Competence and Conservatism Shape the Transparency of Monetary Policy* (Competence Implies Credibility)

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Abstract

The credibility of monetary policy under discretion has been traditionally linked to the Central Bank's conservatism. We show that another trait of the Central Bank (CB), her competence at doing her job, or reputation thereof, can be just as important to make her policy credible and transparent. Like conservatism, competence enhances the CB's ability to successfully explain her intentions to the private sector (P), and works against the effects of time inconsistency. Unlike conservatism, competence directly improves the stabilization power of discretionary monetary policy. We illustrate these points in a static Barro and Gordon (1983)'s game, where the effects of monetary policy depend on an unknown state of the economy. The CB forms her own private and imprecise view of this state, that she may announce to P in a cheap-talk game to explain and motivate her policy intentions. The precision of the noise in the CB's view measures her *competence*: the more accurate her assessment, the clearer an idea she has about what to do. The fineness of the equilibrium message space measures monetary policy's *transparency*. In any equilibrium of the cheap-talk game, the CB credibly announces one of finite inflation targets, that differ by at least a multiple of the inflation bias/competence ratio. If this ratio is too large, the CB can credibly communicate only the direction of monetary policy. As the CB's competence rises, and this ratio falls, her announcements receive increasing weight in P's expectations of inflation, so they must be tailored more carefully, in order not to mislead P excessively, and the equilibrium message space becomes finer. A similar effect originates from an increase in either the CB's conservatism, the weight on inflation in her preferences, or in the nominal rigidity of the economy. Finally, a more competent CB is less likely to erroneously observe extreme realizations of the shocks, so she is more able to fine-tune expectations at times of 'business as usual'.

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1. Introduction

In the 1990's, the Chairman of the FED, Alan Greenspan, acquired a near-magical reputation for competence in the conduct of monetary policy. His reputation survived the burst of the stock market bubble and the ensuing recession. For example, on April 22, 2003 the US stock markets rallied when President Bush announced his endorsement of Greenspan for a fifth term, starting in 2004. At the same time, with Greenspan at the helm, the conduct of monetary policy in the US has become increasingly transparent. In his public briefings and Congress testimonies, Greenspan has consistently been describing the FED's view of the macroeconomic outlook, and suggesting his approach to monetary policy. Minutes of the FOMC meetings are published with decreasing delay. In fact, the April 22 episode is probably a sign of relief by investors, who feel familiar with Greenspan's reasoning (transparency) and trust his judgement (competence).

The increasing transparency in the FED's conduct of monetary policy is part, and certainly a cause, of a worldwide trend (Geraats 2002). Central bankers surveyed by Blinder (1999) answered that credibility is the main asset to their job, and the best ways to build a reputation for credibility are a track record of honesty, namely "matching deeds to words", and transparency. The European Central Bank is no exception to this trend, but her communication with the private sector appears less effective and credible than the Greenspan's FED. At the same time, the ECB does not enjoy such a great reputation for competence, due both to her shorter track record, and to the more complex task that she faces in a newly minted monetary union of heterogeneous economies, lacking fiscal policy coordination. Indeed, the competence of a Central Bank (CB) should be meant in a relative sense, vis-a-vis to the complexity of the task she faces.

In this paper, we argue that the positive correlation between the reputation for competence and the transparency in the conduct of monetary policy, both over time and across central banks, is not a coincidence. Our main goal is to show that (a reputation for) *competence implies credibility and transparency.* A CB that commands the respect of the private sector (P) for her clarity of vision can communicate to P more credibly her intentions and the motivations behind them, because competence compensates for the lack of credibility that originates from an inflation bias, or reputation thereof. Indeed, as we discuss in Section 6, this principle applies to other monetary policy biases, such as the "stabilization bias" (Clarida, Gali and Gertler 1999) and the "CNBC effect" of monetary policy announcements (Morris and Shin 2002, Amato, Morris and Shin 2002). In fact, it extends even beyond monetary policy, to all kinds of public announcements by (perspective) policy-makers, such as electoral campaign promises. Often, and justifiably so, public opinion discounts public statements – especially claims on the effects of particular policies, such as President Bush's current tax cut proposal – as possibly motivated by some kind of bias, be it political or inflationary. But, also, the public pay more attention to such announcements if their sender is considered authoritative and competent on the issues.

Competence is obviously a desirable trait of a CB. For central bankers, standard career concerns à la Hölmstrom (1982) are a powerful motive to acquire and to maintain (a reputation for) competence. We emphasize an additional motive, the gain in credibility. As mentioned, communication and transparency have recently become pillars of the theory and practice of monetary policy, inverting a long tradition of secrecy. But what is transparency? We provide a formal meaning to this term, as the amount of information about her intentions that a CB can and wants to *credibly* convey to P, namely that P believes and incorporates in his expectations.

Following Canzoneri (1985), we extend a static version of Barro and Gordon (1983)'s classic monetary policy game, to allow for CB's private information about the state of the economy.[1] Our main innovation is to make this CB's observation (or "judgment", in Svensson (2003)'s terminology) of the state of the economy noisy, and to study the effect of the precision of this noise, that we refer to as the CB's *competence*. The more competent a CB, the clearer an idea she has about what to do. After observing her private signal, the CB sends to P a message about the required inflation target in a cheap-talk Bayesian game, in the style of Crawford and Sobel (1982)'s classic model of partisan advice. P then formulates his rational expectations of inflation (and accordingly

[1] Albeit central to a vast academic literature, time-inconsistency has received an increasingly cold reception by central bankers. Blinder (1998) states that, during his stint at the FED's Board, never the issue of surprising expectations presented itself on the agenda. Similarly, Otmar Issing, Executive Board Member of the ECB, has written that the ECB is not in the business of surprising expectations. These statements, however, do not diminish in any way the importance of time inconsistency, which truly concerns a counterfactual: if for some reason P were to expect a (say) zero inflation rate, would the CB want to choose a monetary stimulus that accelerates growth and also generates positive inflation? We contend that the answer is likely to be: yes. The point is that, *because of this*, inflation is rarely expected to be zero, as rational expectations cannot be fooled (systematically), and then there is indeed no point in trying to create further surprises. Therefore, we maintain that time inconsistency remains a significant impediment to the credibility of at least some types of statements by CBs.

sets some prices) based on the CB's announcement, if he believes it. Finally, given P's expectations of inflation, the CB takes an action that ultimately determines the rates of inflation and output (gap). The fineness of the equilibrium message space is a measure of monetary policy's *transparency*; in fact, in the absence of credible communication, the motives for monetary policy actions remain unexplained and unverifiable, thus opaque and open to all sorts of interpretations.

We obtain the following results. If and only if the CB has an inflation bias, in equilibrium truthtelling is impossible and communication is coarse. The CB can credibly announce one of finitely many inflation targets, although she observes them from a continuum. Among multiple equilibria of the communication game, one with two messages always exists, where the CB can only announce the qualitative direction of monetary policy. In the equilibrium with the most communication, the fineness of the equilibrium message space increases in competence. That is, *competence implies credibility*.

In fact, competence has two separate effects on communication, each formally identifiable and easily interpreted. The first effect is *the power of the words*, that the CB trades off against her inflation bias. Formally, the CB's *bias/competence ratio* determines the equilibrium message space. The more precise the CB's information, the more weight (relative to prior beliefs) P puts on the CB's announcement in forming inflation expectations, the less the CB needs to lie to surprise expectations and to stimulate output, the more credible her announcements. By reacting strongly to her messages, P forces a competent CB to be careful and accurate in her language. P cares only about anticipating the CB's moves, so P weighs more the words of a competent CB not because they are more informative *per se*, but because the CB thinks they are. In turn, the CB aims to use information efficiently, to align P's expectations to the required inflation target, so she cares about passing accurate information to P about what she will do, modulo the bias.

The second effect is the *credibility of likely announcements*, and works through the distribution of private signals. A more competent CB is relatively less likely to observe and to announce the need for extreme inflation. Therefore, she is less credible when this (rarely) happens, but more credible in fine-tuning the more likely events of small deviations of the inflation target from the long-run mean.

We also find that the CB's conservatism, the weight on inflation in her preferences, and the sensitivity of output to inflation surprises play a similar role as competence. The more the CB cares about inflation, and the less she needs to surprise P to stimulate output, the less worried should P be about CB misrepresenting her outlook and her intentions to surprise inflation expectations. We present an algorithm to compute all equilibria of the communication game, and we provide some numerical examples with realistic parameter values. Technically, Crawford and Sobel (1982)'s algorithm does not apply because they confine their analysis to a bounded set of states of nature, while we rely on Gaussian distributions to obtain a natural parameterization of competence. Nonetheless, also in this Gaussian model the equilibrium message space is always finite.

Our model predicts that a CB's governing body should speak with one voice. Dissenting and conflicting opinions released or leaked by its members to the public are likely to convey the impression of confusion and/or diverging views. In terms of our model, this amounts to "incompetence": the CB would appear to lack a clear and well-focused idea of where the economy is, and of what needs to be done, even if individual members of the board think they do. This public perception undermines the credibility of the CB's announcements, and makes the "management of P's expectations" less effective. In fact the CB, seeing her words carry less weight in P's expectations, must exaggerate her announcements, and communication becomes coarser.

This discussion adds yet a new dimension to the endless debate on rules vs. discretion in monetary policy. Since the world is too complex to be dealt with by a single rule, commitment has a cost in terms of flexibility in addressing each new situation with an appropriate response. This loss of flexibility is behind Svensson (2003)'s stance against the adoption of Taylor-like *instrument* rules, and in favor of explicit targets. But the value of flexibility increases in the CB's competence; the freedom afforded by discretion is useless without competence. Thus, a more competent CB has a stronger incentive, and a more credible reason in the eyes of P, to reject binding rules. This strategic effect of a CB's competence adds to that on transparency to work against commitment of any kind, and in favor of case-by-case evaluation, followed by a transparent explanation to P. Indeed, the Greenspan's FED never adopted any explicit rule.

Section 2 reviews the relevant literature, Section 3 presents the model, Section 4 characterizes equilibrium communication, Section 5 illustrates some numerical examples, Section 6 some extensions, Section 7 concludes.

2. Related Literature

The role of CB's private information and signaling in monetary policy is the subject of an ongoing debate (see Cukierman 1995). Romer and Romer (2000) present evidence that the FED's inflation forecasts contain valuable private information, relative to commercial forecasts. In contrast, Blinder (1998) and others argue that the FED knows privately only her own intentions, because the relevant economic data are publicly and readily available. The CB's substantial research assets certainly afford a superior ability to process the available data. In our model, this issue is moot: *the CB's private information can be interpreted as her own subjective view of the macroeconomic outlook*, which plays a purely strategic role, as P is only interested in correctly anticipating the CB's moves. Communication develops as long as the CB believes that her own opinion is informative about economic fundamentals and that P agrees on this, even if P does not agree.

Canzoneri (1985) argues that the standard solutions to time inconsistency are inadequate when the CB has private information on a state of the economy. He extends in this direction a static monetary policy game à la Barro and Gordon (1983). The timing of events makes cheap talk communication redundant: the inflation/output trade-off originates from a wage-contracting model, where P sets nominal wages *before* the CB observes the state and chooses her policy. But, if the game was repeated, any serial correlation in the state of the economy would create a scope for communication. To keep the analysis of communication tractable, we obtain the same effect by changing the timing of events in the static game.[2]

Our approach to cheap talk follows Crawford and Sobel (1982)'s classic model of partisan advice, and the vast literature that grew from it. A sender observes without noise a state of nature and announces a message to a receiver. The two agents have genuinely incongruent preferences over the state and the receiver's action. In our model, the receiver (P) cares only about the sender (CB)'s final action (monetary policy), not about the state of nature, that the CB observes with noise,[3] and the CB has an inflation

[2] Athey, Atkeson and Kehoe (2002) analyze a repeated version of Canzoneri (1985)'s game, but assume i.i.d. states, which is the only special case precluding communication. Therefore, explicit incentives are needed to extract information from the CB. The best equilibrium of the revelation game, society's optimal "dynamic mechanism" for the CB, appropriately balances flexibility with a simple inflation-cap restriction, to moderate time inconsistency.

[3] Money aggregates, M1-M3, are a good example of a variable that the private sector paid attention to uniquely because the Bundesbank (and then the ECB) did, so they helped forecast the intentions of the monetary authority, not because they were of any real relevance to the economy.

bias, measuring the degree of time inconsistency. Stein (1989) shows, in a different monetary policy game, that time inconsistency induces the sender to lie, just like a genuine preference bias. Our contribution is to identify the two beneficial effects of competence on credibility. The literature on cheap talk advice has left relatively unexplored the comparative statics effects of changing the sender's competence, formally equivalent to changing the distribution of states of nature, on equilibrium communication.[4] Also, this literature has typically made two convenient technical assumptions, that we show to have substantive consequences: a bounded state space, which fails to guarantee existence of an equilibrium with communication, while in our setting a two-message equilibrium always exists; and, in solved examples, an uninformative (uniform) prior, which eliminates by construction the effect of competence on the distribution of private signals, thus on the "credibility of likely announcements."

Two different roles of competence in communication have received attention. The first is the concern that an advisor may have for his reputation for competence (Morris 1997, Ottaviani and Sorensen 2003), which distorts cheap talk communication independently of bias. The second is the coordination of expectations. Morris and Shin (2002) study the effects of public information when, as in Woodford (2001), private information is dispersed among private agents, whose actions are strategic complements. In a monetary policy context, the CB's actions signal her valuable information about the state of the economy, but are also common knowledge and coordinate the beliefs and actions of private agents, who put too much weight on the CB's policy/announcements. The social marginal benefit of the CB's competence is initially negative, because a modest increase amplifies the CB's mistakes. Similarly, in our model the CB's competence is "the power of her words" in P's expectations, but weighed against prior beliefs on the state of nature, because P has no private information of his own. Therefore, competence is always beneficial, because it works against the impediment to communication, time inconsistency. Section 6 discusses policy communication in the Morris-Shin environment.

[4]The seminal contribution is Austen-Smith (1990). Benabou and Battaglini (2003) and Battaglini (2003) allow for multiple noisy experts. A beneficial effect of precision on preference bias, formally related to one of our two effects, appears in Ottaviani (2000). He does not emphasize or interpret this effect, because his focus is quite different, and his game features neither competence nor credibility as meant here. The sender can observe a uniformly distributed state with a precision that she may choose covertly; so the issue is moral hazard, as opposed to our 'adverse selection-reputational' view of precision as competence. The sender just speaks, and the receiver cares only about the objective informational content of the message. So, there is no 'credibility' in the usual time-inconsistency sense, because there are no deeds (monetary policy actions) to match to words (announcements).

3. The Model

Our two players are a Central Bank CB (a "she") and the Private sector P (a "he"). The economy is described by an output-gap version of the natural-rate Phillips curve:

$$y = s\,(\pi - x) \tag{3.1}$$

where y is the growth rate of real output (or of minus the output gap), π is the rate of inflation, x is P's rational expectation of π conditional on his information set $\mathbb{I}_P$

$$x = \mathbb{E}\left[\pi|\mathbb{I}_P\right],$$

and s is a positive parameter measuring the sensitivity of output to inflation forecast errors, namely the degree of nominal rigidity in the economy.

CB controls π to minimize a quadratic social loss function

$$L(y, \pi) = \mathbb{E}\left[(y - b)^2 + \lambda\,(\pi - \pi^* - \omega)^2\,|\mathbb{I}_{CB}\right]$$

where b is desired output growth rate, π^* is the average level of desired inflation, ω is a shock to desired inflation, $\lambda > 0$ is the relative weight on inflation, and the expectation is conditioned on CB's information $\mathbb{I}_{CB}$. P has the same loss function, or at least this is what the benevolent CB believes, but possibly a different information set. P's action is to formulate the expectation of inflation x and to set some nominal prices (e.g. wages) accordingly.[5]

When b and ω are public information, as is well known this game has a unique Nash Equilibrium, with $y = 0$ and

$$\pi = x = \pi^* + \frac{s}{\lambda}b + \omega \tag{3.2}$$

where we denote with b the CB's inflation bias. This equilibrium is inefficient if CB is biased, namely if CB attempts to attain a growth rate b different (we assume larger) than the one $y = 0$ dictated by correct expectations. In fact, in this case the equilibrium loss is positive, $b^2\,(1 + s^2/\lambda) > 0$, while it would equal 0 if the CB were to choose and commit to the Stackelberg inflation rate $\pi = x = \pi^* + \omega$. This is a simple illustration of the

[5] Appendix A derives this model from an island economy and a quantity equation, where the CB controls the growth rate of money rather than inflation. As shown by Canzoneri (1985), ω can represent several shocks affecting the economy, such as money demand shocks, or the effectiveness of monetary policy at that particular juncture.

time-inconsistency or inflation-bias problem that has preoccupied monetary economists in the last decades.

We assume instead that ω is uncertain and drawn from

$$\omega \sim \mathbb{N}(0,1)$$

where the mean and variance of the state ω are normalized to 0 and 1 without loss in generality. The CB privately observes an informative but noisy signal of ω:

$$\theta = \omega + \varepsilon \text{ where } \varepsilon \sim \mathbb{N}\left(0, \sigma_\varepsilon^2\right).$$

Bayesian updating implies

$$\omega|\theta \sim \mathbb{N}(H\theta, 1-H)$$

where we introduce the convenient notation

$$H \equiv \frac{1}{1+\sigma_\varepsilon^2} \in [0,1]$$

to denote CB's "competence". This measures the ability of the CB to correctly observe or interpret the state of the economy ω, relative to the difficulty of the problem as parameterized by $\sigma_\omega^2 = 1$.

The private signal θ captures either private information that the CB has about the state of the economy, or just the CB's "judgement" (in Svensson (2003)'s terminology) of the state of the economy. The economy is hit by different shocks, as summarized by ω, and the more precise CB's observations and/or interpretation of these shocks, namely the lower their variance σ_ε^2 and the higher the competence H, the more precise an idea CB has (or thinks to have) of what to do about it.[6]

The timing of the game is as follows:

1. CB observes θ and announces to P a message A about θ (silence is a message).

2. P formulates rational expectations of inflation x conditional on A;

3. CB chooses the optimal inflation rate $\pi(\theta, A)$, which depends on her private information θ and on her announcement A.

[6] It is immaterial whether P believes θ to be genuinely informative of the state of the economy ω, or θ just represents CB's own view of the economy, which might be totally uninformative about ω from P's viewpoint. The role of θ for P is purely strategic; P cares about what CB believes, even if P has no confidence whatsoever in CB's competence, because P is only concerned with anticipating the CB's moves. (Of course, the CB has to believe that P believes θ to be informative, even if P really does not.) This is another difference from Morris and Shin (2002), where public announcements play both an allocative role, and a signaling role of *genuinely* valuable information about fundamentals.

4. Communication and Transparency of Monetary Policy

In this section we characterize the set of Perfect Bayesian Equilibria of the communication game. A *Perfect Bayesian Nash Equilibrium with Communication* (PBEC) is a measurable partition $\mathcal{A}$ of $\mathbb{R}$ and a set of beliefs about θ with the following properties. For all $A \in \mathcal{A}$, if CB announces A to P, P "believes" her, namely P updates his beliefs about θ in a Bayesian fashion to $\theta|\theta \in A$. If CB makes any other announcement not in $\mathcal{A}$, P assimilates it to some $A' \in \mathcal{A}$. Following an announcement A and the updating of inflation expectation to $x(A)$, the CB chooses the optimal inflation rate π. Finally, a CB who privately observes $\theta \in A$ has no incentives to send any other message than A.

When $\mathcal{A} = \mathbb{R}$ no communication takes place: there always exists a fully non-revealing equilibrium, equivalent to "babbling," where P believes nothing that CB says. When every $A \in \mathcal{A}$ is a singleton we have full communication, or truthtelling.

4.1. Incentive Compatible Announcements

In a PBEC, after CB announces a message A, P reformulates an expectation of inflation

$$x(A) = \mathbb{E}[\pi|\theta \in A].$$

CB chooses the optimal announcement A of her private information θ and the optimal inflation rate π to solve

$$\min_{A,\pi} \left\{ L(A,\pi|\theta) = \mathbb{E}\left[(s\pi - s\mathbb{E}[\pi|\theta \in A] - b)^2 + \lambda (\pi - \pi^* - \omega)^2 |\theta \right].\right\}$$

We proceed by backward induction. First, we fix the announcement A and find the equilibrium inflation rate π. Second, we find the equilibrium announcement given the ensuing inflation rate.

Given A and the resulting P's expectation $x(A) = \mathbb{E}[\pi|\theta \in A]$, the NFOC for an optimal inflation rate π is

$$s^2 [\pi - x(A)] - sb + \lambda (\pi - \pi^* - H\theta) = 0$$

which yields a best response inflation rate to expectations equal to:

$$\pi(\theta, A) = \frac{\lambda\pi^* + sb + s^2 x(A) + \lambda H\theta}{s^2 + \lambda}. \tag{4.1}$$

Therefore, P's rational expectation of inflation in equilibrium is

$$x(A) = \mathbb{E}[\pi(\theta, A)|\theta \in A] = \frac{\lambda\pi^* + sb + s^2 x(A) + \lambda H\bar{\theta}(A)}{s^2 + \lambda} = \pi^* + \frac{s}{\lambda}b + H\bar{\theta}(A) \quad (4.2)$$

where

$$\bar{\theta}(A) = \mathbb{E}[\theta|\theta \in A] = \frac{\int_A \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_A e^{-H\frac{\theta^2}{2}} d\theta}$$

is P's expectation of θ given the believed announcement A (that $\theta \in A$), because the unconditional distribution of the private signal is:

$$\theta = \omega + \varepsilon \sim \mathbb{N}\left(0, 1 + \sigma_\varepsilon^2\right) = \mathbb{N}(0, 1/H).$$

Notice that the effect of the inflation bias b on equilibrium expected inflation x is amplified by s and is dampened by λ: the larger s, the more responsive is output to inflation surprises, the stronger the temptation for the CB to inflate the economy, while the higher λ, the more costly to the CB is (off-target) inflation. Also. notice that the equilibrium expected inflation $x(A)$ from (4.2) with incomplete information equals the one of the complete information game, (3.2), where the estimated inflation target $H\bar{\theta}(A)$ announced by the CB to P and believed by P replaces the true inflation target ω.

Replacing $x(A)$ from (4.2) into the CB's best response (4.1) gives an equilibrium inflation rate

$$\pi(\theta, A) = \frac{\lambda\pi^* + sb + s^2\left[\pi^* + \frac{s}{\lambda}b + H\bar{\theta}(A)\right] + \lambda H\theta}{s^2 + \lambda} = \pi^* + \frac{s}{\lambda}b + H\frac{s^2\bar{\theta}(A) + \lambda\theta}{s^2 + \lambda}$$

so that, adding and subtracting $H\theta = \mathbb{E}[\omega|\theta]$, the deviation of inflation from its overall target equals

$$\pi(\theta, A) - \pi^* - \omega = \frac{s}{\lambda}b + s^2 H\frac{\bar{\theta}(A) - \theta}{s^2 + \lambda} + H\theta - \omega.$$

Replacing equilibrium inflation into the supply curve (3.1) gives an equilibrium growth rate of output

$$y = s\left(H\frac{s^2\bar{\theta}(A) + \lambda\theta}{s^2 + \lambda} - H\bar{\theta}(A)\right) = sH\lambda\frac{\theta - \bar{\theta}(A)}{s^2 + \lambda}.$$

Notice that, if the CB announces a higher inflation target $\bar{\theta}(A)$ than the θ she truly observed, and P believes $\bar{\theta}(A)$, then P expects high inflation and this depresses growth.

Finally, conditional on the actually observed inflation target θ and on the A that the CB announces about it, the expected indirect loss minimized w.r. to inflation equals:

$$\mathcal{L}(A|\theta) = \min_\pi L(A, \pi|\theta) = \mathbb{E}\left[\left(s\lambda H\frac{\theta - \bar{\theta}(A)}{s^2 + \lambda} - b\right)^2 + \lambda\left(\frac{s}{\lambda}b + s^2 H\frac{\bar{\theta}(A) - \theta}{s^2 + \lambda} + H\theta - \omega\right)^2 |\theta\right].$$

Using $\mathbb{E}[\,(H\theta-\omega)^2\,|\theta] = \mathbb{V}\,[\omega|\theta] = 1-H$ and collecting terms finally yields

$$\mathcal{L}\,(A|\theta) = \frac{s^2H^2\lambda}{s^2+\lambda}\left[\theta-\bar{\theta}\,(A)\right]^2 - 2bsH\left[\theta-\bar{\theta}\,(A)\right] + b^2\left(1+\frac{s^2}{\lambda}\right) + \lambda\,(1-H)\,.$$

This function measures the incentives to announce any message A for a CB who observed a signal θ of the state of the economy and anticipates to be believed by P.

We notice a key fact. Since only P's expectation of θ given a message $A \in \mathcal{A}$ matters for payoffs, *we can replace messages A with numbers* $\bar{\theta} = \mathbb{E}\,[\theta|\theta \in A]$, with the interpretation that saying "$\bar{\theta}$" means that "θ belongs to the set A with mean $\bar{\theta}$." This enormously simplifies the analysis. So we will write

$$\mathcal{L}(\bar{\theta}|\theta) = \frac{s^2H^2\lambda}{s^2+\lambda}\left(\theta-\bar{\theta}\right)^2 - 2bsH\left(\theta-\bar{\theta}\right) + b^2\left(1+\frac{s^2}{\lambda}\right) + \lambda\,(1-H)\,. \tag{4.3}$$

Let θ be the observed private signal, A be the element of the candidate equilibrium partition $\mathcal{A}$ which includes θ, and $\bar{\theta} = \mathbb{E}\,[\theta|\theta \in A]$. Then, the Incentive Compatibility constraint that PBEC announcements $\bar{\theta}$ must satisfy is

$$\mathcal{L}(\bar{\theta}|\theta) \leq \mathcal{L}(\bar{\theta}'|\theta)$$

for every $\bar{\theta}' = \mathbb{E}\,[\theta|\theta \in A']\,,\ A' \in \mathcal{A}$. Therefore we require:

$$\frac{s^2H^2\lambda}{s^2+\lambda}\left(\theta-\bar{\theta}\right)^2 - 2bsH\left(\theta-\bar{\theta}\right) \leq \frac{s^2H^2\lambda}{s^2+\lambda}\left(\theta-\bar{\theta}'\right)^2 - 2bsH\left(\theta-\bar{\theta}'\right).$$

Dividing through by $H^2s^2\lambda/\,(s^2+\lambda) > 0$ and rearranging, we finally obtain a quite simple inequality

$$\left(\bar{\theta}-\bar{\theta}'\right)\left(\bar{\theta}+\bar{\theta}'-2\theta+q\right) \leq 0 \tag{4.4}$$

where

$$q \equiv 2\frac{b}{H}\left(\frac{s}{\lambda}+\frac{1}{s}\right) \tag{4.5}$$

is a normalized *bias/competence ratio*.

This composite parameter plays a critical role: it increases with the CB's inflation bias b and decreases with the CB's competence H and, after some algebra, the CB's conservatism (weight on inflation) λ. Intuitively, the CB's incentives to engineer a high unexpected inflation increase in her bias b and decrease in her cost of inflation λ. The dependence of q on the sensitivity of output to inflation surprises s is ambiguous, as

$$\frac{\partial q}{\partial s} = \frac{2b}{\lambda H}\left(1-\frac{\lambda}{s^2}\right). \tag{4.6}$$

This is due to two countervailing effects of nominal rigidity s on the CB's incentives to inflate the economy. A large s makes inflation surprises very "productive" in terms of output gap. Therefore, on one hand P expects CB to be prone to deception, on the other CB does not need to deceive P by much to attain the desired output growth b. Which of the two effects dominates depends on conservatism λ. If $\lambda > s^2$ then P knows that the CB prefers moderate inflation surprises, because inflation is more important than output to the CB; so the second effect dominates and q is decreasing in s. Both s and λ are scale-free parameters. As we will argue in Section (5), s is likely to be significantly below 1, while modern central banks are generally believed to care more about inflation than about real activity ($\lambda > 1$), so (4.6) is generally negative.

Finally, we can manipulate (4.4) to obtain an IC constraint in simple form: for every $A \in \mathcal{A}$, $\theta \in A$, and given $\bar{\theta} = \mathbb{E}[\theta|\theta \in A]$,

$$\begin{cases} \bar{\theta}' \leq 2\theta - \bar{\theta} - q \text{ for every } \bar{\theta}' < \bar{\theta} \\ \bar{\theta}'' \geq 2\theta - \bar{\theta} - q \text{ for every } \bar{\theta}'' > \bar{\theta}. \end{cases} \tag{4.7}$$

4.2. Equilibrium Characterization

The CB would like P to expect low inflation and to surprise him to boost growth closer to the bliss rate b. Thus, the CB observing θ prefers not to tell a lie $\bar{\theta}'$ only if $\bar{\theta}'$ is lower than the true θ by more than q, because then $\bar{\theta}'$ would excessively mislead inflation expectations and weigh on the inflation term of the social loss function.

Proposition 1. (Communication is Coarse) *In any PBEC, if $b > 0$ then truthtelling ($\bar{\theta} = \theta$ for all θ) is impossible.*

Proof. With perfect communication, each θ can announce his type credibly, so P believes any pointwise announcement, i.e. $\bar{\theta}$ can be any real number. But then for $\theta = \bar{\theta}$ the first line in (4.7) reads $0 \geq \bar{\theta}' - \theta + q$ for all real numbers $\bar{\theta}' < \theta = \bar{\theta}$, which is clearly impossible iff $b > 0$, namely $q > 0$. ■

Next, it is immediate from (4.7) that if θ prefers $\bar{\theta}$ over $\bar{\theta}' < \bar{\theta}$, then so do all $\hat{\theta} > \theta$. Conversely, if θ prefers $\bar{\theta}$ over $\bar{\theta}'' > \bar{\theta}$ then so do all $\hat{\theta} < \theta$. Therefore, the set of types who announce the same message in equilibrium must be connected.

Lemma 1. *In any PBEC, all CB types θ who send the same message belong to a connected set, either an interval or a (half-)line.*

In light of these results, a PBEC is a collection of intervals $[\theta_k, \theta_{k+1})$ of the real line such that: (i) all $\theta \in [\theta_k, \theta_{k+1})$ send the message

$$\bar{\theta}_k = \mathbb{E}\left[\theta | \theta_k \leq \theta < \theta_{k+1}\right], \tag{4.8}$$

and (ii) if inflation expectations are based on $\bar{\theta}_k$, then no other message will give a smaller expected loss, conditional on θ.

To satisfy all IC constraints, it suffices to impose that each boundary type θ_k be indifferent between the message $\bar{\theta}_k > \theta_k$ that he and his right-neighbors $\theta_k + \varepsilon$ for $\varepsilon > 0$ are supposed to send, and the message $\bar{\theta}_{k-1} < \theta_k$ that his left-neighbors $\theta_k - \varepsilon$ are supposed to send. Because then, from (4.7), all $\theta > \theta_k$ strictly prefer $\bar{\theta}_k$ to $\bar{\theta}_{k-1}$, so their IC constraint is satisfied when comparing their equilibrium message $\bar{\theta}_k$ to the next-to-small message $\bar{\theta}_{k-1}$, and *a fortiori* to all smaller messages $\bar{\theta}_{k-1-j}$. At the same time, again from (4.7), all $\theta < \theta_k$ strictly prefer $\bar{\theta}_{k-1}$ to $\bar{\theta}_k$, so their IC constraint is satisfied when comparing their equilibrium message $\bar{\theta}_{k-1}$ to the next-to-larger message $\bar{\theta}_k$, and *a fortiori* to all larger messages $\bar{\theta}_{k+j}$. When this holds for all k, all IC constraints are automatically satisfied, strictly so for θ in the interior of each interval, and weakly so for boundary types.

From (4.7), the required indifference condition for the boundary type θ_{k+1} between the two nearby messages $\bar{\theta}_k$ and $\bar{\theta}_{k+1}$ can be written as follows:

$$\bar{\theta}_{k+1} = 2\theta_{k+1} - q - \bar{\theta}_k. \tag{4.9}$$

Lemma 2. (Form of the Equilibrium Message Space) *A PBEC is a partition of the real line into intervals $\{[\theta_k, \theta_{k-1})\}$, where $\{\theta_k\}$ is an increasing, possibly doubly infinite sequence which solves (4.8) and (4.9) with $\theta_k < \bar{\theta}_k < \theta_{k+1}$ for each k.*

Equation (4.9) bounds the coarseness of the equilibrium message space in terms of the critical parameter q. First, for $\theta < \bar{\theta}_k$ who announces $\bar{\theta}_k$, and for every lower message $\bar{\theta}_{k-1} < \bar{\theta}_k$:

$$\bar{\theta}_k > \theta \geq \theta_k = \frac{\bar{\theta}_k + \bar{\theta}_{k-1} + q}{2}.$$

Rearranging, messages have to be at least q apart:

$$\bar{\theta}_k - \bar{\theta}_{k-1} > q.$$

Second,

$$q < \bar{\theta}_{k+1} - \bar{\theta}_k = 2\left(\theta_{k+1} - \bar{\theta}_k\right) - q$$

implies the stronger results

$$\theta_{k+1} - \bar{\theta}_k > q \tag{4.10}$$

so the upper bound of each interval must be at least q larger than its mean. Since $\theta_k < \bar{\theta}_k$, this also implies that intervals must have width at least q:

$$\theta_{k+1} - \theta_k > q.$$

If we interpret the fineness of the equilibrium message space as monetary policy's transparency, we obtain:

Proposition 2. (Maximum Transparency of Monetary Policy). *In any PBEC, messages sent by CB to P about the estimated desired inflation rate are either intervals $[\theta_k, \theta_{k+1})$, of width strictly larger than q, or the means of these intervals w.r. to the measure of the target, $\bar{\theta}_k = \mathbb{E}\left[\theta | \theta_k \leq \theta < \theta_{k+1}\right]$, also spaced more than q apart, namely $\bar{\theta}_{k+1} - \bar{\theta}_k > q$. Therefore, the normalized bias/competence ratio q constrains the transparency of monetary policy.*

Next, we show that the sequence is bounded below, i.e. it is of the form $\{\theta_k\}_{k=1}^{\infty}$ with $\theta_1 > -\infty$ and $\theta_{k+1} > \theta_k$ for all $k = 1, 2..$ First we need:

Lemma 3. *For every H, the function*

$$f(t) = t - \mathbb{E}\left[\theta | \theta \leq t\right] = t - \frac{\int_{-\infty}^{t} \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_{-\infty}^{t} e^{-H\frac{\theta^2}{2}} d\theta}$$

is increasing in t with $\lim_{t\to-\infty} f(t) = 0$, $\lim_{t\to+\infty} f(t) = \infty$, and the function

$$\psi(t) = \mathbb{E}\left[\theta | \theta > t\right] - t = \frac{\int_{t}^{\infty} \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_{t}^{\infty} e^{-H\frac{\theta^2}{2}} d\theta} - t$$

is decreasing with $\lim_{t\to-\infty} \psi(t) = \infty$, $\lim_{t\to+\infty} \psi(t) = 0$.

Proof. By a change of variable $z = \theta\sqrt{H}$, $u = t\sqrt{H}$:

$$f(t) = f\left(\frac{u}{\sqrt{H}} | H\right) = \frac{u}{\sqrt{H}} - \frac{\int_{-\infty}^{u} \frac{z}{\sqrt{H}} e^{-\frac{z^2}{2}} \frac{dz}{\sqrt{H}}}{\int_{-\infty}^{u} e^{-\frac{z^2}{2}} \frac{dz}{\sqrt{H}}} = \frac{1}{\sqrt{H}}\left(u - \frac{\int_{-\infty}^{u} z e^{-\frac{z^2}{2}} z}{\int_{-\infty}^{u} e^{-\frac{z^2}{2}} z}\right) = \frac{f(u|1)}{\sqrt{H}}.$$

Similarly $\psi(t|H) = \psi(u|1)/\sqrt{H}$. Given the definition of u, for any given $H > 0$, it suffices to prove the claims for $f(u|1)$ and $\psi(u|1)$, which can be done numerically. ■

Lemma 4. (Finite Equilibrium Message Space) *Any PBEC is of the form* $\{(-\infty, \theta_1), [\theta_1, \theta_2), [\theta_2, \theta_3)... [\theta_K, \infty)\}$ *where* $\{\theta_k\}_{k=1}^{K}$ *is a strictly increasing and finite sequence.*

Proof. First, we show that the sequence $\{\theta_k\}$ is bounded below. By contradiction: for every N finite there exists $k > -\infty$ such that $N \in [\theta_{k-1}, \theta_k)$ for some $\theta_{k-1} > -\infty$. By definition of PBEC and $\bar{\theta}_{k-1} = \mathbb{E}[\theta|\theta_{k-1} \leq \theta < \theta_k]$ this means that

$$\mathbb{E}[\theta|\theta_{k-1} \leq \theta < \theta_k] = 2\theta_k - q - \mathbb{E}[\theta|\theta_k \leq \theta < \theta_{k+1}] > \mathbb{E}[\theta|\theta < \theta_k]$$

because θ has full support on the real line. Rearranging

$$\theta_k - \mathbb{E}[\theta|\theta < \theta_k] = f(\theta_k|H) > q + \mathbb{E}[\theta|\theta_k \leq \theta < \theta_{k+1}] - \theta_k > q.$$

Since q is strictly positive, and this must be true for every N, hence for every $\theta_{k-1} \leq N$, this implies that $f(x|H)$ is bounded away from zero, contradicting Lemma 3.

Second, we show that the sequence $\{\theta_k\}$ is bounded above. Since each increment in the sequence must exceed $q > 0$, this means that the sequence is finite. Using $\bar{\theta}_k = 2\theta_k - q - \bar{\theta}_{k-1} \geq \theta_k$, $\mathbb{E}[\theta|\theta > \theta_k] \geq \bar{\theta}_k$, and rearranging,

$$\mathbb{E}[\theta|\theta > \theta_k] - \theta_k = \psi(\theta_k|H) \geq \bar{\theta}_k - \theta_k = \theta_k - \bar{\theta}_{k-1} - q \geq 0$$

where the last inequality follows from $\bar{\theta}_k \geq \theta_k$. By contradiction, suppose θ_k grows unbounded with k. Then by Lemma 3 $\lim_{k\to\infty} \psi(\theta_k|H) = 0$, so for every $\varepsilon > 0$ there exists K_ε such that for all $k > K_\varepsilon$ we have $\psi(\theta_k|H) < \varepsilon$, and therefore

$$\varepsilon > \theta_k - \bar{\theta}_{k-1} - q \geq 0.$$

We have concluded

$$\lim_{k\to\infty} \left(\theta_k - \bar{\theta}_{k-1}\right) = q. \tag{4.11}$$

It follows:

$$\lim_{k\to\infty} \left[\theta_k - \bar{\theta}_{k-1} - \left(\theta_{k+1} - \bar{\theta}_k\right)\right] = 0.$$

So for every $\delta > 0$ there is K_δ such that for all $k > K_\delta$

$$\theta_{k+1} - \theta_k < \bar{\theta}_k - \bar{\theta}_{k-1} + \delta = 2\left(\theta_k - \bar{\theta}_{k-1}\right) + \delta - q$$

If we take $k > \max\langle K_\delta, K_\varepsilon\rangle$:

$$\theta_{k+1} - \theta_k < 2(q + \varepsilon) + \delta - q = q + 2\varepsilon + \delta.$$

This fact, along with $\theta_{k+1} - \theta_k > q$ from Lemma 2, finally proves

$$\lim_{k\to\infty} (\theta_k - \theta_{k-1}) = q. \tag{4.12}$$

So we have established that we can always satisfy

$$\theta_{k-1} + q - \varepsilon < \bar{\theta}_{k-1} < \theta_k < \theta_{k-1} + q + \delta$$

the first inequality from (4.11), the second by definition of $\bar{\theta}_{k-1}$, the third from (4.12). So

$$\theta_{k-1}+q-\varepsilon < \bar{\theta}_{k-1} = \mathbb{E}\left[\theta|\theta_{k-1} \leq \theta < \theta_k\right] < \mathbb{E}\left[\theta|\theta_{k-1} \leq \theta < \theta_{k-1} + q + \delta\right] = \frac{\int_{\theta_{k-1}}^{\theta_{k-1}+q+\delta} \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_{\theta_{k-1}}^{\theta_{k-1}+q+\delta} e^{-H\frac{\theta^2}{2}} d\theta}$$

But notice that for $\varepsilon = \delta = 0$ the leftmost term is strictly larger than the rightmost term, so this inequality must be violated for some ε, δ small enough and $k > \max \langle K_\delta, K_\varepsilon \rangle$, the desired contradiction. ■

4.3. Equilibrium Construction

Our equilibrium characterization also offers a recursive algorithm to construct equilibria. First, define the implicit function $g\left(\theta_1, \bar{\theta}_1|H\right)$ by

$$\frac{\int_{\theta_1}^{g\left(\theta_1,\bar{\theta}_1|H\right)} \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_{\theta_1}^{g\left(\theta_1,\bar{\theta}_1|H\right)} e^{-H\frac{\theta^2}{2}} d\theta} = \bar{\theta}_1.$$

Second, notice that from Lemma 4, for $k = 1$ the inequality (4.10) reads

$$\theta_1 - \mathbb{E}\left[\theta|\theta < \theta_1\right] = \theta_1 - \bar{\theta}_0 = f\left(\theta_1\right) > q$$

or

$$\theta_1 > \theta_1^* \equiv f^{-1}\left(q\right).$$

This provides a lower bound to the very first (and possibly only) element of the equilibrium sequence $\{\theta_k\}_{k=1}^K$. So:

1. for the chosen q, H pair, compute $\theta_1^* = f^{-1}\left(q\right)$ and start from $\theta_1 = \theta_1^*$;
2. increase θ_1 by some tiny increment;
3. compute $\bar{\theta}_0 = \mathbb{E}\left[\theta|\theta < \theta_1\right]$ and $\bar{\theta}_1 = 2\theta_1 - q - \bar{\theta}_0$;

4. if $\bar{\theta}_1 \leq \theta_1$, stop: no communication in equilibrium is possible for that value of θ_1, go back to step 2; otherwise:

5. check that $\mathbb{E}[\theta|\theta > \theta_1] = \bar{\theta}_1$ (up to some tolerance level); if so, we have found an equilibrium with two messages; if $\mathbb{E}[\theta|\theta > \theta_1] < \bar{\theta}_1$, go back to step 2; if $\mathbb{E}[\theta|\theta > \theta_1] > \bar{\theta}_1$, proceed to

6. compute $\theta_2 = g\left(\theta_1, \bar{\theta}_1|H\right)$; if $\theta_2 - \theta_1 < q$ go back to step 2, otherwise:

7. compute $\bar{\theta}_2 = 2\theta_2 - q - \bar{\theta}_1$;

8. if $\bar{\theta}_2 \leq \theta_2$, stop, etc...Iterate over the previous steps, until for some $k = K$, we have $\bar{\theta}_K = \mathbb{E}[\theta|\theta > \theta_K]$ and we have found the equilibrium.

4.4. Two-Message Equilibrium

A two-message equilibrium always exists, so the CB is always able to credibly communicate something. This stands in contrast to Crawford and Sobel (1982)'s results, where the bias has to be small enough for communication to occur, due to their assumption of a bounded state space.

In a two-message equilibrium, a CB observing $\theta < \theta_1$ for some real number θ_1 announces $\bar{\theta}_0 = \mathbb{E}[\theta|\theta < \theta_1]$, or "The economy needs low inflation", while for all $\theta \geq \theta_1$ she announces $\bar{\theta}_1 = \mathbb{E}[\theta|\theta \geq \theta_1]$, or "The economy needs high inflation". The only IC constraint to be satisfied with equality by the cutoff type θ_1 is $\bar{\theta}_1 = 2\theta_1 - q - \bar{\theta}_0$, so

$$\phi(\theta_1) \equiv f(\theta_1) - \psi(\theta_1) = q.$$

A solution $\theta_1^{(2)}$ to this equation always exists and is strictly positive, because by Lemma 3 ϕ is strictly increasing and onto with $\phi(0) = 0$. Since $\psi(\cdot) > 0$, the last equation also implies the required condition $f(\theta_1^{(2)}) > q$, or $\theta_1^{(2)} > \theta_1^*$. The superscript "(2)" refers to the number of messages sent in this equilibrium.

Since $\theta_1^{(2)} > 0$ and the normal distribution of θ is symmetric around its mean zero, it follows that $\left|\bar{\theta}_0^{(2)}\right| < \left|\bar{\theta}_1^{(2)}\right|$. When the CB announces that low inflation is needed, P believes her but sets a relatively high expectation $\pi^* + H\bar{\theta}_0^{(2)}$, below but not far from π^*; when announcing that high inflation is needed, P's expectation goes way above π^*, to $\pi^* + H\bar{\theta}_1^{(2)}$. Bias introduces an asymmetry in messages. The higher q, the higher and the more asymmetric the two equilibrium messages.

Since $\theta_1^{(2)} > 0$, $\Pr(\bar{\theta}_0^{(2)}) = \Pr(\theta \leq \theta_1^{(2)}) > 1/2$. That is, somewhat paradoxically, the CB is ex ante more likely to credibly announce that low inflation ($\pi^* + H\bar{\theta}_0^{(2)} < \pi^*$) is appropriate, even if the ex ante chance of high and low inflation is even. The CB may even credibly announce below-average inflation ($\pi^* + H\bar{\theta}_0^{(2)}$) when some moderately above-average inflation is truly required by $\theta \in (0, \theta_1^{(2)})$. *So an inflation-biased CB is relatively likely to speak like a conservative one, and gets away with it!* Credibility forces the CB to be conservative in her announcements. The alternative two-message equilibrium, with $\bar{\theta}_0^{(2)}$ very low and $\bar{\theta}_1^{(2)}$ moderately high but positive, would not be feasible, because the temptation to say $\bar{\theta}_0^{(2)}$ and be believed would be very strong. The more competent the CB is, the less conservative she can be. But then, when she is competent she is less likely to observe outliers. When $\theta_1^{(2)}$ is greater than this threshold, no communication may occur in equilibrium.

4.5. Summary and Intuition

We summarize all of our findings in the following:

Proposition 3. (Equilibrium Communication of Monetary Policy.) *There always exist an equilibrium with no communication (babbling). Every PBEC takes the following form. For every $H \in (0,1]$ and $q \in (0,\infty)$ defined in (4.5), there exists a finite integer $N(q,H) \geq 2$ and, for some integer $K \in \{2, ..N(q,H)\}$, a finite, stricly increasing sequence of $K+1$ extended reals, $\left\{\theta_k^{(K)}\right\}_{k=0}^{K}$, $\theta_0^{(K)} = -\infty = -\theta_K^{(K)}$, which defines a sequence of K adjacent intervals $[\theta_k^{(K)}, \theta_{k+1}^{(K)})$ with conditional expectations $\bar{\theta}_k^{(K)} = \mathbb{E}\left[\theta | \theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}\right]$, $k = 0, 1, ...K-1$, and has the following properties: after privately observing a signal realization $\theta \in [\theta_k^{(K)}, \theta_{k+1}^{(K)})$ of the state ω, the CB sends to P the message $\bar{\theta}_k^{(K)}$. Therefore, there always exists a PBEC with two messages, $N(q,H) = 2 \Rightarrow K = 2$, where the CB credibly announces whether required inflation exceeds or not a unique cutoff dictated by $\theta_1^{(2)} = \phi^{-1}(q)$, increasing in q. For every $H \in (0,1]$ and $q \in (0,\infty)$ such that $N(q,H) \geq 3$, there may (and generically do) exist more than one PBEC, namely more than one integer $K \in \{2, ..N(q,H)\}$. In any PBEC, both the messages $\bar{\theta}_k^{(K)}$ and the interval boundaries $\theta_k^{(K)}$ are spaced at least q apart. For every such K, the lowest finite threshold $\theta_1^{(K)}$ strictly exceeds $f^{-1}(q)$, a lower bound increasing in q. The subsequent thresholds $\theta_k^{(K)}$, $k = 2, 3, ..K$, solve the second-order* *non-linear difference equation*

$$\frac{\int_{\theta_k^{(K)}}^{\theta_{k+1}^{(K)}} \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_{\theta_k^{(K)}}^{\theta_{k+1}^{(K)}} e^{-H\frac{\theta^2}{2}} d\theta} = 2\theta_k - q - \frac{\int_{\theta_{k-1}^{(K)}}^{\theta_k^{(K)}} \theta e^{-H\frac{\theta^2}{2}} d\theta}{\int_{\theta_{k-1}^{(K)}}^{\theta_k^{(K)}} e^{-H\frac{\theta^2}{2}} d\theta}.$$

Our central finding concerns the beneficial effect of the competence of the informed party, the CB, on her ability to communicate information to P. Notice that competence, as measured by H, formally plays two roles: it reduces the normalized bias q, and it determines the probability distribution of the private signal $\theta \sim \mathbb{N}(0, H^{-1})$, thus the function g used by P in assessing the fineness of the equilibrium message space.

The first effect of competence is *the power of the words*, which improves equilibrium communication as follows. H is the weight that P puts on the CB's credible announcement $\bar{\theta}$ in forming an expectation of inflation. In fact, inflation expectations $x(A)$ contain a term $H\bar{\theta}$ which is just P's expectation, conditional on the announcement $\bar{\theta}$, of the CB's updated inflation target $H\theta$: namely, from (4.2),

$$x(A) - \pi^* - \frac{s}{\lambda}b = H\bar{\theta} = \mathbb{E}[H\theta|A, \mathbb{I}_P] = \mathbb{E}[\mathbb{E}[\omega|\theta, \mathbb{I}_{CB}]|A, \mathbb{I}_P].$$

So if P knows that the CB deems her own information θ to be accurate, then P knows that the CB will put a large H weight on θ, relative to the weight $1-H$ that she puts on the prior expectation of ω, normalized to 0. Since P only cares about anticipating the CB's moves, in formulating the expectation of $H\theta$ P will use the same value of H that the CB uses, even if P disagrees on the genuine quality of this information, i.e. even if P thinks that H is really equal to some $H' < H$. So the role of competence is purely strategic. Knowing that P embraces her own competence weight H, the CB knows that P's inflation forecast error $\pi - x$ and the resulting output gap y depend on $H(\theta - \bar{\theta})$. The larger H, the smaller the "lie" $\left|\theta - \bar{\theta}\right|$ has to be to align the output gap to the desired rate b. At the same time, the CB must choose inflation as close to $\pi^* + H\theta$ as possible to respond to the economy's needs. So the larger is the CB's competence H, the smaller the CB's incentives to claim a very different θ than the one she truly observed, and the finer the communication structure that P trusts.

The second effect of competence is the *credibility of likely announcements.* The more competent the CB, the more likely she is to observe a true θ near the average, zero. So, P can believe announcements by CB's very different than zero only if they are made by a larger set of types, because such announcements are unlikely to be truthful. This implies that the equilibrium partition is coarser away from zero, and by the same token is finer near zero, which is the more likely area of observation of the private signal.

4.6. The Value of Equilibrium Communication

Recall the loss function from (4.3). In any PBEC, the unconditional expected loss before seeing θ is

$$\mathcal{L} = \Psi + \frac{s^2 H^2 \lambda}{s^2 + \lambda} \mathbb{E}\left[\left(\theta - \bar{\theta}\right)^2 - q\left(\theta - \bar{\theta}\right)\right].$$

where the expectation is taken over the outcome of the PBEC, and we use the shorthand

$$\Psi \equiv b^2\left(1 + \frac{s^2}{\lambda}\right) + \lambda\left(1 - H\right)$$

for the part of the indirect loss that does not depend on announcements. Notice that this component declines in H, because a more competent CB has more valuable information and is more capable of stabilizing the economy. So, for any equilibrium with K messages:

$$\begin{aligned}
\mathcal{L} &= \Psi + \frac{s^2 H^2 \lambda}{s^2 + \lambda} \sum_{k=0}^{K-1} \mathbb{E}\left[(\theta - \bar{\theta}_k^{(K)})^2 - q(\theta - \bar{\theta}_k^{(K)}) | \theta_k^{(K)} \leq \theta < \theta_{k+1}^{K+1}\right] \Pr(\theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}) \\
&= \Psi + \frac{s^2 H^2 \lambda}{s^2 + \lambda} \sum_{k=0}^{K-1} \mathbb{V}\left[\theta | \theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}\right] \Pr\left(\theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}\right) - H^2 s^4 q \cdot 0 \\
&= \Psi + \frac{s^2 H^2 \lambda}{s^2 + \lambda} \sum_{k=0}^{K-1} \int_{\theta_k^{(K)}}^{\theta_{k+1}^{(K)}} \left(\theta - \bar{\theta}_k^{(K)}\right)^2 \frac{e^{-\frac{\theta^2}{2} H}}{\sqrt{2\pi H}} d\theta \\
&= \Psi + \frac{s^2 H^2 \lambda}{s^2 + \lambda} \mathbb{E}\left[\mathbb{V}\left[\theta | \theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}\right] | q, H\right]
\end{aligned}$$

where $\mathbb{E}[\mathbb{V}[\theta|\theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}|q, H]]$ is the ex ante unconditional expectation of the ex post variance of θ in P's beliefs after CB communicates according to the PBEC. This quantity ranges from 0 when truthtelling is an equilibrium to $\mathbb{V}[\theta] = 1/H$ when no communication take places in equilibrium. So $1 - H\mathbb{E}[\mathbb{V}[\theta|\theta_k^{(K)} \leq \theta < \theta_{k+1}^{(K)}|q, H]]$ is the percentage reduction in the variance due to communication, a measure of credibility.

It remains to establish the following:

Conjecture 1. *For each q, H, there is a PBEC with K messages for every $K = 2, 3 \ldots N(q, H)$, where $N(q, H)$ is decreasing in q and increasing in H. The lowest values of $\mathbb{E}[\mathbb{V}[\theta|\theta_k \leq \theta < \theta_{k+1}] | q, H]$ and of the expected loss are attained by the PBEC with the finest partition $K = N(q, H)$, are increasing in q and decreasing in H.*

5. Numerical Examples

A change in competence H has the two mentioned effects, the "power of the words" (reduction in q) and the "credibility of likely announcements" (reduction in the variance of θ). The first effect is equivalent to that of fixing H and changing either bias b or conservatism λ. A higher reputation for competence is a perfect substitute for more conservatism, as attained e.g. through delegation and CB independence. In addition, a higher H has two unique effects: it enhances credibility also through a second channel, and it makes the CB better at managing the economy.

While our final conjecture awaits formal proof, we substantiate it with some numerical examples, which also illustrate the potentially dramatic effect of bias, competence and conservatism on the extent of communication. For each parameter configuration, we look for the equilibrium with the finest message space, which maximizes the scope for communication.

Since we normalized $\sigma_\omega = 1$, we may express quantities in percentage points, so for example we assume an average desired inflation rate $\pi^* = 2$. This implies:

$$\Pr\left(0 = \pi^* - 2\sigma_\omega \leq \pi^* + \omega \leq \pi^* + 2\sigma_\omega = 4\right) = 0.99$$

that is, the true inflation target is almost always between 0 and 4(%).

A key parameter is s, the degree of nominal rigidity in the economy, and the response of output (growth) to a unit impulse to surprise inflation. Although the debate about this number is open, no empirical evidence points to values exceeding 1, and probably significantly lower. So we set $s = 0.25$: a 1% unexpected inflation raises output growth by 0.25%. From (4.6), this implies that the normalized bias/competence ratio q is decreasing in the nominal rigidity of the economy, s, as long as the relative weight on inflation in the CB's preferences (conservatism) λ is smaller than 16. It seems plausible that any CB, even those publicly committed to price stability only, would care about real activity at least this little.

Consistently with the claims, mentioned in the Introduction, made by central bankers against their own time inconsistency , we allow for a strong conservatism $\lambda = 5$ and a tiny inflation bias, $b = 0.02$ (%). This implies a normalized bias

$$q = \frac{2 \cdot 0.02 \cdot \left(5 + \frac{1}{16}\right)}{H 5\frac{1}{4}} \simeq \frac{1}{6H}.$$

We study the probability distribution of the equilibrium messages $\bar{\theta}$, for a range of values of the competence parameter $H = \left(1+\sigma_{\varepsilon}^{2}\right)^{-1}$. Recall that the unconditional distribution of the CB's private information θ is $\mathbb{N}\left(0, H^{-1}\right)$, so that the ideal inflation target $\mathbb{E}\left[\pi^{*}+\omega|\theta\right] = \pi^{*}+H\theta$ is $\mathbb{N}\left(\pi^{*}, 1\right)$ for every value of H. By the Bayesian nature of the equilibrium and the martingale property of posterior beliefs, communication is on average unbiased:

$$\mathbb{E}\left[\bar{\theta}\right] = \sum_{k=0}^{K-1} \bar{\theta}_{k}^{(K)} \Pr\left(\theta_{k}^{(K)} \leq \theta < \theta_{k+1}^{(K)}\right) = \sum_{k=0}^{K-1} \int_{\theta_{k}^{(K)}}^{\theta_{k+1}^{(K)}} \theta \frac{e^{-\frac{\theta^{2}}{2}H}}{\sqrt{2\pi H}} d\theta = \int_{-\infty}^{+\infty} \theta \frac{e^{-\frac{\theta^{2}}{2}H}}{\sqrt{2\pi H}} d\theta = 0.$$

Figure A.1 reveals a few more, interesting facts.

First, even for the tiny inflation bias chosen, two hundredths of one per cent, communication is fairly coarse. The number of credible messages never exceeds 6, as opposed to the uncountable support of θ. In fact, an inflation bias exceeding 1 results in $q > 2$ even for the perfectly competent CB $H = 1$, and this can be shown to prevent any communication other than the simple two-message equilibrium. Therefore, any meaningful communication between the CB and P requires that P perceives almost no meaningful bias in the CB's preferences. Even if the temptation to surprise inflation is very modest and presents itself very occasionally (when the observe θ is extreme), it stills hampers communication most of the time, especially for incompetent CB's.

Second, for every value of the competence parameter H, the message distribution is skewed above its average. This is a consequence of the inflation bias, which makes announcements of low inflation incredible even when the CB truly observes a signal of low required inflation θ. The CB cannot announce a very low inflation target because P would not believe her.

Third, as communication is unbiased on average, to compensate for skewness the CB must be relatively likely to announce below-average inflation. In order to offset the perception of bias, a CB, especially if not highly competent, must speak as a moderately conservative one most of the time. As we have seen, this result has been proven in general for the two-message equilibrium.

As the CB's competence H rises and the normalized bias/competence ratio q declines, the maximum number of messages that can be sent by CB to P in equilibrium rises from 2 to 6. At the same time, the distribution of messages becomes gradually more symmetric and concentrated around the mean, approaching the distribution $\mathbb{N}\left(0, H^{-1}\right)$

of the private signal θ that the CB truly observes. These phenomena are an illustration of the beneficial effect of competence on credibility.

Figure A.2 quantifies the gains from communication according to a scale-free metric, invariant to affine transformations of payoffs. We plot the percentage reduction in the variance of the private signal θ from the unconditional one H^{-1} to the value $\mathbb{E}\left[\mathbb{V}\left[\theta|\theta_k \leq \theta < \theta_{k+1}\right]|q, H\right]$ attained via the best possible equilibrium communication. This is a rough measure of the amount of information that the CB can fruitfully pass to P. We see that this magnitude is substantial, in relative terms, and increasing in competence.

6. Extensions

6.1. Alternative Impediments to Credibility: Stabilization Bias and the CBNC Effect

The beneficial impact of (a reputation for) competence on credibility illustrated in this paper should extend beyond the inflation bias, and should apply also to other sources of lack of credibility. Two important examples come to mind.

First, the "stabilization bias" (Clarida, Gali and Gertler 1999): when an adverse cost shock hits the economy, a CB would like to commit to raise future nominal interest rates, in order to subdue current inflation expectation. However, once this goal has been attained, the promised rise in interest rates is undesirable, thus time-inconsistent. If the CB has some superior information about either the cost shock or just her own interpretation of its effects, then it has an incentive to downplay the inflation risks, or to announce/signal a tightening bias for the medium run. But P should not believe the CB's announcements and stated intentions, due to the aforementioned time inconsistency. Again, if the CB is benevolent, P knows that the CB wants to convey precise information, correcting for the source of incredibility. Therefore, a more competent CB should again command more of P's attention, and be more credible.

Second, the "CNBC effect" (Morris and Shin 2002): when private agents have a coordination motive that depends on an unobserved state of the economy, and valuable private information on this state is dispersed across the population, then a public announcement that becomes common knowledge serves as a focal point for coordination of expectations and actions. Private agents place a suboptimal weight on their own private information, in favor of the commonly known public information, because they expect the other agents to do the same and to coordinate their actions accordingly. As a consequence, if the public announcement is imprecise relatively to private information, too much of the latter is ignored in formulating expectations and actions, so that the public announcement is socially harmful on average. Morris and Shin do not allow the public announcement to be withheld, but in our context this possibility becomes natural. In their linear Gaussian model, that shares several points of contact with the present model, a CB would always choose a bang-bang solution, either full communication or no communication, depending on the precision of the information in question (see also Hellwig 2002 for a similar conclusion). However, this particular effect, unlike the general idea behind the CNBC effect, appears to be model-specific. It is not too difficult to imagine a different setup where the CB would be happiest if she could make her announcement just of the right precision, unless of course it were already almost perfectly accurate. That is, the bang-bang solution would involve two positive values of precision.

It is then plausible that a relatively incompetent CB, one that is moderately confident in her information, would add noise to her announcements and engage in "opaque" communication, again in the style of the cheap-talk equilibrium analyzed in this paper. Conversely, a very competent CB should be outspoken, because the advantages of the information transmission exceed the harm of expectation coordination on the possibly incorrect outcome. So, again competence and transparency should go hand in hand, at least over some range.

6.2. Reputation for Competence and Reputation for Conservatism.

Prendergast and Stole (1996) first analyzed the effects of a concern for a reputation for competence on costly behavior, and Morris (1997) on cheap talk, while Morris (2001) focuses on the effects of a reputation for bias on cheap talk. In ongoing research, we also allow the CB competence and bias to be private information. Therefore, the CB decision to commit or not to a rule (to resist proposed incentives schemes or delegation) may convey information about the CB's competence and bias, and impact on private sector's expectations about policy and the credibility of announcements. Choosing discretion may be seen not as a sign of bias or lack thereof, but rather as an expression of competence and of the resulting value attached to flexibility. Discretion is not necessarily punished by P with high inflationary expectations, because the CB's motives to choose discretion are multiple.

In this light, the ECB's rigidity, relative to the FED, can be due either to the need to convince the private sector that the ECB did not inherit the historical bias of some of its members, or to its "incompetence" relatively to its formidable task. We might never know the true reason. Conversely, the FED's activism and effectiveness of communication in shaping expectations may be explained with its reputation for competence acquired during Greenspan's tenure, which swamps any doubt on possible motives for discretion stemming from bias, thus any reason to commit.

6.3. Central Bank's Over/Underconfidence

Suppose that the CB believes to be more competent than P truly considers her to be. That is, the CB is relatively overconfident in her own ability, and that the two players disagree on this key parameter H, and know to disagree. This is an interesting possibility in light of the natural tendency of human beings toward overconfidence in their own ability. Fang and Moscarini (2002) discuss the lack of common knowledge arising from overconfidence. Then we can still solve the game in a similar manner, and *the CB's overconfidence has a beneficial effect on communication and welfare.* The reason is that the CB's is benevolent, and her incentives to truthful communication depend on her own perceived degree of competence. A confident CB is self-disciplined, even if she thinks that P incorrectly (in the CB's view) disagrees with her assessment, she acts paternalistically and chooses a transparent approach. In turn, P believes her because he knows that the CB thinks to be right.

Conversely, suppose that competence is CB's private information, and is learned by P over time from monetary policy performance. If the CB gets lucky early on, then she enjoys what she knows to be an excessive reputation for competence, and becomes relatively underconfident. Nonetheless, due to the beneficial effects of (reputation for) competence on credibility, the CB has no reason to reveal her incompetence. This result stands in contrast to that in Morris and Shin (2002), where a benevolent policy-maker who knows to possess valuable but imprecise information would prefer not to announce it publicly. When both bias and CNBC effect are present, a CB would have to trade off the two effects in relaying to P her confidence in her own forecasts.

7. Conclusions

Communication and transparency are taking centerstage in the analysis and the practice of monetary policy. We analyze a simple setting where a central bank has an incentive to communicate to the private sector her intentions, based on her own reading of the state of the economy. Communication finds an impediment in a classic inflation bias, giving rise to time inconsistency. We uncover two separate beneficial effects of the competence of a central bank at doing her job on the credibility and transparency of her monetary policy.

A. Appendix. More Foundations of the Model

Consider the following economy. Supply is described by a Lucas supply curve

$$y^s = s\left(p - p^e\right)$$

where y^s is the log of real output, p is the log of the price level and p_e is the log of the expected price level. Demand is described by a constant-velocity quantity equation

$$y^d = m - p$$

where m is the log of the money stock, directly controlled by the CB. The equilibrium price level and output as a function of expectations are:

$$p = \frac{m + sp^e}{1+s}$$

$$\mathsf{y} = \frac{s}{1+s}\left(m - p^e\right).$$

Taking derivatives w.r. to time of the demand equation

$$\mu = \pi + y$$

and of equilibrium output

$$y = \frac{s}{1+s}\left(\mu - x\right) = \frac{s}{1+s}\left(\pi + y - x\right) = s\left(\pi - x\right)$$

we obtain the Phillips curve used in the paper. Once the CB knows (or anticipates) x, she may control equally well μ or π (and thus y).

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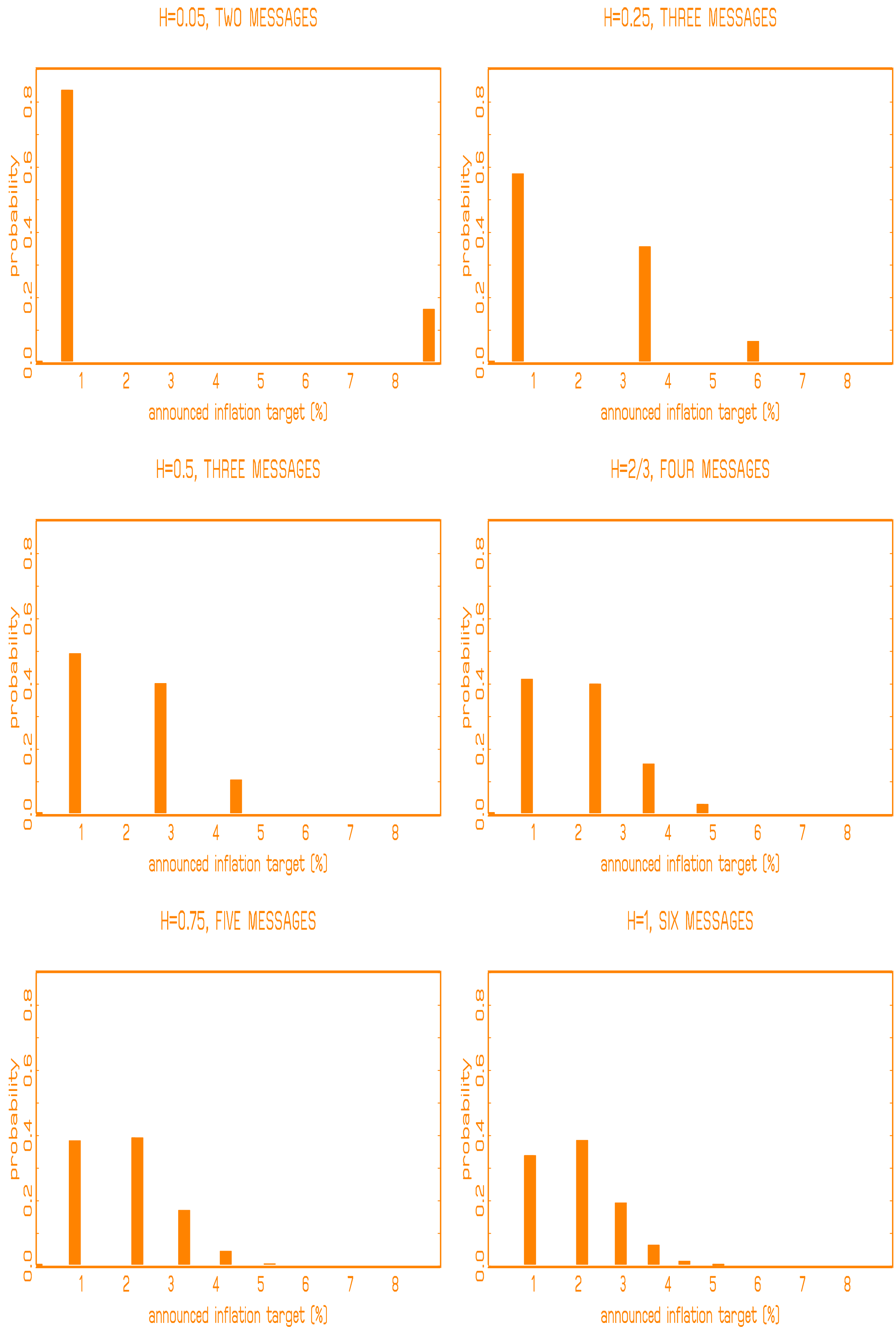
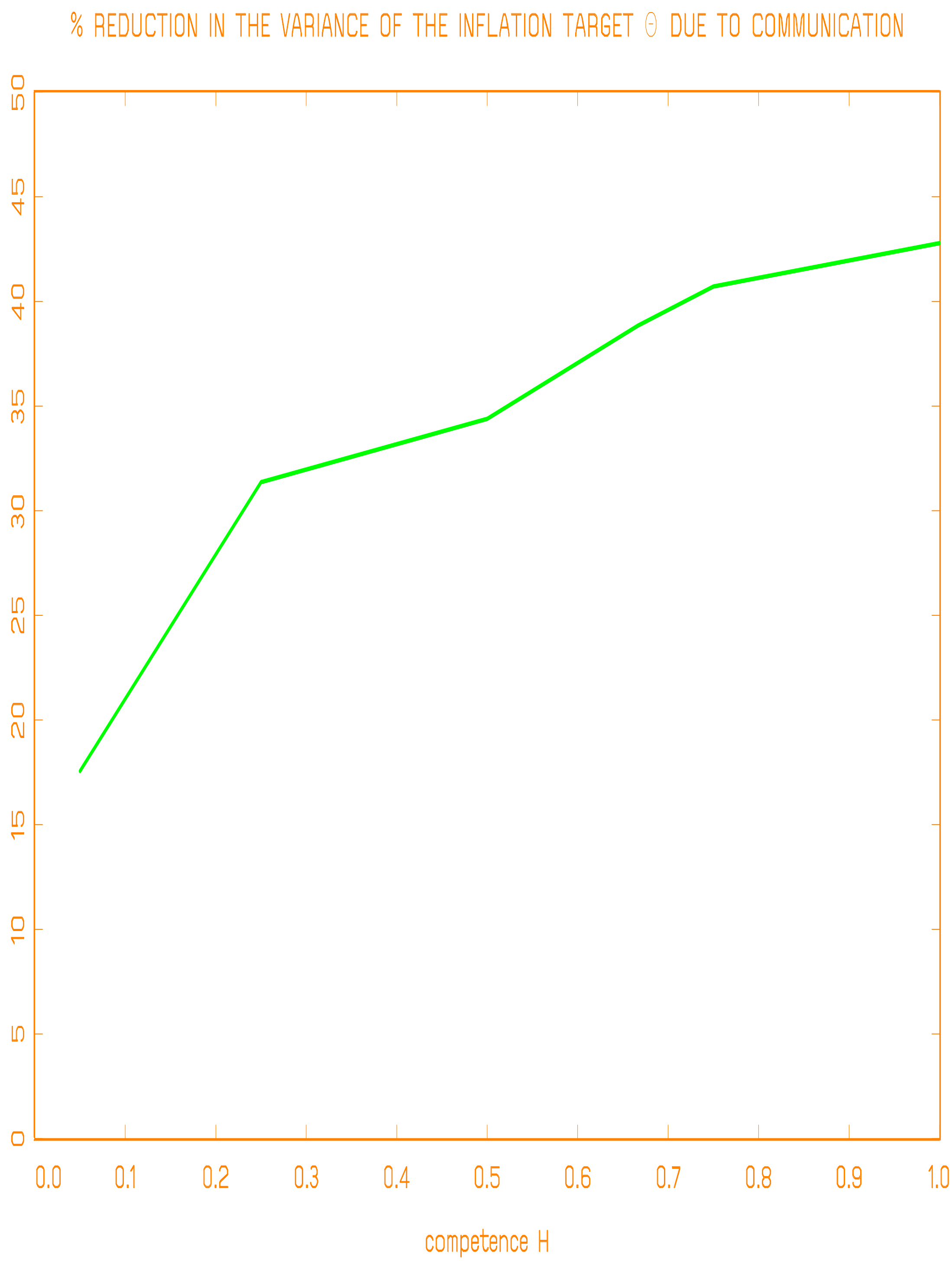


Figure A.1: Examples of equilibrium (finest) message space for different values of competence H. In each graph the private signal θ is $\mathbb{N}(0, 1/H)$. Each graph iluustrates the reports $\bar{\theta}$ of θ that the CB can be credibly announce to P in equilibrium, and the unconditional probability of each announcement.



% REDUCTION IN THE VARIANCE OF THE INFLATION TARGET Θ DUE TO COMMUNICATION
0
5
10
15
20
25
30
35
40
45
50
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
competence H

Figure A.2: