When Is it Optimal to Abandon a Fixed Exchange Rate?*

Sergio Rebelo† and Carlos A. Végh‡

First draft: August 2001
Current draft: April 2002

Abstract

This paper analyzes the optimal time to abandon a fixed exchange rate regime in response to a fiscal shock that renders the peg unsustainable. We consider three variants of an optimization-based first-generation speculative attack model. In the first variant there are exogenous costs of abandoning the fixed exchange rate. These costs may represent a bailout of the banking sector or output loss. The second variant endogenizes the costs of abandoning the fixed exchange rate in an economy with liability dollarization. The third variant incorporates a fiscal reform – which makes the peg sustainable once again – that arrives according to a Poisson process while the exchange rate is fixed. In all three cases, for a sufficiently large fiscal shock it is optimal to abandon the peg as soon as the shock occurs, regardless of the level of international reserves. This represents a sharp departure from the Krugman-Flood-Garber model. In that model the size of the underlying fiscal shock plays no direct role in the decision to abandon the peg and matters only insofar as it affects the speed at which reserves are depleted.

J.E.L. Classification: F31

Keywords: Currency crisis, speculative attacks, optimal policy, fixed exchange rates.

---

*We thank Linda Goldberg, Pierre-Olivier Gourinchas, Michael Klein, Assaf Razin, seminar participants at the 2001 Di Tella Summer Camp, NBER IFM Program Meeting, New York Fed, and, especially, Bob Flood for useful comments and suggestions. Financial support from the National Science Foundation is gratefully acknowledged.

†Northwestern University, NBER and CEPR.

‡UCLA and NBER.

1. Introduction

Suppose that you are the central banker of a country that has a fixed exchange rate. The economy just suffered a "fiscal shock"–an unexpected increase in government expenditures that will have to be financed with seignorage revenues. When should you abandon the fixed exchange rate? This paper discusses the answer to this question in the context of an optimization-based version of the first-generation speculative attack models of Krugman (1979) and Flood and Garber (1984).[1]

The Krugman-Flood-Garber (KFG) model is arguably one of the most influential models in international finance. Its most remarkable feature is that, even in a perfect foresight context, the model generates a speculative attack – a discrete fall in international reserves – at the time of the crisis. Since most currency crises coincide with a large decline in reserves, the model's key prediction is remarkable from both a theoretical and empirical point of view.

One well-known weakness of the KFG framework is that the central bank is not acting optimally. Instead, it follows a mechanical, exogenous rule for abandoning the fixed exchange rate regime. Specifically, the KFG model *assumes* that the central bank will abandon the fixed exchange rate *if and only if* international reserves reach a critical lower bound. The obvious question is: why would central bankers blindly follow such an arbitrary rule?[2] In a perfect foresight model, the

[1] Optimization-based first generation models of speculative attacks include Obstfeld (1986a), Calvo (1987), Drazen and Helpman (1987), Wijnbergen (1991), Lahiri and Végh (2000), and Burnside, Eichenbaum, and Rebelo (2001).

[2] Second generation of models of speculative attacks introduced an optimizing central banker (Obstfeld (1986b,1996)). However, they also changed the nature of the currency crisis. In first-generation models the crisis has a fiscal origin–the government is forced to resort to seignorage to satisfy its intertemporal budget constraint. In contrast, second generation models adopt a Barro-Gordon formulation, which emphasizes the effects of unexpected inflation on the economy. This shift in focus was motivated by 1992 speculative attacks on European countries that did not seem to face a fiscal crisis. At least in simple versions of second generation models there is no reason to observe a loss of reserves coinciding with a devaluation. Once it is optimal to abandon fixed exchange rates, the central bank should do so immediately.

presence of a discrete loss of foreign reserves just before the abandonment of fixed exchange rates suggests that the central bank is acting irrationally. If fixed exchange rates are going to be abandoned, why lose reserves in the process?[3]

This paper tackles this issue head on by modeling the decision to abandon a fixed exchange rate as the outcome of an *optimal* policy decision: policymakers rationally abandon the peg when it is best to do so from a social point of view. This exercise turns out to be much more than a theoretical refinement, as it makes clear that the exogenous KFG abandonment rule often leads to sub-optimal outcomes. In particular, in response to a sufficiently large fiscal shock, we show that it is always optimal to abandon the fixed exchange rate immediately (even if it is costly to do so) *regardless* of the level of international reserves. This result stands in sharp contrast with the KFG model where the peg is abandoned only when reserves have reached a critical threshold. Hence, in our world of rational policymakers, the focus shifts from the level of international reserves (the key variable in the KFG model) to the size of the fiscal shock as the main determinant of the decision of when to abandon the fixed exchange rate.

We begin our analysis in section 2 with a small open economy model in which money is introduced via a cash-in-advance constraint on consumption. We show that in this basic model it is never optimal for the central bank to withstand a speculative attack. The first-best policy is to abandon the peg as soon as the fiscal shock hits, thus avoiding the loss of international reserves. This result holds even when the government faces a borrowing constraint. Hence, contrary to what is often claimed, borrowing constraints *per se* cannot be used to rationalize the KFG abandonment rule.

[3]Some authors (Buiter (1987), Flood, Garber, and Kramer (1996), Flood and Jeanne (2000), and Lahiri and Végh (2000)) have analyzed the feasibility and/or optimality of the central bank delaying the crisis (i.e., "defending the peg") by borrowing and/or raising interest rates. While these models give the central bank a more active role than in the original KFG model, they continue to assume that abandonment of the peg is governed by the exogenous KFG rule.

In section 3 we extend the basic model by introducing costs of abandoning the fixed exchange rate. These costs may reflect, for instance, output losses associated with the currency crises or the cost of bailing out the banking system.[4] Our main result is that the optimal time of abandonment declines with the size of the fiscal shock. In particular, for a sufficiently large fiscal shock it is optimal to abandon the peg as soon as the shock occurs, regardless of the level of reserves. For moderate fiscal shocks it is optimal to delay the abandonment of the fixed exchange rate. In this case the KFG abandonment rule may be actually optimal: the central bank can implement the optimal monetary policy by announcing that the peg will be abandoned when international reserves reach a suitably chosen lower bound. Surprisingly, immediate abandonment of the peg is also optimal when the costs of abandoning the peg are large.

In section 3 the social and fiscal costs of abandoning the peg are modeled as exogenous and assumed to coincide. In section 4 we endogenize these costs by considering an economy characterized by liability dollarization.[5] In this economy production firms borrow in dollars against revenue denominated in local currency. Their foreign borrowing is guaranteed by the government. When a devaluation occurs some firms declare bankruptcy. Bankruptcy creates a social cost because the future profits of these firms are lost. These bankruptcies also create a fiscal cost because the government has to repay the firm's foreign loans. We study numerically the optimal monetary policy for this economy and obtain results that are similar to those of section 3. In particular, for sufficiently large fiscal shocks, it is optimal to abandon the peg immediately. For smaller shocks, the optimal time of abandonment is a decreasing function of the size of the shock.

[4] Kaminsky and Reinhart (1999) persuasively argue that banking and currency crises tend to occur simultaneously.

[5] The recent traumatic abandonment of the 10 year-old fixed exchange rate in Argentina has made clear that liability dollarization is arguably the major cost of a devaluation in emerging countries.

In section 5 we consider a stochastic version of our model to explore a different setting in which to analyze whether the KFG abandonment rule might be optimal. In this version of the model, there are no fiscal costs of abandoning the peg but fiscal fundamentals are random. These fundamentals are governed by a stochastic process that captures the idea that a fiscal reform is more likely to occur while the economy has a fixed exchange rate. Specifically, we assume that, while the exchange rate is fixed, there may be a fiscal reform that restores the sustainability of the fixed exchange rate regime.[6] This reform arrives according to a Poisson process. Once the economy abandons the fixed exchange rate regime there is no hope of a fiscal reform and the initial fiscal shock must be financed with seignorage revenues. There is thus an *option value* of sticking to the peg since it is always possible that by delaying one more instant, a reform will take place. In this context, the cost of abandoning the peg is giving up this option value. We show that there is a close connection between this model and the one of section 3 and that, once again, for sufficiently large shocks it is optimal to abandon the peg right away. Section 6 concludes.

2. The Basic Model

Consider a standard optimizing small open economy model in which money is introduced via a cash-in-advance constraint on consumption. All agents, including the government, can borrow and lend in international capital markets at a constant real interest rate r. There is a single consumption good in the economy and no barriers to trade, so that the law of one price holds: $P_t = S_t P_t^*$, where P_t and P_t^* denote the domestic and foreign price level, respectively. The exchange rate, defined as units of domestic currency per unit of foreign currency, is denoted by

[6] See Flood, Bhandari and Horne (1989) and Rigobon (2002) for analyses that also emphasizes the link between fixed exchange rates and fiscal discipline.

S_t. For convenience we assume that $P_t^* = 1$.

Just before time zero (i.e., at $t = 0^-$), the exchange rate, S_t, is fixed at a value S. For $t < 0^-$, the economy has a sustainable fixed exchange rate regime: the government can satisfy its intertemporal budget constraint without resorting to seignorage. At $t = 0$ the economy learns that it has suffered a 'fiscal shock': an increase in government expenditures that must be financed with seignorage revenues. Denote by T the time at which the fixed exchange rate regime is abandoned. We wish to answer the following question: what is the optimal value of T?

2.1. Households

Households maximize their lifetime utility, V, which depends on their consumption (c_t) path:

$$V \equiv \int_0^\infty \ln(c_t) e^{-\rho t} dt. \tag{2.1}$$

The discount factor is denoted by ρ. To simplify, we assume that $r = \rho$. The household's flow budget constraint is:

$$\begin{array}{ll} \Delta b_t = -(M_t - M_{t^-})/S_t & \text{if } t \in J, \\ \dot{b}_t = r b_t + y - c_t - \dot{m}_t - \varepsilon_t m_t & \text{if } t \notin J. \end{array} \tag{2.2}$$

Throughout the paper $\dot{x}_t$ denotes dx/dt. Here b_t denotes the household's holdings of foreign bonds that yield a real rate of return of r, and y is a constant, exogenous, flow of output. The variable m_t represents real money balances, defined as $m_t = M_t/P_t$, where M_t denotes nominal money holdings. The variable ε_t denotes the rate of devaluation, which coincides with the inflation rate, $\varepsilon_t = \dot{P}_t/P_t = \dot{S}_t/S_t$.

As in Drazen and Helpman (1987), equation (2.2) takes into account the possibility of discrete changes in b_t and M_t at a finite set of points in time, J. We will later see that this set is comprised of time zero and at the time when the peg is abandoned. These jumps are defined as $\Delta b_t \equiv b_t - b_{t^-}$, where b_{t^-} represents the limit from the left. Since at any point in time $t > 0$, the total level of real financial assets cannot change discretely, $b_{t^-} + m_{t^-} = b_t + m_t$.[7] At time $t = 0^-$, just before the households' time zero decisions are made, agents hold an amount b_{0^-} in real bonds. Their holdings of nominal money balances are M_{0^-}, and their real money balances are therefore $m_{0^-} = M_{0^-}/S$.

Consumption is subject to a cash-in-advance constraint:

$$m_t \geq c_t. \tag{2.3}$$

Since we will only consider environments in which the nominal interest rate is positive, (2.3) will always hold with equality.

The flow budget constraint, together with (2.3) and the transversality condition $\lim_{t\to\infty} e^{-rt} b_t = 0$, implies the following intertemporal budget constraint:

$$b_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj} (M_j - M_{j^-})/S_j. \tag{2.4}$$

This constraint can be further simplified by using the cash-in-advance constraint and imposing the condition that $\lim_{t\to\infty} e^{-rt} m_t = 0$:[8]

$$b_{0^-} + \frac{M_{0_-}}{S_0} + y/r = \int_0^\infty c_t (1 + r + \varepsilon_t) e^{-rt} dt. \tag{2.5}$$

This expression makes clear that, as is typical of cash-in-advance models, the *effective* price of consumption is given by $1 + r + \varepsilon_t$.

The first-order condition for the household's problem is:

$$1/c_t = \lambda \left(1 + r + \varepsilon_t\right), \tag{2.6}$$

where λ is the Lagrange multiplier associated with (2.5).

[7] Notice that at $t = 0$ the total level of real financial assets may change discretely due to an unanticipated jump in the exchange rate, which changes the value of real money balances from M_{0^-}/S to M_{0^-}/S_0.

[8] This condition will always be satisfied in equilibrium since (2.3) will hold as an equality.

2.2. Government

The government collects seignorage revenues and carries out expenditures (g_t). To simplify, we assume that these expenditures yield no utility to the representative household. The government's flow budget constraint is given by:

$$\begin{aligned} \Delta f_t &= (M_t - M_{t^-})/S_t && \text{if } t \in J, \\ \dot{f}_t &= r f_t - g_t + \dot{m}_t + \varepsilon_t m_t && \text{if } t \notin J, \end{aligned}$$

where f_t denotes the government's net foreign assets. This constraint, together with the condition $\lim_{t\to\infty} e^{-rt} f_t = 0$, implies the following intertemporal budget constraint:

$$f_{0^-} + \int_0^{\infty} (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj} (M_j - M_{j^-})/S_j = \Gamma_{0^-}, \tag{2.7}$$

where, by definition, Γ_{0^-} is the present value of government spending:

$$\Gamma_{0^-} \equiv \int_0^{\infty} g_t e^{-rt} dt.$$

Notice that when the peg is abandoned at time zero the jump in the money supply $(M_0 - M_{0^-})$ is controlled by the central bank through its choice of M_0. In contrast, when the peg is abandoned at $T > 0$, the jump in the money supply $(M_T - M_{T^-})$ is endogenously determined. As is well known, under perfect foresight the path for the exchange rate has to be continuous for all $t > 0$ to rule out arbitrage opportunities. This means that in equilibrium households will, at time T, reduce their money holdings in anticipation of the higher inflation rate for $t \geq T$.

2.3. Equilibrium Consumption

Combining the households' and government's intertemporal constraints (equations (2.4) and (2.7), respectively), we obtain the economy's aggregate resource constraint:

$$b_{0^-} + f_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Gamma_{0^-}. \tag{2.8}$$

This constraint implies that the present value of output plus the total net foreign assets in the economy must equal the present value of consumption and government expenditures.

2.4. A Sustainable Fixed Exchange Rate Regime

Before $t = 0^-$ the economy was in a sustainable fixed exchange rate regime in which agents expected ε to be permanently zero. This requires that the government's net foreign assets be sufficient to finance the present value of government expenditures. This condition for $t = 0^-$ is:

$$f_{0^-} = \Gamma_{0^-}.$$

In this regime, equations (2.3) and (2.8) imply that consumption and real balances are given by:

$$\begin{aligned} c_{0^-} &= y + rb_{0^-}, \\ m_{0^-} &= c_{0^-}. \end{aligned} \tag{2.9}$$

2.5. Optimal Monetary Policy

Suppose that at time zero there is an unanticipated increase in the present value of government expenditures from Γ_{0^-} to Γ_0 and that this increase in expenditure must be financed with seignorage. Clearly, the peg will be abandoned at some point because Γ_0 cannot be intertemporally financed with $\varepsilon = 0$. When is it optimal to abandon the peg? Throughout the paper we will focus on the perfect commitment solution to this question.

The new aggregate constraint for the economy can be written as:

$$b_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Delta\Gamma, \tag{2.10}$$

where $\Delta\Gamma = \Gamma_0 - \Gamma_{0^-}$ represents the present value of the *new* government expenditures. Suppose that the government could finance this new expenditure with lump sum taxes. Consumption would be constant over time and its level would be given by:

$$c_0 = c_{0^-} - r\Delta\Gamma.$$

Since $\Delta\Gamma > 0$, the new level of consumption is lower than before. The economy has the same resources as before the fiscal shock, so the rise in government spending has to be accommodated by a fall in private consumption. The corresponding fall in real money balances is accommodated through a fall in the nominal money supply at $t = 0$.

The government can replicate the lump sum taxes outcome by either starting to expand the money supply at $t = 0$ at a constant rate or printing money at $t = 0$ (or a combination thereof). Specifically, suppose that the government abandons the fixed exchange rate regime at time zero, keeps $M_0 = M_{0^-}$, and expands the money supply at a constant rate ε such that the government budget constraint is satisfied:

$$\int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \Delta\Gamma.$$

Since the central bank abandons the fixed exchange rate regime as soon as news about the fiscal shock arrives, there is no speculative attack at time zero. Private agents are not given a chance to trade their money balances for foreign reserves at the fixed exchange rate S before the devaluation occurs. The adjustment in the level of real balances occurs through a jump in the exchange rate, rather than through a discrete fall in the nominal money supply at time 0. The aggregate resource constraint (2.10) implies that consumption will be equal to c_0. The cash-in-advance constraint implies that the new level of real balances will be $m_0 = c_0$. This monetary policy is optimal since it replicates the outcome that can be achieved under lump sum taxes.

The fall in real balances from c_{0^-} to c_0 is associated with a jump in the exchange rate from S to:

$$S_0 = Sc_{0^-}/c_0. \tag{2.12}$$

The constant level of money growth is given by: $\varepsilon = r\Delta\Gamma/c_0 > 0$. Thus from time zero on the currency depreciates at rate ε.

There is another optimal policy which consists of abandoning the peg at time zero and printing enough money to finance the new government spending. In this case the resource constraint of the government is given by: $(M_0 - M_{0^-})/S_0 = \Delta\Gamma$. Printing money at time zero is equivalent to taxing existing real balances. This form of financing is therefore equivalent to lump sum taxes. Since all the seignorage revenue is collected at time zero, this policy implies a higher rate of instantaneous depreciation at time zero than that given by (2.12):

$$S_0 = \frac{Sc_{0^-}}{c_{0^-} - (1+r)\Delta\Gamma}.$$

Any combination of the two policies discussed above, expanding the money supply at a constant rate from time zero on and printing money at time zero, is also optimal. Thus, there are multiple ways for monetary policy to achieve the optimal outcome but all these policies require that the fixed exchange rate be abandoned at time zero.

Abandoning the peg at time $T > 0$ yields a lower level of welfare than the solutions just discussed. It is easy to show that once the fixed exchange rate regime is abandoned at time T, it is optimal to expand the money supply at a constant rate, ε:[9]

$$\varepsilon_t = 0, \quad \text{for } 0 \leq t < T \ , \tag{2.13}$$
$$\varepsilon_t = \varepsilon, \quad \text{for } t \geq T \ .$$

We now show that this constant rate will be strictly positive, thus imposing an intertemporal distortion on the household's consumption path. To this effect, notice that the value of ε has to satisfy the government's intertemporal budget constraint, (2.7), which can be rewritten as:

$$\frac{e^{-rT}\varepsilon}{r}\frac{M_T}{S_T} = \Delta\Gamma + \frac{M_{0^-} - M_0}{S} + \frac{M_0 - M_T}{S}e^{-rT}, \tag{2.14}$$

where $(M_{0^-} - M_0)/S + [(M_0 - M_T)/S]e^{-rT}$ represents the net reserve losses incurred by the government as households rearrange their money balances while the exchange rate is fixed in response to the changes in the path for inflation.

The first order condition for the household problem, (2.6), implies that consumption will be constant within the subperiods $t \in [0, T)$ and $t \in [T, \infty)$. Let us denote by c^1 and c^2 the level of consumption in the periods $t \in [0, T)$ and $t \in [T, \infty)$, respectively. Using equations (2.9), (2.10), and the cash-in-advance constraint, (2.3), we can show that, independently of the form of the momentary utility function and the value of T, the net reserve loss incurred by the government is given by:

$$(M_{0^-} - M_0)/S + [(M_0 - M_T)/S]e^{-rT} = r\Delta\Gamma. \tag{2.15}$$

Using this result we can re-write the government budget constraint (2.14) as:

$$\frac{e^{-rT}\varepsilon}{r}\frac{M_T}{S_T} = \Delta\Gamma(1 + r). \tag{2.16}$$

[9] To show this, solve the planner's problem for an economy with no cash-in-advance constraint. Then show that the cash in advance economy with constant ε can replicate the solution to the planner's problem. See Rebelo and Xie (1999) for details of a closed economy version of this result.

This equation implies that $\varepsilon > 0$. The first order condition for the household problem, (2.6), then implies that $c^2 < c^1$. Since the present value of resources that are available for consumption is independent of T, this non-flat path of consumption results in lower welfare compared to the case where the peg is abandoned at time zero.

Finally, notice that introducing a borrowing constraint on the government would not affect the results. Since the optimal policy decision is to abandon immediately it involves no additional borrowing, so the presence of a borrowing constraint is irrelevant.

The net reserve loss described in (2.15) is a cost that the government incurs when the abandonment of the fixed exchange rate regime is delayed. However, since this cost is just a transfer from the government to households (i.e., it is not a *social* cost), it cannot provide a rationale for delaying the abandonment of the peg. The next section considers the case in which there are social costs associated with the abandonment of the peg.

3. Costs of Abandoning the Peg

We now extend the model by assuming that there are social costs associated with abandoning the fixed exchange rate. These costs can be given several interpretations. First, they may reflect declines in output following the abandonment of the peg. Second, since banking crises are typically a by-product of currency crises (see Kaminsky and Reinhart (1999)), these costs could stem from bailing out domestic banks. Third, these costs could result from bailing out foreign creditors. Specifically, a devaluation may make it optimal for domestic firms to default on foreign loans that had been guaranteed by the government. In these circumstances a devaluation creates a fiscal liability for the government (this interpretation is developed in section 4).[10] Finally, this cost could represent the loss of the *option value* of sticking to the peg in an environment in which there is a certain probability that the fixed exchange rate will induce fiscal authorities to undertake a fiscal reform that would make it sustainable once again (we pursue this stochastic interpretation in section 5). As a benchmark, this section focuses on the case in which the cost of abandoning the peg, ϕ, is exogenous and constant over time.[11] Specifically, we assume that when the fixed exchange rate is abandoned, the government incurs a fiscal cost of ϕ. Furthermore, this cost also represents a social loss for the economy as a whole.

3.1. Optimal Monetary Policy

The present value at time zero of the fiscal cost of abandoning the peg at time T is ϕe^{-rT}.[12] Thus the gain, measured in units of output, from a marginal delay in the time of abandonment is $r\phi e^{-rT}$. With this in mind we return to the same question that we asked in the basic model. Suppose that, at time zero, the present value of government spending increases unexpectedly by $\Delta\Gamma$. When is it optimal to abandon the peg in response to this shock?[13]

[10] The model in Burnside, Eichenbaum and Rebelo (2000) suggests an alternative source of fiscal costs stemming from a devaluation. In their model the fact that banks are guaranteed by the government makes it optimal for them to use forward currency markets to increase their exposure to exchange rate risk. As a result these banks go bankrupt when a devaluation occurs, creating a fiscal liability for the government.

[11] Similar results would obtain if ϕ grew at a rate lower than r. Obviously, if ϕ grows at a rate faster than r, the presence of this cost would provide another reason for the devaluation to occur at time zero, so the results of the previous section would continue to hold.

[12] In order to ensure positive consumption, ϕ must be restricted to be less than $y/r + b_{0^-} - \Delta\Gamma e^{rT}$.

[13] The existence of costs of abandoning the peg can give rise to multiple equilibrium if the central bank follows an exogenous KFG abandonment rule. In particular, the peg may collapse in the absence of a fiscal shock ($\Delta\Gamma = 0$). Suppose that agents believe that the peg will be abandoned at some time t and that inflation will be equal to $\varepsilon > 0$ from time t on. Agents will then reduce their money balances at time t. The associated reserve loss can potentially trigger the KFG abandonment rule. When this happens the government incurs a cost ϕ which has to

Since the household's problem remains unaffected by the introduction of the fixed cost of abandoning the peg, the first-order condition (2.6) continues to hold. The optimal path for ε is given by (2.13), so $c_t = c^1$ for $t \in [0, T)$ and $c_t = c^2$ for $t \in [T, \infty)$. To compute the values of c^1 and c^2, we can rewrite the households' intertemporal budget constraint, given by (2.5), as:

$$b_{0^-} + \frac{M_{0^-}}{S_0} + y/r = \frac{c^1(1+r)}{r}(1 - e^{-rT}) + \frac{c^2(1+r+\varepsilon)}{r}e^{-rT}. \tag{3.1}$$

Combining this constraint with the optimality condition (2.6), we obtain:[14]

$$\begin{array}{ll} c_t \equiv c^1 = c_{0^-} = y + rb_{0^-}, & \text{for } 0 \leq t < T, \\ c_t \equiv c^2 = c^1/p = (y + rb_{0^-})/p, & \text{for } t \geq T, \end{array} \tag{3.2}$$

where

$$p \equiv \frac{1+r+\varepsilon}{1+r}, \tag{3.3}$$

denotes the *relative* effective price of consumption across the two regimes. As is typical of cash-in-advance models, consumption is higher before T than afterwards due to the fact that consumption is effectively cheaper before T. Notice also that c^1 is exogenously given by $y + rb_{0^-}$ (i.e., c^1 does not depend on either T or p).

Using (3.2) to replace consumption into (2.1) the household's lifetime utility can be rewritten as:

$$V = \frac{\log(c^1)}{r} - \frac{\log(p)}{r}e^{-rT}. \tag{3.4}$$

We can use (2.16), amended to incorporate the fiscal cost of abandoning the peg, together with (3.2), (3.3), and the fact that the cash-in-advance constraint

be financed with seignorage. This validates the expectations of positive inflation that triggered the attack. These self-fulfilling attacks are, however, eliminated when the government follows an optimal policy. In the absence of a fiscal shock, an optimizing government will never abandon the peg regardless of the behavior of reserves.

[14] To ensure continuity at time zero and thus simplify the optimization problem, we henceforth assume that M_0 is such that $S_0 = S$. This is an innocuous assumption given that when $T = 0$ the central bank is indifferent between collecting seignorage by increasing M_0 or by raising the growth rate of money, ε.

always binds to rewrite the government budget constraint as:[15]

$$e^{-rT}\left[\frac{c^1}{r}\frac{(p-1)}{p}-\phi\right]=\Delta\Gamma. \tag{3.5}$$

This equation implicitly defines p (and, given (3.3), ε) as an increasing function of T, ϕ, and $\Delta\Gamma$:

$$p(T,\phi,\Delta\Gamma)=\frac{c^1}{c^1-r(\Delta\Gamma e^{rT}+\phi)}, \tag{3.6}$$

The derivatives of $p(T,\phi,\Delta\Gamma)$ are:

$$p_T(T,\phi,\Delta\Gamma) = \frac{(pr)^2\Delta\Gamma e^{rT}}{c^1}>0, \tag{3.7}$$

$$p_\phi(T,\phi,\Delta\Gamma) = \frac{p^2 r}{c^1}>0, \tag{3.8}$$

$$p_{\Delta\Gamma}(T,\phi,\Delta\Gamma) = \frac{p^2 r e^{rT}}{c^1}>0. \tag{3.9}$$

Intuitively, a higher T implies that a higher inflation rate, ε, is required once the peg is abandoned because the abandonment takes place at a later date. For a given T, a larger fiscal shock ($\Delta\Gamma$) or a higher cost of abandoning (ϕ) also call for a higher rate of inflation (i.e., a larger p).

After this groundwork, we are now ready to tackle the optimal policy problem. The central bank chooses an optimal T to maximize (3.4), with $p(T,\phi,\Delta\Gamma)$ being given by (3.6). The Kuhn-Tucker condition for this problem implies that:

$$\begin{aligned} e^{-rT}\left[\log p(T,\phi,\Delta\Gamma)-\tfrac{p_T(T,\phi,\Delta\Gamma)}{rp(T,\phi,\Delta\Gamma)}\right] &\leq 0, \quad \text{if } T=0, \\ e^{-rT}\left[\log p(T,\phi,\Delta\Gamma)-\tfrac{p_T(T,\phi,\Delta\Gamma)}{rp(T,\phi,\Delta\Gamma)}\right] &= 0, \quad \text{if } T>0. \end{aligned} \tag{3.10}$$

Intuitively, increasing T (i.e., delaying the abandonment of the peg), confers both a direct utility benefit and an indirect utility cost. The direct benefit is that the

[15] In the presence of a positive ϕ, equation (2.16) becomes $\frac{\varepsilon}{r}\frac{M_T}{S_T}=(1+r)(e^{rT}\Delta\Gamma+\phi)$.

good times (i.e., high consumption times) are prolonged, which yields a marginal increase in lifetime utility given by $e^{-rT}\log(p)$. There is, however, an indirect utility cost. Delaying implies a marginal increase in the post-crisis inflation rate (captured by p_T), which raises the intertemporal distortion and reduces utility, at the margin, by $e^{-rT}p_T/rp$.

3.2. When Is It Optimal to Abandon the Peg Immediately?

Abandoning the fixed exchange rate as soon as the fiscal shock hits (i.e., at $T = 0$) will be the optimal solution when the net marginal benefit of increasing T (given by (3.10)) around $T = 0$ is either negative (in which case there will be a corner solution at $T = 0$) or exactly zero (in which case there is a boundary solution at $T = 0$). If the net marginal benefit around $T = 0$ is positive, then it is optimal to delay the abandonment of the peg until some later date.[16] Naturally, both the benefit and the cost of delaying depend on ϕ and $\Delta\Gamma$ through their effect on p.

To proceed, we therefore need to evaluate (3.10) at $T = 0$ and characterize the resulting expression as a function of ϕ, for a given positive value of $\Delta\Gamma$(hereafter denoted by $\Psi(\phi;\Delta\Gamma)$).[17] Taking into account (3.5) and (3.7), it follows that

$$\Psi(\phi;\Delta\Gamma) \equiv \log p(0,\phi,\Delta\Gamma) - [p(0,\phi,\Delta\Gamma)-1] + \frac{r\phi p(0,\phi,\Delta\Gamma)}{c^1}, \qquad \phi \in [0, \frac{c1}{r} - \Delta\Gamma). \tag{3.11}$$

As a first check, let us evaluate this function at $\phi = 0$. Note that:

$$\Psi(0;\Delta\Gamma) = \log p(0,0,\Delta\Gamma) - [p(0,0,\Delta\Gamma) - 1] < 0, \tag{3.12}$$

[16] This characterizes a "local" solution around $T = 0$. All the solutions discussed below, however, are also "global" solutions, as shown in Appendix 7.1.

[17] Notice that if $\Delta\Gamma = 0$, there are two trivial cases. If $\phi = 0$, then $\Psi(0;0) = 0$ for any $T \geq 0$. Technically, this implies that the optimal T is indeterminate (i.e., since there is no fiscal shock, "abandoning" would still imply a completely stable exchange rate). If $\phi > 0$, then $\Psi(\phi;0) = \log(p) > 0$ because, from (3.6), $p(T,\phi,0) > 1$. This implies, from (3.10), that the optimal T is "infinity" (i.e., it is never optimal to abandon). This is, of course, consistent with the intertemporal fiscal constraint because there has been no fiscal shock.

since $p(0, 0, \Delta\Gamma) > 1$ for any $\Delta\Gamma > 0$ (see (3.6)). Hence, there is a corner solution at $T = 0$, which implies that it is optimal to abandon the peg as soon as the expenditure shock hits, regardless of the size of this shock. This is, of course, the result derived in the previous section.

What happens when the cost of abandoning is positive (i.e., $\phi > 0$)? Remarkably, we first show that, for large fiscal shocks, it is optimal to abandon the fixed exchange rate right away regardless of the value of ϕ.

Proposition 3.1. *Let $\Delta\Gamma \geq c1/er$. Then it is always optimal to abandon the fixed exchange rate at $T = 0$ for any value of ϕ in the admissible range.*

Proof. See Appendix 7.1.

This proposition thus states that, for large fiscal shocks, it is always optimal to abandon the peg immediately. The reason is that, by abandoning the peg right away, policymakers can avoid the intertemporal distortion that would be introduced by delaying the abandonment until some $T > 0$ and then having to raise the inflation rate very sharply. This result illustrates how critical the assumption of rational policymakers is. In contrast with the standard KFG model, in our model the decision to abandon the peg is unrelated to the size of the stock of international reserves. Further, under the mechanical (i.e., "irrational") KFG rule, policymakers would never abandon right away as long as reserves are available. Hence, unlike the KFG model, our model would be capable of rationalizing situations in which a fixed exchange rate is abandoned without first exhausting reserves (as in the EMS crises of 1992).

What happens for smaller values of the fiscal shock? The answer then depends on the value of ϕ, as the following proposition shows.

Proposition 3.2. *Let $\Delta\Gamma < c^1/er$. Then there is a range of values of ϕ, given by $\phi \in [0, \phi^*]$ and $\phi \in [\phi^{**}, \frac{c1}{r} - \Delta\Gamma]$, for which it is optimal to abandon right* *away. For intermediate values of ϕ, that is $\phi \in (\phi^*, \phi^{**})$, the solution is interior (i.e., it is optimal to delay the abandonment of the peg).*

Proof. See Appendix 7.1.

Table 1 offers a convenient way of conveying the results stated in the last two propositions. This 3x2 matrix defines 6 different cases. For high values of $\Delta\Gamma$ ($\Delta\Gamma \geq c^1/er$), it is always optimal to abandon right away regardless of the value of ϕ (Proposition 1). For low values of $\Delta\Gamma$ ($\Delta\Gamma < c^1/er$), there is only one case in which it is optimal to delay the abandonment of the peg (when ϕ is in the intermediate range).

Table 1

Optimal Time For Abandoning a Fixed Exchange Rate

	Low $\Delta\Gamma$ $\Delta\Gamma < c^1/er$	High $\Delta\Gamma$ $\Delta\Gamma \geq c^1/er$
Low ϕ ($0 \leq \phi \leq \phi^*$)	$T = 0$	$T = 0$
Intermediate ϕ ($\phi^* < \phi < \phi^{**}$)	$T > 0$	$T = 0$
High ϕ ($\phi \geq \phi^{**}$)	$T = 0$	$T = 0$

Intuitively, for $\Delta\Gamma < c^1/er$, the intertemporal distortion imposed by abandoning the peg for some $T > 0$ is no longer so large and therefore the decision comes down to a trade-off between the benefit of delaying (which reduces the present discounted value of abandoning) and the cost of delaying (which increases the intertemporal distortion). As one would expect, for small values of ϕ, it is not worth it to delay and it is thus optimal to abandon right away. For intermediate values of ϕ, the costs of abandoning are large enough to make it worth delaying the abandonment of the peg. For values of ϕ larger than ϕ^{**} (that is, values of ϕ which are close to the maximum admissible value of ϕ), both the marginal benefit and marginal cost of delaying become arbitrarily large but the cost of delaying increases faster than the benefit of delaying, making it optimal to abandon immediately.[18] In terms of relating our model to the KFG model, notice that Table 1 can be read as saying that there is only one out of six possible cases in which a mechanical rule a la KFG could be optimal. In all other cases, rational policymakers would abandon the peg right away.

3.3. Behavior of T, ε, and Reserve Loss as a Function of ϕ and Fiscal Shock

Having established when the solution will be corner or interior, we can now analyze how the optimal choice of T (as well as the inflation rate and the reserve loss) depends on ϕ and $\Delta\Gamma$ for the whole admissible range of parameter values (formal proofs are relegated to Appendices 7.2 and 7.3). Figure 1 illustrates the behavior of the optimal T as a function of ϕ, for a *small* fiscal shock (i.e., for $\Delta\Gamma < c^1/er$). Up to $\phi = \phi^*$, the optimal solution is to abandon right away, as already established. When the solution is interior, the optimal T is a non-monotonic function of ϕ. For values of ϕ larger than ϕ^{**}, it becomes again optimal to abandon right away.

Panel B in Figure 1 illustrates the behavior of the optimal inflation rate as a function of ϕ. Notice that, for $T = 0$, it follows from equations (3.3) and (3.5) that:

$$\varepsilon = \frac{r(1+r)}{c^2}(\Delta\Gamma + \phi), \tag{3.13}$$

$$c^2 = rb_{0^-} + y - r(\Delta\Gamma + \phi). \tag{3.14}$$

It follows from (3.13), taking into account (3.14), that, for $T = 0$, the rate of

[18] Formally, one can see that, as ϕ approaches its maximum feasible value, the marginal cost of delaying is linear in p whereas the marginal benefit of delaying is logarithmic in p.

inflation is an increasing function of ϕ. The same is true when the solution is interior. Hence, as Panel B illustrates, the rate of inflation is an increasing function for the whole range of values of ϕ.

Finally, Panel C in Figure 1 illustrates the behavior of the reserve loss at the time of abandonment. By definition, the reserve loss equals $c^1 - c^2$. However, since c^1 is independent of T and ϕ, the reserve loss depends only on the behavior of c^2 (when the solution is interior). Naturally, when it is optimal to abandon at time zero, there are no reserve losses. When the solution is interior, the reserve loss is an increasing function of ϕ.

Figure 2 illustrates the behavior of the same endogenous variables (T, ε, and reserve loss) as a function of the fiscal shock for a given value of ϕ.[19] Panel A shows that, when the solution is interior, the optimal T is a decreasing function of the fiscal shock. In other words, the larger the fiscal shock the sooner it is optimal to abandon the peg. Intuitively, a larger fiscal shock implies that, all else equal, a higher inflation rate will be needed which will impose a larger intertemporal distortion. It is thus optimal to abandon earlier and impose a smaller intertemporal distortion. When the value of the fiscal shock reaches $\Delta\Gamma^*(\equiv c^1/er)$, it becomes optimal to abandon immediately.

With respect to the rate of inflation, notice how it remains constant when the solution for T is interior.[20] This reflects the effect of two forces that cancel each other out. First, for a given T, a larger fiscal shock tends to increase the inflation rate. Second, since the optimal T falls as the fiscal shocks increases, the inflation rate falls. In this case of logarithmic preferences, these two effects exactly cancel each other out.[21] Once it becomes optimal to abandon immediately (i.e.,

[19] The given value of ϕ is $(c^1/r)(1 - 2/e)$. As shown in Appendix 7.3, at that point the Kuhn-Tucker condition is exactly equal to zero.

[20] As noted above, when $\Delta\Gamma = 0$, it is optimal never to abandon the peg and hence the optimal inflation rate is zero.

[21] For the CES case, numerical examples suggest that the inflation rate is an increasing function

for $\Delta\Gamma \geq \Delta\Gamma^*$), the inflation rate is an increasing function of the fiscal shock as follows from (3.13) and (3.14).

Two final observations. First, could this model with rational policymakers replicate the mechanical rule of the KFG model? Yes, provided that the parameter configuration falls in the only cell in Table 1 that ensures an interior solution. In that case, the optimal monetary policy can be implemented using a rule in the spirit of the KFG model. Specifically, instead of announcing T, the central bank can simply announce that it will abandon the fixed exchange rate regime the first time that net government foreign assets fall by $c^2 - c^1$. This would replicate, with optimal behavior on the part of policymakers, the key features of the KFG model.

Second, what would be the consequence of introducing a borrowing constraint on the government? Consider the case where government expenditure is constant at a level g_{0^-} before the fiscal shock and at a level $g_0 > g_{0^-}$ after the fiscal shock. Suppose that there is a binding borrowing constraint that dictates that $f_t \geq \bar{f}$. It can be shown that lifetime utility, V, is an increasing function of T for values of T below the optimal. Note that once the regime is abandoned, f_t becomes constant. The value of f_T is a decreasing function of T. Thus a borrowing constraint will force the economy to abandon the fixed exchange rate regime before the optimal T (assuming, of course, that the unconstrained optimal T was positive to begin with). This situation resembles closely the monetary policy followed in the KFG model, as central bankers maintain the regime for as long as possible and, at the time of abandonment, exhaust their borrowing constraint. In general, appealing to the presence of a borrowing constraint to justify the KFG mechanical rule is misleading because whenever the optimal solution is $T = 0$, borrowing constraints are clearly irrelevant.

---

of the fiscal shock (in the interior solution range) when the intertemporal elasticity of substitution is less than one and a decreasing function of the fiscal shock when the elasticity is larger than one. The same is true for the reserve loss (analyzed below).

4. Liability Dollarization

So far we have assumed that there is both a social and a fiscal cost ϕ of abandoning the peg at time T. We now explore a formulation in which these costs arise endogenously as a result of liability dollarization. In this case the fiscal cost is different from the social cost. In addition, the abandonment costs depend not only on T but also on the path followed by the exchange rate once the peg is abandoned.

One prominent feature of modern currency crises is that banks and corporations borrow in dollars against revenues that are denominated in local currency. When the fixed exchange rate regime is abandoned the dollar value of these revenues falls, leading some firms and banks to declare bankruptcy. Here we study a formulation in which firms borrow in dollars against revenue denominated in local currency and, hence, are exposed to exchange rate risk.[22]

Production Firms In the previous sections we assumed that agents in the economy received an exogenous output endowment. We now set this endowment to zero and assume that production is carried out by a continuum of production firms with names, n, in the interval [0 1]. Each firm produces a constant number of units of output, y, which it sells to domestic retailers. The total cost of firm n's production is ny. There is a lag δ between the time when output is produced and sold to domestic retailers and the time when sales revenue is collected. Production firms finance their costs at a interest rate r by borrowing in dollars against the sales revenues that they receive δ periods later.[23] Production firms charge interest

[22] One natural question is why banks and firms do not hedge their exchange rate risk. Burnside, Eichenbaum and Rebelo (2001) discuss how the presence of government guarantees eliminates the incentive to hedge.

[23] This type of trade credit is in practice an important feature of how firms operate, see Petersen and Rajan (1997).

to the retailers during the lag that occurs between delivery of the product and collection of the sales revenue.[24] We assume that foreign borrowing is implicitly or explicitly guaranteed by the government. Thus, abandoning the fixed exchange rate regime can potentially create a fiscal liability for the government.

As in the previous sections we will study what happens if at time zero there is an unanticipated increase in the present value of government spending ($\Delta\Gamma$) that makes the fixed exchange rate regime unsustainable. Since before time zero production firms expected the exchange rate to be constant, they billed retailers $Sye^{r\delta}$ for the goods that they sold in the period $-\delta \leq t \leq 0$. The retailers will pay for these goods in the period $0 \leq t \leq \delta$. At time zero production firms learn that the exchange rate will no longer be fixed and use the new perfect foresight path for the exchange rate to bill future sales, which will be collected from time δ on. This means that only the dollar value of sales collected by production firms between zero and δ can potentially be affected by the abandonment of the peg. The value of these sales will remain intact if the peg is abandoned only after time δ.

The value in dollars of firm n at time zero is:

$$V_0(n) = \int_0^{\delta} \frac{Sye^{r\delta}}{S_t} e^{-rt} dt + \int_{\delta}^{\infty} ye^{r\delta} e^{-rt} dt - \int_0^{\delta} (nye^{r\delta}) e^{-rt} dt - \int_{\delta}^{\infty} (nye^{r\delta}) e^{-rt} dt. \tag{4.1}$$

The first term in this expression, $\int_0^{\delta} (Sye^{r\delta}/S_t) e^{-rt} dt$, is the present value in dollars of revenues from sales collected between zero and δ. These sales were billed before time zero under the expectation of a constant exchange rate, so their value in dollars will fall if there is a devaluation ($S_t > S$). The second term, $\int_{\delta}^{\infty} ye^{r\delta} e^{-rt} dt$, is the present value of the new sales which will be made from time zero on. The dollar value of these sales is not affected by the change in the exchange rate path

[24] An alternative interpretation is that firms have to import ny units of materials to produce y units of output.

that takes place at time zero. The third term, $\int_0^\delta (nye^{r\delta})e^{-rt}dt$, represents the repayment of the dollar loans incurred to finance production costs between time $-\delta$ and zero. Finally, the term $\int_\delta^\infty (nye^{r\delta})e^{-rt}dt$ represents repayment of loans incurred to finance production from time zero on.

Under a sustainable fixed exchange rate regime the exchange rate remains constant after time zero ($S_t = S$) and the value of firm n reduces to:[25]

$$V_{0^-}(n) = \frac{y(1-n)e^{r\delta}}{r}.$$

The aggregate value of production firms is in this case given by:

$$\int_0^1 V_{0^-}(n)dn = \frac{ye^{r\delta}}{2r}.$$

This expression reflects the fact that the average profit margin in the production sector ($\int_0^1 (1-n)dn$) is 1/2.

The fiscal shock that occurs at time zero implies that the fixed exchange rate regime has to be abandoned at some point. If the exchange rate devalues between time zero and δ the revenue collected by some of the production firms during this period may not be sufficient to repay their international loans. This may render the value of these firms negative, making it optimal for them to declare bankruptcy. The value of firm n at time zero can be rewritten as:

$$V_0(n) = \begin{cases} y(1-n)e^{r\delta}/r - \int_T^\delta [(S_t - S)ye^{r\delta}/S_t]e^{-rt}dt & \text{if } 0 \leq T < \delta, \\ y(1-n)e^{r\delta}/r & \text{if } T \geq \delta. \end{cases} \quad (4.2)$$

Note that $y(1-n)e^{r\delta}/r$ is the value of the firm before time zero, while $\int_T^\delta [(S_t - S)ye^{r\delta}/S_t]e^{-rt}dt$ is the present value measured in dollars of the loss in sales revenue that occurs if the fixed exchange rate is abandoned before time δ.

[25] The term $e^{r\delta}$ in this expression reflects the fact that when the firm started to operate it did not collect profits for the first δ periods.

Let n^* be the name of the production firm whose value at $t = 0$ becomes zero: $V_0(n^*) = 0$. Equation (4.2) implies that n^* is given by:

$$n^* = \begin{cases} 1 - r \int_T^\delta [(S_t - S)/S_t] e^{-rt} dt & \text{if } 0 \leq T < \delta, \\ 1 & \text{if } T \geq \delta. \end{cases} \tag{4.3}$$

When the regime is abandoned after time δ, n^* is equal to one and there are no defaults since $V_0(n) \geq 0$ for all n. When the peg is abandoned before time δ the marginal firm, n^* is such that the profit per unit of output produced (which is foregone upon default) equals the annuity value of the shortfall in the present of value of sales revenue measured in dollars. All firms with $n \geq n^*$ default at time zero.

When firms default the government collects the revenue from their past sales and repays their loans. This bailout operation creates a fiscal liability. We assume that the government cannot operate the firms that default. This creates a loss in future profits that represents a social cost associated with default.[26] We will later characterize in more detail the fiscal and social costs associated with abandoning the peg.

Retailers Retailers can buy goods from either domestic or foreign production firms. These goods take δ periods to arrive to the retailers. Once the merchandise arrives it is sold instantaneously either to domestic consumers or to foreigners. The price of each good sold in local currency is $e^{r\delta} S_t$. To simplify we assume that the retailers operate at zero cost.

Under a sustained fixed exchange rate regime, retail profits are zero and hence the value of the retail sector is also zero. When the unanticipated fiscal shock

[26] This extreme assumption is made for convenience. It would be sufficient to assume that future profits fall if the firm is resold or operated by the government. It is important that there be a loss in future profits associated with the firms that go bankrupt. In the absence of this loss there are no social costs of abandoning the peg and it is optimal to abandon at time zero.

occurs at time zero the value of the retail sector becomes:

$$V_0^R = \left[\int_0^1 ye^{r\delta} dn\right]\left[\int_T^\delta \frac{S_t - S}{S_t} e^{-rt} dt\right].$$

Note that the profits made by retailers mirror the losses incurred by the production sector. Retailers make losses when $S_t < S$. In this case it is optimal for them to declare bankruptcy. We assume that when this occurs the goods in transit from the production firms to the retailers are lost.

The Household's Budget Constraint Households own both the production firms and the retailers. The household's present value budget constraint before the shock, at time $t = 0^-$ is:

$$b_{0^-} + \int_0^1 V_{0^-}(n)dn = \int_0^\infty c_t e^{-rt} dt. \tag{4.4}$$

Recall that the value of the retailers is zero under a sustained fixed exchange rate regime so it does not affect this expression. After the fiscal shock at time zero the present value constraint for the household becomes:

$$b_{0^-} + \int_0^{n^*} V_0(n)dn + V_0^R = \tag{4.5}$$

$$\int_0^\infty c_t e^{-rt} dt + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj}(M_j - M_{j^-})/S_j.$$

The term $\int_0^{n^*} V_0(n)dn$ represents the value of the production firms that did not fail. Recall that V_0^R denotes the aggregate value of the retail sector at time zero.

The Government's Budget Constraint At $t = 0$ (i.e., immediately after the fiscal shock), the government budget constraint is given by:

$$\int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj}(M_j - M_{j^-})/S_j = \Delta\Gamma + \phi^F, \tag{4.6}$$

where ϕ^F denotes the fiscal cost of abandoning the peg, which we characterize next.

The Fiscal Cost of Abandoning the Peg The government takes over the firms that default at time zero, collects the sales revenue that occurs between zero and δ, and pays the firm's foreign loans. This generates a fiscal cost ϕ^F, given by:

$$\phi^F = \int_{n^*}^{1} \int_{0}^{\delta} \left[nye^{r\delta} - \frac{Sye^{r\delta}}{S_t} \right] e^{-rt} dt dn.$$

Note that the path for S_t has two effects on ϕ^F. First, it affects the dollar value of the sales revenue collected by the government. Second, it affects the number of firms that fail (see (4.3)). The path of S_t will be shaped by two factors: the time at which the peg is abandoned (T) and the monetary policy followed after time T.

The Social Cost of Abandoning the Peg Combining the household and the government budget constraints (equations (4.6) and (4.5)) and consolidating the value of production firms and retailers we obtain the following aggregate resource constraint for the economy:

$$b_{0^-} + \int_0^1 V_{0^-}(n) dn = \int_0^\infty c_t e^{-rt} dt + \Delta\Gamma + \phi^S, \tag{4.7}$$

where ϕ^S, which represents the social cost of abandoning the peg, is defined as:

$$\phi^S \equiv \int_{n^*}^{1} \frac{y(1-n)}{r} dn. \tag{4.8}$$

Note that ϕ^S is equal to zero when the peg is only abandoned after time δ since in this case there are no bankruptcies ($n^* = 1$). The social cost of abandoning the peg stems from the loss of the future profits of the firms that fail. This profit flow, which would be collected from time δ on, is given by $y(1-n)e^{r\delta}$. The social cost is thus the present value of this profit flow, $y(1-n)/r$ summed over all the firms that fail ($n \geq n^*$). This cost arises because the private incentives to default do not coincide with the social incentives. The private firms that default forego their future profits in order to avoid paying their loans to foreigners. However, these loans were guaranteed by the government and hence have to be repaid to foreign banks by the government. For this reason it is not optimal, from a social standpoint, for production firms to fail.[27]

4.1. Optimal Monetary Policy

The optimal monetary policy for this economy can be characterized by choosing the paths for M_t and c_t that maximize household's lifetime utility, (2.1) subject to the cash-in-advance constraint (2.3), the government's budget constraint (4.6), the aggregate resource constraint, (4.7), and equation (4.3), which determines which production firms default.

This problem is difficult to study analytically. However, we can rule out the possibility that a revaluation (a fall in S_0) at time zero is optimal. Suppose that the peg is abandoned at $t = 0$ and that M_0 is reduced. In addition, from time zero on money grows at a constant rate ε, chosen so that the government's budget constraint is satisfied. Suppose that M_0 is chosen so that S_0 satisfies the condition: $S_\delta = S_0 e^{\varepsilon\delta} = S$. In this case, the exchange rate at time δ coincides with S so, abstracting from retailer default, $n^* = 1$. Since the rate of inflation is constant there is no intertemporal distortion imposed on the path for consumption chosen by the households. The problem with this policy is that the exchange rate revaluation delivers losses to the retailers, leading them to declare bankruptcy. The fact that the goods in transit to the retailers are lost generates a large social cost. In addition, the sales revenue received by production firms becomes zero between time zero and time δ, maximizing the number of bankruptcies in the

[27] In the text we assume that there is no free entry, so the firms that exit are not replaced. However, the social cost would be the same if we assumed that there is free entry and that it costs $y(1-n)/r$ to set up a firm with productivity n. Entrants would start operating today but would only start receiving profits from period δ on.

production sector.

To make further progress we will characterize numerically the optimal monetary policy within the class of policies where the growth rate of money is constant once the peg is abandoned. Note that for this class of policies n^* is solely a function of ε and T (see (4.3)). As result, both the social and the fiscal costs are also functions of T and ε. Figure 3 shows how ϕ^F and ϕ^S vary with T.[28] As in section 3 these costs are declining functions of T, thus providing the government with an incentive to delay the attack. Note that both costs are equal to zero for $T \geq \delta$.

Figure 4 displays the optimal abandonment time, T, as a function of the size of the fiscal shock, $\Delta\Gamma$. In line with our results in section 3 (recall Figure 2) the optimal value of T is a declining function of $\Delta\Gamma$. For values of $\Delta\Gamma$ greater than certain threshold (call it $\Delta\Gamma^*$), it is optimal to abandon the peg at time zero. The value of ε is an increasing function of the fiscal shock. The higher the value of $\Delta\Gamma$, the higher the money growth rate that is necessary to finance the expenditure increase with seignorage revenues. Reserve losses at the time of abandonment (given by $(M_0 - M_T)/S$) are an increasing function of the fiscal shock for values of $\Delta\Gamma < \Delta\Gamma^*$ because larger fiscal shocks are associated with larger values of ε. When ε is higher, expected inflation is also higher, leading to a larger reduction in real balances on the part of households at the time of the attack. Since for $\Delta\Gamma \geq \Delta\Gamma^*$ it is optimal to choose $T = 0$, the reserve loss is zero in these cases.

For values of $\Delta\Gamma$ such that it is optimal to abandon at time zero the central bank can follow two basic courses of action (as a well as combinations thereof). Policy 1 is to finance the increase in expenditures with an increase in M_0. Policy 2 is to leave $M_0 = M_{0^-}$ and choose a constant growth rate of money ε from time

[28] The numerical example used in this section was constructed with the following parameters: $r = 0.015$, $\delta = 6$, $f_{0^-} = 0$, $b_{0^-} = 0.1$, $y = 1$, and $S = 1$.

zero on. In the model of section 2 these two policies generated the same level of welfare. In contrast, Policy 2 generates a higher level of welfare in the model considered here. This occurs because Policy 2 generates a lower cumulative rate of depreciation between time zero and time δ, leading fewer production firms to declare bankruptcy.

5. Stochastic Fiscal Reform

In sections 3 and 4 we studied the optimal monetary policy in models where there are both fiscal and social costs of abandoning the fixed exchange rate regime. We will now consider an economy where these costs are absent but where government spending is stochastic. As in the previous sections, we assume that before time zero the fixed exchange rate regime was sustainable, so the government's net foreign assets were sufficient to finance the present value of government spending. At time zero the economy learns that the present value of government spending has increased by $\Delta\Gamma$. The new element introduced in this section is that while the exchange rate is fixed there may be a reduction in government spending that makes the peg, once again, sustainable. This expenditure reduction occurs according to a Poisson process with arrival rate λ. If the peg is abandoned the increase in government spending becomes permanent and has to be financed with seignorage revenues. There is thus an option value of holding on to the peg. This formulation captures in a simple way the idea that a fixed exchange rate regime exerts pressure on the fiscal authorities to enact reforms to make the peg sustainable. This pressure disappears once the economy floats. An alternative interpretation is that the country may receive a bailout transfer from abroad that pays for the increase in government spending and renders the peg sustainable. This external bailout arrives according to a Poisson process.

The size of the fiscal reform or of the external bailout that has to occur to make the fixed exchange rate regime sustainable depends, naturally, on the path of government spending. If the reform occurs at time t the present value of government spending from time t on has to be reduced to a value Γ_t given by:

$$\Gamma_t = f_0 e^{rt} - e^{rt} \int_0^t g_s e^{-rs} ds.$$

This expression implies that if there has been no new spending between time zero and time t all that is necessary to make the peg sustainable is to cancel the plans for new government spending in the future. However, if new spending has already taken place in the time interval up to time t the government needs to reduce the present value of government spending below its level before the fiscal shock.

The design of optimal policy reduces to choosing the time T at which the fixed exchange rate regime will be abandoned if a fiscal reform has not, in the meantime, materialized. A higher value of T makes a fiscal reform more likely. However, the longer the horizon T, the larger the intertemporal consumption distortion that the government will have to introduce if reform does not occur.

The Time When Reform Occurs We will start by characterizing the case in which a fiscal reform has just occurred making the fixed exchange rate sustainable. Consumption will be constant and its level, which we denote by c^*, can be computed using the household's budget constraint:

$$b + y/r = c^*/r + (c^* - m).$$

Here b and m denote the levels of net foreign assets and real balances that households had in the period where the reform took place. The term $(c^* - m)$ represents the jump in real balances that occurs when agents learn that the fixed exchange rate regime has become sustainable. Lifetime utility is given by:

$$V^*(b+m) = \frac{\log\left[(rb + rm + y)/(1+r)\right]}{r}.$$

The $t \geq T$ Regime Suppose that we have reached time T and a reform has not occurred. The fixed exchange rate regime will now be abandoned and the growth rate of money will rise to a level ε such that the government's intertemporal resource constraint is satisfied. Consumption will be constant at a level which we denote by c^2. This level can be computed using the household's budget constraint:

$$b + y/r = c^2(1+\varepsilon)/r + (c^2 - m). \tag{5.1}$$

where $(c^2 - m)$ represents the jump in real balances that takes place at time T in response to a permanent increase in inflation from zero to ε. Using (5.1) to solve for c^2, we can compute lifetime utility at time T:

$$V(b+m,T) = \frac{\log\left[(rb + rm + y)/(1+r+\varepsilon)\right]}{r}.$$

For future reference we note that this value function bears a simple relation with the value function associated with the reform regime:

$$V(b+m,T) = V^*(b+m) - \frac{\log(p)}{r},$$

where p is given by (3.3).

The fact that $r = \rho$ and that inflation is constant means that for any time period $t \geq T$ the value function coincides with $V(b+m,T)$:

$$V(b+m,t) = V(b+m,T) \text{ for } t \geq T.$$

The Regime for $t \leq T$ and No Reform The optimality equation for the household's problem during this period is:

$$\begin{aligned} rV(b+m,t) \quad = \quad & \max_{c^1}\{\log(c^1) + V_2(b+m,t) + \\ & [r(b+m) + y - c^1(1+r)]V_1(b+m,t) + \\ & \lambda[V^*(b+m) - V(b+m,t)]\}. \end{aligned}$$

The first order condition with respect to consumption (c^1) is:

$$1/c^1 = V_1(b+m,t)(1+r).$$

It is easy to verify that the value function has the form:

$$V(b+m,t) = \frac{\log\left[(rb+rm+y)/(1+r)\right]}{r} - \frac{e^{-(\lambda+r)(T-t)}\log(p)}{r}. \tag{5.2}$$

This equation has a simple interpretation. Consider first an economy in which a fiscal reform has no chance of occurring ($\lambda = 0$) and which will switch to the floating regime with certainty at time T. Since utility declines by $\log(p)/r$ at time T lifetime utility at time t would be

$$\frac{\log\left[(rb+rm+y)/(1+r)\right]}{r} - \frac{e^{-r(T-t)}\log(p)}{r}.$$

Our value function is similar to this expression but the discount factor applied to $\log(p)/r$ incorporates λ to reflect the fact that there is an ongoing probability of a fiscal reform until time T.

5.1. Optimal Monetary Policy

At time zero, when the economy learns that there has been an increase in the present value of government spending, the lifetime utility of the household declines from $V^*(b+m)$ to $V(b+m,0)$ (given by equation (5.2)).

The central bank chooses T, the maximum number of "periods" that it is willing to wait for a fiscal reform to occur. If the economy reaches time $T > 0$ with no fiscal reform, the central bank will have to print money so that the government's intertemporal budget constraint holds. Since it is optimal to choose a constant growth rate of money, the government's present value resource constraint can be written as:

$$\frac{\varepsilon c^2}{r}e^{-rT} + (c^1 - m_{0^-}) + (c^2 - c^1)e^{-rT} = \Delta\Gamma. \tag{5.3}$$

Note that there are no stochastic elements in this equation. This constraint is only relevant when the economy reaches time T without a fiscal reform, in which case all uncertainty has been resolved.

Using the fact that $c^2 = c^1/p$ together with (3.3) we can rewrite (5.3) as:

$$p = \frac{c^1/r}{c^1/r - (\Delta\Gamma + m_{0^-} - c^1)\, e^{rT}}. \tag{5.4}$$

This equation defines p as a function of T .

The optimal policy can be characterized by maximizing $V(b+m,0)$ (given by (5.2)) subject to (5.4). The optimal value of T is given by condition:

$$(\lambda + r)\log(p) - \frac{dp}{dT}\frac{1}{p} \leq 0, \tag{5.5}$$

which holds with equality whenever the optimal value of T is interior. This equation is similar to the one that characterizes the optimal policy in section 3 (equation (3.10)), showing the close connection between the two problems. In fact, comparing these two equations it is easy to see that for every value of ϕ in the economy of section 3, there is a value of λ in the model of this section such that the two economies choose the same value of p when the peg is abandoned at time T.

Using equation (5.4) to compute dp/dT,we can rewrite equation (5.5) as:

$$(1-p)r + (r+\lambda)\log(p) \leq 0.$$

Using this equation together with (5.4) we can characterize the optimal abandonment time, T. The results are summarized by the following proposition.

Proposition 5.1. *For every $\lambda > 0$ there is a level for the present value of government spending, Γ^* such that for $\Gamma_0 > \Gamma^*$ it is optimal to abandon the peg at time zero ($T = 0$), while for $\Gamma_0 \leq \Gamma^*$ it is optimal to delay abandoning the peg ($T \geq 0$). The value of Γ^* is increasing in λ.*

Proof: See Appendix 7.4.

It may seem counterintuitive that when the fiscal shock is large it is optimal to abandon the peg immediately. Why not wait for a while to see whether a fiscal reform occurs eliminating the effect of this fiscal shock? The problem with waiting is that if there is no reform until time T the government will have to generate very high rates of inflation that will severely reduce household consumption from time T on. In other words, when $\Delta\Gamma$ is high the distortion that will be imposed on the intertemporal allocation of consumption if reform fails is so large that it is preferable to abandon the peg immediately. In contrast, for small values of $\Delta\Gamma$ it is optimal to wait, since if reform fails to materialize the distortion that will be introduced in the economy is relatively minor.

The fact that Γ^* is increasing in λ is intuitive: it means that when the reform arrival rate is higher the range of fiscal shocks for which it is optimal to delay abandoning the peg is larger.

6. Conclusion

Versions of the Krugman-Flood-Garber (KFG) are widely used to think about speculative attack episodes. This class of models has often been criticized for assuming a non-optimizing central banker who follows a mechanical rule for abandoning the fixed exchange rate regime: the peg is abandoned if and only if international reserves reach a pre-specified lower bound.

In this paper we use an optimization-based version of the KFG model to characterize the optimal time for abandoning a fixed exchange rate regime that has become unsustainable due to an unexpected increase in the present value of government spending. We consider two different costs of abandoning the peg: an exogenous cost and an endogenous cost that results from liability dollarization. In addition, we also consider a stochastic case in which a fiscal reform may occur while the peg is in effect. In all three cases, for a sufficiently large shock, it is optimal to abandon the peg right away, independently of the level of reserves. This implies a sharp departure from the KFG model where the size of the underlying fiscal deficits plays no direct role in the decision to abandon the peg (and matters only insofar as it affects the speed at which the stock of reserves is depleted). In principle, therefore, our model predicts that the decision to abandon should be mostly determined by the size of the underlying fiscal shock rather than by the level of reserves (which would be the key variable according to the KFG model). For moderate fiscal shocks, however, the rule postulated in the Krugman-Flood-Garber model may be optimal.

So far we have studied a basic monetary model where the only impact of inflation is that it may distort intertemporal consumption allocations. This analysis provides us with a departure point to study richer environments in which tax revenue and the cost of financing public debt are endogenous and where monetary policy affects the level of output through various channels.

References

[1] Buiter, Willem "Borrowing to Defend the Exchange Rate and the Timing and Magnitude of Speculative Attacks," *Journal of International Economics*, 23: 221-239, 1987.

[2] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo "Government Guarantees and Self-Fulfilling Speculative Attacks," mimeo, Northwestern University, 2000.

[3] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo "Prospective Deficits and the Asian Currency Crisis," *Journal of Political Economy*, 109: 1155-1198, 2001.

[4] Calvo, Guillermo "Balance of Payments Crises in a Cash-in-Advance Economy," *Journal of Money Credit and Banking*, 19: 19-32, 1987.

[5] Drazen, Allan and Elhanan Helpman "Stabilization with Exchange Rate Management," *The Quarterly Journal of Economics*, 835-855, 1987.

[6] Flood, Robert P. and Peter Garber "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics*, 17: 1–13, 1984.

[7] Flood, R., and O. Jeanne, "An Interest Rate Defense of a Fixed Exchange Rate?," IMF Working Paper, 159, 2000.

[8] Flood, Robert., P. Peter Garber, and Charles Kramer, "Collapsing Exchange-Rate Regimes: Another Linear Example," *Journal of International Economics* 41, 223-234, 1996.

[9] Flood, Robert P., Jagdeep S. Bhandari, and Jocelyn Horne, "Evolution of Exchange Rate Regimes," *IMF Staff Papers*, 36: 810-835, 1989.

[10] Kaminsky, Graciela and Carmen Reinhart, "The Twin Crises: The Causes of Banking and Balance-of-Payments," *American Economic Review*, 89: 473-500, 1999.

[11] Krugman, Paul "A Model of Balance of Payments Crises," *Journal of Money, Credit and Banking*, 11: 311–25, 1979.

[12] Lahiri, Amartya and Carlos Végh "Delaying the Inevitable: Optimal Interest Rate Policy and BOP Crises," National Bureau of Economic Research Working Paper 7734, 2000.

[13] Obstfeld, Maurice "Speculative Attack and the External Constraint in a Maximizing Model of the Balance of Payments," *Canadian Journal of Economics*, 29: 1-20, 1986a.

[14] Obstfeld, Maurice "Rational and Self-Fulfilling Balance-of-Payments Crises," *American Economic Review*, 76: 72–81, 1986b.

[15] Obstfeld, Maurice "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review*, 40: 1037-48, 1996.

[16] Petersen, Mitchell A., and Raghuram G. Rajan, "Trade Credit: Theories and Evidence," *Review of Financial Studies*, 10: 661-691, 1997.

[17] Rebelo, Sergio and Danyang Xie "On the Optimality of Interest Rate Smoothing," *Journal of Monetary Economics*, 43: 263-282, 1999.

[18] Rigobon, Roberto "Disinflation and Fiscal Reform: A Neoclassical Perspective," National Bureau of Economic Research Working Paper 8706, 2002.

[19] Wijnbergen, Sweder Van "Fiscal Deficits, Exchange Rate Crises and Inflation," *Review of Economic Studies*, 58: 81-92, 1991.

7. Appendices

7.1. Behavior of $\Psi(\phi;\Delta\Gamma)$.

This appendix contains the proofs of Propositions 3.1 and 3.2 (the results of which are summarized in Table 1). First, notice that, for $\Delta\Gamma > 0$, we can rule out a solution in which $T \to \infty$ because, by construction, such a solution would violate the government's intertemporal constraint. In what follows, therefore, we can ignore the exponential term in (3.10). Let $\Psi(\phi;\Delta\Gamma) \equiv \log p(T,\phi,\Delta\Gamma) - \frac{p_T(T,\phi,\Delta\Gamma)}{rp(T,\phi,\Delta\Gamma)}$. Hence, for $\Delta\Gamma > 0$ (and using (3.7) and (3.8)),

$$\Psi'(\phi;\Delta\Gamma) = \frac{p_\phi}{c^1}\left(c^1 - r\phi - 2r\Delta\Gamma e^{rT}\right). \tag{7.1}$$

We can now establish the following (for the $T = 0$ case):

1. If $\Delta\Gamma \geq c^1/2r$, then $\Psi'(0;\Delta\Gamma) \leq 0$ and $\Psi'(\phi > 0;\Delta\Gamma) < 0$. Since $\Psi(0;\Delta\Gamma) < 0$ (recall (3.12)), it then follows that $\Psi(\phi;\Delta\Gamma)$ is always negative and hence there is always a corner solution at $T = 0$.

2. If $c^1/er \leq \Delta\Gamma < c^1/2r$, then the function $\Psi(\phi;\Delta\Gamma)$ reaches a maximum at $\phi^{\max} = c^1/r - 2\Delta\Gamma$.[29] The value of the function at this point is given by:

   $$\Psi(\phi^{\max};\Delta\Gamma) = \log p(0,\phi^{\max},\Delta\Gamma) - 1 \leq 0,$$

   which implies that the solution is $T = 0$ (either corner or boundary solution).

3. If $0 < \Delta\Gamma < c^1/er$, $\Psi(0;\Delta\Gamma)$ cuts the horizontal axis twice (denote these roots by ϕ^* and ϕ^{**}, $\phi^* < \phi^{**}$). To show this, notice that $\Psi(\phi^{\max};\Delta\Gamma) > 0$, which implies that ϕ^* exists. To establish existence of the second root, ϕ^{**}, it is enough to notice that (i) in light of the resource constraint, $\phi < c^1/r - \Delta\Gamma$ and (ii) the limit of $\Psi(\phi;\Delta\Gamma)$ as ϕ approaches this upper bound is $-\infty$.

[29] Notice that $\phi^{\max}$ is always smaller than the maximum feasible value for ϕ (dictated by the resource constraint) which equals $\frac{c^1}{r} - \Delta\Gamma e^{rT}$.

Points 1 and 2 constitute the proof of Proposition 3.1, while point 3 proves Proposition 3.2.

So far we have established that for some ranges of parameter values, we have "local" corner solutions (i.e., around $T = 0$). To show that these corner solutions are also global (i.e., that they hold for any T), it is enough to show that for any given ϕ, $\Psi(\phi; \Delta\Gamma)$ is strictly decreasing in T for any $T \geq 0$. This is indeed true as this derivative can be shown to be given by $-p_T r \Delta\Gamma e^{rT}/c^1 < 0$ for $T \geq 0$.

7.2. Behavior of T, ε, and Reserve Loss as a Function of ϕ.

Behavior of T. Take as given $\Delta\Gamma \in (0, c^1/er)$ and consider the ranges for ϕ – established above – for which the solution for T is interior. In that case, $\Psi(\phi; \Delta\Gamma) = 0$ implicitly defines the optimal T as a function of ϕ:

$$\phi = \frac{c^1}{rp}\left(p - 1 - \log p\right). \tag{7.2}$$

Hence:

$$\frac{dT}{d\phi} = \frac{\Psi'(\phi; \Delta\Gamma) c^1}{p_T r \Delta\Gamma e^{rT}}.$$

where the behavior of $\Psi'(\phi; \Delta\Gamma)$ has been derived above. Hence, T is an increasing function of ϕ for $\phi \in [\phi^*, \phi^{\max})$ and a decreasing function for $\phi \in [\phi^{\max}, \phi^{**},)$ For all other values of ϕ, the optimal $T = 0$, as established above. Figure 1, Panel A, illustrates optimal T as a function of ϕ.

Behavior of ε. For the range of interior solutions, it follows from (7.2) that:

$$\frac{d\varepsilon}{d\phi} = \frac{r(1+r)p^2}{c^1 \log(p)} > 0.$$

When optimal $T = 0$, ε is also an increasing function of ϕ, as follows from (3.13) and (3.14). Figure 1, Panel B, illustrates the optimal ε as a function of ϕ. Clearly, at $\phi = \phi^* = \phi^{**}$, this function need not be differentiable.

Behavior of reserve loss By definition, the reserve loss at T is equal to $c^1 - c^2$. Since c^1 is independent of both T and ϕ, we just need to check the behavior of c^2 as a function of ϕ for interior solutions (naturally, for $T = 0$, the reserve loss is zero). Since $c^2 = c^1/p$, it follows that

$$\frac{dc^2}{d\phi} = -\frac{c^1}{p^2(1+r)}\frac{d\varepsilon}{d\phi} < 0.$$

Hence, the reserve loss is an increasing function of ϕ when the solution is interior (see Figure 1, Panel C).

7.3. Behavior of T, ε, and Reserve Loss as a Function of $\Delta\Gamma$.

Behavior of T We now derive the behavior of the optimal values of T, ε, and reserve loss as a function of $\Delta\Gamma$ for a given $\phi \in (\phi^*, \phi^{**})$. As shown above, the solution will be interior for $\Delta\Gamma \leq \frac{c1}{er}$. In this range, using (3.10), it follows that:

$$\begin{aligned} \frac{dT}{d\Delta\Gamma} &= -\frac{p_{\Delta\Gamma}}{p_T} < 0. \\ \lim_{\Delta\Gamma \to 0} T &= \infty. \end{aligned}$$

For any $\Delta\Gamma \geq \frac{c1}{er}$, the solution is $T = 0$, as shown above. In Figure 2, and without loss of generality, the given value of ϕ has been taken to be $\phi = \frac{c^1}{r}(1 - \frac{2}{e})$. It can be checked that $\Psi[\frac{c^1}{r}(1 - \frac{2}{e}); \frac{c1}{er}] = 0$ and hence in Panel A, $T(\Delta\Gamma^* = \frac{c^1}{er}) = 0$.

Behavior of ε. Consider now the behavior of the optimal value of ε as a function of $\Delta\Gamma$. Since the RHS of (7.2) is a strictly increasing function of p, it then follows that, when the solution is interior, the optimal value of p (and hence ε) is fully determined by ϕ and is therefore independent of $\Delta\Gamma$. Hence, for $0 < \Delta\Gamma < \frac{c1}{er}$, optimal ε does not depend on $\Delta\Gamma$.[30] For $\Delta\Gamma \geq \frac{c1}{er}$, optimal T is zero. It then follows from (3.13) and (3.14) that ε is an increasing function of $\Delta\Gamma$. (See Panel B in Figure 2).

Behavior of reserve loss Finally, consider the reserve loss ($\equiv c^1 - c^2$). Clearly, for $\Delta\Gamma \geq \frac{c1}{er}$, the reserve loss is zero since the peg is abandoned right away. For $0 < \Delta\Gamma < \frac{c1}{er}$, the reserve loss equals $c^1(p-1)/p > 0$. Since p is independent of $\Delta\Gamma$ when the solution is interior, then the reserve loss is also independent of $\Delta\Gamma$ in this range.

7.4. Proof of Proposition 5.1

Define the function $K(p) = (1-p)r + (r+\lambda)\log(p)$. It is easy to show that this function is concave, that for $\lambda > 0$ it intersects the x-axis twice, at $p = 1$ and at a value of p greater than 1 which we will denote by p^*. The maximum value of K is achieved for $p = (r+\lambda)/r$. To check whether $T = 0$ is optimal we can set $T = 0$ in (5.4) to compute the value of p that would be consistent with the government budget constraint if the peg was abandoned immediately. We denote this value of p by p^0:

$$p^0 = \frac{c^1/r}{c^1/r - (\Delta\Gamma + m_{0^-} - c^1)}.$$

Using the fact that $b_{0^-} + m_{0^-} + y/r = c^1(1+r)/r$ we can rewrite this expression as:

$$p^0 = \frac{c^1/r}{b_{0^-} + y/r - \Delta\Gamma}.$$

[30] Notice that for $\Delta\Gamma = 0$, optimal ε is zero since optimal T is "infinity" (i.e., the peg will never be abandoned).

We can then use this expression for p^0 to evaluate the Kuhn-Tucker condition. If $K(p^0) < 0$, $T = 0$ is optimal, otherwise $T > 0$ is optimal. The variable p^0 is an increasing function of $\Delta\Gamma$ which takes the value 1 when $\Delta\Gamma = 0$ (in this case there in no expenditure shock at time zero and the regime continues to be sustainable). The value of p^0 converges to infinity as $\Delta\Gamma \to b_{0^-} + y/r$. This limiting value of $\Delta\Gamma$ is such that government spending exhausts all the resources of the economy. Define $\Delta\Gamma^*$ as the value of $\Delta\Gamma$ such that $p^0 = p^*$. Then for $\Delta\Gamma > \Delta\Gamma^*$, $K(p^0) < 0$ so it is optimal to abandon immediately. For $\Delta\Gamma < \Delta\Gamma^*$, $K(p^0) > 0$ and the optimal value of T is interior. Finally, it is easy to see that p^* is an increasing function of λ. This implies that $\Delta\Gamma^*$ is also an increasing function of λ.

When Is it Optimal to Abandon a Fixed Exchange Rate?*

Sergio Rebelo[†] and Carlos A. Végh[‡]

First draft: August 2001
Current draft: April 2002

Abstract

This paper analyzes the optimal time to abandon a fixed exchange rate regime in response to a fiscal shock that renders the peg unsustainable. We consider three variants of an optimization-based first-generation speculative attack model. In the first variant there are exogenous costs of abandoning the fixed exchange rate. These costs may represent a bailout of the banking sector or output loss. The second variant endogenizes the costs of abandoning the fixed exchange rate in an economy with liability dollarization. The third variant incorporates a fiscal reform – which makes the peg sustainable once again – that arrives according to a Poisson process while the exchange rate is fixed. In all three cases, for a sufficiently large fiscal shock it is optimal to abandon the peg as soon as the shock occurs, regardless of the level of international reserves. This implies a sharp departure from the KFG model where the size of the underlying fiscal deficit plays no direct role in the decision to abandon the peg (and matters only insofar as it affects the speed at which the stock of reserves is depleted).

J.E.L. Classification: F31

Keywords: Currency crisis, speculative attacks, optimal policy, fixed exchange rates.

*We thank Linda Goldberg, Pierre-Olivier Gourinchas, Michael Klein, Assaf Razin, seminar participants at the 2001 Di Tella Summer Camp, NBER IFM Program Meeting, New York Fed, and, especially, Bob Flood for useful comments and suggestions. Financial support from the National Science Foundation is gratefully acknowledged.

[†]Northwestern University, NBER and CEPR.

[‡]UCLA and NBER.

1. Introduction

Suppose that you are the central banker of a country that has a fixed exchange rate. The economy just suffered a "fiscal shock"–an unexpected increase in government expenditures that will have to be financed with seignorage revenues. When should you abandon the fixed exchange rate? This paper discusses the answer to this question in the context of an optimization-based version of the first-generation speculative attack models of Krugman (1979) and Flood and Garber (1984).[1]

The Krugman-Flood-Garber (KFG) model is arguably one of the most influential models in international finance. Its most remarkable feature is that, even in a perfect foresight context, the model generates a speculative attack – a discrete fall in international reserves – at the time of the crisis. Since most currency crises coincide with a large decline in reserves, the model's key prediction is remarkable from both a theoretical and empirical point of view.

One well-known weakness of the KFG framework is that the central bank is not acting optimally. Instead, it follows a mechanical, exogenous rule for abandoning the fixed exchange rate regime. Specifically, the KFG model *assumes* that the central bank will abandon the fixed exchange rate *if and only if* international reserves reach a critical lower bound. The obvious question is: why would central bankers blindly follow such an arbitrary rule?[2] In a perfect foresight model, the

[1] Optimization-based first generation models of speculative attacks include Obstfeld (1986a), Calvo (1987), Drazen and Helpman (1987), Wijnbergen (1991), Lahiri and Végh (2000), and Burnside, Eichenbaum, and Rebelo (2001).

[2] Second generation of models of speculative attacks introduced an optimizing central banker (Obstfeld (1986b,1996)). However, they also changed the nature of the currency crisis. In first-generation models the crisis has a fiscal origin–the government is forced to resort to seignorage to satisfy its intertemporal budget constraint. In contrast, second generation models adopt a Barro-Gordon formulation, which emphasizes the effects of unexpected inflation on the economy. This shift in focus was motivated by 1992 speculative attacks on European countries that did not seem to face a fiscal crisis. At least in simple versions of second generation models there is no reason to observe a loss of reserves coinciding with a devaluation. Once it is optimal to abandon fixed exchange rates, the central bank should do so immediately.

presence of a discrete loss of foreign reserves just before the abandonment of fixed exchange rates suggests that the central bank is acting irrationally. If fixed exchange rates are going to be abandoned, why lose reserves in the process?[3]

This paper tackles this issue head on by modeling the decision to abandon a fixed exchange rate as the outcome of an *optimal* policy decision: policymakers rationally abandon the peg when it is best to do so from a social point of view. This exercise turns out to be much more than a theoretical refinement, as it makes clear that the exogenous KFG abandonment rule often leads to sub-optimal outcomes. In particular, in response to a sufficiently large fiscal shock, we show that it is always optimal to abandon the fixed exchange rate immediately (even if it is costly to do so) *regardless* of the level of international reserves. This result stands in sharp contrast with the KFG model where the peg is abandoned only when reserves have reached a critical threshold. Hence, in our world of rational policymakers, the focus shifts from the level of international reserves (the key variable in the KFG model) to the size of the fiscal shock as the main determinant of the decision of when to abandon the fixed exchange rate.

We begin our analysis in section 2 with a small open economy model in which money is introduced via a cash-in-advance constraint on consumption. We show that in this basic model it is never optimal for the central bank to withstand a speculative attack. The first-best policy is to abandon the peg as soon as the fiscal shock hits, thus avoiding the loss of international reserves. This result holds even when the government faces a borrowing constraint. Hence, contrary to what is often claimed, borrowing constraints *per se* cannot be used to rationalize the KFG abandonment rule.

[3] Some authors (Buiter (1987), Flood, Garber, and Kramer (1996), Flood and Jeanne (2000), and Lahiri and Végh (2000)) have analyzed the feasibility and/or optimality of the central bank delaying the crisis (i.e., "defending the peg") by borrowing and/or raising interest rates. While these models give the central bank a more active role than in the original KFG model, they continue to assume that abandonment of the peg is governed by the exogenous KFG rule.

In section 3 we extend the basic model by introducing costs of abandoning the fixed exchange rate. These costs may reflect, for instance, output losses associated with the currency crises or the cost of bailing out the banking system.[4] Our main result is that the optimal time of abandonment declines with the size of the fiscal shock. In particular, for a sufficiently large fiscal shock it is optimal to abandon the peg as soon as the shock occurs, regardless of the level of reserves. For moderate fiscal shocks it is optimal to delay the abandonment of the fixed exchange rate. In this case the KFG abandonment rule may be actually optimal: the central bank can implement the optimal monetary policy by announcing that the peg will be abandoned when international reserves reach a suitably chosen lower bound. Surprisingly, immediate abandonment of the peg is also optimal when the costs of abandoning the peg are large.

In section 3 the social and fiscal costs of abandoning the peg are modeled as exogenous and assumed to coincide. In section 4 we endogenize these costs by considering an economy characterized by liability dollarization.[5] In this economy production firms borrow in dollars against revenue denominated in local currency. Their foreign borrowing is guaranteed by the government. When a devaluation occurs some firms declare bankruptcy. Bankruptcy creates a social cost because the future profits of these firms are lost. These bankruptcies also create a fiscal cost because the government has to repay the firm's foreign loans. We study numerically the optimal monetary policy for this economy and obtain results that are similar to those of section 3. In particular, for sufficiently large fiscal shocks, it is optimal to abandon the peg immediately. For smaller shocks, the optimal time of abandonment is a decreasing function of the size of the shock.

[4] Kaminsky and Reinhart (1999) persuasively argue that banking and currency crises tend to occur simultaneously.

[5] The recent traumatic abandonment of the 10 year-old fixed exchange rate in Argentina has made clear that liability dollarization is arguably the major cost of a devaluation in emerging countries.

In section 5 we consider a stochastic version of our model to explore a different setting in which to analyze whether the KFG abandonment rule might be optimal. In this version of the model, there are no fiscal costs of abandoning the peg but fiscal fundamentals are random. These fundamentals are governed by a stochastic process that captures the idea that a fiscal reform is more likely to occur while the economy has a fixed exchange rate. Specifically, we assume that, while the exchange rate is fixed, there may be a fiscal reform that restores the sustainability of the fixed exchange rate regime.[6] This reform arrives according to a Poisson process. Once the economy abandons the fixed exchange rate regime there is no hope of a fiscal reform and the initial fiscal shock must be financed with seignorage revenues. There is thus an *option value* of sticking to the peg since it is always possible that by delaying one more instant, a reform will take place. In this context, the cost of abandoning the peg is giving up this option value. We show that there is a close connection between this model and the one of section 3 and that, once again, for sufficiently large shocks it is optimal to abandon the peg right away. Section 6 concludes.

2. The Basic Model

Consider a standard optimizing small open economy model in which money is introduced via a cash-in-advance constraint on consumption. All agents, including the government, can borrow and lend in international capital markets at a constant real interest rate r. There is a single consumption good in the economy and no barriers to trade, so that the law of one price holds: $P_t = S_t P_t^*$, where P_t and P_t^* denote the domestic and foreign price level, respectively. The exchange rate, defined as units of domestic currency per unit of foreign currency, is denoted by

[6] See Flood, Bhandari and Horne (1989) and Rigobon (2002) for analyses that also emphasizes the link between fixed exchange rates and fiscal discipline.

S_t. For convenience we assume that $P_t^* = 1$.

Just before time zero (i.e., at $t = 0^-$), the exchange rate, S_t, is fixed at a value S. For $t < 0^-$, the economy has a sustainable fixed exchange rate regime: the government can satisfy its intertemporal budget constraint without resorting to seignorage. At $t = 0$ the economy learns that it has suffered a 'fiscal shock': an increase in government expenditures that must be financed with seignorage revenues. Denote by T the time at which the fixed exchange rate regime is abandoned. We wish to answer the following question: what is the optimal value of T?

2.1. Households

Households maximize their lifetime utility, V, which depends on their consumption (c_t) path:

$$V \equiv \int_0^\infty \ln(c_t)e^{-\rho t}dt. \tag{2.1}$$

The discount factor is denoted by ρ. To simplify, we assume that $r = \rho$. The household's flow budget constraint is:

$$\begin{array}{ll} \Delta b_t = -(M_t - M_{t^-})/S_t & \text{if } t \in J, \\ \dot{b}_t = rb_t + y - c_t - \dot{m}_t - \varepsilon_t m_t & \text{if } t \notin J. \end{array} \tag{2.2}$$

Throughout the paper $\dot{x}_t$ denotes dx/dt. Here b_t denotes the household's holdings of foreign bonds that yield a real rate of return of r, and y is a constant, exogenous, flow of output. The variable m_t represents real money balances, defined as $m_t = M_t/P_t$, where M_t denotes nominal money holdings. The variable ε_t denotes the rate of devaluation, which coincides with the inflation rate, $\varepsilon_t = \dot{P}_t/P_t = \dot{S}_t/S_t$.

As in Drazen and Helpman (1987), equation (2.2) takes into account the possibility of discrete changes in b_t and M_t at a finite set of points in time, J. We will later see that this set is comprised of time zero and at the time when the peg is abandoned. These jumps are defined as $\Delta b_t \equiv b_t - b_{t^-}$, where b_{t^-} represents the limit from the left. Since at any point in time $t > 0$, the total level of real financial assets cannot change discretely, $b_{t^-} + m_{t^-} = b_t + m_t$.[7] At time $t = 0^-$, just before the households' time zero decisions are made, agents hold an amount b_{0^-} in real bonds. Their holdings of nominal money balances are M_{0^-}, and their real money balances are therefore $m_{0^-} = M_{0^-}/S$.

Consumption is subject to a cash-in-advance constraint:

$$m_t \geq c_t. \tag{2.3}$$

Since we will only consider environments in which the nominal interest rate is positive, (2.3) will always hold with equality.

The flow budget constraint, together with (2.3) and the transversality condition $\lim_{t\to\infty} e^{-rt} b_t = 0$, implies the following intertemporal budget constraint:

$$b_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j\in J} e^{-rj}(M_j - M_{j^-})/S_j. \tag{2.4}$$

This constraint can be further simplified by using the cash-in-advance constraint and imposing the condition that $\lim_{t\to\infty} e^{-rt} m_t = 0$:[8]

$$b_{0^-} + \frac{M_{0_-}}{S_0} + y/r = \int_0^\infty c_t(1 + r + \varepsilon_t) e^{-rt} dt. \tag{2.5}$$

This expression makes clear that, as is typical of cash-in-advance models, the *effective* price of consumption is given by $1 + r + \varepsilon_t$.

[7] Notice that at $t = 0$ the total level of real financial assets may change discretely due to an unanticipated jump in the exchange rate, which changes the value of real money balances from M_{0^-}/S to M_{0^-}/S_0.

[8] This condition will always be satisfied in equilibrium since (2.3) will hold as an equality.

The first-order condition for the household's problem is:

$$1/c_t = \lambda\left(1 + r + \varepsilon_t\right), \tag{2.6}$$

where λ is the Lagrange multiplier associated with (2.5).

2.2. Government

The government collects seignorage revenues and carries out expenditures (g_t). To simplify, we assume that these expenditures yield no utility to the representative household. The government's flow budget constraint is given by:

$$\begin{array}{ll} \Delta f_t = (M_t - M_{t^-})/S_t & \text{if } t \in J, \\ \dot{f}_t = r f_t - g_t + \dot{m}_t + \varepsilon_t m_t & \text{if } t \notin J, \end{array}$$

where f_t denotes the government's net foreign assets. This constraint, together with the condition $\lim_{t\to\infty} e^{-rt} f_t = 0$, implies the following intertemporal budget constraint:

$$f_{0^-} + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j\in J} e^{-rj}(M_j - M_{j^-})/S_j = \Gamma_{0^-}, \tag{2.7}$$

where, by definition, Γ_{0^-} is the present value of government spending:

$$\Gamma_{0^-} \equiv \int_0^\infty g_t e^{-rt} dt.$$

Notice that when the peg is abandoned at time zero the jump in the money supply $(M_0 - M_{0^-})$ is controlled by the central bank through its choice of M_0. In contrast, when the peg is abandoned at $T > 0$, the jump in the money supply $(M_T - M_{T^-})$ is endogenously determined. As is well known, under perfect foresight the path for the exchange rate has to be continuous for all $t > 0$ to rule out arbitrage opportunities. This means that in equilibrium households will, at time T, reduce their money holdings in anticipation of the higher inflation rate for $t \geq T$.

2.3. Equilibrium Consumption

Combining the households' and government's intertemporal constraints (equations (2.4) and (2.7), respectively), we obtain the economy's aggregate resource constraint:

$$b_{0^-} + f_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Gamma_{0^-}. \tag{2.8}$$

This constraint implies that the present value of output plus the total net foreign assets in the economy must equal the present value of consumption and government expenditures.

2.4. A Sustainable Fixed Exchange Rate Regime

Before $t = 0^-$ the economy was in a sustainable fixed exchange rate regime in which agents expected ε to be permanently zero. This requires that the government's net foreign assets be sufficient to finance the present value of government expenditures. This condition for $t = 0^-$ is:

$$f_{0^-} = \Gamma_{0^-}.$$

In this regime, equations (2.3) and (2.8) imply that consumption and real balances are given by:

$$\begin{aligned} c_{0^-} &= y + rb_{0^-}, \qquad (2.9) \\ m_{0^-} &= c_{0^-}. \end{aligned}$$

2.5. Optimal Monetary Policy

Suppose that at time zero there is an unanticipated increase in the present value of government expenditures from Γ_{0^-} to Γ_0 and that this increase in expenditure must be financed with seignorage. Clearly, the peg will be abandoned at some point because Γ_0 cannot be intertemporally financed with $\varepsilon = 0$. When is it optimal to abandon the peg? Throughout the paper we will focus on the perfect commitment solution to this question.

The new aggregate constraint for the economy can be written as:

$$b_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Delta\Gamma, \tag{2.10}$$

where $\Delta\Gamma = \Gamma_0 - \Gamma_{0^-}$ represents the present value of the *new* government expenditures. Suppose that the government could finance this new expenditure with lump sum taxes. Consumption would be constant over time and its level would be given by:

$$c_0 = c_{0^-} - r\Delta\Gamma.$$

Since $\Delta\Gamma > 0$, the new level of consumption is lower than before. The economy has the same resources as before the fiscal shock, so the rise in government spending has to be accommodated by a fall in private consumption. The corresponding fall in real money balances is accommodated through a fall in the nominal money supply at $t = 0$.

The government can replicate the lump sum taxes outcome by either starting to expand the money supply at $t = 0$ at a constant rate or printing money at $t = 0$ (or a combination thereof). Specifically, suppose that the government abandons the fixed exchange rate regime at time zero, keeps $M_0 = M_{0^-}$, and expands the money supply at a constant rate ε such that the government budget constraint is satisfied:

$$\int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \Delta\Gamma.$$

Since the central bank abandons the fixed exchange rate regime as soon as news about the fiscal shock arrives, there is no speculative attack at time zero. Private agents are not given a chance to trade their money balances for foreign reserves at the fixed exchange rate S before the devaluation occurs. The adjustment in the level of real balances occurs through a jump in the exchange rate, rather than through a discrete fall in the nominal money supply at time 0. The aggregate resource constraint (2.10) implies that consumption will be equal to c_0. The cash-in-advance constraint implies that the new level of real balances will be $m_0 = c_0$. This monetary policy is optimal since it replicates the outcome that can be achieved under lump sum taxes.

The fall in real balances from c_{0^-} to c_0 is associated with a jump in the exchange rate from S to:

$$S_0 = Sc_{0^-}/c_0. \tag{2.11}$$

The constant level of money growth is given by: $\varepsilon = r\Delta\Gamma/c_0 > 0$. Thus from time zero on the currency depreciates at rate ε.

There is another optimal policy which consists of abandoning the peg at time zero and printing enough money to finance the new government spending. In this case the resource constraint of the government is given by: $(M_0 - M_{0^-})/S_0 = \Delta\Gamma$. Printing money at time zero is equivalent to taxing existing real balances. This form of financing is therefore equivalent to lump sum taxes. Since all the seignorage revenue is collected at time zero, this policy implies a higher rate of instantaneous depreciation at time zero than that given by (2.11):

$$S_0 = \frac{Sc_{0^-}}{c_{0^-} - (1+r)\Delta\Gamma}.$$

Any combination of the two policies discussed above, expanding the money supply at a constant rate from time zero on and printing money at time zero, is also optimal. Thus, there are multiple ways for monetary policy to achieve the optimal outcome but all these policies require that the fixed exchange rate be abandoned at time zero.

Abandoning the peg at time $T > 0$ yields a lower level of welfare than the solutions just discussed. It is easy to show that once the fixed exchange rate regime is abandoned at time T, it is optimal to expand the money supply at a constant rate, ε:[9]

$$\begin{aligned} \varepsilon_t &= 0, \quad \text{for } 0 \leq t < T \ , \\ \varepsilon_t &= \varepsilon, \quad \text{for } t \geq T \ . \end{aligned} \tag{2.12}$$

We now show that this constant rate will be strictly positive, thus imposing an intertemporal distortion on the household's consumption path. To this effect, notice that the value of ε has to satisfy the government's intertemporal budget constraint, (2.7), which can be rewritten as:

$$\frac{e^{-rT}\varepsilon}{r}\frac{M_T}{S_T} = \Delta\Gamma + \frac{M_{0^-} - M_0}{S} + \frac{M_0 - M_T}{S}e^{-rT}, \tag{2.13}$$

where $(M_{0^-} - M_0)/S + [(M_0 - M_T)/S]e^{-rT}$ represents the net reserve losses incurred by the government as households rearrange their money balances while the exchange rate is fixed in response to the changes in the path for inflation.

The first order condition for the household problem, (2.6), implies that consumption will be constant within the subperiods $t \in [0, T)$ and $t \in [T, \infty)$. Let us denote by c^1 and c^2 the level of consumption in the periods $t \in [0, T)$ and $t \in [T, \infty)$, respectively. Using equations (2.9), (2.10), and the cash-in-advance constraint, (2.3), we can show that, independently of the form of the momentary utility function and the value of T, the net reserve loss incurred by the government is given by:

$$(M_{0^-} - M_0)/S + [(M_0 - M_T)/S]e^{-rT} = r\Delta\Gamma. \tag{2.14}$$

[9] To show this, solve the planner's problem for an economy with no cash-in-advance constraint. Then show that the cash in advance economy with constant ε can replicate the solution to the planner's problem. See Rebelo and Xie (1999) for details of a closed economy version of this result.

Using this result we can re-write the government budget constraint (2.13) as:

$$\frac{e^{-rT}\varepsilon}{r}\frac{M_T}{S_T} = \Delta\Gamma(1+r). \tag{2.15}$$

This equation implies that $\varepsilon > 0$. The first order condition for the household problem, (2.6), then implies that $c^2 < c^1$. Since the present value of resources that are available for consumption is independent of T, this non-flat path of consumption results in lower welfare compared to the case where the peg is abandoned at time zero.

Finally, notice that introducing a borrowing constraint on the government would not affect the results. Since the optimal policy decision is to abandon immediately it involves no additional borrowing, so the presence of a borrowing constraint is irrelevant.

The net reserve loss described in (2.14) is a cost that the government incurs when the abandonment of the fixed exchange rate regime is delayed. However, since this cost is just a transfer from the government to households (i.e., it is not a *social* cost), it cannot provide a rationale for delaying the abandonment of the peg. The next section considers the case in which there are social costs associated with the abandonment of the peg.

3. Costs of Abandoning the Peg

We now extend the model by assuming that there are social costs associated with abandoning the fixed exchange rate. These costs can be given several interpretations. First, they may reflect declines in output following the abandonment of the peg. Second, since banking crises are typically a by-product of currency crises (see Kaminsky and Reinhart (1999)), these costs could stem from bailing out domestic banks. Third, these costs could result from bailing out foreign creditors. Specifically, a devaluation may make it optimal for domestic firms to default on foreign loans that had been guaranteed by the government. In these circumstances a devaluation creates a fiscal liability for the government (this interpretation is developed in section 4).[10] Finally, this cost could represent the loss of the *option value* of sticking to the peg in an environment in which there is a certain probability that the fixed exchange rate will induce fiscal authorities to undertake a fiscal reform that would make it sustainable once again (we pursue this stochastic interpretation in section 5). As a benchmark, this section focuses on the case in which the cost of abandoning the peg, ϕ, is exogenous and constant over time.[11] Specifically, we assume that when the fixed exchange rate is abandoned, the government incurs a fiscal cost of ϕ. Furthermore, this cost also represents a social loss for the economy as a whole.

3.1. Optimal Monetary Policy

The present value at time zero of the fiscal cost of abandoning the peg at time T is ϕe^{-rT}.[12] Thus the gain, measured in units of output, from a marginal delay in the time of abandonment is $r\phi e^{-rT}$. With this in mind we return to the same question that we asked in the basic model. Suppose that, at time zero, the present value of government spending increases unexpectedly by $\Delta\Gamma$. When is it optimal to abandon the peg in response to this shock?[13]

[10] The model in Burnside, Eichenbaum and Rebelo (2000) suggests an alternative source of fiscal costs stemming from a devaluation. In their model the fact that banks are guaranteed by the government makes it optimal for them to use forward currency markets to increase their exposure to exchange rate risk. As a result these banks go bankrupt when a devaluation occurs, creating a fiscal liability for the government.

[11] Similar results would obtain if ϕ grew at a rate lower than r. Obviously, if ϕ grows at a rate faster than r, the presence of this cost would provide another reason for the devaluation to occur at time zero, so the results of the previous section would continue to hold.

[12] In order to ensure positive consumption, ϕ must be restricted to be less than $y/r + b_{0^-} - \Delta\Gamma e^{rT}$.

[13] The existence of costs of abandoning the peg can give rise to multiple equilibrium if the central bank follows an exogenous KFG abandonment rule. In particular, the peg may collapse in the absence of a fiscal shock ($\Delta\Gamma = 0$). Suppose that agents believe that the peg will be

Since the household's problem remains unaffected by the introduction of the fixed cost of abandoning the peg, the first-order condition (2.6) continues to hold. The optimal path for ε is given by (2.12), so $c_t = c^1$ for $t \in [0, T)$ and $c_t = c^2$ for $t \in [T, \infty)$. To compute the values of c^1 and c^2, we can rewrite the households' intertemporal budget constraint, given by (2.5), as:

$$b_{0^-} + \frac{M_{0^-}}{S_0} + y/r = \frac{c^1(1+r)}{r}(1 - e^{-rT}) + \frac{c^2(1+r+\varepsilon)}{r}e^{-rT}. \tag{3.1}$$

Combining this constraint with the optimality condition (2.6), we obtain:[14]

$$\begin{array}{ll} c_t \equiv c^1 = c_{0^-} = y + rb_{0^-}, & \text{for } 0 \leq t < T, \\ c_t \equiv c^2 = c^1/p = (y + rb_{0^-})/p, & \text{for } t \geq T, \end{array} \tag{3.2}$$

where

$$p \equiv \frac{1+r+\varepsilon}{1+r}, \tag{3.3}$$

denotes the *relative* effective price of consumption across the two regimes. As is typical of cash-in-advance models, consumption is higher before T than afterwards due to the fact that consumption is effectively cheaper before T. Notice also that c^1 is exogenously given by $y + rb_{0^-}$ (i.e., c^1 does not depend on either T or p).

Using (3.2) to replace consumption into (2.1) the household's lifetime utility can be rewritten as:

$$V = \frac{\log(c^1)}{r} - \frac{\log(p)}{r}e^{-rT}. \tag{3.4}$$

abandoned at some time t and that inflation will be equal to $\varepsilon > 0$ from time t on. Agents will then reduce their money balances at time t. The associated reserve loss can potentially trigger the KFG abandonment rule. When this happens the government incurs a cost ϕ which has to be financed with seignorage. This validates the expectations of positive inflation that triggered the attack. These self-fulfilling attacks are, however, eliminated when the government follows an optimal policy. In the absence of a fiscal shock, an optimizing government will never abandon the peg regardless of the behavior of reserves.

[14] To ensure continuity at time zero and thus simplify the optimization problem, we henceforth assume that M_0 is such that $S_0 = S$. This is an innocuous assumption given that when $T = 0$ the central bank is indifferent between collecting seignorage by increasing M_0 or by raising the growth rate of money, ε.

We can use (2.15), amended to incorporate the fiscal cost of abandoning the peg, together with (3.2), (3.3), and the fact that the cash-in-advance constraint always binds to rewrite the government budget constraint as:[15]

$$e^{-rT}\left[\frac{c^1}{r}\frac{(p-1)}{p}-\phi\right]=\Delta\Gamma. \tag{3.5}$$

This equation implicitly defines p (and, given (3.3), ε) as an increasing function of T, ϕ, and $\Delta\Gamma$:

$$p(T,\phi,\Delta\Gamma)=\frac{c^1}{c^1-r(\Delta\Gamma e^{rT}+\phi)}, \tag{3.6}$$

The derivatives of $p(T,\phi,\Delta\Gamma)$ are:

$$p_T(T,\phi,\Delta\Gamma) = \frac{(pr)^2\Delta\Gamma e^{rT}}{c^1}>0, \tag{3.7}$$

$$p_\phi(T,\phi,\Delta\Gamma) = \frac{p^2 r}{c^1}>0, \tag{3.8}$$

$$p_{\Delta\Gamma}(T,\phi,\Delta\Gamma) = \frac{p^2 r e^{rT}}{c^1}>0. \tag{3.9}$$

Intuitively, a higher T implies that a higher inflation rate, ε, is required once the peg is abandoned because the abandonment takes place at a later date. For a given T, a larger fiscal shock ($\Delta\Gamma$) or a higher cost of abandoning (ϕ) also call for a higher rate of inflation (i.e., a larger p).

After this groundwork, we are now ready to tackle the optimal policy problem. The central bank chooses an optimal T to maximize (3.4), with $p(T,\phi,\Delta\Gamma)$ being given by (3.6). The Kuhn-Tucker condition for this problem implies that:

$$e^{-rT}\left[\log p(T,\phi,\Delta\Gamma)-\tfrac{p_T(T,\phi,\Delta\Gamma)}{rp(T,\phi,\Delta\Gamma)}\right]\leq 0, \quad \text{if } T=0, \tag{3.10}$$

$$e^{-rT}\left[\log p(T,\phi,\Delta\Gamma)-\tfrac{p_T(T,\phi,\Delta\Gamma)}{rp(T,\phi,\Delta\Gamma)}\right]= 0, \quad \text{if } T>0.$$

[15] In the presence of a positive ϕ, equation (2.15) becomes $\frac{\varepsilon}{r}\frac{M_T}{S_T}=(1+r)(e^{rT}\Delta\Gamma+\phi)$.

Intuitively, increasing T (i.e., delaying the abandonment of the peg), confers both a direct utility benefit and an indirect utility cost. The direct benefit is that the good times (i.e., high consumption times) are prolonged, which yields a marginal increase in lifetime utility given by $e^{-rT}\log(p)$. There is, however, an indirect utility cost. Delaying implies a marginal increase in the post-crisis inflation rate (captured by p_T), which raises the intertemporal distortion and reduces utility, at the margin, by $e^{-rT}p_T/rp$.

3.2. When Is It Optimal to Abandon the Peg Immediately?

Abandoning the fixed exchange rate as soon as the fiscal shock hits (i.e., at $T = 0$) will be the optimal solution when the net marginal benefit of increasing T (given by (3.10)) around $T = 0$ is either negative (in which case there will be a corner solution at $T = 0$) or exactly zero (in which case there is a boundary solution at $T = 0$). If the net marginal benefit around $T = 0$ is positive, then it is optimal to delay the abandonment of the peg until some later date.[16] Naturally, both the benefit and the cost of delaying depend on ϕ and $\Delta\Gamma$ through their effect on p.

To proceed, we therefore need to evaluate (3.10) at $T = 0$ and characterize the resulting expression as a function of ϕ, for a given positive value of $\Delta\Gamma$(hereafter denoted by $\Psi(\phi; \Delta\Gamma)$).[17] Taking into account (3.5) and (3.7), it follows that

$$\Psi(\phi; \Delta\Gamma) \equiv \log p(0, \phi, \Delta\Gamma) - [p(0, \phi, \Delta\Gamma) - 1] + \frac{r\phi p(0, \phi, \Delta\Gamma)}{c^1}, \qquad \phi \in [0, \frac{c1}{r} - \Delta\Gamma). \tag{3.11}$$

[16] This characterizes a "local" solution around $T = 0$. All the solutions discussed below, however, are also "global" solutions, as shown in Appendix 7.1.

[17] Notice that if $\Delta\Gamma = 0$, there are two trivial cases. If $\phi = 0$, then $\Psi(0; 0) = 0$ for any $T \geq 0$. Technically, this implies that the optimal T is indeterminate (i.e., since there is no fiscal shock, "abandoning" would still imply a completely stable exchange rate). If $\phi > 0$, then $\Psi(\phi; 0) = \log(p) > 0$ because, from (3.6), $p(T, \phi, 0) > 1$. This implies, from (3.10), that the optimal T is "infinity" (i.e., it is never optimal to abandon). This is, of course, consistent with the intertemporal fiscal constraint because there has been no fiscal shock.

As a first check, let us evaluate this function at $\phi = 0$. Note that:

$$\Psi(0; \Delta\Gamma) = \log p(0, 0, \Delta\Gamma) - [p(0, 0, \Delta\Gamma) - 1] < 0, \tag{3.12}$$

since $p(0, 0, \Delta\Gamma) > 1$ for any $\Delta\Gamma > 0$ (see (3.6)). Hence, there is a corner solution at $T = 0$, which implies that it is optimal to abandon the peg as soon as the expenditure shock hits, regardless of the size of this shock. This is, of course, the result derived in the previous section.

What happens when the cost of abandoning is positive (i.e., $\phi > 0$)? Remarkably, we first show that, for large fiscal shocks, it is optimal to abandon the fixed exchange rate right away regardless of the value of ϕ.

Proposition 3.1. *Let $\Delta\Gamma \geq c1/er$. Then it is always optimal to abandon the fixed exchange rate at $T = 0$ for any value of ϕ in the admissible range.*

Proof. See Appendix 7.1.

This proposition thus states that, for large fiscal shocks, it is always optimal to abandon the peg immediately. The reason is that, by abandoning the peg right away, policymakers can avoid the intertemporal distortion that would be introduced by delaying the abandonment until some $T > 0$ and then having to raise the inflation rate very sharply. This result illustrates how critical the assumption of rational policymakers is. In contrast with the standard KFG model, in our model the decision to abandon the peg is unrelated to the size of the stock of international reserves. Further, under the mechanical (i.e., "irrational") KFG rule, policymakers would never abandon right away as long as reserves are available. Hence, unlike the KFG model, our model would be capable of rationalizing situations in which a fixed exchange rate is abandoned without first exhausting reserves (as in the EMS crises of 1992).

What happens for smaller values of the fiscal shock? The answer then depends on the value of ϕ, as the following proposition shows.

Proposition 3.2. *Let* $\Delta\Gamma < c^1/er$. *Then there is a range of values of* ϕ*, given by* $\phi \in [0, \phi^*]$ *and* $\phi \in [\phi^{**}, \frac{c1}{r} - \Delta\Gamma]$*, for which it is optimal to abandon right away. For intermediate values of* ϕ*, that is* $\phi \in (\phi^*, \phi^{**})$*, the solution is interior (i.e., it is optimal to delay the abandonment of the peg).*

Proof. See Appendix 7.1.

Table 1 offers a convenient way of conveying the results stated in the last two propositions. This 3x2 matrix defines 6 different cases. For high values of $\Delta\Gamma$ ($\Delta\Gamma \geq c^1/er$), it is always optimal to abandon right away regardless of the value of ϕ (Proposition 1). For low values of $\Delta\Gamma$ ($\Delta\Gamma < c^1/er$), there is only one case in which it is optimal to delay the abandonment of the peg (when ϕ is in the intermediate range).

Table 1

Optimal Time For Abandoning a Fixed Exchange Rate

	Low $\Delta\Gamma$ $\Delta\Gamma < c^1/er$	High $\Delta\Gamma$ $\Delta\Gamma \geq c^1/er$
Low ϕ ($0 \leq \phi \leq \phi^*$)	$T = 0$	$T = 0$
Intermediate ϕ ($\phi^* < \phi < \phi^{**}$)	$T > 0$	$T = 0$
High ϕ ($\phi \geq \phi^{**}$)	$T = 0$	$T = 0$

Intuitively, for $\Delta\Gamma < c^1/er$, the intertemporal distortion imposed by abandoning the peg for some $T > 0$ is no longer so large and therefore the decision comes down to a trade-off between the benefit of delaying (which reduces the present discounted value of abandoning) and the cost of delaying (which increases the intertemporal distortion). As one would expect, for small values of ϕ, it is not worth it to delay and it is thus optimal to abandon right away. For intermediate values of ϕ, the costs of abandoning are large enough to make it worth delaying the abandonment of the peg. For values of ϕ larger than ϕ^{**} (that is, values of ϕ which are close to the maximum admissible value of ϕ), both the marginal benefit and marginal cost of delaying become arbitrarily large but the cost of delaying increases faster than the benefit of delaying, making it optimal to abandon immediately.[18] In terms of relating our model to the KFG model, notice that Table 1 can be read as saying that there is only one out of six possible cases in which a mechanical rule a la KFG could be optimal. In all other cases, rational policymakers would abandon the peg right away.

3.3. Behavior of T, ε, and Reserve Loss as a Function of ϕ and Fiscal Shock

Having established when the solution will be corner or interior, we can now analyze how the optimal choice of T (as well as the inflation rate and the reserve loss) depends on ϕ and $\Delta\Gamma$ for the whole admissible range of parameter values (formal proofs are relegated to Appendices 7.2 and 7.3). Figure 1 illustrates the behavior of the optimal T as a function of ϕ, for a *small* fiscal shock (i.e., for $\Delta\Gamma < c^1/er$). Up to $\phi = \phi^*$, the optimal solution is to abandon right away, as already established. When the solution is interior, the optimal T is a non-monotonic function of ϕ. For values of ϕ larger than ϕ^{**}, it becomes again optimal to abandon right away.

Panel B in Figure 1 illustrates the behavior of the optimal inflation rate as a function of ϕ. Notice that, for $T = 0$, it follows from equations (3.3) and (3.5) that:

$$\varepsilon = \frac{r(1+r)}{c^2}(\Delta\Gamma + \phi), \tag{3.13}$$

$$c^2 = rb_{0^-} + y - r(\Delta\Gamma + \phi). \tag{3.14}$$

[18] Formally, one can see that, as ϕ approaches its maximum feasible value, the marginal cost of delaying is linear in p whereas the marginal benefit of delaying is logarithmic in p.

It follows from (3.13), taking into account (3.14), that, for $T = 0$, the rate of inflation is an increasing function of ϕ. The same is true when the solution is interior. Hence, as Panel B illustrates, the rate of inflation is an increasing function for the whole range of values of ϕ.

Finally, Panel C in Figure 1 illustrates the behavior of the reserve loss at the time of abandonment. By definition, the reserve loss equals $c^1 - c^2$. However, since c^1 is independent of T and ϕ, the reserve loss depends only on the behavior of c^2 (when the solution is interior). Naturally, when it is optimal to abandon at time zero, there are no reserve losses. When the solution is interior, the reserve loss is an increasing function of ϕ.

Figure 2 illustrates the behavior of the same endogenous variables (T, ε, and reserve loss) as a function of the fiscal shock for a given value of ϕ.[19] Panel A shows that, when the solution is interior, the optimal T is a decreasing function of the fiscal shock. In other words, the larger the fiscal shock the sooner it is optimal to abandon the peg. Intuitively, a larger fiscal shock implies that, all else equal, a higher inflation rate will be needed which will impose a larger intertemporal distortion. It is thus optimal to abandon earlier and impose a smaller intertemporal distortion. When the value of the fiscal shock reaches $\Delta\Gamma^*(\equiv c^1/er)$, it becomes optimal to abandon immediately.

With respect to the rate of inflation, notice how it remains constant when the solution for T is interior.[20] This reflects the effect of two forces that cancel each other out. First, for a given T, a larger fiscal shock tends to increase the inflation rate. Second, since the optimal T falls as the fiscal shocks increases, the inflation rate falls. In this case of logarithmic preferences, these two effects exactly

[19] The given value of ϕ is $(c^1/r)(1 - 2/e)$. As shown in Appendix 7.3, at that point the Kuhn-Tucker condition is exactly equal to zero.

[20] As noted above, when $\Delta\Gamma = 0$, it is optimal never to abandon the peg and hence the optimal inflation rate is zero.

cancel each other out.[21] Once it becomes optimal to abandon immediately (i.e., for $\Delta\Gamma \geq \Delta\Gamma^*$), the inflation rate is an increasing function of the fiscal shock as follows from (3.13) and (3.14).

Two final observations. First, could this model with rational policymakers replicate the mechanical rule of the KFG model? Yes, provided that the parameter configuration falls in the only cell in Table 1 that ensures an interior solution. In that case, the optimal monetary policy can be implemented using a rule in the spirit of the KFG model. Specifically, instead of announcing T, the central bank can simply announce that it will abandon the fixed exchange rate regime the first time that net government foreign assets fall by $c^2 - c^1$. This would replicate, with optimal behavior on the part of policymakers, the key features of the KFG model.

Second, what would be the consequence of introducing a borrowing constraint on the government? Consider the case where government expenditure is constant at a level g_{0^-} before the fiscal shock and at a level $g_0 > g_{0^-}$ after the fiscal shock. Suppose that there is a binding borrowing constraint that dictates that $f_t \geq \bar{f}$. It can be shown that lifetime utility, V, is an increasing function of T for values of T below the optimal. Note that once the regime is abandoned, f_t becomes constant. The value of f_T is a decreasing function of T. Thus a borrowing constraint will force the economy to abandon the fixed exchange rate regime before the optimal T (assuming, of course, that the unconstrained optimal T was positive to begin with). This situation resembles closely the monetary policy followed in the KFG model, as central bankers maintain the regime for as long as possible and, at the time of abandonment, exhaust their borrowing constraint. In general, appealing to the presence of a borrowing constraint to justify the KFG mechanical rule is

[21] For the CES case, numerical examples suggest that the inflation rate is an increasing function of the fiscal shock (in the interior solution range) when the intertemporal elasticity of substitution is less than one and a decreasing function of the fiscal shock when the elasticity is larger than one. The same is true for the reserve loss (analyzed below).

misleading because whenever the optimal solution is $T = 0$, borrowing constraints are clearly irrelevant.

4. Liability Dollarization

So far we have assumed that there is both a social and a fiscal cost ϕ of abandoning the peg at time T. We now explore a formulation in which these costs arise endogenously as a result of liability dollarization. In this case the fiscal cost is different from the social cost. In addition, the abandonment costs depend not only on T but also on the path followed by the exchange rate once the peg is abandoned.

One prominent feature of modern currency crises is that banks and corporations borrow in dollars against revenues that are denominated in local currency. When the fixed exchange rate regime is abandoned the dollar value of these revenues falls, leading some firms and banks to declare bankruptcy. Here we study a formulation in which firms borrow in dollars against revenue denominated in local currency and, hence, are exposed to exchange rate risk.[22]

Production Firms In the previous sections we assumed that agents in the economy received an exogenous output endowment. We now set this endowment to zero and assume that production is carried out by a continuum of production firms with names, n, in the interval [0 1]. Each firm produces a constant number of units of output, y, which it sells to domestic retailers. The total cost of firm n's production is ny. There is a lag δ between the time when output is produced and sold to domestic retailers and the time when sales revenue is collected. Production firms finance their costs at a interest rate r by borrowing in dollars against the

[22] One natural question is why banks and firms do not hedge their exchange rate risk. Burnside, Eichenbaum and Rebelo (2001) discuss how the presence of government guarantees eliminates the incentive to hedge.

sales revenues that they receive δ periods later.[23] Production firms charge interest to the retailers during the lag that occurs between delivery of the product and collection of the sales revenue.[24] We assume that foreign borrowing is implicitly or explicitly guaranteed by the government. Thus, abandoning the fixed exchange rate regime can potentially create a fiscal liability for the government.

As in the previous sections we will study what happens if at time zero there is an unanticipated increase in the present value of government spending ($\Delta\Gamma$) that makes the fixed exchange rate regime unsustainable. Since before time zero production firms expected the exchange rate to be constant, they billed retailers $Sye^{r\delta}$ for the goods that they sold in the period $-\delta \leq t \leq 0$. The retailers will pay for these goods in the period $0 \leq t \leq \delta$. At time zero production firms learn that the exchange rate will no longer be fixed and use the new perfect foresight path for the exchange rate to bill future sales, which will be collected from time δ on. This means that only the dollar value of sales collected by production firms between zero and δ can potentially be affected by the abandonment of the peg. The value of these sales will remain intact if the peg is abandoned only after time δ.

The value in dollars of firm n at time zero is:

$$V_0(n) = \int_0^{\delta} \frac{Sye^{r\delta}}{S_t} e^{-rt} dt + \int_{\delta}^{\infty} ye^{r\delta} e^{-rt} dt - \int_0^{\delta} (nye^{r\delta}) e^{-rt} dt - \int_{\delta}^{\infty} (nye^{r\delta}) e^{-rt} dt. \tag{4.1}$$

The first term in this expression, $\int_0^{\delta}(Sye^{r\delta}/S_t)e^{-rt}dt$, is the present value in dollars of revenues from sales collected between zero and δ. These sales were billed before time zero under the expectation of a constant exchange rate, so their value in

[23] This type of trade credit is in practice an important feature of how firms operate, see Petersen and Rajan (1997).

[24] An alternative interpretation is that firms have to import ny units of materials to produce y units of output.

dollars will fall if there is a devaluation ($S_t > S$). The second term, $\int_\delta^\infty ye^{r\delta}e^{-rt}dt$, is the present value of the new sales which will be made from time zero on. The dollar value of these sales is not affected by the change in the exchange rate path that takes place at time zero. The third term, $\int_0^\delta (nye^{r\delta})e^{-rt}dt$, represents the repayment of the dollar loans incurred to finance production costs between time $-\delta$ and zero. Finally, the term $\int_\delta^\infty (nye^{r\delta})e^{-rt}dt$ represents repayment of loans incurred to finance production from time zero on.

Under a sustainable fixed exchange rate regime the exchange rate remains constant after time zero ($S_t = S$) and the value of firm n reduces to:[25]

$$V_{0^-}(n) = \frac{y(1-n)e^{r\delta}}{r}.$$

The aggregate value of production firms is in this case given by:

$$\int_0^1 V_{0^-}(n)dn = \frac{ye^{r\delta}}{2r}.$$

This expression reflects the fact that the average profit margin in the production sector ($\int_0^1 (1-n)dn$) is 1/2.

The fiscal shock that occurs at time zero implies that the fixed exchange rate regime has to be abandoned at some point. If the exchange rate devalues between time zero and δ the revenue collected by some of the production firms during this period may not be sufficient to repay their international loans. This may render the value of these firms negative, making it optimal for them to declare bankruptcy. The value of firm n at time zero can be rewritten as:

$$V_0(n) = \begin{cases} y(1-n)e^{r\delta}/r - \int_T^\delta [(S_t - S)ye^{r\delta}/S_t]e^{-rt}dt & \text{if } 0 \leq T < \delta, \\ y(1-n)e^{r\delta}/r & \text{if } T \geq \delta. \end{cases} \quad (4.2)$$

[25] The term $e^{r\delta}$ in this expression reflects the fact that when the firm started to operate it did not collect profits for the first δ periods.

Note that $y(1-n)e^{r\delta}/r$ is the value of the firm before time zero, while $\int_T^\delta [(S_t - S)ye^{r\delta}/S_t]e^{-rt}dt$ is the present value measured in dollars of the loss in sales revenue that occurs if the fixed exchange rate is abandoned before time δ.

Let n^* be the name of the production firm whose value at $t=0$ becomes zero: $V_0(n^*) = 0$. Equation (4.2) implies that n^* is given by:

$$n^* = \begin{cases} 1 - r\int_T^\delta [(S_t - S)/S_t]e^{-rt}dt & \text{if } 0 \leq T < \delta, \\ 1 & \text{if } T \geq \delta. \end{cases} \tag{4.3}$$

When the regime is abandoned after time δ, n^* is equal to one and there are no defaults since $V_0(n) \geq 0$ for all n. When the peg is abandoned before time δ the marginal firm, n^* is such that the profit per unit of output produced (which is foregone upon default) equals the annuity value of the shortfall in the present of value of sales revenue measured in dollars. All firms with $n \geq n^*$ default at time zero.

When firms default the government collects the revenue from their past sales and repays their loans. This bailout operation creates a fiscal liability. We assume that the government cannot operate the firms that default. This creates a loss in future profits that represents a social cost associated with default.[26] We will later characterize in more detail the fiscal and social costs associated with abandoning the peg.

Retailers Retailers can buy goods from either domestic or foreign production firms. These goods take δ periods to arrive to the retailers. Once the merchandise arrives it is sold instantaneously either to domestic consumers or to foreigners. The price of each good sold in local currency is $e^{r\delta}S_t$. To simplify we assume that the retailers operate at zero cost.

[26] This extreme assumption is made for convenience. It would be sufficient to assume that future profits fall if the firm is resold or operated by the government. It is important that there be a loss in future profits associated with the firms that go bankrupt. In the absence of this loss there are no social costs of abandoning the peg and it is optimal to abandon at time zero.

Under a sustained fixed exchange rate regime, retail profits are zero and hence the value of the retail sector is also zero. When the unanticipated fiscal shock occurs at time zero the value of the retail sector becomes:

$$V_0^R = \left[\int_0^1 ye^{r\delta}dn\right]\left[\int_T^\delta \frac{S_t - S}{S_t}e^{-rt}dt\right].$$

Note that the profits made by retailers mirror the losses incurred by the production sector. Retailers make losses when $S_t < S$. In this case it is optimal for them to declare bankruptcy. We assume that when this occurs the goods in transit from the production firms to the retailers are lost.

The Household's Budget Constraint Households own both the production firms and the retailers. The household's present value budget constraint before the shock, at time $t = 0^-$ is:

$$b_{0^-} + \int_0^1 V_{0^-}(n)dn = \int_0^\infty c_t e^{-rt}dt. \tag{4.4}$$

Recall that the value of the retailers is zero under a sustained fixed exchange rate regime so it does not affect this expression. After the fiscal shock at time zero the present value constraint for the household becomes:

$$\begin{aligned} & b_{0^-} + \int_0^{n^*} V_0(n)dn + V_0^R = \qquad (4.5) \\ & \int_0^\infty c_t e^{-rt}dt + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t)e^{-rt}dt + \sum_{j\in J} e^{-rj}(M_j - M_{j^-})/S_j. \end{aligned}$$

The term $\int_0^{n^*} V_0(n)dn$ represents the value of the production firms that did not fail. Recall that V_0^R denotes the aggregate value of the retail sector at time zero.

The Government's Budget Constraint At $t = 0$ (i.e., immediately after the fiscal shock), the government budget constraint is given by:

$$\int_0^\infty (\dot{m}_t + \varepsilon_t m_t)e^{-rt}dt + \sum_{j\in J} e^{-rj}(M_j - M_{j^-})/S_j = \Delta\Gamma + \phi^F, \tag{4.6}$$

where ϕ^F denotes the fiscal cost of abandoning the peg, which we characterize next.

The Fiscal Cost of Abandoning the Peg The government takes over the firms that default at time zero, collects the sales revenue that occurs between zero and δ, and pays the firm's foreign loans. This generates a fiscal cost ϕ^F, given by:

$$\phi^F = \int_{n^*}^{1} \int_{0}^{\delta} \left[nye^{r\delta} - \frac{Sye^{r\delta}}{S_t} \right] e^{-rt} dt dn.$$

Note that the path for S_t has two effects on ϕ^F. First, it affects the dollar value of the sales revenue collected by the government. Second, it affects the number of firms that fail (see (4.3)). The path of S_t will be shaped by two factors: the time at which the peg is abandoned (T) and the monetary policy followed after time T.

The Social Cost of Abandoning the Peg Combining the household and the government budget constraints (equations (4.6) and (4.5)) and consolidating the value of production firms and retailers we obtain the following aggregate resource constraint for the economy:

$$b_{0^-} + \int_0^1 V_{0^-}(n) dn = \int_0^\infty c_t e^{-rt} dt + \Delta\Gamma + \phi^S, \tag{4.7}$$

where ϕ^S, which represents the social cost of abandoning the peg, is defined as:

$$\phi^S \equiv \int_{n^*}^{1} \frac{y(1-n)}{r} dn. \tag{4.8}$$

Note that ϕ^S is equal to zero when the peg is only abandoned after time δ since in this case there are no bankruptcies ($n^* = 1$). The social cost of abandoning the peg stems from the loss of the future profits of the firms that fail. This profit flow, which would be collected from time δ on, is given by $y(1-n)e^{r\delta}$. The social cost is thus the present value of this profit flow, $y(1-n)/r$ summed over all the firms that fail ($n \geq n^*$). This cost arises because the private incentives to default do not coincide with the social incentives. The private firms that default forego their future profits in order to avoid paying their loans to foreigners. However, these loans were guaranteed by the government and hence have to be repaid to foreign banks by the government. For this reason it is not optimal, from a social standpoint, for production firms to fail.[27]

4.1. Optimal Monetary Policy

The optimal monetary policy for this economy can be characterized by choosing the paths for M_t and c_t that maximize household's lifetime utility, (2.1) subject to the cash-in-advance constraint (2.3), the government's budget constraint (4.6), the aggregate resource constraint, (4.7), and equation (4.3), which determines which production firms default.

This problem is difficult to study analytically. However, we can rule out the possibility that a revaluation (a fall in S_0) at time zero is optimal. Suppose that the peg is abandoned at $t = 0$ and that M_0 is reduced. In addition, from time zero on money grows at a constant rate ε, chosen so that the government's budget constraint is satisfied. Suppose that M_0 is chosen so that S_0 satisfies the condition: $S_\delta = S_0 e^{\varepsilon\delta} = S$. In this case, the exchange rate at time δ coincides with S so, abstracting from retailer default, $n^* = 1$. Since the rate of inflation is constant there is no intertemporal distortion imposed on the path for consumption chosen by the households. The problem with this policy is that the exchange rate revaluation delivers losses to the retailers, leading them to declare bankruptcy.

[27] In the text we assume that there is no free entry, so the firms that exit are not replaced. However, the social cost would be the same if we assumed that there is free entry and that it costs $y(1-n)/r$ to set up a firm with productivity n. Entrants would start operating today but would only start receiving profits from period δ on.

The fact that the goods in transit to the retailers are lost generates a large social cost. In addition, the sales revenue received by production firms becomes zero between time zero and time δ, maximizing the number of bankruptcies in the production sector.

To make further progress we will characterize numerically the optimal monetary policy within the class of policies where the growth rate of money is constant once the peg is abandoned. Note that for this class of policies n^* is solely a function of ε and T (see (4.3)). As result, both the social and the fiscal costs are also functions of T and ε. Figure 3 shows how ϕ^F and ϕ^S vary with T.[28] As in section 3 these costs are declining functions of T, thus providing the government with an incentive to delay the attack. Note that both costs are equal to zero for $T \geq \delta$.

Figure 4 displays the optimal abandonment time, T, as a function of the size of the fiscal shock, $\Delta\Gamma$. In line with our results in section 3 (recall Figure 2) the optimal value of T is a declining function of $\Delta\Gamma$. For values of $\Delta\Gamma$ greater than certain threshold (call it $\Delta\Gamma^*$), it is optimal to abandon the peg at time zero. The value of ε is an increasing function of the fiscal shock. The higher the value of $\Delta\Gamma$, the higher the money growth rate that is necessary to finance the expenditure increase with seignorage revenues. Reserve losses at the time of abandonment (given by $(M_0 - M_T)/S$) are an increasing function of the fiscal shock for values of $\Delta\Gamma < \Delta\Gamma^*$ because larger fiscal shocks are associated with larger values of ε. When ε is higher, expected inflation is also higher, leading to a larger reduction in real balances on the part of households at the time of the attack. Since for $\Delta\Gamma \geq \Delta\Gamma^*$ it is optimal to choose $T = 0$, the reserve loss is zero in these cases.

For values of $\Delta\Gamma$ such that it is optimal to abandon at time zero the central

[28] The numerical example used in this section was constructed with the following parameters: $r = 0.015$, $\delta = 6$, $f_{0^-} = 0$, $b_{0^-} = 0.1$, $y = 1$, and $S = 1$.

bank can follow two basic courses of action (as a well as combinations thereof). Policy 1 is to finance the increase in expenditures with an increase in M_0. Policy 2 is to leave $M_0 = M_{0^-}$ and choose a constant growth rate of money ε from time zero on. In the model of section 2 these two policies generated the same level of welfare. In contrast, Policy 2 generates a higher level of welfare in the model considered here. This occurs because Policy 2 generates a lower cumulative rate of depreciation between time zero and time δ, leading fewer production firms to declare bankruptcy.

5. Stochastic Fiscal Reform

In sections 3 and 4 we studied the optimal monetary policy in models where there are both fiscal and social costs of abandoning the fixed exchange rate regime. We will now consider an economy where these costs are absent but where government spending is stochastic. As in the previous sections, we assume that before time zero the fixed exchange rate regime was sustainable, so the government's net foreign assets were sufficient to finance the present value of government spending. At time zero the economy learns that the present value of government spending has increased by $\Delta\Gamma$. The new element introduced in this section is that while the exchange rate is fixed there may be a reduction in government spending that makes the peg, once again, sustainable. This expenditure reduction occurs according to a Poisson process with arrival rate λ. If the peg is abandoned the increase in government spending becomes permanent and has to be financed with seignorage revenues. There is thus an option value of holding on to the peg. This formulation captures in a simple way the idea that a fixed exchange rate regime exerts pressure on the fiscal authorities to enact reforms to make the peg sustainable. This pressure disappears once the economy floats. An alternative interpretation is that the country may receive a bailout transfer from abroad that pays for the increase in government spending and renders the peg sustainable. This external bailout arrives according to a Poisson process.

The size of the fiscal reform or of the external bailout that has to occur to make the fixed exchange rate regime sustainable depends, naturally, on the path of government spending. If the reform occurs at time t the present value of government spending from time t on has to be reduced to a value Γ_t given by:

$$\Gamma_t = f_0 e^{rt} - e^{rt} \int_0^t g_s e^{-rs} ds.$$

This expression implies that if there has been no new spending between time zero and time t all that is necessary to make the peg sustainable is to cancel the plans for new government spending in the future. However, if new spending has already taken place in the time interval up to time t the government needs to reduce the present value of government spending below its level before the fiscal shock.

The design of optimal policy reduces to choosing the time T at which the fixed exchange rate regime will be abandoned if a fiscal reform has not, in the meantime, materialized. A higher value of T makes a fiscal reform more likely. However, the longer the horizon T, the larger the intertemporal consumption distortion that the government will have to introduce if reform does not occur.

The Time When Reform Occurs We will start by characterizing the case in which a fiscal reform has just occurred making the fixed exchange rate sustainable. Consumption will be constant and its level, which we denote by c^*, can be computed using the household's budget constraint:

$$b + y/r = c^*/r + (c^* - m).$$

Here b and m denote the levels of net foreign assets and real balances that households had in the period where the reform took place. The term $(c^* - m)$ represents the jump in real balances that occurs when agents learn that the fixed exchange rate regime has become sustainable. Lifetime utility is given by:

$$V^*(b+m) = \frac{\log\left[(rb+rm+y)/(1+r)\right]}{r}.$$

The $t \geq T$ Regime Suppose that we have reached time T and a reform has not occurred. The fixed exchange rate regime will now be abandoned and the growth rate of money will rise to a level ε such that the government's intertemporal resource constraint is satisfied. Consumption will be constant at a level which we denote by c^2. This level can be computed using the household's budget constraint:

$$b + y/r = c^2(1+\varepsilon)/r + (c^2 - m). \tag{5.1}$$

where $(c^2 - m)$ represents the jump in real balances that takes place at time T in response to a permanent increase in inflation from zero to ε. Using (5.1) to solve for c^2, we can compute lifetime utility at time T:

$$V(b+m,T) = \frac{\log\left[(rb+rm+y)/(1+r+\varepsilon)\right]}{r}.$$

For future reference we note that this value function bears a simple relation with the value function associated with the reform regime:

$$V(b+m,T) = V^*(b+m) - \frac{\log(p)}{r},$$

where p is given by (3.3).

The fact that $r = \rho$ and that inflation is constant means that for any time period $t \geq T$ the value function coincides with $V(b+m,T)$:

$$V(b+m,t) = V(b+m,T) \text{ for } t \geq T.$$

The Regime for $t \leq T$ and No Reform The optimality equation for the household's problem during this period is:

$$\begin{aligned} rV(b+m,t) &= \max_{c^1}\{\log(c^1) + V_2(b+m,t) + \\ &\quad [r(b+m) + y - c^1(1+r)]V_1(b+m,t) + \\ &\quad \lambda[V^*(b+m) - V(b+m,t)]\}. \end{aligned}$$

The first order condition with respect to consumption (c^1) is:

$$1/c^1 = V_1(b+m,t)(1+r).$$

It is easy to verify that the value function has the form:

$$V(b+m,t) = \frac{\log\left[(rb+rm+y)/(1+r)\right]}{r} - \frac{e^{-(\lambda+r)(T-t)}\log(p)}{r}. \tag{5.2}$$

This equation has a simple interpretation. Consider first an economy in which a fiscal reform has no chance of occurring ($\lambda = 0$) and which will switch to the floating regime with certainty at time T. Since utility declines by $\log(p)/r$ at time T lifetime utility at time t would be

$$\frac{\log\left[(rb+rm+y)/(1+r)\right]}{r} - \frac{e^{-r(T-t)}\log(p)}{r}.$$

Our value function is similar to this expression but the discount factor applied to $\log(p)/r$ incorporates λ to reflect the fact that there is an ongoing probability of a fiscal reform until time T.

5.1. Optimal Monetary Policy

At time zero, when the economy learns that there has been an increase in the present value of government spending, the lifetime utility of the household declines from $V^*(b+m)$ to $V(b+m,0)$ (given by equation (5.2)).

The central bank chooses T, the maximum number of "periods" that it is willing to wait for a fiscal reform to occur. If the economy reaches time $T > 0$ with no fiscal reform, the central bank will have to print money so that the government's intertemporal budget constraint holds. Since it is optimal to choose a constant growth rate of money, the government's present value resource constraint can be written as:

$$\frac{\varepsilon c^2}{r} e^{-rT} + (c^1 - m_{0^-}) + (c^2 - c^1)e^{-rT} = \Delta\Gamma. \tag{5.3}$$

Note that there are no stochastic elements in this equation. This constraint is only relevant when the economy reaches time T without a fiscal reform, in which case all uncertainty has been resolved.

Using the fact that $c^2 = c^1/p$ together with (3.3) we can rewrite (5.3) as:

$$p = \frac{c^1/r}{c^1/r - (\Delta\Gamma + m_{0^-} - c^1)\, e^{rT}}. \tag{5.4}$$

This equation defines p as a function of T .

The optimal policy can be characterized by maximizing $V(b+m, 0)$ (given by (5.2)) subject to (5.4). The optimal value of T is given by condition:

$$(\lambda + r)\log(p) - \frac{dp}{dT}\frac{1}{p} \leq 0, \tag{5.5}$$

which holds with equality whenever the optimal value of T is interior. This equation is similar to the one that characterizes the optimal policy in section 3 (equation (3.10)), showing the close connection between the two problems. In fact, comparing these two equations it is easy to see that for every value of ϕ in the economy of section 3, there is a value of λ in the model of this section such that the two economies choose the same value of p when the peg is abandoned at time T.

Using equation (5.4) to compute dp/dT,we can rewrite equation (5.5) as:

$$(1 - p)r + (r + \lambda)\log(p) \leq 0.$$

Using this equation together with (5.4) we can characterize the optimal abandonment time, T. The results are summarized by the following proposition.

Proposition 5.1. *For every $\lambda > 0$ there is a level for the present value of government spending, Γ^* such that for $\Gamma_0 > \Gamma^*$ it is optimal to abandon the peg at time zero ($T = 0$), while for $\Gamma_0 \leq \Gamma^*$ it is optimal to delay abandoning the peg ($T \geq 0$). The value of Γ^* is increasing in λ.*

Proof: See Appendix 7.4.

It may seem counterintuitive that when the fiscal shock is large it is optimal to abandon the peg immediately. Why not wait for a while to see whether a fiscal reform occurs eliminating the effect of this fiscal shock? The problem with waiting is that if there is no reform until time T the government will have to generate very high rates of inflation that will severely reduce household consumption from time T on. In other words, when $\Delta\Gamma$ is high the distortion that will be imposed on the intertemporal allocation of consumption if reform fails is so large that it is preferable to abandon the peg immediately. In contrast, for small values of $\Delta\Gamma$ it is optimal to wait, since if reform fails to materialize the distortion that will be introduced in the economy is relatively minor.

The fact that Γ^* is increasing in λ is intuitive: it means that when the reform arrival rate is higher the range of fiscal shocks for which it is optimal to delay abandoning the peg is larger.

6. Conclusion

Versions of the Krugman-Flood-Garber (KFG) are widely used to think about speculative attack episodes. This class of models has often been criticized for assuming a non-optimizing central banker who follows a mechanical rule for abandoning the fixed exchange rate regime: the peg is abandoned if and only if international reserves reach a pre-specified lower bound.

In this paper we use an optimization-based version of the KFG model to characterize the optimal time for abandoning a fixed exchange rate regime that has become unsustainable due to an unexpected increase in the present value of government spending. We consider two different costs of abandoning the peg: an exogenous cost and an endogenous cost that results from liability dollarization. In addition, we also consider a stochastic case in which a fiscal reform may occur while the peg is in effect. In all three cases, for a sufficiently large shock, it is optimal to abandon the peg right away, independently of the level of reserves. This implies a sharp departure from the KFG model where the size of the underlying fiscal deficits plays no direct role in the decision to abandon the peg (and matters only insofar as it affects the speed at which the stock of reserves is depleted). In principle, therefore, our model predicts that the decision to abandon should be mostly determined by the size of the underlying fiscal shock rather than by the level of reserves (which would be the key variable according to the KFG model). For moderate fiscal shocks, however, the rule postulated in the Krugman-Flood-Garber model may be optimal.

So far we have studied a basic monetary model where the only impact of inflation is that it may distort intertemporal consumption allocations. This analysis provides us with a departure point to study richer environments in which tax revenue and the cost of financing public debt are endogenous and where monetary policy affects the level of output through various channels.

References

[1] Buiter, Willem "Borrowing to Defend the Exchange Rate and the Timing and Magnitude of Speculative Attacks," *Journal of International Economics*, 23: 221-239, 1987.

[2] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo "Government Guarantees and Self-Fulfilling Speculative Attacks," mimeo, Northwestern University, 2000.

[3] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo "Prospective Deficits and the Asian Currency Crisis," *Journal of Political Economy*, 109: 1155-1198, 2001.

[4] Calvo, Guillermo "Balance of Payments Crises in a Cash-in-Advance Economy," *Journal of Money Credit and Banking*, 19: 19-32, 1987.

[5] Drazen, Allan and Elhanan Helpman "Stabilization with Exchange Rate Management," *The Quarterly Journal of Economics*, 835-855, 1987.

[6] Flood, Robert P. and Peter Garber "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics*, 17: 1–13, 1984.

[7] Flood, R., and O. Jeanne, "An Interest Rate Defense of a Fixed Exchange Rate?," IMF Working Paper, 159, 2000.

[8] Flood, Robert., P. Peter Garber, and Charles Kramer, "Collapsing Exchange-Rate Regimes: Another Linear Example," *Journal of International Economics* 41, 223-234, 1996.

[9] Flood, Robert P., Jagdeep S. Bhandari, and Jocelyn Horne, "Evolution of Exchange Rate Regimes," *IMF Staff Papers*, 36: 810-835, 1989.

[10] Kaminsky, Graciela and Carmen Reinhart, "The Twin Crises: The Causes of Banking and Balance-of-Payments," *American Economic Review*, 89: 473-500, 1999.

[11] Krugman, Paul "A Model of Balance of Payments Crises," *Journal of Money, Credit and Banking*, 11: 311–25, 1979.

[12] Lahiri, Amartya and Carlos Végh "Delaying the Inevitable: Optimal Interest Rate Policy and BOP Crises," National Bureau of Economic Research Working Paper 7734, 2000.

[13] Obstfeld, Maurice "Speculative Attack and the External Constraint in a Maximizing Model of the Balance of Payments," *Canadian Journal of Economics*, 29: 1-20, 1986a.

[14] Obstfeld, Maurice "Rational and Self-Fulfilling Balance-of-Payments Crises," *American Economic Review*, 76: 72–81, 1986b.

[15] Obstfeld, Maurice "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review*, 40: 1037-48, 1996.

[16] Petersen, Mitchell A., and Raghuram G. Rajan, "Trade Credit: Theories and Evidence," *Review of Financial Studies*, 10: 661-691, 1997.

[17] Rebelo, Sergio and Danyang Xie "On the Optimality of Interest Rate Smoothing," *Journal of Monetary Economics*, 43: 263-282, 1999.

[18] Rigobon, Roberto "Disinflation and Fiscal Reform: A Neoclassical Perspective," National Bureau of Economic Research Working Paper 8706, 2002.

[19] Wijnbergen, Sweder Van "Fiscal Deficits, Exchange Rate Crises and Inflation," *Review of Economic Studies*, 58: 81-92, 1991.

7. Appendices

7.1. Behavior of $\Psi(\phi;\Delta\Gamma)$.

This appendix contains the proofs of Propositions 3.1 and 3.2 (the results of which are summarized in Table 1). First, notice that, for $\Delta\Gamma > 0$, we can rule out a solution in which $T \to \infty$ because, by construction, such a solution would violate the government's intertemporal constraint. In what follows, therefore, we can ignore the exponential term in (3.10). Let $\Psi(\phi;\Delta\Gamma) \equiv \log p(T,\phi,\Delta\Gamma) - \frac{p_T(T,\phi,\Delta\Gamma)}{rp(T,\phi,\Delta\Gamma)}$. Hence, for $\Delta\Gamma > 0$ (and using (3.7) and (3.8)),

$$\Psi'(\phi;\Delta\Gamma) = \frac{p_\phi}{c^1}\left(c^1 - r\phi - 2r\Delta\Gamma e^{rT}\right). \tag{7.1}$$

We can now establish the following (for the $T = 0$ case):

1. If $\Delta\Gamma \geq c^1/2r$, then $\Psi'(0;\Delta\Gamma) \leq 0$ and $\Psi'(\phi > 0;\Delta\Gamma) < 0$. Since $\Psi(0;\Delta\Gamma) < 0$ (recall (3.12)), it then follows that $\Psi(\phi;\Delta\Gamma)$ is always negative and hence there is always a corner solution at $T = 0$.

2. If $c^1/er \leq \Delta\Gamma < c^1/2r$, then the function $\Psi(\phi;\Delta\Gamma)$ reaches a maximum at $\phi^{\max} = c^1/r - 2\Delta\Gamma$.[29] The value of the function at this point is given by:

   $$\Psi(\phi^{\max};\Delta\Gamma) = \log p(0,\phi^{\max},\Delta\Gamma) - 1 \leq 0,$$

   which implies that the solution is $T = 0$ (either corner or boundary solution).

3. If $0 < \Delta\Gamma < c^1/er$, $\Psi(0;\Delta\Gamma)$ cuts the horizontal axis twice (denote these roots by ϕ^* and ϕ^{**}, $\phi^* < \phi^{**}$). To show this, notice that $\Psi(\phi^{\max};\Delta\Gamma) > 0$, which implies that ϕ^* exists. To establish existence of the second root, ϕ^{**}, it

[29] Notice that $\phi^{\max}$ is always smaller than the maximum feasible value for ϕ (dictated by the resource constraint) which equals $\frac{c^1}{r} - \Delta\Gamma e^{rT}$.

is enough to notice that (i) in light of the resource constraint, $\phi < c^1/r - \Delta\Gamma$ and (ii) the limit of $\Psi(\phi; \Delta\Gamma)$ as ϕ approaches this upper bound is $-\infty$.

Points 1 and 2 constitute the proof of Proposition 3.1, while point 3 proves Proposition 3.2.

So far we have established that for some ranges of parameter values, we have "local" corner solutions (i.e., around $T = 0$). To show that these corner solutions are also global (i.e., that they hold for any T), it is enough to show that for any given ϕ, $\Psi(\phi; \Delta\Gamma)$ is strictly decreasing in T for any $T \geq 0$. This is indeed true as this derivative can be shown to be given by $-p_T r \Delta\Gamma e^{rT}/c^1 < 0$ for $T \geq 0$.

7.2. Behavior of T, ε, and Reserve Loss as a Function of ϕ.

Behavior of T. Take as given $\Delta\Gamma \in (0, c^1/er)$ and consider the ranges for ϕ – established above – for which the solution for T is interior. In that case, $\Psi(\phi; \Delta\Gamma) = 0$ implicitly defines the optimal T as a function of ϕ:

$$\phi = \frac{c^1}{rp}\left(p - 1 - \log p\right). \tag{7.2}$$

Hence:

$$\frac{dT}{d\phi} = \frac{\Psi'(\phi; \Delta\Gamma)c^1}{p_T r \Delta\Gamma e^{rT}}.$$

where the behavior of $\Psi'(\phi; \Delta\Gamma)$ has been derived above. Hence, T is an increasing function of ϕ for $\phi \in [\phi^*, \phi^{\max})$ and a decreasing function for $\phi \in [\phi^{\max}, \phi^{**},)$ For all other values of ϕ, the optimal $T = 0$, as established above. Figure 1, Panel A, illustrates optimal T as a function of ϕ.

Behavior of ε. For the range of interior solutions, it follows from (7.2) that:

$$\frac{d\varepsilon}{d\phi} = \frac{r(1+r)p^2}{c^1 \log(p)} > 0.$$

When optimal $T = 0$, ε is also an increasing function of ϕ, as follows from (3.13) and (3.14). Figure 1, Panel B, illustrates the optimal ε as a function of ϕ. Clearly, at $\phi = \phi^* = \phi^{**}$, this function need not be differentiable.

Behavior of reserve loss By definition, the reserve loss at T is equal to $c^1 - c^2$. Since c^1 is independent of both T and ϕ, we just need to check the behavior of c^2 as a function of ϕ for interior solutions (naturally, for $T = 0$, the reserve loss is zero). Since $c^2 = c^1/p$, it follows that

$$\frac{dc^2}{d\phi} = -\frac{c^1}{p^2(1+r)}\frac{d\varepsilon}{d\phi} < 0.$$

Hence, the reserve loss is an increasing function of ϕ when the solution is interior (see Figure 1, Panel C).

7.3. Behavior of T, ε, and Reserve Loss as a Function of $\Delta\Gamma$.

Behavior of T We now derive the behavior of the optimal values of T, ε, and reserve loss as a function of $\Delta\Gamma$ for a given $\phi \in (\phi^*, \phi^{**})$. As shown above, the solution will be interior for $\Delta\Gamma \leq \frac{c1}{er}$. In this range, using (3.10), it follows that:

$$\begin{aligned} \frac{dT}{d\Delta\Gamma} &= -\frac{p_{\Delta\Gamma}}{p_T} < 0. \\ \lim_{\Delta\Gamma \to 0} T &= \infty. \end{aligned}$$

For any $\Delta\Gamma \geq \frac{c1}{er}$, the solution is $T = 0$, as shown above. In Figure 2, and without loss of generality, the given value of ϕ has been taken to be $\phi = \frac{c^1}{r}(1 - \frac{2}{e})$. It can be checked that $\Psi[\frac{c^1}{r}(1 - \frac{2}{e}); \frac{c1}{er}] = 0$ and hence in Panel A, $T(\Delta\Gamma^* = \frac{c^1}{er}) = 0$.

Behavior of ε. Consider now the behavior of the optimal value of ε as a function of $\Delta\Gamma$. Since the RHS of (7.2) is a strictly increasing function of p, it then follows that, when the solution is interior, the optimal value of p (and hence ε) is fully determined by ϕ and is therefore independent of $\Delta\Gamma$. Hence, for $0 < \Delta\Gamma < \frac{c1}{er}$, optimal ε does not depend on $\Delta\Gamma$.[30] For $\Delta\Gamma \geq \frac{c1}{er}$, optimal T is zero. It then follows from (3.13) and (3.14) that ε is an increasing function of $\Delta\Gamma$. (See Panel B in Figure 2).

Behavior of reserve loss Finally, consider the reserve loss ($\equiv c^1 - c^2$). Clearly, for $\Delta\Gamma \geq \frac{c1}{er}$, the reserve loss is zero since the peg is abandoned right away. For $0 < \Delta\Gamma < \frac{c1}{er}$, the reserve loss equals $c^1(p-1)/p > 0$. Since p is independent of $\Delta\Gamma$ when the solution is interior, then the reserve loss is also independent of $\Delta\Gamma$ in this range.

7.4. Proof of Proposition 5.1

Define the function $K(p) = (1-p)r + (r+\lambda)\log(p)$. It is easy to show that this function is concave, that for $\lambda > 0$ it intersects the x-axis twice, at $p = 1$ and at a value of p greater than 1 which we will denote by p^*. The maximum value of K is achieved for $p = (r+\lambda)/r$. To check whether $T = 0$ is optimal we can set $T = 0$ in (5.4) to compute the value of p that would be consistent with the government budget constraint if the peg was abandoned immediately. We denote this value of p by p^0:

$$p^0 = \frac{c^1/r}{c^1/r - (\Delta\Gamma + m_{0^-} - c^1)}.$$

[30] Notice that for $\Delta\Gamma = 0$, optimal ε is zero since optimal T is "infinity" (i.e., the peg will never be abandoned).

Using the fact that $b_{0^-} + m_{0^-} + y/r = c^1(1+r)/r$ we can rewrite this expression as:

$$p^0 = \frac{c^1/r}{b_{0^-} + y/r - \Delta\Gamma}.$$

We can then use this expression for p^0 to evaluate the Kuhn-Tucker condition. If $K(p^0) < 0$, $T = 0$ is optimal, otherwise $T > 0$ is optimal. The variable p^0 is an increasing function of $\Delta\Gamma$ which takes the value 1 when $\Delta\Gamma = 0$ (in this case there in no expenditure shock at time zero and the regime continues to be sustainable). The value of p^0 converges to infinity as $\Delta\Gamma \rightarrow b_{0^-} + y/r$. This limiting value of $\Delta\Gamma$ is such that government spending exhausts all the resources of the economy. Define $\Delta\Gamma^*$ as the value of $\Delta\Gamma$ such that $p^0 = p^*$. Then for $\Delta\Gamma > \Delta\Gamma^*$, $K(p^0) < 0$ so it is optimal to abandon immediately. For $\Delta\Gamma < \Delta\Gamma^*$, $K(p^0) > 0$ and the optimal value of T is interior. Finally, it is easy to see that p^* is an increasing function of λ. This implies that $\Delta\Gamma^*$ is also an increasing function of λ.

Figure 1. Optimal policy as a function of cost of abandoning

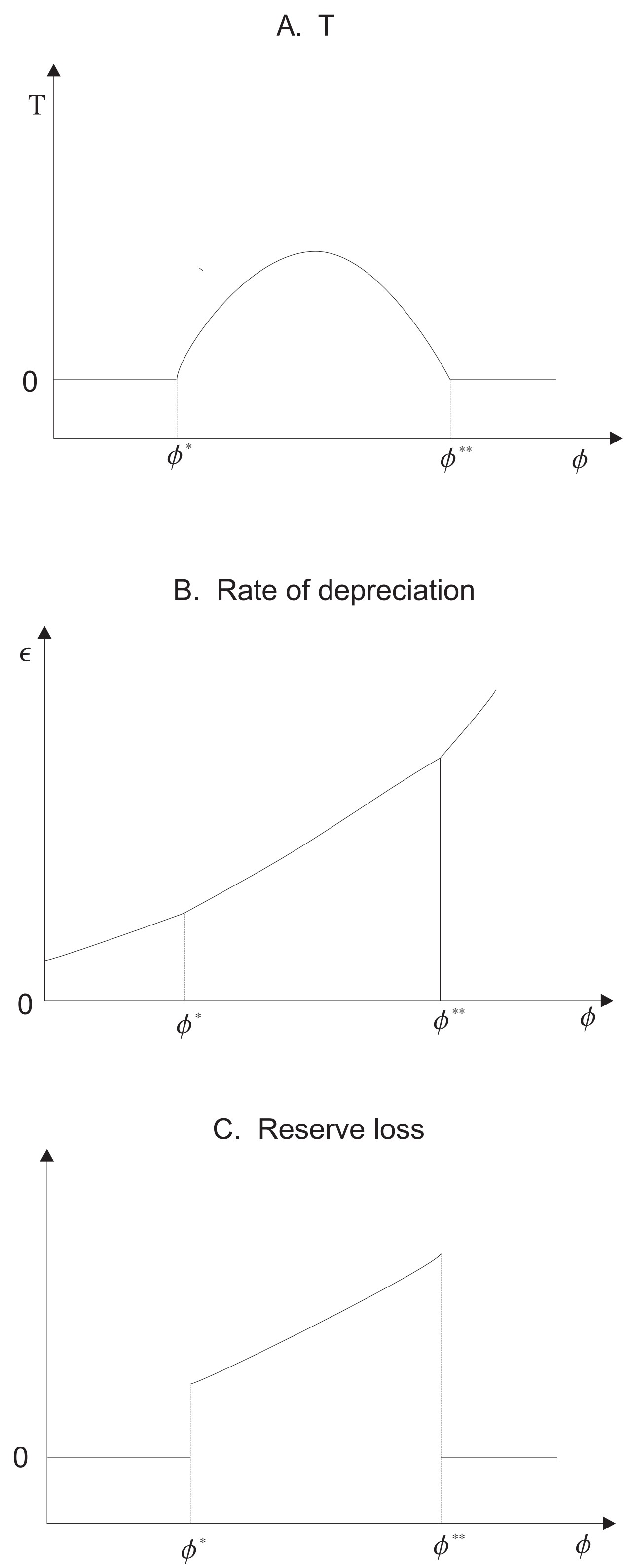

Figure 3. Liability dollarization:: Fiscal and social cost

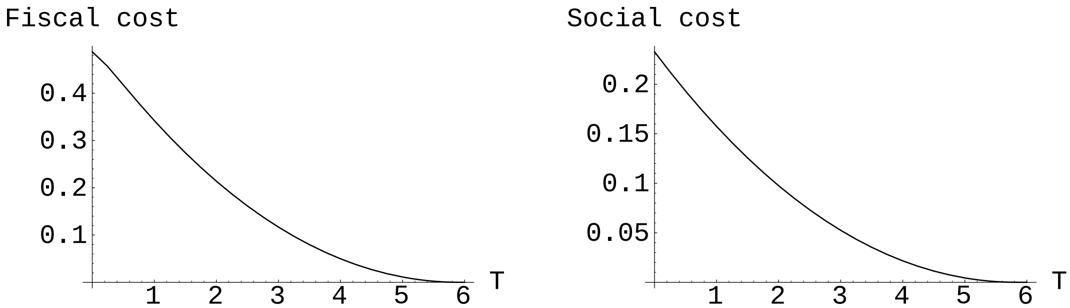

Figure 4. Liability dollarization : Optimal policy