Keynesian Beauty Contest, Accounting Disclosure, and Market Efficiency

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Abstract

This paper examines the market efficiency consequences of accounting disclosure in the context of stock markets as a Keynesian beauty contest, an influential metaphor originally proposed by Keynes (1936) and recently formalized by Allen, Morris, and Shin (2006). In such markets, public information plays an additional coordination role, biasing stock prices away from the consensus fundamental value toward public information. Despite this bias, I demonstrate that provisions of public information always drives stock prices closer to the fundamental value. Hence, as a main source of public information, accounting disclosure enhances market efficiency, and transparency should not be compromised on grounds of the Keynesian-beauty-contest effect.
1 Introduction

I investigate how the quality of accounting disclosure influences market efficiency in the context of stock markets as a Keynesian beauty contest, a metaphor first introduced by Keynes (1936). At that time, a London newspaper was running a beauty contest in which readers were asked to select a set of six “most beautiful” pictures from 100 photographs of women. Whoever picked the most popular pictures was entitled for a raffle prize. To win the competition, players should not naively select six faces they believed the most beautiful; instead, they should use their information to infer which faces other players would believe the prettiest and other players would believe that other players would believe the prettiest and so on. Keynes observed that stock markets shared the essence of this competition, in that many rational but short-horizon investors’ actions were similarly governed by expectations about what other investors believed, rather than by genuine expectations about the true value of a firm.

Allen, Morris, and Shin (2006) rationalize this Keynesian-beauty-contest effect as a consequence of investors’ short horizons. Since a short-horizon investor exits a firm before its fundamental value is known, her payoff depends on how much other investors would like to pay, rather than on how much she expects the fundamental value of the firm will be. Given access to both public and private information, she puts an extra weight on public information due to its dual role. Public information does not only convey information about the fundamental value (hereafter the information-content role), but also anchors an investor’s belief about other investors’ beliefs (hereafter the coordination role). This additional coor-
dination role biases stock prices away from the consensus fundamental value toward public
information.

Having qualitatively formalized the Keynesian-beauty-contest effect, Allen, Morris, and Shin (2006) also leave many questions open, one of which concerns the market efficiency con-
sequences of disseminating public information in a Keynesian-beauty-contest stock market. How does the quality of public information affect market efficiency? How is the intensity
of the dual role of public information related to its quality? Could the Keynesian-beauty-
contest effect justify the withdrawal of some noisy public information?

These questions are important to accounting researchers. The Keynesian-beauty-contest
effect introduces a new perspective to understand accounting disclosure. Accounting discl-
sure, as a main source of public information, is a unique feature of public firms characterized
by dispersed ownership. Understanding accounting disclosure entails examining the conse-
quences of dispersed ownership, among which agency problems and differential information
have been the best known. However, dispersed ownership has other consequences that have
profound impact on disclosure practice but have not received the attention they deserve. One
of such examples is the Keynesian-beauty-contest effect. The effect results from investors’
short horizons, which in turn arise because dispersed ownership of modern corporations
decouples the life span of entrepreneurs and owners from that of their firms.

In the absence of a formal rational interpretation of the Keynesian-beauty-contest effect,
conventional wisdom comes that transparency should be curtailed in a Keynesian-beauty-
contest stock market. The logic goes as follows. As public information becomes noisy, its
information content becomes tenuous. If its coordination role increases or at least does not
decrease as public information becomes less precise, there could exist a threshold precision
under which the diminishing information-content role is dominated by the non-decreasing
coordination role. For example, Shiller (2000) criticizes that, news media, by promulgating
noisy public information and thus creating “similar thinking among large groups of people”,
exert undue influence on market events. The conventional wisdom, which usually claims
support from Shiller (2000)’s criticism, could have immediate prescriptive implications.

This paper formally evaluates the market efficiency consequences of public information
in a rational Keynesian-beauty-contest stock market. Built on Allen, Morris, and Shin
(2006), the paper solves for a closed-form equilibrium of a two-trading-period noisy rational
expectations model. The comparative statics then show that more public information always
drives stock prices closer to the fundamental value, even in the presence of the Keynesian-
beauty-contest effect. Therefore, provisions of more and better accounting disclosure boost
the overall informativeness of stock prices to market participants, and transparency should
not be compromised on grounds of the Keynesian-beauty-contest effect. Prohibition of public
information, however noisy it is, amounts to “throwing the baby out with the bathwater”.

The rationale for the positive market efficiency effect of public information lies in the
endogenous link between the dual role of public information. The information-content role
and the coordination role are connected through the quality of public information. Contrary
to the conventional wisdom, the coordination role decreases as public information becomes
less accurate. That is, when public information becomes noisier, not only do short-horizon
investors use it less, they also overuse it less. The intuition is straightforward. The coor-
dination role occurs because short-horizon investors correctly believe in the first place that
other investors will be using public information due to its information content. When the
information content of public information becomes thinner, investors overuse it less because
they again correctly believe that others will be using it less. In the extreme, if a piece
of public information is completely useless, it will be neither used nor overused. Just as
Morris and Shin (2002) differentiate the Keynesian-beauty-contest effect from the sunspot
phenomenon based on the fact that public information has the information-content role, the
endogenous link between the dual role of public information is a hallmark of the rational
Keynesian-beauty-contest effect.

The endogenous link between the dual role of public information reconciles this study
with Morris and Shin (2002). Assuming that agents have a fixed “beauty contest” component
of utility derived from being closer to other agents’ actions, Morris and Shin (2002) conclude
that noisy public information could be detrimental to social welfare. Despite the modeling
differences, the discussion in the last two paragraphs suggests that their conclusion may be
due to the assumed beauty contest utility, which, by fixing the intensity of the coordination
role, divorces the link between the dual role of public information. Hence, the comparison
of the two studies deepens our knowledge of the Keynesian-beauty-contest effect.

This paper also contributes to the literature on dynamic noisy rational expectations in
capital market by supplying a closed-form equilibrium, which is rare in this literature.1 While

introduce a single-period noisy rational expectations model; Besides Allen, Morris, and Shin (2006), see
employing essentially the same basic setting as that in this paper, Brown and Jennings (1989) and Allen, Morris, and Shin (2006) only conduct qualitative analysis.\textsuperscript{2} Besides making it possible to analyze the market efficiency consequences of public information, the closed-form equilibrium of the Keynesian-beauty-contest model opens many other possibilities. For example, the model could be developed to study firms’ choice between private and public channels to convey information to stock markets. Private information is only partially reflected in stock price while public information is overused in stock markets. Firms may benefit from a balanced combination of both channels. For another example, the relation between information quality and cost of capital has been extensively studied in a single-period model (see Easley and O’hara, 2004; Lambert, Leuz, and Verrecchia, 2006, 2007, e.g.). A tractable dynamic model could help reexamine the relation in the context of stock markets as a Keynesian beauty contest. In addition, the model may also find applications in the vast ERC literature.

The rest of the paper is organized as follows. Part 2 describes the basic setting of the model; part 3 defines and solves for the equilibrium; part 4 examines the market efficiency consequences of accounting disclosure; and part 5 concludes.

\textsuperscript{2}Brown and Jennings (1989) use a similar model to qualitatively prove that technical analysis has value in a myopic-investor economy. The myopic investors are the same as the short-horizon investors in Allen, Morris, and Shin (2006). In addition, Brown and Jennings (1989) assume a more general correlation structure of supply noise across two periods.


2 The Model

I describe the basic setting of the model in this section, which closely follows Brown and Jennings (1989) and Allen, Morris, and Shin (2006). It is a two-trading-period noisy rational expectations economy with short-horizon investors and independent supply shocks.

There are two trading periods. A risk free asset and a firm’s risky stocks are traded in both periods in a competitive market. The risk free asset acts as the numeraire of the economy and its return is normalized to be zero. The per share random payoff of the firm’s stocks, denoted as $\theta$, is unknown to investors until the end of the last period.

Investors’ short horizons are characterized by the overlapping generations assumption. There are two generations of investors and each generation have a continuum of investors indexed by an interval $[0,1]$. Each generation only live in one period and transfer their ownership of the firm to the next generation through trading. Young investors are born with endowment and save through the market; at the end of their life period, they become old, sell their stocks and consume the proceeds. For the purpose of this study, I end the overlapping generations cycle at the end of period 2 by the liquidation of the firm. Figure 1 describes the time line of events.
The firm is liquidated and $\theta$ is revealed.

The firm is liquidated and $\theta$ is revealed.

Investors have access to both private and public information, with the later disclosed by the firm. Conditional on $\theta$, each investor $i$ in period $t$ receives an independent private signal $\tilde{x}_{ti}$, $t \in \{1, 2\}$ and $i \in [0, 1]$, with precision $\beta$. The realization of the private signal is $x_{ti}$.

$$\tilde{x}_{ti} = \theta + \tilde{\epsilon}_{ti}, \tilde{\epsilon}_{ti} \sim N(0, \frac{1}{\beta}) \quad (1)$$

Note that the aggregate private information fully reveals the fundamental value, due to the assumption of a competitive market. This design is only for convenience. The general results hold if the pooling of private information is not perfect. We can either use a finite number of investors or assume a common error in investors’ private signals to prevent the pooling of private information from fully revealing the fundamental value.

At the beginning of period 1, the firm discloses an independent public signal, $\tilde{z}$, with precision $\alpha$. The realization of the public signal is $z$.

$$\tilde{z} = \theta + \tilde{\epsilon}_z, \tilde{\epsilon}_z \sim N(0, \frac{1}{\alpha}) \quad (2)$$
Although the only public information in the model is the firm’s disclosure, the main results still hold if other public information sources are modeled. The linear property of multivariate normal distribution allows other public information to be summarized in \( \alpha \). More other public information indicates a higher lower bound of \( \alpha \). Similarly, allowing the firm to disclose another independent signal at the beginning of period 2 does not affect the main results, either.

While the firm could make its information public according to a pre-announced disclosure policy, investors can not communicate with each other except through stock price.\(^3\) Investors learn about other investors’ private information through stock price. The learning is not perfect because stock prices are contaminated by information irrelevant trading. Following the tradition from Grossman and Stiglitz (1980) and Diamond and Verrecchia (1981), I use supply noise to summarize all forces other than information that affect stock prices. The random per capita supply noise of the firm’s shares in period \( t \), \( \tilde{s}_t \), \( t \in \{1, 2\} \), is normally distributed with mean zero and precision \( \gamma_t \), respectively. The supply shocks are independent of all signals and of each other.\(^4\)

\(^3\)This paper does not explicitly study choices of the firm’s disclosure policy. However, given the definition of information, the firm’s disclosure policy can be characterized by choosing a parameter \( \alpha \) from the interval \([0, \alpha_{max}]\). A choice of \( \alpha = 0 \) means that the firm does not disclose anything, a choice of \( \alpha = \alpha_{max} \) indicates a policy of disclosing everything the firm knows, and a choice of an interior \( \alpha \) is equivalent to a partial disclosure policy by which the firm adds some white noise to its information and discloses the garbled signal.

\(^4\)See Grossman (1995) and Black (1986) for a discussion about the nature and source of supply noise.
A typical investor, $i$, has a CARA utility function

$$U_i(c) = -e^{-\frac{c_i}{\tau}}$$

where $c_i$ is her consumption financed by selling or liquidating her holdings when she becomes old, and $\tau$ is her risk tolerance which is the same across investors. By solving the expected utility maximization problem, we get her demand for stocks, which is linear in information and supply noise.

$$D_i = \frac{\tau(E_i[\tilde{\mu}|I_i] - p)}{Var_i[\tilde{\mu}|I_i]}$$  \hspace{1cm} (3)

where $\tilde{\mu}$ is the random payoff from holding stocks, $E_i[\tilde{\mu}|I_i]$ and $Var_i[\tilde{\mu}|I_i]$ represent investor $i$’s estimates of the mean and variance of the random payoff conditional on her information set $I_i$, and $p$ is the stock price. I denote the stock prices in period 1 and 2 using $p_1$ and $p_2$, respectively. Note that investors in period 1 expect a payoff of $p_2$ while investors in period 2 is rewarded by the firm’s fundamental value $\theta$. Short horizons induce investors in period 1 to be concerned with the interim price $p_2$, rather than the fundamental value $\theta$. For convenience, I also assume that information quality, precisions of supply noise and investors’ risk tolerance are well defined. That is, $\alpha, \beta, \gamma_1, \gamma_2$ and $\tau$ are positive and finite.

3 The Closed-form Equilibrium

In this section, I establish the existence of a unique linear rational expectations equilibrium in Proposition 1. While it is usually difficult to solve for a closed-form equilibrium of a multi-period noisy rational expectations model, this paper makes it possible by relying on
assumptions of both short-horizon investors and independent supply noise. Investors who can trade multiple times establish extra positions to hedge their demands in later periods. The form of the extra hedging demands is often complicated. Brown and Jennings (1989) circumvent this issue by introducing short-horizon investors who only live in one period and have to close their positions at the end of their life period. Moreover, learning from stock price under a general correlation structure of supply noise across periods is highly non-linear. Allen, Morris, and Shin (2006) assume independent supply noise and provide justification for the assumption. Although not explored by Allen, Morris, and Shin (2006), the seemingly innocuous assumption of independent supply noise drastically simplifies the analysis by allowing of a clean expression of the notion of learning from stock price.

A rational expectations equilibrium of this economy is defined as a pair of prices \((p_1, p_2)\) that satisfies

a. The stock market clears in both period 1 and 2;

b. Investors maximize their expected utilities, conditional on all available information, including information gleaned from stock price;

c. Investors have rational expectations. Their beliefs about all random variables are consistent with the true underlying distributions;

d. Prices depend on information only through supply and demand.

In addition, if \((p_1, p_2)\) are linear functions of information and supply noise, then the equilibrium is a linear rational expectations equilibrium.
Proposition 1. There is a unique linear rational expectations equilibrium \((p_1, p_2)\), characterized by

\[
p_1 = bz + c\theta - ds_1 \quad \text{(4)}
\]

\[
p_2 = \frac{1}{\alpha + \beta + \rho + \rho_2} \left[ \alpha z + (\beta + \rho + \rho_2)\theta - \frac{\beta + \rho_2}{\rho_2} \frac{\rho}{\beta \tau} s_1 - \frac{(\beta + \rho_2)}{\beta \tau} s_2 \right] \quad \text{(5)}
\]

where

\[
b = \frac{\alpha}{\alpha + \rho + \beta M} \\
c = \frac{\rho + \beta M}{\alpha + \rho + \beta M} \\
d = \frac{\beta + \rho_2}{\rho_2} \frac{\rho + \beta M}{\alpha + \rho + \beta M} \frac{1}{\beta \tau} \\
M = \frac{\beta + \rho_2}{\alpha + 2\beta + \rho + \rho_2} \\
\rho = k\rho_1 \\
\rho_1 = \beta^2 \tau^2 \gamma_1 \\
\rho_2 = \beta^2 \tau^2 \gamma_2 \\
k = \left( \frac{\rho_2}{\beta + \rho_2} \right)^2
\]

All proofs are placed in the Appendix. The proof of Proposition 1 involves backward induction by solving for \(p_2\) first. The basic idea is to assume a linear price function, plug in the assumed price function to investors’ demand functions, obtain a new price function by equating aggregate demand with aggregate supply, and then compare coefficients of the two price functions to determine the coefficients in the assumed price function.
We will focus on $p_1$ to accentuate the influence of short horizons. In contrast, investors in period 2 receive the liquidation value of the firm. Thus, $p_2$ is only used to terminate the overlapping generations cycle and establish an anchor from which we can apply backward induction to solve for $p_1$. Unless explicitly noted, from now on, by price I mean $p_1$.

The coordination role of public information, the extra weight on public information, and the Keynesian-beauty-contest effect are three equivalent concepts. When investors have short horizons, stock prices depend on investors’ expectations of the average expectation of the fundamental value. Public information, as common knowledge among investors, plays an additional coordination role of anchoring investors’ beliefs about other investors’ beliefs, causing investors to overuse it. As a result, stock prices have an extra weight on public information relative to private information, and the Keynesian-beauty-contest effect occurs.

To characterize the Keynesian-beauty-contest effect, I use as the benchmark the weight on public information in a long-horizon economy in which the firm’s value is revealed at the end of period 1. It is a long-horizon economy because investors’ horizons are as long as the firm’s life span. This long-horizon benchmark, characterized in Lemma 1, emphasizes on the fact that the Keynesian-beauty-contest effect is attributable to investors’ short horizons. I denote all the variables in this long-horizon case using the same notations with a top “^”.

Allen, Morris, and Shin (2006) use as the benchmark the consensus fundamental value in the short-horizon economy. The two benchmarks are only slightly different. Moreover, the difference does not affect the main results but makes the interpretation easier, as we shall see soon. In addition, Allen, Morris, and Shin (2006) define a long-horizon economy as one in which investors live longer than one period, resulting in difficulty in solving for a close-form equilibrium.
For example, the price in this long-horizon economy is “$\hat{p}_1$”.

**Lemma 1.** The long-horizon equilibrium is described by $\hat{p}_1$.

\[
\hat{p}_1 = \hat{b}z + \hat{c}\theta - \hat{d}s_1 \tag{6}
\]

where

\[
\hat{b} = \frac{\alpha}{\alpha + \beta + \rho_1}, \\
\hat{c} = \frac{\beta + \rho_1}{\alpha + \beta + \rho_1}, \\
\hat{d} = \frac{1}{\beta \tau} \frac{\beta + \rho_1}{\alpha + \beta + \rho_1}
\]

Note that $\hat{p}_1$ differs from $p_2$, although both are generated in a long-horizon economy. $p_2$ is not directly comparable to $p_1$ because investors in period 2 have additional access to price $p_1$ which conveys the private information of investors in period 1.

Before proceeding to the main analysis of the role the quality of public information plays in a Keynesian-beauty-contest stock market, I discuss briefly the relation between the short-horizon and long-horizon equilibria, which has been qualitatively discussed in Allen, Morris, and Shin (2006).

Figure 2 summarizes the discussion by comparing the first-best price $p_{fb}$, the expected long-horizon price $E_{s_1}\hat{p}_1$, and the expected short-horizon price $E_{s_1}p_1$. Given the focus on the impact of public information, we use the expected prices with respect to supply noise to get around the confounding effect of supply noise.
\( \bar{E}_1[\cdot] \) is the operator of the consensus by taking the average of investors' expectations in period 1. Since investors learn from different prices in two economies, \( p_1 \) and \( \hat{p}_1 \), \( \hat{I}_1 \) is slightly different from \( I_1 \), resulting in the difference between two metrics of the consensus fundamental value, \( \bar{E}_1[\theta|I_1] = \frac{\alpha}{\alpha + \beta + \rho} z + \frac{\beta + \rho}{\alpha + \beta + \rho} \theta \) and \( \bar{E}_1[\theta|\hat{I}_1] = \frac{\alpha}{\alpha + \beta + \rho_1} z + \frac{\beta + \rho_1}{\alpha + \beta + \rho_1} \theta \). However, because both \( \rho \) and \( \rho_1 \) are independent of \( \alpha \), using either of them as a benchmark for the extra weight does not qualitatively affect the main results. Thus, for ease of exposition, I omit their difference in the consequent discussion.

\( p_{fb} \) is the fundamental value or the first-best price when there are neither supply noise nor short horizons. Supply noise prevents the full expression of private information in stock price.\(^6\) This noise effect drives the expected long-horizon price, \( E_{s_1}\hat{p}_1 \), away from the fundamental value toward public information. One prominent feature of \( E_{s_1}\hat{p}_1 \) is that it coincides with the noise effect.

\(^6\)The inability of private information to be fully reflected in stock price resembles the mechanism underlying rational herding in which a subsequent actor can not fully learn her predecessors' private information because she can only observe their actions but not their private information. However, rational herding usually concerns only about discrete actions (see Bikhchandani, Hirshleifer, and Welch, 1998, e.g.).
with $E_1[\theta|\hat{I}_1]$, the consensus fundamental value. The Keynesian-beauty-contest effect is characterized by the further deviation of $E_{s,t}p_1$ from $E_{s,t}\hat{p}_1$ toward public information. Although both the Keynesian-beauty-contest effect and the noise effect bias stock prices toward public information, the discrepancy between $E_{s,t}p_1$ and the consensus fundamental value provides a useful clue to empirically differentiate the two effects.

Allen, Morris, and Shin (2006) qualitatively prove the existence of the discrepancy between $E_{s,t}p_1$ and the consensus fundamental value. It is unclear, however, how change in the quality of public information affects the discrepancy and ultimately the overall distance between $p_1$ and $\theta$. For example, one extreme case is to eliminate the Keynesian-beauty-contest effect by prohibiting public information. As $\alpha$ goes to zero, weights on public information approach zero in both long- and short-horizon economies and the Keynesian-beauty-contest effect vanishes. Does the withdrawal of public information drive stock prices closer to the fundamental value? The answer is nebulous. While investors put no weight on the noise in the public signal, they are now putting more weight on supply noise.\(^7\) I give a formal investigation of this issue in the next section.

### 4 Keynesian Beauty Contest and Market Efficiency

While there are many metrics measuring market efficiency, I focus on price efficiency, the accuracy with which stock prices reflect the fundamental value (see Tobin, 1984, e.g.). The primary goal of financial reporting is to provide various market participants with information

\(^7\)This is the case because d is decreasing in $\alpha$. 

about a business enterprise’s fundamental value (see FASB, 1978, e.g.). Meanwhile, one basic function of markets, stock markets included, is to aggregate and disseminate value relevant information inherently dispersed among market participants (see Hayek, 1945, e.g.). To the extent that I examine the impact of the quality of accounting disclosure on the price discovery function of stock markets, price efficiency is a proper measure.

By adhering to my definition of market efficiency, I try to avoid unnecessary confusion of terminology. In the theoretical literature on noisy rational expectations in capital markets, price efficiency is sometimes labeled as informational efficiency, as opposed to allocational efficiency based on Pareto-efficiency in a general equilibrium (see Brunnermeier, 2001, e.g.). The label “informational” is inevitably confused with the empirical definition of market efficiency by Fama (1970). Moreover, despite the possible divergence, two types of efficiency are closely related. In a much richer model, more accurate prices could enable market participants to make better informed decisions with respect to resource allocation. To that extent, price efficiency may also be viewed as a reduced-form counterpart of the Pareto-efficiency-based definition of market efficiency, such as the one in Grossman (1995).

Price efficiency is measured using the reciprocal of the mean-squared error (MSE) between a firm’s fundamental value and its stock prices, a traditional measure of the extent to which markets fulfill the price discovery function. In statistical terms, price efficiency emphasizes on the efficiency property of stock prices as an estimator of the fundamental value. Lower MSE implies that stock prices are much closer to the fundamental value, resulting in higher

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8 Hirshleifer (1971) provides an example of the possible divergence.
market efficiency.

\[ PE = \frac{1}{E_{s,z}[p - \theta]^2} \]  

(7)

PE is an ex ante measure. \( E_{s,z}[\cdot] \) means that the expectation is taken with respect to both supply noise and public information.

Given the presence of the Keynesian-beauty-contest effect, how could we improve market efficiency through accounting disclosure? Proposition 2 answers this question.

**Proposition 2.** *More public information uniformly improves market efficiency, even in the presence of the Keynesian-beauty-contest effect.*

In the context of stock markets as a Keynesian beauty contest, transparency is still a worthwhile cause. Accounting disclosure provides information about the future cash flow of a firm. Although short-horizon investors overuse it, provisions of more and better financial reporting still boost the overall informativeness of stock prices to market participants. The fact that stock markets behave like a Keynesian beauty contest does not justify the withdrawal of public information.

Shiller (2000) criticizes that, news media, by promulgating information and thus creating “similar thinking among large groups of people”, exert undue influence on market events. This criticism, although not explicitly intended by Shiller (2000), has been claimed to support the conventional wisdom that elimination or prohibition of some noisy public information may be necessary to enhance market efficiency in a Keynesian-beauty-contest stock market. The logic goes as follows. As public information becomes less precise, its information content
is attenuated. If the coordination role increases or at least does not decrease in the variance of public information, there could exist a threshold of the precision of public information under which the diminishing information-content role is dominated by the non-decreasing coordination role.

Proposition 2 contradicts such a speculation. It implies that while public information plays the dual role, the information-content role always dominates the coordination role. Since the information-content role dissipates as public information becomes less precise, a necessary condition for the dominance is that the coordination role decreases in the variance of public information. That is, the dual role of public information is endogenously linked through the quality of public information. I proceed to verify this intuition by showing that the Keynesian-beauty-contest effect intensifies as public information becomes more accurate.

As discussed, the Keynesian-beauty-contest effect, the coordination role of public information, and the extra weight on public information in stock price are three equivalent concepts. Using as the benchmark the weight on public information in the long-horizon economy, we could quantify the Keynesian-beauty-contest effect as the discrepancy between the weights on public information in \( p_1 \) and \( \hat{p}_1 \).

I define the extra weight on public information, \( R \), as follows.

\[
R = 1 - \frac{1}{\frac{b}{\hat{b}}/\frac{c}{\hat{c}}} \tag{8}
\]

Since the ratio \( \frac{b}{\hat{b}} / (\frac{c}{\hat{c}}) \) reflects the relative use of public and private information when investors have short (long) horizons, the ratio \( \frac{b}{\hat{b}}/\frac{c}{\hat{c}} \) represents the extent to which public information is overused by short-horizon investors. The monotonic transformation in defi-
nition 8 normalizes R to lie between zero and one. The greater R is, the more salient the Keynesian-beauty-contest effect.

**Proposition 3.** The Keynesian-beauty-contest effect intensifies as public information becomes more precise.

Since we have solved for the closed-form equilibrium, we can show here the determinant of the intensity of the Keynesian-beauty-contest effect. The coordination role is connected to the information-content role through the quality of public information. When the quality of public information improves, the Keynesian-beauty-contest effect becomes more salient and the price concentrates further on public information. The intuition for this link is straightforward. The Keynesian-beauty-contest effect occurs because public information has information content in the first place. Anticipating that other investors will be using public information to forecast the fundamental value, a short-horizon investor, who tries to forecast the consensus fundamental value, overuses public information over and above its optimal use in assessing the fundamental value. As it becomes more accurate, not only does she use public information more due to its improved information content, she also overuses it further because of its enhanced coordination role of forecasting the consensus fundamental value. Just as Morris and Shin (2002) differentiate the Keynesian-beauty-contest effect from the sunspot literature based on the fact that public information has the information-content role, the endogenous link between the dual role of public information is a hallmark of the rational Keynesian-beauty-contest effect.

This endogenous link between the information-content and the coordination roles of pub-
lic information plays a crucial role in the market efficiency consequences of public information. Short horizons create interdependency of investors’ demands for stocks and give rise to the dual role of public information. The dual role is endogenously connected to each other in such a way that the coordination role is always secondary to the information-content role in terms of market efficiency.

In contrast, if the dual role of public information is directly assumed, as opposed to being derived from short horizons, the endogenous link is then muted. Consequently, when the information content is tenuous while the coordination role is strong enough, noisy public information could decrease market efficiency. Morris and Shin (2002) prove a similar point. In their game theory model, besides a standard component of utility defined over the distance between her action and the true state, an agent’s loss function has an additional “beauty contest” component with a fixed weight. Defined over the distance between her action and the average action across all agents, this assumed “beauty contest” component of utility not only gives rise to the coordination role of public information, but also fixes the intensity of this coordination role. As a result, when the fixed weight is great and public information is noisy enough, the fixed coordination role dominates the diminishing information-content role and public information becomes detrimental to social welfare, which is defined as the negative of the average mean-squared error between individual action and the true state.

The following exercise gives us a glance at the importance of the endogenous link between the dual role of public information. Recall that \( R \) measures the intensity of the coordination role and that it is increasing in \( \alpha \). Now we fix \( R \) to be a constant, \( r \), so that the coordination
role is decoupled from the information-content role.

**Observation 1.** When the Keynesian-beauty-contest effect is fixed at \( r \), provisions of public information decrease market efficiency if and only if \( r \) and \( \alpha \) are such that

\[
\alpha < (1 - r)(\beta + \rho_1)(1 - \frac{2(\beta + \rho_1)}{\rho}(1 - r))
\]  

(9)

and

\[
1 - \frac{\rho}{2(\beta + \rho_1)} < r < 1
\]

(10)

In the sense that fixing an endogenous variable to a constant inherently has too many degrees of freedom, Observation 1 is more a back-of-the-envelope calculation than a rigorous statement. However, it gives some clues about the importance of the endogenous link between the dual role of public information in determining its market efficiency consequences. When the public information is relatively noisy (condition 9) and the Keynesian-beauty-contest effect is fixed at a high level (condition 10), provisions of public information reduce market efficiency. In this sense, conditions 9 and 10 resemble condition 20 in Morris and Shin (2002). Therefore, the comparison of Proposition 2 and Observation 1 suggests that the detrimental social welfare effect of public information in Morris and Shin (2002) may result from the assumed “beauty contest” utility, an observation that warrants further investigation.

5 Conclusion

I study the market efficiency consequences of accounting disclosure in the context of stock markets as a Keynesian beauty contest. In such markets, public information performs both
an information-content role and a coordination role. Because the dual role of public information is endogenously linked to each other via the quality of public information, disclosure of public information, however noisy it is, always brings stock prices closer to the fundamental value. Transparency should not be compromised on grounds of the Keynesian-beauty-contest effect.

Accounting disclosure is a distinct feature of modern corporations characterized by dispersed ownership. Previous researches on accounting disclosure have mainly focused on agency problems and differential information among investors. This paper provides one example that dispersed ownership has other important consequences for accounting disclosure that have not received the attention they deserve. It opens many new opportunities for future researches. The Keynesian-beauty-contest effect may have substantial implications for the trade-off between public and private channels to disclose firm information, the relation between information quality and cost of capital, and the relation between earnings and stock prices.

Another promising direction is to extend the model to a general equilibrium so as to examine the allocational efficiency consequences of accounting disclosure in a Keynesian-beauty-contest stock market. Since allocational efficiency may diverge from market efficiency, such an extension complements this study and deepens our understanding of the real consequences of accounting disclosure in a Keynesian-beauty-contest stock market.
Appendix

Proof of Proposition 1. I solve for the model in five steps.

Step 1: Previous work has shown that for an economy in which investors have CARA utility functions and the payoff of the security is normally distributed, an investor \(i\)’s demand for the risky security is described by equation 3.

Step 2: The information structure in period 1.

Assume

\[ p_1 = bz + c\theta - ds_1 \quad (A-1) \]

For investor \(i\), she interprets \(p_1\) as an independent signal, \(p^*_1\), with precision \(\rho\).

\[ p^*_1 = \frac{1}{c}(p_1 - bz) = \theta - \frac{d}{c}s_1 \quad (A-2) \]

\[ \rho = (\frac{c}{d})^2 \gamma_1 \quad (A-3) \]

Correspondingly, her information set is \(I_{1i} = (z, p^*_1, x_{1i})\). Notice that \(p^*_1\) is the same for all investors although they have differential private information. Her belief about the fundamental value \(\theta\) is characterized by

\[ E_i[\theta|I_{1i}] = \frac{\alpha z + \rho p^*_1 + \beta x_{1i}}{\alpha + \rho + \beta} \]

and

\[ Var_i[\theta|I_{1i}] = \frac{1}{\alpha + \rho + \beta} \]
The estimate of the variance is independent of the realization of signals and thus the same across investors.

Step 3: The information structure in period 2.

Assume

\[ p_2 = a_2 p_1 + b_2 z + c_2 \theta - d_2 s_2 \] (A-4)

Given the assumption that both \( z \) and \( p_1^* \) are available to investors in period 2, investor \( i \) in period 2 starts with a prior about \( \theta, \frac{\alpha z + \rho p_1^*}{\alpha + \rho} \), with a precision of \( \alpha + \rho \). Moreover, she also learns from \( p_2 \). Given her knowledge about \( z \) and \( p_1^* \), the independent signal she can extract from \( p_2 \) is \( p_2^* \), with precision \( \rho_2 \).

\[ p_2^* = \frac{1}{c_2} (p_2 - a_2 p_1 - b_2 z) = \theta - \frac{d_2}{c_2} s_2 \]

\[ \rho_2 = \left( \frac{c_2}{d_2} \right)^2 \gamma_2 \]

Thus her information set is \( I_{2i} = (p_1^*, z, p_2^*, x_{2i}) \).

Conditional on \( I_{2i} \), she forms her belief of \( \theta \) as follows:

\[ E_i[\theta|I_{2i}] = \frac{\alpha z + \rho p_1^* + \beta x_{2i} + \rho_2 p_2^*}{\alpha + \rho + \beta + \rho_2} \]

\[ Var_i[\theta|I_{2i}] = \frac{1}{\alpha + \rho + \beta + \rho_2} \]

Again the estimate of the variance is independent of the realization of individual investors’ private signals and thus identical across investors.

Step 4: Solve for \( p_2 \).
According to equation 3, investor \( i \)'s demand conditional on \( I_{2i} \) is

\[
D_{2i} = \frac{\tau(E_i[\theta|I_{2i}] - p_2)}{Var_i[\theta|I_{2i}]}
\]

\[
= \tau[\alpha z + \rho p_1^* + \beta x_{2i} + \rho_2 p_2^* - (\alpha + \rho + \beta + \rho_2)p_2]
\]

\[
= \tau[\alpha z + \rho \frac{1}{c}(p_1 - b) + \beta x_{2i} + \rho_2 \frac{1}{c_2}(p_2 - a_2 p_1 - b_2 z) - (\alpha + \rho + \beta + \rho_2)p_2]
\]

\[
= \tau[(\alpha - \frac{b}{c} \rho - \frac{b_2}{c_2} \rho_2)z + (\frac{1}{c} \rho - \frac{a_2}{c_2} \rho_2)p_1 + \beta x_{2i} - (\alpha + \rho + \beta + (1 - \frac{1}{c_2})\rho_2)p_2]
\]

\( p_2 \) is determined by aggregating individual investors’ demands and equating it with the aggregate supply.

\[
p_2 = \frac{(\alpha - \frac{b}{c} \rho - \frac{b_2}{c_2} \rho_2)z + (\frac{1}{c} \rho - \frac{a_2}{c_2} \rho_2)p_1 + \beta x_{2i} - \frac{z_2}{\tau}}{(\alpha + \rho + \beta + (1 - \frac{1}{c_2})\rho_2)}
\]

(A-5)

The coefficients array \((a_2, b_2, c_2, d_2)\) are determined by comparing coefficients in equation A-5 with those in equation A-4.

\[
a_2 = \frac{\frac{1}{c} \rho}{\alpha + \rho + \beta + \rho_2}, \quad b_2 = \frac{\alpha - \frac{b}{c} \rho}{\alpha + \rho + \beta + \rho_2}, \quad c_2 = \frac{\beta + \rho_2}{\alpha + \rho + \beta + \rho_2}, \quad d_2 = \frac{1}{\beta \tau c_2}
\]

(A-6)

and

\[
\rho_2 = \beta^2 r^2 \gamma_2
\]

(A-7)

Step 5: Solve for \( p_1 \).

Investor \( i \) who purchases stocks in period 1 does not hold them until the firm is liquidated; instead, she resells them at the price of \( p_2 \). Her demand, according to equation 3, is shaped by her expectation about \( p_2 \), which is different from her expectation about \( \theta \).
In period 1, investor $i$'s belief about $p_2$ is characterized by

$$E_i[p_2|I_{1i}] = \frac{1}{c} \rho \frac{\alpha - b \rho}{\alpha + \rho + \beta + \rho_2} p_1 + \frac{\beta + \rho_2}{\alpha + \rho + \beta + \rho_2} E_i[\theta|I_{1i}]$$

$$= \frac{1}{c} \rho \frac{\alpha - b \rho}{\alpha + \rho + \beta + \rho_2} p_1 + \frac{\beta + \rho_2}{\alpha + \rho + \beta + \rho_2} \alpha z + \beta \rho_2 \beta x_{1i}$$

$$= \frac{1}{\alpha + \rho + \beta + \rho_2} [(1 + \frac{\beta + \rho_2}{\alpha + \rho + \beta}) (\alpha - b \rho) z + \frac{\beta + \rho_2}{\alpha + \rho + \beta} \beta x_{1i} + (1 + \frac{\beta + \rho_2}{\alpha + \rho + \beta}) \frac{1}{c} \rho p_1]$$

and

$$\text{Var}_i[p_2|I_{1i}] = (\frac{\beta + \rho_2}{\alpha + \rho + \beta + \rho_2})^2 \text{Var}_i[\theta|I_{1i}] + (\frac{\beta + \rho_2}{\alpha + \rho + \beta + \rho_2})^2 \frac{1}{\beta^2 \tau^2 \gamma_2}$$

$$= \frac{(\beta + \rho_2)^2}{(\alpha + \rho + \beta + \rho_2) (\alpha + \rho + \beta) \rho_2}$$

Her demand is $D_i$.

$$D_i = \frac{\tau (E_i[p_2|I_{1i}] - p_1)}{\text{Var}_i[p_2|I_{1i}]}$$

$$= \frac{\tau}{\alpha + \rho + \beta + \rho_2} \left[ (\alpha - b \rho) \frac{1}{M} z + \beta x_{1i} - (\alpha - b \rho) \frac{1}{M} + \beta) p_1 \right]$$

(A-8)

where

$$M = \frac{1}{1 + \frac{\alpha + \rho + \beta}{\beta + \rho_2}} = \frac{\beta + \rho_2}{\alpha + \rho + 2 \beta + \rho_2}$$

(A-9)

$p_1$ is determined by equating aggregate supply with aggregate demand.

$$p_1 = \frac{(\alpha - b \rho) \frac{1}{M} z + \beta \theta - \frac{\beta + \rho_2}{\tau p_2} s_1}{(\alpha - b \rho) \frac{1}{M} + \beta}$$

(A-10)

The coefficient array $(b, c, d)$ are determined by comparing coefficients in equations A-1 and A-10.

$$b = \frac{\alpha}{\alpha + \rho + \beta M}, c = \frac{\rho + \beta M}{\alpha + \rho + \beta M}, d = \frac{\rho + \beta M + \beta + \rho_2}{\alpha + \rho + \beta M} \frac{1}{\beta \tau}$$

(A-11)
and

$$\rho = \left( \frac{p_2}{\beta + p_2} \right)^2 \beta^2 \tau^2 \gamma_1$$  \hspace{1cm} (A-12)$$

So $p_1$ is solved for by plugging equation A-11 to equation A-1.

$$p_1 = \frac{\alpha}{\alpha + \rho + \beta M} z + \frac{\rho + \beta M}{\alpha + \rho + \beta M} \theta - \frac{\beta + p_2}{\rho_2} \frac{\rho + \beta M}{\alpha + \rho + \beta M} \frac{1}{\beta \tau}$$  \hspace{1cm} (A-13)$$

$p_2$ is determined by inserting equation A-11 and A-13 to equation A-6.

$$p_2 = \frac{1}{\alpha + \beta + \rho + p_2} [\alpha z + (\beta + \rho + p_2) \theta - \frac{\beta + p_2}{\rho_2} \frac{\rho + \beta M}{\beta \tau} s_1 - \frac{(\beta + p_2)}{\beta \tau} s_2]$$

Since the coefficients ($b, c, d$) and ($a_2, b_2, c_2, d_2$) are unique, the linear rational expectations equilibrium in Proposition 1 is unique, too.

Proof of Lemma 1. Lemma 1 is proved by using the traditional solution to a single-period noisy rational expectations equilibrium, such as that in Diamond and Verrecchia (1981). The procedure is the same as that in the proof of Proposition 1, except that $E_i[p_2|I_{1i}]$ and $Var_i[p_2|I_{1i}]$ in equation A-8 are replaced by $E_i[\theta|I_{1i}]$ and $Var_i[\theta|I_{1i}]$.

Proof of Proposition 2. Proposition 2 is proved by showing that the derivative of PE with respect to $\alpha$ is positive.

Define $M' = \frac{\partial M}{\partial \alpha}$. Thus, $M' = -\frac{(\beta + p_2)}{(\alpha + \beta \rho + p_2)^2} < 0.$
\[ PE = \frac{1}{E_{s,z}(p_1 - \theta)^2} \]
\[ = \frac{1}{E_{s,z}(b\varepsilon_z - ds_1)^2} \]
\[ = \frac{1}{b^2 \alpha + d^2 \gamma_1} \]
\[ = \frac{1}{\frac{b^2}{\alpha} + \frac{c^2}{\rho}} \]
\[ = \frac{\rho(\alpha + \rho + \beta M)^2}{\alpha \rho + (\beta M + \rho)^2} \]

\[ \frac{\partial PE}{\partial \alpha} = \frac{2\rho(\alpha + \beta M + \rho)(1 + \beta M')(\alpha \rho + (\rho + \beta M)^2) - \rho(\alpha + \beta M + \rho)^2(\rho + 2(\rho + \beta M)\beta M')}{(\alpha \rho + (\rho + \beta M)^2)^2} \]
\[ = \frac{(\alpha + \beta M + \rho)\rho}{(\alpha \rho + (\rho + \beta M)^2)^2} [2\beta^2 M^2 + 3\beta M \rho + \alpha \rho + \rho^2 - 2\alpha \beta^2 MM'] > 0 \]

\[ \square \]

**Proof of Proposition 3.** Proposition 3 is proved by showing that the derivate of \( R \) with respect to \( \alpha \) is positive.

\[ R = 1 - \frac{1}{b/\bar{c}/\bar{c}} = \frac{\beta(1 - M) + \rho_1(1 - k)}{\beta + \rho_1} \]

(A-14)

\[ \frac{\partial R}{\partial \alpha} = -\frac{\beta M'}{\beta + \rho_1} > 0 \]
Proof of Observation 1. Observation 1 is proved by showing that the derivative of \( PE \) with respect to \( \alpha \), evaluated at \( R = r = \frac{\beta(1-M) + \rho(1-k)}{\beta + \rho_1} \), is negative under condition 9 and 10.

For notational convenience, define \( B \) as follows.

\[
B = \frac{\dot{b}}{\dot{c}} = \frac{\alpha}{\beta + \rho_1}
\]

\[
PE = \frac{1}{\frac{b^2}{\alpha} + \frac{c^2}{\rho}} = \frac{1}{\frac{c^2}{\rho}} = \frac{(b/c)^2}{\alpha + \frac{1}{\rho}} = \frac{(1 - R + B)^2}{\frac{B^2}{\alpha} + \frac{(1-R)^2}{\rho}} = \frac{\alpha \rho (1 - R + B)^2}{B^2 \rho + (1 - R)^2 \alpha}
\]

and

\[
\frac{\partial PE}{\partial \alpha}
|_{R=r} = \frac{\rho^2 (1-r+B) \alpha^2}{(\beta + \rho_1)^3((1-r)^2 \alpha + B^2 \rho)^2}
\]

\[
[\alpha - (1-r)(\beta + \rho_1)(1 - \frac{2(\beta + \rho_1)}{\rho} (1-r))] (A-15)
\]

Since \( \alpha, \beta, \tau, \gamma_1, \) and \( \gamma_2 \) are positive and finite, both \( M \) and \( k \) lie between zero and one. As a result, \( 0 < r < 1 \). It could be verified that \( A-15 < 0 \) is equivalent to conditions 9 and 10. \( \square \)
References


