Labor Supply in a Frictional Labor Market*

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Abstract

We develop a model featuring search frictions and a nondegenerate labor supply decision along the extensive margin, and argue that it does a reasonable job of matching labor market flows between employment, unemployment and out of the labor force. Persistent idiosyncratic productivity shocks play a key role in allowing the model to match the persistence of the employment and out of the labor force states found in individual labor market histories. We then use this model to address two questions: how do taxes affect aggregate employment and how do changes in frictions affect aggregate employment. We find that the presence of empirically plausible frictions has virtually no impact on the response of aggregate employment to taxes. The labor supply response present in our model greatly attenuates the employment response to frictions relative to the simplest matching model.

Keywords: Labor Supply, Labor Market Frictions, Taxes

JEL Classifications: E24, J22, J64

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1 Introduction

Analyses of aggregate employment are dominated by two frameworks. One is the frictionless version of the standard growth model with an endogenous labor leisure choice, as in Kydland and Prescott (1982), but modified as in Hansen (1985) to include the indivisible labor formulation of Rogerson (1988). The other is the class of matching models à la Diamond-Mortensen-Pissarides, as described in Pissarides (2000). Loosely speaking, the former can be viewed as a model of labor force participation, while the latter can be viewed as a model of unemployment conditional on a participation rate. Cross country data reveal that there are significant differences across countries along all three margins: employment, unemployment and non-participation. Moreover, it seems reasonable to think that participation rates, employment rates and unemployment rates are all jointly determined, in the sense that any policy that affects one margin is likely to affect both of the other two margins. Additionally, resolving the debate between Prescott (2004) and Ljungqvist and Sargent (2006, 2008) regarding the role of benefits versus taxes in accounting for cross country differences in hours of work requires a model that can capture the differences in benefit levels available to the nonemployed depending upon their history of labor market participation and employment. This suggests that a comprehensive model of the aggregate labor market should explicitly incorporate all three labor market states.

The goal of this paper is to develop a unified model of participation, unemployment and employment. The model is a hybrid of the two models discussed above, extended to
allow for idiosyncratic productivity shocks. Abstracting from labor market frictions, an
individual in our model solves a textbook problem of labor supply in a dynamic setting with
indivisible labor. That is, the individual must decide what fraction of his or her life to spend
in employment, and how to arrange the timing of employment spells. An individual in our
model faces frictions just like a worker in the textbook Pissarides model: when employed
the individual faces a probability of becoming non-employed, and when not employed, the
individual finds an employment opportunity only with some probability. We show that
a reasonably calibrated version of our model captures the distribution of workers across
the three states as well as the salient features of worker flows across these three states.
Idiosyncratic productivity shocks play a critical role in allowing the model to match the
patterns found in the worker flow data. Without idiosyncratic shocks the model is the same
as that studied in Krusell et al (2008), and we show that such a model is unable to match
both the flows and the distribution of workers across states.

Having developed a quantitative general equilibrium model of steady state labor market
dynamics, we then use it to address several questions. First, we use the model to address
the question posed by Prescott (2004), of whether hours of work in Europe relative to the
US can be accounted for by differences in the scale of transfer payments that are financed
by taxes on labor. Prescott addressed this in a model that abstracted from trading frictions.
Our calibrated model predicts that the employment effects are effectively unchanged by
the presence of frictions. However, in our model the presence of frictions also implies that
higher taxes will lead to both higher unemployment as well as higher non-participation. So, although the aggregate effects on employment in our model are effectively those found in the frictionless version of the model, the analysis shows that this does not imply that there are not also effects on both the level and nature of unemployment.

The second issue that we address is the impact of changes in frictions. In the simplest Pissarides style matching model, the level of frictions critically affects the level of both aggregate employment and unemployment. We assess the extent to which this effect is altered by placing the frictional model in a context where individuals also solve a nondegenerate labor supply problem. Intuitively, if employment opportunities are harder to come by, then individuals can adjust lifetime labor supply by extending the length of employment spells when they receive an employment opportunity, or by accepting more employment opportunities when productivity is low. In our calibrated model we find that the labor supply response greatly attenuates the direct effect of frictions on employment levels. Specifically, we find that a decrease in the arrival rate of employment opportunities leads to a large increase in the unemployment rate but only a small decrease in the employment rate. The increase in the unemployment rate in our model is very similar to that implied by a simple matching model. We conclude that the role of changes in frictions for aggregate employment (as opposed to unemployment) is probably overstated in simple matching models.

Our analysis is related to many papers in the literature. In addition to the work cited above, our paper is similar to Merz (1995), Andolfatto (1996), Alvarez and Veracierto (1999),
Gomes et al (2001) and Veracierto (2008) in that these papers all introduce frictions into an otherwise standard version of the growth model. Merz, Andolfatto, and Gomes et al do not consider unemployment and nonparticipation as distinct states. The other two papers do consider all three labor market states but their model only puts restrictions on the stocks of workers in the three states and does not pin down labor market flows. A key distinction between our model and all of these analyses is that these models all have the property that if frictions were removed, the employment rate would be equal to one, with all labor supply decision occurring only along the intensive margin. Beginning with Burdett et al (1984), there are also several papers that have extended the simple matching model to allow for nonparticipation.\footnote{Other examples include Andolfatto and Gomme (1996), Andolfatto et al (1998), Kim (2001), Yip (2003), Garibaldi and Wasmer (2005), and Pries and Rogerson (2008).} These models assume linear utility and therefore implicitly impose assumptions on the income and substitution effects that govern labor supply that are not consistent with standard specifications of labor supply. Additionally, they cannot address issues in which risk sharing plays a key role. Lastly, our work is related to a recent literature that studies labor supply in settings with incomplete markets and idiosyncratic shocks, including papers by Domeij and Floden (2005), Floden and Linde (2001), Chang and Kim (2006, 2007) and Pijoan-Mas (2006). None of these papers allows for trading frictions. Similar to us, Meghir et al (2008) consider a model with frictions and a nondegenerate labor supply decision. They consider a richer model of frictions and income support programs, but their analysis is partial equilibrium. Moreover, the papers address very different issues.
An outline of the paper follows. Section 2 describes the model. Section 3 describes the calibration of the model and presents the implications of the calibrated model for labor market flows. Section 4 considers the role of idiosyncratic shocks and how they are important in order to produce reasonable properties for labor market flows. Section 5 presents the results for analyzing tax and transfer programs. Section 6 analyzes the effects of changes in frictions on steady state outcomes. Section 7 concludes.

2 Model

The economy is populated by a continuum of workers with total mass equal to one. All workers have identical preferences over streams of consumption and time devoted to work given by:

$$\sum_{t=0}^{\infty} \beta^t [\log(c_t) - \alpha e_t]$$

where $c_t \geq 0$ is consumption in period $t$, and $e_t \in \{0, 1\}$ is time devoted to work in period $t$. The restriction that $e_t$ is either zero or one reflects the assumption that labor is indivisible, so that all adjustment occurs along the extensive margin. Workers are subject to idiosyncratic shocks that affect the quantity of labor services that they contribute if working. We denote this value by $p$ and assume that it follows a stochastic process with transition function $Q(p, p')$. This process is the same for all workers, but realizations are iid across workers. We assume that $Q$ displays first order stochastic dominance with regard to its first argument, i.e., that if $p_1 > p_2$ then the distribution $Q(p_1, \cdot)$ first order stochastically dominates the
distribution $Q(p_2, \cdot)$. 

We formulate equilibrium recursively. In each period there are markets for output, capital services and labor services. In particular, there are no insurance markets, so individuals will potentially accumulate assets to self-insure. In what follows we will focus on steady state equilibria, so that factor prices will be constant. We normalize the price of output to equal one in all periods, let $r$ denote the rental price for a unit of capital, and let $w$ denote the rental price for a unit of labor services. If a worker with productivity $p$ chooses to work then he or she would contribute $p$ units of labor services and therefore earn $wp$ in labor income. We assume that individuals are not allowed to borrow, which is equivalent to assuming that capital holdings must be nonnegative. There is a government that taxes labor income at constant rate $\tau$ and uses the proceeds to finance a lump-sum transfer payment $T$ subject to a period-by-period balanced budget constraint. In steady state, the period budget equation for an individual with $k_t$ units of capital and productivity $p_t$ is given by:

$$c_t + k_{t+1} = rk_t + (1 - \tau)wp_te_t + (1 - \delta)k_t + T.$$ 

The production technology is described by a Cobb-Douglas aggregate production function:

$$Y_t = K_t^\theta L_t^{1-\theta}.$$ 

$K_t$ is aggregate input of capital services and $L_t$ is aggregate input of labor services:

$$K_t = \int k_{it}di$$

$$L_t = \int e_{it}p_{it}di.$$
Output can be used either as consumption or investment, and capital depreciates at rate $\delta$.

We let $E_t$ represent aggregate employment:

$$E_t = \int e_t \, di$$

and let $P_t$ represent the average productivity of employed workers, i.e.,

$$P_t = \frac{\int e_t p_i di}{E_t}.$$

It follows that $L_t = P_t E_t$.

To capture frictions in the labor market, we assume that there are two islands, which we refer to as the production island and the leisure island. At the end of period $t - 1$ an individual is either on the production island or the leisure island, depending upon whether they worked during the period. That is, a worker who worked in period $t - 1$ will be on the production island as of the end of period $t - 1$, and an individual who did not work in period $t - 1$ will be on the leisure island as of the end of period $t - 1$. At the beginning of period $t$ each individual will observe the realizations of several shocks. First, each worker receives a new realization for the value of their idiosyncratic productivity shock. Second, each individual on the production island observes the realization of an \textit{iid} separation shock: with probability $\sigma$ the individual is relocated to the leisure island. Third, each individual on the leisure island, including those that have been relocated on account of the separation shock, observes the realization of an \textit{iid} employment opportunity shock: with probability $\lambda_w$ an individual is relocated to the production island. Loosely speaking, $\sigma$ is the exogenous job
separation rate, and $\lambda_w$ is the exogenous job arrival rate. After all of these shocks have been
realized, each worker on the production island decides whether to work and how much to
consume. An individual who is on the production island and chooses not to work will then
be on the leisure island at the end of period $t$ and will therefore not have the opportunity to
return to the production island until receiving a favorable employment opportunity shock.
An individual who is on the leisure island after the realization of all of the shocks is not
allowed to supply labor, so his or her only choice is how much to consume. Note that this
individual still has two sources of income: income from renting out capital services and the
transfer payment from the government. This individual will be on the leisure island at the
end of period $t$.

A worker’s state consists of his or her location at the time that the labor supply decision
needs to be made, the level of asset holdings, and productivity. Let $W(k, p)$ be the maximum
value for an individual who works given that they have productivity $p$ and capital holdings
$k$, and let $N(k, p)$ denote the maximum value for an individual who does not work given that
he or she has productivity $p$ and capital holdings $k$. Define $V(k, p)$ by:

$$
V(k, p) = \max\{W(k, p), N(k, p)\}
$$

The Bellman equations for $W$ and $N$ are given by:

$$
W(k, p) = \max_{c, k'} \{\log(c) - \alpha + \beta E_p\left[(1 - \sigma + \sigma \lambda_w) V(k', p') + \sigma (1 - \lambda_w) N(k', p')\right]\}
$$
\[
s.t. \ c + k' = rk + (1 - \tau)wp + (1 - \delta)k + T
\]

\[
c \geq 0, \ k' \geq 0
\]

and

\[
N(k,p) = \max_{c,k'}\{\log(c) + \beta E_p\left[\lambda_w V(k',p') + (1 - \lambda_w)N(k',p')\right]\}
\]

\[
s.t. \ c + k' = rk + (1 - \delta)k + T
\]

\[
c \geq 0, \ k' \geq 0
\]

Let \(\mu(k,p,l)\) denote the measure of individuals over individual states after all of the idiosyncratic shocks have been realized and before any decisions have been taken, where \(l\) indexes location and can take on the two values 0 and 1, with \(l = 1\) indicating the production island. There are three decision rules: one for \(c\), one for \(k'\), and one for \(e\) (which can only take on the values of 0 or 1).

2.1 Properties of Decision Rules

It is useful to discuss some features of the decision rules in order to gain some intuition about the forces that shape individual choices in the steady state equilibrium. It is trivial to show that the value functions are increasing in both assets and the level of the idiosyncratic
productivity shock. One can then show that the decision rules have some simple reservation properties. Specifically, for a given level of assets, it turns out that the work decision for an individual on the production island is characterized by a reservation rule in terms of the idiosyncratic productivity: work if productivity is above some threshold \( p^*(k) \). Similarly, one can show that for a given productivity level, the work decision for an individual on the production island is also characterized by a reservation rule in terms of assets: work if assets are below some threshold \( k^*(p) \). It also follows that \( p^*(k) \) and \( k^*(p) \) are both increasing functions. The fact that \( p^*(k) \) is increasing reflects the fact that higher assets lead to a positive wealth effect, effectively lowering labor supply. The fact that \( k^*(p) \) is increasing reflects intertemporal substitution effects of optimal labor supply: an individual wants to work when productivity is high and enjoy leisure when productivity is low.

### 2.2 Measurement

In this section we describe how we will connect our model with the data on labor market flows. Statistical agencies such as the BLS divide the noninstitutionalized population into three categories: employed, unemployed and not in the labor force, and collect data that allows one to compute the monthly rate at which individuals move from among these categories. A recent literature has questioned whether the rules by which statistical agencies allocate individuals between these three states is the most useful from the perspective of characterizing economic behavior. Using data from Canada, Jones and Riddell (1999) showed that the active search criterion used by many statistical agencies to determine the allocation
of workers between the unemployed state and the out of the labor force state is potentially misleading because it excludes a group of workers (whom they call marginally attached) who say that they would like a job but have not actively searched in the last four weeks. These workers do have somewhat lower transition rates into employment than do active searchers, by about twenty-five percent. Moreover, the job finding rates for the marginally attached workers are the same as those of active searchers who report reading job ads or visiting a public employment office as their sole method of active search. In contrast, the marginally attached workers have transition rates into employment that are more than 4 times as high as the other non-participants. We conclude from this that the marginally attached workers (i.e., passive searchers) are more similar to unemployed workers than they are to nonparticipants. As further evidence of the blurred distinction between the marginally attached and the unemployed, the group of passive searchers has a rate of transition into either unemployment or employment that is almost an order of magnitude large than the corresponding flow for the other individuals in the out of the labor force category. These same findings have emerged when this analysis has been repeated for many other countries.\footnote{For example, see Brandolini, Cipoline and Viviano (2006) for an analysis of several European countries, Garrido and Toharia (2004) for Spain, Gray, Heath and Hunter (2005) for Australia and Marzano (2006) for the United Kingdom.} A similar point was made much earlier by Clark and Summers (1979), who argued that what the statistical agencies view as a series of short unemployment spells punctuated by spells of nonparticipation are in fact better interpreted as a long spell of unemployment where the individual’s reported search
effort varies over time.\textsuperscript{3} This may occur, for example, when an individual has already applied to many jobs and is simply awaiting responses. It may also reflect the fact that an individual is expecting an offer from earlier search activity and is therefore not currently engaged in active search, even though they are actively pursuing particular employment opportunities.

The issues raised by this literature lead us to conclude that a broader definition of unemployment is preferable to that used by statistical agencies when evaluating our model. This has two key implications for measurement. The first is that the size of the unemployment pool is increased, since we are including those individuals who say that they would like a job but have not actively searched in the last four weeks. The second, and more important implication is that this affects the level of flows between the three states. In particular, the relatively large flows in the data between unemployment and marginally attached that were documented by Jones and Riddell will be measured as flows between unemployment and not in the labor force by conventional methods, but will be viewed as flows from unemployment to unemployment by us.

To summarize, the approach that we take is to define the unemployment state as consisting of those individuals who would like to work at the existing market wage rate but do not have the opportunity. This is easy to operationalize in the context of our model, and also captures what we believe is the essence of the unemployment concept: an individual that would like to work at the market wage rate but is not able to find employment. In terms of connecting to

\textsuperscript{3}Abowd and Zellner (1985) and Poterba and Summers (1986) have devised methods to purge the data of truly spurious transitions between unemployment and not in the labor force.
the data, this means that we will be including passive searchers in our unemployment pool.

Data gathered by the BLS does allow us to compute the counterpart to our notion of unemployment in the data. That is, the BLS does ask people if they would like to work even if they did not engage in active search in the previous four weeks. In what follows, we will use this notion to compute the unemployment rate in the data. For the period 1994-2008, which is the period for which consistent data is available, the standard unemployment rate for the US averages 5.1%, whereas our expanded notion of unemployment averages 8.3%. One problem that we face is that flow rates for the US data based on our measure of unemployment are not readily available. However, the evidence in Jones and Riddell (1999, 2006) does allow one to make some estimates of the extent to which the flow rates would be affected.4 Table 1 shows the results of applying these corrections.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>US 1994-2008</th>
<th>Adjusted</th>
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</thead>
<tbody>
<tr>
<td>FROM FROM TO</td>
<td>F</td>
<td>E U N</td>
</tr>
<tr>
<td>E</td>
<td>0.96</td>
<td>0.01</td>
</tr>
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</tr>
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<td>E</td>
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<td>0.01</td>
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<tr>
<td>U</td>
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<td>0.50</td>
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<tr>
<td>N</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note that the only significant effects is to flows out of the unemployment state. This is because the size of the passive searcher group is small relative to the other non-participants.

While the adjustments are based on Canadian data and should be interpreted cautiously, the

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4Based on the evidence in Jones and Riddell (2006) we compute that the effect of the change in definitions has the following effects on flow rates: the flow from U to E is decreased by 10%, the flow from U to N is increased by 5%, the flow from N to E is reduced by 10% and the flow from N to U is decreased by 10%. We note that these adjustments are for Canadian data, and so should be interpreted cautiously.
key point that we want to stress is that the adjustments are relatively minor.

A final issue that we want to address in this subsection has to do with the cost of active search. The relatively new American Time Use Survey that is conducted as part of the Current Population Survey reveals that active searchers devote very little time to search, typically less than one hour per week. This suggests that the cost of active search is best thought of as being quite small. If the cost of active search is actually very small, and given the dynamic nature of search, the decision of whether to engage in active search at a given point in time may not be very meaningful. This also suggests that allocating individuals to various states on the basis of active search may not be prudent. Consistent with the evidence that the cost of active search is very small, our model has not placed any emphasis on search costs and likewise our definition of unemployment also does not stress search effort.\textsuperscript{5}

3 Calibration

In this section we describe the procedure that we use to calibrate the model and consider how well the calibrated model performs along some dimensions that are not targeted as part of the calibration. The only functional form that we did not yet specify was the one that describes the stochastic process for the idiosyncratic productivity shocks. Motivated by empirical work on individual wage dynamics, we assume that this process is described by an AR(1) process.

\textsuperscript{5}However, if we assumed that the search effort decision is a binary decision, and that the cost of search is positive but arbitrarily small, then it would follow that all individuals who prefer working to nonworking given their current state would engage in active search. In this sense one could connect our model with the active search criterion used by the BLS.
on log of productivity:

\[ \log p_{t+1} = \rho \log p_t + \varepsilon_t \]

where \( \varepsilon_t \) is a normally distributed random variable with mean 0 and standard deviation \( \sigma_e \).\(^6\)

The model has nine parameters that need to be assigned: preference parameters \( \beta \) and \( \alpha \), production parameters \( \theta \) and \( \delta \), idiosyncratic shock parameters \( \rho \) and \( \sigma_e \), frictional parameters \( \sigma \) and \( \lambda_w \), and the tax rate \( \tau \). The length of a period will matter for many of the parameter values. We will be interested in the behavior of worker flows between labor market states and since this data is available at a monthly frequency, we set the length of a period equal to one month. Because our model is a variation of the standard growth model, we can choose some of these parameter values using the same procedure that is typically used to calibrate versions of the growth model. Given that our model assumes incomplete markets and uncertainty, our steady state cannot be represented analytically in the same fashion as the standard growth model, and in particular one cannot isolate the connection between certain parameters and target values. Nonetheless, it is still useful and intuitive to associate particular targets and parameter values. Specifically, we set \( \theta = .3 \) to target a capital share of .3, choose \( \delta \) so that the steady state ratio of investment to output is equal to .2, and choose the discount factor \( \beta \) to target an annual real rate of return on capital equal to 4%. The other preference parameter \( \alpha \), which captures the disutility of working, is set so that the steady state value of employment is equal to .632. This is the value of the employment to population ratio for

\(^6\)The mean of the \( \varepsilon \) shock affects the mean of the invariant distribution for productivity values and therefore is effectively just a choice of units for measuring output.
the population aged 16 and older for the period 1994 – 2008.\textsuperscript{7}

The tax rate is set at $\tau = .30$. Following the work of Mendoza et al (1994) there are several papers which produce estimates of the average effective tax rate on labor income across countries. Examples include Prescott (2004) and McDaniel (2006). There are minor variations in methods across these studies, which do produce some small differences in the estimates, and the value .30 is chosen as representative of these estimates.\textsuperscript{8}

We calibrate the idiosyncratic productivity process based on estimates of this process that are available in the data. In the steady state of our model, log hourly wages are simply a constant plus log productivity, so estimates of the stochastic process for log wages yield estimates for the productivity process. Estimates for employed prime aged male workers have been provided by many researchers, including Card (1994), Floden and Linde (2001), French (2005) and Chang and Kim (2006). Estimates for much wider samples are not available. Though the estimates for a select sample of the population may not carry over to a wider population, we will nonetheless base our calibration on these estimates, and will conduct sensitivity analysis to explore how robust our findings are to the specific values that we use. Even for a fixed sample of individuals, there is still a range of estimates available due to different assumptions about which observables to control for in estimating the idiosyncratic component of the wage process. The common finding is that the idiosyncratic process is very

\textsuperscript{7}We calibrate to values for the period 1994-2008 because this is the period for which we have consistent measures of labor market flows.

\textsuperscript{8}Note that Prescott (2004) makes an adjustment to the average labor tax rate to arrive at a marginal tax rate that is roughly 40%. For purposes of computing the effect of changes in taxes this adjustment plays no role.
persistent, with estimates of $\rho$ ranging from .85 to .97 for annual data. For our benchmark calibration we use the estimates from Floden and Linde, which are $\rho = .92$ and $\sigma_e = .21$, which we convert into a monthly process.\footnote{In solving for equilibrium we approximate the AR(1) process by using 20 grids between $-2\sigma_e \sqrt{1 - \rho^2}$ and $2\sigma_e \sqrt{1 - \rho^2}$ with Tauchen (1986) method.}

The two remaining parameters are the two values that describe the frictions in the model. To understand the role of these frictions, we note that $\lambda_w$ is a key parameter in generating unemployment in our model. Specifically, if $\lambda_w = 1$, then everyone always has the opportunity to work, and as a result there will be no workers unemployed according to the definition that we are using. As $\lambda_w$ is lowered from one there will be more workers in the situation of wanting to work but not having the opportunity. Motivated by this, we choose $\lambda_w$ so that the steady state unemployment rate in our model (i.e., (number unemployed)/(number employed plus number unemployed)) is equal to .083, which is the average value for our notion of the unemployment rate in the US data for the period 1994 – 2008.

The only remaining parameter is $\sigma$. Intuitively, this parameter will play a large role in shaping the transitions out of employment, though the two are not identical since workers may also transition out of employment due to choice. However, individuals who transition out of employment for reasons other than the separation shock will transition to out of the labor force, while workers who transition out of employment due to the separation shock will typically transition to unemployment.\footnote{In a discrete time model this is not exactly true, since a worker can experience both a separation shock and a bad idiosynratic productivity shock in the same period, in which case they would transition to out of the labor force.} This suggests that the flow from employment
to unemployment is a good target for pinning down the value of $\sigma$. One problem with this is that we do not have flow data from E to U that match our notion of unemployment. (The data available from BLS do allow us to compute the stocks that correspond to our notion, but tabulated data on flows do not permit us to redo the calculations for alternative groupings.) As a practical matter this mismatch in concepts might be expected to be not so important, in that workers who have only been out of employment for less than one month are perhaps less likely to report wanting a job but not engaging in search. Given the mismatch in unemployment concepts we will explore how sensitive our results are to this target, but for our benchmark case we target the flow rate from employment to unemployment of .0139, which is again the average value for this flow for the period 1994 – 2008. Note that the value of $\sigma$ will be larger than this value for two reasons. First, because we allow a separated worker to receive a new employment opportunity in the same period as the separation and therefore not experience a period of nonemployment. Second, as noted earlier, a worker may experience both a separation shock and obtain a sufficiently low value of the productivity shock that they transition from E to N rather than from E to U.

Table 2 shows our calibrated parameter values.

<table>
<thead>
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<th>$\theta$</th>
<th>$\delta$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\rho$</th>
<th>$\sigma_e$</th>
<th>$\lambda_w$</th>
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<td>.1017</td>
<td>.364</td>
<td>.0225</td>
<td>.30</td>
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</table>
3.1 Properties of the Steady State

The only flow rate that we targeted in the calibration was the flow from E into U. It is of interest to examine the predictions of the calibrated model for the other flows as well, though in doing this one must keep in mind the previously noted issue that the concepts of U and N in the model and the CPS flow data do not perfectly coincide. In any case, Table 3 shows the flow rates from the data and from our calibrated model.

Table 3
Flows in the Model and Data

<table>
<thead>
<tr>
<th>FROM</th>
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</tbody>
</table>

Given that flows out of state must sum to one, there are really six independent values in the data. Our calibration procedure targeted the flow rate from E to U, leaving five independent flows. Two key features of the data are the very high persistence for both the E and N states. Our calibrated model not only generates a lot of persistence in these two states, but also matches the corresponding values in the data very well. The model generated flows are similar to those found in the data for several of the other cells as well, such as the N to E flow, the E to N flow and the N to U flow.

11 Given that we do target the shares of E, U and N in the data we do implicitly impose some additional restrictions on the flows. But when we consider the case of no productivity shocks we will see that this by itself does not generate a close match for the flows.
There are three flows for which there seems to be a sizable discrepancy between the model and the data, all of which have to do with the flows out of U: the model predicts that the U to U flow is too large, that the U to N flow is too low and that the U to E flow is too large. The relatively large flow between U and N in the data relative to the model can be interpreted in many ways. It is useful to note that this flow is dominated by transitions that are reversed in the following period. Specifically, based on the analysis in Jones and Riddell (2006), one finds that the transition rate from U to N at a five month horizon is only a fraction of what would be predicted on the basis of the monthly numbers. This implies that these flows are transitory in nature. One possibility is that this reflects measurement error, since those individuals nearly indifferent between working and not working might be expected to have noisy answers to the question of whether they would like to work even though they are not working. A second possibility is that there is a transitory shock which induces a lot of high frequency transitions between the two states. While we view both of these as plausible possibilities, we do not pursue either in this paper. Whether these high frequency transitions represent true transitions or measurement errors, in either case they basically reflects transitions for individuals near an indifference boundary, and in view of this we suspect that they are not of first order importance.

To summarize, while the model does not perfectly replicate the flows found in the data, we conclude that this model with a single source of heterogeneity and constant frictions does a reasonable job of accounting for the flow rates observed in the data. Given that our
benchmark specification induces flows from U to E that are somewhat too large, we will also consider results for a lower value of $\lambda_w$.

Another way to represent some of the above information is to report durations of spells for each of the three states. Measured in months, the duration of E, U, and N spells in the model are 22.5, 2.4 and 13.4 respectively. Based on the adjusted data, these values in the data are given by 22.5, 2.0 and 14.3. These values are close.

We also report some additional statistics on flows that are of potential interest. Specifically, in our model we can isolate different reasons for a worker to transition out of employment. Specifically, there are three different reasons why a worker might transition out of employment. One is that the worker might receive a lower value of the idiosyncratic shock and choose not to work. A second possibility is that the worker receives a separation shock and does not simultaneously receive a new employment opportunity shock. And third, it is possible that a worker does not receive any shocks but still chooses to transition out of employment. This last possibility would result if the worker were to accumulate assets beyond the reservation value $k^*(p)$ defined earlier. As a practical matter it is possible that a worker receives multiple shocks, thereby complicating this labelling process. We label the reasons for leaving employment as follows. Conditional on observing a worker that transitions from employment to either U or N, we first ask whether the worker has the opportunity to work. If the answer is no, and the worker would have worked given the current realization of $p$ and their current assets, then we say that the reason for the transition of out E is a separation
shock. If the individual did have the opportunity to work, then we ask if the individual would have worked this period if instead of their current realization of $p$ they instead had the same value for $p$ this period as last period, i.e., would they have worked if there had been no change in the value of their productivity. If they answer yes to this, then we say that the reason for the transition out of employment is due to the productivity shock. The residual is what we label "other". The results are shown in Table 4.

<table>
<thead>
<tr>
<th>Reason for Transitions out of E</th>
<th>$\sigma$ Shock</th>
<th>$p$ Shock</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.33</td>
<td>.55</td>
<td>.12</td>
</tr>
</tbody>
</table>

Note that productivity shocks are the dominant source of transitions out of E. This helps to explain why the model is successful in replicating the high degree of persistence in employment and non-participation. We will see later when we examine the properties of a model without productivity shocks that transitions due to asset accumulation produce far too many movements between E and N, so that persistent productivity shocks play a key role in generating enough persistence in the E and N states. A related statistic concerns reasons for the flow into unemployment. In our model there are two flows into unemployment: one from employment and the other from not in the labor force. Those that flow from employment into unemployment are necessarily individuals who were hit by the $\sigma$-shock. In this sense it is natural to label them as job-losers. In steady state, 38% of the new flows into unemployment represent job losers.
Both of these last two statistics can be compared to measures in the data. Over the period 1994-2008 the fraction of new flows into unemployment accounted for by job losers is around 49%, so our model is a bit low on this measure. In the JOLTS data set, which covers 2000-2008, 38% of all separations are labelled as involuntary. Some care needs to be taken in comparing this statistic to our model since we do not have any job to job flows in our model, and in reality many voluntary separations are associated with job to job flows. Additionally, the work of Davis et al (2008) shows that the JOLTS figures tend to underestimate involuntary separations due to sampling issues. Their revised numbers suggest that the fraction of all separations that are involuntary is closer to 45%.

3.2 Calibrating the Frictions: A Closer Look

Except for the frictions, our model represents a fairly standard growth model extended to allow for idiosyncratic productivity shocks that are not insurable except through self-insurance. Because these models are very standard in the literature we have a good understanding of what role the various parameters play in shaping the properties of the steady state equilibrium. It is somewhat less clear what role the frictions play in influencing the steady state equilibrium, especially at a quantitative level. In this section we report some results that attempt to provide more information about how frictions, specifically the value of $\lambda_{w}$, influences features of the steady state equilibrium.

In our calibration we chose the value of $\lambda_{w}$ so as to target the level of unemployment in the steady state, having argued that there was intuitively a close link between the two. We begin
by displaying the nature of this relationship. Specifically, we set $\lambda_w$ to an arbitrary value and then calibrate all of the remaining parameters in the same fashion as above, except that we no longer match the steady state unemployment rate in the data. We do note, however, that we do continue to set the preference parameter $\alpha$ that gives the disutility of working so that the steady state employment rate does match the target value of .632 found in the data. Table 5 gives the results.

Table 5

<table>
<thead>
<tr>
<th>$\lambda_w$ and the Steady State Unemployment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_w = 1$</td>
</tr>
<tr>
<td>.000</td>
</tr>
</tbody>
</table>

In the extreme case in which $\lambda_w = 1$, any individual who wants to work at the going wage rate can do so, and hence there are no individuals who would like to work but are unable, implying that the unemployment rate is equal to 0. It is important to note that the elasticity of $U$ with respect to $\lambda_w$ is quite large, in the sense that moving from our calibrated value of .364 to a value of .2 leads to more than a fifty percent increase in unemployment, while increasing $\lambda_w$ to .5 leads to a drop of more than 25% in unemployment, so that our calibration procedure pins down the value of $\lambda_w$ quite precisely.

In addition to influencing the level of steady state unemployment, different values of $\lambda_w$ will also influence some other aspects of the steady state equilibrium. In particular, it will influence the nature of labor market flows. Rather than show all of the flows we simply report how different values of $\lambda_w$ influence the duration of employment and unemployment spells. This is done in Table 6.
The table shows that as $\lambda_w$ decreases, implying greater frictions, unemployment durations increase, but so do employment durations. It is important to keep in mind that as $\lambda_w$ changes in this table we are recalibrating the model so as to keep the employment rate constant. (Although we do not report them in the table, spells of nonparticipation actually decrease.) The fact that an increase in frictions (i.e., a decrease in $\lambda_w$) leads to an increase in unemployment durations is perhaps not very surprising. The fact that duration of employment spells also increases reflects in part a response in labor supply. Specifically, if individuals know that it is harder to come into contact with an employment opportunity, then they understand that when they do have one they should hold onto it longer. The way that they do this is by lowering the threshold value of productivity for when they choose not to work.\textsuperscript{12} The flip side of this is that individuals spend less time in nonparticipation, since they become less choosy about when to work, which is why the duration of nonparticipation spells decreases. One can see these effects in productivity statistics. Table 7 reports average productivity of employed workers in the various steady states.

\begin{table}[h]
\centering
\caption{Spell Durations and $\lambda_w$}
\begin{tabular}{lcccccc}
\hline
& $\lambda_w = 1.0$ & $\lambda_w = 0.5$ & $\lambda_w = 0.4$ & $\lambda_w = 0.3$ & $\lambda_w = 0.2$ & $\lambda_w = 0.1$ \\
\hline
Employment & 20.8 & 21.4 & 22.0 & 24.3 & 28.3 & 36.4 \\
Unemployment & 0.0 & 1.9 & 2.3 & 2.9 & 4.0 & 7.3 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{12}We say "in part" because higher values of $\lambda_w$ imply lower values of $\sigma$ due to the lesser probability that a separated worker receives a new employment opportunity immediately. This effect accounts for only part of the increase in employment durations.
As the table shows, average productivity when frictions are high ($\lambda_w = .1$) is almost ten percent lower than when there are no frictions, so the productivity effects of frictions is substantial.

In view of our discussion of how well our benchmark calibration does in terms of accounting for labor market flows, and the results in Tables 5 and 6, it would seem that in our current model there is some tension between matching the U to E flow and the fraction of the population that is in the unemployment state. In particular, our benchmark model seems to have too many transitions from U to E. Table 6 shows that one can obtain a higher duration of unemployment by increasing the degree of frictions, but Table 5 shows that this comes at the cost of having too many individuals in the unemployment state. In the subsequent analysis that we carry out, we will sometimes also report results for the $\lambda_w = .20$ calibration in order to provide a sense of how different this case is from the benchmark case. The flow rate from U to E in this alternative calibration is .188, which is even lower than the corresponding value in the US data, so this seems to be a very conservative lower bound on the set of empirically reasonable values for $\lambda_w$.

Lastly, changes in $\lambda_w$ also influence the reasons for transitions out of employment. As explained above, the greater the frictions the more individuals want to hold on to employment opportunities once they have them. As a result, separation shocks become more important in accounting for transitions out of employment. Table 8 shows this.
Table 8

<table>
<thead>
<tr>
<th>( \lambda_w )</th>
<th>( \sigma ) shock</th>
<th>( p ) shock</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.00</td>
<td>0.70</td>
<td>0.30</td>
</tr>
<tr>
<td>0.5</td>
<td>0.31</td>
<td>0.58</td>
<td>0.11</td>
</tr>
<tr>
<td>0.4</td>
<td>0.32</td>
<td>0.56</td>
<td>0.12</td>
</tr>
<tr>
<td>0.3</td>
<td>0.35</td>
<td>0.55</td>
<td>0.10</td>
</tr>
<tr>
<td>0.2</td>
<td>0.40</td>
<td>0.53</td>
<td>0.07</td>
</tr>
<tr>
<td>0.1</td>
<td>0.51</td>
<td>0.45</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Lastly, we can also see the extent to which frictions affect the level of precautionary savings in the economy. Table 9 shows the steady state level of capital per unit of labor services across steady states that have different values for \( \lambda_w \).

Table 9

<table>
<thead>
<tr>
<th>( \lambda_w )</th>
<th>( K/L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>165.72</td>
</tr>
<tr>
<td>0.5</td>
<td>164.83</td>
</tr>
<tr>
<td>0.4</td>
<td>164.75</td>
</tr>
<tr>
<td>0.3</td>
<td>164.57</td>
</tr>
<tr>
<td>0.2</td>
<td>164.24</td>
</tr>
<tr>
<td>0.1</td>
<td>163.82</td>
</tr>
</tbody>
</table>

Probably the key message from this table is that the effects are relatively small, but it is somewhat surprising that this ratio decreases as frictions increase. One piece of intuition that suggests the opposite effect is the following. When frictions increase, individuals face more uncertainty regarding earnings and as a result precautionary savings will increase. In fact, this effect is operative in our model, and can be seen in the results presented in Krusell et al (2008). But because our model also has heterogeneous productivity and a labor supply margin it turns out that there are some additional effects in operation as well. To begin with, note that the level of precautionary saving is related to the nature of the equilibrium stochastic process for labor income. But this process is in turn determined by not only the exogenous productivity process but also the frictions and the resulting labor.
supply process. We have already noted that the effect of frictions is to spread employment out across productivity realizations. This affects the stochastic properties of the equilibrium process on labor income, and in particular has the effect of making it less persistent. So even though frictions represent an additional source of uncertainty, the net effect on the equilibrium process for labor income is more complicated. In addition to this issue one should also keep in mind that the model is recalibrated as \( \lambda_w \) changes, so that this table does not represent the effect of changes in \( \lambda_w \) holding all else constant. In particular, the value of \( \sigma \) decreases as \( \lambda_w \) increases, thereby offsetting some of the risk.

### 3.3 The Role of Heterogeneity

A prominent feature of our model is that individuals are subject to idiosyncratic productivity shocks. In this section we show that this feature plays an important role in allowing the model to capture the key properties of worker flows found in the data. To do this we consider a model that abstracts from the idiosyncratic shocks. In particular, we consider the benchmark model from Krusell et al (2008), which had no shocks, \( \lambda_w = .2 \), \( \sigma = .02 \) and \( \alpha \) chosen to produce a steady state employment rate of .66. Table 10 shows the results.

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>U</td>
</tr>
<tr>
<td>E</td>
<td>0.85</td>
</tr>
<tr>
<td>U</td>
<td>0.30</td>
</tr>
<tr>
<td>N</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Recall from Table 1 that two of the key features of the actual flow data are the high persistence of the E and N states. Whereas the model with idiosyncratic productivity shocks
matched both of these flows very well, this example misses somewhat with regard to the persistence in the E state, and by a huge margin with regard to persistence in the N state.\textsuperscript{13} Spell durations are now equal to 6.4, 3.3 and 1.0 months respectively for E, U, and N. The short durations of employment and non-participation spells is striking. The model without idiosyncratic productivity shocks seems to produce far too little persistence at the individual level–individuals are transitioning between the various states too frequently. Adding a persistent productivity shock adds a source of persistence at the individual level and serves to produce a substantial quantitative improvement in terms of the models ability to account for the salient features of the underlying flow data.

4 Tax Effects

In this section we analyze what our model predicts regarding the labor market effects of increases in the size of the tax and transfer program.\textsuperscript{14} Prescott (2004) argued that differences in the scale of tax and transfer programs could account for the bulk of the observed differences in hours worked between the US and several European countries. His analysis assumed no frictions and abstracted from the issue of how workers are distributed across labor market states. We can assess the importance of frictions for this exercise by comparing the results

\textsuperscript{13}Because the steady state employment rate is .66, the model must necessarily have a fair amount of persistence in the employment state, in the sense that at least half of those employed this period must also be employed next period.

\textsuperscript{14}While Krusell et al (2008) carry out this analysis in a simpler version of this model, our earlier analysis shows that the Krusell et al (2008) model does not do a good job of accounting for worker flows. A related issue is that the earlier analysis did not really pin down the extent of labor market frictions. Moreover, that paper did not distinguish between unemployment and nonparticipation and so could not be used to assess the consequences for these variables and statistics such as the duration of unemployment.
in our benchmark calibrated economy with the results that emerge from the case in which
\( \lambda_w \) is set equal to 1 and we calibrate the model without targeting the unemployment rate.

Table 11 shows the results for the case of no frictions and our benchmark calibration, as well
as for the case of \( \lambda_w = .2 \).

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( \lambda_w = 1.0 )</th>
<th>( \lambda_w = .364 )</th>
<th>( \lambda_w = .200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.880</td>
<td>.860</td>
<td>.847</td>
</tr>
<tr>
<td>0.15</td>
<td>.799</td>
<td>.783</td>
<td>.775</td>
</tr>
<tr>
<td>0.30</td>
<td>.632</td>
<td>.632</td>
<td>.632</td>
</tr>
<tr>
<td>0.45</td>
<td>.482</td>
<td>.486</td>
<td>.488</td>
</tr>
</tbody>
</table>

The striking result from this table is that the presence of frictions has virtually no effect
on the impact of tax increases on employment. For the case of tax decreases, the presence
of frictions does have some effect, but even when \( \lambda_w = .2 \) the effect of frictions is relatively
small compared to the overall change. For example, when taxes are reduced to zero, the
employment rate increases by roughly 25 percentage points when there are no frictions, and
by roughly 22 percentage points when \( \lambda_w = .2 \). It follows that for evaluating the steady state
effects of tax changes, the presence of reasonable frictions has little impact on the aggregate
response of employment.

Table 12 shows the implications of tax changes for the unemployment rate for the same
three cases considered in Table 11.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( \lambda_w = 1.0 )</th>
<th>( \lambda_w = .364 )</th>
<th>( \lambda_w = .2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.00</td>
<td>.049</td>
<td>.084</td>
</tr>
<tr>
<td>0.15</td>
<td>.00</td>
<td>.061</td>
<td>.099</td>
</tr>
<tr>
<td>0.30</td>
<td>.00</td>
<td>.083</td>
<td>.134</td>
</tr>
<tr>
<td>0.45</td>
<td>.00</td>
<td>.100</td>
<td>.160</td>
</tr>
</tbody>
</table>
An interesting result emerges. Given that the results with frictions are virtually identical to results without frictions (especially for the case of tax increases), one might expect that the changes in the employment rate will be reflected mostly in changes in the participation rate rather than the unemployment rate. But the table shows that changes in taxes do affect the measured unemployment rate in the models with frictions. In particular, when taxes are increased from .30 to .45, and the employment rate drops from .63 to .49, we see that the unemployment rate increases from .08 to .10 and .16 for the cases of \( \lambda_w = .364 \) and .20 respectively. Moreover, it is also the case the spell durations are also affected, as shown in Table 13.

<table>
<thead>
<tr>
<th>( \lambda_w )</th>
<th>( \tau = 0.00 )</th>
<th>( \tau = 0.15 )</th>
<th>( \tau = 0.30 )</th>
<th>( \tau = 0.45 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>97.6/0.0/13.3</td>
<td>36.8/0.0/9.3</td>
<td>20.8/0.0/12.1</td>
<td>16.5/0.0/17.7</td>
</tr>
<tr>
<td>.364</td>
<td>47.1/2.6/12.8</td>
<td>34.8/2.5/12.1</td>
<td>22.8/2.4/13.4</td>
<td>18.0/2.4/18.3</td>
</tr>
<tr>
<td>.2</td>
<td>49.5/4.6/11.8</td>
<td>41.8/4.4/12.9</td>
<td>29.7/4.1/13.9</td>
<td>23.8/3.9/19.2</td>
</tr>
</tbody>
</table>

Some interesting patterns emerge. Specifically, an increase in taxes leads to shorter durations of employment and unemployment spells, and longer durations of nonparticipation. The reason for the decrease in unemployment spell durations is that when taxes are high individuals have a higher reservation productivity level for a given value of assets, and this means that unemployed workers are more likely to experience a negative productivity shock and transition into out of the labor force.\(^{15}\)

To summarize, the main finding of this section is that for reasonably calibrated frictions, the aggregate employment effects of the model with frictions is essentially identical to that

\(^{15}\)It is important to note that we have not included an unemployment insurance system in our model, and that in reality a more generous system may well influence the distribution of individuals between U and N.
of the model without frictions. However, in the model with frictions, changes in taxes do impact on statistics such as the unemployment rate and the duration of employment and unemployment spells. In this sense the model with frictions has a richer set of predictions for the effect of tax changes than the model without frictions. Some researchers argue against the importance of the labor supply channel emphasized in the frictionless model in some contexts by suggesting that it is inconsistent with responses in the unemployment rate. This analysis shows that such a general critique is not compelling in the context of a model with both a nondegenerate labor supply decision and frictions.

5 Changes in Frictions

One of the defining features of the Pissarides matching model and its many variants is that the level of frictions play a key role in determining not only the level of unemployment but also in determining the level of aggregate employment. Intuitively, labor supply considerations will attenuate the impact of changes in frictions on aggregate employment. The reason for this is that if it becomes harder to find employment opportunities, then workers will be more willing to continue with a job opportunity once they find it, or decide to accept employment at lower productivities. The goal of this section is to explore the quantitative importance of these effects in our model.

We begin by exploring the impact of exogenous changes in the level of $\lambda_w$. Unlike the results that we showed in Section 3, where we changed the value of $\lambda_w$ but simultaneously recalibrated the other model parameters so that the steady state employment rate (among
other variables) was unchanged, in this section we change the value of $\lambda_w$ holding all of the other parameters fixed. That is, we evaluate the impact on the steady state of an exogenous change in the level of frictions.

We are primarily interested in the extent to which the responses in our model are different than those that would emerge in a simple version of the Pissarides matching model. In the simple Pissarides model, the match separation rate is exogenous, but the job offer arrival rate is endogenously determined by the volume of vacancy posting. In this model all job offers are accepted, so the job offer arrival rate is also the probability that an unemployed worker becomes employed. If the match separation rate is $\sigma$ and the job offer arrival rate is $\lambda_w$, and we assume that individuals can begin to work in the same period as receiving a job offer, then the law of motion for the unemployment rate is:

$$u_{t+1} = (1 - \lambda_w)u_t + \sigma(1 - \lambda_w)(1 - u_t)$$

It follows that the steady state employment rate is given by:

$$\bar{u} = \frac{\sigma(1 - \lambda_w)}{\lambda_w + \sigma(1 - \lambda_w)}$$

We set $\sigma = .0225$ as in our benchmark calibration, and then set $\lambda_w$ so that the steady state unemployment rate is equal to .083, which was the same target that we matched in our calibration. The implied value of $\lambda_w$ is .20. We will then consider equal proportional changes in the value of $\lambda_w$ in the two models, i.e., we increase or decrease $\lambda_w$ by the same percentage in the two models. Although the value of $\lambda_w$ is endogenously determined in the
Pissarides model, we do not model the source of this change. Rather, we focus simply on the consequences of such a change for employment and unemployment.

Table 14 shows the effects for aggregate employment and unemployment in the two models.

<table>
<thead>
<tr>
<th>$\lambda_w$</th>
<th>Our model</th>
<th>Pissarides model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E/P$</td>
<td>$U/(U+E)$</td>
</tr>
<tr>
<td>0.5</td>
<td>63.6%</td>
<td>5.2%</td>
</tr>
<tr>
<td>0.4</td>
<td>63.3%</td>
<td>7.4%</td>
</tr>
<tr>
<td>0.36</td>
<td>63.2%</td>
<td>8.3%</td>
</tr>
<tr>
<td>0.3</td>
<td>63.0%</td>
<td>10.3%</td>
</tr>
<tr>
<td>0.2</td>
<td>62.4%</td>
<td>14.8%</td>
</tr>
<tr>
<td>0.1</td>
<td>60.1%</td>
<td>24.5%</td>
</tr>
</tbody>
</table>

In reading this table each row represents the same percentage change in $\lambda_w$ relative to the two benchmark calibrations, which by construction each have the same unemployment rate. A striking result emerges. If one looks at the responses of unemployment, one observes that the effects are very similar across the two different models. Moreover, the effects are large—when $\lambda_w$ is decreased from the benchmark setting to the lowest value in the table, the unemployment rate roughly triples in both cases. But when one looks at the employment rate responses one sees dramatic differences. In the Pissarides model, changes in the unemployment rate and changes in the employment rate are necessarily mirror images of each other since by construction all workers are in the labor force. Hence, the Pissarides model also predicts large employment responses as a result of changes in $\lambda_w$. In sharp contrast, our model predicts very small changes in employment rates. The change in the employment rate in our model is only about one-sixth as large as the change in the Pissarides model. For
example, when moving from the benchmark specification to the lowest value of $\lambda_w$ in the table, the employment rate decreases by more than 19 percentage points in the Pissarides model but only by about 3 percentage points in our model.

To see why the two models give such different employment responses it is instructive to examine the durations of employment and unemployment spells.

<table>
<thead>
<tr>
<th>Table 15</th>
<th>Effect of $\lambda_w$ on Spell Durations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Our model</td>
</tr>
<tr>
<td></td>
<td>$E$</td>
</tr>
<tr>
<td>$\lambda_w = 0.5$</td>
<td>22.7</td>
</tr>
<tr>
<td>$\lambda_w = 0.4$</td>
<td>22.4</td>
</tr>
<tr>
<td>$\lambda_w = 0.364$</td>
<td>22.8</td>
</tr>
<tr>
<td>$\lambda_w = 0.3$</td>
<td>23.5</td>
</tr>
<tr>
<td>$\lambda_w = 0.2$</td>
<td>25.7</td>
</tr>
<tr>
<td>$\lambda_w = 0.1$</td>
<td>29.5</td>
</tr>
</tbody>
</table>

In both models a decrease in $\lambda_w$ leads to an increase in the duration of unemployment, and the proportional changes are very similar in the two models. But the changes in employment durations are actually opposite in the two models. In the Pissarides model a decrease in $\lambda_w$ leads to a decrease in the duration of employment spells. The reason for this is that decreases in $\lambda_w$ make it less likely that a separated worker finds a new job during the initial period of the separation, thereby preserving the employment spell. If we had instead assumed that workers necessarily spend one period out of employment following a separation, then we would have found that the duration of employment spells is constant. In contrast to either of these outcomes, in our model the duration of employment spells increases significantly in response to decreases in $\lambda_w$. In moving from the benchmark value of $\lambda_w$ to the lowest value
in the table, the duration of employment in our model increases by more than one third.\textsuperscript{16} 

We can also repeat the above analysis to examine how the two different models respond to exogenous changes in $\sigma$, the separation shock. Proceeding the same as above, Table 16 presents the effects on employment and unemployment.\textsuperscript{17}

<table>
<thead>
<tr>
<th>Effect of $\sigma$ on Employment and Unemployment Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>$\sigma = 0.01$</td>
</tr>
<tr>
<td>$\sigma = 0.02$</td>
</tr>
<tr>
<td>$\sigma = 0.0225$</td>
</tr>
<tr>
<td>$\sigma = 0.03$</td>
</tr>
<tr>
<td>$\sigma = 0.04$</td>
</tr>
</tbody>
</table>

Changes in unemployment rates in response to changes in $\sigma$ are about one half as large in our model as in the Pissarides model. And the employment response is only about one-tenth as large in our model as in the Pissarides model. Table 17 shows that the reason for the large differences in employment rate responses has to do with a labor supply effect.

\textsuperscript{16}It is important to note that one can generate similar types of effects in the context of the simple Pissarides model via different extensions. For example, Mortensen and Pissarides (1994) adds a job destruction margin, and in principle, if matches are harder to find then matches can be destroyed less frequently. Of if there is a match quality draw, as in Pissarides (1986), then when matches are harder to come by, workers will be less choosy about the match qualities that are acceptable. Both of these responses are in the spirit of labor supply responses, and are potentially of interest. But even with these extensions, these models still have the property that absent frictions, workers would always work.

\textsuperscript{17}Because the values of $\sigma$ are the same in the two benchmark economies we now consider equal changes in the two economies.
Table 17

Effect of $\sigma$ on Spell Durations

<table>
<thead>
<tr>
<th></th>
<th>Our model</th>
<th>Pissarides model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.01$</td>
<td>$\mathbf{E}$</td>
<td>27.3</td>
</tr>
<tr>
<td>$\sigma = 0.02$</td>
<td>$\mathbf{E}$</td>
<td>23.5</td>
</tr>
<tr>
<td>$\sigma = 0.0225$</td>
<td>$\mathbf{E}$</td>
<td>22.8</td>
</tr>
<tr>
<td>$\sigma = 0.03$</td>
<td>$\mathbf{E}$</td>
<td>20.6</td>
</tr>
<tr>
<td>$\sigma = 0.04$</td>
<td>$\mathbf{E}$</td>
<td>18.5</td>
</tr>
</tbody>
</table>

In the Pissarides model, changes in $\sigma$ lead mechanically to changes in employment duration. The implication is that the large changes in $\sigma$ are associated with large and proportional changes in employment duration. In contrast, in our model, decreases in $\sigma$ lead to what in comparison are only very moderate increases in employment duration. The reason is due to the response in labor supply. When $\sigma$ is high, it is less likely that an individual has an employment opportunity in any given period, and as a result they respond by being willing to work at lower productivity levels. This in turn implies less "voluntary" separations. But when $\sigma$ decreases, the reverse is true, and individuals become choosier about when to work, leading to more voluntary separations.

6 Conclusion

We have built a model that features search frictions and a nondegenerate labor supply decision along the extensive margin. We argue that the steady state equilibrium of our model does a reasonable job of matching labor market flows between the three labor force states of employment, unemployment and out of the labor force. Persistent idiosyncratic shocks play a key role in allowing the model to match the persistence of the employment and out of the labor
force states found in individual labor market histories. We then use this model to address two questions. The first is the effect of tax and transfer programs on aggregate employment. We find that the presence of frictions has virtually no impact on the response of aggregate employment, but the model also predicts that higher taxes lead to higher unemployment and lower participation. The second issue concerns the effect of changes in frictions on aggregate employment. We find that changes in either the job loss rate of the job finding rate do not have large effects on aggregate employment, though they do have sizable effects on unemployment. In particular, the labor supply response present in our model greatly attenuates the employment response relative to the simplest matching model.

Having developed a reasonable three state model of worker flows, there are several interesting issues that would be of interest to address. One is to carry out a more comprehensive analysis of taxes and transfer programs in order to address the debate between Prescott (2004) and Ljungqvist and Sargent (2006, 2008) concerning the role of benefits. Specifically, Ljungqvist and Sargent argue that once one takes into account the generosity of benefit systems in many European countries that Prescott’s model implies implausibly large responses. But Ljungqvist and Sargent do not distinguish between the unemployed and those out of the labor force, so implicitly assume that all nonemployed individuals can receive unemployment benefits. A model such as ours is required to properly assess these arguments. Second, it would be of interest to examine business cycle fluctuations in our model, and in particular the role of technology shocks and shocks to the exogenous frictions.
References


