EQUILIBRIUM AND STRATEGIC COMMUNICATION IN THE ADVERSE SELECTION INSURANCE MODEL

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Abstract

Shows equilibrium always exists (Rothschild-Stiglitz-Wilson model) when firms enforce policy exclusivity via strategic (profit-maximizing) communication of client purchases. Strategic communication induces two equilibrium types: partial communication of purchase information or non-communication which exhibits a lemon effect (low-risk purchase no insurance). Nonetheless, Jaynes' configuration (Jaynes; Beaudry & Poitevin) allocating both risk-types a low-coverage pooling contract and high-risk supplementary expensive coverage always characterizes equilibrium including Perfect Bayesian Equilibrium in Hellwig's two-stage framework where inter-firm informational asymmetries impose additional "competitive" features. Adverse selection induces salient features of financial markets: Bertrand-Edgeworth competition, latent contracts, strategic exclusivity-policy cancellation tactics, market institutions for sharing information.

Keywords: adverse selection, insurance, communication of information, pooling contract

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[1] I am indebted to Martin F. Hellwig for several stimulating conversations. Of course, any errors are my responsibility.

Introduction

In the canonical model of adverse-selection insurance (Rothschild and Stiglitz (1976); Wilson (1977)), insurance companies compete for consumers of unidentified risk quality by designing partial coverage fixed premium policies. Famously, the model produces either no Nash equilibrium or a "separating equilibrium" inducing consumers to reveal their risk type by self-selecting low premium–low coverage or high premium-high coverage policies. In separating equilibrium, each type's policy is priced fairly for its risk class; high-risk types select full-coverage and low-risk types the maximum partial coverage leaving high-risk types indifferent to switching to the cheaper low-risk policy.

To screen consumers via partial coverage policies, insurance companies impose exclusivity conditions restricting consumers to one policy. Implicitly, this assumes firms share client identities to prohibit multiple purchases. Jaynes (1978) showed separating equilibrium is vulnerable to a profitable deviation by a firm selling coverage above the high-risk fair price and not disclosing buyer identities. High-risk agents prefer purchasing this high price insurance plus the low-risk policy to their separating equilibrium policy. Since issuing high price policies and not divulging purchasers' identities is profitable whenever low-risk types buy insurance at a price (premium per unit coverage) below the high-risk fair price, all policies priced below pooling odds (population average odds of a claim) lose money and cannot be traded. With firms communicating client information strategically, a unique equilibrium is reached; firms selling pooling contracts share client identities, firms selling higher price insurance do not; both risk-types purchase the low-risk's most preferred zero profit pooling contract and high-risk types supplement it with additional high-risk fair price insurance to reach full coverage. By mimicking low-risk types, high-risk types undermine firm screening; except to firms selling high price coverage and not divulging client identities, a consumer's risk type remains private information.

Hellwig (1988) reformulated the model as a two-stage game with firms simultaneously choosing policy offers and communication strategies at stage one. He claimed Jaynes' solution was not a sequential equilibrium of the game, therefore not Nash, because firms must condition communication strategies on competitors' contracts *reacting* to withdraw client lists from a firm innovating a particular deviation from the proposed equilibrium. Then, in an insightful analysis of the strategic richness afforded by endogenous communication of information, he showed Jaynes' solution was a sequential equilibrium for a four-stage game with firms choosing communication strategies after observing competitors' contracts. Clouding the issue, subsequent papers analyzing a variety of adverse selection insurance and credit market models rediscovered Jaynes' solution (see e.g. Beaudry & Poitevin, 1993; 1995; Dubey & Geanakoplos, 2001), Gale, 1991). Reconciling the disparate results, we show (even for Hellwig's two-stage frame, Perfect Bayesian Equilibrium always exists and is always characterized by Jaynes' configuration; even when, under economically intuitive conditions, low-risk types' equilibrium allocation is the null contract and they purchase no insurance. Thus, adverse selection can have a lemon effect driving low-risk types from the market with trade in some financial instruments (policies priced below high-risk fair price) not occurring. Moreover, two-stages is sufficient to explore the strategic dynamics of endogenous communication and equilibrium strategies are richer than previously shown: adverse selection alone induces emergence of latent contracts, specific exclusivity clauses and policy cancellation tactics, and market institutions for sharing information.

Preliminaries

Hellwig's critique *implicitly* expands firm strategy spaces but does not examine if the firm communication strategies he considers remain optimal given the expanded strategy spaces. They do not. His enlarged strategy space creates an informational asymmetry that transforms (at stage two) what firms know about the history of previous firm actions from full information to an information set containing two nodes. Therefore, beliefs about competitors' possibly unobserved actions affect optimal strategies. Jaynes' solution uniquely characterizes the aggregate contract set in Perfect Bayesian Equilibrium. However, different profiles of communication and supply strategies can support it. Strategic communication hybridizes equilibrium combining features of non-strategic models assuming multiple contracting with no firm communication (Bisin & Guaitoli, 2004; Attar & Chassagnon, 2009, Ales & Maziero, 2011) or exclusive contracting with complete inter-firm communication (R-S-W).

Figure 1 depicts the Jaynes configuration. In indemnity-premium space (α, β), with q^i the fair odds of a claim for the designated populations, the rays $\beta = \bar{q}\alpha$; $\beta = q^L\alpha$; $\beta = q^H\alpha$ represent the loci of policies earning zero profit if purchased by respectively, the pooled, the low-risk, and the high-risk populations. The contract p* = (α^*, β^*) maximizes low-risk consumers' utility on the zero profit pooling line. In the J* configuration (see Figure 1):

1. $n \geq 2$ firms offer the set of pooling policies $\lambda \cdot (\alpha^*, \beta^*),\ for\ any\ \lambda,\ 0 \leq \lambda \leq 1$; and require exclusivity – consumers may purchase only one policy at the price $\frac{\beta^*}{\alpha^*}$; and no policy at a different price.

2. To enforce exclusivity, the n firms communicate their client lists only to firms offering (α^*, β^*);

3. $m \geq 2$ firms offer the set of policies $(\alpha, q^H\alpha)$, $\alpha \geq 0$; No Exclusivity conditions;

4. These m firms do not communicate their client lists to any firm.

5. Low-risk consumers buy one contract p* = (α^*, β^*), high-risk consumers also buy p* once, then supplement it with a contract H* = $(\alpha^H, q^H\alpha^H)$ taking them to full-coverage at h*.

Only a policy attracting low-risk types alone could break the proposed equilibrium. In Figure 1, consider the line hh with slope q^H through p* and h*. Right of p* this budget line is the locus of feasible policies attainable to a consumer buying p* then purchasing additional insurance at high-risk fair odds q^H. On this line from p*, high-risk agents obtain their optimal contract h*. Now consider *any* policy γ above hh, below the low-risk indifference curve (LL) tangent to p*, and above the low-risk zero-profit line. Low-risk consumers prefer such policies to p*, high-risk agents do not.[2] Furthermore, high-risk types prefer h* to any policy combining γ and high-risk insurance. The (not shown) budget line of slope q^H through γ defining the final policies available to consumers supplementing γ with high-risk insurance is parallel to and must lie above hh. Trading up this budget line from γ would take high-risk agents left of hh to a best total contract also left of the high-risk indifference curve (HH) through h*. It follows, if some firm offering such a contract γ could prohibit its clients from purchasing pooling coverage on the ray 0p*, γ would only be purchased by low-risk types and earn a profit. However, under the conditions specified, if a firm offered such a γ, both high and low-risk types would purchase γ and supplement it with a contract on 0p*. Trading up the budget line (pp) through γ of slope $\bar{q}$, high-risk consumers now reach a policy to the right of the budget line hh (possibly beyond full-coverage). If necessary, they buy more insurance at price q^H to obtain full coverage. The entry preventing unsold contracts 0p* render all γ policies unprofitable.

Hellwig's critique alters firm strategy spaces by allowing a firm to introduce a policy innovation. He assumes a firm offering the pooling contracts on 0p* (thus receiving client information from competitors offering those policies) can in addition offer consumers a *secret* γ contract on condition they not purchase a policy on 0p*. Armed with client lists, this firm enforces exclusivity cancelling contracts of clients who violate the exclusivity condition. Low-risk agents (who prefer the exclusive contract γ) buy

[2] Rationed at p*, high-risk types desire more coverage at price q^H, therefore their indifference curve through p* must cut hh from below implying they prefer p* to γ.

it alone to avoid having it cancelled. If high-risk types buy p* and γ, their γ contract is cancelled. Then γ earns a positive profit and the pooling contract negative profit, the deviation upsets the equilibrium.

Key to this equilibrium busting move is the assumption either the disingenuous firm's identity is unknown or competitors' are unaware a γ contract is offered allowing the deviator to receive client information from the other firms offering pooling contracts. If communication strategies are given (in the Nash sense that the deviator continues to receive the communications after offering the secret contract), the proposed equilibrium is upset. If 0p* firms' communications are withdrawn after the deviator makes its secret offer, exclusivity conditions on γ could not be enforced and it would earn a negative profit. However, this requires a reactive element in firms' communication strategies because firms would be conditioning their communications on the contract offers made by competitors.

To clarify his argument firm communication strategies must be reactive, Hellwig analyzed a two-stage game using the sequential equilibrium solution concept. At stage one, firms make binding contract offers and announce communication strategies. At stage two, consumers choose optimal contracts, taking into account firms' ability to enforce exclusivity conditions. The crucial stipulation is firms choose contract and communication strategies simultaneously eliminating any reactive element in the communication of client lists. His definition of equilibrium required sequential rationality for both firms and consumers: given constellations of firm contract offers and consumer purchases; 1. No firm can increase its expected profit by deviating from the given constellation of contract offers; 2. For *any* constellation of contract offers, each consumer's portfolio of contracts is utility maximizing.[3]

In this two-stage framework, with firms simultaneously committing to their stage 1 policy and communication strategies and *assuming the communication strategies of* J^* on page 4, the J* contract set and its *specific* communication strategies would be upset by a firm able to offer both $\lambda \cdot (\alpha^*, \beta^*)$, $0 \leq \lambda \leq 1$; *and secretly* an exclusive γ contract preferred to (α^*, β^*) by low but not high-risk consumers.[4]

[3] Firm i's two contract offers (possibly null contracts) have two components: i. Its policies; ii. Exclusivity conditions. Its communication strategy identifies (for each policy) the list of firms it sends its client list. Refer to a policy (α_i, β_i) as a primary policy and any fraction of it a derivative. Firm i's policy offers may be written,

$$\{\varphi_i^j\}, \varphi_i^j = \lambda_i^j(\alpha_i^j, \beta_i^j),\ for\ \ any\ \lambda_i^j,\ 0 \leq \lambda_i^j \leq 1;\ or\ \lambda_i^j = 1,\ (\alpha_i^j \geq 0, \beta_i^j \geq 0,\ j = 1, 2)$$

Restricting λ_i^j to equal one, offers only the primary contract. If $0 \leq \lambda_i^j \leq 1$, the primary contract is divisible.

[4] There is a logical conundrum entailed in this bait and switch offer . We discuss this in the final section of the paper.

However, this result is inconclusive. What it shows is, given the implicit change in the structure of the game (firm strategy sets expanded to allow secret policies and separate communication strategies per policy), the J contract configuration is not an equilibrium given the *specified* communication strategy.

Importantly, the definition of equilibrium adopted did not require sequential rationality with respect to firm communication strategies, nor was this issue investigated. Given expansion of firm strategy spaces, the specified communication strategy is not optimal for firms offering pooling contracts. Allowing a firm to offer a contract secretly, and to select different communication strategies for its policies significantly alters the rules of firm behavior considered in Jaynes or R-S-W. However, if one firm's strategy set allows it to offer secret contracts and contracts with different communication strategies this same strategy must be available to all firms, and, as importantly, all firms must be aware such a strategy is possible. If at stage 1, rational firms know any firm j can prevaricate by making a secret offer and another offer in order to receive communications of client lists, this surely alters optimal firm communication strategies. How does expansion of firm strategy sets affect optimal strategies and equilibrium, how reasonable are the altered assumptions? The paper's final section discusses the reasonableness of various modeling assumptions. Now we focus on the first two questions.

Competitive Equilibrium

The two-period frame is retained taking care to maintain the assumption firms choose strategies simultaneously during period 1 and are unable to condition their communications on the aggregate set of contracts. We use the following two-stage framework.

- At stage-one, firms announce contract offers and communications, consumers choose optimal contracts taking into account exclusivity conditions and firms' ability to enforce them.

- At stage two, stage one communications and consumer insurance claims are revealed, firms maximize expected profit exercising rights to cancel policies of clients violating contracts.

At this point, we observe the terms policy and contract are used distinctively. A policy represents a premium and indemnity. A contract designates a policy, if the policy is divisible, what (if any) exclusivity clauses are attached to the policy, and the cancellation rule used to enforce exclusivity. For example, a firm offering low-risk type's optimal pooling contract with a single purchase exclusivity clause might also announce it will always cancel the contract of clients who purchase another contract. But this cancellation strategy is neither credible nor optimal for different reasons. It is not credible because the firm can cancel a policy only if it actually observes an exclusivity violation. Since firms selling the high-risk subprime contracts will not find it optimal to communicate client identities, consumers know the pooling contract seller cannot enforce its announced cancellation policy against their purchase of supplemental high-risk insurance. Moreover, suppose this pooling contract seller did observe clients had violated its exclusivity condition. A nondiscretionary cancellation strategy would never make sense for the simple reason a profit maximizer should only cancel contracts of clients who actually present claims. Furthermore, if at stage 2 applying forward induction, a firm could infer clients' risk-type from the contract purchased at stage 1, we shall see, selective cancellation dominates a nondiscretionary rule. For this reason, contracts generally contain an exclusivity-cancellation strategy that "reserves the right" to cancel given a violation. Firms cancel only when doing so maximizes profit.

A firm's strategy must specify its contract offer(s) and communication rule(s). A contract consists of three components: a policy, any exclusivity conditions attached to the policy, and the cancellation procedures used to enforce exclusivity conditions. A firm's communication rule for a policy designates which firms will receive that policy's client list. Specifically, we assume each firm offers two contracts with five components to each offered contract. For example, one strategy of a firm offering the low-risk optimal pooling contract and its derivatives is $S^1 = ((\alpha^*, \beta^*)$, divisible, exclusive, cancel all exclusivity violators presenting claims, communicate only with firms for which every policy offer (α, β) satisfies $\frac{\beta}{\alpha} \geq \frac{\beta^*}{\alpha^*}$; 0,0,0,0,0) with the zeros indicating no second contract is offered. Analogously, one strategy of each firm offering unlimited insurance at the high-risk fair price is S^2 = (H, divisible, no exclusivity, no cancellation, no communication, 0,0,0,0,0). One strategy of a firm offering a secret γ deviation is described $S^3 = ((\alpha^*, \beta^*)$, divisible, exclusive, cancel all exclusivity violators presenting claims, communicate only with firms for which every policy offer (α, β) satisfies $\frac{\beta}{\alpha} \geq \frac{\beta^*}{\alpha^*}$; secret γ, indivisible, exclusive, cancel all exclusivity violators presenting claims, no communication).

The possibility a firm may offer a secret γ contract significantly alters the informational structure of the competitive environment by introducing a new informational asymmetry between firms. If this strategy were not available at stage 1, all firms at stage 2 would know the complete history of previous actions. If the strategy is available, firms at stage 2 are uncertain about the complete history of previous actions and face an informational set with two possibilities; in addition to the aggregate set of observable contracts, either a secret γ contract was offered or no firm offered a γ contract. Sequential rationality of firm strategies requires firms respond to this imperfect information optimally, and this requires the introduction of beliefs concerning the probabilities of reaching one or the other of these alternative environments. The equilibrium concept we apply is the Perfect Bayesian Equilibrium (PBE).

Definition of Competitive PBE: Equilibrium requires for firms a profile of contract and communication strategies, and beliefs π and 1-π a γ contract will (will not) be offered, and for consumers a profile of purchases by type satisfying three conditions. Given the profile of contract offers, communication strategies, beliefs π and 1-π, and consumer purchases, 1. No firm can increase its expected profit by deviating from its current contract offer *or* communication strategy at any stage; 2. Firm beliefs are derived from Bayes' Rule conditioned on firm strategies and consumer purchases; 3. Each consumer's portfolio of contracts is utility maximizing at each stage.

Consumer sequential rationality concerns taking firms' communication and cancellation strategies into account at stage one to avoid suboptimal allocations at stage two.

Additional Assumptions

1. Consumers have a twice continuously differentiable strictly concave utility function u(w) over income with Von Neuman-Morgenstern utility $p^i u(w_0) + \left(1 - p^i\right) u(w_1)$ where $w_0 = w - d + \alpha,\ w_1 = w - \beta,$ d is the income loss in state 0, and i denotes risk type. Denote marginal utilities by $u_j', j = 0,1$ to reference the relevant income for evaluating marginal utility.
2. Firms selling an identical contract receive an equal proportional distribution of consumer types demanding the contract.
3. Firms are prohibited (say by law) from cancelling coverage of a traded contract arbitrarily (e.g. the firm must have an *observed* exclusivity violation.
4. Firm communication strategies are part of the contract with clients and are inviolable, e.g. a firm choosing a non-communication strategy makes a contractual promise to its clients and cannot sell the information.
5. Insurance companies only issue nonnegative policies, and until relaxed later, we assume for $(\alpha, \beta) = (0, 0),\ \frac{p^L u_0'}{(1-p^L) u_1'} > \bar{q}$.

Condition 5 says, at their endowment income, low-risk type's marginal rate of substitution between consumption in the two states exceeds the pooling price implying low-risk types' most preferred pooling policy has positive coverage. When this assumption is not satisfied low-risk types' most preferred feasible pooling policy equals the null contract, and they only purchase insurance if a policy priced below pooling odds is available. However, lemma 1 shows equilibrium cannot entail policies priced below pooling odds. Nevertheless, equilibrium exists and is characterized by a special case of the J configuration. When condition 5 is not satisfied, low-risk types' equilibrium allocation is the null contract (they purchase no insurance). Thus, (relative to utility function's risk aversion) when risk type probability spreads are large and low-risk population proportion "small" and/or endowment incomes in the two states close, adverse selection can produce a lemon effect driving low-risk types from a market where trade in some financial instruments (policies priced below the high-risk fair price) does not occur.

Lemma 1: At stage one, firms are in Bertrand-Edgeworth competition. The only possible contract set supporting an equilibrium is the J configuration with unlimited, nonexclusive, non-communicated high-risk fair policies and exclusive pooling contracts capped at low-risk types most preferred pooling coverage.

Proof: Consider any policy (α, β) with implicit price $\frac{\beta}{\alpha}$ less than the high-risk fair price. Whatever this price, if not rationed, high-risk types demand greater coverage than low-risk types. Therefore, restricting coverage to the low-risk demand point maximizes expected profit at that price, and no firm would offer greater coverage at the same price. This means a firm's best response can always be characterized by a price and maximum coverage. Firms are effectively in Bertrand-Edgeworth competition. Each firm may infer the lowest price (with its associated maximum coverage) promising nonnegative profit will prevail. This lowest feasible price is the population-pooling price. If a contract cheaper than the pooling contracts were part of an equilibrium, it could only earn nonnegative profit if no high-risk types buy it. But that implies, its seller is enforcing an exclusivity condition and only low-risk types are purchasing this cheap contract (as their sole purchase). Therefore, only the high-risk buy more expensive contracts implying those contracts must be the high-risk fair contracts or they would earn negative profit. Since the high-risk are not buying the cheap contract, its sellers must be receiving client identities from sellers of the high-risk contracts. But communication of high-risk contracts cannot be optimal (a firm could earn positive profit by switching to non-communication and raising its price slightly to attract all high-risk types who could then also buy the cheap policy). This implies either firms selling high-risk contracts are not optimizing or the firm selling the cheap contract could not enforce exclusivity conditions, both contradict the equilibrium hypothesis. Therefore, equilibrium has both risk types buying the low-risk utility maximizing pooling contract. Then, low-risk types will not buy at a higher price and the only other contracts sellable without negative profit and immune to undercutting are high-risk fair contracts which again must optimally be non-communicated. The only candidates for equilibrium are as claimed with the communication and cancellation strategies on pooling contracts to be determined.

Now consider the possibility a firm offers a secret γ contract. At stage 2, firms exercising cancellation rights do not know the complete history of stage 1 actions with certainty. Instead, each firm's information set contains two nodes. Either a secret γ contract was offered or one was not. If a secret γ contract has been offered and firm i has offered a pooling contract with communication, the deviating firm offering the secret γ contract will receive firm i's client list allowing the deviator to enforce its exclusivity requirements. Therefore, high-risk consumers (who prefer (α^*, β^*) to γ) have bought firm i's pooling contract but the low-risk (preferring γ and avoiding its cancellation) have not. Possessing no information allowing it to cancel pooling contracts of claimants all of whom are high-risk, firm i earns negative profit. Clearly firm i's contract-communication strategy is not sequentially rational.

Lemma 2: If belief's place positive probability on a secret γ contract being offered, no firm offers pooling contracts with communication of client information.

Proof: At stage 1, firms expect a pooling contract with communication to earn zero profit with probability 1-π (no secret γ contract is offered) and negative profit with probability π (a secret γ contract is offered). Given beliefs π > 0, expected profit is negative. Offering pooling contracts with communication is not sequentially rational. Best responses are to offer either null contracts, high-risk contracts without communication, or the pooling contracts without communication.

Lemma 3: No equilibrium has firms believing with positive probability a secret γ contract is offered.

Proof: By lemma 2, firm strategies have only three possibilities. If all firms offer the null contract, offer of a γ contract would attract both risk types and earn negative profit. However, all firms offering the null contract cannot be an equilibrium because firms would deviate with a profitable not communicated contract offer above the pooling price. Moreover, if all firms offer contracts with no communication either they all offer the high-risk fair contracts *or* some offer those contracts while n firms each offer one-nth the low-risk optimal pooling contract (and its derivatives) to prevent high-risk types from purchasing more than one optimal pooling policy. The case with only high-risk fair contracts cannot be an equilibrium because each firm would have incentive to change its strategy to offer a contract between the high-risk and pooling prices to attract both risk types and earn positive profit. The mix of firms offering non-communicated high-risk contracts and n firms offering one-nth the optimal pooling contract without communication could not be an equilibrium because each of the n firms offering pooling contracts without communication possesses monopolistic power and could raise its premium a small amount (retain its clients) and expect to earn positive profit.

Remark 1. If firms believe with positive probability a secret γ contract will be offered, no firm will offer a secret γ contract. Moreover, if firms place zero probability on a secret γ contract being offered, a secret γ contract is profitable only if *all* firms offering pooling contracts communicate clients.

To see the first assertion, note by Lemma 1 if $\pi > 0$, no firm will offer pooling contracts with communication. It follows no firm would offer any policy priced between the high-risk fair price and zero profit pooling policies and communicate clients either, because, by a revealed preference argument, low-risk types would also prefer an exclusive secret γ to such a contract implying the contract could only be sold to high risk types for negative profit. But with no contracts offered with communication, any firm offering a secret γ contract will receive no client information and priced below the pooling price must earn negative profit because both high and low-risk types would buy γ. Hence, $\pi > 0$ implies no secret γ contract would be offered. Alternatively, if $\pi = 0$, some firms may find it sequentially rational to offer pooling contracts with communication. However, a secret γ contract could earn positive profit only if every firm offering pooling contracts communicated clients. Otherwise, high-risk types would purchase γ and a contract from some firm selling pooling contracts without communication.

Remark 2: We conclude, from lemmas 1 and 3 and remark 1 if equilibrium exists, firm beliefs must place zero probability on the offer of a secret γ contract, and there must be firms offering the optimal pooling contract and its derivatives and choosing different communication strategies.

Definition: A communication strategy profile is symmetric if every firm offering the same set of policy prices adopts the same communication strategy. Otherwise, a communication strategy profile is non-symmetric.

The communication strategy profile in J* is symmetric, therefore, not sequentially rational in the two-stage framework allowing secret contract offers. More generally,

Remark 3: If it is assumed firms can offer a fake pooling contract (it cannot actually be sold, see below) to receive client information and secretly offer another contract for sale, no equilibrium can have a symmetric strategy profile for firms offering a pooling contract.

If firm strategy spaces allow offering a pooling policy as a *pretense* to receive client information to enforce exclusivity on a secret γ contract, we showed above it is not optimal for every firm selling a pooling contract to disclose its list of clients allowing entry of a successful secret γ deviation. Below we show there is a non-symmetric communication strategy profile for which the J configuration is an equilibrium for the two-stage framework allowing secret contracts.

Strategic Competition

The communication strategy associated with a policy determines if a firm can actually enforce any exclusivity conditions attached to the policy, an attribute of signal import to consumers and firms. For example, if a firm offers a policy but neither sends nor receives client information, any exclusivity conditions it afixes to its policy are unenforceable allowing consumers to purchase additional contracts. For this reason, the same policy offered with two different communication strategies (and or exclusivity conditions) can amount to two different contracts providing firms very different strategic possibilities. In this regard, Hellwig's four-stage structure revealed important observations related to his insightful point that sharing information involves sending *and* receiving information. In addition to what and to whom to send information, firms must decide what to do with the information they receive. Below we show the two-stage framework is sufficient for analyzing each of these points when firms' full strategy spaces are considered thereby enabling employment of non-symmetric communication strategies.

If a firm deviates from the equilibrium by offering a secret γ contract, it can hardly announce its clients on the γ contract to its competitors. However, although, there is no longer any question of the contract being secret, by adopting the strategy of offering a γ contract and communicating its clients' identities, this firm presents firms selling the pooling contract a decision not confronted if it had offered γ secretly.[5] If it receives the client list of a firm selling a γ contract, what should a firm selling the pooling contract do with the information? Since a pooling firm receiving a list of buyers of the γ contract could cancel the contract of any of its clients on the list, one might suppose profit maximization requires pooling firms cancel contracts of all violators with claims. However, if pooling firms' strategy is to summarily cancel all such contracts, no high-risk type would buy γ and no low-risk consumer

[5] One strategy of a firm offering a communicated γ deviation is $S^4 = (\gamma$, indivisible, exclusive, cancel all exclusivity violators presenting claims, communication with firms offering only contracts (α, β) satisfying $\frac{\beta}{\alpha} \geq \frac{\gamma_2}{\gamma_1}$, 0,0,0,0,0).

would purchase a pooling contract (they prefer γ). In this case, pooling contracts would necessarily earn negative profit and the γ contract positive profit upsetting equilibrium. Demonstration of equilibrium must therefore consider both types of γ contracts, secret (thus, not communicated) and communicated.

Proposition 1: There exists N* such that for $N \geq N^*$ and firm beliefs π = 0, the J configuration with non-symmetric strategies described in 1-7 below is a PBE for the two-stage frame.

1 $N \geq N^*$ firms offer consumers *either* (not both) the pooling policy (α^*, β^*) as an indivisible limited policy or any derivative $\lambda \cdot \frac{(\alpha^*,\beta^*)}{N}$ for any λ satisfying $0 \leq \lambda < 1$. Either purchase requires exclusivity – consumers may purchase only one policy at the price $\frac{\beta^*}{\alpha^*}$ and no policy at a different price. Each firm reserves the right to cancel the contract of a client who violates the exclusivity condition.

2. Each of these N firms communicates its list of clients who purchase (α^*, β^*) to those firms for which every offered contract (α, β) satisfies $\frac{\beta}{\alpha} \geq \frac{\beta^*}{\alpha^*}$.

   Each of these N firms promises not to communicate to any firm its list of clients who purchase a derivative of (α^*, β^*).

3. n ≥ 2 firms offer consumers choice of $\lambda \cdot (\alpha^*, \beta^*)$ for any λ satisfying $0 \leq \lambda \leq 1$. Purchase requires exclusivity – consumers may purchase only one policy at the price $\frac{\beta^*}{\alpha^*}$; and no policy at a different price. Each firm reserves the right to cancel the contract of a client who violates the exclusivity condition.

4. Each of these n firms communicates its list of clients who purchase (α^*, β^*) or a derivative to those firms for which every offered contract (α, β) satisfies $\frac{\beta}{\alpha} \geq \frac{\beta^*}{\alpha^*}$.

5. "Reserves the right to cancel" equates to, at stage 2, the N + n firms selling pooling contracts cancel the contract of any client who is known to have violated exclusivity conditions, presents a claim, and demands more pooling coverage than low-risk types.

6. $m \geq 2$ firms offer the set of policies $\lambda \cdot (\alpha, q^H\alpha),\ \alpha \geq 0, for\ any\ \lambda \geq 0$ with no exclusivity conditions.

7. These m firms promise not to communicate their client lists to any firm.

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The N firms offering one-Nth derivatives of the pooling contract do not communicate client lists on the derivatives, but do communicate clients who purchase the full pooling contract. The stage 1 communication strategy of these N firms with respect to the indivisible contract (α^*, β^*) and the n firms offering the pooling contract *and* its derivatives commits firms to sending client lists only to firms offering contracts priced greater than or equal to the pooling contract price (this includes a firm offering a secret γ contract). Note the only information the firm needs to announce this communication strategy at stage 1 is its own primary contract and unit price, and therefore the strategy is consistent with the firm's information set at stage 1. The strategy is also independent of the contracts other firms choose at stage 1 or any other stage. This communication strategy commits firms (at stage 1) to a specific rule for sharing client lists at stage 2. Condition 5 is an important component of firm strategies. Below, it is shown to be the stage 2 expected profit maximizing strategy of firms offering pooling contracts when firms are in receipt of both other firms' client lists and claims for payment from their own clients. We note immediately this cancellation strategy always leads to cancellation of claimants with pooling contracts observed to have purchased any combination of pooling contracts exceeding (α^*, β^*).

Remark 4: The aggregate set of policies available to consumers is the J-configuration. Furthermore, given this set of policy offers and communication rules, low-risk consumers buy one contract p* = (α^*, β^*), high-risk consumers also buy p* once, then supplement it with a contract H* = $(\alpha^H, q^H \alpha^H)$ taking them to full-coverage at h* in Figure 1.

First, observe the N firms offering the derivative policies without communication can each offer at most one-Nth of the total derivatives purchased, otherwise high-risk types by purchasing enough derivatives would be able to exceed (α^*, β^*) and the derivatives would earn negative profit.[6] Moreover, since each firm would sell only one contract to any one consumer, buyers of the N contracts $\lambda \frac{(\alpha^*, \beta^*)}{N}$ may get arbitrarily close to (α^*, β^*) but cannot attain it because of the restriction $\lambda < 1$. Therefore, anyone purchasing N derivatives will obtain less coverage than (α^*, β^*) implying all consumers prefer the full pooling contract to purchasing any available quantity of derivatives. Next, observe no consumer can obtain both the pooling contract (α^*, β^*) and one of its derivatives. Since all sellers of (α^*, β^*) communicate clients, any firm selling a derivative of (α^*, β^*) will receive client information about purchasers of (α^*, β^*) and correctly inferring only high-risk types would purchase both, will cancel the contract of clients violating its exclusivity condition and turning in a claim. Similarly, with sellers of (α^*, β^*) enforcing the quantity restriction by communicating clients to all sellers of pooling contracts, no consumer could obtain more than one full (α^*, β^*) contract.

Both high and low-risk consumers optimize utility by purchasing (α^*, β^*). Only high-risk consumers supplement (α^*, β^*) to full coverage by purchasing from one of the m firms selling at the

[6] See Bisin and Guaitoli where the exogenous assumption of complete non-communication between firms necessitates the same one-nth supply restriction in a model of moral hazard insurance.

high-risk price. It follows all consumers obtain the same allocation as in J* and firms earn zero expected profit.

Remark 5: All derivatives of (α^*, β^*) are latent policies -- not purchased in equilibrium, these latent policies perform the same entry preventing role as in the J* equilibrium. Their strategic function is analogous to an oligopoly pre-committing stage 1 sunk costs to credibly signal a potential entrant it will earn negative profit. Observe how firm beliefs and the latent policies are mutually reinforcing. Firm beliefs are clearly consistent with Bayes' Rule given strategies and purchase behavior.

We now prove Proposition 1 by showing all agents' strategies are sequentially rational; i.e. (both on and off the equilibrium path) consumers are maximizing utility and no firm can alter any component of its strategy and increase its expected profit. This requires showing no firm could earn a positive profit by offering an exclusive, indivisible γ contract either as a secret offer without communication or as an offer with communication.

- No firm could earn a profit by changing the price or coverage of any contract it offers.

If one of the m firms selling at the high-risk price were to raise its price, it would lose its clients to its $m-1$ competitors; lowering the high-risk fair price toward the pooling price would attract only high-risk agents and earn negative profit, moreover, these firms already supply unlimited coverage. Similarly, if one of the N+ n firms selling (α^*, β^*) and communicating client information were to raise the price on its pooling policy, it would lose its clients to one of its N+ n-1 competitors. Furthermore, if one of the N + n firms communicating buyers of (α^*, β^*) both raised its price (between the pooling and high-risk prices) and switched to not communicating and nonexclusivity, no low-risk agent would purchase but it would attract high-risk agents substituting the new contract for insurance at the high-risk fair price. Since the deviation would be priced below the high-risk fair price, it would earn negative profit. Since low-risk types are sated at (α^*, β^*), increasing coverage at a constant price would only attract high-risk types and earn negative profit. For the same reason, if one of the N firms offering derivatives at the pooling price changed its derivative offers to the same coverage at a higher price, it would attract no additional clients, and if it increased its coverage it could only attract a disproportionate number of high-risk agents and earn negative profit. Clearly, any firm lowering the price of its pooling contracts would earn negative profit.

- Sequential rationality of communication and exclusivity strategies

We now confirm firm communication and exclusivity strategies are sequentially rational. Consider the N+ n firms offering the prime pooling contract. If one unilaterally dropped its exclusivity condition, the relaxed constraint would be of no value to consumers (thus the firm) because continued communication of client identities would bar clients from multiple purchases of insurance at the pooling price. If one of these N + n firms stopped communicating client information, any additional clients would be high risk bringing negative profit. Similarly, dropping its exclusivity condition *and* switching to non-communication could only gain it high-risk consumers and negative profit (low-risk types desiring only one policy would not switch firms). Now consider the N firms' strategies with respect to their pooling derivatives. If one of these N firms dropped exclusivity while retaining non-communication, it could only gain high-risk clients and negative profit. Moreover, communication gains it no additional clients. If it were to switch to communication and non-exclusivity, communication would render the non-exclusivity useless to consumers because other firms would use the information to void unprofitable contracts, and communication of clients would again gain it no additional clients. Finally, if one of the m firms offering high-risk price coverage switched to communication and/or exclusivity it would lose all its clients to its m-1 competitors.

- No secret γ offer is profitable

  Suppose a firm (including an entrant) other than one of the N firms selling the one-Nth pooling derivatives deviates from the equilibrium and at stage 1 offers a secret γ contract while simultaneously pretending to offer the full set of pooling contracts expecting to receive client information from firms selling (α^*, β^*) and communicating their clients. The communicating firms would sell no contracts and would send the deviating firm empty client lists. This follows because both risk types maximize utility purchasing γ plus additional insurance from the N firms selling $\lambda \cdot (\alpha^*, \beta^*)$ derivatives without communicating client lists. Since the deviator could not identify clients who have purchased pooling insurance from the N non-communicating firms, it could not cancel contracts. Selling to both risk types at a price below the pooling price, it would earn negative profit. This type of deviation is not sequentially rational.

  Alternatively, suppose one of the N firms offering (α^*, β^*) with communication or one-Nth its derivatives without communication deviates and pretends to offer the full set of pooling contracts while actually offering a secret γ contract. This firm could expect to cancel the policy of any client who also purchased any pooling contract from one of the n+N-1 firms offering these contracts with communication, but it could not identify clients who purchased a one-Nth derivative from any of the remaining N-1 firms selling derivatives without communicating. Low-risk agents preferring γ to

(α^*, β^*) would purchase γ but are rationed at γ. They optimize by purchasing γ and some amount of pooling coverage from the remaining N-1 firms selling one-Nth derivatives and not communicating client information. Therefore, whatever the amount of pooling coverage purchased by low-risk types, the deviation could earn a positive profit if high-risk types do not purchase γ. This requires that after purchasing γ and supplementing it with a policy arbitrarily close to but not including $\frac{N-1}{N}(\alpha^*, \beta^*)$, their combined policy $\gamma^* = \gamma + \frac{N-1}{N}\lambda(\alpha^*, \beta^*)$ is left of hh for all positive $\lambda < 1$, see Figure 1. With γ* left of the budget line hh through h*, by supplementing γ* with high-risk fair priced coverage, high-risk consumers' best total contract would also be left of hh providing less utility than h*. In that case, their optimal response would be to reject γ, purchase (α^*, β^*) from one of the N-1+ n firms communicating such purchases, and then purchase high-risk insurance to remain at h* allowing γ to earn positive expected profit. However, if N is sufficiently large (depriving the deviating N-firm of sufficient monopolistic power), high-risk types can purchase enough derivative pooling insurance from the remaining N-1 non-communicators to supplement to the right of hh from which they can obtain a total policy preferred to h*. Then, the deviation earns negative profit.

A detailed proof showing N can always be large enough to prevent a profitable secret γ deviation is in the appendix. The intuition is as follows. First observe any potentially profitable γ deviation must be contained within the set of policies circumscribed by the boundary p*$\underline{\gamma}$p* in Figure 2. Denote this set of policies Ω. It is bounded by the low-risk indifference curve through p*, the budget line hh through p*, and possibly a portion of the low-risk zero profit line.[7] Define $\underline{\gamma}$ to be the larger of the two policies defined respectively by the lower intersection of the low-risk indifference curve with hh or the indifference curve's intersection with the low-risk zero profit line (hence $\underline{\gamma}$ is independent of γ). Including its boundary, the set Ω is compact with p* = (α^*, β^*) its least upper bound and $\underline{\gamma}$ its greatest lower bound. If a deviation γ is to be profitable, $\gamma^* = \gamma + \dfrac{N-1}{N}(\alpha^*, \beta^*)$ must be contained within Ω so high-risk consumers supplementing γ with pooling insurance cannot reach a policy right of hh. We now show high-risk types can obtain a policy right of hh for large enough N. In Figure 2, starting from γ, high-risk consumers purchase pooling insurance at price $\frac{\beta^*}{\alpha^*}$ along the line parallel to 0p*. Refer to the policy defined by the

[7] As type probabilities converge, hh cuts indifference curve higher shrinking Ω. Diverging probabilities steepen hh to intersect low-risk zero profit line before second intersection with indifference curve, creating a third boundary.

intersection of this line and hh as $h(\gamma) = \gamma + \lambda(\alpha^*, \beta^*)$ for some $\lambda < 1$. At h(γ), high-risk types can purchase supplemental coverage at the high-risk price to reach h* and are therefore indifferent between supplementing (α^*, β^*) or supplementing h(γ). It follows, if, by supplementing γ with $\frac{N-1}{N}(\alpha^*, \beta^*)$ (i.e. trading at price $\frac{\beta^*}{\alpha^*}$), high-risk types can obtain a total policy to the right of hh (i.e. total coverage greater than the indemnity $h_1(\gamma)$ of h(γ)), they will strictly prefer γ plus some obtainable pooling derivative to (α^*, β^*). For large enough N this is always possible. We have,

Lemma 4: Let N* equal the least integer such that $\frac{(\alpha^*, \beta^*)}{N^*} < \underline{\gamma}$. Then, for $N \geq N^*$ *and* γ in Ω ,

$$\gamma_1 + \frac{N-1}{N}\alpha^* > h_1(\gamma).$$

Because of the strict inequality, since consumers can obtain coverage arbitrarily close to $\frac{N-1}{N}\alpha^*$ buying each of the N-1 firm's derivatives, the result follows. See appendix for proof.

The economic intuition behind Lemma 4 requires the number of firms not communicating and selling up to one-Nth the pooling contract be large enough each such firm's market presence is "small." Here small is understood in the perfect competition sense and made precise by the condition $\frac{(\alpha^*, \beta^*)}{N} < \underline{\gamma}$. At equilibrium, each firm's largest non-communicated derivative policy (effectively $\frac{(\alpha^*, \beta^*)}{N}$) is smaller than any feasible deviation from the equilibrium. If one of the N firms deviates, its γ offer has a lower price (premium per coverage) and greater coverage than the policy $\frac{(\alpha^*, \beta^*)}{N}$ it effectively withdraws from the market. Since γ more than compensates for the loss of $\frac{(\alpha^*, \beta^*)}{N}$ all consumers desire γ as part of their total coverage. No firm's market presence is large enough to create a virtual excess demand for pooling derivatives by withdrawing its own pooling offer.

- No communicated γ offer is profitable

Confirming no firm could earn a positive profit by offering an exclusive, indivisible γ contract with client communication requires careful discussion of firm and consumer behaviors at stages two and one respectively. To set this up, we begin with some inferences assumed common knowledge to all agents.

Both high and low-risk agents are rationed at any feasible γ contract and prefer to purchase more coverage at the price $\frac{\beta^*}{\alpha^*}$. Refer to the supplementary pooling coverage demanded by risk type i as $d\left(\frac{\beta^*}{\alpha^*}, \gamma, p^i\right)$ denoted $d^i(\gamma)$ if no confusion arises.

Remark 6. First observe, Since the γ firm chooses to communicate its clients and offers a contract priced $\frac{\gamma_2}{\gamma_1} < \frac{\beta^*}{\alpha^*}$, it will not receive any client lists and cannot cancel contracts of consumers purchasing supplementary insurance. However, due to the γ firm's communication strategy, any supplementary insurance is at risk of cancellation.

Remark 7. Because $d^H(\gamma) > d^L(\gamma)$ and both lie on the zero profit pooling line, firms selling both contracts sell a disproportionate amount of coverage to high-risk agents and earn negative profit.

Firm Behavior at Stage Two

At stage two, each firm selling a pooling contract and in receipt of the list of clients who purchased γ must decide which if any of its contracts to cancel. We discuss optimal firm behavior on each contract.

Firms offering $\lambda \cdot (\alpha^*, \beta^*)$, $0 \leq \lambda \leq 1$ with communication

- Cancelling contracts of clients who appear on the list of γ clients and present claims greater than $d^L(\gamma)$ is a best response. It guarantees these firms zero profit and they can do no better.

Informational constraints restrict cancellation to contracts of clients known to have also purchased elsewhere; sequential rationality requires only cancelling unprofitable contracts. Although, firms would like to cancel the contract of every client who demonstrably violated exclusivity conditions and submits a claim, competition prevents them from earning a positive profit by doing so. If every firm offering these pooling contracts employs the bulleted cancellation strategy, a firm choosing the strategy of unilaterally cancelling contracts of exclusivity violators presenting claims equal to $d^L(\gamma)$ would attract no buyers and therefore would not increase its expected profit. Alternatively, if any firm deviated and honored contracts greater than $d^L(\gamma)$ it would attract every high-risk consumer and earn negative expected profit. Moreover, below we show at stage 1 forward-looking consumers of both types who desire to purchase γ plus a pooling contract optimize by purchasing $d^L(\gamma)$ giving firms zero expected profits. The bulleted cancellation strategy is the only sequentially rational strategy for an active firm.

Firms offering the indivisible (α^*, β^*) *with communication*

- Cancelling contracts of all clients on the γ buyer list who present a claim is a best-response.

In the presence of a γ contract, only high-risk types would purchase (α^*, β^*), so to avoid negative profit firms must cancel contracts of all clients with observable exclusivity violations and presenting a claim. An important implication of the last two bullets is no consumer can attain both (α^*, β^*) and γ.

Firms offering $\lambda\frac{(\alpha^*,\beta^*)}{N}$, $0 \le \lambda < 1$ *without communication*

- Cancelling contracts of clients appearing on the list of γ clients and presenting claims greater than $\frac{d^L(\gamma)}{N}$ is a best response guaranteeing firms zero profit.

If a communicated γ contract is offered, no consumer not purchasing γ would buy these derivatives (such consumers must be high-risk and prefer the primary (α^*, β^*) offered by the same N firms offering the partial derivatives). Moreover, consumers seeking to supplement γ have the option of purchasing $d^L(\gamma)$ from one of the n firms offering the full complement of pooling policies with communication. Therefore, neither risk type buying γ would purchase from firms offering one-Nth pooling contracts unless their combined availability (given firm cancellation rules) were greater than or equal to $d^L(\gamma)$. In the former case, firms would expect negative profit because high-risk types would buy a disproportionate amount, and in the latter case, they could expect at best zero profit. If all firms adopt the cancellation strategy of cancelling all contracts exceeding $\frac{d^L(\gamma)}{N}$, there are two possibilities for consumers. If the firm offering γ is not one of the N non-communicators, consumers can purchase $d^L(\gamma)$ by purchasing N contracts. If the firm offering γ is one of the non-communicators, consumers will only be able to obtain $\frac{N-1}{N}d^L(\gamma)$ and will not buy since $d^L(\gamma)$ is available elsewhere. In both cases, firms earn zero expected profit, and no alternative cancellation rule increases a firm's profit. A firm unilaterally decreasing the size of the maximum contract it honors would lose its clients. A firm increasing the size of its maximum honored contract could only attract additional high-risk clients insuring above $d^L(\gamma)$ and bringing negative profit.

Consumer Behavior at Stage One

Suppose some firm offers a γ deviation with communication. At stage 1, consumers' choice sets include γ, the pooling contract and its derivatives, and the high-risk subprime priced policies. Low-risk type's utility maximizing behavior is to purchase γ and $d^L(\gamma)$. They guarantee themselves this allocation by obtaining $d^L(\gamma)$ from a communicating firm selling pooling insurance up to and including (α^*, β^*) or by purchasing N partial contracts $\frac{d^L(\gamma)}{N}$ from the firms offering up to one-Nth (α^*, β^*) without communicating provided there are still N firms offering these partial contracts. As for high-risk types, we have already noted, firms offering the (α^*, β^*) contract may infer any consumer purchasing it must be high-risk and will cancel the contract of any client appearing on the γ list and presenting a claim. Therefore, if high-risk types purchase (α^*, β^*) they will not purchase γ depriving sellers of (α^*, β^*) of an exclusivity violation and excuse for cancelling the contract of clients presenting claims at stage two. This means the minimum utility high-risk agents must receive is attained by purchasing (α^*, β^*) and supplementing to their equilibrium allocation h* in Figure 1. If high-risk types find this optimal, only low-risk types will purchase γ allowing it to earn positive expected profit upsetting the proposed equilibrium. However, high-risk agents are also able to buy γ and supplement it with pooling insurance either from one of the communicating firms selling the complete set of pooling contracts or by purchasing several pooling derivatives from the non-communicating firms. The former firms communicate client identities to all sellers of pooling contracts. Therefore, high-risk types can only choose one of these two purchase routes. Moreover, if high-risk agents buying γ reveal their risk type by purchasing $d^H(\gamma)$ from one of the communicating firms offering (α^*, β^*) and its derivatives, they will see their contracts cancelled at stage two should they turn in a claim. Therefore, if high-risk agents' buying γ purchase from one of the communicating firms, their optimal strategy is to mimic low-risk agents and purchase $d^L(\gamma)$. Through similar reasoning, high-risk types may also purchase $d^L(\gamma)$ from the firms offering up to one-Nth (α^*, β^*) without communicating provided their combined (post cancellation) offering allows them to attain $d^L(\gamma)$. This means high-risk types must choose between $\gamma + d^L(\gamma)$ (possibly plus subprime coverage) and (α^*, β^*) + subprime coverage. If the equilibrium is to be sustained, high-risk consumers must prefer to mimic low-risk consumers by purchasing γ + $d^L(\gamma)$.

Lemma 5: High risk types strictly prefer purchasing γ + $d^L(\gamma)$ *to* (α^*, β^*)*.*

To show high-risk types strictly prefer to purchase γ + $d^L(\gamma)$ to purchasing (α^*, β^*), recall any prospective γ deviation must lie below the low-risk indifference curve through (α^*, β^*) and on or above the budget line (now labeled h) representing the locus of zero profit high-risk contracts through h* and (α^*, β^*), see Figure 2. A consumer can supplement any such γ with pooling insurance (α, β) to obtain a total insurance policy on the budget line through γ defined by $\beta = \gamma_2 + \frac{\beta^*}{\alpha^*}(\alpha - \gamma_1)$ for $\alpha \geq \gamma_1$. Low-risk type's total purchase $\gamma^* = \gamma + d^L(\gamma)$ maximizes utility on this budget line, and a low-risk indifference curve is tangent to the budget line at γ^*. For varying γ, the locus of tangency points for parallel budget lines (low-risk type's income-consumption curve for the constant price $\frac{\beta^*}{\alpha^*}$) is depicted as AA in Figure 2. This curve passes through (α^*, β^*) when $\gamma = (\alpha^*, \beta^*)$ and $d^L(\alpha^*, \beta^*) = (0,0)$. Because γ^* is constant for all γ lying on the same budget line, and (α^*, β^*), and (0,0) are on the same budget line, the curve also passes through (α^*, β^*) when $\gamma = (0,0)$ and $d^L(0,0) = (\alpha^*, \beta^*)$. In (α, β) space, constraining β to a γ budget line, the income consumption curve is defined implicitly by low-risk type's FOC;

$$p^L u_0'(w - d + \alpha) - (1 - p^L) u_1'(w - \beta) \frac{\beta^*}{\alpha^*} = 0.$$

Differentiating this condition implicitly (and observing from FOC that $\frac{p^L}{1-p^L} = \frac{u_1'}{u_0'} \cdot \frac{\beta^*}{\alpha^*}$) shows the slope of AA, $\frac{d\beta}{d\alpha} = -\frac{A_0}{A_1}$ where A_i equals the Arrow-Pratt measure of absolute risk aversion computed for income state i. Therefore, the income-consumption-curve has a negative slope and must cut the positively sloped budget line labeled h through (α^*, β^*) from above. Thus, for any feasible γ contract, the corresponding low-risk type demand point $\gamma + d^L(\gamma)$ on AA lies right of the budget line labeled h allowing high-risk consumers mimicking the low-risk to supplement $\gamma + d^L(\gamma)$ by purchasing high-odds insurance up some budget line such as the one labeled h' in Figure 2 to a total contract preferred to h*. Since both risk types purchase $d^L(\gamma)$ and it lies on the zero profit pooling line, the firms selling this contract all earn zero expected profit. But, since the firm selling the lower price γ sells to the same proportion of high and low-risk clients as firms selling $d^L(\gamma)$, the γ seller must earn negative expected profit. No communicated γ deviation breaks the equilibrium. This demonstrates the J configuration is an equilibrium for the two-stage framework.

Features of Proposition 1 are based on the assumption low-risk types most preferred pooling policy is positive. If the optimal pooling policy is the null contract, it follows immediately from Proposition 1:

Corollary to Proposition 1: Suppose $(\alpha^*, \beta^*) = (0, 0)$, then for firm beliefs π = 0, the J configuration described in 1-2 below is a PBE for the two-stage frame.

1. $m \geq 2$ firms offer the set of policies $\lambda \cdot (\alpha, q^H \alpha)$, $\alpha \geq 0, for\ any\ \lambda \geq 0$ with no exclusivity conditions.

2. These m firms promise not to communicate their client lists to any firm.

Remark 8: Under the conditions of the corollary, the aggregate set of policies available to consumers is still the J-configuration: given this set of policy offers and communication strategies, low-risk consumers are allocated their optimal pooling policy (α^*, β^*), (thus, buy no insurance) while high-risk consumers supplement (α^*, β^*) with the full-coverage contract $(\alpha^{H*}, q^H \alpha^{H*})$ priced at high-risk fair odds.

A recent paper by Laurence Ales and Pricila Maziero (2011) with an interesting discussion of public policy implications derived the separating equilibrium of our corollary under the exogenous assumptions that low-risk types prefer their endowment to every strictly positive zero profit pooling policy, and firms share no client information allowing consumers multiple purchases. The corollary shows their result is a special case of the J configuration. Once an assumption implying low-risk types will purchase no positive pooling policy is made, equilibrium with the J configuration follows and firms' non-communication strategies can be derived endogenously and need not be assumed.

A related point concerns the nature of strategic communication itself. Some authors seem to imply it means information will be shared (but not necessarily unanimously). But the corollary shows the endogenous flow of information through strategic communication is flexible and depending on underlying market conditions can result in complete non-communication or a mix of communication strategies. Proposition 1 and its corollary demonstrate conclusively that equilibrium in the adverse selection model is strictly a consequence of strategic communication. Indeed, under conditions inducing all firms not to communicate privately held information, although equilibrium still exists, there is a lemon effect and Pareto improving trades that would be possible were more information shared do not occur.

Remark 8: Although the J configuration is the unique aggregate contract set supporting equilibrium, multiple firm strategy profiles can provide it.

For example, the N firms offering one-Nth (α^*, β^*) without communication could be separated from firms offering (α^*, β^*) as an indivisible contract with communication.

Rules of the Game: Public Versus Private Information

The adverse selection insurance model founds the notion of imperfect information on the asymmetry of information about consumer risk-types. Informed consumers trade with firms uninformed about consumers' risk type which is therefore private information. The remaining model determining aspects of the information structure was implicitly assumed in R-S-W. The set of offered contracts, identity of a firm's list of clients (hence firm's contract offer), and each firm's identical communication strategy divulging its clients to all firms were each public information. Jaynes retained the informational assumptions of R-S-W except for the assumption that a firm's client list should be considered private information divulged strategically to other firms only if doing so is profitable. In his 2-stage game, Hellwig retained Jaynes' assumptions about the public/private classification of firm clients and communication strategies but altered the assumption concerning a firm's contract offer assuming an offer could be either private or public information (or in effect with two offers both) at the discretion of the firm. We have shown the assumption firms can offer secret contracts produces a kind of hybrid model inducing features common to two models, one assuming exogenous unanimous communication and exclusivity (R-S-W) the other complete non-communication and unfettered multiple contracting where N firms each constrain consumers to one-Nth of the same contract see (Bisin & Guatoli, 2004; Attar and Chassagnon, 2009). Complete non-communication with multiple contracting models assume clients' total purchases are unobservable, they do not assume unobservable contracts. There are good reasons for this.

The assumption a firm can offer a secret contract involves the underlying model in a logical conundrum. Earlier, see footnote 3, we suggested a firm offering both a pooling contract with full communication (in order to deceive competitors and receive their client lists) and a secret γ contract clearly has no intention of selling the pooling contract. Since the γ contract is chosen to be preferred by low but not high risk agents, the firm's pooling contract could only be sold to high-risk types and would earn negative profit. Moreover, since the γ contract is smaller, its positive profit could not cover the losses on the pooling contract so the combined profit would also be negative. Therefore the secret γ deviation also adds another implicit assumption, firms may unilaterally refuse to sell agents a contract they are offering. Regardless what one thinks of this implied assumption itself, it leads to questioning how reasonable it is to assume a competitive firm selling to large numbers of consumers could keep its identity and/or contract offers secret from competitors. Although, suspect too, one might argue low-risk agents who contract for γ might have an incentive to keep the firm's offer and identity secret, it is simply not tenable to assume high-risk agents turned away after refusing the bait and switch γ offer would do so.

Our view is it is not reasonable to assume the identity of a firm offering a contract is known to would be clients turned away and forced to seek other firms, but unknown to those other firms. Moreover, a firm fabricating a pooling offer must also be assumed to either send its competitors either no client information or falsified information-- it might plan to send its γ clients' names representing them as purchasers of (α^*, β^*). Our view is the assumption of secret contracts should be dropped. The important contribution of the sequential structure (unprofitability of a communicated γ contract) does not even require it anyway. If firms are not assumed to be able to offer secret contracts, equilibrium no longer requires there be N firms offering partial pooling contracts without communication. And, the equilibrium is sustained by symmetric communication strategies among firms offering the complete set of pooling contracts.

Appendix

Lemma 4: Let N* equal the least integer such that $\frac{(\alpha^*, \beta^*)}{N^*} < \underline{\gamma}$. Then, for $N > N^*$ *and* γ in Ω,

$$\gamma_1 + \frac{N-1}{N}\alpha^* > h_1(\gamma).$$

Proof: First observe any potentially profitable γ deviation must be contained within Ω the set of policies bounded by the low risk indifference curve through p* and the hh budget line through h* and p* (and possibly a section of the low-risk fair profit line). We note for every γ properly in Ω, $(\alpha^*, \beta^*) > \gamma$. Furthermore, for every such γ, the policy $\underline{\gamma}$ defined by the greater of the low-risk indifference curve's first intersection with the hh budget line or the indifference curve's intersection with the low-risk zero profit line satisfies $\underline{\gamma} < \gamma$. Now for γ in Ω let h(γ) equal the policy defined by the intersection of the budget line hh with slope q^H and the budget line with slope $\frac{\beta^*}{\alpha^*}$ through γ, see Figure 2. Equilibrium requires, for every γ in Ω, $\gamma + \frac{N-1}{N}(\alpha^*, \beta^*) > h(\gamma)$. Since the budget line hh has positive slope through (α^*, β^*), for all γ, $h(\gamma) < (\alpha^*, \beta^*)$, so $\gamma + \frac{N-1}{N}(\alpha^*, \beta^*) > (\alpha^*, \beta^*)$ is sufficient for the result. By the hypothesis:

$$\underline{\gamma} > \frac{(\alpha^*, \beta^*)}{N^*} \Longrightarrow \underline{\gamma} + \frac{N^*-1}{N^*}(\alpha^*, \beta^*) > \frac{(\alpha^*, \beta^*)}{N^*} + \frac{N^*-1}{N^*}(\alpha^*, \beta^*) = (\alpha^*, \beta^*) > h(\underline{\gamma}).$$

Then, $\gamma > \underline{\gamma}$ leads to same result for any γ in Ω and the conclusion follows.

Figure 1

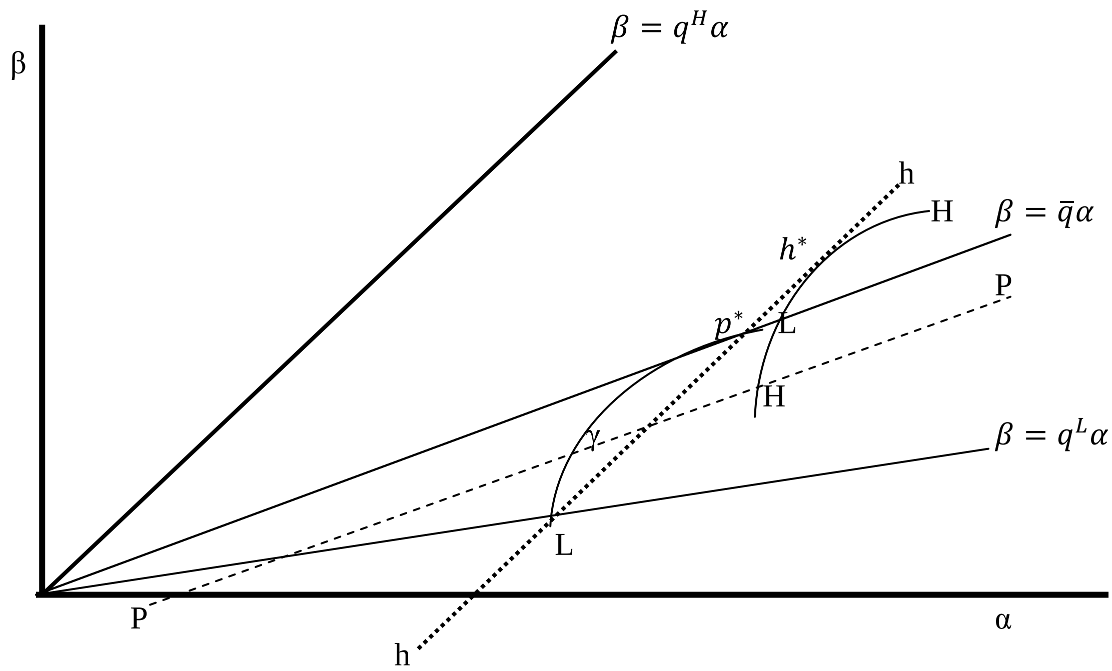
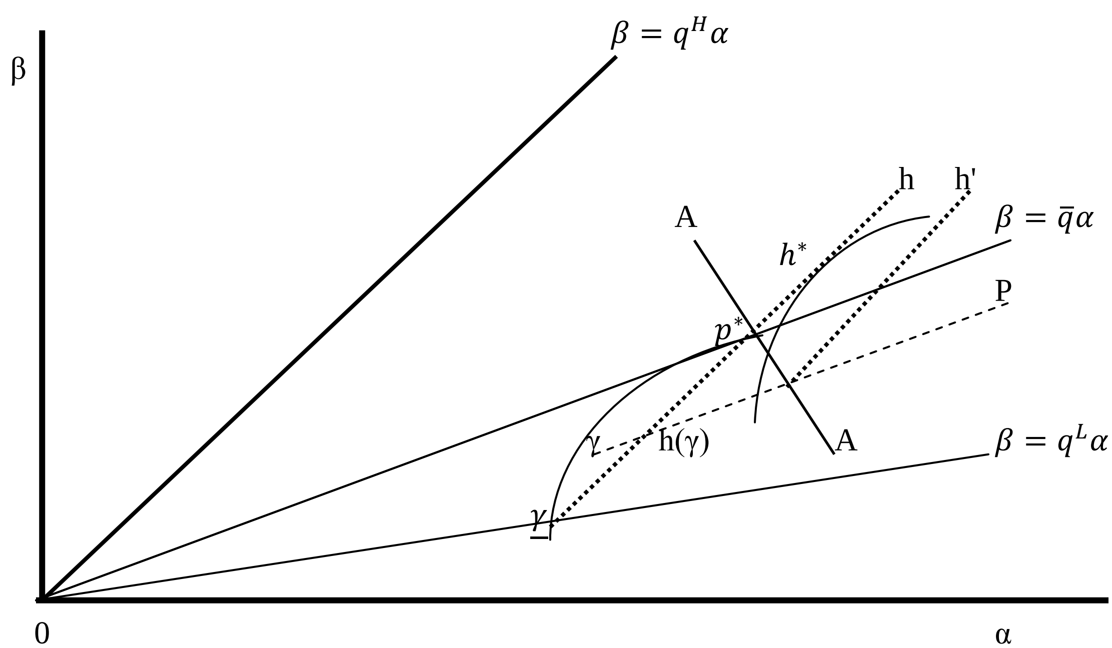


Figure 2



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