Delaying the Inevitable: Optimal Interest Rate Policy and BOP Crises*

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Abstract

The classical model of balance of payments crises implicitly assumes that the central bank sits passively as international reserves dwindle. In practice, however, central banks typically defend pegs aggressively by raising short-term interest rates. This paper analyzes the feasibility and optimality of raising interest rates to delay a potential BOP crisis. Interest rate policy works through two distinct channels. By raising demand for domestic, interest-bearing liquid assets, higher interest rates tend to delay the crisis. Higher interest rates, however, increase public debt service and imply higher future inflation, which tends to bring forward the crisis. We show that, under certain conditions, it is feasible to delay the crisis, but raising interest rates beyond a certain point may actually hasten the crisis. A similar non-monotonic relationship emerges between welfare and the increase in interest rates. It is thus optimal to engage in some active interest rate defense but only up to a certain point. In fact, there is a whole range of interest rate increases for which it is feasible to delay the crisis but not optimal to do so.

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1 Introduction

In the aftermath of the currency crises in Europe (1992), Mexico (1994), Asia (1997), Russia (1998) and, more recently, Brazil (January 1999), there has been a renewed interest in further understanding the mechanics of balance of payments (BOP) crises. For the better part of the last two decades, the profession's view of BOP crises has essentially followed the view espoused by Paul Krugman in his 1979 seminal paper. Under a fixed exchange rate regime, any attempt to maintain a fixed exchange rate for the domestic currency while simultaneously expanding domestic credit to finance a fiscal deficit introduces a fundamental inconsistency into the system. Since real money demand is given, any attempt to monetize a given fiscal deficit under a fixed exchange rate regime will lead to a continuous loss of international reserves. If reserves cannot fall below a certain threshold, then countries pursuing such policies will inevitably be forced to abandon the peg at some point. Krugman (1979) showed how, at the time of the crisis, a speculative attack will deplete the central bank's reserves. This is a remarkable feature since the attack occurs in a context in which the public has rational expectations and can thus perfectly anticipate all future events.[1]

Another notable — but seldom discussed — feature of Krugman-type models is that the central bank is a rather defenseless agent. It sits passively as it watches the public hurrying to the central bank window and taking home its international reserves. In this paradigm, the central bank is – like Gulliver in his unfortunate encounter with the Lilliputians – a sleeping giant caught by surprise and unable to react. In reality, of course, central banks are hardly sleeping giants. Quite to the contrary, they typically fight long and hard before giving up a peg.

The key weapon in the central banks' arsenal is the use of short-term interest rates. When reserves begin to fall, central banks raise interest rates to often very high levels to induce investors to hold on to domestic currency denominated assets rather than switching to foreign currency denominated assets (see IMF (1997)). During the ERM crisis, for example, Sweden's monetary authority pushed overnight interest rates to around 500 percent per year. In October 1997, and in the midst of the Asian turmoil, Brazil's central bank moved forcefully to double already high interest rates to defend the peg. Interest rates were also raised sharply in Brazil in the aftermath

[1] Apart from Krugman (1979), a few other notable papers which develop this line of thinking include Calvo (1987), Flood and Garber (1984), and Obstfeld (1984, 1986). Extensions and modifications of the basic Krugman model are reviewed in Calvo and Végh (1999) and Flood and Marion (1998).

of the Russian crisis. In practice, therefore, rather than sitting still and let events unfold, central banks are actively engaged in defending the peg by raising key short-term interest rates. In fact, higher interest rates to defend and/or strengthen the currency have been a typical component of IMF programs.[2]

Naturally, even when it succeeds in defending a peg – at least temporarily – a high interest rate policy is not without costs. The main costs – as reflected in countless policy discussions – fall into three main areas. First, high interest rates increase the debt service of public deft, thus raising the fiscal deficit. This issue was particularly evident in the case of Brazil. As Brazil pushed interest rates to 50 percent in the fall of 1997 to defend the peg (instituted in July 1994 in the context of the Real stabilization plan), the fiscal deficit jumped from about 4 percent to 8 percent, in large part due to the rapid build-up of interest payments on public debt. Second, higher interest rates are likely to lead to an economic contraction, which could even worsen the fiscal situation by reducing tax revenues. Third, higher interest rates may aggravate the financial situation of an already weak or poorly regulated banking sector.[3] Policy disagreements over the virtues of raising interest rates to defend a peg ultimately come down to an implicit assessment of the benefits (currency stability) versus the costs (typically

[2] On IMF policies, it is worth quoting from Stan Fischer's (1998) remarks on recent IMF-supported programs in Asia: "Are the programs too tough? In weighing this question, it is important to recall that when they approached the IMF, the reserves of Thailand and Korea were perilously low, and the Indonesian rupiah was excessively depreciated. Thus, the first order of business was, and still is, to restore confidence in the currency. To achieve this, countries have to make it more attractive to hold domestic currency, which, in turn, requires increasing interest rates temporarily, even if higher interest costs complicate the situation of weak banks and corporations. This is a key lesson of the tequila crisis in Latin America 1994-95, as well as from the more recent experience of Brazil, the Czech Republic, Hong Kong and Russia, all of which have fended off attacks on their currencies in recent months with a timely and forceful tightening of interest rates along with other supporting policy measures. Once confidence is restored, interest rates can return to more normal levels."

Jeff Sachs, in particular, has been a loud critic of the IMF's high interest rates policies in Asia. In a New York Times article (November 3, 1997), for example, he states that "[the] International Monetary Fund has just announced a second bailout package for the region, about $20 billion for Indonesia. That should, in principal, boost confidence. But if it is tied to orthodox financial conditions, including budget cuts and sharply higher interest rates, the package could do more harm than good, transforming a currency crisis into a rip-roaring economic downturn."

[3] During 1994, Mexican policymakers were very reluctant in raising interest rates to fend off the loss of reserves because of the perceived fragility of the banking system (see, for instance, Sachs, Tornell, and Velasco (1996)).

some combination of the three factors just mentioned).

For all its practical importance, formal analyses of the effectiveness and desirability of using high interest rates to defend an exchange rate peg has received scant attention in the literature. This surprising neglect may be partly explained by the difficulties that the profession has faced when it comes to thinking about interest rate policy (i.e., about central banks using interest rates as policy instruments). As is well-known from Sargent and Wallace (1975), controlling – or targeting – nominal interest rates in standard monetary models leads to price level indeterminacy. Far from "solving" the problem, sticky prices only lead to a higher-order indeterminacy (Calvo (1983)). While the literature has come up with different ways of dealing with this problem (see, for instance, Auernheimer and Contreras (1993), Calvo and Végh (1995, 1996), McCallum (1981), Reinhart (1992), and Woodford (1995)), it is fair to say that the profession lacks a widely-accepted framework to think about interest rates as policy instruments. Still, as we await a wider consensus on this issue, it would seem important to have a simple analytical framework to think about the use of higher interest rates to fight speculative attacks against the domestic currency. We could then pose the following questions: given a peg which is not consistent with fundamentals, can higher interest rates delay a balance of payment crisis? If so, by how long? Is it optimal to delay a crisis?

The main purpose of this paper is to develop such a framework and provide some answers to the questions just raised. To this end, we incorporate interest rate policy in an otherwise standard, optimizing, small open economy prone to Krugman-type crises. Following Calvo and Végh (1995, 1996), we sidestep all indeterminacy problems by thinking of interest rate policy as the central bank's ability to set the interest rate on an interest-bearing liability (say, a liquid bond). If this liquid bond is an imperfect substitute for cash in the public's liquid portfolio, then the central bank can use this interest rate as an *additional* policy instrument. In other words, the central bank can – in addition to setting the exchange rate– set this interest rate and thus affect the total demand for liquid assets. Raising the interest rate on this liquid bond will, other things being equal, reduce the opportunity cost of holding it and thus increase money demand (defined as the demand for cash and the liquid bond). In this framework, therefore, interest rate policy amounts to paying interest on part of the money supply.[4] This seems like

[4] Of course, in practice, this link may be more indirect. For instance, if commercial banks hold a large fraction of their portfolio as interest-bearing liabilities of the government – as is often the case in emerging markets – then the interest rate paid by banks on their liabilities will be heavily affected by the interest rate they receive on government bonds.

a natural way to think about interest rate policy in the context of currency crises since, in practice, the rationale for raising interest rates is precisely to increase the attractiveness of domestic currency denominated assets.

In our framework, higher interest rates work through two main channels: a money demand effect and a fiscal effect. The money demand effect refers to the fact that higher interest rates on domestic liquid bonds lead, other things being equal, to a higher demand for liquid bonds and, hence, for domestic assets. This mechanism enables the monetary authority to postpone the crisis. There is, however, no such a thing as a free lunch. Higher interest rates on liquid public debt imply a higher debt service burden, which will need to be financed by higher inflation in the future (the fiscal effect). The expectation of higher inflation in the future tends to bring forward the crisis by reducing the amount of liquid domestic assets that the public wishes to hold once the crisis occurs. The fiscal consequences of higher interest rates will thus tend to offset the beneficial effect of higher interest rates in delaying the crisis.[5] [6]

The first question that we ask concerns the *feasibility* of defending a peg: can higher interest rates postpone a crisis? And, if so, for how long? We show that – under the most plausible range for money demand elasticities – raising interest rates will indeed postpone the crisis. As interest rates become higher, however, the fiscal effect mentioned above will at some point begin to dominate and further raising interest rates beyond a certain point will actually hasten the crisis. There is thus a *non-monotonic* relationship between the increase in interest rates and the time of the crisis. In other words, there is a certain increase in interest rates that will maximize the delay of the crisis. Hence, our analysis shows that the central bank can in principle buy precious time by raising interest rates which, in practice, would give the fiscal authority the chance to put its house in order. Raising interest rates too much, however, would be self-defeating.

Naturally, the ability to delay a costly crisis by raising interest rates

[5] In this paper, we abstract from the other two potential costs of higher interest rates on output and the banking system mentioned above. Obviously, this is not to say that we believe the other two effects are unimportant. As a methodological question, however, we believe that as a first pass it is necessary to tackle one cost at a time to understand the fundamental issues involved. In a companion paper (Lahiri and Végh (1999)), we abstract from the fiscal effect and focus on the output costs of higher interest rate policy.

[6] It should be noted that our framework falls squarely within the so-called "first-generation" models of BOP crises. We thus totally abstract from other – potentially important – effects of higher interest rates, such as signalling the willingness of policymakers to defend a peg. In recent work, Drazen (1999) focuses on such "second-generation" considerations.

does not necessarily mean that it is optimal to do so. Our second question is thus: is it optimal (i.e., welfare maximizing) to defend a peg and delay the crisis? The answer is "yes, but not too much." We first show that it is optimal for the monetary authority to engage in *some* active interest rate defense, as opposed to sitting still and doing nothing (as implied by the Krugman case). In other words, starting from an equilibrium in which the central bank is totally passive, raising interest rates is welfare improving. However, the relationship between the increase in interest rates and welfare is non-monotonic. Hence, raising interest rates beyond a certain point will begin to reduce welfare. In fact, we show that there is a whole range of interest rate increases for which it is feasible to further delay the crisis but not optimal do so. From a policy point of view, our results thus suggest that it may be optimal for central banks to engage in *some* interest rate defense. However, raising interest rates too much may be counterproductive due to the resulting future inflationary effects.

The paper proceeds as follows. Section 2 presents the model. For conceptual clarity, Section 3 abstract from the fiscal effect and focuses exclusively on the money demand effect by assuming an endogenous path of government transfers. Section 4 derives the key results of the paper by combining the fiscal and the money demand effects. Section 5 concludes.

2 The model

Consider a small open economy that is perfectly integrated with the rest of the world in both goods and capital markets. The economy is inhabited by an infinitely-lived representative consumer who receives a constant flow endowment y of a perishable good. The consumer derives utility from consuming this good and from holding two liquid assets: non-interest bearing money and a (non-traded) liquid bond. The world price of the good in terms of the foreign currency is given and assumed to be unity. Free mobility of goods across borders implies that the law of one price holds. The consumer can also buy and sell a pure (i.e., non-liquid) bond in perfectly competitive world capital markets. The bonds are denominated in terms of the single good and pay r units of the good as interest at every instant.

2.1 Consumer

The representative consumer's lifetime utility (W) is given by

$$W \equiv \int_0^\infty \left[u(c_t) + v(h_t) + w(z_t)\right] e^{-\beta t} dt, \tag{1}$$

where c denotes consumption; h denotes holdings of non-interest bearing money (which we will refer to as "cash"); z are holdings of the liquid bond; $\beta(> 0)$ is the rate of time preference; and $u(.)$, $v(.)$, and $w(.)$ are all strictly increasing and strictly concave functions.[7]

The consumer can also hold an internationally-traded bond (b). Real financial wealth at time t is thus given by $a_t = b_t + h_t + z_t$. The nominal interest rate on the liquid bond is denoted by i^g. The consumer's flow budget constraint is given by

$$\dot{a}_t = ra_t + y + \tau_t - c_t - i_t h_t - (i_t - i_t^g) z_t, \tag{2}$$

where τ_t denotes lump-sum transfers from the government and i_t is the nominal interest rate borne by the non-liquid bond in terms of domestic currency.

Integrating (2) and imposing the transversality condition $\lim_{t \to \infty} a_t e^{-rt} = 0$ yields the consumer's lifetime budget constraint:

$$a_0 + \frac{y}{r} + \int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty \left[c_t + i_t h_t + (i_t - i_t^g) z_t\right] e^{-rt} dt. \tag{3}$$

The consumer chooses perfect foresight paths for $\{c_t, h_t, z_t\}$ to maximize lifetime utility (1) subject to (3), taking as given τ, i^g, i, r, y, and a_0. Assuming interior solutions, the first-order conditions are given by [8]

$$\begin{aligned} u'(c_t) &= \lambda, & (4) \\ v'(h_t) &= \lambda i_t, & (5) \\ w'(z_t) &= \lambda (i_t - i_t^g), & (6) \end{aligned}$$

[7] There is by now abundant evidence on the importance of liquid financial assets other than cash (such as very short-term government debt, indexed bonds, and foreign currency deposits, among others) in satisfying part of households' liquidity needs in developing countries. See, for example, Arrau, De Gregorio, Reinhart, and Wickham (1995), Easterly, Mauro, and Schmidt-Hebbel (1995) and Savastano (1996). Alternatively, as explained below, liquid bonds could be interpreted as interest-bearing demand deposits (say, money market accounts) held in banks.

[8] As usual, we assume that $\beta = r$ to eliminate inessential dynamics.

where λ is the (time-invariant) Lagrange multiplier associated with constraint (3).[9] Equations (4), (5), and (6) implicitly define the demand for cash and liquid bonds, given by:

$$h_t = \tilde{h}(\underset{+}{c_t}, \underset{-}{i_t}), \tag{7}$$

$$z_t = \tilde{z}(\underset{+}{c_t}, \underset{-}{i_t - i_t^g}), \tag{8}$$

where a sign under a variable denotes its partial derivative. Equation (7) is the standard demand for non-interest bearing money, which depends positively on consumption and negatively on the opportunity cost of holding real cash balances, given by i. Equation (8) captures the demands for liquid bonds, whose opportunity cost is given by the interest rate differential $i - i^g$.

In this model, we will think of "money" as the sum of cash and liquid bonds. Formally, $m \equiv h + z$. Hence, using (4), (7), and (8), the demand for money can be written as

$$m_t = \tilde{h}(c_t, i_t) + \tilde{z}(c_t, i_t - i_t^g) \equiv \tilde{m}(\underset{+}{c_t}, \underset{-}{i_t}, \underset{+}{i_t^g}). \tag{9}$$

Notice, from (9), that money demand is a decreasing function of i but an *increasing* function of i^g. Hence, an increase in i^g will, other things being equal, increase the demand for liquid assets denominated in domestic currency. As will become clear below, the ability to affect the demand for liquid assets will enable policymakers to mount an active defense of an exchange rate peg.

2.2 Government constraints

The government comprises the monetary and the fiscal authority. For formal simplicity, it will be assumed that the monetary authority issues both cash and liquid bonds.[10] The monetary authority also pays interest on these

[9] It will be assumed throughout the paper that $i - i^g \geq 0$. This ensures an interior solution in which all three assets – pure bonds, liquid bonds, and cash – are held. Of course, for $i - i^g = 0$ to be a feasible equilibrium, preferences must be such that the consumer can be satiated with liquid bonds.

[10] An alternative (and formally identical) set-up is one in which liquid bonds are interpreted as interest-bearing demand deposits and the monetary authority issues only non-interest bearing money but requires banks to holds 100 percent reserves. By paying interest on these reserves, the central bank would indirectly control the interest rate paid by the banks on demand deposits.

liquid bonds and holds interest-bearing foreign exchange reserves. The fiscal authority makes lump-sum transfers to the public. The government's budget constraint is thus given by

$$\dot{R}_t = rR_t + \dot{h}_t + \dot{z}_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t - \tau_t, \tag{10}$$

where R is the government's (monetary authority's) stock of net foreign assets (i.e., international reserves). Integrating forward (10) and imposing the transversality condition $\lim_{t\to\infty} R_t e^{-rt} = 0$ yields the government's intertemporal budget constraint:

$$\int_0^\infty \tau_t e^{-rt} dt = R_0 + \int_0^\infty [\dot{h}_t + \dot{z}_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t]e^{-rt}dt + \triangle(h_T + z_T)e^{-rT}, \tag{11}$$

where the last term on the right-hand side (RHS) allows for the possibility of a discrete change in real money balances at some time $t = T$.[11] Equation (11) makes clear that the government must finance the present discounted value of transfers (left-hand side (LHS)) with the initial stock of international reserves and the present discounted value of proceeds from money creation (RHS). The inflation tax is given by $\varepsilon_t h_t$ in the case of cash and $(\varepsilon_t - i_t^g)z_t$ in the case of liquid bonds. For further reference, note that constraint (11) can be simplified to read (imposing the additional transversality condition $\lim_{t\to\infty} m_t e^{-rt} = 0$):

$$\int_0^\infty \tau_t e^{-rt} dt = R_0 - (h_0 + z_0) + \int_0^\infty [i_t h_t + (i_t - i_t^g)z_t]e^{-rt}dt. \tag{12}$$

Denoting by μ_t the rate of growth of domestic credit, it follows that:

$$\frac{\dot{D}_t}{D_t} = \mu_t, \tag{13}$$

where D denotes nominal domestic credit. Let E denote the nominal exchange rate; that is, the price of foreign currency in terms of domestic currency. From the central bank's balance sheet, $\dot{R}_t = \dot{m}_t - \dot{d}_t$, where $d = D/E$.

[11] Throughout the paper, we denote a discrete change in, say, variable x as $\Delta x_T \equiv x_T - x_{T^-}$. Of course, if real money balances were to jump at other times as well, this should also be accounted for. Equation (12) below, however, would not change. We also assume that $R_0 > 0$.

Further, note that $\dot{d}_t = (\mu_t - \varepsilon_t)d_t$. Using these two facts, equation (10) yields the path of government transfers:

$$\tau_t = rR_t + (\mu_t - \varepsilon_t)d_t + \varepsilon_t h_t + (\varepsilon_t - i_t^g)z_t. \tag{14}$$

So far, we have only looked at accounting identities. At this point, we would need to take a stand on whether (i) the monetary authority moves first – by setting an exogenous path of μ – and the fiscal authority passively accommodates such a path by letting transfers adjust, or (ii) the fiscal authority moves first – by setting an exogenous path of transfers – and the monetary authority accommodates this policy by letting the rate of domestic credit growth adjust. Clearly, the latter assumption is the most relevant for policy purposes. For conceptual purposes, however, we will begin our analysis in Section 3 by examining the first scenario. Having this simpler scenario as a convenient benchmark, Section 4 will look at the empirically more relevant case in which the fiscal authority sets an exogenous path of transfers.

2.3 Equilibrium conditions

Combining the consumer's and the government's flow constraints (equations (2) and (10), respectively) yields:

$$\dot{k}_t = rk_t + y - c_t, \tag{15}$$

where $k(\equiv b + R)$ denotes the economy's stock of net foreign assets. The RHS of equation (15) is the current account balance. Combining (3) and (12) yields the economy's resource constraint:

$$k_0 + \frac{y}{r} = \int_0^\infty c_t e^{-rt} dt. \tag{16}$$

Equations (4) and (16) imply that, along a perfect foresight equilibrium path, consumption will be constant and equal to permanent income:

$$c_t = rk_0 + y, \qquad t \in [0, \infty). \tag{17}$$

In fact, consumption will remain equal to permanent income *regardless* of the path of i and i^g. Therefore, to economize on notation – and with no loss of generality – we will henceforth choose units such that $u'(rk_0 + y) = 1$, which implies from (4) that $\lambda = 1$. We can then drop the consumption term from all money demands.[12]

Finally, given the assumption of perfect capital mobility, interest parity holds (recall that foreign inflation is assumed to be zero):

$$i_t = r + \varepsilon_t. \tag{18}$$

2.4 Exchange rate and interest rate policy

Exchange rate policy The exchange rate policy followed by the monetary authority is the same as in standard models à la Krugman. As of $t = 0$, the exchange rate is fixed at the level $\bar{E}$. Hence, by (18), $i_t = r$. In addition, it is assumed that there is a known lower bound for international reserves (say, $R_t = 0$). If that level is reached, the central bank ceases to intervene in the foreign exchange market and allows the exchange rate to float freely.

Interest rate policy The key departure of our model relative to the standard Krugman model is that the monetary authority can engage in *active interest rate policy*. In other words, in addition to fixing the exchange rate, the monetary authority can also set the interest rate on liquid bonds, i^g. The monetary authority has thus two policy instruments: the nominal exchange rate and the interest rate on liquid bonds. By setting i^g, the monetary authority lets the *composition* of its liabilities (cash and liquid bonds) be market-determined.[13]

To see more clearly the logic behind active interest rate policy, note that the central bank issues both cash (H) and liquid bonds (Z). Hence, money market equilibrium is given by (taking into account (9) and (18))

$$\frac{H_t + Z_t}{E_t} = \tilde{m}(c_t, r, i_t^g). \tag{19}$$

The second relevant equilibrium condition is the relative demand for cash and liquid bonds, which follows from (5) and (6):

[12] Naturally, if consumption and real money balances did not enter separably into preferences, the path of consumption would not be independent of i and i^g. These effects will be captured in simulations below.

[13] Alternatively, the monetary authority could set the ratio of both liabilities and let i^g be market-determined.

$$\frac{v'(h_t)}{w'(z_t)} = \frac{i_t}{i_t - i_t^g}. \tag{20}$$

Take as given the level of consumption. Then, for a given level of the exchange rate and i^g set by the monetary authority, the private sector chooses real money demand which, through (19), determines the nominal money supply $(H + Z)$. Given this level of the nominal money supply, equation (20) determines the levels of H and Z. Other things being equal, a rise in i^g increases real money demand by inducing a higher real demand for liquid bonds.[14] In this model, therefore, an active interest rate defense of the peg will involve raising the policy-controlled interest rate in order to increase the demand for domestic-currency denominated assets.

An alternative way of thinking about interest rate policy in this model – which is in the spirit of traditional portfolio models (see, for example, Flood, Garber and Kramer (1995)) – is to derive an interest parity condition between liquid bonds and pure bonds by combining (5) and (6) to obtain:

$$i_t^g = i_t - \left[\frac{w'(z_t)}{v'(h_t)}\right] i_t. \tag{21}$$

This condition says that, in equilibrium, the return on the liquid bond issued by the central bank (i^g) must equal the return on the pure bond (i_t) minus a liquidity premium (last term on the RHS). For a given level of cash, the liquidity premium is a decreasing function of the level of liquid bonds.

Condition (21) is illuminating in several respects. First, it makes clear that the assumption that the bonds issued by the central bank provide liquidity services is a critical one. If they did not (i.e., if $w'(z_t) = 0$), then i^g could not be different from r (assuming, of course, an interior solution). Second, given that these bonds provide liquidity services, interest rate policy operates by inducing changes in the liquidity premium. In other words, a higher i^g must be associated with a lower liquidity premium and viceversa.

Interest rate policy rule In practice, the monetary authority raises interest rates in response to shocks that may induce the public to switch from domestic to foreign assets. By making domestic assets more attractive, the

[14]Under non-separability between h and z in the utility function (a case dealt with below), a rise in i^g leads to a rise in z and a fall in h but still increases real money demand (i.e., $h + z$).

monetary authority hopes to protect its stock of international reserves. In this spirit – and to capture an active interest rate defense in as simple a setting as possible – it will be assumed that the central bank announces at time $t = 0$ that if the nominal interest rate i changes at any point in time t, then it will adjust i^g according to the following rule:

$$\triangle i_t^g = \gamma \triangle i_t, \qquad \gamma \in [0, \gamma^*], \tag{22}$$

where γ is a policy parameter that captures the degree of interest rate activism.[15] In particular, $\gamma = 0$ is the standard case analyzed in the literature in which the central bank follows a completely passive interest rate policy. At the other extreme, the monetary authority can set a value of γ, denoted by γ^*, such that money demand will *not* change even if i rises.[16] Intuitively, it should be clear that $\gamma^* > 1$. The reason is that in response to a rise in i, demand for cash will always fall. Hence, to leave real money demand unchanged, the opportunity cost of holding bonds needs to fall in order to induce consumers to hold more liquid bonds. This, in turn, requires that $\gamma > 1$, so that the opportunity cost of holding liquid bonds falls as i rises (i.e., $i_t - i_t^g$ falls). While highly stylized, policy rule (22) provides an easy and convenient way of parameterizing interest rate policy, which will enable us to study the effectiveness and optimality of more active interest rate policies in defending a fixed exchange rate peg.

3 Interest rate policy in the absence of an exogenous fiscal constraint

As a convenient benchmark, this section analyzes the effectiveness and optimality of active interest rate policy in the case in which the monetary authority sets a constant rate of growth of domestic credit, μ, and the fiscal authority passively accommodates such a policy by letting the level of transfers adjust endogenously.

[15] We restrict γ to be non-negative to fix ideas. In principle, γ could be negative. When such a case is economically relevant, it will be discussed below. Also, the initial level of the policy controlled interest rate (i_0^g) is inconsequential for our analysis (as long as it is less than r) because it only affects the initial level of real money demand through its effect on the initial demand for liquid bonds.

[16] As will become clear below, while γ^* provides an upper bound for γ, in some cases it is not possible for the monetary authority to set such a value of γ.

3.1 Equilibrium paths

Consider first the equilibrium path associated with a fixed exchange rate. By interest parity (given by (18)), $i_t = r$. Since i is constant over time, policy rule (22) implies that i^g will also be constant over time. Hence, $i - i^g$ will also be constant over time. This, in turn, implies that real money demand, given by (9) will also be constant over time. From the central bank balance sheet, it follows that $\dot{R}_t = \dot{m}_t - \dot{d}_t$. The fixed exchange rate combined with the domestic credit policy given by (13) implies that $\dot{R}_t = \dot{m}_t - \mu d_t$. Since money demand is constant along paths with a constant i (i.e., $\dot{m}_t = 0$), the equilibrium evolution of international reserves is given by

$$\dot{R}_t = -\mu d_0 e^{\mu t}. \tag{23}$$

Equation (23) shows that along paths with a fixed exchange rate and expanding domestic credit (i.e., $\mu > 0$), international reserves at the central bank will be falling at an increasing rate. Since the lower bound for international reserves will be reached in finite time, the fixed exchange rate regime is unsustainable. The central bank will thus be forced to abandon the peg at some point in time, T (to be determined endogenously). Private agents expect the central bank to allow the exchange rate to float from time T onward, while leaving its domestic credit policy unchanged. Hence they expect that at time T the economy will jump to its long run steady-state with the domestic currency depreciating at the constant rate of monetary expansion, μ.[17] Formally, the expected path for the rate of devaluation/depreciation is given by

$$\varepsilon_t = \begin{cases} 0, & 0 \leq t < T, \\ \mu, & t \geq T. \end{cases}$$

The private sector also knows that from time T onward the nominal interest rate will be given by $i_T = r + \mu$. Furthermore, given policy rule (22), it is also the case that $i_T^g = i_0^g + \gamma\mu$. Hence, $i_T - i_T^g = r - i_0^g + (1 - \gamma)\mu$.

Given the time paths of i_t and i_t^g, we can now determine γ^*; that is, the value of γ for which real money demand at T will not jump. Formally, γ^* satisfies

[17] This is a perfect foresight monetary model with no intrinsic dynamics. It can be easily shown that the perfect foresight equilibrium path under flexible exchange rates is stationary with the rate of currency depreciation, ε, being equal to the rate of monetary expansion, μ.

$$\tilde{h}(r) - \tilde{h}(r+\mu) = \tilde{z}[r - i_0^g + (1-\gamma^*)\mu] - \tilde{z}(r - i_0^g). \qquad (24)$$

For a given $\mu(>0)$,the LHS of equation (24) is a constant, whereas the RHS is a strictly increasing function of γ (and lower than the LHS for $\gamma = 0$). Hence, γ^* exists and is unique.[18]

3.2 The timing of the crisis

In order to tie down the time of collapse, note that at T the exchange rate (E) cannot jump. If it did, there would be infinite arbitrage opportunities. Since the exchange rate cannot jump at T, money market equilibrium at T is given by

$$\tilde{h}(r+\mu) + \tilde{z}[r - i_0^g + (1-\gamma)\mu] = \frac{D_0 e^{\mu T}}{\bar{E}}, \qquad (25)$$

where the LHS of equation (25) denotes real money demand at time T, whereas the RHS indicates real money supply at time T. Equation (25) implicitly defines the time of collapse T as a function of γ. In what follows, we denote all pre-collapse variables by the subscript 0 and post-collapse variables by the subscript T. Letting T^- denote the instant before the run, the discrete change in real money demand at the moment of the crisis T is given by

$$\Delta m_T \equiv m_T - m_{T^-} = \tilde{m}(r+\mu, i_T^g) - \tilde{m}(r, i_0^g) \leq 0, \qquad (26)$$

which corresponds to the loss in international reserves since $\Delta R_T = \Delta m_T$.[19] The weak inequality in (26) follows from the fact that, as discussed above, real money demand will remain unchanged for γ^*. To fix ideas, we now focus on two polar cases: $\gamma = 0$ and $\gamma = \gamma^*$.

[18] This statement assumes that the condition $i_T - i_T^g = r - i_0^g + (1-\gamma^*) \geq 0$ is satisfied. In other words, it assumes that $\gamma^* \leq \hat{\gamma}$, where $\hat{\gamma}$ is the value of γ that satisfies $r - i_0^g + (1-\hat{\gamma}) = 0$ (i.e, $\hat{\gamma} = 1 + \frac{r - i_0^g}{\mu}$). As will be illustrated below with a quadratic example, whether $\gamma^* \leq \hat{\gamma}$ holds depends – in addition to the specification of preferences – on the parameter configuration. If this condition does not hold, all our main results go through anyway, but there is a limit to how much the central bank can raise interest rates. For expositional purposes, this section will assume that this condition holds unless otherwise noticed.

[19] Note that from time T onward, real domestic credit remains constant since D and E both rise at the common rate μ.

Passive interest rate policy: The Krugman case Consider first the standard case analyzed in the literature in which the central bank is completely passive with respect to interest rate policy. This case – hereafter referred to as the Krugman case – arises as a special case of our model when $\gamma = 0$. In this event, the central bank leaves i^g unchanged in response to an increase in market interest rates. The size of the speculative attack at time T is thus given by

$$\Delta m_T = \tilde{m}(r + \mu,\ i_0^g) - \tilde{m}(r,\ i_0^g) < 0.$$

Since the economy locks into its long run steady-state at time T, both consumption and real money balances remain constant from time T onward. At the time of the crisis, the rise in i leads to a fall in both real cash and real liquid bond demand, as follows from first-order conditions (5) and (6). Hence, real money demand falls. As is well-known from Krugman (1979), the speculative attack will thus occur at that point in time – denoted by T^k in Figure 1 – at which the outstanding stock of international reserves exactly matches the desired change in real money demand. At that point, a speculative run depletes the stock of international reserves and forces the central bank to abandon the peg and let the exchange rate float. Clearly, welfare is lower relative to a non-crisis scenario since holdings of both cash and liquid bonds fall at T.[20]

Active interest rate policy: Crisis with no run Consider now the same experiment as before except for the fact that $\gamma = \gamma^*$. Given perfect foresight and the definition of γ^*, it follows that real money demand will not change at T. The fact that m does not jump at the time of the crisis implies that the crisis must take place precisely at the point in time at which international reserves reach zero (time T^* in Figure 1). Hence, the mere announcement of policy (22) with $\gamma = \gamma^*$ at time 0 enables the central bank to delay the crisis by as long as it is possible under the circumstances (from T^k to T^* in Figure 1). In other words, even though there is a crisis and the exchange rate peg is abandoned at time T^*, there is no run on international reserves!

The intuition behind the central bank's ability to delay the crisis is straightforward. By announcing that it will raise i^g in such a way as to keep total demand for money constant, the central bank is ensuring that, at

[20] Of course, if preferences were non-separable in consumption, the fall in consumption would also be welfare-reducing. We will take into account such effects in simulations presented below.

the time of the crisis, there will be no run on international reserves. This implies that the only consistent equilibrium path is for the crisis to occur at point T^* in Figure 1. Since reserves reach zero at that point, the central bank's abandonment of the peg is consistent with no change in real money demand and, hence, no run.

It is clear that welfare is higher in this case than in the Krugman's case. The reason is that the fall in real cash balances at T will be the same but holdings of real liquid bonds will actually increase. Hence, it is optimal for the monetary authority to engage in an active interest rate policy.

Finally, notice that when the crisis occurs the level of government transfers must fall to accommodate the increased debt service. To see this, notice from (14) that, given that reserves do not jump at T, the level of transfers just before the crisis occurs is $\tau_{T^-} = \mu d_T - i_0^g z_0$. After the crisis has occurred, the level of transfers is given by $\tau_T = \mu d_T - z_T(i_0^g + \gamma^* \mu)$ (notice that $i_T^g = i_0^g + \gamma^* \mu$). The change in the level of transfers is thus $\Delta\tau \equiv \tau_T - \tau_{T^-} = -i_T^g z_T + i_0^g z_0 = i_0^g(z_0 - z_T) - \gamma^* \mu z_T < 0$. The level of transfers must fall at T to accommodate the higher fiscal deficit which results from a higher debt service. The debt service increases on two accounts: the stock of debt increases (from z_0 to z_T) and the interest rate raises from i_0^g to i_T^g (notice that $i_T^g - i_0^g = \gamma^* \mu$). From a policy perspective, this implies that the monetary authority's ability to delay the crisis as much as it can critically depends on the fiscal authority being ready to effect a sharp adjustment at the time of the crisis.

Intermediate cases What would happen in the intermediate cases in which γ lies between 0 and γ^* (i.e., $0 < \gamma < \gamma^*$)? An analogous reasoning to the Krugman case leads to the conclusion that, *qualitatively*, the results would be the same as in the Krugman case. In other words, since real money demand will fall at the time of the crisis, a run must accompany the crisis. Welfare, however, will still be higher than in the Krugman case because the post-crisis level of liquid bonds will be larger. The more relevant questions, however, are whether a more aggressive interest rate policy (i.e., a higher γ) will always succeed in postponing the crisis and, if so, whether such a policy is optimal. The next two subsections provide answers to these questions.

3.3 Effectiveness of active interest rate policy

The first question to ask regarding active interest rate policy is whether a tighter interest rate policy always succeeds in postponing the time of the crisis. In other words, is there a monotonic and positive relationship between the time of the crisis (T) and the degree of policy activism (γ)?

The answer to this question follows directly from equation (25). This equation implicitly defines T as a strictly increasing function of γ; that is, $T = \tilde{T}(\gamma)$. The slope of this function is given by

$$\tilde{T}'(\gamma) = \frac{-\tilde{z}'[r - i_0^g + (1-\gamma)\mu]}{d_0 e^{\mu T}} > 0. \tag{27}$$

Furthermore, in terms of the notation used in Figure 1, notice that $\tilde{T}(0) = T^k$ and $\tilde{T}(\gamma^*) = T^*$. We collect these results in the following proposition:

Proposition 1 *In the absence of an exogenous fiscal constraint, the timing of the crisis is a strictly increasing function of the degree of interest rate policy activism (i.e., $\tilde{T}'(\gamma) > 0$) . Furthermore, $\tilde{T}(0) = T^k$ and $\tilde{T}(\gamma^*) = T^k$.*

The intuition behind this result follows the logic discussed above. The more aggressive is interest rate policy (i.e., the higher is γ), the smaller will be the desired change in real money demand when the crisis occurs, because the change in the opportunity cost of holding liquid bonds will be smaller. Since the desired change in real money demand determines the size of the speculative attack – and hence the time of the crisis – a higher γ implies that the size of the speculative attack will be smaller and hence that it will occur later.[21]

3.4 Optimality of active interest rate policy

Having established that a more active interest rate policy always succeeds in postponing the crisis, the natural follow-up question is whether it is optimal to do so. Formally, we ask the following question: is there a monotonic and positive relationship between consumer's welfare and the degree of policy activism (as captured by γ)?

To answer this question, substitute into the welfare function (1) the reduced-form solutions for demand for cash balances and liquid bonds (ignoring the terms involving consumption which are simply a constant) to

[21] As suggested above, γ could be greater than γ^* or negative. If $\gamma < 0$, then the crisis would be brought forward relative to the Krugman case. In terms of Figure 1, it would occur for $T < T^k$. The intuition is simply that the central bank's interest rate policy now *reduces* money demand at the time of the crisis. If $\gamma > \gamma^*$, the crisis would still occur at time T^*, but the central bank would need to allow for a once-and-for-all remonetization of the economy as real money demand would now increase once the crisis occurs.

obtain

$$W[\gamma, \tilde{T}(\gamma)] = \frac{(1 - e^{-r\tilde{T}(\gamma)})}{r}[v(h_0) + w(z_0)] + \frac{e^{-r\tilde{T}(\gamma)}}{r}[v(h_T) + w(z_T)], \tag{28}$$

where

$$\begin{aligned} h_0 &= \tilde{h}(r), && (29) \\ z_0 &= \tilde{z}(r - i_0^g), && (30) \\ h_T &= \tilde{h}(r + \mu), && (31) \\ z_T &= \tilde{z}[r - i_0^g + (1 - \gamma)\mu]. && (32) \end{aligned}$$

Since W is a function of γ directly as well as indirectly through T, it follows that

$$\frac{dW}{d\gamma} = \frac{\partial W}{\partial \gamma} + \frac{\partial W}{\partial \tilde{T}}\tilde{T}'(\gamma). \tag{33}$$

As (33) makes clear, γ affects welfare through two channels. First, for a given T, a higher γ raises the post-crisis demand for liquid bonds and hence increases welfare (first term on the RHS of (33)). In effect, as follows directly from (28), $\frac{\partial W}{\partial \gamma} \geq 0$ (with the equality holding only for $\gamma = \hat{\gamma}$). Second, for a given real demand for bonds, a higher γ postpones the crisis (i.e., $\tilde{T}'(\gamma) > 0$, as established in Proposition 1). In turn, a larger T should increase welfare because consumers enjoy a higher level of real money balances for a longer period of time. This is obvious for the case in which $\gamma \leq 1$, because in that case demand for cash falls at T while demand for bonds either falls or remains constant. It is less obvious for values of $\gamma > 1$, as in this case the demand for cash falls at T but the demand for liquid bonds increases. We show in the appendix, however, that as long as the real demand for money falls (or remains constant) at T, the utility derived from holding liquidity also falls. In other words, we formally show that $\frac{\partial W}{\partial T} > 0$. Given that welfare is a strictly increasing function of γ, the optimal interest rate policy is to set $\gamma = \gamma^*$. In other words, welfare is maximized by delaying the balance of payment crisis as much as possible. Notice, incidentally, that the worst possible policy is not to increase interest rates at all (i.e., the Krugman case).[22]

[22] If $\hat{\gamma} < \gamma^*$, then it is optimal to set $\gamma = \hat{\gamma}$; that is, to raise interest rate as much as possible.

In this light, we can summarize the welfare results in the following proposition:

Proposition 2 *In the absence of an exogenous fiscal constraint, consumer's welfare is a strictly increasing function of the degree of interest rate activism at the time of the crisis. The optimal interest rate policy is to prevent a run from taking place when the peg is abandoned.*

3.5 A quadratic example

To illustrate some of the results just derived, it shall prove useful to consider the following quadratic example. Suppose that $v(h)$ and $w(z)$ are given by

$$v(h) = -\frac{G}{2}(\bar{h} - h)^2, \tag{34}$$

$$w(z) = -\frac{B}{2}(\bar{z} - z)^2. \tag{35}$$

Taking into account first-order conditions (5) and (6) (recalling that units have been defined such that $\lambda = 1$), the demands for cash and liquid bonds are given by:

$$h = \bar{h} - \frac{i}{G}, \tag{36}$$

$$z = \bar{z} - \frac{(i - i^g)}{B}. \tag{37}$$

Furthermore, for simplicity, we will assume that $G = B$ and $i_0^g = 0$. Given (36) and (37), the pre- and post-crisis money demands take the form (notice that in this case $i_T - i_T^g = r + (1-\gamma)\mu$):

$$m_0 = \bar{h} + \bar{z} - 2\frac{r}{G}, \tag{38}$$

$$m_T = \bar{h} + \bar{z} - 2\frac{r}{G} - \frac{\mu}{G}(2 - \gamma). \tag{39}$$

As expected, m_T is an increasing function of γ because a higher γ makes liquid bonds more attractive.

The first payoff of this example is that it provides a clear illustration of the relationship between γ^* and $\hat{\gamma}$. Since γ^* is the value of γ for which real money demand does not change at T (i.e., the value of γ that ensures that $m_0 = m_T$), it follows immediately from (38) and (39) that $\gamma^* = 2$. Recall that $\hat{\gamma}$ is defined by the condition $i_T - i_T^g = r + (1-\hat{\gamma})\mu = 0$, which implies that $\hat{\gamma} = 1 + \frac{r}{\mu}$. Hence, in this example, $\gamma^* \gtreqless \hat{\gamma}$, depending on the relationship between the parameters μ and r. If $\mu \leq r$, then $\gamma^* \leq \hat{\gamma}$, which implies that it is optimal for the monetary authority to set $\gamma = \gamma^*$, and postpone the crisis all the way to the point at which there is no run when the crisis finally takes place (point T^* in Figure 1). If $\mu > r$, then $\gamma^* > \hat{\gamma}$, which implies that the monetary authority will be able to postpone the crisis only until some point T ($T^k < T < T^*$, in terms of Figure 1), at which there will still be a run when the crisis occurs. In either case, however, it is optimal for the monetary authority to postpone the crisis as much as it can.

Let us now derive T. Using equation (25), we can find an explicit expression for T (assuming $d_0 = 1$)

$$\tilde{T}(\gamma) = \frac{\log\left[\bar{h} + \bar{z} - 2\frac{r}{G} - \frac{\mu}{G}(2-\gamma)\right]}{\mu}. \tag{40}$$

It follows that as γ increases, T increases. In other words, the more aggressive is the announced interest rate policy, the more the crisis will be delayed.

Finally, let us look at welfare. Ignoring the consumption term (which is just a constant), we can write welfare as (substituting (36) and (37) into (1), taking into account (34) and (35)):

$$W(\gamma) = -\frac{r}{G} - \frac{e^{-r\tilde{T}(\gamma)}}{2rG}\left\{\mu^2[1+(1-\gamma)^2] + 2r\mu(2-\gamma)\right\}. \tag{41}$$

It thus follows that:

$$\frac{dW(\gamma)}{d\gamma} = e^{-r\tilde{T}(\gamma)}\left\{\tilde{T}'(\gamma)\{\mu^2[1+(1-\gamma)^2] + 2r\mu(2-\gamma)\} + 2\mu\left[r + \mu(1-\gamma)\right]\right\} \geq 0, \tag{42}$$

for $\gamma \leq \min(\gamma^*, \hat{\gamma})$. The right-most inequality follows from the fact that for all $\gamma \leq \min(\gamma^*, \hat{\gamma})$, $\gamma \leq 2$ and $r + \mu(1-\gamma) \geq 0$.[23] This illustrates the fact that, in the absence of an exogenous fiscal constraint, the optimal interest rate policy is to delay the crisis as much as possible.

[23] Notice that $\frac{dW(\gamma)}{d\gamma} = 0$ for $\gamma = \hat{\gamma}$ (for those cases in which $\mu > r$ and therefore $\gamma^* > \hat{\gamma}$).

3.6 Non-separable preferences

So far we have worked with a utility function – given by equation (1) – which is separable in consumption and liquidity services. This greatly simplifies the analytical solution of the model. But, as seems clear from the underlying intuition behind Propositions (1) and (2), the results should go through for more general, non-separable preferences. To check this conjecture, we run simulations of the model with the following non-separable preferences:

$$\int_0^\infty \frac{x_t^{1-1/\sigma} - 1}{1 - 1/\sigma} e^{-\beta t} dt, \tag{43}$$

where x – a CES aggregator for consumption and liquidity services (ℓ) – and ℓ are given by

$$\begin{aligned} x &\equiv \left[qc^{\frac{\rho-1}{\rho}} + (1-q)\ell^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \\ \ell &\equiv K - \frac{G}{2}(\bar{h} - h)^2 - \frac{B}{2}(\bar{z} - z)^2, \end{aligned}$$

where $\sigma(> 0)$ is the intertemporal elasticity of substitution for the CES aggregator, $\rho(> 0)$ is the intratemporal elasticity of substitution between consumption and liquidity services, and A, B, F, G, $\bar{h}$, and $\bar{z}$ are positive constants.[24]

This general formulation includes as a particular case ($\sigma = \rho$) a separable specification like the one analyzed above. In that case, the path of consumption is flat and all our previous results can be directly applied. For $\sigma > \rho$ (which we will assume), it is easy to check analytically that consumption and liquidity services move in the same direction. Hence, when a crisis occurs at time T, both consumption and real money demand fall, as empirical evidence would suggest.

All the simulations that we ran are consistent with the results established in Propositions 1 and 2 for the separable case. As an illustration, Figure

[24] K is an arbitrary constant (which is irrelevant from a conceptual point of view) that ensures that $\ell > 0$.

2 reports one such simulation.[25,26] Panel A shows that T is an increasing function of γ. Hence, the more aggressive is interest rate policy, the more the crisis is delayed. Panel B, in turn, shows that welfare is also an increasing function of γ. Hence, the optimal interest rate policy consists in delaying the crisis for as long as possible. Intuitively, a more aggressive interest rate policy is welfare improving for two reasons. The first, already present in the separable case, is that the fall in real money demand at time T becomes smaller as γ increases (Panel C). This effect is reinforced by the fact that the fall in consumption at time T also becomes smaller (Panel D), as the higher i^g reduces the intertemporal distortion associated with consumption and thus makes the consumption path "flatter" over time.

4 Exogenous fiscal constraint

As a convenient benchmark, Section 3 assumed that the fiscal authority endogenously adjusts government spending (transfers) in response to changes in available revenues. As noted above, this implies that the fiscal authority is willing to effect a sharp contraction in spending at the time of the crisis to accommodate the higher debt service. In practice, of course, such an adjustment is difficult to implement and inflation must typically rise to provide additional revenues to the treasury. To capture such a scenario, this section analyzes the case in which the monetary authority must finance an exogenously-given flow of transfers. The inflationary consequences of higher interest rates will play a critical role in determining the effectiveness and optimality of an active interest rate policy in defense of the peg.

4.1 Preliminaries

The government now faces an exogenous spending constraint given by

$$\tau_t = \bar{\tau}, \qquad t \in [0, \infty), \tag{44}$$

[25] We are only interested in the *qualitative* nature of the solution and there is thus no attempt at replicating a particular economy. The time unit is years. The key parameter values for this simulation are $\sigma = 10$, $\rho = 1.5$, $G = 0.025$, $B = 0.02$, $i_0^g = 0$, $r = 0.03$, and $\mu = 2$. Welfare is normalized as percentage change with respect to the $\gamma = 0$ case. The fall in real money demand and consumption are expressed as percentage of GDP. We also ran simulations with preferences specifications that generate Cagan and CES money demands, and obtained similar qualitative results.

[26] For this parameter configuration, $\hat{\gamma} = 1 + \frac{r}{\mu} = 1.015$. The value of γ^* (which we established numerically) is 1.81. Since $\gamma^* > \hat{\gamma}$, the central bank cannot postpone the crisis all the way to the point at which there is no run.

which says that real government transfers remain fixed at $\bar{\tau}$ at all points in time. Under this assumption, the monetary authority endogenously adjusts the rate of domestic credit expansion (μ) to balance the budget at all times. Hence, unlike in the previous section, μ is now an endogenous variable which must satisfy the fiscal constraint (substituting (44) into (14)):

$$\bar{\tau} - rR_t - \varepsilon_t h_t - (\varepsilon_t - i_t^g)z_t = (\mu_t - \varepsilon_t)d_t. \tag{45}$$

In terms of solving for the equilibrium dynamics of a BOP crisis, note that the introduction of an exogenous path of government transfers does not change the consumer's problem. Hence, the optimality conditions derived earlier for the endogenous τ case (equations (4)-(6)) remain unchanged. The two crucial differences between this specification and the previous one are that (i) μ_T is now an endogenous variable since it is no longer specified by a domestic credit rule; and (ii) the money market equilibrium condition at time T must also take into account the fiscal constraint at time T, i.e., $\bar{\tau} = \mu_T d_T - i_T^g z_T$. Our interest, as before, is in the impact of interest rate policy on the dynamics of a crisis.

The experiment is the same as before. At time 0 the central bank announces a fixed exchange rate for the domestic currency which will persist as long as the central bank's foreign exchange reserves are positive. Once reserves reach zero, the central bank lets the currency float. At time 0, the central bank also announces the interest rate policy given by (22)

As in the previous case, since the exchange rate floats from the date of the crisis, T, and the economy is completely stationary, the rate of depreciation of the domestic currency from time T is given by μ_T, the stationary rate of domestic credit creation from time T onward. Thus, the nominal interest rate is given by $i_t = r$ for $0 \leq t < T$ and $i_t = r + \mu_T$ for $t \geq T$. Given policy rule (22), the implied path for the policy-controlled interest rate on liquid bonds is $i_t^g = i_0^g$ for $0 \leq t < T$ and $i_t = i_0^g + \gamma\mu_T$ for $t \geq T$. (In what follows and, with no loss of generality, we will assume that $i_0^g = 0$.) Hence, the lifetime budget constraint for the government given by equation (11) reduces to

$$\bar{\tau} = rR_0 + e^{-rT}\left[\mu_T h_T + \mu_T(1-\gamma)z_T - r(z_0 - z_T + h_0 - h_T)\right]. \tag{46}$$

This equation says that in order to meet the government's intertemporal budget constraint, the constant level of government spending has to be equal to interest earnings on initial reserves plus the present discounted value of inflation tax revenues (which are zero until time T and positive afterwards) minus the present discounted value of interest earnings on the fall in the stock of reserves at time T due to the reduction in cash and liquid bond holdings.

At $t = 0$, (45) implies (noting that $\varepsilon_t = i_0^g = 0$) that

$$\bar{\tau} - rR_0 = \mu_0 d_0. \tag{47}$$

We will assume, of course, that $\bar{\tau} - rR_0 > 0$, which implies that $\mu_0 > 0$. In other words, for this economy to suffer a balance of payments crisis we need reserves to be falling from time 0 onward which occurs only if there is an initial deficit.[27] Thus, the government must be running a budget deficit from time 0 onward, which will require an expansion of domestic credit.

Since reserves go to zero at the time of speculative attack, the money market equilibrium condition at time T dictates that real domestic credit must equal real money demand at that time. Hence, $d_T = h_T + z_T$ where h_T and z_T are given by equations (31) and (32), respectively. Further, the flow constraint (45) says that at time T we must have $\bar{\tau} = \mu_T d_T - i_T^g z_T$. Substituting the money market equilibrium into this equation , using (31) and (32), and noting that $\mu_T - i_T^g = \mu_T(1-\gamma)$ (given that $i_0^g = 0$) gives

$$\bar{\tau} = \mu_T \tilde{h}(r + \mu_T) + \hat{\mu}_T \tilde{z}(r + \hat{\mu}_T), \tag{48}$$

where $\hat{\mu}_T \equiv \mu_T(1-\gamma)$. The RHS of equation (48) denotes revenues from the inflation tax *net* of interest payments on liquid bonds. To see this, notice that the RHS can be rewritten (taking into account that $i_T^g = \gamma\mu_T$) as $\mu_T \tilde{h}(r+\mu_T)+\mu_T \tilde{z}(r+\hat{\mu}_T)-i_T^g \tilde{z}(r+\hat{\mu}_T)$, where the first two terms capture the revenues from the inflation tax while the last one denotes interest payments on liquid bonds.

Finally, as is standard in BOP crisis models, we wish to rule out the possibility that the economy will operate on the "wrong side" of any relevant Laffer curve. To this end, we will assume that the opportunity-cost elasticity of the demand for both cash and liquid bonds is less than unity.[28] In other words, it will be assumed that, at least in the relevant range, $\eta^h \equiv -\frac{\tilde{h}'(i)i}{h(i)} < 1$

[27] In terms of equation (46), this means that for a balance of payments crisis to be feasible the second term on the RHS must be positive. As we analyze in detail elsewhere (Lahiri and Vegh (1998)), the feasibility of a balance of payment of crisis under an exogenous path of government transfers depends on the particular form of money demand and, hence, on preferences. Since this issue is unrelated to the existence of interest rate policy, we abstract from it in the current analysis and focus on cases in which this feasibility condition holds.

[28] Notice that the opportunity cost of cash and liquid bonds is i and $i - i^g$, respectively.

and $\eta^z \equiv \frac{\tilde{z}'(i-i^g)(i-i^g)}{\tilde{z}(i-i^g)} < 1$.[29] We also need to rule out the possibility that the economy is operating on the "wrong side" of the Laffer-curve for total net revenues from the inflation tax. Specifically, in this framework, and for values of $\gamma > 1$, a Laffer-curve may arise *even* if both the demand for cash and the demand for liquid bonds have less than unitary elasticity. To see this, differentiate total net revenues from the inflation tax (the RHS of 48) with respect to μ_T to obtain the following expression (denoted by F for further reference):

$$F \equiv h_T + \mu_T \tilde{h}'(r + \mu_T) + (1 - \gamma)\left[z_T + \hat{\mu}_T \tilde{z}'(r + \hat{\mu}_T)\right]. \tag{49}$$

It should be clear that, for $\gamma > 1$, this expression could be negative even if η^h and η^z are less than one (which implies that $h_T + \mu_T \tilde{h}'(r + \mu_T) > 0$ and $z_T + \hat{\mu}_T \tilde{z}(r + \hat{\mu}_T) > 0$). The reason is that, if $\gamma > 1$, a higher μ_T *reduces* net inflation tax revenues on liquid bonds and could thus more than offset higher inflation tax revenues on cash. In other words, for $\gamma > 1$, there may be two values of μ_T that yield the same level of net inflation tax revenues. We will therefore impose the restriction that the economy be operating on the upward-sloping portion of this total inflation tax revenues curve (i.e., it will be assumed that $F > 0$)

4.2 Inflationary effects of interest rate policy

One logical concern of policymakers regarding the use of an active interest rate policy to combat speculative pressures is the likely fiscal effect of such policies. In particular, when governments face constraints on enacting spending cuts, the fiscal costs of raising interest rates (in term of the resulting higher debt service) is likely to imply higher inflation. This logic is easy to formalize in the context of this model. Totally differentiating equation (48), we get

$$\frac{\partial \mu_T}{\partial \gamma} = \frac{\mu_T[z_T + \hat{\mu}_T \tilde{z}'(r + \hat{\mu}_T)]}{F} > 0, \tag{50}$$

where the inequality follows directly from the assumptions related to Laffer curve considerations just discussed. We have thus shown the following result:

[29] As is well-known, this is a sufficient, but not necessary, condition to ensure that the economy is operating on the upward sloping portion of the Laffer curve. Such an assumption, however, makes the presentation more clear.

Proposition 3 *Under an exogenous path of fiscal spending, a more aggressive interest rate policy leads to a higher post-crisis rate of inflation.*

The intuition behind this result is straightforward. A more aggressive interest rate policy (i.e., a higher γ) implies a larger debt service once the crisis has occurred. Since non-interest government spending cannot be curtailed, the only way to finance this additional government spending is to resort to higher inflation. This fiscal effect of higher interest rate policy – which was absent in the benchmark case studied in Section 3 – will clearly affect both the feasibility and optimality of an active interest rate defense of the peg.

Naturally, the positive money demand effect isolated in Section 3 is still present here. To see this, totally differentiate $\hat{\mu}_T \equiv \mu_T(1-\gamma)$ with respect to γ (taking into account (50)) to obtain:

$$\frac{\partial \hat{\mu}_T}{\partial \gamma} = -\frac{\mu_T[h_T + \mu_T \tilde{h}'(r + \mu_T)]}{F} < 0. \tag{51}$$

Recall that $i_T - i_T^g = r + \hat{\mu}_T$. Hence, a more active interest rate policy reduces $\hat{\mu}_T$ and thus the opportunity cost of holding liquid bonds after the crisis. As in the previous section, this direct money demand effect tends to increase post-collapse money demand and thus postpone the crisis. Whether interest rate policy succeeds in delaying a crisis will depend on the relative strength of the fiscal and money demand effects. We turn to this issue next.

4.3 Effectiveness of active interest policy

The first question that we wish to ask is whether, once fiscal costs are taken into account, higher interest rates can delay a BOP crisis. To answer this question, we first establish that, as in the previous case, the time path of international reserves prior to the crisis is independent of the announced interest rate policy (i.e., is independent of γ). To see this, notice that from the central bank's balance sheet and the fact that both h and z are constant prior to the crisis, it follows that $\dot{R}_t = -\dot{d}_t$ for $t \in [0, T)$. Furthermore, from (45), we have that, prior to the crisis, $\bar{\tau} = rR_t + \mu_t d_t$. Time-differentiating the latter expression and combining it with the former yields $\dot{\mu}_t = (r - \mu_t)\mu_t$. This differential equation gives the time path of the rate of domestic credit expansion which is consistent with the fiscal constraint (45), given that real money demand is constant. Since this differential equation is independent of the interest rate policy parameter γ, it follows that the time path of international reserves R prior to the crisis does not depend on γ.[30]

Since, by construction, international reserves fall to zero when the crisis occurs, the size of the speculative attack at time T is given by R_{T^-}. (Since $\Delta R_T \equiv R_T - R_{T^-} = -R_{T^-}$, the size of the run is $-\Delta R_T$. In what follows we denote the size of the run by S; i.e., $S_T \equiv -\Delta R_T$) Hence, for a *given* time path for reserves prior to the crisis and an exogenously-given level of initial reserves R_0, the crisis will happen earlier (later), if and only if the size of the run at time T is bigger (smaller). We have already established that starting from any given initial level of reserves, the exogenously-specified fiscal spending $\bar{\tau}$ induces a rate of loss of reserves which is independent of γ. Thus, if we can determine the effect of the interest rate policy parameter γ on the *size* of the speculative attack then we will also have determined the effect of interest rate policy on the *timing* of the crisis.

Formally, the timing of the crisis can be determined from the government's intertemporal budget constraint, equation (46). Solving (46) for T gives (taking into account (48) and recalling that $\Delta m_T = \Delta R_T$)

$$T = \frac{\log(\bar{\tau} - rS_T)}{r} - \frac{\log(\bar{\tau} - rR_0)}{r}.$$

Differentiating this with respect to γ then yields

$$\frac{\partial T}{\partial \gamma} = \frac{-1}{\bar{\tau} + r\Delta R_T}\frac{\partial S_T}{\partial \gamma}, \tag{52}$$

which proves our previous that the time of the run and the size of the run are inversely related.

The size of the run at time T is simply the jump down in total money demand at that time. Hence,

$$S_T = (h_0 + z_0) - (h_T + z_T) \equiv m_0 - m_T. \tag{53}$$

Since aggregate money demand before the collapse, m_0, depends only on r and is thus independent of γ, interest rate policy affects the size of the speculative attack only through its effect on post-collapse money demand m_T. In particular, equation (53) says that the run will be smaller the higher is m_T. Intuitively, the higher the post-collapse money demand the

[30] Notice that the initial condition, μ_0, is also independent of γ. This follows from the fact that (i) $\bar{\tau} - rR_0 = \mu_0 d_0$ and (ii) d_0 is determined by real money demand and the initial stock of international reserves (i.e., $d_0 = m_0 - R_0$) and is thus also independent of γ.

smaller is the size of the run at time T. Thus, if interest rate policy were to increase money demand at time T, then it would also reduce the size of the speculative attack and, hence, delay the crisis.

After substituting into $m_T = h_T + z_T$ equations (31) and (32), it follows that (using (50))

$$\frac{\partial m_T}{\partial \gamma} = \frac{\mu_T h_T z_T}{F(r+\mu_T)(r+\hat{\mu}_T)} \left[(\eta_T^z - \eta_T^h)(r+\mu_T) + \eta_i^h (1-\eta_T^z)\gamma\mu_T \right], \quad (54)$$

where F is given by (49), and η_T^h and η_T^z denote the opportunity-cost elasticities defined above at time T. It is straightforward to verify from equation (54) that

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \gtreqless \frac{(\eta_T^h - \eta_T^z)(r+\mu_T)}{\eta_T^h (1-\eta_T^z)\mu_T}. \quad (55)$$

Recall that we have already imposed the restrictions that η_T^h and η_T^z be less than unity. Since γ is non-negative, equation (55) says that $\eta_T^z > \eta_T^h$ is a sufficient condition for a more aggressive interest rate policy (a higher γ) to reduce the size of the attack and hence postpone the time of the attack. In other words, if the demand for cash is less interest elastic than the corresponding demand for liquid bonds, a more aggressive interest rate policy is successful in delaying a crisis.

Intuitively, a rise in γ reduces the opportunity cost of holding bonds by reducing $i - i^g$ (recall (51)). This effect raises the post-collapse demand for liquid bonds through the direct money demand effect. However, as we showed earlier, a higher γ also increases μ_T through the fiscal effect and thereby raises the pure interest rate i which reduces the demand for cash. If bonds are more interest elastic than cash, then the rise in bond holdings that is induced by the higher γ outweighs the decline in cash demand that occurs due to the associated rise in i. In this event, the overall post-collapse demand for money rises. Hence, the size of the run is smaller and the run occurs later.

Interestingly, equation (55) also shows how interest rate activism can actually hasten a crisis. Suppose that $\eta_T^z < \eta_T^h$ and start from an initial situation in which $\gamma < \frac{(\eta_T^h - \eta_T^z)(r+\mu_T)}{\eta_T^h(1-\eta_T^z)\mu_T}$. Then a marginal increase in γ causes the size of the run to become bigger and, hence, brings the crisis forward in time. The logic behind this result follows the one just described. When cash is more interest elastic than bonds, a higher γ induces a fall in the demand for cash that is larger than the increase in the demand for bonds as long as the direct effect of γ is not too large relative to this differential effect. In this event, greater interest rate activism increases the size of the attack and hastens the crisis. We collect these results in the following proposition:

Proposition 4 *A sufficient condition for a more aggressive interest policy to delay the time of the BOP crisis is that the interest elasticity of demand for cash be smaller than the interest elasticity of demand for liquid bonds. On the other hand, when bonds are less interest elastic than cash, a more aggressive interest rate policy may, for relatively low values of* γ*, bring forward the time of the attack*

In light of these results, two points are worth stressing. First, recall that, in the absence of an exogenous fiscal constraint, a more aggressive interest rate policy *always* succeeds in delaying the crisis (Proposition 1). The introduction of an exogenous fiscal constraint is therefore critical in raising the possibility that a more aggressive interest rate policy may actually bring forward the time of the crisis. This supports the widely-held (but seldom formalized) notion that taking into effect the fiscal costs of higher interest rates is a critical element in assessing the effectiveness of an active interest rate defense of a peg.

Second, Proposition 4 raises the interesting possibility of a non-monotonic relationship between the time of a crisis and the degree of interest rate activism γ. In particular, if $\eta_T^z > \eta_T^h$ for small values of γ but $\eta_T^z < \eta_T^h$ for higher values of γ (which would obviously required money demands with non-constant elasticities), then the timing of the BOP crisis could bear a non-monotonic relationship with γ. For low values of γ, a more aggressive interest rate policy would be successful in delaying the crisis but once γ surpasses a certain threshold greater activism could in fact hasten the crisis. Equation (55) points to the specific conditions that one needs for such a case to occur. For T to be non-monotonic (initially rising and then declining) in γ, η_T^z should be falling in γ while η_T^h should be increasing in γ. Since i rises and $i - i^g$ falls as γ rises, this implies that cash demand should become more elastic as the nominal interest rate rises while bond demand should become less elastic as $i - i^g$ falls. In other words, the interest rate elasticities of both cash and bond demand need to be *non-constant* and, more specifically, rise with the relevant opportunity cost. As an example below will make clear, Cagan-type money demands exhibit precisely such a property.[31]

[31] It should be noted that Cagan-type money demands seem to provide the best econometric fit in developing countries (see, for example, Easterly, Mauro, and Schmidtt-Hebbel (1995)).

We illustrate Proposition 4 by looking at three different money demand specifications.

Example 1: The constant elasticity case To fix ideas, we begin with the case in which the demands for cash and liquid bonds exhibit constant interest rate elasticity. Let preferences for cash and bonds be given by

$$v(h_t) + w(z_t) = \frac{(h_t)^{1-\theta}}{1-\theta} + \frac{(z_t)^{1-\alpha}}{1-\alpha}, \qquad \alpha > 1,\ \theta > 1. \tag{56}$$

Using (4)-(6) (and recalling that we have defined units so that $\lambda = 1$) yields the demands for cash and bonds:

$$h_t = \left(\frac{1}{i_t}\right)^{\frac{1}{\theta}}, \tag{57}$$

$$z_t = \left(\frac{1}{i_t - i_t^g}\right)^{\frac{1}{\alpha}}. \tag{58}$$

It follows that $\eta^h = 1/\theta$ and $\eta^z = 1/\alpha$. Substituting these elasticities into equation (55), we conclude that

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \gtreqless \left(\frac{\alpha-\theta}{\alpha-1}\right)\left(\frac{r+\mu_T}{\mu_T}\right).. \tag{59}$$

For the relevant range (i.e., $\gamma \geq 0$), it is easy to see that when bonds are more interest elastic than cash (i.e., $\alpha < \theta$), a more aggressive interest rate policy always increases money demand post-collapse and, hence, postpones the crisis. In this case, therefore, a more aggressive interest rate policy always succeeds in postponing the crisis in spite of the rising inflationary effects. On the other hand, for $\alpha > \theta$ it can be seen that for γ small enough, a more aggressive interest rate policy (a higher γ) could reduce money demand after the crisis and, hence, bring the crisis forward. However, once γ becomes sufficiently high, the inequality will reverse and greater activism would successfully delay the crisis.[32]. While in this case the relationship between the timing of the crisis and γ is non-monotonic (falling at first and rising later), we dismiss this case as irrelevant in practice because we should expect that, for low levels of the inflation rate, the demand for cash is less elastic than the demand for liquid bonds.

[32] Note that μ_T rises with γ which implies that $\frac{(r+\mu_T)}{\mu_T}$ falls as γ rises.

Example 2: The Cagan case The second example focuses on the well-known Cagan-type money demands. To this end, let preferences be given by

$$v(h_t) + w(z_t) = h(F - G\log h) + z(A - B\log z). \tag{60}$$

Under this specification, the demands for cash and bonds are given by

$$h = Ke^{-\frac{i}{G}}, \quad K = e^{\frac{F-G}{G}}, \tag{61}$$

$$z = Me^{-\frac{i-i^g}{B}}, \quad M = e^{\frac{A-B}{B}}, \tag{62}$$

which are the standard Cagan demand functions. The corresponding interest elasticities for this case are

$$\eta_i^h = \frac{i}{G}, \tag{63}$$

$$\eta_i^z = \frac{i-i^g}{B}. \tag{64}$$

The crucial feature of the Cagan case is that interest elasticities are no longer constant, but rather *increasing* functions of their respective opportunity cost. Moreover, we already know that as γ rises, μ_T (and hence i_T) increases but $\hat{\mu}_T$ (and hence $i_T - i_g^T$) falls. Hence, as γ rises, η^h increases while η^z falls, which is the precisely the feature that may give rise to a non-monotonic relationship between γ and T (with T first rising and then falling)

To see this, substitute (63) and (64) into (55) to obtain

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \lesseqgtr \frac{G-B}{\mu_T}. \tag{65}$$

Consider the case in which $G - B > 0$; that is, for the same opportunity cost, the demand for cash is less elastic than that for liquid bonds. Then, it follows from (68) that, starting from $\gamma = 0$, a small increase in γ increases m_T and hence postpones the crisis. As γ increases, $\frac{G-B}{\mu_T}$ falls because μ_T is an increasing function of γ. Furthermore, while γ rises linearly, the RHS will eventually rise at a very high rate as $\frac{\partial \mu}{\partial \gamma}$ approaches infinity as F falls to zero (i.e., as we approach the maximum of the Laffer curve). Hence, the second inequality in (68) will eventually reverse itself (from less than to greater than). In that case, for high values of γ, a more aggressive interest rate policy would actually hasten the crisis, rather than delay it.

Indeed, it is easy to choose parameter values such that the relationship between m_T and γ (and, hence, between T and γ) is non-monotonic with

m_T initially rising in γ and then falling once γ goes above a certain critical level. Figure 3 shows such a case for $G = 5$ and $B = 3$.[33] As shown in Panel A, the time of the crisis is indeed a non-monotonic function of the degree of interest rate activism γ. T initially rises as γ increases, reaches a maximum, and then falls rapidly. As expected, Panel C tells the mirror story for the fall in real money demand at T.[34] Figure 3 thus makes clear that there is a certain degree of interest rate activism (i.e., some value of γ) that maximizes the delay.

The intuition follows the lines discussed above. For low values of γ, the nominal interest rate is low and, hence, cash is less elastic than bonds. In that range, a marginal increase in γ reduces the demand for cash by less than it increases the demand for liquid bonds. In other words, real money demand (i.e., real demand for cash *and* liquid bonds) increases, and the time of the crisis is postponed. Once γ rises above a certain threshold, the elasticities change their relative magnitudes with cash becoming more elastic than bonds. That, in turn, implies that further increases in γ may at some point reduce cash demand (through its effect on μ_T) more than they increase liquid bond demand. Thus, money demand declines.

Example 3: The quadratic case As a final example, we revisit the quadratic specification given by (34) and (35). In this case, demands for cash and bonds are given by equations (36) and (37). The corresponding interest rate elasticities of cash and bonds are now given by

$$\eta^h = \frac{1}{\frac{G\bar{h}}{i} - 1}, \tag{66}$$

$$\eta^z = \frac{1}{\frac{B\bar{z}}{i - i^g} - 1}. \tag{67}$$

As can be seen from (66) and (67), this case is very similar to the Cagan case just analyzed since the elasticities are rising in their respective opportunity cost. Substituting (66) and (67) into (55) yields

$$\frac{\partial m_T}{\partial \gamma} \gtreqless 0 \qquad \text{as} \qquad \gamma \lesseqgtr \frac{G\bar{h} - B\bar{z}}{2\mu_T}. \tag{68}$$

Comparing equations (65) and (68), it is easy to see that these two cases are very similar. Hence, all the results that we derived for the Cagan case

[33] The remaining parameters are $A = F = 10$, $\bar{\tau} = 10$, and $R_0 = 4$.

[34] Panel D will be discussed later.

carry over to the quadratic case as well. In particular, there always exist parameter values for which the relationship between γ and m_T, and hence the relationship between γ and the time of the crisis T, is non-monotonic. The intuition for this result is the same as that for the quadratic case described above.

Crisis with no run The analysis above shows that when cash is less interest elastic than bonds the central bank can postpone the time of the crisis by announcing higher interest rates in the event of a crisis. This, however, does not necessarily guarantee that policymakers can raise interest rates all the way to the point at which there is a crisis with no run. As discussed in the previous section, a crisis with no run would occur if policymakers can set $\gamma = \gamma^*$(where γ^* is the value of γ for which there is no change in real money demand at T). If it exists, $\gamma^* > 1$ because the demand for liquid bonds at T must increase to compensate for the fall in the demand for cash.

There are two reasons, however, why it may not be possible to achieve a crisis with no run. The first is the one already highlighted in the previous case: even if m_T is always an increasing function of γ, it may be the case that the non-negativity constraint on $i_T - i_T^g$ is reached for a value of γ lower than γ^*. The second reason is that, as made clear by (55), it is not necessarily the case that m_T is always an increasing function of γ. If m_T is a decreasing function of γ or begins to fall beyond a certain positive value of γ, then a crisis with no run cannot be achieved. In sum, whether a crisis with no run is feasible will depend on the specific parameter configuration. In the simulation reported in Figure 3, for example, a crisis with no run cannot be achieved because T is a non-monotonic function of γ.

4.4 Optimality of active interest rate policy

The preceding analysis has analyzed the conditions under which a more aggressive interest rate policy could delay an impending balance of payments crisis. Of course, the fact that policymakers may have the ability to delay a crisis by raising interest rates does not necessarily mean that it is optimal to do so. To address this issue, we now analyze the optimality of raising interest rates to delay a crisis.

The starting point of the welfare analysis is equation (33), which remains valid for this case. Equation (33) decomposes the change in welfare in response to an increase in γ into two terms. The first term indicates, for a given T, how an increase in γ directly affects welfare. The second term captures the indirect effect of γ on welfare through changes in T.

Let us first look at the second term on the RHS of (33). As shown in the Appendix, $\frac{\partial W}{\partial \tilde{T}} > 0$. In other words, other things being equal, a higher T increases lifetime utility by allowing consumers to enjoy the higher pre-crisis liquidity services for a longer period of time. This implies that the sign of the second term on the RHS of (33) depends solely on the sign of $\tilde{T}'(\gamma)$. On this account, therefore, delaying a crisis is welfare improving.

The first term on the RHS of (33), however, indicates that the direct effect of delaying a crisis is not necessarily welfare-improving. For a given T, a higher γ reduces the opportunity cost of holding liquid bonds (recall (51)) but, by raising the post-crisis inflation rate, μ_T, reduces the demand for cash. Using equations (50) and (51) and suitably rearranging, we obtain

$$\frac{\partial W}{\partial \gamma} = \kappa \left[w'(z_T)\eta_T^z \left(r + \mu_T(1 - \eta_T^h) \right) - v'(h_T)\eta_i^h \left(r + \hat{\mu}_T(1 - \eta_T^z) \right) \right] \quad (69)$$

where $\kappa \equiv \frac{\mu_T h_T z_T e^{-rT}}{rF(r+\mu_T)(r+\hat{\mu}_T)} > 0$. As shown by equation (69), the sign of this term depends, among other things, on the relative magnitudes of the two interest elasticities.

As is obvious from the above, the effect of a more aggressive interest rate policy on welfare is, in general, ambiguous. However, even at this level of generality, we can (i) characterize the behavior of welfare for small values of γ (i.e., around $\gamma = 0$) and (ii) establish a relationship between the non-monotonicity of welfare and the non-monotonicity of the time of the crisis

To study the first issue, simply evaluate $\frac{\partial W}{\partial \gamma}$, given by (69), around $\gamma = 0$ to obtain:

$$\left. \frac{\partial W}{\partial \gamma} \right|_{\gamma=0} = \frac{\mu_T h_T z_T e^{-rT} v'(h_T)}{r(r + \hat{\mu}_T)F} (\eta^z - \eta^h),$$

where we have used the fact that the first order conditions for cash and bonds, given by (5) and (6), imply that $v'(h_T) = w'(z_T) = \lambda(r + \mu_T)$ for $\gamma = 0$. We know from Proposition 4 that for the time of the crisis, T, to be rising initially in γ, we must have $\eta_T^z > \eta_T^h$ at $\gamma = 0$. More generally, it is easy to check that $\frac{\partial W}{\partial \gamma}$ and $\frac{\partial T}{\partial \gamma}$ must have the same sign around $\gamma = 0$ and that this sign is given by the sign of the term $\eta_i^z - \eta_i^h$. Thus, if $\eta_T^z > \eta_T^h$ at $\gamma = 0$ then equation (33) implies that we must have $\frac{dW}{d\gamma} > 0$ at $\gamma = 0$ since both terms are positive. We have thus shown the following:

Proposition 5 *Around $\gamma = 0$, a necessary and sufficient condition for welfare to be rising in γ is that the demand for cash be less interest rate elastic than the demand for liquid bonds.*

This is an important result from a policy point of view because it says that, for the more relevant practical case, it is always optimal to engage in *some* defense of the peg. In other words, it is never optimal for the central bank to sit still (as implied by the Krugman case) and do nothing.

We now address the relationship between the non-monotonicity of welfare and the non-monotonicity of the time of the crisis. From equation (55) we know that in the case where T is initially rising in γ but starts falling once γ crosses a critical level (call it $\tilde{\gamma}$), we must have that $\left.\frac{\partial T}{\partial \gamma}\right|_{\gamma=\tilde{\gamma}} = 0$ where $\tilde{\gamma} = \frac{(\eta_i^h - \eta_i^z)(r+\mu_T)}{\eta_i^h(1-\eta_i^z)\mu_T}$. Hence T is falling in γ for all $\gamma > \tilde{\gamma}$. Evaluating $\frac{\partial W}{\partial \gamma}$ around $\gamma = \tilde{\gamma}$ gives, after some algebraic manipulations,

$$\left.\frac{\partial W}{\partial \gamma}\right|_{\gamma=\hat{\gamma}} \gtreqless 0 \qquad \text{as } \frac{r+\mu_T}{\mu_T} \lesseqgtr \eta^h, \tag{70}$$

where we have made use of the fact that, for $\gamma > 0$, $w' < v'$, as follows from the first-order conditions (5) and (6). Since we are restricting our attention to the range for which $\eta^h < 1$, it will always be the case that $\left.\frac{dW}{d\gamma}\right|_{\gamma=\hat{\gamma}} < 0$. Given that $\left.\frac{\partial T}{\partial \gamma}\right|_{\gamma=\hat{\gamma}} = 0$ and that welfare as a function of γ is continuously differentiable in the relevant range, (33) and (70) lead to the following proposition:

Proposition 6 *A sufficient condition for welfare to be non-monotonic in γ is that the time of the crisis T be non-monotonic in γ (initially rising and then falling). Further, in this event, welfare starts to decline before T does.*

Welfare as a function of γ must thus reach a maximum at some $\gamma' < \hat{\gamma}$. This implies that there typically exists a range of interest rate activism where T is rising while W is falling in γ. The interesting feature of this result is that it shows that even when interest rate activism can be successful in delaying a crisis, it may be welfare reducing.[35] The simulation reported in Figure 3 (for the Cagan case examined above) illustrates this proposition. As can be seen comparing panels A and D, welfare begins to decline before the time of the crisis does. There is thus a whole range for which a more aggressive interest rate policy succeeds in delaying the crisis even though

[35] In the special case of constant interest elasticity of demand for both cash and bonds, one can show that a necessary and sufficient condition for welfare to be monotonically rising (falling) in the degree of interest rate activism γ is that the demand for non-interest bearing cash be less (more) interest elastic than liquid bonds. Welfare becomes independent of interest rate policy in the special case where the interest elasticities of cash and bonds are equal.

it is not optimal to do so. Put differently, there is an optimal amount by which interest rates should be raised to defend the peg.

Intuitively, in the case where the interest elasticity of cash is less than the interest elasticity of liquid bonds, the higher inflation rate that is induced by a higher γ causes a relatively small fall in the post-collapse demand for cash compared to the large increase in the demand for bonds that it generates. Hence, the liquidity effect of interest rate policy and the time of crisis effect of γ work in the same direction. Once γ is high enough and $\eta^z < \eta^h$, higher interest rates do not affect the demand for bonds too much since they are relatively interest inelastic. On the other hand, the higher post-collapse inflation rate that is induced by higher interest rates causes a relatively bigger drop in the demand for cash since it is more interest elastic. This causes a big drop in the post-collapse utility from liquidity which swamps the positive effects (if any) on welfare that comes from delaying the crisis.

Finally, Figure 4 illustrates the fact that the non-monotonicity of T is a sufficient but not necessary condition for welfare to be non-monotonic. While T is rising throughout the whole relevant range of γ (Panel A), welfare is non-monotonic (Panel B). This occurs when the financing needs of the fiscal authority are substantially lower than in Figure 3.[36] This reinforces the idea that, even when a more aggressive interest rate policy may succeed in delaying the crisis, it may not be optimal to do so.

4.5 Non-separable preferences

As in the endogenous fiscal policy case studied earlier, we now proceed to study the effects of interest rate policy under an exogenous fiscal constraint when preferences are non-separable between consumption and liquidity. This is crucial in order to check whether the basic insights gained from studying the separable case remain valid for non-separable preferences. As before, we use the preferences given by equation (43) which reduces to the separable case when $\sigma = \rho$ and report the Cagan case given by (60).[37]

Figure 5 reports the simulation. As can be seen, both T and welfare are non-monotonic functions of γ and, in fact, the paths look rather similar to those obtained in the separable case (Figures 3 and 4). While T reaches a maximum at around $\gamma = 0.8$, welfare does so for a substantially lower

[36] The only change in the parameter configuration relative to Figure 3 is that now $\bar{\tau} = 1$.

[37] The parameter configuration is the same as that used for the separable case reported above, except for the new parameters $\sigma = 10$, $\rho = 1.5$, and $q = 0.5$. The new panel, Panel C, reports the fall in consumption at time T as a percentage of GDP.

value (around $\gamma = 0.4$). This suggests that non-separability does not alter the essential logic behind our results. In this case, of course, the path of consumption is no longer flat. As shown by Panel C in Figure 5, the fall in consumption at time T mirrors the behavior of welfare. The fall in consumption is reduced early on as γ increases, reaches a minimum at around $\gamma = 0.4$ and then rises sharply.

5 Conclusions

In the classical approach to balance of payments crises inspired by Krugman (1979), crises are caused by inconsistent policies. In particular, countries which attempt to expand money supply while simultaneously fixing their exchange rate end up losing foreign exchange reserves and, thereby, open the door to speculative attacks. A notable – but rarely discussed – assumption of Krugman-type models is that the central bank sits passively as its international reserves are depleted, and thus takes no measures to fight a speculative attack. Of course, this is rarely the case in practice since central banks actively try to defend the peg by raising short-term interest rates, thus making more attractive domestic currency denominated assets.

This paper has analyzed the feasibility and optimality of delaying a impending crisis by raising interest rates. We have shown that, under certain conditions, raising interest rates succeeds in delaying a crisis and is in fact the optimal (i.e., welfare-maximizing) policy. Higher interest rates, however, increase the service burden of the public debt and imply higher inflation. This fiscal cost of using higher interest rates to defend a peg implies that there is a certain rise in interest rates which maximizes how much the crisis can be delayed. In the same vein, there is a certain increase in interest rates that maximizes welfare. In fact, there is a range of interest rate increases for which the crisis could be further delayed but is not optimal to do so.

Our results help in providing a conceptual framework for one of the more hotly debated policy issues in recent years. As is well-known, IMF-supported programs in Asia and Latin America typically require a policy of high interest rates to defend the currency. This policy recommendation has been attacked by outside analysts, most notably Jeff Sachs. While highly stylized, our model should at least provide some intellectual food for more though on these issues. Our results suggest that *some* active defense of a peg is optimal but that very high interest rate increases are likely to do more harm than good by generating higher inflation in the future, a result in the spirit of Sargent and Wallace's (1981) famous unpleasant monetarist arithmetic.

An important caveat to our analysis is that we have focused exclusively on the *fiscal* costs of higher interest rates and abstracted from potential *output* costs. In work in progress, we abstract from fiscal costs and incorporate output costs into the picture by introducing a banking system and,hence, supply and demand for bank credit. In this set-up, higher interest rates exert a contractionary effect on output by reducing bank credit (i.e., working capital) to firms. This negative output cost will tend to offset the positive money demand effect and thus generate trade-offs in the monetary authority's ability to delay a crisis and in the optimality of using higher interest rates. Preliminary results suggest that it will still be optimal to engage in some active interest rate defense.

Appendix

A Proof of proposition 2

As argued in the text, the fact that welfare is an increasing function of γ for $\gamma > 1$ is not formally obvious. In light of (33) and the fact that $\frac{\partial W}{\partial \gamma} \geq 0$ and $\tilde{T}'(\gamma) > 0$, we need to show that (using (28))

$$\frac{\partial W}{\partial T} = e^{-rT}\left\{v[\tilde{h}(r)] + w[\tilde{z}(r - i_0^g)] - v[\tilde{h}(r+\mu)] - w[\tilde{z}(r - i_0^g + (1-\gamma)\mu)]\right\} > 0. \tag{71}$$

This appendix proves this claim for the case in which $v(h)$ and $w(z)$ are quadratic functions.[38]

Suppose that $v'(h) > 0$, $v''(h) < 0, v'''(h) = 0, w'(h) > 0$, $w''(h) < 0$, and $w'''(h) = 0$. Using a Taylor expansion around $i = r$ for the case of $v\,(h)$and $i - i^g = r - i_0^g$ for the case of $w(z)$, it follows that (taking also into account that $\tilde{h}'(i) = \frac{1}{v''[\tilde{h}(i)]}$ and $\tilde{z}'(i - i^g) = \frac{1}{w''[\tilde{z}(i-i^g)]}$)

[38] A general proof would involve third derivatives of $v(h)$ and $w(z)$. Since theory does not provide us with plausible restrictions on third derivatives, one would need to proceed on a case-by-case basis. Alternatively, this same proof would hold for general separable utility functions for the corresponding quadratic approximations.

$$v[\tilde{h}(r+\mu)] = v[\tilde{h}(r)] + v'[\tilde{h}(r)]\tilde{h}'(r)\mu + \frac{1}{2}\frac{\mu^2}{v''[\tilde{h}(r)]}, \quad (72)$$

$$w[\tilde{z}(r - i_0^g + (1-\gamma)\mu)] = w[\tilde{z}(r - i_0^g)] + w'[\tilde{z}(r - i_0^g)]\tilde{z}'(r - i_0^g)\mu(1-\gamma) + \frac{1}{2}\frac{\mu^2(1-\gamma)^2}{w''[\tilde{z}(r - i_0^g)]}. \quad (73)$$

Using the last two equations, the term in curly brackets in (71) can be written as

$$\mu\left\{-v'[\tilde{h}(r)]\tilde{h}'(r) - w'[\tilde{z}(r - i_0^g)](1-\gamma)\tilde{z}'(r - i_0^g)\right\} - \frac{\mu^2}{2}\left\{\frac{1}{v''[\tilde{h}(r)]} + \frac{(1-\gamma)^2}{w''[\tilde{z}(r - i_0^g)]}\right\}. \quad (74)$$

Since the second term in curly brackets is negative, we need to establish that the first term in curve brackets is non-negative for the whole expression to be positive. We now show that the restriction that real money demand not increase at T (i.e., either fall or remain constant) implies that the first term in curve brackets in (74) is indeed positive.

Notice that if real money demand is not to increase at T, then

$$\tilde{h}(r) + \tilde{z}(r - i_0^g) - \tilde{h}(r+\mu) - \tilde{z}[r - i_0^g + (1-\gamma)\mu] \geq 0. \quad (75)$$

Since $v(h)$ and $w(z)$ are quadratic, it is easy to see that the demands for cash and liquid bonds are linear. Hence, expanding $\tilde{h}(i)$ and $\tilde{z}(i - i^g)$ around $i = r$ and $i - i^g = r - i_0^g$, respectively, and substituting the resulting expressions into (75) implies that

$$-\tilde{h}'(r) \geq (1-\gamma)\tilde{z}'(r - i_0^g). \quad (76)$$

Since, from first-order conditions (5) and (6), $v'(h) \geq w'(z)$, (76) further implies that

$$-v'[\tilde{h}(r)]\tilde{h}'(r) \geq (1-\gamma)w'[\tilde{z}(r - i_0^g)]\tilde{z}'(r - i_0^g). \quad (77)$$

This last inequality implies that the first term in square brackets in (74) is positive, which proves (71).

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Figure 1. Timing of balance of payments crisis

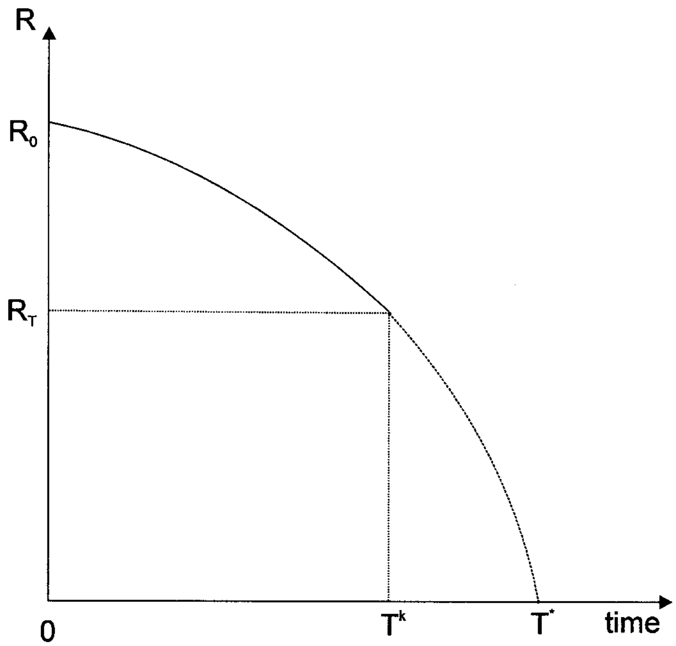

Figure 2. Interest rate policy with no fiscal contraint

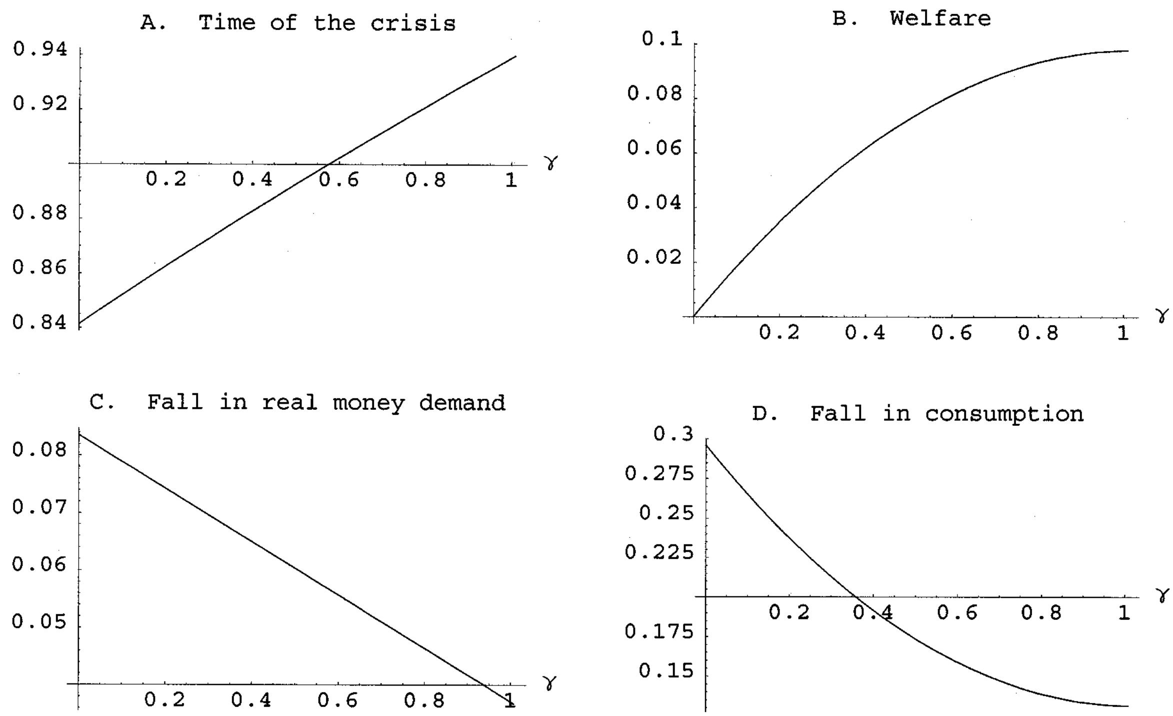

Figure 3. Exogenous fiscal constraint: Separable Cagan case with high fiscal spending

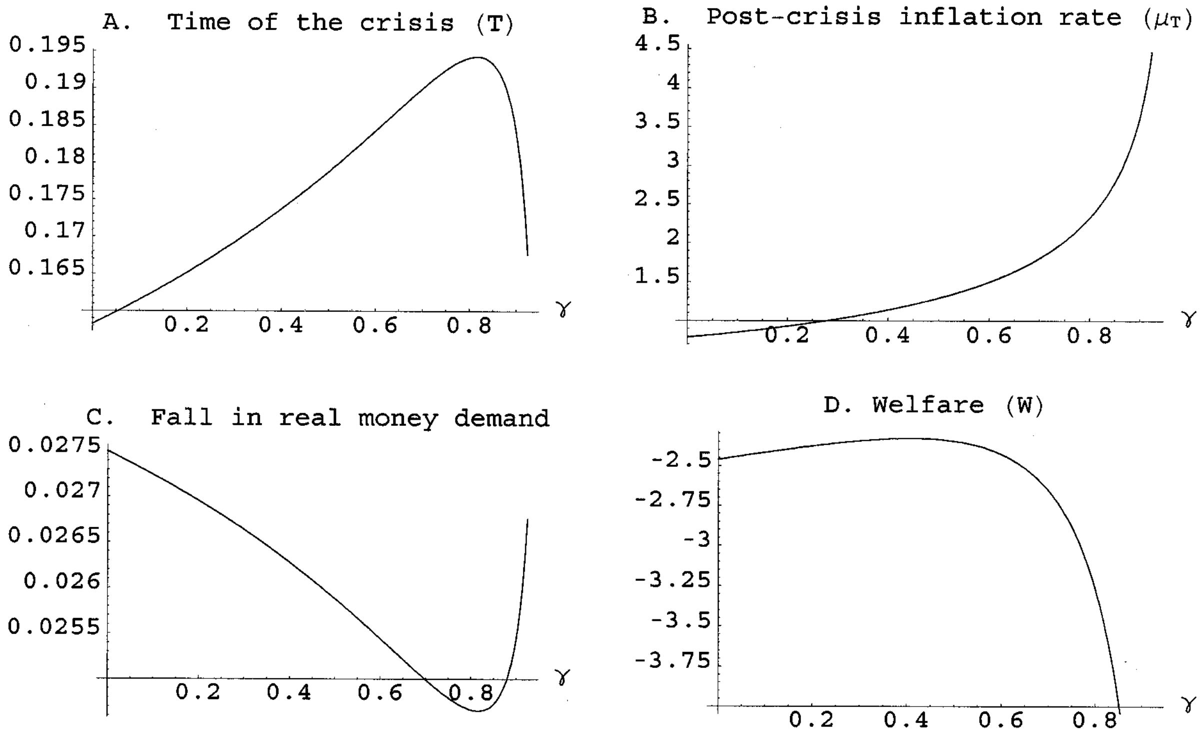

Figure 4. Exogenous fiscal constraint: Separable Cagan case with low fiscal spending

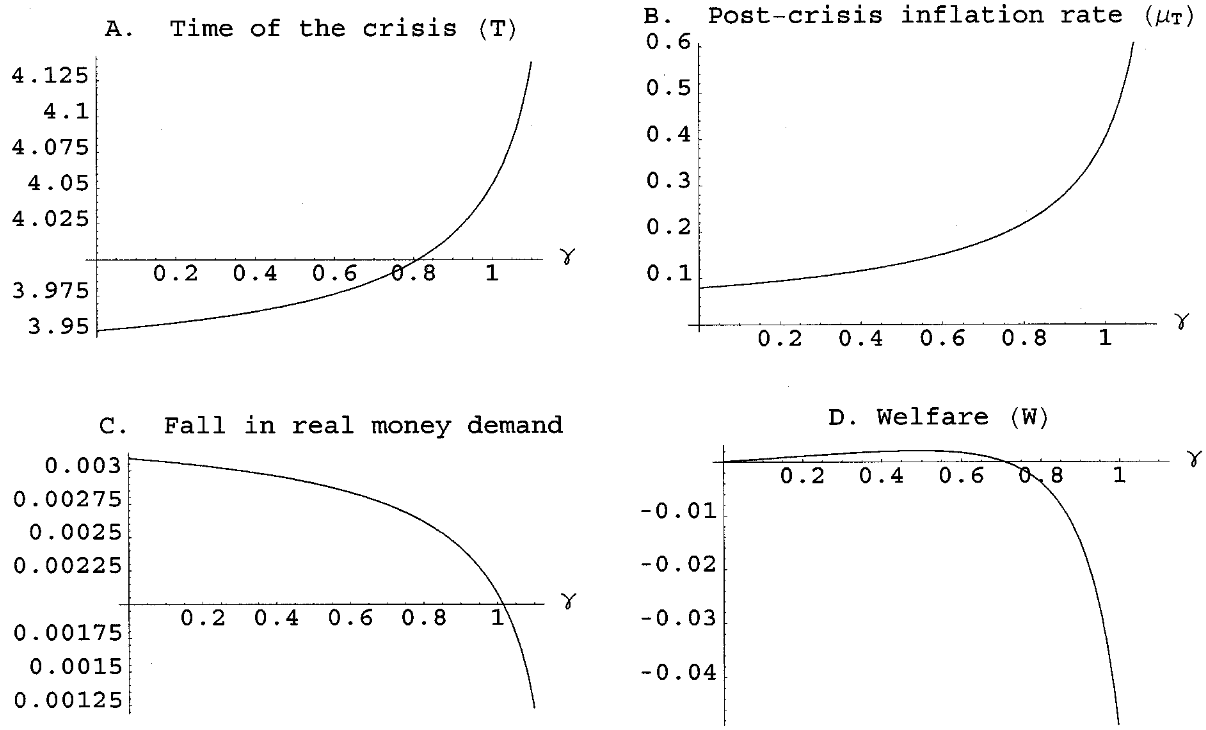

Figure 5. Exogenous fiscal constraint: Non separable case with Cagan money demands