Schumpeterian Restructuring

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Abstract

We develop a theory of cyclical unemployment, which relates the processes of job creation and job destruction over the business cycle, to the processes of restructuring, innovation and implementation arising from a model of endogenous cyclical growth. Due to a moral hazard problem, production workers are paid efficiency wages and experience involuntary unemployment. The model captures two key components of unemployment fluctuations: a broad, but temporary component due to declines in aggregate demand and a sector–specific, but persistent component due to firm turnover. In addition to the standard pro-cyclical forces acting on the wage, our framework highlights the role of countercyclical forces associated with job restructuring that are concentrated in recessions. In particular, our model implies that wages tend to be more pro-cyclical when productivity growth is low, and less pro-cyclical, or even counter–cyclical, when productivity growth is high.

Key Words:
JEL: E0, E3, O3, O4

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1 Introduction

Although recessions are periods of collapsing labor demand and low investment in physical capital, they are also periods of substantial restructuring in the US labor market. As Figure 1 illustrates, for example, while worker flows into unemployment obviously rise during downturns, so also do flows into employment: job creation peaks during upturns, but starts to rise during the preceding recession.[1] Productivity growth ultimately depends on this restructuring process as it determines the rate at which workers are re-allocated from low– to high– productivity jobs. It follows that understanding this process of restructuring is crucial to understanding both the determinants of long run growth and the nature of business cycles. In this paper, we develop a theory of "Schumpeterian restructuring" that relates the forces determining job creation, job destruction and wages over the business cycle, to the process of restructuring, innovation and implementation arising from a model of endogenous, cyclical growth.

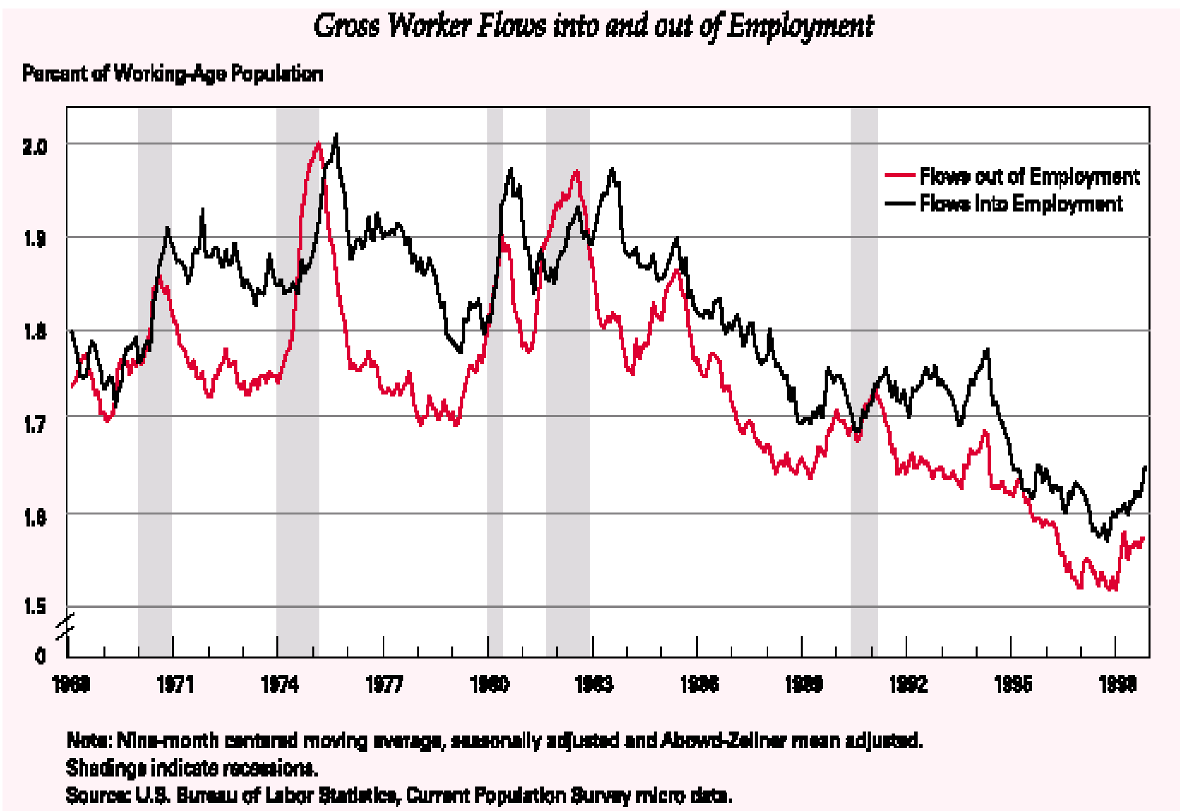

Figure 1: Employment flows — from Bleakley, Ferris and Fuhrer (1999).

Our theory effectively binds together two distinct notions of creation and destruction that have developed simultaneously in the macroeconomics literature. In the growth literature, fol-

[1]See also Blanchard and Diamond (1990), Davis and Haltiwanger (1992) and Davis, Haltiwanger and Schuh (1996).

lowing the work of Schumpeter (1927), Aghion and Howitt (1992) and Grossman and Helpman (1991), "creative destruction" has been used to refer to the process by which newly created ideas destroy the profits of incumbent firms by making them obsolete. This literature rarely considers the implications of this process for unemployment.[2] In the macro/labor literature, the microeconometric analyses of Blanchard and Diamond (1990), Davis and Haltiwanger (1992) and Davis, Haltiwanger and Schuh (1996) have uncovered the process of continuous job creation and job destruction that is observed in the US labor market. This literature does not, however, link the sources of job creation and destruction to the growth process.[3]

According to the Schumpeterian view, productivity growth arises when old ideas and technologies embodied in existing firms are replaced by new ones. Recently, Francois and Lloyd-Ellis (2003) show how aggregate cycles can also be driven by this same process of creative destruction.[4] These cycles occur through the timing of entrepreneurial actions. Entrepreneurs protect the knowledge embedded in their improvements by delaying implementation. Since the profits they receive from innovations are only temporary (in a probablistic sense), they prefer to implement innovations when macroeconomic conditions are favorable. In the presence of imperfect competition, implementation of productivity improvements by some firms increases the demand for others' products by raising aggregate demand, and thus creates favorable conditions. This leads to mutually reinforcing endogenous clustering in implementation, which causes booms in output and productivity together with stretches of stagnation.[5]

A crucial short–coming of the model of Francois and Lloyd-Ellis (2003), however, is that it does not feature aggregate unemployment. It only considers a role for high–skilled workers (e.g. managers) with inelastic labor supply, no labor market frictions, and hence no potential for involuntary unemployment. Here we introduce these features by allowing a role for less skilled workers who are used only in production and whose effort levels are imperfectly monitored. The resulting moral hazard problem is solved through an efficiency wage scheme similar to that developed by Shapiro and Stiglitz (1986). This results in involuntary unemployment and a wage level that depends on labor market factors other than productivity. Consequently, as skilled labor moves in and out of production, aggregate employment of production labor adjusts in proportion.

A key implication of this set up is that, although entrepreneurs implement their improvements at the boom, they choose to enter production during the recession. By doing so, they dissuade other entrepreneurs from targeting the same sector, without revealing the content of their improvements, and thus ensure that profits will not have to be shared with a rival innovating in the same direction. Together, these features lead to the increased churning and worker flows that

[2] An exception is Aghion and Howitt (1994), but the employment effects analyzed there are longer term and do not apply at business cycle frequencies.

[3] An exception is the work of Cabellero and Hammour (1994) and (2004) who endogenize the entry and exit of firms. However, they treat fluctuations as being driven by exogenous productivity shocks, in contrast with the endogenous Schumpeterian productivity increments that arise here.

[4] Matsuyama (1999) and Waelde (2004) offer related models of endogenous business cycles.

[5] The nature of clustering is similar to that described by Shleifer (1986). However, the mechanism is very different and implies a unique equilibrium cycle (with real downturns) that is intimately related to the innovation process.

accompany the recession, and the delay of increased productivity which awaits the expansion. Moreover, because this costly entrepreneurship tends to be endogenously bunched just before the boom, it leads to a downturn in aggregate output. This again leads to endogenous clustering of entrepreneurial actions – in this case with respect to the timing of innovative effort. Since entrepreneurs see the downturn as a period of slack demand, they choose it as the time to allocate their skilled labor to securing productivity improvements that they will implement in the subsequent expansion. However, this has a negative consequence for production workers who are temporarily laid off.

Our framework enables us to simultaneously understand several puzzling observations regarding the US labor market. These observations have been made before, but have typically been analyzed in isolation. We justify our approach by the fact that these observations can best be understood by thinking about the interactions between them:

• *Technological change and business cycles:* Perhaps the most important observation supporting theories of business cycles driven by fluctuations in technology is the fact that, when measured in aggregate data, labor productivity appears to be strongly pro–cyclical (see Wen 2004). However, the validity of this observation is hampered by the existence of serious measurement problems, especially during recessions. In particular, there is considerable evidence of re–allocations of labor across activities (e.g. labor hoarding) which occurs during recessions, but is unmeasured at the aggregate level (see Fay and Medoff, 1985).[6] A recent article by Beaudry and Portier (2004) argues that the "aggregate technology shock" view of business cycles may not be the best way to think about the effects of technological change. They find compelling evidence that aggregate US stock–price movements partially anticipate movements in total factor productivity. Moreover, they find that the economy reacts to this anticipation well before TFP rises.

In the model developed here, expansions are driven by increases in TFP. However, downturns are the result of the restructuring that begins to occur in the economy in anticipation of future movements in TFP. If labor productivity were correctly measured, it would remain constant during the recession. However, because the reallocation of skilled labor to innovative activities is likely to be unmeasured, it would look like labor hoarding.[7] The anticipated growth in TFP starts to be reflected in stock–prices during the recession, prior to implementation. As we argue below, the nature of recessions arising from our framework has important implications for the time of restructuring and cyclical behavior of wages.

• *Restructuring during recessions:* As noted earlier, aggregate flows into employment start to rise during recessions (Blanchard and Diamond, 1990, Davis and Haltiwanger, 1996, Bleakley et al. 1999). Moreover, more detailed analyis, such as that undertaken by Schuh and Triest (1998), uncovers key distinctions in the nature of these job–flows. Permanent (i.e., persistent) changes in job-flows tend to be idiosyncratic (concentrated within relatively few firms that experience large

[6] Moreover, recent attempts to account for variable capital utilization (e.g. Basu, 1996) have called into question the presumed high procyclicality of labor productivity.

[7] As we show in Francois and Lloyd-Ellis (2004), in a version of the model with capital and variable utilization, unmeasured variations in utilization will make labor productivity look like it is falling during recessions.

changes) – two thirds of total jobs created and destroyed occur in plants that expand or contract by more than 25 percent in one year. In contrast temporary changes are spread more diffusely across firms in the economy (Schuh and Triest 1998).

In our model, two distinct forces affect unemployment levels and labor flows through the cycle – one acting broadly and temporarily, the other leading to permanent but idiosyncratic job losses. Since the endogenously created recession occurs when skilled labor is allocated to entepreneurship, it generates low aggregate demand. These falls in aggregate demand affect all firms equally and lead them to suffer temporary job losses. The losses are temporary because the jobs will be replaced when aggregate output turns around. However, Schumpeterian destruction leads to idiosyncratic and concentrated permanent job losses that arise when obsolete firms shut down. Again, these occur intensively in recessions since this is when firms intensively allocate entrepreneurial efforts, but more importantly their concentration corresponds well with what is observed.

An alternative approach to connect restructuring, job flows and aggregate employment would be to link firms with jobs directly and posit an exogenous productivity shock sequence hitting different sectors of the economy. This would, in some ways, be simpler than what we propose here as it would not require endogenizing either cyclical fluctuations or productivity growth. While such an approach could well account for simultaneous flows into employment and unemployment, it would imply that these labor flows would coincide closely with increases in productivity. Given that productivity movements are pro–cyclical, such an explanation would imply that the increased churning would occur during expansions. Thus such an approach is at odds with the broad facts on the timing of job flows particularly with flows into both unemployment and employment increasing in recessions.[8]

• *The mild pro-cyclicality of wages:* Real wages are most commonly characterized as being mildly procyclical.[9] Within reasonable parameters ranges, the canonical Real Business Cycle (RBC) model predicts excessively procyclical real wages (and insuffıcent variation in employment), even when compared with post–1970 data and controlling for the composition bias (see Chang, 2000). The Shapiro–Stigliz efficiency wage model was suggested by some as a way of offsetting this pro–cyclical wage behaviour. However, Gomme (1999) finds that, while the variability of the average wage is reduced, it remains just as pro-cyclical as in the canonical RBC model.[10] His analysis

[8] Blanchard and Diamond (1990) similarly argue that it is precisely the counter–cyclical nature of restructuring which trips up such an approach.

[9] See Abraham and Haltiwanger (1995). In addition, focussing on average real wages may be misleading since several authors (e.g. Stockman, 1983, Bils, 1985, Solon, Barsky, and Parker, 1994) have documented that the entry and exit of low-wage workers creates a counter–cyclical composition bias in aggregate wages in post–1970 data. Assessing the importance of the composition bias prior to 1970 is difficult because PSID date was only available after that date.

[10] More recent contributions have been able to match more closely the average correlation between real wages and output observed in HP–filtered data. One approach has been to account more carefully for occupational flows within the context of a purely neoclassical framework. In Chang (2000), for example, workers are assigned to managerial, production and non-market tasks based on comparative advantage. Another approach has been to argue that labor's reallocation over the cycle is part of an optimal inter-temporal response to fluctuations, and that therefore employment fluctuations and wages do not necessarily reflect current supply and demand conditions. For

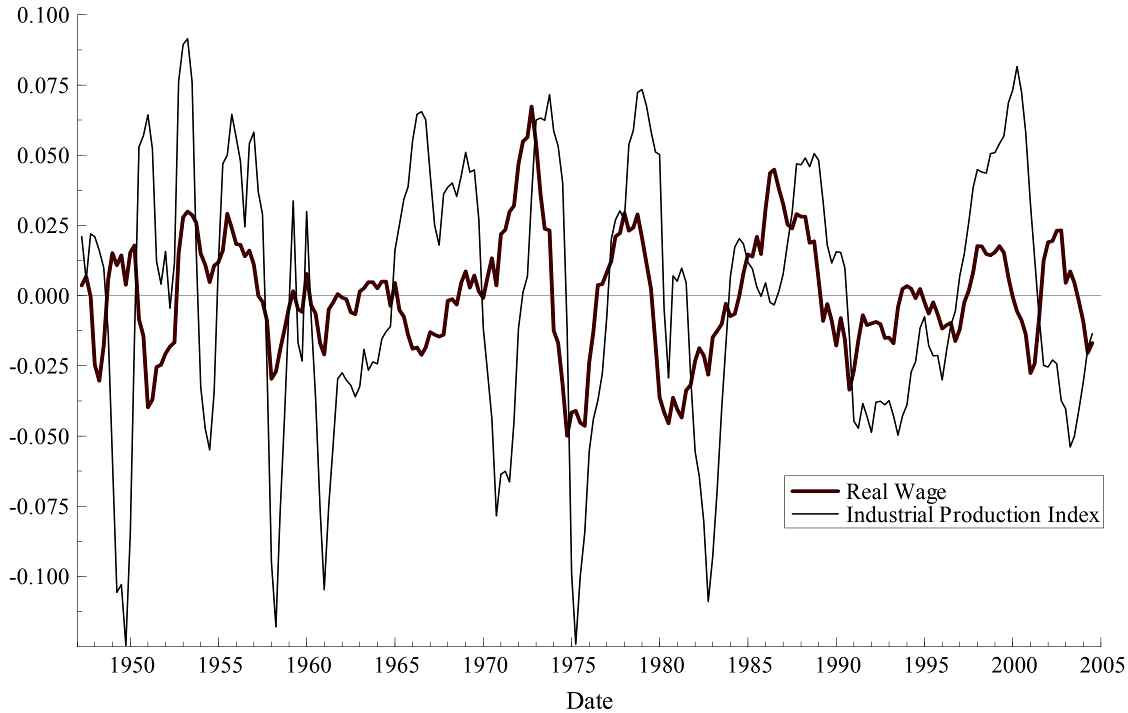

Figure 2: Real Wage and IPI in Manufacturing, 1948–2004 (HP Filtered)

does not, however, allow for the kind of restructuring during recessions discussed here. In his model, firms do not shut down to be replaced by new ones, but rather adjust their rate of hiring downwards as demand falls. In effect his model does not feature a crucial counter-cyclical force implicit in the efficiency wage structure – the fact that an increase in the rate of job destruction requires a compensating increase in current wages to maintain incentive compatibility. This effect can offset, and even outweigh, the downward pressure on wages due to the demand slump. The converse holds in expansions when relationships are relatively stable, but employment is tight.

Once again, it is the fact that much of this restructring is occurring during recessions which drives this counter–cyclical effect on wages. This timing is associated with the early entry, but delayed implementation of productivity improvements, by firms. This behaviour, in turn, is central to our theory of endogenous cyclical growth in which downturns are the result of anticipated booms. The wage behavior implied here would not, in general, arise in a model where productivity movements were treated exogenously.

example, several authors have shown that various types of efficiency wage model can dampen the procyclicality of wages, while also generating involuntary unemployment (see Alexopolous, 2004 and Danthine and Kurman, 2004). Another literature has examined whether search models are consistent with the cyclical behavior of labor markets, and whether the addition of search in the labor market can help the RBC model to account for labor market facts. These approaches have had some success; Shimer (2003 section 5) provides a comprehensive review of this literature.

• *The productivity slowdown and increased wage pro-cyclicality.* The characterization of average post–war wages as being mildly pro–cyclical masks the fact that until 1970, average real wages were acyclical, or even counter cyclical, whereas between the early 1970s and the early 1990s wages became more pro-cyclical (see Figure 2).[11] This is confirmed by Abraham and Haltiwanger (1995) who report that the correlation between real wages and industrial production rose from -0.072 in the period 1949-1969 to 0.259 in the period 1970-1993.[12] As Figure 3 illustrates, the apparent change in the cyclical behavior of average wages after 1970 coincided with a slowdown in its growth rate. The 'productivity slowdown' which extended from the early 1970's until the mid 1990's has been well documented. Trend productivity growth throughout the post-war period in the US had averaged 2.2% from 1950-1972. From 1972-1995 this rate fell to 1% from 1972-1987 and 1.2% from 1987-1994 (see Oliner and Sichel (2000) and Dolmas, Raj and Slottje (1999)). Was this timing a mere coincidence, or were both a function of changes in the same underlying factors?[13] Our model offers a parsimonious explanation for why these two phenomena are likely to occur simultaneously. In particular, when the productivity of innovation is high, this induces greater innovation and fuels faster productivity growth. At the same time, it also induces more obsolescence and more restructuring to occur during recessions. This implies that the counter–cyclical force on wages tends to be greatest when average productivity growth is high, and less during productivity slowdowns.

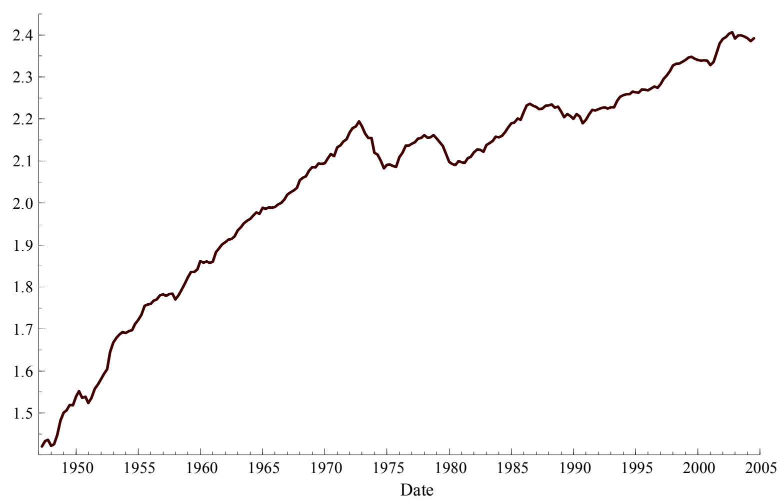

Figure 3: Real Wage in US Manufacturing, 1948–2004

[11] Stock and Watson (1998) find that real wages have essentially no contemporaneous comovement with the business cycle; lagged changes in real wages are only weakly correlated with the cycle; and real wages have no predictive content for output growth.

[12] They also report identical qualitative results for real wages computed with a consumer price index, and for employment as an indicator of cyclicality.

[13] Recently, Comin (2004) argues that the depth of recessions in the 1970s may have been related to the productivity slowdown.

As should be clear, a key feature of our theory is the counter–cyclical allocation of high–skilled labor effort away from production (management) towards longer term productivity enhancing activities (entrepreneurship). The most narrow view of innovation — that which views it as formally measured R&D — suggests that these activities are a small part of the economy's productive base, with movements too trivial to account for any substantial part of aggregate fluctuations (see for example Diego Comin 2004). However, formally measured R&D substantially understates the economy's total productivity improving activity. We view "innovation" as any search for productivity improving ideas that will yield profit. These ideas may be as mundane as the placement of a retail outlet in a new location, or as profound as the introduction of an entirely new good, service or process. Unlike formal R&D workers, who are a small percentage of the economy's productive base, the managers, owners, and other skilled workers, who undertake the search for these new ideas are individuals who also play important roles in the productive process. At certain times, these individuals choose to forego these important roles (typically management) and engage in the search for future oriented productive or competitive advantage.[14] Micro level evidence supports this sort of reallocation (see Francois and Lloyd-Ellis, 2003).

The remainder of the paper is laid out as follows. Section 2 sets up the building blocks of the model. Section 3 posits and describes behavior in the cyclical equilibrium. Section 4 elaborates the dynamics over the phases of the cycle, with particular emphasis on labor flows, and specifies sufficient conditions for existence to be met. Section 5 demonstrates existence and explores the equilibrium's qualitative characteristics. The model's comparative statics are also examined, and applied to the productivity slowdown. Section 6 concludes.

2 The Model and Optimal Behavior

2.1 Final Goods Production

Final output is produced by competitive firms according to a Cobb-Douglas production function utilizing intermediates, x, indexed by i, over the unit interval:

$$Y(t) = \exp\left(\int_0^1 \ln x_i(t)di\right). \tag{1}$$

[14] The view that entrepreneurship 'booms' during recessions can be challenged on the grounds that this seems counter to observed patterns for other investments which are strongly pro-cyclical. But this argument ignores the distinct differences in inputs required to undertake entrepreneurial innovation as compared to more standard (measured) investments. Firstly, since entrepreneurship is more skill intensive than capital accumulation it is much more difficult to pick up with standard measures of investment since it is a within firm, and perhaps even within worker, reallocation. Secondly, since the input mix required for entrepreneurship is intensive in skilled labor, it makes sense that this will occur when returns to skilled labor in production are low, just as investment in education rises when such returns are low — post-secondary educational investments in OECD countries are counter-cyclical, see Sakellaris and Spilimbergo, (2000). Conversely, capital accumulation will tend to be delayed to a point when productivity is higher since it uses the output of production more intensively (see Lloyd–Ellis and Francois, 2004) for such a cyclical model that endogenizes capital accumulation.

Final output is storable (at an arbitrarily small cost), but cannot be converted back into an input for use in production.[15] We let p_i denote the price of intermediate i. Final goods producers choose intermediates to minimize costs. The implied demand for intermediate i is then

$$x_i^d(t) = \frac{Y(t)}{p_i(t)} \tag{2}$$

2.2 Intermediate Goods Production

The output of intermediate i depends upon the state of technology in sector i, $a_i(t)$, and on the labor allocated to production. There are two types of labor — high–skilled manager/entrepreneurs who must be paid $s(t)$ and lower skilled production workers who are paid a wage $w_i(t)$ in sector i. There are two distinct modes which intermediates can use to produce:

• Using small scale production, a skilled worker can set up production on her own and produce a "small" amount of output. Since this individual works alone, there is no need to supervise other workers, the unit cost of production is simply the skilled wage $s(t)$. An individual holding the state of the art technology could produce using this method, but since any individual is of measure zero this would yield negligible profit.

• Using large scale production, an incumbent firm operates a constant returns to scale technology that requires both managers and production workers. As we describe below, the effort provided by production workers is subject to a moral hazard problem. However, assuming that there is no shirking, the firm combines $m_i(t)$ managers and $l_i(t)$ production workers to produce output according to the Leontief production function:[16]

$$x_i^s(t) = a_i(t) \min \left[m_i(t), \frac{l_i(t)}{\theta L} \right]. \tag{3}$$

In equilibrium, firms optimally hire workers so that

$$l_i(t) = \theta L m_i(t) \tag{4}$$

Thus, we effectively assume that managers have a fixed "span of control" with one manager required to supervise $\theta L > 1$ production workers, where $\theta < 1$.[17]

2.3 Innovation

Ongoing marginal improvements in productivity arise in each sector through the diversion of skilled labor effort away from the supervision of production and towards innovation.[18] These

[15] That is, there is no tangible capital in the model. See Francois and Lloyd–Ellis (2004) for a related model with physical capital.

[16] In the presence of shirking, output would be zero.

[17] This production function simplifies the exposition considerably, but could be generalized (allowing for some degree of substitution) without changing our main results.

[18] This process can equivalently be thought of as a search for product improvements, process improvements, organizational advances or anything else in the form of new knowledge which creates a productive advance over the existing state of the art.

activities are financed by selling equity shares to households. The probability of an innovative success over an instant dt is $\delta h_i(t)dt$, where δ is a parameter, and h_i is the management effort allocated to innovation in sector i. The aggregate managerial effort allocated to innovation is given by $H(t) = \int_0^1 h_i(t)di$. The productivity of new innovations are assummed to dominate that of old ones by a factor e^{γ}.

Entrepreneurs with innovations face two choices. (1) They must choose the timing of their entry into production. (2) They must choose whether or not to implement their innovation immediately upon entering production, or delay implementation until a later date. Once they implement, the knowledge associated with the innovation becomes publicly available, even though they can protect a use right, and can be built upon by rival entrepreneurs.[19] However, prior to implementation, the knowledge is privately held by the entrepreneur. By delaying implementation the entrepreneur loses the productive advantage of the new innovation, but gains from ensuring that the content of the innovation is secret, and will thus not be built upon.

An innovator must have control of the productive resources of the firm *prior* to implementation. This is intended to capture the idea that some degree of reorganization is required to take advantage of new approaches or innovations. Once an innovation has been implemented, the entrepreneur with the knowledge of how to implement can costlessly enter or exit production at any time. The information embedded in a new productive technology will be used by the next generation of innovators to design an improvement which will make the current incumbent's technology redundant.

We let the indicator function $Z_i(t)$ take on the value 1 if there exists a successful innovation in sector i which has not yet been implemented, and 0 otherwise. The set of periods in which innovations are implemented in sector i is denoted by Ψ_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing an innovation at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable time in future.

2.4 The Labor Market

In aggregate there is a unit measure of skilled, manager/entrepreurs and a measure L of low–skilled, production workers. We assume that managers' reputations are long–lived, so that if they "shirk" they are forever marked and never again re-hired.[20] In contrast, production workers are working in teams or are otherwise more difficult to monitor. Consequently, "shirking" is a problem that firms must deal with. Specifically, low–skilled workers can choose to shirk by providing zero effort, while potentially retaining their jobs. If they do not shirk, workers in sector i are subject

[19] It is not necessary to assume that implementation reveals all of the information embedded in an improvement as we have done here. Provided there is some information generated through implementation which is useful to entrepreneurs searching for productivity improvements that will supercede the incumbent, then implementation will be 'costly' in terms of hastening redundancy and qualitatively identical results will hold.

[20] An alternative assumption is that the owner of a firm can observe the work effort, or perfectly contract, with managers so that these individuals can be employed at opportunity cost. Our aim is parsimony here, not realism. Introducing a moral hazard problem at this level as well would not change the qualitative behavior of our model.

to a rate of "job destruction" $\mu_i(t)dt$. Jobs may be destroyed for two reasons. First, there is a constant, exogenous "normal" rate of job turnover, $\bar{\mu}dt$, which is independent of the business cycle and which is not assciated with firm shut–downs. Second, there may be job destruction due to firms shutting down completely. If a worker does shirk, the rate of job loss increases to $(\mu_i(t) + q)\,dt$, where q depends on the ability of the firm to detect shirking. Since all workers are identical, firms must pay them an "efficiency wage" that induces them not to shirk, otherwise they will produce no output.

As we shall see, the rate of job destruction may vary across sectors. We denote the equilibrium average rate of job destruction across all sectors by $\mu^A(t)dt$. If workers do lose their job, they enter a pool of unemployed workers who are viewed as homogenous by firms and, hence, face an equal probability of being re–hired over an interval dt equal to $d\Lambda(t)$. Aggregate low–skilled worker flows are such that the change in the level of employment equals the number of jobs created less the number destroyed:

$$dn(t) = (L - n(t))\,d\Lambda(t) - n(t)\mu^A(t)dt \tag{5}$$

2.5 Goods Market Competition

In order to extract rent from the leading edge technology, intermediate producers must utilize the large scale mode of production and hire workers.[21] Given the unit elasticity of demand, the producer holding the state of the art technology wishes to limit price at the marginal cost of his next best competitors. If $\eta(t)$ denotes this marginal cost, the limit price in sector i is then given by

$$p_i(t) = \frac{\eta(t)}{e^{-\gamma}a_i(t)}. \tag{6}$$

We assume that intermediates are completely used up in production, but can be produced and stored for use at a later date. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later.

2.6 Households

There is a unit measure of infinitely–lived hosueholds who are assumed to have preferences given by

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)}u(c(t), n(t))d\tau, \tag{7}$$

[21] In order to sustain meaningful innovation, there must exist some way for successful innovators to extract, at least temporarily, rents from their knowledge. At the most formal end, this may be through the use of patents and the threat of a law suit, more informally, rent dissipation may simply be a matter of time, or may be preserved by the threat that, if copied, the innovator will drive copiers out of the market. We leave unspecified the precise means by which this occurs and simply assume that the leading technology is the exclusive province of the innovator. Once the innovation has been superceded, others may enter and use it at will, since, as it can no longer generate rents, the innovator controlling it no longer has incentive to limit (by whatever means) its use elsewhere.

where ρ denotes the rate of time preference, $c(t)$ denotes household consumption and $n(t)$ denotes the measure of household labor supply that is exerting effort. We assume that the period utility function takes the Cobb–Douglas form

$$u(c(t), n(t)) = c(t)^{1-\sigma} (L - n(t))^{\sigma}. \tag{8}$$

These preferences are a special case of those commonly considered in the RBC literature. We focus on them here for expositional simplicity, but they can be generalized to allow for homogeneity of degree other than one without changing our qualitative conclusions.[22]

Each household is endowed with a measure one of high–skilled human capital (manager/entrepreneur) and a measure $L > 1$ of low–skilled human capital (production workers). High–skilled workers supply labor inelastically. If they are employed, low–skilled workers choose whether or not to supply one unit of labor effort per period so as to maximize their present discounted contribution to household utility, taking current and future household consumption and employment levels as given. Note that this "large" household assumption effectively allows us to abstract from employment uncertainty at the household level. In particular, it implies that total household wage income, $\omega(t)$, is certain and identical across households.

Each household chooses consumption over time to maximize (7) subject to the intertemporal budget constraint

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} c(\tau) d\tau \leq S(t) + \int_t^{\infty} e^{-[R(\tau)-R(t)]} \left[s(\tau) + \omega(\tau) \right] d\tau \tag{9}$$

where $S(t)$ denotes the household's stock of assets at time t and $R(t)$ denotes the discount factor from time zero to t. There are three assets that could potentially be held by households in our economy: claims to the profits of intermediate firms, stored intermediate output and stored final output. As we shall see, in equilibrium only claims to the profits of intermediate firms will be traded — intermediate and final output are never stored. However, the potential for stored output to be traded imposes restrictions on the possible equilibria that can emerge.

The first–order conditions of the household's dynamic optimization require that

$$dR(t) = \rho dt + \sigma \left[\frac{dc(t)}{c(t)} + \frac{dn(t)}{L - n(t)} \right] \qquad \forall \ t \tag{10}$$

and that (9) holds with equality.[23]

A low skilled worker who is offered employment in sector i at the beginning of time t chooses whether to accept the offer or remain unemployed. If he accepts, he chooses whether or not to exert effort in order to maximize his contribution to household utility. If the low–skilled wage in sector i is $w_i(t)$, then if such a worker supplies one unit of labor effort, his marginal contribution

[22] Note that the implied intertemporal elasticity of substitution exceeds unity. This feature can be relaxed in a model with physical capital (see Francois and Lloyd–Ellis, 2004), but is essential here.

[23] The Euler equation is expressed in this form to allow for the possibility of discontinuous jumps in the discount factor.

to household utility in the current period (measured in consumption–equivalent units) is

$$w_i(t) + \frac{u_n(t)}{u_c(t)}, \tag{11}$$

where

$$\begin{aligned} u_c(t) &= (1-\sigma)c\,(t)^{-\sigma}\,(L-n(t))^{\sigma} && (12)\\ u_n(t) &= -\sigma c\,(t)^{1-\sigma}\,(L-n(t))^{\sigma-1}. && (13)\end{aligned}$$

The worker uses the household's equilibrium discount factor, $e^{-dR(t)}$, to discount future wages net of the marginal disutility of providing effort (measured in consumption–equivalent units). As a result the value of the worker's optimal program if he is offered employment is

$$V_i^M(t) = \max\left[V_i^E(t), V_i^S(t), V^U(t)\right] \tag{14}$$

where

$$V_i^E(t) = \left(w_i(t) + \frac{u_n(t)}{u_c(t)}\right)dt + e^{-dR(t)}\left[e^{-\mu_i(t)dt}V_i^M(t+dt) + (1-e^{-\mu_i(t)dt})V^U(t+dt)\right] \tag{15}$$

represents the value of being employed and supplying effort,

$$V_i^S(t) = w_i(t)dt + e^{-dR(t)}\left[e^{-(\mu_i(t)+q)dt}V_i^M(t+dt) + (1-e^{-(\mu_i(t)+q)dt})V^U(t+dt)\right] \tag{16}$$

represents the value of being employed and shirking, and

$$V^U(t) = e^{-dR(t)}\left[e^{-d\Lambda(t)}V_i^M(t+dt) + (1-e^{-d\Lambda(t)})V^U(t+dt)\right], \tag{17}$$

represents the value of being unemployed. If the worker is not offered employment at time t the value of his programme is simply $V^U(t)$.

2.7 General Equilibrium

Given an initial stock of implemented innovations represented by a cross–sectoral distribution of productivities $\{a_i(0)\}_{i=0}^1$ and an initial distribution of unimplemented innovations, $\{Z_i(0)\}_{i=0}^1$, an equilibrium for this economy satisfies the following conditions:

• Households allocate consumption optimally over time, (10).

• Low–skilled workers accept employment offers if and only if their participation constraint is satisfied

$$V_i^E(t) \geq V_i^U(t). \tag{18}$$

• Final goods producers choose intermediates to minimize costs, (2).

• Intermediate producers set prices so as to maximize profits, given demand, (6).

• Intermediate producers choose the mode of production which maximizes their profits.

• The skilled–labor market clears:

$$\int_0^1 m_i(t)di + H(t) = 1. \tag{19}$$

Morover, it follows from (2), (4) and (6), that the skilled work force receives a constant share of output:

$$s(t)(1 - H(t)) = e^{-\gamma} Y(t) \tag{20}$$

• In the face of unemployment, intermediate producers offer a path of low–skilled wages so as to maximize profits subject to the participation constraint and the incentive compatability condition

$$V_i^E(t) \geq V_i^S(t). \tag{21}$$

• Free entry into arbitrage. For all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{R(s)-R(t)}{s-t}$.
• There is free entry into innovation. Entrepreneurs select the sector in which they innovate so as to maximize the expected present value of the innovation. Also

$$\delta \max[V_i^D(t), V_i^I(t)] \leq s(t), \quad h_i(t) \geq 0 \qquad \text{with at least one equality} \tag{22}$$

• Entrepreneurs with innovations choose whether to enter production using the previous technology.
• In periods where there is implementation, entrepreneurs with innovations must prefer to implement rather that delay until a later date

$$V_i^I(t) \geq V_i^D(t) \quad \forall\ t \in \Psi_i \tag{23}$$

• In periods where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } Z_i(t) &= 0, \\ \text{or if } Z_i(t) &= 1,\ V_i^I(t) \leq V_i^D(t) \quad \forall\ t \notin \Psi_i. \end{aligned} \tag{24}$$

3 The Cyclical Equilibrium

In this section we posit a cyclical growth path along which innovations are implemented in clusters rather than in a smooth fashion. We derive the optimal behavior of agents in such a cyclical equilibrium and the evolution of the key variables under market clearing. We then derive sufficient conditions for the existence of such a cyclical equilibrium and show that market clearing implies a unique positive cycle length and long run growth rate.

Suppose that the implementation of entrepreneurial innovations occurs at discrete intervals. An implementation period is denoted by T_ν where $v \in \{1, 2, ..., \infty\}$, and we adopt the convention

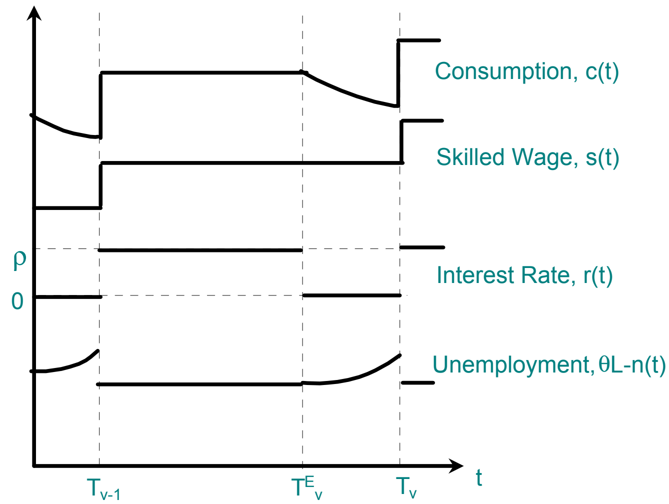

Figure 4: The Cyclical Growth Path

that the vth cycle starts in period T_{v-1} and ends in period T_ν. The evolution of consumption, which equals final output, during a typical cycle between implementation periods is depicted in Figure 4.

A boom occurs when accumulated innovations are implemented at T_{v-1}. After that there is an interval during which no entrepreneurial effort is devoted to improvement of existing technologies and consequently skilled labor is used in production. During this interval, no new innovations are implemented so that growth slows to zero. Moreover, no firms shut down or enter and the only source of job turnover is exogenous. During this phase, job creation is just equal to job destruction and unemployment is at its minimum level. The low skilled wage is constant and identical across all sectors because all workers face the same probability of being displaced.

At some time T_v^E innovation commences again. Successful entrepreneurs enter production and displace existing incumbents in their sectors, however they withhold implementation of their productivity improvement until time T_v, prefering instead to use the existing technology until that time. It is not worthwhile for any entrepreneur to innovate in a sector where this has already happened. Entrepreneurial activity occurs throughout the interval $[T_v^E, T_v]$ and corresponds to a decline in the economy's production, as high skilled workers are diverted away from production towards the search for improvements. As this happens the demand for production workers falls in all sectors putting downward pressure on wages. In sectors where no innovation or restructuring has occurred, the likelihood that a worker will be displaced in the near future, rises. In order to maintain incentives, firms in these sectors must pay a sufficiently high wage, $w^H(t)$. In sectors where restructuring has already occurred, firms can pay a lower wage $w^L(t)$, because no further displacement will occur. The impact on the average wage is the net effect of these forces. At T_v the simultaneous implementation of all successful entrepreneurs leads the $(v+1)$th cycle to start with a boom.

3.1 Optimal Consumption

Over intervals during which neither the discount factor not the level of of employment does not jump, consumption is allocated according to the differential equation

$$\rho + \sigma \frac{\dot{c}(t)}{c(t)} + \sigma \frac{\dot{n}(t)}{L - n(t)} = r(t), \tag{25}$$

where $r(t) = \frac{dR(t)}{dt}$. However, as we will demonstrate here, along the cyclical growth path, the discount factor jumps at the boom, so that consumption exhibits a discontinuity during implementation periods. We therefore characterize the optimal evolution of consumption from the beginning of one cycle to the beginning of the next by the difference equation

$$\sigma \ln \left(\frac{c_0(T_v)}{c_0(T_{v-1})} \right) - \sigma \ln \left(\frac{L - n_0(T_v)}{L - n_0(T_{v-1})} \right) = R_0(T_v) - R_0(T_{v-1}) - \rho \left(T_v - T_{v-1} \right) \qquad \forall \ t, \tau, \tag{26}$$

where the 0 subscript is used to denote values of variables the instant after the implementation boom. As we describe below, it must be the case that $n_0(T_v) = \theta L \ \forall v$, so that the second term is zero. Note that a sufficient condition for the boundedness of the consumer's optimization problem is that $\ln \frac{c_0(T_v)}{c_0(T_{v-1})} < R(T_v) - R(T_{v-1})$ for all v, or that

$$\frac{1}{T_v - T_{v-1}} \ln \frac{c_0(T_v)}{c_0(T_{v-1})} < \frac{\rho}{1-\sigma} \quad \forall \ v. \tag{27}$$

In our analysis below, it is convenient to define the discount factor that will be used to discount from some time t during the cycle to the beginning of the next cycle. This discount factor is given by

$$\beta(t) = R(T_v) - R(t). \tag{28}$$

3.2 Mode of Production

Lemma 1 *In equilibrium, an employer who holds a redundant technology cannot use the large scale mode of production.*

This is because such a firm cannot profitably motivate production workers using an efficiency wage since it cannot promise employment into the indefinite future. This implies that the previous incumbent can produce only when hiring skilled workers. Though still able to use the large scale technology, with skilled workers filling the positions of unskilled workers, this will not be efficient. The lowest cost competitors are therefore the competitive fringe who, though producing at small scale, can in aggregate steal the incumbent's whole market. These competitors avoid the costs of setting up large scale production and hiring multiple workers, but can only produce using the previous state of the art technology.

The marginal cost of this competitive fringe is $\eta(t) = s(t)$. It follows that the limit price is $p_i(t) = \frac{s(t)}{e^{-\gamma} a_i(t)}$, so that incumbent firm profits can be expressed as

$$\pi\left(w_i(t), t\right) = \left(1 - e^{-\gamma} - e^{-\gamma}\theta L \frac{w_i(t)}{s(t)}\right) Y(t). \tag{29}$$

Note that, since all firms face the same skilled salary and revenue, profits vary across sectors only due to differences in the production wage, $w_i(t)$.

3.3 Entrepreneurship

The present value of profits earned in a sector where no future restructuring is anticipated up to the end of the current cycle is

$$V^*(t) = \int_t^{T_v} e^{-\int_t^\tau r(s)ds} \pi(w^L(\tau), \tau) d\tau$$

In the cyclical equilibrium considered here, secrecy (i.e. protecting the knowledge embodied in a new innovation by delaying implementation) can be a valuable option.[24] Since, as will be demonstrated, innovations are withheld until a common implementation time, simultaneous implementation is a possibility. However, as the following Lemma demonstrates, such duplications do not arise in the cyclical equilibrium because successful innovators enter production to displace previous incumbents, sending a credible signal that stops subsequent entrepreneurial efforts in their sector.

Lemma 2 *In a cyclical equilibrium, given that innovations are implemented at the subsequent boom, a successful entrepreneur offers $V^*(t) + \varepsilon$ to an incumbent to stop producing until T_{v+1}, and takes over production in that sector. All innovative activity in their sector then stops until the next round of implementation.*

The transfer of an amount $V^*(t) + \varepsilon$ by a successful entrepreneur to the previous incumbent acts as a credible signal that this entrepreneur has had a research success, but does not reveal the content of that success. If an entrepreneur's announcement is credible, other entrepreneurs will exert their efforts in sectors where they have a better chance of becoming the dominant entrepreneur. One might imagine that unsuccessful entrepreneurs would have an incentive to mimic successful ones by falsely announcing success to deter others from entering the sector. But doing this yields a flow of profits for the interval $t \to T_{v+1}$ which is ε less than paid for it. It

[24] As Cohen, Nelson and Walsh (2000) document, delaying implementation to protect knowledge is a widely followed practice in reality.

follows that the value of an incumbent firm in a sector where no innovation has occurred by time t during the vth cycle can be expressed as

$$V_i^I(t) = \pi_i(w_H(t), t)dt + e^{-r(t)dt}\left[e^{-\delta h_i dt}V^I(t+dt) + (1 - e^{-\delta h_i dt})V^*(t+dt)\right] \tag{30}$$

Let $P_i(t)$ denote the probability that, since time T_{v-1}, no entrepreneurial success has been made in sector i by time t:

$$P_i(t) = \exp\left(-\int_t^{T_v} \delta h_i(\tau)d\tau\right). \tag{31}$$

It follows that the probability of there being no innovation by time T_v conditional on there having been none by time t, is given by $P_i(T_v)/P_i(t)$. Integrating () over time yields

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^\tau [r(s)+\delta h_i(s)]ds}\left[\pi(w^H(\tau), \tau) + \delta h_i(\tau)V_i^*(\tau)\right]d\tau + \frac{P_i(T_v)}{P_i(t)}e^{-\beta(t)}V_{0,i}^I(T_v). \tag{32}$$

The first term here represents the expected discounted profit stream that accrues to the entrepreneur during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter.

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)}V_{0,i}^I(T_v), \tag{33}$$

The symmetry of sectors implies that innovative effort is allocated evenly over all sectors that have not yet experienced an innovation within the cycle. The amount of entrepreneurship varies over the cycle, but at the beginning of each cycle all industries are symmetric with respect to this probability: $P_i(T_v) = P(T_v)\ \forall i$.

4 Within–cycle dynamics

Within a cycle, $t \in [T_{v-1}, T_v)$, the state of technology in use is unchanging. A critical variable is the amount of labor devoted to entrepreneurship, the opportunity cost of which is production. In order to determine this, we first characterize wages paid to skilled human capital in production.

Lemma 3 *The skilled wage for $t \in [T_{v-1}, T_v]$ is pinned down by the level of technology*

$$s(t) = e^{-\gamma}\exp\left(\int_0^1 \ln a_i(T_{v-1})di\right) = s_v. \tag{34}$$

Since they are determined by the level of technology in use, and since this does not change until the subsequent boom, skilled wages are constant within the cycle.

During the cycle, both the discount factor and the rate of job creation are continuous variables, so that the derivative

$$\lambda(t) = \frac{d\Lambda(t)}{dt} \tag{35}$$

is well defined. Utilizing the incentive compatibility condition (21), net present value of each of the three employment states (15), (16) and (17) yields the following binding efficiency wage.

Proposition 1 *: The low–skilled wage in sector i is given by*

$$w_i(t) = \frac{\sigma c(t)}{(1-\sigma)(L-n(t))}\left[1+\frac{1}{q}\left(\rho+\mu_i(t)+\lambda(t)-\frac{\dot{u}_n(t)}{u_n(t)}\right)\right] \tag{36}$$

The standard first–order condition for labor supply in the absence of real rigidity would not include the term in square brackets. The impact of the efficiency wage problem is to drive a wedge into this condition. Note that the implied wage depends positively on $\mu_i(t)$ — the less likely the relationship is to continue, the weaker are the incentives of the promised employment into the future, and the higher the efficiency wage must be. It depends positively on $\lambda(t)$ — the easier it is to find a job once unemployed, the less effective is the threat of being fired, so that the wage must rise to compensate. Finally, the wage depends negatively on $\frac{\dot{u}_n(t)}{u_n(t)}$ — if the marginal cost of supply effort is rising rapidly, workers are less likely to risk unemployment by shirking (effectively trading future for current employment), so the current wage need not be so high.

4.1 The Slowdown: Constant and Low Labor Flows

As a result of the boom, skilled wages rise rapidly. Since the next implementation boom is some time away, the present value of engaging in innovation falls below the skilled wage, $\delta V^D(t) < s(t)$. During this phase, no skilled labor is allocated to innovation and no new ideas are implemented. Since technology is unchanging, final output must be constant

$$g(t) = \frac{\dot{n}(t)}{n(t)} = \frac{\dot{s}(t)}{s(t)} = 0. \tag{37}$$

Note that since all skilled labor is used in production, it must be the case that unskilled employment is at its maximum: $n(t) = \theta L$. There is no firm turnover, so that $\mu_i(t) = \mu^A(t) = \bar{\mu} \;\; \forall\, i$, and so the rate at which unemployed worker are hired is given by

$$\lambda(t) = \frac{\theta\bar{\mu}}{1-\theta} \quad \forall\, t \in [T_{v-1}, T_v^E]. \tag{38}$$

With zero growth and a constant level of employment, the household's Euler equation dictates that the interest rate just equal the rate of time preference,

$$r(t) = \rho. \tag{39}$$

Since the economy is closed, and there is no incentive to store either intermediate or final output when $r(t) \geq 0$, it must be the case that:

$$c(t) = Y(t) = Y_0(T_{v-1}). \tag{40}$$

Substituting these conditions into the (36) we get

Lemma 4 *During the slowdown unemployment is constant at $(1-\theta)L$ and the production wage is constant and given by*

$$w^A(t) = w_0^A(T_v) = \frac{A}{L} Y_0(T_{v-1}) \tag{41}$$

where $A = \sigma\left(\rho + q + \frac{\bar{\mu}}{1-\theta}\right) \Big/ \left[(1-\sigma)(1-\theta)q\right]$.

During this phase, the expected value of entrepreneurship, $\delta V^D(t)$, is necessarily growing at the rate of interest, $r(t) = \rho$, but continues to be dominated by the wage in production. Since the wage is constant during the cycle, $\delta V^D(t)$, must eventually equal $s(t)$. At this point, entrepreneurship commences. The following Lemma demonstrates that it does so smoothly:

Lemma 5 *At time T_v^E, when entrepreneurship first commences in a cycle, $s_v = \delta V^D(t)$ and $H\left(T_v^E\right) = 0$.*

4.2 The Contraction: Increasing Flows into Employment and Unemployment

For positive entrepreneurship to occur under free entry, it must be that $s_v = \delta V^D(t)$. Since the wage is constant throughout the cycle, $\delta V^D(t)$ must also be constant during this phase. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits because implementation is delayed, the instantaneous interest rate must be zero.

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{s}(t)}{s(t)} = 0. \tag{42}$$

With a positive discount rate, $\rho > 0$, a zero interest rate implies that consumption must be declining. Since the economy is closed, it follows once again that, because there is no incentive to store output, (40) holds.[25] Hence, the household's Euler equation can be expressed as

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{n}(t)}{L - n(t)} = -\frac{\rho}{\sigma}. \tag{43}$$

Since consumption is proportional to employment we can express this as a differential equation in $n(t)$, the solution to which is derived in the following Lemma:

Lemma 6 *: Employment during the downturn evolves according to*

$$n(t) = \frac{\theta L e^{-\frac{\rho}{\sigma}(t - T_v^E)}}{1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}}. \tag{44}$$

[25] Although $r = 0$, strict preference for zero storage results from arbitrarily small storage costs.

Note that $n(T_v^E) = \theta L$. This expression implies that $n(t)$ must be declining during the downturn because, as high–skilled labor flows out of production and into entrepreneurship, the demand for low-skilled workers falls in proportion. The implied skilled labor that flows into innovation is therefore

$$H(t) = 1 - \frac{n(t)}{\theta L} = \frac{(1-\theta)\left(1 - e^{-\frac{\rho}{\sigma}(t-T_v^E)}\right)}{1-\theta+\theta e^{-\frac{\rho}{\sigma}(t-T_v^E)}} \tag{45}$$

The proportion of sectors that have not yet experienced an entrepreneurial success by time $t \in (T_v^E, T_v)$ is given by

$$P(t) = \exp\left(-\int_{T_v^E}^{t} \delta h(\tau)d\tau\right). \tag{46}$$

Recalling that labor is only devoted to entrepreneurship in sectors which have not innovated since the start of the cycle, the labor allocated to entrepreneurship in each sector where no innovation has yet occurred is then

$$h(t) = \frac{H(t)}{P(t)}. \tag{47}$$

Differentiating (46), and substituting in (47), we thus obtain the aggregate rate of entrepreneurial success,

$$\dot{P}(t) = -\delta h(t)P(t) = -\delta H(t). \tag{48}$$

Observe that although the rate of decline in the proportion of sectors that have not yet innovated, $P(t)$, is proportional to the amount of entrepreneurship in each sector, the level reductions in P are proportional to the aggregate amount of entrepreneurship. This reflects the fact that as new innovations arise, aggregate innovative effort is allocated across fewer and fewer sectors.[26]

In the measure $(1-P(t))$ of sectors where restructuring has occurred, the only source of job destruction is normal turnover, $\bar{\mu}$. However, in sectors where no restructuring has occurred, the probability of job destruction also includes the probability of a restructuring occurring, $\delta h(t)$, which increases as the downturn proceeds. It follows that the aggregate rate of job destruction is given by

$$\mu^A(t) = (1-P(t))\bar{\mu} + P(t)\left[\bar{\mu} + \delta\frac{H(t)}{P(t)}\right] = \bar{\mu} + \delta H(t). \tag{49}$$

Using (5), the probability of being re–hired once unemployed must then be given by

$$\lambda(t) = \frac{\dot{n}(t) + \mu^A(t)n(t)}{L - n(t)} \tag{50}$$

In sectors where innovation has occured, wages are lower as there is no chance of further restructuring in the contraction, and so

$$w^L(t) = \frac{\sigma c(t)}{(1-\sigma)(L-n(t))}\left[1 + \frac{1}{q}\left(\rho + \bar{\mu} + \lambda(t) - \frac{\dot{u}_n(t)}{u_n(t)}\right)\right]. \tag{51}$$

[26] We characterize an equilibrium in which the cycle is never long enough that all sectors innovate, $P(T_v) > 0$. This is an equilibrium condition that will be imposed below.

When a restructuring has not yet occurred, there is higher probability of job destruction and a higher wage is required to ensure incentive compatibility:

$$w^{H}\left(t\right)=\frac{\sigma c(t)}{(1-\sigma)(L-n(t))q}\left[1+\frac{1}{q}\left(\rho+\bar{\mu}+\delta h\left(t\right)+\lambda(t)-\frac{\dot{u}_{n}(t)}{u_{n}(t)}\right)\right]. \tag{52}$$

Since the unskilled wage in each sector is linearly related to the rate of job destruction, it follows that the average efficiency wage for the unskilled can be expressed as

$$w^{A}(t)=\frac{\sigma c(t)}{\left(1-\sigma\right)\left(L-n(t)\right)q}\left[1+\frac{1}{q}\left(\rho+\mu^{A}(t)+\lambda(t)-\frac{\dot{u}_{n}(t)}{u_{n}(t)}\right)\right] \tag{53}$$

In general, the evolution of the average production wage during this phase reflects the interaction of six forces. Four of these are procyclical:

• Falling aggregate demand for production workers, so that $n(t)$ falls, pushes wages down.

• As consumption $c(t)$ falls the marginal benefit of supplying labor increases, so that firms can lower the wage and still maintain incentives.

• As unemployment rises, $\lambda(t)$ falls, so that the threat of termination becomes stronger and firms can lower the wage.

• As the downturn proceeds, more and more sectors are taken over by new entrants who pay the relatively low wage, $w^{L}(t)$, thereby driving down the average wage.

The other two forces, however, are countercyclical:

• Increased innovation leads to greater turnover in sectors where no innovation has occurred yet, $\bar{\mu}+\delta h(t)$. This exerts upward pressure on the wage in these sectors, $w^{H}(t)$, because workers must be compensated for the increased likelihood of job loss.

• With negative employment growth, the marginal cost of suppling effort is lower tomorrow than today $\frac{\dot{u}_{n}(t)}{u_{n}(t)}>0$. Other things equal, this makes workers more willing to risk unemployment by shirking, so firms must raise wages to compensate.

As we show in the next section, if the rate of innovation is high enough, these latter effects could even dominate, causing low–skilled wages to rise during recessions.

By substituting in for the endogenous variables, the following Proposition derives the time path for the production wage analytically. In general, the evolution of the wage is ambiguous.

Proposition 2 *: During the downturn, the average low–skilled wage is given by*

$$w^{A}(t)=\left[Be^{-\frac{\rho}{\sigma}\left(t-T_{v}^{E}\right)}-Ce^{-\frac{2\rho}{\sigma}\left(t-T_{v}^{E}\right)}+\frac{De^{-\frac{\rho}{\sigma}\left(t-T_{v}^{E}\right)}}{1-\theta+\theta e^{-\frac{\rho}{\sigma}\left(t-T_{v}^{E}\right)}}\right]\frac{Y_{0}(T_{v-1})}{L} \tag{54}$$

where $B=\frac{\sigma(q+\delta+\bar{\mu})}{(1-\sigma)(1-\theta)q}$, $C=\frac{\sigma\left(\delta-\frac{\bar{\mu}\theta}{1-\theta}\right)}{(1-\sigma)(1-\theta)q}$ *and* $D=\frac{\rho}{(1-\sigma)q}$.

4.3 The Boom

We denote the improvement in aggregate productivity during implementation period T_v by e^{Γ_v}, where

$$\Gamma_v = \int_0^1 [\ln a_i(T_v) - \ln a_i(T_{v-1})]\, di \tag{55}$$

Productivity growth at the boom is given by $\Gamma_v = (1 - P(T_v))\gamma$, where $P(T_v)$ is defined by (46). Substituting in the allocation of labor to entrepreneurship through the downturn given by (45) and integrating over the interval

$$\Delta_v^E = T_v - T_v^E. \tag{56}$$

yields the following implication:

Proposition 3 *In an equilibrium where there is positive entrepreneurship only over the interval $(T_v^E, T_v]$, the growth in productivity during the succeeding boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E + \frac{\sigma\delta\gamma}{\rho\theta}\ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta_v^E}\right)\right). \tag{57}$$

Equation (57) tells us how the size of the productivity boom depends positively on the amount of time the economy is in the entrepreneurship phase, Δ_v^E. The amount of innovation in that phase is determined by the movements in the interest rate, so once the length of the entrepreneurship phase is known, the growth rate over the cycle is pinned down. The size of the boom is convex in Δ_v^E, reflecting the fact that as the boom approaches, the labor allocated towards innovation is increasing. This also implies that the boom size is increasing in the depth of the downturn, since the longer the downturn the greater the allocation of innovative effort and hence the larger the decline in output.

Over the boom, the asset market must simultaneously ensure that entrepreneurs holding innovations are willing to implement immediately (and no earlier) and that, for households, holding equity in firms dominates holding claims to alternative assets (particularly stored intermediates). The following Proposition demonstrates that these conditions imply that during the boom, the discout factor must equal productivity growth:

Proposition 4 *Asset market clearing at the boom requires that*

$$\beta(T_v) = \Gamma_v \tag{58}$$

The long run discount factor during the downturn is given by $\beta(T_v) = \sigma\Gamma_v + \rho\Delta_v^E$. Combining this with (58) yields

$$\Gamma_v = \frac{\rho\Delta_v^E}{1-\sigma}. \tag{59}$$

Since the short term interest rate during the downturn is zero, asset market clearing requires that the long term interest rate at the end of the downturn is equal to its value at the beginning.

The value at the end must equal the size of the productivity boom in equilibrium; the value at the beginning reflects the size of the future boom and the time until it occurs. It follows that asset market–clearing yields a unique relationship between the length of the downturn and the size of the subsequent productivity boom.

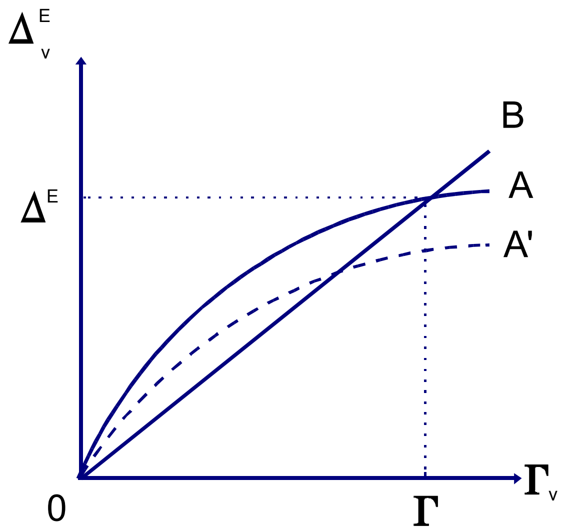

Figure 5: Equilibrium Recession Length and Boom Size

Figure 5 depicts the two conditions (57) and (59) graphically. As shown by the solid lines, combining the two conditions yields a unique (non–zero) equilibrium pair (Γ, Δ^E) that is consistent with the within–cycle dynamics and the asset market clearing condition. Combining them implies that Δ^E must satisfy

$$\left(1 - \frac{\rho}{\delta\gamma(1-\sigma)}\right)\Delta^E = -\frac{\sigma}{\rho\theta}\ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right) \tag{60}$$

Note that although we did not impose any stationarity on the cycles, the equilibrium conditions imply stationarity of the size of the boom and the length of the downturn. For a unique positive value of Δ^E that satisfies this condition to exist it is sufficient that $\rho < \delta\gamma(1-\sigma)$.

The jump in productivity at the boom increases the marginal product of skilled labor induces them to re–allocate their effort back into production. With a fixed span of control and no search frictions in the labor market, this in turn induces a jump in production worker employment given by

$$\Delta \log n(T_v) = (1-\theta)\left(\frac{1-\sigma}{\sigma}\right)\Gamma.$$

Output therefore grows through the boom both because of the rise in TFP and this re–allocation of labor into production:

$$\Delta \log y(T_v) = \left[1 + (1-\theta)\left(\frac{1-\sigma}{\sigma}\right)\right]\Gamma.$$

4.4 Optimal Entrepreneurial Behavior

It has thus far been assumed that entrepreneurs are willing to follow the innovation, entry and implementation sequence hypothesized in the cycle. The equilibrium conditions that we have considered so far effectively assume that entrepreneurs who plan to innovate will implement at T_v and that they start innovation at T_v^E. However, the willingness of entrepreneurs to delay implementation until the boom and to just start engaging in innovative activities at exactly T_v^E depends crucially on the expected value of monopoly rents resulting from innovation, relative to the current labor costs. This is a forward looking condition: given Γ and Δ^E, the present value of these rents depend crucially on the length of the subsequent cycle, $T_{v+1} - T_v$.

Since Lemma 5 implies that entrepreneurship starts smoothly at T_v^E, free entry into entrepreneurship, requires that

$$\delta V^D(T_v^E) = \delta e^{-\beta(T_v^E)} V_0^I(T_v) = s_v \tag{61}$$

Since the increase in the wage across cycles reflects only the improvement in productivity: $s_{v+1} = e^{\Gamma} s_v$, and since from the asset market clearing conditions, we know that $\beta(T_v^E) = \Gamma$, it immediately follows that the increase in the present value of monopoly profits from the beginning of one cycle to the next must, in equilibrium, reflect only the improvements in aggregate productivity:

$$V_0^I(T_v) = e^{\Gamma} V_0^I(T_{v-1}). \tag{62}$$

Equation (62) implies that, given some initial implementation period, and stationary values of Γ and Δ^E, the next implementation period is determined. Notice, once again that this stationarity is not imposed, but is an implication of the equilibrium conditions. Letting $\Delta_v = T_v - T_{v-1}$, we therefore have the following result:

Proposition 5 *Given the boom size, Γ, and the length of the entrepreneurial innovation phase, Δ^E, there exists a unique cycle length, Δ, such that entrepreneurs are just willing to commence innovation, Δ^E periods prior to the boom.*

The equilibrium cycle length is given by

$$\Delta = \Delta^E + \frac{1}{\rho}\ln\left[1 + \alpha\Delta^E + \zeta_1\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right) - \zeta_2\left(1 - e^{-\frac{2\rho}{\sigma}\Delta^E}\right) - \frac{\zeta_3\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)}\right] \tag{63}$$

where α, ζ_1, ζ_2 and ζ_3 are constants defined in the Appendix.

The equilibrium conditions (22), (23) and (24) on entrepreneurial behavior also impose the following requirements on our hypothesized cycle:

• Successful entrepreneurs at time $t = T_v$, must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \tag{E1}$$

• Entrepreneurs who successfully innovate during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{E2}$$

• No entrepreneur wants to innovate during the slowdown of the cycle. Since in this phase of the cycle $\delta V^D(t) < s(t)$, this condition requires that

$$\delta V^I(t) < s(t) \qquad \forall\, t \in (0, T_v^E) \tag{E3}$$

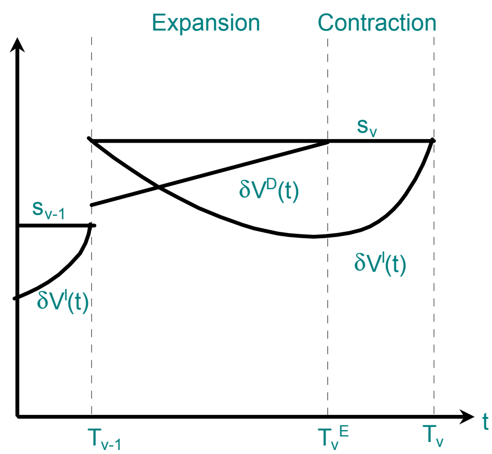

Figure 6: Evolution of Value Functions

Figure 6 illustrates the evolution of the relevant value functions in the cyclical equilibrium, and the productivity adjusted wage s_v/δ. At the beginning of the cycle $s_v = \delta V^I(T_v) > \delta V^D(T_v)$. Since the wage is constant, $\delta V^D(t)$ grows and $\delta V^I(t)$ declines during the first phase of the cycle, this condition implies that $\delta V^D(t)$ and $\delta V^I(t)$ must intersect before $\delta V^D(t)$ reaches $s(t)$. It follows that when entrepreneurship starts, it is optimal to delay implementation, $V^D(T_v^E) > V^I(T_v^E)$. Over time, during the entrepreneurship phase, the probability of not being displaced at the boom if you implement early declines so that $V^I(t)$ rises. Eventually, an instant prior to the boom, $V^I(T_{v+1}) = V^D(T_{v+1})$, but until that point it continues to be optimal to delay. At the boom, the value of immediate implementation rises by more than the value of delay, so that all existing innovations are implemented. However, since the skilled wage increases by at least as much as $V^I(t)$, entrepreneurship ceases and the cycle begins again. Note finally that in constructing the equilibrium above we have implicitly imposed the requirement that the downturn is not long enough that all sectors innovate:

$$P(T) > 0. \tag{E4}$$

This is an additional condition which must be satisfied by the models' solution.

5 Existence and Baseline Example

In this section we demonstrate that there is a non-empty parameter space such that the triple $(\Delta^E, \Delta, \Gamma) > 0$ solving (59), (60) and (63) exists, and the conditions (E1), (E2), (E3) and (E4) are satisfied. We do this by solving the model for a baseline set of parameters given in Table 1. Our choice of parameters is guided by estimated values where possible, but this exercise is not an attempt to assess the quantitative significance of the model. This is because many of the model's fundamental parameters cannot be directly computed so that available empirical estimates do not provide a serious discipline on the model. The aim here is instead to establish that the existence conditions can be simultaneously satisfied for values (for the estimable parameters at least) that are within reasonable bounds. The second aim is to gain some understanding of the model's comparative static properties by varying parameters around this baseline case.

Table 1: Baseline Parameters

Parameter	Value
ρ	0.03
σ	0.30
θ	0.40
L	10.00
γ	0.60
$\overline{\mu}$	0.09
δ	0.90
q	0.40

We calibrate for annual parameters using a rate of time preference $\rho = .03$, and intertemporal substitution parameter $\sigma = 0.3$. As already stated, this implies a high value for the intertemporal elasticity of substitution of around three, which is necessary in this version of the model without capital to obtain existence (see Francois and Lloyd-Ellis (2004) for analysis of the case with capital where rates around unity are obtained[27]). We consider a baseline detection rate of q =0.4, implying that a shirker will be detected with 40% probability within a year of shirking. With 168 hours in a week we assume ten hours per day are needed for basic self maintenance (eating,

[27]The reason for this is that, with capital, variation in the rate of investment, which is strongly pro-cyclical, allow for aggregate output fluctuations that exceed those in consumption. This is not the case here since all output is consumed.

cleaning and sleeping). This leaves 98 hours in total available for work/leisure so that the average work week of 37.5 hours implies a value of θ around 0.4. We set the ratio of managers to workers, $\theta L = 4$.[28] The parameter $\overline{\mu}$ measures the rate that employment is terminated for essentially exogenous reasons. In calibrating this, we take as a guide a rate of exogenous annual voluntary turnover in the US, which is around 20%.[29] Since this data includes transitions into and out of the labor market and job to job transitions, which are excluded from our model, we take as a baseline that half are transitions into exogenous turnover and thus aim for a turnover rate in the model of about 10%. It turns out that the cycle length is quite sensitive to this parameter, so we choose a value of $\overline{\mu} = 0.09$, or annual rate of 9%, which yields a reasonable cycle length, and then vary around this in the comparative statics. The value of the direct technological advance embeded in new innovations, γ, is set equal to 0.6 which yields a mark-up rate of approximately 5% at the end of the expansion and contraction. We choose the unobservable productivity of innovation parameter, $\delta = 0.9$ to yield an annual growth rate close to one and a half percent for most parameter variations.

For these values as the model's baseline we obtain a unique cycle length satisfying conditions (E1), (E2), (E3) and (E4) and solving equations (59), (60) and (63). This yields an average long–run growth rate of $\frac{\Gamma}{\Delta} = 1.5\%$ and virtually acyclical wages — the correlation of de–trended log average wages and de–trended log of output is 0.0001.[30] Note that, since average wages calculated using the aggregate employment rate embody standard composition biases, controlling for these here will yield greater pro-cyclicality, which we discuss further below. The length of expansion, Δ^X, is 5.1 years and contractions, Δ^E, are 2.7 years in this baseline.

5.1 Restructuring: Temporary and Permanent Job Flows

The pattern of flows between unemployment and employment implied by the baseline case are plotted in Figure 7. Flows into and out of employment are equivalent and stable over the expansion while skilled labor is fully utilized in production. Upon entering the contractionary phase the flow into unemployment increases while that into employment drops initially but then tracks upwards through the recession. The gradual increase in flows occurs because of the intense firm restructuring taking place in the recession. Successful entrepreneurs enter then, displacing existing incumbents and leading, in the first instance, to job loss. This matches the pattern of plant destruction summarized by Davis, Haltiwanger and Schuh (1996, p.146), where during recessions it is reported that older and larger plants experience sharply higher job destruction rates. In the cycle depicted here, it is only firms that were incumbents at the previous boom that are displaced, new entrants, in contrast, last until at least the next contraction.

[28] Estimates of this ratio vary considerably depending on industry and precise definition of managerial labor. Our calibration is close to the lower bound of these estimates, but the nature of the cycles we study is not affected by this variable. All that changes is the degree of wage pro-cyclicality.

[29] See the Bureau of Labor Statistics site: http://www.bls.gov/jlt/home.htm#data for estimates.

[30] The trend is computed using the long–run growth rate Γ/Δ.

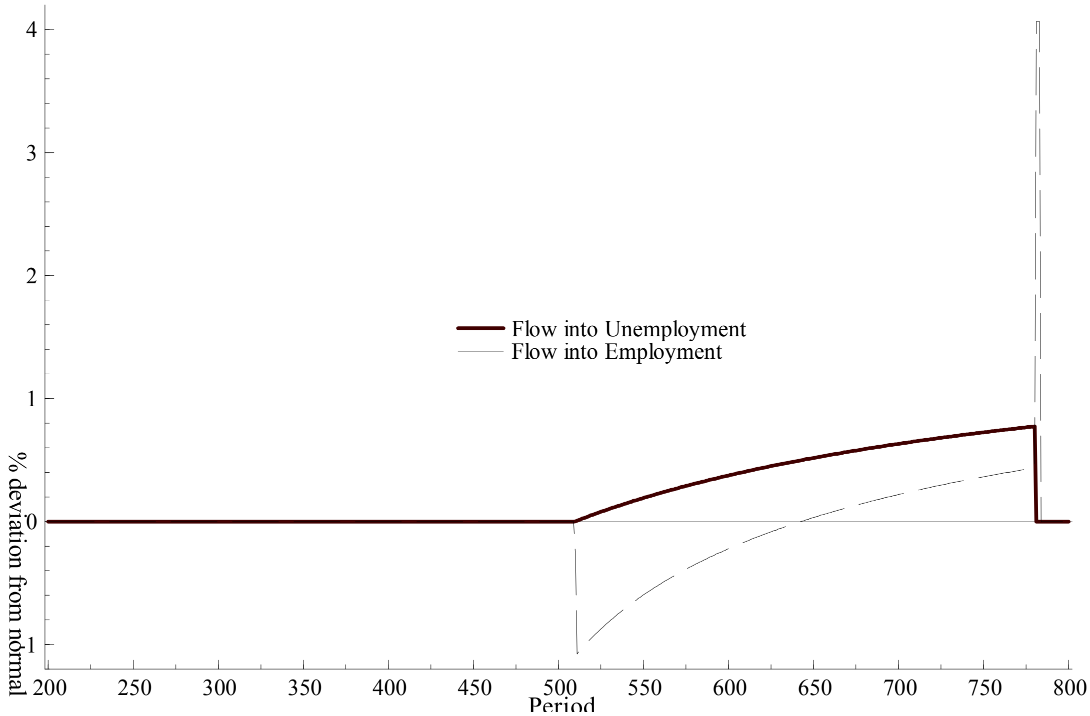

Figure 7: Employment Flows over the Cycle

Though successful entrepreneurs entering in recessions find it profitable to delay implementation of their own innovation until the boom, they start up production and thus hire workers out of the employment pool. Since the economy's output is contracting, and firms are reducing output to meet the lower aggregate demand, the rate of flow into employment, though higher than in the expansion, is less than the rate of flow into unemployment so that unemployment monotonically increases through the recession. This pattern of increased restructuring in recessions qualitatively matches the behavior of actual US flows in Figure 1. As noted earlier, flows into employment increased monotonically through the '70, '74, '80, '82 and 91 recessions.

The model's implied pattern of firm restructuring in recessions endogenously captures two commonly emphasized components of this process, as depicted in Figure 8. One components results from idiosyncratic firm–specific shocks, associated with the Schumpeterian process of creative destruction and firm turnover. This is depicted by the firm turnover rate which increases throughout the downturn with increased entrepreneurship, $H(t)$. The second component results from the aggregate contraction which affects all firms simultaneously. Note that the permanence of reallocation implied by these effects differs. Idiosyncratic, firm-specific shocks lead to permanent changes in employment since workers laid-off by the plants closing down, and workers employed in the new more-productive plants starting up will experience persistent changes. The laid-off workers will not be re-hired into these plants, and the workers hired into newly formed plants can expect their positions to remain with high probability throughout the expansionary phase.

The decline in demand implied by the aggregate downturn – which diffuses broadly and mildly across all firms – leads, in contrast, to temporary reductions in employment. The jobs lost in the recessionary phase will be re-created in the same firms as soon as aggregate conditions change – at the boom.

This pattern of temporary broadly felt changes, coupled with permanent idiosyncratic changes matches well with the pattern of employment adjustment reported by Schuh and Triest (1998). They report that large employment growth rate changes (including start-ups and shut-downs) tend to be permanent and occur late in recessions. These are a large proportion of flows – two thirds of jobs created and destroyed – occur in plants that experience a more than 25% change in employment. In contrast, generalized contractions lead to fluctuations in employment that occur among a large number of plants, and tend to be temporary.

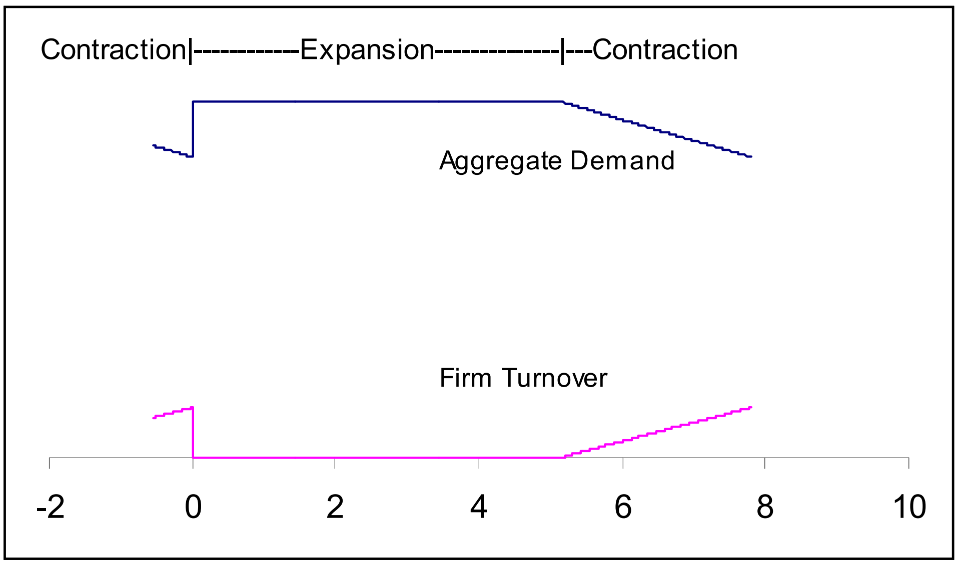

Figure 8: Components of Job Flows

The essence of the argument underlying the timing of this aggregate downturn is strongly linked to the 'opportunity cost' view of recessions as in Caballero and Hammour (1994), Aghion and Saint-Paul (1991) and Hall (1991).[31] The productivity sequence is, however, quite different than what would arise in such models. Both flows into unemployment and flows into employment

[31] Recently, Caballero and Hammour (2004) argue that the data forces an alternative to this "liquidationist" view. Specifically, they argue that the increase in employment in expansions is not driven by increased creation, but instead by reduced destruction then. Note that this is also consistent with the pattern of destruction generated by the model. Destruction falls from $\overline{\mu}+\delta H(t)$ in recessions to $\overline{\mu}$ throughout the expansion. However the present model is not able to replicate the lag between gross destruction — which peaks in the recession — and gross creation — which peaks in the expansion. This is, at least in part, due to the dramatic nature of the boom generated here, and the flat expansion in the absence of capital accumulation. The introduction of capital in Francois and Lloyd-Ellis (2004) leads to a sustained increase in output through the expansion. That model did not feature endogenous employment fluctuations — as all labor was skilled there — but it does suggest that the introduction of capital in the present framework could generate a similar protracted bout of job creation through the expansion.

increase when aggregate demand is falling, in the recession, and is de-coupled from increases in productivity which do not arise until the boom.

5.2 Low Wage Pro-Cyclicality

Figure 9 shows that after rising through the boom, the production wage remains constant through the expansion and thus tracks output through this phase, it does, however, move considerably through the recession. In the baseline case, the unskilled wage rises at the start of the recession and then follows an inverted U shape through the downturn. However, the increase in unemployment of workers leads the model to replicate the counter-cyclical composition bias which emerges in the data for the same reason (see Bils (1984) and Solon et al. (1996)). This is reflected in the continuously increasing average wage through the recession, depicted in Figure 10 below. Consequently, though the baseline correlation of output with average wages is zero, this becomes significantly positive if composition biases are controlled for, or if attention is limited to unskilled workers alone. However, even accounting for this, the degree of pro-cyclicality is mitigated by the effect of increased Schumpeterian destruction in the recession which generates the hump-shaped wage profile then.

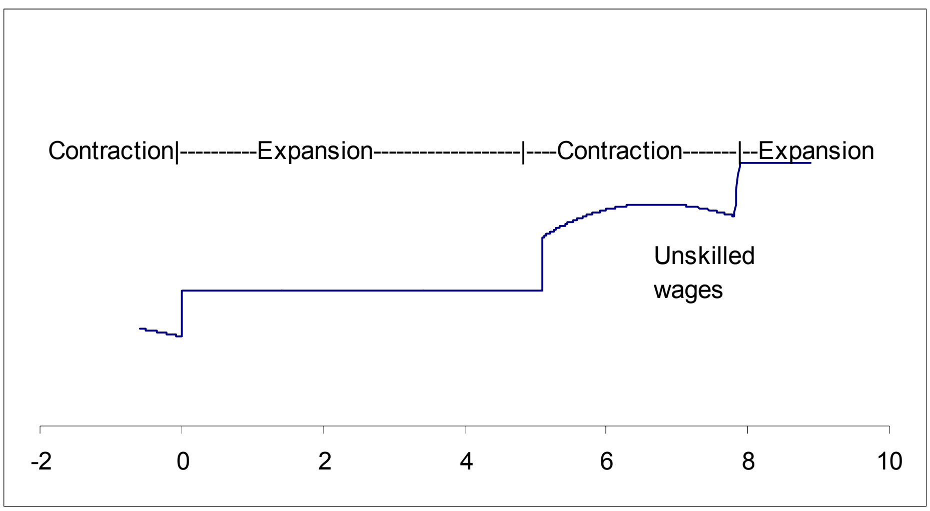

Figure 9: Production Wage (Baseline)

Note that real wages jump upon entering the recession.[32] This is because household employment growth, which was previously zero, becomes negative. Consequently, the marginal cost of

[32]For $\sigma = 1$, the case of log preferences, this would not occur, but, as already mentioned, for a model without capital to generate this sort of Schumpeterian cycle requires elasticities of substitution of $\frac{1}{\sigma} > 1$, so that the jump is unavoidable. Note, however, that there is a jump of the same magnitude at the boom, which occurs for the same reason that the interest rate discretely changes then. This pro-cyclical jump in wages offsets the counter-cyclical one at the start of the recession, so that these discrete changes in wages have no net effect on the measure of wage cyclicality reported here.

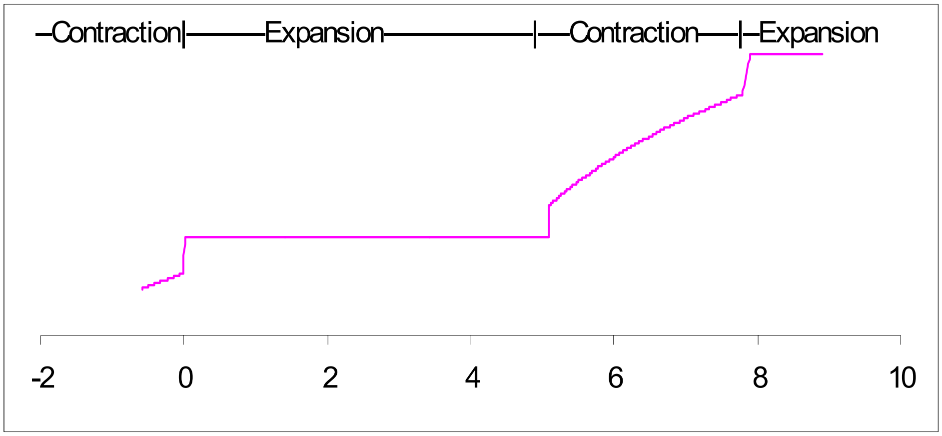

Figure 10: Average Wage (Baseline)

providing effort is expected to be lower tomorrow than today which, on the margin, increases the willingness of workers to risk unemployment by shirking today. To compensate for this effect, firms must raise the efficiency wage in order to maintain incentives. The hump–shaped time profile of wages through the downturn reflects the changing relative strength of the forces act on the wage mentioned above. Initially, the effect of increasing turnover, which effects the wage in most sectors, $w^H(t)$, dominates, so that wages rise to maintain incentives. However, as more and more sectors experience turnover, their wages fall to $w^L(t)$ which also unambiguously falls. Consequently, the average wage falls towards the end of the recession. If the rate of turnover is low enough, this latter effect may dominate throughout. Such a case is depicted in Figure 11 which is computed for the baseline set of parameters, but with a lower value of $\delta = 0.6$.

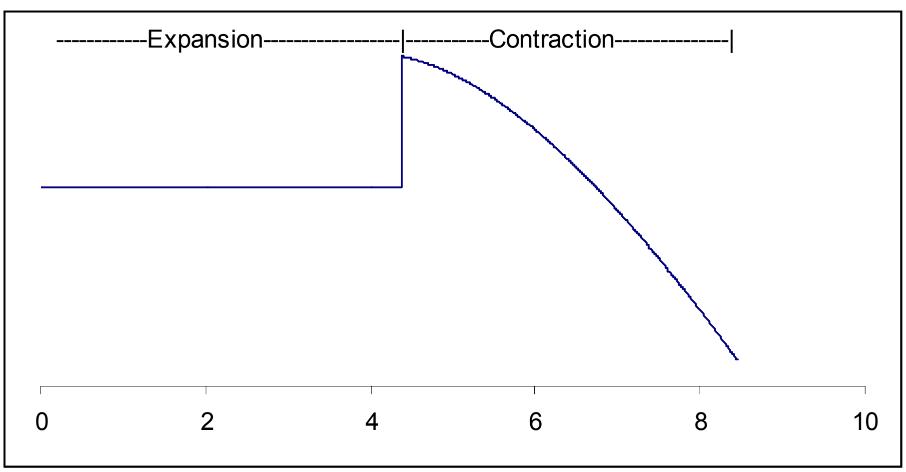

Figure 11: Production Wage ($\delta = 0.6$)

5.3 Comparative Steady States

The following table shows the variation in output/wage correlation, growth, and cycle length for changes in each of the underlying parameters. The table lists the single parameter varied, and its value in the first column with the endogenous results in the columns to the immediate right.

Table 2: Growth, Wage Cyclicality, and Cycle Lengths

Parameter Values	$g(\%)$	Corr(w, y)	Δ^X	Δ^E
Baseline Case	1.49	0.00	5.1	2.7
Variation: $\delta = 1.000$	1.51	-0.28	4.4	2.4
$\delta = 0.800$	1.41	0.30	6.2	3.0
$\sigma = 0.305$	0.78	0.27	12.6	2.8
$\sigma = 0.295$	2.15	-0.22	2.6	2.6
$\overline{\mu} = 0.095$	1.08	0.15	8.0	2.7
$\overline{\mu} = 0.085$	1.88	-0.13	3.5	2.7
$\theta = 0.405$	0.80	0.24	11.9	2.7
$\theta = 0.395$	2.15	-0.16	2.7	2.7
$q = 0.410$	1.75	-0.01	3.9	2.7
$q = 0.390$	1.22	0.04	6.8	2.7
$\gamma = 0.610$	1.90	-0.01	3.3	2.7
$\gamma = 0.590$	1.06	0.13	8.4	2.8
$\rho = 0.040$	0.67	0.44	20.6	2.7
$\rho = 0.025$	1.85	-0.27	2.52	2.7

The intuition for most of the comparative static results in the table is relatively straightforward. Changes in parameters that reduce incentives to engage in entrepreneurship: lowering δ or γ (which have a direct effect on returns to entrepreneurship); lowering q and raising $\overline{\mu}$ (which raises the efficiency wage); raising σ and ρ (which makes consumers less willing to delay consumption) all lower the growth rate, as one would expect in a model of endogenous growth. Most of these changes also leave the length of the economy's contraction relatively unchanged, but imply (sometimes large) changes in the length of expansion. This is because with weaker underlying incentives to invest in entrepreneurship, longer expansionary phases, and therefore a longer reign of incumbency and profit, are required to provide sufficient incentives for entrepreneurship. The robust implication of the model then is that lengthy expansionary phases will tend to correspond to periods of relatively low productivity growth.

5.4 The Productivity Slowdown and Wage Pro-Cyclicality

According to the model, a sensible candidate explanation for a productivity slowdown is a decline in δ; the productivity of labor in research.[33] The movement of average wages in the case of a

[33] This is consistent with a view of the productivity slowdown being caused by a decline in labor productivity due to a GPT, see for example, Galor and Tsiddon (1997), Helpman and Rangel (1999) and Galor and Moav (2000). In

lower $\delta = 0.8$ is depicted in Figure 12. This change leads to a 20% increase in the length of expansion while only a 10% increase in the contraction. As shown in the table of comparative statics, the model can match the qualitative post–1970 pattern — the correlation of output with average wages increases from 0 to 0.3, while the growth rate drops from 1.5% to 1.41%.

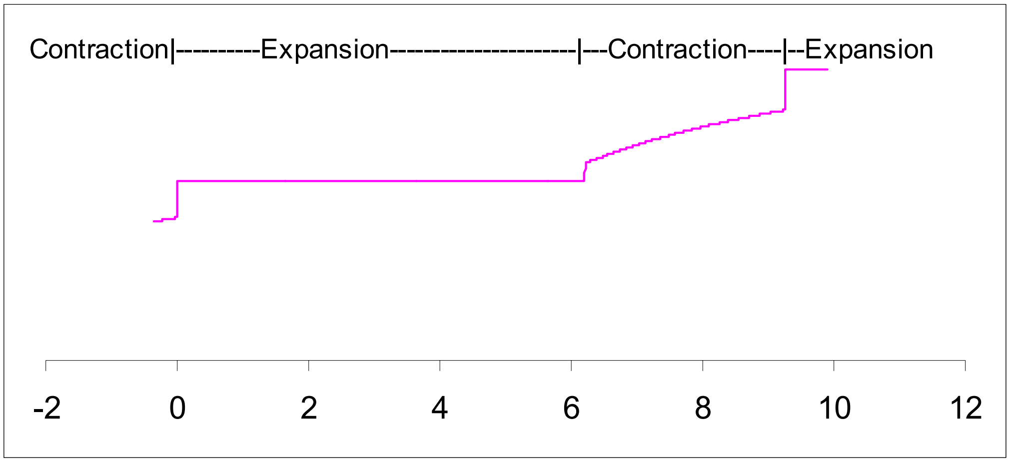

Figure 12: Average Wage ($\delta = 0.8$)

6 Conclusion

A Schumpeterian process of creative destruction implies a pattern of firm turnover, employment flows, wage movements and aggregate demand, that is broadly consistent with much of what is observed in actual data. Specifically, it can generate counter-cyclical restructuring, pro-cyclical productivity and cyclicality in wages which, depending on parameters, may exhibit any form of cyclicality. Moreover, such patterns are yielded in a model where the underlying source of productivity growth is endogenous, as is the clustering of activities across disparate sectors. The Schumpeterian specification implies that the increased turnover integral to the recessionary phase developed here leads to increased volatility in employment. By increasing the fragility of the employment relationship in recessions, the incentive providing effect of efficiency wages is reduced, causing upward pressure on wages. This counter-cyclical mechanism, which offsets the standard pro-cyclical mechanisms that are also present here, leads to the possibility of low wage cyclicality in general. As the model is a unified framework for cycles, growth, and labor flows, it is a natural framework to examine the interaction between changes in cyclical components and changes in the secular. Correlated changes in secular and cyclical aggregates, such as that

that view, since there is a decline in labor's ability to work with the new technology – though it is of fundamentally higher productivity – there is a temporary decline in labor productivity, corresponding to a lowering of δ in our framework.

observed between the productivity slowdown and increased wage pro-cyclicality of the early 70's, can be explained by the current model.

The approach taken here complements recent models of endogenous business cycles which have proved useful in understanding other phenomena associated with the business cycle. For example, Francois and Lloyd–Ellis (2004) illustrate how such a model can help account for the relationship between aggregate investment and Tobin's Q.[34]. Although the model we employ here is somewhat specialized and designed to focus on particular aspects of the labor market, we believe it is possible to generalize the paradigm in a way which can offer a quantitative account of several phenomena simultaneously.

[34]Both Matsuyama (2001) and Waelde (2004) also develop models of endogenous cycles and growth. Both of these models seem better suited to explaining the longer wave, Schumpeterian cycles. Our focus, in contrast, is the NBER length business cycle.

7 Appendix

Proof of Lemma 2 Given that implementation of a new technology is delayed until the subsequent boom, T_v, we show: (1) An entrepreneur with a succesful innovation offering amount $V^*(t) + \varepsilon$ to an incumbent to stop producing until T_{v+1} takes over production in that sector (2) other entrepreneurs stop innovation in that sector;.

Part (1) If a potential entrepreneur offers amount $V^*(t) + \varepsilon$ to an incumbent to stop producing until T_{v+1} and undertakes production itself using the existing technology upto time T_{v+1} then the incumbent correctly conjectures that this entrepreneur has discovered an innovation. This is because the value of the profit stream taken over is only $V^*(t)$. Since, before implementation, a successful entrepreneur must be producing with the technology in order to implement and since buying out the previous incumbent stops all research, for $\varepsilon \to 0$, the entrepreneur is willing to pay this transfer to the previous incumbent. (2) Given (1) then all other entrepreneurs believe that i has a productivity advantage which is e^{γ} times better than the current state of the art. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of e^{γ}. Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production, $s(t) > 0$, are thus strictly higher.An entrepreneur without an innovation would not find it worthwhile to take over production paying $V^*(t) + \varepsilon$ for the interval $t \to T_{v+1}$. At time T_{v+1} the previous incumbent re-enters production so that the total profit stream to an entrepreneur taking over production for the interval would be $V^*(t)$ which is ε less than the amount transferred.

Proof of Proposition 1: Profit maximization implies that $V_i^E(t) = V_i^S(t) \geq V^U(t)$ and so $V_i^M(t) = V_i^E(t)$. It follows that (15) can be expressed as

$$V_i^E(t) = \left(w_i(t) + \frac{u_n(t)}{u_c(t)}\right) dt + e^{-r(t)dt}\left[e^{-\mu_i(t)dt} V_i^E(t+dt) + (1 - e^{-\mu_i(t)dt}) V^U(t+dt)\right]. \tag{64}$$

Subtracting $e^{-(r(t)+\mu_i(t))dt} V_i^E(t)$ from both sides we get

$$\begin{aligned}(1 - e^{-(r(t)+\mu_i(t))dt}) V_i^E(t) &= \left(w_i(t) + \frac{u_n(t)}{u_c(t)}\right) dt + e^{-(r(t)+\mu_i(t))dt}\left[V_i^E(t+dt) - V_i^E(t)\right] \\ &\quad + (1 - e^{-\mu_i(t)dt}) e^{-r(t)dt} V^U(t+dt)\end{aligned} \tag{65}$$

Dividing by dt we get

$$\begin{aligned}\frac{(1 - e^{-(r(t)+\mu_i(t))dt})}{dt} V_i^E(t) &= w_i(t) + \frac{u_n(t)}{u_c(t)} + e^{-(r(t)+\mu_i(t))dt}\left[\frac{V_i^E(t+dt) - V_i^E(t)}{dt}\right] \\ &\quad + \left(\frac{1 - e^{-\mu_i(t)dt}}{dt}\right) e^{-r(t)dt} V^U(t+dt)\end{aligned} \tag{66}$$

Letting $dt \to 0$ and noting that $\lim_{dt\to 0}\left(1-e^{-xdt}\right)=x$ we get

$$[r(t)+\mu_i(t)]\,V_i^E(t)=w_i(t)+\frac{u_n(t)}{u_c(t)}+\dot{V}_i^E+\mu_i(t)V^U(t) \tag{67}$$

Re–arranging yields

$$r(t)V^E(t)=\left(w_i(t)+\frac{u_n(t)}{u_c(t)}\right)-\mu_i(t)\left[V^E(t)-V^U(t)\right]+\dot{V}^E(t) \tag{68}$$

Similar reasoning can be applied to (16) and (17) to show that

$$r(t)V^S(t) = w_i(t)-[\mu_i(t)+q]\left[V^S(t)-V^U(t)\right]+\dot{V}^S(t) \tag{69}$$
$$r(t)V^U(t) = \lambda(t)\left[V_i^E(t)-V^U(t)\right]+\dot{V}^U(t) \tag{70}$$

Profit maximization subject to the incentive compatability condition implies that $V_i^E(t)=V_i^S(t)$ and so subtracting (68) from (69) we get

$$V_i^E(t)-V^U(t)=-\frac{u_n(t)}{qu_c(t)} \tag{71}$$

It follows that $V_i^E(t)=V^E(t)\ \forall i$ and that

$$\dot{V}^E(t)-\dot{V}^U(t)=-\frac{1}{q}\frac{u_n(t)}{u_c(t)}\left(\frac{\dot{u}_n(t)}{u_n(t)}-\frac{\dot{u}_c(t)}{u_c(t)}\right) \tag{72}$$

Subtracting (70) from (68) we get

$$\left(\rho-\frac{\dot{u}_c(t)}{u_c(t)}\right)\left[V^E(t)-V^U(t)\right]=w_i(t)+\frac{u_n(t)}{u_c(t)}-[\mu_i(t)+\lambda(t)]\left[V^E(t)-V^U(t)\right]-\frac{1}{q}\frac{u_n(t)}{u_c(t)}\left(\frac{\dot{u}_n(t)}{u_n(t)}-\frac{\dot{u}_c(t)}{u_c(t)}\right) \tag{73}$$

Substituting in (71) and re-writing yields

$$-\left(\rho-\frac{\dot{u}_c(t)}{u_c(t)}\right)\frac{u_n(t)}{qu_c(t)}=w_i(t)+\frac{u_n(t)}{u_c(t)}+[\mu_i(t)+\lambda(t)]\frac{u_n(t)}{qu_c(t)}-\frac{1}{q}\frac{u_n(t)}{u_c(t)}\left(\frac{\dot{u}_n(t)}{u_n(t)}-\frac{\dot{u}_c(t)}{u_c(t)}\right) \tag{74}$$

$$[\rho+\mu_i(t)+\lambda(t)]\,(-\frac{1}{q}\frac{u_n(t)}{u_c(t)})=w_i(t)+\frac{u_n(t)}{u_c(t)}-\frac{1}{q}\frac{u_n(t)}{u_c(t)}\frac{\dot{u}_n(t)}{u_n(t)}. \tag{75}$$

Re–arranging yields (36).■

Proof of Lemma 3: From the production function we have $\ln y(t)=\int_0^1 \ln\frac{y(t)}{p_i(t)}di$. Substituting for $p_i(t)$ using (6) yields $0=\int_0^1 \ln\frac{s(t)e^{\gamma}}{a_i(T_{v-1})}di\ 0$ which re-arranges to (34).

Proof of Lemma 5: Note that in any preceding no-entrepreneurship phase, $r(t)=\rho$. Thus, since, in a cycling equilibrium, the date of the next implementation is fixed at T_v, the expected value of entrepreneurship, δV^D, also grows at the rate $\rho>0$. Thus, if under $H(T_v^E)=0$, $\delta V^D(T_v^E)>w_v$, then the same inequality is also true the instant before, i.e. at $t\to T_v^E$, since w_v

is constant within the cycle. But this violates the assertion that entrepreneurship commences at T_v^E. Thus necessarily, $\delta V^D(T_v^E) = w_v$ at $H\left(T_v^E\right) = 0$.

Proof of Lemma 6: Since $c(t) = Y(t) = \Omega_v n(t)/\theta L$, the Euler equation can be expressed as

$$\sigma\frac{\dot{n}(t)}{n(t)} + \sigma\frac{\dot{n}(t)}{L - n(t)} = -\rho \tag{76}$$

Integrating, we get

$$\ln\frac{n(t)}{\theta L} - \ln\left(\frac{L - n(t)}{L - \theta L}\right) = -\frac{\rho}{\sigma}(t - T_v^E) \tag{77}$$

Solving for $n(t)$ yields (44).

Proof of Proposition 2: From (5) we can express the rate of job creation as

$$\lambda(t) = \frac{\mu^A(t)n(t) + \dot{n}(t)}{L - n(t)}. \tag{78}$$

Substituting into the expression for the efficiency wage we get

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)\,(L - n(t))q}\left[\rho + q + \frac{\mu^A(t)L + \dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{79}$$

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)\,(L - n(t))q}\cdot \left[\rho + q + \frac{\bar{\mu}L + \delta(L - n(t)/\theta)}{L - n(t)} + \frac{\dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{80}$$

$$w^A(t) = \frac{\sigma c(t)}{(1-\sigma)\,(L - n(t))q}\left[\rho + q + \frac{\delta}{\theta} - \frac{\frac{\delta(1-\theta)}{\theta}L - \bar{\mu}L}{L - n(t)} + \frac{\dot{n}(t)}{L - n(t)} - \frac{\dot{u}_n(t)}{u_n(t)}\right] \tag{81}$$

Differentiating (44) yields

$$\dot{n}(t) = \frac{-\left(1-\theta\right)\frac{\rho}{\sigma}\theta L e^{-\frac{\rho}{\sigma}(t - T_v^E)}}{\left[1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}\right]^2} \tag{82}$$

>From (77) we have

$$\frac{n(t)}{L - n(t)} = \left(\frac{\theta}{1-\theta}\right)e^{-\frac{\rho}{\sigma}(t - T_v^E)} \tag{83}$$

Using (44) it follows that

$$L - n(t) = \frac{\left(1-\theta\right)L}{1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}} \tag{84}$$

Diving (82) by (84) implies

$$\frac{\dot{n}(t)}{L - n(t)} = -\frac{\frac{\rho}{\sigma}\theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}}{1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}} \tag{85}$$

Differenting (13) w.r.t. time yields

$$\frac{\dot{u}_n(t)}{u_n(t)} = (1-\sigma)\,\frac{\dot{c}(t)}{c(t)} + (1-\sigma)\,\frac{\dot{n}(t)}{L - n(t)} \tag{86}$$

Using (43) to substitute we get

$$\frac{\dot{u}_n(t)}{u_n(t)} = \rho - \frac{\rho}{\sigma} \tag{87}$$

It follows that

$$w^A(t) = \frac{\sigma Y(t)}{(1-\sigma)(L-n(t))q}\left[\frac{\rho}{\sigma} + q + \frac{\delta}{\theta} - \frac{\frac{\delta(1-\theta)}{\theta}L - \bar{\mu}L}{L-n(t)} - \frac{\frac{\rho}{\sigma}\theta e^{-\frac{\rho}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{\rho}{\sigma}(t-T_v^E)}}\right]. \tag{88}$$

Note that $Y(t) = Y_0(T_{v-1})n(t)/\theta L$ and using (84) we get

$$w^A(t) = \frac{\sigma Y_0(T_{v-1})e^{-\frac{\rho}{\sigma}(t-T_v^E)}}{(1-\sigma)(1-\theta)qL}\left[q + \frac{\delta}{\theta} - \frac{\frac{\delta}{\theta}(1-\theta)-\bar{\mu}}{(1-\theta)}\left(1-\theta+\theta e^{-\frac{\rho}{\sigma}(t-T_v^E)}\right) + \frac{\frac{\rho}{\sigma}(1-\theta)}{1-\theta+\theta e^{-\frac{\rho}{\sigma}(t-T_v^E)}}\right] \tag{89}$$

$$w^A(t) = \frac{\sigma Y_0(T_{v-1})}{(1-\sigma)(1-\theta)qL}\left[(q+\delta+\bar{\mu})e^{-\frac{\rho}{\sigma}(t-T_v^E)} - \left(\delta - \frac{\bar{\mu}\theta}{1-\theta}\right)e^{-\frac{2\rho}{\sigma}(t-T_v^E)} + \frac{\frac{\rho}{\sigma}(1-\theta)e^{-\frac{\rho}{\sigma}(t-T_v^E)}}{1-\theta+\theta e^{-\frac{\rho}{\sigma}(t-T_v^E)}}\right] \tag{90}$$

Re–arranging yields (54).

Proof of Proposition 3: Long–run productivity growth is given by

$$\Gamma_v = \gamma(1-P(T_v)) = \delta\gamma\int_{T_v^E}^{T_v} H(\tau)d\tau \tag{91}$$

Substituting using (45) implies

$$\Gamma_v = \delta\gamma\int_{T_v^E}^{T_v}\left(\frac{(1-\theta)\left(1-e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}\right)}{1-\theta+\theta e^{-\frac{\rho}{\sigma}(\tau-T_v^E)}}\right)d\tau. \tag{92}$$

Integrating yields (57).[35]

Proof of Proposition 4: Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = s_v. \tag{93}$$

During the boom $V_0^I(T_v) > V_0^D(T_v)$. Thus the return to innovation at the boom is the value of immediate incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq s_{v+1} \tag{94}$$

The opportunity cost to financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no innovations have occurred

$$\beta(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right). \tag{95}$$

[35] Details of integration are available from the authors.

Note that since the short–term interest rate is zero over this phase, $\beta(t) = \beta(T_v)$, $\forall\ t \in (T_v^E, T_v)$. Combined with (93) and (94) it follows that asset market clearing at the boom requires

$$\beta(T_v) \leq \log\left(\frac{s_{v+1}}{s_v}\right) = \Gamma_v. \tag{96}$$

Provided that $\beta(t) > 0$, households will never choose to store final output from within a cycle to the beginning of the next because it is dominated by the long–run rate of return on claims to future profits. However, unlike final output, the return on stored intermediate output in sectors with no innovations, is strictly positive because of the increase in its price that occurs as a result of the boom. Even though there is a risk that the intermediate becomes obsolete at the boom, if the anticipated price increase is sufficiently large, households may choose to purchase claims to intermediate output rather than claims to firm profits.

If innovative activities are to be financed at time t, households cannot be strictly better off buying claims to stored intermediate goods. There are two types of storage that could arise, but the return to each is the same. In sectors with unimplemented innovations, entrepreneurs who hold innovations and are currently producing with the previous technology have the option of implementing the new technology before the boom, storing and then selling at the boom. The instant at which this is most likely to be profitable is the instant prior to the boom; before the discrete rise in wages then, and allowing competitors only an instant to improve on one's technology. The best time to sell the stored output is an instant after the boom, since after the boom interest rates are positive and demand is flat. Any such path of implementation will, however, affect the incentive compatible wage stream offered to unskilled workers. Intuitively, producing and storing today for sale tomorrow implies a higher rate of layoffs tomorrow and hence a higher efficiency wage today. This upward effect on incentive compatible wages is what rules out such storage as a profitable option. Since the stream of revenue is unaffected by such storage, in order for firms' expected discounted profits to rise it must be that the discounted wage bill for unskilled labor falls. To see this clearly, consider the expected value of the firm with an innovation at the beginning of the cycle, $V^I(T_v)$. Since $Y(\cdot), s, P(\cdot)$ and all discount rates are taken as given by the firm, V^I can only rise if the value of being employed at the firm $V^E(T_v)$ falls. But at all instants, the incentive compatible wage, $w^L(\tau)$ is given by the solution to $V_i^E(t) - V^U(t) = -\frac{u_n(t)}{q}$. Thus any such storage which raises profits for the firm will necessarily violates incentive compatibility for unskilled workers and will lead to shirking. The same argument rules out altering the production stream to benefit from discrete jumps in the wage anywhere along the cycle.

Innovators may also wish to benefit from the price increase that will occur at the boom. For instance firms could sell such claims, use them to finance greater current production and then store the good to sell at the beginning of the next boom when the price is higher. Since this leads to an altered revenue stream, this type of storage is not ruled out by the argument above. In this case, since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1}/p_{i,v} = \Gamma_v$.

It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$\beta(T_v) \geq \Gamma_v. \tag{97}$$

Because there is a risk of obsolescence, this condition implies that at any time prior to the boom the expected rate of return on claims to stored intermediates is strictly less than $\beta(t)$. Combining (96) and (97) yields the result.

Proof of Proposition 5: Re-writing (30) yields

$$\begin{aligned}(1-e^{-(r(t)+\delta h_i)dt})V_i^I(t) &= \pi_i(w_H(t),t)dt + e^{-r(t)dt}e^{-\delta h_i dt}\left[V^I(t+dt) - V_i^I(t)\right] \\ &\quad +(1-e^{-\delta h_i dt})e^{-r(t)dt}V^*(t+dt)\end{aligned} \tag{98}$$

Dividing by dt and letting $dt \to 0$ we get

$$(r(t)+\delta h_i(t))V_i^I(t) = \pi_i(w_H(t),t) + \delta h_i(t)V^*(t) + \dot{V}_i^I(t) \tag{99}$$

Given some initial time period t and the final period T_v, the solution to this first–order differential equation is

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^s (r(\tau)+\delta h_i(\tau))d\tau}\left[\pi_i(w_H(s),s) + \delta h_i(s)V^*(s)\right]ds + e^{-\int_t^{T_v}(r(s)+\delta h_i(s))ds}V_0^I(T_v) \tag{100}$$

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^s r(\tau)d\tau}\frac{P(s)}{P(t)}\left[\pi_i(w_H(s),s) + \delta h_i(s)V^*(s)\right]ds + e^{-\beta(t)}\frac{P(T_v)}{P(t)}V_0^I(T_v) \tag{101}$$

If $t \geq T_v^E$ we know that $r(\tau) = 0$ to the end of the cycle. Hence

$$\begin{aligned}V_i^I(t) &= \int_t^{T_v} \frac{P(s)}{P(t)}\left[\pi_i(w_H(s),s) + \delta h_i(s)V^*(s)\right]ds + e^{-\beta(t)}\frac{P(T_v)}{P(t)}V_0^I(T_v) \\ &= \frac{1}{P(t)}\int_t^{T_v}\left[P(s)\pi_i(w_H(s),s) + \delta H(s)V^*(s)\right]ds + e^{-\beta(t)}\frac{P(T_v)}{P(t)}V_0^I(T_v)\end{aligned} \tag{102}$$

Note that by partial integration

$$\int_t^{T_v} \delta H(s)V^*(s)ds = \int_t^{T_v} V^*(s)d(1-P(s)) \tag{103}$$

$$= \left[V^*(s)(1-P(s))\right]_t^{T_v} - \int_t^{T_v}(1-P(s))dV^*(s) \tag{104}$$

But $V^*(T_v) = 0$ and $dV^*(s) = -\pi_i(w_L(s),s)ds$, so that

$$\int_t^{T_v} \delta H(s)V^*(s)ds = \int_t^{T_v}(1-P(s))\pi(w_L(s),s)ds - V^*(t)(1-P(t)) \tag{105}$$

Since profits are a linear function of the production wage we get $\forall\ t \geq T_v^E$:

$$V_i^I(t) = \frac{1}{P(t)}\int_t^{T_v}\left[\pi(w^A(s),s)\right]ds + e^{-\beta(T_v^E)}\frac{P(T_v)}{P(t)}V_0^I(T_v) - V^*(t)\left(\frac{1-P(t)}{P(t)}\right) \tag{106}$$

Now if $t = T_v^E$, $P(T_v^E) = 1$ and this can be expressed as

$$V_i^I(T_v^E) = \int_{T_v^E}^{T_v} \pi(w^A(s), s)ds + e^{-\beta(T_v^E)} P(T_v) V_0^I(T_v). \tag{107}$$

It follows that the discounted monopoly profits from owning an innovation at time T_{v-1} is given by

$$V_0^I(T_{v-1}) = \tag{108}$$
$$= \int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} \left(1 - e^{-\gamma}\right) Y(\tau)d\tau - \int_{T_{v-1}}^{T_v} e^{-\int_{T_{v-1}}^{\tau} r(s)ds} e^{-\gamma}\theta L \frac{w^A(\tau)}{s_v} Y(\tau)d\tau + P(T_v)e^{-\beta(T_v)} V_0^I(T_v)$$
$$= \left(1 - e^{-\gamma}\right) Y_0(T_{v-1}) \left(\int_{T_{v-1}}^{T_v^E} e^{-\rho(\tau - T_{v-1})} d\tau + e^{-\rho(T_v^E - T_{v-1})} \int_{T_v^E}^{T_v} \frac{n(\tau)}{\theta L} d\tau \right) \tag{109}$$
$$- \left(\int_{T_{v-1}}^{T_v^E} e^{-\rho(\tau - T_{v-1})} w^A(\tau)\theta L d\tau + e^{-\rho(T_v^E - T_{v-1})} \int_{T_v^E}^{T_v} w^A(\tau) n(\tau) d\tau \right) + P(T_v)e^{-\beta(T_v)} V_0^I(T_v)$$

Using (62) and re–arranging we get

$$\left(1 - P(T_v)e^{\Gamma - \beta(T_v)}\right) V_0^I(T_{v-1})$$
$$= \left(1 - e^{-\gamma}\right) Y_0(T_{v-1}) \left(\frac{1 - e^{-\rho(T_v^E - T_{v-1})}}{\rho} + e^{-\rho(T_v^E - T_{v-1})} \int_{T_v^E}^{T_v} \frac{e^{-\frac{\rho}{\sigma}(t - T_v^E)}}{1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}} dt \right)$$
$$- \left(\frac{1 - e^{-\rho(T_v^E - T_{v-1})}}{\rho} \right) A\theta Y_0(T_{v-1}) \tag{110}$$
$$- e^{-\rho(T_v^E - T_{v-1})} Y_0(T_{v-1})\theta \int_{T_v^E}^{T_v} \left[\frac{B e^{-2\frac{\rho}{\sigma}(t - T_v^E)} - C e^{-3\frac{\rho}{\sigma}(t - T_v^E)}}{1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}} + \frac{D e^{-2\frac{\rho}{\sigma}(t - T_v^E)}}{\left[1 - \theta + \theta e^{-\frac{\rho}{\sigma}(t - T_v^E)}\right]^2} \right] dt$$

Integrating and dividing through by $Y_0(T_{v-1})$ yields

$$\left(1 - P(T_v)e^{\Gamma - \beta(T_v)}\right) \frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})}$$
$$= \left(1 - e^{-\gamma}\right) \left[\frac{1 - e^{-\rho(T_v^E - T_{v-1})}}{\rho} - e^{-\rho(T_v^E - T_{v-1})} \left(\frac{\sigma}{\rho\theta} \ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right)\right) \right]$$
$$- \left(\frac{1 - e^{-\rho(T_v^E - T_{v-1})}}{\rho} \right) A\theta \tag{111}$$
$$- e^{-\rho(T_v^E - T_{v-1})} \frac{\sigma}{\rho} \left[\begin{array}{c} \left(B + \frac{1-\theta}{\theta}C\right)\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E} + \frac{1-\theta}{\theta}\ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right)\right) \\ -\frac{1}{2}C\left(1 - e^{-\frac{2\rho}{\sigma}\Delta^E}\right) - D\left(\frac{1}{\theta}\ln\left(1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right) + \frac{(1-\theta)\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)}{1 - \theta\left(1 - e^{-\frac{\rho}{\sigma}\Delta^E}\right)}\right) \end{array} \right]$$

Collecting terms

$$
\begin{aligned}
& \left(1-P(T_v)e^{\Gamma-\beta(T_v)}\right)\frac{V_0^I(T_{v-1})}{Y_0(T_{v-1})} \\
= \quad & \left(1-e^{-\gamma}\right)\frac{1-e^{-\rho(T_v^E-T_{v-1})}}{\rho} \\
& -e^{-\rho(T_v^E-T_{v-1})}\left(\frac{\sigma}{\rho\theta}\ln\left(1-\theta\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)\right)\right)\left[\left(1-e^{-\gamma}\right)+\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right] \\
& -\left(\frac{1-e^{-\rho(T_v^E-T_{v-1})}}{\rho}\right)A\theta \qquad (112) \\
& -e^{-\rho(T_v^E-T_{v-1})}\frac{\sigma}{\rho}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}+\right)-\frac{1}{2}C\left(1-e^{-\frac{2\rho}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

Substituting using (59) and (60),

$$
\begin{aligned}
& \left(1-\left(1-\frac{\rho\Delta^E}{\gamma(1-\sigma)}\right)e^{-\rho(\Delta-\Delta^E)}\right)\frac{e^{-\gamma}}{\delta} \\
= \quad & \left(1-e^{-\gamma}\right)\frac{1-e^{-\rho(\Delta-\Delta^E)}}{\rho}+e^{-\rho(\Delta-\Delta^E)}\left[\left(1-e^{-\gamma}\right)+\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{\rho}{\delta\gamma(1-\sigma)}\right)\Delta^E \\
& -\left(\frac{1-e^{-\rho(\Delta-\Delta^E)}}{\rho}\right)A\theta \qquad (113) \\
& -e^{-\rho(\Delta-\Delta^E)}\frac{\sigma}{\rho}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}+\right)-\frac{1}{2}C\left(1-e^{-\frac{2\rho}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \left(e^{\rho(\Delta-\Delta^E)}-1+\frac{\rho\Delta^E}{\gamma(1-\sigma)}\right)\frac{e^{-\gamma}}{\delta} \\
= \quad & \left(1-e^{-\gamma}\right)\frac{e^{\rho(\Delta-\Delta^E)}-1}{\rho}+\left[\left(1-e^{-\gamma}\right)+\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{\rho}{\delta\gamma(1-\sigma)}\right)\Delta^E \\
& -\left(\frac{e^{\rho(\Delta-\Delta^E)}-1}{\rho}\right)A\theta \qquad (114) \\
& -\frac{\sigma}{\rho}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}+\right)-\frac{1}{2}C\left(1-e^{-\frac{2\rho}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}\right]
\end{aligned}
$$

Collecting terms we get

$$
\begin{aligned}
& \left(e^{\rho(\Delta-\Delta^E)}-1\right)\left[\frac{e^{-\gamma}}{\delta}-\left(\frac{1-e^{-\gamma}}{\rho}\right)+\frac{A\theta}{\rho}\right] \qquad (115) \\
= \quad & \left[\left(1-e^{-\gamma}\right)-\frac{\rho}{\delta\gamma(1-\sigma)}+\left[\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{\rho}{\delta\gamma(1-\sigma)}\right)\right]\Delta^E
\end{aligned}
$$

$$-\frac{\sigma}{\rho}\left[\left(B+\frac{1-\theta}{\theta}C\right)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)-\frac{1}{2}C\left(1-e^{-\frac{2\rho}{\sigma}\Delta^E}\right)-D\frac{(1-\theta)\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}{1-\theta\left(1-e^{-\frac{\rho}{\sigma}\Delta^E}\right)}\right]$$

Dividing through by $\left[\frac{e^{-\gamma}}{\delta}-\left(\frac{1-e^{-\gamma}}{\rho}\right)+\frac{A\theta}{\rho}\right]$ and solving for Δ yields (63), where

$$\alpha=\frac{\frac{\rho}{\delta\gamma(1-\sigma)}-(1-e^{-\gamma})-\left[\left(B+\frac{1-\theta}{\theta}C\right)(1-\theta)-D\right]\left(1-\frac{\rho}{\delta\gamma(1-\sigma)}\right)}{\left(\frac{1-e^{-\gamma}}{\rho}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{\rho}}, \tag{116}$$

$$\zeta_1=\frac{\frac{\sigma}{\rho}\left(B+\frac{1-\theta}{\theta}C\right)}{\left(\frac{1-e^{-\gamma}}{\rho}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{\rho}},\ \zeta_2=\frac{\frac{\sigma}{\rho}\frac{1}{2}C}{\left(\frac{1-e^{-\gamma}}{\rho}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{\rho}}\text{ and }\zeta_3=\frac{\frac{\sigma}{\rho}D(1-\theta)}{\left(\frac{1-e^{-\gamma}}{\rho}\right)-\frac{e^{-\gamma}}{\delta}-\frac{A\theta}{\rho}}. \tag{117}$$

References

[1] Abraham, K.G. and J.C. Haltiwanger (1995), “Real Wages and the Business Cycle,” *Journal of Economic Literature*, Vol. 33, No. 3, 1215-1264.

[2] Aghion, Philippe and Peter Howitt (1992) “A Model of Growth Through Creative Destruction,” *Econometrica*, vol. 60, pp. 323–351.

[3] Aghion, Philippe, and Peter Howitt (1994) “Growth and Unemployment,” *Review of Economic Studies*, 61,3,477-94.

[4] Aghion, Phillippe and Giles Saint-Paul (1998), “On the virtue of bad times,” *Macroeconomic Dynamics*, 2-3, 322-344.

[5] Alexopoulos, Michelle (2004), “Unemployment and the Business Cycle,” *Journal of Monetary Economics* (2004) vol. 51, 277-298

[6] Beaudry, Paul, and Franck Portier (2004) Stock Prices, News and Economic Fluctuations, Working Paper, University of British Columbia.

[7] Bils, M. (1985) ”Real Wages over the Business Cycle: Evidence from Panel Data,” *Journal of Political Economy*,

[8] Blanchard, O.J. and P. Diamond (1990), “The cyclical behavior of the gross flows of US workers,” Brookings Papers on Economic Activity, 85-155.

[9] Bleakley, H., A. Ferris and J. Fuhrer (1999), “New Data on Worker Flows During Business Cycles,” *New England Economic Review*, July/August 1999.

[10] Caballero, R. and M. Hammour (1994) “The Cleansing Effect of Recessions” *American Economic Review* 84(5), December 1994, 1350-1368.

[11] Caballero, R. and M. Hammour (2004) “The Costs of Recessions Revisited: A Reverse Liquidationist View,” Working Paper MIT, forthcoming in *Review of Economic Studies*.

[12] Chang, Yongsung (2000) Wages, Business Cycles, and Comparative Advantage,” *Journal of Monetary Economics,* 46 (1) 143-172.

[13] Cohen, W., R. Nelson and J. Walsh (2000) “Protecting Their Intellectual Assets: Appropriability Conditions and Why US firms Patent or Not” *National Bureau of Economic Research Working Paper*, # 7552.

[14] Comin, D. (2004), “R&D: A small contribution to productivity growth,” *Journal of Economic Growth*, December.

[15] Danthine, J.P. and A. Kurman (2004), "Fair wages in a Keynesian model of the business cycle," CIRPEE discussion paper 03-02.

[16] Davis, S.J. and J. Haltiwanger (1992), "Gross job creation, gross job destruction and employment reallocation," *Quarterly Journal of Economics*, 107(3) 819-863.

[17] Davis, S.J., J.Haltiwanger and S. Schuh (1996), "Job creation and destruction," MIT Press, Cambridge.

[18] Dolmas, J., B. Raj and D.J. Slottje (1999), "The US productivity slowdown: a peak through the structural break window," *Economic Inquiry*, 37, 226.

[19] Fay, Jon A. and James L. Medoff (1985), "Labor and Output over the Business Cycle: Some Direct Evidence," *American Economic Review,* vol. 75 (4), pp. 638-55.

[20] Francois, P. and H. Lloyd-Ellis (2003) "Animal Spirits Through Creative Destruction," *American Economic Review*, vol. 93 (3), pp. 530-550

[21] Francois and Lloyd-Ellis (2004) "Investment Cycles," Working Paper, Queen's University.

[22] Galor, O. and O. Moav (2000) "Ability biased technological transition, wage inequality and economic growth," *Quarterly Journal of Economics* 115,2, 465-497.

[23] Galor, O. and D. Tsiddon (1997) "Technological Progress, Mobility and Economic Growth," *American Economic Review*, 87,3, 363-382.

[24] Gomme, Paul (1999), "Shirking, Unemployment and Aggregate Fluctuations," *International Economic Review*, vol. 40 (1), pp. 3–21.

[25] Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy*, Cambridge, MA.: MIT press, 1991.

[26] Hall, R.E. (1991), "Labor Demand, Labor Supply, and Employment Volatility," NBER Macroeconomics Annual, 1991, pp. 17-46.

[27] Haltiwanger, J.C. and S. Schuh (1999), "Gross Job Flows Between Plants and Industries." New England Economic Review, March/April, pp. 41-64.

[28] Helpman, E. and A. Rangel (1999), "Adjusting to a new technology: experience and training," *Journal of Economic Growth,* 4, 359-383.

[29] Matsuyama, Kiminori, "Growing Through Cycles" *Econometrica,* 67 (2), 1999, pp. 335–47.

[30] Oliner, S. and D. Sichel (2000), "The resurgence of growth in the 1990s: is information technology the story?" Federal Reserve Board Finance and Economics Discussion Series, 2000-20.

[31] Sakellaris, P. and A. Spilimbergo "Business Cycles and Investment in Human Capital: International Evidence on Higher Education" *Carnegie Rochester Conference Series on Public Policy*, 52, 2000, 221-56.

[32] Schuh, Scott and Robert K. Triest. 1998. "Job Reallocation and the Business Cycle: New Facts for an Old Debate." In Jeffrey C. Fuhrer and Scott Schuh, eds., Beyond Shocks: What Causes Business Cycles? Federal Reserve Bank of Boston Conference Series No. 42, pp. 271-337.

[33] Schumpeter, J. (1927) The explanation of the business cycle, *Economica*.

[34] Shapiro, K. and J. Stiglitz (1986), "Equilibrium Unemployment as a Worker DisciplineDevice," *American Economic Review* 74, 433-444.

[35] Shimer, R. (2003), "The cyclical behavior of equilibrium unemployment and vacancies," forthcoming *American Economic Review.*

[36] Sill, K. (1998), "Restructuring during recessions, a silver lining in the cloud," Federal Reserve Bank of Philadelphia Review, May/June 15-31.

[37] Solon, Gary, Robert Barsky, and Jonathan A. Parker (1994) "Measuring the Cyclicality of Real Wages: How Important Is Composition Bias?" *Quarterly Journal of Economics*, Vol. 109, No. 1 (February), pp. 1-25

[38] Stock, James and Mark Watson, (1998) "Business Cycle Fluctuations in US Macroeconomic Time-Series" NBER working paper no. 6528.

[39] Waelde, K. (2004) "Endogenous Growth Cycles," *International Economic Review.*

[40] Wen, Yi. (2004)"What Does It Take to Explain Procyclical Productivity?" Contributions to Macroeconomics: Vol. 4: No. 1, Article 5, http://www.bepress.com/bejm/contributions/vol4/iss1/art5