On the Non-Monotonic Relation between Interest Rates and the Exchange Rate[1]

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Revised draft: November 2001

[1]We would like to thank Joshua Aizenman, Bob Flood, Eduardo Moron, Chetan Subramanian, seminar participants at Chicago Business School, Dartmouth College, AEA meetings, the Universidad de Montevideo (Uruguay), the World Bank, the Berkeley - San Francisco Fed Summer Camp, and, especially, Rajesh Sing for helpful comments and suggestions. Rajesh Singh also provided superb research assistance. Both authors gratefully acknowledge research support provided by grants from the UCLA Academic Senate.

Abstract

Central banks typically raise short-term interest rates to defend against currency depreciations. The empirical literature in this area has, however, been unable to detect a clear systematic relationship between interest rates and exchange rates. We use an optimizing model of a small open economy to rationalize the mixed empirical findings. In the model, higher interest rates generate both a negative output effect and a higher fiscal deficit. We show that the relationship between nominal interest rates (both policy-controlled and market-determined) and exchange rates is inherently *non-monotonic*. The results of the model are thus consistent with the inability of non-structural empirical models to find a systematic relationship.

JEL Classification: F3, F4

Keywords: Interest rate policy, flexible exchange rates, currency depreciation

1 Introduction

In the aftermath of the Asian crises, an intense policy debate has erupted regarding the effectiveness of higher interest rates in stabilizing depreciating currencies. On the one hand, higher interest rates have been a key component of IMF programs because, in spite of its associated costs, a stable currency is viewed as a pre-condition for achieving financial and macro-economic stabilization (see Fischer (1998)). On the other hand, prominent economists like Jeffrey Sachs and Joe Stiglitz have vehemently disagreed and argued that higher interest rates may not only do more harm than good but may, in fact, lead to a perverse outcome (i.e., higher interest rates might lead to a *depreciation* of the currency).[1] The debate continues unabated as many emerging countries (with Brazil being a prominent example) currently face yet another round of international financial turmoil and are confronted with the dilemma of whether and by how much to raise interest rates to defend a rapidly depreciating currency. The relation between higher interest rates and exchange rates in developing countries is thus one of the most pressing and outstanding monetary policy issues in the post-Asian crises world.

As expected, such important policy debate has led in a short period of time to a voluminous empirical literature attempting to settle the debate by letting the data speak.[2] Unfortunately, this body of work based on developing country data has failed to find any systematic relationship between interest rates and nominal exchange rates. Results seem to be sensitive to, for example, country in question, sample size, data frequency, and cross-section (panel) versus time series analysis. The typical response of the empirical literature to inconclusive or contradictory empirical findings has been to argue that, if only one could

[1] Commenting on the IMF package for Indonesia (*New York Times*, Nov. 11, 1997), Jeff Sachs noted that although the program should boost confidence, "if tied to orthodox financial conditions, including budget cuts and sharply higher interest rates, the package could do more harm than good, transforming a currency crisis into a rip-roaring economic downturn." Joe Stiglitz, former chief economist of the World Bank, argued in the *New Republic* (April 17, 2000) that the IMF-supported high interest rate policy in Asia was totally misguided as it provided little benefits while plunging the economies into a deep recession by devastating highly indebted firms and causing widespread bankruptcies.

[2] See, among others, Cho and West (2001), Dekle, Hsiao, and Wang (2001), Drazen and Hubrich (2000), Goldfajn and Gupta (1999), Gould and Kamin (2001), Kraay (1999), Tanner (2001) and Zettelmeyer (2000).

control for a host of econometric problems and omitted variables, the "truth" would emerge (typically meaning that higher interest rates would indeed lead to a more appreciated currency, as we teach our undergraduates).

Our take on this debate will be quite different. Based on a simple monetary model, we will argue that there is no theoretical reason to expect a monotonic relationship between interest rates and exchange rates. In fact, our model predicts that the relationship between interest rates and exchange rates is inherently non-monotonic. We will show that it takes some surprisingly small modifications of an otherwise standard optimizing monetary model to generate non-monotonicities. Hence, assuming that our model captures some essential features of the real world (as we will argue), it follows that there is nothing surprising in the fact that the empirical literature has failed to unearth a systematic monotonic relationship between interest rates and exchange rates, since there may be none to begin with.

In building our model, we take as a starting point the fact that most analysts seem to agree that higher interest rates affect key macro variables essentially through three channels. First, higher interest rates raise the demand for domestic-currency denominated assets thus leading, all else equal, to an appreciation of the currency. Henceforth, we shall refer to this as the "money demand effect". Second, higher domestic interest rates induce a contraction in domestic output. This contraction can occur through either a direct credit crunch or an increase in bankruptcies and/or a deterioration of bank balance sheets. We shall refer to this channel as the "output effect". Third, an increase in interest rates increases the debt service burden of the fiscal authority. This in turn increases inflationary expectations and, hence, weakens the currency. We shall call this the "fiscal effect". At a policy level, the disagreement on the net benefits of raising interest rates essentially rests on a differing assessment of the relative importance of these three factors.

We proceed to build a model that captures these three effects. We incorporate the money demand effect by allowing for an independent interest rate instrument. Following Calvo and Végh (1995), interest rate policy is essentially modeled as the central bank's ability to pay interest on part of the money supply. In particular, we assume that households hold interest-bearing money in the form of bank deposits. Commercial banks, in turn, hold as part of their portfolio non-tradable government bonds ("domestic bonds") issued by the central bank. In this set-up, the interest rate on these non-tradable bonds (hereafter referred to as the *policy-controlled* interest rate), is an *additional* policy instrument.[3] Raising this policy-controlled interest rate leads, all else equal, to higher money demand (the money demand effect).[4]

The model also embodies a credit channel à la Bernanke and Blinder (1988, 1992). The presence of this credit channel implies that higher interest rates on domestic bonds crowds out the supply of bank credit to the private sector and causes an output contraction (the output effect). Finally, the model incorporates the fiscal effect by assuming an exogenous path for government spending. This implies that higher interest rates on domestic bonds also have fiscal costs since they lead to a higher debt service burden, which requires a higher inflation rate.

For a given supply of money and a given inflation rate, the positive money demand effect of higher interest rates leads to an appreciated domestic currency. However, both the fiscal and output channels tend to depreciate the currency. The fiscal cost of higher interest rates on domestic bonds implies that the inflation rate needs to be higher to finance the exogenous level of government spending. This tends to increase the market interest rate, thus raising the opportunity cost of holding money and depreciating the currency. In a similar vein, for a given level of deposits, the fall in bank credit associated with the output effect implies that banks will lend more to the government. This higher stock of government liabilities also requires, all else equal, a higher inflation rate. In this set-up, therefore, whether an increase in the policy-controlled interest rate appreciates the currency or not will depend on the interaction between these three effects.

Our main results indicate that the relationship between nominal interest rates and the exchange rate is inherently non-monotonic. In particular, the relation between the *policy-controlled* interest rate and the exchange rate is U-shaped. This implies that, up to certain point, an increase in the policy-controlled interest rate will indeed appreciate the domestic currency. Beyond that point, however, further increases by the central bank of the policy-controlled interest rate will actually begin to depreciate the currency. In the same vein, the

[3] The nominal interest rate on tradable bonds will be referred to as the *market* interest rate.

[4] Based on money demand estimations, Freire (2001) presents convincing evidence that the demand for interest-bearing liquid assets (i.e., quasi-money) depends positively on the corresponding interest rate.

relation between the *market* interest rate and the exchange rate is also non-monotonic. Our model thus predicts that one should not expect to find any systematic monotonic relationship in the data between nominal interest rates (either policy-controlled or market interest rates) and nominal exchange rates. This is fully consistent with the empirical evidence so far and would further suggest that it is simply a futile exercise to try to uncover any linear relationship in the data. In the presence of these inherent non-monotonicities, estimating a structural model would appear to be the only sensible approach. Finally, we also find that a higher policy-controlled interest rate often induces an intertemporal trade-off in the term structure of the exchange rate, as it appreciates today's currency at the expense of a more depreciated currency in the future. This is consistent with the evidence presented in Drazen and Hubrich (2000).

This paper is related to two recent strands in the literature. The first is concerned with the use of higher interest rates to defend a fixed exchange rate, as in Lahiri and Végh (2000), Flood and Jeanne (2000) and Drazen (1999). None of these studies, however, deal with an active interest rate defense of floating exchange rates, which is arguably the more relevant issue in today's world where few pegs remain. The second is related to the "fear of floating" phenomenon identified by Calvo and Reinhart (2000a, 2000b). Calvo and Reinhart (2000a) argue that developing countries are surprisingly reluctant to let their currencies depreciate too much in response to negative shocks and argue that higher interest rates is one key means whereby further depreciation is prevented. This certainly lends credence to our giving interest rate policy an independent role in the model. Furthermore, based on a large sample of episodes of floating exchange rates in developing countries, Calvo and Reinhart (2000a) fail to find a clear pattern for the effects of higher interest rates on exchange rates (see Table 26). Our model is fully consistent with such a finding, as it would predict that no systematic relation should be expected even when central banks are indeed using higher interest rates to defend a weakening currency.

The paper proceeds as follows. Section 2 is an empirical section whose purpose is two-fold. First, it provides an illustration of the lack of a systematic link between interest rates and exchange rates by estimating vector autoregressions (VARs) for ten countries and computing the impulse responses of exchange rates in response to innovations in interest rates. Second, it provides empirical evidence on the existence of both the output and the fiscal effects. Sections 3 and 4, which constitute the core of the paper, develop the model and derive the main results. Section 5 reinterprets our results as applying to the policy response to a negative fiscal shock and briefly discusses the optimality of an active interest rate defense. Section 6 contains concluding remarks.

2 Empirical motivation

We start off by empirically documenting our motivating issue (the lack of a systematic relationship between interest rates and exchange rates) and our modelling choices (the output channel and the fiscal channel) through a look at the data. In order to investigate these relationships, we run unrestricted VARs on a country-by-country basis for a sample of ten countries. Our sample includes six developing countries – Brazil, Korea, Mexico, Thailand, Peru and Philippines – and four developed countries – Canada, Germany, Italy, and the United States.[5]

We estimate country-specific four variable VARs using monthly data on nominal exchange rates (domestic currency units per U.S. dollar), short term interest rate differentials between home and abroad (domestic minus U.S. interest rate), industrial production and government fiscal balance. For the U.S., the exchange rate is dollar per yen while the interest rate differential is the U.S. minus the Japanese short term interest rate. Since monthly fiscal data for all countries in our sample is highly seasonal and volatile, we use the 12-month moving average instead. Our data is from the International Financial Statistics (IFS). Monthly data on industrial production was not available for Brazil and Thailand. Hence, for these two countries we ran three variable VARs involving the exchange rate, short term interest rate

[5] Our sample for developing countries was dictated by data availability and circumstances (i.e., countries and periods during which the exchange rate was floating). Based on this, the sample periods are as follows: Brazil 1998:12 – 2001:03; Mexico 1995:01 – 2001:03; Peru 1992:01 – 2001:03; Korea 1997:07 – 2001:03; Thailand 1997:07 – 2001:03; and Phillipines 1983:01 – 2001:03. We chose four major developed countries as a control group and to ilustrate some interesting differences in terms of the transmission channels that may be involved (see below). The sample period in this case is 1974:01 – 2001:03.

differentials and the fiscal balance.[6]

We use the estimated VARs to calculate the impulse response of the exchange rate, industrial production and the fiscal balance to an orthogonalized one standard deviation innovation in the interest rate differential between home and abroad for each country. Following Eichenbaum and Evans (1995) and Grilli and Roubini (1995), we compute the impulse responses using the following ordering: industrial production, interest rate differential, exchange rate, and fiscal balance.[7]

2.1 The relation between interest rates and exchange rates

Figure 1 depicts the impulse response of the nominal exchange rate (with a one standard deviation significance band) to a one standard deviation orthogonalized innovation in the interest rate differential. The picture reveals mixed results. Within the set of developed countries, in Canada, Germany and Italy there is a significant appreciation of the currency in response to an increase in the interest rate differential. This is the well-known result of Eichenbaum and Evans (1995) and Grilli and Roubini (1995). For the developing group the effect is mostly the opposite. Except for Thailand, in all countries a positive innovation in the interest rate differential between home and the United States induces a significant depreciation of the currency. Thailand, on the other hand, shows a significant appreciation of the currency in response to an interest rate innovation.

However, even for the developed countries the relationship between interest rates and exchange rates is not stable over time. Thus, for the U.S. we split the sample into two sub-periods – 1974:01 – 1990:05 and 1990:06 – 2001:03. Note that the first sub-period corresponds to the period analyzed in Eichenbaum and Evans (1995). As can be seen from the last row of Figure 1, the exchange rate effect of an interest rate innovation is different in the two sub-periods. For 1974:01–1990:05 we see the standard result - a positive innovation in the interest rate differential between the US and Japan causes a significant exchange rate appreciation. However, this relationship is reversed for the latter period in which the dollar depreciates

[6] For the interest rate data, we use short term money market rates. For the U.S., the series used is the federal funds rate.

[7] The Akaike criterion was used to choose the lag length.

relative to the yen in response to an innovation in the same interest rate differential.

Our evidence is thus consistent with the lack of a systematic relationship between interest rates and the exchange rate in the data. As can be seen in Figure 1, this puzzle exists both on a cross-country basis as well as on a time series basis.

2.2 The output effect

In Figure 2 we plot the impulse response of industrial production in each country to a one standard deviation innovation in the relevant interest rate differential. As can be seen from the figure, in all cases but one there is a significant contraction in industrial production in response to a positive interest rate innovation. The only exception is the U.S. for the latter sub-period (1990:06–2001:03). On the whole, therefore, we interpret the evidence as suggesting that the output effect (an output contraction in response to higher domestic interest rates) is mostly present for all countries – both developed and developing. We take this as providing strong support for incorporating an output channel into the model below.

2.3 The fiscal effect

In Figure 3 we plot the impulse responses of the central government's fiscal balance to an innovation in the interest rate differential for each country. Here the evidence is mixed. For the three Latin American countries (Brazil, Mexico, and Peru), the fiscal balance does indeed deteriorate in response to an increase in the interest rate differential with the United States. For Thailand, the effect goes in the wrong direction, while for Philippines it is insignificant. For Korea it is a mixed bag with a significant initial improvement followed by a deterioration six months later. For the developed countries however, the fiscal effect is always insignificant (with the exception of Canada). We interpret this evidence as suggesting that the fiscal effect is not a big issue for developed countries but it can be a significant channel in developing countries.

3 The model

Consider a representative household model of a small open economy that is perfectly integrated with the rest of the world in both goods and capital markets. The infinitely-lived household receives utility from consuming a (non-storable) good and disutility from supplying labor. The world price of the good in terms of foreign currency is fixed and normalized to unity. Free goods mobility across borders implies that the law of one price applies. The consumer can also trade freely in perfectly competitive world capital markets by buying and selling real bonds which are denominated in terms of the good and pay r units of the good as interest at every point in time.

3.1 Households

Household's lifetime welfare is given by

$$W \equiv \int_0^\infty \frac{1}{1-1/\sigma}\left[(c_t - \zeta x_t^\nu)^{1-1/\sigma} - 1\right] e^{-\beta t} dt, \quad \sigma > 0, \quad \zeta > 0, \quad \nu > 1, \tag{1}$$

where c denotes consumption, x denotes labor supply, σ is the intertemporal elasticity of substitution, $\nu - 1$ is the inverse of the elasticity of labor supply with respect to the real wage (as will become evident below), and $\beta(> 0)$ is the exogenous and constant rate of time preference.[8]

Households use cash and demand deposits for reducing transactions costs. Specifically, the transactions costs technology is given by

$$s_t = v(h_t) + \psi(d_t), \tag{2}$$

where s denotes the non-negative transactions costs incurred by the consumer, h denotes cash and d denotes interest-bearing demand deposits (in real terms). We assume that the

[8] These preferences are well-known from the work of Greenwood, Hercowitz and Huffman (1988) and have been widely used in the real business cycle literature, as they provide a better description of consumption and the trade balance for small open economies than alternative specifications (see, for instance, Correia, Neves, and Rebelo (1995)). As will become clear below, the key analytical simplification introduced by GHH preferences is that there is no wealth effect on labor supply.

transactions technology is strictly convex. In particular, the functions $v(h.)$ and $\psi(d)$, defined for $h \in [0, \bar{h}]$, $\bar{h} > 0$, and $d \in [0, \bar{d}]$, $\bar{d} > 0$, respectively, satisfy the following properties:

$$\begin{aligned} v &\geq 0, \quad v' \leq 0, \quad v'' > 0, \quad v'(\bar{h}) = v(\bar{h}) = 0. \\ \psi &\geq 0, \quad \psi' \leq 0, \quad \psi'' > 0, \quad \psi'(\bar{d}) = \psi(\bar{d}) = 0. \end{aligned}$$

Thus, additional cash and demand deposits lower transactions costs but at a decreasing rate. The assumption that $v'(\bar{h}) = \psi'(\bar{d}) = 0$ ensures that the consumer can be satiated with real money balances (i.e., the Friedman rule can be implemented).

In addition to the two liquid assets, households can also hold an internationally-traded bond (b). Real financial wealth at time t is thus given by $a_t = b_t + h_t + d_t$. Further, perfect capital mobility implies that the nominal interest rate is $i = r + \varepsilon$, where ε denotes the rate of currency depreciation. We denote the deposit rate by i^d. Hence, the opportunity cost of holding demand deposits is $I^d \equiv i - i^d$ (the deposit spread) while the corresponding cost of holding cash is i. The flow budget constraint facing the representative household is thus given by

$$\dot{a}_t = ra_t + w_t x_t + \tau_t - c_t - s_t - i_t h_t - I_t^d d_t + \Omega_t^f + \Omega_t^b, \tag{3}$$

where w denotes the real wage, τ are lump sum transfers received from the government, while Ω^f and Ω^b denote dividends received from firms and banks, respectively. Integrating (3) and imposing the standard transversality condition yields the household's lifetime budget constraint:

$$a_0 + \int_0^\infty (w_t x_t + \Omega_t^f + \Omega_t^b + \tau_t) e^{-rt} dt = \int_0^\infty (c_t + i_t h_t + I_t^d d_t + s_t) e^{-rt} dt. \tag{4}$$

The household chooses paths for $\{c_t, x_t, h_t, d_t\}$ to maximize lifetime utility (1) subject to (2) and (4), taking as given $\tau_t, r, i_t, I_t^d, w_t, \Omega_t^f, \Omega_t^b$ and a_0. The first-order conditions for this problem are given by [9]

$$(c_t - \zeta x_t^\nu)^{-1/\sigma} = \lambda, \tag{5}$$

$$\nu \zeta x_t^{\nu - 1} = w_t, \tag{6}$$

[9] As usual, we assume that $\beta = r$ to eliminate inessential dynamics.

$$-v'(h_t) = i_t, \tag{7}$$

$$-\psi'(d_t) = I_t^d, \tag{8}$$

where λ is the (time-invariant) Lagrange multiplier associated with constraint (4). Equation (5) says that along a perfect foresight equilibrium path, the marginal utility from consumption is constant. Equation (6) shows that labor supply depends only on the real wage. Moreover, the assumption $\nu > 1$ implies that labor supply x is an increasing function of the real wage, w. Finally, equations (7) and (8) implicitly define the demand for cash and demand deposits as a decreasing function of their respective opportunity cost:

$$h_t = \tilde{h}(i_t), \qquad \tilde{h}' < 0. \tag{9}$$

$$d_t = \tilde{d}(I_t^d), \qquad \tilde{d}' < 0. \tag{10}$$

3.2 Firms

The representative firm in this economy produces the perishable good using a linear technology

$$y_t = x_t. \tag{11}$$

We also assume that firms face a "credit-in-advance" constraint to pay the wage bill.[10] Formally, this constraint is given by

$$n_t = \phi w_t x_t, \quad \phi > 0, \tag{12}$$

where n denotes bank loans. The assumption that firms must use bank credit to pay the wage bill is needed to generate a demand for bank loans.

The real financial wealth of the representative firm at time t is given by $a_t^f = b_t^f - n_t$, where b^f denotes foreign bonds held by firms. Using i^l to denote the lending rate charged by banks and letting $I^l \equiv i^l - i$ (which is the lending spread), the firm's flow constraint can

[10] Alternatively, we could assume that bank credit is an input in the production function, in which case the derived demand for credit would be interest rate elastic. This would considerably complicate the model without adding any additional insights.

be written as[11]

$$\dot{a}_t^f = ra_t^f + y_t - w_t x_t(1 + \phi I_t^l) - \Omega_t^f. \tag{13}$$

Note that $\phi I_t^l w_t x_t = I_t^l n_t$ is the additional resource cost that is incurred by firms due to the credit-in-advance constraint. Integrating forward equation (13), imposing the standard transversality condition, and using equation (11) gives

$$\int_0^\infty e^{-rt}\Omega_t^f dt = a_0^f + \int_0^\infty \left[x_t - w_t x_t(1 + \phi I_t^l)\right] e^{-rt} dt. \tag{14}$$

The firm chooses a path of x to maximize the present discounted value of dividends, which is given by the right hand side of equation (14), taking as given the paths for w_t, I_t^l, r and the initial stock of financial assets, a_0^f. The first order condition for this problem is given by

$$1 = w_t(1 + \phi I_t^l). \tag{15}$$

Intuitively, at the optimum the firm equates the marginal product of labor to the marginal cost of an additional unit of labor which, as noted above, includes the cost of credit.

3.3 Banks

The banking sector is assumed to be perfectly competitive. The representative bank accepts deposits from consumers and lends to both firms (n) and the government in the form of domestic government bonds (z^b).[12] It also holds required cash reserves, δd, where $\delta > 0$ is the reserve-requirement ratio imposed on the representative bank by the central bank.[13]

[11] We should note that the credit-in-advance constraint given by equation (12) holds as an equality only along paths where the lending spread I^l is strictly positive. We will assume that if $I^l = 0$, this constraint also holds with equality.

[12] Commercial bank lending to governments is particular common in developing countries. Government debt is held not only as compulsory (and remunerated) reserve requirements but also voluntarily due to the lack of profitable investment opportunities in crisis-prone countries. This phenomenon was so pervasive in some Latin American countries during the 1980's that Rodriguez (1991) aptly refers to such governments as "borrowers of first resort". For evidence, see Rodriguez (1991) and Druck and Garibaldi (2000).

[13] Similar results would go through if we allowed banks to hold foreign bonds in world capital markets as long as banks face a cost of managing domestic assets (along the lines of Edwards and Végh (1997) or Burnside, Eichenbaum, and Rebelo (1999)). As is well-known, one needs some friction in the banking sector

Thus, the balance sheet identity of the bank is given by

$$(1-\delta)d_t = n_t + z_t^b. \tag{16}$$

The bank pays depositors an interest rate of i^d while it charges an interest rate of i^l to the firms and earns i^g on the government bonds. The flow constraint faced by the bank is then given by

$$\Omega_t^b = I_t^l n_t + I_t^d d_t + I_t^g z_t^b - i_t \delta d_t. \tag{17}$$

Note that I^l, $I^g (\equiv i^g - i)$, and I^d are the net resource gains per unit of lending to firms, n, lending to the government z, and deposits h, respectively. Moreover, since required reserves are non-interest bearing, the opportunity cost of holding required reserves is the foregone nominal interest rate, i. The term $i_t \delta d_t$ reflects the cost of holding required reserves. Note that since required reserves do not earn any interest, at an optimum the bank will not hold any excess reserves.

The representative bank chooses sequences of n_t, z_t^b, and d_t to maximize profits given by equation (17) subject to equation (16), taking as given the paths of I^l, I^d, I_t^g, δ and i. The first order conditions for this problem are (assuming an interior solution)

$$(1-\delta)\, I_t^l + I_t^d = \delta i_t, \tag{18}$$

$$(1-\delta)\, I_t^g + I_t^d = \delta i_t. \tag{19}$$

Conditions (18) and (19) say that, at an optimum, the representative bank equates the marginal cost of deposits δi with the marginal revenue from an extra unit of deposits, which equals the revenues earned by either the additional loans produced by the incremental unit of deposits, $(1-\delta)\, I_t^l$, or the government bonds that are purchased, $(1-\delta)\, I_t^g$, plus the deposit spread $(I^d = i - i^d)$.

Equations (18) and (19) imply that

$$I_t^g = I_t^l. \tag{20}$$

This also implies that $i^l = i^g$, i.e., the lending rate to firms must equal the interest rate on government bonds. Intuitively, loans and government bonds are perfect substitutes in

for banks to play a non-trivial role in the credit-transmission mechanism. We chose the specification with no foreign borrowing because it is analytically simpler.

the bank's asset portfolio. Since the bank can get i^g by lending to the government it must receive at least as much from firms in order to extend loans to them. Hence, in an interior equilibrium (the only ones we will consider) any change in the policy-controlled interest rate, i^g, will automatically translate into a rise in the loan rate i^l. From equation (19), it is also easy to see that the deposit spread I^d is given by

$$i_t^d = (1-\delta)i_t^g. \tag{21}$$

Thus, *ceteris paribus*, a rise in the policy-controlled interest rate, i^g, must result in a higher deposit rate for consumers and, hence, an increase in demand deposits.

Lastly, we will restrict attention to parameter ranges for which I^d and I^l are non-negative. Thus, we will confine attention to environments where $i^d \leq i \leq i^g$. This restriction is needed to ensure a determinate demand for both loans and demand deposits. Note that this amounts to restricting the relevant interest rates to be in the range $0 \leq i^g - i \leq \delta i^g$.

3.4 Government

The government issues high powered money, $m\,(= h + \delta d)$, and domestic bonds, z^b, makes lump-sum transfers, τ, to the public, and sets the reserve requirement ratio δ on deposits. Domestic bonds are interest bearing and pay i^g per unit. Since we are focusing on flexible exchange rates, we assume with no loss of generality that the central bank's holdings of international reserves are zero. We assume that the government's transfers to the private sector are fixed exogenously at $\bar{\tau}$ for all t. Hence, the consolidated government's flow budget constraint is given by

$$\bar{\tau} + (i_t^g - \varepsilon_t)z_t^b = \dot{m}_t + \dot{z}_t^b + \varepsilon_t m_t. \tag{22}$$

As indicated by the LHS of (22), total expenditures consist of lump-sum transfers and the real debt service. These expenditures may be financed by issuing either high powered money or bonds and by the inflation tax, $\varepsilon_t m_t$. Lastly, the rate of growth of the nominal money supply is given by:

$$\frac{\dot{M}_t}{M_t} = \mu_t, \qquad M_0 \text{ given.} \tag{23}$$

The monetary authority has two policy instruments: (a) monetary policy which entails setting the rate of growth of the nominal money supply; and (b) interest rate policy which involves setting i^g (or alternatively, setting the composition of m and z^b and letting i^g be market determined). Given that lump-sum transfers are exogenously-given, only one of these two instruments can be chosen freely while the second gets determined through the government's flow constraint (equation (22)). Since the focus of this paper is on the effects of interest rate policy, we shall assume throughout that i^g is an actively chosen policy instrument. This implies that the rate of money growth μ adjusts endogenously so that equation (22) is satisfied.

3.5 Resource constraint

By combining the flow constraints for the consumer, the firm, the bank, and the government (equations (3), (13), (17) and (22)) and using equations (11) and (12), we get the economy's flow resource constraint:

$$\dot{k}_t = rk_t + y_t - c_t - v(h_t) - \psi(d_t). \tag{24}$$

where $k = b + b^f$. Note that the right hand side of equation (24) is simply the current account. Integrating forward subject to the no-Ponzi game yields

$$k_0 + \int_0^\infty [y_t - c_t - v(h_t) - \psi(d_t)]e^{-rt}dt = 0. \tag{25}$$

3.6 Equilibrium relations

The firm's optimality condition (equation (15)) implies that, in equilibrium, the real wage is given by

$$w_t = \frac{1}{1+\phi I_t^g}. \tag{26}$$

where we have used the fact that $I^l = I^g$ in equilibrium. Thus, a higher I^g makes bank credit more expensive for firms which increases production costs and, hence, reduces firms' demand for labor thereby lowering the real wage. Combining equations (6) and (26) yields

$$\nu\zeta x_t^{\nu-1} = \frac{1}{1+\phi I_t^g}, \tag{27}$$

which shows that a higher lending spread must reduce employment x. Equation (27) implies that equilibrium employment is given by

$$x_t = \left(\frac{1}{\nu\zeta}\right)^{\frac{1}{\nu-1}} \left(\frac{1}{1+\phi I_T^g}\right)^{\frac{1}{\nu-1}} \equiv \tilde{x}(I^g). \tag{28}$$

Since $n = \phi w x$, one can also derive the equilibrium amount of loans in this economy by combining equations (26) and (28):

$$n_t = \phi \left(\frac{1}{\nu\zeta}\right)^{\frac{1}{\nu-1}} \left(\frac{1}{1+\phi I_t^g}\right)^{\frac{\nu}{\nu-1}} \equiv \tilde{n}(I^g). \tag{29}$$

Thus, a rise in the lending spread induces a fall in both output and bank credit. Hence, a recession in this economy is characterized by a rise in the lending spread which, in turn, is linked one-for-one with the policy-controlled interest rate i^g.

The monetary base in this economy is given by $m = h + \delta d$, i.e., cash plus required reserves held by the banking system. Since $z^b = (1-\delta)d - n$ from the commercial banks' balance sheet, we can use the bank optimality condition $i^d = (1-\delta)i^g$ to rewrite equation (22) as

$$\bar{\tau} - \left(i_t^g - \varepsilon_t\right) n_t = \dot{h}_t + \varepsilon_t h_t + \dot{d}_t + \left(\varepsilon_t - i_t^d\right) d_t - \dot{n}_t. \tag{30}$$

The intuition behind this equation parallels that for equation (22), once it is noticed that, for a given h, n and z^b move in opposite direction. Hence, a flow expansion of loans to firms ($\dot{n} > 0$) decreases revenues as it implies a reduction in the flow expansion of government bonds. Similarly, for given h, a higher n implies a smaller stock of bonds which reduces the real debt service.

3.7 Perfect foresight stationary equilibrium

In what follows, we shall focus on stationary environments in which both the policy-controlled interest rate, i^g, and government transfers, $\bar{\tau}$, are constant for all t. Hence, we first derive the corresponding perfect foresight equilibrium path. Time-differentiating equations (7), (8), and (29), using (9) and (10), and substituting the results into (30) yields an equilibrium differential equation in ε:

$$\dot{\varepsilon}_t = \Gamma\left[\varepsilon_t \tilde{h}(i_t) + [\varepsilon_t - (1-\delta)i^g]\tilde{d}(I^d) + \left(i^g - \varepsilon_t\right)\tilde{n}(I^g) - \bar{\tau}\right], \tag{31}$$

where $\Gamma \equiv \left(-\tilde{n}' + \frac{1}{v''} + \frac{1}{\psi''}\right)^{-1} > 0$. (Recall that $i = r + \varepsilon$, $I^d = r + \varepsilon - (1-\delta)i^g$, and $I^g = i^g - r - \varepsilon$.) In deriving the above, we have used the fact that under stationary policies $\dot{i}^g = 0$.

It is easy to check that, in a local neighborhood of the steady state, equation (31) is an unstable differential equation if and only if $\left(1-\frac{\varepsilon}{r+\varepsilon}\eta_h\right)h+\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n>0$, where $\eta_h \equiv -\tilde{h}'i/h$ is (the absolute value of) the interest elasticity of cash, $\eta_d \equiv -\tilde{d}'I^d/d$ is (the absolute value of) the opportunity-cost elasticity of demand deposits, and $\eta_n \equiv -\tilde{n}'I^g/n$ is (the absolute value of) the interest elasticity of loans by firms (in general equilibrium).[14]

To understand this stability condition, note that in the steady state equation (30) reduces to

$$\bar{\tau} = \varepsilon_t h_t + \left(I^d - r\right)d + \left(I^g + r\right)n. \tag{32}$$

The expression $\left(1-\frac{\varepsilon}{r+\varepsilon}\eta_h\right)h+\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n$ is the effect of a change in ε on *net* government revenues. If all three elasticities are less than unity, then a rise in ε increases inflation tax revenues from cash and deposits (first two terms) and, for given h, increases the real debt service (third term) since an increase in ε reduces I^g. If this overall expression is positive, then equation (31) is unstable around the steady state. Hence, to ensure a unique convergent perfect foresight equilibrium path, we shall restrict attention to parameter ranges for which $\left(1-\frac{\varepsilon}{r+\varepsilon}\eta_h\right)h+\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n>0$. It follows that, along any perfect foresight equilibrium path with constant $\bar{\tau}$ and i^g, ε will also be constant over time. A constant ε and i^g imply that i, I^d and I^g must all be constant over time. This along with equations (5), (9), (10), (19), (27)and (29) imply that consumption, c, output x, cash demand, h, deposit demand, d, demand for loans, n and demand for bonds, z, must all remain constant as well. Lastly, the constancy of both h and d implies that money demand is constant over time.

Before proceeding further it is useful to note that the term $\left(1-\frac{\varepsilon}{r+\varepsilon}\eta_h\right)h+\left(1-\frac{I^d-r}{I^d}\eta_d\right)d$ reflects the well-known possibility of a Laffer curve relationship between revenues from money printing and the opportunity cost of holding money. As is standard, and to ensure that the economy is always operating on the "correct" side of the Laffer curve, we will assume throughout that $\left(1-\frac{I^d-r}{I^d}\eta_d\right)d+\left(1-\frac{\varepsilon}{r+\varepsilon}\eta_h\right)h>0$.

[14] To simplify the derivation of some results below, we will assume that both η_h and η_d are strictly increasing functions of their respective opportunity costs (i and I^d). This property is satisfied by, among others, Cagan money demands, which provide the best fit for developing countries (see Easterly, Mauro, Schmidt-Hebbel (1995)).

4 Exchange rates and interest rate policy

Having described the economy, we now turn to the central issue of the paper – the effects of interest rate policy on the nominal exchange rate. In other words, we want to ask the following questions: how does the nominal exchange rate change when the policy-controlled interest rate, i^g, changes? What is the relationship between the market interest rate, i, and the exchange rate? Our goal is to demonstrate that in the model just described, there is an inherent tendency for the relationship between interest rates (both i^g and i) and the exchange rate to be non-monotonic. In fact, we want to show the minimal elements that are needed to generate such non-monotonicities. To this effect, we will show that (i) in the presence of only one money (interest-bearing deposits), both the fiscal and the output effects are needed to generate a non-monotonic relationship between interest rates and the nominal exchange rate, and (ii) in the presence of two monies (cash and deposits), the fiscal effect is all that is needed to generate a non-monotonic relationship. As will be discussed below, these two cases illustrate the general principle that two sources of fiscal revenues are needed to generate a non-monotonic relationship between interest rates and exchange rates. In (i), the two sources of revenues are demand deposits and, indirectly, banking lending to firms (through its effect on banking lending to the government), whereas in (ii) the two sources of revenues are the two monies.

4.1 Case 1: The one-money case

In this one-money case, higher interest rates have both an output effect and a fiscal effect and both effects are key in generating the non-monotonic relationship between interest rates and exchange rates.[15] Formally, we set $v(h) \equiv 0$ for all h. Hence, the demand for cash is zero at all times (i.e., $\tilde{h}(i) \equiv 0$). This implies that the entire money base is held by the banking sector. In particular, $m = \delta d$. Hence, after setting $\tilde{h}(i) = 0$, the equilibrium differential equation (31) remains valid. It follows that, under our maintained assumption

[15] To show that, in this case, both the output and fiscal effects are needed, the appendix shows that, in the presence of only either one, no non-monotonicities arise. Hence, this case contains minimal elements to generate non-monotonicities in the one-money case.

that $\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-(1-\frac{I^g+r}{I^g}\eta_n)n>0$, the rate of devaluation remains constant along a convergent perfect foresight equilibrium path.[16]

After setting $\tilde{h}(i)=0$, we can totally differentiate (32) to implicitly solve for $I^d = \tilde{I}^d(I^g;\bar{\tau})$, where

$$\frac{\partial \tilde{I}^d}{\partial I^g} = -\frac{\left(1-\frac{I^g+r}{I^g}\eta_n\right)n}{\left(1-\frac{I^d-r}{I^d}\eta_d\right)d}. \tag{33}$$

The sign of this expression is ambiguous. It can be easily checked that $1 \gtreqless \frac{I^g+r}{I^g}\eta_n$ as $1+\phi I^g \lesseqgtr \nu(1-\phi r)$. Hence, if $1<\nu(1-\phi r)$, which is our maintained assumption, then $\frac{\partial \tilde{I}^d}{\partial I^g}<0$ for low values of I^g but $\frac{\partial \tilde{I}^d}{\partial I^g}>0$ for all $I^g > \left(\nu(1-\phi r)-1\right)/\phi \equiv \hat{I}^g$.

Substituting $I^d = \tilde{I}^d(I^g;\bar{\tau})$ into the bank's first order condition (19), we can also solve for the stationary depreciation rate ε as an implicit function of I^g, for a given $\bar{\tau}$, i.e., $\varepsilon = \tilde{\varepsilon}(I^g;\bar{\tau})$ where

$$\frac{\partial \tilde{\varepsilon}}{\partial I^g} = \frac{\left(1-\frac{I^d-r}{I^d}\eta_d\right)(1-\delta)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n}{\left(1-\frac{I^d-r}{I^d}\eta_d\right)\delta d}. \tag{34}$$

The sign of this expression is, in general, ambiguous. We shall return to this issue below.

Lastly, we can substitute $\tilde{\varepsilon}(I^g;\bar{\tau})$ into $I^g = i^g - r - \varepsilon$ to implicitly solve for $I^g = \tilde{I}^g(i^g;\bar{\tau})$ where

$$\frac{\partial \tilde{I}^g}{\partial i^g} = \frac{\left(1-\frac{I^d-r}{I^d}\eta_d\right)\delta d}{\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n} > 0. \tag{35}$$

The sign of this expression follows directly from our assumption $1>\frac{I^d-r}{I^d}\eta_d$ and the stability condition $\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n>0$. The key feature to note from equation (35) is that I^g is monotonically increasing in i^g. Hence, each i^g maps into a unique I^g.

Proposition 1 *For any given level of fiscal spending, $\bar{\tau}$, the stationary level of demand deposits bears a non-monotonic relationship with i^g. For all $i^g < (>)\ \hat{\imath}^g$ demand deposits rise (fall) as i^g rises.*

Proof. Note that demand deposits d are a strictly decreasing function of I^d. Now, $\frac{\partial \tilde{I}^d}{\partial i^g} = \frac{\partial \tilde{I}^d}{\partial I^g}\frac{\partial \tilde{I}^g}{\partial i^g} = \frac{-\left(1-\frac{I^g+r}{I^g}\eta_n\right)\delta n}{\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n}$. Also, $1 \gtreqless \frac{I^g+r}{I^g}\eta_n$ as $I^g \lesseqgtr \hat{I}^g \equiv \left(\nu(1-\phi r)-1\right)/\phi$. Define $\hat{\imath}^g$ such that $\hat{I}^g = \tilde{I}^g(\hat{\imath}^g;\bar{\tau})$. The proof then follows directly from the fact that $\partial I^g/\partial i^g > 0$. ■

[16] Also notice that, in this particular case, the condition ensuring that the economy is operating on the correct side of the Laffer curve reduces to $1-\frac{I^d-r}{I^d}\eta_d>0$.

Proposition 1 has important implications for the relationship between the policy-controlled interest rate, i^g, and the nominal exchange rate, E. In particular, it implies that there are three potential non-monotonicities in this relationship which we describe through the following proposition:

Proposition 2 *Along any perfect foresight equilibrium path, the relationship between the nominal exchange rate and the policy-controlled interest rate, i^g, is non-monotonic along three dimensions: (i) the initial level of the nominal exchange rate is falling or rising with i^g as $i^g \lesseqgtr \hat{\imath}^g$; (ii) the steady-state depreciation rate falls or rises with i^g as $i^g \lesseqgtr \bar{\imath}^g$ where $\bar{\imath}^g < \hat{\imath}^g$; and (iii) in the range $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$, a rise in i^g appreciates the currency on impact but depreciates it at some point in the future.*

Proof. See appendix. ■

Figure 4 illustrates part (i) of this proposition. Specifically, the initial level of the exchange rate, E_0, is a U-shaped function of the policy-controlled interest rate, i^g, with the minimum being reached at $i^g = \hat{\imath}^g$. The intuition is as follows. Remember that the opportunity cost of demand deposits is $I^d \equiv r + \varepsilon - i^d$. A rise in i^g, in and of itself, increases the deposit rate, i^d – recall (21) – and therefore tends to reduce I^d and appreciate the currency. A rising i^g, however, also raises I^g which induces a fall in bank credit to firms, n. This effect tends to reduce fiscal revenues because the counterpart of a falling n is an increase in z^b (i.e., an increase in liabilities of the central bank held by commercial banks), which increases the government's debt service. In order to finance this fall in revenues, the inflation rate (i.e., the rate of depreciation) must increase. This effect tends to increase I^d. For all $i^g > \hat{\imath}^g$, the higher debt service overwhelms the increase in the deposit rate, and further increases in i^g actually raise I^d.

Figure 4 also illustrates part (ii) of this proposition, by showing that the market interest rate, $i(= r + \varepsilon)$, is also a U-shaped function of i^g. For $i^g < \bar{\imath}^g$, the direct effect on revenues of an increase in i^g (due to a higher demand for real demand deposits) is so large that it facilitates a cut in the inflation tax. However, for $i^g > \bar{\imath}^g$ the indirect effect of a fall in n becomes large enough to require an increase in ε (or equivalently, the rate of money growth μ) in order to finance fiscal spending.

Aside from the non-monotonicity of both the initial level of the exchange rate and the steady-state depreciation rate, Proposition 2 also shows that an increase in the policy-controlled interest rate often induces an intertemporal trade-off in the path of the nominal exchange rate. In particular, the instantaneous appreciation of the currency that is generated by a higher i^g comes at the cost of a more depreciated level of the nominal exchange rate at some time in the future (relative to the path with a lower stationary i^g). This occurs for values of $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$ because in that range, as Figure 4 illustrates, a rise in i^g reduces E_0 but increases the rate of depreciation.

The results of Proposition 2 imply that the relationship between the nominal exchange rate and the market interest rate is highly non-linear, as highlighted in the following proposition:

Proposition 3 *For $i^g < \bar{\imath}^g$ and $i^g > \hat{\imath}^g$, changes in i^g induce a positive comovement between the nominal interest rate (i) and the initial level of the nominal exchange rate (E_0). In the range $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$ however, changes in i^g induce a negative comovement between these two variables.*

Proof. The proof follows directly from Proposition 2 once we note that $i = r + \varepsilon$. ■

Proposition 3 can be easily visualized using Figure 4. For "low" values of i^g (i.e., $i^g < \bar{\imath}^g$), both the initial exchange rate (E_0) and the market interest rate (i) fall (positive comovement). For "high" values of i^g (i.e., $i^g > \hat{\imath}^g$), both increase (also positive comovement). In contrast, for "intermediate" values of i^g, (i.e., $\bar{\imath}^g < i^g < \hat{\imath}^g$), E_0 falls while i increases, indicating a negative comovement. This proposition has extremely important implications for tests of the relationship between the nominal exchange rate and the nominal interest rate. In particular, the model predicts that market interest rates and the nominal exchange rate may be positively or negative associated, so one should not, in general, be able to detect any systematic pattern in statistics such as correlations or linear regression coefficients.

4.2 Case 2: The two-money case

We now turn to our second special case of the general model. Here, we reintroduce a transactions role for cash so that $s_t = v(h_t) + \psi(d_t)$. Thus, this economy has two liquid assets – cash and demand deposits. However, we now assume that $\phi = 0$ so that loan demand by firms is zero (i.e., $\tilde{n}(I^g) \equiv 0$). Hence, there is no output effect of higher interest rates. With $\phi = 0$, the real wage is unity at all times (recall that we have assumed a linear production technology) while output is given by (recall equation (28))

$$x_t = \left(\frac{1}{\nu\zeta}\right)^{\frac{1}{\nu-1}}.$$

After setting $\tilde{n}(I^g) = 0$, the equilibrium differential equation (31) continues to be valid. Hence, under the assumption that $\left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right) h + \left(1 - \frac{I^d - r}{I^d}\eta_d\right) d > 0$, the rate of devaluation remains constant along a convergent perfect foresight equilibrium path.[17]

By setting $\tilde{n}(I^g) \equiv 0$, equation (32) can be used to solve for the rate of currency depreciation as a function of i^g, for a given $\bar{\tau}$, i.e., $\varepsilon = \tilde{\varepsilon}(i^g; \bar{\tau})$. Moreover,

$$\frac{\partial \varepsilon}{\partial i^g} = \frac{(1-\delta)\left(1 - \frac{I^d - r}{I^d}\eta_d\right) d}{\left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right) h + \left(1 - \frac{I^d - r}{I^d}\eta_d\right) d} > 0, \tag{36}$$

where the sign follows from our maintained assumptions of $1 > \frac{I^d - r}{I^d}\eta_d$ and $\left(1 - \frac{I^d - r}{I^d}\eta_d\right) d + \left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right) h > 0$. The rate of depreciation, ε, is thus a strictly increasing function of i^g. In particular, equation (36) captures the inflationary consequences of increasing the policy-controlled interest rate (the so-called "fiscal effect").

Next notice that $I^d = r + \varepsilon - (1-\delta)i^g$ implies that

$$\frac{\partial I^d}{\partial i^g} = \frac{-(1-\delta)\left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right) h}{\left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right) h + \left(1 - \frac{I^d - r}{I^d}\eta_d\right) d}. \tag{37}$$

As long as $1 > \frac{\varepsilon}{r+\varepsilon}\eta_h$, the opportunity cost of holding demand deposits, I^d, falls with i^g as the direct effect of paying higher interest on money dominates the indirect effect through a higher ε. However, if $1 < \frac{\varepsilon}{r+\varepsilon}\eta_h$, the inflationary consequences are so large that an increase in i^g actually *increases* the opportunity cost of holding demand deposits.

To determine the effect of higher interest rates on the nominal exchange rate, we start by recalling that money demand is given by $m = h + \delta d$. Using equations (9) and (10), we obtain, after some manipulations,

$$\frac{\partial m}{\partial i^g} = \chi \left[\frac{\eta_d - \eta_h}{\eta_d} - \frac{r(1-\delta)i^g}{(r+\varepsilon)I^d}\eta_h + \frac{(1-\delta)I^g}{I^d}\left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right)\right], \tag{38}$$

[17] To be consistent with the one-money case, we will also maintain the assumption that $\left(1 - \frac{I^d - r}{I^d}\eta_d\right) d > 0$.

where $\chi \equiv \frac{(1-\delta)hd\eta_d}{\left[\left(1-\frac{\varepsilon}{r+\varepsilon}\eta_h\right)h+\left(1-\frac{I^d-r}{I^d}\eta_d\right)d\right]i} > 0$.

Since ε is always rising in i^g, demand for cash (h) is always falling in i^g. Hence, m must necessarily fall with a higher i^g if demand deposits are non-increasing in i^g. Noting that the interest elasticity of cash η_h is a function of the nominal interest rate $i = r + \varepsilon$, define $\bar{\imath}^g$ by the relation $\eta_h(r + \tilde{\varepsilon}(\bar{\imath}^g, \bar{\tau})) = \frac{r+\tilde{\varepsilon}(\bar{\imath}^g,\bar{\tau})}{\tilde{\varepsilon}(\bar{\imath}^g,\bar{\tau})}$. Hence, from equation (37) we have $\left.\frac{\partial I^d}{\partial i^g}\right|_{i^g=\bar{\imath}^g} = 0$. But this implies that $\left.\frac{\partial d}{\partial i^g}\right|_{i^g=\bar{\imath}^g} = 0$ and $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\bar{\imath}^g} < 0$.

We will now assume that the demands for cash and liquid bonds satisfy the following condition:

Condition 1: $\eta_h(r + \tilde{\varepsilon}(0, \bar{\tau})) < \delta\eta_d(r + \tilde{\varepsilon}(0, \bar{\tau}))$.

This condition requires that for "low" inflation rates (i.e., inflation rates corresponding to a non-activist interest rate policy), the interest elasticity of cash be lower than that of liquid bonds (adjusted by the reserve requirement ratio). The idea is that cash is kept mainly for transactions and is therefore relatively interest inelastic for low inflation rates. This intuition is consistent with the evidence for the United States provided in Moore, Porter, and Small (1990).

We can now state the main proposition of this section:

Proposition 4 *Under condition 1, the initial nominal exchange rate is a non-monotonic (U-shaped) function of the policy-controlled interest rate, i^g. In particular there exists an $\hat{\imath}^g \in (0, \bar{\imath}^g)$ such that $\frac{\partial E_0}{\partial i^g} \lesseqgtr 0$ as $i^g \lesseqgtr \hat{\imath}^g$.*

Proof. See appendix. ■

This proposition shows that, as in the previous case, the initial level of the exchange rate is a U-shaped function of the policy-controlled interest rate. Intuitively, for low values of i^g, the positive money demand effect dominates the fiscal effect (i.e., the inflationary consequences of a higher i^g). Beyond a certain point, however, further increases in i^g have such a large impact on the rate of inflation that money demand begins to fall and hence the currency depreciates. The role of condition 1 is to ensure that, around $i^g = 0$, the demand for cash falls by less than the amount by which demand for bank deposits rises, so that overall real money demand increases.[18]

[18] If condition 1 is not satisfied, then an increase in i^g would always lead to a *depreciation* of the currency.

Given (36), Proposition 4 implies that, for $i^g < \bar{\imath}^g$, a rise in i^g appreciates the currency on impact but increases the depreciation rate. Hence, there is again an intertemporal trade-off in the sense that a higher i^g buys a more appreciated currency in the short-run at the expense of a more depreciated currency in the future.

The next proposition addresses the relationship between the initial level of the exchange rate and the market interest rate.

Proposition 5 *For $i^g < \bar{\imath}^g$, changes in i^g induce a negative comovement between the nominal interest rate (i) and the initial level of the nominal exchange rate (E_0). For $i^g > \bar{\imath}^g$, the comovement is positive.*

Proof. Follows immediately from equation (36) and Proposition 4. ■

Since i is a strictly increasing function of i^g (recall (36)), this last proposition indicates that for low values of i, increases in i will be associated with an appreciation of the currency, whereas for high values of i, increases in i will be associated with a depreciation of the currency. Once again, the empirical implication of this proposition is that one should not expect to find a linear relationship between market interest rates and the exchange rate.

5 Interest rate defense in response to a fiscal shock

To sharpen the presentation, the above analysis has focused on how the perfect foresight equilibrium path varies for different values of the policy-controlled interest rate, i^g. In terms of actual policymaking, however, the more natural way to think about this set-up is to assume that, initially (i.e., for $t < 0$), the economy is in some stationary steady state and that at $t = 0$ there is an unanticipated and permanent fiscal shock (i.e., an increase in $\bar{\tau}$).[19] This section reinterprets our previous results in this light and, in addition, briefly discusses the optimal interest rate defense.

[19] Note that since $\bar{\tau}$ can be interpreted as the "permanent" level of fiscal transfers, the fiscal shock can be thought of as referring to an increase in the present discounted value of fiscal transfers at $t = 0$. This could correspond to an increase in *future* fiscal deficits which, according to Burnside, Eichenbaum, and Rebelo (1998), was the relevant shock in the case of the Asian crises.

5.1 Positive implications

Given the instability of differential equation (31), it follows that the rate of depreciation will jump instantaneously to its new steady-state in response to an unanticipated and permanent increase in $\bar{\tau}$. Using (32), it follows that under a passive interest rate policy (i.e., no change in i^g), the change in the rate of depreciation is given by:

$$\frac{\partial \varepsilon}{\partial \bar{\tau}} = \frac{1}{\left(1 - \frac{\varepsilon}{r+\varepsilon}\eta_h\right) h + \left(1 - \frac{I^d - r}{I^d}\eta_d\right) d - \left(1 - \frac{I^g + r}{I^g}\eta_n\right) n} > 0.$$

Since i^g does not change, I^d rises. Hence, both the demand for cash and demand deposits fall, which implies that the exchange rate increases on impact (i.e., the currency depreciates).

The question would then be: can policymakers offset some of the depreciation by raising the policy-controlled interest rate? Here one can apply the previous analysis by just reinterpreting the initial value of the exchange rate, E_0, as the value of the exchange rate that would prevail after the shock if $i^g = 0$. The answer would then be that there is a certain rise in i^g that would maximize by how much policymakers can offset this depreciation (given, in the one-money case, by $\hat{\imath}^g$ in Figure 4). Hence, a small increase in the policy-controlled interest rates *always* lead to the conventional effect. Further increases, however, will lead to a perverse effect, whereby further increases in i^g depreciate the currency.

5.2 Normative implications

Having established that, in response to an unanticipated fiscal shock at $t = 0$, policymakers can "defend" the currency with an active interest rate policy, the natural question is whether it is optimal to do so. It can be shown that, in general, the fact that it is feasible to defend the currency does not necessarily imply that it is optimal to do so. In particular, for the one-money case, it can be shown that the optimal policy response to an unanticipated fiscal shock involves raising the policy-controlled interest rate to partially offset the depreciation while still letting the currency depreciate relative to its pre-shock value. There is in fact a whole range of increases in i^g for which it is *feasible* to appreciate the currency further but *not optimal* to do so.[20] Hence, it is always optimal to engage in some active interest rate defense of the currency in response to a negative shock. This is an interesting result because it provides a formal rationalization of the "fear of floating" phenomenon discussed by Calvo and Reinhart (2000a). In this set-up, however, the "reluctance" to let the exchange fully adjust reflects an optimal monetary policy response on the part of policymakers.

6 Conclusions

In the aftermath of the Asian crises, the relationship between higher interest rates and the exchange rate has become the focus of a spirited policy debate. This paper has developed a simple model to rationalize the mixed and often conflicting results that have been obtained by a large body of empirical work in this area. We have shown – for two particular cases of the general model – how the relationship between changes in interest rates (both the interest rate controlled by policymakers as well as the market-determined interest rate) and the level of the exchange rate is inherently non-monotonic. Hence, there is no reason to expect to find in the data a linear (or even monotonic) relationship between exchange rates and interest rates. This calls into question the usefulness of estimating non-structural models (typically VARs) to assess this relationship and suggests that estimating structural models may be the only sensible avenue.

To make our points as sharply as possible, we have focused on two particular cases of what – as should be clear – is a general property of this class of monetary models. Our two cases illustrate the fact that if the government has two sources of revenues, then a non-monotonic relationship is to be expected as an increase in the policy-controlled interest rate will have fiscal effects, either directly (as in the two-money case) or indirectly (through the output effect, as in the one-money case). The specific way in which these two sources of revenue are modeled might differ, but the same principle should hold. For instance, one could imagine an economy in which the second source of revenues are proceeds from, say, an income tax. Then, an active interest rate defense, which pushes the economy into

[20] The formal derivations can be found in a previous version of this paper, available from the authors upon request.

a contraction, would reduce income tax revenues and necessitate of a higher inflation tax. The same non-monotonic results would then go through. In sum, we believe that these non-monotonic results are quite general and not specific to the particular formulation that we may have chosen.

Appendices

A One money-case

This appendix shows that, in the one-money case discussed in the text, having only the fiscal effect or the output effect is not enough to generate non-monotonicities.

A.1 Output effect only

It is easy to see that there cannot be any non-monotonicity in the exchange rate response to interest rate changes without a fiscal effect in the model. Suppose that the rate of money growth μ is exogenous while fiscal spending is endogenous, i.e., the fiscal authority reacts by adjusting spending τ in order to balance the budget equation (30). Since $i^d = (1-\delta)i^g$, the deposit rate is rising linearly in i^g. Moreover, the rate of currency depreciation is equal to the stationary rate of money growth μ at all times. Hence, the deposit spread I^d is falling linearly in i^g. Thus, money demand is always rising in i^g, while the nominal exchange rate is always falling in i^g (i.e., the currency always appreciates in response to a higher i^g). Hence, the output effect, by itself, is insufficient to generate any non-monotonicity in the relationship between interest rates and exchange rates.

A.2 Fiscal effect only

Suppose firms do not face a credit-in-advance constraint for the wage bill, i.e., $\phi = 0$. Without the loan demand term (and $h \equiv 0$), equation (32) reduces to $\bar{\tau} = \left(I_t^d - r\right) d_t$. Since $d = \tilde{d}(I^d)$ from equation (10), it is immediately obvious that the deposit spread is fully determined by $\bar{\tau}$. Crucially, here i^g does not have any independent effect on I^d. Hence, money demand and the initial level of the nominal exchange rate are independent of i^g. It is easy to check that in this case we must have $\frac{\partial \varepsilon}{\partial i^g} = \frac{\partial i}{\partial i^g} = 1 - \delta > 0$. Hence, raising the policy-controlled interest rate only generates a larger fiscal burden which always increases the *future* exchange rate but leaves the *initial* exchange rate unchanged. The preceding implies that without an output effect the model does not generate any non-monotonicities between the exchange rate and the interest rate.

B Proof of proposition 2

(i) Since the nominal money supply is given at time 0, any change in m_0 due to a change in I_0^d has to be accommodated by a change in the exchange rate E_0 in the opposite direction. The proof then follows directly from Proposition 1.

(ii) $\frac{\partial \varepsilon}{\partial i^g} = \frac{\partial \tilde{\varepsilon}}{\partial I^g}\frac{\partial \tilde{I}^g}{\partial i^g} = 1 - \frac{\left(1-\frac{I^d-r}{I^d}\eta_d\right)\delta d}{\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n}$. From Proposition 1, $\frac{\partial \varepsilon}{\partial i^g}\Big|_{i^g=\hat{\imath}^g} = 1-\delta > 0$. Since I^g and I^d are both rising in i^g for $i^g > \hat{\imath}^g$, equation (19) directly implies that $\frac{\partial \varepsilon}{\partial i^g} > 0$ in this range. It is easy to verify that $\frac{\partial \eta_d}{\partial I^d} > 0$ (a maintained assumption) is a sufficient condition for $\frac{\left(1-\frac{I^d-r}{I^d}\eta_d\right)\delta d}{\left(1-\frac{I^d-r}{I^d}\eta_d\right)d-\left(1-\frac{I^g+r}{I^g}\eta_n\right)n}$ to be decreasing in i^g for $i^g < \hat{\imath}^g$. Hence, by arguments of continuity, there exists an $\bar{\imath}^g < \hat{\imath}^g$ such that $\frac{\partial \varepsilon}{\partial i^g} \gtreqless 0$ for all $i^g \gtreqless \bar{\imath}^g$.[21]

(iii) From (i) and (ii) we know that for $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$ an increase in i^g appreciates the currency on impact but also increases the steady-state depreciation rate.

C Proof of proposition 4

Since $\frac{\partial m}{\partial i^g}\Big|_{i^g=\bar{\imath}^g} < 0$, the proof of the non-monotonicity of m in i^g hinges on showing that $\frac{\partial m}{\partial i^g}\Big|_{i^g=0} > 0$. First, note that since ε is increasing in i^g from equation (36) and $\frac{\partial \eta_h}{\partial i} > 0$, we must have $\eta_h(r+\tilde{\varepsilon}(0,\bar{\tau})) < \frac{r+\tilde{\varepsilon}(0,\bar{\tau})}{\tilde{\varepsilon}(0,\bar{\tau})}$. Hence, $\frac{\partial I^d}{\partial i^g}\Big|_{i^g=0} < 0$. But this implies that $\frac{\partial d}{\partial i^g}\Big|_{i^g=0} > 0$. Noting that $I^d = i = -I^g$ around $i^g = 0$, it is easy to check that equation (38) gives

$$\frac{\partial m}{\partial i^g}\bigg|_{i^g=0} = \chi\left[\delta - \frac{\eta_h(r+\tilde{\varepsilon}(0,\bar{\tau}))}{\eta_d(r+\tilde{\varepsilon}(0,\bar{\tau}))} + (1-\delta)\frac{\tilde{\varepsilon}(0,\bar{\tau})}{r+\tilde{\varepsilon}(0,\bar{\tau})}\eta_h(r+\tilde{\varepsilon}(0,\bar{\tau})\right].$$

Hence, condition 1 is sufficient for $\frac{\partial m}{\partial i^g}\Big|_{i^g=0} > 0$. Moreover, $\frac{\partial \varepsilon}{\partial i^g} > 0$ and $\frac{\partial \eta_h}{\partial i} > 0$ jointly imply that $\frac{\tilde{\varepsilon}(i^g,\bar{\tau})}{r+\tilde{\varepsilon}(i^g,\bar{\tau})}\eta_h(r+\tilde{\varepsilon}(i^g,\bar{\tau}))$ is rising in i^g. Hence, by arguments of continuity, there must exist an $\hat{\imath}^g$ such that $\frac{\partial m}{\partial i^g}\Big|_{i^g=\hat{\imath}^g} = 0$. The proof is completed by noting that $E_0 = M_0/m$ and that nominal money supply at time 0 is given (see equation (23)). Hence, E_0 moves inversely with m.

[21] It is fairly easy to restrict parameters such that $\bar{I}^g > 0$ where $\bar{I}^g = \tilde{I}^g(\bar{\imath}^g, \bar{\tau})$. This restriction is necessary to guarantee a non-monotonicity of ε within the permissible range of I^g.

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Figure 1: Nominal exchange rate response to interest rate innovations

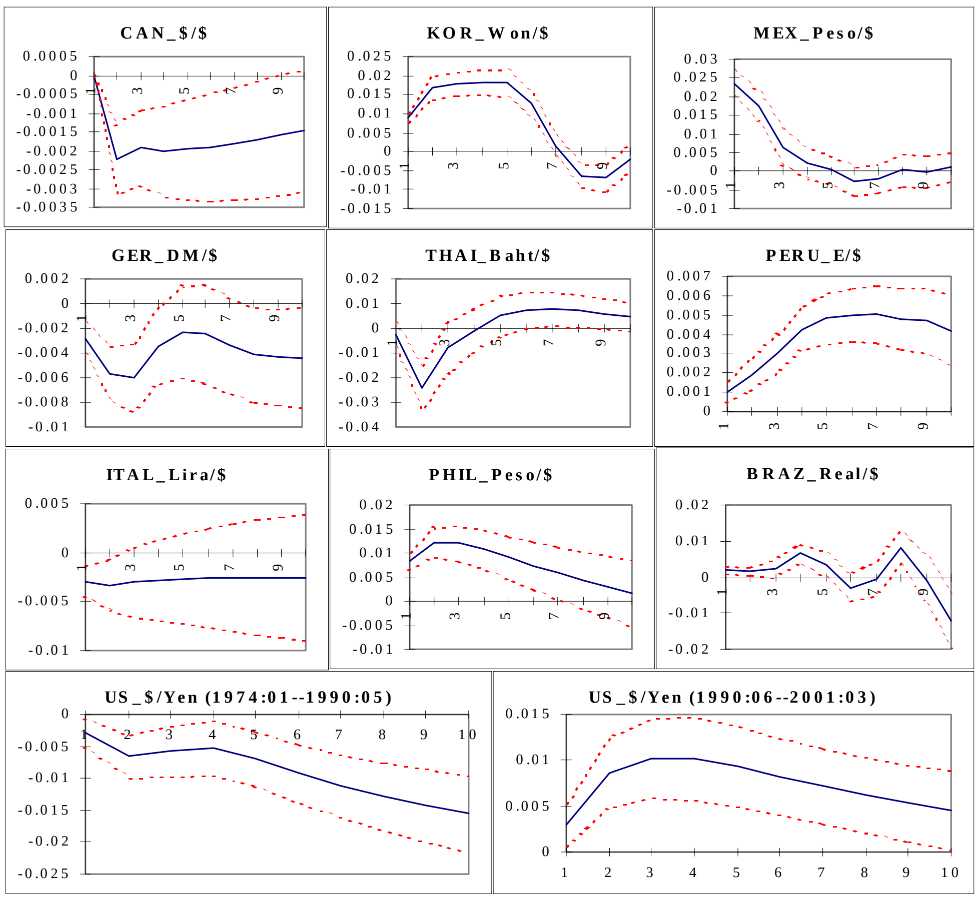

Figure 2: Industrial production response to interest rate innovations

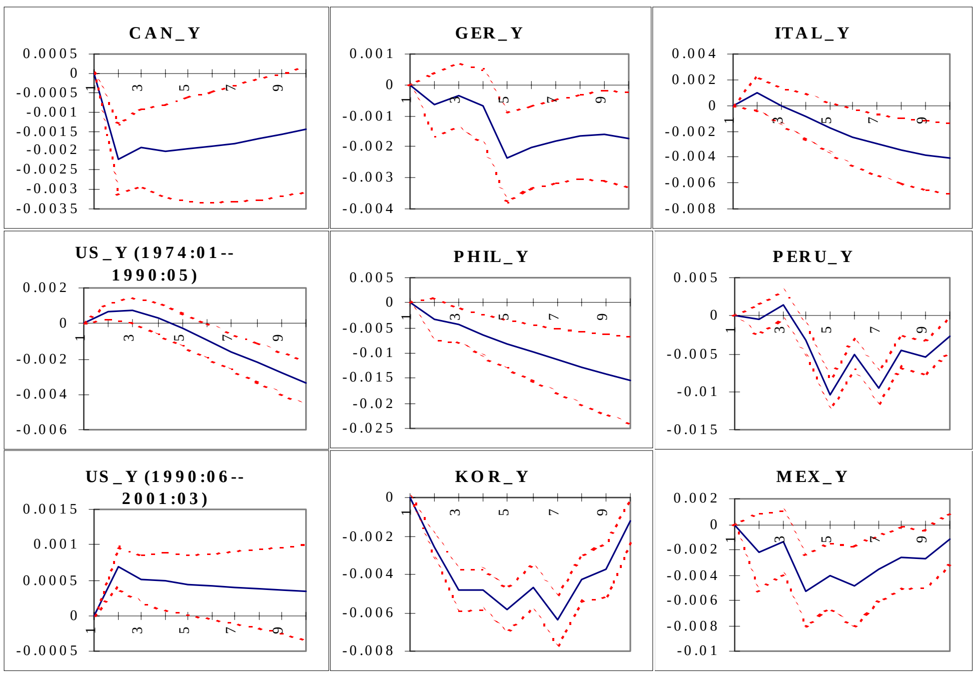

Figure 3: Fiscal balance response to interest rate innovations

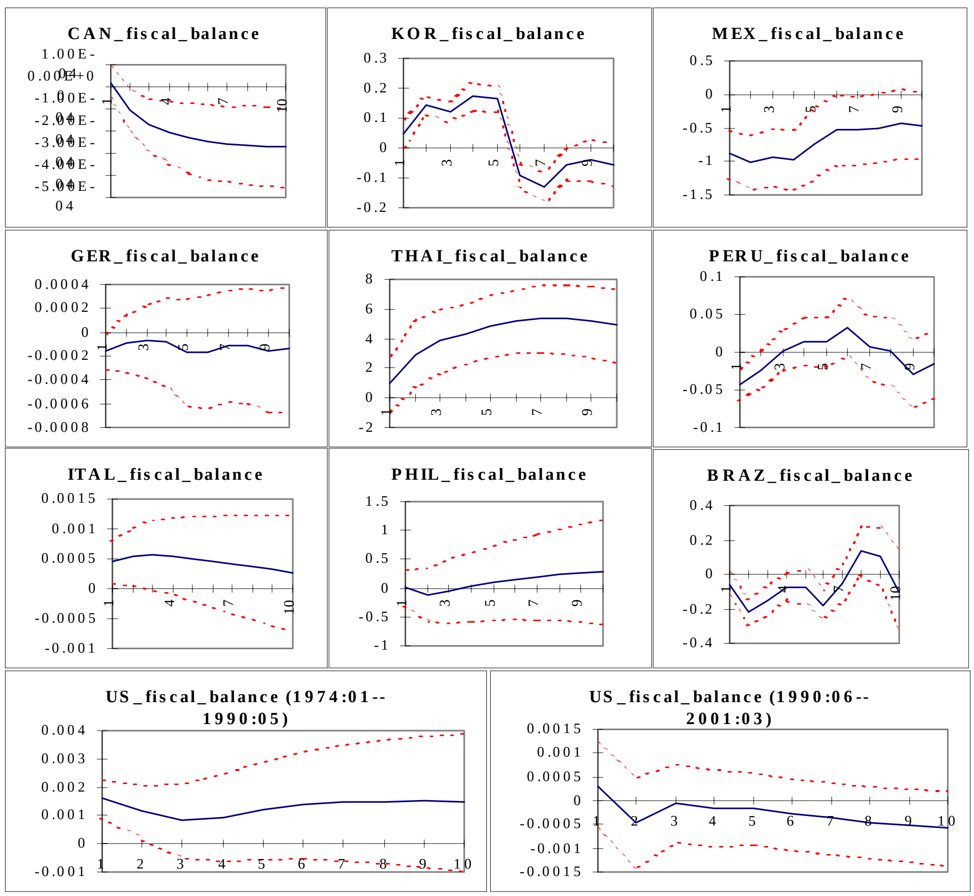

Figure 4: Initial exchange rate and the nominal interest rate