table 6.1,

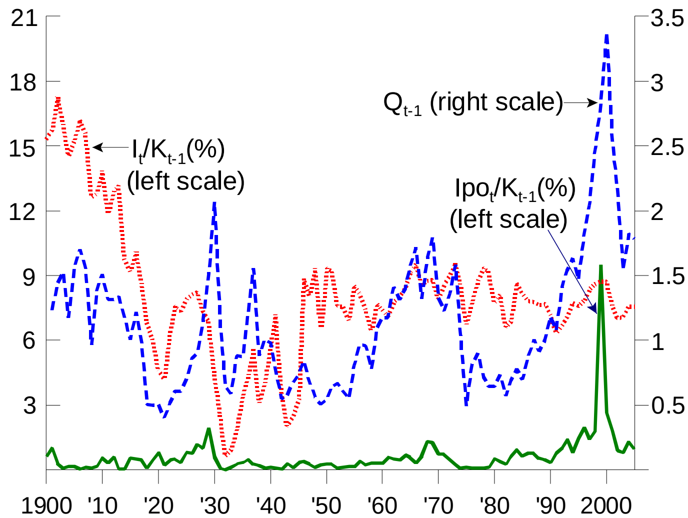

Figure 2: IPOs, AGGREGATE INVESTMENT, AND Q : 1900-2005

Over the past 115 years, and respective correlations are 0.574 and 0.305, while for 1954-2005 the correlations are 0.635 and 0.246.

We shall follow the vintage-capital tradition and stress heterogeneous investment technologies. The heterogeneity will be over different vintages of firms.[5] Instead of a

line 1) for 1925 through 2005. For 1900-1924, we begin with annual estimates from Goldsmith (1955, Vol. 3, Table W-1, col. 2, pp. 14-15) that include reproducible, tangible assets (i.e., structures, equipment, and inventories), and then subtract government structures (col. 3), public inventories (col. 17), and monetary gold and silver (col. 18). We ratio-splice the result to the BEA series. IPOs are measured as the total year-end market value of the common stock of all firms that enter the database developed by the University of Chicago's Center for Research in Securities Prices (CRSP) in each year from 1925 through 2005, excluding American Depository Receipts. The CRSP files include only listings from the New York Stock Exchange (NYSE) from 1925 until 1961, with American Stock Exchange and NASDAQ firms joining in 1962 and 1972 respectively. This generates large entry rates in 1962 and 1972 that for the most part do not reflect initial public offerings. Because of this, we compute the average of the entry rates in 1961 and 1963 and in 1971 and 1973, and assign these averages to the years 1962 and 1972 respectively. For 1900-1924 we obtain market values of firms that list for the first time on the NYSE using the pre-CRSP database of stock prices, par values, and book capitalizations developed in Jovanovic and Rousseau (2001, see footnote 1, p. 1). We continue to use Hall's (2001) fourth quarter data to bring Q_t back to 1950, and then ratio splice the "equity Q" measure underlying Wright (2004) to take the series back to 1900. Note that Hall's measure of Q_t exceeds Wright's by factor of more than 1.5 in 1950, when the splice occurs, producing Qs before 1950 that are considerably higher than Wright's original estimates.

[5] Jovanovic and Rousseau (2001) show that old firms and firms with old capital trade at a discount, especially in sectors where technological progress is rapid. Thus more efficient new capital devalues

full-blown vintage-capital model, however, we model an economy in which capital is homogeneous, but in which entrants and incumbent firms have different investment technologies. Entrants will face convex capital-adjustment costs in the tradition of Lucas (1967) and Lucas and Prescott (1971), whereas incumbents will have constant but random costs of investment. The treatment will be a stochastic version of the 'spin-out' model Prescott and Boyd (1987).

Our model will, however, reverse a line of causation usually present in the vintage-capital model. In our model, as in the vintage capital model, the value of old capital is determined by the state of the investment technology – the technology of the latest vintage determines the market value of capital of earlier vintages. In our model, by contrast, shocks to the investment technology of incumbent firms determine their own Tobin's Q and the value of creating new firms. This small change in the vintage-capital model is important for explaining the pattern shown by Figures 1 and 2.

Our model shares the following feature of the vintage-capital model: High stock prices (i.e., a high value of capital in place) are a signal of an unfavorable shock to the incumbent firms' technology for creating new capital.[6] As a result, aggregate investment is decreasing in Q.

Two papers that stress investment of new productive units in a business-cycle context are Campbell (1998) and Bilbiie, Ghironi and Melitz (2006). Both papers treat the capital of incumbents as fixed at its entering value, however, and so they do not decompose aggregate investment as we do here.

2 Model

Aggregate output is zk. There is one capital good, k, but two ways to augment it: via investment of incumbents, X, and via investment of entrants, Y, so that capital evolves as

$$k' = (1-\delta)\,k + X + Y. \tag{1}$$

The aggregate resource constraint expresses the consumption of the representative agent as

$$C = zk - qX - h\left(\frac{Y}{k}\right)k \tag{2}$$

The RHS of (2) is linear homogeneous in (k, X, Y).

On the RHS of (2), the two forms of investment cost are subtracted from output. The first cost, qX, is interpreted as the investment costs borne by incumbent firms

old capital. Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) have used this logic to argue that technological progress caused Q to fall below 1 in the mid 70s and remain there for 10 years.

[6] This feature would be present in Greenwood, Hercowitz and Krusell's (1997, 'GHK') model if final-goods producers had convex adjustment costs to investment in addition to having to buy their capital from the capital-goods sector.

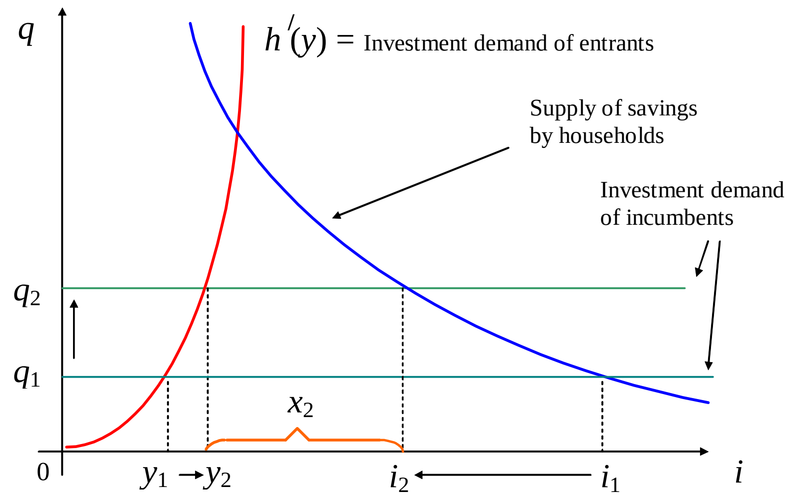

Figure 3: The determination of y via (5)

which face a constant cost, q, per unit of capital created. We assume that q is random. In our model, q reflects the efficiency of entrants relative to incumbents. In the spirit of vintage-capital models, when q is high, the cost of entering capital is low relative to the cost of expanding the capital of incumbents. The parameter q will be regarded as a shock that is not reflected in the BLS measures of capital-goods prices. GHK argue in a two-sector model that $1/q$ is the productivity of the capital-goods sector and is directly related to the measured price of capital. In our model, however, q is a shock *relative* to any price of capital that anyone can measure. The model assumes that the measured cost of capital is unity. Moreover, and in contrast to GHK, k is a capital aggregate, an amalgam of physical and human capital, and so q includes (maybe predominantly so) shocks to the training function.

The other cost, $h\left(\frac{Y}{k}\right)k$, represents the costs of creating entering capital. We follow Prescott and Boyd (1987) and assume that every unit of entering capital is 'spun out' by incumbents. In return for providing the investment needed to create that spinout, the incumbent pays commensurately lower earnings to the workers that will end up managing the spinout. This arrangement can work because, as Becker (1993) explains, general training should be financed by the worker.[7]

The determination of investment.—We shall now show that the ex-dividend price of capital will equal q. The investment rate of entering capital is fully determined by, and increasing in q, as shown in Figure 3. Incumbent investment will take up the slack between desired total investment and the investment of entrants. There will be a second shock, z, and the supply of savings and, hence, the residual incumbent

[7]Other models in this spirit are Chari and Hopenhayn (1991), Franco and Filson (forthcoming) and Chatterjee and Rossi-Hansberg (2007).

investment will depend on both q and z. Figure 3 illustrates the effect of a rise in q when z is held constant. The 'interiority' requirement that $x > 0$ implies that we can determine y from the intersection of the entrants' investment-demand curve $h'(y)$ with q. As q rises while z stays fixed, two things happen: First, savings declines and with it total investment i must fall from i_1 to i_2. Second, the supply of entrants rises from y_1 to y_2, thereby crowding out an even larger amount of incumbent investment. A reading of Gompers (2004) suggests a high degree of substitutability between a corporation's in-house investment in physical and R&D capital and its venturing investments.[8]

The economy has no external effects or monopoly power and equilibrium can be represented by a planner's problem. This problem has a traditional one-sector representation with a single cost of adjusting the economy-wide capital stock. That adjustment cost function will be a reduced form, representing the outcome of a static allocation problem, namely one of minimizing the cost of providing a certain amount of new capital conditional on the realization of q alone.

2.1 The Planner's problem

Preferences are $E_0 \left\{ \sum_0^\infty \beta^t U(C_t) \right\}$. Let $s \equiv (q, z)$ be stochastic with transition function $F(s', s)$. The state of the economy is (s, k), but since returns are constant and preferences homothetic, k will not affect prices or investment rates. The planner's problem is to maximize the representative agent's expected utility by choosing the two kinds of investments X and Y. The planner has no other technology. The Bellman equation is

$$V(s,k) = \max_{X \geq 0, Y \geq 0} \left\{ U\left(zk - qX - h\left(\frac{Y}{k}\right) k \right) + \beta \int V(s', (1-\delta) k + X + Y) \, dF(s', s) \right\}. \tag{3}$$

Let $y = Y/k$ and $x = X/k$. The FOCs are[9]

$$-qU' + \beta \int V_k dF = 0, \tag{4}$$

and

$$-h'(y) U' + \beta \int V_k dF = 0.$$

We shall assume throughout that both FOCs hold with equality. To ensure this, we assume that $h(0) = 0$, which rules out the value $Y = 0$, and that (ii) $h'(y) > q_{\max}$ at a value of y too low to satisfy the demand for saving in any state s, which rules

[8] On the other hand, only about ten percent of venture investments are made by corporations – see Chart 4 of Gompers (2004).

[9] Differentiability of V in k will be shown below.

out $x = 0$.[10] Combining the two FOCs leads to

$$h'(y) = q. \tag{5}$$

as illustrated in Figure 3. Therefore $y(q) \equiv (h')^{-1}(q)$ is increasing in q.

If y always satisfies (5), we can write (3) as[11]

$$V(s,k) = \max_{k' \geq (1-\delta)k} \left\{ U\left(zk - q\left(k' - (1-\delta)k - y(q)k\right) - h\left(y(q)\right)k\right) + \beta \int V(s',k')\, dF(s',s) \right\}$$

whence

$$\begin{aligned} V_k &= U'(C)\left(z + q\left(1-\delta+y(q)\right) - h\left(y(q)\right)\right) = U'(C_t)\left(z_t + q_t\left(1-\delta+y_t\right) - h(y_t)\right) \\ &= U'(C_t)\left(\frac{D_t}{k_t} + q_t\left(\frac{k_{t+1}}{k_t}\right)\right) \end{aligned}$$

because $= 1 - \delta + y_t = \frac{k_{t+1}}{k_t} - x_t$. Therefore the price of a unit of capital satisfies[12]

$$q_t = \beta \int \frac{U'(C_{t+1})}{U'(C_t)} \left(\frac{D_{t+1} + q_{t+1}k_{t+2}}{k_{t+1}}\right) dF(s_{t+1}, s_t). \tag{6}$$

This is the discounted expected value of the firm's earnings and in the standard decentralization as done by Lucas (1978) would be the price of capital that shareholders pay. The replacement cost of capital is unity, and therefore q is also what is known as Tobin's Q. We should think of q_t as the price of capital at the *end* of the period, since a purchase at date t at that price does not entitle the holder to period-t dividends.

Let preferences be

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

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[10] This requirement will be considered explicitly when we solve for the growth rate in the deterministic case in Section 2.2.

[11] Evidently, the model has a standard one-sector representation. Let $I = X + Y$ and let

$$f(i,q) \equiv \min_x \{qx + h(y)\} \qquad \text{subject to} \quad x + y = i.$$

Then the economy is equivalent to one in which, instead of (1) and (2), we have

$$C = zk - f\left(\frac{I}{k}, q\right)k \qquad \text{and} \qquad k' = (1-\delta)k + I.$$

[12] If x_{t+1} and y_{t+1} both were to equal zero, then we would get the more intuitive expression

$$q_t = \beta \int \frac{U'(C_{t+1})}{U'(C_t)} \left(z_{t+1} + (1-\delta)q_{t+1}\right).$$

in which case the value function must satisfy

$$V(s,k) = v(s)\, k^{1-\sigma}, \tag{7}$$

where

$$v(s) = \max_{x,y} \left\{ \frac{(z - qx - h[y])^{1-\sigma}}{1-\sigma} + (1-\delta+x+y)^{1-\sigma} v^*(s) \right\} \tag{8}$$

and

$$v^*(s) = \beta \int v(s')\, dF(s', s).$$

If the processes z and q are positively persistent and if they are mutually independent, v and v^* are strictly increasing in z and strictly decreasing in q.

The investment policies.—By (5) we know that y depends on q alone and is increasing in q.[13] Regarding x, we have

Proposition 1 *x is (i) strictly increasing in z, and (ii) strictly decreasing in q.*

Proof. (*i*) The RHS of (9) is strictly increasing in z directly and increasing through v^*, and it is strictly decreasing in x. The LHS of (9) does not depend on z. Then the implicit function theorem yields the first claim. (*ii*) From (7),

$$V_k(s,k) = (1-\sigma)\, v(s)\, k^{-\sigma},$$

and then (4) reads

$$\begin{aligned} q &= \frac{1-\sigma}{U'} \left([1-\delta+x+y]\, k\right)^{-\sigma} \beta \int v(s')\, dF(s', s) \\ &= (1-\sigma) \left(\frac{z - qx - h(y)}{1-\delta+x+y} \right)^{\sigma} v^*(s) \\ &= (1-\sigma) \left(\frac{z - qx - h\left((h')^{-1}(q)\right)}{1-\delta+x+(h')^{-1}(q)} \right)^{\sigma} v^*(s), \end{aligned} \tag{9}$$

which allows us to solve for x as a function of $v^*(s)$. Since $(h')^{-1}(q)$ is increasing in q, the term $(\cdot)^{\sigma}$ is strictly decreasing in q, and so is v^*. Therefore the RHS of (9) is strictly decreasing in q. As we mentioned under (*i*), the RHS of (9) is strictly decreasing in x. Since the LHS of (9) is increasing in q, the implicit function theorem then delivers the second claim. ■

[13] The simplest way to overeturn this exclusion restriction and to get q to depend on z would be to assume that h depends on z.

2.2 Micro issues

To motivate the two decentralizations, we need to comment on the way the empirical and theoretical literature has handled the concepts we have modeled above.

The literature on entry (Carroll and Hannan 2000) has distinguished three types of firm entry:

- De Novo entry = Entry by a new firm
- Spin-off entry = Technically, entry by a new firm
- De Alio or 'lateral' entry = Entry by an existing firm.

Entry, however, can be into a new industry or into an already established or 'old' industry. Each type of entry carries its own investment. Therefore 'entry investment' is of three types. All investment that is not 'entry investment' will be termed 'incumbent investment.' Incumbent investment is the investment of an existing firm in an industry where it has operated in the past.[14] Things become clearer with the help of the following Table:

		Origin of investment	
		New Firm	Old Firm
Destination	New Industry	a	b
of investment	Old industry	c	d

In this table, the symbols a, b, c, and d denote total amounts of investment of the various kinds. Specifically, then, we may associate these investment types with the types of firms that carry out that investment as follows:

$a + c$ = De Novo + Spin-off investment

b = lateral investment

d = Incumbent investment+ lateral investment

Possible meanings of the paper's symbols:

(i) $X = d$, and $Y = a + b + c$

(ii) $X = b + d$, and $Y = a + c$

We are in doubt about whether b should be part of X or part of Y. Thus we are flexible on how to interpret empirically the model's symbols. Some comments now on how we shall match the data to the concepts of cost in (2):[15]

[14] In fact, some incumbent investment may be hard to classify because of the multiproduct nature of production in most plants. A firm may build a plant that produces some of an 'old' product, and some of a new product, in which case it is ambiguous whether that investment represents item b or item d.

[15] Alongside the sort of macro model we have built here, models of the various entry margins are developed and tested – see Wang (2007) for a systematic step in this direction.

1. *The asymmetry in the cost of new and old capital.*—The cost of an incumbent firm's capital is qX, and that of entering firms is $h\left(\frac{Y}{k}\right)k$. We think of q as the unit cost of routine investment, and of h as the cost of doing new things. New ideas vary in quality and the good ones yield a higher return. The best ideas are exploited first, hence h' is upward sloping. The microfoundations of spinouts involve asymmetric information. Klepper and Thompson (2006) argue that spinouts occur because people disagree on management decisions – people leave firms to develop their ideas on their own because coworkers would otherwise implement the idea suboptimally. Chatterjee and Rossi-Hansberg (2007) is about adverse selection – the best ideas leave the firm. Some evidence for this is in Prusa and Schmitz (1993).

2. *No externalities.*—Neither decentralization allows for externalities. We shall be assuming a representative firm that treats the cost $h\left(\frac{Y}{k}\right)k$ as internal. If Y stands for all new capital in new firms, this means that new firms can be formed only by paying the cost $h\left(\frac{Y}{k}\right)k$. Each new firm, therefore, is a product of activity done in incumbent firms. Some de-novo startups by people with no relevant labor-market experience. Only the first two categories are covered by the RHS of (2), but perhaps the third does not matter quantitatively. This is analytically extremely convenient, for it implies that equilibrium can be calculated via the planner's optimum.[16]

3. *Who owns the claims to earnings that entry-investment creates?*—In the first decentralization the claims to the profits of the new firms will be held by incumbent firms, resembling corporate venturing activity that most large firms undertake.[17]In the second decentralization the claims to the earnings will be owned by ex-employees of incumbent firms. The second decentralization is of greater empirical significance because spinouts are empirically larger than corporate venturing.

[16] One could, instead, treat a part of the costs as external, such as

$$kh\left(\frac{Y}{k}\right)G\left(\frac{\mathbf{Y}}{\mathbf{k}}\right),$$

with k the component internal to the firm, and $(\mathbf{Y}, \mathbf{k})$ as the external part. The firm's problem would remain linearly homogeneous, and equilibrium would remain qualitatively the same. Equilibrium growth, however, would generally no longer be efficient.

[17] Corporate venture investment is remarkably similar to that of independent VCs. The main difference that is relevant here is that corporations have a slight tendency to avoid seed rounds and early rounds (Dushnitsky and Shapira 2007, Table 2). In 1983 Corporations made only 5 percent of all VC investments, but by 1994 this was up to 12 percent (Gompers 2002, Table 3).

2.3 Decentralization 1: A corporate venturing economy

We can think of $h\left(\frac{Y}{k}\right)k$ as the cost of discovering new investment opportunities adding up to Y units of tomorrow's capital. Experience helps in such discovery and therefore k lowers its cost. This fits the activity of venture investment by corporations. These corporations retain ownership of the dividends that these investments will provide in the future. We discuss later the empirical significance of such activities. The component qX represents the costs of reinvesting in routine activities.

We shall assume that the firm maximizes the value of its current shareholders, measured in units of current consumption. As Lucas and Prescott (1971) explain, the firm takes as given the next-period value of its capital, its ex-dividend value today being q per unit of capital created. The firm's maximization problem is then

$$w = \max_{x,y} \left\{z - qx - h(y) + (1 - \delta + x + y)\, q\right\}. \tag{10}$$

This means that the firm's y must solve (5), but x must be obtained from the household savings decision and the identity that savings = investment = $x + y$. The linear technology for creating k does not yield any rents because the unit price of capital adjusts to equal the average and marginal cost of creating it. Therefore (10) reduces to

$$w = z + q(1 - \delta) + \max_{y} \left\{qy - h(y)\right\}.$$

We shall only sketch this (MORE NEEDED HERE) since it straightforwardly adapts Lucas (1978). The household maximizes discounted expected utility subject the constraint

$$qn' + c = (z - x + q)\, n$$

where n is the number of shares of the representative firm that the household owns and where $c = C/k$. The household's FOC and that of the firm then lead to (6).

2.4 Decentralization 2: A spin-out economy

We can, instead, think of $h\left(\frac{Y}{k}\right)k$ as the cost (either direct or in the form of foregone output) of providing the firm's workers with the training needed to discover new investment opportunities, but where the implementation of these opportunities is done by the workers after they leave the firm. The parent corporation now does not own the dividends that these investments will provide. Rather, it will charge for the training that it provides by paying lower wages. This decentralization will be essentially that of Prescott and Boyd (1987, 'PB'), and in the spirit of the analyses of general human capital in Becker (1993). People live for two periods and have preferences $E\left\{U(c_Y) + \beta U(c_O)\right\}$ where c_Y is consumption when young and c_O consumption when old. The young inherit all the capital that the firm creates, but a certain fraction of them, Y_t/k_{t+1}, start new firms. The rest stay and continue to operate the existing entity.

2.4.1 The version with size of firm exogenous

Output can be produced only if an old worker (the 'manager') and a young 'worker' are present. Population is constant and each firm is composed of an equal number of old and young workers. Let k be the human capital of the managers. Let $k' = k + I$ be the total human capital given to each young worker. Net output per unit of k is $z - f(i, q)$ where, as discussed in footnote 10, the firm's investment-cost minimization problem is

$$f(i, q) \equiv \min_{x,y} \{qx + h(y)\} \text{ subject to } x + y = i. \tag{11}$$

That is, the firm provides the capital as cheaply as possible so that y still solves (5). We then say that a fraction

$$\frac{y(q)}{1 - \delta + i(s)}$$

of the workers start new companies, while the rest stay with the same company.

Let k or, rather, the firms be the only asset. A firm is owned by its manager(s). No other assets exist in this economy but because returns are constant there is no borrowing and lending among firms in the equilibrium that we shall describe.

Because the young care about lifetime expected utility, the manager will choose the wage-training package that provides his one worker the equilibrium utility as efficiently as possible. Suppose that the manager consumes $kp(s)$. We take the function $p(s)$ as given for now, and we assume it to be independent of k. The lifetime utility of the worker, ϕ, is

$$\phi(s, k, p(.)) = \max_i \left\{ U(k(z - f(i, q) - p(s))) + \beta \int U(k(1 - \delta + i) p(s')) \, dF(s', s) \right\}.$$

Since $\frac{\partial f}{\partial i} = q$, this gives rise to the investment policy $i(s)$ satisfying

$$q = \frac{1}{U'(C_Y)} \beta \int p(s') U'(C_O) \, dF(s', s) \tag{12}$$

With $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, (12) reads

$$q = \frac{(1 - \delta + i)^{-\sigma}}{(z - f(i, q) - p(s))^{-\sigma}} \beta \int [p(s')]^{1-\sigma} \, dF(s', s), \tag{13}$$

and it is therefore indeed feasible, as we assumed, that $p(s)$ and i should be independent of k. Eq. (13) Therefore it is as if the worker buys the firm from the manager at a fee $kp(s)$ and maximizes his utility with the expectation that he will receive $k'p(s')$ next period.

There appear to be many other equilibria in this OLG economy, each corresponding to a different weight of the young and old in the sharing of output. This is a point that Cozzi (1998) has already made in a similar setting. An equilibrium to this situation is the one that we already displayed, namely, one in which everyone consumes a constant fraction of output in each period. In that case, (12) and (4)

$$-qU' + \beta \int V_k dF = 0, \tag{14}$$

are the same, because when everyone consumes the same amount, $\int V_k dF = E_t U'(C_{t+1})$, and therefore $p_t = q_t$.

Example.—Let $\sigma = 1$. Then (13) reads $q = \beta \frac{z - f(i,q) - p(s)}{1-\delta+i}$, whence we get

$$i + \frac{\beta}{q} f(i,q) = \frac{\beta}{q}(z - p(s)) - (1-\delta). \tag{15}$$

Since $\frac{\partial f}{\partial i} = q$, the LHS has a derivative w.r.t. i equal to $1+\beta$ as long as the solutions for x and y are interior. Therefore, $i = \mu - \frac{\beta}{q(1+\beta)} p(s)$,where $\mu > 0$ is a constant. Therefore, equilibria with a larger share of the old imply less investment and less growth. On reflection we should have expected this, because (i) If this were a savings problem, a change in the return on capital would have exactly offsetting income and substitution effects, and (ii) A larger p implies a subtraction from the output left over for the consumption and investment of the young. Therefore there is a second negative income effect that causes investment to fall because consumption is a normal good.

2.4.2 The version with size of firm endogenous

Endogenizing firm-size.—We follow PB and reach a unique solution by introducing a firm-size margin. To narrow down the equilibrium, PB append a decision about firm size as follows: Let n be the firm's employment and let the firm's output be

$$\text{output} = kQ(n)$$

where k is the quality of the manager, as in Lucas (1978a). Let k, X, and Y be capital and investments per worker. Let the firm's investment costs depend only on the totals accumulated[18], nX and nY, so that these costs are $qnX - h\left(n\frac{Y}{k}\right)k$. The firm's revenue per unit of k then is

$$zQ(n) - qx - h(ny),$$

where Q is increasing and strictly concave. For $n \neq 1$, the firm's subproblem (11) becomes

$$f(i,q,n) \equiv \min_y \{qn(i-y) + h(ny)\} \tag{16}$$

[18] This is different from eq. (1) of PB

and when evaluated at $n = 1$, the FOC is still (5).

Since the firm's output is shared among n workers, the worker-participation constraint now reads

$$\max_i \left\{ U\left(\frac{k}{n}\left(zQ(n) - f(i,q,n) - p(s)\right)\right) + \beta \int U\left(k(1-\delta+i)\,p(s')\right) dF(s',s) \right\}.$$

Normalizing $Q(1) = 1$, when evaluated at $n = 1$, the FOC w.r.t. i is still (12). Finally, the FOC w.r.t. the new choice variable, n, is

$$-\left(z - f(i,q,1) - p(s)\right) + zQ'(1) - q(x+y) = 0,$$

in light of (5). Then since $q(x+y) - f(i,q,1) = qy - h(y)$,

$$p(s) = z\left(1 - Q'(1)\right) + qy - h(y) > 0. \tag{17}$$

The inequality follows because (i) $1 - Q'$ is the average minus the marginal product of a unit measure of workers and (ii) Since $h' = q$, $qy - h$ is the marginal minus average cost.

Example: Once again, let $\sigma = 1$, $Q(n) = n^{\alpha}$, and $h(y) = \gamma y^2/2$. Then

$$p(s) = (1-\alpha)\,z + \frac{q^2}{2\gamma}$$

$Q'(1) = \alpha$, and $h(y) = \gamma y^2/2$. In that case Since (15) which is still valid, we substitute into it for $p(s)$ from (17) to get

$$i + \frac{\beta}{q} f(i,q) = \alpha\frac{\beta}{q} z - (1-\delta) - \beta\frac{q}{2\gamma}$$

which has exactly one solution for i whenever the parameters are chosen so that RHS is always positive.

3 Simulation and data fitting

3.1 Regressions with data in Figures 1 and 2

Measurement.—To take our theory to the data, we need measures of the state variables q and z, and of x and y. In light of (6), we measure q by Tobin's Q. Since output is zk, we measure z by the ratio of private output over the course of a given year to private capital at the start of that year. This measure is not accurate for the period from the start of U.S. involvement in World War II until several years after the war because, as Gordon (1969) explains, capital used by private firms was sometimes classified as Government capital, and this would cause $\hat{z}$ to be biased upwards. For this reason we exclude the 1941-1952 period from our regression analysis.

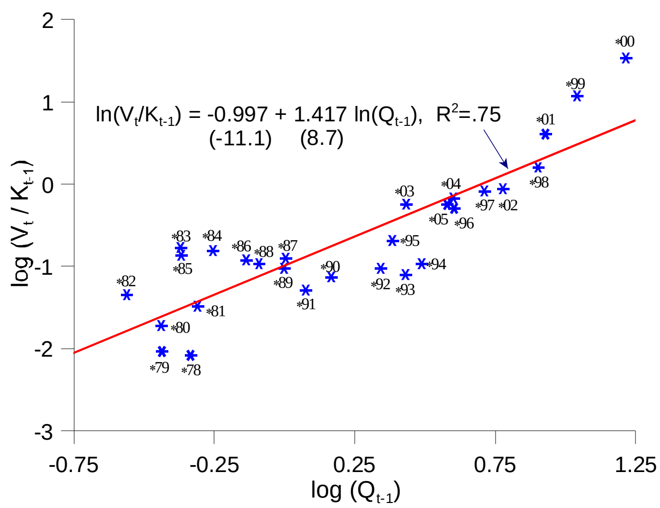

Figure 4: The relation between venture investment and Q, 1978-2005

We entertain two measures for y_t. The first is total investment of venture capital funds. It covers a significant fraction of startup investment, and very little else. Occasionally, venture capital (VC) funds are involved in taking mature companies public, but this is rare and the investment involved is small. Thus, while our series for VC-investment series excludes new firms that are not backed by venture capital, it does not include anything that we would call incumbent investment. Figure 4 shows the strong and positive univariate relation between the log of V_t, defined as the ratio of total VC investment per \$1000 of beginning-of-year capital, and the log of Q at the start of each year from 1978 to 2005.

Our second measure for y_t is the total year-end market value of firms that had an IPO during year t, which we denote by IPO capitalization. Though with a lag, this measure will include not only VC investments but also corporate venturing investments (Gompers 2002).[19] It is remarkable how similar the corporate-venturing portfolio firms are to the independent VC-backed portfolio companies. They are concentrated only slightly in investment rounds that come later than the typical rounds of independent-VC-backed investments, but are otherwise quite similar in terms of the size or the investments and industries covered. Corporate-backed ventures are more likely to reach IPO (Gompers 2002, Table 6), which runs counter to popular perception that corporations invest in young startups in order to acquire them once they reach a viable stage. Recalling that q_t is the end-of period price of capital,

[19]During the late 1960s and early 1970s, more than 25 percent of the Fortune 500 firms set up divisions that emulated venture capitalists.

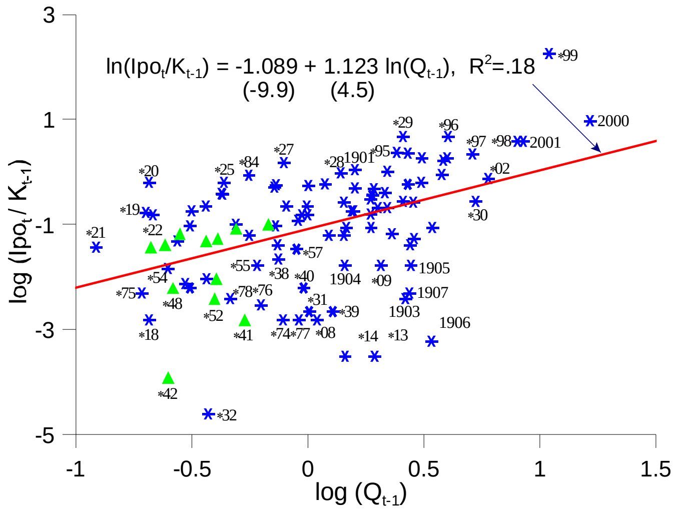

Figure 5: The relation between IPOs values and Q

the date-t value of entering-firms' capital relative to the value of *beginning*-of-period capital is

$$\varepsilon_t^* = \frac{q_t y_t k_t}{q_{t-1} k_t} = \frac{q_t}{q_{t-1}} h'^{-1}(q_t) \tag{18}$$

(from (5)). Then the model says that z should not enter this equation once q_t and q_{t+1} are held fixed. If adjustment costs are quadratic in y: $h(y) = \gamma y^2/2$, then $y_t = q_t/\gamma$, and so (18) reads

$$\varepsilon_t^* = \frac{q_t^2}{\gamma q_{t-1}}. \tag{19}$$

or, taking logs, $\ln \varepsilon_t^* = -\ln \gamma + 2 \ln q_t - q_{t-1}$. We also note that z_t does not enter this regression. We measure $q_{t+1} y_t k_t$ is the end-of-t value of all the firms that listed during year t, and $q_t k_t$ is the value of firms listed at the beginning of year t. Figure 5 once again shows a strong and positive univariate relation between this second measure of y and Q.[20]

We also entertain two measures for x. The first is private investment, I_t, deflated by the private capital stock at the start of the year, K_{t-1}. This would be the right measure if all entering investment was in the form of foregone output. But venture investment, for one, is measured investment. The second measure subtracts a measure of Y from I to arrive at an estimate of X that we denote by $\hat{X}_t$. For the period 1978-2005 we define $\hat{X}_t = I_t - 2V_t$ because roughly half of the firms that have IPOs in the

[20] The years from 1941 to 1953 appear as light triangles in Figures 6 and 8, and are not included in fitting the regression relationships shown.

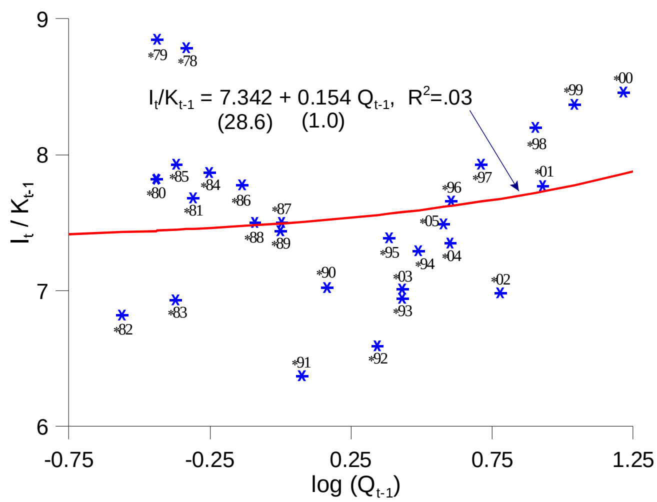

Figure 6: The relation between aggregate investment and Q for the period 1978-2005

United States are VC backed. Since far fewer non-VC-backed firms ever go public, investment in these companies is probably higher than that in VC-backed firms; i.e., $\hat{X}_t$ is probably larger than X_t. The regressions in Table 1 use measures that exclude R&D spending. Figure 6 shows the weak cross-section relation between I_t/K_{t-1} and the log of Q_{t-1} for the 1978-2005 period (i.e., the period covered by Figure 4). The relation is not much stronger for the century as a whole, as Figure 7 shows. As we shall show by means of a regression, the relation that this Figure portrays is illusory in that it is caused by the the omitted variable, z_t, which pulls I_t/K_{t-1} up (especially early on in the 20th Century), and which is positively related to Q_{t-1}. Our model predicts a negative partial correlation coefficient between Q_{t-1} and I_t/K_{t-1} when z_t is held constant, and the regression results will bear that out.

Functional form.—The functional form for adjustment costs will be assumed to be quadratic: $h(y) = \frac{\gamma}{2}y^2$, in which case $y = \frac{q}{\gamma}$. Although we do not have x in closed form as a function of q, we shall assume that it, too, is adequately represented by a linear function of q.

Proposition 1 tells us that the two investment policies, x and y, differ qualitatively in their dependence on q and z. The investment of entrants, y, should depend positively on q, and not at all on z. Investment of incumbents, x, should be decreasing in q and increasing in z. Table 1 reports the regressions that test these implications.

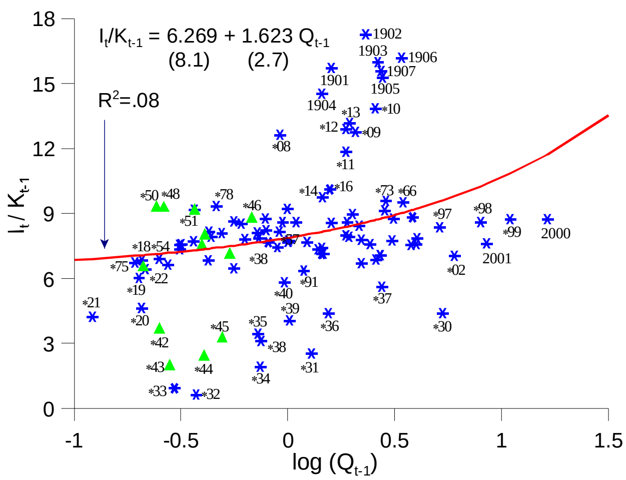

Figure 7: The relation between aggregate investment and Q for the period 1900-2005

Table 1
Entering Capital and Aggregate Investment Regressions

	1978-2005				1901-2005			
	$\ln V_t/K_{t-1}$		$\widehat{X}_t/K_{t-1}$	I_t/K_{t-1}	$\ln Ipo_t/K_{t-1}$		$\widehat{X}_t/K_{t-1}$	I_t/K_{t-1}
const.	-0.997	-6.667	2.315	3.040	-1.089	-9.417	-10.073	-10.656
	(-11.07)	(-3.54)	(0.92)	(1.11)	(-9.90)	(-2.40)	(-3.23)	(-3.87)
Q_{t-1}	1.417	1.879	-0.344	-0.082	1.123	0.911	-0.650	0.460
	(8.72)	(8.98)	(-1.74)	(-0.38)	(4.53)	(3.46)	(-1.09)	(0.87)
z_t		-5.780	0.146	0.121		2.288	0.477	0.474
		(-3.01)	(2.08)	(1.58)		(2.12)	(5.61)	(6.33)
R^2	.75	.81	.15	.12	.18	.22	.26	.36
N	28	28	28	28	93	93	93	93

Note: T-statistics in parentheses. All variables are scaled as in Figures 1 and 2. The regressions for 1901-2005 exclude the years 1941-53.

3.2 Simulations and fitting data

We fit two sets of data. For the entire 20th Century we use IPOs as the measure of new investment. This is Simulation 1. Because we prefer VC investment as a measure of the investment of new firms, and since this measure is available for a shorter, more recent period, we do a second fit of the model to the post-1978 data. That is Simulation 2.

Estimation routine.—The simulated policies are optimal for a discretized version of the (q, z) process. The parameters of the (q, z) precesses were chosen to be AR(1) and fitted via the Tauchen-Hussey procedure. We let z take on 5 values and q take on 15 values. The statistical properties of the discretized processes were then presented to the Planner who chose the optimal policies. In each simulation we chose the parameters $\{\beta, \delta, \sigma, \gamma\}$ to solve the following problem:

$$\begin{aligned}\min_{\{\beta,\delta,\sigma,\gamma\}} \quad & \lambda_x(\bar{x}_{\text{Fitted}} - \bar{x}_{\text{Data}})^2 + \\ & \lambda_\delta(\delta - 0.085)^2 + \\ & \lambda_\sigma(\sigma - 3)^2 + \\ & \lambda_y(\bar{y}_{\text{Fitted}} - \bar{y}_{\text{Data}})^2 + \\ & \lambda_g\left(\sum_q \sum_z [x_t(q,z) + y_t(q,z) - \delta - 0.019]^2\right)\end{aligned}$$

We chose the weights λ_i in such a way that each of the terms in the penalty function are of the same order. Note that the λ_i that accomplished this differed depending on whether we used the data for $1978 - 2005$ or $1901 - 2005$. The estimated parameters were also different.

The estimation routine leaves β unconstrained. If we were to insist on values of β above 0.9, say, the model would overpredict x by a factor of at least two. This is probably because we have a linear production technology and because we understate the capital stock by including only physical capital in the measure of k. This causes us to overstate the estimate of the z series by a factor of about 3 or 4. Rather than introduce another parameter that expresses this overstatement, we shall fit the data with an unreallistically low value of β. This low value of β serves to discourage investment and reduce it to levels that are found in the data. Future drafts of this paper will construct a measure of broad capital that includes human capital.

The results are plotted in Figure 8.

The discretized processes track the actual values extremely well.[21] In both simulations the model fits y pretty well, but it exaggerates the negative relation between Q; the data seem to show an incumbent investment that is flat.

[21] The simulations use the data from Figures 1 and 2 directly. We proxy z_t with private GDP as a percent of K_t, with the latter constructed as described in footnote 2. Private domestic product for 1929 to 2005 is from the BEA (2006, Table 1.1.5, line 1 less line 20). For 1900-1929, we multiply

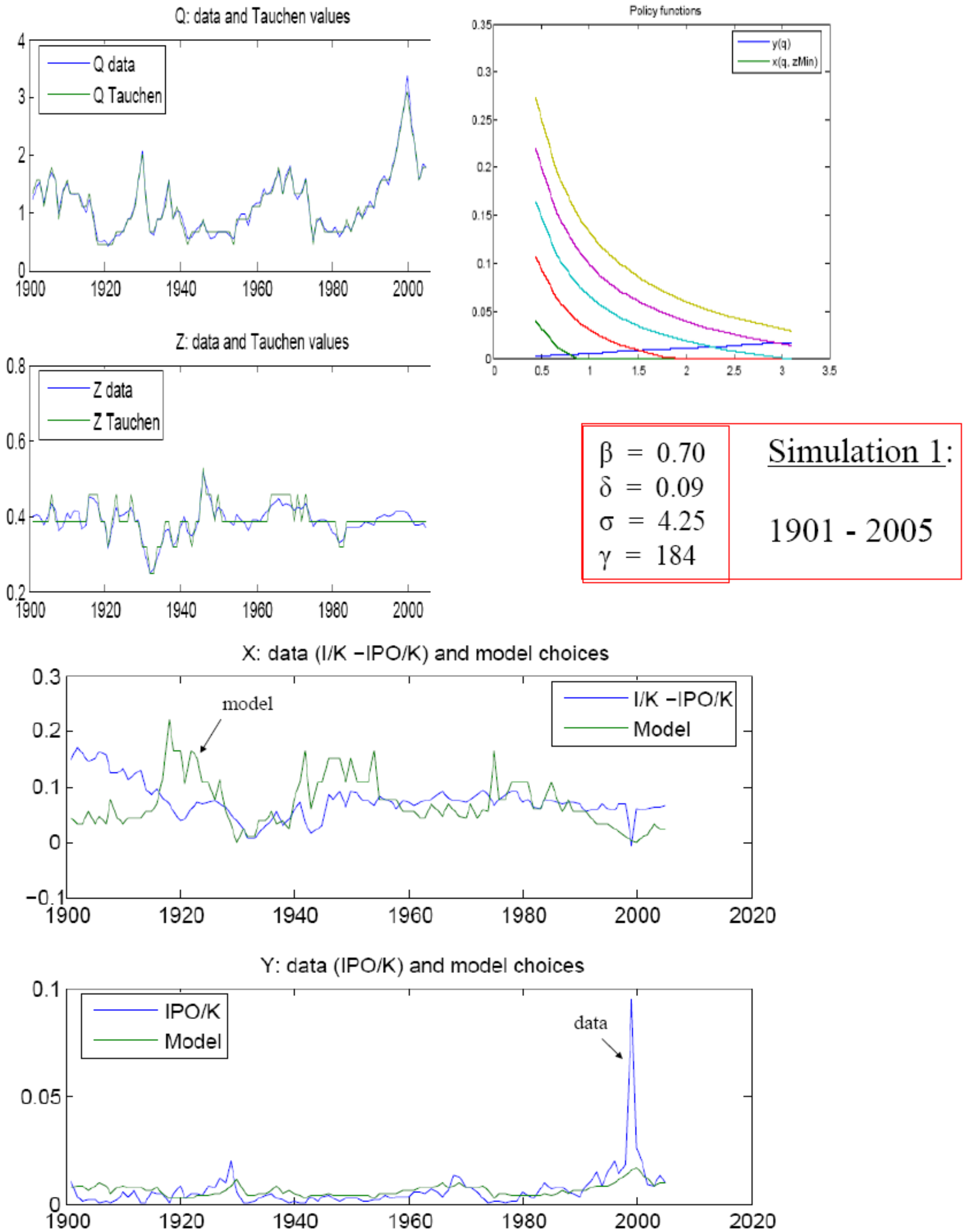

Figure 8: SIMULATION 1: 1900-2005

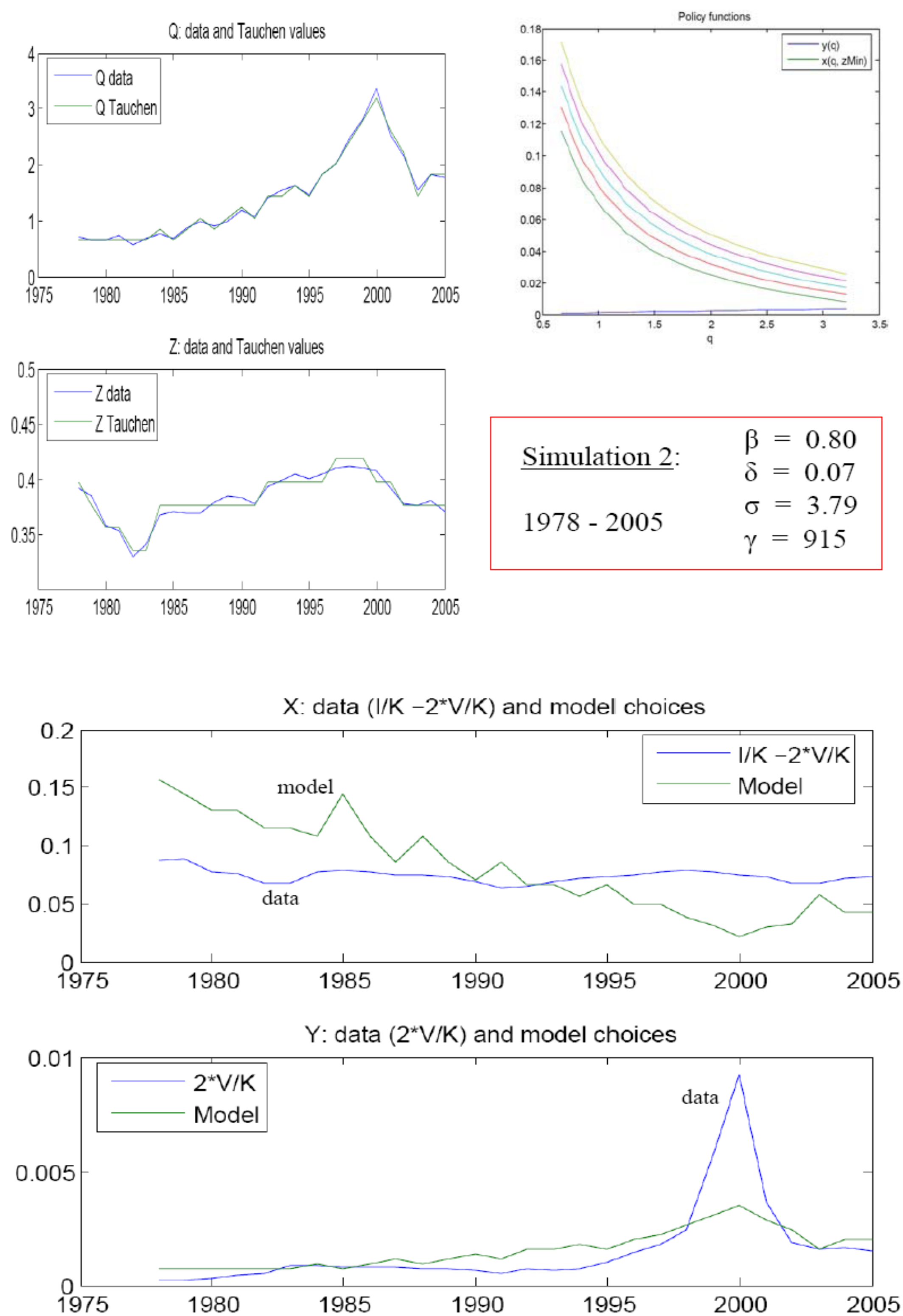

Figure 9: Simulation 2: 1978-2005

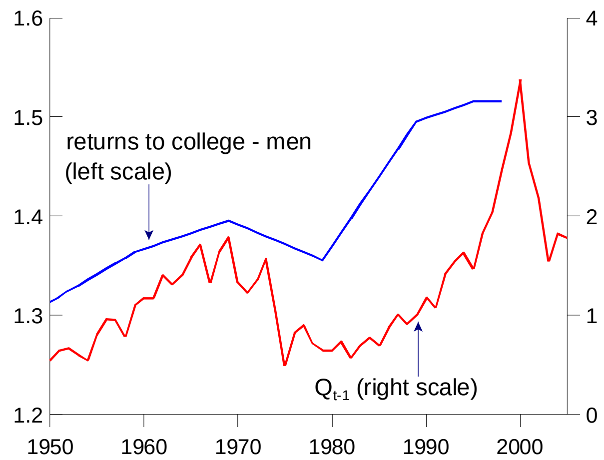

Figure 10: Q AND THE SKILL PREMIUM SINCE 1950

3.2.1 Relation to the micro evidence

The model argues that high values of Tobin's Q are caused by spontaneous rises in the cost of capital formation. If we take a broad view of capital then we must include human capital as part of k. Since it is mainly skilled labor that participates in the creation of new k, it seems that the q should be related to the cost of skilled labor, transformed so that it is stationary. The most famous transformation of the cost of skilled labor is the skill premium which one arrives at by dividing that cost by the cost of unskilled labor. Figure 10 shows the relation between Q and the skill premium for the post-WW2 period, while Figure 11 shows it for the entire century.[22]

private GDP from Kendrick (1961, Table A-III, col. 6, pp. 298-9) in constant 1929 dollars by the GNP deflator, formed as the ratio of nominal GNP from Kendrick (1961, Table AII-b, col. 11, pp. 296-7) to real GNP in constant 1929 dollars from Kendrick (1961, Table AII-a, col. 11, pp. 293-4). The GNP deflator was the closest index that we could construct for private GDP with Kendrick's data.

[22] We construct the century-long skill premium in Figure 11 by joining two quite different series. For 1939-1995, it is the "returns to college" for all men from Goldin and Katz (1999, Figure 1, p. 32, and Table 7, p. 45), which are available on a decadal basis from 1939 to 1989 with a final observation in 1995. These returns are constructed as differences in mean log wages adjusted for the age and experience composition of the workforce. To these data we splice the ratio of the earnings of all clerical workers, excluding supervisors, to the earnings of production workers in manufacturing, which are available in 1895, 1909, 1914, 1919, 1923-29, and 1939 from Goldin and Katz (1999, Table 2, p. 38). When we make the ratio splice in 1939, after linearly interpolating between the available observations to form a continuous series, the return to college is 40 percent, while the clerical to manufacturing wage premium is 15 percent. This means that our synthetic "skill premium" is about 22 percent higher for the 1900-1939 period than the actual ratio of clerical to manufacturing wages.

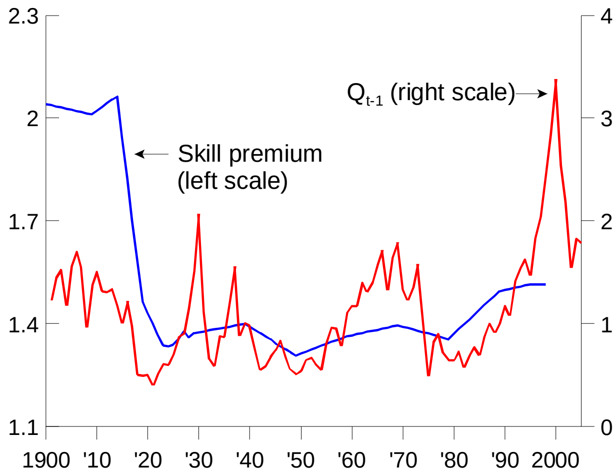

Figure 11: Q AND THE SKILL PREMIUM SINCE 1900

Our measures of "entering investment" do not seem to show the same pattern as the "Class-1" (i.e., low payout) sample of Fazzari, Hubbard and Petersen (1988) or the "immature firms" sample of Chirinko and Schaller (1995). These studies work entirely with publicly-owned firms, and the authors chose their sub-samples in order to identify firms that are likely to be liquidity constrained. These authors found a smaller Q-elasticity of investment among the low-payout and immature firms than among high-payout and mature firms. It is clear that our data on entering investment cover firms that are not so liquidity constrained. That is certainly so for the VC-backed investment data. A VC-backed firm may feel that it is liquidity constrained, but the VC funds backing it usually are not, because the investment "overhang" (i.e., the amount by which monies committed to the VC fund exceed those actually used to fund their investments) typically exceeds an entire year's worth of investment (Thompson Venture Economics). Roughly speaking, then, VC-backed firms have indirect and immediate access to about one-year's worth of investment. Indirectly then, one may be tempted to infer liquidity constraints, but this would be unwarranted. VC-backed firms are usually much more focused on state-of-the-art technologies than other small firms, and much more likely to eventually go public and for that reason if for no other, much more likely to be sensitive to variation in Tobin's Q.

The volume of IPOs, on the other hand, is a transition rate from one financing

In Figure 10 we show only the segment of the full series that includes the returns to college from 1950-1995.

status to another. It arguably measures the rate at which firms acquire easier financing, since one motive for IPOs, it is said, is to gain access to liquidity. It is apparent that there is a "timing" aspect to the IPO decision; The owners of the firm will want to exercise the option to have an IPO when the market values are high and the owners can get the most for their shares. Rather than a measure of real investment, one can argue that what we measure is simply the exercise of a financial option. On the other hand, evidence also shows (Chemmanur, He and Nanda 2005, Figure 3) that a firm's investment rises by the non-trivial factor of 1.4 around the time of IPO ($\pm$ 2 years), which means that IPOs also measure a rise in investment. But such investment is perhaps less "entry investment" than VC-backed investment is because the firms are often a lot older – see Figure 1 of Jovanovic and Rousseau (2001).

The model assumes that firms are homogeneous. If firms had different (q, z) realizations and if no other change was made to the model, it would imply much more turbulence among firms that we observe: All X would be undertaken exclusively by those firms that had the highest-z and lowest-q combination. To prevent this from happening we would need to add adjustment costs to x. This would introduce a positive slope to the supply curves of x in Figure 3 which would now be firm specific. But the model's aggregate implications would remain much the same.

3.2.2 Vintage capital and the homogeneity of k

UNFINISHED SECTION The model assumes that capital is homogeneous. Only the cost of creating it differs among agents. After entry, the capital of different vintages does not change relative to that of other vintages. The loss of market share to entrants is therefore shared by incumbents of all vintages equally. This subsection checks how well this assumption fits the facts.

Figure 12 shows how well over time the IPO-ing firms of each decade performed relative to firms that existed before them.[23] There is some downward trend in these series because the stock market has grown over time. Thus, for example, the IPOs of the 1890s which included AT&T and General Electric, were large relative to the value of the stock market in 1890 partly because fewer firms were listed in 1890.

These measures look at surviving value, and not returns. If dividend policies differed by cohort so that if, for example, the 1890's cohort tended to pay less dividends and reinvest more of its earnings, changes in value would reflect partly such a difference in reinvestment policies. An alternative measure is cohort returns. Gerdes (1999) has studied the returns patterns by stock-market cohort and found that older (but not very old) vintages have a higher return than the representative rebalanced index. Our vintage value plot in Figure 12 shows no tendency for the value of any particular decade to appreciate more than the others, with the exception of the 1890s

[23] Figure 12 is based on end-of-year market capitalizations from the CRSP files for 1925 to 2006, and our backward extension of the CRSP files for 1890 to 1924. IPO years are recorded as those in which firms enter our database.

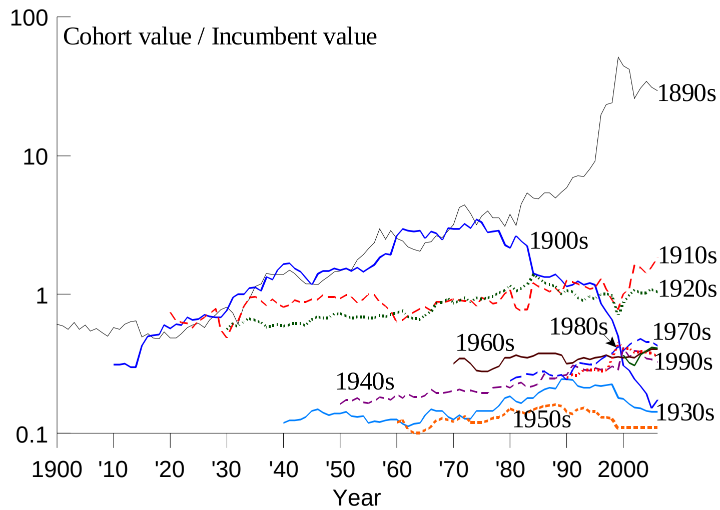

Figure 12: Shares of decadal vintages relative to earlier vintages

cohort: The 1890s cohort includes GE and AT&T and has, since the 1970s, done much better the other cohorts, especially the 1900-1910 cohort.

This relates to two sets of findings in the Finance literature. Fama and French (2000) say there is substantial mean reversion in earnings at the firm level. This could be true and yet earnings could be stable at the level of the cohort as our model implies. For instance, z could have a mean-reverting but firm-specific component, but the law of large numbers would remove the influence of this component on the relative valuation of cohorts. When divided by k_t, aggregate earnings, zk_t, are also mean reverting in the model because z is stationary. The second is the systematic IPO underpricing which implies that new firms are initially undervalued when they IPO, a phenomenon that inflates the stock-market returns of young capital relative to old capital.

4 Conclusion

We analyzed various measures of investment by new firms and we found that such investment responds to Tobin's Q much more elastically than does investment by incumbent firms, which responds hardly at all. We argue that this is because investment of new firms crowds out investment by incumbents more when Q is high than when Q is low. Paradoxically, the investment of incumbents is highly elastic and, for that very reason, easy to crowd out with little effect on stock prices.

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5 Appendix: Growth in the deterministic case

Lucas (1988) solved explicitly for the optimal and the equilibrium rate of growth. We can do that too, but only for some parameter values. When q and z are constant, the rate of growth of capital and output is $g \equiv x + y - \delta$. We can solve for g with the help of the following result:

Proposition 2 *When (q, z) are constant, y still solves (5) and x satisfies the implicit function*

$$q = \beta (1 - \delta + x + y)^{-\sigma} (z - qx - h(y) + q[1 - \delta + x + y]). \tag{20}$$

Proof. When q and z are constant, (6) becomes

$$q = \beta \left(\frac{C'}{C}\right)^{-\sigma} (z - qx - h(y) + q(1 - \delta + x + y)).$$

But C must grow at the same rate as k, namely $x + y - \delta$, and this leads to (20). ■

Solving for g in a special case.—For the case in which $\sigma = 1$ and $h(y) = \frac{\gamma}{2}y^2$, we have $y = \frac{1}{\gamma}q$ and $h[y(q)] = \frac{1}{2\gamma}q^2$, and later on this Appendix shows that

$$x = \beta\frac{z}{q} - \frac{1}{2\gamma}(2 - \beta) q - (1 - \beta)(1 - \delta) \tag{21}$$

which is declining in q and increasing in z, β, and δ, and that

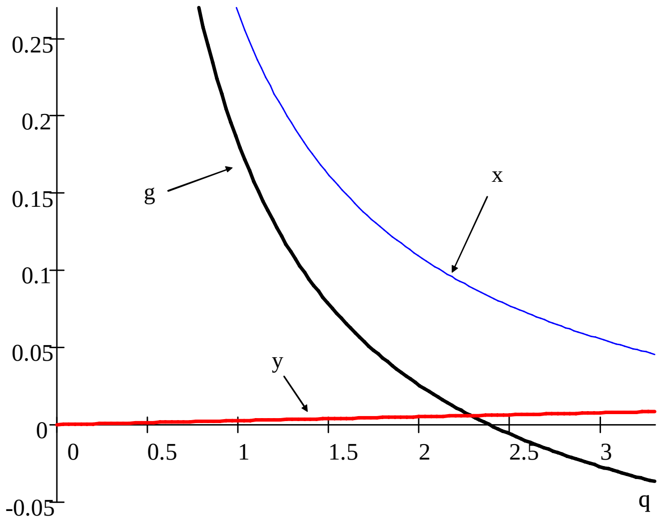
$$g = \beta\left(\frac{z}{q} + \frac{q}{2\gamma}\right) - (1 - \beta(1 - \delta)) \tag{22}$$

which is increasing in z and β, and decreasing in δ and in q, the latter being confined to the 'admissible' range in which $x > 0$.

Calibrated case.—If entrants investment is roughly one percent of GDP, then

$$\frac{Y}{zk} = \frac{y}{z} = \frac{q}{\gamma z} = 0.01. \tag{23}$$

Second, let the capital-output ratio be 3 so that $k/zk = 10$, implying that $z = 0.33$. The average value of q is 1.3. Then (23) implies that $\gamma = \frac{1.3}{(0.33)(0.01)} = 394$. We set $\beta = 0.95$, and $\delta = 0.10$ and obtain the following plot:



LONG-RUN GROWTH AND Q, γ=394, β=0.95, AND δ=0.10.

The growth rate is the rate is the black line, and we also plot the investment of entrants, q/γ as the red line and of incumbents $x = i - y = \delta + g - y$ as the blue line and plot the result in the Figure.

Derivation of (21).—When $\sigma = 1$ and $h(y) = \frac{\gamma}{2}y^2$, (20) reads

$$q = \beta\left(1 - \delta + x + \frac{1}{\gamma}q\right)^{-\sigma}\left(z - qx - \frac{1}{2\gamma}q^2 + q\left[1 - \delta + x + \frac{1}{\gamma}q\right]\right)$$

so that

$$1 - \delta + x + \frac{1}{\gamma}q = \frac{\beta}{q(1-\beta)}\left(z - qx - \frac{1}{2\gamma}q^2\right),$$

which, since $1 + \frac{\beta}{1-\beta} = \frac{1}{1-\beta}$, implies that

$$\begin{aligned} x &= \beta\left(\frac{z}{q} - \frac{1}{2\gamma}q\right) - (1-\beta)\left(1 - \delta + \frac{1}{\gamma}q\right) \\ &= \beta\frac{z}{q} + \beta\frac{1}{2\gamma}q - (1-\beta)(1-\delta) - \frac{1}{\gamma}q \end{aligned}$$

i.e., (21).

Derivation of (22).—Since $g = x + \frac{q}{\gamma} - \delta$,

$$\begin{aligned} g &= \frac{2q}{2\gamma} + \beta\frac{z}{q} - \frac{1}{2\gamma}(2-\beta)q - (1-\beta)(1-\delta) - \delta \\ &= \beta\frac{z}{q} + \frac{\beta}{2\gamma}q - (1-\beta) + \delta(1-\beta) - \delta, \end{aligned}$$

i.e., (22).

The 'admissible' range of qs for which $x > 0$.—Since x is decreasing in q, (21) states that the upper bound on q, call it q^u, solves the equation $x = 0$, i.e.,

$$\frac{1}{2\gamma}(2-\beta)q^2 + (1-\beta)(1-\delta)q - \beta z = 0,$$

which means that (letting $b = (1-\beta)(1-\delta)$),

$$q^u = \frac{\gamma}{2-\beta}\left(-b + \sqrt{b^2 + \frac{2(2-\beta)\beta z}{\gamma}}\right) < \frac{\gamma}{2-\beta}\sqrt{\frac{2(2-\beta)\beta z}{\gamma}} < \gamma\sqrt{\frac{2\beta z}{\gamma}} < \sqrt{2\gamma z},$$

where the first inequality follows because for any $n > 0$, $\sqrt{b^2+n} < b + \sqrt{n}$. Now differentiating in (22), this implies that

$$\frac{\partial g}{\partial q} = -\frac{z}{q^2} + \frac{1}{2\gamma} < -\frac{z}{2\gamma z} + \frac{1}{2\gamma} = 0.$$