A Revealed-Preference Test of the Correlation Between Risk-Aversion and Ambiguity-Aversion

Donald Brown[1], Chandra Erdman[2], Kirsten Ling[1] and Laurie Santos[3]
[1]Department of Economics, Yale University
[2]U.S. Bureau of the Census
[1]Department of Economics, Yale University
[3]Department of Psychology, Yale University

August 12, 2010

Abstract

We replicate the Huettel, et al experiment on choice under uncertainty (2006) with 30 Yale undergraduates, where each subject makes 200 pair-wise comparisons between risky and ambiguous lotteries.

Inferences about the independence of economic preferences for risk and ambiguity are derived from estimation of a mixed logit model, where the choice probabilities are functions of two random effects: the proxies for risk-aversion and ambiguity-aversion.

Our principal empirical finding is that we cannot reject the null hypothesis that risk- aversion and ambiguity-aversion are uncorrelated.This finding is consistent with the independence of the neural mechanisms governing choice under risk and choice under ambiguity, as suggested by the double dissociation-fMRI study in Huettel et al (2006)

These two empirical results refute the conventional wisdom held by economists that decision-making under ambiguity is simply a more complicated instance of decision-making under risk. That is, economists believe that the respective cognitive processes differ in degree, but not in kind as first suggested by Huettel et al (2006)

Acknowledgement 1 *We are pleased to acknowledge the helpful suggestions of Joe Altonji and Phil Haile and the research assistance of Oliver Bunn.*

1 Introduction

Knight and Keynes in their classic monographs offer two independent but overlapping discussions of estimating probabilities for decision-making under risk and uncertainty. In this regard Keynes is probably best known for his chapter on "The state of long-run expectation" in *The General Theory* (1936) and Knight for his chapter on "The meaning of risk and uncertainty " in *Risk, Uncertainty and Profit* (1921). A central contribution in the cited works of Knight and Keynes is the distinction between risk and uncertainty. Here is a quotation from Keynes(1937):

"By uncertain knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever.We simply do not know."

This distinction is absent in the expected utility (EU) model of decision- making under risk, due to Von Neumann and Morgenstern (1944), and Savage's (1954) model of decision-making under uncertainty, but it is the genisis of Ellsberg's (1961) seminal critique of Savage's theory of subjective expected utility (SEU). In his analysis, Ellsberg proposes ambiguity or "irreducible uncertainty" as it is called by Keynes, as another aspect of decision-making under uncertainty. In Ellsberg's two-color thought experiment, subjects make pair-wise choices between a risky urn, where the relative frequencies of the two outcomes are 1/2, and an ambiguous urn, where the relative frequencies are unknown. In the first trial, if the subject chooses an urn and draws a black ball then she receives \$100, the "good" outcome, but if she draws a white ball then she receives zero dollars, the "bad outcome". In the second trial the payoffs are reversed. Subjects that choose the ambiguous urn on both trials are said to be ambiguity- seeking and subjects that choose the risky urn on both trials are said to be ambiguity- averse. Ambiguity-seeeking subjects in the Ellsberg experiment "act as if ", the affective probability of the "good" outcome is greater than the relative frequency of the "good" outcome. Ambiguity-averse subjects in the Ellsberg experiment "act as if " the affective probability of the "bad" outcome is greater than the relative frequency of the "bad" outcome

The dependence of subjective probabilities on payoffs is inconsistent with Savage's axiomatic model of decision-making under uncertainty, i.e., subjective expected utility (SEU) theory. See Savage (1954), pg 68: "...the...view sponsored here does not leave room for optimisim or pessimisim...to play any role in the person's judgement."or Ellsberg's (1961) explanation of the two color Ellsberg paradox: " $\cdots$ we would have to regard the subject's subjective probabilities as being dependent upon his payoffs, his evaluation of the outcomes $\cdots$ it is impossible to infer from the resulting behavior a set of probabilities for events independent of his payoffs".

In an interesting and provocative experiment, Huettel et al (2006) test a new model of decision-making under uncertainty with proxies for risk-aversion and ambiguity-aversion,consistent with Ellsberg's explanation of the two color Ellsberg paradox, where agents choose actions and beliefs. The proxies are β for risk-aversion, where β is the coefficient of relative risk-aversion for the utility function $u(w) = w^{\beta}$, and α for ambiguity-aversion in the $\alpha-$maxmin expected utility model. The $\alpha-$maxmin expected utility of an ambiguous lottery, $x = (x_1, x_2)$ is $(1-\alpha)u(x_1 \vee x_2) + \alpha u(x_1 \wedge x_2)$. If $\beta \in (0, 1)$ then u is concave and the subject is risk-averse. If $\beta = 1$ then u is linear and the subject is risk -neutral. Finally, if $\beta > 1$ then u is convex and the subject is risk-loving. Huettel et al (2006) interpret α as a measure of ambiguity-aversion, where $\alpha \in [0, 0.5)$ denotes ambiguity-seeking, $\alpha = 0.5$ is ambiguity-neutral, and $\alpha \in (0.5, 1]$ denotes ambiguity-averse. The utility of an ambiguous lottery in the Huettel et al (2006) model is the $\alpha-$maxmin expected utility and the utility of a risky lottery is the expected utility. Huettel et al (2006)assume that subjects maximize utility in choosing between a pair of lotteries. Before reviewing their experiment, we show that the Huetell et al model is consistent with Ellsberg's explanation of the two- color paradox. The utility of the risky urn is $[u(0) + u(100)]/2$ and the utility of the ambiguous urn is $(1-\alpha)u(100) + \alpha u(0)$, where $u(0) = 0$. If the agent is ambiguity-averse then $(1-\alpha) \in [0, 0.5)$. Hence $u(100)/2 > (1-\alpha)u(100)$ and the agent chooses the risky urn on both trials. If the agent is ambiguity-seeking then $(1-\alpha) \in (0.5, 1]$. Hence $u(100)/2 < (1-\alpha)u(100)$ and the agent chooses the ambiguous urn on both trials.

Returning to the experiment of Huettel et al (2006). Using $fMRI$ data from pair-wise comparisons of risky lotteries, where the relative frequencies of the outcomes are known to the subjects, and ambiguous lotteries, where the relative frequencies of the outcomes are unknown to the subjects, Huettel et al (2006) conclude that the neural mechanisms governing choice under risk and choice under ambiguity are independent. Briefly, they asked 13 subjects to make pair-wise comparisons between risky lotteries, and used the $fMRI$ data to identify a region in the brain that was activated during the choice process; call it region R. For each subject, β is chosen to maximize the number of correct predictions in the risky-risky and risky-certain trials. Subjects are then asked to make pair-wise comparisons between risky and ambiguous lotteries. Using the estimated value of β, Huettel et al (2006)estimate α for each subject, however, their estimation method is unstated. Here the $fMRI$ data identified a different region of the brain that is activated during the choice process, call it region A. Moreover, A is inactive when R is active and R is inactive when A is inactive. As is common in the neural science literature, this double dissociation-$fMRI$ study is interpreted as independence of the two choice behaviors.

This finding refutes the conventional wisdom held by economists that decision-making under ambiguity is simply a more complicated instance of decision-making under risk. That is, economists believe that the respective cognitive processes differ in degree, but not in kind as first suggested by Huettel et al (2006). It is therefore important for economists to replicate the experiment of Huettel et al (2006) and test the hypothesis that economic preferences for risk and ambiguity are independent.

To test the independence of economic preferences for risk and ambiguity using revealed-preference data, we recast the Huettel et al (2006) model as a random utility model, more specifically a mixed logit model. The mixed logit model allows us to estimate a parametric, bivariate distribution over α and β from pair-wise comparisons in risky and ambiguous lotteries made by subjects randomly selected from the population. The random utility model was first proposed in psychology byThurstone (1927) in a form now called the binomial probit model, and subsequently introduced in economics by Marshak (1960) who investigated the properties of choice probabilities for utility functions subject to random perturbations. McFadden (1974) introduced the conditional logit model. In the binomial case, this is the well-studied logistic model in biostatistics. See McFadden's Nobel Lecture for a brief history of the origins of the random utility model .

The proxies for ambiguity-aversion and risk-aversion, α and β, are treated as random effects, i.e., random variables uncorrelated with the explanatory variables, in the mixed logit model presented in this paper. For a detailed discussion of the mixed logit model see chapter 6 of Train (2009). The following example from McCulloch et al (2008) illustrates the differences between fixed and random effects:

> Consider a clinical trial to treat epileptics, in which a drug is administered at four different dose levels. y_{ij} is the number of seizures experienced by patient j receiving dose i, where $E[y_{ij}] = \mu + \alpha_i$, μ is a general mean and α_i is the effect on the number of seizures due to treatment i. In this model of the expected value of y_{ij}, μ and each α_i are considered fixed and unknown constants, that we wish to estimate. These are the only treatments being used and we are considering no others, thus the α_i are fixed effects.
>
> Suppose now the clinical trials were conducted at 20 different clinics in New York City, where y_{ij} is the number of seizures experienced by patient j receiving treatment at the i th clinic. Now $E[y_{ij}] = \mu + a_i$. The clinics have been choosen randomly with the object of treating them as a representation of the population of of all clinics in New York City and inferences can and will be made about that population. This is characteristic of random effects, thus the a_i are random effects.

There are two criteria for using a random effects model in lieu of a fixed effects model. First, the data is generated by taking a random sample from some fixed population. Our sample is randomly selected from the population of Yale students, matriculating in the summer session and fall term of 2009. Second, the explanatory variables – the payoffs and probabilities defining the lotteries – must be uncorrelated with the random effects, α and β. This is certainly true in our experiment in which the payoffs and probabilites defining the lotteries in the pair-wise comparisons are generated randomly and independently for each subject.

We replicate the Huettel et al (2006) experiment with 30 randomly chosen Yale undergraduates. In our experiment, each subject makes 200 pair-wise choices between risky and ambiguous lotteries. In the Huettel et al (2006) analysis, α and β are interpreted as parameters and the choice probability, p_A, for x_A in the pair-wise comparison between lotteries x_A and x_B is defined as the percent correctly predicted. This is not the case for the mixed logit model that we present. In our model, the choice probability, p_A, for x_A in the pair-wise comparison between lotteries x_A and x_B is interpreted as the proportion of individuals in the population, with the same preferences for risk and ambiguity, that choose x_A or is interpreted as the proportion of times that a single individual chooses x_A in repeated pairwise comparisons between options x_A and x_B.

We interpret α and β as random effects with a bivariate log-normal distribution, parameterized by unknown hyper-parameters Θ. Using the Bayesian perspective, we can first estimate Θ by simulated maximum likelihood and then estimate the individual random effects α_j and β_j for each subject $j = 1, 2, ..., 30$, by simulating the posterior distribution of α_j and β_j conditional on the subject's pair-wise comparisons of risky and ambiguous lotteries, using Bayes theorem. The posterior means are consistent estimates of the individual-level random effects, α_j and β_j. See chapter 11 of Huettel et al (2006) for the details. The Bernstein-von Mises theorem in chapter 12 of Train (2009) provides an alternative classical method of estimating the individual-level random effects. That is, maximum likelihood estimation of α_j and β_j. The Bernstein-von Mises theorem shows that the Bayesian and classical estimates of the individual-level random effects α_j and β_j are asympototically equivalent.

We estimate α_j and β_j by maximizing the log-likelihood of each subject's pair-wise choices in risky and ambiguous lotteries. Following Huettel et al (2006), we use a two-step procedure to estimate α_j and β_j for each subject $j = 1, 2, ..., 30$. That is, our estimator is two-step maximum likelihood estimation. To estimate β_j, we ask each subject to choose between 40 risky-certain pairs and 40 risky-risky pairs of lotteries. Here we assume that each subject is maximizing expected utility, which only depends on β_j. To estimate α_j, we ask each subject to choose between 40 ambiguous-certain pairs and 40 pairs of ambiguous-ambiguous lotteries. Here we assume that each subject is maximizing $\alpha - \max\min$ expected utility, which depends on both α_j and β_j, where we use the previously estimated value of β_j and need only estimate α_j. It is well known that these estimates are consistent under the standard conditions for maximum likelihood estimation, but any standard estimator of the asymptotic covariance matrix for asymptotic normality of the maximum likelihood estimate of α requires a correction. For specifics, see Theorem 17.8 in Greene (2003) due to Murphy and Topel (1985). This correction is not necessary under the null hypothesis that ambiguity-aversion and risk-aversion are statistically independent.

Treating these estimates as realizations of the random variables α and β, we examine the correlation, ρ, between α and β by regressing α on β. Our results are consistent with the hypothesis suggested by the double dissociation-$fMRI$ experiments on economic preferences for risk and ambiguity reported in Huettel et al (2006). that the neural processes governing choice under risk are independent of the neural processes governing choice under ambiguity. That is, we cannot reject the reject the null hypothesis that $\rho = 0$ at the 5% significance level.

2 The Experimental Design

To replicate the Huettel et al (2006) experiment, we consider pair-wise choices in 200 monetary gambles made by 30 randomly chosen Yale undergraduates in 2009. As in the Huettel et al (2006) experiment, each gamble involves choices between a known outcome, outcomes with known probabilities, and outcomes with unknown probabilities. We refer to these gambles as certain, risky and ambiguous, respectively. In our experiment, each subject chooses between 40 risky-certain pairs, 40 risky-

risky pairs, 40 ambiguous-certain pairs, 40 ambiguous-ambiguous pairs and 40 risky-ambiguous pairs. All ambiguous gambles have two positive outcomes and all certain gambles have one positive outcome. In the risky-certain pairs and the risky-risky pairs, all risky gambles have one zero outcome and one positive outcome, but in the risky-ambiguous pairs both ambiguous and risky gambles have two positive outcomes. Expected values of gambles are chosen as random, whole-dollar amounts between $5 and $25, and expected values of pairs of gambles are matched within 20 percent. The probability of winning the amount presented in a certain gamble is always 1, and the probabilities of winning amounts presented in risky and ambiguous gambles are chosen randomly between 0.25 and 0.75, and varied across gambles.

At the start of each trial, subjects are given a pair-wise choice between gambles, represented by two pie charts. Subjects are instructed to choose the gamble on the left or right by typing 'L' or 'R.' Once a choice is made, a box appears around the chosen gamble and the other gamble disappears. Finally, the payoff of the lottery is displayed at the bottom of the screen. Figure 1displays potential risky, certian and ambiguous gambles. After completion of 200 trials, subjects are paid winnings from 4 randomly selected trials. Winnings ranged from $0 to $93 in a single trial, and $35 to $99 overall. The results of the experiment are summarized in Table 2

Here is a brief description of the mixed logit models we use to analyze our data. For each pair of risky lotteries: $x = (x_1, x_2; \pi_1, \pi_2)$ and $y = (y_1, y_2; \eta_1, \eta_2)$, p_X is the probability of choosing x, and p_Y is the probability of choosing y in a pair-wise comparison between x and y, where $p_X + p_Y = 1$

To estimate β, we use the multiplicative random utility model, where the expected utility of the risky option x is given by β

$$EU(x) = \pi_1(x_1)^\beta + \pi_2(x_2)^\beta.$$

and the choijce probability is

$$p_X(\beta) = Prob[\ln EU(x) + \varepsilon_X > \ln EU(y) + \varepsilon_Y]$$

where ε_X and ε_Y are i.i.d. extreme value. The logit choice probability is

$$p_X(\beta) = \exp[\ln EU(x) - \ln EU(y)]/(1 + \exp[\ln EU(x) - \ln EU(y)]).$$

See Fosgerau and Bielaire (2009) and the references therein for the details. In our data set of pairs of risky-certain and risky-risky lotteries, $x_2 = y_2 = 0$. Hence$_j$

$$p_X(\beta) = Prob[\beta \ln(x_1) + \varepsilon_X > \beta \ln(y_1) + \varepsilon_Y]$$

$$p_X(\beta) = \exp[\beta \ln(x_1) - \beta \ln(y_1)]/(1 + \exp[\beta \ln(x_1) - \beta \ln(y_1)])$$

For each pair of lotteries $\{(X^j, Y^j)\}_{j=1}^{j=40}$, if we denote the chosen lotteries as X^j then the likelihood of the observed risky choices as a function of β is $\Pi_{j=1}^{j=80} p_{X^j}(\beta)$.

McFadden (1974) has shown that the log-likelihood function with these choice probabilities is globally concave in β. We estimate β for each subject with the pair-wise comparisons of 40 risky-certain and 40 risky-risky lotteries, by numerically maximizing the log-likelihood of the choice probabilities.

Our null hypothesis is that economic preferences for risk and ambigity are independent, where β is a measure of the subject's tolerance for risk and α is a measure of the subject's attitude towards ambiguity. The alternative hypothesis is that economic preferences for risk and ambiguity are correlated. Under the null hypothesis, every function of α and every function of β are independent. In particular, the LL for β and risky-certain or risky-risky data is independent of the LL for α and ambiguous-certain or ambiguous-ambiguous data, for every fixed value of β, e.g., $\widehat{\beta}$, the estimate of β.

To estimate α, we use the additive random utility model, where subjects evaluate ambiguous lotteries, using $\alpha - \max\min$ expected utility. Given the pair of ambiguous lotteries $w = (w_1, w_2)$ and $z = (z_1, z_2)$, the choce probability for w is

$$p_W(\alpha, \widehat{\beta}\,) = Prob[\alpha(w_1 \wedge w_2)^{\widehat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\widehat{\beta}} + \varepsilon_W > \alpha(z_1 \wedge z_2)^{\widehat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\widehat{\beta}} + \varepsilon_Z]$$

where

$$\begin{aligned} p_X(\alpha, \widehat{\beta}) \;=\; & \exp\{[\alpha(w_1 \wedge w_2)^{\widehat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\widehat{\beta}}\,] - [\alpha(z_1 \wedge z_2)^{\widehat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\widehat{\beta}}]\} \\ & /[1 + (\exp\{[\alpha(w_1 \wedge w_2)^{\widehat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\widehat{\beta}}] - [\alpha(z_1 \wedge z_2)^{\widehat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\widehat{\beta}}]\} \end{aligned}$$

is the logit choice probability. See Train for details. For each pair of lotteries $\{(X^j, Y^j)\}_{j=1}^{j=40}$, if we denote the chosen lotteries as X^j then, for fixed $\widehat{\beta}$, the likelihood of the observed ambiguous choices as a function of α is $\Pi_{j=1}^{j=80} p_{X^j}(\alpha, \widehat{\beta})$. The log-likelihood function is globally concave in α, if $\widehat{\beta} \neq 0$. $\widehat{\alpha}$ is not identified for subjects where $\widehat{\beta} = 0$. That is, if $\widehat{\beta} = 0$, then for all $\widehat{\alpha} \in [0, 1]$: $p_X(\widehat{\alpha}, \widehat{\beta}) = \frac{1}{2}$. Hence $\widehat{\alpha}$ is indeterminate and the six subjects with indeterminate $\widehat{\alpha}$ "act as if "they flip a fair coin to choose between any pair of risky or ambiguous lotteries. We estimate α for each subject with the 40 ambiguous-certain and the 40 ambiguous-ambiguous lotteries, by numerically maximizing the log-likelihood of the choice probabilities.

To estimate the correlation between risk and ambiguity, we consider several specifications. In the five specifications, where we exclude the six subjects with indeterminate $\widehat{\alpha}$, the slope coefficient of the regression is not significantly different from 0 at the 0.05 level, indicating linear independence between risk and ambiguity. Hence we cannot reject the null hypothesis of independence of economic preferences for risk and ambiguity. The regression coefficients, regression statistics confidence intervals and plots of the data are in the section on data analysis. All the statistical and numerical analysis was done with Matlab.

3 Conclusion

4 Data Analysis