Asymmetric Information and Employment Fluctuations

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Abstract

Shimer (2005) showed that a standard search and matching model of the labor market fails to generate fluctuations of unemployment and vacancies of the magnitude observed in US data in response to shocks to average labor productivity of plausible magnitude. He also suggested that wage determination through Nash bargaining may be the culprit.

In this paper we pursue two objectives. First we identify those properties of Nash bargaining that limit the ability of the model to generate a large response of unemployment and vacancies to a shock to average labor productivity. Second, we examine whether two classic models of wage determination share these properties of Nash bargaining. Asymmetric information has been suggested as a route of escaping the tight limits on labor fluctuations associated with Nash bargaining. Thus, we assume that the firm has private information about the job's productivity, the worker about the amenity of the job, and aggregate labor productivity shocks do not change the distribution of private information around their mean. In this environment, we consider the monopoly (or monopsony) solution, namely a take-it-or-leave-it offer, and the constrained efficient allocation. We find that the properties of wage determination that limit unemployment fluctuations are satisfied for the first model essentially under all circumstances. They frequently (for commonly used specific distributions of beliefs) also apply to the constrained efficient allocation. Essentially, all of these solutions imply that the worker loses no surplus when productivity rises. The high empirical volatility of the job finding rate then makes the outside option of the worker also comove strongly with productivity, so as to absorb most of its beneficial effects on firm's incentives to create jobs.

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1 Introduction

The search-and-matching framework (Pissarides (2000)) is the workhorse of the analysis of aggregate labor markets and an important component of many quantitative business cycle models. Shimer (2005) recently pointed out that a plausible calibration of a baseline, representative agent version of the search and matching model driven by labor productivity shocks of plausible magnitude and persistence grossly fails to account for the observed volatility of unemployment and vacancies. Therefore, in spite of its many successes, to explain business cycles the search-and-matching model fares no better than a simple demand-and-supply representative-agent competitive model of the labor market.

Shimer suggests that the weakness of the search-and-matching model may lie in the assumption of wage determination by Nash bargaining. In response, some authors (e.g. Hall (2005), Hall and Milgrom (2005)) have considered alternatives to Nash bargaining that produce a larger response of unemployment and vacancies to labor productivity shocks. Other authors have taken alternative routes and introduced on-the-job search and heterogeneity of either firms (Krause and Lubik (2004)) or workers (Nagypal (2004)).

In this paper we focus again on wage determination, but we address the problem from the opposite angle. We investigate the extent to which the failure of the model generalizes to other models of wage determination beyond Nash bargaining. Our analysis proceeds in two steps. First, we ask: what is wrong with Nash bargaining? That is, we identify the qualitative features of Nash bargaining that limit the ability of the model to produce large fluctuations in unemployment and vacancies in response to shocks to average labor productivity. Second, we examine several classic models of wage determination and ask whether they share those properties, and thus are equally incapable of producing the large labir market fluctuations that we observe in US data.

Asymmetric information has been repeatedly suggested as a natural direction to escape the tight limits on fluctuations associated with Nash bargaining. We follow this lead, and we extend the basic model to allow for (potentially bilateral) asymmetric information. In particular, we assume that, upon being matched, the firm draws a match specific productivity and the worker draws a match specific amenity value of the job. We allow for the possibility that the former is private information of the firm and the latter is private information of the worker. This innovation raises a new issue. With heterogenous productivity, a given increase in average labor productivity can come about through various changes in the (belief) distribution of productivity across jobs. For example, an increase in average labor productivity could be associated with more or less dispersion in productivity. Kennan (2005) provides an example of substantial amplification through such interactions. We ask whether introducing asymmetric information can provide amplification without interactions between average labor productivity and the distribution of private information. Thus we assume that a shock to average labor productivity does not alter the distribution of productivity around its mean, as well as the distribution of worker's amenity from the job.

In pursuing our first objective−identifying the features of Nash bargaining that are responsible for the limits on labor market fluctuations−we take a methodological shortcut. A fully specified dynamic model of wage determination can be simulated, to compare the predicted volatility of unemployment and vacancies to the volatility of average labor productivity. This is the exercise that Shimer (2005) performs for Nash bargaining. However, as a preliminary exercise, he computes the elasticity of the steady state v/u ratio (ratio of vacancies to unemployment) to a permanent shock to average labor productivity. His results suggest that this exercise provides a remarkably close approximation to the relative volatilities obtained from the dynamic simulations. It appears that the quality of this approximation is related to the high persistence of average labor productivity and the rapid transitional dynamics of the search-and-matching model. We do not want to fully specify a model of wage determination that one could then subject to simulations. Rather, we only uncover qualitative properties. Thus, we will use the shortcut of focussing on the elasticity of the steady state v/u ratio with respect to average labor productivity.

Specifically, we show that in any a model of wage determination which shares with Nash bargaining a certain set of qualitative properties, the steady state elasticity of the v/u ratio with respect to a shock to average labor productivity must be less than an upper bound of the form

$$\left(\frac{\text{average productivity (or average wage)}}{\text{average productivity (or average wage)} - \text{flow utl. of non market activity}}\right)$$
$$\times \text{ function(parameters not related to wage determination, data)}$$

This is the product of two terms. The first term measures the inverse of the gains from market activity. In a recent paper Hagedorn and Manovskii (2005) have made the case that these gains are tiny, so this term should be calibrated to be perhaps as large as 20, in which case even the model with Nash bargaining could deliver satisfactory fluctuations in unemployment and vacancies driven by shocks to average labor productivity. If this term is indeed large, then our bounds will not be tight, and indeed unemployment is almost equivalent to employment, so not worth studying. However, other calibrations such as Shimer's assign to this term a much lower value, between 1 and 2. If one prefers the latter calibration, then it becomes crucial whether the second factor of the bound is large. The second term, that we will occasionally refer to as the *multiplier*, is small in any model of wage determination that shares certain properties with Nash bargaining. Importantly, for all models in this class, the multiplier only depends on parameters of the model not associated with wage determination (such as the matching function, the interest rate and the rate of exogenous separations), and on some empirical magnitudes that the model is usually calibrated to match (such as the job finding rate).

While simulating a specific model is neither difficult nor costly, it does require choosing some distribution of private information. Our general approach reveals the key properties of wage determination that mute the response of aggregate employment statistics to productivity shocks, a theme that has dominated quantitative macroeconomics for more than two decades. Our key finding is as follows. Assume that wage determination is such that firms' profits comove positively with productivity. As long as workers expect to receive positive rents from new matches, the cyclical variations in the job finding rate observed in the data are sufficiently strong to make the value of search, the worker's outside option, also very volatile. This volatility is sufficient to absorb a large part of productivity shocks and leaves no room for the large variations in profits that are necessary to rationalize the observed volatility of job creation. The key point is that this is true even if the worker's net gain from employment (share of match surplus) is totally acyclical in absolute value, i.e. if gaining employment yields the same returns at any point in time. In order to generate strong profit fluctuations, the worker's net gain must be countercyclical. With Nash bargaining, it is procyclical, and the model fails.

After deriving these bounds on the elasticity of the v/u ratio with respect to shocks to average labor productivity, we verify whether they apply to two classic wage determination models under asymmetric information: (i) the monopoly (or monopsony) solution, where either the firm or the worker makes a take-it-or-leave-it wage proposal to the other privately informed party; (ii) the constrained efficient allocation obtained with the help of a mediator (e.g. an arbitrator in wage contracting), as in Myerson and Satterthwaite (1983).

In the case of monopoly our bounds apply under very weak assumptions, particularly in the firm offer case. Our analysis of the constrained efficient allocation is in progress. So far, we showed that the bounds apply to a model where the distribution of private information is the same for workers and firms. We also analyze in some detail the case of uniform distributions, the canonical example in the literature on two-sided asymmetric information, and an asymmetric example. From these applications, we draw the following conclusion. The properties of Nash bargaining that are responsible for the failure of the search model as a business cycle tool are fairly weak. In other words, for the purpose of business cycle analysis, Nash bargaining is an excellent approximation to a large class of wage determination mechanisms even in a more general environment.

In Section 2 we introduce the economy, in Section 3 we define our notion of a model of wage determination. We discuss Nash bargaining and define its properties that mute the response of the steady state v/u ratio to a permanent shock to average labor productivity. We also discuss some models of wage determination that have been shown to imply large fluctuations in unemployment and vacancies, and we illustrate which of these properties of Nash bargaining they violate. The bounds are derived in Section 4. We then consider whether these bounds apply to two classic models of wage determination in the presence of asymmetric information. Section 5 is devoted to monopoly, and 6 to the constrained efficient allocation. Section 7 reviews our results and concludes.

2 The Economy

We consider a search-and-matching model of the labor market à la Pissarides (1985). We extend it to allow for bilateral asymmetric information about match-specific values: the worker may ignore how much output she is producing for the firm, and the employer how much the worker likes the job.

The economy is populated by a measure 1 of workers and a much larger measure of firms. All agents are infinitely-lived, risk neutral and share the discount rate $r > 0$. Workers can either be employed or unemployed. An unemployed worker receives flow utility b and searches for a job. Employed workers receive endogenously determined wage payments from their employers and cannot search for other jobs. Firms can search for a worker by maintaining an open vacancy at flow cost c. Free entry implies that the value of an open vacancy is zero. Unemployed workers and vacancies are matched at rate $m(u, v)$ where m is a constant returns to scale matching function. Let $\theta \equiv \frac{v}{u}$ denote the vacancy/unemployment ratio. Then workers are matched at rate $f(\theta) \equiv m(1, \theta)$ and vacancies are matched at rate $q(\theta) \equiv m(1/\theta, 1)$.

Upon being matched, the worker draws a match specific amenity value z from the distribution F_Z and the firm draws a match specific productivity component y from the distribution F_Y. The draws are once and for all until the match dissolves. Without loss in generality, the two distributions have mean zero. Output of the match is given by

$p + y$, so p is ex ante average labor productivity. However, in general, not all matches are formed and p will not equal labor productivity averaged across existing matches. We will refer to p as the aggregate component of labor productivity. The amenity value z adds to the wage to determine the flow value of employment for the worker. This value z may be private information of the worker, and the idiosyncratic productivity component y may be private information of the firm. Matches are destroyed exogenously at rate δ.

Shimer (2005) considers the complete information version of this model. He simulates the dynamics of the economy driven by a first order Markov process for labor productivity p. He shows that fluctuations in p of plausible magnitude cannot generate observed business-cycle-frequency fluctuations in unemployment and vacancies if wages are determined by Nash bargaining (from now on: NB). As a preliminary exercise, he computes the steady state of the model for constant labor productivity p, and examines the comparative statics of the model with respect to p. In particular, he computes the elasticity of the v/u ratio with respect to labor productivity p under the assumption that wages are determined by NB. He argues that this elasticity is small for plausible parameter values. In this paper we focus on the latter exercise. We argue that this elasticity is small for plausible parameter values not only for NB, but for a much larger class of models of wage determination that share some of the properties of NB. We conjecture that models in which this comparative statics elasticity is small will also be unable to generate substantial fluctuations in simulations with a stochastic process for labor productivity. An accurate quantitative evaluation of the full stochastic effects of aggregate shocks requires specifying the wage-setting rule, while we are mainly concerned with the implications of a broad class of such rules.

3 Models of Wage Determination

We think of a model of wage determination as pinning down the value of the match and how it is split between the worker and the firm. We are interested in the general equilibrium effects of changes in productivity p on the division of rents and, consequently, on unemployment. Each match takes the outside options, the utility of unemployment U for the worker and zero for the firm by free entry, as given, and internalizes the direct effects of changes in p on the rents. In equilibrium, the outside option U also changes, and we capture this effect through the flow value $n = rU$.

We allow the outcome of wage determination to depend on the aggregate component of labor productivity p, the flow value of unemployment n, and the match specific values

y and z. Let $W(y, z, p, n)$ denote the value of employment to the worker given the flow outside option n, $G(y, z, p, n) = W(y, z, p, n) - U$ the capital gain from the job obtained by the worker, and $J(y, z, p, n)$ the corresponding capital gain for the firm (which is the value of the job, since the outside option of the firm is zero). These values are conditional on private information draws y, z, that is, on trade (on the match forming). Let $x(y, z, p, n)$ be the probability that the match is formed given an outcome y, z. Then we can define the unconditional counterparts, namely, the ex ante chance of trading and the expected gains from trade to workers and firms, taking into account the possibility that the match will not form:

$$\begin{aligned}
\xi(p, n) &\equiv \iint x(y, z, p, n) dF_Y(y) dF_Z(z), \\
\mathcal{G}(p, n) &\equiv \iint G(y, z, p, n) dF_Y(y) dF_Z(z), \\
\mathcal{J}(p, n) &\equiv \iint J(y, z, p, n) dF_Y(y) dF_Z(z).
\end{aligned} \tag{1}$$

A *model of wage determination* is then a a triple $\Omega = \{\mathcal{G}, \mathcal{J}, \xi\}$. We could define it in terms of conditional values, $\{G, J, x\}$, but our key properties will be in terms of objects in Ω. Notice that by adopting this formulation we implicitly assume that the outcome of the wage determination model is unique. Multiplicity of equilibria is one way that has been considered to escape the tight bounds on labor market fluctuations associated with NB (see the wage norm example below).

Our first objective is to identify those properties of NB that are responsible for the limited ability of the model to generate large fluctuations in unemployment and vacancies. The generalized NB solution selects a wage to maximize $G^\beta J^{1-\beta}$ for some $\beta \in [0, 1]$. As is standard, this implies that the total surplus $G + J$ is shared between the worker and the firm with shares β and $1 - \beta$, respectively: in flow terms

$$\begin{aligned}
G(y, z, p, n) &= x(y, z, p, n)\beta \frac{p - n + y + z}{r + \delta}, \\
J(y, z, p, n) &= x(y, z, p, n)(1 - \beta) \frac{p - n + y + z}{r + \delta}.
\end{aligned}$$

The probability of trade $x(y, z, p, n)$ is equal to one if the match has a positive surplus, otherwise it is zero:

$$x(y, z, p, n) = \mathbb{I}\{p - n + y + z \geq 0\} \tag{2}$$

where $\mathbb{I}$ is an indicator function. Notice that the functions G, J and x depend on p and n only through their difference $p - n$. Since y and z have mean zero, and the flow gains from trade are $p + y + z - n$, we can think of $p - n$ as the mean gains from trade. If

p and n increase by the same amount, this leaves the gains from trade unchanged, and only changes the location of the bargaining problem. With NB, a change in p and n that does not change the gains from trade also leaves the division of the gains from trade unchanged. Therefore, also $\mathcal{G}$, $\mathcal{J}$ and ξ depend only on $p-n$. This property motivates the first definition.

Definition 1 ***Location Invariance.*** *A model of wage determination $\Omega = \{G, J, x\}$ satisfies* Location Invariance *if the functions $\mathcal{G}$, $\mathcal{J}$ and ξ depend on p and n only through their difference $p-n$.*

Each of the upper bounds that we will derive in Section 4 requires this property. Indeed, some of the other properties that we will rely on are only defined for location invariant models of wage determination.

A feature of the trading rule (2) is that the probability of trade is non-decreasing in both y and z. That is, trade is more likely if the firm draws a high productivity or the worker draws a higher amenity value of the job. This suggests that existing matches are better than the average match draw. Specifically, for a location invariant model, define average

Definition 2 ***Positive Selection.*** *A location invariant model of wage determination $\Omega = \{\mathcal{G}, \mathcal{J}, x\}$ satisfies* Positive Selection *if the average match specific productivity and the average match specific amenity value z conditional on trade (observed among active jobs) exceed their unconditional counterparts, hence are non-negative*

$$\mathcal{Y}(p-n) \equiv \frac{\iint x(p,n,y,z) y dF_Y(y) dF_Z(z)}{\xi(p-n)} \geq 0 = \frac{\iint y dF_Y(y) dF_Z(z)}{\xi(p-n)}, \tag{3}$$

$$\mathcal{Z}(p-n) \equiv \frac{\iint x(p,n,y,z) z dF_Y(y) dF_Z(z)}{\xi(p-n)} \geq 0 = \frac{\iint z dF_Y(y) dF_Z(z)}{\xi(p-n)}. \tag{4}$$

In order to obtain bounds on an elasticity we need to take derivatives. So for each model of wage determination we will make sufficient assumptions (usually concerning smoothness of the distribution functions F_Z and F_Y) to guarantee that the functions $\xi(p-n)$, $\mathcal{G}(p-n)$ and $\mathcal{J}(p-n)$ are differentiable. For NB, one then obtains from the envelope theorem

$$\begin{aligned} (r+\delta)\,\mathcal{G}'(p-n) &= \beta\xi(p-n) \\ (r+\delta)\,\mathcal{J}'(p-n) &= (1-\beta)\xi(p-n). \end{aligned}$$

Since the trading decision is privately efficient, at the margin it is not affected by a change in $p-n$. Only the direct effect remains, which is to increase expected surplus by the fraction of matches where it is positive, namely $\xi(p-n)$. This property of NB motivates:

Definition 3 ***Increasing Gains From Trade.*** *A location invariant model of wage determination* $\Omega = \{\mathcal{G}, \mathcal{J}, x\}$ *satisfies Increasing Worker's (Firm's) Gains From Trade if* $\mathcal{G}' \geq 0$ *(*$\mathcal{J}' \geq 0$*).*

Definition 4 ***Regular Gains from Trade.*** *A location invariant model of wage determination* $\Omega = \{\mathcal{G}, \mathcal{J}, x\}$ *satisfies Regular Firm's Gains from Trade if* $(r+\delta)\mathcal{J}' \leq \xi$*.*

The wording "regular" refers to the following fact. In the NB model $(r+\delta)\left[\mathcal{G}'(p-n)+\mathcal{J}'(p-n)\right] = \xi(p-n)$, that is, total surplus rises with the flow gains from trade $p-n$ at a rate equal to the chance of trade. Then overall gains from trade are regular if, given the wage determination model, the firm's share of the surplus does not rise faster than the total surplus.

Before analyzing how these properties of NB are related to the limited ability of the model to generate large fluctuations in unemployment and vacancies, we discuss two examples of models of wage determination that have been suggested as a remedy of this limited ability and that violate some of the properties introduced above.

Example: Constant Wage. Consider the model that simply specifies a constant exogenous wage. If p and n increase by the same amount, the wage would have to move along in order for the split of the gains from trade to remain unchanged, so this model violates Location Invariance.

Example: Double Auction (Hall (2005)). Hall (2005) considers a more sophisticated model of wage determination with implications similar to a constant wage, namely a double auction. With symmetric information any split of the surplus is an equilibrium of the double auction. As p and n rise by the same amount, say Δ, the set of equilibria, an interval of the real line, also shifts up by the same Δ. So the productivity-wage wedge and the wage-outside option wedge for the same job are unchanged. So, in this sense there is Location Invariance, although strictly speaking the multiplicity of equilibria does not allow to apply its formal definition. However, the presence of multiplicity can be exploited to select different splits of the gains from trade for different values of p and n, even if overall gains from trade $p-n$ are the same. This is what Hall's equilibrium selection of a constant wage accomplishes.

Example: Outside Option Principle (Hall and Milgrom (2005)) Hall and Milgrom (2005) replace the standard NB assumption of the Mortensen-Pissarides model with the bargaining theory of Binmore, Rubinstein and Wolinsky (1986). According to this theory, the relevant threat point of the worker is not unemployment but delay of bargaining. Now suppose p and n increase by the same amount but the cost of delay to the worker remains unchanged (it does not fall one for one with the increase in n). Then the split of the gains from trade will not remain the same, so this model of wage determination fails Location Invariance, and in fact can generate large unemployment fluctuations in response to plausible productivity shocks.

4 Bounds on Labor Market Fluctuations

In this section we present four upper bounds on the the elasticity of the steady state v/u ratio with respect to the aggregate component of labor productivity p. As discussed in the Introduction, we focus on the steady state elasticity $\varepsilon_\theta = \theta_p p/\theta$, as previous research suggests that this yields a good approximation to the relative volatility of the v/u ratio and labor productivity obtained from dynamic simulations.

Whether a particular bound applies depends on whether the model of wage determination satisfies a corresponding set of the four properties discussed in the previous section (Definition 1-4). Location Invariance is always in the picture, so we will simplify notation by already using this property when writing the steady state conditions.

The steady state values of the two endogenous variables θ and n are determined by two equations. First, the free entry condition, equating the flow cost of posting a vacancy c to the expected capital gain, which is the rate $q(\theta)$ at which open vacancies receive applications, times the expected value to the firm of an acceptable job

$$c = q(\theta)\mathcal{J}(p-n). \tag{5}$$

Second, the Bellman equation determining the flow value of unemployment as the flow value of leisure b plus the expected capital gain:

$$n = b + f(\theta)\mathcal{G}(p-n) \tag{6}$$

where recall that $f(\theta)$ is the rate at which unemployed workers contact open vacancies. We log-differentiate the system of equations (5)–(6) and evaluate the derivatives equations at steady state values, indicated by bars. Let $\bar{n}_p$ be the derivative of the flow value of unemployment with respect to p, and $\eta = \bar{f}'(\bar{\theta})\bar{\theta}/\bar{f}(\bar{\theta})$ denote the elasticity of the matching function with respect to vacancies, both evaluated at the steady state. Then

$$(1-\eta)\varepsilon_\theta = \frac{\bar{\mathcal{J}}'(\bar{p}-\bar{n})}{\bar{\mathcal{J}}(\bar{p}-\bar{n})}(1-\bar{n}_p)\bar{p}, \tag{7}$$

$$\frac{\bar{n}_p\bar{p}}{\bar{n}-b} = \eta\varepsilon_\theta + \frac{\bar{\mathcal{G}}'(\bar{p}-\bar{n})}{\bar{\mathcal{G}}(\bar{p}-\bar{n})}(1-\bar{n}_p)\bar{p}. \tag{8}$$

Define the average payment that workers receive conditional on trade,

$$\bar{w} \equiv \frac{(r+\delta)\bar{\mathcal{G}}}{\bar{\xi}} + \bar{n} - \bar{\mathcal{Z}},$$

(notice that the average amenity value must be deducted from the average flow utility of an employed worker in order to obtain observable wage payments), the job finding rate

$$\bar{h} \equiv \bar{f}\bar{\xi},$$

the product of the matching rate $\bar{f}$ and the probability of match formation $\bar{\xi}$,and finally observed average labor productivity, the average of $\bar{p}+y$ conditional on trade

$$\bar{A} \equiv \bar{p} + \bar{\mathcal{Y}}.$$

Notice that Positive Selection implies $\bar{A} \geq \bar{p}$: since only relatively good matches are implemented, average labor productivity conditional on trade is higher than its unconditional counterpart. We use these equations and definitions to derive our bounds on the elasticity ε_θ of interest.

The First Bound. Combining equations (6) and (8) we obtain

$$1 - \bar{n}_p = \frac{1}{1+\bar{f}\bar{\mathcal{G}}'}\left[1 - \eta\varepsilon_\theta\frac{\bar{h}}{r+\delta+\bar{h}}\frac{(r+\delta)\bar{\mathcal{G}} + \bar{\xi}(\bar{n}-b)}{\bar{\xi}\bar{p}}\right] \tag{9}$$

The left hand side is the derivative of $p-n$ with respect to p evaluated at the steady state. If this derivative is negative, the worker's flow outside option n responds more than one for one to the increase in labor productivity p. Now consult equation (7). If the firm's gains from trade are increasing ($\mathcal{J}' \geq 0$) then a negative right hand side would imply $1-\bar{n}_p < 0$, an increase in average labor productivity p makes firms worse off, which is clearly inconsistent with a positive elasticity ε_θ. Hence, to maintain ε_θ as observed in the data, we assume that the wage determination model satisfies $\mathcal{J}' \geq 0$ and, from (7), $1-\bar{n}_p \geq 0$.

To derive a bound, assume further Increasing Worker's Gains from Trade $\bar{\mathcal{G}}' \geq 0$, so the term in square brackets in Equation (9) must be positive. Finally, Positive Selection implies that we can replace the unobservable magnitudes $\bar{p}$ and $(r+\delta)\bar{\mathcal{G}}/\bar{\xi} + \bar{n}$ with the observable magnitudes $\bar{A} \geq \bar{p}$ and $\bar{w} \leq (r+\delta)\bar{\mathcal{G}}/\bar{\xi} + \bar{n}$, respectively, to keep the term in square brackets positive. So we obtain:

Proposition 1 *If the model of wage determination satisfies (i) Location Invariance, (ii) Increasing Firm's Gains from Trade (iii) Increasing Worker's Gains from Trade and (iv) Positive Selection, then*

$$\varepsilon_\theta \leq \frac{\bar{w}}{\bar{w}-b}\frac{\bar{A}}{\bar{w}}\frac{1}{\eta}\frac{r+\delta+\bar{h}}{\bar{h}}. \tag{10}$$

This bound has the general structure illustrated in the Introduction, an inverse gains from market activity times a multiplier. Notice that even if the worker's gains from trade do not rise, $\mathcal{G}' = 0$, an increase in average labor productivity has a positive effect on the worker's outside option n, through the higher job-finding rate: $\bar{n}_p = \eta\varepsilon_\theta(\bar{n}-b)$. If the job finding rate responds strongly to labor market tightness (high η), labor market tightness responds strongly to productivity (high ε_θ), and the flow value of unemployment $\bar{n}$ is much larger that the flow utility b, then n will respond strongly to an increase in productivity. The strength of this effect depends on the wedge $\bar{n}-b$ between the flow outside options of the worker, endogenous $\bar{n}$ minus exogenous (value of leisure) b. In turn, from equation (6), $\bar{n}-b = \frac{\bar{h}}{r+\delta+\bar{h}}\frac{(r+\delta)\bar{\mathcal{G}}+\bar{\xi}(\bar{n}-b)}{\bar{\xi}\bar{p}}$, so this wedge is large (and the outside option is very sensitive to the job-finding rate) if on average employment is a lot better than non market activity and if the job finding rate is high.

To put a number on this bound, we follow the calibration of Shimer (2005). We take from him values for the exogenous parameters are r, δ and η (Panel A of Table 1). The model should match two empirical values, the job finding rate $\bar{h}$ and the average productivity to average wage ratio $\bar{A}/\bar{w}$. Panel B of the table reports the value 1.35 of the job finding rate found by Shimer for US data. We have not yet constructed a careful empirical analog of the "markup" $\bar{A}/\bar{w}$. To be on the safe side, we pick a value of 1.2. Panel C of the table computes the resulting components of the bound in Proposition 1. The term $\frac{\bar{A}}{\bar{w}}\frac{1}{\eta}\frac{r+\delta+\bar{h}}{f\bar{h}}$ takes the value 4.64. Since both $\bar{A}/\bar{w}$ and $\frac{r+\delta+\bar{h}}{\bar{h}}$ are not much larger than one, the magnitude of this term is mainly due to η^{-1}.

If b is set to 40 percent of the wage, then the overall upper bound on the elasticity ε_θ equals 7.74. This value is very sensitive to the elasticity of matching to job creation η. In particular for $\eta = 0.5$ the bound drops to 4.33. Contrast these values with Shimer's finding that, in the US, the v/u ratio is roughly 20 times as volatile as average labor

Table 1

A.	Parameter	Value
	r	0.012
	δ	0.1
	η	0.28

B.	Data	Value
	$\bar{h}$	1.35
	$\frac{\bar{A}}{\bar{w}}$	1.2

C.	Bound Factor	Value
	$\frac{1}{\eta}$	3.57
	$\frac{r+\delta+\bar{h}}{\bar{h}}$	1.08
	$\frac{1}{\eta}\frac{r+\delta+\bar{h}}{\bar{h}}$	3.87
	$\frac{\bar{A}}{\bar{w}}\frac{1}{\eta}\frac{r+\delta+\bar{h}}{\bar{h}}$	4.64
	$\frac{1}{1-\eta}$	1.39

productivity. The bound is not very sensitive to $r + \delta$ nor $\bar{A}/\bar{w}$, so relaxing the bound towards the desired empirical target of 20 requires a much lower value of η than the 0.28 chosen by Shimer, or lower gains for workers from market activity $\bar{w} - b$. We should add that $\eta = 0.28$ is at the lower end of the range of estimates in the literature (see Petrongolo and Pissarides (2001) for a survey. "Looking into the Black Box: A Survey of the Matching Function". Petrongolo, Barbara; Pissarides, Christopher A, Journal of Economic Literature, vol. 39(2), June 2001, pp. 390-431).

The Second Bound. Equation (7) can also be solved explicitly for a specific positive value of the elasticity ε_θ. Substituting equation (9) into equation (7) yields

$$\varepsilon_\theta = \frac{\bar{\mathcal{K}}}{\bar{\mathcal{K}} - b}\frac{p}{\bar{\mathcal{K}}} \cdot \frac{1}{\frac{\bar{\xi}\left(1+\bar{f}\bar{\mathcal{G}}'\right)}{(r+\delta)\bar{\mathcal{J}}'}(1-\bar{\beta})\frac{1}{\frac{1}{1-\eta}} + \bar{\beta}\frac{1}{\frac{1}{\eta}\frac{r+\delta+\bar{\xi}\bar{f}}{\bar{\xi}\bar{f}}}} \tag{11}$$

where

$$\bar{\mathcal{K}} \equiv \frac{(r+\delta)\left[\bar{\mathcal{J}} + \bar{\mathcal{G}}\right] + \bar{\xi}\bar{n}}{\bar{\xi}} \tag{12}$$

is the mean of $p + y + z$ conditional on trade, a measure of average labor productivity that includes the amenity value of the job to the worker, and

$$\bar{\beta} \equiv \frac{(r+\delta)\bar{\mathcal{G}} + \bar{\xi} \cdot (\bar{n} - b)}{\bar{\xi}(\bar{\mathcal{K}} - b)} = 1 - \frac{(r+\delta)\bar{\mathcal{J}}}{\bar{\xi}(\bar{\mathcal{K}} - b)}$$

is the share of the flow gain from market activity $\bar{\mathcal{K}} - b$ that goes to the worker.

Proposition 2 *If the model of wage determination satisfies (i) Location Invariance, (ii) Increasing Firm's and Worker's Gains from Trade, (iii) Positive Selection and (iv) Regular Firm's Gains from Trade, then*

$$\varepsilon_\theta \leq \frac{\bar{A}}{\bar{A} - b} \max \left\langle \frac{1}{1-\eta}, \frac{1}{\eta} \frac{r + \delta + \bar{h}}{\bar{h}} \right\rangle. \tag{13}$$

Proof.

$$\begin{aligned}
\varepsilon_\theta &\leq \frac{\bar{\mathcal{K}}}{\bar{\mathcal{K}} - b} \frac{1}{\frac{\bar{\xi}\left(1+\bar{f}\bar{\mathcal{G}}'\right)}{(r+\delta)\bar{\mathcal{J}}'}\left(1-\bar{\beta}\right)\frac{1}{\frac{1}{1-\eta}} + \bar{\beta}\frac{1}{\frac{1}{\eta}\frac{r+\delta+\bar{\xi}\bar{f}}{\bar{\xi}\bar{f}}}} \\
&\leq \frac{\bar{\mathcal{K}}}{\bar{\mathcal{K}} - b} \frac{1}{\left(1-\bar{\beta}\right)\left(\frac{1}{1-\eta}\right)^{-1} + \bar{\beta}\frac{1}{\frac{1}{\eta}\frac{r+\delta+\bar{\xi}\bar{f}}{\bar{\xi}\bar{f}}}} \\
&\leq \frac{\bar{A}}{\bar{A} - b} \left\{ \left(1-\bar{\beta}\right)\left(\frac{1}{1-\eta}\right)^{-1} + \bar{\beta}\left(\frac{1}{\eta}\frac{r+\delta+\bar{\xi}\bar{f}}{\bar{\xi}\bar{f}}\right)^{-1} \right\}^{-1}
\end{aligned}$$

where the first line follows from (11) and Positive Selection ($\bar{\mathcal{K}} \geq p$), the second line from Increasing Worker's Gains from Trade ($1 + \bar{f}\bar{\mathcal{G}}' \geq 1$) and from Increasing and Regular Firm's Gains from Trade ($0 \leq (r+\delta)\bar{\mathcal{J}}' \leq \bar{\xi}$), the third line from Positive Selection again, which allows us to replace the unobservable quantity $\bar{\mathcal{K}}$ with the observable average labor productivity $\bar{A} \leq \bar{\mathcal{K}}$.$\blacksquare$

Notice the structure of Equation (11): up to the "mark-up" factor $\frac{\bar{\mathcal{K}}}{\bar{\mathcal{K}}-b}$, the multiplier is almost the harmonic weighted average of the two terms $\frac{1}{1-\eta}$ and $\frac{1}{\eta}\frac{r+\delta+\bar{h}}{\bar{h}}$, with weights equal to the shares $\bar{\beta}, 1-\bar{\beta}$. The second term is familiar from the bound of Proposition 1, and a low value of this term is associated with a low value of the elasticity ε_θ for the reasons discussed earlier. The first term of the average $(1-\eta)^{-1}$ captures congestion effects. A low η implies that an increase in labor market tightness has a strong negative effect on the rate at which vacancies are matched with workers. Holding constant the increase in the value of a match to the firm due to the increase in productivity, labor market tightness cannot respond much if vacancy congestion is severe.

If all the gains from market activity go to the firm ($\bar{\beta} \to 0$), then finding a job entails no capital gain for the worker, and consequently an increase in the job finding rate does not help her outside option. In this case, only congestion effects limit the value of the elasticity ε_θ. At the other extreme, if firms receive only very little of the gains from market activity ($\bar{\beta} \to 1$), a given absolute increase in the firm's gains from trade will be very large in percentage terms, so the scope for an increase in labor market tightness is large even if congestion effects are strong. Notice that a large discount factor $r+\delta$ makes firm's returns even smaller in absolute terms, and their increase even larger in percentage terms. In this case, vacancy congestion $(1-\eta)^{-1}$ is not an important limiting factor for the magnitude of ε_θ. Finally, a high response of the worker's value (high $\bar{\mathcal{G}}'$) must occur at least in part the expense of the firm, amplifying the importance of congestion. A similar effect obviously stems from a low $\bar{\mathcal{J}}'$.

In contrast to the first bound (Proposition 1), which of course still applies, this second bound also has to reckon with congestion effects. If congestion effects are strong so that the maximum operator in (13) yields $(1-\eta)^{-1}$, the second bound may be less tight than the previous one. This is not the case for the parameter values of Table 1 since $(1-\eta)^{-1} = 1.39$ is much smaller than $\frac{1}{\eta}\frac{r+\delta+\bar{h}}{\bar{h}} = 4.64$, so the second bound (when it does apply) significantly sharpens the first one. Once again, this is mainly due to the low value of the elasticity of matching to vacancy, $\eta = 0.28$. For $\eta = 0.5$ the two numbers are much closer, at 2 and 2.17, respectively. Notice also from Table 1 that $r + \delta << \bar{h}$, so the second bound is approximately

$$\varepsilon_\theta \leq \frac{\bar{A}}{\bar{A}-b} \max \left\langle \frac{1}{1-\eta}, \frac{1}{\eta} \right\rangle$$

and the "multiplier" is determined uniquely by congestion effects on either side of the market. The bound is tightest when the two effects are equal, at $\eta = 0.5$.

Example: Nash bargaining without heterogeneity Without heterogeneity, $\bar{\mathcal{K}} = \bar{p}$, so our second bound in Proposition 2 is $\varepsilon_\theta \leq \frac{\bar{p}}{\bar{p}-b} \max \left\langle \frac{1}{1-\eta}, \frac{1}{\eta}\frac{r+\delta+\bar{h}}{\bar{h}} \right\rangle$. For the baseline model with NB wages and without heterogeneity, Shimer (2005) calculates the elasticity

$$\varepsilon_{\theta,NB} = \frac{\bar{p}}{\bar{p}-b} \frac{r+\delta+\bar{h}\beta}{(r+\delta)(1-\eta)+\bar{h}\beta} \tag{14}$$

Setting the worker's NB share β to zero yields $\varepsilon_{\theta,NB} = \bar{p}(\bar{p}-b)^{-1}(1-\eta)^{-1}$. The reason is that with $\beta = 0$ workers do not participate in the gains from market activity, so as discussed above only congestion effects limit labor market fluctuations. This is the best-case scenario for amplification with NB. Hence, with $\eta = 0.5$ our first two bounds are essentially as tight as what can be attained with NB, and models of wage determination that satisfy the properties used in Propositions 1 and 2 cannot deliver substantially larger labor market fluctuations than NB.

The Third Bound. While the second bound is more appealing than the first one, the additional properties that it requires may be restrictive. As we shall see in the case of the constrained efficient allocation considered in Section 6, this is particularly true for the property of Regular Firm's Gains from Trade. The latter condition can be dispensed with in the special case of symmetry.

Definition 5 ***Symmetry.*** *A model of wage determination* $\Omega = \{\mathcal{G}, \mathcal{J}, x\}$ *is Symmetric if* $\mathcal{G} = \mathcal{J}$.

Proposition 3 *If the model of wage determination satisfies (i) Location Invariance, (ii) Increasing Firm's Gains from Trade, (iii) Positive Selection and (iv) Symmetry, then*

$$\varepsilon_\theta \leq \frac{\bar{A}}{\bar{A} - b} \frac{1}{\bar{h}} \max \left\langle \frac{r+\delta}{1-\eta}, \frac{r+\delta+\bar{h}}{\eta} \right\rangle \tag{15}$$

Proof. The proof is analogous to that of Proposition 3, noting that under symmetry and increasing gain from trade

$$\bar{\xi} \frac{1+\bar{f}\bar{\mathcal{G}}'}{(r+\delta)\bar{\mathcal{J}}'} = \bar{\xi} \frac{1+\bar{f}\bar{\mathcal{G}}'}{(r+\delta)\bar{\mathcal{G}}'} \geq \bar{\xi} \frac{\bar{f}}{r+\delta} = \frac{\bar{h}}{r+\delta}. \blacksquare \tag{16}$$

It is immediate to verify that the multiplier of the third bound is usually pinned down by the second term in the maximum, as $\bar{h} >> r+\delta$ and η is not too close to either 0 or 1. That is, for plausible parameter values, in the symmetric case what really binds is the magnitude of the firm's steady state profits and not vacancy congestion.

Heterogeneity. We conclude this section on upper bounds by turning to an issue that we have glossed over so far. In Shimer's (2005) setup matches are homogenous, so p is average labor productivity. Thus ε_θ is the elasticity of the v/u ratio with respect to average labor productivity, the appropriate comparative statics counterpart of the empirical values of relative standard deviations of the v/u ratio and average labor productivity. The fact that in Shimer's setup the elasticity also provides a good quantitative approximation of the relative standard deviation is our justification for studying upper bounds on this elasticity.

However, in our setup with heterogeneity p is the ex ante, not the ex post, average labor productivity. The actual average productivity $\bar{A}$ is endogenous, due to selection. Thus the appropriate comparative statics counterpart for the relative standard deviation—in the sense of relating the same economic concepts—is not ε_θ, but rather the ratio between ε_θ and the elasticity $\varepsilon_{\bar{A}}$ of average labor productivity $\bar{A}$ with respect to p. The bounds on ε_θ that we have obtained apply, strenghtened, to $\varepsilon_\theta/\varepsilon_{\bar{A}}$ if $\varepsilon_{\bar{A}} \geq 1$, or, $d\bar{A}/dp \geq \bar{A}/p$. Notice that, by positive selection, $\bar{A}/p = 1 + \mathcal{Y}/p \geq 1$. This means that, when aggregate productivity is higher, the quality of implemented new matches must not worsen, and in fact improve sufficiently. This is typically not the case in all the models of wage determination that we analyze.

One observation, however, soothes this problem. When p changes by a small Δp, the change in labor productivity that we observe in the data is equal to Δp for existing matches, where selection has already taken place, and to $\Delta \bar{A}$ for new matches. So the total change in average labor productivity is a weighted average of the two. Since the overwhelming majority of jobs that are active at each point in time in the US economy existed before this quarter, this weighted average is dominated by Δp, thus our bounds should be appropriate.

This discussion suggests an alternative avenue to resolve the shortcoming of the search model as a tool of analysis of business cycles. The existing literature uniformly assumes that labor productivity shocks affect all jobs, pre-existing and new. But the model tells us that job creation is driven only by the productivity of new jobs. If, for some reason, existing jobs's productivity does not change, and all movement is at the margin, as in a vintage model, a 2% change in empirical labor productivity implies a many-fold change in the productivity of new jobs. More generally, a strong procyclicality in the quality of new matches, relative to the existing ones (as for example in Moscarini (2001)'s Roy model with search frictions), could be enough to explain the empirically observed fluctuations in average productivity and in unemployment. Brügemann (2005) explores this avenue.

Having found several bounds, our strategy consists of going through some particularly interesting extensive forms of the bargaining game, specifically, the monopoly solution, and the efficient mechanism. For each extensive form, we verify whether and under what conditions the equilibrium is unique and satisfies the assumptions of one of our earlier Propositions.

5 Monopoly

In this section we consider the game in which the privately informed party makes a take-it-or-leave-it offer to the uninformed party. If accepted, the offer is binding for both parties until exogenous separation. This game has a unique equilibrium, which is constrained ex ante efficient in the sense that the offer-making party's welfare cannot be improved further given information asymmetry (Satterthwaite and Williams (1989)). This equilibrium does not, however, maximize ex ante gains from trade, due to the monopoly distortion. We analyze separately the two cases of unilateral wage offer by the firm and wage request by the worker, because the properties used to derive the second bound are not symmetric for firms and workers.

5.1 Unilateral Wage Offer by the Firm

The Optimal Wage Offer. Consider a firm of type y. If it offers a wage w_M, then the worker is indifferent between taking the job and staying unemployed if his amenity value is $z_M = n - w_M$. Thus the offer is accepted for amenity values $z \geq z_M$. One can equivalently think of the firm choosing the threshold z_M or the wage w_M, and adopting the former approach the objective of of the firm is to maximize

$$[1 - F_Z(z_M)](p - n + y + z_M). \tag{17}$$

The second term is the payoff of the firm after paying $w_M = n - z_M$, and the first term is the probability of trade. The first order condition is

$$p - n + y + z_M = \frac{1 - F_Z(z_M)}{F_Z'(z_M)}. \tag{18}$$

The left hand side is the gain from trading with an additional worker. However, if the firm wants to trade with more workers, it has to pay higher informational rents to the workers (types, values of z) it is already trading with. The right hand side gives the number of workers that receive higher rents relative to the number of workers gained from reducing z_M.

We now introduce an assumption about private information that will allow us to verify all the properties in Definitions 1-4.

Assumption 1 *a. The distributions F_Y and F_Z have support $[\underline{y}, \bar{y}]$ and $[\underline{z}, \bar{z}]$, respectively, with $\underline{y}$, $\underline{z} \in \mathbb{R} \cup \{-\infty\}$ and $\bar{y}$, $\bar{z} \in \mathbb{R} \cup \{+\infty\}$.*

b. The "virtual valuations" $y - \frac{1-F_Y(y)}{F_Y'(y)}$ and $z - \frac{1-F_Y(z)}{F_Y'(z)}$ are strictly increasing and continuously differentiable on $[\underline{y}, \bar{y}]$ and $[\underline{z}, \bar{z}]$, respectively.

We allow for finite lower and upper bounds. Thus the solution to the firm's problem could be at a corner, and one may expect that corner solutions may generate sufficient wage rigidity to escape the bounds. We will show that this is not the case. Part (b) of the assumption insures that if the first order condition has an interior solution, it is unique, differentiable, and the global maximizer. Let $z_M(p - n + y)$ denote the optimal amenity threshold, such that a worker accepts the wage offer iff she draws an amenity $z \geq z_M$ for the job. This threshold equals the lower bound $\underline{z}$ (the offer is accepted for sure) if $p-n+y+\underline{z} \geq \frac{1-F_Z(\underline{z})}{F_Z'(\underline{z})}$, that is if the gain from trading with more workers always outweighs the cost of higher informational rents.It equals the upper bound $\bar{z}$ (the offer is rejected for sure) if $p - n + y + \bar{z} \leq \frac{1-F_Z(\bar{z})}{F_Z'(\bar{z})}$. In this case no trade takes place and the model is trivial, so we rule this case out by assumption.

It is now straightforward to map this model of wage determination into the notation of Section 3: let $\mathbb{I}$ be an indicator function, and

$$x(y, z, p, n) = \mathbb{I}\left\{z \geq z_M(p - n + y)\right\} \tag{19}$$

$$G(y, z, p, n) = x(y, z, p, n)\frac{z - z_M(p - n + y)}{r + \delta}, \tag{20}$$

$$J(y, z, p, n) = x(y, z, p, n)\frac{p - n + y + z_M(p - n + y)}{r + \delta}. \tag{21}$$

We now verify that this model of wage determination satisfies the properties introduced in Section 3.

Location Invariance. It is immediate from equations (19)–(21) that the functions x, G and J depend on p and n only through the difference $p - n$. As with NB, an increase in p and n by the same amount just shifts the location of the firm's problem, and leaves the division of the gains from trade unaffected.

Positive Selection. Inspecting the firm's objective in (17), an increase in $p - n + y$ raises the marginal gain from trade by lowering the threshold z_M. By a monotone comparative statics argument, or by the implicit function theorem, $z_M(p-n+y)$ is weakly decreasing (and strictly so over the range where the solution is interior). Consulting equation (19), this implies that $x(p, n, y, z)$ is non-decreasing in both y and z.

Increasing Worker's Gains from Trade. As a first step, it is convenient to define the worker's average gains from trading with a firm of type y:

$$(r+\delta)\mathcal{G}(p-n|y) \equiv \int_{z_M(p-n+y)}^{\bar{z}} (z - z_M(p-n+y))dF_Z(z)$$

This function is differentiable except possibly at the two threshold values where the first order condition holds with equality for the corners $\underline{z}$ and $\bar{z}$, with

$$(r+\delta)\mathcal{G}'(p-n|y) = -z_M'(p-n+y)[F_Z(z_M(p-n+y))] \geq 0.$$

The firm expands the range of workers it is trading with by $-z_M'(p-n+y)$, so the informational rents of all worker types that it is already trading with have to increase by exactly this amount. By definition $\mathcal{G}(p-n) = \int \mathcal{G}(p-n|y)dF_Y(y)$, so that

$$\mathcal{G}'(p-n) = \int \mathcal{G}'(p-n|y)dF_Y(y)$$

which establishes differentiability. Since $\mathcal{G}'(p-n|y) \geq 0$, also $\mathcal{G}'(p-n) \geq 0$, that is worker's gains from trade are increasing.

Regular Firm's Gains from Trade. The maximized value for firm type y is

$$(r+\delta)\mathcal{J}(p-n|y) = (1 - F_Z(z_M(p-n+y)))(p-n+y+z_M(p-n+y)).$$

Differentiation yields

$$(r+\delta)\mathcal{J}'(p-n|y) = (1 - F_Z(z_M(p-n+y))).$$

If the firm is at a corner this follows immediately, as $z_M(p-n+y)$ does not respond to a change in $p-n$. If the solution to the firm's problem is interior this relationship follows from the envelope theorem. Since the threshold z_M is chosen optimally, the firm cannot gain at the margin from adjusting the threshold, so the benefit from an increase in $p-n$ is just the direct effect on the gains from trade with worker's the firm already trades with.

It follows that $\mathcal{J}(p-n|y)$ is continuously differentiable, and differentiation under the integral sign yields

$$(r+\delta)\mathcal{J}'(p-n) = \int (r+\delta)\mathcal{J}'(p-n|y)dF_Y(y) = \int (1-F_Z(z_M(p-n+y)))dF_Y(y) = \xi(p-n).$$

This proves differentiability of $\mathcal{J}'(p-n)$, Increasing Firm's Gains from Trade, as well as Regular Firm's Gains from Trade. Notice that with NB match formation is ex post efficient, and the envelope theorem applies to the overall gains from trade, that is $(r+\delta)(\mathcal{J}'+\mathcal{G}')=\xi$. Here the envelope theorem delivers $(r+\delta)\mathcal{J}'=\xi$. Due to the monopoly inefficiency, one generally has $(r+\delta)(\mathcal{J}'+\mathcal{G}')>\xi$.

We summarize these results in the following proposition.

Proposition 4 *Under Assumption 1 the firm offer monopoly model satisfies (i) Location Invariance, (ii) Positive Selection, (iii) Increasing Firm's and Worker's Gains from Trade and (iv) Regular Firm's Gains from Trade.*

Thus, under weak assumptions, this model of wage determination satisfies those properties which are sufficient for the bounds of Propositions 1 and 2.

5.2 Unilateral Wage Request by the Worker

By symmetry with the firm offer model, the worker offer monopoly model satisfies Location Invariance, Positive Selection and Increasing Firm's and Worker's Gains from Trade. These are all the properties needed to insure that the bound of Proposition 1 applies.

However, for the firm offer model we only established that the gains from trade of the offer-making party are regular. Now the firm is at the receiving end of the offer. To apply the second bound, we need regular gains from trade of the offer-receiving party. Using notation symmetric to the firm offer model, in the worker offer model

$$(r+\delta)\mathcal{J}'(p-n|z)=-y_M'(p-n+z)[1-F_Y(y_M(p-n+z))] \tag{22}$$

at points of differentiability of $y_M(p-n+z)$. Here $y_M(p-n+z)$ is the threshold productivity level chosen by the worker with amenity value z. Only firm types that the worker has already been trading with experience an increase in their informational rent, which is why the probability of trade $1-F_Y(y_M(p-n+z))$ appears in equation (22). How large the increase in the informational rent is for these firm types depends on how many more firm types the worker wants to trade with, that is the drop in the threshold $-y_M'(p-n+z)$. If the worker lowers the threshold substantially, then the increase in the firm's informational rent will be large. Now suppose the worker reduces the threshold less than one for one with an increase in $p-n$, that is $-y_M'(p-n+z)\leq 1$. Then

$$\begin{aligned}(r+\delta)\mathcal{J}'(p-n) &= \int -y_M'(p-n+z)[1-F_Y(y_M(p-n+z))]dF_Z(z)\\ &\leq \int [1-F_Y(y_M(p-n+z))]dF_Z(z)=\xi(p-n),\end{aligned}$$

enough to insure Regular Firm's Gains from Trade. The following strengthening of the second part of Assumption 1 insures that $-y'_M \leq 1$.

Assumption 2 *The hazard rate* $\frac{F'_Y(y)}{1-F_Y(y)}$ *is weakly increasing and continuously differentiable on* $[\underline{y}, \bar{y}]$.

To understand the role of a monotone hazard rate, consult the worker's first order condition for an optimal wage request to the firm:

$$p - n + y_M + z = \frac{1 - F_Y(y_M)}{F'_Y(y_M)}. \tag{23}$$

If in response to an increase in $p - n$ the worker would reduce y_M one for one, then the left hand side, which is the marginal benefit from trading with another firm type, would be unchanged. However, under the monotone hazard assumption the worker would end up at a point with a lower hazard rate, that is the loss of trade associated with a more aggressive wage request is smaller relative to the number of firms that would pay the higher wage. It follows that it is optimal to reduce the threshold less than one for one. Thus we obtain the following proposition.

Proposition 5 *Under Assumption 1 the worker request monopoly model satisfies (i) Location Invariance, (ii) Positive Selection and (iii) Increasing Firm's and Worker's Gains from Trade. If part (b) of Assumption 1 is strengthened to Assumption 2, then this model also satisfies Regular Firm's Gains from Trade.*

The stronger Assumption 2 of a monotone hazard is sufficient to apply Propositions 2. We emphasize that it is not needed for the bound of Proposition 1.

6 The Constrained Efficient Allocation

We now turn to the constrained efficient allocation in the presence of bilateral asymmetric information, as in Myerson and Satterthwaite (1983). Parties have access to a mediator, who receives announcements about the draws of private information, y and z, and recommends a binding trading decision and wage. This allocation is of great interest because it also features the maximal expected gains from trade in the equilibrium of any unmediated bargaining game. In this wage negotiation context, the mediator enforcing the rules of the game can be thought of as an arbitrator of a labor dispute. This allocation is always unique and, for some classes of belief distributions, can be implemented through a sealed-bid double auction. Therefore, the indeterminacy of the set of efficient equilibria of the double auction under complete information, exploited by Hall (2005) to generate sufficient wage rigidity, breaks down under any modicum of asymmetric information. Under these circumstances, it is then natural to verify the properties of the unique best possible balanced budget allocation that parties can attain with a mediator.

It is straightforward to verify Location Invariance and Positive Selection. However, we have not yet been able to uncover simple sufficient conditions for properties such as Increasing Firm's and Worker's Gains from Trade and Regular Firm's Gains from Trade. So far we can only show that the bounds of Section 4 apply to some special cases, which are considered at the end of this section. Specifically, under the assumption of symmetric beliefs one can also establish increasing gains from trade, so Proposition 3 applies. We also specialize further to the case of uniform symmetric beliefs. This case has received particular attention due to the fact that the constrained efficient allocation can be implemented through an equilibrium of the $\frac{1}{2}$-double auction analyzed by Chatterjee and Samuelson (1983). It is also of particular interest here because in this case the property Regular Firm's Gains from Trade holds, so the conditions of Proposition 2 are satisfied. Finally, we verify that Proposition 1 applies to asymmetric beliefs of the exponential class.

The Mechanism Design Problem. A mediator, or principal, receives reports $\hat{y}$ and $\hat{z}$ by the two parties and enforces a probability of trade $x(\hat{y}, \hat{z}, p, n)$ and a wage $w(\hat{y}, \hat{z}, p, n)$ so as to maximize the sum of expected values to the two parties. The reports are a Bayesian Nash equilibrium of this optimal mechanism. That is, the efficient mechanism is a direct revelation game whose Bayesian Nash equilibrium produces the constrained efficient allocation.

Given a pair of reports $\hat{y}, \hat{z}$ and realizations y, z, the firm's value is

$$J(\hat{y}, \hat{z}, y, p, n) = \frac{p + y - w(\hat{y}, \hat{z}, p, n)}{r + \delta} \tag{24}$$

and the worker's value

$$W(\hat{y}, \hat{z}, z, p, n) = \frac{z + w(\hat{y}, \hat{z}, p, n) - \delta U}{r + \delta} = \frac{z + w(\hat{y}, \hat{z}, p, n) - \delta n/r}{r + \delta}. \tag{25}$$

The constrained efficient allocation obtained through a direct revelation mechanism maximizes the total expected value to firm and worker

$$\max_{x,w} \int_{\underline{y}}^{\bar{y}} \int_{\underline{z}}^{\bar{z}} \left[J(y, z, y, p, n) + W(y, z, z, p, n)\right] x(y, z, p, n) dF_Z\left(z\right) dF_Y(y)$$
$$+ \int_{\underline{y}}^{\bar{y}} \int_{\underline{z}}^{\bar{z}} U\left[1 - x(y, z, p, n)\right] dF_Z\left(z\right) dF_Y(y) \tag{26}$$

subject to IR and IC of the firm: for all $y, \hat{y} \in$supp$\{F_Y\}$

$$\int_{\underline{z}}^{\bar{z}} J(y,z,y,p,n)x(y,z,p,n)dF_Z(z) \geq \max\left\langle 0, \int_{\underline{z}}^{\bar{z}} J(\hat{y},z,y,p,n)x(\hat{y},z)\,dF_Z(z)\right\rangle \quad (27)$$

and of the worker: for all $z, \hat{z} \in$supp$\{F_Z\}$

$$\begin{aligned} & \int_{\underline{y}}^{\bar{y}} \{W(y,z,z,p,n)x(y,z,p,n) + U[1 - x(y,z,p,n)]\}\,dF_Y(y) \\ \geq \;& \max\left\langle U, \int_{\underline{y}}^{\bar{y}} \{W(y,\hat{z},z,p,n)x(y,\hat{z}) + U[1 - x(y,\hat{z},p,n)]\}\,dF_Y(y)\right\rangle. \quad (28) \end{aligned}$$

We can rewrite the problem as follows. Subtract $n(r+\delta)/r$ from both sides of (28), use (24) and (25), ignore constant terms independent of choice variables, to transform the original problem into that of maximizing the ex ante flow surplus

$$\max_{x,w} \int_{\underline{y}}^{\bar{y}} \int_{\underline{z}}^{\bar{z}} (p+y+z-n)x(y,z,p,n)dF_Z(z)\,dF_Y(y) \quad (29)$$

subject to

$$\int_{\underline{z}}^{\bar{z}} [p+y-w(y,z,p,n)]\,x(y,z,p,n)dF_Z(z) \geq \max\left\langle 0, \int_{\underline{z}}^{\bar{z}} [p+y-w(\hat{y},z,p,n)]\,x(\hat{y},z,p,n)dF_Z(z)\right\rangle,$$

$$\int_{\underline{y}}^{\bar{y}} [z+w(y,z,p,n)-n]\,x(y,z,p,n)dF_Y(y) \geq \max\left\langle 0, \int_{\underline{y}}^{\bar{y}} [z+w(y,\hat{z},p,n)-n]\,x(y,\hat{z},p,n)dF_Y(y)\right\rangle.$$

Notice that this is *not* a constrained efficient allocation for society: here parties take the outside option $n = rU$ as given, and just mind the division of rents. The objective is independent of the wage w, which only plays the role of a transfer function to induce parties to truthfully reveal their valuations.

Existence and Uniqueness of the Optimal Mechanism. Next, we show that, under a standard assumption on distributions, the optimal mechanism exists and is unique. We begin by transforming the problem. Let $\upsilon \equiv p + y$, $\zeta \equiv n - z$, $\Phi(\zeta) \equiv 1 - F_Z(n-\zeta)$, so $\Phi'(\zeta)\,d\zeta = F_Z'(n-\zeta)\,d\zeta = -F_Z'(z)\,dz$, $\Gamma(\upsilon) \equiv F_Y(\upsilon - p)$, so $\Gamma'(\upsilon)\,d\upsilon = F_Y'(\upsilon-p)d\upsilon = F_Y'(y)dy$. Then the efficient mechanism maximizes expected gains from trade subject to IC, IR and budget balance.

$$\max_{x,w} \int\int (\upsilon - \zeta)\,x(\upsilon,\zeta)\,d\Phi(\zeta)\,d\Gamma(y)$$

$$\int [\upsilon - w(\upsilon,\zeta)]\, x(\upsilon,\zeta)\, d\Phi(\zeta) \;\geq\; \max\left\langle 0, \int [\upsilon - w(\hat{\upsilon},\zeta)]\, x(\hat{\upsilon},\zeta)\, d\Phi(\zeta) \right\rangle$$
$$\int [w(\upsilon,\zeta) - \zeta]\, x(\upsilon,\zeta)\, d\Gamma(\upsilon) \;\geq\; \max\left\langle 0, \int \left[w(\upsilon,\hat{\zeta}) - \zeta\right] x\left(\upsilon,\hat{\zeta}\right) d\Gamma(\upsilon) \right\rangle.$$

This is the same formulation as in MS. We apply their terminology and results. Let the "virtual types" be

$$Q_f(\upsilon,\alpha) \equiv \upsilon - \alpha \frac{1-\Gamma(\upsilon)}{\Gamma'(\upsilon)} \text{ and } Q_w(\zeta,\alpha) \equiv \zeta + \alpha \frac{\Phi(\zeta)}{\Phi'(\zeta)} \tag{30}$$

which are, respectively, increasing in υ and decreasing in ζ by Assumption 1. Then IR, IC and budget balance are equivalent to

$$\int\int [Q_f(\upsilon,1) - Q_w(\zeta,1)]\, x(\upsilon,\zeta)\, d\Phi(\zeta)\, d\Gamma(y) \geq 0 \tag{31}$$

with equality if there is positive probability of no gains from trade (which we will assume to avoid trivialities).

Form a Lagrangian

$$\max_x \int\int \{\upsilon - \zeta + \mu [Q_f(\upsilon,1) - Q_w(\zeta,1)]\}\, x(\upsilon,\zeta)\, d\Phi(\zeta)\, d\Gamma(y) \tag{32}$$

where μ is the multiplier. The FOC is

$$\begin{aligned} x^*(\upsilon,\zeta) &= \mathbb{I}\{\upsilon - \zeta + \mu [Q_f(\upsilon,1) - Q_w(\zeta,1)] > 0\} \\ &= \mathbb{I}\left\{ \upsilon - \zeta > M \left[\frac{1-\Gamma(\upsilon)}{\Gamma'(\upsilon)} + \frac{\Phi(\zeta)}{\Phi'(\zeta)}\right]\right\} \\ &= \mathbb{I}\{Q_f(\upsilon,M) > Q_w(\zeta,M)\} \end{aligned}$$

where $\mathbb{I}$ is the indicator function and

$$M \equiv \frac{\mu}{1+\mu} \in [0,1]. \tag{33}$$

This, in particular, implies that trade occurs iff $\upsilon \geq \zeta$. More precisely, let the trading cutoff $\upsilon^*(\zeta, M)$ solve

$$Q_f(\upsilon^*(\zeta,M), M) = Q_w(\zeta, M)$$

so that trade occurs iff $\upsilon > \upsilon^*(\zeta, M)$.

Proposition 6 *There exists a unique optimal mechanism.*

Proof. Assumption 1 implies that $Q_f(.,1)$ and $Q_w(.,1)$ are increasing. Then $Q_f(\upsilon, M)$ and $Q_w(\zeta, M)$ are also increasing for every $M \in [0,1]$ (see MS who state this without proof; there is a simple proof by contradiction). It follows (MS Theorem 2) that an efficient mechanism exists, and the efficient rule is: trade iff $\upsilon > \upsilon^*(\zeta, M)$ for a cutoff function υ^* defined implicitly by

$$\upsilon^*(\zeta, M) - \zeta = M\left\{\frac{1-\Gamma(\upsilon^*(\zeta, M))}{\Gamma'(\upsilon^*(\zeta, M))} + \frac{\Phi(\zeta)}{\Phi'(\zeta)}\right\}. \tag{34}$$

To show uniqueness, proceed by contradiction. Suppose that there exist two distinct efficient allocations $\{x_i^*\}_{i=1,2}$. Given the nature of the optimal rule (trade if $\upsilon > \upsilon^*(\zeta, M)$) these two mechanisms must be associated to two different values of the Lagrange multiplier, M_1 and $M_2 > M_1$. Then

$$M_2 > M_1 \Leftrightarrow \upsilon^*(\zeta, M_2) > \upsilon^*(\zeta, M_1) \tag{35}$$

which imply

$$\begin{aligned} x_2^*(\upsilon, \zeta) &= x_1^*(\upsilon, \zeta) = 1 \text{ for all } \upsilon > \upsilon^*(\zeta, M_2) \\ x_1^*(\upsilon, \zeta) &= 1 > 0 = x_2^*(\upsilon, \zeta) \text{ for all } \upsilon \in (\upsilon^*(\zeta, M_1), \upsilon^*(\zeta, M_2)] \\ x_1^*(\upsilon, \zeta) &= x_2^*(\upsilon, \zeta) = 0 \text{ for all } \upsilon \leq \upsilon^*(\zeta, M_1) \end{aligned}$$

Therefore

$$\int\int (\upsilon - \zeta)\, x_1^*(\upsilon, \zeta)\, d\Phi(\zeta)\, d\Gamma(y) > \int\int (\upsilon - \zeta)\, x_2^*(\upsilon, \zeta)\, d\Phi(\zeta)\, d\Gamma(y) \tag{36}$$

so that the second mechanism, associated to the higher Lagrange multiplier, yields a strictly smaller objective function, and cannot be optimal. The uniqueness of the payment function w^* then follows from the fact that the IR constraint is binding for the lowest types $\underline{z}$ and $\underline{y}$ (MS Theorem 2). ■

Uniqueness is, of course, a key property when discussing comparative statics effects of changes in the aggregate component of labor productivity p.

Location Invariance and Differentiability. Let the cutoff

$$y^* \equiv \upsilon^*(\zeta, M) - p \tag{37}$$

solve uniquely $Q_f(y^*, M) + Q_w(z, M) = n - p$, or

$$y^* + p - n + z = M\left\{\frac{1 - F_Y(y^*)}{F_Y'(y^*)} + \frac{1 - F_Z(z)}{F_Z'(z)}\right\}.$$

Notice that $y^* = y^*(z, p-n)$. Trade occurs iff the productivity of the match for the firm exceeds this threshold: $y \geq y^*(z, p-n)$. We can also state the efficient trading rule in terms of the worker's private value: trade occurs iff $z \geq z^*(y, p-n) = y^{*-1}(y, p-n)$,where the threshold z^* solves

$$y + p - n + z^* = M \left\{ \frac{1 - F_Y(y)}{F_Y'(y)} + \frac{1 - F_Z(z^*)}{F_Z'(z^*)} \right\}. \tag{38}$$

By the implicit function theorem, these cutoff functions are also differentiable in $p - n$. Since $Q_f(\upsilon, M)$ and $Q_w(\zeta, M)$ are increasing in υ and ζ respectively, then $y^*(z, p-n)$ is decreasing in z and $z^*(y, p-n) = y^{*-1}(y, p-n)$ is decreasing in y: the higher the valuation a party has for the match, the more likely she expects trade to be. Furthermore, $x^*(\upsilon, \zeta) = x^*(y, z, p-n, M)$, and by the Implicit Function Theorem $x^*(y, z, p-n, M)$ is differentiable in $p - n$.

The probability of trading conditional on private information is therefore

$$\begin{aligned} \int x^*\left(\upsilon, \zeta\right) d\Gamma(\upsilon) &= \Pr(\upsilon > \upsilon^*\left(\zeta, M\right) | \zeta) = 1 - \Gamma(\upsilon^*\left(\zeta, M\right)) = 1 - F_Y(y^*(z, p-n)) \\ \int x^*\left(\upsilon, \zeta\right) d\Phi(\zeta) &= \Pr(\upsilon > \upsilon^*\left(\zeta, M\right) | \upsilon) = 1 - F_Z\left(z^*(y, p-n)\right) \end{aligned}$$

for a worker of type $\zeta = n - z$, and for a firm of type $\upsilon = p + y$, respectively. These chances are decreasing in ζ and increasing in υ (increasing in z and y) respectively.

The probability of trade unconditional on private information is

$$\xi^*(p-n) = \int_{\underline{z}}^{\bar{z}} [1 - F_Y(y^*(z, p-n))] dF_Z(z) = \int_{\underline{y}}^{\bar{y}} [1 - F_Z(z^*(y, p-n))] dF_Y(y). \tag{39}$$

The expected value to each party, unconditional on trade but conditional on private information, is

$$\mathcal{G}^*(z|p, n) = \mathcal{G}^*(\underline{z}|p, n) + \int_{\underline{z}}^{z} \left[1 - F_Y(y^*(z', p-n))\right] dz'$$

$$\mathcal{J}^*(y|p, n) = \mathcal{J}^*(\underline{y}|p, n) + \int_{\underline{y}}^{y} [1 - F_Z\left(z^*(y', p-n)\right)] dy'.$$

By Theorem 2 in MS, the values of the least happy worker and of the least productive firm are zero (the IR constraint binds for those extreme types):

$$\mathcal{G}^*(p, n|\underline{z}) = \mathcal{J}^*(p, n|\underline{y}) = 0.$$

Therefore, we can write $\mathcal{G}^*(z|p-n)$ and $\mathcal{J}^*(y|p-n)$, depending only on $p - n$, for the conditional values. Notice that $\mathcal{G}^*(z|p-n)$ is increasing in z and $\mathcal{J}^*(y|p-n)$ is increasing in y, so the IR constraints $\mathcal{G}^*(z|p-n) \geq 0$ and $\mathcal{J}^*(y|p-n) \geq 0$ for each type of worker and firm are satisfied.

Taking expectations w.r. to private information, we can finally obtain the expected values to each party unconditional on trade and on private information:

$$(r+\delta)\mathcal{G}^*(p-n) = \int_{\underline{z}}^{\bar{z}} \int_{\underline{z}}^{z} \left[1 - F_Y(y^*(z', p-n))\right] dz' dF_Z(z)$$

integrating by parts

$$\begin{aligned}
&= -\left\{\int_{\underline{z}}^{\bar{z}} \left[1 - F_Y(y^*(z', p-n))\right] dz'\right\} \left[1 - F_Z(\bar{z})\right] + \left\{\int_{\underline{z}}^{\underline{z}} \left[1 - F_Y(y^*(z', p-n))\right] dz'\right\} \left[1 - F_Z(\underline{z})\right] \\
&\quad + \int_{\underline{z}}^{\bar{z}} \left[1 - F_Y(y^*(z, p-n))\right] \left[1 - F_Z(z)\right] dz \\
&= \int_{\underline{z}}^{\bar{z}} \left[1 - F_Y(y^*(z, p-n))\right] \left[1 - F_Z(z)\right] dz
\end{aligned}$$

Similarly

$$(r+\delta)\mathcal{J}^*(p-n) = \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{y} [1 - F_Z(z^*(y', p-n))] dy' dF_Y(y) = \int_{\underline{y}}^{\bar{y}} [1 - F_Z(z^*(y, p-n))] \left[1 - F_Y(y)\right] dy$$

The total maximized expected gains from trade are

$$\begin{aligned}
(r+\delta)\mathcal{S}^*(p-n) &= (r+\delta)\left[\mathcal{J}^*(p-n) + \mathcal{G}^*(p-n)\right] \\
&= \int_{\underline{z}}^{\bar{z}} \int_{\underline{y}}^{\bar{y}} x^*(y, z, p-n, M) \left\{F_Y'(y) \left[1 - F_Z(z)\right] + F_Z'(z) \left[1 - F_Y(y)\right]\right\} dydz.
\end{aligned}$$

By inspection, all of these values are differentiable with respect to $p-n$. Therefore, Location Invariance and differentiability hold.

Positive Selection. By definition, the maximized expected flow gains from trade can be written as follows:

$$(r+\delta)\mathcal{S}^*(p-n) = \int_{\underline{z}}^{\bar{z}} \int_{\underline{y}}^{\bar{y}} x^*(y, z, p-n, M)(p+y+z-n) dF_Y(y) dF_Z(z)$$

using the form of the efficient trading rule

$$
\begin{aligned}
&= \xi^*(p-n)\cdot(p-n) + \int_{\underline{z}}^{\bar{z}}\int_{y^*(z,p-n)}^{\bar{y}} (y+z)\, dF_Y(y) dF_Z(z) \\
&= \xi^*(p-n)\cdot(p-n) + \int_{\underline{z}}^{\bar{z}} \{z + \mathbb{E}[y|y \geq y^*(z,p-n)]\}\, [1 - F_Y(y^*(z,p-n))] dF_Z(z) \\
&= \xi^*(p-n)\{(p-n) + \int_{\underline{z}}^{\bar{z}} \{z + \mathbb{E}[y|y \geq y^*(z,p-n)]\}\, \frac{1 - F_Y(y^*(z,p-n))}{\int_{\underline{z}}^{\bar{z}} [1 - F_Y(y^*(z',p-n))]\, dF_Z(z')} dF_Z(z)\} \\
&= \xi^*(p-n)\cdot\left\{(p-n) + \int_{\underline{z}}^{\bar{z}} \mathbb{E}[y|y \geq y^*(z,p-n)] dH^*(z) + \int_{\underline{z}}^{\bar{z}} z dH^*(z)\right\}
\end{aligned}
$$

where H^* is the cdf of the worker's valuation conditional on trade. Then notice that

$$
\mathbb{E}[y|y \geq y^*(z,p-n)] \geq \mathbb{E}[y] = 0
$$

so the inequality is also true when averaging over $dH^*(z)$. Next,

$$
\int_{\underline{z}}^{\bar{z}} z dH^*(z) \geq \int_{\underline{z}}^{\bar{z}} z dF_Z(z) = 0
$$

if $H^* \succeq_{FSD} F_Z$. This follows from the fact that the cutoff $y^*(z, p-n)$ is decreasing in z.

We conclude that Positive Selection holds: $(r+\delta)\mathcal{S}^*(p-n) \geq (p-n)\cdot\xi^*(p-n)$.

Increasing Gains from Trade. To apply the first bound from Proposition 1, using the above expression, it remains to show

$$
\begin{aligned}
\mathcal{G}^{*\prime}(p-n) &= \int_{\underline{z}}^{\bar{z}} -\frac{dy^*(z,p-n)}{d(p-n)} F_Y'(y^*(z,p-n))\, [1 - F_Z(z)]\, dz \geq 0 \\
\mathcal{J}^{*\prime}(p-n) &= \int_{\underline{y}}^{\bar{y}} -\frac{dz^*(y,p-n)}{d(p-n)} F_Z'(z^*(y,p-n))\, [1 - F_Y(y)]\, dy \geq 0
\end{aligned}
$$

where

$$
\frac{dy^*(z,p-n)}{d(p-n)} = \frac{-1 + \frac{dM}{d(p-n)}\left\{\frac{1-F_Y(y^*(z,p-n))}{F_Y'(y^*(z,p-n))} + \frac{1-F_Z(z)}{F_Z'(z)}\right\}}{Q_f'(y^*(z,p-n))} = \frac{1}{\frac{dz^*(y^*(z,p-n),p-n)}{d(p-n)}}.
$$

Since the denominator is positive by Assumption 1, a sufficient condition is that $M = \mu/(1+\mu)$, thus the Lagrange multiplier μ, be non-increasing in $p-n$. This implies that, as the average gains from trade $p-n$ rise, the critical trading cutoff $y^*(z, p-n)$ declines for all z, or $z^*(y, p-n)$ declines for all y, so the trading set becomes larger and both parties gain. While we have not yet been able to sign this derivative in general, we can establish it for some special cases.

A Special Case: Symmetric Beliefs. In the special case $F_Y = F_Z$ the third bound of Proposition 3 applies provided also that the firm has increasing gains from trade. By the envelope theorem $(r+\delta)\mathcal{S}^{*\prime}(p-n) = (1+\mu)\cdot\xi^*(p-n) > 0$ so, by symmetry, $\mathcal{G}^*(p-n) = \mathcal{J}^*(p-n) = \mathcal{S}^*(p-n)/2$ and

$$\mathcal{G}^{*\prime}(p-n) = \mathcal{J}^{*\prime}(p-n) = \frac{\mathcal{S}^{*\prime}(p-n)}{2} > 0. \tag{40}$$

Now we further specialize to symmetric uniform beliefs $[\underline{y},\bar{y}] = [\underline{z},\bar{z}] = \left[-\frac{1}{2},\frac{1}{2}\right]$. Since beliefs are symmetric, this is a special case of the preceding special case, and the bound of Proposition 3 applies. Nevertheless it provides an instructive example since it also satisfies the assumptions of Proposition 2, in particular Regular Firm's Gains from Trade. We restrict variation in $p-n$ to the interval $\left[0,\frac{1}{3}\right]$. Over this range the Langrange multiplier is constant at $\mu = \frac{1}{2}$, so the worker has increasing gains from trade as desired. One also obtains the simple closed form solutions

$$\xi(p-n) = \frac{1}{2}\left(\frac{3}{4}\right)^2 (1+(p-n))^2,$$
$$(r+\delta)\mathcal{J}(p-n) = \frac{1}{6}\left(\frac{3}{4}\right)^3 (1+(p-n))^3.$$

Thus one can directly verify that the increase in the gains from trade of the firm are bounded by the probability of trade, hence they are regular:

$$(r+\delta)\mathcal{J}'(p-n) = \frac{3}{4}\xi(p-n).$$

A Special Case: Asymmetric Exponential Beliefs Now suppose that y and z are positive real numbers with $F_Y(y) = 1-e^{-\gamma y}$, $F_Z(z) = 1-e^{-\lambda z}$ for some $\lambda,\gamma > 0$. Then the constrained efficient cutoff is linearly decreasing

$$y^*(z) = \frac{\mu}{1+\mu}\left(\frac{1}{\gamma}+\frac{1}{\lambda}\right) - z - (p-n) \equiv k - z.$$

The principal's objective, the expected gains from trade, after some manipulations, is

$$\begin{aligned}
&\int_0^\infty\int_0^\infty \left[p-n+y+z+\mu\left(p+y-\frac{1}{\gamma}+z-n-\frac{1}{\lambda}\right)\right]x(y,z)\gamma e^{-\gamma y}dy\lambda e^{-\lambda z}dz \\
= \;& (1+\mu)\int_0^\infty\int_0^\infty [y-y^*(z)]\,x(y,z)\gamma e^{-\gamma y}dy\lambda e^{-\lambda z}dz \\
= \;& (1+\mu)\frac{\gamma^2 e^{-\lambda k} - \lambda^2 e^{-\gamma k} - 2e^{-\lambda k}\lambda^2}{\lambda\gamma(\gamma-\lambda)} \\
= \;& (1+\mu)\frac{\gamma^2 e^{-\frac{\mu}{1+\mu}\left(\frac{\lambda}{\gamma}+1\right)+\lambda(p-n)} - \lambda^2 e^{-\frac{\mu}{1+\mu}\left(1+\frac{\gamma}{\lambda}\right)+\gamma(p-n)}}{\lambda\gamma(\gamma-\lambda)}
\end{aligned}$$

which is, evidently, symmetric in γ and λ. We can now maximize the objective w.r. to μ. The FOC is

$$0 = \gamma^2 e^{-\frac{\mu}{1+\mu}\left(\frac{\lambda}{\gamma}+1\right)+\lambda(p-n)} \frac{\mu - \frac{\lambda}{\gamma}}{1+\mu} - \lambda^2 e^{-\frac{\mu}{1+\mu}\left(1+\frac{\gamma}{\lambda}\right)+\gamma(p-n)} \frac{\mu - \frac{\gamma}{\lambda}}{1+\mu}.$$

Since $\mu \geq 0$, this reads

$$e^{-\frac{\mu}{1+\mu}\left(\frac{\lambda}{\gamma}-\frac{\gamma}{\lambda}\right)+(\lambda-\gamma)(p-n)} = \frac{\lambda\mu - \gamma}{\gamma\mu - \lambda}\frac{\lambda}{\gamma}.$$

If $\lambda = \gamma$, a symmetric model, this equation, thus μ, is independent of $p-n$. In any event, the RHS is positive, so the LHS must be positive, namely, $(\lambda\mu - \gamma) / (\gamma\mu - \lambda) > 0$, or $(\lambda\mu - \gamma)(\gamma\mu - \lambda) > 0$.

Now take an asymmetric model. WLOG let $\lambda > \gamma$. We can solve for $p - n$ as a function of μ

$$p - n = \frac{\ln(\lambda\mu - \gamma) - \ln(\gamma\mu - \lambda) + \ln(\lambda/\gamma) + \frac{\mu}{1+\mu}\left(\frac{\lambda}{\gamma} - \frac{\gamma}{\lambda}\right)}{\lambda - \gamma}.$$

and

$$(\lambda - \gamma)\frac{d(p-n)}{d\mu} = -\left(\lambda^2 - \gamma^2\right)\mu \frac{2\gamma\lambda + \lambda^2 + \gamma^2}{(\mu\lambda - \gamma)(\mu\gamma - \lambda)(1+\mu)^2 \gamma\lambda}$$

so finally, dividing through by $\lambda - \gamma$, and taking the inverse derivative,

$$\frac{d\mu}{d(p-n)} = -\frac{(\mu\lambda - \gamma)(\mu\gamma - \lambda)(1+\mu)^2 \gamma\lambda}{\mu(\lambda+\gamma)^3} < 0.$$

It follows that, as average gains from trade $p-n$ rise, the incentive problem is lessened, the cutoff y^* declines with $p-n$, both parties gain, and the first bound from Proposition (1) applies.

7 Discussion and Conclusions

The analysis of various different wage-determination mechanisms uncovers important similarities. In every case, trade occurs according to a cutoff rule (trade if the valuation for the match is large enough). Therefore, positive selection holds: the matches that are actually implemented are weakly better than the average quality of the matches drawn by nature.

Next is the property of increasing gains from trade. Both parties must benefit from an increase in the average gains from trade $p - n$. In general, the envelope theorem implies that the player who is maximizing an objective function gains from $p - n$. In the monopoly case, this gain accrues to the offer-making party and is exactly equal to the probability of trade. The initial optimal wage offer/request must appropriately balance the chance of trade and the returns conditional on trade. This implies that an increase in $p - n$ must be spread on both the offer and the chance of trade. The offer recipient also benefits, because trade becomes more likely. In the efficient mechanism, the envelope theorem applies to the principal. The maximized expected gains from trade rise in $p - n$ even faster than the chance of trade, due to the incentive constraints. In the (near-)symmetric case, this overall gain to the match is shared by firm and worker, who are then both better off.

Finally, albeit not strictly necessary for the main results, is the property of regular gains from trade: as mean flow gains from trade $p-n$ rise, we require that the expected firm's profits rise less than the chance of trade. If not, the response of job creation could be strong enough to generate unemployment fluctuations of plausible magnitude. This is the hardest property to verify. As said, in the monopoly case, the payoffs to the offer-making party rise exactly like the chance of trade, by an envelope theorem argument. In the efficient mechanism, an additional opposing force comes into play. If incentive constraints are binding and severely limit trade, an increase in aggregate labor productivity can relax them so as to boost the chance of trade, with a multiplier effect on the rents of the firm and the worker. That is, more favorable business conditions may help circumvent the inefficiency due to asymmetric information, which manifests itself in failed wage negotiations. Formally, the Lagrange multiplier M may be strongly decreasing in $p-n$. In this case, the firm's profit gain following a productivity boom could be sufficiently large to offset the indirect impact of job creation on wages through the worker's outside option. In the aggregate, job creation may surge enough to produce a sharp fall in unemployment. We remark that this would obtain not through wage rigidity but via a change in the "quantity" dimension of matching, namely the probability of a mutually acceptable agreement.

To distill the essence of our results, we can say that the key property is Increasing Gains from Trade. As the average gains from trade $p-n$ (which contain an endogenous component n) increase with productivity p, both the firm and the worker enjoy a weakly larger surplus over their outside options. If the worker's share of the surplus is constant in absolute terms, still the model fails. The reason is that the outside option of an employed worker, the continuation value of search, depends on the product of this share and of the job finding rate. As the latter is very procyclical, so is this product, even if the surplus share does not vary. Under Nash bargaining, this surplus share is even covarying with the job-finding rate.

In conclusion, to reconcile the representative agent equilibrium search model with the empirical evidence on employment fluctuations, while maintaining that unemployment is costly for society, we either need to break the link between wages and value of unemployed search or to make the worker's surplus from employment strongly countercyclical in absolute value, so as to offset the effect of the job finding rate. Otherwise, we need to abandon the representative agent and to introduce heterogeneity. In this paper, we showed that two natural attempts in this direction fail, and yet they new shed light on the internal mechanism of the standard search model.

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