Storable good monopoly: the role of commitment

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PRELIMINARY DRAFT

Abstract

We study dynamic monopoly pricing of storable goods in an environment where demand changes over time.

The literature on durables has focused on incentives to delay purchases. Our analysis focuses on a different intertemporal demand incentive for consumers: the key force on the consumer side is advance purchases or stockpiling. This stockpiling motive can be present in the case of standard durables as well, in environments unmodeled in the prior literature.

We show that if the monopolist cannot commit, then prices are higher in all periods, and social welfare is lower, than in the case in which the monopolist can commit. This is in contrast with the durable goods analysis in the literature on the Coase conjecture.

1 Introduction

According to most standard models in industrial organization a change in the price of a good at some future date has no effect on current incentives for producers because current consumer behavior is only affected by current prices. However, a large fraction of production involves goods for which intertemporal demand incentives may play a large role. A large literature, considers one important consequence of durability. Durability can generate a special kind of intertemporal demand incentives, namely the incentive to postpone purchases in the expectation of better deals in the future. This emerges most starkly in the Coase conjecture (Coase 1972, Gul, Sonneschein, and Wilson 1986) where, under certain circumstances,

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durability and intertemporal demand incentives combine to generate a striking contrast with textbook monopoly analysis: if consumers are patient, or transactions can occur quickly, the power of the monopolist to extract surplus is completely undermined, and the monopoly distortion disappears. In other environments with durable goods and intertemporal demand incentives, (e.g., Sobel 1991) durability does not lead to such extreme outcomes. However, a consistent picture that has emerged from this literature: when goods are durable, and intertemporal demand incentives are taken into account, by undermining monopoly power, lack of commitment leads to lower prices and higher welfare.

The incentive to delay purchases is not the only relevant intertemporal demand incentive. In particular, there are several goods for which there can be incentives to stockpile in anticipation of higher prices. Some recent empirical literature (in particular, Erdem, Keane, and Imai (2002) and Hendel and Nevo 2004, a and b) has found evidence that the timing and size of purchases by consumers responds to the timing of price changes in a quantitatively important way that must be attributed at least in part to stockpiling motives.[1] This evidence pertains mostly to groceries, but there are a large number of other goods for which it is at least plausible to think that stockpiling motives may be important (oil, various intermediate goods ...).

This paper shows that once we consider environments where there are incentives to stockpile, the picture about the effect of commitment on the exercise of market power can be very different from the one that has been presented in the previous literature related to the Coase conjecture. We provide an analysis of monopoly pricing in an environment where goods are storable and consumers have incentives to stockpile. We characterize price dynamics when demand varies deterministically over time. We consider both the case in which the monopolist commits to a sequence of prices and the case in which the monopolist does not commit. We show that under certain conditions, prices are uniformly higher (in all periods), and that welfare is lower when the monopolist cannot commit; commitment leads to a Pareto-improving outcome: higher monopoly profits, higher consumer surplus and a reduction of wasteful storage.

The analysis of this paper may lead to a more cautious evaluation of contracts that enhance a firm's commitment ability: the policy advice that emerges from the literature on the Coase conjecture is to be suspicious of any contractual arrangements, such as rental or leasing contracts, that enhance commitment since these may restore monopoly power, and lead to higher prices and lower welfare. In contrast, in our model, enhancing a monopolist's ability to commit may lead to lower prices and reduce wasteful storage.

We have focused most of our analysis on storable goods that have the feature that they are perishable in consumption but can be stored for future consumption (a classic example

[1]Hendel and Nevo point out that accounting properly for purchases due to stockpiling is important for distinguishing between short-run and long-run elasticities.

See also Aguirregabiria (1999), Pesendorfer (2002), and Rust (2002).

is laundry detergent). This provides a particularly stark scenario because the *only* intertemporal demand incentive is stockpiling. However, stockpiling incentives can be present in durables as well. Incentives for advance purchase can be generated by demand seasonality. We show that even in the case of durables there are plausible environments in which lack of commitment can lead to higher prices. Furthermore, in a rich durable good environment both incentives (for demand postponement and advancement) might be present at different times. In those situations the overall effect of commitment on prices and welfare is more difficult to assess since it depends on the exact nature of the cycle. However, in such rich environments with demand fluctuations, which are common in many markets, Coase's stark predictions are likely to be significantly altered.

To gain an initial intuition for our main result, consider a two period problem. Suppose that the marginal cost of storage is constant and is smaller than the difference between the static monopoly prices. If the monopolist charged the static monopoly price in each period, consumers would have an incentive to purchase more in the first period and stockpile for second period consumption. In the equilibrium with commitment the monopolist counters this consumer incentive by announcing current and future prices such that the difference between prices is small enough to ensure that consumers do not stockpile. The intuition is that the monopolist makes sure that consumers pay at the second period what that they would otherwise spend for storage. Subject to the requirement that the difference in prices is equal to the marginal cost of storage, the monopolist chooses in the first period the price at which the sum of the marginal revenues is equal to zero.

Suppose now that the monopolist lacks commitment. In equilibrium the difference between the two prices is still equal to the marginal cost of storage. Yet, if consumers did not store in the first period the monopolist would charge the static monopoly price in the second period. Hence, in equilibrium consumers stockpile in order to reduce future demand to a point where the monopolist will have no incentive to charge a high price. In the first period the monopolist chooses the price that equates the sum of marginal revenue to the marginal loss in future profits due to an increase in stockpiling. The first period price (and consequently the second period price) is then higher than under commitment.

The key effect is that, absent commitment, in the second period, the monopolist does not take into account the fact that an increase in the second period price increases storage, thereby shifting sales to the first period when prices are lower. Thus, every increase in the second period price leads to a loss in profit margin proportional to the difference between second and first period prices. When the monopolist can commit, this effect is taken into account, leading to lower prices, higher profits, lower storage, and higher welfare.

2 Related Literature

Anton and Das Varma (2003) study a two period duopoly model in which consumers can store first period purchases. They study the impact of storability on the intertemporal price path. The main result is that prices increase (if consumers are patient and storage affordable). The low initial prices are a consequence of the firms' incentive to capture future market share from their rival. In contrast to the duopoly case, the demand shifting incentives do not show up under monopoly or competition. Under these market structures there is no incentive to capture future market share, so the price dynamics are absent.

Several theoretical papers offer models of price dispersion (Varian (1980), Salop and Stiglitz (1982), Narasimhan (1988) and Rao (1991)), interpreted as sales, however, these are competitive models and they do not capture the dynamics of demand generated by sales. Hong, McAfee and Nayyar (2000) is a competitive industry model, where consumers are assumed to chose a store based on the price of a single item can store up to one unit.

There is a vast literature on durable goods.[2] The literature that is most related to our paper is the one on the Coase conjecture. This started with a paper by Coase (1972). Bulow (1982) and Gul, Sonnenschein, and Wilson (1986) are two of the early papers that provided a formal analysis of Coase's conjecture. These papers discuss models in which a good is perfectly durable, consumers have unit demands, and differ in their valuations for the good. The Coase conjecture states that the seller's ability to extract surplus from a buyer can be completely undermined by an inability to commit not to make more attractive offers in the future. Specifically, if buyers are very patient, or offers can be made very quickly, the seller will, in equilibrium, offer very low prices from the beginning of the game. In the limit, the initial price (and hence the profits of the seller) converge to the lower bound of the valuations of the buyer, and almost all consumers purchase almost immediately. This implies that the equilibrium is asymptotically efficient.

Sobel (1991) (see also Conlisk, Gerstner, and Sobel 1984, Sobel 1984, and Board 2004) describes a model of a market with a durable good monopolist in which, at every date a mass of new consumers enter. Consumers have unit demands and two possible valuations for the good. Sobel (1991) characterizes the set of equilibria under the assumption that the monopolist cannot commit. Board (2004) assumes that the monopolist commits and allows for a more general time path of entry of consumers. An important feature of the analysis in this strand of the literature is the possibility of price cycles, namely sales. There are several dimensions in which our analysis differs from this literature. In particular, in those papers, inability to commit reduces prices and raises welfare. In our model the opposite is true.

Jeuland and Narasimhan (1985) present a model in which storability may allow a monopolist to price discriminate among consumers. Their analysis is based on a negative correlation between demand and cost of storage. This is not very appealing because it is not

[2] For a recent survey see Waldman (2003).

clear why high demand consumers should find it harder to store goods, in fact, if storability is chosen endogenously, as discussed below, the reverse should be true. Furthermore, in their model consumer behavior is not fully optimal.

3 Example

We first present a simple example to give an initial intuition for the basic forces at play in this environment. The next section considers the general environment.

Assume that there are two periods: the first period is a low demand period: $D_1(p) = 1-p$. The second period is a high demand period: $D_2(p) = 2 - p$. Assume that a consumer can store between periods 1 and 2 at a cost of $c(s) = cS$ when he stores S units. Note that absent storage, the optimal solution is $p_1 = \frac{1}{2}$, $p_2 = 1$. If $c > \frac{1}{2}$ this solution is sustainable because at those prices no consumer would choose to store. Assume that costs of production are zero.

In the remainder of this section we assume that $c < \frac{1}{2}$.

3.1 Commitment

Consider the following maximization problem.

$$\begin{aligned}\max_{p_1,p_2} \pi(p_1, p_2) &= (1 - p_1 + S) p_1 + (2 - p_2 - S) p_2 \\ s.t.\ p_2 - p_1 &= c\end{aligned}$$

We will show that the solution to this problem, with $S = 0$ is the monopoly solution under commitment

The solution of this problem is

$$\begin{aligned}p_1^c &= \frac{3}{4} - \frac{1}{2}c \\ p_2^c &= \frac{3}{4} + \frac{1}{2}c\end{aligned}$$

At these prices consumers are indifferent between storing and not storing. However, in equilibrium, storage must be zero. To see this, observe first that profits are decreasing in S: if the consumer stores less he consumes the same amounts and pays the same total cost but buys more at p_2 instead of p_1 thus the monopolist captures c more for every reduction in S. However, the monopolist can induce zero storage by reducing p_2 slightly below $\frac{3}{4} + \frac{1}{2}c$ while keeping $p_1 = \frac{3}{4} - \frac{1}{2}c$. Thus, it must be the case that storage is zero in an equilibrium with commitment.

3.2 No commitment

Note first that the commitment solution cannot be an equilibrium without commitment. To see this, assume that $p_1 = \frac{3}{4} - \frac{1}{2}c$, and that $S = 0$. Then, in the second period the monopolist will charge the static monopoly price $p_2 = 1$. Furthermore, for $c < \frac{1}{2}$, we then have $p_2 - p_1 > c$ implying that S cannot be zero. Thus, absent commitment by the monopolist, in making their storage decisions, consumers must take into account the fact that second period prices depend on the storage level. The monopolist has the incentive to raise prices higher in circumstances in which storage is low.

Let us then construct an equilibrium without commitment.

Suppose that all consumers have stored $S \geq 0$ units in period 1. Then, in equilibrium, the monopolist will choose the second period price to maximize

$$V_2(p_2, S) = (2 - p_2 - S)\, p_2.$$

The solution is

$$p_2(S) = 1 - \frac{1}{2}S. \tag{1}$$

Given p_1, in equilibrium, if S is interior, it must be the case that $p_2(S) = p_1 + c$ so that consumers are indifferent between storing and not storing. Thus, in equilibrium, we must have:

$$p_2(S) = p_1 + c \tag{2}$$

implying that

$$S(p_1) = 2 - 2p_1 - 2c \tag{3}$$

and that second period profits are $V_2(p_1) = (p_1 + c)^2$.

Thus, the present value of period profits in the first period is:

$$V_1(p_1) = (1 - p_1 + 2 - 2p_1 - 2c)\, p_1 + (p_1 + c)^2.$$

which are maximized by

$$p_1^{nc} = \frac{3}{4}. \tag{4}$$

This allows us to obtain the equilibrium amount of storage from equation (3)

$$S^{nc} = \frac{1}{2} - 2c. \tag{5}$$

Since $S^{nc} \geq 0$ this equation implies that a necessary condition for this construction to characterize an equilibrium is $c \leq \frac{1}{4}$. Substituting into equation (1) we obtain the equilibrium second period price.

$$p_2^{nc} = \frac{3}{4} + c. \tag{6}$$

Equations (4),(6), and (3) characterize the equilibrium when $0 < c < \frac{1}{4}$.

Let us now consider the region in which $\frac{1}{4} < c < \frac{1}{2}$. Here the boundary condition that $S \geq 0$ is binding, implying that $S_b^{nc} = 0$. Thus, the second period price must be the static monopoly price $p_{2b}^{nc} = 1$. The first period price is given by $p_{1b}^{nc} = 1 - c$. To see why this must be the case, note first that $p_2 - p_1 \geq c$ otherwise consumers will purchase all their consumption in the first period (which clearly cannot be optimal). Furthermore, for $c < \frac{1}{2}$, $1 - c > \frac{1}{2}$ implying that p_{1b}^{nc} is larger than the static monopoly price corresponding to first period demand. Thus, the monopolist clearly has no incentive to choose $p_1 > p_{1b}^{nc}$.

3.3 Comparison

Because under both commitment and lack of commitment second period prices are exactly c higher than first period prices, it is enough to compare first period prices. When $c \leq \frac{1}{4}$,

$$p_1^{nc} - p_1^c = \frac{1}{2}c.$$

When $\frac{1}{4} < c < \frac{1}{2}$,

$$p_1^{nc} - p_1^c = \frac{1}{4} - \frac{1}{2}c > 0 \text{ for } c < \frac{1}{2}.$$

Thus, prices are lower under commitment. It is interesting to note that the difference in prices between the two scenarios is not monotonic in c: it is maximal at $c = \frac{1}{4}$ which is the boundary value for positive storage under no commitment.

Profits are clearly higher under commitment. It is also clear that, because prices are lower under commitment, and wasteful storage is lower under commitment, consumer surplus must also be higher under commitment. Thus, welfare is higher under commitment.

4 Analysis

In this section we develop and analyze dynamic pricing incentives by a monopolist with changing demand for a storable good under two scenarios: commitment and lack of commitment.

4.1 The model

A monopolist faces a demand for a storable good in each one T periods. For simplicity we assume that there is no cost of production and no discounting.

At each period t the monopolist can take two actions: post a price $p_t \geq 0$ and be willing to sell at that price, or post no price ($\emptyset_t$) and shut down.

At each period t demand comes from a continuum of identical consumers[3] whose utility is quasi-linear in the consumption x_t of the good and money

$$U_t(x_t, m_t) = u_t(x_t) + m_t.$$

We initially assume that the cost of storage is linear $c(S) = cS$.[4] At each date t, given any sequence of prices $p_t, ..., p_T$ consumers choose purchases q_t (if the market is open) consumption levels c_t and storage levels S_t to maximize

$$\sum_{i=t}^{T} [U_i(x_i, m_i) - q_i p_i - cS_i]$$

subject to

$$q_i = x_i + S_i - S_{i-1}.$$

Let $D_t(p_t)$ be the static demand function, i.e., the maximizer of $u_t(q) - qp_t$. Preferences are assumed to be sufficiently regular that the resulting demand functions $D_t(p_t)$ are such that the revenue functions $TR_t(p_t) = D_t(p_t)p_t$ are twice continuously differentiable and the marginal revenue functions $MR_t(p_t)$ are strictly decreasing. Denote by p_t^m the static monopoly price at period t (the maximizer of $TR_t(p)$. We assume that $p_t^m < p_{t+1}^m$ for all $t = 1, ..., T-1$. Thus, demand is such that static monopoly prices are increasing over time. We consider the case of fluctuating demand in Section 5.1. We also assume that $D_t(p_T^m) > 0$ for every t and that $c < \min_t \{p_t^m - p_{t+1}^m\}$ for all $t = 1, ..., T-1$. The first assumption merely guarantees that the monopolist never shuts down any market. The second assumption ensures that the incentives of the monopolist are affected by the presence of storage. Dealing with the case in which this condition is not satisfied in every period is straightforward but tedious. Nothing of substance is affected by this assumption.

In equilibrium, it turns out that the storage decision of the consumer at period t only depends on the prices at periods t and $t+1$: the current and next period prices. However, in order to characterize the equilibrium, optimal storage decisions must be defined for all possible prices, in which case, period t storage decisions can depend on the sequence of all future prices.

At $t = 1$ storage S_1 is determined as follows:

If $p_2 - p_1 < c$ then $S_1 = 0$. If $p_2 - p_1 \geq c$, let τ^* be the lowest period t with $t \geq 3$ such that $p_{\tau^*} - p_1 < (\tau^* - 1)c$. Furthermore, define $D(2, \tau^*) \equiv \sum_{t=2}^{\tau^*} D_t(p_1 + (t-1)c)$. Then

$$S_1 = \begin{cases} [0, D(2, \tau^*)] & \text{if } p_2 - p_1 = c \\ D(2, \tau^*) & \text{if } p_2 - p_1 > c \end{cases}$$

[3] The case of heterogeneous consumers raises a number of complications because of the aggregation of storage decisions. We discuss this case briefly in Section 5.

[4] This assumption is relaxed in Section 5.2.

Given S_{t-1}, we can then obtain the optimal S_t, for $t = 2, ..., T-1$, as follows. Let $V(S_{t-1}, \tau)$ be the value of the following maximization problem.

$$\max_{\{c_k\}_t^{\tau-1}} \sum_{k=t}^{\tau} (u(x_k) - c(k-t)x_k) \tag{7}$$
$$s.t. \quad \sum_{k=t}^{\tau-1} x_k \leq S_{t-1}$$

and denote by τ^* the first $\tau \geq t$ such that $p_\tau \leq V'(S_t, \tau) + (\tau - t)c$. Thus, τ^* is the first period in which the consumer purchases a positive quantity (note that $\tau^* = t$ is allowed). At period k such that $t \leq k \leq \tau^*$ storage is determined by the following condition

$$S_k = S_{k-1} - x_k$$

where x_k is obtained from the solution to the problem in equation 7.Next, denote by $\tau^{**} > \tau^*$ the first period such that $p_{\tau^{**}} - p_{\tau^*} < (\tau^{**} - \tau^*)c$. Furthermore, define $D(\tau^{**}, \tau^*) \equiv \sum\limits_{k=\tau^*+1}^{\tau^{**}} D_k(p_{\tau^*} + (k - \tau^*)\, c)$. Then

$$S_{\tau^*} = \begin{cases} [0, D(\tau^{**}, \tau^*)] & \text{if } p_{\tau^*+1} - p_{\tau^*} = c \\ D(\tau^{**}, \tau^*) & \text{if } p_{\tau^*+1} - p_{\tau^*} > c \end{cases}$$

Note that, for $p_{\tau^*+1} - p_{\tau^*} > c$, S_{τ^*} is continuous and differentiable in p.

Finally, for $j > \tau^*$, we have

$$S_j = \sum_{k=j+1}^{\tau^{**}} D_k(p_{\tau^*} + (k - \tau^*)\, c).$$

4.2 Commitment

Under commitment, the monopolist chooses a sequence of either prices p_t or shut down decisions $\emptyset_t$ to maximize total profits

$$V(\sigma) = \sum_{t=1}^{T} [D_t(p_t) - S_{t-1} + S_t] p_t$$

where $S_0 \equiv S_T \equiv 0$ and, for $t = 1, ..., T-1$, S_t is specified as in section 4.1.

Although the reasoning is more elaborate, the appendix shows that the essential properties of the commitment equilibrium generalize beyond the example presented in Section 3:

prices must increase at rate c and storage is zero. The intuition is that in equilibrium consumers should not anticipate stockpile and pay for storage. If they did the monopolist could announce a slight reduction of future prices and induce consumers to postpone purchases and pay a higher price rather than the cost of storage.

Furthermore, given our assumption that $D_1(p_T^m) > 0$, it is never optimal to shut down any market.

These properties imply that in an equilibrium with commitment $p_t^c = p_1^c + (t-1)c$ for $t = 1, ..., T$ and $S_t = 0$ for $t = 1, ..., T-1$.

Thus, the entire strategy of the monopolist is identified by p_1^c. In particular, under commitment the problem of the monopolist is to choose price p_1 to maximize total profit

$$V_1(p_1) = \sum_{t=1}^{T} D_t(p_1 + (t-1)c)(p_1 + (t-1)c).$$

This leads to the following proposition.

Proposition 1 *Let p_1^c be the unique solution of*

$$\sum_{t=1}^{T} MR_t(p_1^c + (t-1)c) = 0. \tag{8}$$

Then, the sequence $\{p_t^c = p_1^c + (t-1)\,c\}_{t=1}^{T}$ is the unique equilibrium sequence of prices under commitment. In equilibrium storage is zero for all t.

An obvious consequence of this characterization is that $p_1^c > p_1^m$ and $p_T^c < p_T^m$.

Observe that in the subgame starting at each period t the sequence of prices $\{p_t^c\}$ is not optimal. Consider for instance the last period. Given that $S_{T-1} = 0$ the monopolist has an incentive to increase his profits by charging p_T^m instead of p_T^c. Thus, the optimal sequence of prices under commitment never constitutes an equilibrium of the game without commitment.

4.3 No Commitment

We will first construct the equilibrium in the case in which storage is interior. As in the example (Section 3) this holds in equilibrium if c is not too high. This construction is necessary to then extend the analysis to the case in which storage may be zero.

Consider the final period problem. Define

$$V_T(p, S) \equiv [D_T(p) - S_{T-1}]p.$$

Given S_{T-1}, in period T the monopolist chooses a price p_T to maximize $V_T(p, S_{T-1})$. The optimal price $p_T^{nc} = p_T(S_{T-1})$ must satisfy the following necessary conditions

$$\left.\frac{\partial V(p, S_{T-1})}{\partial p}\right|_{p_T^{nc}} = 0$$
$$\left.\frac{\partial^2 V(p, S_{T-1})}{\partial^2 p}\right|_{p_T^{nc}} \leq 0.$$

In particular the first order condition becomes

$$MR_T(p_T^{nc}) = S_{T-1}. \tag{9}$$

Let us consider the storage decision at time $T-1$ and define $S_{T-1}(p)$ the equilibrium amount of storage at period $T-1$ as a function of the price at period $T-1$. Since, whenever S_{T-1} is positive, $p_T - p_{T-1} = c$, in equilibrium of a subgame starting with p_{T-1}, we must have that $S_{T-1}(p_{T-1})$ solves

$$p_T(S_{T-1}(p_{T-1})) = p_{T-1} + c.$$

Substituting into $V_T(p_T^{nc}, S)$ we obtain $V_T(p_{T-1})$: last period profits as a function of the previous period price.

Given S_{T-2}, the value of profits in period $T-1$ is given by

$$V_{T-1}(p_{T-1}, S_{T-2}) = (D_{T-1}(p_{T-1}) - S_{T-2} + S_{T-1}(p_{T-1}))\, p_{T-1} + V_T(p_{T-1}).$$

We can then obtain recursively the value of profits at period t:

$$V_t(p_t, S_{t-1}) = (D_t(p_t) - S_{t-1} + S_t(p_t))\, p_t + V_{t+1}(p_t)$$

where $S_t(p_t)$ must be such that, given $S_t(p_t)$, $p_{t+1}(S_t(p_t))$ satisfies $p_{t+1}(S_t(p_t)) = p_t + c$, i.e., the optimal price at period $t+1$ given S_t is exactly c higher than p_t.

The first order conditions for an optimum at period t are

$$MR_t(p_t^{nc}) - S_{t-1} + S_t(p_t^{nc}) - p_t \frac{\partial S_t(p_t^{nc})}{\partial p} + \frac{\partial V_{t+1}(p_t^{nc})}{\partial p} = 0.$$

Lemma 1 *In equilibrium,*
(i) $\frac{\partial V_{t+1}(p_t^{nc})}{\partial p} = -\frac{\partial S_t(p_t^{nc})}{\partial p} p_{t+1}^{nc}$,
(ii) $\frac{\partial S_t(p_t^{nc})}{\partial p} = \frac{\partial^2 V_{t+1}(p_{t+1}^{nc} S_t)}{\partial^2 p}$

Proof. Property (i) follows immediately from the envelope theorem. To prove property (ii), note that, because of property (i) for any price p (not necessarily the optimal one), the first order condition at period $t+1$ implies that

$$S_t(p) \equiv MR_{t+1}(p_{t+1}^{nc}) + S_{t+1}(p_{t+1}^{nc}) - c \left.\frac{\partial S_{t+1}(p_{t+1})}{\partial p_{t+1}}\right|_{p_{t+1}^{nc}}.$$

As a consequence

$$\begin{aligned}\frac{\partial S_t(p)}{\partial p} &\equiv \frac{\partial}{\partial p}\left(MR_{t+1}(p^{nc}_{t+1}) + S_{t+1}(p^{nc}_{t+1}) - c\,\frac{\partial S_{t+1}(p)}{\partial p}\bigg|_{p^{nc}_{t+1}}\right) \\ &\equiv \frac{\partial}{\partial p_{t+1}}\left(MR_{t+1}(p_{t+1}) + S_{t+1}(p_{t+1}) - c\frac{\partial S_{t+1}(p_{t+1})}{\partial p_{t+1}}\right)\bigg|_{p^{nc}_{t+1}} \frac{\partial p^{nc}_{t+1}}{\partial p}.\end{aligned}$$

Recalling that for any price p the optimal price $p^{nc}_{t+1} = p+c$ we have that $\frac{\partial p^{nc}_{t+1}}{\partial p_t} = 1$. Moreover,

$$\frac{\partial}{\partial p}\left(MR_{t+1}(p) + S_{t+1}(p) - c\,\frac{\partial S_{t+1}(x)}{\partial x}\bigg|_{x=p}\right) \equiv \frac{\partial^2 V_{t+1}(p, S_{t-1})}{\partial^2 p}.$$

■

Because of part (i) of Lemma 1, the first order conditions for period t can be written as

$$MR_t(p^{nc}_t) = S_{t-1} - S_t(p^{nc}_t) + c\frac{\partial S_t(p^{nc}_t)}{\partial p} \quad t = 1, ..., T. \tag{10}$$

(For periods 1, and T, recall that $S_0 \equiv S_T \equiv 0$.)

This allows us to state the following proposition

Proposition 2 *Assume that the monopolist cannot commit, and that $c < c^*$, in equilibrium, the first price p^{nc}_1 is must satisfy the following equation*

$$\sum_{t=1}^{T} MR_t(p^{nc}_1 + (t-1)c) = c\sum_{t=1}^{T-1} \frac{\partial S_t(p)}{\partial p}\bigg|_{p^{nc}_1+(t-1)c} \tag{11}$$

and at all other $t = 2, ..., T$,

$$p^{nc}_t = p^{nc}_1 + (t-1)c.$$

Furthermore, in all periods, prices under commitment p^c_t are lower than the corresponding prices without commitment p^{nc}_t.

Proof. To obtain equation (11), sum equations (10) over all t's and recall that $p^{nc}_{t+1} = p^{nc}_t + c$.

To compare with the commitment solution, characterized in Proposition 1 we need to sign $\frac{\partial S_t(p)}{\partial p}\Big|_{p^{nc}_1+(t-1)c}$. By part (ii) of Lemma 1,

$$\frac{\partial S_t(p)}{\partial p_t} = \frac{\partial^2 V_{t+1}(p, S_{t-1})}{\partial^2 p}\bigg|_{p^{nc}_{t+1}} \leq 0 \;\; \forall t = 1, T-1 \tag{12}$$

where the inequality holds because of optimality of p^{nc}_{t+1}. Furthermore, it is easy to show that for time T, the inequality is strict:

$$\frac{\partial S_{T-1}(p)}{\partial p_{T-1}} = MR'\left(p^{nc}_T\right) < 0.$$

Since the right-hand sides of equations (11) and (8) are the same decreasing functions of p_1, it must be the case that $p^{nc}_1 > p^c_1$. Since in both scenarios prices increase at rate t, prices must be lower under commitment in all periods. ■

We now consider the case in which the non negativity constraint for storage is binding.

For any $\tau = 2, ...T$, consider an artificial problem in which the monopolist only faces consumers between periods τ and T. Denote by $\left\{p^{nc}_{t,\tau}\right\}^T_{t=\tau}$ the corresponding equilibrium price sequence. Specifically, $p^{nc}_{\tau,\tau}$ must satisfy the following equation

$$\sum_{t=\tau}^{T} MR_t(p^{nc}_{\tau,\tau} + (t-\tau)c) = c\sum_{t=\tau}^{T-1} \left.\frac{\partial S_t(p)}{\partial p}\right|_{p^{nc}_{\tau,\tau}+(t-\tau)c}. \tag{13}$$

Furthermore, set

$$p^{nc}_{t,\tau} = p^{nc}_{\tau,\tau} + (t-\tau)c \text{ for all } \ t = 1, ..., T. \tag{14}$$

We can now state a result for the case in which storage may be zero.

Proposition 3 *Assume that the monopolist cannot commit, and that $c \geq c^*$. Let $\tau - 1$ be the last period in which storage is zero. Then,*

(i) Storage is zero for all $t < \tau - 1$.

(ii) In equilibrium, prices are given by equations (13) and (14).

(iii) In all periods, prices under commitment p^c_t are lower than the corresponding prices without commitment p^{nc}_t.

Proof. If storage is zero at period $\tau - 1$, the monopolist enters period τ as if the prior periods did not exist. Thus, it is as if period τ is the same as period 1 in the equilibrium characterized in Proposition 2. Thus, for $t \geq \tau$ equations (13) and (14) must characterize equilibrium prices. For $t < \tau$, clearly $p^{nc}_{t,\tau} \geq p^{nc}_{\tau,\tau} + (t-\tau)c$ otherwise storage could not be zero. But it cannot be the case that $p^{nc}_{t,\tau} > p^{nc}_{\tau,\tau} + (t-\tau)c$. The proof of this case is analogous to a similar step in the proof of Lemma 8 in the appendix.

(iii) Note first that the solution under commitment is unchanged by the fact that we are considering the binding storage case for no commitment. Storage was binding in any event in the case of commitment. For any $\tau = 2, ...T$, consider the analogous artificial problem in which the monopolist only faces consumers between periods τ and T. Denote by $\left\{p_{t,\tau}^c\right\}_{t=\tau}^T$ the corresponding equilibrium price sequence. Specifically, $p_{\tau,\tau}^c$ solves the following equation

$$\sum_{t=\tau}^{T} MR_t(p_{\tau,\tau}^c + (t-\tau)c) = 0 \tag{15}$$

and $p_{t,\tau}^c$ is given by

$$p_{t,\tau}^c = p_{\tau,\tau}^c + (t-\tau)c \text{ for all } t = \tau, ..., T.$$

By Proposition 2, $p_{t,\tau}^c < p_{t,\tau}^{nc}$ for all $t = \tau, ..., T$. Furthermore, it is easy to see that $p_{\tau,\tau}^c > p_{\tau,1}^c$ since the latter is given by equation (8). Thus, prices under commitment are lower than prices without commitment. ■

5 Extensions

We now consider a number of extensions of the analysis. We first discuss the case in which demand may also decrease. Then we discuss the case of convex storage costs.

5.1 T periods Cycles

Consider now a deterministic demand cycle $D_1(p)$,... $D_T(p)$, $D_{T+1}(p)$,... $D_{2T-1}(p)$. Assume that this cycle is symmetric in the sense that $D_2(p) = D_{2T-1}(p)$, $D_3(p) = D_{2T-2}(p)$,... For each demand call p_t^m the static monopoly price. Assume that $p_1^m < ... < p_T^m$ and assume also that the marginal cost of storage c is such that $c < \min\{p_{t+1}^m - p_t^m\}$. We want to study and compare prices along the equilibrium path of the game under commitment and absent commitment.

If the jumps in demand between periods T and $T+1$ and between periods $2T-1$ and $2T$ are sufficiently large, then the characterization for this problem is straightforward given the analysis in the previous section: when demand is increasing apply the previous characterization. When demand is decreasing, set prices to be static monopoly prices.

However, in general, the analysis can be complicated by the fact that when demand starts falling at period $T+1$ the static monopoly price p_{T+1}^m might be so high that consumers set positive storage at period T. Similarly it might happen that the static monopoly price p_{2T-1}^m is so low that the consumers store for period 1. In other words, the peak and bottom of the cycle of static monopoly prices may not coincide with that of the equilibrium prices.

In order to solve for the equilibrium we propose an algorithm that can be used to obtain equilibrium prices both in the case of commitment and when there is no commitment. For this reason in the description of the algorithm we will talk about "equilibrium" without specifying whether we will be referring to the equilibrium under commitment or to equilibrium absent commitment. Similarly, without further specification we will use the notation p_t^{eq} to indicate an equilibrium price at period t.

As we pointed out, the upward trend of equilibrium price does not necessarily coincide with the upward trend in static monopoly prices. Hence we use notation t_1 to indicate the period in the cycle at which equilibrium prices start rising. We will also call t_N the period in the cycle after which equilibrium prices start declining. Finally we will call t_n the period at which the n^{th} ascending price is observed in equilibrium. Finally we will call $\{t_n\}$ a generic sequence demands.

Solution algorithm:
Start the algorithm by considering the sequence $\{t_n\}$ where $n = 1, ..., T$ and $t_1 = 1$.

Step 1: compute the equilibrium for the sequence $\{t_n\}$ and then go to step 2.

Step 2: compare $p_{t_N+1}^m$ with $p_{t_N}^{eq}$ and compare p_{2T-1}^m with $p_{t_1}^{eq}$:

Step 2.1: if $p_{t_N+1}^m \leq p_{t_N}^{eq} + c$ and $p_{t_1-1}^m \geq p_{t_1}^{eq} - c$ stop[5]: the sequence of price is an equilibrium;

Step 2.2: if $p_{t_N+1}^m \leq p_{t_N}^{eq} + c$ and $p_{t_1-1}^m < p_{t_1}^{eq} - c$ create a new sequence $\{t'_n\}$ where $n = 1, ..., N+1$, $t'_1 = t_1 - 1$ and $t'_N = t_N$. Then start again from step 1;

Step 2.3: if $p_{t_N+1}^m > p_{t_N}^{eq} + c$ and $p_{t_1-1}^m \geq p_{t_1}^{eq} - c$ create a new sequence $\{t'_n\}$ where $n = 1, ..., N+1$, $t'_n = t_1$ and $t'_N = t_N + 1$. Then start again from step 1;

Step 2.4: if $p_{t_N+1}^m > p_{t_N}^{eq} + c$ and $p_{t_1-1}^m < p_{t_1}^{eq} - c$ create a new sequence $\{t'_n\}$ where $n = 1, ..., N+2$, $t'_n = t_1 - 1$ and $t'_n = t_N + 1$. Then start again from step 1.

Because the cycle is of finite length, the algorithm must converge.

Once the process has stopped set the equilibrium price for all the remaining periods not included in $\{t_n\}$ equal to p_t^m.

Lemma 2 *At all periods t except, possibly, t_1 and t_N, the increase in equilibrium prices is lower than the decrease in equilibrium prices.*

Proof. When prices increase they increase by c, when they drop they drop by $|p_{t+1}^m - p_t^m|$ which, by assumption, is greater than c for all t. ■

[5] When searching for a no commitment equilibrium this condition becomes

$$\begin{aligned} p_{t_N}^m &< p_{t_N}^{eq} + c \\ p_{t_1-1}^m &> p_{t_1}^{eq} - c. \end{aligned}$$

5.1.1 Commitment

In case of commitment this algorithm delivers the unique solution. Notice first that the conditions of optimality of the previous section do not depend on the fact that demand is increasing. Even within a cycle the optimality condition is that

$$\sum_{t=1}^{2T-1} MR_t(p_1 + (t-1)c) = 0. \tag{16}$$

The algorithm above divide the cycle into two sequences. The first sequence is of length N. In this sequence prices are increasing and distanced by c one from the other. Moreover, when performing Step 1 of the algorithm we have that for all the t_n in this sequence

$$\sum_{n=1}^{N} MR_{t_n}(p_{t_1} + (n-1)c) = 0.$$

The other sequence is of length $2T-1-N$. In this sequence prices are decreasing and equal to the static monopoly prices so that $MR_t(p_t^m) = 0$ for all the periods in this sequence. Summing over all periods we have that condition (16) is satisfied. This show that the algorithm identifies the equilibrium.

5.1.2 No commitment

Within a cycle, when prices increase

$$p_{t+1} = p_t + c$$

whereas when prices decrease

$$\begin{aligned} p_t &= p_t^m \\ S_{t-1} &= 0 \\ \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t-1}^{eq}} &= 0 \end{aligned}$$

The optimality condition that equilibrium prices must satisfy is

$$\sum_{t=1}^{2T-1} MR_t(p_t) = c \sum_{t=1}^{2T-1} \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t_n-1}^{eq}}. \tag{17}$$

Upon convergence the proposed algorithm breaks the cycle into two sections. One is of length N. In this section prices are increasing and distanced by c one from the other. Moreover

$$\sum_{n=1}^{N} MR_{t_n}(p_{t_n}^{nc}) = c\sum_{n=1}^{N} \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t_n}^{nc}}.$$

The other part is of length $2T-1-N$, prices $p_t = p_t^m$ are decreasing, $S_{t-1} = 0$ and $\left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t-1}^{eq}} = 0$. Combining these two we obtain condition (17) above.

5.1.3 Comparison

Before we compare prices along the equilibrium paths it is useful to state the following two lemmas

Lemma 3 *At any iteration, if at stage* 2 *only period* $t_N + 1$ *is added to a sequence* $\{t_n\}$ *then equilibrium prices* p_t *increase for all* $t = t_1, ..., t_N$. *At any iteration, if at stage* 2 *only period* $t_1 - 1$ *is added to a sequence* $\{t_n\}$ *then equilibrium prices* p_t *decrease for all* $t = t_1, ..., t_N$.

Proof. Consider first using the algorithm to find an equilibrium with commitment.

Suppose then that at step 2 only period $t_N + 1$ is added to a previous sequence $\{t_n\}$. Consider then the first order condition computed at the equilibrium price p_{t_1} and notice that

$$\sum_{n=1}^{N} MR_{t_n}(p_{t_1} + (n-1)c) + MR_{t_N+1}(p_{t_1} + Nc) > 0.$$

This means that the new equilibrium price $p'_{t_1} > p_{t_1}$. Because in equilibrium $p_{t_n} = p_{t_1} + (n-1)c$, $p'_t > p_t$ for all $t = t_1, ..., t_N$.

Similarly, suppose that at step 2 only period $t_1 - 1$ is added to the sequence $\{t_n\}$. Consider the first order condition computed at $p_{t_1-1} = p_{t_1} - c$ and notice that

$$MR_{t_1-1}(p_{t_1} - c) + \sum_{n=1}^{N} MR_{t_n}(p_{t_1} + (n-1)c) < 0.$$

Hence the equilibrium price $p_{t_1-1} > p_{t_1} - c$. As a consequence new equilibrium prices $p'_t > p_t$ for all $t = t_1, ..., t_N$

Consider now using the algorithm to find an equilibrium without commitment.

Suppose that at step 2 only period $t_N + 1$ is added to a previous sequence $\{t_n\}$. Because $p_{t_N+1}^{eq} = p_{t_1} + Nc$ and because $MR_{t_N+1}(p)$ is decreasing, if the new equilibrium price $p'_{t_1} < p_{t_1}$ then $S'_{t_N} > S_{t_N}$. It is easy to verify that $V_{t_1}(p'_{t_1}, 0) < V_{t_1}(p_{t_1}, 0)$. This means that the new equilibrium price $p'_{t_1} > p_{t_1}$. Because in equilibrium $p_{t_n} = p_{t_1} + (n-1)c$, $p'_t > p_t$ for all $t = t_1, ..., t_N$.

Suppose now that at step 2 only period $t_1 - 1$ is added to the sequence. Notice that the previous equilibrium price p_{t_1} had to satisfy

$$MR_{t_1}(p_{t_1}) + S_{t_1} - c \left. \frac{\partial S_{t-1}(p)}{\partial p} \right|_{p_{t_1}} = 0.$$

After adding demand $t_1 - 1$ we have that $S_{t_1-1} \geq 0$. The first order condition at period t_1 computed at p_{t_1} implies that

$$MR_{t_1}(p_{t_1}) - S_{t_1-1} + S_{t_1} - c \left. \frac{\partial S_{t-1}(p)}{\partial p} \right|_{p_{t_1}} \leq 0.$$

As a consequence, when period $t_1 - 1$ is added price $p'_{t_1} < p_{t_1}$. Because in equilibrium $p_{t_n} = p_{t_1} + (n-1)c$, $p'_t > p_t$ for all $t = t_1, ..., t_N$. ■

We can then use this lemma to prove that:

Lemma 4 *In the equilibrium under commitment prices start to increase and start to decrease no sooner than in the equilibrium without commitment. Moreover, prices are higher when the monopolist lacks commitment.*

Proof. Call t_N^c and t_1^c the last and first period of ascending prices in equilibrium and when the monopolist can commit. Call t_N^{nc} and t_1^{nc} the corresponding periods in equilibrium when the monopolist lacks commitment.

Moreover at each iteration of the algorithm call $\{t_n\}^c$ and $\{t_n\}^{nc}$ the sequence related to the search for a commitment and a no commitment equilibrium respectively.

At *Step 1* of the very first iteration the two equilibria are computed over the same sequence $\{t_n\}$ and we find two sequences $\{p_{t_n}^c\}$ and $\{p_{t_n}^{nc}\}$.

By the result of the previous section prices under commitment are lower than prices under no commitment. This means that at *Step 2* if either $p_{t_N}^{nc} + c \leq p_{t_N+1}^m$ and/or $p_{t_1-1}^m < p_{t_1}^c - c$ then, at the next iteration, the sequences $\{t_n\}^c$ and $\{t_n\}^{nc}$ will be the same. If instead $p_{t_N}^c + c \leq p_{t_N+1}^m$ and $p_{t_N}^{nc} + c > p_{t_N+1}^m$ then period $t_N + 1$ will be added only to the sequence $\{t_n\}^{nc}$ and eventually, upon convergence, $t_N^c < t_N^{nc}$.

If instead $p_{t_1-1}^m < p_{t_1}^c - c$ and $p_{t_1-1}^m \geq p_{t_1}^c - c$ then period $t_1 - 1$ will be added only to the sequence $\{t_n\}^c$ and eventually, upon convergence, $t_1^{nc} < t_1^c$.

Moreover, by lemma 3 whenever only period $t_N + 1$ is added to a sequence the new equilibrium prices will decrease. Because in equilibrium $p_{t_N}^c < p_{t_N}^m$ if $p_{t_N}^c + c \leq p_{t_N+1}^m$ and $p_{t_N}^{nc} + c > p_{t_N+1}^m$ then even if prices p_t^{nc} are on a descending phase they are still higher than commitment prices.

Similarly, by lemma 3 whenever only period $t_1 - 1$ is added to a sequence the new equilibrium prices will increase. Because in equilibrium $p_{t_1}^{nc} \geq p_{t_1}^{m}$, if $p_{t_1-1}^{m} < p_{t_1}^{c} - c$ and $p_{t_1-1}^{m} \geq p_{t_1}^{c} - c$ then prices p_t^{nc} remains above commitment prices even if the latter are equal to the static monopoly prices. ■

5.2 Convex Cost of Storage

Assume that the cost of storage is given by a twice continuously differentiable function $c(S)$ with $c'(S) > 0, c''(S) > 0$, and $c(0) = c'(0) = 0$.

There are two main differences with the previous analysis: (1) storage is now positive under commitment as well. (2) it is no longer possible to characterize the equilibrium price sequence simply by obtaining the first price. As a result, we no longer obtain as crisp a result comparing commitment and no commitment. However, we show that prices cannot be uniformly higher under commitment, and we have computed examples with specific functional forms in which the result of the previous section generalizes.

Consider any fixed sequence of prices $\{p_t\}_{t=1}^{T}$. Suppose that the buyer begins date t with a stock S_{t-1} of the good. Let $S_t^*(p_1, ..., p_T)$ be the optimal storage choice by the consumer. The following Lemma provides a simple characterization of the solution of the buyer's problem.

Lemma 5 *Assume that $p_t < p_\tau$ for $t < \tau$, and that $p_t \leq p_m^T$ for all t. Then, the buyer always purchases a positive amount at every date and store a positive amount at every date except for date T. At date t the consumer stores quantity S_t that solves*

$$c'(S_t) = p_{t+1} - p_t \tag{18}$$

and consumes

$$x_t = D_t(p_t) + S_{t-1}.$$

Thus, at date t the consumer purchases $b_t = D_t(p_t) + S_t - S_{t-1}$ units.

By Lemma 5, we can write the consumer's optimal storage decision at period t as a function of period t and period $t+1$ prices only. Define $S_t(p_t, p_{t+1})$ denote optimal storage decisions at period t as defined by equation (18).

5.2.1 Commitment

By Lemma 5, given a sequence of increasing prices $p_1, ..., p_T$, monopoly profits can be written as

$$\begin{aligned} \pi(p_1, ..., p_T) &= [D_1(p_1) + S_1(p_1, p_2)]p_1 \\ &+ \sum_{t=2}^{T-1} [D_t(p_t) - S_{t-1}(p_{t-1}, p_t) + S_t(p_t, p_{t+1})]p_t + [D_T(p_T) - S_{T-1}(p_{T-1}, p_T)]p_T \end{aligned} \tag{19}$$

Let us assume for the moment that prices form an increasing sequence. Then, under commitment, the monopolist chooses $(p_1, ..., p_T)$ at period 1 to maximize the right-hand side of equation (19).

Recall that $MR_t(p_t) = D_t(p_t) + D_t'(p_t)p_t$ and write the first order conditions as:

$$\begin{array}{rcll} MR_1(p_1) + S_1(p_1, p_2) - \dfrac{\partial S_1(p_1, p_2)}{\partial p_1}(p_2 - p_1) & = & 0 & \\ MR_t(p_t) - S_{t-1}(p_{t-1}, p_t) + S_t(p_t, p_{t+1}) & & & \\ -\dfrac{\partial S_{t-1}(p_{t-1}, p_t)}{\partial p_t}(p_t - p_{t-1}) - \dfrac{\partial S_t(p_t, p_{t+1})}{\partial p_t}(p_{t+1} - p_t) & = & 0 & t = 2, ..., T-1 \\ MR_T(p_T) - S_{T-1}(p_{T-1}, p_T) - \dfrac{\partial S_{T-1}(p_{T-1}, p_T)}{\partial p_T}(p_T - p_{T-1}) & = & 0 & \end{array}$$

Summing these rows we obtain

$$\sum_{t=1}^{T} MR_t\left(p_t^c\right) = 0. \tag{20}$$

This equation is the counterpart of equation (8) that we obtained in the case of linear costs of storage. Note however, that equation (20) is not as informative: because prices are now not necessarily rising at a constant rate c, we need T conditions to obtain each price.

5.2.2 No Commitment

The construction of the equilibrium absent commitment is quite similar to the analysis in Section 4. The main difference is that equilibrium storage $S_t\left(p_t\right)$ at date t must satisfy

$$c'\left(S_t\left(p_t\right)\right) = p_{t+1} - p_t.$$

Appropriately modifying the of the analysis of Section 4, we obtain that equilibrium is characterized by the system:

$$\begin{cases} MR_1(p_1^{nc}) = -S_1^{nc} + (p_2 - p_1)\frac{\partial S_1(p_1)}{\partial p_1} \\ \ldots \\ MR_t(p_t^{nc}) = S_{t-1}^{nc} - S_t^{nc} + (p_{t+1}^{nc}(p_t) - p_t)\frac{\partial S_t^{nc}(p_t)}{\partial p_t} \\ \ldots \\ MR_T(p_T^{nc}) = S_T^{nc} \end{cases}$$

Summing these rows we obtain

$$\sum_{t=1}^{T} MR_t(p_t^{nc}) = \sum_{t=1}^{T}\left((p_{t+1}^{nc} - p_t)\frac{\partial S_t^{nc}(p_t)}{\partial p_t}\right).$$

Going through similar steps as in the proof of Lemma 1 we can show that

$$\frac{\partial S_t^{nc}(p_t)}{\partial p_t} = \frac{\frac{\partial^2 V_{t+1}}{\partial p_{t+1}^2}}{1 - c''(S_t^{nc}) \frac{\partial^2 V_{t+1}}{\partial p_{t+1}^2}} \leq 0.$$

Because $p_{t+1}^{nc} > p_t^{nc}$, and because (as in the previous section) we can prove that $\frac{\partial S_{T-1}^{nc}(p_{T-1})}{\partial p_{T-1}} < 0$, we can conclude that $\sum_{t=1}^{T} MR_t(p_t^{nc}) < \sum_{t=1}^{T} MR_t(p_t^{c})$. Because MR_t are decreasing functions for all t, we can conclude that prices under commitment cannot be uniformly higher. There is also a sense in which they have to be lower "on average." We have computed equilibria with several specific functional forms and we have always found that prices are uniformly lower under commitment.

6 Appendix

Lemma 6 *Consider a commitment equilibrium in which that $S_i = 0$ for all $i = 1, ..., t-1$. Suppose also that market t and k are open. If $p_{t+k} - p_t = kc$ then $S_{t+k-1} = 0$.*

Proof. Assume by way of contradiction that there exists an equilibrium in which $S_i^c = 0$ for all $i = 1, ..., t$, market t and market $t+k$ are open at prices p_t and p_{t+k} respectively (in case $k > 1$ this means that all markets between $t+1$ and $t+k-1$ are closed), $p_{t+k} - p_t = kc$ but $S_{t+k-1} > 0$.

Call τ the first period after $t+k$ at which either market $\tau+1$ is closed or, if market $\tau+1$ is open, $p_{\tau+1} - p_\tau > c$. If such τ does not exist because all markets after period $t+k$ are open and prices are distanced by no more than c, then set $\tau = T$. Consider the sequence σ^* where, with abuse of notation

$$\begin{array}{rl} p_i^* = p_i & \text{for } i = 1, ..., t+k-1 \\ p_i^* = p_i - (i-t)\varepsilon & \text{for } i = t+k, ..., \tau \\ p_i^* = p_i & \text{for } i = \tau+1, ..., T \end{array}$$

(by definition of k, under σ markets $i = t+k, ..., \tau$ are all open at price p_i). We will show that $\pi(\sigma^*) > \pi(\sigma)$.

Notice that under sequence σ^* actions at periods $i = 1, ..., t+k-1$ and at periods $i = \tau+1, ..., T$ are the same as under sequence σ. On the contrary, under sequence σ^* price $p_{i+1}^* - p_i^* < c$ at all the periods $i = t+k, ..., \tau-1$. Hence the optimal storage decision of consumers will be

$$\begin{array}{rl} S_i^* = 0 & \text{for } i = 1, ..., \tau-1 \\ S_\tau^* = S_\tau & \text{for } i = \tau+1, ..., T. \end{array}$$

and for period τ

$$S_\tau^* = S_\tau - (\tau - t)\varepsilon \left.\frac{\partial S_\tau}{\partial p_\tau}\right|_{p\tau}$$

The difference between $\pi(\sigma)$ and $\pi(\sigma^*)$ accrues only from period t and from periods $i = t+k, ..., \tau+1$. Specifically

$$\begin{aligned}
\pi(\sigma^*) - \pi(\sigma) &= -\varepsilon \sum_{i=t+k}^{\tau} (i-t) MR_i(p_i) + kcS_{t+k-1} \\
&\quad + \sum_{i=t+k}^{\tau} S_i(p_{i+1} - p_i) + [S_\tau^* p_\tau^* - S_\tau p_\tau - S_\tau^* p_{\tau+1} + S_\tau p_{\tau+1}] \\
&= -\varepsilon \sum_{i=t+k}^{\tau} (i-t) MR_i(p_i) + kcS_{t+k-1} \\
&\quad + \sum_{i=t+k}^{\tau} S_i(p_{i+1} - p_i) - (\tau - t)\varepsilon S_\tau^* + (\tau - t)\varepsilon \left.\frac{\partial S_\tau^c}{\partial p_\tau}\right|_{p_\tau} (p_{\tau+1} - p_\tau)
\end{aligned}$$

Notice that if ε is small enough all terms containing ε in the right hand side of the above equality become negligible, so that $\pi(\sigma^*) - \pi(\sigma) > 0$. ■

Lemma 7 *In a commitment equilibrium the monopolist opens all markets.*

Proof. Suppose that all markets $i = 1, .., t-1$ are closed whereas market t is open at price p_t. Suppose also that $D_i(p_t) > 0$ for all $i = 1, ..., t-1$. If the monopolist opens all markets $i = 1, .., t$ at price $p_i = p_{t+1} - (t-i)c$ then, by Lemma 6, $S_i = 0$ for all $i = 1, ..., t$ and profits would increase by

$$\sum_{i=1}^{t} D_i(p_i) p_i.$$

Because the monopolist will open at least market T at a price $p_t \leq p_T^m$ and because $D_1(p_T^m) > 0$, it follows that all markets are open. ■

Lemma 8 *In a commitment equilibrium $p_{t+1}^c = p_t^c + c$ for all t.*

Proof. The proof proceeds by induction on t. Starting from $t = 1$ we will first show that in equilibrium it must be that $p_2^c - p_1^c \geq c$. Then, we will show that it cannot be that $p_2^c - p_1^c > c$. We will then use a similar argument to prove the statement for a generic t.

Consider an equilibrium price sequence $\sigma = \{p_t\}_{t=1}^T$ and assume by way of contradiction that $p_2 - p_1 < c$. If $p_1 > p_1^m$ then $MR_1(p_1) < 0$. The monopolist could then decrease p_1 and, by doing so, increase first period revenues and hence total profits. If instead $p_1 \leq p_1^m$, call τ the earliest period at which $S_\tau > 0$. By the lemma 6 if $S_\tau > 0$ and $S_i = 0$ for all $i = 1, ..., \tau - 1$ it must be the case that $p_{\tau+1} - p_\tau > c$.

Consider now $\varepsilon < \min_{1 \le t \le \tau} \{|p_{t+1} - p_t - c|\}$ and consider the price sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{ll} p_t^* = p_t + \frac{\varepsilon}{t-1} & \text{for } 2 \le t \le \tau \\ p_t^* = p_t & \text{for } \tau + 1 \le t \end{array}$$

Then

$$\begin{array}{ll} S_t^* = 0 & \text{for } 2 \le t \le \tau - 1 \\ S_t^* = S_t + \frac{\varepsilon}{t-1} \left.\frac{\partial S_t}{\partial p_t}\right|_{p_t} & \text{for } t = \tau \\ S_t^* = S_t & \text{for } \tau + 1 \le t \end{array}$$

Hence

$$\begin{aligned} \pi(\sigma^*) - \pi(\sigma) &= \varepsilon \sum_{t=\tau+1}^{\tau} \frac{MR_t(p_t)}{t-1} + [S_\tau^* p_\tau^* - S_\tau p_\tau - S_\tau^* p_{\tau+1} + S_\tau p_{\tau+1}] \\ &= \varepsilon \sum_{t=\tau+1}^{\tau} \frac{MR_t(p_t)}{t-1} + \frac{\varepsilon}{t-1} S_{\tau'}^* - \frac{\varepsilon}{t-1} \left.\frac{\partial S_t}{\partial p_t}\right|_{p_t} (p_{\tau+1} - p_\tau) \end{aligned}$$

Since $p_1 \le p_1^m$ and because $c < p_{t+1}^m - p_t^m$ for all t, then $MR_t(p_t) > 0$ for all t. Moreover, $\left.\frac{\partial S_t}{\partial p_t}\right|_{p_t} \le 0$. Hence $\pi(\sigma^*) > \pi(\sigma)$. This concludes the proof that $p_2 - p_1 \ge c$.

Now, assume by way of contradiction that $p_2 - p_1 > c$. Consider τ the lowest period at which $p_{\tau+1} - p_1 \le (\tau + 1)c$. Consider the sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{ll} p_t^* = p_1 + tc - c & \text{for } 1 \le t \le \tau \\ p_t^* = p_t & \text{for } \tau + 1 \le t. \end{array}$$

By lemma 6,

$$\begin{array}{ll} S_t^* = 0 & \text{for } t \le \tau \\ S_t^* = S_t & \text{for } \tau + 1 \le t \end{array}.$$

Hence,

$$\begin{aligned} \pi(\sigma^*) - \pi(\sigma) &= \sum_{t=1}^{\tau} D_t(p_1 + tc - c)(p_1 + tc - c) - \left[\sum_{t=1}^{\tau} D_t(p_1 + tc - c)\right] p_1 \\ &= c \sum_{t=1}^{\tau} D_t(p_1 + tc - c)(t - 1) > 0 \end{aligned}$$

and the fact that $\pi(\sigma^*) > \pi(\sigma)$ concludes the proof.

Assume now that the statement holds for any period $t - 1$ and consider period t. We will first prove that $p_{t+1} - p_t \ge c$ and then that $p_{t+1} - p_t \le c$.

Assume by way of contradiction that $p_{t+1} - p_t < c$ and suppose that $MR_t(p_t) \leq 0$. Since $p_{\tau+1} - p_\tau = c$ for all $\tau < t$ and because $c < \min\{p_{t+1}^m - p_t^m\}$ this implies that $MR_i(p_i) < 0$ for all $i = 1, ..., t-1$. Hence, the monopolist could increase all revenues by decreasing all price p_i with $i = 1, ..., t-1$ by ε.

Suppose now that $MR_t(p_t) > 0$ and call τ the lowest period at which $S_\tau > 0$. By the lemma 6 it must be the case that $p_{\tau+1} - p_\tau > c$. Moreover, consider $\varepsilon < \min\limits_{t \leq i \leq \tau}\{|p_{i+1}^c - p_i^c - c|\}$ and consider the price sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{cc} p_i^* = p_i & \text{for } i = 1, ..., t \\ p_i^* = p_i + \frac{\varepsilon}{i-t} & \text{for } i = t+1, ..., \tau \\ p_i^* = p_i & \text{for } i = \tau+1, ..., T \end{array}$$

Then

$$\begin{array}{cc} S_i^* = 0 & \text{for } i = 1, ..., \tau-1 \\ S_t^* = S_t + \frac{\varepsilon}{t-1} \left.\frac{\partial S_t}{\partial p_t}\right|_{p_t} & \text{for } \tau = \tau' \\ S_i^* = S_i & \text{for } i = \tau+1, ..., T \end{array}$$

Hence, similarly to the argument used above

$$\pi(\sigma^*) > \pi(\sigma)$$

and this proves that $p_{t+1} - p_t \geq c$.

Now, assume by way of contradiction that $p_{t+1} - p_t > c$. Consider τ the lowest period for which $p_{\tau+1} - p_1 \leq (\tau+1)c$. Consider the sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{cc} p_i^* = p_i & \text{for } i = 1, ..., t-1 \\ p_i^* = p_t + (i-1)c & \text{for } i = t, ..., \tau \\ p_i^* = p_i & \text{for } i = \tau+1, ..., T \end{array}$$

so that, by lemma 6,

$$\begin{array}{cc} S_i^* = 0 & \text{for } i = 1, ..., \tau \\ S_i^* = S_i & \text{for } i = \tau+1, ..., T \end{array}.$$

Hence, similarly to what seen above,

$$\pi(\sigma^*) > \pi(\sigma).$$

This means that $p_{t+1}^c - p_t^c \leq c$ and this concludes the proof. ■

7 References

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