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Information Acquisition, Noise Trading, and Speculation in Double Auction Markets*

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Abstract

This paper analyzes information acquisition in double auction markets and shows that for any finite information cost, if the number of traders and the units a trader is allowed to trade are sufficiently large, then an efficient equilibrium allocation fails to exist. For a large set of parameter values any equilibrium with positive volume of trade has the following properties. Ex ante identically informed, rational traders evolve endogenously to noise traders, speculators, and defensive traders. Because of defensive trading the allocation is inefficient, i.e. not all gains from trade are realized. Because of endogenous noise trading the price is not fully revealing.

Journal of Economic Literature Classification Numbers: G14, D82, D83

Key words: double auction, endogenous lemons problem, information acquisition, noise trading, speculation

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1. Introduction

A prototype of a centralized market is a large double auction in which traders submit limit orders to buy and sell some units of an asset. The buy and sell orders are ranked according to the bid and ask prices, respectively, which generates an aggregate demand and supply schedule. The market price is set to equalize demand and supply. Two central questions arise. Does such a trading mechanism lead to an equilibrium allocation that is efficient, i.e. do the traders with the highest valuations of the asset obtain the asset? Does such a trading mechanism lead to an equilibrium price that is informationally efficient, i.e. does the price fully reveal the aggregate information of all traders?

For the case where the traders have exogenous private information about the (common) value of the asset as well as private information about their own valuation of the asset, Reny and Perry (2006) provide a positive answer to both questions. They show that a large double auction market is allocative and informationally efficient. The present paper does not assume exogenous private information, but assumes that ex ante all traders have identical information about the uncertain value of the asset and analyzes the implications of endogenous information acquisition for allocative and informational efficiency in small and large double auction markets.

The demand for financial analysts' coverage, rating services, Bloomberg's and Reuters' financial services suggest that information acquisition is a prevalent activity in financial markets. In secondary markets a seller does not necessarily posses better information than a potential buyer but the traders can acquire information about the risky cash flow stream of the asset before they trade. Therefore, the analysis of double auction markets with endogenous information can provide insights into the functioning of real financial markets since a double auction mimics the working of a call market such as the overnight market on the New York Stock Exchange.[1]

This paper shows that for any finite information cost, if the number of traders and the units a trader is allowed to trade are sufficiently large then an efficient equilibrium allocation in double auction markets fails to exist. For a large set of parameter values any equilibrium with positive volume of trade has the following properties. Ex ante identically informed, rational traders evolve endogenously to informed speculators, uninformed defensive traders, and noise traders. Because of defensive trading the allocation is inefficient, i.e. not all gains from trade are realized. Because of endogenous noise trading the price is not fully revealing

[1] Also, the opening price and allocation of the electronic trading system, Xetra and floor trading on Frankfurt Stock Exchange are determined by a double auction type mechanism.

of the traders' aggregate information. This paper provides a strategic foundation for the Grossman and Stiglitz (1980) impossibility result of informationally efficient large (double auction) markets as well as shows that equilibrium allocations in such a market are not efficient if information is endogenous.

In the present model there are high and low valuation traders of a risky asset.[2] It is common knowledge that the asset is worth $v+\Delta$ to a high valuation trader and $v-\Delta$ to a low valuation trader, where Δ is a constant, and v is a random variable and either v_L or v_H. Each trader maximizes his expected payoff. To illustrate the strategic consequences of information acquisition, the paper first analyzes a double auction with two traders.[3] The paper shows that if the information cost is low, a trader is concerned about an endogenous lemons problem. For example, given that a trader submits a price around the expected value E[v] of the asset, then the best response of the other trader is to acquire information and speculate. Trade only occurs with probability one if both traders are informed. The motive of information acquisition is driven by the desire to trade without being exploited.

If the information cost is larger than the total trading surplus but smaller than the potential speculative profit, then no pure strategy equilibrium with efficient trade exists although the traders maintain symmetric information in equilibrium and the gains from trade are common knowledge. This paper shows that an endogenous lemons problem, i.e. the concern of suffering a potential speculative loss due to the mere possibility of information acquisition by the other trader, can already render efficient trade unattractive.[4]

In any mixed strategy equilibrium in which trade occurs with positive probability a trader randomizes between being informed or not. An uninformed trader is sometimes a *defensive trader*, i.e. a high (low) valuation trader only wants to buy (sell) at a price around v_L (v_H). An uninformed trader is sometimes a *noise-type trader*, i.e. he submits an order at a price around E[v], so that he may suffer a lemons problem when trading with an informed

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[2] A low valuation trader is a trader who has low liquidity (that is a need for cash) or hedging reasons to sell. Also, portfolio rebalance needs, tax-induced trades, or dividend-captured trades give rise to mutually beneficial transactions. See the discussion in section 6. If rational agents have the same private or marginal valuation of the asset, then the No-trade Theorem applies. See Milgrom and Stokey (1982).

[3] A small double auction or simultaneous offer bargaining can be interpreted as a model of over-the-counter-trading. Bargaining is a standard feature in many decentralized markets, such as those for mortgage-backed securities, collateral debt obligations, derivatives, corporate and municipal bonds. See Duffie et al. (2005).

[4] This no trade result is different from Myerson and Satterthwaite (1983) because the gains from trade are common knowledge in the present model. This result is also different from Akerlof (1970) and Gresik (1991) since the traders possess symmetric information about the common valuation in the no trade equilibrium.

trader. An informed trader is always a *speculator* and only trades in his "preferred" state, i.e. a high (low) valuation trader only buys (sells) at a price around E[v] if the true state is v_H (v_L). Consequently, trade only occurs under two circumstances: (i) the two traders are noise traders or (ii) one is a noise trader and the other one is informed and the state of nature is the preferred one of the informed trader. Therefore, the price is always around E[v] and not uninformative.

The second part of the paper analyzes a double auction with many high and low valuation traders and shows that for any finite information cost, if the number of traders and the units a trader is allowed to trade are sufficiently large, then an efficient equilibrium allocation fails to exist. In decentralized trading, if the information cost is large, the traders face no potential lemons problem and there exist equilibria where trade occurs with probability one.

But as the number of traders increases, the potential speculative profit of an informed trader increases because there are potentially more uninformed traders to exploit. Therefore, the speculative threat an uninformed trader faces exists not only for low but also for large information costs. In such a case only trading equilibria in mixed strategies exist where some uninformed traders behave like noise traders, some uninformed traders are defensive traders, and some traders become informed speculators. Because of endogenous noise trading the price is not fully revealing and because of defensive trading the allocation is inefficient.

The remainder of the paper is organized as follows. The next section relates this paper to the literature. Section 3 introduces the model. Section 4 analyzes information acquisition in a small double auction. Section 5 analyses information acquisition in large double auctions. Section 6 discusses the assumptions, and section 7 concludes with a discussion of some market microstructure implications. Omitted proofs in the text are given in Appendix.

2. Relation to the Literature

This paper is most closely related to Reny and Perry (2006) who analyze a large limit double auction market where the traders have exogenous private information and show that such a market is allocative and informationally efficient. They provide a strategic foundation for a rational expectations equilibrium (REE) outcome. This paper analyzes information acquisition in double auction markets and shows that if the number of traders and the units a trader is allowed to trade are sufficiently large, then a double auction market is neither allocative nor informationally efficient.

Reny and Perry (2006) have a much more general information and valuation structures. Yet the key economic reason why their result in the double auction stage does not apply to this setting is the following. In the present model private information is endogenous and an informed trader has to cover his information cost. If the price was fully revealing, then some traders would have profitable deviations: (i) An informed trader chooses not to acquire costly information since there is no speculative profit to make. (ii) Since there is no lemons problem, no uninformed trader submits defensive offers. Consequently, some of these traders deviate to noise traders. On the other hand, if there are too many noise traders and very few informed, then an informed trader can move prices and make speculative profits. Therefore, in an equilibrium with positive volume of trade the price is not fully revealing. Because of the potential lemons problem some traders behave defensively and the allocation is not efficient.

This paper provides a strategic foundation for a noisy REE outcome as well as the behavior assumptions in the noisy REE framework which constitutes a workhorse in financial economics. Market microstructure models typically assume that there are three exogenous types of agents: (i) informed traders (speculators), (ii) uninformed traders without real trading motives (such as market makers), and (iii) uninformed traders with real trading motives or different private valuations of the asset (liquidity traders). An assumption in REE models with exogenous noise is that the trading behavior of liquidity traders is inelastic. These agents just want to trade some exogenous units of the asset irrespective of prices. Grossman and Stiglitz (1980), Hellwig (1980), Kyle (1985, 1989), and Glosten and Milgrom (1985) are influential papers that are based on the REE framework with exogenous noise.

Glosten (1989), Admati and Pleiderer (1988), Chowdhry and Kanda (1991), Spiegel and Subrahmanyam (1992) replace the exogenous noise assumption by the assumption that the liquidity traders account for the existence of speculators and adjust their trading behavior to the expected information asymmetry by choosing different amounts or different markets to trade. Yet in these so-called endogenous noise trading models, the liquidity traders do not consider or are not allowed to acquire information.

The key difference between the present model and most papers on information acquisition in financial markets, such as Verrecchia (1982), Jackson (1991), Barlevy and Veronesi (2000), Mendelson and Tunca (2004), and Veldkamp (2006) is that these papers assume that a subset of traders (liquidity traders) is either not maximizing or not allowed to acquire information. A standard feature of REE models with both exogenous and endogenous noise is that the equilibrium price is typically not informationally efficient. Given the noise assumption, there is no meaningful discussion of allocative efficiency in noisy REE models.

The present model assumes that *all* traders can acquire information. In some sense this paper endogenizes both the number k of informed speculators and the number n of uninformed liquidity traders in noisy REE models such as Spiegel and Subrahmanyam (1992). In the present paper there are *N* high valuation traders and *N* low valuation traders. Out of the 2*N* ex ante uninformed traders, in equilibrium *k* traders become informed speculators, and the remaining $2N-k$ traders stay uniformed and are comparable to the *n* liquidity traders in Spiegel and Subrahmanyam (1992). In contrast to their symmetric linear equilibrium in which the *n* liquidity traders all behave identically, some of the uninformed traders in this model become noise traders while others become defensive traders and do not trade. The behavior of the traders and the expected number of the three types of traders are endogenous and depends on the information cost, the severity of the lemons problem, and the gains from trade.[5]

A second important difference between this paper and many market microstructure models concerns the trading environment. In this model there is no market maker who observes the order flow and determines the price. The inefficiency results of the present paper gives rise to the following question. Can the exogenous presence of a fourth type of traders, such as designated market makers who are forbidden to speculate, facilitate allocative and informational efficiency in double auction markets? Section 7 discusses this question and further market microstructure implications.

This paper is also related to the auction literature. Milgrom (1981), Matthews (1984), Hausch and Li (1993), Persico (2000), and Bergemann and Pesendorfer (2007) analyze information acquisition in auctions where only the buyers' side considers information acquisition. The seller is typically non-strategic or noisy and just wants to sell the asset. In contrast, this model assumes that all traders behave strategically and can acquire information. Because of the endogenous lemons problem a strategic buyer (seller) may not want to buy (sell) and forgoes the trading gain. The two-sided strategic behavior (even with exogenous private information) gives rise to some technical difficulties since the random variables or so-called order statistics are not affiliated. Reny and Perry (2006) provide a new technique to solve for an equilibrium in the large limit market.

For the existence of a mixed strategy equilibrium with positive volume of trade in double auctions with exogenous information see Jackson and Swinkels (2005). They establish the proof by introducing a noise trader and show that as the probability for the existence of the noise trader vanishes, there is still some trade. The present paper assumes that information is endogenous and shows that in a trading equilibrium some strategic traders may endogenously

[5] Trueman (1988) and Dow and Gorton (1997) provide a theory of noise trading based on agency considerations in a delegated portfolio management setting. In the present model, all agents trade on their own behalf.

behave like noise traders. Therefore, this paper suggests that an equilibrium with positive volume of trade also always exists in double auctions with endogenous information.

3. The Model

There are *2N* risk neutral traders in a market for a single asset. The first *N* traders are high valuation traders and the traders $N+1$ to $2N$ are low valuation traders. It is common knowledge that the asset is worth $v+\Delta$ to a high valuation trader and $v-\Delta$ to a low valuation trader, where Δ is the private value component and a constant, and v is the uncertain common value component of the asset. v is either v_L or v_H with equal probability and $0<\Delta<\frac{1}{8}(v_H-v_L)$.[6] Each trader has a trading need of one unit. If each of the *N* pairs of high and low valuation traders have traded one unit with each other, then all traders have the same (marginal) valuation of v for the next units bought or sold. The total trading gain is therefore $2N\Delta$.

Section 6 argues that in an economy with one riskless asset and one risky asset, the specific valuation of $v+\Delta$ ($v-\Delta$) can be interpreted as a shortcut for the marginal valuation of a risk averse trader with low (high) endowment of the risky asset. Having hedged their positions and realized the gains from trade, the traders have equalized their marginal valuations and are only willing to pay at most or accept at least v for further units bought or sold. An alternative story is dividend and tax-motivated trade. A trader facing a low (high) dividend tax rate has a high (low) valuation of the asset. 2Δ represents the tax gains. A high valuation trader is willing to pay $v+\Delta$, while a low valuation trader is willing to accept $v-\Delta$. Having exhausted the tax gains, the traders have a valuation of v for the next units.

The sequence of moves is as follows. In the first stage, a trader can obtain a perfect signal about the true value of the asset by incurring the cost $c>0$. Information acquisition is *not* observable by the other traders. In the second stage the traders play a double auction, i.e. they submit limit orders to buy and sell up to m units of the asset. Short selling is allowed. The exact trading, allocation and pricing rule is specified in the subsequent sections.

[6] This paper assumes that the private valuations are common knowledge. Otherwise, one also has to deal with strategic rent extraction discussed in Chatterjee and Samuelson (1983) and the well-known Myerson and Satterthwaite (1983) problem. This further uncertainty may cause additional allocative inefficiencies. The focus here is on common value uncertainty which might be more important than private value uncertainty in financial transactions. The assumption that $\Delta<\frac{1}{8}(v_H-v_L)$ makes the problem interesting. If the trading gain Δ is large, the potential lemons problem may have no adverse allocative consequences which will become clear in the analysis.

A pure strategy of trader i is denoted with $t_i = (n_i, (b_i, u_i^b), (s_i, u_i^s))$ where $n_i \in \{0,1\}$ denotes information acquisition, b_i (s_i) is the bid (ask) price, and u_i^b (u_i^s) the size of the buy (sell) order. If trader i chooses n_i=0 and is uninformed, then $b_i(n_i) \in R_+$ and $u_i^b(n_i) \in \{0,..,m\}$. For n_i=1, then $b_i(n_i)=(b_{iL}, b_{iH})$ and $u_i^b(n_i)=(u_{iL}^b, u_{iH}^b)$ are vectors with two components, where $b_{iL}, b_{iH} \in R_+$ and $u_{iL}^b, u_{iH}^b \in \{0,..,m\}$ and denote the bid price and the size of the buy order of trader i when the true value of the asset is v_L and v_H. Analogously for $s_i(n_i)$ and $u_i^s(n_i)$. A mixed strategy is a probability distribution over pure strategies and denoted with σ_i.

The following examples illustrate this notation.[7] (i) n_i=0, b_i=v_L, u_i^b=1, s_i=u_i^s=0 is a pure strategy where trader i does not acquire information and submits a bid price of v_L to buy one unit and no sell order. (ii) n_i=1, b_i=(v_L,E[v]), u_i^b=(1,m), s_i=u_i^s=(0,0) is a pure strategy where trader i acquires information, submits a bid price of v_L to buy one unit if v=v_L and a bid price of E[v] to buy m units if v=v_H, and no sell order. (iii) A mixed strategy is e.g. a randomization that puts probability 0.4 on the pure strategy (i), probability 0.6 on the pure strategy (ii), and zero probability on any other pure strategy.

Since information acquisition is not observable, the solution concept is Bayesian Nash equilibrium (BNE). A BNE in pure strategies in this game is a profile $\{t_i^*\}_{i=1}^{2N}$, such that $EU^i(t_i^*, t_{-i}^*) \geq EU^i(t_i, t_{-i}^*)$, for all potential pure strategies t_i of trader i where i=1,..,2N. A BNE in mixed strategies is a profile $\{\sigma_i^*\}_{i=1}^{2N}$ of probability distributions over pure strategies, such that $EU^i(\sigma_i^*, \sigma_{-i}^*) \geq EU^i(\sigma_i, \sigma_{-i}^*)$, for all potential probability distributions σ_i of trader i where i=1,..,2N. Equilibrium always refers to a BNE.

This paper discusses two types of efficiencies: allocative efficiency and informational efficiency. (i) Allocative efficiency has two notions in this model. An allocation is efficient if all low and high valuation traders trade with each other and the total trading gain, $2N\Delta$ is realized. The (overall) outcome is socially efficient if the trading gain $2N\Delta$ is realized *and* no risk neutral agent acquires (socially useless) information. The focus of the paper is on the first

[7] It is assumed that each trader has a trading need of one unit. However, in a large market if a trader becomes informed, he may want to speculate and try to exploit uninformed traders by trading a lot of units. Two allow for this possibility, a trader can trade up to m units in this model. One result of this paper shows that once a trader (or initial hedger) is informed he speculates rather than hedging his position.

notion of allocative efficiency.[8] (ii) The price is informationally efficient or fully revealing if it reflects the joint information of the traders.

Remark 1

There always exist no trade equilibria. A set of no-trade equilibria is given by the following strategies: No trader acquires information and all high valuation traders (potential buyers) only choose to buy at very low bid prices (e.g. $b \leq v_L - \Delta$), while all low valuation traders (potential sellers) only choose to sell at very high ask prices (e.g. $s \geq v_H + \Delta$).

4. Information Acquisition in a Small Double Auction

This section analyzes the two trader (N=1) case. In order to find a trading equilibrium, one can focus on strategies where the low valuation traders (potential seller) only submits an ask price to sell one unit and the high valuation trader (potential buyer) only submits a bid price to buy one unit. To save on notation, in this section a pure strategy of the buyer and seller is just denoted with $t_B=(n_B,b)$ and $t_S=(n_S,s)$, respectively.

Having made their information acquisition decision, $n_i \in \{0,1\}$, the buyer submits a bid price b and the seller submits an ask price s simultaneously (for trading one unit). If $b \geq s$, then the asset is traded at the price $p=\frac{1}{2}(b+s)$, the surplus 2Δ is realized, $U^B=v+\Delta-p$ and $U^S=p-(v-\Delta)$. Otherwise no trade occurs and the payoffs are normalized to zero. If information is acquired, the information cost c is subtracted from the payoff.

The efficient outcome is trade with probability one and without costly information acquisition. If both traders are uninformed, then the set of mutually acceptable prices is $p \in [E[v]-\Delta,\ E[v]+\Delta]$. Therefore, a set of potentially best responses without information acquisition is $(0,b)$ and $(0,s)$ with $b=s=E[v]+k$ and $k \in [-\Delta,\ \Delta]$. In such a (*k-sharing*) outcome the buyer gets $EU^B=\Delta-k$, and the seller gets $EU^S=\Delta+k$.

When do these strategies constitute best responses? Given the above strategy $(0,s)$ of the seller, suppose the buyer acquires information and speculates. In state v_L he chooses a bid price $b_L<s$ and no trade occurs. In state v_H he chooses $b_H=s$ and makes some speculative profits since he pays less than the true value of the asset. This response yields $EU^B= \frac{1}{2}[(v_H+\Delta)-(E[v]+k)]-c=\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c$. So if $\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c>\Delta-k$, speculation is the best response, and the seller suffers an endogenous lemons problem since $EU^S=\frac{1}{2}(\Delta+k)$

[8] If information is bought from an information provider, then the payment of information cost is only a transfer.

$-\frac{1}{4}(v_H-v_L)<0$. (It is assumed that $\Delta<\frac{1}{8}(v_H-v_L)$.) Analogously, if $\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta+k)-c>\Delta+k$, the seller's best response to $(0,b)$ with $b=E[v]+k$ is to choose $(1,s_L,s_H)$ with $s_L=b$ and $s_H>b$. Consequently, a k-sharing outcome without information acquisition can only be established as a BNE in pure strategies, if $c>max\{\pi-\frac{1}{2}(\Delta-k), \pi-\frac{1}{2}(\Delta+k)\}=\pi-\frac{1}{2}(\Delta-|k|)$ where $\pi\equiv\frac{1}{4}(v_H-v_L)$.[9]

However, if $c<\pi-\frac{1}{2}(\Delta-|k|)$, then no efficient k-sharing BNE in pure strategies exists. This condition has a simple economic interpretation. π is the expected speculative profit an informed trader makes and $\frac{1}{2}(\Delta-|k|)$ is the expected opportunity cost of speculation. If the buyer acquires information and speculates, he does not trade in state v_L and he forgoes the surplus $(\Delta-k)$ with probability 0.5. If the seller speculates, his opportunity cost of speculation is $\frac{1}{2}(\Delta+k)$. So if the information cost is smaller than the speculative profit net the opportunity cost of speculation, then trade at a price $p=E[v]+k$ with $k\in[-\Delta, \Delta]$ is not an equilibrium outcome since one trader has an incentive to speculate. The next proposition characterizes for the full range of information costs, when a pure strategy BNE with trade exists.[10]

Proposition 1

(a) If $c\geq\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, there exist a set of pure strategy BNE where trade occurs with probability 1 and where no information is acquired.

(b) If $\Delta<c<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, there exists no pure strategy BNE with positive probability of trade.

(c) If $c\leq\Delta$, there exist pure strategy BNE where trade occurs with probability 1. In any such BNE both traders acquire information and the price fully reveals the traders' information (to a third party).

Proof

(a) See the analysis above.

(b) It remains to show that there is also no pure strategy BNE with one-sided or two-sided information acquisition. It is easy to see that if $c>\Delta$, then no pure strategy equilibrium exists

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[9] If $c\geq\frac{1}{4}(v_H-v_L)$, any k-sharing outcome is attainable as a BNE. As in a "standard" double auction, a continuum of trading equilibria exists. The set of efficient equilibria "shrinks" as the information cost decreases. If $c=\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, only the equal-split ($k$=0) outcome is attainable as an efficient equilibrium, i.e. the efficient BNE is unique.

[10] Note, the assumption $\Delta<\frac{1}{8}(v_H-v_L)$ implies that $\Delta<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$.

in which both traders acquire information. Suppose only the seller acquires information. The assumption $\Delta<\frac{1}{8}(v_H-v_L)$ implies that $v_H-\Delta>E[v]\pm\Delta>v_L+\Delta$. A standard lemons argument shows that given the seller is informed, the best response of an uninformed buyer is to offer at most $v_L+\Delta$. Trade only occurs in state v_L, and the seller's payoff is at most $EU^S=\Delta-c<0$. In such a case, no trader acquires too expensive and non-exploitable information, but because of the endogenous lemons problem the buyer proposes $b\leq v_L+\Delta$ and the seller proposes $s\geq v_H-\Delta$. So no pure strategy BNE with trade exists.
(c) See Appendix.

Proposition 1(c) shows that if the information cost is low, the traders face an information acquisition dilemma. Since the information cost can be covered by the trading gains, the desire of the agents to trade without being exploited induces both traders to acquire information. The price fully reveals the two traders' information (to a third party).[11]

Remark 2

If $c\leq\Delta$, there also exist pure strategy equilibria in which trade occurs with probability 0.5. In any such BNE one trader acquires information and the uninformed trader accounts for the lemons problem. When trade occurs the price is fully revealing.

Proposition 1 (b) shows that if the information cost is in an intermediate range, then no pure strategy BNE with trade exists. This inefficiency result is different from Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983) since the trading gains are common knowledge. It is also different from Akerlof (1970) and Gresik (1991) since there is no asymmetric information about the common valuation in equilibrium. The concern about a potential lemons problem due to the mere possibility of information acquisition by the other trader can cause no trade. Dang (2008) shows that this result also holds in ultimatum and alternating offer bargaining. Before proceeding to the analysis of mixed strategy equilibria the following terms are defined.

Definition

(i) A trader plays a *defensive strategy*, if he chooses $(0,b)$ with $b\leq v_L+\Delta$ or $(0,s)$ with $s\geq v_h-\Delta$. Such a trader is called a *defensive trader*.

[11] Jackson (1991) shows that with imperfect competition fully revealing prices exist despite costly information. In his model the seller is noisy, i.e. his behavior is insensitive to prices.

(ii) A trader plays a *noise-type strategy*, if he chooses $(0,b)$ or $(0,s)$ with $b,s \in [E[v]-\Delta, E[v]+\Delta]$. Such a trader is called a *noise trader*.

(iii) A trader plays a *speculative strategy*, if he chooses $(1,b_L,b_H)$ with $b_L \leq v_L+\Delta$ and $b_H \in [E[v]-\Delta, E[v]+\Delta]$ or $(1,s_L,s_H)$ with $s_L \in [E[v]-\Delta, E[v]+\Delta]$ and $s_H \geq v_H-\Delta$. Such a trader is called an *informed speculator*.

In other words, a trader is called a *defensive trader* if he is uninformed and his offer accounts for the potential lemons problem. A trader is called a *noise trader* if he is uninformed and proposes a price around the expected value of the asset. A trader is called an *informed speculator* if he only buys (sells) at a price around E[v] when the true state is v_H (v_L). The next proposition shows that depending on the outcome of the equilibrium randomization, a trader may become a noise trader, a defensive trader, or an informed speculator.

Proposition 2

Suppose $\Delta < c < \frac{1}{4}(v_H - v_L) - \frac{1}{2}\Delta$.

(a) In any mixed strategy BNE, in which trade occurs with positive probability, the traders randomize over defensive strategies, noise-type strategies, and speculative strategies.

(b) The outcome in any such BNE has the following properties. (i) Trade does not occur if both traders are informed, or at least one trader is a defensive trader. (ii) The price is not fully revealing. (iii) Both traders have zero expected payoffs.

The following example highlights the intuition behind Proposition 2. Suppose the traders are only allowed to choose three offer prices, namely $b,s \in \{v_L, E[v], v_H\}$. Appendix shows that under *this* assumption in the unique (non-degenerated) mixed strategy equilibrium the buyer randomizes over the strategies $(0,v_L)$, $(0,E[v])$ and $(1,v_L,E[v])$. The seller randomizes over the strategies $(0,v_H)$, $(0,E[v])$ and $(1,E[v],v_H)$. Trade only occurs in the following circumstances. (i) Both traders do not acquire information and choose E[v]. (ii) The seller is uninformed and chooses $s=E[v]$ and the buyer is informed and the true state is v_H. (iii) The buyer is uninformed and chooses $b=E[v]$ and the seller is informed and the true state is v_L. The probability of trade is $\frac{16c^2}{(v_H-v_L)^2-4\Delta^2}$, and the price is $p=E[v]$ and not fully revealing.

(a) Why is there no trade if both traders are informed?[12] Equivalently, why does an informed buyer never chooses $b=v_H$ in state v_H, i.e. why does he choose $(1,v_L,v_H)$ with

[12] This is in contrast to Proposition 1(c) where trade only occurs with probability 1, if both traders are informed.

probability zero? If he chooses $b=v_H$ in state v_H then trade also occurs in the event where the seller is informed since an informed seller choose $s=v_H$ in state v_H. The "problem" is that if the buyer is indifferent between the strategies $(1,v_L,E[v])$ and $(1,v_L,v_H)$, then both strategies with information acquisition is strictly dominated by the strategy $(0,v_L)$. If the informed buyer trades at the price v_H in state v_H, then his expected payoff net information cost is negative. So not acquiring information would be a best response. In other words, if a trader acquires information in a mixed strategy equilibrium, he speculates since he expects to meet a noise trader with positive probability.

(b) In the mixed strategy equilibrium an uninformed trader proposes the offer price $E[v]$ with positive probability. In other words, he proposes an offer which is prone to speculation and may suffer a speculative loss. Although his behavior exhibits noise trading, his equilibrium payoff is non-negative since he meets an uninformed trader with positive probability. In such a case he realizes the trading gain without suffering a speculative loss.

(c) Why is the equilibrium payoff non-positive? Put it differently, since the minimum price the seller demands is $s=E[v]$, why does the buyer choose $(0,v_L)$ with positive probability? In order to make the seller indifferent between acquiring and not acquiring information, an uninformed buyer chooses not to trade (i.e. bids v_L) frequently enough so as to discourage too "much" information acquisition by the seller. Since $U^B(0,v_L)=0$ and the buyer is indifferent between this and other strategies, his expected payoff is zero.

(d) In contrast to Proposition 1(c), why is the price not fully revealing? There are three reasons. (i) There is no trade between two informed traders. (ii) There is no trade if one trader plays a defensive strategy. (iii) Suppose the seller does not acquire information and observes trade at $p=E[v]$. In this case he does not know whether the buyer has chosen $(0,E[v])$ or $(1,v_L,E[v])$. Although his posterior belief for $v=v_H$ increases, it is strictly below one. Otherwise he would know for sure that he has made a bad deal and this cannot be an equilibrium outcome.

(e) The probability that a trader becomes a defensive trader, a noise trader, and an informed speculator is given as follows: $1-\frac{4c}{v_H-v_L-2\Delta}$, $\frac{4c}{v_H-v_L+2\Delta}$, $\frac{16c^2}{(v_H-v_L)^2-4\Delta^2}$.

(f) The difference v_H-v_L captures the riskiness of the asset and the importance of the endogenous lemons problem. As the asset becomes more uncertain, this exerts two effects. (i) The information cost range which implies no pure strategy trading equilibrium increases, i.e. even for high information costs there is an endogenous lemons problem and no trade may occur. (ii) The equilibrium probability of trade $\frac{16c^2}{(v_H-v_L)^2-4\Delta^2}$ decreases because the probability that an uninformed trader chooses the offer price $E[v]$ decreases.

5. Information Acquisition in Large Double Auctions

This section analyzes the $2N$ trader case where $N>1$. Each trader can submit one bid price to buy up to m units of the asset as well as one ask price to sell up to m units. The buy and sell orders of the traders are ranked according to the bid and ask prices, respectively. This generates an aggregate demand and supply schedule. The market price is set to equalize aggregate demand and supply. (i) If there are multiple-market clearing prices, the mean price of these prices is chosen. (ii) If there is excess demand (supply) at the market clearing price, the orders with the highest bid prices (lowest ask price) are executed first. The remaining units are allocated with equal probability to the traders who propose the same offer price. This type of trading rules is adopted from Reny and Perry (2006, section 4.1).

A natural question is whether enlarging the size of the market can mitigate the inefficiencies in the small double auction. This section shows that a common market place does not resolve the inefficiencies. In contrast, the next Lemma shows that a large double auction market performs worse in the following sense. In the small (N=1) double auction, there exists a critical information cost $c^{\text{crit}}=\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, such that if $c\geq c^{\text{crit}}$, then a trading equilibrium without information acquisition exists. As both the number N of traders and the units m a trader is allowed to trade increase, c^{crit} also increases. In other words, in a large market a trading equilibrium without information acquisition fails to exist even though the information cost is large.

Lemma

Suppose $N>1$. Define $q=\min[m,N]$ and $c^{\text{crit}}=\frac{1}{2}q(v_H-v_L)-\frac{1}{2}\Delta$. If $c<c^{\text{crit}}$, then there exists no pure strategy BNE with trade and where no information is acquired.

Proof

Case 1: q=1. The proof for q=1 follows almost directly from Proposition 1. Suppose the H-traders (L-traders) only submit buy (sell) orders and the offer price profiles $B=(b_1,\ldots,b_N)$ and $S=(s_{N+1},\ldots,s_{2N})$ yield $p=E[v]$. This price is less prone to speculation because the opportunity cost of speculation is highest for *both* the L- and H-traders. Since there are additional traders, an informed trader can potentially make more profit in a large market although he can only buy or sell one unit. For example, an informed L-trader becomes a speculator in state v_H and buys one unit of the asset which is not possible in a small (N=1) double auction. In state v_L, as in a small double auction, he sells one unit of the asset. Therefore, his expected payoff with information acquisition is $\frac{1}{2}(v_H-v_L)+\frac{1}{2}\Delta-c$. If

$c<\frac{1}{2}(v_H-v_L)-\frac{1}{2}\Delta$, this strategy dominates trade at $p=E[v]$ with expected payoff Δ, and no pure strategy trading BNE without information acquisition exists.

Case 2: $q>1$. The proof is based on three arguments. (a) If a pure strategy trading BNE without information acquisition exists, then N units are traded, i.e. all traders are "satisfied". (b) If a pure strategy trading BNE without information acquisition exists, then trade is executed at the price $p=E[v]$. (c) No pure strategy trading BNE without information acquisition exists where trade is executed at the price $p=E[v]$.

The following arguments prove claim (a). Suppose all traders do not acquire information, and each H-trader submits an order to buy one unit and no sell orders, while each L-trader submits an order to sell one unit and no buy orders. Suppose the bid and ask profiles $B=(b_1,\ldots,b_N)$ and $S=(s_{N+1},\ldots,s_{2N})$ yield a market clearing price, $p\in(E[v]-\Delta,\ E[v]+\Delta)$. Suppose $b_i<p$ and H-trader i does not get the asset. Given (B,S), H-trader i can do better by choosing $b_i\geq p$ and gets one unit with positive probability and $EU>0$. Any H-trader or L-trader who does not get to buy or sell one unit of the asset at the resulting price, has not played a best response. (If $p=E[v]-\Delta$, "unsatisfied" H-trader will deviate. If $p=E[v]+\Delta$, "unsatisfied" L-trader will deviate.) This reasoning implies that if a trading equilibrium without information acquisition exists, then all traders are "satisfied", i.e. N units are traded. Therefore, a candidate offer profile (B,S) for being part of a pure strategy BNE must have $b_i\geq p$ and $s_j\leq p$ for $i=1,..,N$ and $j=1+N,..,2N$, where p is the resulting market price given (B,S).

The proof of claim (b) is as follows. Suppose each trader trades one unit and the bid ask profile (B,S) gives rise to the price $p\in[E[v]-\Delta,E[v]+\Delta]$. Case (i) Suppose $p<E[v]$. Consider H-trader i who chooses b_i and gets one unit of the asset. For example, a profitable deviation is to choose the same offer price but submits an order of q units. The expected payoff is $EU=(E[v]+\Delta-p)+prob(\text{trade})(q-1)(E[v]-p)>\Delta$. Therefore, if $p<E[v]$, then a H-trader who gets one unit has not played a best response. In some sense there is incentive to buy more and overbid the other buyers. (ii) If $p>E[v]$, then a L-trader who sells one unit has not played a best response. There is incentive to sell more and underbid the other sellers. Consequently, only if $p=E[v]$, then an uninformed trader who trades one unit has no profitable deviation.

The proof of claim (c) is similar to the proof of Proposition 1. Suppose that no trader acquires information and the bid ask profile (B,S) yields the market price $p=E[v]$ and all traders trade one unit each. Then some traders have a profitable deviation. For example, H-trader i acquires information. In state v_H, he chooses $b_i=E[v]+\gamma_b$ to buy q units where γ_b is chosen such that b_i is larger than the q-th highest bid prices given $B=(b_1,..,b_N)$. The (additional) demand of H-trader i may increase the market clearing price from $p=E[v]$ to at most $p=E[v]+\gamma_b$. Suppose $p=E[v]+\gamma_b$. His payoff in this state is $(v_H+\Delta)+(q-1)v_H-qp-c = \frac{1}{2}q(v_H-v_L-2\gamma_b)+\Delta-c$.[13] In state v_L, he forgoes the surplus Δ and chooses $b_i=E[v]-\gamma_s$ to sell (short) q units where γ_s is chosen such that s_i is smaller than the q-th lowest ask prices given $S=(s_{N+1},..,s_{2N})$. Suppose $p=E[v]-\gamma_s$. His payoff in this state is $\frac{1}{2}q(v_H-v_L-2\gamma_s)-c$.

Since $q\leq N$, the price impact is zero in the following sense. For a given bid ask profile (B,S) and the market clearing price $p=E[v]$, there always exists the same market clearing price such that H-trader i gets to buy or sell q units.[14] Therefore, the expected payoff of H-trader i with information acquisition is $EU_i^B=\frac{1}{2}q(v_H-v_L)+\frac{1}{2}\Delta-c$. Speculation is the best response if $\frac{1}{2}q(v_H-v_L)+\frac{1}{2}\Delta-c>\Delta$. Analogously for a L-trader. Consequently, if $c<\frac{1}{2}q(v_H-v_L)-\frac{1}{2}\Delta$, there exists no pure strategy trading equilibrium without information acquisition. **QED**

Proposition 3

For any finite information cost c, there exists an integer q^*, such that if $\min[m,N]\geq q^*$, then (i) an efficient equilibrium allocation fails to exist and (ii) the price is typically (i.e. for all $c\neq q\Delta$) not fully revealing.

A notion of a competitive or close to competitive market is that there are many traders and a trader can trade as many units of the asset as he likes without having much price impact. Proposition 3 establishes the main result of the paper and states that if information is endogenous and costly, there exists no efficient equilibrium allocation in such large double auction markets. More precisely, for any finite information cost, if the both the number of traders and the units a trader is allowed to trade are sufficiently large, then there is no equilibrium in a double auction in which the total trading gains of $2N\Delta$ are realized.

In a large market there are additional incentives effects that are not present in the small double auction. In contrast, to Proposition 1(c) which shows that if the information cost is low, then there exists a pure strategy BNE in which the trading gain 2Δ is realized. In a large market, even for low information costs, there exists no BNE where the full trading gains are realized. The reason is free-riding. In a bilateral double auction an uninformed trader must account for the lemons problem and this reduces the probability of trade. If he wants trade to

[13] The H-trader's valuation for the first unit bought is $v+\Delta$ and v for the other units.

[14] Consider the two extreme cases. (i) All L-traders choose $E[v]-\Delta$ and all H-traders choose $E[v]+\Delta$ and the price is $E[v]$. If the true value is high, an informed H-trader chooses $E[v]+\Delta+\varepsilon$ and buys q units. For $q<N$, there exists a market clearing price $p=E[v]$. (ii) All $b=s=E[v]$. If a H-trader chooses $E[v]+\Delta+\varepsilon$, a market price $p=E[v]$ exists.

occur with probability one, he "must" become informed if the other trader is informed. Proposition 1(c) shows that the price is fully revealing any such trading equilibrium. But in a large market where there is a single price for all transactions and if that price is fully revealing, there is free riding, i.e. some traders do not want to acquire costly information.

Proposition 1(a) shows that if $c \geq \frac{1}{4}(v_H - v_L) - \frac{1}{2}\Delta$, then an efficient allocation exists. In a large market there are potentially more uninformed traders to exploit. Therefore, even if the information cost is large, some traders have an incentive to speculate so that uninformed traders are concerned about the endogenous lemons problem. This mere concerns suffices to "destroy" the efficient equilibrium allocation. For any finite information cost, if the number of traders submitting orders around the expected value of the asset is large, some traders have an incentive to speculate.

Free riding and the increased incentives to speculate (or the increased concern about the potential lemons problem) are reinforcing reasons why for any finite information cost, there exists no pure strategy equilibrium if the market is sufficiently large. Suppose that some traders are informed and the price is fully revealing, then both an informed trader and a defensive trader have profitable deviations. (i) An informed trader chooses not to acquire costly information since there is no speculative profits to make. (ii) Since there is no lemons problem, no uninformed trader submits defensive offers. Consequently, some of these traders deviate to noise traders. On the other hand, if there are a lot of noise traders and very few informed, then an informed trader can move prices and make speculative profits. Therefore, a fully revealing price is typically (i.e. for all $c \neq q\Delta$) not consistent with equilibrium behavior.

Technically speaking, any profile of pure strategies leads to a market clearing price that is fully revealing. Proposition 3(ii) is a strategic version of Grossman and Stiglitz (1980) impossibility result of informationally efficient (double auction) markets. In addition, since the price is not fully revealing, there is a potential lemons problem, some traders behave defensively and the allocation is inefficient. The next result restates the non-existence of an efficient equilibrium in term of the riskiness of the asset.

Corollary

For any given set of parameter values (c, N, m, Δ), if the asset is sufficiently risky (i.e. $v_H - v_L$ is large), then no equilibrium exists in which the allocation is efficient.

6. Discussion

Risk aversion

The following arguments show that risk neutrality is not a crucial assumption for all qualitative implications of the paper. Suppose there is a riskless asset S, and a risky asset R, and the agents have concave utility functions u and endowments $\omega=(\omega_S,\omega_R)$ of the assets. In general, if $\omega_{iS}>\omega_{jS}$ and $\omega_{iR}<\omega_{jR}$, then agent i and j can realize gains from trade by reallocation of risks because agent i has a higher marginal valuation of asset R than agent j. To simplify the analysis this paper assumes that agent i and j have a valuation of $v+\Delta$ and $v-\Delta$ for the first unit of asset R traded.[15]

The strategic incentive of information acquisition under risk neutrality also arises under risk aversion because a risk averse agent also has to evaluate the gains from trade (or hedging of risks), the potential speculative loss of being uninformed, as well as the potential speculative gains from being informed. In particular, if the agents are risk averse, the information cost is low, and the signal perfectly reveals the true value of the asset, then is no trade at all because perfect information prevents hedging of risks. So the Proposition 1(c) ceases to exists.[16]

Many heterogenous agents

Suppose there are three types of traders with valuations $u_L=v-\Delta$, $u_M=v$, and $u_H=v+\Delta$. If agent L and M trade with each other, then the total trading gain is Δ and none of the qualitative implications change. For example, Proposition 1(b) would state that if $\frac{1}{2}\Delta<c<\frac{1}{4}(v_H-v_L)-\frac{1}{4}\Delta$, then no pure strategy BNE with trade between agent L and M exists. Also, in a large market any of these agents can become a speculator.

Costless Information

If $c=0$, then there exists a BNE with the following properties. All traders acquire information and trade one unit each. (i) If $q=1$, the equilibrium price is $p\in[v-\Delta,v+\Delta]$. (ii) If $q>1$, then the equilibrium price is $p=v$.

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[15] For example, the liquidity traders in Mendelson and Tunca (2004) have a similar utility function. Duffee et al. (2005) assume that the traders have high or low valuations of the form: $v\cdot\Delta_H$ and $v\cdot\Delta_L$ where $\Delta_H>\Delta_L$.

[16] Similarly, if the signal is noisy, this only changes the expected speculative profit and the critical value of the information cost for the different types of equilibria to arise, but not the qualitative implications.

Proof: (i) Suppose $k \in [-\Delta, \Delta]$. It is easy to see that the following strategy profile constitute a BNE: All H-traders choose n=1, $b=(v_L+k,\ v_H+k)$, $u^b=(1,1)$, $s=u^s=(0,0)$; and all L-traders choose n=1, $b=u^b=(0,0)$, $s=(v_L+k,\ v_H+k)$, $u^s=(1,1)$. The price is $p=v+k$ and $EU^B=\Delta-k$ and $EU^S=\Delta+k$. (ii) For $q>1$, in the unique pure strategy trading BNE the H-traders choose n=1, $b=(v_L,v_H)$, $u^b=(1,1)$, $s=u^s=(0,0)$; the L-traders choose n=1, $b=u^b=(0,0)$, $s=(v_L,\ v_H)$, $u^s=(1,1)$. The outcome is $p=v$ and $EU^B=EU^S=\Delta$. Note if $p\neq v$, then a trader may want to trade more units. There is overbidding and underbidding which can not be an equilibrium.[17]

Observability of information acquisitions

The next proposition shows that if information acquisitions are observable prior to the trading stage, then endogenous information acquisition has no adverse allocative consequences.

Proposition 4

Suppose N=1, information acquisition is observable, and $k \in [-\Delta,\ \Delta]$. Any efficient k-sharing outcome is attainable as a perfect BNE irrespective of information cost.

The intuition is as follows.[18] Since information acquisition is observable, a trader can also condition his offer strategy on the fact whether the other party is better informed or not. Suppose ex ante the traders "agree" to trade at $p=E[v]$, but the buyer acquires information. In the auction stage the seller knows that the buyer is informed. So the seller does not submit the price $s=E[v]$ anymore, but demands a high price. Since the buyer anticipates the lemons problem he himself creates by acquiring information, his best response is not to acquire information. No trader has an incentive to acquire more information than the counter party. However, if information acquisition is not observable, the traders cannot target their offer strategies appropriately and are concerned about the endogenous lemons problem. Private information acquisition is unlikely to be publicly observable in a large market.

[17] If there is asymmetric demand and supply, then the agents on the short side of the market capture the full surplus. Suppose there are more H-traders than L-traders, then the unique equilibrium price is $p=v+\Delta$.

[18] Formally, it is easy to show that for $k \in [-\Delta,\Delta]$, $r \leq k$, and $z \geq k$, the strategies $t_B=(0,b)$ with $b=E[v]+k$ if $n_S=0$, and $b=v_L+r$ if $n_S=1$; and $t_S=(0,s)$ with $s=E[v]+k$ if $n_B=0$, and $s=v_H+z$ if $n_B=1$ constitute a perfect BNE with $EU^B=\Delta-k$ and $EU^S=\Delta+k$. Analogously, one can show that for $N>1$, a trading equilibrium exists in which no trader acquires information, and all traders chooses $E[v]$ in the auction stage if no trader has acquired information. Otherwise the uninformed traders choose a defensive strategy.

7. Conclusion

This paper analyses information acquisition in double auction markets populated with high and low valuation traders of an asset and shows that for any finite information cost, an efficient equilibrium allocation fails to exist if the number of traders and the units a trader is allowed to trade are sufficiently large or the asset is sufficiently risky. There is a large set of parameter values where in any equilibrium with positive volume of trade the traders play mixed strategies and ex ante identically informed, rational traders evolve endogenously to noise traders, speculators, and defensive traders. Because of defensive trading the allocation is inefficient, i.e. not all gains from trade are realized. Because of endogenous noise trading the price is not fully revealing.

This paper provides a strategic foundation for the Grossman and Stiglitz (1980) impossibility result of informationally efficient (double auction) markets and derives some implications. Might the behavior one observes in large double auction markets be understood as the realization of a mixed strategy equilibrium? This paper provides a rationale for the restriction of short selling in financial markets and highlights a potential benefit of over-the-counter markets.[19] In decentralized trading, if the information cost is either low or high, then there exist equilibria where trade occurs with probability one. These equilibria ceases to exist in a large market.

However, this paper ignores the potential cost of finding a trader with the opposite trading need. An important function of a large market may be the bundling of liquidity. In addition, this paper analyzes a one-period trading game and highlights potential inefficiencies in a static large double auction market. A second important function of a centralized market is the transmission of information through prices and sequential trading. This can mitigate the duplication of costly information acquisitions.[20] An interesting extension is to allow the traders to endogenously choose whether to trade in a centralized or decentralized market.

From an mechanism design point of view, one can analyze an optimal multi-stage (direct) mechanism where a trader first announces his buy or sell preference, then whether he has acquired information, and finally his information about the asset value. Alternatively, the

[19] Common stocks are traded in centralized markets with high price transparency and high daily trading volume, while many other financial instruments are traded in decentralized dealership markets with low transparency and relatively low daily trading volume. For example, the trading of mortgage-back securities, derivatives, structured debt products, credit default swaps, and corporate bonds in over-the-counter markets are said to be opaque.

[20] For example, if the traders are heterogeneous then traders facing large trading gains or a continuous inflow of trading needs (such as the execution of incoming orders of costumers) may be the ones who acquire information and determine prices. The small traders wait, observe the price and trade without costly information acquisition.

traders may write (complex) state contingent contracts. However, as mentioned, the simple linear utility function of the traders should be regarded as a shortcut for the marginal utility of risk adverse traders possessing different endowments of the risky asset. If a trader receives or has to make additional post transaction payments after the realization of the cash flow, then this type of arrangements undermines the idea of risk sharing in financial markets.

Market microstructure models typically assume and real financial markets often have designated market makers. These agents are supposed to provide liquidity and explicitly forbidden to speculate. Madhavan and Panchapagesan (2000) provide an empirical analysis of the role of market specialists for price discovery in the overnight market on the NYSE, and state that "*there is strong evidence that the NYSE's designated dealer (specialist) sets a more efficient price than the price that would prevail in a pure call market using only public orders*." (p.656)[21]

It is interesting to explore the implication for this model of the exogenous presence of a fourth category of players, designated market makers who attempt to earn profits from exclusively facilitating the encounter of buyers and sellers, as well as to identify the set of conditions under which such players would endogenously emerge in equilibrium. In other words, can market makers facilitate allocative and informational efficiency in double auction markets with endogenous information? Further research may provide additional insights on the working of different trading institutions as well as the competition between trading platforms when all traders are strategic and can acquire information before they trade.

[21] Madhavan and Panchapagesan (2000) also analyse in their noisy REE type setting with exogenous information a single-price call auction where a market maker sets the opening price after observing the limit order book and they state (p.657) "*The process by which investors' latent demands are translated into realized prices and volumes is a highly complex process that we are only now starting to understand. Our results add to a growing body of evidence that highlights the crucial roles of information and market structure in determining price efficiency, but there are still many important questions to be answered before we fully understand the inner workings of the black box of trading mechanisms.*"

Appendix

Proof of Proposition 1(c)

Statement: (i) If c=Δ, then t_B=t_S=(1, v_L+Δ, v_H–Δ) is the only BNE in which trade occurs with probability one.[22] (ii) If c<Δ, then there is a continuum of *full* trade equilibria. [23] (iii) In any full trade BNE both agents acquire information and the price is fully revealing. (iv) In particular, a full trade k-sharing BNE with $EU^B=\Delta-k-c$ and $EU^S=\Delta+k-c$ exists only if $c\leq\Delta-|k|$ where $k\in[-\Delta, \Delta]$.

Claim A: Suppose $r,z\in[-\Delta, \Delta]$. If $c\leq\frac{1}{2}(\Delta+min\{r,-z\})$, then $t_B^*= t_S^*=(1, v_L+r, v_H+z)$ are best responses and $EU^B=\Delta-\frac{1}{2}(r+z)-c$ and $EU^S=\Delta+\frac{1}{2}(r+z)-c$.

Proof: Given t_S^*, if the buyer chooses t_B^*, then $EU^B=\frac{1}{2}[v_L+\Delta-(v_L+r)]+\frac{1}{2}[v_H+\Delta-(v_H+z)]-c=\Delta-\frac{1}{2}(r+z)-c\geq0$ since $\Delta-\frac{1}{2}(r+z)\geq\frac{1}{2}(\Delta+min\{r,-z\})\geq c$. If the buyer chooses (0,b) with $b=v_H+z$ then $EU^B=\frac{1}{2}[\mathrm{v_L}+\Delta-\frac{1}{2}(v_L+r+v_H+z)]+\frac{1}{2}[v_H+\Delta-(v_H+z)]=\Delta-\frac{1}{4}(v_H-v_L)-\frac{1}{4}(r+3z)$. For $r=z=-\Delta$, the buyers' payoff is maximal and yet $EU^B=-\frac{1}{4}(v_H-v_L)+2\Delta<0$. If the buyer chooses (0,b) with $b=v_L+r$ then $EU^B=\frac{1}{2}(\Delta-r)$. This response is weakly dominated by response t_B^* since $c\leq\frac{1}{2}(\Delta-z)$. So t_B^* is a best response to t_S^*.[24]

Analogously for the seller, if he chooses (0,s) with $s=v_L+r$ then $EU^S=\Delta+\frac{1}{4}(3r+z)-\frac{1}{4}(v_H-v_L)<0$. If the seller chooses (0,s) with $s=v_H+z$ then $EU^S=\frac{1}{2}(\Delta+z)\leq\Delta+\frac{1}{2}(r+z)-c$ since $c\leq\frac{1}{2}(\Delta+r)$. So t_S^* is a best response to t_B^*. For $c\leq min\{\frac{1}{2}(\Delta-z),\frac{1}{2}(\Delta+r)\}=\frac{1}{2}(\Delta+min\{r,-z\})$, (t_B^*,t_S^*) constitutes a BNE.

Claim B: If $c\leq\Delta-|k|$ for $k\in[-\Delta,\Delta]$, there exists a two-sided information acquisition and full trade equilibrium with the payoffs $EU^B=\Delta-k-c$ and $EU^S=\Delta+k-c$.

Proof: For a fixed $k\in[-\Delta,\Delta]$, define $k=\frac{1}{2}(r+z)$ then $r=2k-z$ and $z=2k-r$. From Claim A, the maximal allowable information cost c_k for the payoffs $EU^B=\Delta-\frac{1}{2}(r+z)-c$ and $EU^S=\Delta+\frac{1}{2}(r+z)-c$ being equilibrium payoffs is $c_k=0.5\cdot(\Delta+min\{r,-z\})$. The maximum allowable cost is given by

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[22] There also exist pure strategy equilibria where trade occurs with probability 0.5. For example, the buyer chooses n_B=1 and b=(v_L–Δ,v_H–Δ) and the seller chooses n_S=0 and s=v_H–Δ. From a social point of view, a full trade BNE dominates a partial trade BNE if c<Δ.

[23] Full trade equilibrium means an equilibrium where trade occurs with probability one.

[24] It is easy to see that B^* also dominates the strategies (0,b) where $b<s_L$, $b\in(s_L,s_H)$ or $b>s_H$.

$$\max_{\substack{r,z\in[-\Delta,\Delta] \\ s.t.\ r+z=2k}} 0.5\cdot(\Delta+min\{r,-z\})= \max_{\substack{r,z\in[-\Delta,\Delta] \\ s.t.\ r+z=2k}} 0.5\cdot(\Delta+min\{2k-z, -2k+r.\}). \qquad (*)$$

Since the $min\{r,-z\}$ in (*) is non-decreasing in r and $-z$, set $r=\Delta$ and $z=-\Delta$. Then

$$\max_{\substack{r,z\in[-\Delta,\Delta] \\ s.t.\ r+z=2k}} 0.5\cdot(\Delta+min\{2k-z, -2k+r\})= 0.5\cdot(\Delta+min\{2k+\Delta, -2k+\Delta\})=0.5\cdot(2\Delta-2|k|).$$

For $c=c_k=\Delta-|k|$, *one* equilibrium strategies pair $t_B=t_S=(1,v_L+r, v_H+z)$ leading to the payoffs $EU^B=\Delta-k-c$ and $EU^S=\Delta+k-c$ is given as follows: If $k\in[-\Delta,0]$ then set $r=\Delta+2k$ and $z=-\Delta$. If $k\in[0,\Delta]$ then set $r=\Delta$ and $z=-\Delta+2k$. (Note, for $c=\Delta$, only the (k=0) trading equilibrium exists and $t_B=t_S=(1,v_L+\Delta, v_H-\Delta)$.)

Claim C: If $c\leq\Delta-|k|$, no full trade equilibrium exists in which (i) no trader acquires information or (ii) only one trader acquires information.

Proof: (i) The assumption $\Delta<\frac{1}{8}(v_H-v_L)$ implies that $\Delta-|k|<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$. So no trading equilibrium without information acquisition exists. (ii) Suppose that only the buyer acquires information. He is willing to choose any (b_L,b_H) with $b_L=v_L+r$ and $b_H=v_H+z$ where $r,z\in[-\Delta, \Delta]$. If there is to be full trade the seller must choose $(0,s)$ with $s=v_L+r$. For any $r,z\in[-\Delta, \Delta]$, $EU^S=\Delta+\frac{1}{4}(3r+z)-\frac{1}{4}(v_H-v_L)<0$. Analogously for $n_B=0$ and $n_S=1$. So no full trade occurs if only one trader acquires information. **QED**

Proof of Proposition 2

The set of mixed strategies may be very large. The proof contains two parts and proceeds as follows. Part A derives a mixed strategy BNE under the assumption that the traders can only choose three offer prices. Part B shows that the equilibrium properties in part A also hold without this restriction.

Part A

Assumption A

The traders can only choose offer prices from the set $b,s\in\{v_L,E[v],v_H\}$.

Remark A1

An informed buyer does not choose $b>v_L+\Delta$ at v_L, while an informed seller does not choose $s<v_H-\Delta$ at v_H. An uninformed buyer does not bid more than his expected valuation, i.e. $b>E[v]+\Delta$, and an uniformed seller does not choose $s<E[v]-\Delta$. These actions are weakly dominated choices.

Step 1

Given Assumption A and Remark A1, one can focus on the following pure strategies that are not weakly dominated. For the buyer, these are $(0,v_L)$, $(0,E[v])$, $(1,v_L,E[v])$, $(1,v_L,v_H)$, and $(1,v_L,v_L)$. For the seller, these are $(0,E[v])$, $(0,v_H)$, $(1,v_L,v_H)$, $(1,E[v],v_H)$, and $(1,v_H,v_H)$. In a mixed strategies, the buyer and seller choose probability

σ_{B1} on $(0,v_L)$,	σ_{S1} on $(0,v_H)$,
σ_{B2} on $(0,E[v])$,	σ_{S2} on $(0,E[v])$,
σ_{B3} on $(1,v_L,E[v])$,	σ_{S3} on $(1,E[v],v_H)$,
σ_{B4} on $(1,v_L,v_H)$,	σ_{S4} on $(1,v_L,v_H)$,
σ_{B5} on $(1,v_L,v_L)$,	σ_{S5} on $(1,v_H,v_H)$,

respectively. The expected payoffs of the buyer are given as follows.

$$EU^B(0,v_L) = \tfrac{1}{2}\sigma_{S4}\Delta$$

$$EU^B(0,E[v]) = \tfrac{1}{2}\sigma_{S4}(\Delta-\tfrac{1}{4}(v_H-v_L))+\tfrac{1}{2}\sigma_{S3}(\Delta-\tfrac{1}{2}(v_H-v_L))+\sigma_{S2}\Delta$$

$$EU^B(1,v_L,E[v]) = \tfrac{1}{2}\sigma_{S4}\Delta+\tfrac{1}{2}\sigma_{S2}(\Delta+\tfrac{1}{2}(v_H-v_L))-c$$

$$EU^B(1,v_L,v_H) = \tfrac{1}{2}\sigma_{S4}\Delta+\tfrac{1}{2}(\sigma_{S3}+\sigma_{S4})\Delta+\tfrac{1}{2}\sigma_{S1}\Delta+\tfrac{1}{2}\sigma_{S2}(\Delta+\tfrac{1}{4}(v_H-v_L))-c$$

$$EU^B(1,v_L,v_L) = \tfrac{1}{2}\sigma_{S4}\Delta-c$$

The expected payoffs of the seller are given as follows.

$$EU^S(0,v_H) = \tfrac{1}{2}\sigma_{B4}\Delta$$

$$EU^S(0,E[v]) = \tfrac{1}{2}\sigma_{B4}(\Delta-\tfrac{1}{4}(v_H-v_L))+\tfrac{1}{2}\sigma_{B3}(\Delta-\tfrac{1}{2}(v_H-v_L))+\sigma_{B2}\Delta$$

$$EU^S(1,E[v],v_H) = \tfrac{1}{2}\sigma_{B4}\Delta+\tfrac{1}{2}\sigma_{B2}(\Delta+\tfrac{1}{2}(v_H-v_L))-c$$

$$EU^S(1,v_L,v_H) = \tfrac{1}{2}\sigma_{B4}\Delta+\tfrac{1}{2}(\sigma_{B3}+\sigma_{B4})\Delta+\tfrac{1}{2}\sigma_{B1}\Delta+\tfrac{1}{2}\sigma_{B2}(\Delta+\tfrac{1}{4}(v_H-v_L))-c$$

$$EU^S(1,v_H,v_H) = \tfrac{1}{2}\sigma_{B4}\Delta-c$$

Since the pure strategy $(1,v_L,v_L)$ is strictly dominated by the pure strategy $(0,v_L)$, the buyer chooses $\sigma_{B5}=0$. Since $(1,v_H,v_H)$ is strictly dominated by $(0,v_H)$, the seller chooses $\sigma_{S5}=0$.

Step 2

(a) This step analyses the best responses of the buyer.

(i) The strategy $(1,v_L,E[v])$ weakly dominates $(1,v_L,v_H)$ if $EU^B(1,v_L,E[v]) \geq EU^B(1,v_L,v_H)$

$$\Leftrightarrow \quad \tfrac{1}{2}\sigma_{S2}(\Delta+\tfrac{1}{2}(v_H-v_L))-c \geq \tfrac{1}{2}(\sigma_{S3}+\sigma_{S4})\Delta+\tfrac{1}{2}\sigma_{S1}\Delta+\tfrac{1}{2}\sigma_{S2}(\Delta+\tfrac{1}{4}(v_H-v_L))-c$$

$$\Leftrightarrow \quad \tfrac{1}{8}\sigma_{S2}(v_H-v_L)) \geq \tfrac{1}{2}(\sigma_{S1}+\sigma_{S3}+\sigma_{S4})\Delta$$

$$\Leftrightarrow \quad \sigma_{S2}(v_H-v_L)) \geq 4(1-\sigma_{S2})\Delta$$

$$\Leftrightarrow \qquad \sigma_{S2} \geq \frac{4\Delta}{(4\Delta + v_H - v_L)} \equiv I_{(v_L,v_H)}^{(v_L,E[v])}$$

(ii) The strategy $(1,v_L,E[v])$ weakly dominates the strategy $(0,v_L)$ if

$$\sigma_{S2} \geq \frac{4c}{(2\Delta + v_H - v_L)} \equiv I_{v_L}^{(v_L,E[v])}$$

(iii) The strategy $(1,v_L, E[v])$ weakly dominates the strategy $(0,E[v])$ if

$$\sigma_{S2} \geq \frac{8c - (\sigma_{S3} + \sigma_{S4})(v_H - v_L) - \sigma_{S3}(v_H - v_L - 4\Delta)}{2(v_H - v_L - 2\Delta)} \equiv I_{E[v]}^{(v_L,E[v])}$$

(iv) The strategy $(0,E[v])$ weakly dominates the strategy $(0,v_L)$ if

$$\sigma_{S2} \geq \frac{(2\sigma_{S3} + \sigma_{S4})(v_H - v_L)}{8\Delta} - \frac{1}{2}\sigma_{S3} \equiv I_{v_L}^{E[v]}$$

(v) The strategy $(1,v_L,v_H)$ weakly dominates the strategy $(0,v_L)$ if

$$\sigma_{S2} \geq \frac{4(2c - \Delta)}{v_H - v_L} \equiv I_{v_L}^{(v_L,v_H)}$$

(vi) The strategy $(1,v_L,v_H)$ weakly dominates the strategy $(0,E[v])$ if

$$\sigma_{S2} \geq \frac{8c - 4\sigma_{S1}\Delta + 2\sigma_{S3}(v_H - v_L)}{(v_H - v_L - 4\Delta)} + \sigma_{S4} \equiv I_{E[v]}^{(v_L,v_H)}$$

(b) Analogously for the seller. E.g., if $\sigma_{B2} \geq I_{(v_L,v_H)}^{(v_L,E[v])}$,then $EU^S(1,E[v],v_H) \geq EU^S(1,v_L,v_H)$.

<u>Step 3</u>

<u>Claim:</u> There exists no mixed strategy equilibrium in which the buyer and the seller choose the strategy $(1,v_L v_H)$ with positive probability.

<u>Proof:</u> For the buyer, $(1,v_L,v_H)$ and $(1,v_L,E[v])$ are the two potential strategies with information acquisition for being a candidate in a mixed strategy equilibrium. The buyer does not choose $(1,v_L,v_H)$ with positive probability if it is strictly dominated by $(1,v_L,E[v])$. Suppose that the strategy $(1,v_L,v_H)$ weakly dominates $(1,v_L,E[v])$, i.e. $\sigma_{S2} \leq I_{(v_L,v_H)}^{(v_L,E[v])}$. It is easy to see that $I_{(v_L,v_H)}^{(v_L,E[v])} < I_{v_L}^{(v_L,v_H)}$. Consequently, if the strategy $(1,v_L,v_H)$ weakly dominates $(1,v_L,E[v])$, then $(1,v_L,v_H)$ is strictly dominated by the strategy $(0,v_L)$ because in this case $\sigma_{S2} < I_{v_L}^{(v_L,v_H)}$.[25] Therefore, if the seller randomizes such that the buyer is indifferent between $(1,v_L,v_H)$ and $(1,v_L,E[v])$; or $(1,v_L,v_H)$ dominates $(1,v_L,E[v])$, then the buyer chooses $\sigma_{B1}=1$, i.e. he does not

[25] If the buyer is indifferent between $(1,v_L ,E[v])$ and $(1,v_L,v_H)$, then $(0,v_L)$ strictly dominates both $(1,v_L,E[v])$ and $(1,v_L,v_H)$, since $\sigma_{S2} = I_{(v_L,v_H)}^{(v_L,E[v])}$ and $c>\Delta$ imply $\sigma_{S2} < I_{v_L}^{(v_L,E[v])}$.

acquire information. Analogously for the seller, if the strategy $(1,v_L,v_H)$ weakly dominates $(1,E[v],v_H)$, then $(1,v_L,v_H)$ is strictly dominated by the strategy $(0,v_H)$.

Consequently, in a mixed strategy equilibrium (where information must be acquired with positive probability), the strategy $(1,v_L,v_H)$ must be a strictly dominated strategy and one must have $\sigma_{B4}=\sigma_{S4}=0$ (or $\alpha_3=\beta_3=1$).[26]

Step 4

Claim: In a mixed strategy equilibrium the traders get zero expected payoff.

Proof: For $\sigma_{B4}=\sigma_{S4}=0$, $EU^B(0,v_L)=EU^S(0,v_H)=0$. In other words, if the buyer is indifferent between $(1,v_L,E[v])$ and $(0,v_L)$ or indifferent between $(0,E[v])$ and $(0,v_L)$, then his expected payoff is zero. In order to find a mixed strategy equilibrium in which the traders get positive expected payoffs, the following is required: For the buyer, he should be indifferent between $(1,v_L,E[v])$ and $(0,E[v])$; and $(0,E[v])$ should strictly dominate $(0,v_L)$, i.e. the buyer chooses $\sigma_{B1}=0$.[27] The buyer is indifferent between $(1,v_L,E[v])$ and $(0,E[v])$ if $\sigma_{S2}=I_{E[v]}^{(v_L,E[v])}$. For $\sigma_{S4}=0$,

$$\sigma_{S2}=\frac{8c-\sigma_{S3}(v_H-v_L)-\sigma_{S3}(v_H-v_L-4\Delta)}{2(v_H-v_L-2\Delta)}$$

$$\Leftrightarrow \qquad \sigma_{S2}=\frac{4c}{(v_H-v_L-2\Delta)}-\sigma_{S3}.$$

Since $c<\frac{1}{4}(v_H-v_L)-\frac{1}{2}\Delta$, this implies that $\frac{4c}{v_H-v_L-2\Delta}<1$. Therefore $\sigma_{S2}+\sigma_{S3}<1$, which means that there is some probability "left", i.e. σ_{S1} must be larger than zero. In order to make the buyer indifferent between $(1,v_L,E[v])$ and $(0,E[v])$, the seller must choose $(0,v_H)$ with positive probability.

On the other hand, if the seller chooses $(0,v_H)$ with positive probability he must be indifferent between $(0,E[v])$ and $(0,v_H)$. Since $EU^S(0,v_H)=0$, $EU^S(0,E[v])$ must be zero, too. Otherwise, the seller is not indifferent. Consequently, the expected payoff of the seller must be zero in a mixed strategy equilibrium.[28]

[26] Suppose the buyer just randomizes over $(0,v_L)$ and $(0,E[v])$. If the seller also does not acquire information, then he chooses s=E[v] with probability 1. Given the seller's response, the buyer's best response is $(1,v_L,E[v])$. So there exists no mixed strategy equilibrium in which information is acquired with zero probability.

[27] For the seller, he should be indifferent between $(1,E[v],v_H)$ and $(0,E[v])$; and $(0,E[v])$ should strictly dominate $(0,v_H)$, i.e. he chooses $\sigma_{S1}=0$.

[28] Analogously for the buyer, he must choose $(0,v_L)$ with positive probability in order to make the seller indifferent between $(1,E[v],v_H)$ and $(0,E[v])$, i.e. his expected payoff is zero in a mixed strategy equilibrium.

Step 5

Claim: In the only (non-degenerated) mixed strategy equilibrium the buyer randomizes over $(0,v_L)$, $(0,E[v])$ and $(1,v_L,E[v])$ according to σ_B and the seller randomizes over $(0,v_H)$, $(0,E[v])$ and $(1,E[v],v_H)$ according to σ_S where

$$\sigma_B = \sigma_S = \left(1-\frac{4c}{v_H - v_L - 2\Delta},\ \frac{4c}{v_H - v_L + 2\Delta},\ \frac{16c\Delta}{(v_H - v_L)^2 - 4\Delta^2}\right).$$

Proof:

In order to make the buyer indifferent between $(1,v_L,E[v])$ and $(0,v_L)$, the seller chooses σ_{S2} such that $\sigma_{S2} = I_{v_L}^{(v_L,E[v])}$; and to make the buyer indifferent between $(1,v_L,E[v])$ and $(0,E[v])$, the seller chooses σ_{S2} and σ_{S3} such that $\sigma_{S2} = I_{E[v]}^{(v_L,E[v])}$.[29] So $I_{v_L}^{(v_L,E[v])} = I_{E[v]}^{(v_L,E[v])}$ implies

$$\frac{4c}{v_H - v_L + 2\Delta} = \frac{4c}{v_H - v_L - 2\Delta} - \sigma_{S3}$$

$$\Leftrightarrow \qquad \sigma_{S3} = \frac{16c\Delta}{(v_H - v_L + 2\Delta)(v_H - v_L - 2\Delta)} = \frac{16c\Delta}{(v_H - v_L)^2 - 4\Delta^2}.$$

In addition, the seller chooses

$$\sigma_{S1} = 1 - \sigma_{S2} - \sigma_{S3} = 1 - \frac{4c}{v_H - v_L - 2\Delta}.$$

Step 6

Claim: The outcome in a mixed strategy BNE has the following properties. (i) The probability of trade is $\frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}$. (ii) The only trading price is $E[v]$ and not fully revealing.

Proof :

The buyer randomizes over $(0,v_L)$, $(0,E[v])$ and $(1,v_L,E[v])$.

The seller randomizes over $(0,v_H)$, $(0,E[v])$ and $(1, E[v],v_H)$.

(i) Trade occurs in the following events: (a) both the buyer and the seller choose $(0,E[v])$; (b) the buyer chooses $(0,E[v])$ and the seller chooses $(1,E[v],v_H)$ and the true state is v_L; and (c) the buyer chooses $(1,v_L,E[v])$, the seller chooses $(0,E[v])$ and the true state is v_H. The probability of trade is given as follows:

$$prob(trade) = \sigma_{B2}\sigma_{S2} + \tfrac{1}{2}\sigma_{B2}\sigma_{S3} + \tfrac{1}{2}\sigma_{B3}\sigma_{S2} = \sigma_{B2}\sigma_{S2} + \sigma_{B2}\sigma_{S3}$$

[29] Alternatively, the buyer should be indifferent between $(0,E[v])$ and $(0,v_L)$. This requires $\frac{4c}{(2\Delta + v_H - v_L)} = \frac{2\sigma_{S3}(v_H - v_L)}{8\Delta} - \frac{1}{2}\sigma_{S3}$ and yields the same condition.

$$\Leftrightarrow \qquad prob(trade) = \frac{16c^2}{(v_H - v_L + 2\Delta)^2} + \frac{64c^2\Delta}{(v_H - v_L + 2\Delta)^2(v_H - v_L - 2\Delta)}$$

$$\Leftrightarrow \qquad prob(trade) = \frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}.$$

(ii) The buyer bids at most $E[v]$ and the seller demands at least $E[v]$. Therefore, no trade occurs at the prices v_L and v_H. Trade only occurs if at least one trader is uninformed. If the uninformed trader observes trade, he cannot distinguish whether he makes a fair deal and realizes Δ, or suffers a speculative loss. Although the uninformed trader updates his belief, he does not know the true state when observing $p=E[v]$.

Remark A2

σ_{i3} is the equilibrium probability of information acquisition. It decreases in $v_H - v_L$ and increases in the information cost c which seems unintuitive. As the potential speculative profit increases and the cost of becoming informed decreases, one might expect that a trader has a higher incentive to acquire information. But in order to make the other trader indifferent between his pure strategies, equilibrium randomization requires it.

Remark A3

If $c=\frac{1}{4}(v_H - v_L)-\frac{1}{2}\Delta$, then in the non-degenerated mixed strategy BNE the buyer randomizes over $(0,E[v])$ and $(1,v_L,E[v])$ according to σ_B and the seller randomizes over $(0,E[v])$ and $(1,E[v],v_H)$ according to σ_S where

$$\sigma_B = \sigma_S = \left(\frac{v_H - v_L - 2\Delta}{v_H - v_L + 2\Delta}, \ \frac{4\Delta}{v_H - v_L + 2\Delta} \right),$$

and trade occurs with probability σ_{i1}, and both traders have zero expected payoff.[30]

Remark A4

(a) For $c=\frac{1}{4}(v_H - v_L)-\frac{1}{2}\Delta$, as $(v_H - v_L) \rightarrow \infty$, then $c \rightarrow \infty$ and the probability that both traders choose $(0,E[v])$ converges to one and yet $EU^B=EU^S=0$ (prior to the randomization outcome).
(b) For $c<\frac{1}{4}(v_H - v_L)-\frac{1}{2}\Delta$, as $(v_H - v_L) \rightarrow \infty$, then the probability that the buyer chooses $(0,v_L)$ and the seller chooses $(0,v_H)$ converges to one.

[30] In the unique pure strategy trading BNE both traders choose $(0,E[v])$ and $EU^B=EU^S=\Delta$.

Part B

Now it is assumed that the traders can choose any real number as an offer price.

Remark B1

From part A, a set of mixed strategy BNE from a three points distribution is the following. The buyer randomizes over $(0,b)$, $(0,E[v])$ and $(1,b,E[v])$ where $b\leq v_L$. The seller randomizes over $(0,s)$, $(0,E[v])$ and $(1,E[v],s)$ where $s\geq v_H$.

The considerations in Remark A1 imply that in search for a candidate for an equilibrium randomization with potentially positive payoffs, it suffices to focus on bid and ask prices within the following three subintervals: $I_L=[v_L-\Delta,v_L+\Delta]$, $I_M=[E[v]-\Delta,E[v]+\Delta]$, and $I_H=[v_H-\Delta,v_H+\Delta]$. See Figure 1. In particular, one can focus on the sets $\{t_B\}$ and $\{t_S\}$ of pure strategies with the following properties. For the buyer and seller these are:

(i) $T_L^B=\{(0,b): b\in I_L\}$, $T_H^S=\{(0,s): s\in I_H\}$,

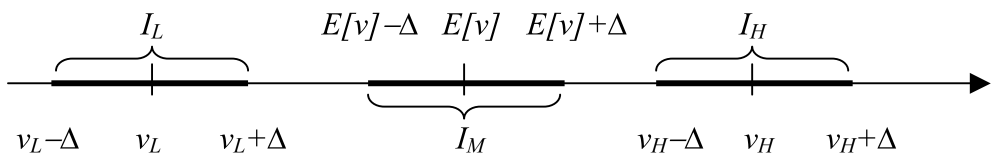
(ii) $T_M^B=\{(0,b): b\in I_M\}$, $T_M^S=\{(0,s): s\in I_M\}$,

(iii) $T_{LM}^B=\{(1,b_L,b_H): b_L\in I_L, b_H\in I_M\}$, $T_{MH}^S=\{(1,s_L,s_H): s_L\in I_M, s_H\in I_H\}$,

(iv) $T_{LH}^B=\{(1,b_L,b_H): b_L\in I_L, b_H\in I_H\}$, $T_{LH}^S=\{(1,s_L,s_H): s_L\in I_L, s_H\in I_H\}$,

(v) $T_{LL}^B=\{(1,b_L,b_H): b_L,b_H\in I_L\}$, $T_{HH}^S=\{(1,s_L,s_H): s_L,s_H\in I_H\}$.

Figure 1



Denote f (and g) as the density over the set $\{t_B\}$ and $\{t_S\}$ of pure strategies.

Claim 1: The equilibrium payoff is zero.

Proof: Observation

$\int_{t_B\in T_L^B} f(t_B)dt_B = prob(t_B\in T_L^B) = prob((0,b)$ with $v_L-\Delta\leq b\leq v_L+\Delta)$ which is "equivalent" to σ_{B1}. Analogously for the other cases.

Suppose the seller actually chooses $(0,s)$ with $s=v_H$ (a $s\in I_H$) as the randomization outcome, then trade only occurs if the buyer actually chooses a strategy $t_B\in T^B_{LH}$ with $b_H\geq s$ and the true state is $v=v_H$. The seller's expected payoff is

$$EU^S(0,s)=\tfrac{1}{2}\cdot 0+\tfrac{1}{2}\cdot \int_{\substack{t_B\in T^B_{LH}\\ b_H\geq s}} f(t_B)\cdot\left((\tfrac{1}{2}(b_H-s)+\Delta\right)dt_B$$

which is the analogous expression to $EU^S(0,v_H)=\frac{1}{2}\sigma_{B4}\Delta$.

From Step 3 in Part A, a similar argument shows that in no mixed strategy equilibrium does the buyer choose a positive density over $T^B_{LH}=\{(1,b_L,b_H)\text{: } b_L\in I_L,\ b_H\in I_H\}$. If he is indifferent between $(1,b_L',b_H')$ and $(1,b_L'',b_H'')$ where $b_L',b_L''\in I_L$, $b_H'\in I_M$ and $b_H''\in I_H$, then these strategies are strictly dominated by $(0,b)$ with $b\in I_L$. Therefore $EU^S(0,s)=0$ when $s\in I_H$.

From Step 4, in order to make the buyer indifferent between choosing strategies without information acquisition (e.g. $T^B_L=\{(0,b)\text{: } b\in I_L\}$) and strategies with information acquisition ($T^B_{LM}=\{(1,b_L,b_H)\text{: } b_L\in I_L,\ b_H\in I_M\}$), the seller must choose a positive density on the strategies from the set $T^S_H=\{(0,s)\text{: } s\in I_H\}$. Consequently, the equilibrium payoff of the seller is zero.

Claim 2: If trade occurs, then the price is $p\in(E[v]-\Delta,E[v]+\Delta)$ and not fully revealing.

Proof:

The previous analysis shows that the buyer chooses a density f and the seller a density g that only have positive mass on the following pure strategies:

For the buyer, $t_B\in\{(0,b)\text{: } b\in I_L, I_M\}\cap\{(1,b_L,b_H)\text{: } b_L\in I_L \text{ and } b_H\in I_M\}$.

For the seller $t_S\in\{(0,s)\text{: } s\in I_M, I_H\}\cap\{(1,s_L,s_H)\text{: } s_L\in I_M \text{ and } s_H\in I_H\}$.

Therefore, trade only occurs if the buyer actually chooses $b\in I_M$ and the seller actually chooses $s\in I_M$ and $b\geq s$. In any case $p\in I_M$. **QED**

Proof Proposition 3

An equivalent statement of Proposition 3 is the following. Define $q=\min[m,N]$. Suppose $N>1$ and $c<\frac{1}{2}q(v_H-v_L)-\frac{1}{2}\Delta$. (i) The allocation in any (pure and mixed) strategy BNE is inefficient. (ii) If $c\neq q\Delta$, only mixed strategy equilibria with positive volume of trade exist and the price is not fully revealing. (iii) If $c=q\Delta$, pure strategy equilibria with positive volume of trade exist. In any such equilibrium the price is fully revealing.

Remark

(a) From Lemma, since $c<\frac{1}{2}q(v_H-v_L)-\frac{1}{2}\Delta$, if there is positive volume of trade in equilibrium, some traders acquire information. Note also that $q\Delta<\frac{1}{2}q(v_H-v_L)-\frac{1}{2}\Delta$.

(b) The proof proceed by showing the following statements. (i) If prices are fully revealing, then the allocation is inefficient. (ii) In any profile of pure strategies, prices are fully revealing. (iii) Prices are typically not fully revealing. (iv) Since prices are not fully revealing, there is a potential lemons problem and the allocation is not efficient.

Step 1:

Case (i): $c<\Delta$: An informed trader can cover his information cost by trading one unit. Suppose the price is fully revealing and all agents trade one unit in both states. (Otherwise some traders have a profitable deviation.) Then the only candidate price for an equilibrium price is $p=v$. If $p<v$ ($p>v$), then both an informed and uninformed buyer (seller) has a profitable deviation by overbidding (underbidding) the other traders and buy (sell) m units instead of one unit.[31] To save on notation, if a trader does want to buy (sell), then (b_i,u_i^b) ((s_i,u_i^s)) are omitted.

Consider the following profile of pure strategies where k H-traders and r L-traders acquire information and all agents trade: $n_i=1$, $(b_i,u_i^b)=((v_L,v_H),(1,1))$; $n_l=0$, $(b_l,u_l^b)=(v_H,1)$, where $i=1,..,k$ and $l=k+1,..,N$, and $n_j=1$, $(s_j,u_j^s)=((v_L,v_H),(1,1))$; $n_n=0$, $(s_N,u_n^s)=(v_L,1)$ where $j=N+1,..,r$ and $n=r+1,..,2N$.

(i) Suppose $k=r=1$. The market clearing price is $p=v_L$ in state v_L and $p=v_H$ in state v_H and each agent trades one unit. The following arguments show that these strategies do not constitute a BNE. A profitable deviation of trader i is $t_i'=\{n_i=1, (b_i,u_i^b)=((v_L,v_H+\varepsilon),(1,q))\}$. Suppose $v=v_H$. If $p=v_H$ is the price, then $(N-1)$ units are traded. The order of trader 1 is not executed. If $p=v_L$ is the price, then also $(N-1)$ units are traded. Trader 1 receives q units. (Trader $N+1$ does not trade). So there is equal probability that trade is executed at one of these prices. It is easy to see that $EU_1(t_1')>EU_1(t_1)$. Analogously, the informed L-trader has incentives to speculate.[32]

(ii) So for any given number (k,r)-informed traders, if an informed trader can move the price (with positive probability), then the informed trader speculates. If the price is fully revealing, the uninformed traders on the other side of the market know that they suffer a

[31] For $q=1$, the price can be $p\in[v-\Delta,v+\Delta]$.

[32] The same arguments apply if the traders choose any offer $v\pm\Delta$.

speculative loss. This cannot be an equilibrium outcome. (iii) Suppose that the number (k,r)-informed traders is such that an informed trader cannot influence the price. If the price is fully revealing, then e.g. an informed H-trader deviates to n=0, $(b,u^b)=(v_H,1)$. He realizes his trading needs without paying the information cost.

Consequently, in any pure strategy profile with positive volume of trade as well as in any (pure or mixed) strategy profile with a fully revealing price, some traders have profitable deviations. Only mixed strategy trading equilibria exist and where the price $p\in[E[v]-\Delta,E[v]+\Delta]$ and not fully revealing.

Case (ii): $\Delta<c<q\Delta$:

The argument is similar. Suppose the price is fully revealing (i.e. $p\in[v-\Delta,v+\Delta]$) and trade occurs in both states. No pair of informed H- and L-trader can jointly cover their information cost by trading one unit. An informed trader must trade more than one unit. For an informed H-trader to cover his cost, the price must be $p<v$. The following arguments show that there is no pure strategy equilibrium with fully revealing prices. Suppose $p<v$, then some uninformed H- traders also wants to buy m unit rather than one unit. Given there is a shortage of supply, these traders overbid each other until $p=v$, but in this case the informed trader has a negative payoff. Similarly, if a L-trader would acquire information.

This also means that some uninformed traders do not realize their trading needs. Suppose the offer profiles (B,S) yields a fully revealing price $p=v\pm\Delta$. An argument similar to the one given in the proof of Lemma shows that an uninformed trader who does trade has not played a best response. Given the profile (B,S), an "unsatisfied" buyer (seller) would choose a sufficiently high (low) offer price so as to get one unit for sure without being concern about the lemons problem since the price is fully revealing. So no BNE with fully revealing prices exists. Therefore, some traders behave defensively.

Case (iii): $c>q\Delta$.

Suppose the price is fully revealing. No informed trader can cover his information cost at all. If only one side of the market acquires information, the other side of the market behave defensively.

Case (iv): $c=q\Delta$.

The following is a pure strategy BNE with positive volume of trade. No L-traders acquire information and k H-traders acquire information where $k>N/q$. In particular, n_i=1,

(b_i, u_i^b)=((v_L–Δ,v_H–Δ),(q,q)) for i=1,..,k; and the remaining H-traders choose n_i=0, (b_i, u_i^b)=(v_L–Δ,1) for i=k+1,..,N. All L-traders choose n_j=0 and (s_j, u_j^s)=(v_H–Δ,1) where j=N+1,..,2N. All traders have zero expected payoffs and no profitable deviations. If H-trader i>k+1 chooses b=v_H–Δ+ε to buy m units, he always get m units but with probability 0.5 he pays too much. Since $\Delta<\frac{1}{8}(v_H-v_L)$, this deviation yields a negative payoff. It is easy to see that no L-trader has a profitable deviation. In any pure strategy BNE at most half of the total trading gains of 2NΔ is realized and the price is fully revealing.

Step 2

This step characterizes a mixed strategy BNE. In a mixed strategy BNE because of the potential lemons problem, some traders behave defensively and do not trade. Denote λ as the fraction of uninformed liquidity traders who do not trade. Given q, if λ is bounded away from zero as N converges to infinity, then the equilibrium allocation is neither efficient nor asymptotically efficient. The following arguments show that this is the case. In a mixed strategy equilibrium the expected payoff of an informed and an uninformed trader must be the same. Otherwise either an informed or uninformed trader has a profitable deviation. An uninformed trader only trades if he proposes an offer $E[v]\pm\Delta$. As the fraction (1−λ) of uninformed traders with this offer increases, the probability that an informed trader makes speculative profits increases. Consequently, their expected payoff increases while the these uninformed traders decreases. In a mixed strategy BNE λ is strictly bounded away from zero.
QED

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