

*Preliminary and Incomplete*

## **Private Information, Wage Bargaining and Employment Fluctuations**

John Kennan<sup>1</sup>

University of Wisconsin-Madison and NBER

September 2005

---

<sup>1</sup>Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison, WI 53706; jkennan@ssc.wisc.edu. I thank Rob Shimer and Bob Hall for useful comments. I am particularly grateful to Rob Shimer for pointing out a serious mistake in previous versions of the paper. The National Science Foundation provided research support.

## 1. Introduction

Shimer (2003) pointed out that the basic Mortensen-Pissarides (1994) model does not generate nearly enough volatility in unemployment and vacancies, for plausible parameter values. Hall (2005) argued that this problem can be fixed if the Nash bargaining component of the model is dropped: Hall assumed that wages are sticky in the sense that the wage level in a previous contract establishes a “social norm” that largely determines the wage in the next contract. In the absence of a theory of social norms, this solution effectively requires the introduction of a free parameter. The question in this paper is whether an extension of the Mortensen-Pissarides model to allow for informational rents can explain the volatility of unemployment in a more parsimonious way. A much more elaborate treatment of private information in this context is given by Menzio (2004). Nagypál (2004) has shown that heterogeneity in workers’ (private) evaluations of nonpecuniary job characteristics can substantially increase the volatility of unemployment.

## 2. A Model of Sticky Wages with Private Information and Aggregate Shocks

The model is a simplified version of the model analyzed in Kennan (2003). A successful job match generates a surplus to be divided between the worker and the employer. The value of the worker’s output is modeled as a binary random variable whose realization (“L” for low or “H” for high) is observed privately by the employer when the match is made. The probability of drawing a high surplus,  $p_s$ , is a publicly observed Markov pure jump process with two states ( $s = 1$  in the bad state and  $s = 2$  in the good state), and exit hazards  $\lambda_1$  and  $\lambda_2$ . The probability of the high surplus is assumed to be higher in the good state.<sup>2</sup> Job and worker flows are modeled in the standard way, following Mortensen and Pissarides (1994). When the joint continuation value from a match falls below the joint opportunity cost, the match is destroyed. The job destruction hazard rate is a constant,  $\delta$ , and there is a constant returns matching function that generates a flow of new matches  $M(N_U, N_V)$  from unemployment and vacancy stocks  $N_U$  and  $N_V$ . There is an

---

<sup>2</sup>In the numerical examples, the probability of the high surplus is assumed to be zero in the bad state. Tawara (2005) considers the quantitative implications of allowing for informational rents in both aggregate states.

infinitely elastic supply of potential vacancies, and the actual number of vacancies posted is such that the expected profit from a vacancy is zero.

The match surplus is divided in the following way. Either the employer or the worker is randomly selected to make an offer, and if this offer is rejected the match dissolves. Clearly, the employer's offer will just match the worker's reservation level, which is the value of searching for another match. The worker effectively has two choices: an offer that exhausts the low surplus, with a sure acceptance, or an offer that exhausts the high surplus, with acceptance only if the high surplus has actually been realized. It is assumed that the parameters are such that the worker always finds it optimal to demand the low surplus. Brügemann and Moscarini (2005) show that the volatility of unemployment remains implausibly low for a broad class of surplus-sharing rules: the Nash Bargaining rule is not the source of the problem. On the other hand if there is some stickiness in wages, the employers' incentive to create vacancies is magnified when the economy improves, and this increases unemployment volatility, as Hall pointed out. Brügemann and Moscarini (2005) rule out wage stickiness by assuming that the division of the surplus should be invariant to a change in the location of the productivity distribution. This assumption is very appealing in the case of complete information. But when the employer has private information, it is optimal for workers to ignore small changes in the productivity distribution, and this gives rise to a kind of wage stickiness.

The match surplus depends on whether the employer draws a high or low value from the output distribution, and it also depends on the aggregate state. Let  $y_s^L$  and  $S_s^L$  be the flow surplus and the continuation value of the match when the output value is low, and the aggregate state is  $s$ , and similarly when the output value is high. For simplicity, it is assumed that the difference between the low and high output values does not depend on the aggregate state. That is,

$$y_2^H - y_2^L = y_1^H - y_1^L = \Delta y.$$

Let  $U$  denote the state-dependent continuation value of an unmatched worker, and let  $G$  denote the joint continuation value of a matched worker-employer pair. In the low-output state, the joint continuation values are determined by the following asset pricing equations

$$\begin{aligned}
rG_1^L &= y_1^L - \delta G_1^L + \delta U_1 + \lambda_1(G_2^L - G_1^L) \\
rG_2^L &= y_2^L - \delta G_2^L + \delta U_2 - \lambda_2(G_2^L - G_1^L)
\end{aligned} \tag{1}$$

This specification assumes that there is no possibility of switching from low to high output, once the match has been made, although the flow of output in a low-quality match is allowed to depend on the aggregate state variable. Even in the absence of informational rents, this would tend to increase unemployment volatility, by strengthening the incentive to create vacancies when the aggregate state is good. Brügemann (2005) analyzes the magnitude to this “vintage productivity” effect.

It is assumed that there is free entry of employers, so that the continuation value of an unmatched employer is zero in all states, so the (state-dependent) match surplus is the difference between the gross continuation value  $G$  and the joint continuation value of an unmatched worker,  $U$ . Thus

$$\begin{aligned}
(r + \delta)(S_1^L + U_1) &= y_1^L + \delta U_1 + \lambda_1(S_2^L - S_1^L + \Delta U) \\
(r + \delta)(S_2^L + U_2) &= y_2^L + \delta U_2 - \lambda_2(S_2^L - S_1^L + \Delta U)
\end{aligned} \tag{2}$$

where  $\Delta U = U_2 - U_1$ . This implies

$$S_2^L - S_1^L + \Delta U = \frac{y_2^L - y_1^L + \delta \Delta U}{r + \delta + \Lambda} \tag{3}$$

where  $\Lambda = \lambda_1 + \lambda_2$ . Substituting this in (2) gives

$$\begin{aligned}
(r + \delta)S_1^L &= y_1^L - rU_1 + \frac{\lambda_1(y_2^L - y_1^L + \delta\Delta U)}{r + \delta + \Lambda} \\
(r + \delta)S_2^L &= y_2^L - rU_2 - \frac{\lambda_2(y_2^L - y_1^L + \delta\Delta U)}{r + \delta + \Lambda}
\end{aligned} \tag{4}$$

Similarly, for a high-output match, the surplus values are given by

$$\begin{aligned}
(r + \delta)S_1^H &= y_1^H - rU_1 + \frac{\lambda_1(y_2^H - y_1^H + \delta\Delta U)}{r + \delta + \Lambda} \\
(r + \delta)S_2^H &= y_2^H - rU_2 - \frac{\lambda_2(y_2^H - y_1^H + \delta\Delta U)}{r + \delta + \Lambda}
\end{aligned} \tag{5}$$

The effect of the aggregate state on the match surplus is given by

$$S_2^H - S_1^H = S_2^L - S_1^L = \frac{y_2^L - y_1^L - (r + \Lambda)\Delta U}{r + \delta + \Lambda} \tag{6}$$

Thus if an unmatched worker has better prospects when the aggregate state is good, the match surplus might be lower when the aggregate state is good, for a given output draw. On the other hand there is a higher probability of drawing a high output value in the good aggregate state.

The effect of the output draw on the match surplus is given by

$$S_2^H - S_2^L = S_1^H - S_1^L = \frac{\Delta y}{r + \delta} \tag{7}$$

The rate at which unemployed workers find new matches is  $M(N_U, N_V)/N_U = m(\theta)$ , where  $\theta = N_V/N_U$  represents market tightness, and  $m(\theta) = M(1, \theta)$ . The job-finding rate function  $m(\theta)$  is

assumed to be strictly increasing, and concave. When a match is made, the worker is selected to make an offer with probability  $v$ . In this case, the worker gets the low-output surplus, and the employer gets an informational rent if the realized match value is high. If the employer is selected to make an offer, the worker gets the reservation level  $U$  and the employer gets the whole surplus. Thus an unmatched worker's continuation values are determined by the asset pricing equations

$$\begin{aligned} rU_1 &= w_0 + m(\theta_1)vS_1^L + \lambda_1(U_2 - U_1) \\ rU_2 &= w_0 + m(\theta_2)vS_2^L - \lambda_2(U_2 - U_1) \end{aligned} \quad (8)$$

where  $w_0$  is the flow value of unemployment (including unemployment benefits and the value of leisure). Thus

$$\begin{aligned} rU_1 &= w_0 + \frac{r + \lambda_2}{r + \Lambda} m(\theta_1)vS_1^L + \frac{\lambda_1}{r + \Lambda} m(\theta_2)vS_2^L \\ rU_2 &= w_0 + \frac{r + \lambda_1}{r + \Lambda} m(\theta_2)vS_2^L + \frac{\lambda_2}{r + \Lambda} m(\theta_1)vS_1^L \end{aligned} \quad (9)$$

Employers post new vacancies to the point where the net profit from doing so is zero. When a match is made, the employer gets an informational rent if the match value is high, and also gets a fraction  $1-v$  of the low-output surplus (in expectation). Thus the zero-profit conditions implied by free entry are

$$\begin{aligned} 0 &= -c + \frac{m(\theta_1)}{\theta_1} \left( (1-v)S_1^L + p_1(S_1^H - S_1^L) \right) \\ 0 &= -c + \frac{m(\theta_2)}{\theta_2} \left( (1-v)S_2^L + p_2(S_2^H - S_2^L) \right) \end{aligned} \quad (10)$$

where  $c$  is the flow cost of maintaining a vacancy, and  $p_s$  is the probability of drawing the high match value, for  $s = 1, 2$ .

It is convenient to let  $d = \theta/m(\theta)$  denote the expected duration of a vacancy. Then the free-entry conditions can be written as

$$\begin{aligned} cd_1 &= (1 - \nu)S_1^L + p_1(S_1^H - S_1^L) \\ cd_2 &= (1 - \nu)S_2^L + p_2(S_2^H - S_2^L) \end{aligned} \quad (11)$$

The model can be solved as follows. For given values of  $d_1$  and  $d_2$ , the free entry conditions determine the low-state surplus values:

$$S_s^L = \frac{c(d_s - \alpha_s)}{1 - \nu} \quad (12)$$

where

$$\alpha_s = \frac{p_s \Delta y}{c(r + \delta)} \quad (13)$$

for  $s = 1, 2$ .

Equation (2) can be rearranged to give  $U_1$  and  $U_2$  as linear functions of  $S_1^L$  and  $S_2^L$ , and  $U_1$  and  $U_2$  can then be expressed in terms of  $d_1$  and  $d_2$  as

$$\begin{aligned} rU_1 &= y_1^L + \frac{c(\delta + r)(d_1 - \alpha_1)}{1 - \nu} + \frac{\lambda_1(y_2^L - y_1^L)}{\Lambda + r} + \frac{\lambda_1 \delta c(d_2 - \alpha_2 - (d_1 - \alpha_1))}{(1 - \nu)(r + \delta)} \\ rU_2 &= y_2^L + \frac{c(\delta + r)(d_2 - \alpha_2)}{1 - \nu} - \frac{\lambda_2(y_2^L - y_1^L)}{\Lambda + r} + \frac{\lambda_2 \delta c(d_1 - \alpha_1 - (d_2 - \alpha_2))}{(1 - \nu)(r + \delta)} \end{aligned} \quad (14)$$

Next (12) can be substituted in (9), giving

$$\begin{aligned} rU_1 &= w_0 + \frac{r + \lambda_2}{r + \Lambda} \frac{vcm(\theta_1)}{1 - \nu} (d_1 - \alpha_1) + \frac{\lambda_1}{r + \Lambda} \frac{vcm(\theta_2)}{1 - \nu} (d_2 - \alpha_2) \\ rU_2 &= w_0 + \frac{r + \lambda_1}{r + \Lambda} \frac{vcm(\theta_2)}{1 - \nu} (d_2 - \alpha_2) + \frac{\lambda_2}{r + \Lambda} \frac{vcm(\theta_1)}{1 - \nu} (d_1 - \alpha_1) \end{aligned} \quad (15)$$

After eliminating  $U_1$  and  $U_2$  and rearranging, this gives the following equations determining  $d_1$  and  $d_2$

$$\begin{aligned} \psi_1(d) &= Z_1 + \left( \frac{\alpha_1}{d_1} - 1 \right) \nu H(d_1) - (r + \delta + \lambda_1)(d_1 - \alpha_1) + \lambda_1(d_2 - \alpha_2) = 0 \\ \psi_2(d) &= Z_2 + \left( \frac{\alpha_2}{d_2} - 1 \right) \nu H(d_2) - (r + \delta + \lambda_2)(d_2 - \alpha_2) + \lambda_2(d_1 - \alpha_1) = 0 \end{aligned} \quad (16)$$

where  $H(d) = \theta$ , and

$$Z_s = \frac{(1 - \nu)(y_s^L - w_0)}{c}$$

for  $s = 1, 2$ .

It is assumed that the function  $d(\theta) = \theta/m(\theta)$  is invertible, and that the inverse function  $H$  is convex.

### Proposition 1

If the function  $\theta = H(d)$  is convex, and if  $H(0) = 0$ , then there is a unique vector  $d^* = (d_1^*, d_2^*)$  such that  $\psi(d^*) = 0$ .

The proof uses the following result.



**Lemma**

Suppose  $\alpha$  is a positive number, and  $H$  is a twice differentiable function, with  $H(0) = 0$ ,  $H'(x) > 0$  and  $H''(x) > 0$ , for  $x > \alpha$ . Define the function  $h$ , on the domain  $[\alpha, \infty)$ , as

$$h(x) = \left( \frac{\alpha}{x} - 1 \right) H(x) \quad (17)$$

Then  $h'(x) < 0$  and  $h''(x) < 0$ .

**Proof**

The first and second derivatives of  $h$  are as follows

$$\begin{aligned} h'(x) &= \left( \frac{\alpha}{x} - 1 \right) H'(x) - \frac{\alpha}{x^2} H(x) \\ h''(x) &= \left( \frac{\alpha}{x} - 1 \right) H''(x) - 2 \frac{\alpha}{x^2} H'(x) + 2 \frac{\alpha}{x^3} H(x) \\ &= \left( \frac{\alpha}{x} - 1 \right) H''(x) + 2 \frac{\alpha}{x^3} (H(x) - xH'(x)) \end{aligned} \quad (18)$$

Since  $x \geq \alpha$ , and  $H'(x) > 0$ , it is clear that  $h$  is decreasing. The function  $H(x)/x$  is increasing. In fact if  $x_1 < x_2$  then  $x_1 = \gamma x_2$  and  $H(x_1) \leq \gamma H(x_2) + (1-\gamma)H(0) = \gamma H(x_2)$ , because  $H$  is convex and  $H(0) = 0$ , so  $H(x_1)/x_1 \leq H(x_2)/x_2$ . The derivative of  $H(x)/x$  is  $(xH'(x) - H(x))/x^2$ , so  $H(x) - xH'(x) \leq 0$ . Thus  $h$  is concave.

**Proof of Proposition 1**

First it will be shown that  $\psi(d^*) = 0$  implies  $d^* > \alpha$ . If  $d_1 \leq \alpha_1$  and  $d_2 \geq \alpha_2$  then  $\psi_1(d) > 0$ ; and if  $d_1 \geq \alpha_1$  and  $d_2 \leq \alpha_2$  then  $\psi_2(d) > 0$ . If  $d \leq \alpha$ , write  $\psi(d)$  as

$$\begin{aligned}
\psi_1(d) &= Z_1 + (\alpha_1 - d_1)v \frac{H(d_1)}{d_1} - (r + \delta)(d_1 - \alpha_1) + \lambda_1[(d_2 - \alpha_2) - (d_1 - \alpha_1)] \\
\psi_2(d) &= Z_2 + (\alpha_2 - d_2)v \frac{H(d_2)}{d_2} - (r + \delta)(d_2 - \alpha_2) - \lambda_2[(d_2 - \alpha_2) - (d_1 - \alpha_1)]
\end{aligned} \tag{19}$$

These equations show that either  $\psi_1(d)$  or  $\psi_2(d)$  is a sum of four positive terms: the first three terms are positive in both equations, and if the last term is negative in the first equation, it must be positive in the second, and vice versa. Thus  $\psi(d) \neq 0$  if  $d \leq \alpha$ .

Next it will be shown that a solution exists. Note that  $\psi(\alpha) = Z > 0$ . Define  $A$  as the solution of the linear equations obtained by setting  $H = 0$ . Then

$$A_s = \alpha_s + \frac{\bar{Z}_s}{r + \delta} \tag{20}$$

where

$$\begin{aligned}
\bar{Z}_1 &= \frac{(r + \delta + \lambda_2)Z_1 + \lambda_1 Z_2}{r + \delta + \lambda_1 + \lambda_2} \\
\bar{Z}_2 &= \frac{(r + \delta + \lambda_1)Z_2 + \lambda_2 Z_1}{r + \delta + \lambda_1 + \lambda_2}
\end{aligned} \tag{21}$$

Thus  $A > \alpha$  and  $\psi(A) < 0$ .

Since  $\psi_1$  is increasing in  $d_2$  and decreasing in  $d_1$ , the equation  $\psi_1(d) = 0$  can be solved to obtain  $d_2$  as an increasing function of  $d_1$ . Write this as  $d_2 = \Upsilon_1(d_1)$ . Since  $\psi_2$  is increasing in  $d_1$  and decreasing in  $d_2$ , the equation  $\psi_2(d) = 0$  can also be solved to obtain  $d_2$  as an increasing function of  $d_1$ . Write this as  $d_2 = \Upsilon_2(d_1)$ . Define the function  $\xi(x) = \Upsilon_2(x) - \Upsilon_1(x)$ . Since  $\psi_1(\alpha_1, \Upsilon_1(\alpha_1)) = 0$ , and  $\psi_1(\alpha_1, \alpha_2) > 0$ , and  $\psi_1$  is increasing in  $d_2$ , it follows that  $\Upsilon_1(\alpha_1) < \alpha_2$ . Also, since  $\psi_2(\alpha_1, \Upsilon_2(\alpha_1)) = 0$ , and  $\psi_2(\alpha_1, \alpha_2) > 0$ , and  $\psi_2$  is decreasing in  $d_2$ , it follows that

$\Upsilon_2(\alpha_1) > \alpha_2$ . Therefore  $\xi(\alpha_1)$  is positive. By a similar argument,  $\xi(A_1)$  is negative. Also,  $\xi$  is continuous (since  $\psi_1$  is linear in  $d_2$  and  $\psi_2$  is linear in  $d_1$ ). So by the intermediate value theorem  $\Upsilon_2(x) = \Upsilon_1(x)$  for some  $x \in [\alpha_1, A_1]$ . This means that  $\psi(x, \Upsilon_1(x)) = 0$ , showing that a solution exists.

To show uniqueness, define the function  $g(z) = \psi(\alpha + z)$ . Then  $g_1$  is increasing in  $z_2$  and  $g_2$  is increasing in  $z_1$ , and both  $g_1$  and  $g_2$  are concave, and  $g(0) > 0$ . Therefore, by Theorem 1 in Kennan (2001),  $g$  has at most one positive root, meaning that  $\psi$  has at most one root above  $\alpha$ . Since it has already been shown that  $\psi$  does have a root above  $\alpha$ , and no roots anywhere else, the proof is complete.

### Optimality of Pooling Offers

It is assumed that when a match is made in the good aggregate state, and the worker is selected to make an offer, it is optimal to demand the low surplus, rather than demand the high surplus at the risk of destroying the match. Thus the equilibrium surplus values must satisfy the following no-screening conditions

$$\begin{aligned} S_1^L &\geq p_1 S_1^H = p_1 \left( S_1^L + \frac{\Delta y}{r + \delta} \right) \\ S_2^L &\geq p_2 S_2^H = p_2 \left( S_2^L + \frac{\Delta y}{r + \delta} \right) \end{aligned} \tag{22}$$

which can be written as

$$S_s^L \geq \frac{p_s}{1 - p_s} \frac{\Delta y}{c(r + \delta)} \tag{23}$$

for  $s = 1, 2$ . Using the free entry conditions, this reduces to

$$d_s \geq \bar{\alpha}_s \quad (24)$$

where

$$\bar{\alpha}_s = \left(1 + \frac{1 - \nu}{1 - p_s}\right) \left(\frac{p_s \Delta y}{c(r + \delta)}\right) = \left(1 + \frac{1 - \nu}{1 - p_s}\right) \alpha_s \quad (25)$$

Since  $\alpha_s = 0$  for  $p_s = 0$ , Proposition 1 implies that a unique equilibrium satisfying the no-screening conditions exists if  $p_s$  is small enough. Conversely, the no-screening condition fails as  $p_s$  approaches 1, as of course it should.

### Theorem 1

If  $H(d)$  is a convex function, with  $H(0) = 0$ , and if  $\psi(\bar{\alpha}) \geq 0$ , then a unique equilibrium exists.

### Proof

By Proposition 1, there is a unique vector  $d^*$  such that  $\psi(d^*) = 0$ . Since  $\psi(\bar{\alpha}) \geq 0$  and  $\psi(A) < 0$ , the argument used in the proof of Proposition 1 can be used to show that  $\psi$  has a root in the rectangle  $[\bar{\alpha}, A]$ , and since there is only one root above  $\bar{\alpha}$ , this root is  $d^*$ . The no-screening conditions are satisfied because  $d^* \geq \bar{\alpha}$ . Therefore  $d^*$  is the unique equilibrium.

Theorem 1 fully characterizes the set of parameter values for which an equilibrium exists, and it shows that if the parameters lie in this set, the equilibrium is unique.

### 3. Unemployment Volatility

Standard parameter values are used as far as possible, following Shimer (2003) and Hall (2003). The simplest choice for the matching function is a constant-returns Cobb-Douglas function that is symmetric in unemployment and vacancies. This implies  $m(\theta) = a\sqrt{\theta}$ , and  $a$  is set at 6.8, per annum (Shimer uses  $a = 1.7$  for quarterly data). The job destruction rate  $\delta$  is set at .42 per annum, so that the quarterly rate is  $\exp(-.25\delta) = 0.1$ . In the NBER postwar data, the average duration of a recession is 10 months, and the average duration of an expansion is 57 months. This implies that the exit hazards are  $\lambda_2 = 12/57$  and  $\lambda_1 = 12/10$ .

It is assumed that all matches produce low output in the bad aggregate state, and the low output value is normalized to 1; thus the aggregate output level in the bad state is also 1. Let  $Y_1$  and  $Y_2$  denote aggregate state-contingent productivity levels. The invariant distribution has mass  $\lambda_2/\Lambda$  on the bad state, and  $\lambda_1/\Lambda$  on the good state. Expected productivity is

$$\mu_Y = Y_1 + \frac{\lambda_1}{\Lambda} \Delta Y$$

where  $\Delta Y = Y_2 - Y_1 = y_2^L - y_1^L + p_2 \Delta y$ . The variance is given by

$$\begin{aligned} \sigma_Y^2 &= \frac{\lambda_1}{\Lambda} (Y_2 - \mu_Y)^2 + \frac{\lambda_2}{\Lambda} (Y_1 - \mu_Y)^2 \\ &= \frac{\lambda_1 \lambda_2}{\Lambda^2} \Delta Y^2 \end{aligned}$$

This implies

$$\frac{\Delta Y}{\sigma_Y} = \sqrt{\frac{\lambda_1}{\lambda_2}} + \sqrt{\frac{\lambda_2}{\lambda_1}}$$

If the process is symmetric, the standard deviation is half of the difference between  $Y_1$  and  $Y_2$ . Otherwise, the standard deviation is less than half of the difference. If the ratio of the transition rates is far from 1, the standard deviation can be made arbitrarily small, for any fixed difference (because the process spends virtually all of its time in one state). Setting  $\Delta Y = .042$ , with  $\lambda_2 = 12/57$  and  $\lambda_1 = 12/10$  and  $Y_1 = 1$  gives  $\sigma_Y/\mu_Y = .014$ . According to Shimer (2003), the coefficient of variation of U.S. aggregate labor productivity is .018. Since the basic question is whether informational rents can explain why unemployment is much more volatile than the underlying shocks, a process that understates the volatility of productivity errs on the side of caution.

The parameter values are summarized in Table 1.

<b>Table 1: Parameter Values</b>			
<b>Parameter</b>	<b>Notation</b>	<b>Value</b>	<b>Comments</b>
matching function	$m(\theta)$	$6.8\sqrt{\theta}$	Shimer
recession exit hazard	$\lambda_1$	12/10	recession duration (10 months)
expansion exit hazard	$\lambda_2$	12/57	expansion duration (57 months)
unmatched flow payoff	$w_0$	0.4	Shimer
low output	$y_L$	1	normalization
informational rent	$p\Delta y$	0.042	volatility of labor productivity
vacancy flow cost	$c$	.54	Shimer
separation rate	$\delta$	.42	Shimer (see text)
interest rate	$r$	.05	

The steady-state unemployment levels are determined in the usual way as

$$u_s^* = \frac{1}{1 + \frac{m(\theta_s)}{\delta}}$$

The equilibrium values of  $d_1$  and  $d_2$  for the parameters in Table 1 can be obtained from the following equations:

$$\begin{aligned}\psi_1(d) &= \frac{151}{423} - \frac{167}{100}d_1 - \frac{578}{25}(d_1)^2 + \frac{6}{5}d_2 = 0 \\ \psi_2(d) &= \frac{53701}{80370} + \frac{2528021}{803700}d_2 - \frac{578}{25}(d_2)^2 + \frac{4}{19}d_1 = 0\end{aligned}$$

The solution is ( $d_1 = .1369563271$ ,  $d_2 = .2545047770$ ).

In this example,  $\alpha$  and  $\bar{\alpha}$  are given by

$$\begin{aligned}\bar{\alpha}_1 &= \alpha_1 = 0 \\ \alpha_2 &= \frac{70}{423} \\ \bar{\alpha}_2 &= \left(1 + \frac{1}{2(1-p_2)}\right)\alpha_2\end{aligned}\tag{26}$$

Since there is no informational rent in the bad state, the no-screening condition is irrelevant in that state. In the good state the no-screening condition holds if  $d_2 \geq \bar{\alpha}_2$ . The equilibrium depends on  $p_s$  only through the effect of  $p_s$  on  $\alpha_s$  (provided that the no-screening condition holds), and with  $p_1 = 0$ ,  $\alpha_2$  depends on  $p_2$  only through the product  $p_2\Delta y$ , which is set to 0.042. The no-screening condition then holds provided that  $p_2 \leq 0.0705$ .

Table 2 shows that these parameter values can generate realistic variations in the unemployment rate. To illustrate the importance of informational rents in generating this result, the table includes the steady state unemployment rates for a baseline parameter set that matches the variance of aggregate productivity by letting the match surplus depend on the aggregate state, with no idiosyncratic variation. The parameter values are as in Table 1, but with  $y_1^L = 1$ ,  $y_2^L = 1.042$ , and  $p_2\Delta y = 0$ . In this case, the unemployment rate is virtually constant.

<b>Table 2: Unemployment Volatility</b>			
		Baseline	Informational Rent
Productivity Variation	$y_2^L$	1.042	1.0
	$p_2\Delta y$	0	.042
Steady State Unemployment Rates	$u_1^*$	5.86%	<b>6.22%</b>
	$u_2^*$	5.7%	<b>3.45%</b>

Table 3 shows results for some alternative values of the vacancy cost, the separation rate, and the flow value of unemployment. Large changes in these parameters have virtually no effect on volatility. Thus the ability of the model to explain unemployment volatility is based almost entirely on the presence of informational rents. The key point is that the informational rent can be large enough to amplify the underlying productivity shocks, without being too large to sustain a pooling equilibrium.

<b>Table 3: Unemployment Volatility (no informational rent)</b>					
		Baseline	Low c	Low $\delta$	Low $w_0$
Baseline			0.54	.42	.40
Variant			0.27	.21	.20
Steady State	$u_1^*$	<b>5.86%</b>	4.14%	2.934%	5.08%
Unemployment Rates	$u_2^*$	<b>5.70%</b>	4.02%	2.852%	4.97%

Recently, Hagedorn and Manovskii (2005) have argued that the Mortensten-Pissarides model can generate realistic unemployment fluctuations if the value of the worker's outside option is close to the value of production. In the model considered here, this means setting  $w_0$  near 1. Hagedorn and Manovskii calibrated  $w_0$  as .943, with  $v = .061$ . Table 4 explores the implications of these parameter values, in the model with no informational rents.



<b>Table 4: Unemployment Volatility (no informational rent)</b>						
		Baseline	High $w_0$	Low $v$	High $w_0$ low $v$	Higher $w_0$ low $v$
Variant		$w_0 = .40$ $v = .5$	$w_0 = .943$ $v = .5$	$w_0 = .40$ $v = .061$	$w_0 = .943$ $v = .061$	$w_0 = y_L^b = 1$ $v = .061$
Steady State	$u_1^*$	<b>5.86%</b>	17.28%	1.67%	5.80%	33.9%
Unemployment Rates	$u_2^*$	<b>5.70%</b>	14.65%	1.62%	4.97%	22.9%

When the workers' outside opportunities are almost as good as their market production opportunities, it makes sense to reduce the number of vacancies. Moving workers into jobs raises the value of their output, but not by much, and in order to move workers into jobs, it is necessary to expend resources on vacancy costs. Reducing the number of vacancies economizes on the vacancy costs (because it reduces congestion); workers spend more time out of employment, but that is not very costly. Even if the value of the outside opportunity is the same as the value of production in the bad aggregate state, it still makes sense to move workers into jobs. This is because there may be a transition to the good aggregate state, and when that happens, employed workers are more productive than unemployed workers. If this transition is unlikely, the unemployment rate in the bad state will be high. But in the data, recessions are relatively short-lived, so although the Hagedorn and Manovskii calibration yields high unemployment rates, there is not much difference between the level of unemployment in different states.<sup>3</sup>

---

<sup>3</sup>Mortensen (2005) argues that the value of  $w_0$  used by Hagedorn and Manovskii is unrealistically high. He recomputes the elasticity of  $\theta$  with respect to productivity in an extended model that allows for capital costs, and finds plausible unemployment volatility even with wages set by Nash bargaining.

## References

- Brügemann, Björn, “Vintage Productivity Shocks and Employment Fluctuations,” Yale University, August 2005.
- Brügemann, Björn and Giuseppe Moscarini, “[Asymmetric Information and Employment Fluctuations](#),” Yale University, August 2005.
- Hagedorn, Marcus and Iourii Manovskii, “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” University of Pennsylvania, April 2005.
- Hall, Robert E., “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, March 2005.
- Kennan, John, “[Informational Conflict and Employment Fluctuations](#),” unpublished, November 2003; <http://www.ssc.wisc.edu/~jkennan/research>.
- Kennan, John, “[Uniqueness of Positive Fixed Points for Increasing Concave Functions on  \$\mathbb{R}^n\$ : An Elementary Result](#),” *Review of Economic Dynamics*, 4, October 2001, 893-899 (<http://dx.doi.org/doi:10.1006/redo.2001.0133>).
- Menzio, Guido, “High Frequency Wage Rigidity,” Northwestern University, December 2004.
- Mortensen, Dale T., “More on Unemployment and Vacancy Fluctuations,” Northwestern University, July 2005.
- Mortensen, Dale T., and Christopher A. Pissarides, “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 61, July 1994, 397-415.
- Nagypál, Éva, “Amplification of Productivity Shocks: Why Vacancies Don’t Like to Hire the Unemployed?”, Northwestern University, August 2004.
- Shimer, Robert E., “The Cyclical Behavior of Equilibrium Unemployment and Vacancies (2005) *American Economic Review*, 95(1): 25-49.
- Tawara, Norikazu, “Search and Matching with Wage Bargaining under Asymmetric Information,” University of Chicago, May 2005.