

Assortative Matching

In a marriage market the competition for spouse leads to sorting of mates by characteristics such as wealth, education, and other characteristics. Positive assortative matching refers to a positive correlation in sorting between the values of the traits of husbands and wives (matching of likes); negative assortative matching refers to a negative correlation (matching of unlikes). While it has been long recognized that sorting of husbands and wives by characteristics occurs in all cultures and societies, economists have tried to understand sorting patterns in the marriage market and other matching markets by focusing on the nature of the gain from match and the mechanism of the market force of competition.

A simple framework to illustrate the economic approach to sorting in matching markets is a two-sided marriage market with an equal number of men and women, who differ in one-dimensional characteristics called “type” and have common preferences for higher types over lower types. In positive assortative matching, the highest type man mates the highest type woman, and the second highest type man mates the second highest type woman, and so on. Negative assortative matching is the highest type man with the lowest type woman, and the second highest type man with the second lowest type woman, and so on. We assume transferable utility and zero reservation utility from remaining single for each market participant. Then, the gain from match can be represented by an increasing, positive-valued function f , which gives the match output $f(x, y)$ of any pair of type x man and type y woman. Consider two men, with types $x_H > x_L$, and two women, with types $y_H > y_L$. If type x_H and type x_L command the same price in terms of the utility transfer they demand from the wife for the match, then both type y_H and type y_L would prefer the higher type man because f is increasing in male type. Competition for type x_H naturally leads to a higher price for type x_H than for type x_L . Whether the higher female type y_H can outbid type y_L for type x_H or vice versa depends on whether male type and female type are complements or substitutes in the match output function f . If

$$f(x_H, y_H) - f(x_H, y_L) > f(x_L, y_H) - f(x_L, y_L), \quad (1)$$

then male type and female type are complementary, because the marginal product of female type is greater when matched with a higher male type (the left-hand-side of inequality 1) than with a lower male type (the right-hand-side of 1). In this case, type y_L is willing to offer type x_H at most $f(x_H, y_L) - f(x_L, y_L)$ more than what she offers type x_L , but by inequality (1) this difference is smaller than $f(x_H, y_H) - f(x_L, y_H)$, which is the most type y_H is willing to offer. Thus, type y_L will be outbid by type y_H for type x_H when male type and female type are complements. Since the argument is valid for any two pairs of men and women, the competition for spouse must lead to positive assortative matching. Conversely, if inequality (1) is reversed, male type and female type are substitutes. A lower female type can outbid a higher type for any male type, and the competition for spouse leads to negative assortative matching.

The differentiable version of inequality (1) is

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} > 0. \quad (2)$$

Conditions (1) and (2) are commonly referred to as the (strict) “supermodularity” condition of the match output function f . See Topkis (1998) for a comprehensive mathematical treatment of supermodularity, and Milgrom and Roberts (1990) and Vives (1990) for applications in game theory and economics.

Inequality (1) can be rewritten as

$$f(x_H, y_H) + f(x_L, y_L) > f(x_L, y_H) + f(x_H, y_L). \quad (3)$$

Condition (3) suggests that positive assortative matching maximizes the sum of match outputs in the marriage market when male type and female type are complements in the match output function. This result is a direct application of Koopmans and Beckmann’s (1957) theorem of equivalence between the efficient matching, which maximizes the sum of match outputs among all feasible pairwise matchings, and the competitive equilibrium matching, which obtains when each woman y takes as given a schedule of utility transfers $u(x)$ to men and chooses the male type that maximizes her utility. The competitive equilibrium matching can also be obtained as each man x takes as given a schedule of utility transfers $v(y)$ to women and chooses the female type that maximizes his utility. Shapely and Shubik (1972) model the marriage market with transferable utilities as a cooperative game. They show that a pair of transfer schedules that support an equilibrium matching corresponds to the core of the game, so that no pair of a man and a woman not matched in equilibrium can form a blocking coalition that produces a match output greater than the sum of their respective transfers.

The results of Koopmans and Beckmann (1957), and Shapley and Shubik (1972) are obtained in a matching market without any hierarchical ordering of types. By introducing one-dimensional, heterogeneous types, Becker (1973) seeks to explain why sorting of mates by wealth, education and other characteristics is similar in the marriage market. He constructs a household production function and derives condition (1) for each of the characteristics separately by considering how the characteristic affects the household output while holding other characteristics fixed. Becker’s model can accommodate dissimilar sorting of mates by some characteristics as well; for example, negative assortative matching by wage rates may arise because the benefits from the division of labor within a household can make the earning abilities of the man and the woman substitutes for each other.

Sattinger (1980) uses condition (2) to explain why the distribution of earnings of workers is skewed to the right relative to the distribution of their measured skills. In a market that matches a continuum of workers with different skills to a continuum of positions of different capital investment, the distribution of earnings would have the same shape as the distribution of skills if matching is random. In Sattinger’s theory of differential rents, positive assortative matching of worker skill and job capital investment occurs because skill and capital investment are complements. In this case, the distribution of earnings will not resemble the distributions of outputs at a job with the average capital investment. Instead, workers with higher skills are paid more than those with lower skills both because they are more productive at any job and because they occupy positions with greater capital investments. Formally, in equilibrium the wage schedule u satisfies the first order condition of type y ’s maximization problem of choosing x to maximizing $f(x, y) - u(x)$

$$\frac{\partial f(x, m(x))}{\partial x} = u'(x),$$

where $m(x)$ is the capital investment of the job occupied by the worker with skill x in equilibrium. It can be shown that condition (2) and positive assortative matching imply that $f(x, y) - u(x)$ is concave in x at $x = m^{-1}(y)$, so the second order condition is satisfied for each y . The first order condition implies that worker's wage increases at the rate of marginal product of the worker's skill x at his equilibrium job, so that the rate of increase of u is augmented by the complementarity (condition 2) and positive assortative matching ($m'(x) > 0$). Thus, with positive assortative matching, the distribution of earnings will therefore be positively skewed relative to the distribution of skills.

Kremer (1993) highlights the role of positive assortative matching in economic development. In his model of one-sided, many-to-many matching market, each firm consists of a fixed number of workers each employed for a production task. Workers have different skills, with a higher-skilled worker less likely to make mistakes in performing his task. Condition (1) is assumed to capture the complementarity among worker skills that the production process of a firm requires completion of each task without mistakes. Self-matching obtains in equilibrium where each firm employs workers of identical skills. Kremer uses this form of positive assortative matching to explain the large wage and productivity differences between developing and developed countries that cannot be accounted for by their differences in levels of physical or human capital.

Self-matching will generally be inefficient and will not occur in equilibrium if production tasks in a firm differ in skill requirements. In Kremer and Maskin (1996), a firm consists of two workers with a match output function $f(x, y)$ that satisfies the supermodularity conditions (1) and (2) but is asymmetric in that $f(x, y) > f(y, x)$ for any $x > y$. The interpretation of the asymmetry is that the first argument in f represents the skill of the worker who does the manager's job while the second argument represents the skill of the worker who performs the assistant's job. In any given firm, it is optimal to make the higher-skilled worker the manager and the lower-skilled worker the assistant, but it is no longer generally true that self-matching maximizes the total match outputs. Indeed, we can have

$$2f(z_H, z_L) > f(z_H, z_H) + f(z_L, z_L) \quad (4)$$

for some $z_H > z_L$, so that two firms each with the higher type z_H as the manager and the lower type z_L as the assistant produce more in total than two firms with the manager and the assistant having the same skill level. Note that inequality (4) does not contradict inequality (3) due to the asymmetry in f . Mixed matching may do better than self-matching because it can be more important to exploit the asymmetry in the match output function and have each high-skill worker as the manager of a firm than to exploit the complementarity in f and have one high-skill worker as the assistant to the other high-skill worker. Kremer and Maskin find that the efficient matching in their model depends on the skill distribution in the matching market, because the trade-off between the asymmetry and the complementarity in the match output function depends on the relative scarcity of high-skilled workers.

Assortative matching may be hindered by the presence of frictions in the matching market. For example, if there is a moral hazard problem in producing the match output by each matched pair, transferability of utilities will be restricted by incentive compatibility constraints. Legros and Newman (2002) discuss this and other examples of transaction

costs, and find that equilibrium matching in these examples can be inefficient. Frictions can also arise due to incomplete information about type. Roth and Xing (1994) provide detailed descriptions of labor markets for entry-level professionals (e.g., lawyers, medical interns) in which early matches are sometimes made before complete information about matching characteristics such as qualifications of job candidates and desirability of job positions becomes available. The complementarity in the match output function between the type of the applicant and the type of the job implies that there will be matching efficiency loss if matches are formed before the uncertainty about types is resolved. If all market participants are risk-neutral, this efficiency loss is sufficient to rule out early matches as applicants compete for job positions. However, when some participants are risk-averse, early matches provide them with some insurance against the payoff risks associated with late matches formed after complete information about types becomes available. Li and Suen (2000) apply competitive equilibrium analysis to the early matching market to determine the pattern of early matching, the terms of early matches, and the distribution of benefits in the early market. Early matching need not be positive assortative in terms of expected type. Higher expected types of workers may face greater payoff risks from late matches due to the complementarity in the match output function. In this case, they may be willing to match with lower expected types of jobs to insure against the risks, while owners of higher expected types of jobs are content with waiting for late matches if they are risk-neutral.

Private information about type may also result in frictions in the matching market. For example, many users of Internet dating agencies complain about the problem of misrepresentation and exaggerations by some users in the information they provide to the agencies. This problem arises because current matching services adopt the uniform pricing policy, and this in practice results in almost random matching. Damiano and Li (2004) point out that the complementarity in the match output function implies a version of the standard single-crossing condition in mechanism design problems, and an intermediary can use price discrimination to improve matching efficiency and generate greater revenue. They consider the problem of a monopoly matchmaker that uses a pair of fee schedules to sort different types of agents on the two sides into exclusive meeting places. The revenue-maximizing sorting need not be positive assortative (i.e., efficient in the first-best sense). Conditions necessary and sufficient to recover positive assortative matching require that the complementarity in the match output function to be sufficiently strong to overcome the incentive cost to the matchmaker of eliciting private type information.

Matching frictions can arise also because finding type information about potential partners takes time or involves costly effort. In the search and matching framework, each market participant randomly meets a currently unmatched agent from the other side of the market, decides whether to form a match or to search again in the next period. Search is costless, but agents must trade off the benefit from starting to produce with the encountered partner right away and the opportunity cost of waiting for a better partner. With an exogenous probability of separation of matched agents who then reenter the market, Shimer and Smith (2000) characterize the stationary search and matching equilibrium where the matching decisions of each type and the type distributions of unmatched agents are time-invariant. Types x and y in an agreeable match are assumed to use the Nash

bargaining solution to split the net surplus, defined as the match output $f(x, y)$ minus sum of the (endogenous) continuation payoffs $g(x)$ to x and $h(y)$ to y as unmatched agents. Shimer and Smith modify the definition of positive assortative matching in the frictionless world to allow for set-valued mutually agreeable matches. The match set of a type x is the intersection of the set of types that type x agrees to match with and the set of types that agree to match with x . In Shimer and Smith's definition, matching is positive assortative if for any male types $x_H > x_L$ and female types $y_H > y_L$ such that y_H is in the match set of x_L and y_L is in the match set of x_H , then y_H is in the match set of x_H and y_L is in the match set of x_L . When match sets are convex, positive assortative matching requires the lowest and the highest type of the match set to be increasing in x . However, match sets need not be convex even though the match output function is supermodular. This is because the net surplus $f(x, y) - g(x) - h(y)$ is not necessarily quasi-concave in y for fixed x , so one cannot say anything about how match sets vary across different x . Shimer and Smith provide conditions on f in addition to supermodularity to ensure convexity of match sets and reestablish positive assortative matching in a stationary equilibrium.

The stationary search and matching equilibrium does not capture the dynamics of matching in markets where there is no entry of a new cohort in each period and each matched pair receives their match output after the market closes for all participants. For example, many entry-level markets for professionals (e.g., academic economists) are organized around annual recruitment cycles. In these markets, matches are formed sequentially without centralized matching procedures. Damiano, Li and Suen (forthcoming) consider such markets by constructing a two-sided, finite-horizon search and matching model with heterogeneous types and complementarity between types. The quality of the pool of potential partners deteriorates as agents who have found mutually agreeable matches exit the market. When search is costless and all agents participate in each matching round, the market performs a sorting function in that high types of agents have multiple chances to match with their peers. The matching efficiency measured by the total expected match outputs improves as the number of matching rounds increases; positive assortative matching is achieved if there are as many matching rounds as are types. However, this sorting function is lost if agents incur an arbitrarily small cost in order to participate in each round. With a sufficiently rich type space relative to the number of matching rounds, the market unravels as almost all agents rush to participate in the first round and match and exit with anyone they meet.

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See also **assignment problems, matching markets.**

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