Demand Shocks as Productivity Shocks*

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Abstract

We provide a macroeconomic model where *demand* for goods has a productive role. A search friction prevents perfect matching between potential customers and producers, and larger demand – in the sense of more search – increases output in the economy. Consequently, when viewed through the lens of a standard neoclassical aggregate production function, an increase in demand will appear as an increase in the Solow residual. We estimate the model using standard Bayesian techniques, allowing for business cycles being driven by both demand shocks and true technology shocks. Demand shocks account for more than 95% of the fluctuations in output and the measured Solow residual, while true technology shocks account for less than 2% of these fluctuations. Our model also provides a novel theory for important macroeconomic variables such as the relative price of consumption and investment, the stock market, and capacity utilization.

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1 Introduction

In the standard neoclassical model, output is a function of inputs such as labor and capital. There is no explicit role for *demand*, in the sense that (Walrasian) prices will adjust so that whatever firms can produce will eventually be utilized. In reality, customers and producers must meet in order for the produced good to be consumed. Consider for example a restaurant. According to the standard model, the output of a restaurant should be a function of its employees, building, equipment, and raw material, irrespective of market conditions. However, the restaurant's production takes place only when customers show up to buy meals. Without customers no meals will be served, so the actual value added is zero. The larger the demand for the restaurant's meals, the more customers will be served and the larger the value added will be. Thus, the demand for goods plays a direct role. The spirit of this example extends to many forms of production: car dealers need shoppers, hospitals need patients, all producers need buyers.

This paper provides a theory where *demand* for goods has a productive role. The starting point is that potential customers search for producers, and a standard matching friction prevents neoclassical market clearing in the sense that all productive capacity does translate into value added. Clearly, for households and firms, the acquisition of goods is an active process that involves costs that are not measured in NIPA. Technically, we resolve the search friction by building on the competitive-search model. Firms post prices and customers trade off good prices versus congestion when searching for the goods: prices are higher for goods that are easier to find.

Allowing such explicit role for demand has direct implications for business-cycle analysis, especially for our understanding of the driving factors of business cycles.[1] A striking consequence of this type of demand-driven business-cycle model is that changes in demand will increase output even if inputs, and the intensity with which they are used, remain constant. If viewed through the lens of a standard neoclassical aggregate production function that ignores demand, an increase in demand would appear as an increase in the total factor productivity (TFP), i.e., shocks to the Solow residual.

The focus of our paper is precisely on how the (search-based) model of demand alters the role of productivity shocks in business-cycle analysis. Such shocks to TFP feature prominently in both real business cycle (RBC) models and in New-Keynesian DSGE models. To study the

[1] There is a long tradition of attributing a role for demand in business cycle analysis, starting with Keynes' theories and, more recently, in New-Keynesian versions of the Dixit-Stiglitz monopolistic competition model. However, in none of these earlier approaches has demand had a directly productive role. This is a key contribution of this paper.

role of shocks to demand and productivity, we embed the search model in an otherwise standard neoclassical growth model. We show how a variety of simple demand shocks (to preferences, to the shopping technology of firms) are capable of generating movements in TFP that mimic those in the data. In fact, we estimate processes for each one of those shocks off the Solow residual in the data. However, the implied business cycle comovements of those univariate economies are not like the ones in the data. But a simple combination of consumption and investment demand shocks generate business cycle statistics that rival and, in fact, beats those generated by standard RBC models without any changes in technologies.

We then proceed to pose an economy where demand shocks coexist with TFP shocks and use Bayesian estimation techniques targeting time series for output, consumption, investment, and the measured Solow residual to tease out the contribution of each of those shock to the variance of aggregate variables. We consider shocks to consumption and investment demand, to the marginal rate of substitution between consumption and leisure (MRS) and to TFP. Our findings are that demand shocks to consumption and investment account for almost two thirds of the variance of output, the shock to the MRS for one third and the technology shock accounts for only one percent of the variance of output. Yet the Solow residual fluctuates as in the data. According to our estimated model, what appears as technology shocks from the eyes of a standard neoclassical growth model are increases in capacity utilization arising from more effective search on the part of consumers and investors.

In addition to these findings, the mechanics of our model relate a lot more closely to the popular notions of what makes business *go well*, this is that there is high demand for the product and not that there is a technological improvement. All these taken together, we think, makes a strong case for demand sources being a major source of economic fluctuations while being within the most complete neoclassical orthodoxy.

In our model the role for demand is intrinsic to the process of production and is not arbitrarily imposed: markets clear, and no agent has incentives to deviate. While New-Keynesian models generate demand induced shocks, they do so by making agents trade at prices that are not equilibrium prices, in the sense that agents would, ex post, prefer to change the prices and quantities in order to achieve better allocations. In this paper there is no involuntary trade and the equilibrium allocation is efficient. We see our paper pursuing Keynes' central idea of the role of demand. However, this is neither done in the fixed-price tradition of the New-Keynesian literature nor in the coordination-problem tradition that sees a recession as a bad outcome within environments susceptible to multiple equilibria. Instead, our model follows a tradition where failures of demand generate recessions via infrautilization of productive capacity. In our environment "animal spirits," modeled as shocks to agents' forecasts, can generate recessions.

We have posed demand shocks in the most simple fashion, as shocks to preferences and to shopping ability, because we wanted to illustrate their ability to generate fluctuations in a manner as stark as possible. It is straightforward to extend our environment to contexts where the demand shocks are generated by financial frictions, government expenditures, and foreign demand shocks. Moreover, it is also straightforward to embed our structure within the the New-Keynesian and the Mortensen-Pissarides approaches to fluctuations which assume frictions in either price setting or labor markets to generate large fluctuations in output and hours worked. K: This is in my mind too strong: Ultimately, these two traditions build on technology shocks as a major source of fluctuations. We think that our ideas will allow to toss productivity shocks out of those models and substitute them for demand shocks.

Additional contributions Besides the proposal of a novel model of aggregate demand to study aggregate fluctuations, our work has other contributions. First, it shows how in models with production and competitive search, to achieve optimality it is necessary to index markets, not only by price and market tightness, but also by the quantity of the good traded. Second, we provide a theory of the cyclical changes of the relative price between investment and consumption goods that is not based on exogenous technology shocks[2]. Third, we provide a theory of endogenous capacity utilization, different from the early capacity-utilization literature (see below for further discussion). Fourth, we also provide a theory of stock market movements that are associated not to capital adjustment costs or shocks productivity or production costs, but rather to aggregate demand and to how well firms can match with customers.

Literature Our exercise of exploring endogenous sources of fluctuations in the Solow residual is related to the capacity-utilization literature. For example, Greenwood, Hercowitz, and Huffman (1988), Kydland and Prescott (1988), Bils and Cho (1994), Cooley, Hansen, and Prescott (1995) Basu (1996), and Licandro and Puch (2000) consider variable capital utilization and Burnside, Eichenbaum, and Rebelo (1993) introduce variable worker utilization in the form of labor hording during periods with low aggregate activity. In periods when productivity and/or profits are high, firms will use the input factors more intensively, and this will drive a wedge between "true" technology shocks and the measured Solow residual. A key insight is that through varying capacity

[2] See for example Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Fisher (2006) for papers that use exogenous technical shocks as the source of changes in the relative price of investment.

utilization, true technology shocks magnify the shocks to the Solow residual. Moreover, Wen (2004) argues that with variable capacity utilization, preference shocks that change the desired timing of consumption will cause changes in the utilization of input factors and, hence, changes in the measured Solow residual. Our theory provides a source of fluctuations in capacity utilization which is fundamentally different from and a complement to this former literature.

Our paper is also related to Petrosky-Nadeau and Wasmer (2011), which is developed independently from our paper. They also model costly search for goods in final-goods markets, and study how this interacts with search in the labor market and influences the business-cycle properties of the model. There are also some papers (Lagos (2006), Faig and Jerez (2005) and Alessandria (2005)) that emphasize the effects of search frictions in shaping TFP, although they do not focus on business cycles. Moreover, Diamond (1982) and Guerrieri and Lorenzoni (2009) show that due to a search friction, the difficulty of coordination of trade can give rise to multiple equilbria and aggregate fluctuations.

There are papers with a very different approach that also consider how demand changes could affect productivity and capacity utilization. In Fagnart, Licandro, and Portier (1999) monopolistic firms with putty-clay technology are subject to idiosyncratic demand shocks, which causes fluctuations in capacity utilization. Floetotto and Jaimovich (2008) consider changes in markup rates due to the number of firms changing over the business cycle. In their model, changes in markups cause changes in the measured Solow residual. Swanson (2006) uses a heterogeneous sector model and shows that shocks from government demand can increase aggregate output, consumption, and investment.

The paper is organized as follows. Section 2 lays out the main mechanism in a simple Lucas-tree version of the economy where we show how increases in demand are partially accommodated by an increase in productivity via more search and by an increase in prices, while in the original Lucas (1978) all the adjustment occurs in prices. The full neoclassical growth model is analyzed in Section 3. We then map the model to data in Section 4. In Section 5 we analyze the properties of the model when restricting attention to univariate shock processes (one shock at the time) and when we have only demand shocks. In Section 6 we estimate the full model with various shocks simultaneously to gauge their relative contribution. We explore other implications of the model, for the relative price of investment, for asset prices, and for capacity utilization in Section 7. Section 8 concludes while an Appendix provides the proofs, additional tables, and computational and data details.

2 The simple Lucas-trees version of the model

We start by illustrating the workings of the model in a simple search model where output is produced by trees instead of capital and labor. We show that the search process impacts aggregate output in a way that appears as a level effect on the Solow residual. In an example we show how shocks to preferences are partly accommodated by increase in prices and interest rates and partly accommodated by increases in quantities, and demonstrate that absent the search friction, the same shocks translate only to price increases.

2.1 Technology and preferences

There is a continuum of trees (i.e., suppliers), with measure $T = 1$. Each tree yields one piece of fruit every period. A standard search friction makes it difficult for consumers to find trees. To overcome this friction the consumer sends out a number of shoppers searching for fruit. The aggregate number of fruits found, Y, is given by the Cobb-Douglas matching function:

$$Y = A\,D^{\alpha}\;T^{1-\alpha}, \tag{1}$$

where D is the aggregate measure of shoppers searching for fruit (we sometimes call it aggregate demand) and A and α are parameters of the matching technology.

Following Moen (1997), we assume a competitive search protocol where agents can choose to search in specific locations indexed by both the price and market tightness, defined as the ratio of trees per shopper, $Q = T/D$. The probability that a tree is found (i.e., matched with a shopper) is $\Psi_T(Q) = A\,Q^{-\alpha} = A\,D^{\alpha}/T^{\alpha}$. Once a match is formed, then the fruit is traded at the posted price p. By the end of the period, all fruit which is not found is lost. The trees pay out sales revenues as dividends and the expected dividend is $\varsigma = p\Psi_T(Q)$.

The economy has a continuum of identical, infinitely lived households of measure one. Their preferences are given by

$$E\left\{\sum_{t}\;\beta^{t}\;u\left(c_t, d_t, \theta_t\right)\right\}, \tag{2}$$

where c_t is consumption, d_t is the measure of shopping units (search effort) by the household, and θ is a preference shock that follows a stochastic Markov process. The probability that an individual shopping unit is successful is given by the ratio of matches to the aggregate number of shoppers. Given the matching technology (1), this can be expressed in terms of market tightness as $\Psi_d(Q) = A\ Q^{1-\alpha}$, so c_t is given by

$$c = d\ \Psi_d(Q) = d\ A\ Q^{1-\alpha}. \tag{3}$$

Trees are owned by households and are traded every period. We normalize the price of the tree to unity and use it as the numéraire good. Let s denote the number of shares owned by the household. The aggregate number of shares is unity. Consequently, with identical households the aggregate state of the economy is just θ, while the individual state includes also individual wealth s.[3] The representative household problem can then be expressed recursively as

$$v(\theta, s) = \max_{c,d,s'} u(c, d, \theta) + \beta\ E\{v(\theta', s')|\theta\} \tag{4}$$

$$\text{s.t.} \quad c = d\ \Psi_d[Q(\theta)] \tag{5}$$

$$P(\theta)\ c + s' = s\ [1 + \varsigma(\theta)] \tag{6}$$

It is easy to see that this is a simple extension of the original Lucas (1978) economy, which is one where $\alpha = 0$ and A is large enough to make consumption worthwhile in equation (3).

2.2 Competitive search in the market for goods

There are differentiated markets indexed by the price and market tightness (number of trees or firms per shopper). Let ς denote the outside value for firms of going to the most attractive market, yet to be determined. Clearly a market can attract trees only if it offers them at least ς. This constraints the feasible combinations of prices and market tightness that shoppers can offer:

$$\varsigma \leq p\ \Psi_T(Q) \tag{7}$$

[3] Throughout the paper we take advantage of the perfect correlation between the idiosyncratic and aggregate shocks to preferences and we write only one of them as a state variable.

The expected contribution to household utility of a shopper that chooses the best price–tightness pair is[4]

$$\Phi = \max_{Q,p} \{u_d(\theta, s) + \Psi_d(Q) \, (u_c(\theta, s) - p \, \widehat{m})\} \qquad \text{s.t. } \varsigma \leq p \, \Psi_T(Q), \tag{8}$$

where $u_d(\theta, s)$ and $u_c(\theta, s)$ are the marginal utility of increases in d and c, respectively. Moreover, $\widehat{m} = m[\theta, s'(\theta, s)]$ is the expected discounted marginal utility of an additional unit of savings:

$$m(\theta, s') = \beta \, E\left\{ \frac{P(\theta) \, [1 + \varsigma(\theta')]}{P(\theta')} \, \frac{\partial v(\theta', s')}{\partial s'} | \theta \right\}$$

Solve (8) by substituting (7) at equality and take the first-order condition w.r.t. Q. This yields the unique equilibrium price p and value of the tree ς as functions of market tightness Q;

$$p = (1 - \alpha) \, \frac{u_c(\theta, s)}{\widehat{m}}, \tag{9}$$

$$\varsigma = pA \, Q^{-\alpha}. \tag{10}$$

2.3 Equilibrium

A *competitive search equilibrium* is defined by a set of individual decision rules, $c(\theta, s)$, $d(\theta, s)$, and $s'(\theta, s)$, the aggregate allocations $D(\theta)$ and $C(\theta)$, good prices $P(\theta)$, and the rate of return on trees $\varsigma(\theta)$ so that

1. The individual decision rules, $c(\theta, s)$, $d(\theta, s)$, and $s'(\theta, s)$ solve the household problem (4).
2. The individual decision rules are consistent with the aggregate functions

$$C(\theta) = c(\theta, 1) \qquad D(\theta) = d(\theta, 1) \qquad s'(\theta, 1) = 1$$

3. Shoppers and firms search optimally in the market for goods, i.e. $(p(\theta), Q(\theta), \varsigma(\theta))$ satisfy the search conditions (9) and (10), where market tightness is $Q(\theta) = 1/D(\theta)$.

[4]To derive this, take the first-order condition of (4) with respect to d and consider the price-tightness posting problem when the cost $\overline{u}_d$ of sending an additional shopper has been borne. The idea is that each shopper is equipped with a credit card and if no fruit is found, then utility is not affected over and above the sunk search cost.

4. Good market clears

$$C(\theta) = A\ D(\theta)^{\alpha}. \tag{11}$$

The competitive search equilibrium and its efficiency properties can then be characterized by the following proposition.

Proposition 1. *1. Aggregate search D is determined by the functional equation*

$$0 = \alpha\ A\ D(\theta)^{\alpha-1}\ \ u_c\left[A\ D(\theta)^{\alpha}, D(\theta), \theta\right] + u_d\left[A\ D(\theta)^{\alpha}, D(\theta), \theta\right]. \tag{12}$$

The functions C, Q and ς then follow directly from the equilibrium conditions $C(\theta) = A\ D(\theta)^{\alpha}$, $Q(\theta) = 1/D(\theta)$, and $\varsigma(\theta) = p(\theta)\ \ A\ D(\theta)^{\alpha}$.

2. The equilibrium price is defined by the functional equation

$$u_c\left[C(\theta), D(\theta), \theta\right] = \ \beta\ E\left\{\frac{P(\theta)\ [1+\varsigma(\theta')]}{P(\theta')}\ u_c\left[C(\theta'), D(\theta'), \theta'\right] \mid \theta\right\}. \tag{13}$$

3. The competitive equilibrium is efficient.

The proof of the first two items is straightforward, just derive the first order conditions of households and combine them with the competitive search conditions (see the Appendix for details). For efficiency, we consider a planner solving $\max_{C,D}\left\{u(C, D, \theta)\right\}$ subject to the aggregate resource constraint $C = A\ D^{\alpha}$. The solution to equation (12) solves this planner problem, which establishes efficiency. Interestingly, the Euler equation (13) is the same as the one in the standard Lucas tree model.

We now turn to our focus, the measured total factor productivity Z, which is defined as $C = Z\ T$. The Solow residual Z is a function of the search effort, and fluctuates in response to preference shocks. A direct application of the equilibrium formulation of aggregate consumption from Proposition 1 and $T = 1$, yields the following corollary.

Corollary 1. *In equilibrium, the Solow residual is a function of preference shocks and given by*

$$Z(\theta) = A\ \left(D(\theta)\right)^{\alpha}. \tag{14}$$

2.4 An example

The explicit consideration of an example allows us to show with the aid of closed form solutions how preference shocks that increase the desire to consume are accommodated in part by an increase in the price of consumption today relative to consumption later and in part by an increase in search effort that translates into squeezing more output out of the economy making it more productive. Consider a version of this economy with preferences given by

$$u(c, d, \theta) = \theta_c \ \log c \ - \ d,$$

where θ_c is i.i.d. with $E\{\theta_c\} = 1$ and $\theta_c > 0$. Given these preferences, the equilibrium conditions (12) and (11) yield the equilibrium allocation:

$$D(\theta) = \alpha \ \theta_c, \qquad C(\theta_c) = A \ \alpha^\alpha \ \ \theta_c^\alpha.$$

It is straightforward to verify that the equilibrium price and interest rate (in terms of the consumption good) are

$$P(\theta) = \left(\frac{1}{\beta} - 1\right) \frac{1}{A\alpha^\alpha} \ \theta_c^{1-\alpha}, \qquad\qquad 1 + r(\theta) = \frac{\theta_c^{1-\alpha}}{\beta \ E\{(\theta_c')^{1-\alpha}\}}.$$

An increase in the desire to consume today, translates in an increase in consumption proportional to the shock to the power of α and an increase in the gross interest rate proportional to the shock to the power of $1-\alpha$. As $\alpha \to 0$, the shopping economy converges to the standard Lucas tree model. In this case aggregate consumption is invariant to the demand shock, and all the adjustment to the shock takes place through prices.

3 The stochastic growth model version of the economy

We now extend the search model to an otherwise standard growth model suitable for quantitative business cycle analysis. We add capital which requires for its installation both investment goods and professionals to shop for those goods, and a disutility of working. We start with describing technology and preferences. We then analyze the problems faced by households and firms and price determination in the presence of competitive search for consumption and investment goods. Along the way we prove a few results that guarantee that all firms make the same choices of labor and investment. We also establish that the equilibrium is Pareto optimal. Finally, we discuss how the labor share and the Solow residual can be estimated using National Accounts data.

3.1 Technology

There is a unit measure of firms. Each firm has a "location," i.e., equivalent to the tree in Section 2. The firm has certain amount of capital installed in that location. There is a technology that transforms capital and labor services into goods that is described by a standard (differentiable and strictly concave) production function $f(k, n)$. To install new capital for the following period, the firm, like the households, has to shop. The firm has a technology that transforms one unit of labor (that cannot be used for production) into ζ shopping units.

As in Section 2, we assume a competitive search protocol in specific locations. Markets are indexed by a triplet (Q, P, F) of market tightness, price, and the quantity of the good produced which in turn is a function of the firm's pre-installed capital stock and labor. Note that, the quantity produced was not indexing market in the simple Lucas tree model.

Potentially at the beginning of each period, there is a distribution of firms with different pre-installed capital, perhaps specializing in the consumption or investment good. We proceed by guessing and verifying below that if firms have the same capital today, they chose the same capital for tomorrow, allowing us to look only at a representative firm, and thus drastically reducing the state space of the economy from a distribution of firms to an aggregate level of capital. To simplify our presentation of the problems of firms and households, we use (θ, K), the preference shock and aggregate capital, to denote the state of the economy. Choosing the same capital stock, firms however may choose to produce different goods, consumption or investment goods, and charge different prices and market tightness. In equilibrium, there are two markets, one for consumption with index (Q^c, P^c, F^c) and one for investment with index (Q^i, P^i, F^i). These goods are identical from the point of view of production but not from the point of view of search and prices. The share of firms producing consumption goods is given by $T(\theta, K)$.

3.2 Households

There is a measure one of households who have preferences over consumption c, shopping d, and working n and is affected by preference shocks θ perfectly correlated across households. This is summarized in the utility function $u(c, d, n, \theta)$.

The state variable for the household is the state of the economy (θ, K) and the individual wealth, i.e., the number of shares s. The households take a number of aggregate variables as given; market tightness Q^c, price P^c, and quantity F^c in the consumption good market, the rate of return, the wage, and the law of motion of capital denoted $G(\theta, K)$. These objects are equilibrium functions of the state variable (θ, K).

Household problem The representative household solves

$$
\begin{aligned}
v(\theta, K, s) \quad &= \quad \max_{c,d,n,s'} \; u(c, d, n, \theta) + \beta \, E\left\{v(\theta', K', s')|\theta\right\} && (15)\\
\text{s.t.} \quad & \quad c = d \; \Psi_d[Q^c(\theta, K)] \; F^c(\theta, K), && (16)\\
& \quad P^c(\theta, K) \; c + s' = s \; [1 + R(\theta, K)] + n \; w(\theta, K), && (17)\\
& \quad K' = G(\theta, K). && (18)
\end{aligned}
$$

Equation (16) shows that consumption requires the household's shopping effort, but it also depends on market tightness and the amount produced by consumption produced firms. Equation (17) is the budget constraint in terms of shares.

The solution to this problem is a set of individual decision rules $d(\theta, K, s)$, $c(\theta, K, s)$, $n(\theta, K, s)$, and $s'(\theta, K, s)$. Anticipating equilibrium conditions, we introduce the aggregate counterparts of these functions:

$$
\begin{aligned}
C(\theta, K) &= c(\theta, K, 1) && (19)\\
D^c(\theta, K) &= d(\theta, K, 1), && (20)\\
N(\theta, K) &= n(\theta, K, 1) && (21)\\
s'(\theta, K, 1) &= 1. && (22)
\end{aligned}
$$

The last condition stems from the fact that stock-market shares is the only asset in positive net supply and the equilibrium condition that these share add up to unity. Abusing notation we use these aggregate conditions to write marginal utility as a function only of aggregate state variables yielding $u_c(\theta, K) = u_c\left[C(\theta, K), D(\theta, K), N(\theta, K), \theta\right]$. Using these aggregate conditions, the stochastic discount factor can be expressed as

$$
\Pi(\theta, \theta', K) = \beta \frac{P^c(\theta, K)}{P^c(\theta', G(\theta, K))} \; \frac{u_c\left[\theta', G(\theta, K)\right] + \frac{u_d[\theta', G(\theta, K)]}{\Psi_d[Q^c(\theta', G(\theta, K))] \;\; F^c(\theta', G(\theta, K))}}{u_c(\theta, K) + \frac{u_d(\theta, K)}{\Psi_d[Q^c(\theta, K)] \;\; F^c(\theta, K)}}. \qquad (23)
$$

So the intertemporal Euler becomes

$$1 = E\left\{[1 + R(\theta', G(\theta, K))]\ \Pi(\theta, \theta', K)\ \ |\ \theta\right\}, \tag{24}$$

while the intratemporal first-order condition is

$$u_c(\theta, K) + \frac{u_d\,(\theta, K)}{\Psi_d\,[Q^c(\theta, K)]\ \ F^c(\theta, K)} = u_n\,(\theta, K)\ \ \frac{P^c(\theta, K)}{w(\theta, K)}. \tag{25}$$

To further simplify notation, let $m(\theta, K, \theta, s)$, denote the value in terms of marginal utility of an additional unit of savings and let $M(\theta, K)$ be its aggregate counterpart,

$$M(\theta, K) = \beta\ E\left\{\frac{[1 + R(\theta', G(\theta, K))]}{P^c(\theta', G(\theta, K))}\ \ \left(u_c\,[\theta', G(\theta, K)] + \frac{u_d\,[\theta', G(\theta, K)]}{\Psi_d\,[Q^c(\theta', G(\theta, K))]\ \ F^c(\theta', G(\theta, K))}\right)|\theta\right\}. \tag{26}$$

3.3 Firms

Given the state of the economy (θ, K) and its individual state k, each firm has to choose three things in a particular order: first, whether to produce for investment or consumption, second, the specific submarket to which to go, and third, how much to invest. Firms choose to produce whichever good that gives higher value, i.e.

$$\Omega(\theta, K, k) = \max\{\Omega^c(\theta, K, k), \Omega^i(\theta, K, k)\}, \tag{27}$$

where $\Omega^j(\theta, K, k)$ is the best value for producing consumption goods, $j = c$ or investment goods, $j = i$, this is they choose (Q^j, P^j, F^j), among those available (a still to be determined set).

$$\Omega^j(\theta, K, k) = \max\ \ \widetilde{\Omega}^j(\theta, K, k, Q^j, P^j, F^j) \qquad \text{for all available } (Q^j, P^j, F^j).$$

A firm in a (Q^c, P^c, F^c) consumption goods submarket chooses labor for shopping n^k, investment i and capital to install tomorrow k' to solve the following problem,

$$\widetilde{\Omega}^c(\theta, K, k, Q^c, P^c, F^c) \;=\; \max_{n^k, k', i} \; \frac{\Psi_d(Q^c)}{Q^c} \, P^c F^c - w(\theta, K) \, [n(k, F^c) + n^k] \\ -P^i(\theta, K) \, i + E \left\{ \Omega(\theta', K', k') \, \Pi(\theta, \theta', K) | \, \theta \right\} \tag{28}$$

$$\text{s.t.} \qquad i = n^k \;\; \zeta \;\; \Psi_d[Q^i(\theta, K)] \;\; F[K, N^i(\theta, K)] \tag{29}$$

$$k' = i + (1 - \delta) k \tag{30}$$

$$K' = G(\theta, K) \tag{31}$$

where $n(k, y)$ is the inverse function of the production function $y = f(k, n)$ for a given k; ζ is a technological requirement specifying how many shopping workers is needed to provide one unit of shopping service. Finally $N^i(\theta, K)$ is the equilibrium amount of production workers in investment goods production, so $f(K, N^i(\theta, K))$ is the amount of investment good produced by investment producing firms.

As for investment-good producers, a firm delivering to an investment-good market (Q^i, P^i, F^i) chooses labor for shopping n^k, investment i, and future capital stock k' to solve the following problem

$$\widetilde{\Omega}^i(\theta, K, k, Q^i, P^i, F^i) \;=\; \max_{n^k, k', i} \; \frac{\Psi_d(Q^i)}{Q^i} \, P^i \, F^i - w(\theta, K) \, [n(k, F^i) + n^k] \\ -P^i(\theta, K) \, i + E \left\{ \Omega(\theta', K', k') \, \Pi(\theta, \theta', K) | \, \theta \right\} \tag{32}$$

subject to (29), (30), and (31).

The first-order condition over investment is given by

$$E \left\{ \Omega_k(\theta', K', k') \, \Pi(\theta, \theta', K) | \, \theta \right\} = \frac{w(\theta, K)}{\zeta \, \Psi_d[Q^i(\theta, K)] \, f[K, N^i(\theta, K)]} + P^i(\theta, K). \tag{33}$$

Let $\varsigma(\theta, K, k)$ denote the firm's revenue induced by selling to the best market (expressed in units of shares), be it a consumption-good market or an investment-good market:

$$\varsigma(\theta, K, k) = \max\{\varsigma^c(\theta, K, k), \varsigma^i(\theta, K, k)\},$$

where $\varsigma^j(\theta, K, k)$ is the maximum over the available submarkets of $\varsigma^j(\theta, K, k, Q^c, P^c, F^c)$,

$$\varsigma^j(\theta, K, k, Q^j, P^j, F^j) \;\; = \;\; P^j \;\; \frac{\Psi_d(Q^j)}{Q^j} \, F^j - w(\theta, K) \;\; n(k, F^j), \tag{34}$$

for j being both consumption and investment.

Before analyzing the search equilibrium, it is useful to establish that firms in all submarkets have the same expected revenue.

Lemma 1. $\Omega^c(\theta, K, k) = \Omega^i(\theta, K, k)$ *implies* $\varsigma^c(\theta, K, k) = \varsigma^i(\theta, K, k)$.

A direct consequence of this lemma is that it allows firms to consider only the current revenue (and so the current labor demand) instead of lifetime revenue (and so the lifetime labor demands) in deciding which markets to enter.

3.4 Competitive search in the market for consumption goods

In addition to the decisions made by the households with respect to the main elements of the allocation (how much to consume, shop, work and save), its shoppers choose how to conduct their shopping, i.e., which market to go to. In our environment with competitive search this means choosing a triplet (Q^c, P^c, F^c) of market tightness, price, and quantity. These choices will give us two conditions to be satisfied by these 3 variables.

We can write the contribution to the utility of a household of a shopper that chooses the best price-tightness-quantity triplet as

$$\Phi = \max_{Q^c, P^c, F^c} \quad u_d(\theta, K) + \Psi_d(Q^c) F^c \;\; (u_c(\theta, K) - P^c m(\theta, K, s'))$$

subject to the constraint

$$\varsigma^c \leq P^c \frac{\Psi_d(Q^c)}{Q^c} F^c - w(\theta, K) n(k, F^c). \tag{35}$$

This constraint reflects the fact that the only relevant markets are those that guarantee certain expected revenue for the firms. Substituting (35) with $k = K$ and the definition for $\Psi_d(Q)$ and replacing $m(\theta, K, s')$ with $M(\theta, K)$, we rewrite the problem as

$$\Phi = \max_{Q^c, F^c} \left\{ u_d(\theta, K) + A \, (Q^c)^{1-\alpha} \, F^c \left(u_c(\theta, K) - \frac{\varsigma^c + w(\theta, K) \; n(k, F^c)}{A(Q^c)^{-\alpha} F^c} \; M(\theta, K) \right) \right\}. \tag{36}$$

The first-order condition over Q^c yields an equation for equilibrium P^c an equation for price P^c

$$
\begin{aligned}
0 &= (1-\alpha)\frac{AF^c}{(Q^c)^\alpha}u_c(\theta,K) - \left[\varsigma^c + w(\theta,K)n(k,F^c)\right]\ M(\theta,K) \\
&= (1-\alpha)\,\frac{AF^c}{(Q^c)^\alpha}u_c(\theta,K) - \frac{AP^c}{(Q^c)^\alpha}F^c\ M(\theta,K)
\end{aligned}
$$

which yields an equation for the equilibrium price P^c

$$
P^c(\theta,K) = (1-\alpha)\frac{u_c(\theta,K)}{M(\theta,K)}. \tag{37}
$$

The first-order condition over F^c together with the relation of $\partial n/\partial F^c = 1/f_n$ and $k = K$ gives us

$$
0 = A(Q^c)^{1-\alpha}u_c(\theta,K) - Q^c\ M(\theta,K)\ w(\theta,K)\frac{\partial n}{\partial F^c}.
$$

Substituting equilibrium price from equation (37) and the relation $\partial n/\partial F^c = 1/f_n$ for the given $k = K$, we have

$$
\frac{w(\theta,K)}{P^c(\theta,K)} = \frac{1}{1-\alpha}\ A(Q^c)^{-\alpha}\ \ f_n\left[K, N^c(\theta,K)\right],
$$

where $N^c(\theta,K) = n(K, F^c(\theta,K))$ is the labor associated with $k = K$ and proposed F^c. The market tightness Q^c is a function of the equilibrium value for producing consumption goods $\varsigma^c(\theta,K)$,

$$
Q^c(\theta,K) = \left[\frac{A\ P^c(\theta,K)\ f(K, N^c(\theta,K))}{\varsigma^c(\theta,K) + w(\theta,K)\ N^c(\theta,K)}\right]^{\frac{1}{\alpha}}. \tag{38}
$$

3.5 Competitive search in the market for investment goods

In the same way as households, firms shop investment goods by sending shoppers to markets offering the best triplet of tightness, price, and quantity, (Q^i, P^i, F^i). These choices yield two conditions to be satisfied by these 3 variables.

A shopper for the firm chooses the best price-tightness pair $\{Q^i, P^i, F^i\}$ to solve

$$
\begin{aligned}
\Phi_F &= \max_{Q^i,P^i,F^i}\ -w(\theta,K) + \zeta\ \Psi_d(Q^i)F^i\left[E\left\{\Omega(\theta',K',k')\ \Pi(\theta,\theta',K)\right|\theta\right\} - P^i\right] \\
\text{s.t.}\quad \varsigma^i &\le P^i\frac{\Psi_d(Q^i)}{Q^i}F^i - w(\theta,K)n(k,F^i).
\end{aligned}
$$

Substituting P^i, we can rewrite the problem as

$$\max_{Q^i, F^i} -w(\theta, K) + \zeta\, A(Q^i)^{1-\alpha} F^i \left[E\left\{ \Omega(\theta', K', k')\, \Pi(\theta, \theta', K) | \, \theta \right\} - \frac{\varsigma^i + w(\theta, K) n(k, F^i)}{A(Q^i)^{-\alpha} F^i} \right]$$

The first order condition over Q^i is given by

$$0 = \frac{(1-\alpha)\zeta A}{(Q^i)^\alpha}\, F^i\, E\left\{ \Omega(\theta', K', k')\, \Pi(\theta, \theta', K) | \, \theta \right\} - \zeta(\varsigma^i + w(\theta, K) n(k, F^i)),$$

which implies that the equilibrium price is

$$P^i(\theta, K) = (1-\alpha)\, E\left\{ \Omega(\theta', K', k')\, \Pi(\theta, \theta', K) | \, \theta \right\}. \tag{39}$$

The first order condition over F^i is given by

$$0 = \zeta\, A\, (Q^i)^{1-\alpha}\, E\left\{ \Omega(\theta', K', k')\, \Pi(\theta, \theta', K) | \, \theta \right\} - \zeta\, Q^i\, w(\theta, K) \frac{\partial n}{\partial F^i}.$$

Substituting equation (39) for price and $\partial \widetilde{n} / \partial F^i = 1/f_n$ for given $k = K$, we have

$$\frac{w(\theta, K)}{P^i(\theta, K)} = \frac{1}{1-\alpha} A(Q^i)^{-\alpha} f_n(K, N^i(K, \theta)).$$

where $n^i(\theta, K) = n(K, F^i(\theta, K))$ is the labor associated with $k = K$ and the amount produced by investment firms proposed F^i. The equilibrium market tightness Q^i as a function of ς^i is given by

$$Q^i(\theta, K) = \left[\frac{A\, P^i(\theta, K)\, F[K, N^i(\theta, K)]}{\varsigma^i(\theta, K) + w(\theta, K) n^i(\theta, K)} \right]^{\frac{1}{\alpha}}. \tag{40}$$

3.6 Equilibrium

We have now established the necessary conditions for equilibrium that arise from the households' and the firm's problems and from the shoppers' problems under competitive search.

Before formally defining equilibrium we provide a set of results – stated in a series of lemmas – that allows us to verify our conjecture that (θ, K) is a sufficient aggregate state variable. They amount to show that if all firms start with the same capital, they will make the same choice of labor and investment and will, hence, have identical capital K' next period.

Lemma 2. *All firms with $k = K$ choose markets with the same quantity $F = F^c = F^i$ and the same labor input for production.*

We therefore use $N^y(\theta, K)$ to denote the aggregate labor input, $N^y(\theta, K) = N^c(\theta, K) = N^i(\theta, K)$ for any (θ, K).

Lemma 3. *The expected revenue per unit of output is the same in both sectors:*

$$P^c(\theta, K)\frac{\Psi_d[Q^c(\theta, K)]}{Q^c(\theta, K)} = P^i(\theta, K)\frac{\Psi_d[Q^i(\theta, K)]}{Q^i(\theta, K)}. \tag{41}$$

Lemma 4. *Firms with the same k chooses the same k' as future capital stock.*

Two more lemmas that will prove useful later follow. The first states properties of investment prices unveiling a relation between the direct and the indirect costs of installing capital.

Lemma 5. *The investment price is proportional to the ratio of the wage and the amount of shopping that a worker can carry out.*

$$\frac{w(\theta, K)}{\zeta\Psi_d[Q^i(\theta, K)]f(K, N^y(\theta, K))} = \frac{\alpha}{1-\alpha}P^i(\theta, K) \tag{42}$$

The last lemma characterizes firms' optimal choice of capital accumulation. When making decisions for future capital, firms face an explicit cost of investment (the price paid) and an implicit cost (the wages of shoppers). Interestingly, the Euler equation in equilibrium looks almost exactly like the one in a standard RBC model in that the implicit wage cost disappears. The reason is that competitive search links the implicit wage cost and explicit cost together. In equilibrium, the following equation holds

Lemma 6. *The Euler equation of a firm equates the price of investment to the value of capital tomorrow.*

$$E\left\{P^i(\theta', K')\,\Pi(\theta, \theta', K)\,\left[\frac{\Psi_d(Q^i)}{Q^i}f_k(K', N^y(\theta', K')) + (1-\delta)\right]\right\} = P^i(\theta, K). \tag{43}$$

We are now ready to define equilibrium in this economy.

Definition 1. *Equilibrium is a set of decision rules and values for the household $\{c, d, n, s', v\}$ as functions of its state (θ, K, s), firms decisions and values $\{n^y, n^k, i, k', \Omega\}$ as functions of its* *state (θ, K, k), and aggregate functions for shopping for investment D^i, shopping for consumption D^c, consumption C, labor N, labor for production N^y, labor for shopping N^k, investment I, aggregate capital G, expected revenues ς, the measure of consumption producing firms T, wages w, consumption good prices P^c, consumption market tightness Q^c, production of firms F^c and F^i, investment good prices P^i, investment market tightness Q^i, and the rate of return of the economy R as functions of the aggregate state (θ, K) that satisfy*

1. Households choices and values $d(\theta, K, s)$, $c(\theta, K, , s)$, $n(\theta, K, s)$, $s'(\theta, K, s)$, and $v(\theta, K, s)$ satisfy (15-17) and (24-25).

2. Firms choose $n^k(\theta, K, k)$, $i(\theta, K, k)$, $k'(\theta, K, k)$ and $\Omega(\theta, K, k)$ to solve their problem (27). They satisfy conditions (29-30) and (33).

3. Competitive Search Conditions. Shoppers and sellers go to the appropriate submarkets, i.e., equations (37-38) and (39-40).

4. Representative Agent and Equilibrium Conditions: individual decisions are consistent with aggregate variables.

5. Market Clearing Conditions:

$$
\begin{aligned}
s'(\theta, K, 1) &= 1, && (44)\\
I(\theta, K) &= D^i(\theta, K)\ \Psi_d(Q^i(\theta, K))\ F^i(\theta, K), && (45)\\
C(\theta, K) &= D^c(\theta, K)\ \Psi_d(Q^c(\theta, K))\ F^c(\theta, K), && (46)\\
N(\theta, K) &= N^y(\theta, K) + N^k(\theta, K), && (47)\\
Q^c(\theta, K) &= \frac{T(\theta, K)}{D^c(\theta, K)}, && (48)\\
Q^i(\theta, K) &= \frac{1 - T(\theta, K)}{D^i(\theta, K)}. && (49)
\end{aligned}
$$

Note that since the numéraire is the stock market pre-dividends, Ω must be given by $\Omega(\theta, K, K) = 1 + R(\theta, K)$. The financial wealth of the household is $s(1 + R)$ and is equal to the stock market today including the dividends. Note also that the share of consumption expenditure equals the fraction of firms producing consumption goods T, i.e. $\frac{P^c C}{P^c C + P^i I} = T$.

3.7 The Equilibrium is efficient

This section analyzes the efficiency properties of the competitive search equilibrium. To this end, we start by characterizing the efficient allocation arising from the problem of a social planner, who also faces the technology constraints that searching efforts have to be exerted for consumption goods and investment goods to be found. We then prove that the competitive search equilibrium is efficient.

Definition 2. *An allocation $\{T, D^c, D^i, N^y, C, K'\}$ is said to be efficient if it solves the following social-planner problem:*

$$V(\theta, K) = \max_{T, D^c, D^i, N^y, C, K'} \quad u\left(C, D, N^y + \frac{D^i}{\xi}, \theta\right) + \beta E\{V(\theta', K')|\theta\}$$

subject to

$$C \leq A(D^c)^{\alpha}(T)^{1-\alpha} f(K, N^y) \quad (50)$$

$$K' - (1-\delta)K \leq A(D^i)^{\alpha}(1-T)^{1-\alpha} f(K, N^y) \quad (51)$$

Proposition 2. *The competitive search equilibrium is efficient.*

Note that efficiency requires that markets are indexed by per-unit price, tightness, *and* quantity. This is necessary to avoid a hold-up problem between firms and consumers. Recall that once the consumers' search cost has been sunk and a consumer has been matched with a firm, a trade between them would be carried out regardless of the quantity offered. Therefore, if markets were characterized only by tightness and price, firms might find it optimal to deviate from the efficient quantity. However, once firms are allowed to index their market also on quantity, this hold-up problem disappears, and the competitive equilibrium allocation is efficient.[5]

3.8 Understanding Solow residual and labor share

We can use our model economy to compute the Solow residual provided that we specify a particular production. Let such production function be $f(k, n) = z\ k^{\gamma_k} n^{\gamma_n}$ where z is a parameter that determines units and $\gamma_k + \gamma_n < 1$, (see Section 4 for a discussion). Measured with base year prices,

[5]See Faig and Jerez (2005) for a related argument in economies with private information. They find that to restore efficiency, it is necessary to index markets by a non-linear price-quantity schedule. In the case of symmetric information, their efficient indexation simplifies to our triplet index of price, tightness, and quantity.

GDP is given by

$$Y = P_0^c C + P_0^i I \tag{52}$$

where P_0^c is the base year consumption price and P_0^i is the base year investment price. Replacing C and I with the aggregate production function the matching terms, we can write GDP as

$$Y = \left[P_0^c\, A(D^c)^{\alpha} T^{1-\alpha} + AP_0^i\,(D^i)^{\alpha}(1-T)^{1-\alpha}\right] zK^{\gamma_k}(N^y)^{\gamma_n}. \tag{53}$$

The Solow residual $\overline{Z}$ is defined as $\overline{Z} = \frac{Y}{K^{1-\gamma}N^{\gamma}}$, where γ denotes average labor share of output. In our model the labor share in steady state is

$$\gamma = \frac{1}{1-\alpha}\gamma_n + \frac{\alpha\delta}{(1-\alpha)(1/\beta - 1 + \delta)}\gamma_k, \tag{54}$$

If we use this steady state labor share to compute the Solow residual in our model we have

$$\overline{Z} = A\, z\, \underbrace{\left[P_0^c\,(D^c)^{\alpha} T^{1-\alpha} + P_0^i (D^i)^{\alpha}(1-T)^{1-\alpha}\right]}_{\text{Demand Effect}} \overbrace{\left(\frac{N^y}{N}\right)^{\gamma_n}}^{\text{Effective Work}} \underbrace{K^{\gamma_k-(1-\gamma)} N^{\gamma_n-\gamma}}_{\text{Share's Error}}. \tag{55}$$

This is, the Solow residual depends on the demand, increases in demand without changes in technology increase the Solow residual. The two additional terms are due to the mismeasure of productive labor and to the imputation of constant returns to scale. In empirical applications $N - N^y$ is small and γ_n is close to γ. This implies that the last two terms in equation (55) are almost constant over the business cycle and that all movements in the Solow residual are due to movements in demand.

4 Mapping the model to data

to map the model to data, we start by making the case for the functional forms that we use, and by describing the 11 implied parameters. We then discuss the targets for the steady state of the model economy.

Preferences (4 parameters). We pose separable preferences to be able to clearly discuss the role of the Frisch labor elasticity and also to isolate the role of shopping. The per period utility function or felicity is given by (56)

$$u(c,n,d) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - d \tag{56}$$

where we omit the shocks to preferences for simplicity. The involved parameters are the discount rate β, the coefficient of risk aversion, σ, the Frisch elasticity of labor, ν, and the parameter that determines average hours worked, χ. That the disutility of shopping is linear is not important as the shocks that affect it will shape its properties.

Production Technology (5 parameters). Firms have a decreasing returns Cobb-Douglas production function

$$f(k,n) = z \;\; k^{\gamma_k} \, (n^y)^{\gamma_n} \tag{57}$$

There is no need to impose constant returns to scale given the fact that there are a limited number of locations that are valuable.[6] Note also that z is a parameter to determine units (in later sections we consider it as shock). In addition, a worker devoted to shop for investment goods produces ζ units of shopping services, allowing for the possibility that firms could be a lot more efficient shoppers than people. Capital depreciates at rate δ.

Matching Technology(2 parameters). The matching technology is Cobb-Douglas indexed by A and α.

$$A \, D^{\alpha} \; T^{1-\alpha}. \tag{58}$$

4.1 Calibration

In this economy most of the targets for the steady state (and associated parameters) are the standard ones in business-cycle research, while others are specific to this economy. Table 1 reports the targets and also the parameter most associated with those targets. The targets are defined in yearly terms even though the model period is a quarter.

The first group of parameters are set independently of the equilibrium allocation. The intertemporal elasticity of substitution is set to two, and the real rate of return is 4.%. The Frisch elasticity is more controversial. We choose a value .72, based on Heathcote, Storesletten, and Violante

[6] The model could be easily extended to accommodate the costly creation of such locations.

Table 1: Calibration Targets, Implied Aggregates and (quarterly) Parameter values

Targets	Value	Parameter	Value
First Group: Parameters set exogenously			
Risk aversion	2.	σ	2.
Real interest rate	4.%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\nu}$	0.72
Second Group: Standard Targets			
Fraction of time spent working	30%	χ	16.81
Physical Capital to Output Ratio	2.75	δ	0.07
Consumption Share of Output	0.80	γ_k	0.23
Labor Share of income	0.67	γ_n	0.59
Steady-state output	1	z	2.03
Third Group: Targets specific to this economy			
Share of production workers	97.%	ζ	3.16
Capacity Utilization of Consumption Sector	0.81	A	0.97
Capacity Utilization of Investment Sector	0.81	α	0.09

Implications over other Aggregate Variables	
Percentage of GDP payable to Shoppers	2.%
Percentage of Cost of New Capital that is internal	9%
Relative price of investment in terms of consumption	1
Wealth to output ratio	3.33

(2008) who take into account the response of hours worked for both men and women in a model that explicitly incorporates households with husbands and wives.[7]

The targets in the second group are standard in the business-cycle literature. Note that the consumption to output ratio is set to 0.8 since investment in our model is strictly business investment. We exclude consumption durables since business investment and consumer durables use different shopping technologies.

The third group of targets are specific to our model. Our notion of capacity utilization is related to the series published by the Federal Reserve Board.[8] We target a steady-state capacity utilization of 81 percent in both sectors, which corresponds to the postwar average of the official data series (see Corrado and Mattey (1997)). We target 3% of the workforce as being involved in investment shopping even we do not have direct measurements of such a variable. As the last panel of Table 1 shows, this choice implies that 2% of GDP is spent on search activities, or that 9% of the cost of installing new capital is internal to the firm. We view this as a plausible magnitude for the adjustment costs associated with finding the right investment goods.

The last panel of Table 1 reports that the relative price of consumption and investment is 1 (a direct implication of equal capacity utilization in both sectors) and the wealth to output ratio which turns out to be 3.33. This result has the nice feature that the book value of firms is only 80% of their stock market value.

5 Demand shocks in univariate economies

We now study economies with univariate shocks, i.e., economies where all fluctuations are driven by one shock. We analyze two types of shocks to preferences, shocks to the shopping technology of firms, and shocks to total factor productivity. In Section 5.1 we estimate univariate shock processes with Bayesian methods[9] using only Solow residual data. We pose AR(1) shocks and we assume that the persistence follows a Beta distribution while the volatility follows an inverse

[7]Table A-3 of the Appendix, reports results for an economy with a Frisch elasticity of 1.1.

[8]The Federal Reserve Board's Industrial Production and Capacity Utilization series is based on estimates of capacity and capacity utilization for industries in manufacturing, mining, and electric and gas utilities. For a given industry, the capacity utilization rate is equal to an output index (seasonally adjusted) divided by a capacity index. The purpose of the capacity indexes is to capture the concept of sustainable maximum output-the greatest level of output a plant can maintain within the framework of a realistic work schedule, after factoring in normal downtime and assuming sufficient availability of inputs to operate the capital in place.

[9]See An and Schorfheide (2007) or Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulalia-Llopis (2009) for details.

Gamma distribution. We show that each one of these shocks can on their own generate fluctuations in the Solow residual like those in the U.S. data. Section 5.2 studies the quantitative business-cycle properties of the univariate shock economies showing that these economies have strongly counterfactual implications. In particular, the demand shock to consumption induces a reduction in investment while the reverse occurs with a demand shock to investment. This indicates that a shock that is joint to both consumption and investment is more likely to generate the correct comovements. In fact, this is the case and is studied in Section 5.3.

5.1 Estimating shock processes of univariate economies

We pose a shock to disutility of shopping θ_d, another to the disutility of work, θ_n, so we have $\frac{c^{1-\sigma}}{1-\sigma} - \theta_n \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \theta_d\ d$, a shock to the firm's shopping technology ζ, and a technology shock z. The estimates are in Table 2. The last column of the table reports the results from a standard

Table 2: Estimates of the Processes for Univariate Shocks

	Univariate Versions of the Shopping Model				Standard RBC
Shocks to	Shop. Disut	Labor Disut	Firm's Shop.	Tech	Tech
	θ_d	θ_n	ζ	z	z
ρ	0.946	0.720	0.985	0.960	0.945
Para(1)	0.94	0.72	0.95	0.95	0.94
Para(2)	0.05	0.05	0.03	0.05	0.05
90% Intv	[0.91, 0.98]	[0.66, 78]	[0.98,0.99]	[0.93, 0.99]	[0.91, 0.98]
σ	0.086	0.171	0.334	0.006	0.006
Para(1)	0.09	0.17	0.30	0.004	0.006
Para(2)	0.20	0.20	0.20	0.20	0.20
90% Intv	[0.08, 0.09]	[0.16, 0.18]	[0.31, 0.36]	[0.006, 0.007]	[0.006, 0.007]
Likelihood	735.13	737.09	732.62	733.98	735.05
Var of $\bar{Z}$	3.44	3.02	2.08	3.36	3.45
Autocorr of $\bar{Z}$	0.95	0.96	0.91	0.95	0.95

RBC economy (without shopping) where the technology shock *is* the Solow-residual. As is evident from the table, all these shocks can generate a process for the Solow residual like that in the data, without having to resort to shocks to technology. Moreover, the estimates are quite precise and in terms of the likelihood, they are as good as those of the RBC economy.[10] The standard deviations

[10] Table A-2 in the Appendix shows the details of its calibration which is designed to be as close as possible as those of the shopping economy.

of these shocks are, however, quite larger than those of the productivity shock. Interestingly the process of the technology shock in the shopping economy does not have a smaller variance than that of the shock in a standard RBC model, if anything (due to the larger autocorrelation) technology shocks as the only shocks of the shopping economy have a larger variance. This property indicates that the shopping economy does not amplify TFP shocks.

5.2 Business cycle properties of univariate economies

Whether demand shocks provide a good rationale or not for business cycles does not depend only on their ability to generate movements in the Solow residual. Indeed, what has made the model with technology shocks so popular over the years is its ability to generate the right comovements: the Solow residual, output, the components of output and hours worked are all strongly correlated and investment is much more volatile than output which in turn is more volatile than consumption. Table 3 shows how this is the case and how the standard RBC economy displays the right comovements. In this economy, however, the variance of hours is quite small. This is due to the relatively low Frisch elasticity of substitution that we use in our calibration (that we are using variances gives an additional optical illusion of being small).

Table 3: Main Business Cycle Moments: U.S. Data and Standard RBC Model

	U.S. Data			Standard RBC		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.19	0.43	0.94	3.45	0.99	0.95
Y	**2.38**	1.00	0.86	**0.82**	1.00	0.71
N	**2.50**	0.87	0.91	**0.04**	0.96	0.72
C	1.55	0.87	0.87	0.05	0.95	0.76
I	34.15	0.92	0.80	13.74	0.99	0.71
cor(C, I)	**0.74**			**0.93**		

All variables except the Solow residual are HP-filtered.

The business cycle statistics of the shopping economies are reported in Table 4. Clearly, they differ significantly from each other and from the RBC economy and the U.S. data. The only feature they really have in common is that output and the Solow residual move together, something that follows immediately from the way the Solow residual is constructed.

Table 4: Main Business Cycle Moments: Various Economies with Demand Shocks

	(a) Shop Disut θ_d			(b) Labor Disut θ_n		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.44	1.00	0.94	3.02	**-0.85**	0.94
Y	**0.37**	1.00	0.71	**35.49**	1.00	0.58
N	**0.07**	**-1.00**	0.72	**97.13**	0.99	0.55
C	0.51	1.00	0.72	2.90	0.76	0.84
I	0.03	0.99	0.69	626.37	0.98	0.55
cor(C, I)	**0.99**			**0.63**		
	(c) Firms' Shopping Tech ζ			(d) Technology Shock z		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	2.08	0.73	0.91	3.45	0.99	0.95
Y	**1.73**	1.00	0.75	**0.59**	1.00	0.73
N	**0.54**	0.78	0.69	**0.01**	**-0.52**	0.96
C	0.45	**-0.53**	0.74	0.22	0.98	0.78
I	69.22	0.96	0.70	4.08	0.98	0.70
cor(C, I)	**-0.74**			**0.92**		

All variables except the Solow residual are HP-filtered.

In the economy with *the shopping disutility shock* (Panel (a)), labor is negatively correlated with output. This is counterfactual. These shocks generate a positive wealth effect: consumption goes up and work goes down. Consumer shoppers are more effective and consumption producing firms operate at higher capacity allowing for lower work effort. So shocks of this type by themselves cannot be the trigger of fluctuations.

The Working disutility shock (Panel (b)) generates the volatility of the Solow residual by attracting more search effort when labor is low, making productivity and output negatively correlated. The variance of output required for this too happen is tremendous, six times that of the data, and that of labor is even larger, about 15 times that of the data. This economy generates comovements that are dramatically different from those associated to business cycles in the data.

The shock to the firm's shopping technology (Panel (c)) has to generate all the movements of the Solow residual from a small part of GDP and hence its variance is immense. It makes hours worked quite volatile and positively correlated with output but consumption is negatively correlated with output.

In the shopping economy posing *a technology shock* (Panel (d)) has different implications than in the standard RBC. The existence of the search margin implies that households do not take advantage of the higher productivity by working longer hours, in fact, hours go down a tiny bit. The comovement of consumption and investment is the correct one. In the shopping model, a positive productivity shock induces enough of a wealth effect and decreases consumer's shopping effort. It, however, makes firms allocate more labor into searching due to a lower investment goods price arising from the positive productivity shock. According to the Solow residual decomposition shown in Section 3.8, both lower consumer shopping and fewer production workers decrease the Solow residual, while higher firm shopping increases the Solow residual. In sum, these effects cancel out somehow and generate almost no amplification of the productivity shock. This also explains why the estimated volatility of innovation in our model is similar as that in the standard RBC model.

To summarize, the shopping economies with only demand shock do not display the business cycle properties of the data in terms of the comovements of the major variables. In economies subject to TFP shocks or to shocks to the MRS between consumption and labor, there is a negative relation between labor and output. More importantly, in the economies with pure demand shocks (to either consumption demand or investment demand) the increase in the expenditures of the variable affected by the shock generates an increase in labor and the Solow residual but a reduction in the other component of expenditures indicating that if demand shocks are to play apart they have to affect simultaneously consumption and investment and this, we explore next.

5.3 Joint Demand Shocks

We now pose an economy with only one source of uncertainty but one that affects simultaneously consumption and investment. Specifically, we pose that θ_d follows an AR(1) process to be estimated and that ζ is proportional to θ_d with the constant also to be estimated. Like before we use Bayesian methods for the estimation, with the new parameter's prior assumed to be normal. The top panel of Table 5 shows the estimates. As we can see, the standard deviation of the consumption demand shock is 60% of what was needed in the pure univariate economy while the standard deviation of the demand shock, while being two and a half times the standard deviation of the consumption shock, is about one third of what it was in the pure univariate version.

The lower panel of Table 5 shows the business cycle statistics of this economy and compares

Table 5: The Shopping Economy with perfectly correlated demand shocks

Priors and Posteriors for the Shock Parameters

Shopping model with (θ_d, ζ), likelihood = 734

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	0.968	[0.946, 0.992]
σ_d	Inv. Gamma	0.05	0.20	0.052	[0.044, 0.059]
σ_ζ/σ_d	Normal	2.60	0.50	2.643	[1.854, 3.471]

Main Business Cycle Statistics

	Variance			Correlation with Y		
	Data	(θ_d, ζ)	RBC	Data	(θ_d, ζ)	RBC
Solow	3.19	3.33	3.45	0.43	0.98	1.00
Y	2.38	0.84	0.82	1.00	1.00	1.00
N	2.50	0.05	0.04	0.87	0.57	0.96
C	1.55	0.10	0.05	0.87	0.75	0.95
I	34.15	13.92	13.74	0.92	0.97	1.00

them with those of the standard RBC economy. Now the comovements of the variables are the appropriate ones. Consumption and investment are positively correlated with investment a lot more volatile. Hours while being positively correlated display a very small volatile, as does the RBC model. This is an implication of the low Frisch elasticity and of the assumption that all employment is voluntary. Table A-3 in the Appendix shows how a Frisch elasticity of 1.1 doubles the volatility of hours worked in both models. Moreover, Section B of the Appendix compares versions of the shopping and RBC economies with a higher hours volatility due to an additional shock to the MRS that increases the volatility of hours. The findings remain unaltered. The shopping economy with a shock to demand that affects jointly consumption and investment performs as satisfactorily as the RBC economy in terms of generating business cycles statistics.

6 Estimating the contribution of all shocks

So far we have compared the shopping model *against* the RBC model. What we do now is to estimate within the shopping model with four shocks which allows us to impute the contribution of each shock to aggregate fluctuations. These shocks are the two demand shocks, a shock to the MRS and a productivity shock within the shopping model. We use again Bayesian methods.

We assume that, except for the consumption and investment demand shocks that are correlated, the shocks are independent. The data that we use are the Solow residual, output, hours, and consumption, all of them linearly detrended. We assume that the autocorrelations follow a Beta distribution, that the standard deviation of the innovations follow an inverse-Gamma distribution, and that the correlation between demand shocks follows a normal distribution. Because of its importance for identification, we explore below in more detail the role of the correlation of the demand shocks.

Table 6 shows the priors and posteriors for all shock parameters. The 90% intervals are tight except for the correlation between the demand shocks. The estimates for the standard deviations of all shocks are much smaller than in the univariate shock economies (a factor of 8 for the shock to the MRS, a factor of 3 for the TFP shock, a factor of 2.5 for the investment demand shock) except for the the consumption demand shock (a factor of 1.2), indicating that the role of the demand shocks is likely to be the strongest. This is confirmed by the variance decomposition of the major aggregate variables also reports in Table 6. Demand shocks are much more important than the productivity shock, not only in terms of its contribution to output (63% relative to 1%) but also in terms to its contribution to the Solow residual itself (95% relative to 2%).

The volatility of hours are still dependent mostly on shocks to the MRS, but the demand shocks contribute 14% while productivity shocks have no effects on hours. Demand shocks also account for more than half of consumption and 90% of investment while the contribution of TFP shocks is less than 5%.

An interesting, and somewhat surprising feature of the estimates, is the estimate of the correlation between the two demand shocks. While when we consider only demand shocks to consumption and investment the correlation has to be very large to get the right comovements between consumption and investment and labor and output,[11] this is not the case when we take into account all shocks. Now the shocks are essentially orthogonal and the correct comovements of the main variables arise from the joint response to all shocks.

It is hard to say exactly where the identification comes from, given that it is such a complicated system. We think it has to do with the timing of the relative moment of the variables. In addition, the 90% interval of the estimated demand-shock correlation is not very tight. To see how sensitive are the variance decompositions to this parameter, we re-estimated the shocks while for values

[11] Estimating the Solow residual out of orthogonal consumption and investment shocks with the same persistence yields an hp-filtered correlation between consumption and investment of -.13 and between hours and output of 0.45.

of the correlation between the demand-shocks within the confidence interval varying from -0.1 to 0.1. The log-likelihood ratios of the estimates for the various values of the correlation are not significantly different from each other. As the correlation increases, the contribution of the demand shocks to the Solow residual goes from 97% to 65%, while that of output goes from 65% to 41%. (See Table A-4 in the Appendix for details).

Table 6: Full Estimation of the Shopping Model

Priors and Posteriors for the Shock Parameters (Likelihood = 2244.29)					
Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.96	0.05	**0.953**	[0.930, 0.979]
σ_d	Inverse Gamma	0.072	0.20	**0.073**	[0.066, 0.084]
ρ_ζ	Beta	0.96	0.05	**0.953**	[0.930, 0.979]
σ_ζ	Inverse Gamma	0.13	0.20	**0.128**	[0.109, 0.144]
ρ_z	Beta	0.93	0.05	**0.918**	[0.840, 0.997]
σ_z	Inverse Gamma	0.002	0.20	**0.002**	[0.001, 0.003]
ρ_n	Beta	0.95	0.05	**0.996**	[0.992,1.000]
σ_n	Inverse Gamma	0.02	0.20	**0.022**	[0.020, 0.024]
Cor(θ_d, ζ)	Normal	-0.10	0.20	**-0.055**	[-0.225, 0.080]

	Variance Decomposition (%)				Business Cycle Statistics	
	θ_d	ζ	z	θ_n	Variances	Cor w Y
Y	32.44	30.46	1.21	35.89	1.07	1.00
Solow	81.66	14.23	2.15	1.97	12.88	0.63
N	2.91	10.88	0.02	86.18	1.29	0.61
C	52.34	15.52	0.75	31.39	0.69	0.62
I	1.84	87.76	0.52	9.88	16.44	0.77

Lastly, we report the major business cycle statistics for the estimations conducted in this Section in the last panel of Table 6. The shopping economy gets the right comovements between the main variables. With respect to the variances, it is too high for the Solow residual and too low for output and labor.

7 Other business cycle implications of the shopping economies

In this section, we explore additional interesting implications of the shopping economies that are absent from the standard RBC model. In the shopping model the relative price of investment, the stock market price, and capacity utilization are all endogenous variables that move over the cycle. Table 7 reports the properties of the data and the two main versions of the shopping model that we have explored, that with perfectly correlated demand shocks and the economy estimated with all shocks.

1. *The relative price of investment.* The role of the relative price of investment in shaping economic performance has been studied in various contexts, from its role in shaping the skill premia (Krusell, Ohanian, Rios-Rull, and Violante (2000)) to its role as a direct source of business cycles (Fisher (2006)). In most of these cases such relative price is taking to be an exogenous object that depends purely on technological considerations (an exception is Valles (1997) that uses a non-linear production possibility frontier). In our economy the relative price of consumption and investment is purely an economic object since consumption and investment are perfect substitutes in production. In the shopping environment a reduction in the disutility (or cost) of shopping translates into a willingness to shop longer while facing a cheaper price for consumption (or the investment) good. In the data, the relative price of investment is countercyclical and less volatile than output. The model economies also pose a countercyclical relative price of investment. The economy with only demand shocks have a volatility that is twice as volatile as that in the data and, like the data, is negatively correlated with output, albeit much more than the data. The multiple shock economy has an extremely volatile relative price of investment with the correct correlation. We find this very encouraging as we think that in the data consumption and investment goods are only partially substitutable and hence the model is likely to exaggerate the volatility of the relative price.

2. *The stock market price.* Given our normalization, the stock market is just the inverse of the price of consumption. In the shopping economy the value of firms changes not only because of changes in the cost of shopping for new capital, but also because the value of locations capable of matching with shoppers changes. As is well known, the stock market price in the data is extremely volatile with a volatility about 40 times that of output. It is also procyclical. Our model does indeed generate a procyclical stock price. Not surprisingly, the variance of the price is much smaller than that of the data, but still sizeable especially given the low risk aversion that we have posed. The correlation is similar to that of the data in both economies.

3. *Capacity utilization.* In the shopping economies the ratio of output to potential output is

constantly changing, with higher utilization resulting in higher measured productivity. In the U.S., the Federal Reserve Board has constructed a series of capacity utilization that while does not coincide with our notion, is clearly related. In the data, the capacity utilization is about twice as volatile as output, and very procyclical. Our model without frictions in production also has procyclical utilization although with lower volatility than the data. Perhaps, this tells us that there are more fixed factors in production that just capital, and this accounts for larger value of the volatility in the data.

Table 7: Other Business Cycle Statistics for the Full Estimation Shopping Model

	Variance			Correlation with Y		
	Data	θ_d, ζ	θ_d, ζ z, θ_n	Data	θ_d, ζ	θ_d, ζ z, θ_n
p_i/p_c	0.47	0.98	2.71	-0.23	-1.00	-0.30
Stock Market (S&P 500)	42.64	0.26	1.82	0.41	0.26	0.33
Capacity Utilization	10.02	0.68	0.57	0.89	0.99	0.71

Overall, we think that the additional implications for business cycles of the shopping economies produce very satisfactory outcomes. All the correlations are the correct ones and the difference in the sizes of the variances are easy to understand.

8 Conclusions and Extensions

In this paper we have developed a model where demand shocks generate procyclical productivity via a cyclical use of production capacity that implies inconveniences for shoppers. We have developed the theory using search frictions under competitive search protocols that imply optimality (and hence existence and uniqueness) of equilibrium. We have shown how demand shocks can replicate the movements of the Solow residual that the RBC literature typically identifies with technology shocks. We show that correlated shocks to the demand of consumption and investment replicate the properties of the standard real business cycle models in terms of the comovements of macroeconomic variables. If anything, our model performs marginally better. When allowing in our model economy for the coexistence of demand shocks and technology shocks, a full Bayesian estimation imputes essentially no role to technology shocks even in shaping the properties of the Solow residual. In addition, our shopping economies have implications for other macroeconomic variables (relative price of consumption and investment, the stock market, capacity utilization) over which standard models are silent. Such implications are remarkably consistent with the data.

Our assessment of the findings is that our modeling structure provides an alternative to technology shocks as a source of fluctuations. We think that these findings can and should be naturally applied to the other most popular lines of business cycle research (New-Keynesian and Mortensen-Pissarides type of models) to accommodate demand shocks as a substitute to technology shocks as the main source of fluctuations. We also think that our findings may provide a rationale for fiscal stimulus packages, a feature that is hard to rationalize in standard macro models. The research agenda we think is clear and has two directions: to pose deep models of demand shocks: financial shocks, shocks to real exchange rate, wealth shocks, monetary policy and so on; and to accommodate mechanisms that substitute the frictionless labor market.

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APPENDIX

A Proofs

Proof of Proposition 1.

Proof. Substituting $d = c/\Psi_d(D)$ into the period utility and differentiating u w.r.t. c, we get

$$\frac{du}{dc} = u_c + \frac{\partial u}{\partial d}\frac{\partial d}{\partial c} = u_c + \frac{u_d}{\Psi_d(D)}$$

The Euler equation can then be expressed as

$$u_c + \frac{u_d}{\Psi_d(D)} = p(\theta)m.$$

Impose the representative-agent conditions $c = C = AD^\alpha$ and $d = D$, and use (9) to substitute out the term $p(\theta)m$. This gives one functional equation:

$$u_c\left(C(\theta), D(\theta), \theta\right) + \frac{u_d\left(C(\theta), D(\theta), \theta\right)}{\Psi_d\left(D(\theta)\right)} = (1-\alpha)\, u_c\left(C(\theta), D(\theta), \theta\right).$$

Rearranging this equation and using (11) yields the functional equation (12). The functional equation (13) is derived from the equilibrium price equation (9) and the definition of m, where we exploit the envelope theorem and (5) to express $\partial v/\partial s$ as

$$\frac{\partial v\left(\theta, s\right)}{\partial s} = u_c\left(C\left(\theta\right), D(\theta), \theta\right) + \frac{u_d\left(C\left(\theta\right), D(\theta), \theta\right)}{\Psi_d[Q(\theta)]}$$

At the equilibrium, the agents' budget constraint (6) is satisfied. Given (12)-(13), the first-order conditions of (4) hold, which guarantees individual optimization.

Consider the planner problem $\max_{C,D}\left\{u(C, D, \theta)\right\}$ subject to the aggregate resource constraint $C = A\ D^\alpha$. It is straightforward to verify that the solution to equation (12) solves this planner problem, which establishes efficiency. □

Lemma 1. $\Omega^c(\theta, K, k) = \Omega^i(\theta, K, k)$ *implies* $\varsigma^c(\theta, K, k) = \varsigma^i(\theta, K, k)$.

Proof. From the firms' first order condition over k' (33) it is clear that both marginal return and marginal cost of capital are independent of firm's current choice over which goods to produce and choice of labor for production. This implies that firms simply search for the markets that give them the best current revenue. Thus $\Omega^c(\theta, K, k) = \Omega^i(\theta, K, k)$ implies $\varsigma^c(\theta, K, k) = \varsigma^i(\theta, K, k)$. $\square$

Lemma 2. *All firms with $k = K$ choose markets with the same output $F = F^c = F^i$ and also the same labor input for production.*

Proof. Let's define $n^c(K, \theta)$ as the necessary labor for a consumption-producing firm with capital $k = K$ to produce output $y^c(\theta, K)$, namely $n^c(K, \theta) = n(K, y^c(\theta, K))$. Similarly, we define $n^i(K, \theta) = n(K, y^i(\theta, K))$ for investment-producing labor. In equilibrium, firms are indifferent of producing consumption goods or investment goods, i.e. $\varsigma^c = \varsigma^i$. By definition, $\varsigma^c = P^c(\theta, K)A[Q^c(\theta, K)]^{-\alpha}y^c(\theta, K) - w(\theta, K)n^c(\theta, K)$. We can further rewrite ς^c using equilibrium conditions from the competitive search $\frac{w(\theta,K)}{P^c(\theta,K)} = \frac{1}{1-\alpha}A(Q^c)^{-\alpha}f_n[K, n^c(\theta, K)]$,

$$\begin{aligned}
\varsigma^c &= P^c(\theta, K)A[Q^c(\theta, K)]^{-\alpha}y^c(\theta, K) - w(\theta, K)n^c(\theta, K) \\
&= w(\theta, K)\left[\frac{P^c(\theta, K)A[Q^c(\theta, K)]^{-\alpha}y^c(\theta, K)}{w(\theta, K)} - n^c(\theta, K)\right] \\
&= w(\theta, K)\left[(1-\alpha)\frac{A[Q^c(\theta, K)]^{-\alpha}y^c(\theta, K)}{A[Q^c(\theta, K)]^{-\alpha}f_n[K, n^c(\theta, K)]} - n^c(\theta, K)\right] \\
&= w(\theta, K)\left[(1-\alpha)\frac{f[K, n^c(\theta, K)]}{f_n[K, n^c(\theta, K)]} - n^c(\theta, K)\right].
\end{aligned}$$

Similarly, we have

$$\varsigma^i = w(\theta, K)\left[(1-\alpha)\frac{f[K, n^c(\theta, K)]}{f_n[K, n^c(\theta, K)]} - n^c(\theta, K)\right].$$

Equalizing ς^i and ς^c implies that $n^c(\theta, K) = n^i(\theta, K)$ under the assumption that the production function is concave and strictly increasing in labor. Thus, the labor inputs are the same for the firms with the same capital $k = K$. Their outputs must be the same too. $\square$

Lemma 3. *The expected revenue per unit of output is the same in both sectors:*

$$P^c(\theta, K)\frac{\Psi_d[Q^c(\theta, K)]}{Q^c(\theta, K)} = P^i(\theta, K)\frac{\Psi_d[Q^i(\theta, K)]}{Q^i(\theta, K)}. \tag{A-1}$$

Proof. Under Lemma 2, firms with the same $k = K$ have the same labor input for production, the same output, and the same ς. This implies equation (A-1). □

Lemma 4. *Firms with the same k chooses the same k' as future capital stock.*

Proof. According to Lemma 2, firms with the same k choose the same labor input. For a firm that considers to produce consumption tomorrow, the first order condition over k' is given by

$$E\left\{\left[-w(\theta',K')n_k(k',y(\theta',K'))+(1-\delta)\left(\frac{w(\theta',K')}{\zeta\,\Psi_d[Q^i(\theta',K')]\,f[(\theta',K')]}+P^i(\theta',K')\right)\right]\Pi(\theta,\theta',K)\middle|\,\theta\right\}$$
$$=\frac{w(\theta,K)}{\zeta\,\Psi_d[Q^i(\theta,K)]\,f[(\theta,K)]}+P^i(\theta,K).$$

For a firm that considers to produce investment tomorrow, the first order condition over k' is

$$E\left\{\left[-w(\theta',K')n_k(k',y(\theta',K'))+(1-\delta)\left(\frac{w(\theta',K')}{\zeta\,\Psi_d[Q^i(\theta',K')]\,f[K'^i(\theta',K')]}+P^i(\theta',K')\right)\right]\Pi(\theta,\theta',K)\middle|\,\theta\right\}$$
$$=\frac{w(\theta,K)}{\zeta\,\Psi_d[Q^i(\theta,K)]\,f[(\theta,K)]}+P^i(\theta,K). \quad \text{(A-2)}$$

With Lemma 2, it is easy to see that the first order conditions for k' of future consumption producing firms and investment producing firms are identical. Thus, all the firms with the same current capital choose the same future capital. □

Lemma 5. *The investment price is proportional to the ratio of the wage and the amount of shopping that a worker can carry out.*

$$\frac{w(\theta,K)}{\zeta\Psi_d[Q^i(\theta,K)]f(K,N^y(\theta,K))}=\frac{\alpha}{1-\alpha}P^i(\theta,K)$$

Proof. From investment-producing firms' first order condition over k', we have

$$E\left\{\Omega(\theta',K',k')\,\Pi(\theta,\theta',K)\right|\theta\} \;=\; \frac{w(\theta,K)}{\zeta\,\Psi_d[Q^i(\theta,K)]\,f[K,N^i(\theta,K)]}+P^i(\theta,K)$$

The equilibrium search in the investment goods market implies

$$P^i(\theta, K) \;=\; (1-\alpha)\, E\left\{\Omega(\theta', K', k')\, \Pi(\theta, \theta', K)|\, \theta\right\}.$$

Combining the above two equations proves the Lemma. □

Lemma 6. *The Euler equation of a firm equates the price of investment to the value of capital tomorrow.*

$$E\left\{ P^i(\theta', K') \left[\frac{\Psi_d(Q^i)}{Q^i} f_k(K', N^y(\theta', K')) + (1-\delta)\right] \Pi(\theta, \theta', K) \bigg| \theta \right\} = P^i(\theta, K).$$

Proof. Recall that an investment-producing firms choose future capital stock evaluated at K' according to equation (A-2). According to Lemma 5, we can replace $\frac{w(\theta,K)}{\zeta\Psi_d[Q^i(\theta,K)]f(K,N^y(\theta,K))}$ with $\frac{\alpha}{1-\alpha}P^i(\theta, K)$. Similarly for the future variables. The Euler becomes

$$E\left\{ \frac{-w(\theta', K')n_k(K', y(\theta', K')) + (1-\delta)P^i(\theta', K')}{1-\alpha} \Pi(\theta, \theta', K) \bigg| \theta \right\} = \frac{P^i(\theta, K)}{1-\alpha}$$

Multiplying $1-\alpha$ on both hand side and reorganizing the equation, we have

$$E\left\{ P^i(\theta', K')\Pi(\theta, \theta', K) \left[-(1-\alpha)\frac{w(\theta', K')n_k(K', y(\theta', K'))}{P^i(\theta', K')} + (1-\delta)\right] \bigg| \theta \right\} = P^i(\theta, K).$$

Substituting $w(\theta', K')/P^i(\theta', K')$ with $\frac{1}{1-\alpha}\frac{\Psi_d(Q^i)}{Q^i} f_n(K', N^y(\theta', K'))$ from the competitive search problem, we can rewrite the Euler as

$$E\left\{ P^i(\theta', K')\Pi(\theta, \theta', K) \left[-\frac{\Psi_d(Q^i)}{Q^i} f_n(K', N^y(\theta', K'))n_k(K', y(\theta', K')) + (1-\delta)\right] \bigg| \theta \right\} = P^i(\theta, K).$$

According to the implicit function theorem, $n_k \equiv \frac{dn}{dk} = -\frac{f_k}{f_n}$. Thus, $f_k = -f_n n_k$. Substituting $-f_n n_k$ with f_k in the Euler equation, we have

$$E\left\{ P^i(\theta', K') \left[\frac{\Psi_d(Q^i)}{Q^i} f_k(K', N^y(\theta', K')) + (1-\delta)\right] \Pi(\theta, \theta', K) \bigg| \theta \right\} = P^i(\theta, K).$$

□

A.1 Proof of Proposition 2.

Proof. Let λ be the multiplier for condition (50) and μ be the multiplier for condition (51). The first order conditions are given by

$$\begin{aligned}
u_C &= \lambda \quad \text{(over } C\text{)} \\
u_D &= \lambda \alpha A (D^c)^{\alpha-1}(T)^{1-\alpha} f \quad \text{(over } D^c\text{)} \\
\frac{u_N}{\zeta} &= \mu \alpha A (D^i)^{\alpha-1}(1-T)^{1-\alpha} f \quad \text{(over } D^i\text{)} \\
u_N &= \lambda A (D^c)^{\alpha}(T)^{1-\alpha} f_n + \mu A (D^i)^{\alpha}(1-T)^{1-\alpha} f_n \quad \text{(over } N^y\text{)} \\
\lambda(1-\alpha) A (D^c)^{\alpha}(T)^{-\alpha} &= \mu (1-\alpha) A (D^i)^{\alpha}(1-T)^{-\alpha} \quad \text{(over } T\text{)} \\
\mu &= \beta E \left\{ \lambda' A' (D^{c\prime})^{\alpha}(T')^{1-\alpha} f_{k'} + \mu' A' (D^{i\prime})^{\alpha}(1-T')^{1-\alpha} f_{k'} + \mu'(1-\delta) | \theta \right\}
\end{aligned}$$

After simplifying, the efficient allocation $\{T,\ D^c,\ D^i,\ N^y,\ C,\ K'\}$ can be characterized by the following 6 equations,

$$\frac{u_N}{u_C} = A (D^c)^{\alpha}(T)^{-\alpha} f_n \tag{A-3}$$

$$\frac{u_D}{u_C} = \alpha A (D^c)^{\alpha-1}(T)^{1-\alpha} f \tag{A-4}$$

$$f_n = \frac{\alpha \zeta (1-T) f}{D^i} \tag{A-5}$$

$$u_C \frac{(D^c)^{\alpha}(T)^{-\alpha}}{(D^i)^{\alpha}(1-T)^{-\alpha}} = \beta E \left\{ u_{c'} \frac{(D^{c\prime})^{\alpha}(T')^{-\alpha}}{(D^{i\prime})^{\alpha}(1-T')^{-\alpha}} \left[A' (D^{i\prime})^{\alpha}(1-T')^{1-\alpha} f_{k'} + (1-\delta) \right] \right\} \tag{A-6}$$

$$C \le A (D^c)^{\alpha}(T)^{1-\alpha} f(K, N^y) \tag{A-7}$$

$$K' - (1-\delta) K \le A (D^i)^{\alpha}(1-T)^{1-\alpha} f(K, N^y) \tag{A-8}$$

Equation (A-3) implies that the marginal rate of substitution between consumption and leisure equals the marginal product of labor. Equation (A-4) implies that the marginal rate of substitution between consumption and shopping effort equals the marginal product of shopping in the consumption-goods producing sector. Equation (A-5) implies that the marginal products of production labor and search labor are the same. Equation (A-6) is the Euler equation for capital.

Equation (A-7) and (A-8) are the resources constraints.

To show that the competitive search equilibrium is efficient, we must prove that the equilibrium allocation satisfies equation (A-3)-(A-8). Clearly, the resource constraints (A-7)-(A-8) are satisfied. In equilibrium, wage is equal to both the marginal product of labor of consumer-producing firms and the marginal rate of substitution between leisure and consumption of households, i.e

$$\begin{aligned}\frac{w}{P^c} &= \frac{1}{1-\alpha}A(D^c)^{\alpha}(T)^{-\alpha}f_n,\\ \frac{w}{P^c} &= \frac{u_N}{(1-\alpha)u_C}.\end{aligned}$$

Combining these two equations implies that the equilibrium allocation satisfies equation (A-3).

Equation (A-4) is also satisfied through the following two conditions in equilibrium,

$$u_c - \frac{u_D}{A(D^c)^{\alpha-1}(T)^{1-\alpha}f} = P^c\ M,$$

$$P^c = (1-\alpha)\frac{u_c}{M}.$$

where M is the expected discounted marginal utility of an additional unit of savings. The first equation is from consumer's optimal choice between consumption and shopping effort. The second equation comes from optimal consumer search.

Similarly, combining consumer's first order condition over labor, firm and consumer's search problem, we can get equation (A-5).

Lastly, we show that the Euler equation for capital, equation (A-6), is satisfied. According to Lemma 6 in the paper,

$$E\left\{\Pi(\theta,\theta',K)P^{i\prime}\left[\frac{\Psi_d(Q^{i\prime})}{Q^{i\prime}}f_{k'}+(1-\delta)\right]\right\} = P^i. \tag{A-9}$$

Substituting the definition for Π and $\Psi_d(Q^i)/Q^i = A(D^i)^{\alpha}(1-T)^{-\alpha}$ into equation (A-9), we have

$$\beta E\left\{\frac{P^c u_{C'}}{P^{c\prime}u_C}P^{i\prime}\left[A'(D^{i\prime})^{\alpha}(1-T')^{-\alpha}f_{k'}+(1-\delta)\right]\right\} = P^i.$$

Reorganizing the above equation, we have

$$\beta E\left\{\frac{P^{i\prime}}{P^{c\prime}}u_{C'}\left[A'(D^{i\prime})^{\alpha}(1-T')^{-\alpha}f_{k'}+(1-\delta)\right]\right\}=\frac{P^{i}}{P^{c}}u_{c}.$$

Recall that

$$\frac{P^{i}}{P^{c}}=\frac{(D^{c})^{\alpha}(T)^{-\alpha}}{(D^{i})^{\alpha}(1-T)^{-\alpha}}.$$

Thus the Euler equation in the competitive search equilibrium can be written as

$$\beta E\left\{u_{C'}\frac{(D^{c\prime})^{\alpha}(T')^{-\alpha}}{(D^{i\prime})^{\alpha}(1-T')^{-\alpha}}\left[A'(D^{i\prime})^{\alpha}(1-T')^{-\alpha}f_{k'}+(1-\delta)\right]\right\}=\frac{(D^{c})^{\alpha}(T)^{-\alpha}}{(D^{i})^{\alpha}(1-T)^{-\alpha}}u_{c},$$

which is exactly the Euler equation from the social planner's problem, equation (A-6).

□

B Main Shocks plus shocks to the MRS in the Shopping and RBC Economies

As a further comparison of the shopping and the RBC economy's performance we estimate versions of both economies where, in addition to the main shock that moves the Solow residual (a shock to both the demand of consumption and investment in the shopping economy and a shock to TFP in the RBC economy), we pose a shock to the MRS that moves the willingness to work. We assume that both the main shocks and the MRS shocks are uncorrelated and, like in Section 5.3, we estimate the relative size of the demand shocks to consumption and investment (they are perfectly correlated). The estimations for both models use Bayesian methods over the Solow residual and output data. We assume that the autocorrelations follow Beta distributions and that innovations of the shocks follow Inverse-Gamma distributions. The priors, posteriors and the estimated parameters are reported in Table A-1. The log likelihood for the shopping model is 1492, significantly higher than that in the standard RBC model of 1484. The 90% intervals are tight, indicating significant estimated parameters. All shocks are highly persistent. The volatility of the demand shocks is much larger than that of the technology shock of the RBC model, and the volatility and persistence of the shock to the MRS is a bit lower. Table A-1 also reports the variance decomposition over the major business cycle variables. We see that the demand shocks play a more central role in the shopping model than the TFP shock in the RBC model: the variance of the MRS shock is lower in the shopping model and the contribution of the demand shock to the variance of the endogenous variables is larger. The last panel of Table A-1 presents the business cycle statistics for the major variables, for both the shopping model and the standard RBC model. They are very similar, with the volatility of hours being slightly higher in the RBC model.

C Computational Details

Data We used 7 raw data series in the paper, GDP, consumption, investment, labor, capacity utilization, stock market price, and relative price of investment. The data series of GDP, consumption, investment, and labor are from the Bureau of Economic Analysis (BEA) for the period of 1948:Q1-2009:Q4. The data series of capacity utilization is the Industrial Production and Capacity Utilization published by the Federal Reserve Board. The series is from 1967:Q1 to 2009Q4 and is based on estimates of capacity utilization for industries in manufacturing, mining, and electric and gas utilities. For a given industry, the capacity utilization rate is equal to an output index (seasonally adjusted) divided by a capacity index. The raw data of S&P 500 stock market price is a monthly data series compiled by Robert Shiller based on the daily price of S&P 500 stock market price, available at *http://www.econ.yale.edu/∼shiller/data.htm*. Our quarterly data of stock price is the monthly average of the raw data in each quarter. Lastly, the relative price of investment is from 1948Q1 to 2009Q4, constructed in Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulalia-Llopis (2009). We detrend all data series with Hodrick-Prescott filter.

Computation. The model is estimated using Dynare, which adopts Metropolis-Hastings algorithm for the Bayesian estimation. We put into Dynare 12 endogenous variables $\{C$, I, N^y N, T,

Table A-1: Comparison of Shopping Model and RBC Model: Main Shocks plus Shocks to the MRS

Priors and Posteriors for the Shock Parameters

Shopping model (likelihood = 1499)

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	**0.926**	[0.900, 0.952]
σ_d	Inverse Gamma	0.20	0.20	**0.061**	[0.053, 0.068]
σ_ζ/σ_d	Normal	0.50	0.50	**1.861**	[1.131, 2.320]
ρ_n	Beta	0.95	0.05	**0.982**	[0.969, 0.999]
σ_n	Inverse Gamma	0.06	0.20	**0.019**	[0.017, 0.020]

RBC model (likelihood = 1484)

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_z	Beta	0.93	0.05	**0.952**	[0.925, 0.981]
σ_z	Inverse Gamma	0.006	0.20	**0.006**	[0.0055, 0.0065]
ρ_n	Beta	0.94	0.05	**0.993**	[0.981,1.000]
σ_n	Inverse Gamma	0.02	0.20	**0.021**	[0.018, 0.023]

Variance Decomposition (%)

	Shopping model		RBC model	
	θ_d, θ_ζ	θ_n	z	θ_n
Y	74.62	25.38	63.13	36.87
Solow	98.47	1.53	100.00	0.00
N	6.17	93.83	3.08	96.92
C	44.36	55.64	19.71	80.29
I	89.19	10.81	86.02	13.98

Business Cycle Statistics

	Shopping model		RBC model	
	Variance	Cor w Y	Variance	Cor w Y
Y	1.23	1.00	1.27	1.00
Solow	2.45	0.78	4.19	0.79
N	0.94	0.63	1.09	0.73
C	0.26	0.82	0.31	0.88
I	16.34	0.96	14.58	0.96

D^c, D^i, P^c, P^i, R, $w\}$ with the following 12 equations.

$$
\begin{aligned}
P^i \frac{C^{-\sigma}}{P^c} &= \beta E \left\{ P^{i\prime} \frac{(C')^{-\sigma}}{P^{c\prime}} \left[\gamma_k \frac{P^{c\prime} C'}{P^{i\prime} K' T'} + (1-\delta) \right] \right\}, \\
\frac{C^{-\sigma}}{P^c} &= \beta E \left\{ \frac{(1+R')(C')^{-\sigma}}{P^{c\prime}} \right\}, \\
(1-\alpha) \frac{w}{P^c} &= \chi \theta_n N^{\frac{1}{\nu}} C^{\sigma}, \\
(1-\alpha) \frac{w}{P^c} &= \gamma_n \frac{C}{N^y T}, \\
\theta_d D^c &= \alpha C^{1-\sigma}, \\
P^c (D^c)^{\alpha} T^{-\alpha} &= P^i (D^i)^{\alpha} (1-T)^{-\alpha}, \\
P^i &= \frac{1-\alpha}{\alpha} \frac{w D^i}{\zeta I}, \\
N &= N^y + D^i/\zeta, \\
C &= A (D^c)^{\alpha} (T)^{1-\alpha} z K^{\gamma_k} (N^y)^{\gamma_n}, \\
I &= A (D^i)^{\alpha} (1-T)^{1-\alpha} z K^{\gamma_k} (N^y)^{\gamma_n}, \\
I &= K' - (1-\delta) K, \\
R &= P^c C - wN.
\end{aligned}
$$

D Additional Tables

Table A-2 presents the calibration for the Standard RBC model. The intertemporal elasticity of substitution σ is set to two, and the real rate of return is 4.%. We choose the Frisch elasticity as .72. We calibrate depreciation rate to match the observed consumption to output ratio of 0.8. Labor share is 0.67 in the data, which implies $\gamma_n = 0.67$. The disutility parameter χ is calibrated to match the average time spent at working of 30%. We normalize the mean of productivity shock such that aggregate output is 1 at steady state.

Table A-2: Calibration for the Standard RBC model

Targets	Value	Parameter	Value
Risk aversion	2.	σ	2.
Real interest rate	4.%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\nu}$	0.72
Fraction of time spent working	30%	χ	18.49
Consumption Share of Output	0.80	δ	0.06
Labor Share of income	0.67	γ_n	0.67
Units of output	1.	z	0.94

Table A-3: Perfectly Correlated Demand Shocks in Shopping Economy and TFP shock in RBC Economy with Frisch =1.1

Priors and Posteriors for the Shock Parameters

Shopping model with (θ_d, ζ), likelihood = 733.43

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	0.979	[0.962, 0.995]
σ_d	Inv. Gamma	0.05	0.20	0.049	[0.036, 0.061]
σ_ζ/σ_d	Normal	2.60	0.90	2.860	[1.414, 4.328]

RBC model with z, likelihood = 734.8

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_z	Beta	0.95	0.05	0.948	[0.916,0.984]
σ_z	Inv. Gamma	0.006	0.20	0.006	[0.0055, 0.0065]

Main Business Cycle Statistics

	Variance			Correlation with Y		
	Data	(θ_d, ζ)	RBC	Data	(θ_d, ζ)	RBC
Solow	3.19	3.51	3.59	0.43	0.98	1.00
Y	2.38	0.93	0.92	1.00	1.00	1.00
N	2.50	0.10	0.08	0.87	0.61	0.96
C	1.55	0.09	0.05	0.87	0.73	0.95
I	34.15	13.46	15.40	0.92	0.98	1.00

Table A-4: Sensitivity Analysis over the Correlation between the Consumption and Investment Demand Shocks

	Correlation			
	-0.2	-0.1	0	0.1
		Estimates		
Likelihood	2244.02	2244.87	2244.48	2244.52
ρ_d	0.958	0.956	0.954	0.958
σ_d	0.076	0.074	0.072	0.067
ρ_n	0.997	0.997	0.996	0.996
σ_n	0.022	0.022	0.022	0.022
ρ_ζ	0.958	0.956	0.954	0.954
σ_ζ	0.133	0.131	0.126	0.126
ρ_z	0.924	0.918	0.886	0.888
σ_z	0.001	0.001	0.002	0.004
		Variance decomposition for output		
θ_d	36.63	32.30	27.77	15.06
ζ	28.76	30.75	31.97	25.97
z	0.65	0.74	2.18	22.89
θ_n	33.96	36.21	38.08	36.08
		Variance decomposition for the Solow residual		
θ_d	83.65	82.43	79.89	52.23
ζ	13.44	14.35	14.82	12.60
z	1.07	1.23	3.18	33.18
θ_n	1.84	1.99	2.10	1.99