

# On the extent of job-to-job transitions

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PRELIMINARY VERSION

## Abstract

The rate of job-to-job transitions is twice as large today as the rate at which workers move from employment to unemployment. I demonstrate that, under plausible specifications, the basic job-ladder model — the workhorse model of the literature on on-the-job search — has no chance of matching the extent of job-to-job transitions. Moreover, it cannot account for the low search effort exerted by most employed workers and the observation that on-the-job search is a means to “escape” unemployment: it is undertaken exactly by those workers who are facing the threat of becoming unemployed.

I develop an alternative theoretical framework that can quantitatively match salient features of job-to-job transitions. The model incorporates a stochastic process that causes the value of a job to the worker to decrease at times, predicting that workers with a lower job value have a higher probability of entering unemployment. This natural feature is not in standard models. A second important element of the model is endogenous search effort, explaining the low search effort exerted by many employed workers and the correlation between search effort and the probability of becoming unemployed observed in the data. Calculating the equilibrium of the model shows that it can successfully account for the stylized facts on job-to-job transitions. I also demonstrate that the model can account for observed differences in the extent of job-to-job transitions across demographic groups.

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# 1 Introduction

The rate of job-to-job transitions, moves of workers between employers without an intervening unemployment spell, are twice as large today as the rate at which workers move from employment to unemployment (Fallick and Fleischman (2004) and Nagypál (2005b)).<sup>2</sup> Moreover, the relative importance of job-to-job transitions has increased dramatically in recent decades: between 1975 and 2000, the rate of job-to-job transitions increased by 59% and the rate of employment-to-unemployment transitions declined by 47% (Stewart (2002)). This means that job-to-job transitions today are at least as important a means of reallocating labor towards its more productive uses as are the transitions of workers through unemployment. Despite their quantitative significance, studying the nature and extent of job-to-job transitions has been rarely a focus of equilibrium search and matching models, which have been for the past two decades the primary and most successful tool used to analyze worker reallocation and the phenomenon of equilibrium unemployment.

After reviewing the stylized facts on job-to-job transitions, in the first part of this paper, I demonstrate some serious and previously unrecognized quantitative shortcomings of the workhorse model of on-the-job search that is used in the literature with respect to recently documented stylized facts. In this model, the so-called job-ladder model, job-to-job transitions are modelled as upwards steps in a distribution of job values, much like steps up the rungs of a ladder. Of course, as long as job-to-job transitions are viewed as outcomes of a decision taken by the worker, the idea of the job-ladder model that these transitions increase the present discounted utility of workers is a simple implication of optimizing behavior. The problem with most existing job-ladder models lies in the fact that they also adopt the assumption that the present discounted utility of the worker is either not changing while the worker remains on the job, or is increasing with tenure. It is this specification, in fact, that I call the basic job-ladder model. Using a simple but novel derivation, I show that the basic job-ladder model has no chance of matching the extent of job-to-job transitions under plausible specifications. The basic intuition for this is as follows. The only way that a higher level of job-to-job transitions can come about in the basic job-ladder model is by employed workers getting offers more frequently. More frequent offers, however, help employed workers reach higher and higher job values in the distribution of job values and thereby make workers less likely to accept a new offer. This offsetting effect means that there is a concave relationship between offer arrival rates and the predicted job-to-job transition rate. Simple calculations show that in order for the model to predict the observed extent of job-to-job transitions, for several demographic groups, employed workers need to contact new offers at rates higher than unemployed workers. There is no evidence for such high offer arrival rates and for the implied low acceptance rates for employed workers in survey data. It is also something that would never arise if search effort was allowed to be chosen optimally by workers, a natural extension to consider if one is to match the evidence on search effort by employed workers.

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<sup>2</sup>For expositional purposes, in this paper I do not make a distinction between a job and an employer. Hence job-to-job transitions are equivalent to employer-to-employer transitions.

I then consider some extensions of the basic job-ladder model and show how they affect the performance of the model. I argue that the only extension found in the literature that has the potential to resolve the contradiction between the model and the data is to allow for shocks to employment relationships such that the value of a job to the worker decreases at times in a way that does not necessarily lead to separation into unemployment but leads to a lower job value and a corresponding increase in the likelihood of accepting a new job offer. I show that this extension in its existing form, however, cannot fully explain the relevant stylized facts. The most important reason for this is that the way these shocks are incorporated into existing models leads to the probability of separation to be constant for all workers, while in the data this probability is strongly correlated with the search effort of a worker.

Highlighting this failure of the basic job-ladder model to quantitatively match the magnitude of job-to-job transitions is one contribution of this study. One reason that this failure was not better understood earlier was a shortage of direct empirical evidence on job-to-job transitions, meaning that authors could not confront their model with the data. Moreover, the limited amount of indirect evidence on job-to-job transitions generally underestimated their extent. It is only with the availability of more recent and detailed datasets that the extent of job-to-job transitions has been fully gauged and has been found to be significantly greater than previously believed. The basic job-ladder model fails to explain this higher level of job-to-job transitions due to a non-linearity in the model that I alluded to above and explain in detail in Section 4.

In the second part of the paper, I develop an alternative theoretical framework that can quantitatively match salient features of job-to-job transitions. Moreover, this framework gives a novel view of job-to-job transitions as a way to “escape” unemployment. In standard models of on-the-job search, it is assumed that the transition into unemployment takes place at an exogenous rate that workers cannot influence. The proposed model gives a different view of job-to-job transitions and implies that these transitions are not only a way to get ahead in the distribution of job values but also a way to avoid unemployment in the face of adverse shocks to a worker’s employment relationship. This view of job-to-job transitions examines seriously the incentives workers face to make such transitions. This new view of job-to-job transitions has potentially wide-ranging policy implications, since it implies that labor market policies affect not only the search behavior of the unemployed but also that of employed workers.

The first key element that I incorporate into the model is a stochastic process that causes the value of a job to the worker to decrease at times. Unlike in existing models of on-the-job search, the stochastic process has a Brownian motion component which leads to the prediction of the model that workers with a lower job value have a higher probability of entering unemployment. While this is a natural feature, it is not a feature of standard models of on-the-job search where this probability is the same for all employed workers. This specification of the productivity process also allows one to explain the observed phenomena of the probability of separation decreasing with tenure and the wage increasing with tenure. A second important element of the model is endogenous search effort. This element helps in explaining the low search effort exerted by many employed workers and the correlation

between search effort and the probability of becoming unemployed observed in the data. Calculating the equilibrium of the model shows that the model can successfully account for the stylized facts on job-to-job transitions, significantly outperforming existing models of on-the-job search. I also demonstrate through comparative statics exercises that the model can account for observed differences in the extent of job-to-job transitions across demographic groups.

## 2 Related literature

Clearly, search models intended to explain job-to-job transitions need to incorporate some notion of on-the-job search, so the notion of job-to-job transitions and on-the-job search are intimately related. Interestingly, while there exists a series of models in the search literature that incorporate on-the-job search, explaining the extent of job-to-job transitions have not been a main focus of this literature. One obvious reason for this has been the lack of reliable data on such transitions.

The first models of on-the-job search due to Parsons (1973) and Burdett (1978) were constructed to explain the negative correlation between quit probability and a worker's tenure and age. Mortensen (1994) used a model of on-the-job search to reconcile evidence on the cyclical behavior of job flows with that on the cyclical behavior of worker flows. Pissarides (1994) argued that incorporating on-the-job search into a matching model results in a decrease in the responsiveness of the unemployment rate to productivity shocks. Barlevy (2002) has emphasized that incorporating on-the-job search into a model of the labor market can explain the decline in the quality of new jobs that takes place in a recession. Nagypál (2006) has shown that incorporating on-the-job search can improve the cyclical properties of search models given that a mechanism exists in the model that ensures that firms have more of an incentive to hire employed workers as opposed to unemployed workers. Nagypál (2005a) explores such mechanisms and studies the efficiency properties of models incorporating them.

Probably the best-known model of on-the-job search is due to Burdett and Mortensen (1998), who have been able to give a search-theoretic explanation of why similar workers are paid differently. They show that the so-called Diamond paradox of a degenerate equilibrium wage distribution does not survive the introduction of job-to-job transitions. With workers engaging in on-the-job search, firms do not rush to offer the lowest wage possible, since they realize that offering a higher wage can help to retain workers and to attract searching employed workers. In fact, the surprising main finding of Burdett and Mortensen (1998) is that one can generate equilibrium wage dispersion in an economy with ex-ante identical agents. In recent years, many authors have used the Burdett-Mortensen framework to estimate models of on-the-job search (for a review see Mortensen (2003)). It is important to note, however, that the Burdett-Mortensen framework is a version of the basic job-ladder model, hence the criticism of this paper applies to it. This is not to say that the Burdett-Mortensen framework could not be possibly extended along the lines suggested here, in fact the key insight of the Burdett-Mortensen model would most likely carry over to such an extension. This extension

does not exist in the literature, however.

The task of developing a model that can quantitatively match relevant features of the data on job-to-job transitions is a crucial one, for at least three reasons. First, an empirically grounded model of job-to-job transitions is needed to gauge the quantitative importance of the many qualitative insights derived from such models. Second, to the extent that job-to-job transitions are an alternative means of reallocating labor to its more productive uses, developing new models to think about job-to-job transitions will invariably lead to a reevaluation of existing insights about unemployment and the policies to influence unemployment. Third, in order to address policy questions relating to job-to-job transitions, one needs to start with an equilibrium model, since the characterization of the counter-factual, socially optimal outcome is possible only in this context. Moreover, one would want to use a model that is capable of quantitatively explaining the magnitude of job-to-job transitions and their variation across groups of workers and across stages of the business cycle.

### 3 Stylized facts on job-to-job transitions

The point of departure of this paper is a set of stylized facts about job-to-job transitions and on-the-job search that a satisfactory model should match. These can be summarized as follows.

1. Job-to-job transitions are twice as large in their magnitude than flows of workers from employment to unemployment, as documented in the work of Fallick and Fleischman (2004) and Nagypál (2005b) based on data from the Current Population Survey. These findings are also borne out by recent data from the Survey of Income and Program Participation (Nagypál (2005b)).

The above measures based on recent empirical studies of the extent of job-to-job transitions are larger than earlier estimates, partially due to the increasing importance of job-to-job transitions over the past decades (Stewart (2002)) and partially due to measurement issues inherent in the data used in previous studies.<sup>3</sup>

2. Better-educated and middle-aged workers are much more likely to transit into new employment without experiencing a spell of unemployment than less educated and

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<sup>3</sup>Lacking direct measures of job-to-job transitions, previous studies, such as Blanchard and Diamond (1990), used indirect estimates. These indirect estimates were too low since they only considered a fraction of job-to-job transitions, those that were reported to be preceded by a quit and where the worker reported to have a job lined up prior to quitting. It turns out that not all job-to-job transitions are preceded by a reported quit, since a significant part of them are triggered by the threat of separation, in which case the worker is likely not report the event to be a quit. Also, the quit rates used were based on manufacturing data which, as we know from recently available data, have a significantly lower number of quits than the economy as a whole. Fallick and Fleischman (2004) provide a detailed description of earlier works and their limitations.

younger workers (again, see Fallick and Fleischman (2004) and Nagypál (2005b)). This explains why studies using the National Longitudinal Survey of Youth (such as Parsons (1991) and Royalty (1998)) understate the importance of job-to-job transitions relative to employment-to-unemployment transitions. (Notice that this statement is about the *relative* importance of job-to-job transitions, which will play a crucial role in the analysis below. The absolute rate of *all* transitions — job-to-job, employment-to-unemployment, employment-to-out-of-labor-force — is higher for younger and less educated workers.)

It is a feature of both the basic job-ladder model and the alternative model proposed in this paper that the ratio of job-to-job to employment-to-unemployment transitions decreases as the unemployment rate increases. To the extent that more educated and older workers have lower unemployment rates, qualitatively these models can match the fact that such workers have a higher ratio of job-to-job to employment-to-unemployment transitions. Quantitatively, however, as I show in Section 4, the basic job-ladder model shows the largest failure for more educated workers. It turns out that the alternative model introduced in Section 5 can generate the observed job-to-job transition rate of more educated workers by making the natural assumption that such workers have a higher expected productivity compared to their value of leisure.

3. Only a small fraction of employed workers report to be actively searching on the job, and actively searching employed workers account for a small portion of job-to-job transitions. Based on the 1980 panel of the National Longitudinal Survey of Youth, Parsons (1991) reports that 19% of men and 16% of women between the ages of 17 and 23 are actively searching on the job. Using data from the Current Population Survey, which provides a representative sample of the U.S. population, Fallick and Fleischman (2004) report the fraction of employed workers actively searching for a job to be 4.4%. They also report that active searchers make up only 20% of all job-to-job transitions. In addition, the probability of actively searching on the job increases with education and declines with age.

To the extent that unemployed workers are actively searching, by definition, this fact implies that most employed workers search less than unemployed workers. An obvious interpretation of this fact is that search effort of employed workers is lower than that of unemployed workers. This is a natural prediction of models of on-the-job search that allow workers to choose their search effort in the presence of a search cost. In fact, many models of on-the-job search starting with Burdett (1978) introduce of a search cost in order to ensure that only some of the employed workers search in the model. Allowing for search effort to vary only along the extensive margin has the shortcoming that it cannot match the small fraction of employed searchers and the large extent of job-to-job transitions at the same time. For this reason, in the model of Section 5, I introduce an intensive margin for search effort and interpret active search as search effort that is close in its magnitude to the search effort exerted by unemployed workers (which in the model is always larger than the search effort of employed workers).

4. Active search on the job is a good predictor not only of a job-to-job transition (11.3% per month among active searchers as opposed to 2.1% among the other employed), but also of becoming unemployed (5.6% per month among active searchers as opposed to 0.9% among the other employed), according to numbers reported by Fallick and Fleischman (2004).

This observation states that search on the job is most intense among workers who have a high likelihood of becoming unemployed, who I refer to as marginal workers. This is a crucial observation that motivates the model I develop in Section 5, since it implies that on-the-job search serves as a means of avoiding unemployment by marginal workers and not just a means of securing a higher payoff job in face of a fixed probability of separation into unemployment. In other words, this observation highlights that job-to-job transitions are indeed an alternative to unemployment of reallocating workers towards their most productive uses.

While the above observation probably corresponds to one's casual observation of how the labor market works, existing models of on-the-job search cannot explain this phenomenon. This is because existing models of on-the-job search imply a fixed transition rate into unemployment, meaning that they do not distinguish marginal workers. As a consequence, they cannot explain the positive correlation between search intensity and the probability of transition into unemployment.

5. Using data from the National Longitudinal Survey of Youth, Holzer (1987) reports for male workers between the ages of 16 and 23 that actively searching employed workers (who make up two fifths of the sample of active searchers) receive offers 13% less often than unemployed workers and accept these offers half as often then their unemployed counterparts. Based on data from the Employment Opportunities Pilot Project conducted in 1979-1980 that includes workers of all ages, Blau and Robbins (1990) report that, employed active searchers (who make up a third of the sample of active searchers) get an offer around 19% more often than unemployed workers, and accept these offers around 1% less often than unemployed searchers. (Unfortunately, the exact numbers are not possible to back out from the numbers reported by Blau and Robbins (1990).)

These numbers indicate that, while the offer arrival rate of employed and unemployed searchers are not equal, they are quite similar, both among young workers and the population as a whole. More importantly, employed searchers have lower acceptance rates than unemployed searchers, but even for young workers, the difference is less than a factor of 2. In fact, for the population as a whole, the acceptance rate of employed workers is only slightly lower than for unemployed workers.<sup>4</sup> The evidence on higher contact rates for employed

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<sup>4</sup>A caveat is that the Employment Opportunities Pilot Project was not explicitly designed to give a representative sample of the U.S. population, though its average characteristics seem to match well with those from representative datasets.

workers led Blau and Robbins (1990) and subsequent authors to hypothesize that firms have a higher incentive to hire employed workers. Nagypál (2005a) examines the mechanisms that could give rise to such an outcome.

It is interesting to note that the implied ratio of employed searchers to unemployed searchers is two-thirds in the datasets used by Holzer (1987), half in the dataset used by Blau and Robbins (1990), while it is over 1 in the Current Population Survey studied by Fallick and Fleischman (2004). This difference could be due to the differences in the datasets or in the time period covered. The latter interpretation would be consistent with the growth in the importance of job-to-job transitions reported by Stewart (2002).

6. The probability that workers separate from a job declines with tenure, as is documented in many empirical studies of job durations (for an overview, see Farber (1994)). At the same time, wages of workers is increasing with tenure, although there is a large disagreement in the literature regarding the size of this increase (see Topel (1991), Altonji and Shakotko (1987), and Altonji and Williams (1997)).

Declining separation rate with tenure is a prediction of the basic job-ladder model: the probability of quitting to take a better job declines with the length of time since a worker's last unemployment spell, which, in turn, is positively correlated with tenure. This prediction of the model was in fact what motivated Burdett's early study of the job-ladder model (Burdett (1978)). Beyond the duration dependence introduced by heterogeneity, the baseline model does not feature true duration dependence. Whether introducing such true duration dependence (as was done, for example, by allowing for the accumulation of job-specific capital in Pissarides (1994)) is necessary to explain the duration dependence observed in the data is a model-specific quantitative question. One of the strengths of the the model proposed in Section 5 is that, contrary to the basic job-ladder model, it easily accommodates such true duration dependence while maintaining its ability to match the extent of job-to-job transitions.

As for wages, rising wages with tenure arise in the basic job-ladder model to the extent that wages rise with tenure conditional on the worker being on their first job after unemployment (see the discussion in Barlevy (2005)). Again, whether a model generates quantitatively enough rise in wages with tenure depends on the specific model considered and one's view of the empirical literature on the returns to seniority. In any event, the model proposed in Section 5 is less restrictive than the basic job-ladder model when it comes to accounting for the change in wages with tenure while maintaining its ability to match the extent of job-to-job transitions.

## 4 Job-to-job transitions in the basic job-ladder model

In this section, I develop the argument that the basic job-ladder model cannot match the extent of job-to-job transitions for reasonable specifications. To begin with, notice that

a simple accounting formula for the ratio of job-to-job to employment-to-unemployment transitions is given by

$$r_{EE} = \frac{\pi_e(1-u)}{\pi_u u}, \quad (1)$$

where  $\pi_e$  and  $\pi_u$  are the average transition rates into new employment for already employed and for unemployed workers and  $u$  is the unemployment rate. Given an unemployment rate of 5%, a ratio of 2 implies that the relative transition rates are 10.53%. For the theoretical exercise below, it is useful to decompose the relative transition rate into two components, the ratio of contact rates with available jobs and the ratio of acceptance probabilities upon contact. Formally,

$$\frac{\pi_e}{\pi_u} = \frac{\lambda_e A_e}{\lambda_u A_u}, \quad (2)$$

where  $\lambda_e$  and  $\lambda_u$  are the contact rate of employed and unemployed workers with available jobs and  $A_e$  and  $A_u$  are the probability of employed and unemployed workers accepting these new jobs. It turns out that different models of job-to-job transitions have different implications for how the relative transition rate decomposes into its two components. This decomposition then can be confronted with evidence on the two components, giving a way to evaluate the different models. It is exactly through this decomposition that one can conclude that the basic job ladder model cannot fit the empirical evidence, which is what I show in this section.

## 4.1 Benchmark case

In order to derive my benchmark result, I now consider the basic job-ladder model of on-the-job search. I will specify only the elements of the model that are necessary to derive the desired result. All other elements of the model can be left unspecified, and, in particular, can be chosen to be as general as possible, as long as the stated assumptions are satisfied.

The model is set in continuous time and is populated by a fixed measure of infinitely-lived ex-ante homogeneous workers who can be either employed or unemployed.<sup>5</sup> These workers have standard preferences and could be risk-averse or risk-neutral. In addition, there could be firms in the economy, whose objective function is left unspecified.

An important restriction is that I consider only the stationary equilibrium of the model. In particular, it is assumed that the technology and wage setting mechanisms are specified

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<sup>5</sup>Considering three states, employment, unemployment, and out of the labor force, and the flow across these states is more than most existing models of the labor market do. Since there is not yet a quantitatively (or even qualitatively) accurate three-state model of worker flows in existence, I maintain the assumption of two states both in the description of the basic job-ladder model and in my proposed model. This, of course, necessitates that empirical estimates are also appropriately adjusted. I do this wherever possible. This is, for example, the reason that I find it more useful to talk about the ratio of job-to-job to employment-to-unemployment transitions as opposed to simply the job-to-job transition rate.

so that in the stationary equilibrium the (present discounted) value of job opportunities is distributed according to the stationary continuous distribution function:  $F : [\underline{V}, \bar{V}] \rightarrow [0, 1]$  (with corresponding survival function  $\bar{F}(V) = 1 - F(V)$ ), where  $V$  is determined by the payoff-relevant attributes of jobs, such as the wage they pay and their desirability to workers.<sup>6</sup>  $F$  could, of course, be an endogenous object, reflecting optimization by firms. A key assumption is that as long as the worker remains on the same job, the value of the job remains the same. Moreover, assume that jobs are destroyed at rate  $\delta > 0$  which leads workers to enter the unemployment pool.

In terms of the search technology, unemployed and employed workers are assumed to contact job opportunities at a constant rate  $\lambda_u > 0$  and  $\lambda_e > 0$ , respectively. For now, these contact rates are treated as exogenous parameters, though endogenizing them will be an important extension considered below. The only restriction (that I relax below) is that all employed workers have the same contact rate, independent of  $V$ .

In this model, the only decision that workers make is whether to accept a job when they make a contact. Optimizing behavior by workers implies that this decision is characterized by a reservation property. In particular, let the reservation value of unemployed workers be  $R \geq \underline{V}$ , while the reservation value of a worker working on a job with value  $V$  is  $V$  itself as the worker is willing to take any job with a higher value than that of their current job.

Let the stationary distribution of employed workers across job values be  $G$ , where  $G$  is unnormalized, i.e.  $G(\bar{V}) = 1 - u$ , where  $u$  is the fraction of workers who are unemployed (the unemployment rate in this two state model). This distribution can be derived from the balance equations that equate the flow into and out of  $G(V)$ . The flow into  $G(V)$  per unit of time consists of unemployed workers who contact a job with a value that is above the reservation value  $R$  but below  $V$ :

$$\lambda_u(F(V) - F(R))u. \tag{3}$$

The flow out of  $G(V)$  consists of two parts, those workers who suffer an exogenous separation and those who succeed in contacting a job with a value that is above  $V$ :

$$\delta G(V) + \lambda_e \bar{F}(V)G(V). \tag{4}$$

Equating Equation (3) and (4) gives

$$G(V) = \frac{\lambda_u u (F(V) - F(R))}{\delta + \lambda_e \bar{F}(V)}, \tag{5}$$

implying that  $G(R) = 0$ . The unemployment rate in this model can also be derived from a balance equation that equates the flow into and out of unemployment. Since the flow

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<sup>6</sup>The assumption of a discrete distribution for  $F$  gives the same results as the assumption of a continuous distribution except that the derived expression is an upper bound on  $r_{EE}$  as opposed to being an exact expression for it. This case is treated in the Appendix.

from employment to unemployment is  $m_{EU} = \delta(1 - u)$  and the flow out of unemployment is  $\lambda_u \bar{F}(R)u$ , we get that

$$u = \frac{\delta}{\delta + \lambda_u \bar{F}(R)}. \quad (6)$$

To calculate the measure of flows from employer to employer,  $m_{EE}$ , notice that workers with value  $V$  make a job-to-job transition at rate  $\lambda_e \bar{F}(V)$ . Integrating over the distribution of employed workers gives that

$$\begin{aligned} m_{EE} &= \int_R^{\bar{V}} \lambda_e \bar{F}(V) dG(V) = \lambda_e \left[ [\bar{F}(V)G(V)]_R^{\bar{V}} - \int_R^{\bar{V}} G(V) d\bar{F}(V) \right] = \\ &= \lambda_e \int_R^{\bar{V}} \frac{\lambda_u u (F(V) - F(R))}{\delta + \lambda_e (1 - F(V))} dF(V) = \lambda_e \lambda_u u \int_{\bar{x}}^1 \frac{x - \bar{x}}{\delta + \lambda_e (1 - x)} dx, \end{aligned} \quad (7)$$

where I have first used integration by parts and then used a change of variables  $x = F(V)$  with  $\bar{x} = F(R)$ . Carrying out the integration gives

$$m_{EE} = \frac{\lambda_u}{\lambda_e} u \left[ (\delta + \lambda_e (1 - \bar{x})) \log \left( \frac{\delta + \lambda_e (1 - \bar{x})}{\delta} \right) - \lambda_e (1 - \bar{x}) \right]. \quad (8)$$

Next, notice from Equation (6) that  $\frac{1-u}{u} = \frac{\lambda_u (1-\bar{x})}{\delta}$ . Let  $y = \frac{\lambda_e (1-u)}{\lambda_u u} = \frac{\lambda_e (1-\bar{x})}{\delta}$  be the ratio of the number of contacts with job offers that employed searchers make to the number of such contacts that unemployed searchers make. Given the measure of employment-to-unemployment transitions and substituting for  $y$ , we get that the ratio of job-to-job to employment-to-unemployment transitions,  $r_{EE}$ , is

$$r_{EE} = \frac{m_{EE}}{m_{EU}} = \frac{(1+y) \log(1+y) - y}{y}. \quad (9)$$

This expression states that the ratio of job-to-job to employment-to-unemployment transitions is uniquely determined by the ratio of contact probabilities,  $\lambda_e/\lambda_u$ , and the unemployment rate. The other parameters of the model influence  $r_{EE}$  only through their impact on the unemployment rate. This is a very powerful result, since, given that  $r_{EE}$  and  $u$  are observable, it allows one to calculate the relative contact probability  $\lambda_e/\lambda_u$  that is consistent with the data without needing to specify any other aspect of the model, which could be very rich, conditional on the above stated assumptions holding.

To fully appreciate and understand the above result, the following observations are useful. First, the expression in Equation (9) holds independently of the distribution function  $F$ , which itself could be endogenously determined by firms' optimizing decisions. In the model, what matters for the determination of worker transitions is the rank of an offer in the

distribution  $F$ , not its actual value. Since ranks are, by definition, uniformly distributed, the independence from the functional form of  $F$  follows.

Second, Equation (9) states that  $r_{EE}$  is a strictly increasing and strictly concave function of  $y$ , and hence of  $\lambda_e/\lambda_u$ . The intuition for this is as follows. If all new job opportunities were accepted, then  $r_{EE}$  would be in fact equal to  $y$ , since then the relative transition rates would be equal to the relative contact rates. In the basic job ladder model, however, not all contacts result in a transition into a new job. Indeed, the higher an employed worker is in the  $F$  distribution, the less likely she is to accept a new offer. With the unemployment rate held constant, increasing  $\lambda_e/\lambda_u$  in this model has two effects. For a given distribution of workers, which pins down the average acceptance probability of employed workers, it increases  $r_{EE}$  one-to-one. But when the relative contact rate of employed workers increases, the distribution of employed workers becomes more highly concentrated in the upper tail of the  $F$  distribution, thereby making employed workers less likely to accept a new offer. This decrease in the acceptance rate has a negative impact on  $r_{EE}$ . Moreover, the higher the relative contact rate  $\lambda_e/\lambda_u$  is, the more pronounced this negative effect is, resulting in  $r_{EE}$  being concave in  $\lambda_e/\lambda_u$  for a fixed value of  $u$ . This explains why, given an unemployment rate of 5%, the value of  $\lambda_e/\lambda_u$  needed to explain  $r_{EE} = 1$  is only 0.206, while the value needed to explain  $r_{EE} = 2$  is 0.831. Figure 1 plots the implied value of  $r_{EE}$  as a function of  $\lambda_e/\lambda_u$  for different unemployment rates. As is clear from Equation (9),  $r_{EE}$  is decreasing in the unemployment rate for a given value of relative contact rates.

Third, given Equation (1), an alternative way to write down a relationship between  $r_{EE}$  and  $y$  is

$$r_{EE} = \frac{\lambda_e A_e (1 - u)}{\lambda_u A_u u} = \frac{A_e}{A_u} y. \quad (10)$$

Therefore observations on  $r_{EE}$  and  $u$  also allow one to derive the value  $A_e/A_u$  implied by Equations (9) and (10).

In Table 1, I perform the calculation of the values of  $\lambda_e/\lambda_u$  and  $A_e/A_u$  implied by the model for five education groups, based on two alternative measures of  $r_{EE}$  for prime age workers (between age 25 and 60) from the Current Population Survey matched across two subsequent months. The first measure (reported in panel a) of Table 1) is derived from the Current Population Survey between 1994 and 2004. It identifies workers undergoing a job-to-job transition as those workers who are employed in the two consecutive months and report to be working with a different employer in the second month than their employer in the first month. It identifies employment-to-unemployment transitions based on the reported labor-market status of the worker in the two subsequent months. This measure of  $r_{EE}$  suffers from two measurement problems. On the one hand, the calculated job-to-job transition rate overstates the true extent of job-to-job transitions to the extent that it incorrectly labels as a job-to-job transition those transitions where a worker had a short unemployment spell that both started and ended between the two interview dates. Correspondingly, the calculated employment-to-unemployment transition rate understates the true extent of such transitions. On the other hand, the calculated job-to-job transition rate understates the true extent of

a) Based on first measure of  $r_{EE}$  (see text) and corresponding unemployment rates from the Current Population Survey between 1994 and 2004.

	observed $u$	observed $r_{EE}$	implied $y$	implied $\lambda_e/\lambda_u$	implied $A_e/A_u$
High school dropout	8.60%	1.25	5.9	0.56	0.21
High school graduate	4.63%	1.88	13.7	0.67	0.14
Some college	3.66%	2.38	24.8	0.94	0.10
College graduate	2.45%	3.48	82.7	2.08	0.04
Advance degree holder	1.91%	4.50	238.2	4.64	0.02

b) Based on second measure of  $r_{EE}$  (see text) and corresponding unemployment rates from the December 2003 Basic Monthly Survey and the January 2004 Job Tenure Supplement of the CPS.

	observed $u$	observed $r_{EE}$	implied $y$	implied $\lambda_e/\lambda_u$	implied $A_e/A_u$
High school dropout	11.00%	2.14	18.5	2.29	0.12
High school graduate	5.39%	2.86	42.5	2.42	0.07
Some college	4.46%	2.57	31.0	1.45	0.08
College graduate	3.03%	2.71	36.0	1.12	0.08
Advance degree holder	2.40%	3.07	53.5	1.32	0.06

Table 1: Calculation of relative contact rates and acceptance rates implied by the basic job-ladder model.

job-to-job transitions since it ignores transitions where the worker has a new job lined up at the time she stops working for her old employer but decides not to start that job immediately and is recorded as not in the labor force in the second month. From the point of view of the theory, such transitions should be counted as job-to-job transitions. Nagypál (2005b) argues that this second source of bias is at least as important quantitatively as the first bias more widely discussed in the literature and devises a statistical model to eliminate these biases. The preliminary findings of that paper imply that on net, if anything, the raw measure of  $r_{EE}$  understates somewhat the true measure.

The second measure (reported in panel b) of Table 1) is calculated by matching the December 2003 Basic Monthly Survey and the January 2004 Job Tenure Supplement in the CPS. This measure identifies  $r_{EE}$  by the ratio of employed to unemployed workers in December among workers with less than one month of tenure in January. Clearly, this measure also suffers from the same measurement issues discussed above. Moreover, it is based on a much smaller sample of workers, so should be treated with more caution than the first measure. Interestingly, this second measure of  $r_{EE}$  shows much less variation with education than the first one, even though their average levels are the same.

The calculations based on the first measure imply that, in order for the basic job-ladder model to match observed levels of job-to-job transitions, less-educated employed workers need to contact new job opportunities roughly two thirds as often as their unemployed counterparts and need to accept new offers at a rate that is five times lower than the acceptance rate of unemployed workers. For the two most educated groups, there is an even more severe problem: in order to explain the extent of job-to-job transitions, employed workers need to contact new job opportunities at least twice as often as unemployed workers do and accept new offers at a rate more than twenty times lower than the acceptance rate of unemployed workers. These numbers are greatly at odds with the low fraction of employed workers that report to be actively searching on the job cited above.<sup>7</sup> They are also inconsistent with the much higher empirical estimates of  $A_e/A_u$  discussed in Section 4. The implied  $\lambda_e/\lambda_u$  ratios are also much higher and the implied  $A_e/A_u$  ratios are much lower than intuition would suggest, and are higher than the rates that have been used to calibrate models of job-to-job transitions. For example, Mortensen (1994) uses a value 0.2 for  $\lambda_e/\lambda_u$ .

The calculations based on the second measure tell a similar story, albeit with this measure it is less educated workers that present a larger puzzle for the model. Based on the second measure, all education groups have implied relative contact rates above 1 and relative acceptance rates below 12%. As I argue below, a relative contact rate of 1 is the maximum that can be accommodated by the basic job-ladder model once one allows workers to make optimal search effort decisions.

In conclusion, these simple calculations imply that the basic job-ladder model cannot account for the extent of job-to-job transitions under empirically plausible parameter values. These simple calculations have significant implications, since all existing models of on-the-job search (including the work of Burdett and Mortensen (1998), Burdett and Coles (2003), Pissarides (1994), Mortensen (1994), Barlevy (2002), and Barlevy (2003)) are either a particular specification of the basic job-ladder model or one of the generalizations that I turn to next.

## 4.2 More general specifications

In this section, I examine what happens to the fit of the model if some parts of the model are further specified or some extensions are allowed. Notice that, conditional on the basic environment, the part of the model that needed to be specified to derive the benchmark result of the previous section consists of three elements. The first element is optimizing behavior of workers which implies that when workers make job-to-job transitions they accept jobs that increase their present discounted utility. The second element concerns contact rates

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<sup>7</sup>A common response to such observations by economists is to doubt whether workers correctly report whether they are undertaking job search. This criticism is a double-edged sword in this case, since if one refuses to believe that workers correctly report whether they are undertaking job search, that makes all measures of unemployment meaningless, since the definition of unemployment is itself based on reported job search. In fact, several studies have established that reporting to be searching for a (new) job is a good predictor of making a transition into one (Jones and Riddell (1999), Fallick and Fleischman (2004)).

which in the benchmark case were assumed to be constant given the employment status of a worker. The third element was the assumption that the present discounted value of a worker while remaining on the same job stays the same. In this section, I will maintain the assumption of optimizing behavior and scrutinize the other two elements. (In a recent paper, Jolivet, Postel-Vinay, and Robin (2004) depart from the assumption of optimizing behavior and introduce forced job-to-job transitions in order to match the data.)

### 4.2.1 Contact rates

**Contact rates generated by a matching function** First, consider what happens when, as it is commonly assumed, contact rates are generated by a matching function. Assume that the number of matches created per unit of time is a function of the number of vacancies in the economy,  $v$ , and the amount of aggregate search effort,  $\bar{e}$ :  $m = m(v, \bar{e})$ . Assume moreover, that unemployed and employed workers exert exogenous search effort  $e_u$  and  $e_e$ , respectively. Then  $\lambda_e = \frac{m(v, \bar{e})}{\bar{e}} e_e$ , and  $\lambda_u = \frac{m(v, \bar{e})}{\bar{e}} e_u$ , and the result in Equation (9) applies unaltered, with the additional restriction that  $\frac{\lambda_e}{\lambda_u} = \frac{e_e}{e_u}$ . Hence in order to explain observed ratios of job-to-job to employment-to-unemployment transitions, for some education groups we need to posit that employed workers search more intensively than unemployed workers, which is greatly at odds with the data on employed search.

Notice that, while the above calculations may seem trivial, assuming a matching function of the above form in the standard matching model (generally considered to be the Mortensen and Pissarides (1994)) imposes a serious restriction on the job-ladder model. In particular, it implies that even if firms have a larger incentive to hire already employed workers, this does not translate into a higher contact rate for employed workers per unit of search effort. This is because firms in the standard matching model are always willing to form an employment relationship upon contact.

**Endogenous search intensity** A second important extension regarding contact rates is to allow not only these rates to depend on the search effort of workers but also to allow workers to optimally choose their search effort level. This is a very natural extension to consider given the abundance of empirical evidence that shows that search effort influences contact probability (Jones and Riddell (1999), Fallick and Fleischman (2004)).

Assume that all other elements of the model unaltered and assume that workers can choose their search effort  $e$  at cost  $c(e)$ , where  $c(\cdot)$  is a strictly increasing and strictly convex function with  $c(0) = 0$  (independent of the employment state). Let the offer arrival rate per unit of search intensity be  $\lambda$ , which, if a matching function is evoked as above, equals  $\frac{m(v, \bar{e})}{\bar{e}}$ . In this model, the search effort of unemployed workers is  $e_u$  and the search effort of employed searchers is simply a function  $e(\cdot)$  of the job value  $V$ . The function  $e(\cdot)$  can be shown to be a strictly decreasing function. Moreover, since an unemployed worker has the same expected discounted value and the same opportunities for search as a worker on a marginal job with  $V = R$ , we get that  $e(R) = e_u$ .

In this model, the distribution of employed workers can again be derived using balance equations equating the flow into and out of  $G(V)$ . The flow into  $G(V)$  per unit of time still consists of unemployed workers who contact a job with a value that is above the reservation value  $R$  but below  $V$ :

$$\lambda e_u (F(V) - F(R))u. \quad (11)$$

The flow out of  $G(V)$  per unit of time still comprises of two parts, those workers that separate into unemployment and those workers that find a job with value above  $V$ , where in the latter we need to take into account that workers at different values search with different intensity:

$$\delta G(V) + \lambda \bar{F}(V) \int_R^V e(V') dG(V'). \quad (12)$$

First, since  $\delta G(V) + \lambda \bar{F}(V) \int_R^V e(V') dG(V') \leq \delta G(V) + \lambda \bar{F}(V) e(R) G(V)$ , this implies that

$$G(V) \geq \frac{\lambda e_u (F(V) - F(R))u}{\delta + \lambda e(R) \bar{F}(V)}. \quad (13)$$

Second, equating the flows directly gives

$$\lambda e_u (F(V) - F(R))u = \delta G(V) + \lambda \bar{F}(V) \int_R^V e(V') dG(V'). \quad (14)$$

Differentiating with respect to  $V$  and rearranging gives

$$\lambda \bar{F}(V) e(V) g(V) = \lambda e_u u f(V) - \delta g(V) + f(V) \frac{\lambda e_u (F(V) - F(R))u - \delta G(V)}{\bar{F}(V)}. \quad (15)$$

Then the measure of job-to-job transitions is

$$\begin{aligned} m_{EE} &= \int_R^{\bar{V}} \lambda e(V) \bar{F}(V) g(V) dV = \int_R^{\bar{V}} \left[ \lambda e_u u f(V) - \delta g(V) + f(V) \frac{\lambda e_u (F(V) - F(R))u - \delta G(V)}{\bar{F}(V)} \right] dV \quad (16) \\ &\leq \lambda e_u u \bar{F}(R) - \delta(1-u) + \int_R^{\bar{V}} \frac{\lambda e_u (F(V) - F(R))u - \delta \frac{\lambda e_u (F(V) - F(R))u}{\delta + \lambda e(R) \bar{F}(V)}}{\bar{F}(V)} dF(V) = \\ &= \int_R^{\bar{V}} \frac{(\lambda e_u)^2 (F(V) - F(R))u}{\delta + \lambda e_u \bar{F}(V)} dF(V) = \lambda e_u u \int_R^{\bar{V}} \left[ \frac{\delta + \lambda e_u \bar{F}(R)}{\delta + \lambda e_u \bar{F}(V)} - 1 \right] dF(V) = \\ &= \lambda e_u u \int_{\bar{x}}^1 \left[ \frac{\delta + \lambda e_u (1 - \bar{x})}{\delta + \lambda e_u (1 - x)} - 1 \right] dx = u \delta \left[ \left( 1 + \frac{\lambda e_u (1 - \bar{x})}{\delta} \right) \log \frac{\delta + \lambda e_u (1 - \bar{x})}{\delta} - \lambda e_u (1 - \bar{x}) \right], \end{aligned}$$

where I have first used the inequality in (13), substituted in  $e(R) = e_u$ , then used that  $\lambda e_u u \bar{F}(R) = \delta(1 - u)$  (flow into and out of unemployment are equal in steady state), and finally used a change of variables  $x = F(V)$  with  $\bar{x} = F(R)$ .

In this model,  $u = \frac{\delta}{\delta + \lambda e_u(1 - \bar{x})}$  so  $\frac{1-u}{u} = \frac{\lambda e_u(1 - \bar{x})}{\delta}$ . Let  $\tilde{y} = \frac{1-u}{u} = \frac{\lambda e_u(1 - \bar{x})}{\delta}$ . Given the measure of employment-to-unemployment transitions and substituting for  $\tilde{y}$ , we get that the ratio of job-to-job to employment-to-unemployment transitions,  $r_{EE}$ , is

$$r_{EE} = \frac{m_{EE}}{m_{EU}} \leq \frac{(1 + \tilde{y}) \log(1 + \tilde{y}) - \tilde{y}}{\tilde{y}}. \quad (17)$$

Notice that  $\tilde{y}$  is nothing more than  $y$  when  $\lambda_e/\lambda_u = 1$ . In this extension, employed workers never choose to have a higher contact rate than unemployed workers, simply because they have less of an incentive to search. This restricts the values of  $\lambda_e/\lambda_u$  to be less than 1. This realistic extension of the model thus cannot match the observed data on more educated workers for any parameter values.

Notice that above I assumed that the search technology available for employed and unemployed workers is the same. Many authors have argued that one way to interpret the empirical evidence in Blau and Robbins (1990) reviewed in Section 3 is to assume that employed workers have access to a better search technology than unemployed workers. One natural justification for this could be that employed workers encounter job opportunities in their day-to-day business. This is an appealing assumption, and one that deserves further study, but it is important to note that without alterations in the other parts of the model, a model with this assumption will still imply an implausibly low level of relative acceptance rates if it is to match the observed extent of job-to-job transitions.

#### 4.2.2 Job values

Next, I consider generalizations regarding the assumption on the value of a job to a worker. Recall that a key assumption of the model was that the value of the job remains the same over the life of the job. I will now turn to alternatives to this assumption.

**Growth of job value with tenure** Several authors have developed models that are versions of the basic job-ladder model with the extension that the value of a job to the worker grows with tenure. This could be due to delayed compensation that arises in an optimal contract in the presence of risk-averse workers and on-the-job search, as in Burdett and Coles (2003). It could also be motivated by the desire to allow for the accumulation of job-specific capital, as in Pissarides (1994).

When the value of a job is allowed to increase with tenure on the job, once again, it is possible to show (see Appendix) that

$$r_{EE} < \frac{(1 + y) \log(1 + y) - y}{y}, \quad (18)$$

where  $y$  is as defined above. This result is rather intuitive, since growth of job value with tenure discourages job-to-job transitions. With such growth, a worker becomes more and more selective with tenure as to what job offers to accept. Correspondingly, the rate of job-to-job transitions declines.

**Shocks to job value** The above result implies that a potential way to resolve the contradiction between the data and the model is to consider occasional declines in job value. If such declines in job value take place without triggering a transition into unemployment, then they could increase the incentive of workers in the model to make job-to-job transitions, thereby increase the extent of such transitions.

A simple way to formalize this idea is along the lines of Mortensen and Pissarides (1994) (which is the model extended to incorporate on-the-job search by Mortensen (1994)). Consider the following extension of the benchmark case. Workers draw job opportunities from the distribution  $F(V)$ . Assume that unemployed and employed workers contact job opportunities at rate  $\lambda_u$  and  $\lambda_e$ , respectively. While employed, a new value of  $V$  is drawn from  $F$  at rate  $\phi$ . Let the reservation value of unemployed workers be  $R$ . Then the flow into  $G(V)$  consists of the unemployed workers as before and the employed workers who had a value above  $V$  but who suffered a  $\phi$  shock and drew a new value between  $R$  and  $V$ :

$$[\lambda_u u + \phi(G(\bar{V}) - G(V))] (F(V) - F(R)). \quad (19)$$

The flow out of  $G(V)$  has a component due to  $\phi$  shocks and a component due to on-the-job search

$$\phi (\bar{F}(V) + F(R)) G(V) + \lambda_e \bar{F}(V) G(V). \quad (20)$$

Given that  $G(\bar{V}) = 1 - u$ , this gives

$$G(V) = \frac{[\lambda_u u + \phi(1 - u)] (F(V) - F(R))}{\phi + \lambda_e \bar{F}(V)}. \quad (21)$$

The measure of job-to-job transitions is then

$$\begin{aligned} m_{EE} &= \int_R^{\bar{V}} \lambda_e \bar{F}(V) dG(V) = \lambda_e \left[ [\bar{F}(V) G(V)]_R^{\bar{V}} - \int_R^{\bar{V}} G(V) d\bar{F}(V) \right] = \lambda_e \int_R^{\bar{V}} G(V) dF(V) = \quad (22) \\ &= \lambda_e [\lambda_u u + \phi(1 - u)] \int_R^{\bar{V}} \frac{F(V) - F(R)}{\phi + \lambda_e (1 - F(V))} dF(V) = \lambda_e [\lambda_u u + \phi(1 - u)] \int_{\bar{x}}^1 \frac{x - \bar{x}}{\phi + \lambda_e (1 - x)} dx, \end{aligned}$$

where I have used a change of variables  $x = F(\mu)$  with  $\bar{x} = F(R)$ . Then

$$m_{EE} = \frac{\phi [\lambda_u u + \phi(1 - u)]}{\lambda_e} \left[ \left( 1 + \frac{\lambda_e (1 - \bar{x})}{\phi} \right) \log \left( 1 + \frac{\lambda_e (1 - \bar{x})}{\phi} \right) - \frac{\lambda_e (1 - \bar{x})}{\phi} \right]. \quad (23)$$

Notice that in this model equating the flow into and out of unemployment gives

$$u = \frac{\phi F(R)}{\phi F(R) + \lambda_u \bar{F}(R)} = \frac{\phi \bar{x}}{\phi \bar{x} + \lambda_u (1 - \bar{x})}. \quad (24)$$

Let  $y = \frac{\lambda_e(1-\bar{x})}{\phi \bar{x}} = \frac{\lambda_e}{\lambda_u} \frac{\lambda_u(1-\bar{x})}{\phi \bar{x}} = \frac{\lambda_e}{\lambda_u} \frac{1-u}{u}$ , as before. Then, since  $\frac{u}{1-u} = \frac{\phi \bar{x}}{\lambda_u(1-\bar{x})}$  and  $m_{EU} = \phi \bar{x}(1-u)$ , with appropriate substitutions we get that the ratio of job-to-job to employment-to-unemployment transitions is

$$r_{EE} = \frac{m_{EE}}{m_{EU}} = \frac{1}{\bar{x}} \frac{(1 + y\bar{x}) \log(1 + y\bar{x}) - y\bar{x}}{y\bar{x}}. \quad (25)$$

It is easy to show that this object is decreasing in  $\bar{x}$ , hence, for a given unemployment rate, a higher  $r_{EE}$  results if the probability that a shock leads to unemployment,  $\bar{x}$ , is smaller and the arrival rate of shocks,  $\phi$ , is larger (which is necessary to maintain the same unemployment rate). The case with no changes in  $V$  corresponds to the  $\bar{x} = 1$  case here (in terms of the magnitude of job-to-job transitions). An application of l'Hôpital's rule gives that as  $\bar{x} \rightarrow 0$ ,

$$r_{EE} \rightarrow \lim_{\bar{x} \rightarrow 0} \frac{(1 + y\bar{x}) \log(1 + y\bar{x}) - y\bar{x}}{y\bar{x}^2} = \lim_{\bar{x} \rightarrow 0} \frac{y \log(1 + y\bar{x})}{2y\bar{x}} = \lim_{\bar{x} \rightarrow 0} \frac{y^2}{2y} = \frac{y}{2}. \quad (26)$$

The intuition for this limit result is clear. As the probability that a shock leads to separation goes to zero, in order to maintain a given unemployment rate, the arrival rate of shocks has to be converging to infinity. This means that workers are experiencing these reallocation shocks all the time, implying that the expected position of the worker is at the middle of the distribution. Since a worker at the middle of the distribution has an acceptance rate of 50%, while, as  $\bar{x} \rightarrow 0$ , an unemployed worker has a 100% acceptance rate, the above formula follows.

Clearly, the introduction of shocks to the value of the relationship along these lines helps resolve the puzzle outlined in Section 4.1. In Table 2, I present two simple calculations based on the above formulas. In panel a), I assume a relative contact rate of 50% and, using Equation (25), calculate the value of  $\bar{x}$  and  $\phi$  that is consistent with observed values of  $r_{EE}$ ,  $m_{EU}$ , and unemployment rate from the CPS. Recall that  $\bar{x}$  is the probability of separation upon the arrival of a shock to the value of the job, while  $\phi$  determines the rate at which shocks arrive to the job value. These calculations explain the much lower transition rate of more educated workers into unemployment by positing that such workers experience shocks to their job value almost as often as less educated workers (though high school dropouts are an exception, since they experience distinctly higher arrival rate of shocks), but such shocks lead to a separation into unemployment with a much lower probability.

In panel b), I use Equation (26) to calculate the lowest relative contact rate (the one corresponding to  $\bar{x} = 0$ ) that could possibly be consistent with the data given the above model. Interestingly, the lowest relative contact rate consistent with the data is quite similar across groups of workers, around 18%, once again with the exception of high-school dropouts.

a) Implied separation probability upon shock arrival and shock arrival probability in the job-ladder model with Poisson shocks to the value of a job assuming a relative contact rate of 50%.

	observed $u$	observed $r_{EE}$	observed $m_{EU}$	assumed $\lambda_e/\lambda_u$	implied $\bar{x}$	implied $\phi$
High school dropout	8.60%	1.25	2.30%	0.5	0.88	2.61%
High school graduate	4.63%	1.88	1.24%	0.5	0.78	1.59%
Some college	3.66%	2.38	0.98%	0.5	0.63	1.56%
College graduate	2.45%	3.48	0.65%	0.5	0.44	1.48%
Advance degree holder	1.91%	4.50	0.46%	0.5	0.34	1.35%

b) Lowest value of relative contact rate consistent with observed data in the job-ladder model with Poisson shocks to the value of a job.

	observed $u$	observed $r_{EE}$	implied minimum $\lambda_e/\lambda_u$
High school dropout	8.60%	1.25	0.238
High school graduate	4.63%	1.88	0.185
Some college	3.66%	2.38	0.184
College graduate	2.45%	3.48	0.179
Advance degree holder	1.91%	4.50	0.181

Table 2: Calculation of implied separation probability upon shock arrival in the job-ladder model with Poisson shocks to the value of a job assuming a relative contact rate of 50%.

Despite the significant improvement that this extension provides over the basic job-ladder model, it does not give a satisfactory explanation of all the stylized facts outlined in Section 3. Its most important shortcoming is that it does not account for the fact that marginal workers are both more likely to be searching on the job and more likely to experience a transition into unemployment. In fact, in the above model there is no search effort decision (which is clearly an important dimension of the data given facts #3 and #4) and all workers are equally likely to transit into unemployment (contradicting fact #4). In the remainder of this paper, I turn to developing a model that overcomes these shortcomings.

## 5 An alternative model of job-to-job transitions

Motivated by the above findings, in the remainder of this paper, I develop a model of job-to-job transitions that has the capability to better match the stylized facts of Section 3. To be able to explain the findings on the extent of employed search, I allow for endogenous search effort. I also allow for two types of shocks to affect the value of a job. Small, diffusion-driven shocks allow the model to generate differences in the probability of separation for different values of a job. In the model, marginal workers — those with a job value close to the reservation value — are more likely to separate from their employers. This is in stark contrast with the Mortensen and Pissarides (1994) specification for the stochastic productivity process, where the probability of separation of workers is independent of their current productivity. Together with endogenous search intensity, this feature of the model allows me to match the observation that marginal workers are more likely both to search for a new job and to experience a separation into unemployment. It turns out that the introduction of small diffusion-driven shocks with a non-negative drift is not sufficient to match the observed extent of job-to-job transitions. Therefore, I allow for large Poisson shocks to the value of the match along the lines discussed in Section 4.2.2. The interaction of the two types of shocks produces a model with desirable properties, as I show to be the case in Section 6, where I study the quantitative performance of the proposed model.

### 5.1 Model environment

Consider an infinite-horizon economy populated by ex-ante identical workers and firms. There is a unit measure of workers who can be either employed or unemployed. The objective of workers is to maximize

$$E_0 \int_{t=0}^{\infty} e^{-rt} u_t dt, \tag{27}$$

where

$$u_t = \begin{cases} w_t - c(e_t) & \text{if employed,} \\ b - c(e_t) & \text{if unemployed.} \end{cases} \tag{28}$$

Here  $w_t$  is the wage received when employed at time  $t$ . In addition, workers (both unemployed and employed) can choose to engage in search for a new job at a flow cost of  $c(e_t)$ , where  $c(\cdot)$  is a strictly increasing, strictly convex, twice continuously differentiable function with  $c(0) = c'(0) = 0$ . Workers exerting search effort  $e_t$  encounter new job opportunities at Poisson rate  $\lambda_t e_t$ , where  $\lambda_t$  is the meeting rate per unit of search intensity. Finally,  $b$  denotes the constant utility flow (derived from leisure and/or from unemployment insurance benefits) that a worker receives while unemployed.

There is a large measure of ex-ante identical firms. Any firm's objective is to maximize

$$E_0 \int_{t=0}^{\infty} e^{-rt} \pi_t dt, \quad (29)$$

where

$$\pi_t = \begin{cases} 0 & \text{if the firm is inactive} \\ -\kappa & \text{if the firm is active with a vacant job} \\ p_t - w_t & \text{if the firm is active with a filled job.} \end{cases} \quad (30)$$

This specification captures the idea that any firm can enter the market and become active by posting a vacancy at flow cost  $\kappa$ . If a firm posts a vacancy, then it participates in the matching market for creating new matches. After a firm creates a match, it receives a flow profit of  $p_t - w_t$  until the match dissolves. Here  $p_t$  is the idiosyncratic productivity of the match, which follows a Brownian motion

$$dp_t = \mu dt + \sigma dW_t, \quad (31)$$

where  $W$  is a Wiener process and  $\sigma > 0$ . The initial productivity of the match is drawn from the distribution  $F : [\underline{p}, \bar{p}] \rightarrow [0, 1]$ .

Matches dissolve for one of three reasons. First, the worker and the firm can decide to terminate the relationship at any time, which, as I show below, happens if the productivity of the match becomes low enough. Second, a match is terminated when the worker decides to form a new employment relationship as a consequence of on-the-job search. Third, matches occasionally experience a discrete change in their productivity. This is represented by assuming that a new value of  $p$  is drawn from the distribution  $\tilde{F} : [\underline{p}, \bar{p}] \rightarrow [0, 1]$  at Poisson rate  $\phi$  after which the Brownian motion restarts from the new value. Such new draws occasionally lead to separations, if the new draw is low enough. If  $\mu > 0$ , then such a feature is necessary to ensure that a stationary distribution exists.

Wages are determined by the sharing of the surplus from the relationship (as in Mortensen (1994) and Pissarides (1994)), where workers and firm receive share  $\beta$  and  $(1 - \beta)$ , respectively. It is assumed that the workers' outside option is unemployment, which arises if matches cannot be "recalled" and there is scope for renegotiation between the worker and the firm immediately after the worker has moved from one employer to another employer.

Notice that for the surplus and hence surplus sharing to be well defined, the necessary assumption is that at each instant first the worker decides how much to search in anticipation of the wage outcome and then the sharing of the surplus takes place. In other words, sharing of the surplus does not allow for the wage to be used to influence the search behavior of the worker. This is the principle shortcoming of this wage-setting mechanism, and it explains why in models with on-the-job search, sharing of the surplus is not equivalent to Nash bargaining or strategic bargaining with alternating offers.<sup>8</sup>

There is a single meeting market with a meeting function determining the number of meetings ( $m_t$ ) as a function of the total amount of search effort of workers ( $\bar{e}_t$ ) and the number of vacancies posted ( $v_t$ ):

$$m_t = m(\bar{e}_t, v_t), \quad (32)$$

such that  $m_e(\bar{e}, v) > 0$ ,  $m(0, v) = 0$  for any  $v$ ,  $m_v(\bar{e}, v) > 0$ ,  $m(\bar{e}, 0) = 0$  for any  $\bar{e}$ . I assume that  $m(\bar{e}, v)$  has constant returns to scale, so that the meeting rate per unit of search effort for workers can be written as

$$\lambda_t = \lambda(\theta_t) = \frac{m(\bar{e}_t, v_t)}{\bar{e}_t} = m\left(1, \frac{v_t}{\bar{e}_t}\right) = m(1, \theta_t), \quad (33)$$

where  $\theta_t = \frac{v_t}{\bar{e}_t}$  is market tightness in this model at time  $t$ . Similarly, the meeting rate for firms can be written as

$$\eta_t = \eta(\theta_t) = \frac{m(\bar{e}_t, v_t)}{v_t} = \frac{\frac{m(\bar{e}_t, v_t)}{\bar{e}_t}}{\frac{v_t}{\bar{e}_t}} = \frac{\lambda(\theta_t)}{\theta_t}. \quad (34)$$

## 5.2 Definition of stationary equilibrium

To keep the analysis tractable, I consider the stationary equilibrium of the above model. The stationary equilibrium is characterized by a constant unemployment rate and vacancy rate and a constant distribution of workers across productivities. Hence the meeting rates are constant in a stationary equilibrium, so that the only state variable that determines the payoffs of workers and firms is the idiosyncratic productivity  $p$ .

I let the value of unemployment be  $U$ , the value of a worker employed in a match of productivity  $p$  be  $W(p)$ , the value of a vacancy for the firm be  $V$ , and the value of an employment match of productivity  $p$  for the firm be  $J(p)$ .

**Definition** A recursive stationary search equilibrium is unemployment rate  $u$ , vacancy rate  $v$ , total search effort  $\bar{e}$ , asset values  $\{U, V, W(p), J(p)\}$ , wage function  $w(p)$ , unemployed searchers' effort  $e_u$ , employed searchers' effort function  $e(p)$ , and a distribution of employed workers  $G(p)$ , such that

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<sup>8</sup>Shimer (2006) is the first attempt to think about wage setting with on-the-job search in a strategic bargaining framework. Expanding his framework to incorporate a stochastic idiosyncratic productivity process is beyond the scope of this paper, however.

1.  $U$  and  $W(\cdot)$  are the value of unemployment and of working for workers making optimal searching and matching decisions, given  $u, v, \bar{e}, w(\cdot)$ , and  $G(\cdot)$ .  $e_u$  and  $e(\cdot)$  are the corresponding optimal search effort policies.
2.  $V$  and  $J(\cdot)$  are the value of a vacancy and of a filled job for firms making optimal vacancy creation decisions, given  $u, v, \bar{e}, w(\cdot), e_u, e(\cdot)$ , and  $G(\cdot)$ .
3. There is free entry of vacancies.
4. Wages are set by sharing of the surplus of the employment relationship in fraction  $\beta$  and  $1 - \beta$  given  $e(p)$ .
5. The distribution  $G(\cdot)$ , the unemployment rate  $u$ , the vacancy rate  $v$ , and the total search effort  $\bar{e}$  are consistent with the decisions of the agents in the economy.

## 5.3 Characterization of optimal policies

### 5.3.1 Deriving the surplus function

The Hamilton-Jacobi-Bellman equation of an employed worker with current productivity  $p$  is

$$rW(p) = \max_{e \geq 0} \left\{ w(p) + \phi \int_{\underline{p}}^{\bar{p}} (\max [W(p'), U] - W(p)) d\tilde{F}(p') - c(e) + \right. \\ \left. + \lambda(\theta)e \int_{\underline{p}}^{\bar{p}} I_{W(p') > W(p)} (W(p') - W(p)) dF(p') + \frac{E[dW]}{dt} \right\}, \quad (35)$$

where  $I$  is an indicator function. The flow payoff of a worker with productivity  $p$  is determined by several factors. First, the worker receives wage  $w(p)$ . Second, the match draws a new value of productivity at rate  $\phi$ , in which case the worker loses the current asset value  $W(p)$  and gains the asset value associated with working at the new productivity level  $p'$  or being unemployed, whichever is greater. Third, the worker optimally chooses her search effort at cost  $c(e)$  and enjoys the benefit of meeting new job opportunities at rate  $\lambda(\theta)e$ . If the worker makes a contact with a new job, optimizing behavior requires that she accept any job that has a higher asset value  $W(p')$  than the worker's current asset value  $W(p)$ . Finally, the asset value of the match is expected to change over time due to diffusion-induced changes in the underlying idiosyncratic productivity.

Similarly, given the search effort function of the worker, the Hamilton-Jacobi-Bellman equa-

tion of an employing firm with current productivity  $p$  is

$$\begin{aligned}
rJ(p) &= p - w(p) + \phi \int_{\underline{p}}^{\bar{p}} (\max [J(p'), V] - J(p)) d\tilde{F}(p') + \\
&+ \lambda(\theta)e(p) \int_{\underline{p}}^{\bar{p}} I_{W(p') > W(p)} dF(p') (V - J(p)) + \frac{E[dJ]}{dt}.
\end{aligned} \tag{36}$$

Similarly to the worker, the flow payoff of a firm with productivity  $p$  is determined by several factors. First, the firm receives flow revenue  $p - w(p)$ . Second, the match draws a new value of productivity at rate  $\phi$ , in which case the firm loses the current asset value  $J(p)$  and gains the asset value associated with employment at the new productivity level  $p'$  or having an open vacancy, whichever is greater. Third, the firm suffers a separation at rate  $\lambda(\theta)e(p) \int_{\underline{p}}^{\bar{p}} I_{W(p') > W(p)} dF(p')$  because the worker succeeds in making a job-to-job transition, in which case he loses asset value  $J(p) - V$ . Finally, the asset value of the match is expected to change over time due to diffusion-induced changes in the underlying idiosyncratic productivity.

The assumption of surplus-sharing implies that the incentives of the worker and the firm are aligned (conditional on the worker's search behavior), since both are making separation decisions to maximize the surplus of the relationship. It turns out that it is analytically more convenient to work with the surplus function defined as

$$S(p) = W(p) + J(p) - U - V. \tag{37}$$

Surplus sharing implies that

$$W(p) - U = \beta S(p) \tag{38}$$

$$J(p) - V = (1 - \beta)S(p). \tag{39}$$

Denoting the optimal search effort choice of an employed worker with a job of productivity  $p$  by  $e(p)$  and using surplus sharing, an equation characterizing  $S(p)$  can be derived by adding the Bellman for the worker in Equation (35) and for the firm in Equation (36):

$$\begin{aligned}
(r + \phi)S(p) &= p - rU - rV + \phi \int_{\underline{p}}^{\bar{p}} \max [S(p'), 0] d\tilde{F}(p') - c(e(p)) + \\
&+ \lambda(\theta)e(p) \int_{\underline{p}}^{\bar{p}} I_{S(p') > S(p)} (\beta S(p') - S(p)) dF(p') + \frac{E[dS]}{dt}.
\end{aligned} \tag{40}$$

This surplus equation clearly displays the negative impact that employed search has on the employing firm. The acceptance decision of the worker is such that any match whose total surplus is greater than the current surplus is accepted by the worker, even though the actual surplus accruing to the partners in the current relationship is only  $\beta S(p') - S(p)$ , since the surplus generated to the new employer  $(1 - \beta)S(p')$  is not taken into account.

### 5.3.2 Optimal search decision

**Lemma 1**  $S(p)$  is strictly increasing in  $p$ . As a result,  $W(p)$  and  $J(p)$  are strictly increasing in  $p$ . (Proof in the Appendix.)

It follows from Lemma 1 that an employed worker accepts all newly encountered jobs that have a higher initial productivity than their current job's productivity, and reject all other newly encountered jobs. Hence for  $p \geq \bar{p}$ ,  $\int_p^{\bar{p}} I_{W(p') > W(p)} (W(p') - W(p)) dF(p') = 0$ , and for  $p < \bar{p}$

$$\int_p^{\bar{p}} I_{W(p') > W(p)} (W(p') - W(p)) dF(p') = \int_p^{\bar{p}} [W(p') - W(p)] dF(p'). \quad (41)$$

Given (41) and the assumptions on  $c(\cdot)$ , a necessary and sufficient condition that the optimal search decision  $e(p)$  has to satisfy for  $p < \bar{p}$  is given by the first-order condition

$$c'(e(p)) = \lambda(\theta) \int_p^{\bar{p}} [W(p') - W(p)] dF(p') = \lambda(\theta)\beta \int_p^{\bar{p}} [S(p') - S(p)] dF(p'). \quad (42)$$

Given the strict convexity of  $c(\cdot)$  and the fact that  $S(p)$  is strictly increasing, this optimality condition implies that search effort by employed workers is strictly decreasing in  $p$  for  $p < \hat{p}$ . Moreover, given the convexity of  $c(\cdot)$  and  $c'(0) = 0$ ,  $e(p) = 0$  for  $p \geq \bar{p}$ . It is intuitive that there is no search above  $\bar{p}$ , since for such high productivity levels there exist no jobs with a higher initial productivity.

Given that  $W(p)$  is strictly increasing, the optimal acceptance policy of an unemployed worker is to accept any job with productivity  $\hat{p}$ , where  $\hat{p}$  is implicitly defined by

$$W(\hat{p}) = U \quad \iff \quad S(\hat{p}) = 0. \quad (43)$$

Hence the Bellman equation of an unemployed worker can be written as

$$rU = \max_{e \geq 0} \left\{ b - c(e) + \lambda(\theta)e \int_{\hat{p}}^{\bar{p}} (W(p') - U) dF(p') \right\}, \quad (44)$$

Denote the optimal search effort choice of an unemployed worker by  $e_u$ , where  $e_u$  satisfies

$$c'(e_u) = \lambda(\theta) \int_{\hat{p}}^{\bar{p}} (W(p') - U) dF(p') = \lambda(\theta)\beta \int_{\hat{p}}^{\bar{p}} S(p') dF(p'). \quad (45)$$

By comparing with the optimality condition in Equation (42), we can see that  $e_u = e(\hat{p})$ . This is rather intuitive, since an unemployed worker and an employed worker with productivity  $\hat{p}$  have the same current value and the same search technology, thereby implying that they have the same incentive to search.

### 5.3.3 Optimal separation decision

Given that  $S(p)$  is strictly increasing, the worker and the firm will find it optimal to sever a relationship for sufficiently low level of productivity. In particular, once the surplus drops below zero, the optimal policy dictates that a separation take place. By Equation (43), this happens at  $\hat{p}$ . The following Lemma establishes the existence of such a threshold and derives an additional optimality condition that needs to hold at  $\hat{p}$ .

**Lemma 2.** For sufficiently low  $\underline{p}$  (and appropriately scaled  $F$  and  $\tilde{F}$ ), there exists  $\hat{p} > \underline{p}$  such that  $S(\hat{p}) = 0$  and  $S'(\hat{p}) = 0$ . (Proof in the Appendix.)

The second condition is the so-called smooth pasting condition (see Dixit and Pindyck (1994)). Intuitively, it states that there can be no kink in the surplus function at  $\hat{p}$  because if there was a kink then a better policy could be constructed either by waiting a short period of time before deciding whether to separate or not (if  $S'(\hat{p}) > 0$ ) or by severing the relationship already for higher values of  $p$  (if  $S'(\hat{p}) < 0$ ).

### 5.3.4 Optimal vacancy creation

The Bellman equation of an active firm with an open vacancy is

$$rV = -\kappa + \eta(\theta) \int_{\underline{p}}^{\bar{p}} (J(p) - V) A(p) dF(p), \quad (46)$$

where  $A(p) : [\underline{p}, \bar{p}] \rightarrow [0, 1]$  is the probability that a searching worker accepts a job with productivity  $p$ . This is simply the ratio of search effort by workers who are willing to accept a match with initial productivity  $p$  to the total amount of search effort  $\bar{e}$  exerted by all workers. Since  $\bar{e}$  can be expressed as

$$\bar{e} = ue_u + (1 - u) \int_{\hat{p}}^{\bar{p}} e(p') dG(p'), \quad (47)$$

an expression for the acceptance probability is

$$A(p) = \begin{cases} 0 & \text{if } p < \hat{p} \\ \frac{ue_u + (1-u) \int_{\hat{p}}^p e(p') dG(p')}{ue_u + (1-u) \int_{\hat{p}}^{\bar{p}} e(p') dG(p')} & \text{if } p \geq \hat{p} \end{cases} \cdot \quad (48)$$

Free entry implies that the value of a vacancy is 0 in equilibrium, hence  $V = 0$ .

## 5.4 Solving for the surplus and the search effort function

The last term in (40) can be obtained from Itô's lemma:

$$\frac{E[dS]}{dt} = S'(p)\mu + \frac{1}{2}S''(p)\sigma^2. \quad (49)$$

Given  $V = 0$  implied by free entry, and given the optimal acceptance policy of employed workers, Equation (40) can be written as

$$\begin{aligned} (r + \phi)S(p) &= p - rU + \phi \int_{\hat{p}}^{\bar{p}} S(p')d\tilde{F}(p') - c(e(p)) + \\ &+ \lambda(\theta)e(p) \int_p^{\bar{p}} [\beta S(p') - S(p)] dF(p') + S'(p)\mu + \frac{1}{2}S''(p)\sigma^2. \end{aligned} \quad (50)$$

Totally differentiating this equation, using (38), (39), the optimality condition in (42), and collecting terms results in an ordinary differential equation for  $S(p)$

$$(r + \phi + \lambda(\theta)e(p)\bar{F}(p)) S'(p) = 1 + [e(p)f(p) - e'(p)\bar{F}(p)] \lambda(\theta)(1-\beta)S(p) + S''(p)\mu + \frac{1}{2}S'''(p)\sigma^2. \quad (51)$$

Notice that, given an effort function  $e(p)$ , this is a third-order linear differential equation in  $S(p)$ . Since its coefficients are variable, it does not generically have a finite closed-form solution. This is due to the presence of  $e(p)$  in the coefficients, which itself depends on  $S(p)$ . Even without this complication, however, for instance, assuming a constant search effort for employed workers, the differential equation would have variable coefficients due to the presence of the  $\bar{F}(p)$  term. This term depends on  $p$  for all specifications, since it is a survival function of a distribution. This means that in a model with on-the-job search and payoff uncertainty modelled using a diffusion process, no restrictions on functional forms exist that would result in a closed form solution.

Totally differentiating the optimality condition in (42) results in an ordinary differential equation for  $e(p)$

$$e'(p) = -\frac{\lambda(\theta)\beta S'(p)\bar{F}(p)}{c''(e(p))}. \quad (52)$$

We thus have a third-order differential equation for  $S(p)$  which requires three boundary conditions in order to be solved coupled with a first-order differential equation for  $e(p)$  which requires one boundary condition in order to be solved. The boundary condition for (52) is  $e(\bar{p}) = 0$ .

To derive boundary conditions for (51), let  $\tilde{S}(p)$  be the surplus associated with the policy of never endogenously separating and the worker not searching on the job prior to the arrival of the next  $\phi$  shock. Upon the arrival of this shock, assume that the policy reverts to the optimal one and thus reaps  $\bar{Q} = \int_{\bar{p}}^{\infty} S(p') d\tilde{F}(p')$ . The surplus resulting from this alternative policy can be expressed as

$$\begin{aligned}\tilde{S}(p_t) &= E_t \int_{T=0}^{\infty} \left[ \int_{\tau=0}^T (p_{t+\tau} - rU) e^{-r\tau} d\tau + e^{-rT} Q \right] \phi e^{-\phi T} dT = \\ &= E_t \int_{\tau=0}^{\infty} (p_t + \mu\tau - rU + \phi Q) e^{-(r+\phi)\tau} d\tau = \frac{p_t - rU + \phi Q + \frac{\mu}{r+\delta}}{r + \phi}.\end{aligned}\quad (53)$$

Notice that while the policy supporting  $\tilde{S}$  is never optimal for a finite  $p$ ,

$$\lim_{p \rightarrow \infty} \tilde{S}(p) = \lim_{p \rightarrow \infty} S(p), \quad (54)$$

since the probability that the worker and firm endogenously separate or that the worker chooses a positive search effort or makes a job-to-job transition goes to 0 as  $p$  goes to  $\infty$ .

Next, notice that for  $p \geq \bar{p}$ ,  $e(p) = 0$ , hence the Bellman equation in (50) determining the surplus function is

$$(r + \phi) S(p) = p - rU + \phi Q + S'(p)\mu + \frac{1}{2} S''(p)\sigma^2. \quad (55)$$

The solution to this differential equation is

$$S(p) = k_1 e^{a_1 p} + k_2 e^{a_2 p} + \frac{p - rU + \phi Q + \frac{\mu}{r+\delta}}{r + \delta}, \quad (56)$$

where

$$a_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2(r + \delta)\sigma^2}}{\sigma^2} \quad (57)$$

and  $k_1$  and  $k_2$  are two undetermined constants. Let  $a_1 < 0$  and  $a_2 > 0$ . The limit condition in Equation (54) implies that  $k_2 = 0$ . Therefore, for  $p \geq \bar{p}$ , the surplus function is

$$S(p) = k_1 e^{a_1 p} + \frac{p - rU + \phi Q + \frac{\mu}{r+\delta}}{r + \phi}. \quad (58)$$

The fact that  $S(p)$  is a twice continuously differentiable function gives three terminal conditions at  $\bar{p}$  as a function of  $k_1$ . In turn, we can use the two optimality conditions in Lemma 2 to solve for  $\hat{p}$  and  $k_1$ .

## 5.5 Equilibrium distribution of workers

### 5.5.1 Distribution of employed workers

Next, I turn to deriving the stationary distribution of employed workers across productivities,  $G(p)$ , with density  $g(p)$ . As I show in the Appendix, for  $\hat{p} \leq p \leq \bar{p}$ ,  $g(p)$  satisfies the differential equation

$$f(p)\lambda(\theta) \left( e_u \frac{u}{1-u} + \int_{\hat{p}}^p e(p')g(p')dp' \right) + \phi \tilde{f}(p) - (\phi + \lambda(\theta)e(p)\bar{F}(p))g(p) - \mu g'(p) + \frac{\sigma^2}{2}g''(p) = 0, \quad (59)$$

with  $g(\hat{p}) = 0$ . Totally differentiating with respect to  $p$  and rearranging results in a third-order differential equation, so given a search effort function  $e(p)$  and distribution functions  $F(p)$  and  $\tilde{F}(p)$ , we need three boundary conditions to solve it. The first is given by  $g(\hat{p}) = 0$ . The second is given by evaluating Equation (59) at  $\hat{p}$  and using  $g(\hat{p}) = 0$  to get

$$f(\hat{p})\lambda(\theta)e_u \frac{u}{1-u} + \phi \tilde{f}(\hat{p}) - \mu g'(\hat{p}) + \frac{\sigma^2}{2}g''(\hat{p}) = 0. \quad (60)$$

Finally, to get a third boundary condition, notice that for  $p > \bar{p}$ , there is no entry from unemployment, from on-the-job search, or from new draws of  $p$  and there is no exit into on-the-job search, so the density function satisfies the differential equation:

$$-\phi g(p) - \mu g'(p) + \frac{\sigma^2}{2}g''(p) = 0. \quad (61)$$

The solution to this differential equation is

$$g(p) = j_1 e^{d_1 p} + j_2 e^{d_2 p}, \quad (62)$$

where

$$d_{1,2} = \frac{-\mu \pm \sqrt{\mu^2 + 2\phi\sigma^2}}{\sigma^2} \quad (63)$$

and  $j_1$  and  $j_2$  are two undetermined constants. Let  $d_1 < 0$  and  $d_2 > 0$ . Since the total mass of matches above  $\bar{p}$  is finite, we need to have  $j_2 = 0$ , so  $g(p) = j_1 e^{d_1 p}$ . For a given value of  $j_1$ , the fact that  $g$  is continuous at  $\bar{p}$  results in a final boundary condition for Equation (59). In turn,  $j_1$  can be determined from the fact that  $g$  is a density function hence  $\int_{\hat{p}}^{\infty} g(p)dp = 1$ .

### 5.5.2 Equilibrium unemployment and job-to-job transitions

The equilibrium level of unemployment can be determined by noticing that the rate at which employment matches hit the separation barrier  $\hat{p}$  is  $\frac{\sigma^2}{2}g'(\hat{p})(1-u)$  per unit of time (shown

in the Appendix). Hence the evolution of the measure of unemployed is given by

$$\frac{du}{dt} = \left( \phi \tilde{F}(\hat{p}) + \frac{\sigma^2}{2} g'(\hat{p}) \right) (1 - u) - \lambda(\theta) u e_u \bar{F}(\hat{p}). \quad (64)$$

The stationary level of unemployment can be determined by setting  $\frac{du}{dt} = 0$ , which gives

$$u = \frac{\phi \tilde{F}(\hat{p}) + \frac{\sigma^2}{2} g'(\hat{p})}{\lambda(\theta) e_u \bar{F}(\hat{p}) + \phi \tilde{F}(\hat{p}) + \frac{\sigma^2}{2} g'(\hat{p})}. \quad (65)$$

In turn, the rate of job-to-job transitions per unit of time in this model is given by

$$m_{EE} = (1 - u) \int_{\hat{p}}^{\bar{p}} \lambda(\theta) e(p) \bar{F}(p) g(p) dp, \quad (66)$$

hence

$$r_{EE} = \frac{m_{EE}}{m_{EU}} = \frac{\int_{\hat{p}}^{\bar{p}} \lambda(\theta) e(p) \bar{F}(p) g(p) dp}{\phi \tilde{F}(\hat{p}) + \frac{\sigma^2}{2} g'(\hat{p})}. \quad (67)$$

## 6 Quantitative performance of the model

In this section, I calculate the equilibrium of the above model for different parameter values using numerical methods, since as I argued above, it is not possible to solve the above model analytically. First, I will consider a benchmark parametrization that produces a good fit of the model with the data on several dimensions. Then, I perform quantitative comparative static exercises by studying the impact of varying each of the relevant parameter values. Recall that I have only characterized the stationary equilibrium of the model above, and correspondingly all calculations in this section maintain that the economy is in a stationary equilibrium.

### 6.1 Benchmark parametrization

For the distribution of initial productivity,  $F$ , I use a truncated exponential distribution over  $[0, 1]$ . In particular, I assume that  $F(p) = \frac{1 - e^{-\gamma p}}{1 - e^{-\gamma}}$ . This distribution is useful since a single parameter  $\gamma$  controls the extent to which new draws are concentrated towards the lower end of the distribution. In particular, as  $\gamma$  goes to 0, this distribution approaches the uniform distribution over  $[0, 1]$ . Notice that setting the support of the distribution to be  $[0, 1]$  is simply a normalization, since the model scales linearly. In the benchmark case, I set  $\gamma = 0$ . Moreover, I set the distribution from which new draws of  $p$  are drawn after a  $\phi$  shock to be the same as  $F$ .

For the cost of search function I use the functional form  $c(e) = c_1 e^{1+\rho}$  for some  $\rho > 0$ . In the benchmark case, I set  $\rho = 1$ , so that the cost function is quadratic.

The model is set to generate monthly series, so  $r$  is chosen to be 0.4%, giving an annual discount rate of 4.8%. The value of leisure or of unemployment insurance,  $b$ , is set to 0.5, although varying this parameter has important effects in the model, as I show in the next section.

The benchmark values of the diffusion process are set to be  $\mu = .01/12$  and  $\sigma^2 = .005$ . This value of  $\mu$  implies that the productivity of the match is expected to grow .01 annually conditional on the match not receiving a  $\phi$  shock, which translates into an average 1% productivity growth for a match at  $\bar{p}$  and gives growth in wages with tenure that are in line with empirical estimates.

The rest of the parameters are set as follows. I follow the literature and use a Cobb-Douglas matching function

$$m(\bar{e}_t, v_t) = m_0 \bar{e}_t^\alpha v_t^{1-\alpha}. \quad (68)$$

Then  $\lambda(\theta) = m_0 \theta^{1-\alpha}$  and  $\eta(\theta) = m_0 \theta^{-\alpha}$ , so that the expected time for a firm to make a contact with a worker is  $\frac{1}{\eta(\theta)} = \frac{1}{m_0} \theta^\alpha = \frac{1}{m_0} \left( \frac{\lambda(\theta)}{m_0} \right)^{\frac{\alpha}{1-\alpha}} = m_0^{\frac{1}{\alpha-1}} \lambda(\theta)^{\frac{\alpha}{1-\alpha}}$ .

For the numerical implementation of the model, it is useful to note that  $c_1$  can be eliminated from the equilibrium conditions. This can be achieved by letting  $\hat{e}(p) = e(p) c_1^{\frac{1}{1+\rho}}$ ,  $\hat{\lambda}(\theta) = \lambda(\theta) c_1^{-\frac{1}{1+\rho}}$ , and  $\hat{\kappa} = \kappa c_1^{\frac{1}{1+\rho} \frac{\alpha}{1-\alpha}}$ . These normalizations are possible because the arrival rate of contacts for workers is determined by both  $\lambda(\theta)$  and  $e(p)$ . An economy with high  $\lambda(\theta)$  and low  $e(p)$  is not distinguishable from one with low  $\lambda(\theta)$  and high  $e(p)$  as long as  $\kappa$  is adjusted appropriately, since that is the parameter that determines the relative costliness of posting a vacancy compared to the parameter,  $c_1$ , determining the marginal cost of search for the worker. In other words,  $c_1$  is inherently undetermined, so I set it equal to 1.

As is the case in matching models with a constant returns to scale matching function, the only variable that links the worker and the vacancy side of the model is the offer arrival rate per unit of search effort,  $\lambda$ . Given the other choice of the production and worker parameters set above, I set  $\lambda$  and  $\phi$  to generate a job finding rate of 30% and an unemployment rate of 4%, the empirical values for these quantities in the CPS for prime age workers for the period 1994-2004.

I then set the remaining parameters by assuming that the elasticity of the matching function with respect to search effort  $\alpha$  is equal to the worker's share of the surplus. Then I set  $\kappa$  and  $m_0$  to match the required value of  $\lambda$  and the observed vacancy-unemployment ratio, which is reported by Faberman (2005) to be 0.5 in the data during the period of study.

Table 3 summarizes the relevant baseline parameter values of the model.

For these parameter values, the model matches exactly the observed  $r_{EE}$ , which is 2.20 in the aggregate data for workers between age 25 and 60. The threshold productivity in the model

parameter	baseline value
$r$	0.048/12
$\gamma$	0
$b$	0.5
$\rho$	1
$\mu$	0.01/12
$\sigma^2$	0.005
$\beta$	0.5
$\phi$	0.0229
$\alpha$	0.5
$\lambda$	0.783

Table 3: Baseline values of parameters for calculations.

is 0.3. This is significantly lower than the value of leisure. Workers are willing to create matches at such low levels of productivity due to the substantial option value of working that is present because of the stochastic nature of productivity.

The search effort function that arises in the model is plotted in Figure 2. The figure also shows the probability of separation into unemployment for workers as a function of their productivity over a one month and a one quarter horizon. As we can see, both the search effort of workers and the probability of separation is decreasing with productivity. This matches the observed strong correlation between employed search and separation probability into unemployment observed in the data. Workers with a productivity above 0.67 search less than half as much as unemployed workers. In the equilibrium of the model, 86% of employed workers are above this productivity. This means that this model can match the observation that employed workers search significantly less than unemployed workers while matching the observed extent of job-to-job transitions.

To match the observation that only 4.4% of employed workers are actively searching, we need to define active search as search at an intensity level of at least 80% as much as the intensity level of unemployed workers, a definition that seems reasonable. With this definition, workers with productivity lower than 0.515 are active searchers in the model and they account for 34% of job-to-job transitions, which is somewhat higher than in the data. Also, with this definition of active search, active searchers experience a separation into unemployment over a one month horizon with probability 6.1%, closely matching the data. The relative acceptance rate of active searchers compared to unemployed workers is 79.9%, which is again not too far from the number observed in the limited amount of available data reviewed in Section 3. Summing over all productivity levels, the relative acceptance rate of employed workers is 52.7%.

Figure 3 plots the distribution of employed workers across productivity levels. Notice that 42% of employed workers are at a productivity level above  $\bar{p}$  and do not search at all. The small amount of drift and the reasonable amount of dispersion in productivity is enough to

push a large fraction of workers above  $\bar{p}$ .

As for the composition of separations into unemployment, 55% of these separations are due to the arrival of a  $\phi$  shock with such a low productivity that separation follows, while 45% is due to the Brownian motion crossing the separation barrier.

## 6.2 Sensitivity analysis

Next, I study how the ratio of job-to-job to employment-to-unemployment transitions changes when I allow different parameters of the model to vary. In Figure 4, I report predicted values of  $r_{EE}$  for two different values of the value of leisure,  $b$ , and as a function of the variance of the Brownian motion,  $\sigma^2$ . The calculations are carried out so that the value of  $\phi$  and  $\lambda$  (and correspondingly  $m_0$ ) are reset so that the model keeps matching an unemployment rate of 4% and a job-finding rate of 30%. As can be seen, using a lower value of leisure for a given unemployment rate increases the predicted value of  $r_{EE}$  substantially. The reasons for this can be understood using the same type of argument as was used above for the simple job-ladder model with Poisson shocks. A lower value of leisure leads to a lower threshold productivity, which in turn means that the given extent of separations now takes place by  $\phi$  shocks occurring more frequently but being less likely to lead to a separation into unemployment. This then means that there is more reallocation of workers within the distribution of acceptable productivities, implying a higher level of job-to-job transitions. This result sheds light on how this model can explain the higher  $r_{EE}$  for more educated workers. As I argue below, simply varying the calibrated unemployment rate goes in the right direction, but is not enough to quantitatively explain the variation in  $r_{EE}$  reported in Table 1. It is a very reasonable assumption to make, however, that more educated workers have a lower value of leisure compared to their productive opportunities. Together with their lower unemployment rate, this can then explain their observed higher level of  $r_{EE}$ .

It is also clear from Figure 4, that allowing for the variance of the Brownian motion to vary does not have a significant impact on the predicted level of  $r_{EE}$  conditional on the unemployment rate remaining the same. This is not to say that the value of  $\sigma^2$  is irrelevant. It has a large impact on the distribution of employed workers and on the value of the different statistics reported for the benchmark case above (such as acceptance rates, probability of separation into unemployment, etc).

Figure 5 reports the predicted values of  $r_{EE}$  for different values of the Brownian motion drift assuming that the value of  $\phi$  and  $\lambda$  are reset to maintain an unemployment rate of 4% and a job-finding rate of 30%. The same comments as for the variance term apply. Unlike in the basic job-ladder model, varying the drift of the Brownian motion does not have a large impact on the predicted level of  $r_{EE}$ , but does have a large impact on the distribution of employed workers and hence on the other statistics discussed above.

Figure 6 reports the predicted values of  $r_{EE}$  for different values of the cost function parameter  $\rho$  assuming that the value of  $\phi$  and  $\lambda$  are reset as above. Recall that a higher value of  $\rho$  increases the convexity of the cost function. Correspondingly, a higher value of  $\rho$  makes

the search effort function more concave leading to more on-the-job search and job-to-job transitions. It should be noted, however, that this increase in the concavity of the cost of search results in the worsening of the model's correspondence with the data on employed search. (In the limit as  $\rho$  goes to infinity, all workers search with the same effort).

Figure 7 reports the predicted values of  $r_{EE}$  for different values of the distribution parameter  $\gamma$  assuming that the value of  $\phi$  and  $\lambda$  are reset as above. Recall that a higher value of  $\gamma$  implies that the distribution of productivities becomes more concentrated towards the lower end of the distribution. Not surprisingly, this change lowers the predicted level of  $r_{EE}$ , since it makes it harder to get a high productivity offer. This result shows that, unlike in the basic job-ladder model, in this model the shape of the underlying distribution is relevant. This is an important observation if one wants to match the distribution of this model with data on wages.

Finally, Figure 8 shows that varying the calibrated unemployment rate increases the value of  $r_{EE}$  predicted by the model, just as in the basic job-ladder model. This is simply because a lower unemployment rate implies a lower extent of employment-to-unemployment transitions, thereby increasing  $r_{EE}$ . Simply varying the unemployment rate goes only part way to explaining the observed differences in  $r_{EE}$  across education groups. Varying the unemployment rate jointly with the value of leisure, however, can give a good quantitative correspondence with the data across education groups.

## 7 Conclusion

TO BE COMPLETED.

## 8 Appendix

### 8.1 Derivation of bound on $r_{jj}$ when $F$ is a discrete distribution

Consider now the case when the distribution  $F$  is a right-continuous step-function with steps at  $\{V_1, V_2, \dots, V_N\}$ , i.e. the value of different jobs has a discrete distribution. Let us denote its value at  $V_i$  by  $F_i$ , hence  $F_i$  is the probability that the realization is  $i$  or lower. The reservation value is  $V_R$ , where it is defined such that  $V_R$  is the smallest offer accepted by unemployed workers. Otherwise, the setup of the model is the same. Then the distribution of employed workers can be derived by equating the flow into  $G_i$  (the measure of workers at  $i$  or lower):

$$\lambda_u u (F_i - F_{R-1})$$

and the flow out of  $G_i$

$$\delta G_i + \lambda_e G_i (1 - F_i).$$

This gives

$$G_i = \frac{\lambda_u u (F_i - F_{R-1})}{\delta + \lambda_e (1 - F_i)}.$$

The measure of job-to-job transitions is then

$$\begin{aligned} JJ &= \lambda_e \sum_{i=R}^N (1 - F_i) (G_i - G_{i-1}) = -\lambda_e G_{R-1} (1 - F_R) + \lambda_e (1 - F_N) G_N + \lambda_e \sum_{i=R}^{N-1} G_i (F_{i+1} - F_i) = \\ &= \lambda_e \sum_{i=R}^{N-1} G_i (F_{i+1} - F_i) = \lambda_e \sum_{i=R}^{N-1} \frac{\lambda_u u (F_i - F_{R-1})}{\delta + \lambda_e (1 - F_i)} (F_{i+1} - F_i) = \\ &= \lambda_e \lambda_u u \sum_{i=R}^{N-1} \left[ \frac{\delta/\lambda_e + 1 - F_i}{\delta + \lambda_e (1 - F_i)} - \frac{1}{\lambda_e} + \frac{F_i - F_{R-1}}{\delta + \lambda_e (1 - F_i)} \right] (F_{i+1} - F_i) = \\ &= \lambda_e \lambda_u u \sum_{i=R}^{N-1} \left[ \frac{\delta/\lambda_e + 1 - F_{R-1}}{\delta + \lambda_e (1 - F_i)} - \frac{1}{\lambda_e} \right] (F_{i+1} - F_i) = \\ &= \lambda_e \lambda_u u \left[ (\delta/\lambda_e + 1 - F_{R-1}) \sum_{i=R}^{N-1} \frac{F_{i+1} - F_i}{\delta + \lambda_e (1 - F_i)} - \frac{1}{\lambda_e} (1 - F_R) \right] = \\ &= \lambda_u u \left[ (\delta + \lambda_e (1 - F_{R-1})) \sum_{i=R}^{N-1} \frac{F_{i+1} - F_i}{\delta + \lambda_e (1 - F_i)} - (1 - F_R) \right] \leq \\ &\leq \lambda_u u \left[ (\delta + \lambda_e (1 - F_{R-1})) \int_{V_R}^{V_N} \frac{1}{\delta + \lambda_e (1 - \tilde{F}(V))} d\tilde{F}(V) - (1 - \tilde{F}(V_R)) \right], \end{aligned}$$

where I have introduced the distribution function  $\tilde{F}$  that has the property that it is continuous, increasing and  $\tilde{F}(V) = F(V)$  if  $V \in \{V_1, V_2, \dots, V_N\}$ . Using a change of variables  $x = \tilde{F}(V)$  and  $\bar{x} = \tilde{F}(V_R)$  we get

$$\begin{aligned} JJ &\leq \lambda_u u \left[ (\delta + \lambda_e (1 - F_{R-1})) \int_{\bar{x}}^1 \frac{1}{\delta + \lambda_e (1 - x)} dx - (1 - \bar{x}) \right] = \\ &= \lambda_u u \left[ (\delta + \lambda_e (1 - F_{R-1})) \left[ -1/\lambda_e \log(\delta + \lambda_e (1 - x)) \right]_{\bar{x}}^1 - (1 - \bar{x}) \right] = \\ &= \frac{\lambda_u u}{\lambda_e} \left[ (\delta + \lambda_e (1 - F_{R-1})) \log \frac{\delta + \lambda_e (1 - \bar{x})}{\delta} - \lambda_e (1 - \bar{x}) \right] \end{aligned}$$

Now let  $\hat{x} = \tilde{F}(V_{R-1}) = F_{R-1}$ . Notice that then

$$\begin{aligned} JJ &\leq \frac{\lambda_u u}{\lambda_e} \left[ (\delta + \lambda_e (1 - \hat{x})) \log \frac{\delta + \lambda_e (1 - \bar{x})}{\delta} - \lambda_e (1 - \bar{x}) \right] \leq \\ &\leq \frac{\lambda_u u}{\lambda_e} \left[ (\delta + \lambda_e (1 - \hat{x})) \log \frac{\delta + \lambda_e (1 - \hat{x})}{\delta} - \lambda_e (1 - \hat{x}) \right], \end{aligned}$$

since  $(\delta + \lambda_e (1 - \hat{x})) \log \frac{\delta + \lambda_e (1 - \bar{x})}{\delta} - \lambda_e (1 - \bar{x})$  is decreasing in  $\bar{x}$  given that  $\hat{x} < \bar{x}$ .

Next notice that  $\frac{1-u}{u} = \frac{\lambda_u (1-\hat{x})}{\delta}$ . Again introduce  $y = \frac{\lambda_e (1-u)}{\lambda_u u} = \frac{\lambda_e (1-\hat{x})}{\delta}$ . Substituting gives

$$JJ \leq \frac{\lambda_u u}{\lambda_e} \delta [(1+y) \log(1+y) - y] = \frac{\delta(1-u)}{y} [(1+y) \log(1+y) - y].$$

Hence the ratio of job-to-job transitions to employment-to-unemployment transitions is

$$r_{jj} = \frac{JJ}{\delta(1-u)} \leq \frac{(1+y) \log(1+y) - y}{y}.$$

## 8.2 Derivation of bound on $r_{jj}$ with growth of job value with tenure

Assume now that job value grows with tenure on the job, so that the value of a job with initial value  $V$  and tenure  $t$  is  $v(t, V)$ , where the function  $v$  has the feature that it is increasing in  $t$  and  $v(0, V) = V$  and that for any  $t, a \geq 0$   $v(t+a, V) - v(t, V) = v(a, v(t, V)) - v(0, v(t, V))$ . Then the distribution of employed workers can be derived by considering the equation of motion for  $G$ :

$$\frac{dG(V, t)}{dt} = -g(V, t)\dot{V} + \lambda_u u(F(V) - F(R)) - \delta G(V) - \lambda_e G(V)\bar{F}(V).$$

In steady state  $\frac{dG(V, t)}{dt} = 0$ , so

$$G(V) = \frac{\lambda_u u(F(V) - F(R)) - g(V)\dot{V}}{\delta + \lambda_e \bar{F}(V)} \leq \frac{\lambda_u u(F(V) - F(R))}{\delta + \lambda_e \bar{F}(V)}.$$

The same derivation as for the benchmark case gives that the ratio of job-to-job transitions to employment-to-unemployment transitions is

$$r_{jj} = \frac{JJ}{\delta(1-u)} \leq \frac{(1+y)\log(1+y) - y}{y}.$$

### 8.3 Proofs of Lemmas

Proof of Lemma 1: Based on optimizing behavior and the fact that, conditional on following a given policy for all values of  $p$ ,  $S(p + \varepsilon) = S(p) + \frac{\varepsilon}{r+\phi}$ .

Proof of Lemma 2: Standard application of result from optimal stopping problems.

## 8.4 Derivations of equilibrium distributions

### 8.4.1 The differential equation for the density $g(p)$

To derive the appropriate differential equations characterizing the density of workers,  $g(p)$ , I proceed by using a discrete approximation of the Brownian motion process and then taking limits (see Dixit-Pindyck). I divide time into short intervals of length  $dt$  and the  $p$  space into segments of length  $dh = \sigma\sqrt{dt}$ . The approximation of the Brownian motion is then a process that in a time interval  $dt$  moves up one segment with probability  $z$  and moves down one segment with probability  $q$ , where

$$z = \frac{1}{2} \left[ 1 + \frac{\mu}{\sigma} \sqrt{dt} \right] \quad \text{and} \quad q = \frac{1}{2} \left[ 1 - \frac{\mu}{\sigma} \sqrt{dt} \right].$$

Let us now consider the distribution  $g(p)$  under this discretization. Consider a segment centered on  $p > \hat{p} + dh$ . Over an interval of length  $dt$ , all matches located in that segment,  $g(p)(1-u)dh$ , move out of it. The arrivals to the segment over the time interval  $dt$ , on the other hand, consist of three parts: new flows into the segment from non-neighboring

segments and from unemployment, flows into the segment from one segment below, and flows into the segment from one segment above. The first of these is given by  $a(p)dtdh$ , where  $a(p) = \lambda(\theta) \left( ue_u + (1-u) \int_{\hat{p}}^p e(p')g(p')dp' \right) f(p) + (1-u)\phi\tilde{f}(p)$ . To get the second and the third flow, notice that of the matches located in a segment centered on  $p'$ , a proportion  $l(p')dt$  will leave that segment in the interval of length  $dt$ , where  $l(p') = \phi + \lambda(\theta)e(p')\bar{F}(p')$ . Of the rest, a fraction  $z$  will move up one segment and a fraction  $q$  will move down one segment. Stationary equilibrium flow balance then requires that

$$g(p)(1-u)dh = a(p)dtdh + z(1-l(p-dh)dt)g(p-dh)(1-u)dh + q(1-l(p+dh)dt)g(p+dh)(1-u)dh.$$

Expanding the terms  $g(p-dh)$ ,  $g(p+dh)$ ,  $l(p-dh)$ , and  $l(p+dh)$  using Taylor series gives

$$\begin{aligned} g(p) &= \frac{a(p)}{1-u}dt + z[1-l(p)dt] \left( g(p) - g'(p)dh + \frac{1}{2}g''(p)dh^2 \right) + \\ &+ q(1-l(p)dt) \left( g(p) + g'(p)dh + \frac{1}{2}g''(p)dh^2 \right) + O(dh^3) \\ g(p) &= \frac{a(p)}{1-u}dt + (1-l(p)dt) \left( g(p) + \frac{1}{2}g''(p)dh^2 \right) - \frac{\mu}{\sigma}\sqrt{dt}(1-l(p)dt)g'(p)dh + O(dh^3) \\ 0 &= \frac{a(p)}{1-u} - l(p)g(p) + \frac{1}{2}g''(p)\sigma^2 - \mu g'(p) + O(dh^3)/dt. \end{aligned}$$

Letting  $dt \rightarrow 0$ ,  $O(dh^3)/dt \rightarrow 0$ , so we get

$$\frac{a(p)}{1-u} - l(p)g(p) - \mu g'(p) + \frac{\sigma^2}{2}g''(p) = 0.$$

Substituting in for  $e(p)$  and  $l(p)$ , we get Equation (59) in the text.

Since once  $\hat{p}$  is reached, an employment relationship ceases to continue, flow balance for the segment centered on  $\hat{p} = \hat{p} + dh$  implies

$$g(\hat{p})(1-u)dh = e(\hat{p})dtdh + q(1-l(\hat{p}+dh)dt)g(\hat{p}+dh)(1-u)dh.$$

Expanding the term  $g(\hat{p}+dh)$  and  $l(\hat{p}+dh)$  around  $\hat{p}$  using Taylor series gives

$$\begin{aligned} g(\hat{p}) &= \frac{e(\hat{p})}{1-u}dt + q(1-l(\hat{p})dt) \left( g(\hat{p}) + g'(\hat{p})dh + \frac{1}{2}g''(\hat{p})dh^2 \right) + O(dh^3) \\ g(\hat{p}) &= \frac{e(\hat{p})}{1-u}dt + \frac{1}{2} \left[ 1 - \frac{\mu}{\sigma}\sqrt{dt} \right] (1-l(\hat{p})dt) \left( g(\hat{p}) + g'(\hat{p})dh + \frac{1}{2}g''(\hat{p})dh^2 \right) + O(dh^3) \\ g(\hat{p}) &= \frac{e(\hat{p})}{1-u}dt + \frac{1}{2} \left( g(\hat{p}) + g'(\hat{p})dh + \frac{1}{2}g''(\hat{p})dh^2 \right) - \frac{\mu}{2\sigma}\sqrt{dt} \left( g(\hat{p}) + g'(\hat{p})dh \right) - \frac{1}{2}l(\hat{p})g(\hat{p})dt + O(dh^3) \\ 2zg(\hat{p}) &= 2\frac{e(\hat{p})}{1-u}dt + \left( g'(\hat{p})\sigma\sqrt{dt} + \frac{1}{2}g''(\hat{p})\sigma^2dt \right) - \mu g'(\hat{p})dt - l(\hat{p})g(\hat{p})dt + O(dh^3). \end{aligned}$$

As  $dt \rightarrow 0$ , the left-hand side converges to  $g(\hat{p})$ , while the right-hand side converges to 0 hence  $g(\hat{p}) = 0$ .

#### 8.4.2 The rate of exit at $\hat{p}$

To derive the rate of exit at  $\hat{p}$ , again I proceed by using a discrete approximation of the Brownian motion process and then taking limits. Using the discretization the flow out at  $\hat{p}$  during the interval of length  $dt$  can be expressed as

$$q(1 - l(\hat{p} + dh)dt)g(\hat{p} + dh)(1 - u)dh.$$

Expanding the term  $g(\hat{p} + dh)$  and  $l(\hat{p} + dh)$  around  $\hat{p}$  using Taylor series gives

$$\begin{aligned} q(1 - l(\hat{p} + dh)dt)g(\hat{p} + dh)(1 - u)dh &= \frac{1}{2} \left[ 1 - \frac{\mu}{\sigma} \sqrt{dt} \right] (1 - l(\hat{p})dt) (g(\hat{p}) + g'(\hat{p})dh) (1 - u)dh + O(dh^3) = \\ &= \frac{1}{2} \left[ 1 - \frac{\mu}{\sigma} \sqrt{dt} \right] (1 - l(\hat{p})dt)g'(\hat{p})\sigma^2(1 - u)dt + O(dh^3) = \\ &= \frac{1}{2}g'(\hat{p})\sigma^2(1 - u)dt + O(dh^3). \end{aligned}$$

The flow out per unit of time is then  $\frac{1}{2}g'(\hat{p})\sigma^2(1 - u) + O(dh^3)/dt$  which converges to  $\frac{1}{2}g'(\hat{p})\sigma^2(1 - u)$  as  $dt \rightarrow 0$ .

## References

- Altonji, J. G. and R. A. Shakotko (1987). Do wages rise with job seniority? *Review of Economic Studies* 54, 437–459.
- Altonji, J. G. and N. Williams (1997). Do wages rise with job seniority? a reassessment. NBER Working Paper, 6010.
- Barlevy, G. (2002). The sullyng effect of recessions. *Review of Economic Studies* 69(1), 65–96.
- Barlevy, G. (2003). Estimating models of on-the-job search using record statistics. NBER Working Paper, 10146.
- Barlevy, G. (2005). Identification of search models using record statistics. Federal Reserve Bank of Chicago, mimeo.
- Blanchard, O. J. and P. Diamond (1990). The cyclical behavior of the gross flows of U.S. workers. *Brookings Papers on Economic Activity* 2, 85–143.
- Blau, D. and P. Robbins (1990). Job search outcomes for the employed and unemployed. *Journal of Political Economy* 98(3), 637–655.
- Burdett, K. (1978). A theory of employee job search and quit rates. *American Economic Review* 68(1), 212–220.
- Burdett, K. and M. Coles (2003). Equilibrium wage-tenure contracts. *Econometrica* 71(5), 1377–1404.
- Burdett, K. and D. Mortensen (1998). Equilibrium wage differentials and employer size. *International Economic Review* 39(2), 257–274.
- Dixit, A. and R. Pindyck (1994). *Investment Under Uncertainty*. Princeton University Press.
- Faberman, R. J. (2005). Studying the labor market with the Job Openings and Labor Turnover Survey. Bureau of Labor Statistics mimeo.
- Fallick, B. and C. Fleischman (2004). The importance of employer-to-employer flows in the U.S. labor market. Federal Reserve Bank Board of Governors mimeo.
- Farber, H. S. (1994). The analysis of interfirm worker mobility. *Journal of Labor Economics* 12(4), 554–593.
- Holzer, H. J. (1987). Job search by employed and unemployed youth. *Industrial and Labor Relations Review* 40, 601–611.
- Jolivet, G., F. Postel-Vinay, and J.-M. Robin (2004). The empirical content of job search models: Labor mobility and wage distributions in Europe and the U.S. Université de Paris I mimeo.
- Jones, S. and C. Riddell (1999). The measurement of unemployment: An empirical approach. *Econometrica* 67(1), 147–161.
- Mortensen, D. T. (1994). The cyclical behavior of job and worker flows. *Journal of Economic Dynamics and Control* 18(6), 1121–1142.

- Mortensen, D. T. and C. A. Pissarides (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies* 61(3), 397–415.
- Nagypál, E. (2005a). Job-to-job transitions and labor market fluctuations. Northwestern University mimeo.
- Nagypál, E. (2005b). Worker reallocation over the business cycle: The importance of job-to-job transitions. Northwestern University mimeo.
- Nagypál, E. (2006). Amplification of productivity shocks: Why don't vacancies like to hire the unemployed? In *Structural Models of Wage and Employment Dynamics*, Contributions to Economic Analysis. Elsevier Science B.V.
- Parsons, D. O. (1973). Quit rates over time: A search and information approach. *American Economic Review* 63(3), 390–401.
- Parsons, D. O. (1991). The job search behavior of employed youth. *Review of Economics and Statistics* 73(4), 597–604.
- Pissarides, C. A. (1994). Search unemployment with on-the-job search. *Review of Economic Studies* 61(3), 457–75.
- Royalty, A. (1998). Job-to-job and job-to-nonemployment turnover by gender and education level. *Journal of Labor Economics* 16(2), 392–443.
- Shimer, R. (2006). On-the-job search and strategic bargaining. In *Structural Models of Wage and Employment Dynamics*, Contributions to Economic Analysis. Elsevier Science B.V.
- Stewart, J. (2002). Recent trends in job stability and job security: Evidence from the March CPS. U.S. Bureau of Labor Statistics.
- Topel, R. H. (1991). Specific capital, mobility and wages; wages rise with job seniority. *Journal of Political Economy* 99(1), 145–176.

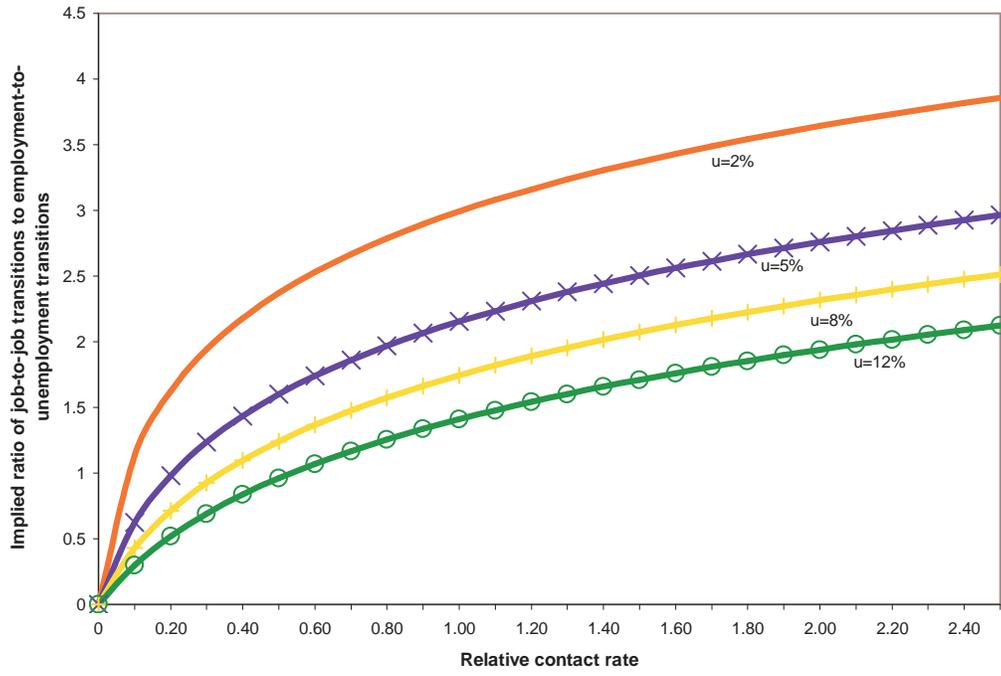


Figure 1:

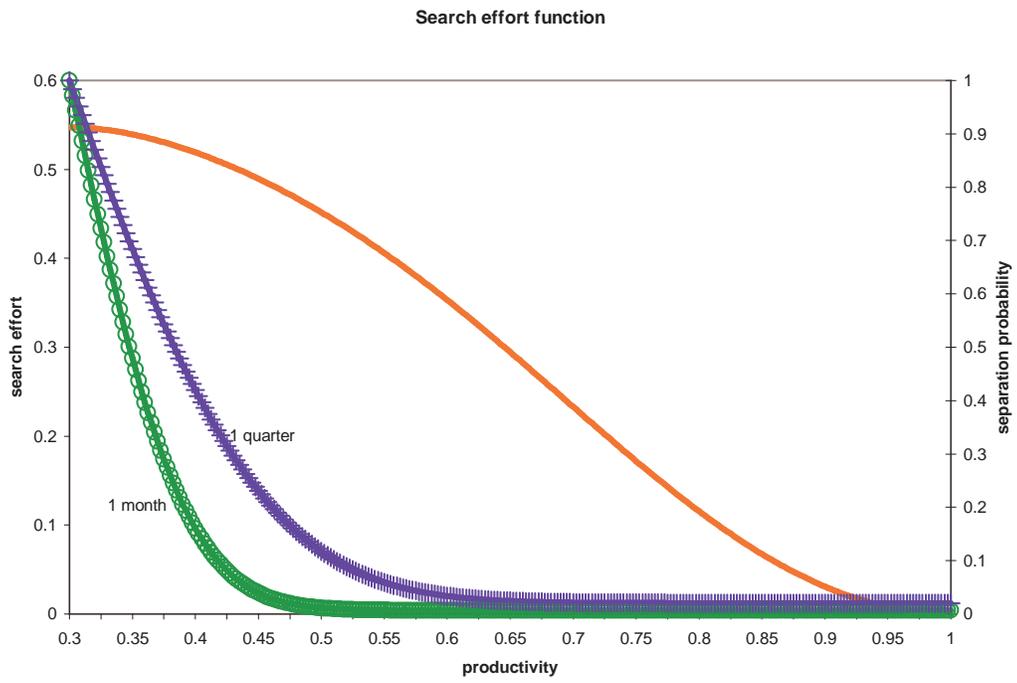


Figure 2:

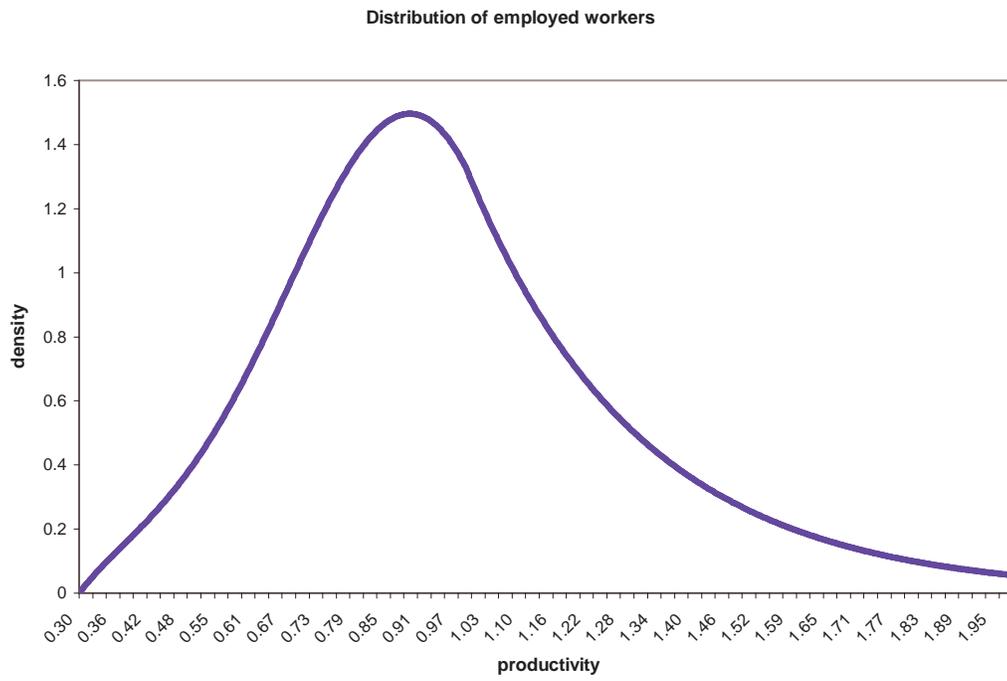


Figure 3:

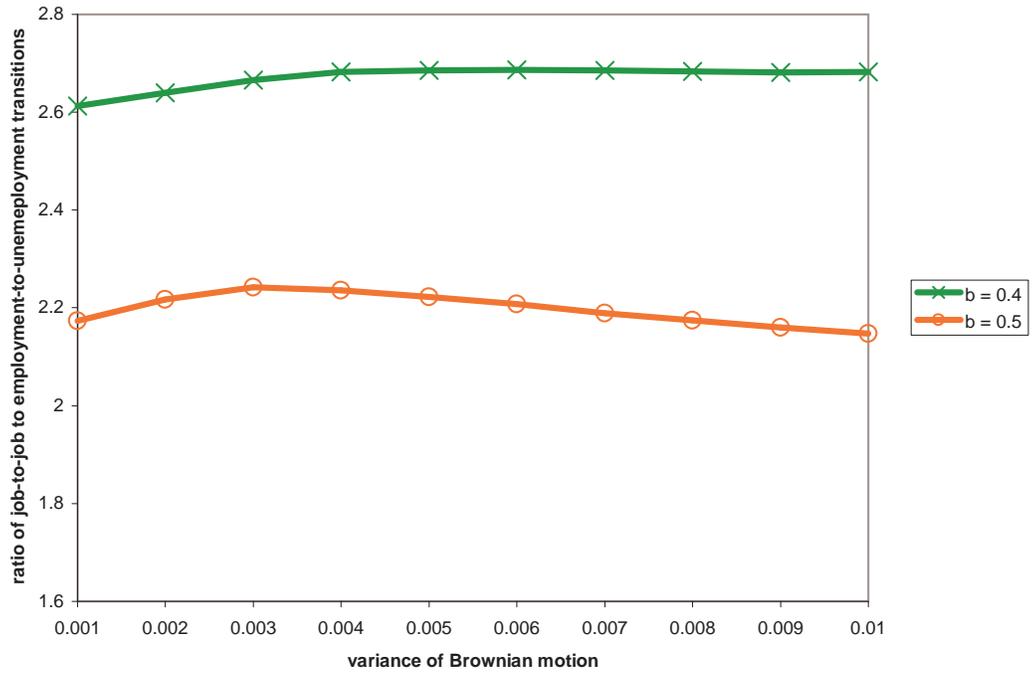


Figure 4:

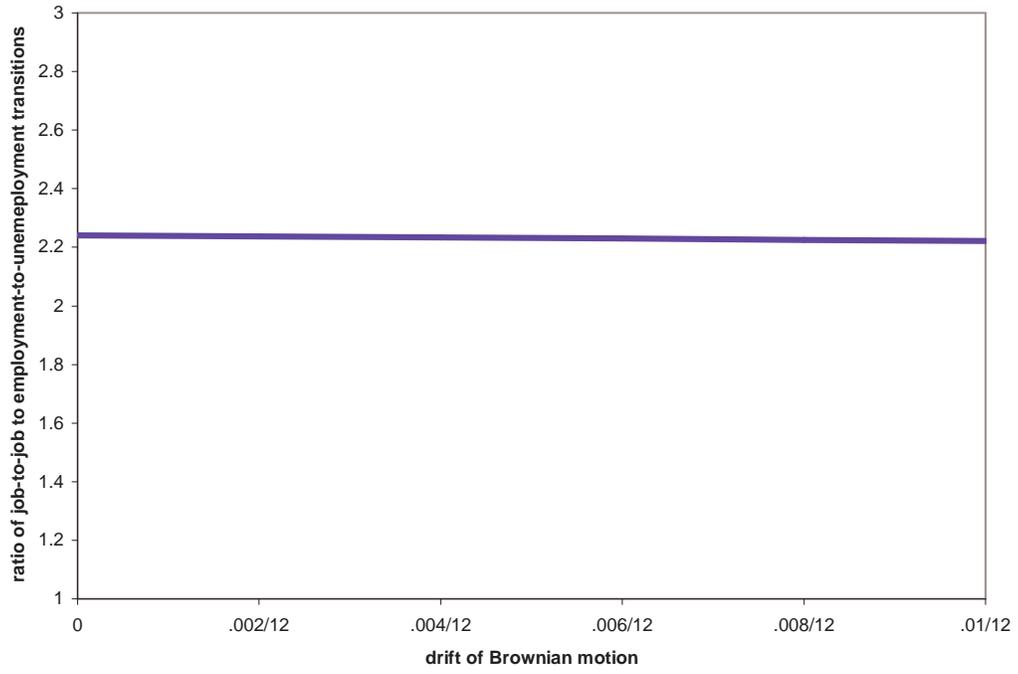


Figure 5:

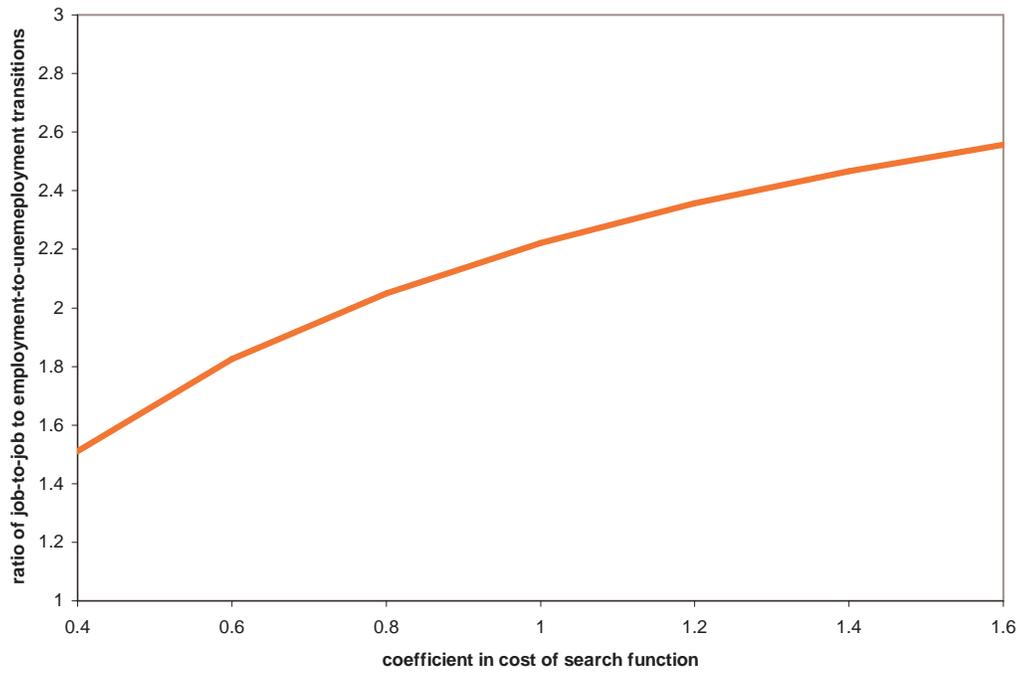


Figure 6:

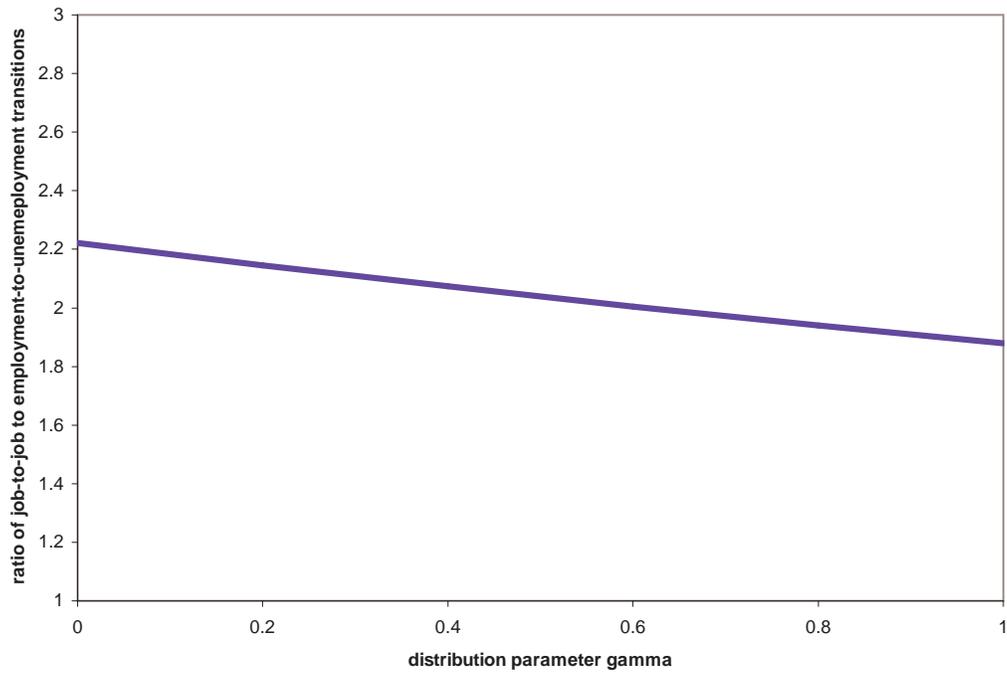


Figure 7:

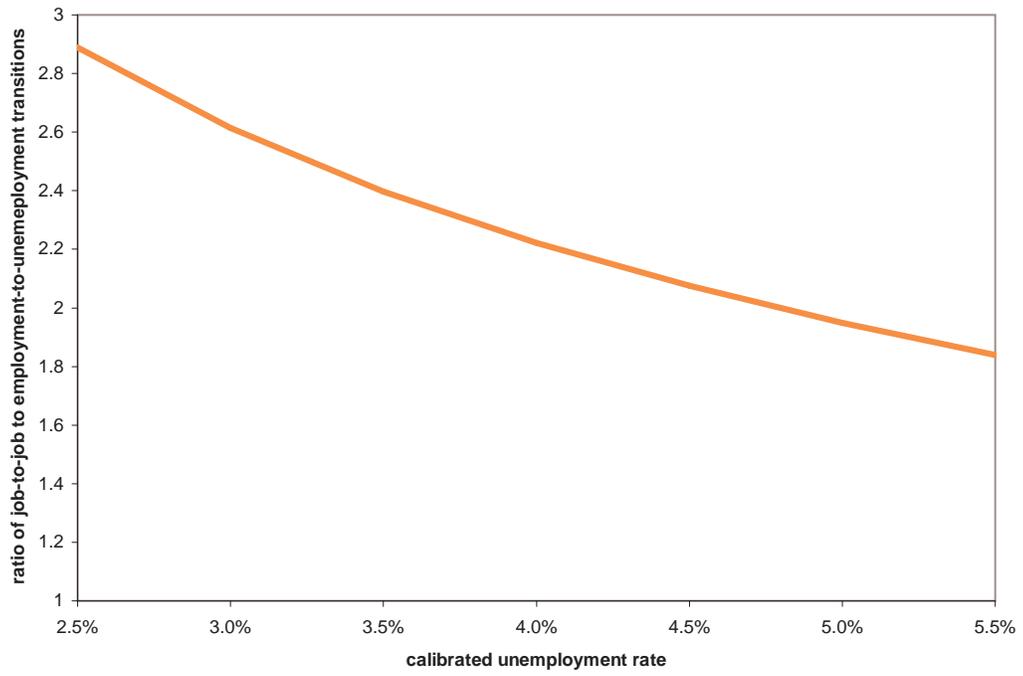


Figure 8: