Should Courts Always Enforce What Contracting Parties Write?*

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Abstract. We find an economic rationale for the common sense answer to the question in our title — courts should not always enforce what the contracting parties write.

We describe and analyze a contractual environment that allows a role for an active court. An active court can improve on the outcome that the parties would achieve without it. The institutional role of the court is to maximize the parties’ welfare under a veil of ignorance.

We study a buyer-seller multiple-widget model with risk-neutral agents, asymmetric information and ex-ante investments. The court must decide when to uphold a contract and when to void it.

The parties know their private information at the time of contracting, and this drives a wedge between ex-ante and interim-efficient contracts. In particular, if the Court enforces all contracts, pooling obtains in equilibrium. By voiding some contracts the Court is able to induce them to separate, and hence improve ex-ante welfare. In some cases, an ambiguous Court that voids and upholds both with positive probability may be able to increase welfare even further.

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1. Introduction

It is self-evident that courts are active players in contractual relationships between economic agents. They routinely intervene in contractual disputes, excusing performance called for in the contract because of intervening events. Yet, in most of modern economic theory courts are treated (often not even modelled, but left in the background) as passive enforcers of the will of the parties embodied in their contractual agreements.

This simplistic view of the role of courts stems from the fact that in a world with complete contracts, to behave as a passive enforcer is clearly the best that a court that is interested in maximizing contracting parties’ welfare can do. In the “classical” world of modern economic theory, contracts are complete.

In a world in which complete contracts are not feasible it is no longer obvious that a court should be a passive enforcer, and in fact it is no longer true. For example, the contracting parties may face some uninsurable risk and the court may improve their welfare if it is able to use some information available ex-post and excuse performance in some eventualities.\footnote{This is the case, for instance, in Anderlini, Felli, and Postlewaite (2005).}

Once the way for an active court is open, a host of related questions naturally arise. The aim of this paper is to address the following question. Suppose that the Court cannot condition ex-post on any variable that cannot be contracted on by the parties themselves. Is it then the case that the Court can play any welfare-enhancing role?

The answer to the question above is “yes” if the parties are asymmetrically informed at the time they contract and the court maximizes their ex-ante welfare, that is, their expected welfare before either party gets information not available to the other. Asymmetry in parties’ information at the time they contract can lead to a “lemons-like” situation in which adverse selection leads to inefficient contracts. Courts that do not simply enforce contracts as they are written can sometimes ameliorate the inefficiency that results from asymmetric information.
We show in the paper that in a world where contracting parties are asymmetrically informed this is indeed the case. We also derive the optimal decision rule for an active welfare-maximizing court. This rule implies that the court in equilibrium voids contracts that the contracting parties (at the contracting stage) would like the court to enforce.

The potential benefit of a court’s voiding explicit contractual clauses stems from asymmetry of information between the parties at the time they contract. Because of asymmetric information, when the Court does not intervene, inefficient trades may take place: in equilibrium some pooling obtains. By intervening and voiding some contractual clauses, The Court may be able to negate the incentives for some types to hide their private information, thus making the pooling no longer viable in equilibrium. In other words, voiding contracts in some cases will decrease the expected gain from withholding private information, thereby promoting disclosure. Clearly voiding some contractual clauses will come at a cost: some surplus-generating trades will no longer take place. However, there will be a net welfare gain when the improvement from the additional disclosure outweighs the inefficiency from voiding.

The view that courts should maximize ex-ante welfare is a compelling one. If the parties were able to meet at the ex-ante stage (when they are symmetrically informed) agreements could be reached that circumvent inefficiencies that are unavoidable at the interim stage when the parties have private information. A court that maximizes ex-ante expected welfare will choose the same contingent rules of behavior as the parties would have chosen at that stage, were it possible. In other words, if the parties could meet at that point, they might instruct the court to void some contracts they might subsequently write. They will do this precisely because the parties will understand that while they may regret this in some circumstances, it may promote the disclosure of private information. This disclosure may increase the efficiency of contracting to an extent that more than outweighs any negative consequences of the court’s intervention. The problem that the court is solving is that the parties are often unable to meet before the arrival of their private information. A court that maximizes ex-ante welfare acts as a commitment device that remedies the parties’
inability to contract at the ex-ante stage.

1.1. The Role of Courts in Promoting Disclosure of Information

Courts have had an interest in promoting disclosure of information at least since the English case of Hadley vs. Baxendale in 1854. The court held in that case that a defendant who breached a contract was liable only for damages that might reasonably have arisen given the known facts rather than the higher damages that were actually suffered because of circumstances known only to the plaintiff. As argued in Adler (1999), the limitation on damages implicit in the Hadley rule is a default that is often viewed as promoting disclosure: “A party who will suffer exceptional damages from breach need only communicate her situation in advance and gain assent to allowance so that the damages are unmistakably in the contemplation of both parties’ at the time of contract.”

The discussion of the role of courts in promoting information disclosure, to our knowledge, focusses primarily on the benefit of disclosure to the contracting parties. In the absence of disclosure, resources will be wasted in writing needless waiver clauses and inefficient precaution.

Courts will have an interest in promoting disclosure of information in our model, but for a very different reason, and with very different consequences. Courts will affect the amount of information that is revealed by informed parties through their treatment of contracts that reveal little information. While contracts may reveal little information simply because the parties have little information, courts will treat such contracts more harshly than they otherwise might because of the incentive effects such treatment will have on informed parties. Those with relevant information will reveal it in order that courts will more likely enforce the agreements that are made. Thus, courts are not examining a contract brought before them solely to uncover the parties’ intent. They also take into consideration how the treatment of the contract will affect contracting parties different from the ones before them.

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3See Ayres and Gertner (1989) and Bebchuk and Shavell (1991) for a discussion of the Hadley rule and its role in promoting disclosure.
1.2. Related Literature

There is a growing literature that explicitly models the role of courts in contractual relationships. Bond (2003) and Usman (2002) model the agency problems (moral-hazard) that stem from hidden actions that the court itself can take, while Levy (2005) models the effect on the court’s decision of the judge’s career concern. Bond (2003) analyzes optimal contracting between parties when judges can impose an outcome other than the contracted outcome in exchange for a bribe. Bond shows that in a simple agency model, this possibility will make the contracting parties less likely to employ high-powered contracts. Usman (2002) lays out a model in which contracts contain variables that are not observable to courts unless a rational and self-interested judge exerts costly effort. Usman (2002) analyzes contracting behavior and the incentive to breach when judges value the correct ruling but dislike effort. Levy (2005) analyzes the trade-off that arises when the judge in ruling on a dispute is, at the same time, trying to influence the perception of the public (or an evaluator) about his own ability. This trade-off can induce the judge to distort his decision to avoid this same decision being appealed and possibly reversed.

The courts in these papers are governed by a judge who maximizes his or her personal utility. In contrast to these papers, there is a literature that analyzes courts that maximize the expected welfare of the contracting parties. Posner (1998) analyzes whether a court should consider information extrinsic to the contract in interpreting the contract. Closer to the current paper, Ayres and Gertner (1989) and Bebchuk and Shavell (1991) analyze the degree to which courts’ interpretation of contracts affect incentives to reveal private information. The focus of this work is the effect of different court rules regarding damages for breach of contract on the incentives for parties to disclose information regarding the costs of breach at the time of contracting. Shavell (2003) presents a general examination of the role of courts in interpreting contracts. The primary focus of this paper is on how courts should interpret contracts that have specific terms compared to the interpretation of more general terms.

The present paper analyzes the role of a welfare-maximizing court that can affect the type of contracts that are written by excusing performance (voiding the contract)
in some circumstances. The possibility of welfare improvements are a consequence of the effect of the Court’s rules for enforcing contracts on the parties’ incentives to reveal private information. Unlike Ayres and Gertner (1989), Bebchuk and Shavell (1991) and Shavell (2003), our focus is on the externality that informed contracting parties may impose on uninformed contracting parties, which is absent from these papers.

1.3. Outline

The plan of the rest of the paper is as follows. We present the model in Section 2, and our results in Sections 3, 4, 5 and 6. Section 7 concludes the paper. For ease of exposition all proofs have been gathered in the Appendix.\footnote{In the numbering of Propositions, Lemmas, equations and so on, a prefix of “A” indicates that the relevant item can be found in the Appendix.}

2. The Model

2.1. Passive Courts

Our main aim in this paper is to investigate the role of Courts as active players in contractual situations. However, for expository purposes it is simpler to begin the description of our model focusing on the trading opportunities faced by the parties in a world in which the Court enforces all contracts that it can verify.

A buyer $B$ and a seller $S$ face a potentially profitable trade of three widgets, denoted $w_1$, $w_2$ and $w_3$ respectively.

Widgets $w_1$ and $w_2$ are “specific.” They require a widget- and relationship-specific investment $I > 0$ on $B$’s part. The buyer can only undertake one of the two widget-specific investments, The value and cost of both $w_1$ and $w_2$ zero in the absence of investment, so that only one of them can possibly be traded profitably.

Widget $w_3$ is not specific. Its cost an value does not depend on any investment. We assume that $w_3$ is not contractible at the ex-ante stage. This is with little loss of generality, except for the case in which “menu contracts” are allowed.\footnote{We devote Section 6 below to this case.}
can be traded regardless of any ex-ante decision. In practice, we can think of \( w_3 \) as being traded (or not) at the ex-post stage.

The buyer has private information at the time of contracting. He knows his type, which can be either \( \alpha \) or \( \beta \). Each type is equally likely, and the seller does not know \( B \)'s type.

To complete the description of our trading set-up, it is now sufficient to specify the cost and value of each widget, for each possible \( B \) type, when investment takes place. We specify them using the smallest number of parameters that we believe are necessary for our results to hold. This is for the sake of simplicity only. The costs and values in the six combinations of of types and widgets could be specified independently without affecting our results provided that the appropriate assumptions hold. The model would be a lot less transparent as a result. With this in mind, we take the cost and value of the three widgets to be as in the table below, where they are represented net of the cost of investment \( I > 0 \).\(^6\) In each cell of the table, the left entry represents surplus, and the right entry represents cost (obviously the sum of the two gives the value, net of investment cost).

\[
\begin{array}{ccc}
\text{Type} & w_1 & w_2 & w_3 \\
\text{\( \alpha \)} & \Delta_M, c_L & \Delta_H, c_L & -\Delta_H, c_H \\
\text{\( \beta \)} & 0, 0 & \Delta_L, c_L & \Delta_S, 0 \\
\end{array}
\] (1)

For the remainder of the paper, we take these parameters to satisfy the following.

**Assumption 1. Parameter Values:** The values of cost and surplus in the matrix in (1) satisfy\(^7\)

(i) \( 0 < \Delta_L < \Delta_M < \Delta_H \)

and

\(^6\)The gross value is therefore computed as the sum of cost, surplus and \( I \), while the gross cost is the cost value reported in table (1).

\(^7\)To fix ideas, it might be useful to consider one possible set of values that satisfy all the conditions needed. These are \( \Delta_L = 2, \Delta_M = 20, \Delta_H = 24, \Delta_S = 60, c_L = 1 \) and \( c_H = 95 \).
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(ii) \[ \Delta_M + \Delta_H < \Delta_S \]

and

(iii) \[ \Delta_H + \Delta_S + \frac{\Delta_M}{2} < c_H < \Delta_S + 2 \Delta_M \]

and

(iv) \[ 0 < c_L < \Delta_H - \Delta_M - \Delta_L \]

The costs and values of the three widgets are not contractible. Any contract between \( B \) and \( S \) can only specify the widget(s) to be traded, and price(s).

We interpret this contractibility assumption in the following way. The Court can only observe (verify) which one of \( w_1 \) or \( w_2 \) is specified in any contract, and whether the correct widget is traded or not as prescribed, and the appropriate price paid.

It is important to notice that the Court never has information that is superior to the trading parties. In fact, ex-ante the Court does not know \( B \)'s type, and hence has information that matches the seller’s. Ex-post the Court has information that is inferior to both trading parties, since \( S \) will eventually discover his cost of production and hence \( B \)'s type.\(^8\)

To keep matters simple, we assume that \( B \) has all the bargaining power at the ex-ante contracting stage, while \( S \) has all the bargaining power ex-post. The flavor of our results would be preserved under less extreme assumptions about bargaining power. What is needed is that \( B \) does not have full bargaining power ex-post since this would eliminate the need for an ex-ante contract. Even without a contract \( B \) would invest in one of the specific widgets (depending on his type), and all prices could be determined ex-post.

To sum up, the timing and relevant decision variables available to the trading parties are as follows.

The buyer learns his type before meeting the seller. Then \( B \) and \( S \) meet at the ex-ante contracting stage. At this point \( B \) makes a take-it-or-leave-it offer of a contract

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\(^8\)See the timing structure of the model described in detail below.
to $S$, which $S$ can accept or reject. A contract consists of a pair $s_i = (w_i, p_i)$, with $i = 1, 2$ specifying a single widget to trade and at which price. After a contract (if any) is signed, $B$ decides whether to invest or not, and in which of the specific widgets.

After investment takes place (if it does), the bargaining power shifts to the seller and we enter the ex-post stage. At this point $S$ makes a take-it-or-leave-it offer to $B$ on whether to trade any widget not previously contracted on and at which price, which $B$ can accept or reject. Without loss of generality, we can restrict $S$ to make a take-it-or-leave-it offer to $B$ on whether to trade $w_3$ and at which price $p_3$. After $B$ decides whether to accept or reject $S'$s ex-post offer (if any), production takes place. First $S$ produces the relevant widgets and then he learns his cost.\footnote{The reason to assume that production costs are sunk before $S$ learns what they are is to prevent the possibility of ex-post revelation games a la Moore and Repullo (1988) and Maskin and Tirole (1999). We return to this point at some length below.} Finally, delivery and payment occur according to contract terms.

\subsection*{2.2. Active Courts}

The trading setup described in Subsection 2.2 is effectively a two-player game between $B$ and $S$. The Court is a “dummy” player whose strategy is fixed. It simply enforces the contract terms, by imposing large penalties if they are not observed. As a result, delivery and payment occur in the last stage of the game, exactly as agreed.

In the reminder of the paper we model the active Court as a third player, $C$, who makes a non-trivial choice before any contract is signed at the ex-ante stage. In particular, $C$ can credibly announce that it will enforce some contracts, but not others. This announcement is known to both $B$ and $S$ at the time of contracting.

The information of $B$, $S$ and $C$ and their bargaining power remain as described above. The timing, investment requirements and all the elements of the matrix in (1) also stay the same.

The Court announces a set of ex-ante contracts $U$ which will be “upheld” and a set of ex-ante contracts $V$ which will be “voided.” There are two contracts in all to be considered, one of the type $s_1 = (w_1, p_1)$ and another of the type $s_2 = (w_2, p_2)$.\footnote{The reason to assume that production costs are sunk before $S$ learns what they are is to prevent the possibility of ex-post revelation games a la Moore and Repullo (1988) and Maskin and Tirole (1999). We return to this point at some length below.}
We restrict $C$ to be able to announce that certain contracts will be upheld or voided, *only according to the widget involved*. Therefore $U$ and $V$ are two mutually exclusive subsets of $\{s_1, s_2\}$ with $U \cup V = \{s_1, s_2\}$, so that effectively the Court’s strategy set consists of a choice of $V \subseteq \{s_1, s_2\}$.

For the moment we restrict $C$ to make deterministic announcements; each contract is either in $V$ or not with probability one.

If $V = \emptyset$ so that all contracts are enforced, then the model is exactly as described in Subsection 2.1 above. If on the other hand one or two contracts are in $V$, in the final stage of the game $B$ and $S$ are free to renegotiate the terms (price and delivery) of any widget in the voided contract, regardless of anything that was previously agreed. Notice that, by our assumptions on bargaining power, this means that $S$ is free to make a take-it-or-leave-it offer to $B$ of a price $p_i$ at which any $w_i$ with voided contract terms is to be delivered.

The Court is a welfare-maximizing player. It chooses $V$ so as to maximize its payoff which equals the sum of the payoffs of $B$ and $S$.

Before proceeding with the equilibrium analysis, for completeness, we identify the efficient investment and trading outcome. The following is stated without proof since it is an obvious consequence of the costs and surplus matrix (1).
**Remark 1. Efficient Trade:** The unique efficient investment and trading outcome is as follows. Both types of $B$ invest and trade $w_2$. The type $\beta$ buyer trades $w_3$, while the type $\alpha$ buyer does not.

Since the two types of $B$ are equally likely, the total amount of expected surplus (net of investment) in this case is $\frac{\Delta_S}{2} + \frac{\Delta_H}{2} + \frac{\Delta_L}{2}$. By definition, this is also the Court’s payoff.

Efficiency is the benchmark to evaluate the equilibria of the model, which we are now ready to characterize in the two cases of passive and active Courts.\(^{13}\)

### 3. Passive Courts

As we anticipated, when all contracts are enforced, inefficient pooling obtains in equilibrium

**Proposition 1. Equilibrium With A Passive Court:** Suppose the Court enforces all contracts, and that Assumption 1 holds. Then the unique equilibrium outcome of the model is that the two types of buyer pool with probability one: they both invest and trade $w_2$ at a price $p_2 = c_L$, and they both trade $w_3$ at a price $p_3 = \Delta_S$.

The total amount of expected surplus (net of investment) in this case is given by $\frac{\Delta_S}{2} + \frac{\Delta_L}{2}$. By definition, this is also the Court’s payoff.

The equilibrium outcome in Proposition 1 is inefficient in the sense that, in equilibrium $w_3$ is traded by the type $\alpha$ buyer; this trade generates a net surplus of $-\Delta_H$.

The reason separation is impossible to sustain as an equilibrium outcome with passive Courts is not hard to outline. In any separating equilibrium, it is clear that the type $\beta$ buyer would trade $w_3$ ex-post for a price $p_3 = \Delta_S$. The type $\beta$ buyer would

\(^{13}\)Throughout the paper, by equilibrium we mean a Sequential Equilibrium (Kreps and Wilson 1982), or equivalently a Strong Perfect Bayesian Equilibrium (Fudenberg and Tirole 1991), of the game at hand. We do not make use of any further refinements. However, it should be pointed out that whenever we assert that something is an equilibrium outcome, then it is the outcome of at least one Sequential Equilibrium that passes the Intuitive Criterion test of Cho and Kreps (1987).
also trade $w_2$ for a price $p_2 = c_L$ (this is in fact true in any equilibrium in which the Court does not void contracts for $w_2$). Given that the type $\beta$ buyer trades both $w_2$ and $w_3$, the type $\alpha$ will always gain by deviating and pooling with the the type $\beta$ buyer.

4. Active Courts

As we mentioned above, a Court that actively intervenes and voids contracts for $w_2$ will be able to induce separation between the two type of buyer and increase expected welfare.

**Proposition 2. Equilibrium With An Active Court:** Suppose the Court is an active player that can choose $V$ as above, and that Assumption 1 holds. Then the unique equilibrium outcome of the model is that $C$ sets $V = \{s_2\}$ and the two types of buyer separate: the type $\alpha$ buyer invests and trades $w_1$ at a price $p_1 = c_L$ and does not trade $w_3$; the type $\beta$ buyer does not invest and only trades $w_3$ at a price $p_3 = \Delta S$.

The total amount of expected surplus (net of investment) in this case is given by \( \frac{\Delta S}{2} + \frac{\Delta M}{2} \). By definition, this is also the Court’s payoff.

When the Court voids contracts for either $w_1$ or $w_2$, the corresponding widget will not be traded in equilibrium. This would be true for completely obvious reasons if the Court’s voiding makes the trade not feasible. It is also true when the Court allows in principle the trade of the widget ex-post acting as a minimal enforcement agency (see footnote 11 above). This is because a classic hold-up problem obtains in our model, driven by the relationship- and widget-specific investment. Given that the seller has all the bargaining power ex-post, unless an ex-ante contract is in place the buyer will be unable to recoup the cost of his relationship- and widget-specific investment.

To see why the Court’s intervention induces the two types of buyer to separate at the contract offer stage consider the incentives of the type $\alpha$ buyer to deviate from the separating equilibrium described in Proposition 2. With a passive Court, pooling
with the type $\alpha$ buyer involved positive payoffs both in the trade of $w_2$ and in that of $w_3$ ex-post. Now that the Court renders the trade of $w_2$ impossible in equilibrium, the payoff to the type $\alpha$ buyer from deviating to pool with the type $\beta$ buyer comes only from the ex-post trade of $w_3$. This decrease is enough to sustain the separating equilibrium of Proposition 2.

The Court’s intervention has two direct effects. One is separation, so that the type $\alpha$ buyer no longer inefficiently trades $w_3$, and the other is the lack of trade of $w_2$. While the first increases expected welfare, the second reduces it. Overall expected welfare increases.

5. Ambiguous Courts

Propositions 1 and 2 together say that while inefficient pooling obtains when the Court enforces all contracts, this can be avoided when the Court credibly announces that it will void any contract for $w_2$.

In the equilibrium in Proposition 2 the Court effectively forbids a profitable investment and trade. The surplus, net of $I$, generated by $w_2$ is strictly positive for both types of buyer. A natural question then arises at this point. Can any of this lost surplus be recovered by the Court, without loosing the advantage gained by inducing separation as in Proposition 2.

The answer is that, provided that $I$ is not too large, some of this surplus can in fact be recovered by inducing an equilibrium that is like the one in Proposition 2, but in which the trade of $w_2$ is sometimes allowed. Suppose the Court voids contracts for $w_2$ with probability strictly between zero and one. If this probability is too small, then using the logic of Proposition 1, inefficient pooling will obtain in equilibrium. The probability that the Court voids contracts for $w_2$ must be high enough for the parties to ever separate.

If the probability that the Court voids contracts for $w_2$ is too large then neither type of buyer will invest in $w_2$. This is because, the investment in $w_2$ yields a return of zero in case the contract is voided. So, if the contract is voided with too high a probability, the expected return from investment will not cover the cost.
To sum up, some of the lost surplus from $w_2$ may be recovered in an equilibrium that is similar to the one in Proposition 2 but in which $w_2$ is sometimes traded if the probability that the Court voids contracts for $w_2$ is neither too low nor too high. However, this range shrinks as investment becomes more expensive. As it turns out $I$ has to be sufficiently low for this range to exist.

What is the interpretation of a stochastic Court? At first sight, this may seem an exotic possibility. We believe instead that this is a natural case to consider with an appealing interpretation: ambiguous Courts. In many instances the state of the Law or of the body of precedents, or both, will be ambiguous, creating genuine uncertainty for the contracting parties as to what the Court ruling will be.

Denote by $\mu_1$ and $\mu_2$ respectively the probabilities that the Court voids contracts for $w_1$ and $w_2$ respectively. Let $F = (\mu_1, \mu_2)$.

**Proposition 3. Equilibrium With A Stochastic Court:** Suppose Assumption 1 holds and that the Court is an active player that can choose $F$ as above.

Assume also that $I > 0$ satisfies:

$$I < \frac{\Delta L (\Delta M + \Delta H + \Delta S - c_H)}{c_H - \Delta L - \Delta M - \Delta S} \quad (2)$$

Then the unique equilibrium outcome of the model is that $C$ sets $F = (0, \mu^*)$ where

$$\mu^* = \frac{c_H - \Delta M - \Delta S}{\Delta H + I} \quad (3)$$

and the two types of buyer separate: the type $\alpha$ buyer invests and trades $w_1$ at a price $p_1 = c_L$ and does not trade $w_3$ while the type $\beta$ buyer invests and trades $w_2$ with probability $(1 - \mu^*)$ at a price $p_2 = c_L$ and trades $w_3$ at a price $p_3 = \Delta_S$.

The total amount of expected surplus in this case is given by $\frac{\Delta S^2}{2} + \frac{\Delta M^2}{2} + \frac{(1 - \mu^*) \Delta L - \mu^* I}{2}$. By definition, this is also the Court’s payoff.

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14Using the numbers mentioned in footnote 7, the left-hand side of (2) equals $18/13$. If we set $I = 1$ to satisfy (2), from (3) we obtain $\mu^* = 60/100$. 
6. Menu Contracts?

In two separate papers, Maskin and Tirole (1990, 1992) examine the general case of an “Informed Principal” problem. Among other insights, they point out that, under certain conditions a “menu contract” equilibrium may Pareto improve over other types of arrangements.

A menu contract, roughly speaking is a pooling contract offered by different types of Principal which the Agent can accept or reject, before any of the Principal’s private information is revealed. The menu contains an array of different contractual arrangements, one for each possible type of Principal. After the Agent accepts the contract, which immediately becomes binding, the Principal announces his type to the Agent, and hence determines which part of the menu array will regulate their relationship from that point on.

The buyer in our model has private information and, ex-ante, makes a take-it-or-leave-it offer to the seller. Therefore he is an informed Principal. Since our Proposition 1 asserts that with a passive Court the equilibrium outcome is inefficient, it is a legitimate concern whether allowing for menu contracts can yield superior investment and trading outcomes relative to what we have identified above.

Allowing menu contracts changes the terms on which our model justifies Court intervention, but still provides a robust rationale for active Courts.

The effect of allowing menu contracts depends critically on whether we maintain the assumption that $w_3$ is not contractible ex-ante. If we do, our conclusions of Sections 3 and 4 hold essentially unchanged.

If on the other hand we allow ex-ante contracting on $w_3$, as well as menu contracts the picture changes. When menu contracts and ex-ante contracting on $w_3$ are

\[15\text{In the terminology of Maskin and Tirole (1990, 1992) we are in the case of “Common Values” examined in more detail in Maskin and Tirole (1992).}
\[16\text{When } w_3 \text{ is contractible ex-ante, the prices at which each widget is traded, when } w_3 \text{ is traded as well } w_1 \text{ or } w_2, \text{ become indeterminate. The equilibrium trading and investment outcomes are as before. See Proposition 4 below.}
both allowed, if the Court enforces all contracts, *multiple* equilibrium outcomes obtain. Pooling as in Proposition 1 is an equilibrium. However, the model also has an equilibrium in which a (non-trivial) menu contract is offered and the same separating outcome as in Proposition 2 obtains. Clearly, even in this case an active Court has a role in eliminating any possibility for the party to inefficiently pool in equilibrium.

In order to proceed, we need to be precise about two new elements of the model: the set of possible contracts when ex-ante contracting on $w_3$ is allowed, and the set of possible menu contracts built on the basis of these.

When $w_3$ can be contracted ex-ante, two types of contracts need to be considered (still abstracting from menu ones). For want of a better term we label them *simple* and *bundle*. A simple contract is as before and consists of a pair $s_i = (w_i, p_i)$, with $i = 1, 2, 3$, specifying a single widget to trade and at which price. The only change is the addition of $s_3$.

A bundle contract consists of an offer to trade a specific widget $w_i$ $i = 1, 2$ and the regular widget $w_3$ at prices $p_i$ and $p_3$ respectively; a bundle contract is denoted by a triplet $b_{1,3} = (w_i, p_i, p_3)$. So, as well as possible offers of $s_3$, $b_{1,3}$ and $b_{2,3}$, we now need to consider any possible choice of $V \subseteq \{s_1, s_2, s_3, b_{1,3}, b_{2,3}\}$.

We also need to specify what a menu contract is. This is not hard to define. A menu ex-ante contract is a pair $(m^\alpha, m^\beta)$ with both $m^\alpha$ and $m^\beta$ elements of $\{s_1, s_2, s_3, b_{1,3}, b_{2,3}\}$ if ex-ante contracting on $w_3$ is allowed, and just elements of $\{s_1, s_2\}$ if ex-ante contracting on $w_3$ is not allowed. The interpretation is that $m^\alpha$ is the contract that rules if the Buyer announces that he is of type $\alpha$ after the contract is accepted and becomes binding, while $m^\beta$ is the relevant arrangement if the Buyer announces that he is of type $\beta$.

With little loss of generality, we take $V \subseteq \{s_1, s_2, s_3, b_{1,3}, b_{2,3}\}$ and $V \subseteq \{s_1, s_2\}$.

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17There is no need to consider any other possible bundles since trading both $w_1$ and $w_2$ is never profitable. The two specific widgets are mutually exclusive since, by assumption, the buyer can only undertake one widget-specific investment.

18For simplicity, we restrict attention to pure strategy equilibria when menu contracts are allowed. That is, we do not allow the buyer to randomize across different menu contracts.
depending on whether ex-ante contracting on \( w_3 \) is allowed or not, even when menu contracts are allowed. In essence, we are restricting the Court to uphold or void on the basis of the applicable (based on the Buyer’s declaration) part of the menu.

**Proposition 4. Menu Contracts and Non-Contractible \( w_3 \):** Assume that menu contracts are allowed and that \( w_3 \) is not ex-ante contractible. Suppose that Assumption 1 holds.

Then Propositions 1 and 2 still hold. In particular the equilibrium payoff of a passive Court is \( \Delta_S + \Delta_L \) while the equilibrium payoff of an active Court is \( \Delta_S + \Delta_M \).

**Proposition 5. Menu Contracts and Contractible \( w_3 \) – Passive Court:** Let menu contracts be allowed and assume that \( w_3 \) is ex-ante contractible. Let Assumption 1 hold, and assume that the Court upholds all contracts. Then:

(i) There is an equilibrium of the model in which the trading and investment outcome is as in Proposition 1. The menu contract in this equilibrium is degenerate in the sense that both types of buyer offer the same menu contract and \( m^\alpha = m^\beta \). Both types of buyer invest in and trade trade \( w_2 \) and both types of buyer trade \( w_3 \). The total amount of expected surplus (net of investment) in this case is given by \( \Delta_S + \Delta_L \).

(ii) There is an equilibrium of the model in which the trading and investment outcome is the same as in Proposition 2: the type \( \alpha \) buyer invests in and trades \( w_1 \), and the type \( \beta \) buyer trades \( w_3 \). The menu contract is this equilibrium is non-degenerate in the sense that both types of buyer offer the same contract and \( m^\alpha \neq m^\beta \). The type \( \alpha \) buyer invests in and trades \( w_1 \), while the type \( \beta \) buyer does not invest in either \( w_1 \) or \( w_2 \), and trades \( w_3 \). The total amount of expected surplus (net of investment) in this case is given by \( \Delta_S + \Delta_M \).

(iii) There is no equilibrium of the model in which the total amount of expected surplus (net of investment) exceeds \( \Delta_S + \Delta_M \).
Proposition 6. Menu Contracts and Contractible $w_3$ – Active Court: Assume that menu contracts are allowed and that $w_3$ is ex-ante contractible. Suppose that Assumption 1 holds. Suppose that the Court voids all contracts involving $w_2$. In other words suppose that $V = \{s_2, b_{2,3}\}$.

Then the unique equilibrium trading and investment outcome of the ensuing subgame is the same as in Proposition 2: the type $\alpha$ buyer invests in and trades $w_1$, and the type $\beta$ buyer trades $w_3$.

Any equilibrium that sustains this equilibrium outcome is non-degenerate in the sense that both types of buyer offer the same menu contract and $m^\alpha \neq m^3$. The type $\alpha$ buyer invests in and trades $w_1$, while the type $\beta$ buyer does not invest in either $w_1$ or $w_2$, and trades $w_3$.

In equilibrium, the total expected surplus (net of investment) is the maximum possible when menu contracts are allowed and the Court enforces all contracts, namely $\Delta S_2 + \Delta M_2$.

7. Conclusions

Our main result (Propositions 1 and 2) can be viewed as identifying a kind of “second best” phenomenon in an incomplete contract world. We start with a model in which some degree of contractual incompleteness is assumed (the costs and values of each widget are not verifiable and hence not contractible). In this world it is in fact welfare-improving to impose further incompleteness by making some contracts effectively impossible in equilibrium. This is what our active Court does.\(^{19}\)

Appendix

Lemma A.1: Consider either the model with passive Courts or any subgame of the model with active Courts following the Court’s choice of $V$. In any equilibrium of the model with passive

\(^{19}\)Bernheim and Whinston (1998) find that when coarse contracts are assumed in the first place then, under some conditions, equilibrium contracts will be even coarser than the constraints impose. Our main result does not assert that contracts will be coarse (or incomplete) in equilibrium. Rather it asserts that imposing incompleteness will increase expected welfare.
Courts, or of the subgame, \( w_3 \) is traded with positive probability by the type \( \beta \) buyer. Moreover, the equilibrium price of \( w_3 \) is \( p_3 = \Delta_S \).

**Proof:** We distinguish four, mutually exclusive, exhaustive cases.

Consider first a possible separating equilibrium in which the two \( \mathcal{B} \) types each offer a distinct contract at the ex-ante stage. In this case, at the ex-post stage it is a best reply for type \( \beta \) buyers to accept offers to trade \( w_3 \) at a \( p_3 \leq \Delta_S \). Their unique best reply is instead to reject any offers to trade \( w_3 \) at any \( p_3 > \Delta_S \). By standard arguments it then follows that in equilibrium it must be that \( w_3 \) is traded between \( \mathcal{S} \) and type \( \beta \) buyers at a price \( p_3 = \Delta_H \).

The second case is that of a possible pooling equilibrium in which both types of \( \mathcal{B} \) offer the same ex-ante contract to \( \mathcal{S} \) with probability 1. In this case the beliefs of \( \mathcal{S} \) at the ex-post stage are that \( \mathcal{B} \) is of either type with equal probability. The type \( \beta \) buyer best reply to offers to trade \( w_3 \) at the ex-post stage is as in the previous case. It is a best reply for type \( \alpha \) buyers to accept offers to trade \( w_3 \) at a \( p_3 \leq c_H - \Delta_H \). Their unique best reply is to reject any offers to trade \( w_3 \) at any \( p_3 > c_H - \Delta_H \). Since Assumption 1 (parts ii and iii) implies that \( c_H - \Delta_H > \Delta_S \), it now follows by standard arguments that only two outcomes are possible in equilibrium: either \( w_3 \) is traded between \( \mathcal{S} \) and both types of \( \mathcal{B} \) at a price \( p_3 = \Delta_S \), or \( w_3 \) is not traded at all because \( \mathcal{S} \) does not make an offer that is accepted by either type of \( \mathcal{B} \). The seller’s expected profit from trading \( w_3 \) at \( p_3 = \Delta_S \) is given by \( \Delta_S - c_H / 2 \), which is positive by Assumption 1 (parts i and iii). Therefore, \( \mathcal{S} \) will choose to offer to trade \( w_3 \) at \( p_3 = \Delta_S \). Hence the conclusion follows in this case.

The third case is that of a possible semi-separating equilibrium in which the type \( \beta \) buyer offers a separating contract at the ex-ante stage with probability strictly between zero and one. In this case, the same logic of the first case applies to show that in equilibrium it must be the case that \( \mathcal{S} \) and the type \( \beta \) buyers who offer the separating contract trade \( w_3 \) at \( p_3 = \Delta_S \).

The fourth and last case is that of a possible semi-separating equilibrium in which the type \( \beta \) buyer offers a separating contract at the ex-ante stage with probability zero. Since some type \( \alpha \) buyers are separating, there must be some contract that the type \( \beta \) buyer offers in equilibrium which is offered by the type \( \alpha \) buyer with a strictly lower probability. Since the ex-ante probabilities of the two types of buyer are the same, there is some contract offered in equilibrium by the type \( \beta \) buyer such that the seller’s beliefs after receiving the offer are that he is facing a type \( \alpha \) buyer with probability \( \nu \in (0, 1/2) \). After this contract is offered and accepted, in any Perfect Bayesian Equilibrium, the seller’s beliefs when he contemplates making an offer to trade \( w_3 \) must also be that he faces the type \( \alpha \) buyer with probability \( \nu \). Using the same logic as in the second case, only two possibilities remain. Either \( w_3 \) is traded at \( p_3 = \Delta_S \), or \( \mathcal{S} \) makes an offer that is not accepted. Given the beliefs we have described, the seller’s expected profit from trading \( w_3 \) at \( p_3 = \Delta_S \), is \( \Delta_S - \nu c_H \).
which is positive using \( \nu < 1/2 \) and Assumption 1 (parts ii and iii). Hence \( S \) will choose to trade \( w_3 \) at \( p_3 = \Delta_S \), and the conclusion follows in this case.

**Lemma A.2:** Suppose that \( C \) enforces all contracts. Then in any equilibrium of the model \( w_2 \) is traded with probability one by the type \( \beta \) buyer at a price \( p_2 = c_M \).

**Proof:** Since the cost of \( w_2 \) is independent of \( B \)'s type it is obvious that if it is traded, then it is traded at \( p_2 = c_L \).

Suppose by way of contradiction that there exists an equilibrium in which with positive probability \( w_2 \) is not traded by the type \( \beta \) buyer. From Lemma A.1 we know that in this equilibrium some type \( \beta \) buyers trade \( w_3 \) at a price \( p_3 = \Delta_S \). Therefore, type \( \beta \) buyers have a payoff of at most 0. (This follows from the fact that their expected profit from the \( w_3 \) trade is zero, and the maximum profit they can possibly make by trading \( w_1 \) is 0.) Consider now a deviation by the type \( \beta \) buyer to offering \( w_2 \) at \( p_2 = c_L + \varepsilon \) with probability one. It is a unique best response for the seller to accept offers to trade \( w_2 \) at any \( p_2 > c_L \). It then follows that the type \( \beta \) buyer can deviate to such offer and achieve a payoff of \( \Delta_L - \varepsilon \). For \( \varepsilon \) sufficiently small this is clearly a profitable deviation for the type \( \beta \) buyer.

**Lemma A.3:** Suppose \( C \) enforces all contracts. Then in any equilibrium of the model the type \( \alpha \) buyer offers a contract to trade \( w_1 \) with probability zero.

**Proof:** Notice that by Lemma A.2 in any equilibrium the type \( \beta \) buyer trades \( w_2 \) with probability one. Suppose by way of contradiction that there exists an equilibrium in which the type \( \alpha \) buyer separates with positive probability and offers a contract to trade \( w_1 \). In this case, the type \( \alpha \) buyer’s payoff must be \( \Delta_M \). This follows from the fact that, by separating, the type \( \alpha \) buyer must be trading \( w_1 \) at a price \( p_1 = c_L \) and, since he separates, \( S \) will not trade \( w_3 \) with him.

Suppose now that the type \( \alpha \) buyer deviates to pool with the type \( \beta \) buyers who trade \( w_2 \) at \( p_2 = c_L \) and then trade \( w_3 \) at \( p_3 = \Delta_S \). By Lemmas A.1 and A.2 we know that the type \( \beta \) buyer behaves in this way with positive probability. Following this deviation the type \( \alpha \) buyer’s payoff is \( \Delta_H + c_H - \Delta_H - \Delta_S \). The latter, by Assumption 1 (parts i, ii and iii) is greater than \( \Delta_M \). Hence this is a profitable deviation for the type \( \alpha \) buyer.

**Lemma A.4:** Suppose \( C \) enforces all contracts. Then in any equilibrium of the model \( w_2 \) is traded with probability one by the type \( \alpha \) buyer at a price \( p_2 = c_L \).
Proof: Since the cost of $w_2$ is independent of $B$’s type it is obvious that if it is traded, then it is traded at $p_2 = c_L$.

Suppose that the claim were false. Using Lemma A.3 we then know that, in some equilibrium, with positive probability the type $\alpha$ buyer trades neither $w_1$ nor $w_2$. By Lemma A.2 a type $\alpha$ buyer who does not trade $w_2$ actually separates from the type $\beta$ buyer. Hence in any equilibrium in which with positive probability the type $\alpha$ buyer trades neither $w_1$ nor $w_2$ the type $\alpha$ buyer’s payoff is at most zero. (The seller will not trade $w_3$ with him because of separation, and he makes no profit on either $w_1$ or $w_2$ since he does not trade them.)

As in the proof of Lemma A.3 the type $\alpha$ buyer has a profitable deviation from this putative equilibrium. He can pool with the type $\beta$ buyers who trade $w_2$ at $p_2 = c_L$ and then trade $w_3$ at $p_3 = \Delta_S$. After this deviation the type $\alpha$ buyer’s payoff is $\Delta_H + c_H - \Delta_H - \Delta_S$, which is positive by Assumption 1 (parts i and iii).

Lemma A.5: Suppose that $C$ enforces all contracts. Then in any equilibrium of the model $w_3$ is traded with probability one by both types of $B$ at a price $p_3 = \Delta_S$.

Proof: From Lemmas A.2 and A.4 we know that the two types of $B$ pool with probability one at the ex-ante stage. The same reasoning as in the second case considered in the proof of Lemma A.1 now ensures that in equilibrium $w_3$ is traded with probability one by both types of $B$ at a price $p_3 = \Delta_S$. ■

Proof of Proposition 1: The claim is a direct consequence of Lemmas A.2, A.4 and A.5. ■

Lemma A.6: Consider the model with an active Court, and any of the subgames following $C$ choosing a $V$ that contains $w_i$, $i = 1, 2$. In any equilibrium of such subgames neither type of $B$ invests in $w_i$, and hence it is not traded.

Proof: If $w_i \in V$ then the terms of its trade can be freely re-negotiated at the ex-post stage, when $S$ makes a take-it-or-leave-it offer to $B$, regardless of anything previously agreed.

Now suppose that in any equilibrium both types of $B$ invest in $w_i \in V$ with positive probability. Then by standard arguments in any equilibrium it must be that $S$ offers to trade $w_i$ at a price $p_i$ that makes one of the two $B$ types indifferent between accepting and rejecting the offer at the ex-post stage. But this since $I > 0$ this must mean that one of the $B$ types has an overall payoff equal to $-I$. Since either type of buyer can always guarantee a payoff of zero (by not investing and not trading) we can then conclude that in any equilibrium of any of these subgames it cannot be the case that both types of $B$ invests in $w_i \in V$ with positive probability.
Suppose then in any equilibrium only one type of $B$ invests in $w_i \in V$ with positive probability. Then by standard arguments in any equilibrium it must be that $S$ offers to trade $w_i$ at a price $p_i$ that makes the type of buyer who is trading $w_i$ indifferent between accepting and rejecting the offer at the ex-post stage. But this since $I > 0$ this must mean that this type of $B$ has an overall payoff equal to $-I$. Since, as before, this type of buyer can always guarantee a payoff of zero we can now conclude that in any equilibrium of any of these subgames it must be that neither type of $B$ invests in $w_i \in V$ with positive probability. ■

**Lemma A.7:** Consider the model with an active Court. In any equilibrium of the subgame following $C$ setting $V = \{w_2\}$ the type $\alpha$ buyer trades $w_1$ with probability one.

**Proof:** From Lemma A.6 we know that in this case neither type of $B$ invests in $w_2$, and hence it is not traded.

Suppose that the type $\alpha$ buyer invests in $w_1$ and trades it. His payoff in this case is at least $\Delta_M$. This is because clearly, in any equilibrium, $p_1$ can be at most $c_L$, and at worst he is unable to trade $w_3$.

Suppose that instead the type $\alpha$ buyer does not invest in $w_1$ and hence does not trade it. Then his payoff is at most $c_H - \Delta_H - \Delta_S$. This is because, using Lemma A.1, at best he will be able to trade $w_3$ at a price $p_3 = \Delta_S$. Using Assumption 1 (part iii) we know that $\Delta_M > c_H - \Delta_H - \Delta_S$, and hence the argument is complete. ■

**Lemma A.8:** Consider the model with an active Court. In any equilibrium of the subgame following $C$ setting $V = \{w_2\}$ the type $\beta$ does not invest in either $w_1$ or $w_2$, separates from the type $\alpha$ buyer, and only trades $w_3$ at a price $p_3 = \Delta_S$.

**Proof:** From Lemma A.6 we know that in this case neither type of $B$ invests in $w_2$, and hence it is not traded.

Suppose that the type $\beta$ buyer invests in $w_1$. Then his payoff must be negative. This is because, using Lemma A.1, he either trades $w_3$ at a price $p_3 = \Delta_S$ or does not trade $w_3$ (in either case the profit is zero), and using Lemma A.7 he trades $w_1$ at a price $p_1$ which is at least $c_L/2$.

Since either type of buyer can always guarantee a payoff of zero (by not investing, making offers that must be rejected, and rejecting all ex-post offers) we can then conclude that the type $\beta$ buyer does not invest in $w_1$.

Therefore, we know that the type $\beta$ buyer does not invest in either $w_1$ or $w_2$. Using Lemma A.7 and the same reasoning as in the first case of Lemma A.1 we can now conclude that the type $\beta$ buyer trades $w_3$ at a price $p_3 = \Delta_S$. ■
Lemma A.9: Consider the model with an active Court. Suppose that $C$ sets $V = \{w_2\}$, then the two types buyer separate: the type $\alpha$ buyer invests in $w_1$ and only trades $w_1$ at a price of $p_1 = c_H$; the type $\beta$ buyer does not invest in either $w_1$ or $w_2$ and only trades $w_3$ at a price $p_3 = \Delta_S$.

By choosing $V = \{w_2\}$ the Court achieves a payoff of $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

Proof: The claim is a direct consequence of Lemmas A.7 and A.8.

Lemma A.10: Consider the model with an active Court. Suppose that $C$ sets $V = \{w_1\}$. Then the unique equilibrium outcome is that the two types of buyer pool with probability one: they both invest and trade $w_2$ at a price $p_2 = c_H$, and they both trade $w_3$ at a price $p_3 = \Delta_S$.

By choosing $V = \{w_1\}$ the Court achieves a payoff of $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

Proof: The proof essentially proceeds in the same way as the proof of Proposition 1. In fact by setting $V = \{w_1\}$, the Court simply takes away the possibility that the parties may trade $w_1$ via Lemma A.6. The details are omitted for the sake of brevity.

Lemma A.11: Consider the model with an active Court. Suppose that $C$ sets $V = \{w_1, w_2\}$. Then the two types of buyer pool: they do not invest in either $w_1$ or $w_2$ and they trade $w_3$ at $p_3 = \Delta_S$.

By choosing $V = \{w_1, w_2\}$ the Court achieves a payoff of $\frac{\Delta_S}{2} - \frac{\Delta_H}{2}$.

Proof: The claim follows immediately from Lemma A.6 using the same reasoning as in the second case of the proof of Lemma A.1.

Proof of Proposition 2: Using Assumption 1 (part i), the claim is an immediate consequence of Lemmas A.9, A.10 and A.11.

Lemma A.12: Consider the model with a stochastic Court, and any of the subgames following $C$ choosing any feasible $F$. In any equilibrium of any such subgame, $w_3$ is traded with positive probability by the type $\beta$ buyer. Moreover, the equilibrium price of $w_3$ is $p_3 = \Delta_S$.

Proof: The argument is identical to the proof of Lemma A.1. We do not repeat the details.

Lemma A.13: Consider the model with a stochastic Court, and any of the subgames following $C$ choosing a $F$ that contains $\mu_1 \in (0,1]$. In any equilibrium of such subgames the type $\beta$ buyer does not invests in $w_1$, and hence he does not trade $w_1$. 
**Proof:** Assume by way of contradiction that in some equilibrium the type $\beta$ buyer invests in $w_1$ and $w_1$ is traded at $p_1 \geq 0$. Then the type $\beta$ buyer’s payoff in this equilibrium would be $-(1-\mu_1)p_1-\mu_1 I$. Clearly for every $\mu_1 \in (0,1]$ this payoff is negative. Therefore the type $\beta$ buyer has a profitable deviation by not offering any contract and not investing. This deviation yields a zero payoff. ■

**Lemma A.14:** Consider the model with a stochastic Court, and any of the subgames following $C$ choosing a $\mathcal{F}$ that contains $\mu_1 = 0$. In any equilibrium of such subgames the type $\beta$ buyer does not invests in $w_1$, and hence he does not trade $w_1$.

**Proof:** Recall that by Lemma A.12, in equilibrium the type $\beta$ buyer trades $w_3$ for a price $p_3 = \Delta_S$ with positive probability.

Suppose that the outcome of the statement of the Lemma did obtain in some equilibrium. Clearly, it would also have to be the case that the type $\beta$ buyer trades $w_1$ for a price $p_1 \in [0,c_L]$.

Suppose that, in this putative equilibrium the type $\alpha$ buyer trades $w_1$ with strictly positive probability. Then it must be that $p_1 > 0$. Hence the payoff of the type $\beta$ buyer would be $-p_1 < 0$. Therefore the type $\beta$ buyer has a profitable deviation by not offering any contract and not investing. This deviation yields a zero payoff.

Suppose that in this putative equilibrium the type $\alpha$ buyer trades $w_2$ with positive probability. His payoff then would have to be $(1-\mu_2)-\mu_2 I$. If instead he deviates and pools with the type $\beta$ buyer he gets $\Delta_M + c_L + c_H - \Delta_H - \Delta_S$. By Assumption 1 (part iii) this is a profitable deviation for the type $\alpha$ buyer.

Lastly, suppose that in this putative equilibrium the type $\alpha$ buyer does not invest and trade either $w_1$ or $w_2$. Then he would be unable to trade $w_3$ ex-post since he would be separating at the contract offer stage. Hence his payoff would have to be 0. By deviating and pooling with the the type $\beta$ buyer and trading $w_1$ and $w_3$, his payoff would be $\Delta_M + c_L + c_H - \Delta_H - \Delta_S$ By Assumption 1 (parts i and iii) this is a profitable deviation for the type $\alpha$ buyer. ■

**Lemma A.15:** Consider the model with a stochastic Court, and any of the subgames following $C$ choosing any feasible $\mathcal{F}$. Then it is not possible that in any equilibrium of such subgames the type $\alpha$ buyer invests and trades $w_2$ while the type $\beta$ does not invest in (and hence does not trade) $w_2$. 
Should Courts Always Enforce What Contracting Parties Write?

Proof: Recall that by Lemma A.12, in equilibrium the type $\beta$ buyer trades $w_3$ for a price $p_3 = \Delta_S$ with positive probability.

Suppose that the outcome of the statement of the Lemma did obtain in some equilibrium. By Lemmas A.13 and A.14 we know that it must be the case that the type $\alpha$ buyer does not invest in and trade either $w_1$ or $w_2$. Therefore he must be trading $w_3$ ex-post at $p_3 = \Delta_S$.

In this putative equilibrium the type $\alpha$ buyer obtains a payoff of $(1 - \mu_2)\Delta_H - \mu_2 I$. If instead he deviates and pools with the type $\beta$ buyer in trading only $w_3$ ex-post he gets $c_H - \Delta_H - \Delta_S$. By Assumption 1 (parts i, iii and iv), using (2), provided that $\mu_2 \geq \mu^{**} = \Delta_L/(\Delta_L + I)$ this is a profitable deviation for the type $\alpha$ buyer. Suppose then that $\mu_2 < \mu^{**}$. In this putative equilibrium the type $\beta$ buyer gains a payoff of 0. If instead he deviates to pooling with the type $\alpha$ buyer and trades $w_2$ at $p_2 = c_L$ he obtains a payoff of $(1 - \mu_2)\Delta_L - \mu_2 I$. Since $\mu_2 < \mu^{**}$, this is a profitable deviation for the type $\beta$ buyer.

Lemma A.16: Consider the model with a stochastic Court, and any of the subgames following $C$ choosing any feasible $F$. Then it is not possible that in any equilibrium of such subgames the type $\alpha$ buyer does not invest and trade either $w_1$ or $w_2$, while the type $\beta$ invests in and trades $w_2$.

Proof: Recall that by Lemma A.12, in equilibrium the type $\beta$ buyer trades $w_3$ for a price $p_3 = \Delta_S$ with positive probability.

Suppose that the outcome of the statement of the Lemma did obtain in some equilibrium. Then the payoff to the type $\alpha$ buyer would be 0 since he would unable to trade $w_3$ ex-post after separating at the contract offer stage.

Notice that if the type $\beta$ buyer invests in and trades $w_2$ then it must be that $\mu_2 \leq \mu^{**}$, otherwise the type $\beta$ buyer would obtain a negative payoff in equilibrium. Whenever $\mu_2 \leq \mu^{**}$, Assumption 1 (parts i and iii) implies that the type $\alpha$ buyer would obtain a positive payoff by deviating and pooling with the type $\beta$ and investing and trading $w_2$.

Proof of Proposition 3: We deal separately with three different possible scenarios.

The first scenario is that of possible equilibria of subgames following the Court’s choice of $F$ in which the type $\alpha$ buyer invests in and trades $w_1$. Therefore in this scenario the type $\beta$ buyer trades $w_3$ at $p_3 = \Delta_S$ with probability 1. Using Lemmas A.13 and A.14 we know in this scenario only two cases are possible. In equilibrium either the type $\beta$ buyer invests in and trades $w_2$ and $w_3$, or the type $\beta$ buyer only trades $w_3$ ex-post. Consider the first case. For this type of equilibrium to be viable we need the following two sets of conditions to be satisfied. The first set of conditions guarantee that both types of buyer are willing to make their respective investments. These are:

\[ (1 - \mu_1)\Delta_M - \mu_1 I \geq 0 \]
The second set of conditions guarantee that neither type of buyer wants to deviate from the separation that the equilibrium prescribes. These are:

\[(1 - \mu_1) \Delta_M - \mu_1 I \geq (1 - \mu_2) \Delta_H - \mu_2 I + c_H - \Delta_H - \Delta_S \quad (A.3)\]

\[(1 - \mu_2) \Delta_L - \mu_2 I \geq -(1 - \mu_1) c_L - \mu_1 I \quad (A.4)\]

Using Assumption 1 (part iv) it is immediate to see that (A.2) implies (A.4). Therefore, we can safely ignore (A.4). The Court’s payoff in this type of equilibrium is given by

\[\frac{(1 - \mu_1) \Delta_M - \mu_1 I}{2} + \frac{(1 - \mu_2) \Delta_L - \mu_2 I + \Delta_S}{2} \quad (A.5)\]

Observe next that (A.5) tells us that the Court’s payoff is strictly monotonically decreasing in both \(\mu_1\) and \(\mu_2\). It is also easy to verify that if any of the inequalities (A.1), (A.2), (A.3) are satisfied for a pair \((\mu'_1, \mu_2)\) then they will also be satisfied for any pair \((\mu'_1, \mu_2)\) with \(\mu'_1 \in [0, \mu_1]\). Therefore, choosing among the equilibria in this case, the Court would choose an \(F\) such that \(\mu_1 = 0\), and the lowest \(\mu_2\) that guarantees that (A.2) and (A.3) are satisfied. For these two inequalities to be both satisfied we need \(\mu_2\) to be such that

\[\frac{c_H - \Delta_H - \Delta_S}{\Delta_H + I} = \mu^* \leq \mu_2 \leq \mu^{**} = \frac{\Delta_L}{\Delta_L + I} \quad (A.6)\]

It follows from Assumption 1 (parts i and iii) that whenever \(I\) satisfies the inequality in (2), then \(\mu^* \leq \mu^{**}\), so that the range in (A.6) is not empty. Therefore the Court’s payoff in this case is maximized by setting \(F = (0, \mu^*)\). The corresponding Court payoff is

\[\frac{\Delta_M}{2} + \frac{(1 - \mu^*) \Delta_L - \mu^* I}{2} + \frac{\Delta_S}{2} \quad (A.7)\]

Consider now the second case in the first scenario. Recall that in this type of equilibrium the type \(\beta\) buyer does not invest in and trade \(w_2\). It follows that for this type of equilibrium to be viable we must have that

\[\frac{(1 - \mu_2) \Delta_L - \mu_2 I}{2} \leq 0 \quad (A.8)\]

since otherwise he would find it profitable to deviate and trade \(w_2\) at \(p_2 = c_L\). For this type of equilibrium to be viable also need (A.1) to be satisfied, as well as conditions that guarantee that
neither type of buyer wants to deviate from the separation that the equilibrium prescribes. These are

\[(1 - \mu_1)\Delta_M - \mu_1 I \geq c_H - \Delta_H - \Delta_S \quad \text{(A.9)}\]

and

\[0 \geq -(1 - \mu_1)c_L - \mu_1 I \quad \text{(A.10)}\]

The Court’s payoff in this type of equilibrium is given by

\[\frac{(1 - \mu_1)\Delta_M - \mu_1 I}{2} + \frac{\Delta_S}{2} \quad \text{(A.11)}\]

Observe next that (A.11) tells us that the Court’s payoff is strictly monotonically decreasing in \(\mu_1\). It is also easy to verify that if any of the inequalities (A.9), (A.10), (A.11) are satisfied for a pair \((\mu_1, \mu_2)\) then they will also be satisfied for any pair \((\mu_1', \mu_2)\) with \(\mu_1' \in [0, \mu_1]\). Therefore, choosing among the equilibria in this case, the Court would choose an \(F\) such that \(\mu_1 = 0\), and the lowest \(\mu_2\) that guarantees that (A.8) is satisfied. For this inequality to be satisfied we need \(\mu_2\) to be such that

\[\mu_2 \geq \mu^{**} = \frac{\Delta_L}{\Delta_L + I} \quad \text{(A.12)}\]

Therefore the Court’s payoff in this case is maximized by setting \(F = (0, \mu_2)\) with \(\mu_2 \in [\mu^{**}, 1]\). The corresponding Court payoff is

\[\frac{\Delta_M}{2} + \frac{\Delta_S}{2} \quad \text{(A.13)}\]

The second scenario we analyze is that of possible equilibria of subgames following the Court’s choice of \(F\) in which the type \(\alpha\) buyer invests in and trades \(w_2\). From Lemma A.15 we know that it must be that the type \(\beta\) buyer pools with the type \(\alpha\) buyer in trading \(w_2\) at \(p_2 = c_L\) and trades \(w_3\) ex-post at \(p_3 = \Delta_S\) with probability 1.

For this type of equilibrium to be viable we need the following two sets of conditions to be satisfied. The first set of conditions guarantee that both types of buyer are willing to make their respective investments. These are:

\[(1 - \mu_2)\Delta_H - \mu_2 I + c_H - \Delta_H - \Delta_M \geq 0 \quad \text{(A.14)}\]

\[(1 - \mu_2)\Delta_L - \mu_2 I \geq 0 \quad \text{(A.15)}\]
The second set of conditions guarantee that neither type of buyer wants to deviate from the pooling that the equilibrium prescribes. These are:

\[(1 - \mu_2)\Delta_H - \mu_2 I + c_H - \Delta H - \Delta S \geq (1 - \mu_1)(\Delta_M + c_L - p_1) - \mu_1 I \quad (A.16)\]

\[(1 - \mu_2)\Delta_L - \mu_2 I \geq -(1 - \mu_1)p_1 - \mu_1 I \quad (A.17)\]

where \(p_1 \in [0, c_L]\) is the lowest price that the seller will accept for \(w_1\), depending on his beliefs following a deviation to an offer to trade \(w_1\). Notice that \(A.16\) implies that this type of equilibrium is viable only if

\[\mu_2 \leq \frac{c_h - \Delta_S - (1 - \mu_1)\Delta_M + \mu_1 I}{\Delta_H + I} \quad (A.18)\]

The Court’s payoff in this type of equilibrium is given by

\[\frac{(1 - \mu_2)\Delta_H - \mu_2 I}{2} + \frac{(1 - \mu_2)\Delta_L - \mu_2 I}{2} + \frac{\Delta S}{2} - \frac{\Delta H}{2} \quad (A.19)\]

Observe next that \(A.19\) tells us that the Court’s payoff is strictly monotonically decreasing in \(\mu_2\). It is also easy to verify that if any of the inequalities \(A.14\), \(A.15\), \(A.16\), \(A.17\) and \(A.19\) are satisfied for a pair \((\mu_1, \mu_2)\) then they will also be satisfied for any pair \((\mu_1, \mu'_2)\) with \(\mu'_2 \in [0, \mu_2]\). Therefore, choosing among the equilibria in this case, the Court would choose an \(F\) such that \(\mu_2 = 0\), and, since the right-hand side of \(A.19\) is always positive, any \(\mu_1 \in [0, 1]\). Therefore the maximum payoff for the Court in this scenario is

\[\frac{\Delta L}{2} + \frac{\Delta S}{2} \quad (A.20)\]

The third scenario we analyze is that of possible equilibria of subgames following the Court’s choice of \(F\) in which the type \(\alpha\) does not invest in and therefore does not trade either \(w_1\) or \(w_2\). From Lemmas A.13, A.14 and A.16 we know that in this scenario it must be the case that in equilibrium the type \(\beta\) buyer pools with the type \(\alpha\) buyer. Clearly in this type of equilibrium both types of buyer trade \(w_3\) ex-post at \(p_3 = \Delta_S\).

For this type of equilibrium to be viable we need the following conditions to be satisfied. The first guarantees that the type \(\beta\) buyer does not want to deviate and invest in and trade \(w_2\) (it is straightforward to check that he cannot profit from deviating and investing in and trading \(w_1\)) we know that he does not want to deviate and invest in and trade \(w_1\). This condition reads

\[(1 - \mu_2)\Delta_L - \mu_2 I \leq 0 \quad (A.21)\]
The other two conditions guarantee that the $\alpha$ type buyer does not want to deviate and invest in and trade either $w_1$ or $w_2$. These are:

$$c_H - \Delta_H - \Delta_S \geq (1 - \mu_1)(\Delta_M + c_L - p_1) - \mu_1 I$$  \hspace{1cm} (A.22)

and

$$c_H - \Delta_H - \Delta_S \geq (1 - \mu_2)\Delta_H - \mu_2 I$$  \hspace{1cm} (A.23)

where $p_1 \in [0, c_L]$ is the lowest price that the seller will accept for $w_1$, depending on his beliefs following a deviation to an offer to trade $w_1$. Notice that (A.21) implies that this type of equilibrium is viable only if

$$\mu_2 \geq \mu^{**} = \frac{\Delta_L}{\Delta_L + I}$$  \hspace{1cm} (A.24)

By Assumption 1 (parts i, iii and iv), using (2), if (A.24) holds then (A.23) is also satisfied. Using (A.22) we can then conclude that this type of equilibrium is viable only if (A.24) holds together with

$$\mu_1 \geq \frac{\Delta_M + \Delta_H + \Delta_S - c_H}{\Delta_M + I}$$  \hspace{1cm} (A.25)

The Court’s payoff in this type of equilibrium is given by

$$\frac{\Delta_S}{2} - \frac{\Delta_H}{2}$$  \hspace{1cm} (A.26)

We can now compare the three scenarios to complete the proof of the proposition. Comparing the Court’s payoff in the two cases of the first scenario and in the second and third scenario, as given by (A.5), (A.11), (A.19) and (A.26) it is clear that the Court’s payoff is highest in the first case of the first scenario. Recall that in this equilibrium the Court sets $F = (0, \mu^*)$.

To conclude the proof we need to argue that the second case of the first scenario as well as the second and third scenarios are ruled out when $F = (0, \mu^*)$.

To see that an equilibrium of the type in the second case of the first scenario is ruled out notice that by Assumption 1 (parts i and iii), using (2), (A.12) is not compatible with $\mu_2 = \mu^*$.

To see that an equilibrium of the type in the second scenario is ruled out notice that the payoff in (A.19) is only available to the Court when $\mu_2 = 0$.

To see that an equilibrium of the type in the third scenario is ruled out notice that the payoff in (A.26) is only available to the Court when $\mu_1 > 0$. ■
Proof of Proposition 4: Throughout the proof, we let $M_\alpha = (m_\alpha^\alpha, m_\alpha^\beta)$ and $M_\beta = (m_\beta^\alpha, m_\beta^\beta)$ denote the menu contract offers of the type $\alpha$ and the type $\beta$ buyer respectively. We first show that Proposition 1 still holds. The two types of buyer must pool and trade both $w_2$ and $w_3$, yielding an equilibrium payoff for a passive Court of $\frac{\Delta S}{2} + \frac{\Delta L}{2}$.

There are three main cases to consider. The first is a possible equilibrium in which $M_\alpha \neq M_\beta$. In this case the two types of buyer would separate at the contract-offer stage. The same argument as in Proposition 1 can be used to establish that this cannot happen in any equilibrium of the model when the Court enforces all contracts. In other words, we conclude that there is no equilibrium of the model with passive Courts when menu contracts are allowed and $w_3$ is not contractible ex-ante in which $M_\alpha \neq M_\beta$.

The second case is that of a possible equilibrium in which $M_\alpha = M_\beta$ and $m_\alpha^\alpha = m_\alpha^\beta = m_\beta^\alpha = m_\beta^\beta$. In this case, the same argument as in Proposition 1 can be used to establish that the only possibility is that of an equilibrium in which the two types of buyer pool and trade both $w_2$ and $w_3$, yielding a Court equilibrium payoff of $\frac{\Delta S}{2} + \frac{\Delta L}{2}$.

The third case is that of $M_\alpha = M_\beta$, and $m_\alpha^\alpha \neq m_\alpha^\beta$ and $m_\alpha^\beta \neq m_\beta^\beta$. Let $m_\alpha^\alpha = m_\alpha^\beta$ and $m_\beta^\alpha = m_\beta^\beta$. Clearly, in equilibrium we need the “truth-telling” constraints to be satisfied: $m^\alpha$ and $m^\beta$ must be such that the type $\alpha$ buyer does not prefer to declare that he is of type $\beta$, and, symmetrically, the type $\beta$ buyer does not prefer to declare that he is of type $\alpha$. We will show that these constraints are in fact impossible to satisfy.

Since $m_\alpha^\alpha \neq m_\beta^\beta$, after declaring $\alpha$, the buyer will be unable to trade $w_3$ since the seller’s beliefs must be that he is facing a type $\alpha$ buyer with probability one. Moreover, after declaring $\beta$ the buyer will trade $w_3$ ex-post at a price $p_3 = \Delta S$. This is because the seller’s beliefs in this case are that he is facing a type $\beta$ buyer with probability one. There are four sub-cases to consider.

The first sub-case is that of $m_\alpha^\alpha$ and $m_\beta^\beta$ both being contracts for $w_1$, so that $m_\alpha^\alpha$ and $m_\beta^\beta$ differ only in the proposed prices. Let these be $p_1^\alpha$ and $p_1^\beta$ respectively. Hence by declaring $\alpha$, the type $\alpha$ buyer receives a payoff of $\Delta M + c_L - p_1^\alpha$, while if he declares $\beta$ he receives a payoff of $\Delta M + c_L - p_1^\beta + c_H - \Delta H - \Delta S$. Therefore, to satisfy the truth-telling constraint for the type $\alpha$ buyer we need

$$p_1^\beta - p_1^\alpha \geq c_H - \Delta H - \Delta S$$

By declaring $\beta$, the type $\beta$ buyer obtains a payoff of $-p_1^\beta$. If instead he declares to be of type $\alpha$ he obtains a payoff of $-p_1^\alpha$. Hence to satisfy the truth-telling constraint for the type $\beta$ buyer we need

$$0 \geq p_1^\beta - p_1^\alpha$$
However, (A.27) and (A.28) cannot both be satisfied because of Assumption 1 (parts i and iii).

The second sub-case we consider is that of $m^\alpha$ and $m^\beta$ both being contracts for $w_2$, so that $m^\alpha$ and $m^\beta$ differ only in the proposed prices. Let these be $p^\alpha_2$ and $p^\beta_2$ respectively. Reasoning in the same way as for the first case, the truth-telling constraint for the type $\alpha$ buyer implies

$$p^\beta_2 - p^\alpha_2 \geq c_H - \Delta_H - \Delta_S$$

(A.29)

while the truth-telling constraint for the type $\beta$ buyer implies that

$$0 \geq p^\beta_2 - p^\alpha_2$$

(A.30)

However, just as in the first case, (A.29) and (A.30) cannot both be satisfied because of Assumption 1 (parts i and iii).

The third sub-case is that of $m^\alpha$ and $m^\beta$ being contracts for $w_1$ and $w_2$ respectively, with prices offered $p^\alpha_1$ and $p^\beta_2$. The truth-telling constraint for the type $\alpha$ buyer implies

$$p^\beta_2 - p^\alpha_1 \geq c_H - \Delta_M - \Delta_S$$

(A.31)

while the truth-telling constraint for the type $\beta$ buyer tells us that

$$\Delta_L + c_L \geq p^\beta_2 - p^\alpha_1$$

(A.32)

However, (A.31) and (A.32) cannot both be satisfied because of Assumption 1 (parts i, iii and iv).

The fourth sub-case is that of $m^\alpha$ and $m^\beta$ being contracts for $w_2$ and $w_1$ respectively, with prices offered $p^\alpha_2$ and $p^\beta_1$. The truth-telling constraint for the type $\alpha$ buyer can be written as

$$p^\beta_1 - p^\alpha_2 \geq c_H + \Delta_M - 2\Delta_H - \Delta_S$$

(A.33)

while the truth-telling constraint for the type $\beta$ says that

$$-\Delta_L - c_L \geq p^\beta_1 - p^\alpha_2$$

(A.34)

However, (A.33) and (A.34) cannot both be satisfied because of Assumption 1 (part i, iii and iv).

We conclude that there is no equilibrium of the model with passive Courts when menu contracts are allowed and $w_3$ is not contractible ex-ante in which $M^\alpha = M^\beta$, and $m^\alpha = m^\beta$ and $m^\alpha = m^\beta$. Therefore, we have shown that Proposition 1 still holds. In any equilibrium of the model with passive Courts when menu contracts are allowed and $w_3$ is not contractible ex-ante the two types
of buyer must pool and trade both \( w_2 \) and \( w_3 \), yielding an equilibrium payoff for a passive Court of 
\[
\frac{\Delta_S}{2} + \frac{\Delta_L}{2}.
\]

There remains to show that Proposition 2 still holds. When menu contracts are allowed and \( w_3 \) is not contractible ex-ante, in equilibrium, an active Court chooses \( V = \{s_2\} \) and its payoff is 
\[
\frac{\Delta_S}{2} + \frac{\Delta_M}{2}.
\]

Notice that the logic of Lemma A.6 still applies to this case. In any of the subgames following \( C \) choosing a \( V \) that contains \( w_i \), \( i = 1, 2 \), in equilibrium, neither type of \( B \) invests in \( w_i \), and hence it is not traded.

It follows that without loss of generality whenever \( V \) equals either \( \{s_1\} \) or \( \{s_2\} \) we can restrict attention to menu contracts that specify the same widget in both components. Incentive-compatibility then ensures that any equilibrium menu contract would have to specify the same price for the single widget appearing in both menu entries. In other words, the only candidates for equilibrium are *degenerate* menus in which \( m^\alpha = m^\beta \). Given this, the claim can be proved using the same argument used to prove Proposition 2 above. The details are omitted.

**Proof of Proposition 5 (i):** Take the degenerate menu offered by both types of buyer to be one that specifies \( m^\alpha = m^\beta = s_2 = (w_2, c_L) \). In other words, the candidate equilibrium has the degenerate menu specifying that \( w_2 \) will be traded at a price \( p_2 = c_L \), regardless of the buyer’s announcement. Moreover, in the proposed equilibrium both types of buyer trade \( w_3 \) ex-post at a price \( p_3 = \Delta_S \).

In the proposed equilibrium the type \( \alpha \) buyer obtains a payoff of \( c_H - \Delta_S \), the type \( \beta \) buyer obtains a payoff of \( \Delta_L \), and the seller obtains an expected payoff of \( \Delta_S - c_H/2 \).

The argument proceeds in two steps. The first step is to show that neither type of buyer can profitably deviate from the proposed equilibrium by making an offer of a contract of the type \( s_1, s_2, s_3, b_{1,3} \) or \( b_{2,3} \). The second is to show that neither type of buyer can profitably deviate from the proposed equilibrium by offering a menu contract different from the equilibrium one.

The first step involves several cases.

Using the same argument as in the proof of Proposition 1 we already know that no type of buyer can profit from a unilateral deviation to offering any other simple contract of the type \( s_1 \) or \( s_2 \). Therefore, it only remains to show that no type of buyer can profit from a unilateral deviation to offering a contract of type \( s_3, b_{1,3} \) or \( b_{2,3} \).

Following a similar logic to the one we used to prove Lemma A.1 it is easy to see that, regardless of his beliefs, the seller will reject any off-path offer of an \( s_3 \) contract specifying a price \( p'_3 < \Delta_S \). (This is because \( c_H - \Delta_H > \Delta_S \) by Assumption 1 (parts i and iii), and hence the seller will either trade \( w_3 \) ex-post at a price \( p_3 = \Delta_S \) or will not trade it at all, depending on his beliefs.)
Now consider a possible deviation by the type \( \alpha \) buyer to offering \( s_3 \) with a price \( p'_3 \geq \Delta_S \). In this case (following the same logic we used to prove Lemma A.6), he will not trade either \( w_2 \) or \( w_1 \). Hence his payoff after the deviation would be \( c_H - \Delta_H - p'_3 \). Therefore for this to be a profitable deviation we need \( c_H - \Delta_H - p'_3 > c_H - \Delta_S \). Since \( p'_3 \geq \Delta_S \), this implies \( \Delta_H < 0 \), which is false by Assumption 1 (part i). We can therefore conclude that the type \( \alpha \) buyer cannot profit from any deviation to offering a contract of the \( s_3 \) variety.

Next, consider a possible deviation from the type \( \beta \) buyer to offering \( s_3 \) with a price \( p'_3 \geq \Delta_S \). In this case (following the same logic we used to prove Lemma A.6), he will not trade either \( w_2 \) or \( w_1 \). Hence his payoff after the deviation would be \( \Delta_S - p'_3 \leq 0 \). Since his payoff in the candidate equilibrium is positive, we conclude that the type \( \beta \) buyer cannot profit from any deviation to offering a contract of the \( s_3 \) variety.

The next case is to consider a possible deviation by the type \( \alpha \) buyer to a bundle contract of the type \( b_{2,3} \). Let the prices specified by the contract be denoted by \( p'_2 \) and \( p'_3 \). For this to be a profitable deviation for the type \( \alpha \) buyer we need \( \Delta_H + c_L - p'_2 + c_H - \Delta_H - p'_3 > c_H - \Delta_S \), which implies \( c_L + \Delta_S > p'_2 + p'_3 \). Let the seller’s off-path beliefs, after receiving the offer of \( b_{2,3} \), be that he is facing a type \( \alpha \) buyer with probability \( \nu \in [0, 1] \). For the seller to accept \( b_{2,3} \) we need \( p'_2 + \nu c_H \geq \max\{0, \Delta_S - \nu c_H\} \). This is because if he rejects the \( b_{2,3} \) offer, then either \( w_3 \) will be traded at a price \( p_3 = \Delta_S \), or will not be traded at all, depending on the seller’s beliefs. But the last inequality implies \( p'_2 + p'_3 \geq c_L + \Delta_S \). Hence we conclude that the type \( \alpha \) buyer cannot profit from any deviation to offering a contract of the \( b_{2,3} \) variety.

Consider now a possible deviation by the type \( \beta \) buyer to a bundle contract of the type \( b_{2,3} \). Let the prices specified by the contract be denoted by \( p'_2 \) and \( p'_3 \). For this to be a profitable deviation for the type \( \alpha \) buyer we need \( \Delta_L + c_L - p'_2 + c_H - \Delta_L - p'_3 > \Delta_L \), which implies \( c_L + \Delta_S > p'_2 + p'_3 \). Let the seller’s off-path beliefs, after receiving the offer of \( b_{2,3} \), be that he is facing a type \( \alpha \) buyer with probability \( \nu \in [0, 1] \). For the seller to accept \( b_{2,3} \) we need \( p'_2 + \nu c_H \geq \max\{0, \Delta_S - \nu c_H\} \). This is because if he rejects the \( b_{2,3} \) offer, then either \( w_3 \) will be traded at a price \( p_3 = \Delta_S \), or will not be traded at all, depending on the seller’s beliefs. But the last inequality implies \( p'_2 + p'_3 \geq c_L + \Delta_S \). Hence we conclude that the type \( \beta \) buyer cannot profit from any deviation to offering a contract of the \( b_{2,3} \) variety.

The next case we consider is that of a possible deviation by the type \( \alpha \) buyer to offering a bundle contract of the \( b_{1,3} \) variety. Let the prices specified by the contract be denoted by \( p'_1 \) and \( p'_3 \). For this to be a profitable deviation for the type \( \alpha \) buyer we need \( \Delta_M + c_L - p'_1 + c_H - \Delta_H - p'_3 > c_H - \Delta_S \), which implies \( \Delta_M + c_L + \Delta_S - \Delta_H > p'_2 + p'_3 \), which using Assumption 1 (part iv) in turn implies \( \Delta_S - \Delta_L > p'_2 + p'_3 \). Let the seller’s off-path beliefs, after receiving the offer of \( b_{1,3} \), be that he is facing a type \( \alpha \) buyer with probability \( \nu \in [0, 1] \). For the seller to accept \( b_{1,3} \) we need \( p'_1 + \nu c_L + p'_3 - \nu c_H \geq \max\{0, \Delta_S - \nu c_H\} \). This is because if he rejects the \( b_{2,3} \) offer, then
either $w_3$ will be traded at a price $p_3 = \Delta_S$, or will not be traded at all, depending on the seller’s beliefs. But the last inequality implies $p_1' + p_3' \geq \Delta_S + \nu c_L$. Hence we conclude that the type $\alpha$ buyer cannot profit from any deviation to offering a contract of the $b_{1,3}$ variety.

The last case we need to consider to conclude the first step in the proof is that of a possible deviation by the type $\beta$ buyer to offering a bundle contract of the $b_{1,3}$ variety. Let the prices specified by the contract be denoted by $p_1'$ and $p_3'$. For this to be a profitable deviation for the type $\beta$ buyer we need $-p_1' + \Delta_S - p_3' > \Delta_L$, which implies $\Delta_S - \Delta_L > p_2' + p_3'$. Let the seller’s off-path beliefs, after receiving the offer of $b_{1,3}$, be that he is facing a type $\alpha$ buyer with probability $\nu \in [0,1]$. For the seller to accept $b_{1,3}$ we need $p_1' - \nu c_L + p_3' - \nu c_H \geq \max\{0, \Delta_S - \nu c_H\}$. This is because if he rejects the $b_{2,3}$ offer, then either $w_3$ will be traded at a price $p_3 = \Delta_S$, or will not be traded at all, depending on the seller’s beliefs. But the last inequality implies $p_1' + p_3' \geq \Delta_S + \nu c_L$. Hence we conclude that the type $\beta$ buyer cannot profit from any deviation to offering a contract of the $b_{1,3}$ variety.

We have now ruled out the possibility that either type of buyer could profitably deviate from the proposed equilibrium by making an offer of a contract of the type $s_1, s_2, s_3, b_{1,3}$ or $b_{2,3}$. The second step in the argument rules out the possibility that either type of buyer can profitably deviate from the proposed equilibrium by offering a menu contract different from the equilibrium one. It involves considering several cases again.

Consider first the possibility that either type of buyer deviates to offering a degenerate menu with $m^\alpha = m^\beta$. In this case, the same argument we used in the first step clearly suffices to prove the claim.

Therefore, there remains to consider the case of some type of buyer deviating to offering a non-degenerate menu contract $M = (m^\alpha, m^\beta)$ with $m^\alpha \neq m^\beta$. Clearly in this case, without loss of generality, we can take it to be the case that the menu $M$ satisfies the truth-telling constraints: $m^\alpha$ and $m^\beta$ must be such that the type $\alpha$ buyer does not prefer to declare that he is of type $\beta$, and, symmetrically, the type $\beta$ buyer does not prefer to declare that he is of type $\alpha$. If this were not the case, the seller would believe that one of the two menu items will be chosen with probability one when the buyer announces his type. Therefore, the same argument as in the case of a degenerate menu would suffice to prove the claim.

It is convenient to classify the possible deviations to non-degenerate menus $M$ that satisfy the truth-telling constraints into three mutually exclusive subsets. We say that a menu contract is of class $\alpha$ if it has the property that, if accepted, it constitutes a strictly profitable deviation (given truth-telling) from the proposed equilibrium for the type $\alpha$ buyer, but not for the type $\beta$ buyer. The class of such menu contracts is denoted by $M^\alpha$. We say that a menu contract is of class $\beta$ if it has the property that, if accepted, it constitutes a strictly profitable deviation (given truth-telling) from the proposed equilibrium for the type $\beta$ buyer, but not for the type $\alpha$ buyer. The class of such
menu contracts is denoted by $M^{\beta}$. We say that a menu contract of class $\omega$ if it has the property that, if accepted, it constitutes a strictly profitable deviation (given truth-telling) from the proposed equilibrium for both the type $\alpha$ and the type $\beta$ buyer. The class of such menu contracts is denoted by $M^{\omega}$. Clearly, to conclude the proof it suffices to show that no type $\alpha$ buyer can profitably deviate by offering a menu $M \in M^{\alpha}$, no type $\beta$ buyer can profitably deviate by offering a menu $M \in M^{\beta}$, and no buyer of either type can profitably deviate by offering a menu $M \in M^{\omega}$.

Consider a possible deviation by a type $\alpha$ buyer to a menu $M \in M^{\alpha}$. In this case, we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type $\alpha$ with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 13 above). The seller believes that the $m^{\alpha}$ component of $M$ will apply with probability one after the buyer declares his type. It follows that the same argument used in the first step of this proof to show that the type $\alpha$ buyer cannot profitably deviate to a contract of type $s_1, s_2, s_3, b_{1,3}$ or $b_{2,3}$ now suffices to show that he cannot profit from a deviation to a menu $M \in M^{\alpha}$.

Next, consider a possible deviation by a type $\beta$ buyer to a menu $M \in M^{\beta}$. In this case, we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type $\beta$ with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 13 above). The seller believes that the $m^{\beta}$ component of $M$ will apply with probability one after the buyer declares his type. It follows that the same argument used in the first step of this proof to show that the type $\beta$ buyer cannot profitably deviate to a contract of type $s_1, s_2, s_3, b_{1,3}$ or $b_{2,3}$ now suffices to show that he cannot profit from a deviation to a menu $M \in M^{\beta}$.

Consider now a possible deviation by a type $\alpha$ buyer to a menu $M \in M^{\omega}$. In this case, we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type $\alpha$ with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 13 above). The seller believes that the $m^{\alpha}$ component of $M$ will apply with probability one after the buyer declares his type. It follows that the same argument used in the first step of this proof to show that the type $\alpha$ buyer cannot profitably deviate to a contract of type $s_1, s_2, s_3, b_{1,3}$ or $b_{2,3}$ now suffices to show that he cannot profit from a deviation to a menu $M \in M^{\alpha}$.

Lastly, consider a possible deviation by a type $\beta$ buyer to a menu $M \in M^{\omega}$. As we specified above, in this case we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type $\alpha$ with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 13 above). The seller believes that the $m^{\alpha}$ component of $M$ will apply with probability one after the buyer declares his type.

Recall that the argument used in the first step of this proof to show that the type $\beta$ buyer cannot profitably deviate to a contract of type $s_1, s_2, s_3, b_{1,3}$ or $b_{2,3}$ applies regardless of the seller’s off-path beliefs following the deviation. Therefore, that argument also suffices to now show that he cannot profit from a deviation to a menu $M \in M^{\omega}$. ■
Proof of Proposition 5 (ii): Take the equilibrium non-degenerate menu contract to be $M = (m^\alpha, m^\beta)$ with $m^\alpha$ of the $s_1$ variety with a price $p_1 = \Delta_M + c_L - c_H + \Delta_S$ and $m^\beta$ of the $s_3$ variety with a price $p_3 = \Delta_S - \Delta_M$.

In this candidate equilibrium the type $\alpha$ buyer gets a payoff (under truth-telling) of $\Delta_M + c_L - p_1 = \Delta_M + c_L - \Delta_M - c_L + c_H - \Delta_S = c_H - \Delta_S$, while the type $\beta$ buyer obtains a payoff (under truth-telling) of $(p_1 - c_L)/2 + p_3/2 = \Delta_S - c_H/2$. Crucially, notice that the type $\beta$ buyer has a payoff strictly greater than the one he obtains in the equilibrium constructed in the proof of Proposition 5 (i). The type $\alpha$ buyer and the seller have the same payoffs as the ones they obtain in the equilibrium constructed in the proof of Proposition 5 (i).

We begin by verifying that the proposed equilibrium contract satisfies the necessary truth-telling constraints. The truth-telling constraint for the type $\alpha$ buyer can be written as

$$p_3 - p_1 \geq c_H - \Delta_H - c_L - \Delta_M$$

which is satisfied for $p_1 = \Delta_M + c_L - c_H + \Delta_S$ and $p_3 = \Delta_S - \Delta_M$ by Assumption 1 (part i).

The truth-telling constraint for the type $\beta$ buyer can be written as

$$\Delta_S \geq p_3 - p_1$$

which is satisfied for $p_1 = \Delta_M + c_L - c_H + \Delta_S$ and $p_3 = \Delta_S - \Delta_M$ by Assumption 1 (part iii).

Consider now a possible deviation by the type $\alpha$ buyer to offering a simple contract of the $s_2$ variety. At best, he would be able to get a payoff of $c_H - \Delta_S$. This is because the seller will not accept any offer to trade $w_2$ for a price below $c_L$, and the type $\alpha$ buyer, at best (depending on the seller’s beliefs) will be able to trade $w_3$ ex-post for a price of $\Delta_S$. Since $c_H - \Delta_S$ is also his payoff in the proposed equilibrium, we conclude that the type $\alpha$ buyer cannot profit from a deviation to offering a simple contract of the $s_2$ variety.

Next, consider now a possible deviation by the type $\beta$ buyer to offering a simple contract of the $s_2$ variety. At best, he would be able to get a payoff of $\Delta_L$. This is because the seller will not accept any offer to trade $w_2$ for a price below $c_L$, and the type $\beta$ buyer, at best (depending on the seller’s beliefs) will be able to trade $w_3$ ex-post for a price of $\Delta_S$. Since $\Delta_L < \Delta_M$, we conclude that the type $\beta$ buyer cannot profit from a deviation to offering a simple contract of the $s_2$ variety.

All other possible deviations can be ruled out using the computations (including the off-path beliefs that they use) in the proof of Proposition 5 (i). This is because the equilibrium payoffs to both types of buyer in the equilibrium proposed here are at least as large as the payoffs that they receive in the equilibrium constructed there. ■
Proof of Proposition 5 (iii): Suppose that there were an equilibrium in which expected net surplus exceeds \( \frac{\Delta_S}{2} + \frac{\Delta_M}{2} \). Then using Assumption 1 (parts i and ii) the equilibrium would have to be of one of the following three varieties. The first variety involves type \( \alpha \) buyer trading \( w_2 \) only and the type \( \beta \) buyer trading \( w_1 \) and \( w_3 \). The second variety involves the type \( \alpha \) buyer trading \( w_2 \) only and the type \( \beta \) buyer trading \( w_3 \) only. The third variety involves the type \( \alpha \) buyer trading \( w_1 \) only and the type \( \beta \) buyer trading \( w_2 \) and \( w_3 \).

As in the proof of Proposition 4, throughout the argument we let \( M_\alpha = (m_\alpha^1, m_\alpha^2) \) and \( M_\beta = (m_\beta^1, m_\beta^2) \) denote the menu contract offers of the type \( \alpha \) and the type \( \beta \) buyer respectively.

There are three main cases to consider. The first is a possible equilibrium in which \( M_\alpha \neq M_\beta \). In this case the two types of buyer would separate at the contract-offer stage. Because of separation at the contract-offer stage we can take it to be the case that both \( M_\alpha \) and \( M_\beta \) are degenerate menus, with \( M_\alpha = (m_\alpha, m_\alpha) \) and \( M_\beta = (m_\beta, m_\beta) \).

There are two possible ways to obtain an equilibrium of the first variety when \( M_\alpha \neq M_\beta \). The first is that \( m_\alpha = s_2 \) and \( m_\beta = s_1 \), with the type \( \beta \) buyer trading \( w_3 \) ex-post. This possibility can clearly be ruled out in the same way as in the proof of Proposition 1. The second way is to have \( m_\alpha = s_2 \) and \( m_\beta = b_{1,3} \). In such putative equilibrium, the type \( \alpha \) buyer would obtain a payoff of \( \Delta_H \), since clearly the \( s_2 \) contract would have to specify \( p_2 = c_L \). Notice also that, given separation, the seller can trade \( w_3 \) ex-post for a payoff of \( \Delta_S \) if he rejects the type \( \beta \) buyer offer of \( b_{1,3} \). It follows that the contract \( b_{1,3} \) contain prices \( p_1 \) and \( p_3 \) such that \( p_1 + p_3 = \Delta_S \). Therefore, by deviating to pooling with the type \( \beta \) buyer, the type \( \alpha \) buyer would obtain a payoff of \( c_H - \Delta_H - \Delta_S + \Delta_M \). Using Assumption 1 (part iii) this is a profitable deviation. Therefore we can conclude that the putative equilibrium is not viable.

A possible equilibrium of the second variety when \( M_\alpha \neq M_\beta \) can be ruled out by noticing that in any case this will involve trading \( w_2 \) at a price \( p_2 = c_L \) and \( w_3 \) at a price \( p_3 = \Delta_S \). Therefore this possibility can clearly be excluded out in the same way as in the proof of Proposition 1.

There are two possible ways to obtain an equilibrium of the third variety when \( M_\alpha \neq M_\beta \). The first is that \( m_\alpha = s_1 \) and \( m_\beta = s_2 \), with the type \( \beta \) buyer trading \( w_3 \) ex-post. This possibility can clearly be ruled out in the same way as in the proof of Proposition 1. The second way is to have \( m_\alpha = s_1 \) and \( m_\beta = b_{2,3} \). In such putative equilibrium, the type \( \alpha \) buyer would obtain a payoff of \( \Delta_M \), since clearly the \( s_1 \) contract would have to specify \( p_1 = c_L \). Notice also that, given separation, the seller can trade \( w_3 \) ex-post for a payoff of \( \Delta_S \) if he rejects the type \( \beta \) buyer offer of \( b_{2,3} \). It follows that the contract \( b_{1,3} \) contain prices \( p_1 \) and \( p_3 \) such that \( p_2 + p_3 = \Delta_S \). Therefore, by deviating to pooling with the type \( \beta \) buyer, the type \( \alpha \) buyer would obtain a payoff of \( c_H - \Delta_H - \Delta_S \). Using Assumption 1 (parts i and iii) this is a profitable deviation. Therefore we can conclude that the putative equilibrium is not viable.
The second case is that of a possible equilibrium in which $M_A = M_B$ and $m^\alpha_A = m^\beta_A = m^\beta_B$. Clearly, no equilibria of the first, second or third variety can be sustained in this case. This is because in all three varieties, the two types of buyer do not trade the same widget $w_1$ or $w_2$.

The third case is that of $M_A = M_B$, and $m^\alpha_A \neq m^\beta_A$ and $m^\beta_A \neq m^\beta_B$. Let $m^\alpha_A = m^\beta_A$ and $m^\beta = m^\alpha_B = m^\beta_B$.

As in the proof of Proposition 4, in equilibrium we need the “truth-telling” constraints to be satisfied: $m^\alpha$ and $m^\beta$ must be such that the type $\alpha$ buyer does not prefer to declare that he is of type $\beta$, and, symmetrically, the type $\beta$ buyer does not prefer to declare that he is of type $\alpha$. We will show that these constraints are in fact impossible to satisfy in any of the three varieties of equilibria.

Notice that, since $m^\alpha \neq m^\beta$, whenever $m^\alpha$ is a simple contract for either $w_1$ or $w_2$, after declaring $\alpha$, the buyer will be unable to trade $w_3$ since the seller’s beliefs must be that he is facing a type $\alpha$ buyer with probability one. Moreover, whenever $m^\beta$ is a simple contract for either $w_1$ or $w_2$, after declaring $\beta$ the buyer will trade $w_3$ ex-post at a price $p_3 = \Delta_S$. This is because the seller’s beliefs in this case are that he is facing a type $\beta$ buyer with probability one.

There are two ways to support a possible equilibrium of the first variety when $M_A = M_B$, and $m^\alpha_A \neq m^\beta_A$ and $m^\beta_A \neq m^\beta_B$. The first is with $m^\alpha_A$ and $m^\beta_B$ being simple contracts for $w_2$ and $w_1$ respectively, with prices offered $p^\alpha_2$ and $p^\beta_1$. The truth-telling constraint for the type $\alpha$ buyer can be written as

$$p^\beta_1 - p^\alpha_2 \geq c_H + \Delta_M - 2\Delta_H - \Delta_S \quad (A.37)$$

while the truth-telling constraint for the type $\beta$ says that

$$-\Delta_L - c_L \geq p^\beta_1 - p^\alpha_2 \quad (A.38)$$

However, $(A.37)$ and $(A.38)$ cannot both be satisfied because of Assumption 1 (part iii). The second is with $m^\alpha$ being a simple contract of the $s_2$ variety and $m^\beta$ being a bundle contract of the $b_{1,3}$ variety with prices $p^\alpha_2$, $p^\beta_1$ and $p^\beta_3$ respectively. The truth-telling constraint for the type $\alpha$ buyer can be written as

$$p^\beta_1 + p^\beta_3 - p^\alpha_2 \geq c_H + \Delta_M - 2\Delta_H \quad (A.39)$$

while the truth-telling constraint for the type $\beta$ says that

$$\Delta_S - \Delta_L - c_L \geq p^\beta_1 + p^\beta_3 - p^\alpha_2 \quad (A.40)$$

However, $(A.39)$ and $(A.40)$ cannot both be satisfied because of of Assumption 1 (parts i, iii and iv).
When $M_\alpha = M_\beta$, and $m^\alpha_\alpha \neq m^\beta_\alpha$ and $m^\alpha_\beta \neq m^\beta_\beta$, to support an equilibrium of the second variety we would have to have $m^\alpha$ and $m^\beta$ being simple contracts for $w_2$ and $w_3$ respectively, with prices offered $p^\alpha_2$ and $p^\beta_3$. The truth-telling constraint for the type $\alpha$ buyer implies

$$p^\beta_3 - p^\alpha_2 \geq c_H - c_L - 2\Delta_H$$  \hspace{1cm} (A.41)

Using Assumption 1 (parts iii and iv), (A.41) implies that $p^\beta_3 \geq p^\alpha_2$. If the seller rejects the menu contract, he will trade $w_3$ ex-post at a price of $\Delta_S$ with equal probability with either type of buyer. Hence by rejecting the offer the seller obtains an expected profit of $\Delta_S - c_H/2$. By standard arguments the menu contract will leave $S$ indifferent between accepting an rejecting. Hence

$$\frac{1}{2}(p^\beta_2 - c_L) + \frac{1}{2}p^\beta_3 = \Delta_S - \frac{1}{2}c_L$$  \hspace{1cm} (A.42)

which together with $p^\alpha_1 \geq p^\alpha_2$ implies that $p^\beta_3 > \Delta_S$. However, the latter implies that the type $\beta$ buyer would get a negative profit from the putative menu contract equilibrium. This is not possible since he can always not invest and not trade and guarantee a payoff of zero.

There are two ways to support a possible equilibrium of the third variety when $M_\alpha = M_\beta$, and $m^\alpha_\alpha \neq m^\alpha_\beta$ and $m^\beta_\alpha \neq m^\beta_\beta$. The first is with $m^\alpha$ and $m^\beta$ being simple contracts for $w_1$ and $w_2$ respectively, with prices offered $p^\alpha_1$ and $p^\beta_2$, and the type $\beta$ buyer trading $w_3$ ex-post. The truth-telling constraint for the type $\alpha$ buyer implies

$$p^\beta_2 - p^\alpha_1 \geq c_H - \Delta_M - \Delta_S$$  \hspace{1cm} (A.43)

while the truth-telling constraint for the type $\beta$ buyer tells us that

$$\Delta_L + c_L \geq p^\beta_2 - p^\alpha_1$$  \hspace{1cm} (A.44)

However, (A.43) and (A.44) cannot both be satisfied because of Assumption 1 (parts i, iii and iv). The second is with $m^\alpha$ being a simple contract of the $s_1$ variety and $m^\beta$ being a bundle contract of the $b_{2,3}$ variety with prices $p^\alpha_1$, $p^\beta_2$ and $p^\beta_3$ respectively. The truth-telling constraint for the type $\alpha$ buyer can be written as

$$p^\beta_2 + p^\beta_3 - p^\alpha_1 \geq c_H - \Delta_M$$  \hspace{1cm} (A.45)

On the other hand, the truth-telling constraint for the $\beta$ type buyer implies that

$$c_L + \Delta_L + \Delta_S \geq p^\beta_2 + p^\beta_3 - p^\alpha_1$$  \hspace{1cm} (A.46)
However, inequalities (A.45) and (A.46) cannot be both satisfied because of Assumption 1 (parts i, iii and iv).

**Proof of Proposition 6:** We begin by arguing that the equilibrium constructed in the proof of Proposition 5 (ii) is still viable when the Court sets \( V = \{s_2, b_{2,3}\} \). This is straightforward since the Court now makes some deviations impossible. The remaining deviations can be shown not to be profitable in the same way as in the proof of Proposition 5 (ii).

Given that \( V = \{s_2, b_{2,3}\} \), using the same logic as in the proof of Lemma A.6 we can be sure that in no equilibrium of the model will it be the case that either (or both) types of buyer will invest in \( w_2 \), and hence it will not be traded.

To show that the type \( \alpha \) buyer investing in and trading \( w_1 \) and the type \( \beta \) buyer trading \( w_3 \) is the unique equilibrium outcome the following variety of equilibrium outcomes need to be ruled out. The first variety is one in which both types of buyer invest in and trade \( w_1 \). The second variety is one in which both types of buyer trade \( w_3 \). The third variety is one in which the type \( \alpha \) buyer trades \( w_3 \), while the type \( \beta \) buyer invests in and trades \( w_1 \).

Consider an equilibrium of the first variety. This outcome cannot be sustained without using menu contracts in equilibrium. This can be proved using the same argument as in the proof of Proposition 2. For the same reason, this outcome cannot be sustained using menu contracts in an equilibrium in which the two types of buyer separate at the contract-offer stage by offering \( M_\alpha \neq M_\beta \). Suppose that \( M_\alpha = M_\beta \) and both menus are degenerate in the sense that \( m_\alpha^\alpha = m_\alpha^\beta = m_\beta^\alpha = m_\beta^\beta \). In this case clearly we must have that the menu contracts specify \( p_1 = c_L/2 \). Hence, just as in the proof Proposition 2, the type \( \beta \) buyer has an incentive to deviate. Lastly, suppose that \( M_\alpha = M_\beta \), and \( m_\alpha^\alpha \neq m_\beta^\alpha \) and \( m_\alpha^\beta \neq m_\beta^\beta \). Then, since both menu items must be simple contracts for \( w_1 \) the truth telling constraints trivially imply that \( p_1^\alpha = p_1^\beta \). Hence, in equilibrium \( p_1^\alpha = p_1^\beta = c_L/2 \), and therefore the type \( \beta \) buyer has an incentive to deviate as before.

Any equilibrium of the second variety can be ruled out in a completely analogous way as any equilibrium of the first variety. The details are omitted.

Consider now an equilibrium of the third variety. From the surplus and cost matrix in (1) it is evident that the sum of the payoffs of the two types of buyer and of the seller in any such equilibrium is negative. Hence at least one of the players will have a profitable deviation to not trading at all.

**References**


