Promises Promises

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September 1996

Abstract

In the classical general equilibrium model, agents keep all their promises, every good is traded, and competition prevents any agent from earning superior returns on investments in financial markets. In this paper I introduce the age-old problem of broken promises into the general equilibrium model, and I find that a new market dynamic emerges.

Given the legal system and institutions, market forces of supply and demand will establish the collateral levels which are required to secure promises. Since physical collateral will typically be scarce, these collateral levels will be set so low that there is bound to be some default. Many kinds of promises will not be traded, because that also economizes on collateral. Scarce collateral thus creates a mechanism for determining endogenously which assets will be traded, thereby helping to resolve a long standing puzzle in general equilibrium theory. Finally, I shall show that under suitable conditions, in rational expectations equilibrium, some investors will be able to earn higher than normal returns on their investments.

The legal system, in conjunction with the market, will be under constant pressure to expand the potential sources of collateral. This will lead to market innovation.

I illustrate the theoretical points in this paper with some of my experiences on Wall Street as director of fixed income research at the firm of Kidder Peabody.

*The analysis in this talk is based on the paper Geanakoplos–Zame–Dubey [1995].

1 Introduction

Every talk is to some extent autobiographical. But when big changes occur in one's life, one must step back and ask what one is really doing. I spent five years as director of fixed income research at Kidder Peabody & Co., until the firm closed in January 1995 after 130 years in business, following a scandal in its Treasury bond department and the collapse of the new-issue mortgage market. In my last two months there I, like all the other managers, went through a ritual firing of each one of my subordinates; I was then fired myself. After leaving Kidder, I became a partner along with some of the former Kidder traders in a hedge fund called Ellington Capital Management that we set up to invest in mortgages and especially mortgage derivatives. At Kidder we had dominated the collateralized mortgage obligation (CMO) market, issuing about 20% of the trillion dollars of CMOs created between 1990 and 1995. What we do now at Ellington is buy the very securities we sold while at Kidder. For a time our motto seemed to be "We made the mess, let us clean it up." What good was I doing society at Kidder Peabody? Is there any logic in switching from the sell side to the buy side, or am I just drifting with the tide of irrational market passions?

Not surprisingly, given man's natural inclination to rationalization, I shall try to argue that Kidder Peabody was part of a process that improved welfare, and that this process makes it inevitable, even if everybody is rational, that there should be periodic episodes of extraordinary investment opportunities in moving from the sell side to the buy side and back. What might be interesting about my talk is therefore not my conclusion, but the framework I shall try to describe in which to argue my claims, and from which I derive some other consequences of Wall Street's main activity.

I shall suggest that the main business of Wall Street is to help people make and keep promises. Over time, as more people have been included in the process, punishment and reputation have been replaced by collateral. This enabled a proliferation of promises, but has led to a scarcity of collateral. The ensuing efforts of Wall Street, in conjunction of course with the government and in particular the courts and the tax code, to stretch the available collateral has significant and surprising effects on the working of the economy, including the cyclical or volatile behavior of prices, which is the subject of my talk here today.

Promises play a vital role in society, and this has been recognized by the great philosophers for a long time. For example, Plato's first definition of justice in the Republic is keeping promises. Socrates eventually rejects that definition because he says that it is sometimes more just not to keep one's promises.

Nietzsche begins the second essay in the *Genealogy of Morals* by asking "To breed an animal with the right to make promises — is this not the paradoxical task that nature has set itself in the case of man?" His idea is that before man could be expected to keep promises he had to develop a conscience, and the capacity to feel guilt. This capacity was bought with generations of torture and cruel punishments for transgressors. Though this history is forgotten, modern man retains the guilt. Thus "the major moral concept guilt (Schuld) has its origin in the very material concept of debts (Schulden)."

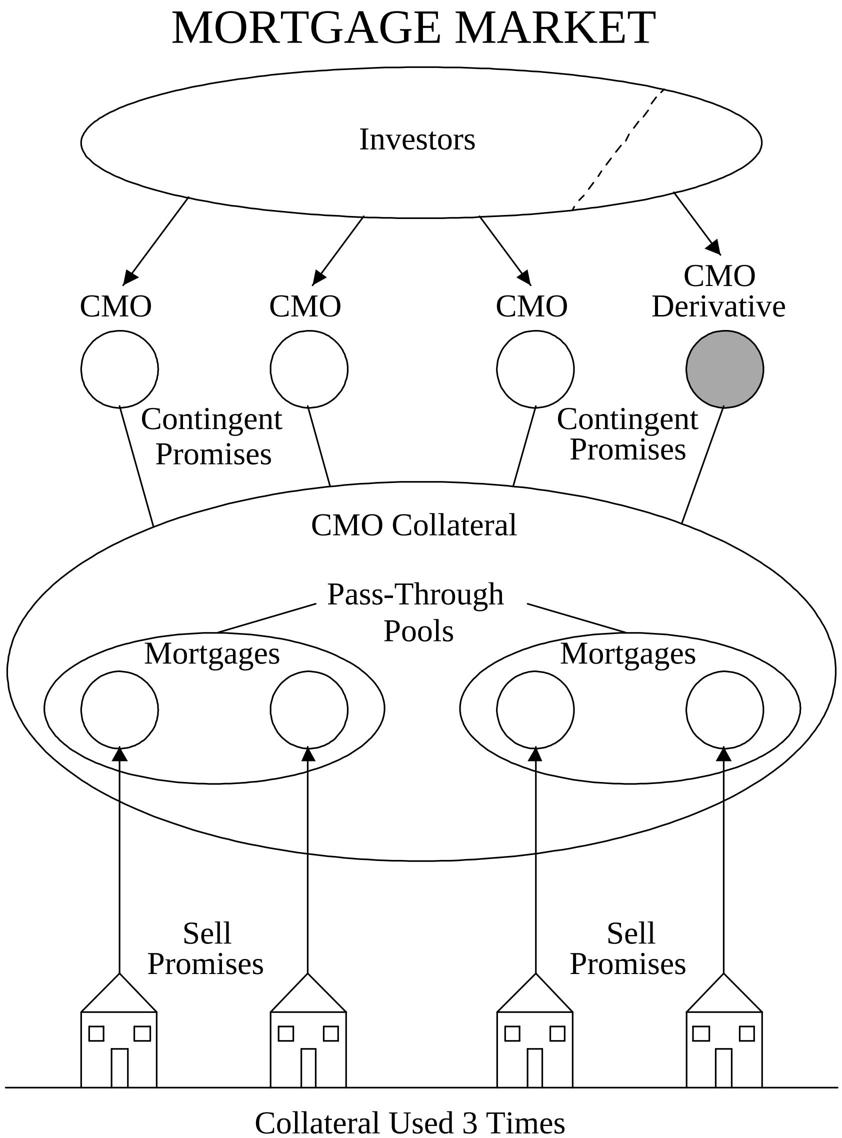
Wall Street has emancipated the debtor relationship from guilt and reduced the amount of suffering by permitting default without punishment, but attaching collateral. One should expect that such a conversion would have important consequences for the workings of the economy and society. I hope to provide a framework for understanding some of them.

2 Mortgages, Kidder Peabody, and Me

My subject today is promises secured by collateral. The vast majority of promises, especially if they extend over a long period of time, are not guaranteed only by oaths and good intentions, but by tangible assets called collateral. Anything that has durable value can in principle serve as collateral, but the less sure its value, and perhaps even more importantly, the more hazardous its ownership, the less likely it will in fact be used as collateral. Short term US government bonds are an ideal collateral for short term loans, because their value is guaranteed, and it is easy for the lender to claim the collateral if there is a default (by holding onto it from the beginning of the loan in what is called a REPO). Long term government bonds are less appropriate for short term loans because their value is very volatile when inflation and real interest rates are variable. Equipment, plants, and whole companies are often used as collateral for corporate bonds, though they are far from ideal, both because their market values are volatile and because much of this value can be dissipated when (or just before) there is a default. Perhaps the most plentiful and useful collateral for short and long term loans are residential homes.

A residential mortgage is a promise to deliver a specified amount of money (perhaps over many periods) in the future, that is secured by a house. If the promise is broken, the house can be confiscated and sold, with the proceeds needed to make the promise whole turned over to the lender. The surplus, if there is any, is returned to the borrower, and the shortfall, if there is any, is typically a loss for the lender. Sometimes the defaulter faces further penalties, including liens on future income or even perhaps jail. Over time these penalties have declined in severity. Since the house cannot run away, there is little opportunity for the borrower to steal part of its value if he anticipates defaulting. Indeed the only effective way to appropriate part of the value is to refrain from making repairs over a long period of time. But defaults usually occur in the first few years of the mortgage.

The depression in the 1930s led to a huge number of defaults, especially on farm mortgages. As a result the amortizing mortgage was invented, whereby the debtor not only makes regular interest payments, but also pays back part of the principal each month. After some years, the principal has been paid down to the point where any default can easily be covered by selling off the farm. In the early years of the mortgage, the danger of default depends critically on the loan to value ratio, that is on how much is borrowed relative to the value of the farm. This ratio is set by supply and demand. By contrast, the pre-depression mortgages had regular payments of interest, and a large balloon payment at the end in which the debtor repaid the



whole of the principal. In balloon mortgages, the probability is relatively high that the farm value will not cover the loan at the balloon payment date.

There are approximately 70 million homes in America, worth on average about $100,000. If we add to this various other kinds of multifamily and commercial real estate, we get a real estate market worth over $10 trillion, giving rise to well over $1 trillion of actively traded mortgage promises.

Until the 1970s, the banks which issued the mortgages bought and held those promises from homeowners. They therefore were subject to two large risks: that inflation would erode the value of their promises, and that higher real interest rates would make them regret tying up their money in a low return instrument. After the 1970s deregulation they could rid themselves of these risks by selling the mortgages to other investors, while continuing to make money by servicing the mortgages. (The homeowner is oblivious to the fact that his mortgage promise has been sold because the same bank services his account by collecting his checks, and writing him reminders when he is late with his payments, but forwarding the money on to the new owner of the promise).

Various government agencies called Fannie Mae, Ginnie Mae, and Freddie Mac were created to make the purchase of these securities more attractive to investors. The agencies provided insurance against homeowner default, and tough (but uniform) criteria for homeowners to get their mortgages included in the program. Most importantly, the agencies bundled together many mortgages into composite securities that investors could buy, in effect getting a small share of many different individual mortgages. As a result, the investors did not have to worry about the adverse selection problem of getting stuck with only the unreliable homeowner promises, nor did they need to spend time investigating the individuals whose promises they were buying. (Of course they did spend millions of dollars calculating average default rates and repayment rates; but that is small compared to the cost they would have incurred if they had access to the personal credit history of each homeowner.)

Wall Street took the whole operation a step further by buying big mortgage pools and then splitting them into different pieces or "tranches", which summed up to the whole. These derivative pieces are called collateralized mortgage obligations (CMOs) because they are promises secured by pools of individual promises, each of which is backed by a physical house. As the following diagram makes clear, there is a pyramiding of promises in which the CMO promises are backed by pools of promises which are backed by individual promises which are backed by physical homes. The streams of promises sometimes come together when many promises are pooled to back other promises, and sometimes split apart when one (derivative) promise is tranched into many smaller promises.

It is interesting to note that the investors buying the CMO tranches also use collateral, when they buy on margin. The investor pays a fraction of the price, borrowing the remainder, and putting up the CMO tranche itself as collateral. The fraction it is necessary to put down (the margin) is determined by the forces of supply and demand, and occasionally by government regulation.

The splitting of mortgage promises into CMO tranches increases the number of buyers in two ways. In the first place, differentiating the cash flows enables investors to purchase the flows that best suit their needs. For example, a pension plan might want cash far in the future, whereas a large winter clothing manufacturer with excess cash might be interested in finding a short term investment with an enhanced yield. Similarly, a bank which is already committed to many long term loans might want money when interest rates go up, whereas a credit card company that makes only short term loans might want to invest its cash reserves in securities that pay when interest rates go down. CMO tranches accordingly differ in their temporal cash flows, and some pay more when interest rates go up (floaters), while others pay more when interest rates go down (inverse floaters). The large menu of potential investments enables the investor to better hedge his business. By spreading risk to those best able to bear it, the creation of new asset markets increases the productive capacity of the economy.

Splitting mortgage promises into pieces with cash flows contingent on exogenous variables like the interest rate increases demand because it enables investors to bet on which way interest rates and the other exogenous variables will go. Though this may make each investor feel richer, it cannot be said to be good for the economy. It does however raise the value of a mortgage to the bank by making it easier to sell after it is split up into pieces, thereby reducing the interest rate homeowners must pay on their mortgages.

The first CMO was created by Salomon Brothers and First Boston in 1983. It consisted of four tranches, and was worth just over $20 million. Shortly thereafter most of the Wall Street firms joined the CMO market, led by the biggest firms Lehman Brothers, Goldman Sachs, and Merrill Lynch, and joined later by Bear Stearns, Drexel (before it went bankrupt in the Michael Milken scandal), DLJ, Nomura, and others. By 1993 Kidder Peabody was tranching pools worth in excess of $3 billion into 100 pieces.

In the spring of 1990 when I went on leave from Yale I decided that it would be enlightening for me, as a mathematical economist, to see first hand how the practitioners in finance were using the mathematical models that academic economists had created. In finance, more than in any other field of economics, the vocabulary economic theorists invented to study the world has become the vocabulary of the practitioners themselves. It is never easy for a theorist to get an insider's look at the markets; I was especially afraid that if I joined a large firm for a short term visit I would be relegated to some very specialized subdepartment. In the winter of 1989 I had spent one summer month at Kidder Peabody working on a small mathematical problem in their tiny mortgage research department. This I thought would be the perfect firm in which to spend a semester: small, yet a participant in nearly every market, and with a venerable 125 year history beginning in the last year of the Civil War. Little did I realize at the time both how lucky and how unlucky I would be. Toward the end of my stay, the firm's management decided they needed to reorient the research department in line with the growing mathematical emphasis of their competitors. They turned to me for some advice, which I did my best to give, and in October of 1990, three months after I ended my sabbatical there and returned to Yale, they offered me the job of Director (part-time) of Fixed Income Research.

During the time I spent at Kidder, the firm went from being a negligible player in the mortgage business to the greatest CMO powerhouse on the street. For each of the five years between 1990 and 1994, Kidder Peabody underwrote more CMOs then any other firm, averaging nearly 20% in a market with forty competitors. The next biggest firm averaged under 10% in that time. In those five years we created more than $200 billion of CMO tranches.

Suddenly in April of 1994 the firm was rocked by a trading scandal in the government bond department, where the head trader was accused of fraudulently manipulating the accounting system into crediting him with $300 million of phantom profits. Two weeks later the Fed raised interest rates, and then did so again four more times the rest of the year. The bond market plummeted, and activity on Wall Street shrank markedly. Many firms had terrible years. The worst hit perhaps were those who had bet on interest rates continuing their downward trajectory by buying bonds such as inverse floaters. CMO tranches, like bonds in general, did terribly, but some of the more esoteric pieces called mortgage derivatives, such as the inverse floaters, did the worst of all. The newspapers were filled with stories about the dangers of investing in mortgage derivatives, and the CMO market suddenly dried up. The great Kidder money machine ground to a halt. At the same time, the trading scandal became more lurid. In January 1995, General Electric officially sold the firm to Paine Webber, and Kidder Peabody ceased to exist as an investment bank after 130 years.

The leading Kidder mortgage traders founded a new company called Ellington Capital Management, which I subsequently joined as a small partner. Instead of creating mortgage derivatives, as we did at Kidder, now we buy them. In one and a half years of operation, we have earned an extraordinarily high return on our capital. So we return to our puzzles: what good did Kidder Peabody do, and how can it be that in a world of rational investors it is possible to make money on the sell side of a market, and then on the buy side of the same market?

3 The Model

We now try to formalize some of the observations we have made about the mortgage market, and we investigate the logical implications of our view of the world.

3.1 States of the World

Probably the most important ingredient in our model is the idea of a state of the world, representing a complete description of what will happen to every asset and commodity and preference in the economy over its life. Furthermore, we posit a set S representing all possible states of the world.

Investors have always been aware that there are risks, and they have always sought to understand these risks. Until recently, however, much of this took the form of a qualitative analysis of important factors that might bear on their returns. For example, ten years ago an investor in the mortgage market would be told that demographics and macroeconomic business cycles affect the demand for new housing. He might have been shown graphs of previous interest rate cycles, and if he was really sophisticated, he might have been shown a history of mortgage prepayments. Nowadays, an investor will demand a computer calculated description of all his future cash flows under each of several scenarios that specify all relevant events for the next 30 years (including vectors of future interest rates and mortgage prepayments). These scenarios correspond to our states of the world.

Needless to say, there is an infinity of potential scenarios, so no matter how many the investor sees, he has at best anecdotal information about what might happen to his investment. Moreover, he cannot really make use of this information unless he also makes the additional heroic assumption that the states of the world he asked about constitute all possible states of the world. Thus we arrive at the same point of view we take in our modeling. Our set S is in effect the union over all the investors of the states each thinks are possible.

For expository simplicity, we shall suppose that there are only two time periods, today and the future.

3.2 Agents

Agents act on a desire to consume as much as possible. Of course there is a variety of say L goods over which agent tastes differ, and agents must also take into account that their consumption might be different in different states of the world. The consumption space is therefore $R_+^L \times R_+^{SL}$, meaning that agents realize that they must choose over consumption plans that specify what they will consume of each of the L goods today, and what they will consume of each of the L consumption goods in every state next period. For each good $s\ell$ there is a market on which the good is traded at a price $p_{s\ell}$.

As we said earlier, investors trade assets primarily to bet or to hedge. We incorporate the betting motive into our model by differences in the probabilities investors assign to the states in S. Hedging is represented by supposing agents want to maximize not the expectation of consumption, but the expectation of the utility of consumption. We suppose utility is a concave function of consumption, which means that additional consumption increases utility more when there is less to start with. Thus agents would prefer to give up some consumption in states where they will be rich in order to obtain the same amount extra in equally probable states in which they expect to be poor.

We subsume both betting and hedging by allowing agents $h \in H$ to have arbitrary concave, continuous, and monotonic utilities

$$u^h : R_+^L \times R_+^{SL} \rightarrow R$$

and arbitrary (but strictly positive) endowments

$$e^h \in R_{++}^L \times R_{++}^{SL}.$$

Utility may, but need not, take the special form

$$u^h(x^h) = v^h(x_0^h) + \sum_{s=1}^{s=S} \gamma_s^h v^h(x_s^h)$$

where γ_s^h represents the probability agent h assigns to state s. An agent h who does not trade will be forced to consume his endowment. He may well prefer to exchange wealth from some state s where his marginal utility of consumption $\partial u^h(e^h)/\partial x_{s\ell}$ is relatively low, either because state s is regarded to be of low probability or because consumption e_s^h is already so high there, for wealth in another state s' where his marginal utility of consumption is relatively high.

3.3 Promises

From the foregoing it is evident that economic welfare can be improved if agents can exchange consumption goods across states of the world. The purpose of the financial markets is to make such exchanges feasible.

Agents trade across states by making promises before the states are realized. There is evidently no point offering to give up goods in a state once everybody knows it will never occur. The receiver of a promise must provide something in return, usually a payment today. Thus it makes sense to think of a promise as an asset sold by the promisor to the receiver in exchange for a price today. (More generally one could allow for the exchange of promises against promises, which indirectly are permitted in our model, as we shall see.) A promise $j \in J$ is denoted by a vector $A_j \in R_+^{SL}$ specifying the basket of goods that are promised for delivery in each state. Equivalently, $p_s \cdot A_{sj}$ is the money promised for delivery in state s. Notice that the promise may be contingent; there is no reason (at least a priori) that the vector A_{sj} should be the same in each state of the world. Indeed agents trade across states of the world by buying promises that are supposed to deliver in some states and selling promises that are supposed to deliver in other states.

We restrict the collection of promises to a finite set J. This is done mostly for technical reasons, so that we can work in finite dimensional Euclidean spaces. Of course we can take $\#J$ so large that the assets in J come close to approximating every conceivable asset on some bounded region.

3.4 Deliveries and Default

An agent who makes a promise has the option of delivering less than he promised. We denote by D_{sj}^h the deliveries of money agent h actually makes in state s. The shortfall $[p_s \cdot A_{sj} - D_{sj}^h]^+$ between what is promised and what is delivered is the amount of default, measured in dollars. Note that there is no point in delivering more than is promised.

Some borrowers are more reliable than others, either because they are more honest, or because the penalties to defaulting affect them more acutely, or because they simply have more wealth. A full blown model of default and penalties, as presented for example in Dubey–Geanakoplos–Shubik [1990], would need to take into account the adverse selection facing lenders who should be wary that anybody who wants to borrow from them is likely to be a worse than average risk. Collateral has the advantage that the lender need not bother with the reliability or even the identity of the borrower, but can concentrate entirely on the future value of the collateral. Collateral thus retains anonymity in market transactions. If there are no penalties to defaulting, then every borrower will deliver the minimum of what he owes in every state and the value of the collateral he put up to secure his promise.

A remarkable theorem of Ken Arrow [1953] shows that if penalties can be taken to be infinite, so that nobody dares default, then any competitive equilibrium allocation will be efficient (i.e., optimal from the social point of view) provided there are at least as many independent promises as there are states of nature. Of course when penalties are infinite for everybody, there is no adverse selection and anonymity is restored. The trouble is that in the real world it is impossible to make penalties too harsh; the punishment must in some sense fit the crime if it is to be politically feasible. More troubling, it is inconceivable that there will be as many assets as there are states of nature, particularly if these assets must be traded in liquid (high volume) markets. When the number of assets falls short of the number of states, then it is generally better from the social point of view to keep the penalties at intermediate levels of harshness, as shown in Dubey–Geanakoplos–Shubik [1990]. If the harshness of penalties is limited, some people will default, and adverse selection becomes an issue. Collateral will again become useful together with the penalties in order to guarantee more promises. In the interest of simplicity, we are led to preserve anonymity and restrict attention to collateral in place of penalties.

3.5 Collateral

The difficulty with promises is that they require some mechanism to make sure they are kept. This can take the form of penalties, administered by the courts, or collateral. As we mentioned at the outset, more and more often collateral has displaced penalties. In this talk we shall exclusively deal with collateral, by supposing that there is no penalty, legal or reputational, to defaulting. Of course, even collateral requires the courts to make sure the collateral changes hands in case of default.

The simplest kind of collateral is pawn shop collateral — valuable goods like watches or jewelry left with third parties (warehoused) for safekeeping. Financial markets have advanced as the number of goods that could function as collateral has increased, from watches and jewelry, to stocks and bonds. A further advance occurred when lenders (instead of warehouses) held collateral, like paintings, that afforded them utility. This required a more sophisticated court system, because the lender had to be obliged to return the collateral if the promise was kept. The biggest advance, however, was in allowing the borrower himself to continue to hold the collateral. This enabled houses, and later cars, to be used as collateral, which again is only possible because of a finely tuned court system that can enforce the confiscation of collateral.

More recently the complexity of collateral has taken several more giant steps forward. Pyramiding occurs when an agent A puts up collateral for his promise to B, and then B in turn uses A's promise to him, and hence in effect the same collateral for a promise he makes to C, who in turn reuses the same collateral for a promise he makes to D. Mortgage pass through securities offer a classic example of pyramiding. Pyramiding naturally gives rise to chain reactions, as a default by Mr. A ripples through, often all the way to D.

Still more complex is tranching, which arises when the same collateral backs several promises to different lenders. Needless to say, the various lenders will be concerned about whether their debts are adequately covered. Tranching usually involves a legal trust which is assigned the duty of dividing up the collateral among the different claims according to some contractual formula. Again collateralized mortgage obligations offer a classic example of tranching.

Every one of these innovations is designed to increase or to stretch the available collateral to cover as many promises as possible. We shall see later that active default is another way of stretching the available collateral.

For the formal analysis in this talk I shall avoid pyramiding and tranching. All collateral will be taken by assumption to be physical commodities. Collateral must be put up at the moment the promise is sold, even if the delivery is not scheduled for much later. Agents are not allowed to pledge their future endowment as collateral, because that would raise questions in the minds of lenders about whether the borrowers actually will have the endowments they pledged, and therefore it would once again destroy the anonymity of markets.

3.6 Assets

To each promise j we must formally associate levels of collateral. Any good can potentially serve as collateral, and there is no reason why the single promise j cannot be backed by a collection of goods. The bundle of goods that is required to be warehoused for asset j is denoted $C_j^W \in R_+^L$, the vector of goods that the lender is allowed to hold is denoted $C_j^L \in R_+^L$, and the vector of goods the borrower is obliged to hold is denoted $C_j^B \in R_+^L$. An asset j is defined by the promise it makes *and* the collateral backing it, $(A_j, C_j^W, C_j^L, C_j^B)$. It is quite possible that there will be many assets which make the same promises $A_j = A_{j'}$, but trade at different prices because their collateral levels are different $(C_j^W, C_j^L, C_j^B) \neq (C_{j'}^W, C_{j'}^L, C_{j'}^B)$. Similarly the two assets might require exactly the same collaterals, but trade at different prices because their promises are different.

The price of asset j is denoted by π_j. A borrower sells asset j, in effect borrowing π_j, in return for which he promises to make deliveries according to A_j.

3.7 Production

Collateral is useful only to the extent that it is still worth something when the default occurs. Durability is a special case of production, so we introduce production into our model, and allow all goods to be durable, to varying degrees.

For ease of notation we shall suppose that production is of the fixed coefficient, constant returns to scale variety. One unit of commodity ℓ becomes a vector of commodities next period. A house may become a house that is one year older, wine may become a wine that is one year older, grapes may become wine one year later and so on. In these examples, one good became a different good the next period, but there is no reason not to permit one good to become several goods if it splits.

The transformation of a commodity depends of course on how it is used. Again for simplicity, we suppose that each commodity ℓ is transformed into a vector $Y_{s\ell}^0 \in R_+^L$ in each state s if it is used for consumption (e.g., living in a house, or using a light bulb). If ℓ is warehoused, then we assume that it becomes a vector $Y_{s\ell}^W \in R_+^L$ in each state s. Likewise, if it is held as collateral by the lender it becomes a vector $Y_{s\ell}^L \in R_+^L$ in each state s, while if it is held by the borrower it becomes the vector $Y_{s\ell}^B \in R_+^L$ in each state s. The $SL \times L$ dimensional matrices Y^0, Y^W, Y^L, Y^B summarize these different durabilities.

Observe that we have allowed for differential durability depending on the use to which the commodity is put. But we have not allowed the durability to be affected by the identity of the user, or by the intensity of its use (which would amount to the same thing). In this way the anonymity of markets is maintained, and our modeling problem becomes easier. In Geanakoplos–Zame–Dubey [1995] the more general case of penalties and individual differences in the treatment of collateral is permitted.

Given the collateral requirements (C_j^W, C_j^L, C_j^B) for each promise j, the security they provide in each state s is

$$p_s \cdot [Y_s^W C_j^W + Y_s^L C_j^L + Y_s^B C_j^B].$$

No matter who holds the collateral, it is owned by the borrower but may be confiscated by the lender (actually by the courts on behalf of the lender) if the borrower does not make his promised deliveries. Since we have assumed that the borrower has nothing to lose but his collateral from walking away from his promise, it follows that the actual delivery by every agent h on asset j in state s will be:

$$D_{sj}^h = \min\{p_s \cdot A_s^j,\ p_s \cdot [Y_s^W C_j^W + Y_s^L C_j^L + Y_s^B C_j^B]\}.$$

4 Collateral Equilibrium

We are now ready to put together the various elements of our model. An economy E is defined by a vector

$$E = ((u^h, e^h)_{h \in H}, (A^j, C_j^W, C_j^L, C_j^B)_{j \in J}, (Y^0, Y^W, Y^L, Y^B))$$

of agent utilities and endowments, asset promises and collateral levels, and the durability of goods kept by buyers, warehouses, lenders, and borrowers, respectively.

In keeping with the standard methodological approach of general equilibrium and perfect competition, we suppose that *in equilibrium* agents take the prices (p, π) of commodities and assets as given. We do not mean to suggest by this that the prices are actually set somewhere else, but only that competition among many agents guarantees that no one of them has the power to set the prices himself, so that paradoxically it appears to each and every one of them that the prices are set elsewhere.

These equilibrium prices cannot be known in advance, independent of the preferences and endowments of the agents, because they must be precisely chosen so that when agents believe in them, their optimizing trades will lead to a balance of supply and demand in all the commodity markets and all the asset markets.

Our price taking hypothesis also has the implication that agents have rational expectations about future prices, for these are taken as given as well. Agents in our model have perfect conditional foresight, in that they anticipate at time 0 what the prices p_s will be, depending on which state s prevails at time 1. Since they know the collateral that has been put up, and they know the production technology, they also understand in each state how much each asset will actually pay.

It might seem therefore that we could simply replace each asset promise A_j with an actual delivery vector, and thereby bypass the complications of collateral. But this is not possible, since whether an asset defaults or not in state s depends on whether the promise or the collateral is worth more. Since both are vectors, this cannot be known in advance until the prices $p_s \in R^L_+$ have been determined in equilibrium.

4.1 The Budget Set

Given the prices (p, π), each agent h decides what net trades of commodities $(x_0 - e^h_0)$, what asset purchases θ, and what asset sales φ he will make at time 0. Note that for every promise φ_j that he makes, he must put up the corresponding collateral $(C^W_j, C^L_j, C^B_j)\varphi_j$. The value of all his net trades at time 0 must be less than or equal to zero, that is, the agent cannot purchase anything without raising the money by selling something else (initial endowments of money are taken to be zero).

After the state of nature is realized in period 1, the agent must again decide on his net purchases of goods $(x_s - e^h_s - Y^0_s x_0)$. Recall that the goods x_0 whose services he consumed at time zero may be durable, and still available (in the form $Y^0_s x_0$) for consumption at time 1 in each state s. These net expenditures on goods can be financed out of sales of the collateral that the agent put up in period 0, and from the receipts from assets j that he purchased at time 0, less the deliveries the agent makes on the assets he sold at time 0. Putting all these transactions together, and noting again that the agent cannot buy anything without also selling something else of at least equal value, we derive the budget set for agent h:

$$
\begin{aligned}
&B^h(p,\pi) = \{(x,\theta,\varphi) \in R^L_+ \times R^{SL}_+ \times R^J_+ \times R^J_+ : \\
&p_0(x_0 - e^h_0) + \pi(\theta - \varphi) + p_0 \sum_{j\in J}(C^W_j + C^L_j + C^B_j)\varphi_j \leq 0 \text{ and for all } s \in S, \\
&p_s(x_s - e^h_s - Y^0_s x_0) \leq \sum_{j\in J} \varphi_j p_s \cdot [Y^W_s C^W_j + Y^L_s C^L_j + Y^B_s C^B_j] + \\
&\sum_{j\in J}(\theta_j - \varphi_j)\min\{p_s \cdot A^j_s,\ p_s \cdot [Y^W_s C^W_j + Y^L_s C^L_j + Y^B_s C^B_j]\}\}
\end{aligned}
$$

4.2 Equilibrium

The economy $E = ((u^h, e^h)_{h\in H}, (A_j, C^W_j, C^L_j, C^B_j)_{j\in J}, (Y^0, Y^W, Y^L, Y^B))$ is in equilibrium at macro prices and individual choices $((p,\pi), (x^h, \theta^h, \varphi^h)_{h\in H})$ if supply equals demand in all the goods markets and asset markets, and if given the prices, the designated individual choices are optimal, that is if:

$$
\begin{aligned}
&1) \quad \sum_{h\in H}(x^h_0 - e^h_0 - \sum_{j\in J}(C^W_j + C^L_j + C^B_j)\varphi^h_j) = 0, \\
&1') \quad \sum_{h\in H}(x^h_s - e^h_s - Y^0 x_0 - \sum_{j\in J}\varphi^h_j[Y^W_s C^W_j + Y^L_s C^L_j + Y^B_s C^B_j]) = 0
\end{aligned}
$$

2) $\sum_{h\in H}(\theta^h - \varphi^h) = 0$

3) $(x^h, \theta^h, \varphi^h) \in B^h(p, \pi)$

4) $(x, \theta, \varphi) \in B^h(p, \pi) \Rightarrow u^h(x_0 + \sum_{j\in J}[C_j^B \varphi_j + C_j^L \theta_j], \bar{x})$

$\leq u^h(x_0^h + \sum_{j\in J}[C_j^B \varphi_j^h + C_j^L \theta_j^h], \bar{x}^h).$

We write $x^h = (x_0^h, \bar{x}^h)$, so consumption at time 0 is $x_0^h + \sum_{j\in J}[c_j^B \varphi_j^h + c_j^L \theta^h]$.

5 Benchmark Economies

It is easiest to see the role collateral plays in the economy and the realism of our model by considering three alternative benchmark economies in which collateral is absent, either because there is no need to worry about default, or because the court system is so backward that there is no available provision for recovering collateral in case of default. We can simultaneously cover all these benchmark cases with one formal model that drops collateral and default, by reinterpreting in various ways the set J of assets.

Consider an economy in which the promises are exactly as before, but in which agents automatically keep all their promises (perhaps because implicitly they recognize that there would be a horrible punishment if they did not). Without default or collateral, only net asset trades matter and the budget set would simply become

$$B^h(p, \pi) = \{(x, \theta, \varphi) \in \mathbb{R}_+^L \times \mathbb{R}_+^{SL} \times \mathbb{R}_+^J \times \mathbb{R}_+^J : p_0(x_0 - e_0^h) + \pi(\theta - \varphi) \leq 0,$$
$$p_s(x_s - e_s^h - Y_s^0 x_0) \leq p_s A(\theta - \varphi) \text{ for all } s \in S\}.$$

In equilibrium demand would have to balance supply for commodities and assets, and every agent would have to be choosing optimally in his budget set. The equilibrium conditions then are:

1) $\sum_{h\in H}(x_0^h - e_0^h) = 0, \sum_{h\in H}(x_s^h - e_s^h - Y^0 x_0) = 0$

2) $\sum_{h\in H}(\theta^h - \varphi^h) = 0$

3) $(x^h, \theta^h, \varphi^h) \in B^h(p, \pi)$

4) $(x, \theta, \varphi) \in B^h(p, \pi) \Rightarrow u^h(x) \leq u^h(x^h)$

We describe each of our benchmark economies as special cases of this model below. In a later section we shall consider two examples which further illuminate the difference between these models and our collateral model. Perhaps the most important difference is that without explicitly considering the possibility of default and the safeguards the social system designs to counteract it, one has no good theory to explain which asset markets are actively traded and which are not.

5.1 Arrow–Debreu Complete Markets Equilibrium

In our first benchmark economy we presume that every conceivable promise is available to be traded in J. It can easily be shown, as Arrow and Debreu noted in the 1950s, that equilibrium efficiently spreads risks and that there is nothing any government agency can do to improve general welfare. A government which observes a complete set of asset markets should therefore concentrate on punishing people who default by setting draconian penalties so that there will be no default. There is no need for a government in such a world of complete markets to make any arrangements for compensating lenders against whom there is default.

In Arrow–Debreu equilibrium, the value of durable commodities will be at a peak. Durable goods can have tremendously high potential value because they provide services over a long period of time. If there were sufficient assets available, agents could in principle sell promises to pay their entire future endowments in order to get the money to buy the durable good today. An agent can thus buy a house for say $200,000 even though he has only a few thousand dollars of savings, because he can borrow the rest of the money by selling promises to pay people later out of his future endowment.

The value of a durable good can fluctuate over time or across states of nature because tastes for the good have changed across the states, or because people expect tastes to change (in a multiperiod model). There is no reason for prices to change because relative income changes across states of nature, since with complete markets, agents will insure themselves so that relative incomes will not change dramatically across states of nature.

In Arrow–Debreu equilibrium there is no connection between the value of promises outstanding, and the value of durable goods. An agent can for example sell a promise to deliver a large number of goods tomorrow for a very high price today, even though he owns no durable goods of any kind.

Arrow, Debreu, and McKenzie noted that a complete markets equilibrium always exists, given the assumptions we have already made on endowments and utilities. In fact, as Arrow [1953] noted, a complete markets equilibrium is always isomorphic to an equilibrium of the same underlying economy in which there is an asset $A_{s\ell}$ for each state contingent commodity, $s\ell$, $s \in S$, $\ell \in L$, promising delivery of exactly one unit of that good. The asset prices $\pi_{s\ell}$ can then be taken to be the same as the commodity price $p_{s\ell}$.

5.2 Assetless Equilibrium

At the other extreme from the Arrow–Debreu complete markets economy is an economy in which no provision is made for enforcing promises. In that case investors would rationally anticipate that all promises would be broken, and the economy would effectively reduce to one without any assets: $J = \phi$. In these circumstances it is easy to see that durable goods might have very low prices, since the potential buyers might not be able to give much immediately in return, and by hypothesis they are forbidden from promising anything in the future. Holding durable goods would also be very risky, because it would be impossible to hedge them except by holding other durable goods. Not only might prices change across states of nature because tastes changed or because tastes were expected to change, but more importantly, changes in the distribution of income could dramatically affect durable goods prices.

Equilibrium would always exist, but risk would not be distributed efficiently. The government in such an assetless economy should work hard at creating new markets, perhaps by subsidizing them, or even by guaranteeing delivery if an agent should default. Only at a later stage of economic development can a country expect to benefit by constructing a court system capable of recovering collateral and so on.

5.3 The GEI Incomplete Markets Model

Lastly we turn to an intermediate and more realistic setting in which there are some assets, but far fewer than the astronomical number it would take to complete the asset markets. At first glance one would expect the equilibrium behavior of this kind of economy to be intermediate between the two just described. In some respects it is. For example, GEI equilibria are typically (though not always) better from a welfare point of view than assetless economies, but less good than complete markets economies. On the other hand, the presence of some but not all asset markets creates the possibility of new phenomena that are smoothed over when markets are complete or completely absent. For example, the volatility of durable goods prices may be greater with incomplete markets than it is with either complete markets or assetless economies. And the government may be able to intervene by taxes or subsidies to improve welfare in a GEI economy, but not in an assetless or Arrow–Debreu economy. Some of the rough edges of a GEI economy may be rounded out by replacing the (implicit) infinite penalties for default with collateral.

If asset markets are missing (that is if $\#J$ is taken to be smaller than S but greater than 1 by assumption), then the equilibrium GEI allocation does not share risk in the ideal way, despite the absence of default. Somewhat more surprising, it is possible in some GEI equilibria to make everybody better off without adding any assets simply by letting people default. The prohibition against default is presumably enforced by draconian and inflexible penalties. By substituting collateral or weaker penalties, the system may be improved despite the fact that defaults appear. For example, if there were one highly improbable state of the world in which an agent could not pay, then he could never sell an asset that promised something in every state. The loss to the lenders from his default in just this one state, plus any penalty he might be required to pay for defaulting, could well be less important than the gains to both the lenders and to him from being allowed to borrow the money and pay it back in the other states. (See Dubey–Geanakoplos–Shubik [1990]). The moral is: when asset markets are complete, set high penalties, but when some asset markets are missing, set low or intermediate levels of default penalties, when penalties are available, or better yet, use collateral whenever possible. We shall see this in our example.[1]

[1] When some asset markets are missing it can also be shown (Geanakoplos–Polemarchakis [1986]) that an outside agency that knows everybody's utilities can almost always make every agent better off, without introducing any new assets, if it has the power to coerce agents into trading a little

A shortage of available assets will tend to reduce the value of durable goods. If assets are missing, then the agents only have available the income from today's endowment and a few promises to spend on the durable good; the demand for the durable good is effectively curtailed, and its price will typically be between the complete markets price and the assetless economy price. More surprising, the volatility of durable goods prices will often be higher than in complete or assetless economies on account of the possibility of leverage. We discuss this in the next section.

Unless the assets in J are redundant, typically they will all be traded in the GEI equilibrium. The reason is that in equilibrium the marginal utility to each agent of buying or selling a little more of the asset must equal the price. The chances that every agent will happen to have the same marginal utility at zero trade are infinitesimal. The GEI model therefore cannot explain why we observe so few assets actually traded. In order to match reality, it must simply assume that many assets are missing from J. That is how the theory gets its name (general equilibrium with incomplete markets). But it begs the question: which asset markets are missing and why?

The GEI theory, like the complete markets economy, allows for large trades in the asset markets. An agent might take a huge positive position in one asset, and a huge negative position in another asset that nearly offsets all his debts. In general the gross value of the promises requiring delivery in each state might be much larger than the value of all the outstanding goods in that state, which in turn is greater than the value of the durable goods held over from the previous period. Agents would be able to keep their promises because other agents kept their promises to them. If each agent were forced to pay out of his own income before receiving payments from others, the equilibrium would break down. Of course, some promise structures give rise to larger promises than others, even for the same underlying economic preferences and endowments. This possibility for arbitrarily large asset transactions sometimes compromises the existence of GEI equilibrium. (See Hart [1974].)

6 Properties of Collateral Equilibrium

6.1 The Orderly Function of Markets

We return now to our model of equilibrium with default. Compared to the benchmark model of general economic equilibrium, the agents we have described must anticipate not only what the prices will be in each state of nature, and not only what the assets promise in each state of nature, but also what they will actually deliver in each state of nature. The hypothesis of agent rationality is therefore slightly more stringent in this model than in the conventional models of intertemporal perfect competition. Nevertheless, equilibrium always exists in this model, (under the assumptions made so far), yet in the standard model of general equilibrium with incomplete asset markets, equilibrium may not exist. The following theorem is taken from GZD.

---

differently on each asset than they would choose left to their own devices.

Theorem 1 (Geanakoplos, Zame, Dubey, 1995) *Under the assumptions on endowments and utilities already specified, equilibrium must exist, no matter what the structure of assets and collateral.*

If an asset contains no provision for collateral whatsoever, then of course everybody will rightly anticipate that it will deliver nothing, and its equilibrium price will be 0. Indeed the economy would function exactly the same way if it were not available at all. For assets with some nonzero collateral, agents will not be able to sell arbitrarily large quantities, because they will not be able to obtain the required collateral. This limiting factor helps to guarantee the existence of equilibrium.

6.2 Endogenous Assets

One of the major shortcomings of the GEI model is that it leaves unexplained which assets are traded. Generically, all the assets exogenously allowed into the model will be traded. When default can only be avoided by collateral, the situation is different and much more interesting.

The crucial idea is that without the need for collateral, the marginal utility $m_j^h(B)$ to an agent h of buying the first unit of an asset j is almost exactly the same as the marginal utility loss $m_j^h(S)$ in selling the first unit of the asset; we can call both m_j^h. Only by an incredible stroke of luck will it turn out that $m_j^h = m_j^{h'}$ for different agents h and h', and hence asset j will almost surely be traded in a GEI equilibrium. When collateral must be provided by the seller, the disutility of making a promise goes up, sometimes by as much as the consumption foregone by buying the collateral. If the required collateral is borrower held, and if it is something that agent h planned to hold anyway, then there is no extra utility loss from selling the first unit of asset j. But if agent h did not plan to hold the collateral for consumption, or if all that he intended to hold as consumption has already been allocated as collateral for other promises, then the loss in utility from selling even the first unit of asset j would be larger than the marginal utility from buying the first unit of asset j, $m_j^h(S) > m_j^h(B)$. It might well transpire that

$$\min_{h \in H} m_j^h(S) > \pi_j > \max_{h \in H} m_j^h(B)$$

and hence that asset j does not trade at all in equilibrium.

This situation can be most clearly seen when the value of the Arrow–Debreu promises in some state exceeds the salvage value of all the durable goods carried over into that state. It is then physically impossible to collateralize every socially useful promise up to the point that every delivery is guaranteed without exception. The market system, through its equilibrating mechanism, must find a way to ration the quantity of promises. This rationing is achieved by a scarcity of collateral. The resulting wedge between the buying marginal utility of each asset and the selling marginal utility of the asset, however, not only serves to limit the quantity of each promise, but more dramatically, it chokes off most promises altogether, so that the subset of assets that are actually traded is endogenous and potentially much smaller than the set of available assets.

Consider a set $\Pi = \{a \in Z_+^{SL} : a_{s\ell} \leq 10^{10}\}$ of potential promises, restricted to integer coordinates. Note that Π is a finite set, but it includes virtually every conceivable promise (since only the relative size of the promises matter). Let $\mathcal{C} = \{(c^W, c^L, c^B) \in Z_+^L \times Z_+^L \times Z_+^L : c_\ell^i \leq 10^{100}\}$ be a finite set of (virtually) all potential collateral levels. We can then take $J = \Pi \times \mathcal{C}$. In equilibrium all of these many assets will be priced, but only a very few of them will actually be traded. The rest will not be observable in the marketplace, and therefore the appearance will be given of many missing markets. The untraded assets will lie dormant not because their promises are irrelevant to spreading risk efficiently, but because the scarce collateral does not permit more trade.

It would be interesting to catalogue the rules by which the market implicitly chooses one promise over another, or one level of collateral over another. This issue is more fully developed in Geanakoplos–Zame–Dubey [1995], but let us note some things here. The easiest way of economizing on collateral is by allowing default in some states of nature. Moreover, if one vector of collaterals guarantees full delivery in every state of nature, there is no point in trading the same promise collateralized by greater levels of collateral. Finally, if a vector of promises is very different from the vector of its collateral values across the states of nature, the asset is not well drawn. In some states there will be too much collateral, and in others not enough. One might suspect that such an asset would also not be traded. The general principle is that the market chooses assets that are as efficient as possible, given the prices. We make this precise in the next section.

6.3 Constrained Efficiency

It is to be expected that an increase in available collateral, either through an improvement in the legal system (e.g., borrower held collateral), or through the increased durability of goods, will be welfare improving. More subtly, we might wonder whether government intervention could improve the functioning of financial markets given a fixed level of available collateral. After all, the unavailability of collateral might create a wedge that prevents agents from trading the promises in J that would lead to a Pareto improving sharing of future risks. If the government transferred wealth to those agents unable to afford collateral, or subsidized some market to make it easier to get collateral, could the general welfare be improved? What if the government prohibited trade in assets with low collateral levels? The answer, surprisingly, is no, at least under some important restrictions.

Constrained Efficiency Theorem (Geanakoplos–Zame–Dubey, 1995) *Each collateral equilibrium is Pareto efficient among the allocations which* (1) *are feasible and* (2) *given whatever period 0 decisions are assigned, respect each agent's budget set at every state s at time 1 at the old equilibrium prices, and* (3) *assume agents will deliver no more on their asset promises than they have to, namely the minimum of the promise and the value of the collateral put up at time* 0, *given the original prices.*

In particular, no matter how the government redistributes income in period 0, and taxes and subsidizes various markets at time 0, if it allows markets to clear on their own at time 1, then we can be sure that if the time 1 market clearing relative prices are the same as they were at the old equilibrium, then the new allocation cannot Pareto dominate the old equilibrium allocation. This will be illustrated in our examples.

6.4 Volatility, the Distribution of Wealth, and Collateral

In any general economic equilibrium, the price of a good depends on the utilities of the agents and the distribution of wealth. If the agents who are fondest of the good are also relatively wealthy, the good's price will be particularly high. Any redistribution of wealth away from these agents toward agents who like the good less will tend to lower the price of the good.

To a large extent, the value of durable goods depends on the expectations, and, when markets are incomplete, on the risk aversion of potential investors, as well as on intrinsic utility for the good. These multiple determinants of value make it quite likely that there will be wide divergences in the valuations different agents put on durable goods.

For example, farms in 1929 could be thought of as an investment, available to farmers and bankers, but to farmers there is a superior intrinsic value that made it sensible for them to own them and use them at the same time. Since the farmers did not have enough money to buy a farm outright, they typically borrowed money and used the farm as collateral. Similarly mortgage derivatives in the 1990s are worth much more to investors who have the technology and understanding to hedge them than they are to the average investor.

Needless to say, the value of many durable assets will be determined by the marginal utilities of those who like them the most. (This is literally true if one cannot sell the asset short).

Since the 1929 stock market crash it has been widely argued that low margin requirements can increase the volatility of stock prices. The argument is usually of the following kind: when there is bad news about the stocks, margins are called and the agents who borrowed against the stocks are forced to put them on the market, which lowers their prices still further.

The trouble with this argument is that it does not quite go far enough. In general equilibrium theory, every asset and commodity is for sale at every moment. Hence the crucial step in which the borrowers are forced to put the collateral up for sale has by itself no bite. On the other hand the argument is exactly on the right track.

We argue that indeed using houses or stocks, or mortgage derivatives as collateral for loans (i.e., allowing them to be bought on margin) makes their prices more volatile. The reason is that those agents with the most optimistic view of the assets' future values, or simply the highest marginal utility for their services, will be enabled by buying on margin to hold a larger fraction of them than they could have afforded otherwise.

The initial price of those assets will be much higher than if they could not be used as collateral for two reasons: every agent can afford to pay more for them by promising future wealth, and second, the marginal buyer will tend to be somebody with a higher marginal utility for the asset than would otherwise be the case.

As a result of the margin purchases, the investment by the optimistic agents is greatly leveraged. When the asset rises in value, these agents do exceedingly well, and when the asset falls in price, these agents do exceedingly badly. Thus on bad news the stock price falls for two reasons: the news itself causes everyone to value it less, and this lower valuation causes a redistribution of wealth away from the optimists and toward the pessimists who did not buy on margin. The marginal buyer of the stock is therefore likely to be someone less optimistic than would have been the case had the stock not been purchased on margin, and the income redistribution not been so severe. Thus the fall in price is likely to be more severe than if the stock could not have been purchased on margin.

The properties of collateral equilibrium are illustrated in two examples.

7 Example: Borrowing Across Time

We consider an example with two agents[2] $H = \{A, B\}$, two time periods, and two goods F (food) and H (housing) in each period. For now we shall suppose that there is only one state of nature in the last period.

We suppose that food is completely perishable, while housing is perfectly durable. Thus the consumption-durability technology is $Y_1^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, meaning that 1 unit of F becomes 0 units of both goods, and 1 unit of H becomes 1 unit of H in period 1.

We suppose that agent B likes living in a house much more than agent A,

$$u^A(x_{0F}, x_{0H}, x_{1F}, x_{1H}) = x_{0F} + x_{0H} + x_{1F} + x_{1H},$$
$$u^B(x_{0F}, x_{0H}, x_{1F}, x_{1H}) = 9x_{0F} - 2x_{0F}^2 + 15x_{0H} + x_{1F} + 15x_{1H}.$$

Furthermore, we suppose that the endowments are such that agent B is very poor in the early period, but wealthy later, while agent A owns the housing stock

$$e^A = (e^A_{0F}, e^A_{0H}, e^A_{1F}, e^A_{1H}) = (20, 1, 20, 0),$$
$$e^B = (e^B_{0F}, e^B_{0H}, e^B_{1F}, e^B_{1H}) = (4, 0, 50, 0).$$

7.1 Arrow–Debreu Equilibrium

If we had complete asset markets, so that $J = \{A_{1F}, A_{1H}\}$, with infinite default penalties and no collateral requirements, then it is easy to see that there would be a unique equilibrium (in utility payoffs):

$$\begin{aligned} p &= (p_{0F}, p_{0H}, p_{1F}, p_{1H}) = (1, 30, 1, 15), \ \pi = (1, 15) \\ x^A &= (x^A_{0F}, x^A_{0H}, x^A_{1F}, x^A_{1H}) = (22, 0, 48, 0), \\ x^B &= (x^B_{0F}, x^B_{0H}, x^B_{1F}, x^B_{1H}) = (2, 1, 22, 1), \\ u^A &= 70 \ ; \ \ u^B = 62. \end{aligned}$$

[2] In order to maintain our hypothesis of many agents, we shall actually suppose that there are a great many agents of type A, and the *same* large number of agents of type B. Agents of the same type will take identical actions.

Housing has a very high price in period 0 because it is a durable good which provides a lot of utility to agent B in period 0 and in period 1. Even though agent B is relatively poor in period 0, he can afford to pay the high price of 30 for the house because he can sell a promise to deliver 28 units of food in period 1 for a price of 28 today, which he then repays out of his endowment of food in period 1. The price of housing naturally falls over time as it ages. The fact that the relative endowments of A and B change over time has no bearing on the fluctuation of housing prices.

Notice that the equilibrium is welfare efficient: the agents who most like housing own the whole housing stock.

7.2 GEI Equilibrium

Suppose now that instead of the two Arrow–Debreu state contingent commodity assets there is just the first asset $A_j = \binom{1}{0}$ which promises 1 unit of food in period 1, and no housing. Suppose in addition that there are infinite default penalties for this asset, so that full delivery is assured even with no collateral. As Arrow [1953] pointed out, since there are as many assets as states of nature, the GEI equilibrium allocation will be identical to the Arrow–Debreu allocation computed above. To describe the equilibrium, we need only add to the Arrow–Debreu equilibrium the portfolio trades $\theta^A = 28, \varphi^A = 0, \theta^B = 0, \varphi^B = 28$. The difference between Arrow–Debreu equilibrium and GEI equilibrium will not emerge until there are more states than assets.

7.3 No Collateral and no Penalties Equilibrium, i.e., Assetless Equilibrium

Without the sophisticated financial arrangements involved with collateral or default penalties, there would be nothing to induce agents to keep their promises. Recognizing this, the market would set prices $\pi_j = 0$ for all the assets. Agents would therefore not be able to borrow any money. Thus agents of type B, despite their great desire to live in housing, and great wealth in period 1, would not be able to purchase much housing in the initial period. Again it is easy to calculate the unique equilibrium:

$$
\begin{aligned}
&\pi_j = 0\\
&p = (p_{0F},\, p_{0H},\, p_{1F},\, p_{1H}) = (1,\, 16,\, 1,\, 15),\\
&x^A = (x^A_{0F},\, x^A_{0H},\, x^A_{1F},\, x^A_{1H}) = \left(20 + \tfrac{71}{32},\, 1 - \tfrac{71}{32\cdot 16},\, 35 - \tfrac{71\cdot 15}{32\cdot 16},\, 0\right),\\
&x^B = (x^B_{0F},\, x^B_{0H},\, x^B_{1F},\, x^B_{1H}) = \left(\tfrac{57}{32},\, \tfrac{71}{32\cdot 16},\, 35 + \tfrac{71\cdot 15}{32\cdot 16},\, 1\right),\\
&u^A = 56; \quad u^B = 50 + \tfrac{71\cdot 87}{32\cdot 16} \approx 62.6.
\end{aligned}
$$

Agent A, realizing that he can sell the house for 15 in period 1, is effectively paying only $16 - 15 = 1$ to have a house in period 0, and is therefore indifferent to how much housing he consumes in period 0. Agents of type B, on the other hand, spend their available wealth at time 0 on housing until their marginal utility of consumption of x_{0F} rises to $\frac{30}{16}$, which is the marginal utility of owning an extra dollar's worth of housing stock at time 0. (The marginal utility of an extra unit of housing is worth

$15+15$, and at a cost of \$16 this means marginal utility of 30/16 per dollar). That occurs when $9-4x^B_{0F}=\frac{30}{16}$, that is, when $x^B_{0F}=\frac{57}{32}$.

The assetless equilibrium is Pareto inferior, since the agents of type B do not get to consume the bulk of the housing stock even though it matters relatively more to them. They simply cannot use their last period wealth to command more buying power over the initial period housing stock. For the same reason the price of housing in the first period is much lower in this equilibrium than it is in Arrow–Debreu equilibrium.

7.4 Collateral Equilibrium

We now introduce the possibility of collateral in a world where there are no penalties. As in the GEI model, we assume there is only one asset j that promises one unit of food in period 1 and nothing else. We suppose that the collateral requirement for each unit of asset j is $\frac{1}{15}$ unit of housing and the borrower holds the collateral,[3] $C^B_j=\begin{pmatrix}0\\\frac{1}{15}\end{pmatrix}$, $C^L_j=C^W_j=\binom{0}{0}$. We also suppose that borrower held collateral has the same durability as consumption $Y^B_{1j}=Y^0_1=\begin{bmatrix}0&0\\0&1\end{bmatrix}$. The unique equilibrium is then:

$$
\begin{aligned}
&\pi_j=1,\\
&p=(p_{0F},\,p_{0H},\,p_{1F},\,p_{1H})=(1,\,18,\,1,\,15),\\
&x^A=(x^A_{0F},\,x^A_{0H},\,x^A_{1F},\,x^A_{1H})=(23,\,0,\,35,\,0)\\
&\theta^A_j=15;\quad \varphi^A_j=0;\\
&x^B=(x^B_{0F},\,x^B_{0H},\,x^B_{1F},\,x^B_{1H})=(1,1,\,35,\,1),\\
&\theta^B_j=0;\quad \varphi^B_j=15\ ;\\
&u^A=58;\quad u^B=72.
\end{aligned}
$$

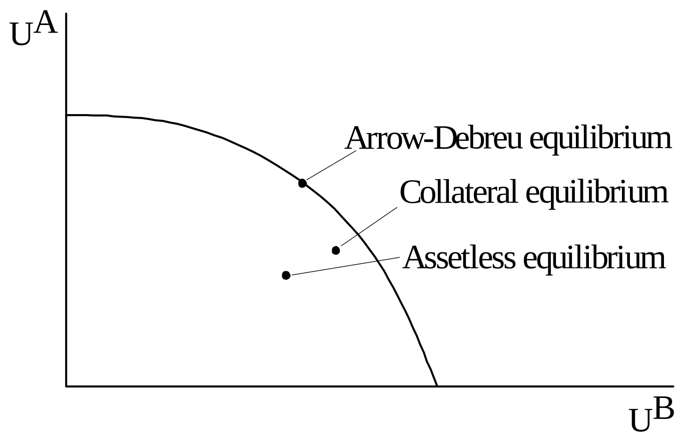
Agent B borrows 15 units of x_{0F} and uses the 15 units of x_{0F} plus 3 he owns himself to buy 1 unit of x_{0H}, which he uses as collateral on the loan. He returns 15 units of x_{1F} in period 1, which is precisely the value of the collateral. Since, as borrower, agent B gets to consume the housing services while the house is being used as collateral, he gets final utility of 72. Agent A sells all his housing stock, since the implicit rental of $3=18-15$ is more than his marginal utility (expressed in terms of good F) for housing services for a single period. Agent B is content with purchasing exactly one unit of housing stock at the implicit rental of 3, since at $x^B_{0F}=1$, his marginal utility of consumption of x_{0F} is $9-4(1)=5$, and $5=15/3$, his marginal utility of a dollar of housing for a single period. (Recall that by giving up 3 units of x_{0F} he could effectively own the house for one period.)

Notice that the collateral equilibrium is Pareto superior to the assetless equilibrium, but not on the Pareto frontier. Although the housing stock goes to the agents of type B who value it the most, they are forced to give up too much food consumption in period 0 to get the housing stock. Their marginal utility of food consumption

[3] It is important to note that without the invention of borrower held collateral (e.g., if we supposed collateral had to be warehoused, there would be no improvement on the assetless economy.

in period 0 relative to period 1 is higher than the agents of type A. Both could be made better off if the type A agents consumed δ less in period 0 and $\delta/(1-\delta)$ more in period 1 while the agents of type B did the reverse. This cannot be arranged in the collateral economy because agents of type B do not have the ability to borrow more money than \$15 because they do not have the collateral to put up for it. To borrow more money they would have to get more housing to use as collateral, which they could only do if they gave up still more food consumption in time 0.

The difficulty of borrowing when collateral must be put up keeps the price of the durable housing down to 18, above the assetless price but well below the Arrow–Debreu price. Even though there is no default in the collateral equilibrium, the shadow of default dramatically changes the equilibrium. The inconvenience of putting up collateral cuts the borrowing down from 28 to 15.



It is interesting to observe that although neither the assetless equilibrium nor the collateral equilibrium is Pareto efficient, neither is dominated by the Arrow–Debreu equilibrium. The B agents who are buyers of durable goods are paradoxically helped by limitations in the asset markets because then they cannot borrow as easily, so they cannot compete with each other as fiercely over the housing stock, hence the price is lower, hence they end up better off. In general it is the owners of durable goods who will have the most to gain by loosening credit controls.

7.4.1 Endogenous Collateral Requirements

We can extend our example by adding more assets j that promise the same 1 unit of food in period 1, but which require $1/j$ units of housing as borrower held collateral. Say $J = \{1, ..., 10^{10}\}$. (With this notation, our previous economy consisted of a single asset, namely $j = 15$.) A collateral equilibrium will now have to specify a price π_j for each of these assets j. It is easy to see that the equilibrium is essentially unique: $\pi_j = \min\{1, 15/j\}$. Each asset $j < 15$ forces agents to put up more than 1/15th of a house as collateral for each unit of food owed, and hence will not be traded, since it fetches no higher price than $j = 15$ and causes the borrower–seller additional hardship. Assets $j > 15$ which enable borrowers to put up less collateral fetch proportionately lower prices, and hence amount to exactly the same assets as $j = 15$. Thus we see that the market forces of supply and demand pick out of the multiplicity of assets a single one which agents trade. The others are all priced, but not traded.

8 Example: Borrowing Across States of Nature, with Default

We consider almost the same economy as before, with two agents A and B, and two goods F (food) and H (housing) in each period. But now we suppose that there are two states of nature $s = 1$ and 2 in period 1. The extra state allows for uncertainty and permits a distinction between complete markets and GEI equilibrium. The example turns on the assumption that in state 2 agents of type B have smaller endowments than in state 1. This objective change in the economy would tend to lower the price for houses in state 2, since it is agents of type B who especially like houses. The interesting phenomenon illustrated in this example is that the drop in state 2 housing prices is much greater in the GEI equilibrium and the collateral equilibrium than it is in either the complete markets equilibrium or the assetless equilibrium. The price volatility is less in the collateral equilibrium than in the GEI equilibrium, and welfare is higher.

As before, we suppose that food is completely perishable and housing is perfectly durable,

$$Y_s^0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{for} \quad s = 1, 2$$

meaning that 1 unit of F becomes 0 units of both goods, and 1 unit of H becomes 1 unit of H, in both states in period 1.

We assume

$$\begin{aligned} & u^A(x_{0F}, x_{0H}, x_{1F}, x_{1H}, x_{2F}, x_{2H}) \\ = \; & x_{0F} + x_{0H} + (1-\varepsilon)(x_{1F} + x_{1H}) + \varepsilon(x_{2F} + x_{2H}), \\ & u^B(x_{0F}, x_{0H}, x_{1F}, x_{1H}, x_{2F}, x_{2H}) \\ = \; & 9x_{0F} - 2x_{0F}^2 + 15x_{0H} + (1-\varepsilon)(x_{1F} + 15x_{1H}) + \varepsilon(x_{2F} + 15x_{2H}). \end{aligned}$$

Furthermore, we suppose that

$$\begin{aligned} e^A &= (e^A_{0F}, e^A_{0H}, (e^A_{1F}, e^A_{1H}), (e^A_{2F}, e^A_{2H})) = (20, 1, (20, 0), (20, 0)), \\ e^B &= (e^B_{0F}, e^B_{0H}, (e^B_{1F}, e^B_{1H}), (e^B_{2F}, e^B_{2H})) = (4, 0, (50, 0), (3, 0)). \end{aligned}$$

To complete the model, we suppose as before that there is one asset A_j with $A_{sj} = \binom{1}{0}$, $\forall s \in S$ promising one unit of good F in every state $s = 1$ and 2. We suppose that the collateral requirement is $C_j^B = \begin{pmatrix} 0 \\ \frac{1}{15} \end{pmatrix}$, as before, and that borrower held houses are perfectly durable $Y_s^B = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right]$ for $s = 1, 2$.

It turns out that it is very easy to calculate the various equilibria for arbitrary ε.

8.1 Arrow–Debreu Equilibrium

The unique (in utility payoffs) Arrow–Debreu equilibrium is:

$$
\begin{aligned}
&p = ((p_{0F},\, p_{0H}),\, (p_{1F},\, p_{1H}),\, (p_{2F},\, p_{2H})) = ((1,\, 30),\, ((1-\varepsilon)(1,15),\, \varepsilon(1,15)),\\
&x^A = ((x^A_{0F}, x^A_{0H}),\, (x^A_{1F}, x^A_{1H}),\, (x^A_{2F}, x^A_{2H})) = \left((22,0),\, \left(20+\tfrac{28}{(1-\varepsilon)},\, 0\right),\, (20,0)\right),\\
&x^B = ((x^B_{0F}, x^B_{0H}),\, (x^B_{1F}, x^B_{1H}),\, (x^B_{2F}, x^B_{2H})) = \left((2,1),\, \left(50-\tfrac{28}{1-\varepsilon},\, 1\right),\, (3,1)\right),\\
&u^A = 70; \quad u^B = 62 - 47\varepsilon\ .
\end{aligned}
$$

Notice that agent B transfers wealth from period 1 back to period 0 (i.e., he borrows), and also transfers wealth from state 1 to state 2. The great drop in agent B wealth in state 2 has a small effect on the economy (if ε is not too large). The reason is that with the complete markets, agent B is able to completely insure himself against this loss in wealth. The price of the durable good housing does not change at all. The only effect is a diminution in utility for agents of type B reflecting the probability that their wealth might be smaller. They are forced to forego more consumption in state 1 in order to insure against this eventuality.

8.2 No-Collateral and No-Penalties, Equilibrium, i.e., Assetless Equilibrium

$$
\begin{aligned}
\pi_j &= 0\\
p &= ((p_{0F},\, p_{0H}),\, (p_{1F},\, p_{1H}),\, (p_{2F},\, p_{2H})) = \left((1,16),\, (1,15),\, \left(1,\, \frac{3}{1-\frac{71}{32\cdot 16}}\right)\right)\\
&\approx ((1,16),\, (1,15),\, (1,3.6))\ ,\\
x^A &= ((x^A_{0F},\, x^A_{0H}),\, (x^A_{1F},\, x^A_{1H}),\, (x^A_{2F},\, x^A_{2H}))\\
&= \left(\left(20+\tfrac{71}{32},\, 1-\tfrac{71}{32\cdot 16}\right),\, \left(35-\tfrac{15\cdot 71}{32\cdot 16},\, 0\right),\, (23,0)\right),\\
x^B &= ((x^B_{0F},\, x^B_{0H}), (x^B_{1F},\, x^B_{1H}), (x^B_{2F},\, x^B_{2H})) = \left(\left(\tfrac{57}{32},\, \tfrac{71}{32\cdot 16}\right),\, \left(35+\tfrac{15\cdot 71}{32\cdot 16},\, 1\right),\, (0,1)\right),\\
u^A &= 56 + \varepsilon\tfrac{15\cdot 71}{32\cdot 16} - 12\varepsilon \approx 56 - 9.9\varepsilon; \quad u^B = 50 + \tfrac{87\cdot 71}{32\cdot 16} - \left(35 + \tfrac{15\cdot 71}{32\cdot 16}\right)\varepsilon\\
&\approx 62 - 37.1\varepsilon\ .
\end{aligned}
$$

In the assetless equilibrium there is a large drop from 15 to 3.6 in the price of the durable good in state 2 because the agents of type B cannot insure against their loss of wealth. They carry over part of the housing stock, and in state 2 spend all their wealth acquiring more housing.

8.3 Collateral Equilibrium

We can exactly calculate the unique collateral equilibrium by noting that agents of type B will default in state 2 and then spend all of their endowment e^B_{2F} on good $2H$, giving a price $p_{2H} = 3$. The asset price is then $\pi_j = (1-\varepsilon) + \varepsilon\frac{3}{15}$

$$\pi_j = 1 - \tfrac{4}{5}\varepsilon,$$
$$((p_{0F}, p_{0H}), (p_{1F}, p_{1H}), (p_{2F}, p_{2H})) = ((1, 18{-}12\varepsilon), (1, 15), (1, 3)),$$
$$x^A = ((x^A_{0F}, x^A_{0H}), (x^A_{1F}, x^A_{1H}), (x^A_{2F}, x^A_{2H})) = ((23, 0), (35, 0), (23, 0)),$$
$$\theta^A_j = 15; \quad \varphi^A_j = 0;$$
$$x^B = ((x^B_{0F}, x^B_{0H}), (x^B_{1F}, x^B_{1H}), (x^B_{2F}, x^B_{2H})) = ((1, 1), (35, 1), (0, 1)),$$
$$\theta^B_j = 0; \quad \varphi^B_j = 15;$$
$$u^A = 58 - 12\varepsilon; \; u^B = 72 - 35\varepsilon.$$

To see that this is the correct equilibrium, note that agent B can sell 15 units of asset j in period 0, use the money $(1 - \frac{4}{5}\varepsilon)15 = 15 - 12\varepsilon$ he receives in exchange, plus three units of $0F$ from his endowment to purchase one housing unit, which gives him consumption utility in period 0 of 15 and enables him to put up the whole of the necessary collateral for his loan. Thus on the margin, 3 units of good $0F$ can be transformed into 15 utiles. But the marginal utility of consumption of good $0F$ is also $9 - 4x^B_{0F} = 5$.

8.3.1 Excess Volatility

One striking aspect of this example is that the volatility of housing prices is much higher when there is scarce collateral than when there are complete markets or totally incomplete markets. When the housing stock can be purchased on margin (i.e., used as collateral), agents of type B are enabled to purchase the entire housing stock, raising its price from 16 (where it would have been without collateral) to 18. In the bad state these agents default and all their holdings of the housing stock are seized. Although they end up buying back the entire housing stock, their wealth is so depleted that they can only bid up the housing prices to 3.

When there is no collateral the agents of type B can afford to purchase only a fraction $\alpha = 71/(32)(16)$ of the housing stock at time 0. But they own that share free and clear of any debts. Thus when bad news comes, they do not lose any more on account of past debts. They can apply their wealth to purchasing the remaining $1{-}\alpha$ of the housing stock, which forces the price up to approximately 3.6. Thus when there is no collateral (and no other penalty for defaulting), the housing prices are never as high nor never as low as when the housing stock can be used as collateral.

The volatility could have been even more extreme if agents could not walk entirely away from their debts after their collateral was seized. In that case agents of type B would lose even more wealth in state 2, and the price of housing would drop still further. Of course anticipating this further loss of wealth, agents of type B would probably borrow less, but that would mitigate and not prevent their wealth from going down, at least for robust examples. We demonstrate this in our next example of GEI equilibrium.

8.3.2 Endogenous Collateral and Endogenous Assets

Consider again the collateral example, but with assets A_j, for $j = 1, ..., J$, defined as before so that each asset j promises 1 unit of good sF in each state $s = 1, 2$, and requires $1/j$ units of good $0H$ as borrower-held collateral. Any asset j with $j < 15$ is more secure than asset A_{15}, paying the same amount in state 1 and strictly more in state 2. The question is, will lenders insist on the more secure assets, or will they be content in equilibrium to loan money on terms that are more likely to lead to default?

The reader can check that actually the unique equilibrium of this enlarged economy is identical to the equilibrium we just calculated: none of the assets j with $j < 15$ will be traded at all, and the assets of type $j > 15$ are identical to asset $j = 15$. In equilibrium, each asset j will be priced at $\pi_j = (1-\varepsilon)\min\left\{1, \frac{15}{j}\right\} + \varepsilon \min\left\{1, \frac{3}{j}\right\}$, since at this price agents of type A are just indifferent to buying or not buying the asset. For $j > 15$, agents who sell asset j will default in both states and so the deliveries of the asset are proportional to $1/j$, hence their prices must be as well, and the assets are all effectively the same. Any asset j with $j < 15$ will bring only a miniscule increase in price (if ε is small), but cause a huge inconvenience to agents of any type who might want to sell it. Thus it will not be traded in equilibrium.

Thus we see that the free market will not choose levels of collateral which eliminate default. Borrowers, for obvious reasons, and lenders, because it increases demand for loans and for the durable goods they wish to sell, like surprisingly low collateral levels. We are left to wonder whether the collateral levels are in any sense optimal for the economy: does the free market arrange for the optimal amount of default?

In our example with the augmented list of assets the government could mandate that loans be carried out only at a collateral level of $1/14$, thereby reducing default. Since the resulting equilibrium would maintain the same price ratios of 1:15 in state 1 and 1:3 in state 2 between the two goods F and H, our constrained efficiency theorem implies that the resulting equilibrium could not Pareto dominate the original equilibrium at which trade took place exclusively at collateral levels $1/15$.

Our theorem suggests that the free market does not expose this economy to unnecessary risks from default. However, it relies on the hypothesis that future relative prices are not affected by current collateral requirements. In general this will prove to be false. Indeed, if the collateral requirement were set high enough in our example, there would be no financial market transactions, and the future relative prices would change. In the example this only hurts the economy still more, but in general the relative price change caused by government mandated, stringent collateral requirements, may be beneficial.

The reader can also check that if two new assets i (an Arrow security paying food only in state 1) and k (an Arrow security paying food only in state 2) were introduced with promises $A_{1i} = \binom{1}{0}, A_{2i} = \binom{0}{0}, A_{1k} = \binom{0}{0}, A_{2k} = \binom{1}{0}$, then no matter what the collateral requirements were for assets i and k, they would not be traded. Suppose for example that without loss of generality we take the collateral on asset i to be $1/15$ housing units. Its price would then have to be $(1 - \varepsilon)$. Then an agent could borrow $15(1 - \varepsilon)$ units of food, put up $3 + 3\varepsilon$ units of his own endowment to attain the housing price of $18 - 12\varepsilon$, gain 15 utiles by living in the house and putting it up as collateral to back his loan, and another 15ε utiles from living in the house in state 2. This is no improvement. The ideal arrangement would be for B to sell j and buy asset i. But A would never sell asset i because he would not want to put up the collateral, since he gains little utility from living in the house. Both assets i and k waste collateral in the sense that the deliveries in some states are less than the value of the collateral there. In this example that suffices to eliminate them from trade.

Thus our example also illustrates how scarce collateral rations the number and kind of promises that actually get made in equilibrium.

8.3.3 Superior Returns

Consider an infinitesimal agent of type B who preserved wealth equal to 50 units of food in state 2 instead of 3 units of food. Then in state 2 he would spend all his wealth on housing, getting a fantastic return on his money, and doing much better than in state 1 even though objectively speaking to him, the housing stock was no different. Let us now try to interpret this parable.

The agents of type B are responsible for maintaining the price of housing. They may care more about housing because they get special utility from living in houses, as our model explicitly presumes, (and like farmers did from farms in the 1930s), or because they are more optimistic about the future of housing in some unmodeled future, or because they are less risk averse about the future, or because they know how to hedge this kind of asset better than others, as perhaps Ellington Capital Management and other hedge funds do with respect to mortgage derivatives. Needless to say, a sudden drop in the wealth of this class of people will lower the price of housing in the short run. But at the same time it will make the return to each dollar invested in housing earn a higher rate of return. If somebody in the class has not suffered a wealth hit, then he can earn a higher total return as well. Incomplete markets permit leverage, which magnify exogenous shocks. We see a more dramatic example next.

8.4 GEI Equilibrium

Suppose as before that there is just one asset j with $A_{sj} = \binom{1}{0}$ for each $s \in S$, which promises 1 unit of food in period 1 in both states, and no housing. Suppose in addition that there are infinite default penalties for this asset, so that full delivery is required even with no collateral.

It turns out remarkably that the price volatility of houses is even greater than when there was collateral. The reason is that in state 2 the type B agents not only lose their housing stock, but since default is not permitted, they also must make up all of their debts from their endowment. This reduces their wealth even more, and forces type A agents to become the marginal buyers of housing in state 2, which of course lowers the housing price all the way to 1. We see here the full force of leverage, without any ameliorating notion of limited liability.

Of course the agents of type A and B anticipate that all B's wealth will be confiscated in state 2 to pay the debt, and this limits the amount of money that B

can borrow at time 0.

The price of the asset is clearly 1, since delivery is guaranteed. Moreover, the price of housing in period 0 can be expressed exactly as $1+(1-\varepsilon)15+\varepsilon 1 = 16-14\varepsilon$, since the marginal buyer in period 0 will be a type A agent, who realizes that he gets only 1 utile from the house in period 0 and can then sell it off at prices 15 or 1 in states 1 and 2, respectively. Given this price for the asset, agent B will realize that borrowing is a great deal for him as long as $\varepsilon < 1/30$ and he has the wealth to pay back the loan in the last period, since by borrowing 1 unit of food he gets $30/(16-14\varepsilon)$ utiles of consumption and gives up only $(1-\varepsilon)1+\varepsilon 15$ utiles in the last period to pay back the loan. Therefore he will continue to borrow until the point at which he entirely runs out of wealth in state 2. The closed form formula for the rest of the equilibrium with $\varepsilon > 0$ is messy, so we give an approximate calculation for ε very small.

$$\pi_j = 1$$
$$((p_{0F}, p_{0H}), (p_{1F}, p_{1H}), (p_{2F}, p_{2H})) = ((1, 16-14\varepsilon), (1, 15), (1, 1)),$$
$$x^A = ((x^A_{0F}, x^A_{0H}), (x^A_{1F}, x^A_{1H}), (x^A_{2F}, x^A_{2H})) \approx ((22.2, \tfrac{10.5}{16}), (33.1, 0), (23, 1)),$$
$$\theta^A_j \approx 3.3; \quad \varphi^A_j = 0;$$
$$x^B = ((x^B_{0F}, x^B_{0H}), (x^B_{1F}, x^B_{1H}), (x^B_{2F}, x^B_{2H})) \approx ((\tfrac{57-63\varepsilon}{32-28\varepsilon}, \tfrac{5.5}{16}), (36.9, 1), (0, 0)),$$
$$\theta^B_j = 0; \quad \varphi^B_j \approx 3.3;$$
$$u^A \approx 58; \; u^B \approx 61.6.$$

It is interesting to note that the GEI equilibrium is also Pareto dominated by the collateral equilibrium. (Had we made agents of type A risk averse this would have been more dramatic). The reason is that with infinite default penalties, there is much less flexibility in the credit markets, since once an agent sees that he cannot pay off in some state, no matter how unlikely, he must forego further borrowing.[4]

9 Conclusion

We have argued that Wall Street directs much of its activity to the stretching of collateral to cover as many promises as possible. We saw concretely in our example how making more collateral available made it possible to make more promises, and improve social welfare. It also typically increases the prices of houses. Perhaps with some justice, Kidder Peabody can claim to have played a role in this enterprise.

We also saw how the scarcity of collateral rations the kind of promises that are made, and fixes the levels of collateral that are required for any given promise. The kinds of promises and collateral levels that are traded in the market are not made at the whim of one seller like Kidder Peabody, but are the result of forces of supply and demand.

[4] Indeed it is clear that there is a discontinuity in the GEI equilibrium, since for $\varepsilon = 0$, presumably we would not require full delivery in a state that has zero probability, and we would be back in the no uncertainty situation from the last section in which we got a vastly different equilibrium.

We saw how incomplete markets leads agents to leverage, which in some states can change the distribution of income, or magnify changes that would have occurred anyway. When agents have heterogeneous valuations of durable assets, these redistributions of income can create large fluctuations in prices. Inevitably this in turn will create situations where some agents have much greater than normal opportunities to invest, at least compared to the opportunities they might have taken had events been different, or had collateral requirements been different.

There are two aspects of this investigation that call for further work. In this lecture I have presumed that there was no tranching or pyramiding of promises. Only physical commodities were allowed to collateralize promises. In Geanakoplos–Zame–Dubey [1995] the model allows for both tranching and pyramiding, but further work is probably merited. This line of research touches on the Modigliani–Miller theorem, for example.

Finally, we have limited ourselves to two periods. With more time periods, we might have discovered an interesting dynamic. In states in which the physical collateral suffers a precipitous drop in price, the agents who most like it suddenly have their best investment opportunity. Assuming some of them are still around to take advantage of the opportunity, their wealth will tend to grow over time. But of course they will tend to use this increasing wealth to bid up the price of the very same physical collateral. This dynamic suggests a cyclical behavior to prices, and suggests that the opportunity to earn extraordinary returns will eventually disappear.

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