Inflation and Unemployment: Lagos-Wright meets Mortensen-Pissarides*

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Abstract

Inflation and unemployment are central issues in macroeconomics. While progress has been made on these issues recently using models that explicitly incorporate search-type frictions, existing models analyze either unemployment or inflation in isolation. We develop a framework to analyze unemployment and inflation together. This makes contributions to disparate literatures, and provides a unified model for theory, policy, and quantitative analysis. We discuss optimal fiscal and monetary policy. We calibrate the model, and discuss the extent to which it can account for salient aspects of a half century's experience with inflation, unemployment, interest rates, and velocity. Depending on some details concerning how one calibrations certain parameters, the model can do a good job matching the data.

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There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, provided allowance is made for the effect of price change on the real cost of holding money balances. Friedman (1977).

1 Introduction

Inflation, unemployment, and relation between the two are central policy concerns and classic topics for macroeconomic analysis. In recent years, much progress has been made studying unemployment and inflation using theories that incorporate frictions explicitly using search theory.[1] It is not surprising that models with frictions are useful for understanding dynamic labor markets and hence unemployment, as well as for understanding the role of money and hence inflation. However, existing models along these lines analyze either unemployment or inflation in isolation. The goal of this project is to integrate and extend these disparate theories, in order to develop a new framework that can be used to analyze unemployment and inflation together.

We think this makes contributions to two different literatures. One can learn a lot by extending the standard labor market model to include the exchange of commodities, even without money playing an explicit role, but perhaps especially when money does play such a role. Similarly, one can learn a lot by extending the standard model of monetary exchange to have a more interesting labor market. Our model provides a unified framework for theory, policy, and quantitative analysis. It has some novel qualitative predictions concerning e.g.

[1] In terms of unemployment, we have in mind search-based macro models of the labor market along the lines of Mortensen and Pissarides (1994), but also going back to work by Diamond (1982), Mortensen (1982), and Pissarides (1990), and continuing up to recent contributions by Shimer (2005), Hall (2005), Hagedorn and Manovskii (2006) and others; see Rogerson et al. (2005) for a survey. In terms of inflation, we have in mind search-based models of monetary economies along the lines of Lagos and Wright (2005), but also going back to work by Kiyoatki and Wright (1989,1993), Shi (1995,1997) Trejos and Wright (1995), Kochelakota (1998), Wallace (2001) and many others.

the interaction between fiscal and monetary policy. Taking fiscal policy as given, or at least taking as given certain constraints on fiscal policy (e.g. we cannot tax the unemployed too much), we show optimal monetary policy entails inflation in excess of the Friedman rule. If we can choose both fiscal and monetary policy with no constraints, however, the Friedman rule is optimal and fiscal policy can be used to correct any inefficiencies in the labor market.

We calibrate the model, and discuss the extent to which it can account quantitatively for salient aspects of inflation, unemployment, interest rates, and velocity over the last half century. First, we think the model is most appropriate for studying medium- to long-term observations because, in this paper, we abstract from imperfect information, price stickiness, and other features that may be important at higher frequencies. Hence we focus on the HP (Hodrick-Prescott) trends in the data, which show a clear positive relation between inflation and unemployment – i.e. an upward sloping Phillips curve – and between velocity and interest rates – i.e. a downward sloping money demand curve. The model generates both of these qualitative features for simple and natural reasons: higher inflation (or nominal interest) rates consitute a tax on money holdings, and hence a tax activities that use money, including goods market trade, and this reduces profit, discourages job creation, and raises unemployment.

How much? The framework easily can be calibrated using textbook methods. However, it well known there is more than one way to calibrate the standard Mortensen and Pissarides (1986) labor market model. Shimer (2005) e.g. proposes a stratgey that implies the model can match steady states, but does a poor job explaining cyclical behavior; Hagedorn and Manovskii (2006) propose an alternative method that implies the model matches cyclical behavior well. We remain agnostic, and report results for both. The Shimer method implies our long-run Phillips curve is nearly vertical; the Hagedorn and Manovskii (2006) method implies a bigger elasticity of unemployment with respect to inflation, very much consistent with the data. In all cases the model matches money demand data fairly well. Although more empirical work could be done, from these results, it seems clear there is potential for a theory of inflation and unemployment that has microfoundations and that works well empirically.[2]

2 The Basic Model

Time is discrete and continues forever. Each period, there are three distinct markets where economic activity takes place: a labor market, in the spirit of Mortensen-Pissarides; a goods market, in the spirit of Kiyotaki-Wright; and a general market, in the spirit of Arrow-Debreu. For brevity we call these the MP, KW and AD markets. While it does not especially matter if they meet sequentially, simultaneously, or in some combination, for concreteness let us say they meet sequentially.[3] There are two basic configurations: after any meeting of the MP market, we can convene either the KW or AD market. Since it actually does not matter for any interesting results, we arbitrarily choose the configuration shown in Figure 1. In general, agents can discount between one market and the next at any rate, as shown in the Figure, but since all that matters is $\beta = \beta_1\beta_2\beta_3$, we set $\beta_1 = \beta_2 = 1$ to reduce notation.

There are two types of private agents, plus government. What one calls them depends on which market (or which literature) one looks at; e.g. they could be called firms and workers in the MP market, or buyers and sellers in the KW

[2] Some recent attempts to pursue similar ideas include Farmer (2005), Blanchard and Gali (2005), and Gertler and Trigari (2006), but they all take a very different tact by assuming nominal rigidities. Our model does not need any such stickiness to generate interesting feedback from money to real variables.. Moreover, in this project, we are more interested in intermediate- to long-run phenomena, at which frequency we find wage or price stickiness less compelling. Lehmann (2006), Lehmann and van der Linden (2006), and Kumar (2006) are recent attempts more in line with our approach, although the details are different. Rocheteau, Rupert and Wright (2006) integrate modern monetary theory into an alternative model of unemployment, Rogeron's (1988) indivisible labor model; there are reasons to prefer the Mortensen-Pissarides framework.

[3] See Williamson (2005) for a model in which a search-based market where money is essential and a perfectly competitive market meet simultaneously.

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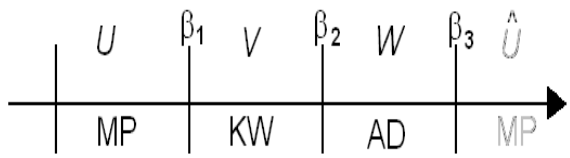

Figure 1: Timing

market. We call them *firms* and *households*, and index them by f and h. The set of households is $[0, 1]$; the set of potential firms has arbitrarily large measure, although not all will be active at any point in time. Households work, consume and enjoy utility; firms maximize profit and pay out dividends to households.[4] As in the standard MP model, a household and a firm can combine to create a job that produces output y. Let e index employment status: $e = 1$ indicates that a household (firm) is matched with a firm (household); $e = 0$ indicates otherwise. As seen in Figure 1, we introduce three value functions for the three markets, U_e^i, V_e^i and W_e^i, which generally depend on type $i \in \{h, f\}$, employment status $e \in \{0, 1\}$, and possibly other state variables as specified below; note $\hat{U}_f^i$ in the figure is the MP value function next period (in our notation, $\hat{a}$ is the value of any variable a next period).

2.1 Households

Let us analyze one round of three markets, starting with AD. Household h with money holdings m chooses a vector of consumption goods $\mathbf{x}$ and money for next period $\hat{m}$, to solve

$$W_e^h(m) = \max_{\mathbf{x},\hat{m}} \left\{ \Upsilon_e(\mathbf{x}) + \beta \hat{U}_e^h(\hat{m}) \right\} \tag{1}$$
$$\text{st } \mathbf{px} = \mathbf{p\bar{x}} + ew_n(1-\tau) + (1-e)b_n + \Delta - T + m - \hat{m},$$

[4] This is different from the textbook MP model, where firms are interpreted as consuming profits (but see Merz 1995, Andolfatto 1996, or Fang and Rogerson 2006, e.g.). This is not important for what we do – everything interesting goes through if firms are consumers.

where Υ_e is instantaneous utility conditional on e, $\mathbf{p}$ is the price vector, $\bar{\mathbf{x}}$ is the endowment vector, w_n is the (nominal) wage, b_n is unemployment income, Δ is dividend income, T is a lump sum tax, and τ is a wage tax. We assume quasi-linear utility: $\Upsilon_e(\mathbf{x}) = x + \tilde{\Upsilon}(\tilde{\mathbf{x}})$ is linear in x, where $\tilde{\mathbf{x}}$ is a vector of goods other than x. Although this is not necessary for the theory as long as one is willing to proceed numerically, quasi-linear utility allows us to make a lot of progress analytically. As a benchmark, we often assume $\tilde{\Upsilon}_e(\tilde{\mathbf{x}}) = 0$ (i.e. a single consumption good traded in the AD market).

It is useful to provide a few results about the AD market before specifying the rest of the model. First, substitute x from the budget equation into the objective function in (1) and rearrange to write

$$W_e^h(m) = \frac{I_e + m}{p} + \max_{\tilde{\mathbf{x}}} \left\{ \tilde{\Upsilon}_e(\tilde{\mathbf{x}}) - \frac{\tilde{\mathbf{p}}\tilde{\mathbf{x}}}{p} \right\} + \max_{\hat{m}} \left\{ -\frac{\hat{m}}{p} + \beta \hat{U}_e^h(\hat{m}) \right\}, \qquad (2)$$

where p is the price of x, $\tilde{\mathbf{p}}$ is the price vector for other goods, and nominal income given e is $I_e \equiv \mathbf{p}\bar{\mathbf{x}} + e w_n(1-\tau) + (1-e)b_n + \Delta - T$. Notice W_e^h is linear: $\partial W_e^h / \partial m = 1/p$. Moreover, the choice of $\hat{m}$ is independent of m and I_e, although in general it depends on employment status e. However, if the KW utility function introduced below is independent of e, as we assume in the benchmark model, then the derivative of $\hat{U}_e^h(\hat{m})$ is independent of e and hence so is $\hat{m}$ (see below). Therefore all households exit the AD market and with the same $\hat{m}$.[5]

We now move back to the KW market, where a commodity q different from those in the AD market is sold by firms to households. We assume that house-

[5] This assumes a unique $\hat{m}$ solves (2), which we can check, and an interior solution for x, which can be guaranteed e.g. by assuming the endowment $\bar{\mathbf{x}}$ is big. Analogous results (the AD value function is linear in m and all households choose the same $\hat{m}$) hold in Lagos-Wright (2005), and are precisely what make the framework tractable. The analysis would be a little harder, but not much, if the KW utility funciton and hence $\hat{m}_e$ depended on e – it would generate a two-point distribution of money across h – but we did not think this was worth the complication. If we dispense with quasi-linearity or free access to an AD market, as in Molico (2006) or Chiu and Molico (2006), the problem really is a lot harder and one can only proceed by numerical methods.

holds are *anonymous* in this market, as is standard in monetary theory, in order to generate an essential role for a medium of exchange. See Kocherlakota (1997), Wallace (2001), Corbae et al. (2003), or Aliprantis et al. (2006) for formal discussions of anonymity and essentiality, but to convey the basic idea, suppose firms cannot identify households by name. Then any h that asks f for q now, and promises to pay later (in the next AD market, say), he can renege without fear of repercussions. Hence, f must insist on quid pro quo. If we assume consumption goods are not storable by households, then fiat money will step into the role of medium of exchange.[6]

For h with m dollars and employment status e in the KW market,

$$V_e^h(m) = \alpha_h \left[\upsilon(q) + W_e^h(m-d) \right] + (1-\alpha_h) W_e^h(m), \tag{3}$$

where $\upsilon(q)$ is utility, α_h is the probability of trade, and q and d are the quantity and dollars exchanged, as determined below. The probability of trade is given by a matching function $\alpha_h = \mathcal{M}(B,S)/B$, where B and S are the measures of buyers and sellers in the market. Assuming $\mathcal{M}$ satisfies the usual assumptions, including constant returns, $\alpha_h = \mathcal{M}(Q,1)/Q$, where $Q = B/S$ is the *queue length* or *market tightness*. All households participate in the KW market, so $B = 1$; only firms with $e = 1$ participate, so $S = 1-u$ where u is unemployment. Thus, $\alpha_h = \mathcal{M}(1, 1-u)$. This exact relation depends on details, but the very general idea this: it is better to be a buyer when there are more sellers, and more sellers means less unemployment. One reason it is better to be a buyer

[6] Modern monetary theory goes into much detail about specialization and other features of the environment that give rise to a role for a medium of exchange; we see no need to repeat all of that here. We do need to say something about why *money* is the medium of exchange, and not e.g. claims to real assets, like shares in firms. This can be addressed formally as in Lester, Postlewaite and Wright (2006) by assuming agents may not recognize counterfeit claims in the KW market, even if they can be authenticated in the AD market, while currency cannot be counterfeited (or it can always be recognized). The model still works if we allow credit in the KW market, or allow real claims to circulate, although the implications for monetary policy may be a little different; actually, the case with credit or real claims circulating can be interretted as a special case of the baseline model when we run the Friedman rule, since this makes cash and credit prefect substitutes.

when there are more sellers is that the probability of trade is higher, but even if this were not the case, the terms of trade can be better (more on this in Section 4.3).

For h in the MP market,

$$U_1^h(m) = \delta V_0^h(m) + (1-\delta) V_1^h(m) \tag{4}$$

$$U_0^h(m) = \lambda_h V_1^h(m) + (1-\lambda_h) V_0^h(m), \tag{5}$$

where δ is the exogenous rate at which matches are destroyed, and λ_h the endogenous rate at which they are created. The latter is determined by another standard matching function, $\lambda_h = \mathcal{N}(u,v)/u$, where u is unemployment and v is the number of vacancies posted by firms. By constant returns, $\lambda_h = \mathcal{N}(1, v/u)$, where v/u is labor market tightness. We assume wages are determined when firms and households meet in the MP market (see below), although they are not actually paid until the next AD market. This is not especially important, but it avoids having to specify how w is paid – e.g. in dollars or in goods – since all that matters in AD is the implied purchasing power.

This completes the specification of the household problem. Before moving to firms, we mention that one can collapse the three value functions for h into one Bellman equation. Substituting (3) into (4) we get

$$\begin{aligned} U_1^h(m) &= \delta \left[\alpha_h \upsilon(q) + \alpha_h W_0^h(m-d) + (1-\alpha_h) W_0^h(m) \right] \\ &\quad + (1-\delta) \left[\alpha_h \upsilon(q) + \alpha_h W_1^h(m-d) + (1-\alpha_h) W_1^h(m) \right] \\ &= \alpha_h \left[\upsilon(q) - \frac{d}{p} \right] + \frac{m}{p} + \delta W_0^h(0) + (1-\delta) W_1^h(0), \end{aligned}$$

using the linearity of W_e^h. Something similar can be done for U_0^h. Inserting these into (2), the AD problem becomes

$$\begin{aligned} W_e^h(m) &= \frac{I_e + m}{p} + \max_{\tilde{\mathbf{x}}} \left\{ \tilde{\Upsilon}_e(\tilde{\mathbf{x}}) - \frac{\tilde{\mathbf{p}}\tilde{\mathbf{x}}}{p} \right\} \qquad (6) \\ &\quad + \max_{\hat{m}} \left\{ -\frac{\hat{m}}{p} + \beta \hat{\alpha}_h \left[\upsilon(\hat{q}) - \frac{\hat{d}}{\hat{p}} \right] + \beta \frac{\hat{m}}{\hat{p}} \right\} + \beta \mathbb{E} \hat{W}_{\hat{e}}^h(0), \end{aligned}$$

where expectation $\mathbb{E}$ is with respect to next period's employment status $\hat{e}$. As shown below – see e.g. condition (12) – the terms of trade $(\hat{q}, \hat{d})$ depend on $\hat{m}$ but not e, I_e or m, so (6) makes clear that $\hat{m}$ is independent of e, I_e and m.

2.2 Firms

First, since firms do not need money in the KW market, they obviously choose $\hat{m} = 0$. Then, in the MP market,

$$U_1^f = \delta V_0^f + (1-\delta) V_1^f$$
$$U_0^f = \lambda_f V_1^f + (1-\lambda_f) V_0^f$$

where $\lambda_f = \mathcal{N}(u,v)/v$, as is standard. However, we now depart from the textbook MP model as follows: rather than having f and h each consume some share of their output when they are matched, in our setup, f takes y to the KW market and tries to sell it. The idea, which should be uncontroversial, is this: agents do not necessarily want to consume what they made that day at work. Hence there is a role for a separate goods market, or retail sector, which is our KW market.

Firm f participates in the goods market iff he produced that period, which requires $e = 1$. If f with output y makes sells q in the KW market, we assume that $y - q$ is transformed into $x = \zeta(y-q)$ units of the quasi-linear good in the next AD market, where $\zeta' > 0$ and $\zeta'' \leq 0$ (there is implicitly a nonnegativity constraint on inventories, $q \leq y$, but it is easy to give conditions guaranteeing that this is not binding). It is useful to define the opportunity cost of a sale as $c(q) = \zeta(y) - \zeta(y-q)$, which satisfies $c' > 0$ and $c'' \geq 0$. Unless otherwise specified, we assume ζ is linear. Then we can write $x = y - q$, without loss in generality, and $c(q) = q$. This means one could interpret x and q as the same physical good that f can store across markets, bearing in mind that consumers generally value it differently in the two markets.[7]

For convenience we consolidate production and retail activities within the same firm (although it might be interesting to proceed differently). Thus, for f with $e = 1$ in the KW market,

$$V_1^f = \alpha_f W_1^f \left[\zeta\left(y-q\right), d\right] + (1-\alpha_f) W_1^f \left[\zeta\left(y\right), 0\right], \tag{7}$$

where $\alpha_f = \mathcal{M}(B,S)/S$, and $W_1^f(x,m)$ is the value of entering the AD market with x units of the quasi-linear good in inventory and m in cash receipts. That is,

$$W_1^f(x,m) = x + m/p - w + \beta \hat{U}_1^f, \tag{8}$$

where $w = w_n/p$ is the real wage, which as we said is paid in AD. Combing these expressions,

$$V_1^f = R - w + \beta\left[\delta \hat{V}_0^f + (1-\delta)\hat{V}_1^f\right], \tag{9}$$

where

$$R = \zeta\left(y\right) + \alpha_f\left[d/p - c(q)\right] \tag{10}$$

is expected real revenue.

As in the standard model, a firm with $e = 0$ has no current revenue or wage obligations, but if it pays a real cost k in the current AD market, it enters the next MP market with a vacancy that might match with a worker. Thus,

$$W_0^f = \max\left\{0, -k + \beta\lambda_f \hat{V}_1^f + \beta(1-\lambda_f)\hat{V}_0^f\right\},$$

where $\hat{V}_0^f = \hat{W}_0^f = 0$ under the usual free entry assumption. In steady state $k = \beta\lambda_f V_1^f$, which by (9) can be written

$$k = \frac{\beta\lambda_f\left(R-w\right)}{1-\beta(1-\delta)}. \tag{11}$$

[7] We could also assume unsold output simply vanishes between the KW and AD markets, or $\zeta' = 0$, but we like the idea of having an opportunity cost in the KW market. One could assume $y - q$ is carried forward to the next KW market, but then we would need to track the distribution of inventories across firms. Having firms liquidate inventories in the AD market allows us to capture the notion of opportunity cost while avoiding this technical problem, just like the AD market allows us to avoid a distribution of money across households.

Average real profit across all firms is $(1-u)(R-w)-vk$. As we said, our firms pay out profits as dividends to households. If we assume the representative household holds the representative portfolio, their real dividend is $\Delta/p=(1-u)(R-w)-vk$.

2.3 Government

Government consumes G in real terms, levies lump sum and proportional wage taxes T and τ, and prints money at rate π so that $\hat{M}=(1+\pi)M$. It also pays out a real UI (unemployment insurance) benefit b, which is taxed, to households with $e=0$. Hence, the nominal value of unemployment income in (1) is $b_n = pb(1-\tau)+p\ell$, where ℓ is the real value of leisure plus home production, which is not taxed. The government budget constraint is $pG+pbu=T+\tau w_n(1-u)+\pi M$. We can describe monetary policy in terms of setting the nominal interest rate i, or equivalently in terms of the growth rate of the money supply π, because of the Fisher equation $1+i=(1+\pi)/\beta$. We always assume $i>0$, although we do consider the limit as $i\to 0$, which is the Friedman rule.

3 Equilibrium

Various assumptions can be made concerning price determination in the different markets, including (Walrasian) price taking, bargaining, and price posting with or without directed search. We think the most reasonable scenario is the following: price taking in the AD market, bargaining in the MP market, and posting with directed search in the KW market. We like price taking in the AD market because it is simple, and in any case the AD market is not the prime focus of our analysis. In the MP market, which is a key part of the theory, we opt for bargaining because it seems realistic for many labor markets and because it is standard in the related literature. The choice is less clear for the KW market, so we consider several options. We actually start here with bargaining version because it is slightly easier to present. In the next section we present a Walrasian price-taking version, which might feel more comfortable to mainstream macroeconomists, and our most preferred option, price posting with directed search in this market.

Price posting with directed search – also known as competitive search equilibrium, after Moen (1997) and Shimer (1996) – is our preferred approach for several reasons. First, it is fairly convenient after some initial set-up cost. Second, directed search should seem like a big step forward to those like Howitt (2005) who criticize monetary theory with random matching for the assumption of randomness per se.[8] It should also appease those like Phelan (2005), who don't like much modern monetary theory because they "don't like bargaining." More seriously, bargaining models typically need an unpalatable assumption that agents see each others' money holdings to avoid the technical difficulties of bargaining with private information. Finally, using competitive search eliminates bargaining power as a free parameter. Hence, competitive search deflects several critiques, is convenient, and seems realistic.

Whatever the pricing mechanism, we break the analysis of equilibrium into three parts. First, following Lagos-Wright (2005), we determine the value of money as measured by q, taking unemployment u as given. We then determine u, taking q as given, as in Mortensen-Pissarides (1996). It is convenient to depict these two relationships graphically in (u, q) space by what we call the *LW curve* and the *MP curve*. Their intersection determines the equilibrium unemployment rate and value of money (u, q), from which all of the other endogenous variables easily follow.

[8] As Howitt (2005) puts it, "In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus, when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters." (p. 405). The do exist several other directed (as opposed to random) search models of money, such as Corbae et al. (2003) or Julien et al. (2007).

3.1 The LW Curve

Imagine for now f and h meet and bargain bilaterally over (q, d) in the KW market, st $d \leq m$, where m is money held by h. Because it is easy, we use the generalized Nash solution, with threat points equal to continuation values from not trading and θ denoting the buyer's bargaining power.[9] The surplus for h is $\upsilon(q) + W_e^h(m-d) - W_e^h(m) = \upsilon(q) - d/p$, by the linearity of $W_e^h(m)$. Similarly, the surplus for f is $d/p - c(q)$. Then is is easy to show the following: maximizing the Nash product st $d \leq m$ implies $d = m$ and q solves exactly the condition in Lagos and Wright (2005):[10]

$$\frac{m}{p} = g(q, \theta) \equiv \frac{\theta \upsilon'(q) c(q) + (1-\theta) \upsilon(q) c'(q)}{\theta \upsilon'(q) + (1-\theta)\, c'(q)} \tag{12}$$

As we said above, in this paper we usually assume $c(q) = q$, which simplifies (12) a little.

Now recall (6), which in terms of the choice $\hat{m}$ is summarized by

$$\max_{\hat{m}} \left\{ -\frac{\hat{m}}{p} + \beta \hat{\alpha}_h \left[\upsilon(\hat{q}) - \frac{\hat{m}}{\hat{p}} \right] + \beta \frac{\hat{m}}{\hat{p}} \right\},$$

where we inserted $\hat{d} = \hat{m}$ and it is understood that $\hat{q}$ is a function of $\hat{m}$, given implicitly by (12). By the Fisher equation $1 + i = \hat{p}/p\beta$, this problem is equivalent to

$$\max_{\hat{m}} \left\{ \upsilon(\hat{q}) - \frac{\hat{m}}{\hat{p}} \frac{i + \hat{\alpha}_h}{\hat{\alpha}_h} \right\}. \tag{13}$$

The first order condition for an interior solution is $\upsilon'(\hat{q}) \partial \hat{q} / \partial \hat{m} = (i + \hat{\alpha}_h) / \hat{\alpha}_h \hat{p}$ (second order conditions are discussed below). Inserting $\partial \hat{q} / \partial \hat{m} = 1/\hat{p} g_1(\hat{q}, \theta)$, which we get by differentiating (12), as well as $\hat{\alpha}_h = \mathcal{M}(1, 1 - \hat{u})$, and then imposing steady state, we arrive at

$$\frac{\upsilon'(q)}{g_1(q, \theta)} - 1 = \frac{i}{\mathcal{M}(1, 1-u)}. \tag{14}$$

[9] Rocheteau and Waller (2004) analyze several alternative bargaining solutions in monetary models, any of which would do for our purposes.

[10] Basically, $d = m$ follows from a simple arbitrage argument – why bring more money than you are ever going to spend? – and (12) follows directly from the first order condition.

This is the LW curve, determining q for a given u exactly as in Lagos and Wright (2005). An increase in u affects q because, as discussed above, higher unemployment makes it less attractive to be a buyer by adversely affecting the probability and/or terms of trade. This reduces the demand for m/p and hence reduces q via the bargaining solution $m/p = g(q, \theta)$. From known results we can describe the properties of the LW curve. First, it is not automatic that the LHS of (14) is decreasing in q, but one can impose conditions to guarantee it is, and hence to guarantee a unique $q > 0$ solving this condition for any $u \in (0, 1)$ with $\partial q/\partial u < 0$.[11] But even if υ'/g_1 is not globally decreasing, it is decreasing at any q such that the second-order condition is satisfied; hence, whenever the first- and second-order conditions hold $\partial q/\partial u < 0$. Also, letting q^* be the efficient quantity, given by $\upsilon'(q^*) = 1$, q is bounded by q^* for any u. Also, $u = 1$ implies $q = 0$. Summarizing:

Proposition 1 *For all $i > 0$ the LW curve slopes downward in (u, q) space, with $u = 0$ implying $q \in (0, q^*)$ and $u = 1$ implying $q = 0$. It shifts down with i and up with θ. In the limit as $i \to 0$, $q \to q_0$ for all $u < 1$, where q_0 is independent of u, and $q_0 \leq q^*$ with $q_0 < q^*$ unless $\theta = 1$.*

3.2 The MP Curve

When unmatched f and h meet in the labor market, they bargain over w according to the generalized Nash solution with threat points equal to continuation values from remaining unmatched and η the bargaining power of f. The surplus for h is $S^h = V_1^h(m) - V_0^h(m) = W_1^h(0) - W_0^h(0)$. Inserting W_e^h from (6) and simplifying,

$$S^h = (w - b)(1 - \tau) - \ell + \beta\,(1 - \delta - \lambda_h)\,\hat{S}^h.$$

[11] Examples of conditions that can be used to make υ'/g_1 globally decreasing in q are: (i) u' log-concave; or (ii) $\theta \approx 1$.

The surplus for f is $S^f = V_1^f - V_0^f = R - w + \beta(1-\delta)\hat{S}^f$ by virtue of (7) and free entry. One solves this problem in the standard way: (i) first maximize the Nash product taking as given future surpluses $\hat{S}^i$; (ii) then insert the steady state values

$$S^h = \frac{(w-b)(1-\tau) - \ell}{1-\beta\,(1-\delta-\lambda_h)} \text{ and } S^f = \frac{R-w}{1-\beta\,(1-\delta)},$$

and rearrange for

$$w = \frac{\eta\,[1-\beta\,(1-\delta)]\,z + (1-\eta)\,[1-\beta\,(1-\delta-\lambda_h)]\,R}{1-\beta\,(1-\delta) + (1-\eta)\,\beta\lambda_h}, \tag{15}$$

where $z = b + \ell/(1-\tau)$ is the value to h of not working, adjusting for taxes.

If we substitute w from (15) and R from (10) into the free entry condition (11), we arrive at

$$k = \frac{\lambda_f \eta\,[y - z + \alpha_f\,(d/p - q)]}{r + \delta + (1-\eta)\lambda_h},$$

where $r = 1/\beta - 1$. To reduce this to one equation in (u, q) we do three things: (i) insert the arrival rates from the matching functions $\lambda_f = \mathcal{N}(u,v)/v$, $\lambda_h = \mathcal{N}(u,v)/u$ and $\alpha_f = \mathcal{M}(1, 1-u)/\,(1-u)$; (ii) insert

$$\frac{d}{p} - q = g(q,\theta) - q = \frac{(1-\theta)\,[u(q) - q]}{\theta u'(q) + 1 - \theta}$$

from the bargaining solution; and (iii) use the so-called Beveridge curve (the steady state condition for unemployment) $(1-u)\delta = \mathcal{N}(u,v)$ to solve for and insert $v = v(u)$. The final answer is

$$k = \frac{\mathcal{N}\,[u, v(u)]\,\eta}{v(u)} \frac{y - z + \frac{\mathcal{M}(1,1-u)}{1-u}\frac{(1-\theta)[u(q)-q)]}{\theta u'(q)+1-\theta}}{r + \delta + (1-\eta)\frac{\mathcal{N}[u,v(u)]}{u}}. \tag{16}$$

This is the *MP curve*, determining u as in Mortensen and Pissarides (1996), except that they effectively have $q = 0$. It is a matter of routine calculation to show that this curve is downward sloping. Intuitively, there are three effects from an increase in u, two from the usual MP model and one that is new, all of which encourage entry: (i) it is easier for firms to hire; (ii) it is harder for households to get hired, which lowers w; (iii) it is easier for firms to compete in the KW market, which is the new effect. Again, these three effects go in the same direction and the MP curve unambiguously slopes downward. It is easy to derive other properties of the MP curve. Summarizing:

Proposition 2 *The MP curve slopes downward in (u, q) space. It shifts in with y, η or θ, and out with k, r, δ or $z = b + \ell/(1-\tau)$.*

3.3 LW meets MP

Propositions 1 and 2 imply both LW and MP slope downward in the box $\mathcal{B} = [0,1] \times [0, q^*]$ in (u, q) space, as show in Figure 2. LW enters $\mathcal{B}$ from the upper left when $u = 0$ at $q_0 \leq q^*$, and exits at $(0, 0)$. MP enters $\mathcal{B}$ when $q = q^*$ at some $\underline{u} > 0$, with $\underline{u} < 1$ iff k is not too big, and exits $\mathcal{B}$ by either hitting the horizontal axis at some $u_0 \in (0, 1)$, or hitting the vertical axis at some $q_1 \in (0, q^*)$. It is easy to check MP hits the horizontal axis $u_0 \in (0, 1)$, as shown by the curve labeled 1 in the Figure, iff $\eta(y - z) > k(r + \delta)$. This inequality simply says there would be entry into the MP market even if we shut down the KW market. In this case, there exists a nonmonetary steady state equilibrium at $(u_0, 0)$, which is exactly the standard MP equilibrium, and there exists at least one monetary steady state with $q > 0$ and $u < u_0$.

The Figure also shows two cases, labeled 2 and 3, where the MP curve intersects the vertical axis at some q_1. In these case, there either exist multiple monetary steady states, as shown by curve 2, or there exist no monetary steady states, as shown by curve 3. In either case there also exists a nonmonetary steady state at $u = 1$ and $q = 0$. In these nonmonetary equilibria, which occur iff $\eta(y - z) < k(r + \delta)$, the KW market shuts down, and this means the MP market also shuts down. In case 3, this is the only possible equilibria; in case 2, however, there are also monetary equilibria with the KW and MP markets and Settings/rwright/Desktop/Docs/graphics/BMW-slides-4.bmp

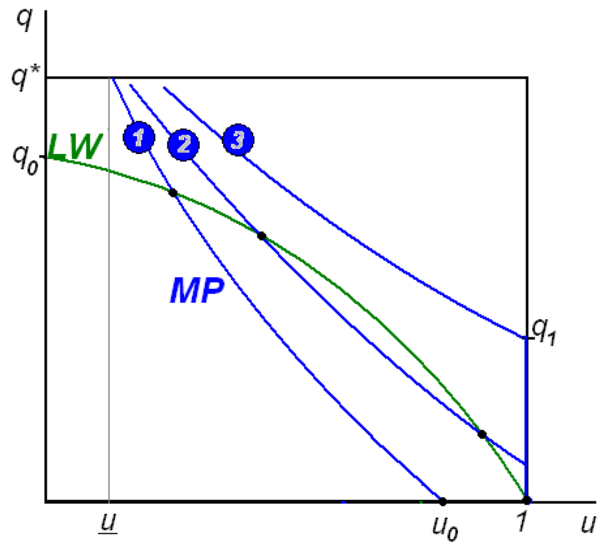

Figure 2: LW meets MP

both open and $u < 1$. In case 1, recall, even if $q = 0$ there is still the standard MP equilibrium with $u < 1$.

To understand which case is more likely to occur, simply look at Propositions 1 and 2 concerning the effects of changes in parameters. In every case we have established the existence of steady state equilibrium. We do not generally get uniqueness, as is clear from the Figure, but it is possible for the monetary steady state to be unique, as shown with curve 1. If there exists any steady state with $u < 1$ then there exists a monetary steady state. We also know that a sufficient condition for a steady state with $u < 1$, and hence for a monetary steady state, is $\eta (y - z) > k(r + \delta)$, which is also required for a steady state with $u < 1$ in the standard MP model. Given (u, q), we can recover all of the other endogenous variables, including vacancies v, the arrival rates α_j and λ_j, real balances $m/p = g(q, \theta)$, and the nominal price level $p = M/g(q, \theta)$.[12]

[12] Given these variables, the AD budget equation yields x for every individual as a function

A convenient result from Propositions 1 and 2 is that changes in i shift only the LW curve, while changes in y, η, r, k, δ or z shift only the MP curve. This makes it easy to analyze changes in parameters. For example, an increase in the nominal interest rate i shifts the LW curve toward the origin, reducing q and u if equilibrium is unique (or, in the 'natural' equilibria if we do not have uniqueness). The result $\partial q/\partial i < 0$ holds in the standard LW model, with a fixed α_h, but now there is a general equilibrium (multiplier) effect via u that reduces α_h and further reduces q. Similarly, an increase in z shifts the MP curve out, increasing u and reducing q if equilibrium is unique (or, in the 'natural' equilibria). The result in $\partial u/\partial z > 0$ u holds in the standard MP model with fixed R, but now there is an effect via q that reduces R and further increases u. Other experiments can be analyzed similarly.

Proposition 3 *Steady state equilibrium always exist. One steady state is a nonmonetary equilibrium, which has $u < 1$ iff $\eta(y-z) > k(r+\delta)$. If this inequality holds, there also exists a monetary steady state. Assuming the monetary steady state is unique, a rise in i decreases q and increases u, while a rise in y or η, or a fall in k, r, δ or z, increases q and decreases u.*

4 Alternative Pricing Mechanisms

While the model works fine with random matching and bargaining, we want to consider some alternatives, as discussed above. We first consider competitive

of the m with which he enters. In the general case where there is a vector $\tilde{x}$ of other AD goods, maximization determines an individual demand as a function of employment status and $\tilde{\mathbf{p}}$ (plus p which has already been determined). Write this as $\tilde{\mathbf{x}} = \mathbf{D}_e(\tilde{\mathbf{p}})$. Market demand is $\mathbf{D}(\tilde{\mathbf{p}}) = u\mathbf{D}_0(\tilde{\mathbf{p}}) + (1-u)\mathbf{D}_1(\tilde{\mathbf{p}})$. Equating this to the endowment vector yields a standard system of general equilibrium equations $\mathbf{D}(\tilde{\mathbf{p}}) = \bar{\mathbf{x}}$ that solve for $\tilde{\mathbf{p}}$. The model displays classical neutrality: if we change M, we can change p and $\tilde{\mathbf{p}}$ proportionally without affecting the AD equilibrium conditions or the values of the other real variables (q, u, v). This does not mean money does not matters: a change in i (or equivalently π) shifts the LW curve, which affects q, u and the rest of the system. When AD utility Υ does not depend on e, however, neither does demand, so $\mathbf{D}(\tilde{\mathbf{p}})$ is independent of u and hence $\tilde{\mathbf{x}}$ is independent of monetary policy – a version of the neoclassical dichotomy.

search equilibrium, which involves price posting and directed search, and then competitive equilibrium, with Walrasian price-taking behavior.

4.1 Price Posting

There are several ways to formalize the notion of competitive search equilibrium. One is to have sellers first post the terms of trade, then have buyers direct their search to the most favorable sellers, taking into account that they may not get served if more buyers show up than a seller has capacity to handle. Or we can have buyers post the terms of trade to attract sellers. Or we can imagine market makers that set up submarkets and try to attract buyers and sellers by posting the terms of trade, so they can charge them an entrance fee (which is 0 in equilibrium), where in each submarket buyers and sellers match randomly according to $\mathcal{M}(B, S)$ but are bound by the posted terms. It is known in the literature than these different stories all lead to the same set of equilibrium conditions.[13]

Given this, we proceed by assuming sellers post the terms of trade, but trade is still probabilistic, in the sense that if a group of B buyers direct their search towards a group of S sellers, the number of meetings is given by $\mathcal{M}(B, S)$. Hence, agents need to know the queue length $Q = B/S$ to determine the relevant probabilities, $\alpha_f = \mathcal{M}(Q, 1)$ and $\alpha_h = \mathcal{M}(Q, 1)/Q$. We imagine the firm posting in the AD market the following message: "If I have anything to sell (i.e. if $e = 1$) next period, then I commit to sell q units for d dollars in the KW market, but I can serve at most one customer, and you should expect a queue length Q."

[13] Papers that model different versions of these stories, in addition to Moen (1997) and Shimmer (1996), include Acemoglu and Shimer (1999), Burdett, Shi and Wright (2001), Mortensen and Wright (2002), and Julien, Kennes and King (2000). There is one detail. While these different models are equivalent in nonmonetary economies, Faig and Huangfu (2005) show that a monetary economy can do better with market makers than with sellers or buyers posting. The idea is that market makers can in principle charge buyers and pay sellers (i.e. charge them a negative fee) to enter their submaket, and then have them trade if they meet at price 0. Effectively, this insures agents against the possibility of not trading and allows buyers to each bring less money. However, one can assume market makers cannot tell if an agent is a buyer or a seller, which precludes differential fees and recovers the equivalence.

Formally, the problem of a firm is

$$\max_{q,d,Q} \mathcal{M}(Q,1)\left(\frac{d}{p}-q\right) \tag{17}$$
$$\text{st } \frac{\mathcal{M}(Q,1)}{Q}\left[\upsilon(q)-\frac{d}{p}\right]-i\frac{d}{p}=\hat{Z},$$

where $\hat{Z}$ is the terms offered by the best alternative seller.[14]

Assume for now that $\hat{Z}\in(0,\bar{Z})$ where $\bar{Z}$ is not too big (see below), so that the firm wants some buyers to show up, $Q>0$. Then use the constraint to eliminate d and rewrite (17) as

$$\max_{q,Q} \mathcal{M}(Q,1)\left[\frac{\mathcal{M}(Q,1)\upsilon(q)-\hat{Z}Q}{\mathcal{M}(Q,1)+iQ}-q\right]. \tag{18}$$

One can show this problem has a solution with $Q>0$. Indeed, over the interval $(0,\bar{Z})$ there is at most a finite number of values for $\hat{Z}$ with multiple solutions, as shown in Figure 3. We assume here that the solution $Q(\hat{Z})$ is convex valued – i.e. we rule out the situation at $\hat{Z}_2$ in the Figure – and give sufficient conditions for this to be valid below.[15]

To indicate where this is leading, think of Q as the demand for buyers by a seller, and $\hat{Z}$ as the price (in terms of utility) that the seller has to pay in order to get buyers to show up. In equilibrium $\hat{Z}$ will be determined so that the supply of buyers per firm $1/(1-u)$ equals the demand Q as long as $\hat{Z}>0$; if $\hat{Z}=0$ then buyers get no surplus in the KW market, and any given buyer is indifferent whether or not he shows up, so we can get any number $B\in[0,1]$. Let $Q_0=Q(0)$ and define u_0 by $Q_0=1/(1-u_0)$. Then as long as $u<u_0$, there is a $Z>0$ such that the market clears with $Q(Z)=1/(1-u)$. When $u\geq u_0$,

[14] The objective function is expected revenue net of opportunity cost, ignoring constants that do not affect the maximization, and the constraint says the seller has to offer buyers at least $\hat{Z}$, since this problem is conditional on wanting some buyers to actually show up. It is easy to derive this simplified version of a firm's problem from the underlying dynamic model.

[15] Indeed, we will give sufficient conditions for Q to be single-valued, and strictly decreasing. When $Q(\hat{Z})$ is not convex valued, as is the case in the Figure when $Z=Z_2$, we need to have different sellers posting different Q's to clear the market. This is not a big deal, but the situation is obviously simpler when $Q(\hat{Z})$ is convex valued.

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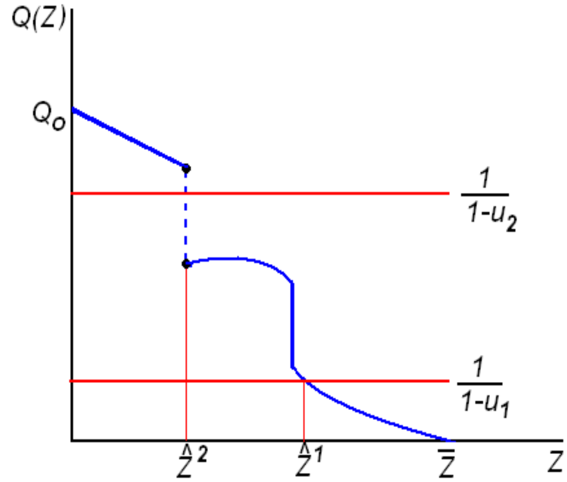

Figure 3: Solution for Q to Firm Posting Problem

the market equilibrates at $Z = 0$, in which case $B < 1$ buyers show up and the rest stay home, so that $Q = Q_0 < 1/(1-u)$. Clearly, the market clears at $Z > 0$ iff u is not too big. Hence, in equilibrium $Q = Q(u) = \min\{Q_0, 1/(1-u)\}$.

Given this, take the first order conditions to (17):

$$\frac{\upsilon'(q)\mathcal{M}}{\mathcal{M}+iQ} - 1 = 0 \tag{19}$$

$$\mathcal{M}_1\left[\frac{\mathcal{M}\upsilon(q) - \hat{Z}Q}{\mathcal{M}+iQ} - q\right] - \mathcal{M}\mathcal{M}_2\left[\frac{i\upsilon(q)+\hat{Z}}{(\mathcal{M}+iQ)^2}\right] = 0 \tag{20}$$

From (19) we determine q by:

$$\upsilon'(q) - 1 = \frac{iQ(u)}{\mathcal{M}[Q(u),1]} = \begin{cases} i/\mathcal{M}(1, 1-u) & \text{if } u < u_0 \\ i/\mathcal{M}(1, Q_0^{-1}) & \text{if } u > u_0 \end{cases} \tag{21}$$

Now eliminate from (17) $\mathcal{M} + iQ$ using (19) and $\hat{Z}$ using the constraint, to rewrite (20) as

$$\frac{d}{p} = \frac{Q\mathcal{M}_1\upsilon'(q)q) + \mathcal{M}_2\upsilon(q)}{Q\mathcal{M}_1\upsilon'(q) + \mathcal{M}_2}.$$

Letting $\varepsilon(Q) \equiv Q\mathcal{M}_1(Q,1)/\mathcal{M}(Q,1)$ be the elasticity of $\mathcal{M}$ with respect to B, this reduces to

$$\frac{d}{p} = g[q, \varepsilon(Q)], \tag{22}$$

where $g(\cdot)$ is defined in (12). Hence, as is well-known in this literature, posting and bargaining give similar solutions, except bargaining power θ is replaced by the elasticity $\varepsilon(Q)$.

Let us collect some results. First, substitute (22) into the constraint in (17) and use (19) to write

$$F(q, Q) \equiv \frac{\mathcal{M}(Q, 1)}{Q} \{\upsilon(q) - \upsilon'(q) g[q, \varepsilon(Q)]\} - Z = 0. \tag{23}$$

Then rewrite (19) as

$$G(q, Q) \equiv \upsilon'(q) - 1 - \frac{iQ}{\mathcal{M}(Q, 1)} = 0. \tag{24}$$

Given Z, (q, Q) solves the first order conditions iff it solves (23)-(24). This is a candidate solution to (18), but we need to check the second order conditions. To this end we make the following assumptions.

$$\text{Assumption 1: } \frac{\partial \Phi(q, Q)}{\partial q} \geq 0 \tag{25}$$

$$\text{Assumption 2: } \frac{\partial \varepsilon(Q)}{\partial Q} \leq 0, \tag{26}$$

where $\Phi(q, Q) \equiv \upsilon(q) - \upsilon'(q) g[q, \varepsilon(Q)]$ is the term in braces in (23). A sufficient condition for (25) is that $\varepsilon(Q)$ is not too small; (26) holds for all the standard matching functions used in the literature.

Given (25)-(26), the locus of points in (Q, q) space satisfying (23) is upward sloping. Since the locus satisfying (24) is always downward sloping, there is at most one solution to (23)-(24). Moreover, if a non-zero solution exists, it must be the global maximizer since it yields $\mathcal{M}(Q, 1) \{g[q, \varepsilon(Q)] - q\} > 0$, and $Q = 0$ or $q = 0$ yields payoff 0. It is easy to check that, at least for i and Z not too big, a non-zero solution (Q, q) does exist. It is also easy to check that $\partial Q / \partial Z < 0$

under (25)-(26). All of this implies we can rule out the complications shown in Figure 3, and the situation is as in Figure 4.

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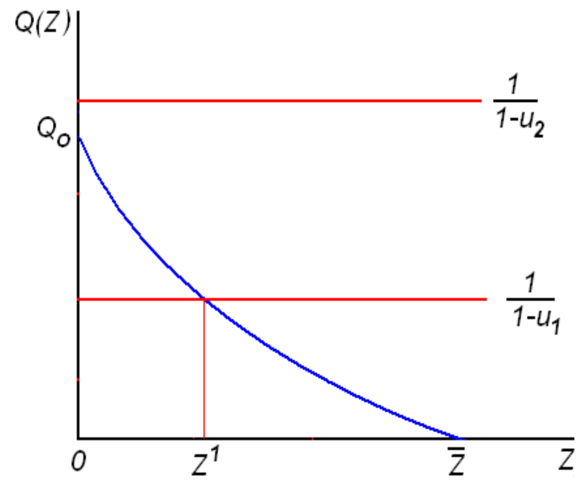

Figure 4: Solution for Q to Firm Posting Problem with A1 and A2

So under assumptions (25)-(26), there is a unique solution (q, Q) and it is non-zero iff $Z < \bar{Z}$. Given an unemployment rate $u < u_0$, we have $Q = 1/(1-u)$, and q is pinned down by (19). If we increase u, as Figure 4 shows, we reduce Z, which reduces q, as one can check by differentiating (23)-(24). This traces out a downward-sloping locus of points in (u, q) space. Intuitively, as u increases the KW queue length Q increases, reducing α_h and q. The only complication is that when we increase u beyond u_0, there is no $Z > 0$ that equates $Q(Z) = 1/(1-u)$, so in equilibrium $Q = Q_0$ and households get $Z = 0$ in the KW market. For $u > u_0$, increases in u do not change $Q = Q_0$ and hence do not change $q = q_0$.

The LW curve is the value of money q solving (21), shown in Figure 5 for two different levels of i and $i' < i$. From (21), under competitive search with $i = 0$, the LW curve goes through q^* at $u = 0$. This is true under bargaining iff $\theta = 1$. Also notice that there is a kink in LW at $u = u_0$ and $q = \psi(u_0)$, where we get $\psi(\cdot)$ by solving (23) with $Z = 0$, or $v(q_0) = v'(q_0)g[q_0, \varepsilon(Q_0)]$. Solving this for q_0 and inserting $Q_0 = 1/(1-u_0)$ implies $q_0 = \psi(u_0)$ with $\psi' \geq 0$ by (26); if $\mathcal{M}(B, S)$ is Cobb-Douglas, q_0 is independent of u_0 and $\psi(u_0)$ horizontal. Thus the LW curve is qualitatively similar to what we had under bargaining.

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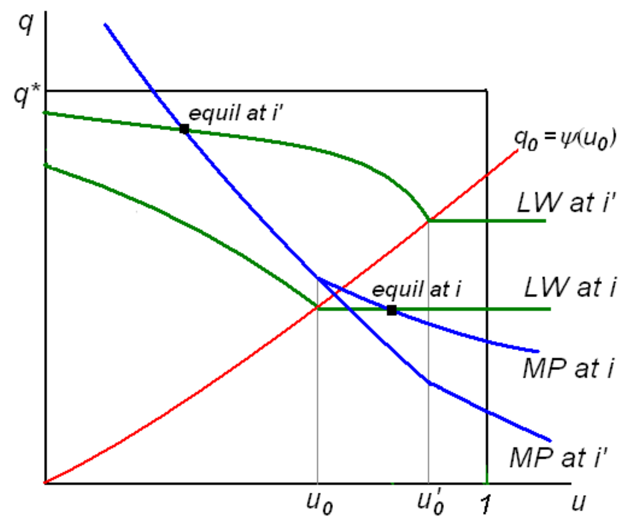

Figure 5: LW and MP with Competitive Search

The MP curve also needs to be modified, as follows. For $u < u_0$ we have

$$k = \frac{\mathcal{N}[u, v(u)]}{v(u)} \frac{\eta \left\{ y - z + \frac{\mathcal{M}(1,1-u)}{1-u} \frac{(1-\varepsilon)[u(q)-q)]}{\varepsilon u'(q)+1-\varepsilon} \right\}}{r + \delta + (1-\eta)\frac{\mathcal{N}[u,v(u)]}{u}}, \tag{27}$$

which is identical to (16), except that we replace bargaining power θ with the elasticity $\varepsilon = \varepsilon(Q) = \varepsilon(\frac{1}{1-u})$. For $u > u_0$, the result is the same except we replace $\frac{\mathcal{M}(1,1-u)}{1-u}$ with $\mathcal{M}(Q_0, 1)$ and $\varepsilon = \varepsilon(Q_0)$. Hence, the MP curve is still downward sloping, but now has a kink at u_0, because R is independent of u for $u > u_0$. Notice that in Figure 5, at the low interest rate i' equilibrium occurs at $u < u_0$, while at the higher interest rate i it occurs at $u > u_0$. Despite these modifications, the properties of the model are qualitatively the same as those derived under bargaining.

4.2 Walrasian Price Taking

We now consider Walrasian price taking in the KW market. Search models with price taking go back to the Lucas and Prescott (1974) model of unemployment, where although it may take time to get from one local labor market to another, each one is composed of large numbers of workers and firms who behave perfectly competitively. This idea is similar in the KW market here, except a medium of exchange is essential for the usual reasons – a double coincidence problem and anonimity. Agents take the dollar price ϕ of KW goods parametrically. Also, as in Rocheteau and Wright (2005), we generalize Lucas-Prescott by allowing for frictions in the sense that agents get into the market only probabilistically. We also allow a potentially nonlinear opportunity cost $c(q)$ in the KW market to illustrate certain points.

Every f with $e = 1$ wants to get into the KW market. Those that do choose q^f to maximize $\phi q^f/p - c(q^f)$, which implies $c'(q^f) = \phi/p$. Then in the AD market, with the usual MP manipulations, free entry implies

$$k = \frac{\lambda_f \eta \left\{ y - z + \alpha_f \left[q^f c'(q^f) - c(q^f) \right] \right\}}{r + \delta + (1-\eta)\lambda_h}, \tag{28}$$

where α_f is the probability f gets into the KW market. Every h wants to get into the KW market, and those that do choose q^h to maximize $\upsilon(q^h) + W_e^h(m - \phi q^h)$ st $\phi q^h \leq m$, and in equilibrium as usual we get $q^h = m/\phi$. In the AD market, h chooses $\hat{m}$ to maximize $\upsilon(\hat{q}^h) - \frac{\hat{m}}{\hat{p}} \frac{i + \hat{\alpha}_h}{\hat{\alpha}_h}$, where $\hat{\alpha}_h$ is the probability h gets into the KW market. Since Walrasian pricing implies $\partial q^h/\partial m = 1/\phi$, and $\phi = pc'(q^f)$ from the firm problem, the usual LW manipulations yield

$$\frac{i}{\alpha_h} = \frac{\upsilon'(q^h)}{c'(q^f)} - 1. \tag{29}$$

Search-type frictions are introduced by letting the measures of h and f that get in to the KW market be a function of the measures that want to get in,

$\mathcal{M}^h = \mathcal{M}^h(1, 1-u)$ and $\mathcal{M}^f = \mathcal{M}^f(1, 1-u)$, which means $\alpha^h = \mathcal{M}^h(1, 1-u)$ and $\alpha^f = \mathcal{M}^f(1, 1-u)/(1-u)$. A special case has equal measures, $\mathcal{M}^h = \mathcal{M}^f$, as is necessary in models of bilateral trade but obviously not in Walrasian models. If we do assume $\mathcal{M}^h = \mathcal{M}^f$, however, we can always interpret agents as trading bilaterally even though they take prices parametrically. Another special case is the *frictionless* version of the model where everyone who wants to can trade, $\mathcal{M}^h = 1$ and $\mathcal{M}^f = 1 - u$, or $\alpha^h = \alpha^f = 1$; we cannot possibly have this in a model with bilateral trade unless $u = 0$. In any case, the goods market clearing condition is $\mathcal{M}^h q^h = \mathcal{M}^f q^f$. Inserting $q^h = q$ and $q^f = q\mathcal{M}^h/\mathcal{M}^f$, as well as the usual MP arrival rates λ_h and λ_f, into (29) and (28) gives us the LW and MP curves with Walrasian pricing in the KW market.

In the special case with no frictions, $\alpha^h = 1$ and $\alpha^f = 1$, we have $q^f = q/(1-u)$. In this special case the LW and MP curves become

$$
\begin{aligned}
i &= \frac{v'(q)}{c'\left(\frac{q}{1-u}\right)} - 1 \\
k &= \frac{\frac{\mathcal{N}[u,v(u)]}{v(u)}\eta\left\{y - z + \left[\frac{q}{1-u}c'\left(\frac{q}{1-u}\right) - c\left(\frac{q}{1-u}\right)\right]\right\}}{r + \delta + (1-\eta)\frac{\mathcal{N}[u,v(u)]}{u}}
\end{aligned}
$$

If we additionally impose linear cost, $c(q) = q$, then something moderately spectacular occurs: u vanishes from LW and q vanishes from MP. That is, the LW curve is horizontal and the MP curve vertical in (u, q) space, so the model dichotmozies. Intutively, with linear cost profit is 0 in a Walrasian market, which means that any surplus generated by KW trade goes to households, and therefore firm entry decisions and employment are independent of q. The MP cure is vertical. Also, since all households get into the goods market and the price (equals marginal cost) is independent of the number of firms when $c(q)$ is linear, the value of money does not depend on u. The LW curve is horizontal.

This dichotomy arises in the special case where (i) there are no fricitons and (ii) cost is linear. Actually, while both of these conditions are needed to solve

LW for q independently of u, only the latter is needed to solve MP independently of q. Based on this insight, one can reinterpret the standard MP model as one where firms do indeed sell their output in a market to households other than their own employees (for cash or credit, it is irrelevant in this case), since as long as the cost of sales in this market is linear and pricing is Walrasian, households get all and firms get none of the gains from trade, and u is determined as in the textbook model. There may or may not be monetary exchaneg lurking behind the labor market scene, but it does not affect vacancy creation or unemployment. of course we could also get the standard MP model as a special case in the bargaining model, say, but giving buyers all the bargaining power $\theta = 1$.

One might say that when $\alpha^h = \alpha^f = 1$ the model "looks like" a standard cash-in-advance economy, since there are no search-type frictions and no non-competitive pricing issues. We think it is good to allow for the possibility of frictions and alternative pricing mechanisms. We also firmly believe it is better to have a deeper model of monetary exchange – to be explicit and logically consistent about things like the double coincidence problem and role of anonimity – rather than to cavalierly impose cash-in-advance (this is not the place to discuss the issue in detail, but we do want to mention that the explicit and logically consistent model is no harder to solve). If however the reader is firmly committed to shallow models of money, we can offer little more than sympathy and the frictionless version of our model with Walrasian pricing – it is equivalent (in terms of algebra) to cash-in-advance model, and still has LW and MP curves like the benchmark model (as long as we do not have linear cost).

5 Efficiency

Consider a planner who seeks to maximize the welfare (expected utility) of the representative household, subject to several constraints. First, output y is produced in employment relations that form in the first subperiod, subject to the law of motion for u. Second, if firms and households meet in the second subperiod, which occurs according to the technology $\mathcal{M}(\cdot)$, the former can transfer q to the latter for a payoff of $\upsilon(q)$, and bring the remainder to the third subperiod, where it can be allocated to any household for a payoff of $y-q$. Each period, the planner takes unemployment u as a state variable, and chooses how many vacancies v to post, as well as q. He also chooses how to allocate remaining output in the third subperiod, but with quasi-linear utility this does not effect average welfare, and in paticular UI payments b do not show up in the planner's calculations.[16]

To reduce the notation, let $s(q) \equiv \upsilon(q) - q$ be the surplus from a KW trade. Then after simplification the planner's problem reduces to the following dynamic program:

$$
\begin{aligned}
J(u) &= \max_{q,v} \left\{ -vk + u\ell + (1-u)y + \mathcal{M}(1, 1-u)\, s(q) + \beta \hat{J}(\hat{u}) \right\} \\
&\text{st } \hat{u} = u + (1-u)\,\delta - \mathcal{N}(u, v)
\end{aligned}
$$

Instantaneous utility subtracts vacancy costs from output, including the output of the unemployed and the employed, plus the suplus generated by meetings in the KW market. It is not hard to show this is a well-behaved problem, and J is concave. The FOC for q is $s'(q) = 0$, which implies $q = q^*$ at every date. The FOC for v is $-k - \beta \hat{J}'(\hat{u})\, \mathcal{N}_2(u, v) = 0$. This together with the law of motion $\hat{u} = u + (1-u)\,\delta - \mathcal{N}(u, v)$ generates a decision rule for v as a function of u, determining the path for (u, v).

The envelope condition is

$$
J'(u) = -\mathcal{M}_2(1, 1-u)\, s(q) - y + \ell + \beta \hat{J}'(\hat{u}) \left[1 - \delta - \mathcal{N}_1(u, v)\right].
$$

[16] To ease the presentation we assume there are no goods $\tilde{\mathbf{x}}$ in the third subperiod other than x; if there were, the allocation would satisfy the obvious additional marginal conditions.

Using the FOC for v to eliminate J', we get the Euler equation

$$\frac{k}{\beta\mathcal{N}_2(u,v)} = y - \ell + \mathcal{M}_2(1, 1-\hat{u})\, s(\hat{q}) + \frac{k\left[1 - \delta - \mathcal{N}_1(\hat{u},\hat{v})\right]}{\mathcal{N}_2(\hat{u},\hat{v})}.$$

In steady state, the planner's solution generates unemployment u^* satisfying

$$k = \frac{\mathcal{N}_2\left[u^*, v(u^*)\right]\left[y - \ell + \mathcal{M}_2(1, 1-u^*)\, s(q^*)\right]}{r + \delta + \mathcal{N}_1\left[u^*, v(u^*)\right]}, \tag{30}$$

where we inserted $v = v(u)$, which we get from the Beveridge curve $(1-u)\delta = \mathcal{N}(u,v)$.[17]

We want to compare this outcome to the steady state of the competitive search equilibrium model (one could do something similar assuming other KW pricing mechanisms, including bargaining, but as is well known bargaining can generate its own inefficiencies). To facilitate the comparison, rewrite the MP curve (27) after inserting the elasticity $\sigma \equiv v\mathcal{N}_2/\mathcal{N}$, which generally depends on u and v, and $\mathcal{M}[g(q,\varepsilon) - c(q)] = \frac{\mathcal{M}_2 s(q)}{\varepsilon s'(q) + 1 - \varepsilon}$, to get

$$k = \frac{\frac{\eta}{\sigma}\mathcal{N}_2\left[u, v(u)\right]\left[y - b - \frac{\ell}{1-\tau} + \mathcal{M}_2(1, 1-u)\frac{s(q)}{\varepsilon s'(q) + 1 - \varepsilon}\right]}{r + \delta + \frac{1-\eta}{1-\sigma}\mathcal{N}_1\left[u, v(u)\right]}. \tag{31}$$

We also assume for now that $u < u_0$ (we are to the left of the kink), which will certainly be true for small i. Also, we only consider the limiting case where $r \to 0$.

From the LW curve, $q = q^*$ iff $i = 0$. Then, given q^*, a comparison of (30) and (31) with $r \approx 0$ implies (after some algebra) that $u = u^*$ iff

$$b + \frac{\tau}{1-\tau}\ell = \frac{\eta - \sigma}{1-\sigma}\frac{\mathcal{N}_1(u^*, v^*)\left[y - \ell + \mathcal{M}_2(1, 1-u^*)s(q^*)\right]}{\delta + \mathcal{N}_2(u^*, v^*)} \tag{32}$$

Any combination of b and τ that makes $b + \frac{\tau}{1-\tau}\ell$ equal to the RHS of (32) makes the planner's solution and equilibrium coincide. If $\eta = \sigma$ (the Hosios

[17] Dynamics are more intricate than in the standard LW model, because here u is a state variable, and more intricate than in the standard MP model, where v/u is independent of u, because here u enters $\mathcal{M}(1, 1-u)$ (these observations are true for both the planner's problem and for equilibrium). We focus on steady states for now, relegating dynamic analysis to a companion paper.

1990 condition), then $u = u^*$ iff $b + \frac{\tau}{1-\tau}\ell = 0$, which certainly holds in laissez-faire equilibrium, $b = \tau = 0$. If $\eta > \sigma$, then we need to set $b + \frac{\tau}{1-\tau}\ell = 0 > 0$, which means either market activity should be taxed or unemployment subsidized; and if $\eta < \sigma$, then we need to set $b + \frac{\tau}{1-\tau}\ell = 0 < 0$, which means either market activity should be subsidized or unemployment taxed. Hence, if policy is unconstrained, we can always achieve efficiency by running the Friedman rule to get q^* and setting labor market policy to get u^*.

Suppose there is a restriction, for whatever reason, that $b + \frac{\tau}{1-\tau}\ell$ is strictly less than the RHS of (32). Then we claim monetary policy should compensate with $i > 0$. To verify this, note that if $i > 0$ is small the LW curve implies only a second order welfare loss by the envelope theorem, while the MP curve implies a first order gain since $\frac{s(q)}{\varepsilon s'(q)+1-\varepsilon}$ is strictly increasing at $q = q^*$. Hence, if labor market policy is constrained so that b or τ are too low, monetary policy should compensate with inflation above the Friedman rule. The intuition is clear: given $\eta > \sigma$, e.g., there is too much entry in laissez-faire, and if we are restricted in directly taxing market activity or subsidizing unempolyment, the next best is to inflate and tax market activity indirectly. This generates a welfare gain in terms of u and loss in terms of q, but the net gain is unambiguouly positive.[18]

Proposition 4 *Given no constraints on policy, the optimum is to set* $i = 0$ *and set* $b + \frac{\tau}{1-\tau}\ell$ *to equal the RHS of (32), which takes the same sign as* $\eta - \sigma$*. Given* $b + \frac{\tau}{1-\tau}\ell$ *is too low, the optimal constrained policy is* $i > 0$*, which means inflation above the Friedman rule.*

[18] If $b + \frac{\tau}{1-\tau}\ell$ is constrained to be too high, the same logic might suggest we should set $i < 0$ – except that $i < 0$ is *not* consistent with equilibrium. One can also ask how labor market policy should respond in the second best scenario when i is constrained to be above the Friedman rule. Intuition and examples indicate that we should set $b + \frac{\tau}{1-\tau}\ell < 0$ – i.e. subsidize market activity or tax unemployment directly to increase profit and encourage entry – but this is hard to prove in general because the envelope theorem does not apply (with $i = 0$ we get $q = q^*$ for any $b + \frac{\tau}{1-\tau}\ell$, but with $b + \frac{\tau}{1-\tau}\ell = 0$ we do not get $u = u^*$ for any i).

6 Quantitative Analysis

6.1 Some Observations

A first-order prediction of the theory in this paper is that inflation increases unemployment, because inflation is a tax on cash-intensive activty, including the goods market, which reduces profit, vacancy creation, and employment. Our priors were that this would be a problem empirically, since the conventional wisdom is that inflation and unemployment are if anything negatively related; our thinking was that one would need to be somewhat clever to twist the theory to fit the stylized facts. Before getting carried away with twisting, however, the first thing to do is to check the data.

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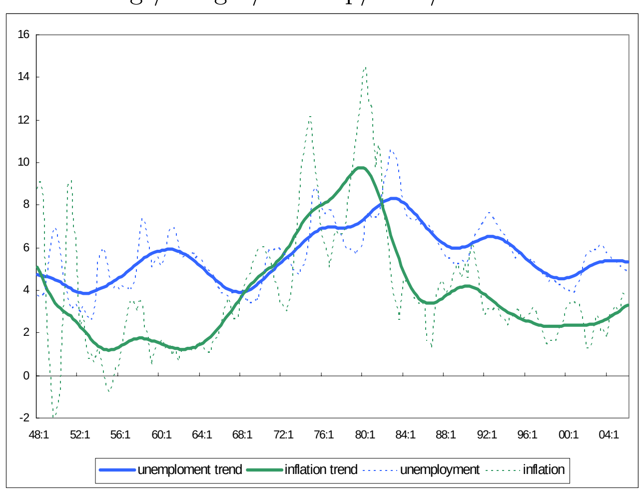

Figure 6: Quarterly Inflation and Unemployment, 1948-2006

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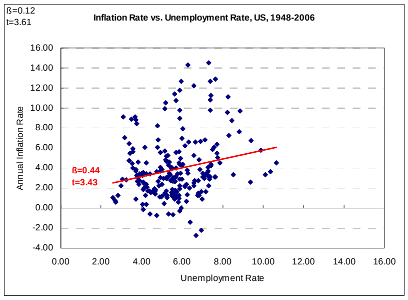

Unfiltered u vs. π

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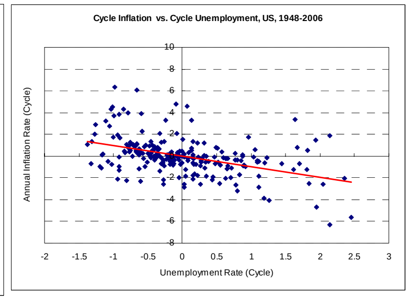

Cyclical u vs. π

In Figure 6, the dotted lines depict quarterly US unemployment and inflation, 1948-2006. The solid lines depict the HP trends. We also show the scatter plots of the raw series and the deviations from trend. Whether there is something interesting in these Figures may be in the eye of the beholder, but we think it is important to emphasize that our model is probably better suited to address lower frequency fluctuations. At business-cycle frequencies there might be any number of complications, such as imperfect information (including real-nominal signal extraction), rigidities (including sticky prices), etc. from which we in this analysis abstract. Our model is about the "natural" rate of unemployment – what Friedman had in mind in the epigraph. In Figure 7 we plot the data after filtering out cyclical components – i.e. the HP trend in u vs. the HP trend in π. Different colors represent different episodes in US inflation-unemployment history, clearly displaying e.g. the 1960s downward-sloping Phillips curve, the stagflation of the 1970s, and the subsequent disinflation. On the whole, there is clearly a positive relation.[19]

Before asking what our model has to say about π and u, we also want to be sure that it does a reasonable job accounting for the usual money demand

[19] There are three main subperiods where the relation seems negative: 53:1-56:2; 58:2-67:4; and 80:1-83:1.

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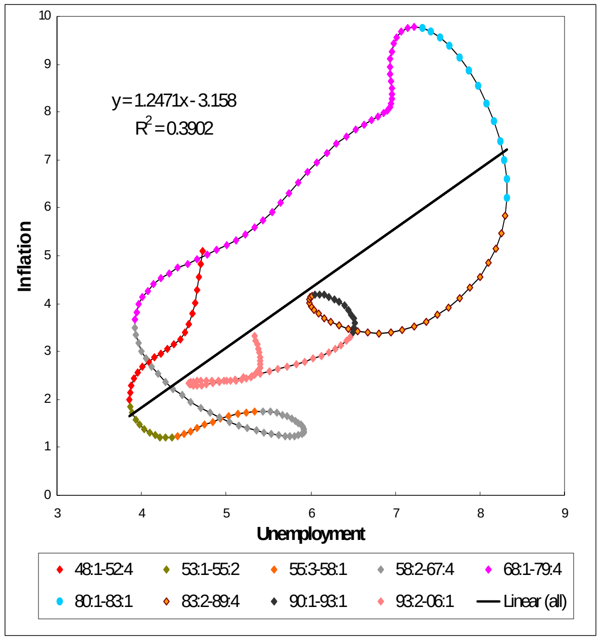

Figure 7: High-Frequency-Filtered Inflation and Unemployment

observations. We depicty in the next Figures a century of annual time series observations and the scatter of the HP trends of M/PY (the inverse of velocity) and the nominal rate (these are Aaa Corporate Bond rates; the chart is similar using T-Bill rates). There is a clear negative relation, depsite the well-known problem that from the early 1980s to the late 1990s interest rates dropped and money demand did not increase – i.e. there seems to have been a structural shift. This is something we will not model here, although at an informal level it may not be hard to understand given innovations in transactions technologies. The question we ask is, how does our model account for the low-frequency dynamics in inflation and unemployment, given we try to also match money and interest rates?

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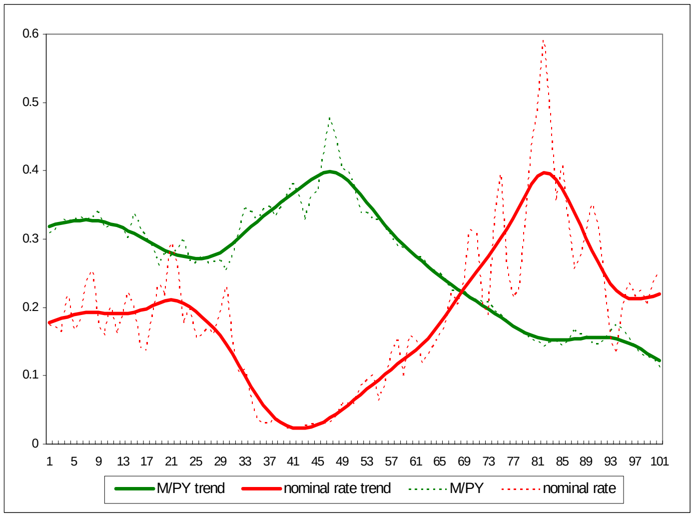

Annual Money and Interest, 1901-2001

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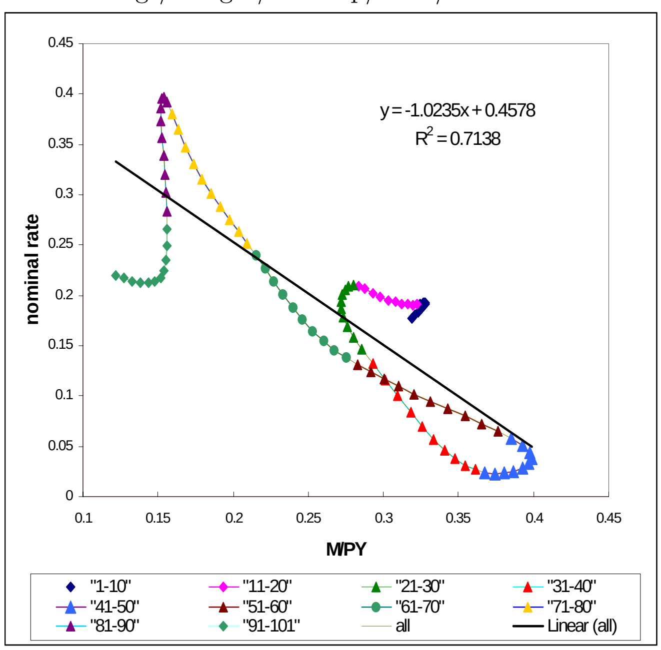

Money and Interest, High-Frequency Filtered, 1901-2001

6.2 Parameters and targets

We choose a quarter as the model period, and take 1959-2005 as the sample to be studied. We need to calibrate: (i) preferences as described by β, ℓ and $\upsilon(q)$; (ii) technology as described by y, δ, k, $\mathcal{N}(u,v)$ and$\mathcal{M}(B,S)\}$, although we can normalize $y=1$; (iii) bargaining power η and θ (the latter is relevant only if we assume bargaining in the KW market); and (iv) policy τ, i and b. Following most of the macro literature, we assume $\upsilon(q)=Aq^{1-\alpha}/(1-\alpha)$. Following most of the labor literature, we take the MP matching function to be $\mathcal{N}(u,v)=Zu^{1-\sigma}v^{\sigma}$, and normalize $Z=1$ without loss in generality (it merely determines the units in which we measure vacancies). We take the KW matching function to be $\mathcal{M}(B,S)=B(1-e^{-S/B})$, the so-called urn-ball matching technolgy, which is a parsimonious specification, and one that can be derived endogenously with by price posting and directed search.[20]

There are 11 parameters to calibrate, some of which are relatively "obvious." We take an average annual nominal interest rate on Aaa corporate bonds is $i=0.072$, and an average CPI inflation rate of $\pi=0.037$, which implies a real rate of $r=0.035$, and hence $\beta=0.966$ per year. We use the average effective marginal labor tax rate measured by McGrattan et al. (1997), $\tau=0.242$. We target a UI replacement rate of $b/w=0.5$, consistent with Shimer (2005). Also following Shimer (2005), we set the elasticity of $\mathcal{N}$ wrt v to the regression coefficient of the job-finding probability on labor market tightness, $\sigma=0.72$, and set the job-destruction rate to $\delta=0.051$ to match an average unemployment rate of $u=0.059$.[21] Table 1 shows the values for all of these "obvious" parameters,

[20] Burdett, Shi and Wright (2001) e.g. consider a market with finite numbers of sellers and buyers, where the former first post prices and the latter then decide where to go knowing that each seller can only serve one buyer. The "natural" equilibrium has all sellers post the same price and buyers select one at random, making the number of matches (buyers who get served) a random variable. As the numbers of buyers and sellers increases holding the ratio fixed, the number of matches converges to specification in the text.

[21] Shimer estimates an average monthly job-finding rate of $\lambda_h=0.45$, which implies a monthly job-destruction rate of $\delta=0.028$, given u. At quarterly frequencies these estimates

and five others to be determined, as we now discuss.

First, as is well known, the values of leisure ℓ and firm bargaining power η are hard to calibrate. Shimer (2005) somewhat arbitrarily sets $\ell = 0$ and $\eta = \sigma$ (as we saw in the previous section, the latter choice guarantees efficiency under laissez-faire in the basic MP model). Hagedorn and Manovskii (2006) alternatively identify ℓ and η using evidence on recruitment costs and wage volatility over the business cycle, which in their benchmark model implies $z = b+\ell/(1-\tau) = 0.955$ and $\eta = 0.94$. To say these alternative calibration strategies are controversial is an understatement; we remain agnostic and report findings for both. Then we set the remaining preference parameters A and α to match average money demand (or velocity) and its interest elasticity, which in our data are 0.169 and -0.3.[22] However, since the results are somewhat sensitive to the elasticity, for good economic reasons, and estimates are sensitive to e.g. how one treats the apparent structural shift in money demand, we also consider -0.5 and -0.8. Finally, using the free entry condition, we set k to match the average job-finding rate λ_h, since λ_h pins down market tightness and hence the expected duration of a vacancy.

Table 1 reports the calibrated parameter values when we use $\eta = 1/2$ and two alternative values of leisure. In the "Shimer" calibration, we set $\ell = 0$, which implies a ratio $\rho = z/w$ between the household's value of non-market and market activities of 0.5, as in Shimer (2005). In the "HM" calibration, we set $\ell = 0.386$, which implies $\rho = 0.998$, as in Hagedorn-Manovskii (2006). The

imply $\lambda_h = 0.807$ and $\delta = 0.051$ (Simer uses a slighty different sample, 1951-2003, but this should not matter much). Also, values for σ in the literature vary from 0.6 (Andolfatto 1996), 0.5 (Farmer 2004), 0.4 (Merz 1995) and 0.235 (Hall 2005a); we can report results for some of theses values as well.

[22] It is standard to use some version of this method for calibrating monetary models – see e.g. Cooley and Hansen (1989), Lucas (2000) or Lagos and Wright (2005). See Aruoba et al. (2007) e.g. for alternative ways to estimate the elasticity (that all give about the same number) and for details about picking parameters to match the money demand observations. In our model, M/pY is easily measured, since $M/p = g(q)$ and $Y = \mathcal{M}(1, 1-u)\,[g(q) - q] + (1-u)y$, where in equilibrium u and q depend on i.

value $\eta = 1/2$ is somewhat arbitrary, but it turns out not to matter – a change in η modifies k, but does not affect any interesting results.

Table 1a: Baseline calibration "obvious" parameters

Parameter	β	i	b/w	τ	σ	δ
Target	0.987	0.055	0.50	0.242	0.72	0.05

Table 1b: Baseline calibration remaining parameters

Parameter	ℓ	η	A	α	k
Target Shimer	0	0.50	0.988	0.07	0.999
Target HM	0.386	0.50	0.988	0.07	0.005

6.3 Results

Our approach is to trace out how the steady state competitive search equilibrium varies with monetary policy, as measured by either the nominal interest or inflation rate, and compare this to the data.[23] We are fully aware that there are alternative approaches; we choose this rather than, say, looking at the dynamic-stochastic rational expectations equilibrium path for an economy subjected to monetary policy (and possibly other) shocks *not* due to laziness – we have solved the dynamic-stochastic version, and plan to use it in future work. It is however the case that in the current version of the model, with no capital, as in the basic MP model, it does not take long to reach steady state, so ignoring these dynamics is not a big deal. Moreover, it is not at all obvious that one wants to impose rational expectations anyway. If we want to think about longer-term phenomena – say, comparing the inflation-unemployment experience of the 1960s with that of the 1970s – as captured by the HP trends, it arguably may be better to interpret each observation as a different regime that we entered unexpectedly. We have no desire to delve deeper into this here, and instead simply report results.

[23] For our calibrated parameter values, the competitive search equilibrium has a unique steady state – i.e. the situation looks like Figure 5 with the low interest rate i'. Later we will also consider the bagaining version, which may not to have a unique steady state; in this case we select the best steady state (the one with the lowest u and highest q).

Figure 8 shows the scatter plot between the HP trends in M/pY and i in the data between 1959-2006, and the same variables generated by the model, under the two benchmark calibrations. From the Figure we draw two basic conclusions. First, it seems that the model matches the standard money demand observations about as well as one could hope, given the structural shift in the latter part of the sample mentioned above. And, second, there is little to distinguish between the "Shimer" and "HM" calibrations based on these observations.

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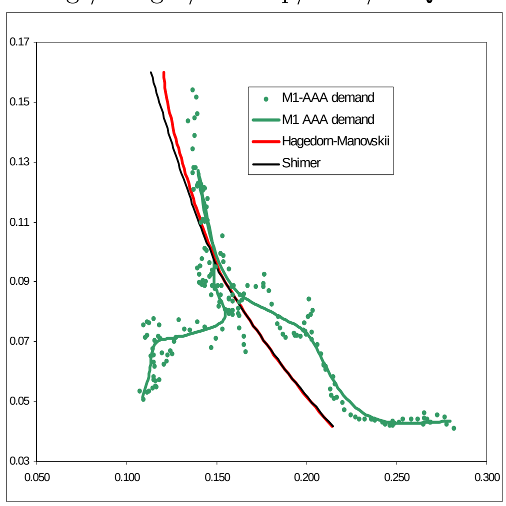

Figure 8: 'Fit' of M/pY vs. i, 1959-2006

Figure 9 reports the scatter betwen u and π over the same period, and the same variables generated by the model. Under the "Shimer" calibration, the relation between u and π is positive, but barely perceptable: u increases by only 0.1% when π goes from 1% to 10%, much less than in the data. But under the "HM" calibration, the model is quantitatively quite close to the data: as π goes from 1% to 10%, u increases from around 5% to around 9%, roughly the same as in the data. The difference between the two versions is easy to understand. Under the "HM" calibration, the flow value of non-market activity is nearly twice as high as under the "Shimer" calibration. In turn, equilibrium wages are higher and profits lower. Therefore, in the "HM" calibration, an increase in inflation implies a much larger percentage fall in profit, and hence a more pronounced drop in vacancies and ultimately employment. This is of course the very same economics that explains why the Hagedorn-Manovskii model generates realistically large, and the Shimer model generates very small, responses of u to technology shocks.

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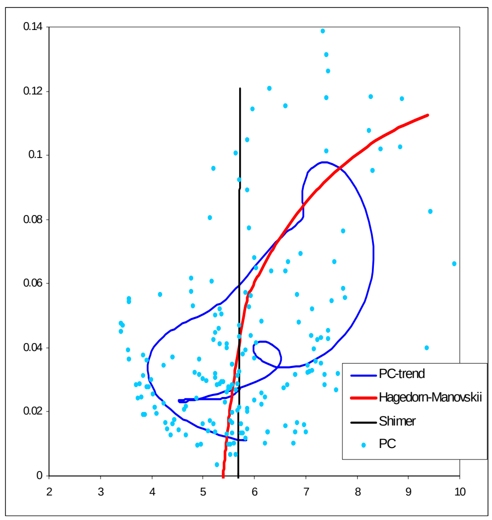

Figure 9: 'Fit' of u vs. π, 1959-2006

We present another view of the same results in Figure **??**. In the model it does not matter if we specify monetary policy in terms of π or i since they are connected via the Fisher equation. Thie Fisher equation however does not hold exactly in the data. So Figure **??** plots the scatter betwen u and i rathern than between u and π. If anything, the empirical pattern is even more remarkable. The results of comparing the data and models are the same: using the "HM" calibration we can account for much of the HP trend in unemployment in the sample by changes in monetary policy; using the "Shimer" calibration we can account for very little. To be clear, our goal is *not* to account for as much as possible of the pattern observed in the data – we rather ask "how much" we can account for by monetary policy, abstracting counterfactually from other real shocks that presumably were also relevant, in the tradition of e.g. Prescott (1985) who asks how much of the business cycle we can account for by technology shocks abstracting counterfactually from other shocks. Our answer is, "it depends."

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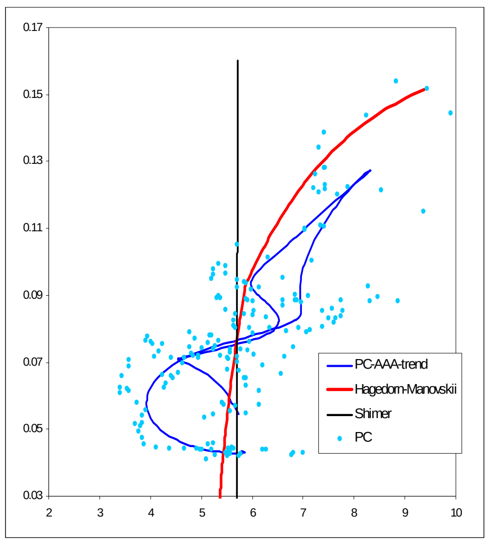

Figure 10: 'Fit' of u vs. i, 1959-2006

To provide further intuition for the results, Tables 2 and 3 give the elasticities of u and $R - w$ (profit) wrt i, y, k, ℓ, τ and b/w for the "Shimer" and "HM" calibrations. There are several points of interest. First, all elasticities are much larger in the "HM" calibration. Second, in either case the elasticity of u wrt y dominates the other elasticities, including the elasticity wrt i. Note that the elasticity of u wrt y is too low under the "Shimer" calibration and too high under the "HM" calibration, since in the data this elasticity is approximately

−20. In table 4, we use this number as a target to pin down $\ell = 0.379$, which is quite close to the "HM" value. Recalibrating in this way, the results obviously fall somewhere in between the two benchmark calibrations.

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	i	y	k	l	t	p
unemployment	0.000941946	-0.516584	0.272363	Indeterminate	0	0.264132
R - w	-0.000899419	1.4115	0.255805	Indeterminate	0	-0.721705

Table 2: 'Shimer" Calibration

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	i	y	k	l	t	p
unemployment	0.177375	-97.2885	0.25432	51.0352	16.2931	51.2819
R - w	-0.246446	265.796	0.305105	-139.442	-44.5185	-140.116

Table 3: "HM" Calibration

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	i	y	k	l	t	p
unemployment	0.0403422	-22.1247	0.268333	11.3966	3.63849	11.6568
R - w	-0.0422313	60.4524	0.266815	-31.1396	-9.94168	-31.8507

Table 4: Elasticity wrt y targeted to −22.

One can also ask how well we do with alternative versions of the model, such as the version with bargaining in the goods market instead of posting. The parameters in table 1a are the same, but we need to recalibrate the other parameters to match the same tragets, given any particular value of the buyer's bargaining power θ. The results can be seen in Figure 8 for $\theta = 0.5$, which shows that even under the "HM" calibration we account for less of the changes in u based on changes in π. This, however, depends on θ. The case with $\theta = 2/3$ is shown in Figure 9. The basic message is similar – how much we can account for depends on which calibration strategy we adopt – but with bargaining there is an additional free parameter that also matters.

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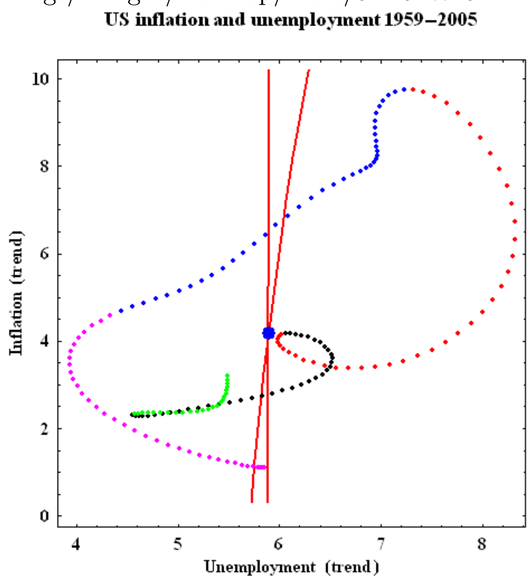

Figure 8: bargaining with $\theta = 0.5$

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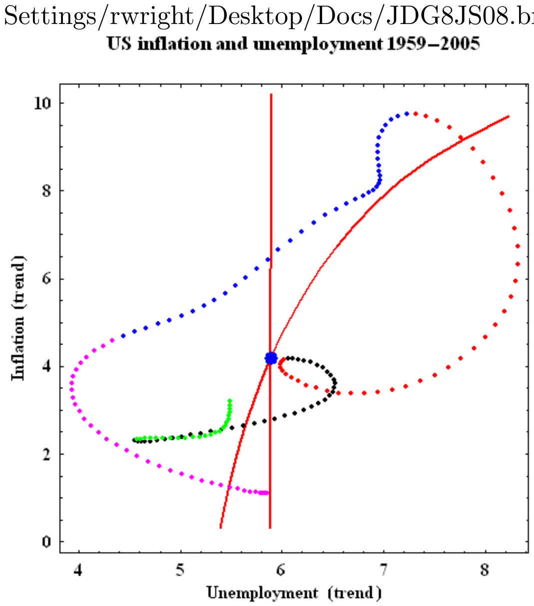

Figure 9: bargaining with $\theta = 2/3$.

6.4 Welfare Cost of Inflation

To be added.

7 Conclusions

To be added.

References

Aliprantis, C.D., Camera, G. and D. Puzzello (2006) "Anonymous Markets and Monetary Trading." *Journal of Monetary Economics*, forthcoming.

Andolfatto, David (1996) "Business Cycles and Labor-Market Search." *American Economic Review,* March 1996, 86(1): 112-132.

Acemoglu, Daron and Robert Shimer (1999) "Holdups and efficiency with search frictions," *International Economic Review* 40, 827–849.

Aruoba, S. Boragan, Chris Waller and Randall Wright (2006) "Money and Capital." *Mimeo.*

Berentsen, Aleksander, Guillaume Rocheteau and Shouyong Shi (2001) "Friedman Meets Hosios: Efficiency in Search Models of Money." *Economic Journal*, forthcoming.

Blanchard, Olivier and Gordi Gali (2005) "Real Wage Rigidities and the New Keynesian Model." *Mimeo*, MIT.

Burdett, Kenneth, Shouyong Shi and Randall Wright (2001) "Pricing and matching with frictions." *Journal of Political Economy* 109, 1060-1085.

Cooley, Thomas F. and Gary D. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *American Economic Review* 79, 733-748.

Corbae, Dean, Ted Temzelides and Randall Wright (2003) "Directed matching and monetary exchange." *Econometrica* 71, 731-756.

Diamond, Peter (1982) "Aggregate Demand Management in Search Equilibrium."

Faig, Miquel, and Xiuhua Huangfu (2005) "Competitive Search Equilibrium in Monetary Economies." *Journal of Economic Theory*, forthcoming.

Fang, Lei and Richard Rogerson (2006) "Policy Analysis in a Matching Model With Intensive and Extensive Margins." *Mimeo*, Arizona State.

Farmer, Roger (2005) "Shooting the Auctioneer." *Mimeo*, UCLA.

Friedman, Milton (1977) "Inflation and Unemployment." *Journal of Political Economy.*

Gertler, Mark and Antonella Trigari (2006) "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Mimeo*, NYU.

Hosios, Arthur J. (1990) “On the Efficiency of Matching and Related Models of Search and Unemployment.” *Review of Economic Studies* 57, 279-298.

Howitt, Peter (2005) “Beyond Search: Fiat Money in Organized Exchange,” *International Economic Review* 46, 405-436.

Johri, Alok (1999) “Search, money and prices.” *International Economic Review* 40, 439-54.

Julien, Benoit, John Kennes and Ian King (2007) “Bidding for Money,” *Journal of Economic Theory* in press.

Kiyotaki, Nobuhiro and Randall Wright. (1989) “On money as a medium of exchange.” *Journal of Political Economy* 97, 927-954.

Kiyotaki, Nobuhiro, and Randall Wright (1993) “A search-theoretic approach to monetary economics.” *American Economic Review* 83, 63-77.

Kocherlakota, Narayana (1998) “Money is memory.” *Journal of Economic Theory* 81, 232-251.

Kumar, Alok (2006)

Lagos, Ricardo and Randall Wright (2005) “A unified framework for monetary theory and policy analysis.” *Journal of Political Economy* 113, 463–488.

Lagos, Ricardo and Randall Wright (2003) “Dynamics, Cycles, and Sunspot Equilibria in ‘Genuinely Dynamic, Fundamentally Disaggregative’ Models of Money.” *Journal of Economic Theory* 109, 156-171.

Leman, Etienne (2006) “A Search Model of Unemployment and Inflation.” *Mimeo.*

Lehmann, Etienne and Bruno van der Linden (2006) “Search Frictions on Product and Labor Markets.” *Mimeo.*

Lester, Benjamin, Andrew Postlewaite and Randall Wright (2006) “Information and Liquidity.” *Mimeo.*

Li, Victor (1995) “The optimal taxation of fiat money in search equilibrium.” *International Economic Review* 36, 927-942.

Lucas, Robert E., Jr. (2000) “Inflation and Welfare,” *Econometrica* 68, 247-274.

Lucas, Robert E., Jr. and Edward C. Prescott (1974). "Equilibrium search and unemployment." *Journal of Economic Theory* 7, 188-209.

Merz, Monika (1995) "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics* 36, 269-300.

Moen, Espen R. (1997) "Competitive search equilibrium." *Journal of Political Economy* 105, 385-411.

Molico, Miguel (2006) "The distribution of money and prices in search equilibrium." *International Economic Review*, forthcoming.

Molico, Miguel and Johnathon Chiu (2006)

Mortensen, Dale (1982)

Mortensen, Dale and Christopher Pissarides (1994) "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies.*

Mortensen, Dale and Randall Wright (2002) "Competitive pricing and efficiency in search equilibrium." *International Economic Review* 43, 1-20.

Petrongolo, Barbara and Christopher Pissarides (2001) "Looking into the black box: A survey of the matching function." *Journal of Economic Literature* 39, 390-431.

Phelan, Christopher (2005) "Discussion [of Huberto Ennis]." Philadelphia Conference on Macro and Monetary Economics, 2005.

Pissarides, Christopher (2000) *Equilibrium unemployment theory*, 2nd edition, MIT Press, Cambridge.

Rocheteau, Guillaume and Randall Wright (2005) "Money in Competitive Equilibrium, in Search Equilibrium, and in Competitive Search Equilibrium." *Econometrica* 73 (2005), 175-202.

Rocheteau, Guillaume, Peter Rupert and Randall Wright (2006) "The Phillips Curve in General Equilibrium." *Mimeo.*

Rocheteau, Guillaume and Chris Waller (2005) *Mimeo.*

Rogerson, Richard (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21, 3-16.

Rogerson, Richard, Robert Shimer and Randall Wright (2005) "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature* 43, 959-988.

Shi, Shouyong (1995) "Money and prices: A model of search and bargaining", *Journal of Economic Theory* 67, 467-496.

Shi, Shouyong (1997) "A divisible search model of fiat money." *Econometrica* 65, 75-102.

Shimer, Robert (1996) "Contracts in frictional labor market." *Mimeo*, Princeton University.

Trejos, Alberto and Randall Wright (1995) "Search, bargaining, money, and prices." *Journal of Political Economy* 103, 118-141.

Williamson, Steve (2005) *International Economic Review*, forthcoming.

Wallace, Neil (2001) "Whither monetary economics?" *International Economic Review* 42, 847-869.