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On the Desirability of Campaign Contribution Limits

Abstract

This paper argues that limits on campaign contributions may well be Pareto improving even under the most optimistic assumptions concerning the role of campaign advertising and the rationality of voters. The argument assumes that candidates use campaign contributions to convey truthful information to voters about their qualifications for office and that voters update their beliefs rationally on the basis of the information they have seen. It also assumes that campaign contributions are provided by interest groups and that candidates can offer to provide policy favors for their interest groups to attract higher contributions. The argument is developed in the context of a novel model of political competition with campaign contributions and informative advertising.

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1 Introduction

This paper argues that limits on campaign contributions could well be Pareto improving even under the most optimistic assumptions concerning the role of campaign advertising and the rationality of voters. The argument assumes that candidates use campaign contributions to convey *truthful information* to voters about their qualifications for office and voters update their beliefs *rationally* on the basis of the information they have seen. It also assumes that campaign contributions are provided by interest groups and that candidates can offer to provide policy favors for their interest groups to attract higher contributions.

The argument is developed in a simple model of electoral competition. There are two political parties representing opposing ideologies. Parties put forward candidates who represent their ideologies, but may have difficulty finding qualified candidates. Thus each party's candidate may be qualified or unqualified. Voters know a candidate's party affiliation but not whether he is qualified. Advertising allows a candidate to provide voters with this information. Such advertising can be advantageous for a qualified candidate because it may attract swing voters. Resources for campaign advertising are obtained by candidates from interest groups consisting of citizens of opposing ideologies. If elected, candidates are able to implement policy favors for their interest groups and, before the election, they can offer to implement such favors to extract larger contributions.

The starting point for the argument is the observation that the potential social benefit of contributions lies in giving qualified candidates an electoral advantage over unqualified opponents. With no contributions, there would be no mechanism for qualified candidates to get out the word to voters. Giving qualified candidates an electoral advantage potentially benefits all citizens, as it results in better leaders.

In order for campaign contributions to have this benefit, campaign advertising must be *effective* in that learning that a candidate is qualified will induce a non-trivial fraction of swing voters to switch their votes from unadvertised candidates. If advertising induces no voters to switch their votes then qualified candidates obviously have no electoral advantage. However, *when campaign contributions are unrestricted and candidates are sufficiently power-hungry, campaign advertising must be close to ineffective.* For if campaign advertising were effective, power-hungry candidates would promise a large number of favors to their interest groups to extract more resources for campaigning. Voters would rationally become cynical about candidates they learn are qualified, anticipating that they would implement large amounts of favors when in office. This cynicism would negate the effectiveness of campaign advertising.

Accordingly, when campaign contributions are unrestricted and candidates are sufficiently power-hungry, resources will be spent on campaigning but qualified candidates will not have much of an electoral advantage over unqualified opponents. Moreover, if elected, qualified candidates will implement *some* favors for their interest groups. This must be the case for advertising to be close to ineffective. It follows that banning campaign contributions would only result in a negligible reduction in the likelihood that leaders would be qualified, while eliminating the favors they would implement. This means that all regular citizens benefit from a contribution ban. The only possible losers are interest group members who no longer receive favors. But their expected gains from favors are dissipated by the contributions they make, meaning they are also better off. Thus, *banning contributions creates a Pareto improvement when candidates are sufficiently power-hungry.*

When candidates are less power-hungry, campaign advertising will be effective even with unrestricted contributions and, accordingly, contributions will give qualified candidates an edge over unqualified opponents. In such circumstances, *banning* contributions will reduce the probability that qualified candidates defeat their unqualified opponents. However, *limiting* contributions need not necessarily reduce this probability. This is because a limit reduces the level of favors qualified candidates provide and this may raise the effectiveness of campaign advertising. This increase in the *effectiveness* of advertising can compensate for the reduction in the *level* of advertising. In such circumstances, contribution limits again have the potential to be Pareto improving. Finally, even when limits necessarily reduce the probability that qualified candidates defeat their unqualified opponents, they may be Pareto improving if the reduction in this probability is compensated for by a large enough reduction in favors.

The organization of the remainder of the paper is as follows. The next section discusses the relationship of the paper to previous work on the regulation of campaign advertising and the more general literature on campaign finance. Section 3 presents the model. Section 4 characterizes equilibrium with unrestricted contributions and shows that when candidates are sufficiently power-hungry, campaign advertising is close to ineffective. The impact of contribution limits is analyzed in Section 5. It is first shown that banning contributions will be Pareto improving when candidates are sufficiently power-hungry. It is then argued that, when candidates are less power-hungry, limits need not necessarily reduce the probability that qualified candidates are elected and that in such circumstances there will exist Pareto improving limits. Section 6 concludes with a summary of the argument and some suggestions for further research.

2 Related Literature

Despite the manifest policy significance of the topic, there have been few papers studying the welfare economics of campaign finance regulation. Partly this reflects the difficulty of incorporating campaign contributions into theories of electoral competition in a tractable way. Most efforts simply assume that campaign advertising buys the votes of "noise" voters, implying that it has no social benefit (see, for example, Baron (1994), Besley and Coate (2000), and Grossman and Helpman (1996)). Such an assumption obviously precludes a serious analysis of the case for contribution limits.

Work in which campaign advertising has a social benefit falls into two categories. First, there are those papers that assume that campaign advertising is *directly* informative (Austen-Smith (1987), Coate (2001), Ortuno-Ortin and Schultz (2000), and Schultz (2001)). The idea is that candidates can use advertising to provide voters with hard information about their policy positions, ideologies, or qualifications for office, thus permitting more informed choices. Second, there are those who argue that campaign advertising may best be understood as providing information *indirectly* (Potters, Sloof, and Van Winden (1997), Prat (1999) and (2000)). The idea is that candidates have qualities that interest groups can observe more precisely than voters and the amount of campaign money a candidate collects signals these qualities to voters.

Coate (2001) addresses the desirability of contribution limits in a world of directly informative advertising. The model used in this paper builds on Coate (2001), but differs in two key ways. First, voters are uninformed about candidates' "qualification for office" which is a "valence" characteristic that all voters value. In Coate (2001) voters are uninformed about candidates' ideologies. This makes Coate's analysis more intricate, because with ideology, parties' candidate selection strategies must be modelled. With a valence characteristic, it is natural to presume that all parties would field a candidate with a high value of the characteristic if they could find one, so it seems reasonable to treat the probability that parties select qualified candidates as exogenous. The second key difference is that in this analysis, candidates can offer policy favors to attract more contributions from the interest groups that support them. In Coate (2001) interest groups only give to help elect candidates whose ideologies they favor. This feature is key to explaining the difference in policy conclusions concerning the desirability of contribution limits. While in this paper limits can be Pareto improving, in Coate (2001) limits redistributes welfare *from* moderate voters *to* interest group members.

Prat (1999) addresses the case for limiting contributions in a world of indirectly informative advertising. In his analysis, two office-seeking candidates, who may differ in competence, compete by staking out positions in a one dimensional policy space. A single interest group with non-median policy preferences offers contributions to candidates in exchange for them moving their platforms towards its preferred policy position. Candidates the interest group believes to be more competent are offered larger contributions because they are more likely to win. This is because voters observe a noisy signal of competence and hence, ceteris paribus, are more likely to vote for the more competent candidate. In equilibrium, therefore, the more a candidate advertises, the higher is his competence. Campaign contributions are good for voters in the sense that they provide information about competence, but bad in that they lead candidates to distort policy. Banning contributions can raise voters' aggregate welfare when the losses in terms of information about competence are smaller than the costs of policy distortion. This is different from our argument which stresses that there need be no such trade off - banning contributions need not significantly impact the probability that competent candidates are elected. While Prat does not consider the distributional consequences of banning contributions, it seems likely that in his model banning is either Pareto inefficient or redistributes from citizens on the side of the interest group to those on the other side of the political spectrum.

While the literature on the specific topic of the welfare economics of campaign finance regulation is sparse, the general topic of campaign contributions has attracted much more attention. A significant strand of the literature is devoted to assessing the empirical relationship between campaign spending and votes - how effective is campaign advertising in delivering votes? (see, for example, Abramowitz (1988), Green and Krasno (1988), Jacobson (1980), (1985), Levitt (1994), and Palfrey and Erikson (2000).) In the model of this paper the effectiveness of campaign advertising is derived endogenously as part of the equilibrium (see also Coate (2001)). Moreover, a major lesson of the paper is that rules governing elections may be expected to have implications for the effectiveness of campaign advertising. In particular, ceteris paribus, campaign advertising may be more effective when limits are tighter. This has interesting implications for future empirical studies.

A further theme in the literature is the distinction between *service* and *position-induced* contributions (Morton and Cameron (1992)). The latter are contributions that are given because the donor shares some of the candidate's policy positions and wants to enhance his/her chances of winning. The former are contributions that are given in the expectation that the candidate will provide services for the donor if elected to office. Prior theoretical work has assumed *either* that contributions are position induced *or* that they are service-induced. In the model of this paper, the degree to which contributions are service or position induced is determined endogenously and is again affected by the rules governing elections.

3 The Model

3.1 Basics

A community must elect a representative. Citizens differ in their ideology which is measured on a 0 to 1 scale. The population is divided into three groups: leftists, rightists, and swing voters. Swing voters make up a fraction γ of the community and the remaining $1-\gamma$ are evenly divided between leftists and rightists. Leftists and rightists have ideologies d and $1-d$ respectively, where $d < \frac{1}{2}$. Swing voters come in two types: left-leaning and right-leaning with ideologies x and $1-x$ respectively where $x \in (d, \frac{1}{2})$. The fraction of swing voters who are left-leaning, denoted μ, is ex ante uncertain, reflecting the fluid nature of these voters' attitudes. Specifically, μ is the realization of a random variable uniformly distributed on $[0,1]$.

Candidates for community representative are put forward by two political parties: Party L - the leftist party, and Party R - the rightist party. Candidates are citizens and hence are characterized by their ideologies. Each party must select from the ranks of its membership, so that Party L selects a leftist and Party R a rightist. Candidates differ in their qualifications for office, denoted by q. They are either "qualified" ($q = 1$) or "unqualified" ($q = 0$). All citizens, including party members, prefer a qualified candidate. Thus parties will always select qualified candidates if they are available. The probability that each party can find a qualified candidate is σ.

A citizen with ideology i enjoys a payoff from having a leader of ideology i' and qualifications q given by $\delta q - \beta |i - i'|$ where $|i - i'|$ is the *distance* from i to i' and $\beta > 0$. It is assumed that swing voters prefer a qualified candidate of the opposing ideology to an unqualified candidate of their own ideology implying that $\delta > \beta(1-2x)$. Leftists and rightists, however, prefer a candidate of their own ideology even if he is unqualified which implies that $\delta < \beta(1 - 2d)$. Candidates have the same payoffs as citizens except that the winning candidate enjoys an ego-rent r. This measures how "power-hungry" candidates are.

Swing voters do not have perfect information about candidates, in the sense of not knowing whether each party's candidate is qualified. Such information could be acquired, but swing voters are not politically engaged and choose to remain "rationally ignorant". However, candidates can convey information concerning their qualifications via advertising. Swing voters cannot ignore such advertising because it is bundled with radio or television programming.

Campaign advertising is governed by the following rules. First, candidates can only advertise their own characteristics; i.e., whether they are qualified. This rules out negative advertising. Second, candidates can only advertise the truth. The idea is that candidates have records which reveal their qualifications and that candidates cannot lie about their records. The advertising technology is such that if a candidate spends an amount C, his message reaches a fraction $\lambda(C) = C/(C + \alpha)$ of the population, where $\alpha > 0$.

Candidates' advertising is financed by campaign contributions from interest groups. There are two such groups - a leftist group that contributes to Party L's candidate and a rightist group that contributes to Party R's. A fraction θ of partisans belong to each interest group. The interest groups behave so as to maximize the expected payoff of their representative members.

After he has been selected, each party's candidate requests a contribution from his interest group. To obtain a larger contribution, a candidate may offer to implement policy favors. When a candidate provides a level of favors f each interest group member enjoys a monetary benefit $b(f)$ at the expense of a uniform monetary cost of f to each citizen. The function b is increasing and strictly concave, satisfying the conditions that $b(0) = 0$ and $b'(\delta) > 1$. The interest group agrees to a candidate's request if and only if it benefits it to do so.

In terms of timing, it is assumed that candidates make their requests before they or their interest group knows the type of their opponent. Needless to say, swing voters do not observe the interaction between candidates and interest groups and hence do not observe the favors a candidate has promised.

Parties choose the best candidate they can find. Candidates approach their interest group and decide what contribution to request and how many favors to offer. Interest groups decide whether or not to accept candidates' offers. Leftists and rightists always vote for the candidate put forward by the party representing their ideology. Swing voters, having possibly observed one or both candidates' advertisements update their beliefs about candidates' qualifications and vote. They may vote for either party's candidate and are assumed to vote probabilistically. All these behaviors are now described in greater detail.

3.2 Behavior of swing voters

At the time of voting, each swing voter may have seen advertisements from both, one, or neither candidate. Let (I_L, I_R) denote a swing voter's information where $I_K = 1$ if he has seen an advertisement from Party K's candidate and $I_K = 0$ if not. Let $\rho_K(I_L, I_R)$ denote his belief that Party K's candidate is qualified conditional on informational state (I_L, I_R). Since only qualified candidates advertise, both $\rho_L(1, I_R)$ and $\rho_R(I_L, 1)$ must equal 1. The beliefs $\rho_L(0, I_R)$ and $\rho_R(I_L, 0)$ will be derived as part of the equilibrium.

Swing voters will also have beliefs about the amount of favors that each party's candidate, if qualified, will provide to the interest group. In equilibrium, the amount of favors that voters think that candidates will implement must equal the amount that they actually will. Accordingly, we will not employ a separate notation to distinguish voters' beliefs from the actual levels promised. We let f_K denote the amount of favors that Party K's candidate, if qualified, will provide to the interest group. Letting $v_K(J; I_L, I_R)$ denote the expected payoff of a swing voter with leaning $K \in \{L, R\}$ from Party J's candidate being elected when the voter has information (I_L, I_R), we have that

$$v_K(K; I_L, I_R) = \rho_K(I_L, I_R)(\delta - f_K) - \beta(x - d),$$

and for $J \neq K$

$$v_K(J; I_L, I_R) = \rho_J(I_L, I_R)(\delta - f_J) - \beta(1 - d - x).$$

To create heterogeneity in the voting behavior of swing voters, they are assumed to vote probabilistically.[1] Specifically, a swing voter with leaning K in informational state (I_L, I_R) votes for Party L's candidate if and only if $v_K(L; I_L, I_R) + \varepsilon \geq v_K(R; I_L, I_R)$ where ε is the realization of a random variable with range $[-\overline{\varepsilon}, \overline{\varepsilon}]$ ($\overline{\varepsilon} > 0$) and symmetric and increasing cumulative distribution function $H(\varepsilon)$. The fraction of swing voters with leaning K in informational state (I_L, I_R) voting for Party L's candidate is therefore

$$\xi_K(I_L, I_R) = 1 - H(v_K(R; I_L, I_R) - v_K(L; I_L, I_R)),$$

where we adopt the convention that $H(\varepsilon) = 0$ for all $\varepsilon \leq -\overline{\varepsilon}$ and $H(\varepsilon) = 1$ for all $\varepsilon \geq \overline{\varepsilon}$.

It is assumed that $\overline{\varepsilon}$ is smaller than $\beta(1 - 2x)$ but larger than $\beta(1 - 2x) - (1 - \sigma)\delta$. The former assumption implies that swing voters with leaning K who believe that Party K's candidate is at least as likely to be qualified as the opposing Party's candidate will always vote for him provided that, if qualified, he will not implement more favors than his opponent. In particular, this means

[1] Without this assumption, it would be necessary to allow swing voters to use mixed strategies.

that $\xi_L(1, I_R) = 1$ when f_L is no greater than f_R and $\xi_R(I_L, 1) = 0$ when f_L is no smaller than f_R.

3.3 Election probabilities

Given this voting behavior, the probability that each party's candidate will win may be computed. Suppose first that the two candidates are qualified and that they receive contributions C_L and C_R. Then, the fraction of left-leaning swing voters voting for Party L's candidate is

$$\begin{aligned}\delta_L(C_L, C_R) &= \xi_L(1,1)\lambda(C_L)\lambda(C_R) + \xi_L(1,0)\lambda(C_L)(1-\lambda(C_R)) \\ &\quad +\xi_L(0,1)(1-\lambda(C_L))\lambda(C_R) + \xi_L(0,0)(1-\lambda(C_L))(1-\lambda(C_R)).\end{aligned}$$

This includes a fraction $\xi_L(1,1)$ of those who have seen both candidates' advertisements; a fraction $\xi_L(1,0)$ of those who have seen only the advertisement of Party L's candidate; a fraction $\xi_L(0,1)$ of those who have seen only the advertisement of Party R's candidate; and a fraction $\xi_L(0,0)$ of those who have seen neither candidate's advertisement. Similarly, the fraction of right-leaning swing voters voting for Party R's candidate is

$$\begin{aligned}\delta_R(C_L, C_R) &= \xi_R(1,0)(1-\lambda(C_R))\lambda(C_L) + \xi_R(0,0)(1-\lambda(C_R))(1-\lambda(C_L)) \\ &\quad +\xi_R(1,1)\lambda(C_R)\lambda(C_L) + \xi_R(0,1)\lambda(C_R)(1-\lambda(C_L)).\end{aligned}$$

The fraction of swing voters voting for Party L's candidate is $\mu\delta_L + (1-\mu)\delta_R$. Given the assumption that leftists and rightists are equally numerous, Party L's candidate will win if this fraction exceeds 1/2 or, equivalently, if $\mu \geq (1/2 - \delta_R)/(\delta_L - \delta_R)$. This implies that the probability that Party L's candidate wins is $\pi(C_L, C_R)$, where the *probability of winning function* π is defined as follows:

$$\pi(C_L, C_R) = \begin{cases} 0 \; if \; \frac{1/2-\delta_R}{\delta_L-\delta_R} > 1 \\ \frac{\delta_L(C_L,C_R)-1/2}{\delta_L(C_L,C_R)-\delta_R(C_L,C_R)} \; if \; \frac{1/2-\delta_R}{\delta_L-\delta_R} \in (0,1) \\ 1 \; if \; \frac{1/2-\delta_R}{\delta_L-\delta_R} < 0 \end{cases} .$$

If only Party L's candidate is qualified, he wins with probability $\pi(C_L, 0)$. Similarly, if only Party R's candidate is qualified, the probability that Party L's candidate wins is $\pi(0, C_R)$. If both candidates are unqualified, then no contributions are given and Party L's candidate wins with probability $\pi(0, 0)$.

3.4 Campaign contributions

Each candidate, not knowing his opponent's type, must decide the level of favors to offer its interest group and how much to ask it for. Each interest group, must decide whether to accept the request. If it does so, it hands over the contribution and the candidate, if elected, will implement the agreed level of favors. If it does not, then we assume that it makes no contribution. Interest groups observe the types of their party's candidate but not that of the opposing candidate's. Clearly, unqualified candidates will neither solicit nor receive contributions.

Recalling that C_K denotes the contribution a qualified candidate of Party K receives from his interest group and f_K the amount of favors he promises, interest group L's expected payoff from accepting Party L's candidate's request is

$$\begin{aligned}&\sigma[\pi(C_L, C_R)(\beta(1-2d) + b(f_L) - f_L + f_R) + \delta - f_R] \\ &+(1-\sigma)[\pi(C_L, 0)(\beta(1-2d) + \delta + b(f_L) - f_L)] - \beta(1-2d) - \frac{2C_L}{(1-\gamma)\theta}. \quad (1)\end{aligned}$$

If the interest group does not accept the request, it would make no contributions and obtain a payoff:

$$\sigma[\pi(0, C_R)(\beta(1-2d) + f_R) + \delta - f_R] + (1-\sigma)[\pi(0,0)(\beta(1-2d) + \delta)] - \beta(1-2d). \quad (2)$$

Thus, in order for the interest group to accept the request, (1) must exceed (2). Similar remarks apply to interest group R.

When Party L's candidate's request is accepted, his expected payoff is:

$$\sigma[\pi(C_L, C_R)(r+\beta(1-2d)+f_R-f_L)+\delta-f_R]+(1-\sigma)\pi(C_L, 0)(r+\beta(1-2d)+\delta-f_L)-\beta(1-2d). \quad (3)$$

Party L's candidate's request (C_L, f_L) maximizes his expected payoff subject to the constraint that the interest group will agree to it. Thus, (C_L, f_L) maximizes (3) subject to the constraint that (1) exceeds (2). Similarly, for Party R's candidate.

3.5 Political equilibrium

A *political equilibrium* consists of (i) candidate requests $((C_L, f_L), (C_R, f_R))$; (ii) voting behavior functions $(\xi_L(I_L, I_R), \xi_R(I_L, I_R))$ describing swing voters' voting behavior as a function of the information they have received in the campaign; and (iii) voter belief functions $(\rho_L(I_L, I_R), \rho_R(I_L, I_R))$ describing swing voters' beliefs concerning the likelihood that candidates are qualified. Candidate strategies must be mutual best responses given voter behavior and the constraint of interest group acceptance. Voter behavior must be consistent with their beliefs and these beliefs must be consistent with candidates' strategies.

The analysis will focus on political equilibria that are *symmetric* in the sense that candidates make the same request to their interest groups (i.e., $(C_L, f_L) = (C_R, f_R) = (C, f)$). In such an equilibrium, swing voters who have observed an advertisement from their party's candidate will vote for him; (i.e., $\xi_L(1, I_R) = 1 - \xi_R(I_R, 1) = 1$). In addition, swing voters who have observed neither candidate advertise are loyal to their party's candidate (i.e., $\xi_L(0, 0) = 1- \xi_R(0, 0) = 1$). Thus, letting $\xi = \xi_R(1, 0)$ and $\rho = \rho_R(1, 0)$, a symmetric political equilibrium may be described by just four variables (C, f, ξ, ρ).

The variable ξ represents the fraction of right-leaning (left-leaning) swing voters who vote for Party L's candidate (Party R's candidate) when they have only observed an advertisement from this candidate. It measures the *effectiveness* of campaign advertising in inducing swing voters to switch from their natural allegiances. When ξ is high, voters are easily swayed and when ξ is low, campaign advertising is ineffective. Together with the two interest groups' contributions, ξ determines the fractions of left and right-leaning swing voters voting for their party's candidate.

We recognize this dependence by writing the probability of winning function as $\pi(C_L, C_R; \xi)$.

The effectiveness of advertising is determined by ρ - voters beliefs concerning the likelihood that their party's candidate is qualified when they have only seen an advertisement from the opposing candidate and by f - the level of favors a qualified candidate will enact. Assuming that $C \neq 0$, ρ is tied down by Bayes Rule. However, if $C = 0$, then the event of observing one candidate's advertisement does not arise along the equilibrium path and ρ is not tied down. Since it seems unreasonable to suppose that ρ is anything other than σ when candidates are not expected to advertise, we focus only on symmetric equilibria which have the property that $C = 0$ implies that $\rho = \sigma$. Henceforth, a symmetric equilibrium is understood to be an equilibrium satisfying this additional requirement.

3.6 Welfare

For the purposes of welfare analysis, the equilibrium payoffs of the various types of citizens can be calculated. Assuming a symmetric equilibrium, we can divide the population into just three types: partisans (i.e., leftists and rightists), interest group members, and swing-voters. We deal with each in turn.

Consider a representative partisan. Given symmetry, the elected candidate is equally likely to be from either party. The expected payoff of the partisan is therefore $\delta - f - \beta(1-2d)/2$ if the elected candidate is qualified and $-\beta(1-2d)/2$ if not. Recall that both parties select a qualified candidate with probability σ^2 while only one party selects a qualified candidate with probability $2\sigma(1-\sigma)$. In the latter case, the qualified candidate wins with probability $\pi(C, 0; \xi)$ and hence the probability that a qualified candidate is elected is $\eta(\pi(C, 0; \xi))$ where $\eta(\pi) = \sigma^2 + 2\sigma(1-\sigma)\pi$. The expected payoff of the partisan is therefore

$$\eta(\pi(C, 0; \xi))(\delta - f) - \frac{\beta(1-2d)}{2}.$$

Notice that partisans are better off when qualified candidates are more likely to defeat unqualified ones (assuming $\delta > f$) and worse off when qualified candidates implement more favors.

Interest group members provide campaign contributions to qualified candidates and also get policy favors enacted when their candidate wins. The expected payoff of a representative interest group member is therefore

$$\eta(\pi(C,0;\xi))(\delta - f + \frac{b(f)}{2}) - \frac{\beta(1-2d)}{2} - \frac{2\sigma C}{(1-\gamma)\theta}.$$

The fact that $b(f)$ is divided by two reflects the fact that the interest group only gets its favors implemented if the qualified candidate it is backing is elected.

The payoffs of swing voters are more complicated to compute because of the correlation between which party's candidate wins and the likelihood that a swing voter is left or right-leaning. To illustrate, suppose that both parties select unqualified candidates. A representative swing voter will be left-leaning with probability μ and right-leaning with probability $1-\mu$. If μ is less than 1/2 then the majority of swing voters are right-leaning and Party R's candidate wins. Accordingly, the representative swing voter's expected payoff is $-\mu\beta(1-x-d)-(1-\mu)\beta(x-d)$. If μ exceeds 1/2 then the majority of moderates are left-leaning and Party L's candidate wins. The representative swing voter's expected payoff is therefore $-\mu\beta(x-d)-(1-\mu)\beta(1-x-d)$. Taking expectations over the realization of μ, the representative swing voter's expected payoff is

$$\begin{aligned} & -\int_0^{\frac{1}{2}}[\mu\beta(1-x-d)+(1-\mu)\beta(x-d)]d\mu - \int_{\frac{1}{2}}^{1}[\mu\beta(x-d)+(1-\mu)\beta(1-x-d)]d\mu \\ = & -(\frac{\beta(1-x-d)}{4}+\frac{3\beta(x-d)}{4}). \end{aligned}$$

The key point is that *states in which the representative swing-voter is more likely to be left-leaning are states in which Party L's candidate will win.* Indeed, ensuring that this is the case is a key function of elections in this community.

Pursuing this logic for the cases in which both parties select partisans and only one party selects a moderate, the expected payoff of a representative swing voter can be shown to equal

$$\eta(\pi(C,0;\xi))(\delta - f) - \varphi(\pi(C,0;\xi))$$

where,

$$\begin{aligned}\varphi(\pi) &= [(1-\sigma)^2+\sigma^2](\frac{\beta(1-x-d)}{4}+\frac{3\beta(x-d)}{4}) + 2\sigma(1-\sigma)\{\pi[(1-\frac{\pi}{2})\beta(x-d)+\frac{\pi}{2}\beta(1-x-d)] \\ &\quad +(1-\pi)[\frac{1-\pi}{2}\beta(1-x-d)+\frac{1+\pi}{2}\beta(x-d)]\}.\end{aligned}$$

The first and second terms are, respectively, the expected ideological payoffs when parties select candidates with the same qualifications and when they select candidates with different qualifications.

It is clear from this expression that swing voters are worse off when qualified candidates implement more favors. The impact of changes in the probability a qualified candidate defeats an unqualified one is more complicated. Differentiating, an increase in this probability will be beneficial if $\delta - f$ exceeds $2(\pi - \frac{1}{2})\beta(1-2x)$. This is necessarily the case if $f = 0$ but may not be true more generally. Imagine, for example, that a qualified candidate would implement a level of favors almost equal to δ. Then a qualified candidate is not much better than an unqualified one but raising the probability that a qualified candidate wins reduces the sorting benefit of elections.

4 Equilibrium with Unrestricted Contributions

This section discusses the equilibrium that would arise with no restrictions on the amount interest groups could contribute to candidates. It first provides a general characterization of equilibrium. It then shows what happens in the limit as candidates become increasingly power-hungry.

4.1 Preliminaries

As the first step towards characterizing equilibrium, we study the offers that candidates will make to their interest groups, taking as given the effectiveness of campaign advertising ξ. Let

$U(C_L, f_L, C, f; \xi)$ be the expected utility of Party L's candidate if he is qualified and offers his interest group (C_L, f_L) when his qualified opponent offers his group (C, f); that is,

$$U = \sigma(\pi(C_L, C; \xi)(r+\beta(1-2d)+f-f_L)+\delta-f)+(1-\sigma)\pi(C_L, 0; \xi)(r+\beta(1-2d)+\delta-f_L)-\beta(1-2d).$$

Note that this is decreasing in f_L and increasing in C_L when advertising is effective.

Now let $G(C_L, f_L, C, f; \xi)$ denote the gain (gross of the contribution) to the leftist interest group from accepting the offer of Party L's candidate; that is,

$$\begin{aligned} G &= \sigma(\pi(C_L, C; \xi) - \pi(0, C; \xi))(\beta(1-2d) + f) + (1-\sigma)(\pi(C_L, 0; \xi) - \frac{1}{2})(\beta(1-2d) + \delta) \\ &\quad + (b(f_L) - f_L)(\sigma\pi(C_L, C; \xi) + (1-\sigma)\pi(C_L, 0; \xi)). \end{aligned}$$

Provided that advertising is effective, this gain is positive even when the interest group is promised no favors. This reflects the interest group's pure policy preference for a qualified candidate who shares its ideology. The gain is increasing in favors as long as b' exceeds 1 and increasing in the size of the contribution when advertising is effective.

Party L's candidate will optimally demand a contribution from his interest group sufficient to exhaust its gain from contributing. The level of favors will balance the gains of the interest group to the candidate's personal policy cost. In equilibrium, (C, f) must solve the problem:

$$\max_{(C_L, f_L) \in \Re_+^2} U(C_L, f_L, C, f; \xi) \text{ } s.t. \text{ } G(C_L, f_L, C, f; \xi) \geq \frac{2C_L}{(1-\gamma)\theta}.$$

Henceforth, we refer to this as *Problem P*. It will be studied in more detail below.

Turning to the effectiveness of campaign advertising, we know that, in equilibrium, ξ is given by:

$$\xi = \xi_R(1, 0) = 1 - H(v_R(R; 1, 0) - v_R(L; 1, 0)).$$

Since $v_R(R; 1, 0) = \rho(\delta - f) - \beta(x - d)$ and $v_R(L; 1, 0) = \delta - f - \beta(1 - d - x)$, the above equation implies that ξ is related to voters' beliefs about the probability an unadvertised candidate is qualified ρ in the following way:

$$\xi = 1 - H(\beta(1-2x) - (1-\rho)(\delta - f)). \quad (4)$$

Obviously, the higher is ρ, the lower is the effectiveness of campaign advertising. *Bayes Rule* implies that voters' beliefs are given by:

$$\rho = \frac{\sigma[1-\lambda(C)]}{\sigma[1-\lambda(C)] + (1-\sigma)} = \frac{\sigma\alpha}{\alpha + C(1-\sigma)}. \quad (5)$$

Observe that this formula holds even if $C = 0$ under our assumption that $C = 0$ implies that $\rho = \sigma$. Note that ρ is decreasing in C, reflecting the logic that when contributions are plentiful, not having observed a candidate advertise increases the likelihood that he is unqualified.

We may conclude that (C, f, ξ, ρ) is an equilibrium if and only if (i) (C, f) solves *Problem P* given ξ and (ii) ξ and ρ satisfy equations (4) and (5). We may substitute the expression for ρ from (5) into the expression for ξ in (4) to obtain

$$\xi = 1 - H(\beta(1-2x) - \frac{(1-\sigma)(\alpha + C)}{\alpha + C(1-\sigma)}(\delta - f)). \quad (6)$$

An equilibrium can then be defined more compactly as a triple (C, f, ξ) such that (C, f) solves *Problem P* given ξ and ξ satisfies equation (6). The associated equilibrium beliefs may then be recovered from (5). Intuitively, equilibrium requires first that the offers qualified candidates make to interest groups must be optimal for them given the effectiveness of campaign advertising, and second that the effectiveness of advertising must be consistent with the amount of contributions qualified candidates receive and the favors they promise.

4.2 Characterization of equilibrium

Further progress necessitates a more detailed study of *Problem P*. Figure 1 presents a diagrammatic treatment. The family of convex curves represents the candidate's indifference map. The candidate dislikes favors and likes contributions, so that moving in a north-easterly direction increases the candidate's utility. The convexity of the indifference curves follows from the fact that the function $U(.,.,C,f;\xi)$ is quasi-concave on $\Re^2_+$.

The concave curve is the set of (C_L, f_L) pairs with the property that the interest group's gain $G(C_L, f_L, C, f; \xi)$ exactly equals the per-capita contribution $\frac{2C_L}{(1-\gamma)\theta}$. The constraint set for *Problem P* is the set of pairs on or below this curve. As drawn, this is a convex set. This will necessarily be the case when ξ is small, but is not obviously true when ξ is large.[2] In equilibrium, the optimal choice for the candidate will be $(C_L, f_L) = (C, f)$ as illustrated in Figure 1. This optimal choice occurs at the tangency of the candidate's indifference curves and the constraint set and this fact may be used to characterize (C, f).

Using first order conditions to characterize (C, f) in this way is complicated by the fact that, if ξ exceeds $\frac{1}{2}$, the probability of winning $\pi(C_L, 0; \xi)$ is not differentiable at the point at which $C_L = \frac{\alpha}{2\xi-1}$. At this point the probability of winning reaches 1. It is increasing in C_L for levels below $\frac{\alpha}{2\xi-1}$ and constant for levels above it. This creates kinks in the indifference curves and the boundary of the constraint set at $C_L = \frac{\alpha}{2\xi-1}$.

To sidestep the difficulties this creates, we will impose an assumption that ensures that, *in equilibrium*, the interest groups never contribute so much that their qualified candidate defeats an

[2] For all $f_L \in [0, \delta]$ let $\widetilde{C}_L(f_L)$ be the non-negative solution to $G(C_L, f_L, C, f; \xi) = \frac{2C_L}{(1-\gamma)\theta}$ (when $f_L = 0$ there might be two such solutions - let $\widetilde{C}_L(0)$ be the largest). Differentiating, we have that

$$\frac{d\widetilde{C}_L(f_L)}{df_L} = \frac{\frac{\partial G}{\partial f_L}}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}} > 0$$

and

$$\frac{d^2\widetilde{C}_L(f_L)}{df_L^2} = \frac{[\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}]\frac{\partial G}{\partial f_L}\frac{\partial^2 G}{\partial f_L \partial C_L} + [\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}]^2 \frac{\partial^2 G}{\partial f_L^2} + \frac{\partial^2 G}{\partial C_L^2}(\frac{\partial G}{\partial f_L})^2}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}}.$$

The second and third terms of the denominator are negative, but the first term is positive because $\frac{\partial^2 G}{\partial f_L \partial C_L} > 0$. Thus, for the whole expression to be negative, we need that the second and third terms outweigh the first. This will necessarily be the case when ξ is small because $\frac{\partial^2 G}{\partial f_L \partial C_L} = 0$ when $\xi = 0$. However, it is not obviously true for large ξ. Thus, to ensure that $\widetilde{C}_L(f_L)$ is concave it is necessary to assume that $[\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}]\frac{\partial G}{\partial f_L}\frac{\partial^2 G}{\partial f_L \partial C_L}$ is smaller than $-[\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}]^2 \frac{\partial^2 G}{\partial f_L^2} - \frac{\partial^2 G}{\partial C_L^2}(\frac{\partial G}{\partial f_L})^2$. Simulations suggest that this is not overly restrictive.

unqualified opponent with probability 1. To formally state the assumption, for all $f \in [0, \delta]$ let

$$\widetilde{C}(f) = \frac{\theta(1-\gamma)[\beta(1-2d) + \sigma f + (1-\sigma)\delta + (b(f) - f)(2-\sigma)]}{4}.$$

Notice that $G(C, f, C, f; \xi) = \frac{\theta(1-\gamma)}{2}\widetilde{C}(f)$ when $\pi(C, 0; \xi) = 1$, implying that $\widetilde{C}(f)$ would be the contribution in an equilibrium in which qualified candidates defeat their unqualified opponents with probability 1. In addition, for all $f \in [0, \delta]$, let

$$\widetilde{\xi}(f) = 1 - H(\beta(1-2x) - \frac{(1-\sigma)(\alpha + \widetilde{C}(f))}{\alpha + \widetilde{C}(f)(1-\sigma)}(\delta - f)).$$

From (6), we can see that $\widetilde{\xi}(f)$ would be the equilibrium effectiveness of advertising in an equilibrium in which interest groups gave $\widetilde{C}(f)$ in exchange for favors f. We now make:

Assumption 1: *For all $f \in [0, \delta]$ either $\widetilde{\xi}(f) \leq \frac{1}{2}$ or $\widetilde{C}(f) < \frac{\alpha}{2\widetilde{\xi}(f)-1}$.*

This assumption requires that if the interest groups gave $\widetilde{C}(f)$ in exchange for favors f, then the effectiveness of advertising must be such that qualified candidates defeat their unqualified opponents with a probability of less than one. Thus, it rules out the possibility that qualified candidates defeat their unqualified opponents with probability one in equilibrium. Basically, it requires that at low levels of favors (when the effectiveness of advertising is high) the amount that the interest groups are willing to give is not large enough to finance an advertising campaign sufficient to guarantee a qualified candidate victory. For any given values of the other parameters, the assumption will be satisfied for α above some critical level.

We also impose an assumption that guarantees that equilibrium involves candidates receiving a positive amount of contributions. Define the function

$$\Psi(C_L, f_L, C, f; \xi) = \frac{-\partial U/\partial f_L}{\partial U/\partial C_L} - \frac{\partial G/\partial f_L}{\frac{2}{\theta(1-\gamma)} - \partial G/\partial C_L}.$$

The function Ψ is simply the difference between the candidate's and interest group's marginal rate of substitution between contributions and favors.

Assumption 2: $\Psi(0,0,0,0;\xi') < 0$ *where* $\xi' = 1 - H(\beta(1-2x) - (1-\sigma)\delta)$.

Diagrammatically, this assumption rules out the possibility illustrated in Figure 2. It ensures that when $(C, f, \xi) = (0, 0, \xi')$, the slope of the candidate's indifference curve at the point $(C_L, f_L) = (0, 0)$ is flatter than the slope of the boundary of the constraint set, when the latter emanates from the origin. This assumption is more likely to be satisfied the larger is the size of the interest group, the greater is the candidates' ego-rent, and the greater is the marginal value of favors to the interest groups.[3]

With these assumptions, we obtain the following useful characterization result.

Lemma 1: *Let (C, f, ξ) be an equilibrium. Then, under Assumptions 1 and 2, the level of contributions C is positive and (C, f, ξ) satisfies the following pair of equations:*

$$\Psi(C, f, C, f; \xi) \geq 0 \ (= \text{ if } f > 0), \quad (7)$$

and

$$G(C, f, C, f; \xi) = \frac{2C}{(1-\gamma)\theta}. \quad (8)$$

These equations are simply the first order conditions for *Problem P* when either $\xi \leq \frac{1}{2}$ or $C < \frac{\alpha}{2\xi - 1}$. They say that, in equilibrium, the candidate's indifference curve cannot be flatter than the slope of the constraint set and must be tangent to it if the level of favors is positive.

Assumptions 1 and 2 are not sufficient to guarantee that candidates will offer to implement favors in equilibrium. It is possible that the equilibrium looks like that depicted in Figure 3, in which case contributions are purely position induced. In this case, the gain to candidates from extracting further contributions is offset by the costs of implementing favors. The following strengthening of Assumption 2 ensures that contributions are service induced.

[3] The assumption amounts to the requirement that $\frac{\xi'[b'(0)(r+\beta(1-2d)+(1-\sigma)\delta)-r]}{2\alpha}$ exceeds $\frac{2}{\theta(1-\gamma)}$.

Assumption 3: $\Psi(\widetilde{C}(0), 0, \widetilde{C}(0), 0; \xi') < 0$ *where* $\xi' = 1 - H(\beta(1-2x) - (1-\sigma)\delta)$.

Observe that Assumption 3 is equivalent to Assumption 2 when $\widetilde{C}(0) = 0$.

We now have:

Proposition 1: *Under Assumptions* 1 *and* 3*, in any equilibrium qualified candidates offer to implement favors for their interest groups to extract larger contributions. The contributions they receive allow them to defeat unqualified opponents with a probability between* $\frac{1}{2}$ *and* 1*.*

Thus, with unrestricted contributions, qualified candidates will offer favors to extract more contributions from their supporters. These contributions are used to finance campaign advertising that gives them an electoral advantage over their unqualified opponents. The campaign contributions play the social role of raising the likelihood of qualified leaders. However, qualified candidates implement favors which reduces the benefits to non-interest group members from electing them. The level of favors is not sufficient to exhaust the benefits of a more qualified candidate (i.e., $f < \delta$), because otherwise advertising could not be effective. Nonetheless, voters pay a price. Moreover, the favors granted do not ultimately benefit the interest groups, because interest group members pay for them up front through their contributions.

It follows from Proposition 1 and Lemma 1 that if (C, f, ξ) is an equilibrium, then it must satisfy equations (6) and (8) and equation (7) with equality. This gives three equations that may be solved for the three unknowns (C, f, ξ) and enables the numerical computation of equilibrium.[4]

If (C, f, ξ) satisfies equations (6) and (8) and equation (7) with equality then it will be an equilibrium provided that equations (8) and (7) with equality are sufficient to imply that (C, f) solves *Problem P*. Provided that the constraint set in Figure 1 is convex, they will be sufficient.

[4] It is easy to show that $\Psi(C, f, C, f; \xi) = 0$ if and only if

$$\sigma \frac{\partial \pi(C, C; \xi)}{\partial C_L}[(b'(f) - 1)r + \beta(1-2d)b'(f) + b(f)]$$
$$+(1-\sigma)\frac{\partial \pi(C, 0; \xi)}{\partial C_L}[(b'(f) - 1)r + (\beta(1-2d) + \delta - f)b'(f) + b(f)] = \frac{2}{(1-\gamma)\theta}$$

As noted above, the constraint set will necessarily be convex when ξ is small. Thus, if (C, f, ξ) satisfies equations (6) and (8) and equation (7) with equality and ξ is small then it will be an equilibrium. More generally, conditions under which satisfaction of (6), (8) and (7) with equality imply an equilibrium can be found by imposing assumptions which imply that the constraint set is convex.[5] Under such assumptions, the issue of the existence of equilibrium boils down to the existence of a triple (C, f, ξ) satisfying (6), (8) and (7) with equality. We return to this issue in Section 5.2.

4.3 Power-hungry candidates

The logic of the equilibrium is that the effectiveness of advertising determines the incentives of candidates to offer favors and the level of favors feeds back into the determination of the effectiveness of advertising. Intuitively, when candidates are very power-hungry one might expect them to be desperate to obtain more contributions and thus willing to promise large amounts of favors to extract more money. But the level of favors must be less than the benefits of being qualified if campaign advertising is to be effective. One would therefore expect equilibrium to involve a low level of advertising effectiveness to dampen candidates' propensity to offer favors. Thus, as candidates become more and more power-hungry, the effectiveness of campaign advertising should become smaller and smaller. This logic is confirmed in:

Proposition 2: *Suppose that Assumption 1 is satisfied. For all r, let $(C(r), f(r), \xi(r))$ be the equilibrium (or an equilibrium) that would arise with no limits when ego-rents are r. Then,*

$$\lim_{r\to\infty} (C(r), f(r), \xi(r)) = (\frac{\theta(1-\gamma)(b(\widehat{f}) - \widehat{f})}{4}, \widehat{f}, 0),$$

where $\widehat{f} > 0$ is implicitly defined by the equation:

$$\overline{\varepsilon} = \beta(1-2x) - \frac{(1-\sigma)(\alpha + \frac{\theta(1-\gamma)(b(\widehat{f})-\widehat{f})}{4})}{\alpha + \frac{\theta(1-\gamma)(b(\widehat{f})-\widehat{f})}{4}(1-\sigma)}(\delta - \widehat{f}).$$

[5] See footnote 4.

The conclusion that the effectiveness of advertising must go to zero may be understood diagrammatically. An increase in r raises the candidate's marginal value of contributions, thereby flattening his indifference curves.[6] For given $\xi > 0$, the candidate's indifference curves become horizontal as r goes to infinity. On the other hand, a reduction in ξ reduces the candidate's marginal value of contributions, steepening his indifference curves. Indeed, for given r, the candidate's indifference curves become vertical as ξ goes to zero. As r increases, the candidate's indifference curves become flatter and he is prepared to offer more and more favors. Since the level of favors must be strictly less than the gains from qualifications (δ) in any equilibrium and the slope of the boundary of the constraint set is positive over this range, the only way that the tangency condition (7) may hold as r gets larger and larger is for ξ to get smaller and smaller.

To get the effectiveness of advertising to be zero requires that the level of favors that qualified candidates are expected to implement be sufficiently high to deter swing voters from switching their votes. Thus, the level of favors $\widehat{f}$ is positive under our assumption that $\beta(1-2x) - (1-\sigma)\delta < \overline{\varepsilon}$. Note also that the Proposition implies that the equilibrium probability that a qualified candidate defeats an unqualified one tends to 1/2 as candidates become more power-hungry (i.e., $\lim_{r\to\infty} \pi(C(r), 0; \xi(r)) = 1/2$). Accordingly, *while resources are expended on campaign advertising, these resources do not make qualified candidates more likely to be elected.* This observation has important implications for the desirability of contribution limits.

5 Contribution Limits

This section analyzes the impact of contribution limits. It first characterizes equilibrium with contribution limits. It then shows that when candidates are sufficiently power-hungry, banning

[6] The slope of the candidate's indifference curves $\frac{-\partial U/\partial f_L}{\partial U/\partial G_L}$ is given by $\frac{\sigma(\pi(C_L,C;\xi)+(1-\sigma)\pi(C_L,0;\xi)}{\sigma\frac{\partial \pi(C_L,C;\xi)}{\partial C_L}[r+\beta(1-2d)+f-f_L]+(1-\sigma)\frac{\partial \pi(C_L,0;\xi)}{\partial C_L}[r+\beta(1-2d)+\delta-f_L]}$.

contributions is Pareto improving. Finally, it argues that a similar logic may imply that limiting contributions is Pareto improving when candidates are more policy-motivated.

5.1 Equilibrium with contribution limits

Suppose now that the laws governing elections limit the amount of money an interest group can contribute. Let the limit be denoted by L. Candidates are now constrained in what they can obtain from their interest groups. In equilibrium, (C, f) must solve the problem:

$$\max_{(C_L, f_L) \in [0,L] \times \Re_+} U(C_L, f_L, C, f; \xi) \ s.t. \ G(C_L, f_L, C, f; \xi) \geq \frac{2C_L}{(1-\gamma)\theta}.$$

We will refer to this as *Problem P′*. An equilibrium is then a triple (C, f, ξ) such that (C, f) solves *Problem P′* given ξ and ξ satisfies equation (6).

We may follow the strategy of the previous section and use the first order conditions for *Problem P′* to characterize equilibrium. Figure 4 illustrates how the introduction of a limit changes a candidate's constraint set. Note that while the Figure may suggest that such a limit should reduce the level of favors, the limit will also impact the effectiveness of advertising. Without understanding this impact, nothing may be concluded. We now have:

Lemma 2: *Let (C, f, ξ) be an equilibrium under contribution limit L such that the limit binds (i.e., $C = L$). Then, under Assumptions 1 and 3, (f, ξ) satisfies the following pair of equations:*

$$\Psi(L, f, L, f; \xi) \leq 0 \qquad (9)$$

and

$$G(L, f, L, f; \xi) \geq \frac{2L}{(1-\gamma)\theta} \ (= \textit{if } f > 0). \quad (10)$$

The constraint that interest groups cannot contribute more than the limit, prevents the level of favors from being driven to the level where the slope of the candidate's indifference curve equals the slope of the boundary of the constraint set. Effectively, when the limit is binding, the candidate's indifference curve at the optimal choice can be flatter than the boundary of the constraint set as illustrated in Figure 4. This explains equation (9). With limits, it is possible that no favors are offered and contributions are purely "position induced" even under Assumption 3. This arises when interest groups would obtain a net gain from contributing the maximal level of contributions when the effectiveness of advertising is that which would arise if $(C, f) = (L, 0)$. Diagrammatically, the situation is as illustrated in Figure 5. In such an equilibrium, interest groups may obtain some surplus because candidates are unable to extract more contributions from them or offer them fewer favors. This explains equation (10).[7]

5.2 Contribution limits and welfare

To understand the welfare implications of limits, it is necessary to understand both how the equilibrium is impacted by limits and how changes in the equilibrium impact citizens' payoffs. Using the expressions for the payoffs of the various types of citizens established earlier and equation (8), we may establish:

Lemma 3: *If imposing a limit moves the community from some status quo* (C, f, ξ) *to a new equilibrium* (C', f', ξ') *such that (i)* $\pi(C', 0; \xi') \approx \pi(C, 0; \xi)$ *and (ii)* $f' < f$*, then it makes all types of citizens strictly better off.*

Thus if introducing a limit does not appreciably change the probability a qualified candidate defeats an unqualified one and reduces the level of favors, it will create a Pareto improvement. That these conditions imply that partisans and swing voters are better off, follows directly from the expressions for their payoffs developed in Section 3.6. That they imply that interest group members are better off is less obvious. The key is to note that the *equilibrium* payoff of interest

[7] Lemma 2 suggests the following procedure for computing equilibrium with limits. First, we use (6) to compute the effectiveness of advertising that would arise if $(C, f) = (L, 0)$. If at this level of ξ equation (10) is satisfied, there exists an equilibrium in which interest groups simply provide candidates with contribution L and receive no favors. If there is no such position induced equilibrium, one looks for solutions to equation (10) with equality and (6). If a solution involves a positive level of favors, one then checks to see if equation (9) is satisfied. If so, then the solution satisfies the equilibrium conditions.

group members is decreasing in f. Intuitively, this is because interest group members pay for their own favors up front with their contributions and must also share the burden of favors granted to the other interest group.

Combining Lemma 3 with Proposition 2 enables us to easily establish our main result:

Proposition 3: *Suppose that Assumption 1 is satisfied. Then, if candidates are sufficiently power-hungry (i.e., r is sufficiently large), banning contributions (i.e., setting $L = 0$) will create a Pareto improvement.*

To understand this result, note that if contributions were banned entirely then no favors would be promised and the probability that a qualified candidate defeats an unqualified one is just $1/2$. The result now follows from the fact that, with no limits, as candidates become more power-hungry the probability that a qualified candidate defeats an unqualified one approaches $1/2$ while the level of favors remains strictly positive.

The logic of the above argument is that when candidates are sufficiently power-hungry, banning contributions will have a negligible impact on the probability that a qualified candidate defeats an unqualified one, while reducing favors. This implies a Pareto improvement. With less power-hungry candidates, it is clear that *banning* contributions could lead to a significant reduction in the probability that qualified candidates defeat unqualified ones and hence this argument will not imply. We now argue that *limiting* contributions, while reducing favors, need not appreciably reduce the probability that qualified candidates win, in which case the same logic implies that limits could be Pareto improving.

Establishing this requires us to understand more completely the impact of limits on the equilibrium variables, which is a challenging task. Our strategy will be to first develop a way to understand more deeply the determination of equilibrium with unrestricted contributions. We then utilize this understanding to assess the impact of limits.

Let $C_o(f;\xi)$ be the level of contributions that qualified candidates must receive to generate an effectiveness of advertising ξ when qualified candidates provide an amount of favors f. Formally, $C_o(f;\xi)$ is implicitly defined by equation (6). Clearly, $C_o(f;\xi)$ will not be defined for all pairs $(f;\xi)$ - for example, there will exist no amount of contributions that will generate a high level of effectiveness when the level of favors is very high. In the proof of Proposition 1, we show that for given ξ, $C_o(f;\xi)$ is well-defined for f values between $\min\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}$, and $\delta - \beta(1-2x) + H^{-1}(1-\xi)$. On this interval, $C_o(\cdot;\xi)$ is increasing at an increasing rate, approaching infinity as the level of favors approaches the upper limit of the interval. Intuitively, as the level of favors qualified candidates provide increases, the amount of contributions necessary to generate a given level of effectiveness increases.[8] The function $C_o(\cdot;\xi)$ is depicted in Figure 6 under the assumption that $\delta > \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}$. For a given level of favors, it takes a higher level of contributions to generate a higher level of effectiveness, so that an increase in ξ shifts this curve to the left.

Let $C_i(f;\xi)$ be the level of contributions that would make interest groups indifferent between accepting candidates offers when the level of favors promised is f and the effectiveness of advertising is ξ. Formally, $C_i(f;\xi)$ is implicitly defined by equation (8). For all ξ, $C_i(\cdot;\xi)$ is an increasing function, reflecting the fact that interest groups value favors. It may or may not be positive at $f = 0$ depending on the strength of the position-induced incentive to give and the effectiveness of campaign advertising.[9] The function $C_i(\cdot;\xi)$ is depicted in Figure 6 under the assumption that $C_i(0;\xi) = 0$. For a given level of favors, the gain from contributing is higher the more effective is advertising, so that an increase in ξ shifts this curve to the left.

For any given level of advertising effectiveness ξ, one can ask the question of whether there

[8] As observed earlier, when contributions are plentiful, not having observed a candidate's advertisement increases the likelihood that he is unqualified and thus increases the effectiveness of advertising.

[9] There may be two non-negative solutions to equation (8) when $f = 0$. One solution is always $C = 0$, since the gain from giving no contributions in exchange for no favors is obviously zero. But there will be a positive solution if $\partial G(0,0,0,0;\xi)/\partial C > 0$. $C_i(0;\xi)$ is the positive solution when it exists.

exists a level of favors f such that $C_i(f;\xi) = C_o(f;\xi)$. If so, then at this level of favors, the amount that interest groups would contribute given ξ just equals the amount that would generate the effectiveness ξ. Thus, both equations (6) and (8) are satisfied. If the curve $C_i(\cdot;\xi)$ lies above $C_o(\cdot;\xi)$ over any part of the relevant range, there must exist at least one level of favors such that $C_i(f;\xi) = C_o(f;\xi)$. This is because $C_i(\cdot;\xi)$ is bounded above and $C_o(\cdot;\xi)$ increases without limit as the level of favors increase.

When ξ is small, the situation is as depicted in Figure 6. The function $C_i(\cdot;\xi)$ is strictly concave and satisfies $C_i(0;\xi) = 0$. The function $C_o(\cdot;\xi)$ is not defined for small levels of favors. Thus, there exists a unique level of favors at which $C_i(f;\xi) = C_o(f;\xi)$. At higher levels of ξ there may exist multiple favor levels at which $C_i(f;\xi) = C_o(f;\xi)$ because $C_o(\cdot;\xi)$ is positive for small levels of favors (Figure 7). Alternatively, there may exist no favor levels (Figure 8) because $C_i(\cdot;\xi)$ lies below $C_o(\cdot;\xi)$ over the relevant range. Intuitively, the latter corresponds to a situation where the amount of contributions necessary to generate advertising effectiveness ξ is always greater than the amount that interest groups would contribute.

Let $\overline{\xi}$ denote the maximum level of effectiveness for which there exists a level of favors f such that $C_i(f;\xi) \geq C_o(f;\xi)$.[10] Then, we make the following assumptions on the functions C_i and C_o which serve to make the problem well-behaved.

Assumption 4: *(i)* $\overline{\xi} < 1$.

(ii) For all $\xi \in [0, \overline{\xi}]$, $C_i(\cdot;\xi)$ *is strictly concave.*

(iii) For all $f \in (\min\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}, \delta - \beta(1-2x) + H^{-1}(1-\xi))$, $\frac{\partial C_i(f;\xi)}{\partial \xi} < \frac{\partial C_o(f;\xi)}{\partial \xi}$.

(iv) $\frac{\partial C_i(0;\underline{\xi})}{\partial f} > \frac{\partial C_o(0;\underline{\xi})}{\partial f}$ *where* $\underline{\xi}$ *is defined by* $C_i(0;\underline{\xi}) = C_o(0;\underline{\xi})$.

Part (i) simply says that at any level of favors the contribution necessary to make advertising maximally effective exceeds the contribution that would be made by the interest groups. Part (ii)

[10] More precisely, $\overline{\xi}$ is the maximum value of $\xi \in [0,1]$ for which there exists a value of f between $\min\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}$, and $\delta - \beta(1-2x) + H^{-1}(1-\xi)$ such that $C_i(f;\xi) \geq C_o(f;\xi)$.

is self-explanatory. Part (iii) says that a marginal increase in advertising effectiveness necessitates a larger increase in C_o than it generates in the interest groups' contribution C_i. Finally, part (iv) requires that $C_i(\cdot;\underline{\xi})$ has a steeper slope than $C_o(\cdot;\underline{\xi})$ at $f = 0$. While it is hard to find simple conditions on the primitives that guarantee that all parts of Assumption 4 will be satisfied, simulations suggest that they are not overly restrictive.[11]

Under Assumption 4, the solutions to the equation $C_i(f;\xi) = C_o(f;\xi)$ have a simple structure. If ξ exceeds $\overline{\xi}$, the situation is as illustrated in Figure 8 and there is no solution. If ξ lies in the interval $[\underline{\xi}, \overline{\xi}]$ the situation is as depicted in Figure 7 and there are two solutions, which we denote $f_-(\xi)$ and $f_+(\xi)$ respectively. At the former solution, $C_o(\cdot;\xi)$ cuts $C_i(\cdot;\xi)$ from above, at the latter from below. In the case in which ξ exactly equals $\overline{\xi}$, we have that $f_-(\xi) = f_+(\xi)$ and $C_o(\cdot;\xi)$ is tangent to $C_i(\cdot;\xi)$. If ξ lies in the interval $[0, \underline{\xi})$ the situation is as depicted in Figure 6 and there is a unique solution, which we denote $f_+(\xi)$. At this solution, $C_o(\cdot;\xi)$ cuts $C_i(\cdot;\xi)$ from below. Under Assumption 4, the function $f_-(\cdot)$ is increasing on the interval $[\underline{\xi}, \overline{\xi}]$ and $f_+(\cdot)$ is decreasing on $[0, \overline{\xi}]$.

For all ξ in the interval $[0, \overline{\xi}]$, let $C_+(\xi) = C_o(f_+(\xi);\xi)$ and for all ξ in the interval $[\underline{\xi}, \overline{\xi}]$ let $C_-(\xi) = C_o(f_-(\xi);\xi)$. Since C_o is increasing in ξ, $C_-(\xi)$ is increasing on $[\underline{\xi}, \overline{\xi}]$. However, the sign of $C_+(\xi)$ is indeterminate. The functions (C_+, f_+) and (C_-, f_-) are illustrated in Figure 9.

We know from the previous section that if (C^*, f^*, ξ^*) is an equilibrium with unrestricted contributions then it must satisfy equation (7) with equality; i.e., the slope of the candidate's indifference curve must equal the slope of the boundary of the constraint set. For all ξ in the interval $[0, \overline{\xi}]$ let $\Psi_+(\xi) = \Psi(C_+(\xi), f_+(\xi), C_+(\xi), f_+(\xi);\xi)$ and for all ξ in the interval $[\underline{\xi}, \overline{\xi}]$ let $\Psi_-(\xi) = \Psi(C_-(\xi), f_-(\xi), C_-(\xi), f_-(\xi);\xi)$. Then, if (C^*, f^*, ξ^*) is an equilibrium *either* the effectiveness of advertising must lie in the interval $[\underline{\xi}, \overline{\xi}]$, the candidates offer their interest groups $(C_-(\xi^*), f_-(\xi^*))$, and $\Psi_-(\xi^*) = 0$, *or* the effectiveness of advertising lies in the interval $[0, \overline{\xi}]$,

[11] The Appendix describes the conditions that are required for the various parts of Assumption 4 in more detail.

candidates offer $(C_+(\xi^*), f_+(\xi^*))$, and $\Psi_+(\xi^*) = 0$. In the former case we say that the equilibrium is in *Case 1*, in the latter it is in *Case 2*. Since $\underline{\xi} > 0$, the equilibrium must be in Case 2 for r sufficiently large.[12]

We know that the candidate's indifference curve is vertical when $\xi = 0$, so that $\Psi_+(0) > 0$. In addition, Assumption 3 implies that the candidate's indifference curve must be flatter than the boundary of the constraint set when $\xi = \underline{\xi}$ so that $\Psi_-(\underline{\xi}) < 0$. We also know that $\Psi_+(\overline{\xi}) = \Psi_-(\overline{\xi})$ and that Ψ_+ and Ψ_- are continuous. Thus, under Assumptions 1, 3 and 4, there must exist (C, f, ξ) which satisfies equations (6), (8) and (7) with equality.

Assuming that there exists a unique equilibrium, it follows that if the equilibrium is in Case 1 it must be that Ψ_- is negative on the interval $[\underline{\xi}, \xi^*)$ and positive on the interval $(\xi^*, \overline{\xi}]$, while Ψ_+ is positive on its entire range. If the equilibrium is in Case 2 it must be that Ψ_- is negative on its entire range, while Ψ_+ is negative on the interval $(\xi^*, \overline{\xi}]$ and positive on $[0, \xi^*)$.

We are finally in a position to understand the impact of limits. Suppose that there exists a unique equilibrium with unrestricted contributions given by (C^*, f^*, ξ^*) and consider a limit $L < C^*$. Suppose first that the status quo equilibrium is in Case 1 and that $L \geq C_-(\underline{\xi})$. The situation is as illustrated in Figure 10.

Observe that the limit *reduces* the effectiveness of advertising to ξ_L where $L = C_-(\xi_L)$ and reduces the level of favors to $f_-(\xi_L)$.[13] The effectiveness of advertising is reduced, despite the fact that the level of favors decreases, because not seeing an advertisement is less likely to mean that a candidate is unqualified because there is less advertising. Thus, while the benefit of electing a qualified candidate has increased, swing voters are less likely to switch their votes from unadvertised candidates because unadvertised is less likely to imply unqualified. It follows that the probability that a qualified candidate defeats an unqualified candidate must fall since *both*

[12] The fact that $\underline{\xi} > 0$ follows from the assumption that $\beta(1 - 2x) - (1 - \sigma)\delta < \overline{\varepsilon}$.

[13] Since $\Psi_-(\xi_L) > 0$, equation (9) of Lemma 2 holds.

the level of contributions *and* the effectiveness of advertising falls. If $L < C_{-}(\underline{\xi})$, then the limit reduces the level of favors to 0 and contributions become purely position-induced.[14] Again, the probability that a qualified candidate defeats an unqualified candidate falls.

Now suppose that the status quo equilibrium is in Case 2. First note that, if there is a unique equilibrium with unrestricted contributions, the limit must reduce the level of favors. Even though it might be the case that there exists $\widetilde{\xi} < \xi^*$ such that $C_+(\widetilde{\xi}) = L$, there could not be an equilibrium with limit L under which $(f, \xi) = (f_+(\widetilde{\xi}), \widetilde{\xi})$. This is because $\Psi_+(\widetilde{\xi}) < 0$ and hence equation (9) could not be satisfied.[15] The next point to note is that the limit may well increase the effectiveness of campaign advertising. Figure 11 illustrates a situation in which this occurs. Intuitively, swing voters are more likely to be responsive to learning a candidate is qualified because they know that qualified candidates will implement lower levels of favors. This offsets the fact that not seeing an advertisement is less likely to mean that a candidate is unqualified because there is less advertising. It follows that the limit need not reduce the probability that a qualified candidate defeats an unqualified candidate because the increase in the responsiveness of swing voters could compensate for the smaller fraction reached as a result of reduced campaign spending.

A binding limit that leaves unchanged the probability that a qualified candidate wins will create a Pareto improvement by Lemma 3. A sufficient condition for the existence of such a limit is that $\pi(C_+(\xi), 0; \xi)$ is increasing at $\xi = \xi^*$. If this is the case, there must exist a limit $L < C^*$ that will reduce favors and leave the probability that a qualified candidate defeats an unqualified one unchanged. Obviously, this sufficient condition will be satisfied if C_+ is increasing at $\xi = \xi^*$. If C_+ is decreasing, it will still be satisfied provided that the rate of decrease $(-C_+^{'})$ is smaller than $\partial\pi/\partial\xi/\partial\pi/\partial C_L$.

[14] The effectiveness of advertising is given by $1 - H(\beta(1-2x) - \frac{(1-\sigma)(\alpha+L)}{\alpha+L(1-\sigma)}\delta)$.

[15] There is no guarantee that there is a unique equilibrium under a particular limit in this case. Because $C_+(\xi)$ is not monotonic on $[0, \overline{\xi}]$, there may be more than one $\xi > \xi^*$ such that $C_+(\xi) = L$.

Unfortunately, the problem is sufficiently complex that it is difficult to get useful sufficient conditions on the primitives (other than that r be large!) to ensure that equilibrium will be in Case 2 and that there will exist a limit that does not reduce the probability that a qualified candidate wins. However, numerical examples suggest that there is nothing paradoxical about the possibility. We now present such an example.

Example

Assume that $d = 0.15$, $x = 0.4$, $\beta = 100$, $\gamma = 0.5$, $\sigma = 0.5$, $\theta = 0.2$, $\alpha = 1.5$, $r = 200$, $\delta = 40$, $b(f) = 6f - (0.05)f^2$, $H(\varepsilon) = (\varepsilon + \overline{\varepsilon})/2\overline{\varepsilon}$, and $\overline{\varepsilon} = 20$. Plugging these values into equations (6), (7), and (8) and using the formulas for the probability of winning function, we obtain a system of three equations in three unknowns. This system has a unique solution given by $(C, f, \xi) = (3.\,221\,8, 19.\,903, 0.381\,3)$. Note that campaign advertising is reasonably effective. The equilibrium probability that a qualified candidate defeats an unqualified one is $0.675\,83$.

Now suppose that the constitution were to specify a limit of 2. Then, the solution to equations (10) and (6) is $(f, \xi) = (5.\,013\,7, 0.612\,26)$. It is readily checked that (9) is satisfied, so that the equilibrium with limit 2 is $(C, f, \xi) = (2, 5.\,013\,7, 0.612\,26)$. Note that campaign advertising is much more effective and the level of favors is reduced. The probability that a qualified candidate is elected is 0.76907. Thus the limit raises the probability that a qualified candidate is elected and reduces the number of favors

It may be verified that the equilibrium with limit 3 is $(C, f, \xi) = (3, 12.915, 0.50784)$ and the probability that a qualified candidate defeats an unqualified one is 0.75593. With a limit 2.5, $(C, f, \xi) = (2.5, 7.\,598\,9, 0.589\,11)$ and $\pi = 0.79138$. With a limit of 1.5, $(C, f, \xi) = (1.5, 3.336, 0.61107)$ and $\pi = 0.71998$. With a limit of 1, $(C, f, \xi) = (1, 2.0706, 0.59265)$ and $\pi = 0.65536$. With a limit of 0.5, $(C, f, \xi) = (0.5, 1.0009, 0.55713)$ and $\pi = 0.58091$.

The picture this suggests is that the probability that a qualified candidate wins first rises and then falls as the limit becomes more stringent. It follows that there must exist a limit that reduces favors and leaves unaffected the probability a qualified candidate wins. Thus, by Lemma 3, there must exist a Pareto improving contribution limit. This limit is approximately 1.

The final point to note is that even when imposing a limit implies a reduction in the probability that qualified candidates are elected, it still maybe the case that the limit is Pareto improving. Let (C, f, ξ) be the status quo and suppose that a limit leads to a new equilibrium (C', f', ξ') such that $\pi(C', 0; \xi') < \pi(C, 0; \xi)$. Then, provided that $\eta(\pi(C', 0; \xi'))(\delta - f') > \eta(\pi(C, 0; \xi))(\delta - f)$, it makes all types of citizens strictly better off. This condition ensures that any reduction in the probability a qualified candidate defeats an unqualified one is compensated by a reduction in the favors that such a candidate will provide.[16] That these conditions imply that partisans and swing voters are better off, follows directly from the expressions for their payoffs. That they imply that interest group members are better off follows from the fact that the *equilibrium* payoff of interest group members is decreasing in π as well as f.

This admits a simple sufficient condition for limits to be Pareto improving. Under a contribution ban, we have that

$$\eta(\pi(C', 0; \xi'))(\delta - f') = \sigma^2 + \sigma(1 - \sigma)\delta.$$

Since $\pi(C, 0; \xi) \leq 1$ it must be the case that

$$\eta(\pi(C, 0; \xi))(\delta - f) \leq [\sigma^2 + 2\sigma(1 - \sigma)](\delta - f).$$

Thus, if the status quo level of favors is such that

$$f > \frac{(1 - \sigma)}{2 - \sigma}\delta,$$

there must exist a Pareto improving limit.[17]

[16] Recall that $\eta(\pi)$ is the probability that a qualified candidate is elected when a qualified candidate defeats an unqualified one with probability π.

[17] It is possible to use this observation to develop a sufficient condition for the existence of Pareto improving limits in terms of the primitives. Suppose it were the case that $f \leq \frac{(1-\sigma)}{2-\sigma}\delta$. Then the maximum contribution

6 Conclusion

The basic logic of the argument presented in this paper is easily summarized. When candidates use campaign contributions to finance advertising that conveys truthful information to voters about their qualifications for office, contributions have the potential social benefit of helping elect more qualified leaders. But for contributions to have this benefit, voters who are informed that a candidate is qualified through campaign advertising must be induced to switch their votes from unadvertised candidates. However, when contributions are unrestricted, voters will rationally be cynical about qualified candidates, anticipating that they will implement favors for their contributors when elected. This cynicism will reduce the likelihood of voters switching their votes and, despite the fact that resources are spent on advertising, qualified candidates will not have much of an electoral advantage over unqualified opponents.

When campaign contributions are limited, candidates' incentive to offer favors to extract more contributions is dampened. While less money is available for campaign advertising, voters now anticipate that advertised candidates will implement fewer favors than in the unrestricted case and this may increase the likelihood that they will vote for them. In this way, limits can actually raise the likelihood that qualified candidates get elected. Moreover, if elected such candidates will implement lower levels of favors than in the unrestricted case. Thus, all regular citizens can be better off when contributions are limited. The only possible losers are contributors who receive lower levels of favors. But their expected gains from favors will be dissipated by the contributions they make, meaning they may also be better off.

While the underlying logic seems quite general, the argument has been formally developed in an

would be $\widetilde{C}(f)$ as defined earlier. The minimum contribution would be $\underline{C}(f) = \frac{\theta(1-\gamma)(b(f)-f)}{4}$. The minimum level of advertising effectiveness would therefore be

$$\underline{\xi}(f) = 1 - H(\beta(1-2x) - \frac{(1-\sigma)(\alpha + \underline{C}(f))}{\alpha + \underline{C}(f)(1-\sigma)}(\delta - f)).$$

If it were the case that $\Psi(\overline{C}(f), f, \overline{C}(f), f; \underline{\xi}(f)) < 0$ for all $f \leq \frac{(1-\sigma)}{2-\sigma}\delta$, the equilibrium level of favors must exceed $\frac{(1-\sigma)}{2-\sigma}\delta$.

undeniably simple model. It would be well worth investigating the robustness of the argument to alternative or more general specifications. One obvious assumption to change is that the candidates present interest groups with "take it or leave it" offers that allow them to extract all their surplus. One could alternatively follow Grossman and Helpman (1994) in assuming the opposite; i.e., that interest groups make "take it or leave it" offers. It seems likely that the conclusion that even interest group members would benefit from contribution limits might need modification. That said, even when interest group members obtain some surplus from the favors they are given, they must still bear their share of the collective cost of granting other groups favors.

It would also be interesting to allow for a richer set of candidate types. For example, one could introduce multiple levels of qualifications. Alternatively, one could assume that candidates differed in their willingness to take favors - some were more honest than others. Under the latter assumption, the number of times a voter had seen a candidate's advertisement might have some significance. There might be a penalty for advertising too heavily, because voters would take it as a signal of a candidate being dishonest. This may limit the incentive to offer favors even when contributions are unrestricted.

More generally, from an empirical perspective, it would be extremely interesting to exploit the cross-state variation in U.S. campaign finance regulations, to see if there is indeed systematic differences in the effectiveness of campaign advertising as our argument would suggest. It would also be interesting to consider whether the type of argument developed here has implications for the case for public financing of campaigns. In some U.S. states (for example, Massachusetts), candidates for statewide offices are entitled to public financing if (i) they have raised some minimum level of contributions from private citizens or groups and (ii) they forego taking further private contributions. Such a scheme would seem to have the potential of reducing favors and increasing the effectiveness of advertising, while not reducing the level of advertising. However, the downside is that public contributions must be financed via tax hikes.

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7 Appendix

Properties of the probability of winning function: In a number of the proofs, we will appeal to properties of the probability of winning function. We establish these here. We first derive an explicit expression for the probability of winning function. Using the functional form for $\lambda(\cdot)$, we obtain

$$\delta_L(C_L, C_R) = \frac{C_L(\alpha + C_R) + \alpha((1-\xi)C_R + \alpha)}{(\alpha + C_L)(\alpha + C_R)},$$

and

$$\delta_R(C_L, C_R) = \frac{\xi \alpha C_L}{(\alpha + C_L)(\alpha + C_R)}.$$

Note that $\delta_L(C_L, C_R) < \frac{1}{2}$ if and only if $C_L < \frac{\alpha[C_R(2\xi-1)-\alpha]}{\alpha+C_R}$ and that $\delta_R(C_L, C_R) > \frac{1}{2}$ if and only if $C_L[\alpha(2\xi-1) - C_R] > \alpha(\alpha + C_R)$. Thus, we have that

$$\pi(C_L, C_R; \xi) = \left\{ \begin{array}{ll} 0 & if \;\; C_L < \frac{\alpha[C_R(2\xi-1)-\alpha]}{\alpha+C_R} \\ \frac{C_L(\alpha+C_R)/2+\alpha(1/2-\xi)C_R+\alpha^2/2}{C_L(\alpha(1-\xi)+C_R)+\alpha(1-\xi)C_R+\alpha^2} & otherwise \\ 1 & if \;\; C_L[\alpha(2\xi-1)-C_R] > \alpha(\alpha+C_R) \end{array} \right. .$$

We can now establish the required properties. Observe that $\pi(\cdot, C_R; \xi)$ is differentiable at C_L if C_L exceeds $\frac{\alpha[C_R(2\xi-1)-\alpha]}{\alpha+C_R}$ and $C_L[\alpha(2\xi-1) - C_R]$ exceeds $\alpha(\alpha + C_R)$. The derivative is given by:

$$\frac{\partial \pi}{\partial C_L} = \frac{\alpha\xi(C_R \alpha 2(1-\xi) + C_R^2 + \alpha^2)}{2\{C_L(\alpha(1-\xi) + C_R) + \alpha(1-\xi)C_R + \alpha^2\}^2}.$$

Observe also that $\frac{\partial^2 \pi}{\partial C_L^2} < 0$ so that $\pi(\cdot, C_R; \xi)$ is a concave function over this range.

Note that

$$\frac{\partial \pi(C, C; \xi)}{\partial C_L} = \frac{\alpha\xi}{2\{C^2 + 2\alpha(1-\xi)C + \alpha^2\}},$$

so that $\frac{\partial \pi(C,C;\xi)}{\partial C_L}$ is a decreasing function of C and $\frac{\partial^2 \pi(C,C;\xi)}{\partial C_L \partial \xi} > 0$. In addition,

$$\frac{\partial \pi(C, 0; \xi)}{\partial C_L} = \frac{\alpha\xi}{2\{C(1-\xi) + \alpha\}^2},$$

so that $\frac{\partial^2 \pi(C,0;\xi)}{\partial C_L \partial \xi} > 0$. Finally, note that

$$\frac{\partial \pi(C,0;\xi)}{\partial \xi} = \frac{C(C+\alpha)}{2\{C(1-\xi)+\alpha\}^2}.$$

Proof of Lemma 1: Recall that (C,f) must solve the problem

$$\max_{(C_L,f_L)\in\Re^2_+} U(C_L,f_L,C,f;\xi) \ s.t.\ G(C_L,f_L,C,f;\xi) \geq \frac{2C_L}{(1-\gamma)\theta}$$

and that

$$\xi = 1 - H(\beta(1-2x) - \frac{(1-\sigma)(\alpha+C)}{\alpha+C(1-\sigma)}(\delta - f)).$$

Observe first that it must be the case that $\xi > 0$. If not, then $\xi = 0$ which implies that $(C,f) = (0,0)$ and hence that

$$0 = 1 - H(\beta(1-2x) - (1-\sigma)\delta).$$

This is inconsistent with our assumption that $\overline{\varepsilon}$ is larger than $\beta(1-2x)-(1-\sigma)\delta$. It follows that $G(C,f,C,f;\xi) = \frac{2C}{(1-\gamma)\theta}$. If not, then the candidate could ask for a slightly larger contribution and make himself better off, since $\xi > 0$. This proves (8).

Next observe that either $\xi \leq \frac{1}{2}$ or $C < \frac{\alpha}{2\xi-1}$. Suppose to the contrary that $\xi > \frac{1}{2}$ and $C \geq \frac{\alpha}{2\xi-1}$. Then, it is the case that $\pi(C,0;\xi) = 1$. It follows from the fact that $G(C,f,C,f;\xi) = \frac{2C}{(1-\gamma)\theta}$ that $C = \widetilde{C}(f)$ and that $\xi = \widetilde{\xi}(f)$. But, by Assumption 1, we know that for all $f \in [0,\delta]$ either $\widetilde{\xi}(f) \leq \frac{1}{2}$ or $\widetilde{C}(f) < \frac{\alpha}{2\xi-1}$.

It follows from this claim that $U(\cdot,C,f;\xi)$ and $G(\cdot,C,f;\xi)$ are differentiable at (C,f). Thus, by the Kuhn Tucker Theorem, there exists $\mu \geq 0$ such that

$$\frac{\partial U}{\partial C_L} - \mu(\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}) \quad \leq \quad 0 \ (= \text{ if } C > 0) \ \text{(A.1)}$$

$$\text{and} \quad \frac{\partial U}{\partial f_L} + \mu\frac{\partial G}{\partial f_L} \quad \leq \quad 0 \ (= \text{ if } f > 0) \ \text{(A.2)}$$

We can now show that $C > 0$. If not, then $(C, f) = (0, 0)$ and $\xi = 1 - H(\beta(1-2x) - (1-\sigma)\delta)$ which was defined as ξ' in Assumption 2. (A.2) implies that

$$\mu \leq \frac{-\partial U/\partial f_L}{\partial G/\partial f_L}.$$

(A.1) implies that

$$\mu(\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}) \geq \frac{\partial U}{\partial C_L}.$$

Since $\xi' > 0$, $\frac{\partial U}{\partial C_L} > 0$ and hence $\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L} > 0$. Thus, this equation implies that

$$\mu \geq \frac{\partial U/\partial C_L}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}}.$$

This means that

$$\frac{-\partial U/\partial f_L}{\partial G/\partial f_L} \geq \frac{\partial U/\partial C_L}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}}$$

or, equivalently,

$$\Psi(0, 0, 0, 0; \xi') \geq 0.$$

Assumption 2 implies that this inequality cannot hold.

Finally, we can establish (7). Since $C > 0$, (A.1) implies that $\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L} > 0$ and that

$$\mu = \frac{\partial U/\partial C_L}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}}.$$

(A.2) implies that

$$\mu \leq \frac{-\partial U/\partial f_L}{\partial G/\partial f_L} \ (= \text{ if } f > 0)$$

It follows that

$$\frac{\partial U/\partial C_L}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}} \leq \frac{-\partial U/\partial f_L}{\partial G/\partial f_L} \ (= \text{ if } f > 0)$$

Thus,

$$\Psi(C, f, C, f; \xi) \geq 0 \ (= \text{ if } f > 0).$$

QED

Proof of Proposition 1: Let (C, f, ξ) be an equilibrium. From the proof of Lemma 1, we already know that $\xi > 0$, that $C > 0$, and that either $\xi \leq \frac{1}{2}$ or $C < \frac{\alpha}{2\xi - 1}$. These facts together imply that qualified candidates defeat unqualified opponents with a probability between $\frac{1}{2}$ and 1. It remains to show that $f > 0$. Suppose not. Then $f = 0$ and by Lemma 1, $\Psi(C, 0, C, 0; \xi) \geq 0$. It is straightforward to show that for all (C, f, ξ), we have that $\Psi(C, f, C, f; \xi) < 0$ if and only if

$$\sigma \frac{\partial \pi(C, C; \xi)}{\partial C_L}[(b'(f) - 1)r + \beta(1 - 2d)b'(f) + b(f)]$$
$$+(1 - \sigma)\frac{\partial \pi(C, 0; \xi)}{\partial C_L}[(b'(f) - 1)r + (\beta(1 - 2d) + \delta - f)b'(f) + b(f)] > \frac{2}{(1 - \gamma)\theta}.$$

Thus, we must have

$$\sigma \frac{\partial \pi(C, C; \xi)}{\partial C_L}[(b'(0) - 1)r + \beta(1 - 2d)b'(0)]$$
$$+(1 - \sigma)\frac{\partial \pi(C, 0; \xi)}{\partial C_L}[(b'(0) - 1)r + (\beta(1 - 2d) + \delta - f)b'(0)] \leq \frac{2}{(1 - \gamma)\theta}.$$

We know that $\pi(C, 0; \xi) < 1$ and hence $C < \widetilde{C}(0)$. In addition, since $C > 0$,

$$\xi \geq 1 - H(\beta(1 - 2x) - (1 - \sigma)\delta) = \xi'.$$

As shown above, $\frac{\partial \pi(C,C;\xi)}{\partial C_L}$ is a decreasing function of C, $\frac{\partial^2 \pi(C,C;\xi)}{\partial C_L \partial \xi} > 0$ and $\frac{\partial^2 \pi(C,0;\xi)}{\partial C_L \partial \xi} > 0$. Thus, we have that

$$\sigma \frac{\partial \pi(C, C; \xi)}{\partial C_L}[(b'(0) - 1)r + \beta(1 - 2d)b'(0)]$$
$$+(1 - \sigma)\frac{\partial \pi(C, 0; \xi)}{\partial C_L}[(b'(0) - 1)r + (\beta(1 - 2d) + \delta)b'(0)] \geq$$
$$\sigma \frac{\partial \pi(\widetilde{C}(0), \widetilde{C}(0); \xi')}{\partial C_L}[(b'(0) - 1)r + \beta(1 - 2d)b'(0)]$$
$$+(1 - \sigma)\frac{\partial \pi(\widetilde{C}(0), 0; \xi')}{\partial C_L}[(b'(0) - 1)r + (\beta(1 - 2d) + \delta)b'(0)]$$

By Assumption 3, it follows that

$$\sigma \frac{\partial \pi(\widetilde{C}(0), \widetilde{C}(0); \xi')}{\partial C_L}[(b'(0) - 1)r + \beta(1 - 2d)b'(0)]$$
$$+(1 - \sigma)\frac{\partial \pi(\widetilde{C}(0), 0; \xi')}{\partial C_L}[(b'(0) - 1)r + (\beta(1 - 2d) + \delta)b'(0)] > \frac{2}{(1 - \gamma)\theta}.$$

Hence

$$\sigma\frac{\partial\pi(C,C;\xi)}{\partial C_L}[(b'(0)-1)r+\beta(1-2d)b'(0)]$$
$$+(1-\sigma)\frac{\partial\pi(C,0;\xi)}{\partial C_L}[(b'(0)-1)r+(\beta(1-2d)+\delta)b'(0)]>\frac{2}{(1-\gamma)\theta},$$

which is a contradiction. QED

Proof of Proposition 2: We prove the result via a sequence of claims.

Claim 1: Let (C,f,ξ) be an equilibrium, then $C<\overline{C}$ where

$$\overline{C}=\frac{\theta(1-\gamma)}{4}(\beta(1-2d)+\delta)+\frac{\theta(1-\gamma)}{2}(b(\delta)-\delta)((1-\sigma)+\frac{\sigma}{2})$$

Proof: Since $f<\delta$, we know that $b(f)-f<b(\delta)-\delta$, because (by assumption) $b'(\delta)>1$ and b is concave. Using this and Lemma 1, we have that

$$\begin{aligned}\frac{2C}{(1-\gamma)\theta} &= G(C,f,C,f;\xi)\\ &= (\pi(C,0;\xi)-\frac{1}{2})(\beta(1-2d)+\sigma f+(1-\sigma)\delta)\\ &\quad +(b(f)-f)(\frac{\sigma}{2}+(1-\sigma)\pi(C,0;\xi))\\ &\leq \frac{1}{2}(\beta(1-2d)+\delta)+(b(\delta)-\delta)((1-\sigma)+\frac{\sigma}{2}).\end{aligned}$$

Multiplying through by $\frac{\theta(1-\gamma)}{2}$ yields the result.

Claim 2: $\lim_{r\to\infty}\xi(r)=0$.

Proof: We need to show that for all $\varepsilon>0$, there exists r_ε such that if $r\geq r_\varepsilon$ it is the case that $\xi(r)\leq\varepsilon$. Let ε be given. Let r_ε be any value of r satisfying both Assumption 2 and the inequality

$$\sigma\frac{\partial\pi(\overline{C},\overline{C};\varepsilon)}{\partial C_L}[(b'(\delta)-1)r_\varepsilon+\beta(1-2d)b'(\delta)]\geq\frac{2}{(1-\gamma)\theta}.$$

Clearly, such an r_ε exists. Now let $r\geq r_\varepsilon$. By Lemma 1, we know that

$$\Psi(C(r),f(r),C(r),f(r);\xi(r))\geq 0.$$

As pointed out in the proof of Proposition 1, this implies that

$$\sigma\frac{\partial\pi(C(r),C(r);\xi(r))}{\partial C_L}[(b'(f(r))-1)r+b(f(r))+\beta(1-2d)b'(f(r))]\leq\frac{2}{(1-\gamma)\theta}.$$

However, if $\xi(r)>\varepsilon$, then because $\frac{\partial\pi(C,C;\xi)}{\partial C_L}$ is decreasing in C, $b''<0$, and $\frac{\partial^2\pi(C,C;\xi)}{\partial C_L\partial\xi}>0$

$$\begin{aligned}&\sigma\frac{\partial\pi(C(r),C(r);\xi(r))}{\partial C_L}[(b'(f(r))-1)r+b(f(r))+\beta(1-2d)b'(f(r))]\\ >\;&\sigma\frac{\partial\pi(\overline{C},\overline{C};\varepsilon)}{\partial C_L}[(b'(\delta)-1)r_\varepsilon+\beta(1-2d)b'(\delta)]\geq\frac{2}{(1-\gamma)\theta}.\end{aligned}$$

Thus, it must be the case that $\xi(r)\leq\varepsilon$.

Claim 3: There exists $\widehat{\xi}>0$ such that for all $\xi\in[0,\widehat{\xi})$ the pair of equations (6) and (8) have a unique solution $(C_+(\xi),f_+(\xi))$ in the domain $\Re_+\times[0,\delta]$. Moreover, the functions $C_+(\cdot)$ and $f_+(\cdot)$ are continuous on $[0,\widehat{\xi})$.

Proof: This claim may be established graphically by computing the loci of (C,f) combinations satisfying equations (6) and (8). Consider first equation (6). Let $C_o(f;\xi)$ be the level of contributions that qualified candidates must receive to generate an effectiveness of advertising ξ when qualified candidates provide an amount of favors f. Clearly, $C_o(f;\xi)$ will not be defined for all pairs $(f;\xi)$ - for example, there will exist no amount of contributions that will generate a high level of effectiveness when the level of favors is very high. When it is defined, C_o satisfies

$$H(\beta(1-2x)-\frac{(1-\sigma)(\alpha+C_o)}{\alpha+C_o(1-\sigma)}(\delta-f))=1-\xi.$$

Solving this for C_o, we obtain

$$C_o(f;\xi)=\frac{\alpha[\beta(1-2x)-H^{-1}(1-\xi)-(1-\sigma)(\delta-f)]}{(1-\sigma)[\delta-f-(\beta(1-2x)-H^{-1}(1-\xi))]}$$

Thus, for given ξ, $C_o(f;\xi)$ is well-defined for f values between $\min\{0,\delta-\frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}$, and $\delta-\beta(1-2x)+H^{-1}(1-\xi)$. On this interval, $C_o(\cdot;\xi)$ is increasing at an increasing rate, approaching infinity as the level of favors approaches the upper limit of the interval.

Now consider equation (8). Let $C_i(f;\xi)$ be the level of contributions that would make interest groups indifferent between accepting candidates offers when the level of favors promised is f and the effectiveness of advertising is ξ. Formally, C_i is implicitly defined by the equality:

$$G(C_i, f, C_i, f; \xi) = \frac{2C_i}{(1-\gamma)\theta}.$$

Note that there may be two non-negative solutions to this equation when $f = 0$. One solution is always $C = 0$, since the gain from giving no contributions in exchange for no favors is obviously zero. But there will be a positive solution if $\partial G(0,0,0,0;\xi)/\partial C > 0$. We will let $C_i(0;\xi)$ be the positive solution when it exists.

It is possible to explicitly solve for $C_i(f;\xi)$. Let

$$\chi(f;\xi) = \frac{t(f,\xi) + (t(f,\xi)^2 + 4a(\xi)e(f,\xi))^{1/2}}{2a(\xi)},$$

where:

$$\begin{aligned}
a(\xi) &= 4(1-\xi) \\
t(f,\xi) &= (1-\gamma)\theta\{\xi(\beta(1-2d) + \sigma f + (1-\sigma)\delta) + ((1-\xi)\sigma + (1-\sigma))(b(f) - f)\} - 4\alpha \\
e(f,\xi) &= (1-\gamma)\theta\alpha(b(f) - f).
\end{aligned}$$

Then for $f > 0$,

$$C_i(f;\xi) = \min\{\widetilde{C}(f), \chi(f;\xi)\}$$

while for $f = 0$

$$C_i(0;\xi) = \{ \begin{array}{ll} 0 & \text{if} \quad (1-\gamma)\theta\{\xi(\beta(1-2d) + (1-\sigma)\delta)\} \leq 4\alpha \\ & \min\{\widetilde{C}(0), \chi(0;\xi)\} \quad \text{otherwise,} \end{array}$$

where $\widetilde{C}(f)$ is as defined in Section 4.2.

Note that C_i is increasing in f and bounded above on $[0,\delta]$. Note also that when $C_i(f;\xi) = \widetilde{C}(f)$ it is the case that $\pi(C_i, 0; \xi) = 1$. Thus, when $\xi \leq 1/2$, $C_i(f;\xi) = \chi(f;\xi)$. Since

$$\chi(f;0) = \frac{(1-\gamma)\theta\{(b(f) - f)\}}{4},$$

it follows that $C_i(\cdot;\xi)$ is strictly concave on $[0,\delta]$ for sufficiently small ξ.

Given ξ, $(C,f) \in \Re_+ \times [0,\delta]$ is a solution of the pair of equations (6) and (8) if and only if $f \in [\max\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}, \delta - (\beta(1-2x) - H^{-1}(1-\xi)))$, $C = C_o(f,0)$ and $C_i(f,\xi) = C_o(f,\xi)$. To formally explore the existence and uniqueness of a solutions, define the function $\phi(\cdot;\xi)$ as follows:

$$\phi(f;\xi) = C_o(f,\xi) - C_i(f,\xi).$$

Clearly, f is a solution if and only if $\phi(f;\xi) = 0$. We know that $\phi(f;\xi)$ approaches infinity as f approaches $\delta - \beta(1-2x) - H^{-1}(1-\xi)$. Thus, by continuity, there exists a solution if $\phi(f;\xi)$ is negative for $f = \max\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}$. Moreover, if $C_i(\cdot,\xi)$ is strictly concave, then this solution must be unique. To see this assume that $\phi(f;\xi)$ is negative for $f = \max\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}$ and let f_o denote the smallest solution. Since $C_i(\cdot,0)$ is strictly concave and $C_o(\cdot;0)$ is strictly convex it must be that $\frac{\partial\phi(f,\xi)}{\partial f}$ is increasing in f. It follows that $\frac{\partial\phi(f_o,\xi)}{\partial f}$ is positive. If not, then $\frac{\partial\phi(f,\xi)}{\partial f}$ is negative for all $f \in [\max\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}, f_o)$ which implies that $\phi(f_o,\xi)$ is negative. But if $\frac{\partial\phi(f_o,\xi)}{\partial f}$ is positive then it must be the case that $\frac{\partial\phi(f,\xi)}{\partial f}$ is positive for all $f > f_o$ which implies that $\phi(f,\xi)$ is positive. Thus, f_o is the unique solution.

We now claim that for ξ sufficiently small, we have that $\phi(f;\xi)$ is negative for $f = \max\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\}$. When $\xi = 0$, $H^{-1}(1-\xi) = \overline{\varepsilon}$ implying that $0 < \delta - \frac{\beta(1-2x)-H^{-1}(1)}{1-\sigma}$. Thus, for ξ sufficiently small, we have that $\max\{0, \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}\} = \delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma}$. Since $C_i(\delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma};\xi)$ is positive and $C_o(\delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma};\xi) = 0$, it is clear that $\phi(\delta - \frac{\beta(1-2x)-H^{-1}(1-\xi)}{1-\sigma};\xi)$ is negative.

As noted above, for ξ sufficiently small, $C_i(\cdot;\xi)$ is strictly concave on $[0,\delta]$. It therefore follows that for sufficiently small ξ the pair of equations (6) and (8) have a unique solution $(C_+(\xi), f_+(\xi))$ in the domain $\Re_+ \times [0,\delta]$. That these solutions are continuous in ξ follows from the Implicit Function Theorem.

It follows from Claims 2 and 3 that $\lim_{r\to\infty} C(r) = C_+(0)$ and $\lim_{r\to\infty} f(r) = f_+(0)$. From the proof of Claim 3 we know that

$$C_+(0) = C_i(f_+(0), 0) = \frac{(1-\gamma)\theta(b(f_+(0)) - f_+(0))}{4}$$

and that $f_+(0)$ is defined by the equality

$$\frac{(1-\gamma)\theta(b(f_+(0)) - f_+(0))}{4} = \frac{\alpha[\beta(1-2x) - \overline{\varepsilon} - (1-\sigma)(\delta - f_+(0))]}{(1-\sigma)(\delta - f_+(0) - \beta(1-2x) + \overline{\varepsilon})}.$$

The result now follows. QED

Proof of Lemma 2: We know that (L, f) must solve the problem

$$\max_{(C_L, f_L)\in[0,L]\times\Re_+} U(C_L, f_L, L, f; \xi) \ s.t. \ G(C_L, f_L, L, f; \xi) \geq \frac{2C_L}{(1-\gamma)\theta}$$

and that

$$\xi = 1 - H(\beta(1-2x) - \frac{(1-\sigma)(\alpha+L)}{\alpha + L(1-\sigma)}(\delta - f)).$$

Observe first that it must be the case that $\xi > 0$. If not, then $\xi = 0$ which implies that $(L, f) = (0, 0)$ and hence that

$$0 = 1 - H(\beta(1-2x) - (1-\sigma)\delta).$$

This is inconsistent with our assumption that $\overline{\varepsilon}$ is larger than $\beta(1-2x) - (1-\sigma)\delta$.

Next observe that $G(L, f, L, f; \xi) = \frac{2L}{(1-\gamma)\theta}$ if $f > 0$. If not, then the candidate could reduce the amount of favors he promises. This yields (10).

We now claim that either $\xi \leq \frac{1}{2}$ or $L < \frac{\alpha}{2\xi - 1}$. Suppose to the contrary that $\xi > \frac{1}{2}$ and $L \geq \frac{\alpha}{2\xi-1}$. Then, it is the case that $\pi(L, 0; \xi) = 1$. It follows from the fact that $G(L, f, L, f; \xi) \geq \frac{2L}{(1-\gamma)\theta}$ that $L \leq \widetilde{C}(f)$ and that $\xi \leq \widetilde{\xi}(f)$. But, by Assumption 1, we know that for all $f \in [0, \delta]$ either $\widetilde{\xi}(f) \leq \frac{1}{2}$ or $\widetilde{C}(f) < \frac{\alpha}{2\xi - 1}$.

We can now establish (9). If $L = 0$ then $f = 0$ and (9) follows from Assumption 3. Thus, suppose that $L > 0$. If $f = 0$, then we know that $\pi(L, 0; \xi) < 1$ and hence $L < \widetilde{C}(0)$. In addition, since $L > 0$,

$$\xi \geq 1 - H(\beta(1-2x) - (1-\sigma)\delta) = \xi'.$$

As noted earlier, $\frac{\partial \pi(C,C;\xi)}{\partial C_L}$ is a decreasing function of C, $\frac{\partial^2 \pi(C,C;\xi)}{\partial C_L \partial \xi} > 0$ and $\frac{\partial^2 \pi(C,0;\xi)}{\partial C_L \partial \xi} > 0$. Thus, we have that

$$\begin{aligned}
&\sigma \frac{\partial \pi(L,L;\xi)}{\partial C_L}[(b'(0)-1)r + \beta(1-2d)b'(0)] \\
&+(1-\sigma)\frac{\partial \pi(L,0;\xi)}{\partial C_L}[(b'(0)-1)r + (\beta(1-2d)+\delta)b'(0)] \geq \\
&\sigma \frac{\partial \pi(\widetilde{C}(0),\widetilde{C}(0);\xi')}{\partial C_L}[(b'(0)-1)r + \beta(1-2d)b'(0)] \\
&+(1-\sigma)\frac{\partial \pi(\widetilde{C}(0),0;\xi')}{\partial C_L}[(b'(0)-1)r + (\beta(1-2d)+\delta)b'(0)]
\end{aligned}$$

By Assumption 3, it follows that

$$\begin{aligned}
&\sigma \frac{\partial \pi(L,L;\xi)}{\partial C_L}[(b'(0)-1)r + \beta(1-2d)b'(0)] \\
&+(1-\sigma)\frac{\partial \pi(L,0;\xi)}{\partial C_L}[(b'(0)-1)r + (\beta(1-2d)+\delta)b'(0)] > \frac{2}{(1-\gamma)\theta},
\end{aligned}$$

which implies that $\Psi(L,0,L,0;\xi) \leq 0$.

If $f > 0$, then, since $U(\cdot,L,f;\xi)$ and $G(\cdot,L,f;\xi)$ are differentiable at (L,f), there exists $\mu \geq 0$ such that

$$\begin{aligned}
\frac{\partial U}{\partial C_L} - \mu\left(\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}\right) &\geq 0 \text{ (A.3)} \\
\textit{and} \quad \frac{\partial U}{\partial f_L} + \mu \frac{\partial G}{\partial f_L} &= 0 \text{ (A.4)}
\end{aligned}$$

(A.4) implies that

$$\mu = \frac{-\partial U/\partial f_L}{\partial G/\partial f_L}.$$

It follows from (A.3) that

$$\mu \leq \frac{\partial U/\partial C_L}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}}.$$

Thus,

$$\frac{-\partial U/\partial f_L}{\partial G/\partial f_L} \leq \frac{\partial U/\partial C_L}{\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L}}.$$

Multiplying this expression through by $\frac{\partial G/\partial f_L}{\partial U/\partial C_L}$ yields (9). QED

Proof of Lemma 3: Recall that there are three types of citizens: partisans, interest group members, and swing voters. That partisans and swing voters will be strictly better off follows directly from the expressions for their payoffs derived in Section 3.6. Thus, we need only deal with interest group members. The expected payoff of an interest group member is

$$\eta(\pi(C,0;\xi))(\delta - f + \frac{b(f)}{2}) - \frac{\beta(1-2d)}{2} - \frac{2\sigma C}{(1-\gamma)\theta}.$$

In an equilibrium with unrestricted contributions, we know that (8) holds. We can use this to express the expected payoff of an interest group member as:

$$\delta[\sigma^2 + \frac{\sigma(1-\sigma)}{2} + (1-\sigma)\sigma\pi(C,0;\xi))] - f\sigma\pi(C,0;\xi) - \frac{\beta(1-2d)}{2}[1-\sigma+2\sigma\pi(C,0;\xi)]. \quad (A.5)$$

Observe that this payoff is decreasing in the level of favors and in the probability that a qualified candidate defeats an unqualified one. In an equilibrium with limits, then (10) holds and $(A.5)$ is a lower bound on the expected payoff of an interest group member. If imposing a limit moves the community from some status quo (C, f, ξ) to a new equilibrium (C', f', ξ') such that $\pi(C', 0; \xi') \simeq \pi(C, 0; \xi)$ and $f' < f$, then it is clear that interest group members will be better off. QED

Proof of Proposition 3: For all r, let $(C(r), f(r), \xi(r))$ be the equilibrium (or an equilibrium) that would arise with no limits when ego-rents are r. Then, from Proposition 2 we know that $\lim_{r\to\infty} \pi(C(r), 0; \xi(r)) = 1/2$ and that $\lim_{r\to\infty} f(r) > 0$. This implies the result, since banning contributions would eliminate favors and make the probability that a qualified candidate defeats an unqualified one equal 1/2. QED

Assumption 4: Part (i) requires that for all f between $\min\{0, \delta - \frac{\beta(1-2x)+\overline{\varepsilon}}{1-\sigma}\}$ and $\delta - (\beta(1-2x)+\overline{\varepsilon})$ it is the case that $C_i(f;1) < C_o(f;1)$. A sufficient condition is obviously that $\delta \leq (\beta(1-2x)+\overline{\varepsilon})$.

Part (ii) is necessarily satisfied for small ξ. Additional conditions are necessary to ensure that it is concave for higher values of ξ. Specifically, we require that the following expression

$$
\begin{aligned}
&(\frac{\partial^2 G}{\partial f_L^2} + 2\frac{\partial^2 G}{\partial f_L \partial f} + \frac{\partial^2 G}{\partial f^2})(\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L} - \frac{\partial G}{\partial C})^2 \\
&+(\frac{\partial^2 G}{\partial C_L^2} + 2\frac{\partial^2 G}{\partial C_L \partial C} + \frac{\partial^2 G}{\partial C^2})(\frac{\partial G}{\partial f_L} + \frac{\partial G}{\partial f})^2 \\
&+2(\frac{\partial G}{\partial f_L} + \frac{\partial G}{\partial f})(\frac{2}{(1-\gamma)\theta} - \frac{\partial G}{\partial C_L} - \frac{\partial G}{\partial C})(\frac{\partial^2 G}{\partial C_L \partial f_L} + \frac{\partial^2 G}{\partial C_L \partial f} + \frac{\partial^2 G}{\partial C \partial f_L} + \frac{\partial^2 G}{\partial C \partial f})
\end{aligned}
$$

be negative, where the derivatives are evaluated at $(C_L, f_L, C, f, \xi) = (C_i, f, C_i, f, \xi)$. The first two terms are negative but the last is positive. The first two terms must therefore outweigh the first.

Part (iii) requires that

$$\frac{\partial G/\partial \xi}{\frac{2}{\theta(1-\gamma)} - (\frac{\partial G}{\partial C_L} + \frac{\partial G}{\partial C})} < \frac{\sigma\alpha(\delta - f)}{H'(H^{-1}(1-\xi))(1-\sigma)(\delta - f - \beta(1-2x) + H^{-1}(1-\xi))^2}$$

where the derivatives on the left hand side are evaluated at $(C_L, f_L, C, f, \xi) = (C_i, f, C_i, f, \xi)$.

Part (iv) requires that

$$\frac{(\frac{\partial G}{\partial f_L} + \frac{\partial G}{\partial f})}{\frac{2}{\theta(1-\gamma)} - (\frac{\partial G}{\partial C_L} + \frac{\partial G}{\partial C})} < \frac{\sigma\alpha(\beta(1-2x) - H^{-1}(1-\underline{\xi}))}{(1-\sigma)(\delta - \beta(1-2x) + H^{-1}(1-\underline{\xi}))^2}$$

where the derivatives on the left hand side are evaluated at $(C_L, f_L, C, f, \xi) = (C_i, 0, C_i, 0, \underline{\xi})$. Note also that $\underline{\xi}$ is well-defined given the other parts of the assumption. Let $\delta(1-\sigma) = \beta(1-2x) - H^{-1}(1-\widetilde{\xi})$. Then, $C_o(0;\widetilde{\xi}) = 0$. There are two cases: (i) $C_i(0;\widetilde{\xi}) = 0$, and (ii) $C_i(0;\widetilde{\xi}) > 0$. Consider case (i). In that case, for any smaller ξ, $C_o(0;\xi)$ is not defined. For any larger ξ $C_i(0;\xi) < C_o(0;\xi)$ by part (iii). Thus, letting $\underline{\xi} = \widetilde{\xi}$ does the job. Consider case (ii). In that case, on the interval $[\widetilde{\xi}, 1]$ define the function $\phi(\xi) = C_i(0;\xi) - C_o(0;\xi)$. By part (iii) this function is decreasing. By part (i) $\phi(1) < 0$ and, by hypothesis, $\phi(\widetilde{\xi}) > 0$. Thus, there is a unique $\underline{\xi}$ such that $\phi(\underline{\xi}) = 0$.