RELATIONAL TEAM INCENTIVES AND OWNERSHIP

LUIS RAYO

Abstract. This paper develops a stylized theory of internal organization of the firm based on the interplay between *explicit* (court-enforced) and *implicit* (self-enforced) incentives. The firm is modeled as a team that meets repeatedly, and is confronted with a problem of moral hazard. Two instruments are used to induce effort: court-enforced *ownership shares* over the stream of profits, and self-enforced *voluntary transfers* contingent on a non-verifiable performance measure. These transfers are sustained by the surplus created through repeated interaction. I consider several environments differing in the level of informational asymmetry *between* team members. When information is fully symmetric, ownership shares will be dispersed so that no single player is the full residual claimant of output, and every player receives implicit incentives. However, when information is sufficiently asymmetric due to either hidden actions or hidden information, ownership will be concentrated in the hands of one player to the degree that she receives no implicit incentives. Moreover, this player can always be charged with paying all voluntary transfers to the remaining members, and is therefore viewed as an *endogenously chosen principal*. Thus, the model endogenizes the principal-agent relationship which the existing repeated agency literature takes as given, and in this sense suggests a theory of hierarchy.

1. Introduction

In market economies, substantial resource allocation occurs within firms. Following the work of Coase [1937] and Williamson [1975,1985], much effort has been devoted to understand the precise means by which firms outperform market contracting. Key to materializing these efficiency gains is the *internal organization* of enterprise. Indeed, the structures of ownership and hierarchy display wide diversity across firms together with a strong correlation within industries (e.g., Hansmann [1996]), which prompts inquiry regarding the conditions that favor particular forms of organization.

This paper views organizational design as a response to the problem of incentive creation, much as Alchian and Demsetz [1972], but takes the position that a primary source of incentives is the surplus, or "relational capital", created by the repeated interaction among the members of the firm. Although myriad factors influence organizational

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structure, the present analysis focuses on the exploitation of relational capital while abstracting from other well known determinants of organization.[1] Such an approach allows for a detailed consideration of the subtleties behind repeated interaction while retaining tractability. Thus, rather than considering a general theory of the firm, the paper merely seeks to add a new element to the discussion.

The firm is modeled as a team of infinitely lived players who repeatedly interact with each other, and are confronted with a problem of moral hazard regarding their levels of *effort* towards joint production. In the spirit of Alchian and Demsetz [1972] and Holmstrom [1982], moral hazard arises because efforts are non-contractible due to a lack of observability by third parties. Joint output (i.e., profits), however, will be contractible. Moreover, due to the nature of their interaction, team members will have an informational advantage over courts. In particular, they will publicly observe a (non-contractible) noisy performance measure. Accordingly, the team will have two instruments to induce effort: *ownership shares* and *voluntary transfers* (e.g., productivity bonuses). Ownership shares will simply be court-enforced property rights over the stream of profits.[2] Voluntary transfers, on the other hand, can be made contingent on the non-contractible performance measure. As in Bull [1987] and MacLeod and Malcomson [1989], these transfers will be self-enforced through the use of relational capital.

The analysis is centered around the interplay between these explicit (court-enforced) and implicit (self-enforced) incentives. In fact, provided there is only a limited amount of relational capital (e.g., players are impatient), neither type of incentive alone can induce first-best efforts, which calls for their combined use. Thus, *the organizational design problem concerns the allocation of ownership shares, together with the implicit agreement to carry out performance payments, in a way that best promotes effort.* Formally, this translates into the dual problem of minimizing the amount of (scarce) relational capital required to implement the desired effort profile.

The shadow cost of providing implicit incentives stems from their need for relational capital. In particular, the amount of relational capital demanded to sustain (i.e., self-enforce) a given contingent performance bonus is equal to the *power* of such bonus, i.e., the difference between its highest and lowest prescribed values. In other words, the power of implicit incentives corresponds to the level of *discretion* that must be given to the player ultimately in charge of paying the bonus. Discretion, in turn, will only be exercised against short-run opportunism to the extent that such player faces a threat of losing future surplus. Consequently, she must be allocated a corresponding fraction

[1] Most conspicuously risk-sharing, capital raising, hold-up, and collective governance.

[2] Thus, ownership shares will correspond to a restricted notion of "ownership": the receipt of residual earnings. A second notion, that of residual rights of control, will be mostly absent from the analysis (albeit somewhat captured by the exercise of discretion regarding voluntary transfers).

of relational capital. Ownership shares, on the other hand, can substitute for implicit incentives and hence save on relational capital. But since shares are limited (e.g., not every player can be the full residual claimant of earnings) they will be allocated where they best serve this purpose.

The gist of optimal design is the marginal rate of substitution between ownership shares and the power of implicit incentives, given the desired effort schedule; i.e., the rate at which explicit and implicit incentives can be traded off, for each player, while inducing the same levels of effort. Equivalently, this *incentive trade-off* represents the marginal savings in relational capital brought on by increases in ownership shares, and will hence dictate the distribution of ownership in the dual minimization problem. In other words, within this dual problem, the incentive trade-off represents the marginal benefit of allocating ownership in hands of each player, while the (shadow) cost derives from an adding up constraint that restricts allocated shares to not exceed 100% of ownership.

A key determinant of the incentive trade-off is the degree of informational asymmetry *between* team members. Whenever a player's deviations are hard to detect by her peers (e.g., when her work is complex and hard to assess), her implicit incentives must be high powered, and thus relatively expensive. In addition, when a player's marginal contribution to output is high (e.g., when she plays an important role in the firm), explicit incentives will be particularly effective towards increasing her effort. In this way, the combination of high difficulty to assess performance and high marginal contribution to output, for a given player, yields a favorable incentive trade-off and therefore promotes allocating ownership in her hands.[3]

The model considers several environments differing in the level of informational asymmetry between peers. When information is fully symmetric, ownership shares will be dispersed so that no single player is the full residual claimant of output, and every player receives implicit incentives. However, when information is sufficiently asymmetric due to either hidden actions or hidden information, ownership will be concentrated in the hands of one player to a degree that she receives no implicit incentives. Moreover, this player can always be charged with paying all voluntary transfers to the remaining members, and is thus viewed as an *endogenously chosen principal*. In this sense, the model endogenizes the principal-agent relationship, suggesting a theory of hierarchy.

Intuitively, the variation in optimal ownership arises from the specific shape taken by the incentive trade-off across information environments. Under full symmetry, since any deviation is detectable, if a player deviates she will necessarily be punished, and

[3] A high marginal productivity would also promote ownership in a static model (e.g., Holmstrom [1982]). The novelty of the analysis is in the *combination* of marginal productivity and informational asymmetry (in relation to an efficient use of continuation surplus).

therefore she might as well select a *global* deviation towards her "static best response" (i.e., her optimal effort level in the absence of implicit incentives). Formally, the binding effort-incentive constraint will be a global one. In accordance, two effects arise when increasing ownership in her hands: (i) the incentive to perform any given deviation is reduced, and (ii) the effort corresponding to the optimal static deviation is increased, reducing the *size* of this global deviation. As a consequence, the incentive trade-off, representing the marginal benefit of increasing ownership, will be positive due to (i), and *decreasing* due to (ii). Decreasing marginal returns will then imply that spreading ownership is beneficial.

In contrast, under sufficient informational asymmetry, small deviations become especially tempting because they are the hardest to detect: now the binding effort-incentive constraints will be *local*. As a result, an increase in ownership has only effect (i) described above, and therefore marginal returns to ownership will be positive and *constant*. Consequently, it is optimal to adopt a corner solution that concentrates ownership in hands of a player with the most favorable incentive trade-off. (The paper also discusses intermediate cases of informational asymmetry, giving rise to intermediate ownership patterns.)

Section 10 compares the above results with patterns of ownership and hierarchy (within relatively simple firms) discussed in Alchian and Demsetz [1972], and Hansmann [1996] . For example, shared ownership and shallow hierarchy are the norm in service partnerships wherein "the quantity and quality of each individual's inputs and outputs can be observed with relative ease", Hansmann [1996, p. 70]. On the other hand, Alchian and Demsetz associate concentrated ownership and marked hierarchy (i.e., the "classical firm") with an underlying shirking problem that stems from a difficulty to detect behavior.

Section 2 relates the paper to the literature. Section 3 discusses the contracts considered throughout. Sections 4 through 9 develop the formal analysis. Finally, Section 11 summarizes the results and comments on possible extensions of the model.

2. Relation to the Literature

The paper builds primarily on two strands of the literature, those concerning team moral hazard (e.g., Alchian and Demsetz [1972], Holmstrom [1982]), and repeated agency with implicit contracting (e.g., Bull [1987], Spear and Srivastava [1987], MacLeod and Malcomson [1988,1989,1998], Pearce and Stacchetti [1993], Baker, Gibbons and Murphy [1994], Levin [2000], Che and Yoo [2001]).[4] The distinctive feature of the present model is that it introduces relational considerations to the team incentive problem *while* endogenizing the principal-agent relationship, which the above work takes as given (e.g.,

[4]Implicit contracts, and the interaction between implicit and explicit contracts have been widely studied outside agency relationships, e.g., Klein and Leffler [1981], Bernheim and Whinston [1998].

these models include a pre-determined principal who is the residual claimant of output and is not subject to moral hazard).

The structure of the model is closest to Levin's [2000] repeated agency, and indeed capitalizes on many of his results. I extend his work in two dimensions. The first is relatively straightforward and concerns the addition of multiple agents (Levin [1998] and Dewhurst [2000] also include results in this direction). The second, and more novel, involves endogenizing ownership. In this sense, the analysis shifts from the study of *optimal agent compensation* to the broader theme of *organizational design*.

Also related is the literature on ownership and integration spawned by Grossman and Hart [1986], and Hart and Moore [1990]. They view ownership as the possession of residual control rights over assets, which play a key role in the allocation of ex-post bargaining power and thus provide incentives for ex-ante investments.[5] Their theory is designed to study the boundaries of the firm, rather than its internal organization, and thus serves a complementary purpose (while focusing on a complementary notion of ownership). In this line of inquiry, Baker, Gibbons and Murphy [2001,2002] view the interplay between explicit and implicit contracts as key to understanding integration. Their work also stresses the issue of allocating discretion (and trust) across players.[6]

More generally, the present work fits into the theory of incentives in organizations (centered around the principal-agent paradigm) that highlights the role of the firm as a means to induce productivity. Gibbons [1998], and Baron and Kreps [1999] present comprehensive discussions of this field. Papers related to particular aspects of the analysis will be mentioned in turn.

3. Contract Space

The paper limits attention to a class of simple contracts with two goals in mind. The first is tractability. The combination of multi-sided moral hazard and repeated interaction imposes technical challenges that can nevertheless be surmounted through suitable simplifications. The second goal is potential applicability. These contracts will have a clear empirical counterpart, together with multiple practical advantages (in terms of robustness and informational efficiency) that are not embedded in the model but are nevertheless likely to be relevant for real-world design.

The initial simplification involves the use of *stationary contracts,* which is without loss of optimality under the time-invariant environment I employ (see also Levin [2000]).[7]

[5] In fact, the entitlement to profit streams (i.e., the notion of ownership adopted herein) remains fixed in their analysis.

[6] See also Bragelien [1998] and Halonen [2001].

[7] Such invariance, however, will preclude consideration of important matters such as reputation building and job-training.

Next, explicit contracts are assumed to be linear functions of output x (i.e., profits) of the form $\alpha_i x + \omega_i$, where α_i is the ownership share in hands of player i, and ω_i is her court-enforced wage. Notice that proportional profit sharing, $\alpha_i x$, is at the core of every for-profit enterprise. Moreover, complex sharing rules are rarely observed.[8]

Finally, implicit contracts will take an additively separable form. Associated to each player, there will be a noisy performance signal y_i that serves as an imperfect measure of effort. The self-enforced voluntary transfer received by player i, denoted by τ_i, can in principle be any arbitrary function of the (non-contractible) signal vector y. However, in the contracts considered, $\tau_i(y)$ will be of the form: $b_i(y_i) + v_i(y_{-i})$. That is, the transfer will only depend on y_i through the function $b_i(\cdot)$ (e.g., a performance bonus), which in turn does not depend on other players' signals y_{-i}. The second function $v_i(y_{-i})$ will simply be used to balance the budget across players.[9]

Additive separability generates the robustness property that each player will exert the prescribed level of effort *regardless* of the signal produced by her peers. Equivalently, observing y_{-i} in advance will not alter player i's effort incentives. Conversely, whenever such robustness property is imposed, additive separability can be adopted without loss of optimality. This kind of robustness will be especially desirable (in a practical environment) whenever actions are not taken at precisely the same time, or if the evaluation period is long, allowing for an information flow concerning peer performance before all actions are taken. (Further advantages of separability are discussed once the details become clear.)

4. Model

N infinitely lived risk-neutral players, denoted by $i \in \{1, 2, ..., N\}$, interact each period $t = 0, 1, ...$ At date t, they take four kinds of actions: (i) the decision whether or not to *participate* in joint production for that period; (ii) in case they do participate, the level

[8]A notable exception is the use of derivative instruments such as stock options. Consideration of these instruments is left for future research. However, their use is by no means pervasive, and in any event it is convenient to first focus on the underlying asset α_i.

[9]Separable transfers rule out *relative performance* compensation schemes in which marginal incentives are affected by peer performance. Nonetheless, the main value of such schemes derives from the fact that, when the uncertainly behind y is not independent across players, measuring relative performance provides a statistical advantage, granting superior incentives (e.g., Mookherjee [1984]). In the model below, however, the signals will be independent and in this sense relative performance will be unnecessary. Moreover, as discussed at length by Baron and Kreps [1999, pp. 225-31], relative compensation schemes present abundant practical disadvantages (e.g., they would generally call for a complex process of handicapping that easily entails haggling and corruptibility), especially pronounced in the present framework when additive separability is relaxed.

of *effort* exerted towards production; (iii) *monetary transfers*; and (iv) the selection of *court enforced contracts* for the following period.[10]

Any subset of players can engage in production at date t, and each player that decides not to participate receives a fixed reservation (flow) payoff $\overline{u}_i(1-\delta)$, where $\delta \in (0,1)$ is the players' common discount factor, e.g., the players engaged in production may exclude the non-participating players from the proceeds of their work. If player i is the only one that decides to participate, she also receives $\overline{u}_i(1-\delta)$. However, I assume it is optimal for *every* player to participate in production. Consequently, for expositional purposes and (essentially) without loss of generality, it will be convenient to assume that whenever at least *one* player decides not to participate, joint production does not take place and *every* player receives her min-max payoff $\overline{u}_i(1-\delta)$.[11]

Each period has three stages, as shown in Figure 1. In the first, the players simultaneously take their participation decisions. Let $\pi_i^t \in \{0,1\}$ denote player i's decision to participate ($\pi_i^t = 1$) or not ($\pi_i^t = 0$) in period t; and define $\pi^t := \prod_i \pi_i^t$, which is simply the joint participation indicator function. If $\pi^t = 0$ each player gets her reservation value $\overline{u}_i(1-\delta)$ as assumed above. Nature then selects a vector of *types* $\theta^t \in \mathbb{R}^N$, where θ^t is i.i.d. across time, and each entry θ_i^t belongs to the interval $[\underline{\theta}_i, \overline{\theta}_i] \in \mathbb{R}$. Two cases will be considered: (i) θ_i^t is only privately known to player i, and (ii) θ^t is publicly observed.

In the second stage, provided $\pi^t = 1$, after (partially or totally) observing nature's selection, players simultaneously select effort levels $e_i^t \in \mathbb{R}_+$.[12] Each e_i^t will be private information, but it will produce a noisy signal y_i^t commonly observed by all players (but not by courts).

[10] MacLeod [1984] uses repeated team production to model a labor cooperative, but allows neither for an endogenous selection of ownership shares within the team, nor for the use of self-enforced performance payments.

[11] More precisely, although production by a proper subset of players may arise in a non-stationary optimal contract, such contract can be replaced without loss of optimality with a stationary contract that must involve participation by *all* players at every t on the path of play (by the above assumption that full participation is optimal). Furthermore, this contract can always be sustained by threat of *full* separation (i.e., the worst possible punishment), and thus *partial* participation will never play a role. In this sense, given that I focus on stationary contracts, the above simplification will be (essentially) without loss of generality (i.e., it will not alter the path of play). On the other hand, if we require renegotiation proofness (which can be done without loss of optimality as shown below), then any optimal stationary contract will involve participation by all players for every t and *every history*. Thus, partial participation will *never* play a role, and the simplification will be (fully) without loss of generality.

[12] Notice that a message stage regarding types θ_i, before the selection of e, is ruled out. Allowing for such message games would raise a broad set of issues beyond the scope of the paper (e.g., truth telling constraints would need to be added, interacting in non-trivial ways with the effort selection constraints that follow). In any event, only Theorem 6 below would be affected by type communication.

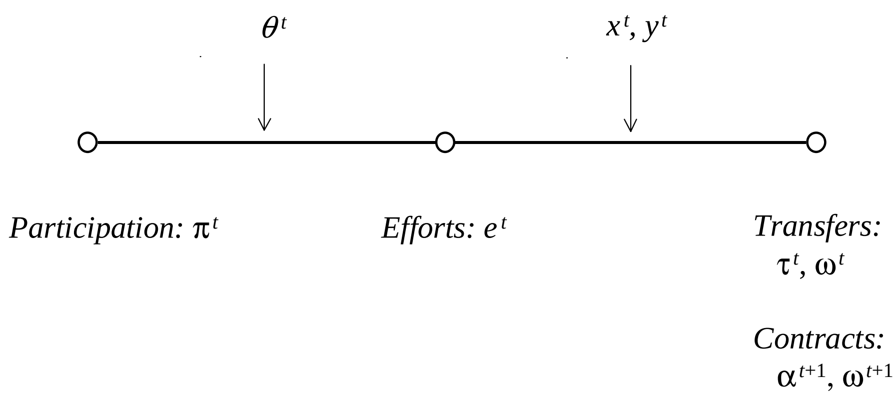

Figure 1. Period t Timing

In the third stage, they transfer money and select the court enforced contract for period $t+1$. There will be two kinds of monetary transfers: *voluntary* and *court-enforced*. Let $\tau_i^t, \omega_i^t \in \mathbb{R}$ denote the *net* voluntary and court enforced monetary transfer respectively, received by i in period t, and assume a balanced budget: $\sum_i \tau_i^t \leq \sum_i \omega_i^t = 0$.[13] A *court-enforced contract* for period t, denoted by $\langle \omega^t, \alpha^t \rangle$, will specify a vector ω^t of court enforced monetary transfers conditional on participation (i.e., wages), together with a vector of *ownership shares* α^t. Each α_i^t will represent the fraction of period t output (i.e., profits) to which i is entitled, so that $\alpha_i^t \in [0,1]$ and $\sum_i \alpha_i^t = 1$.[14]

The selection of $\langle \omega^{t+1}, \alpha^{t+1} \rangle$ works as follows. Each player simultaneously proposes a value for $\langle \omega^{t+1}, \alpha^{t+1} \rangle$ denoted by Γ_i^t, if they all agree (i.e., $\Gamma_1^t = ... = \Gamma_N^t$) then

[13] One could, more precisely, specify what transfer is made between each pair of agents. But such extra detail is unnecessary for the analysis.

Although in principle $\sum_i \tau_i^t$ may be negative (corresponding to payments to third parties or money burning), this will not arise in the optimal contracts studied below.

[14] Court-enforced severance payments and bonding (i.e., payments conditional on no participation) will be of no use provided they do not alter total reservation surplus $\sum_i \overline{u}_i$.

$\langle \omega^{t+1}, \alpha^{t+1} \rangle$ takes such value, and otherwise it remains equal to the previous period contract $\langle \omega^t, \alpha^t \rangle$.[15] In period $t = 0$ only the third stage is played, so the game begins with voluntary transfers and the selection of $\langle \omega^1, \alpha^1 \rangle$.

Provided joint participation takes place, period t joint output is given by $x^t \in \mathbb{R}_+$, a random variable stochastically determined by the effort vector e^t, and accrues to the players according to fractions α_i^t. The cost of effort for each player is given by $c_i(e_i^t, \theta_i^t) \in \mathbb{R}_+$. Throughout, I assume the expected value of per-period output conditional on effort levels, $E[x \mid e]$, and the cost functions c_i, do not change across time and satisfy, for all i:

Assumption (A1):

a. $E[x \mid e]$ *is smooth, strictly increasing, and concave in* e.

b. $c_i(e_i, \theta_i)$ *is smooth in both arguments, strictly increasing and strictly convex in* e_i, *and has decreasing differences in* (e_i, θ_i). *Also,* $c_i(0, \theta_i) = 0$ *for all* θ_i.

c. $E[x \mid e] - c_i(e_i, \theta_i) \to -\infty$ *as* $e_i \to \infty$ *for all* e_{-i} *and all* θ_i.

d. $\frac{\partial}{\partial e_i} c_i(0, \overline{\theta}_i) < \frac{\partial}{\partial e_i} E[x \mid 0, e_{-i}]$ *for all* e_{-i}, and $\frac{\partial}{\partial e_i} c_i(0, \theta_i) \to \infty$ as $\theta_i \to -\infty$.[16]

Smoothness is used for analytical convenience. Concavity of $E[x \mid \cdot]$ and strict convexity of $c_i(\cdot, \theta_i)$ will imply unique solutions. Decreasing differences for $c_i(\cdot, \cdot)$ means the cross-partials $\partial c_i^2 / \partial e_i \partial \theta_i$ are negative (e.g., single-crossing), so that higher types have a lower marginal cost. (A1c) guarantees that optimal efforts remain bounded. Finally, (A1d) implies that first-best effort is positive for (at least) the highest type $\overline{\theta}_i$, and zero for sufficiently low θ_i.

I assume payoffs are quasilinear in money and the cost of effort is additive, so that the period t expected payoff for player i, once nature has selected types, is given by

$$g_i^t = \pi^t \left\{ \alpha_i^t E\left[x \mid e^t\right] - c_i(e_i^t, \theta_i^t) + \omega_i^t \right\} + (1 - \pi^t)\overline{u}_i(1 - \delta) + \tau_i^t.$$

Let u_i^t denote the present value of future payoffs as of the beginning of period t (before the selection of θ^t), i.e.,

$$u_i^t = E_\theta \sum_{\tau=t}^{\infty} \delta^{\tau - t} g_i^\tau .$$

Players will publicly observe the entire history of output, signals y^t (and possibly θ^t), and all actions except e^t. But courts can only verify output levels, monetary transfers, and the decisions to participate (e.g., employment). Consequently, any actions that are

[15] Alternatively, it could be assumed that only the agreement of the affected parties is required to change a contract, but any non-unanimous procedure could be replaced by a unanimous one that uses the threat of permanent separation if agreement is not reached. On the other hand, the results would not change if we allowed for explicit contracting over future values of $\langle \omega, \alpha \rangle$ several periods in advance.

[16] Although $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$, I assume $c_i(e_i, \cdot)$ is well defined over $\mathbb{R}$.

contingent on y^t (and possibly θ^t), can only be enforced through implicit contracts, i.e., through equilibria of the repeated game.

In order to allow for a dynamic programming approach (Abreu, Pearce and Stacchetti [1990]) I will focus on (pure strategy) *perfect public equilibria* (Fudenberg, Levine and Maskin [1994]), where the period t actions of each player can only be conditioned on the commonly observed history, except for effort levels, which can also be conditioned on the (possibly) private information θ_i^t of that period alone.[17] Let h^t denote the commonly observed history at the beginning of period t. An *action profile* for player i at time t, $a_i^t(\cdot)$, is a collection of functions

$$\left\langle \pi_i^t(\cdot), e_i^t(\cdot), \tau_i^t(\cdot), \Gamma_i^t(\cdot) \right\rangle.$$

Where $\pi_i^t(\cdot)$ is a function of h^t. $e_i^t(\cdot)$ is a function of h^t, type θ_i^t, and possibly the type vector θ_{-i}^t. And $\tau_i^t(\cdot)$, $\Gamma_i^t(\cdot)$ depend on h^t as well as on the period t participation decisions, output levels, signals y^t, and possibly on the vector θ^t. However, I assume that the realization of x^t contains no information about e^t beyond what is conveyed by y^t, and therefore it will be optimal to restrict attention to contracts that do not condition the transfers $\tau^t(\cdot)$ on x^t (see also Holmstrom [1979] and Shavell [1979]).[18] In what follows, I consider only such contracts.

A *strategy* for player i, σ_i, is simply a collection of action profiles, one for each t: $\left(a_i^t(\cdot)\right)_{t=0}^{\infty}$. A *relational contract* is a strategy profile $\sigma := \langle \sigma_1, ..., \sigma_N \rangle$, and we say that a relational contract is self-enforcing *iff* it constitutes a Nash equilibrium following every history (which is equivalent to a perfect public equilibrium given the above restrictions on the strategy sets).

Section 5 develops some preliminary results that dramatically simplify the analysis. In particular, the use of stationary contracts will allow us to express the problem of optimal contract design as a "static" problem with an additional dynamic enforcement constraint, described in Section 6.

[17]One might wonder if an equilibrium that uses mixed and/or private strategies can yield a higher payoff. However, the only possible gain from randomizations will concern mixtures of effort levels, which can potentially improve effort-incentive compatibility. In particular, the marginal explicit incentives faced by player i, i.e., $\alpha_i \frac{\partial}{\partial e_i} E\left[x \mid e_i, e_{-i}\right]$, can be increased using a lottery over e_{-i} provided this derivative is convex in e_{-i}. (Concavity of $E\left[x \mid \cdot\ \right]$ and convexity of the cost functions will nevertheless make such randomization costly). Thus, weak concavity of the expected marginal products will provide a sufficient condition for pure strategies to be optimal, and consequently for public equilibria to convey no loss of optimality. Randomizations for monitoring purposes of the type considered in Kandori and Obara [2000] are of no value in the present setup since the public signal y has a product structure (Fudenberg, Levin and Maskin [1994]).

[18]The use of x as a possible public randomization device will not be valuable, since adding variability over voluntary transfers will only make their enforcement more difficult.

5. Preliminaries

As a benchmark, consider a single period of the team game taken in isolation: the stage game. To fix ideas, suppose θ is publicly observed. Since there is no future, after production takes place there is no incentive to make monetary transfers, and therefore $\tau_i = 0$ for all i. Thus, the only source of incentives is ownership α. Denote player i's static best responses to e_{-i} by

$$e_i^s(\alpha_i, e_{-i}, \theta_i) := \arg\max_{e_i} \{\alpha_i E[x \mid e_i, e_{-i}] - c_i(e_i, \theta_i)\}.$$

Also let $e^*(\theta)$ represent the unique first-best effort levels given θ (i.e., those that maximize joint surplus). Notice that, provided $e_i^*(\theta) > 0$ for two or more players, first-best cannot be achieved in this one-shot game because at least one player will not receive the full return to her effort. This is a special case of the teams problem in Holmstrom [1982, Theorem 1]. Finally, observe that no participation in production always constitutes a Nash equilibrium of the stage game.

We now return to the original infinite horizon game. Since players are always able to walk away from the relationship, we must have

$$(P) \qquad u_i^t \geq \overline{u}_i \text{ for all } i \text{ and all } t.$$

Namely, the worst possible equilibrium payoffs from any period onward are given by the present value of the worst stage game Nash equilibrium payoffs $\overline{u}_i(1-\delta)$. These inequalities will be referred to as *participation constraints* (P). Now let $\overline{s} := (1-\delta)\sum_i \overline{u}_i$, which represents the total reservation flow surplus. Likewise, let $s^t := (1-\delta)\sum_i u_i^t$ denote the average per-period surplus from date t onward. (i.e., "s" will be used to denote flow payoffs, while "u" represents present values.)

Lemma 1 is a straightforward extension of Levin [2000, Theorem 1] to the present multi-sided moral hazard setup. It states that the joint surplus created by any self-enforcing contract can be divided arbitrarily across peers provided their participation constraints are satisfied. Thus, the objective of contract design will be the maximization of (expected) joint surplus. All proofs are in Appendix 1.

Lemma 1. *Suppose some self-enforcing relational contract σ generates a total surplus $s^0/(1-\delta)$ larger than the reservation surplus $\delta\overline{s}/(1-\delta)$ (from the viewpoint of $t=0$). Then, for any $\lambda \in [0,1]^N$ with $\sum_i \lambda_i = 1$, there exists a self enforcing relational contract $\widetilde{\sigma}$ that generates surplus $s^0/(1-\delta)$ and payoffs $\widetilde{u}_i^0 = \overline{u}_i + \lambda_i\left(s^0 - \delta\overline{s}\right)/(1-\delta)$.*

The Lemma implies that the (individually rational) utility possibility frontier is linear, a consequence of the fact that payoffs are quasilinear in money, and utility can be transferred using court-enforced payments (which in turn are agreed upon through the threat of separation). In what follows, in the spirit of Levin [2000] I focus on *stationary* relational contracts, which will turn out to be optimal. A stationary contract will be defined conditional on no deviations regarding past voluntary transfers and contract proposals:

Definition 1. *A relational contract σ is* ***stationary*** *iff*

(a) for all t, i, and all histories, $\Gamma_i^t(\cdot) = \langle \omega, \alpha \rangle$ for some $\langle \omega, \alpha \rangle$.

(b) for all $t \geq 1$ and all i, conditional on no deviations in past voluntary transfer and contract proposal stages, we have that $a_i^t(\cdot)$ depends neither on h^t nor on t, i.e., $a_i^t(\cdot) = a_i(\cdot)$.

Notice that in a stationary relational contract, $\tau_i^t(\cdot)$ can still depend on participation decisions of the current period, on signals y^t, and possibly on θ^t, which will be key for the creation of incentives. Also, the above definition of stationarity differs from that of Levin [2000] in the sense that he conditions on no deviations in *any* past stage (i.e., the history must be on the equilibrium path). Thus, the above definition is more restrictive. Lemma 2 establishes the optimality of stationary contracts.

Lemma 2. *Suppose an optimal relational contract σ exists. Then, there exists an optimal relational contract $\widetilde{\sigma}$ that is stationary.*

Although the proof of the Lemma is lengthy, the driving force is straightforward. In a non-stationary contract, incentives will be provided (in part) through contingent continuation values u_i^{t+1} that deter shirking. However, these contingent continuation values can always be replaced by the combination of voluntary transfers and (possibly) money burning (i.e., $\sum_i \tau_i^t < 0$) at the end of each period, while maintaining u_i^{t+1} fixed. In other words, any "punishment" or "prize" following production can be settled on the spot, instead of through changes in future actions.

In addition to possible money burning, the optimal stationary contract $\widetilde{\sigma}$ built in the proof of the Lemma uses Nash reversion (i.e., permanent separation) to punish deviations at the money payment stage. Thus, the contract may be susceptible to *renegotiation*, which in turn could undermine incentives and potentially destroy the equilibrium. However, this can be avoided. On one hand, the contracts considered below will never involve money burning. Informally, this is because voluntary payments will be additively separable: $\tau_i(y) = b_i(y_i) + v_i(y_{-i})$, and the function $v_i(y_{-i})$ can be used to balance the budget without affecting effort incentives. (In what follows, I drop the redundant period t superscripts).

Nash reversion, on the other hand, can also be dispensed with. In fact, it is possible to replace any optimal stationary contract, that generates continuation surplus levels $\widehat{s}/(1-\delta)$ *on the path of play,* with a new self-enforcing contract that implements the same actions and that produces surplus levels of at least $\delta\widehat{s}/(1-\delta)$ *following any history.* This is achieved by punishing any deviation by player i, in period t, with the following actions in $t+1$: (i) no joint production, (ii) voluntary transfers and the agreement towards a contract that together produce a continuation surplus $\widehat{s}/(1-\delta)$, but only a continuation value of $\overline{u}_i$ for player i (which is possible due to Lemma 1). The fact that players must loose one period of production, following a deviation, is needed so that a new stage of transfers and agreement towards court enforced contracts is reached. Thus, an amount $\widehat{s}$ of total surplus is destroyed in the process. Nevertheless, it is fair to assume that any contract renegotiation among players also consumes at least one period, and therefore the above contract can never be renegotiated in favor of one that produces a higher surplus.

6. "Static" Design

It will be useful to informally describe the structure of the contract design problem, and discuss the general methodology used in what follows. Once stationary contracts are employed, the problem faced by the team can be expressed as a static one with an additional *dynamic enforcement constraint.* In particular, the problem is to select a stationary contract $\widehat{\sigma}$, composed of an effort schedule $\widehat{e}\,(\theta)$, an explicit contract $\langle\widehat{\alpha},\widehat{\omega}\rangle$, and contingent voluntary transfers $\widehat{\tau}(y)$ (or $\widehat{\tau}(y,\theta)$ if θ is public information), that maximizes the joint flow surplus

$$\widehat{s} := E_\theta \left\{ E\left[x \mid \widehat{e}(\theta)\right] - \sum_i c_i\left(\widehat{e}_i(\theta_i),\theta_i\right) \right\},$$

subject to three constraints (corresponding to each of the three stages of the period t game).

The first is the *participation constraint* (P) derived above, i.e., $\widehat{u}_i \geq \overline{u}_i$, where the continuation surplus $\widehat{u}_i$ is simply the present value of the expected per-period payoff under $\widehat{\sigma}$. The second is an *effort-incentive constraint*, denoted by (E), which requires that each player i finds it optimal to select the prescribed effort $\widehat{e}_i(\theta_i)$ (or $\widehat{e}_i(\theta)$ in the case of public information) *given* that: (i) the remaining players select $\widehat{e}_{-i}(\theta_{-i})$, and (ii) voluntary transfers $\widehat{\tau}(y)$ are credible. The third is a *voluntary transfer constraint* that requires transfers to be credible, denote this constraint by (T).[19] In particular, each player i must prefer to pay $-\widehat{\tau}_i(y)$ (whenever $\widehat{\tau}_i(y)$ is negative) over loosing her future net surplus $\delta\left[\widehat{u}_i - \overline{u}_i\right]$, regardless of the value of y. Thus, the constraint is given by

$$(T) \qquad -\inf_y \widehat{\tau}_i(y) \leq \delta\left[\widehat{u}_i - \overline{u}_i\right] \text{ for all } i.$$

[19] No additional constraint concerning the selection of the period $t+1$ court enforced contract will be required. Due to the threat of separation, constraint (P) will be sufficient for such purpose.

An important simplification will occur when transfers are additively separable, i.e., $\widehat{\tau}_i(y) = \widehat{b}_i(y_i) + \widehat{v}_i(y_{-i})$, and no money burning takes place. Indeed, the $2 \cdot N$ inequalities in (P) and (T) can be expressed as an aggregate *dynamic enforcement* inequality:[20]

$$(DE) \qquad \sum_i \left\{ \sup_{y_i} \widehat{b}_i(y_i) - \inf_{y_i} \widehat{b}_i(y_i) \right\} \leq \delta \sum_i \left[\widehat{u}_i - \overline{u}_i \right] = \frac{\delta}{1-\delta} \left[\widehat{s} - \overline{s} \right].$$

The right hand side of (DE) is the total amount of relational capital available to enforce voluntary transfers, while the left hand side is the *cost* of enforcing the bonus payments (in terms of relational capital), i.e., the minimum amount of relational capital needed to enforce them. Now let

$$\Delta b_i := \sup_{y_i} b_i(y_i) - \inf_{y_i} b_i(y_i),$$

which will be called the *power* of player i's implicit incentives. Informally, this power will require allocating discretion in hands of the team members j that *pay* such voluntary bonus. However, this discretion will only be used as dictated by the contract (and against short-run opportunism) if these members j fear the loss of future surplus $\delta \left[\widehat{u}_j - \overline{u}_j \right]$. Indeed, the higher the power of Δb_i, the higher the level of discretion in hands of players j, and the higher the values of $\delta \left[\widehat{u}_j - \overline{u}_j \right]$ must be, thus consuming more relational capital. Consequently, the *cost* of implicit incentives is precisely their *power*.

The next step will be the consolidation of constraints (E) and (DE) into a single *relational capital* inequality (RC), off of which we can read the optimal ownership structure. This is done in Sections 7 through 9 while specializing the model to different information environments.

7. Symmetric Information

Assume the vector of types θ is commonly observed by all players, and $y \equiv e$. Namely, there is no informational asymmetry across peers. As suggested in the previous section, the problem reduces to the static maximization of flow surplus, subject to constraints (E) (effort incentives) and (DE) (dynamic enforcement).

[20]To illustrate why this is the case, suppose $\widehat{v}_i(y_{-i})$ is equal to

$$-\frac{1}{N-1} \sum_{j \neq i} \widehat{b}_j(y_j),$$

i.e., the payment of each bonus $\widehat{b}_i(y_i)$ is shared equally across the remaining players (in fact, this as well as *any* other linear sharing rule will be optimal). Then, (T) will imply (DE) by adding up both sides of (T) across i. Conversely, whenever (DE) and (E) hold, net future surplus $\left[\widehat{s} - \overline{s} \right] / (1-\delta)$ can always be distributed across players (using ω) in a way that both (P) and (T) hold, while maintaining effort incentives (E). (The formal derivation is shown below.)

Under symmetric information, additive separability of voluntary transfers can be adopted without loss of optimality, and their use does not change the optimal ownership pattern (all formal derivations are in Appendix 1).[21] These transfers will depend on both e and θ: $\tau_i(e,\theta) = b_i(e_i,\theta) + v_i(e_{-i},\theta)$, and only the first function b_i will affect i's effort incentives. Thus, constraint (E) is given by

$$(E) \qquad \alpha_i E\left[x \mid \widehat{e}(\theta)\right] - c_i(\widehat{e}_i(\theta),\theta_i) + \widehat{b}_i(\widehat{e}_i(\theta),\theta) \geq \max_{e_i}\left\{\alpha_i E\left[x \mid e_i, \widehat{e}_{-i}(\theta)\right] - c_i(e_i,\theta_i) + \widehat{b}_i(e_i,\theta)\right\} \text{ for all } \theta.$$

(Where the variables with a " $\widehat{\cdot}$ " are those prescribed by the contract.) Constraint (DE), on the other hand, must account for the variability of θ, and in this case becomes

$$(DE) \qquad \sup_\theta \sum_i \left\{\sup_{e_i} \widehat{b}_i(e_i,\theta) - \inf_{e_i} \widehat{b}_i(e_i,\theta)\right\} \leq \frac{\delta}{1-\delta}\left[\widehat{s} - \overline{s}\right].$$

As shown in Proposition 1 below, these two constraints can be unified in a single inequality (RC). To see how this is done, consider again the power of implicit incentives (in this case, a function of θ):

$$\Delta b_i(\theta) := \sup_{e_i} b_i(e_i,\theta) - \inf_{e_i} b_i(e_i,\theta).$$

Given θ, it turns out that the least expensive way to induce player i to exert effort $\widehat{e}_i(\theta)$ is to set $\Delta b_i(\theta)$ equal to the short-term gain from the optimal static deviation. This short-term gain is obtained using the terms in (E) while ignoring the bonus payments, and is equal to

$$\varphi_i(\alpha_i,\widehat{e}(\theta),\theta_i) := \max_{e_i}\left\{\alpha_i E\left[x \mid e_i,\widehat{e}_{-i}(\theta)\right] - c_i(e_i,\theta_i)\right\} - \left\{\alpha_i E\left[x \mid \widehat{e}(\theta)\right] - c_i(\widehat{e}_i(\theta_i),\theta_i)\right\}.$$

Notice that if we set player i's bonus following a deviation an amount $\varphi_i(\alpha_i,\widehat{e}(\theta),\theta_i)$ lower than her bonus following cooperation, then she will be induced to follow the prescribed effort with the smallest possible bonus power. Once $\Delta b_i(\theta) = \varphi_i(\alpha_i,\widehat{e}(\theta),\theta_i)$, constraints (E) and (DE) reduce to constraint (RC) below, which depends only on $\widehat{e}_i(\theta)$ and α, and says that the sum of short-term gains from deviating cannot exceed relational capital:

Proposition 1. *Effort schedule $\widehat{e}(\cdot)$ can be implemented using a stationary relational contract with ownership structure α if and only if*

$$(RC) \qquad \sup_\theta \sum_i \varphi_i(\alpha_i,\widehat{e}(\theta),\theta_i) \leq \frac{\delta}{1-\delta}\left[\widehat{s} - \overline{s}\right],$$

[21]As formalized by Corollary 1 in the Appendix, additive separability will imply that money burning can be avoided.

where $\widehat{s}$ is the (expected) per-period surplus achieved under efforts $\widehat{e}(\cdot)$ when no money burning takes place:

$$\widehat{s} = E_\theta \left\{ E\left[x \mid \widehat{e}(\theta)\right] - \sum_i c_i(\widehat{e}_i(\theta), \theta_i) \right\}.$$

The analysis will be separated into two cases, according to whether or not first-best can be achieved. Let s^* denote the flow of first-best surplus, i.e., achieved when first-best efforts $e^*(\cdot)$ are implemented, and no money burning take place. Whenever $s^* > \overline{s}$ (which is assumed throughout), first-best can be achieved under *any* ownership structure provided δ is sufficiently close to 1. However, it will be relevant to derive the ownership structure that implements $e^*(\cdot)$ in the least expensive way (in terms of relational capital). Equivalently, this efficient ownership structure will implement first-best for the *lowest* possible δ, or the *highest* possible $\overline{s}$, for which first-best can be achieved.

On the other hand, when first-best cannot be achieved we have:

Lemma 3. *Under any second-best stationary relational contract, constraint* (RC) *must bind.*

Consequently, regardless of whether first-best can be achieved or not, I search for the efficient ownership structure that minimizes the left hand side of (RC). Theorems 1 and 2 are based on the solution to this minimization problem, and contain the main results of the section.

Theorem 1. *Suppose first-best surplus can be achieved. Then, there exists a first-best stationary relational contract under which:*

- ***(a)*: *ownership is non-extreme, i.e.,* $\alpha_i < 1$ *for all* i,**
- ***(b)*: *all players receive implicit incentives, i.e.,* $\sup_\theta \Delta b_i(\theta) > 0$ *for all* i.**

Theorem 1 follows from the shape of the (short-run) incentives to deviate $\varphi_i(\cdot)$.[22] These functions will be decreasing and convex in α_i. In fact, application of the envelope theorem yields:

$$\frac{\partial}{\partial \alpha_i}\varphi_i(\alpha_i, e^*(\theta), \theta_i) = E\left[x \mid e_i^s, e_{-i}^*(\theta)\right] - E\left[x \mid e^*(\theta)\right],$$

which is negative for all $\alpha_i < 1$ because the static best response e_i^s will be smaller than $e_i^*(\theta)$. Since e_i^s increases with α_i, this derivative will also be increasing in α_i, and equal to *zero* when $\alpha_i = 1$.

[22] Garvey [1995] presents a result in a sense related to Theorem 1. In a two-agent simple example, he compares the optimal distribution of ownership under a one-shot vs. a repeated game. Under the specific technology considered, ownership becomes more symmetric in the repeated case. In his model, since no monetary transfers are possible, non-extreme ownership occurs by assumption.

Recall that $\varphi_i(\alpha_i, e^*(\theta), \theta_i)$ represents the *cost* of implicit incentives, and equals the bonus power Δb_i. Thus, the negative of the above derivative equals the *incentive trade-off*

$$(*) \qquad -\frac{\partial \Delta b_i}{\partial \alpha_i},$$

which portrays the marginal *savings* in relational capital due to increased ownership in hands of player i. But since these marginal savings are *decreasing*, fully concentrated ownership will be inefficient. Informally, when player i possesses all shares, a marginal reduction in α_i will only cause a *second-order* increase in Δb_i, i.e., $\partial \Delta b_i / \partial \alpha_i|_{\alpha_i = 1} = 0$. But allocating these additional shares in hands of any other player j for which $e_j^*(\theta) > 0$ (and thus $-\partial \Delta b_j / \partial \alpha_j > 0$) will yield a *first-order* decrease in Δb_i. The combined effect will be a reduction in the need for relational capital, rendering $\alpha_i = 1$ inefficient. (The formal argument, shown in Appendix 1, is more subtle because it requires considering the effect of a changing θ, but follows an analogous method.)

The decreasing incentive trade-off $(*)$ can be used to say more about the efficient value of α for a fixed θ. For example, if all players are identical, they will hold equal shares. On the other hand, notice that $(*)$ equals the reduction in total output when a player reduces effort to her static best response. Thus, if a player is relatively "productive", in the sense that a deviation to her static best response causes a higher loss in output than a deviation from each of her peers (under equal ownership shares), then she will receive a higher share of profits. (Under a changing value of θ the analysis is more involved, but the same type of pattern emerges.)

The results in Theorem 1 can be extended to second-best contracts provided two conditions hold:

C1-Joint Effort: $e(\cdot)$ *is such that* $\sup_\theta e_i(\theta) > 0$ *for at least two players.*

C2-Complementarity: For all i, $\frac{\partial}{\partial e_i} E[x \mid e_i, e_{-i}]$ *is non-decreasing in* e_{-i}.

Condition (C1) says that at least two players exert positive effort (for some θ), and it will be sufficient to guarantee that ownership is never concentrated in hands of a single player. (If only one player j exerts positive effort, i.e., (C1) is violated, then it will be optimal to set $\alpha_j = 1$.)

Condition (C2) states that individual efforts are complementary, and implies that they can only produce positive externalities across peers. Under (C2) implicit incentives will be provided to all (productive) players.[23] Intuitively, analogous to the reasoning above,

[23](C2) is not required when type θ plays no role, e.g., when $\underline{\theta}_i = \overline{\theta}_i$ for all i. It can also be dispensed with whenever the efforts prescribed by the contract are at least as large as the static best responses:

$$\widehat{e}_i(\theta) \geq e_i^s(\widehat{\alpha}_i, \widehat{e}_{-i}(\theta), \theta_i) \text{ for all } i \text{ and all } \theta.$$

if a given player receives only explicit incentives, a marginal amount of ownership shares can be taken away from her while causing only a *second-order* increase in her incentive to deviate.[24] These shares can then be transferred to a player who previously received implicit incentives while *first-order* reducing her incentive to deviate. The result will be a relaxation of (RC), violating Lemma 3. However, this *first-order* reduction in the second player's incentive to deviate will only occur if she is prescribed a *higher* effort than her static best response, which in turn will be guaranteed by (C2).[25]

Theorem 2. *Suppose first-best surplus cannot be achieved. Then, any second-best stationary relational contract σ satisfies:*

- ***(a)*:** *whenever (C1-Joint Effort) holds, ownership is non-extreme,*
- ***(b)*:** *whenever (C2-Complementarity) holds and σ delivers a per-period surplus strictly higher than $\overline{s}$, implicit incentives are provided for every player that exerts positive effort (for some θ).*

In contrast to Theorems 1 and 2, the two sections that follow develop extreme ownership results under asymmetric information.

8. Hidden Actions

Consider now the case of *hidden actions* where individual efforts are only privately observed, but stochastically influence the publicly observed vector y. Each $y_i \in [\underline{y}, \overline{y}] \subset \mathbb{R}$ will be a noisy signal of effort e_i, and will be distributed according to the smooth conditional cumulative function $F(y_i \mid e_i)$ with full support, which will not depend on e_{-i}, i.e., conditional on e_i, each y_i will be i.i.d. across players.[26] Let f denote the density of F. The two assumptions below will be imposed throughout the section. Following Rogerson [1985], they allow for a first-order approach to effort-incentive compatibility.

Assumption (A2): The likelihood ratio $\frac{f_e}{f}(y_i \mid e_i)$ is increasing in y_i for all e_i.
Assumption (A3): $F(y_i \mid e_i = c_i^{-1}(z))$ is convex in z.

(A2) states that a higher e_i always stochastically increases the value of y_i (in the sense of first-order stochastic dominance). (A3) implies that the marginal impact on y_i of increasing the cost of effort, z, is stochastically decreasing, i.e., the net marginal returns to effort in terms of y_i are decreasing. Thus, (A3) is a joint condition on F and c_i, and will hold whenever c_i is sufficiently convex in e_i.

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[24]This again is a consequence of the envelope theorem: if a player receives no implicit incentives, then her prescribed effort level $\widehat{e}_i$ must equal her static best response e_i^s, and thus $\partial\varphi_i/\partial\alpha_i = 0$.

[25]This issue did not arise under first-best because $\alpha_i \leq 1$ implies that the static best responses will never be higher than first-best effort levels.

[26]y will also be (conditionally) i.i.d. across time. The analysis would not change if we allowed for a different function $F_i(y_i \mid e_i)$ for each player, this is avoided for notational simplicity.

In order to focus on the effect of hidden actions, I initially assume that type θ plays no role, so that $c_i(e_i, \theta_i) = c_i(e_i)$ for all i and θ_i, e.g., $\underline{\theta}_i = \overline{\theta}_i$. The case where θ does play a role is discussed in Appendix 2. Voluntary transfers will be required to be additively separable: $\tau_i(y) = b_i(y_i) + v_i(y_{-i})$, which is without loss of optimality when instead we require that the choice of e_i is "robust" to the value of y_{-i}.[27] Formally:

Lemma 4. *Let σ be a self-enforcing stationary contract that implements efforts $\widehat{e}$ using transfer schedule $\tau(\cdot)$. Suppose $\tau(\cdot)$ is such that the effort-incentive constraints for all players are also satisfied whenever $\tau_i(y)$ is replaced by $\tau_i(y_i, \widetilde{y}_{-i})$ for any fixed $\widetilde{y}_{-i}$. Then, there exists a self-enforcing stationary contract that implements efforts $\widehat{e}$ using additively separable transfers, and that yields a (weakly) higher surplus than σ.*

As shown in Lemma 5 below, a further advantage of separable bonuses is that they never call for money burning. The Lemma also shows that each budget-balancing function $v_i(y_{-i})$ can be expressed, with no loss, as a linear combination of the functions $b_j(y_j)$, for $j \neq i$. The coefficients of these linear combinations will be indeterminate, implying that any player can be made the (partial or total) residual claimant of her peer's bonuses if and only if she is endowed with a large enough continuation surplus $\delta[u_i - \overline{u}_i]$. Since the utility possibility frontier is linear (i.e., Lemma 1), the players will therefore be perfect substitutes in terms of such bonus-paying (or budget-balancing) task.

Lemma 5. *Let σ be a stationary self-enforcing contract that implements effort levels $\widehat{e}$ using additively separable voluntary transfers $\tau(\cdot)$. Also, fix constants $\gamma_i^j \in [0,1]$ such that $\sum_{i \neq j} \gamma_i^j = 1$ for all j (i.e., γ_i^j will be the fraction of bonus b_j paid by player i). Then, there exists a stationary self-enforcing relational contract $\widetilde{\sigma}$ that implements effort levels $\widehat{e}$ using transfers $\widetilde{\tau}$ such that:*

$$(\gamma) \qquad \widetilde{\tau}_i(y) = b_i(y_i) - \sum_{j \neq i} \gamma_i^j b_j(y_j) \text{ for all } i \text{ and all } y,$$

for some functions $b_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}$. Thus, no money burning takes place.

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[27] Under a two-sided moral hazard environment similar to the present one (but with fixed ownership shares), Dewhurst [2000] solves for the optimal non-separable voluntary transfer schemes. These have the "bang-bang" property that, for each realization of y, one party (e.g., the one with the lowest y_i) transfers all her future surplus to the other party. When applied to a multi-agent setup, this extreme form of comparative performance compensation will be especially subject to the practical disadvantages discussed by Baron and Kreps [1999], e.g., in this case, for each y, $N-1$ players will transfer all their future surplus to the single remaining player according to a pre-determined handicapping rule (of doubtful plausibility). Moreover, these schemes call for money burning, which further subtracts realism.

The contract design problem now becomes

$$\max_{\widehat{e},\alpha,b(\cdot)} \left\{ E\left[x \mid \widehat{e}\,\right] - \sum_i c_i(\widehat{e}_i) \right\} \; s.t.$$

$$(E) \qquad \widehat{e}_i \in \arg\max_{e_i} \left\{\alpha_i E\left[x \mid e_i, \widehat{e}_{-i}\right] + E\left[b_i(y_i) \mid e_i\right] - c_i(e_i)\right\} \text{ for all } i,$$

$$(DE) \qquad \sum_i \Delta b_i \leq \frac{\delta}{1-\delta}\left[s - \overline{s}\right].$$

This problem is greatly simplified if we replace constraints (E) with the corresponding first-order conditions

$$(FOC) \qquad \alpha_i \frac{\partial}{\partial e_i} E\left[x \mid \widehat{e}\,\right] + \frac{\partial}{\partial e_i} E\left[b_i(y_i) \mid \widehat{e}_i\right] - c_i'(\widehat{e}_i) = 0 \text{ for all } i.$$

In fact, under (A2)–(A3), it turns out that a cost-effective way (in terms of (DE)) to induce effort is to use "one-step" non-decreasing bonus payments, under which (FOC) and (E) are equivalent.[28] These one-step bonuses take only two values, according to whether y_i is above or below a certain cut-off level $\widehat{y}_i$. Given the desired effort $\widehat{e}_i$, this cut-off is defined by

$$f_{e_i}(\widehat{y}_i \mid \widehat{e}_i) = 0.$$

Under (A2), $f_{e_i}(y_i \mid \widehat{e}_i)$ will be negative for all $y_i < \widehat{y}_i$, and positive for all $y_i > \widehat{y}_i$. Since signals below $\widehat{y}_i$ will convey "bad" news in terms of effort, player i will be punished by receiving only $\inf_{y_i} b_i(y_i)$, while "good" signals $y_i > \widehat{y}_i$ will be rewarded by $\sup_{y_i} b_i(y_i)$. (When $y_i = \widehat{y}_i$ either bonus can be paid.)

Although during the voluntary transfer stage (in equilibrium) it will be common knowledge that the prescribed efforts $\widehat{e}$ were indeed selected, it is useful to interpret the one-step schemes as implicitly performing a statistical test. In fact, a (uniformly most powerful) Neyman–Pearson test of the null hypothesis "$e_i = \widehat{e}_i$", versus the alternative "$e_i < \widehat{e}_i$" (i.e., it is the *downward* local effort constraints that will be binding) will have a rejection region of the form $\{y_i < \widetilde{y}_i\}$ for some $\widetilde{y}_i$. Thus, the optimal test for these hypotheses of size $F(\widehat{y}_i \mid \widehat{e}_i)$ has precisely $\{y_i < \widehat{y}_i\}$ as its rejection region. (The scheme then pays the low bonus when the test "concludes" that an effort $e_i < \widehat{e}_i$ was selected.)

The next step is to combine (FOC) and (DE) into a single inequality (RC) that no longer depends on voluntary payments. Notice that under the above one-step bonuses, the first-order conditions become

$$(FOC) \qquad \alpha_i \frac{\partial}{\partial e_i} E\left[x \mid \widehat{e}\,\right] - \Delta b_i F_{e_i}(\widehat{y}_i \mid \widehat{e}_i) - c_i'(\widehat{e}_i) = 0.$$

[28]This result generalizes the optimality of one-step schemes derived by Levin [2000].

From (FOC) we can solve for Δb_i, and substitute into (DE), to obtain

$$(RC) \qquad \sum_i \Delta b_i = \sum_i \{p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e})\} \leq \frac{\delta}{1-\delta}\left[\widehat{s} - \overline{s}\right],$$

where the functions $p_i(\widehat{e}_i)$ and $q_i(\widehat{e})$ contain the terms in (FOC):

$$p_i(\widehat{e}_i) := \frac{c_i'(\widehat{e}_i)}{-F_{e_i}\left(\widehat{y}_i \mid \widehat{e}_i\right)}, \text{ and } q_i(\widehat{e}) := \frac{\frac{\partial}{\partial e_i} E\left[x \mid \widehat{e}\,\right]}{-F_{e_i}\left(\widehat{y}_i \mid \widehat{e}_i\right)}.$$

(RC) can then be used to derive the optimal ownership structure. However, the optimal contract must satisfy N additional conditions. Since the equivalence of (FOC) and (E) requires the bonuses to be *non-decreasing,* we need $\Delta b_i = p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e}) \geq 0$ for all i. The above reasoning is formalized in Proposition 2.

Proposition 2. *Effort vector $\widehat{e}$ can be implemented using a stationary contract σ with ownership structure α if and only if (RC) holds and*

$$(ND) \qquad \Delta b_i = p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e}) \geq 0 \textit{ for all } i.$$

The analysis is again separated into two cases, depending on whether or not first-best can be achieved. In either case, as in the previous section, I focus on ownership structures that minimize the left hand side of (RC) (subject to (ND)). These will correspond to the *most efficient* structures under first-best (i.e., Theorem 3), and to the *only* optimal structures under second-best because (RC) will bind (i.e., Theorem 4).

Theorem 3. *Suppose first-best surplus can be achieved. Then, there exists a first-best stationary contract with extreme ownership $\alpha_k = 1$ concentrated over some player $k \in \arg\max_i q_i(e^*)$, and therefore $\Delta b_k = 0$.*

Under first-best efforts e^*, $p_i(e_i^*) = q_i(e^*)$ (i.e., $c_i'(e_i^*) = \frac{\partial}{\partial e_i} E\left[x \mid e^*\right]$) and therefore the inequalities in (ND) will be redundant. Thus, an efficient contract solves the linear problem $\max_\alpha \sum_i \alpha_i q_i(e^*)$, achieved by concentrating ownership in hands of the players in $\arg\max_i q_i(e^*)$, or in hands of any single one among them. (Indeed, in a "generic" environment we can expect $\arg\max_i q_i(e^*)$ to be a singleton.) Thus, a high level of $q_i(e^*)$ promotes ownership. In fact, $q_i(e^*)$ is equal to the incentive trade-off $-\partial \Delta b_i / \partial \alpha_i > 0$, representing the marginal savings in relational capital brought on by increased ownership in hands of player i.

The incentive trade-off does not depend on α_i (intuition for this is provided below), and will be high when two conditions are combined: (i) a *high* marginal productivity $\frac{\partial}{\partial e_i} E\left[x \mid e^*\right]$, and (ii) a *low* $-F_{e_i}\left(y_i^* \mid e_i^*\right)$, where y_i^* is the optimal bonus cut-off value. The quantity $-F_{e_i}\left(y_i^* \mid e_i^*\right)$ will be positive due to (A2), and proportional to the *power* of the above Neyman–Pearson test against local deviations $e_i < e_i^*$, given by $F\left(y_i^* \mid e_i\right)$.

In particular, if we set $e_i = e_i^* - \epsilon$ for some small $\epsilon > 0$, and Taylor-expand this power $F(y_i^* \mid e_i)$ around e_i^*, we obtain:

$$F(y_i^* \mid e_i) \cong F(y_i^* \mid e_i^*) - F_{e_i}(y_i^* \mid e_i^*)\,\epsilon.$$

Thus, a low value for $-F_{e_i}(y_i^* \mid e_i^*)$ corresponds to a *poor* test, and implies that deviations by player i are hard to detect. Therefore, ownership will be placed in hands of a player when she possesses a high *marginal productivity* combined with a high level of *informational asymmetry* with respect to her peers, in the sense that *her* actions are hard to assess.

Theorem 4 deals with second-best contracts. As before, given efforts $\widehat{e}$, there will be an incentive to concentrate ownership in hands of the players within $\arg\max_i q_i(\widehat{e})$. But, due to Constraint (ND), this can only be done as long as $\Delta b_i = p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e}) \geq 0$. Since under second-best we will typically have $c_i'(\widehat{e}_i) < \frac{\partial}{\partial e_i} E[x \mid \widehat{e}\,]$ (and therefore $p_i(\widehat{e}_i) < q_i(\widehat{e})$), Δb_i may equal zero for $\alpha_i < 1$, and hence full concentration may not arise. Nonetheless, concentration over some player $k \in \arg\max_i q_i(\widehat{e})$ to the point where she receives *zero implicit incentives* will always be optimal.

Theorem 4. *Suppose first-best cannot be achieved. Then, there exists an optimal contract such that some player* $k \in \arg\max_i q_i(\widehat{e})$ *receives no implicit incentives, i.e.,* $\Delta b_k = 0$ *(where* $\widehat{e}$ *denotes the optimal effort levels).*

Moreover, assume $\arg\max_i q_i(\widehat{e})$ *is a singleton. Then, under any optimal contract that implements* $\widehat{e}$*, player* $k = \arg\max_i q_i(\widehat{e})$ *receives no implicit incentives.*

The above extreme ownership results greatly diverge from the efficiency of dispersed ownership under symmetric information (section 7). This difference is accounted for by the way ownership impacts the incentive trade-off, in turn a consequence of which effort-incentive constraints bind. Under symmetric information, any effort deviation will be detected with certainty and can be punished with equal strength. As a consequence, the most tempting deviation will be the global one towards the static best response e_i^s. Formally, it is this *global* constraint that binds. Moreover, the size of the most tempting deviation $[\widehat{e}_i - e_i^s]$ is decreasing in α_i. Thus, an increase in ownership not only reduces the incentive to incur in a given deviation, but also reduces the *size* of the optimal deviation. This second effect, responsible for the curvature of φ_i, implies that the incentive trade-off is decreasing.

Under asymmetric information, in contrast, small deviations are the hardest to detect and therefore become especially tempting. Together with the fact that players face a concave problem, this implies that only *local* constraints will bind. Therefore, changes in α_i will not alter the size of the most tempting deviation (i.e., it is always a local one), so the above curvature effect will disappear, resulting in a constant incentive-trade off. This intuition will also apply to the case of hidden information that follows.

9. Hidden Information

This section considers the case where each type θ_i is only privately known to player i. In order to focus on the effect of hidden information, y is set equal to e (as in Section 7). For expositional simplicity, I also assume that output takes an additively separable form, i.e., Assumption (A4) below. However, the results that follow also hold for the more general technologies in (A1). This generalization and some caveats are discussed in Appendix 3.

Assumption (A4): $E[x \mid e] = \sum_i x_i(e_i)$, for some functions $x_i(\cdot)$.[29]

Due to (A1a), each $x_i(\cdot)$ will be smooth, increasing, and concave. Also normalize $x_i(0) = 0$.

If we consider the effort selection stage in isolation, (A4) implies that any effort schedule $e(\cdot)$ that is implementable using a Bayesian mechanism, will also be implementable in *dominant strategies* (while balancing the budget), adding considerable robustness to the contract (e.g., the details of the distribution of θ will be irrelevant, and learning peers' efforts in advance will not alter incentives).

Indeed, in order to motivate the use of separable transfers, I will impose this requirement of dominant strategy implementation for the effort selection stage, i.e., the prescribed effort $\widehat{e}_i(\theta_i)$ is required to be optimal for player i *regardless* of the choice of e_{-i}. (Notice that such requisite is analogous to that imposed in the previous section: robustness to y_{-i}.) As shown in Lemma 6, under dominant strategy implementation, additively separable bonuses can be adopted without loss of optimality.[30] Conversely, any additively separable bonuses will implement efforts in dominant strategies, while eliminating the need to burn money.

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[29](A4) will represent an environment where tasks are relatively independent from each other. But one may then wonder why the team is together in the first place, i.e., why doesn't each player create a firm on her own? A possible reason would be the existence of a fixed set-up cost that is shared across players. As argued by Williamson [1975], this cost may refer to a physical asset or even to the acquisition of information. Radner [1970, p.457] notes that "the acquisition of information often involves a 'setup cost'; i.e., the resources needed to obtain the information may be independent of the scale of the production process". Furthermore, Williamson [1975, p. 49] submits that: "(1) there are reasons other than nonseparabilities for internal organization to appear; (2) nonseparabilities are much less widespread than is commonly believed".

[30]Complex comparative performance schemes may save some relational capital even in this case of separable output, but would loose robustness.

Lemma 6. *Let σ be a stationary self-enforcing relational contract that implements effort levels $\widehat{e}(\cdot)$ in dominant strategies (for the effort selection stage). Then, there exists a self-enforcing relational contract $\widetilde{\sigma}$ that implements effort levels $\widehat{e}(\cdot)$ in dominant strategies using voluntary transfers $\widetilde{\tau}$ such that:*
(a) No money burning takes place:

$$\sum_i \widetilde{\tau}_i(e) = 0 \text{ for all } e.$$

(b) Transfers are additively separable:

$$\widetilde{\tau}_i(e) = b_i(e_i) - v_i(e_{-i}), \text{ for all } i \text{ and } e,$$

for some functions b_i and v_i.

Once additive transfers are adopted, the key step of the analysis (as in previous sections) will be the consolidation of constraints (DE) (i.e., $\sum_i \Delta b_i \leq \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}]$) and (E) (i.e., the effort selection constraint) into a single inequality, off of which we can read the optimal ownership structure. This amounts to solving for the power of bonuses Δb_i in terms of α and the desired effort schedule $\widehat{e}(\cdot)$.

Consider initially the case where first-best can be sustained. In order to implement $e^*(\cdot)$, the power Δb_i must be large enough to discourage two kinds of deviations: those that are *observed* by peers and those that are not. In particular, whenever player i selects an effort within $e_i^*([\underline{\theta}_i, \overline{\theta}_i])$ (i.e., the range of efforts occurring on the path of play), her peers (who do not know θ_i) cannot distinguish whether she is indeed following the prescribed effort $e_i^*(\theta_i)$ or she is deviating to an effort prescribed for some other type. In contrast, when $e_i \notin e_i^*([\underline{\theta}_i, \overline{\theta}_i])$, it will become public knowledge that she indeed deviated and can be punished accordingly.

The second sort of deviations will be analogous to those considered in Section 7 (under full information), so Δb_i must be at least as large as the gain from performing the optimal static deviation e_i^s. Since $e_i^s < e_i^*(\theta_i)$ (provided $\alpha_i < 1$), the only profitable deviations outside $e_i^*([\underline{\theta}_i, \overline{\theta}_i])$ will be towards some $e_i < e_i^*(\underline{\theta}_i)$. Moreover, under single-crossing, the type that most gains from this kind of deviation is $\underline{\theta}_i$, and her gain is given by

$$\varphi_i(\alpha_i, e_i^*(\underline{\theta}_i), \underline{\theta}_i) := \max_{e_i} \{\alpha_i x_i(e_i) - c_i(e_i, \underline{\theta}_i)\} - \{\alpha_i x_i(e_i^*(\underline{\theta}_i)) - c_i(e_i^*(\underline{\theta}_i), \underline{\theta}_i)\},$$

representing the power of b_i required to hinder such observable deviations.

Deviations within $e_i^*([\underline{\theta}_i, \overline{\theta}_i])$, on the other hand, will be deterred through an adequate selection of the *slope* of $b_i(e_i)$. Using a first-order approach to the incentive problem (e.g., Mirrlees [1971], Milgrom and Segal [2002]), Appendix 1 shows that this slope must be equal to the *marginal externality* caused on peers:

$$(S) \qquad (1 - \alpha_i)x_i'(e_i),$$

i.e., $b_i(e_i)$ will align private and social marginal benefits by internalizing such external effect. By integrating (S), and letting $x_i^*(\theta) := x_i(e_i^*(\theta))$, it follows that

$$b_i(e_i^*(\overline{\theta}_i)) - b_i(e_i^*(\underline{\theta}_i)) = (1-\alpha_i)\left[x_i^*(\overline{\theta}_i) - x_i^*(\underline{\theta}_i)\right],$$

which defines the incentive power necessary to deter deviations within $e_i^*([\underline{\theta}_i, \overline{\theta}_i])$. Let $\Delta x_i^*(\underline{\theta}_i, \overline{\theta}_i) := \left[x_i^*(\overline{\theta}_i) - x_i^*(\underline{\theta}_i)\right]$.

As formalized in Proposition 3, by combining the analysis for both kinds of deviations, we obtain the total necessary power of implicit incentives:

$$\Delta b_i = \varphi_i(\alpha_i, e_i^*(\underline{\theta}_i), \underline{\theta}_i) + (1-\alpha_i)\,\Delta x_i^*(\underline{\theta}_i, \overline{\theta}_i).$$

Proposition 3. *There exists a self-enforcing contract that achieves first-best surplus using ownership structure α if and only if*

$$(RC) \qquad \sum_i \varphi_i(\alpha_i, e_i^*(\underline{\theta}_i), \underline{\theta}_i) + \sum_i (1-\alpha_i)\,\Delta x_i^*(\underline{\theta}_i, \overline{\theta}_i) \le \frac{\delta}{1-\delta}\left[s^* - \overline{s}\right].$$

Under first-best, the most efficient ownership structure will minimize the left hand side of (RC). The first term, $\sum_i \varphi_i$, corresponding to observable deviations, has the properties derived in Section 7 (under symmetric information). Thus, according to Theorem 1, it provides an incentive to disperse ownership. The new term, $\sum_i (1-\alpha_i)\,\Delta x_i^*$, associated with unobservable deviations, indicates that a high Δx_i^* promotes ownership in hands of i. Δx_i^* will be related to both the *productivity* of i and the level of *informational asymmetry* she bears with respect to her peers.

High informational asymmetry can be though of as a large dispersion in type θ_i, i.e., a large interval $\left[\underline{\theta}_i, \overline{\theta}_i\right]$. As $\underline{\theta}_i$ decreases and $\overline{\theta}_i$ increases, the dispersion in efforts will increase and so will Δx_i^*. In other words, a large Δx_i^* will endow i with a large range of deviations that will go undetected.

Furthermore, when $\underline{\theta}_i$ is sufficiently low, $e_i^*(\underline{\theta}_i)$ and $x_i^*(\underline{\theta}_i)$ will be zero, and thus *any* desirable deviation (i.e., $e_i < e_i^*(\overline{\theta}_i)$) will go undetected. Condition (C3), representing *extreme* informational asymmetry, describes this case.

C3-Asymmetry: $e_i(\underline{\theta}_i) = 0$ for all i.

Under (C3), the range Δx_i^* will correspond to its upper limit $x_i^*(\overline{\theta}_i)$. Moreover, a high marginal productivity $x_i'(e_i)$ will imply a high value for $x_i^*(\overline{\theta}_i)$, and therefore it promotes ownership as well.

Notice also that (C3) implies $\sum_i \varphi_i = 0$ (i.e., $e_i^s = e_i^*(\underline{\theta}_i) = 0$ for all i), so the incentive trade-off becomes

$$-\frac{\partial \Delta b_i}{\partial \alpha_i} = \Delta x_i^*.$$

These returns will be independent of α_i because the binding effort constraints are always local. Thus, according to (RC), any efficient ownership structure will concentrate ownership over the players in $\arg\max_i \Delta x_i^*$, or in hands of any single one of them:

Theorem 5. *Suppose $e^*(\cdot)$ is implementable and (C3-Asymmetry) holds. Then, it is optimal to concentrate ownership over any single player $k \in \arg\max_i \Delta x_i^*$, and therefore $\Delta b_k = 0$.*

When (C3) does not hold, extreme ownership will still be efficient whenever: $\Delta x_k^* - \Delta x_j^* \geq x_j^*(\underline{\theta}_j)$ for all $j \neq k$. This condition simply says that the marginal advantage of increasing α_k, i.e., $\Delta x_k^* - \Delta x_j^*$ (captured by the second term in (RC)), will be larger than the marginal benefit of increasing ownership in hands of some other player (captured by the second term in (RC)), which is at most $-\partial\varphi_j/\partial\alpha_j \mid_{\alpha_j=0} = x_j^*(\underline{\theta}_j)$. On the other hand, when Δx_i^* is the same across players, Theorem 1 applies and ownership will be dispersed.

Consider now the case of second-best contracts. Although ownership in general will not be fully concentrated,[31] a result in the lines of Theorem 4 would say that it is optimal to concentrate ownership in hands of a player k to the point where she receives no implicit incentives. However, such a result need not be true. The reason is that explicit incentives, which are linear in x_i, cannot fully substitute for implicit incentives when these are non-linear. (Recall that, under first-best, implicit incentives are indeed linear.) Thus, increased ownership may not fully eliminate the need for bonuses. This nonetheless suggests that ownership concentration will result if we restrict bonuses to be linear in x_i, which is confirmed by Theorem 6.[32]

Let $\widehat{x}_i(\theta_i) := x_i(\widehat{e}_i(\theta_i))$, where $\widehat{e}(\cdot)$ denotes the second-best effort schedule. For technical reasons, linearity will only be imposed over the equilibrium domain $\widehat{x}([\underline{\theta}_i, \overline{\theta}_i])$, because otherwise bonuses would be unbounded and thus not credible for some histories.

Theorem 6. *Suppose (C3-Asymmetry) holds for any optimal contract, i.e., $\Delta\widehat{x}_i = \widehat{x}_i(\overline{\theta}_i)$, and voluntary transfers must be linear on the path of play, i.e., for all i we have $b_i(\widehat{x}_i(\theta_i)) = \lambda_i + \beta_i \widehat{x}_i(\theta_i)$ for some constants λ_i and β_i, and all θ_i. Then, there exists an optimal contract under which some player $k \in \arg\max_i \Delta\widehat{x}_i$ receives no implicit incentives, i.e., $\beta_k = 0$.*

Moreover, assume $\arg\max_i \Delta\widehat{x}_i$ is a singleton. Then, under any second-best contract that implements $\widehat{x}(\cdot)$, player $k = \arg\max_i \Delta\widehat{x}_i$ receives no implicit incentives.

[31] i.e., $\alpha_i = 1$ would imply that $e_i(\cdot) = e_i^*(\cdot)$, which need not be desirable under second-best.

[32] Linear schemes, i.e., piece rates, are commonly adopted. A possible "robustness" reason for this is given by Holmstrom and Milgrom [1987].

Since instruments α_i and β_i are perfect substitutes in terms of inducing effort (i.e., $\partial\alpha_i/\partial\beta_i \,|_{\widehat{e}_i}= -1$), the incentive trade-off will be equivalent to that under first-best:

$$-\frac{\partial \Delta b_i}{\partial \alpha_i} = -\frac{\partial \left\{\beta_i \Delta \widehat{x}_i\right\}}{\partial \alpha_i} = \Delta \widehat{x}_i.$$

As above, $\Delta\widehat{x}_i$ will be associated to marginal productivity and informational asymmetry.

Finally, when we do not restrict bonuses to be linear, a high $\Delta\widehat{x}_i$ will also promote ownership. In particular, to the extent that α_i can substitute for (the linear part of) b_i, we will have $-\partial\Delta b_i/\partial\alpha_i = \Delta\widehat{x}_i$, and thus ownership concentration will be optimal to such an extent.[33]

10. Partnerships and the "Classical Firm"

This section relates the previous results to patterns described by Alchian and Demsetz [1972], and Hansmann [1996]. The above model would best resemble relatively simple closely-held firms (as opposed to large publicly owned corporations).[34] Within this class, which constitute the overwhelming majority of all enterprises, shared ownership is dominant in service professions (e.g., law, accounting, investment banking, management consulting, advertising, architecture, engineering, and medicine) where "the productivity of individual employees can be, and generally is, monitored remarkably closely", Hansmann [1996, p. 70]. Such firms are typically composed of a relatively small number of individuals of the same profession, who understand well what their peers are doing. Their work is also relatively independent from each other and thus easier to assess.[35] And in terms of adverse selection, "employee ownership tends to appear in precisely those settings in which management is likely to have relatively little difficulty understanding employees' preferences", Hansmann [1996, p. 73].

Employee ownership, on the other hand, rarely occurs in the industrial sector and in non-professional services (e.g., hotels and retailing), where a larger task diversity is likely to entail informational asymmetries. Indeed, centralized ownership and hierarchy are the trade-mark of what Alchian and Demsetz call the "classical firm", wherein ownership

[33] As an illustration, using Milgrom and Segal's [2002] envelope Theorem 2 we obtain:

$$b_i(x_i(\overline{\theta}_i)) - b_i(x_i(\underline{\theta}_i)) = c_i(e_i(\overline{\theta}_i),\overline{\theta}_i) - c_i(e_i(\underline{\theta}_i),\underline{\theta}_i) + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial}{\partial\theta} c_i(e_i(z),z)dz - \alpha_i\Delta x_i,$$

which only depends on α_i through the last term, and its derivative w.r.t. α_i is precisely Δx_i. (The full derivation can be seen from the proof of Proposition 3.)

[34] At least one feature of the model that precludes an adequate consideration of very large firms is that any team member can be made the sole residual claimant of output.

[35] Alchian and Demsetz [1972, p. 779] conclude that for a "cooperative productive activity [...] measuring *marginal* productivity and making payments in accord therewith is more expensive by an order of magnitude than for separable production functions".

and control are concentrated in the hands of the manager/monitor. Underlying this widespread form of organization, they observe, is a difficulty to detect performance. In fact, they view clustered ownership and control as a response to the problem of asymmetric information, i.e., the owner monitors employees, and her incentive emerges from the residual claim of earnings.[36] The above analysis also identifies informational asymmetry with hierarchy, but through a different channel: the optimal use of relational capital.

11. Summary and Conclusions

In the above the analysis, the unifying principle behind the optimal allocation of ownership is the *incentive trade-off*

$$-\left.\frac{\partial \Delta b_i}{\partial \alpha_i}\right|_{\widehat{e}},$$

i.e., the marginal rate of substitution between the power of implicit incentives Δb_i, and explicit incentives α_i, under the desired effort profile $\widehat{e}$. By indicating the marginal savings in *relational capital* achieved by increasing ownership, this trade-off isolates two key factors behind organizational design: *marginal productivity* and *informational asymmetry*. The following table summarizes the specific value for the incentive trade-off under the three information environments considered.

Information Environment	$-\partial \Delta b_i / \partial \alpha_i \|_{\widehat{e}}$
Symmetric Information	$E[x \mid \widehat{e}\,] - E[x \mid e_i^s, \widehat{e}_{-i}]$
Hidden Actions	$-\frac{\partial}{\partial e_i} E[x \mid \widehat{e}\,] / F_{e_i}(\widehat{y}_i \mid \widehat{e}_i)$
Hidden Information	$\Delta \widehat{x}_i$

When information is fully symmetric (Section 7), the trade-off is given by the difference between expected output under the prescribed effort $\widehat{e}_i$ and expected output under the optimal static deviation e_i^s. This difference, which depends positively on marginal productivity, is decreasing in α_i. Thus, marginal returns to ownership shares are decreasing, and consequently it will be optimal to disperse ownership (while partly favoring productive members). Under hidden actions, on the other hand, the trade-off depends positively on marginal productivity $\frac{\partial}{\partial e_i} E[x \mid \widehat{e}\,]$ and negatively on $-F_{e_i}(\widehat{y}_i \mid \widehat{e}_i)$, the power of an optimal statistical test regarding deviations by member i. Therefore, when a player's actions are both productive and hard to assess, the marginal benefits of ownership (in her hands) are high. Similarly, under hidden information, the returns to ownership will be associated with member i's productivity and the difficulty (experienced by i's peers) of

[36] In a context of information gathering (regarding the optimal course of a firm's action) and costly communication, Segal [1996] argues that concentrating the managerial task also improves incentives.

detecting her deviations, both of which increase $\Delta\widehat{x}_i$. Under both asymmetric information environments, the incentive trade-off will not depend on α_i, implying that extreme ownership (to the point where $\Delta b_i = 0$) will be efficient. Moreover, extreme ownership will be the *only* efficient structure when (even infinitesimal) differences exist in the above idiosyncracies.

The second aspect of organizational design concerns the payment of voluntary transfers. In all three cases, there is considerable liberty in allocating the task of paying bonuses, together with the *discretion* required. In fact, discretion can be given to *any* member as long as she is also allocated sufficient relational capital (in the form of continuation surplus). But since the utility possibility frontier for the team is linear, the available relational capital can be freely distributed across members, making them perfect substitutes in performing this task.[37] Nonetheless, an important difference across information environments does arise.

When information is symmetric, every (productive) player will receive implicit incentives and therefore cross-monitoring is inevitable. It is also possible to cast bonuses in terms of *punishments* following deviations instead of *prizes* following high effort (or any combination of both). As a consequence, transfers can be limited to punishment stages of the game, and thus never occur on the equilibrium path of play.

Under asymmetric information, in contrast, transfers *will* occur in equilibrium. However, since ownership is concentrated in hands of a single player to a degree that she receives no implicit incentives, this player need not be monitored by her peers. In addition, she can be charged with the task of paying *all* of her peers' voluntary bonuses, and thus serves as an endogenously selected *principal*. This organizational structure resembles Alchian and Demsetz's [1971] classical firm, where the metering activity is focused in the hands of a single owner-manager.

The concentration of ownership and metering has an important advantage. Since transfers are additively separable, once the principal handles the metering task, cross-monitoring between peers is unnecessary, i.e., the only relevant information for the (endogenously selected) agents is their *own* effort measure y_i, needed to asses if they indeed received the stipulated bonus. Thus, the unique cross-informational requisite is the principal's knowledge of all agents signals y_i, which involves $N-1$ "pieces" of information, and hence this structure constitutes the most efficient one in terms of information requirements. (In the other extreme, if the payment task is fully dispersed across the team, each member i will need to learn y_{-i}, and therefore $N(N-1)$ pieces of cross-information must be processed.)

[37] MacLeod and Malcomson [1998] consider a repeated agency model where the form of the labor market determines how future surplus must be allocated, thus eradicating such flexibility.

I conclude with a possible direction for future work. Two important simplifications in the model are that the quality of information is exogenous (i.e., investments towards improving information are not considered), and the information structure across team members is "homogeneous" in the sense that the performance vector y is *publicly* observed by the team (e.g., any two players have the same information regarding their $N-2$ peers). Allowing for non-homogeneous and/or endogenous information structures may produce multi-layer hierarchies (and multiple divisions) where subsets of players with relatively homogeneous information are grouped under principals, which in turn are subject to the metering discretion of yet higher ranked players. If successful, such an approach would provide insights for more complex corporations, based again on the subtle interplay between implicit and explicit contracting.

12. Appendix 1: Proofs

Proof of Lemma 1. Let $\widetilde{\sigma}$ be identical to σ except for the fact that the new period zero monetary transfers, $\widetilde{\tau}_i^0$, are now equal to $\tau_i^0 - u_i^0 + \delta\overline{u}_i + \lambda_i \left(s^0 - \delta\overline{s}\right)\frac{1}{1-\delta}$ (which maintain a balanced budget), and non-payment of these transfers is punished by permanent separation. Notice that $\widetilde{\sigma}$ is self-enforcing from period $t=1$ onward by construction. Moreover, if the $t=0$ transfers are made, the payoffs under the new contract will be $\widetilde{u}_i^0 = \delta\overline{u}_i + \lambda_i\left(s^0 - \delta\overline{s}\right)\frac{1}{1-\delta}$, as required by the Lemma. Thus, it suffices to verify that these transfers are indeed credible, which will occur whenever:

$$-\widetilde{\tau}_i^0 \leq \delta\left[\widetilde{u}_i^1 - \overline{u}_i\right] \text{ for all } i,$$

where $\widetilde{u}_i^1$ are the continuation values under the new contract. But observe that:

$$\begin{aligned}
-\widetilde{\tau}_i^0 &\leq -\widetilde{\tau}_i^0 + \lambda_i\left(s^0 - \delta\overline{s}\right)\frac{1}{1-\delta} \\
&= -\widetilde{\tau}_i^0 + \widetilde{u}_i^0 - \delta\overline{u}_i \\
&= \delta\left[\widetilde{u}_i^1 - \overline{u}_i\right],
\end{aligned}$$

where the second equality follows from the fact that $\widetilde{u}_i^0 = \widetilde{\tau}_i^0 + \delta\widetilde{u}_i^1$. ∎

Proof of Lemma 2. Let $s^0/(1-\delta)$ be the expected surplus generated under σ (recall that $s^t/(1-\delta)$ is the surplus generated from period t onward, which may depend on h^t), and assume $s^0 > \delta\overline{s}$ (otherwise any stationary contract that implements the no-participation stage game Nash equilibrium outcome every period would be optimal).[38] Assume also that, under σ, $\sum_i \tau_i^0 = 0$, because otherwise the contract could be improved by setting these transfers to zero (which would not alter incentives).

[38]Notice that any surplus occurring at period 1 or later is discounted when comparing it with s^0 because production does not take place at $t=0$.

Observe that s^0 must be at least as large as δs^2, for any history h^1. Because otherwise there would exist a history $h^{1\prime}$ and a contract σ' that specifies the same actions as σ, following $h^{1\prime}$, but one period in advance, creating a higher surplus than $s^0/(1-\delta)$. Moreover, this implies joint production must take place at $t = 1$.

Consider first the case where θ is publicly observed. The proof will build a stationary contract $\widetilde{\sigma}$ that creates expected flow surplus $\widetilde{s}^0 = s^0$ and that implements, in every period, the $t = 1$ effort levels specified by σ; denote these by $\widehat{e}_i(\theta)$. Also let $\tau_i(y,\theta)$ (with $\sum_i \tau_i(y,\theta) \leq 0$) and $u_i(y,\theta)$ denote, respectively, the $t = 1$ monetary transfers and continuation payoffs (from period 2 onward) achieved under the original contract after type vector θ and signals y are realized at $t = 1$. Finally, let α denote the $t = 1$ shares under σ. Since σ is self-enforcing by hypothesis, we must have:

$$(P) \qquad \begin{gathered} E_\theta \left\{\alpha_i E_x \left[x \mid \widehat{e}(\theta)\right] - c_i(\widehat{e}_i(\theta), \theta_i) + E_y \left[\tau_i(y,\theta) \mid \widehat{e}(\theta)\right] + \delta E_y \left[u_i(y,\theta) \mid \widehat{e}(\theta)\right]\right\} \\ \geq \overline{u}_i \text{ for all } i, \text{ and} \\ u_i(y,\theta) \geq \overline{u}_i \text{ for all } i, \ y \text{ and } \theta, \end{gathered}$$

which are required for participation in production;

$$(E) \qquad \begin{aligned} &\alpha_i E_x \left[x \mid \widehat{e}(\theta)\right] - c_i(\widehat{e}_i(\theta), \theta_i) + E_y \left[\tau_i(y,\theta) \mid \widehat{e}(\theta)\right] + \delta E_y \left[u_i(y,\theta) \mid \widehat{e}(\theta)\right] \\ &\quad \geq \alpha_i E_x \left[x \mid e_i, \widehat{e}_{-i}(\theta)\right] - c_i(e_i, \theta_i) + E_y \left[\tau_i(y,\theta) \mid e_i, \widehat{e}_{-i}(\theta)\right] \\ &\quad\quad + \delta E_y \left[u_i(y,\theta) \mid e_i, \widehat{e}_{-i}(\theta)\right] \text{ for all } i, \ \theta, \text{ and } e_i, \end{aligned}$$

which is required so that efforts $\widehat{e}_i(\theta)$ are indeed selected; and

$$(T) \qquad -\tau_i(y,\theta) \leq \delta \left[u_i(y,\theta) - \overline{u}_i\right] \text{ for all } i, \ y \text{ and } \theta,$$

so that transfers are credible.

Now let the new stationary contract $\widetilde{\sigma}$ have, for all i and all $t \geq 1$: $\widetilde{\pi}_i^t = 1$ if and only if no deviations have occurred in the past (i.e., any deviation is punished with permanent separation); $\widetilde{e}_i^t(\theta) = \widehat{e}_i(\theta)$; $\left\langle \widetilde{\omega}^t, \widetilde{\alpha}^t \right\rangle = \langle \widetilde{\omega}, \alpha \rangle$ for some $\widetilde{\omega}$ specified below (i.e., $\Gamma_i^t(\cdot) \equiv \langle \widetilde{\omega}, \alpha \rangle$ for all $t \geq 1$); and $\widetilde{\tau}_i^t(\cdot) = \widetilde{\tau}_i(\cdot)$, where $\widetilde{\tau}_i(\cdot)$ (a function of current variables only) satisfies

$$(i) \qquad \widetilde{\tau}_i(y,\theta,\pi) = 0 \text{ whenever } \pi = 0,$$

and, whenever $\pi = 1$,

$$(ii) \qquad \widetilde{\tau}_i(y,\theta) = \tau_i(y,\theta) + \delta u_i(y,\theta) - \delta \widetilde{u}_i.$$

Where the values of $\widetilde{u}_i$ in (ii) are given by:

$$\widetilde{u}_i = E_\theta E_y \left[u_i(y,\theta) \mid \widehat{e}(\theta)\right] \text{ for } i < N, \text{ and} \qquad (iii)$$

$$\widetilde{u}_N = \frac{s^0}{\delta(1-\delta)} - \sum_{i<N} \widetilde{u}_i, \qquad (iv)$$

which correspond to the stationary expected continuation payoffs, under the new contract, from any period onward. These values are achieved by selecting the court enforced payments $\widetilde{\omega}$ such that:

$$\widetilde{\omega}_i = (1-\delta)\widetilde{u}_i - E_\theta\left\{\alpha_i E_x\left[x \mid \widehat{e}(\theta)\right] - c_i(\widehat{e}_i(\theta),\theta_i) + E_y\left[\widetilde{\tau}_i(y,\theta) \mid \widehat{e}(\theta)\right]\right\},$$

which add up to zero due to (iv), since the flow surplus created under $\widetilde{\sigma}$ is given by:

$$\widetilde{s}^0 = s^0 = \delta E_\theta\left\{E_x\left[x \mid \widehat{e}(\theta)\right] - \sum_i c_i(\widehat{e}_i(\theta),\theta_i) + \sum_i E_y\left[\widetilde{\tau}_i(y,\theta) \mid \widehat{e}(\theta)\right]\right\}.$$

It must now be shown that the above actions indeed constitute an equilibrium. Let (P'), (E'), and (T') denote the inequalities corresponding to (P), (E), and (T), but under the new contract. From the reasoning in the second paragraph of this proof, we must have $s^0/(1-\delta) \geq \delta s^2/(1-\delta) = \delta\sum_i u_i(y,\theta)$ for all y and θ, so that $s^0/(1-\delta) \geq \delta\sum_i E_\theta E_y\left[u_i(y,\theta) \mid \widehat{e}(\theta)\right]$. Thus, from (iii) and (iv), $\widetilde{u}_N \geq E_\theta E_y\left[u_N(y,\theta) \mid \widehat{e}(\theta)\right]$. Which, together with (P), (ii) and (iii), implies (P'). On the other hand, (E), (T) and (ii) directly imply (E') and (T').

Finally, (ii) and the fact that $\sum_i \widetilde{u}_i \geq \sum_i u_i(y,\theta)$, for all y and θ, imply a balanced budget:

$$\sum_i \widetilde{\tau}_i(y,\theta) = \sum_i\left[\tau_i(y,\theta) + \delta u_i(y,\theta) - \delta\widetilde{u}_i\right] \leq 0.$$

On the other hand, the proof for the case where the values of θ_i^t are private information is a special case of the above proof, except for one difference mentioned below. This special case is one in which the above functions $\widehat{e}_i(\theta)$, $\tau_i(y,\theta)$, and $u_i(y,\theta)$, now take the form $\widehat{e}_i(\theta_i)$, $\tau_i(y)$, and $u_i(y)$. The only difference arises from the fact that, in condition (E), every function that depends upon θ_{-i} must be replaced by its conditional expectation over θ_{-i} given θ_i, after which the proof applies to the case of private information. ∎

Proof of Proposition 1. For necessity, suppose $\widehat{e}(\cdot)$ is implemented by σ using shares α. Then, for all i and all θ we must have:

$$\begin{aligned} & \alpha_i E\left[x \mid \widehat{e}(\theta)\right] - c_i(\widehat{e}_i(\theta),\theta_i) + \tau_i(\widehat{e}(\theta),\theta) \\ \geq\ & \max_{e_i}\left\{\alpha_i E\left[x \mid e_i, \widehat{e}_{-i}(\theta)\right] - c_i(e_i,\theta_i) + \tau_i(e_i,\widehat{e}_{-i}(\theta_{-i}),\theta)\right\}, \end{aligned} \qquad (E)$$

in order for $\widehat{e}(\cdot)$ to be selected given transfers $\tau_i(\cdot)$; and

$$-\inf_{e,\theta}\tau_i(e,\theta) \leq \delta\left[u_i - \overline{u}_i\right] \text{ for all } i, \qquad (T)$$

$$\sum_i \tau_i(e,\theta) \leq 0 \text{ for all } e \text{ and all } \theta, \qquad (B)$$

so that transfers are credible and the budget is balanced (where u_i denote the continuation payoffs under σ). But (E) implies that for all i and all θ:

$$\varphi_i(\alpha_i,\widehat{e}(\theta),\theta_i) \leq \tau_i(\widehat{e}(\theta),\theta) - \inf_{e_i}\tau_i(e_i,\widehat{e}_{-i}(\theta_{-i}),\theta).$$

Thus, summing the above equations across i we obtain:

$$\begin{aligned}\sum_i \varphi_i(\alpha_i, \widehat{e}(\theta), \theta_i) &\leq -\sum_i \inf_{e_i} \tau_i(e_i, \widehat{e}_{-i}(\theta_{-i}), \theta) \\ &\leq \delta \sum_i [u_i - \overline{u}_i] \\ &= \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}] + \frac{\delta}{1-\delta} E_\theta \sum_i \tau_i(\widehat{e}(\theta), \theta) \\ &\leq \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}] \text{ for all } \theta,\end{aligned}$$

where the first inequality follows from (B); the second inequality from (T); the equality from the fact that, due to the stationarity σ, the continuation surplus $\sum_i u_i$ must equal the present value of $\widehat{s}$ minus the expected level of money burning per period; and the last inequality is again a consequence of (B). From which (RC) follows directly.

For sufficiency, suppose (RC) holds for shares α and efforts $\widehat{e}(\cdot)$, and let σ be a stationary contract that prescribes $\widehat{e}(\cdot)$ using the following continuation values u_i (achieved under appropriate selection of ω) and transfer scheme $\tau_i(\cdot)$:

$$u_i = \overline{u}_i \text{ for all } i < N, \text{ and } u_N = \frac{\widehat{s}}{1-\delta} - \sum_{i<N} \overline{u}_i, \qquad (i)$$

$$\tau_i(e, \theta) = b_i(e_i, \theta) - \frac{1}{N-1} b_N(e_N, \theta) \text{ for all } i < N, \text{ and} \qquad (ii)$$

$$\tau_N(e, \theta) = b_N(e_N, \theta) - \sum_{i<N} b_i(e_i, \theta), \text{ where} \qquad (iii)$$

$$b_i(e_i, \theta) = \begin{cases} \varphi_i(\alpha_i, \widehat{e}(\theta), \theta_i) & \text{if } e_i = \widehat{e}_i(\theta), \\ 0 & \text{otherwise,} \end{cases} \quad \text{for all } i < N,$$

$$b_N(e_N, \theta) = -\begin{cases} 0 & \text{if } e_i = \widehat{e}_i(\theta), \\ \varphi_N(\alpha_N, \widehat{e}(\theta), \theta_N) & \text{otherwise.} \end{cases}$$

By construction the budget will be balanced (indeed no money burning will take place following any history), and effort levels $\widehat{e}(\cdot)$ will be selected given $\tau_i(\cdot)$. It thus remains to show that such transfers will be credible. Since $\tau_i(e, \theta) \geq 0$ for all e, θ, and all $i < N$, it suffices to show that player N will find it optimal to make her corresponding transfers, for whom we have:

$$\begin{aligned}-\inf_{e,\theta} \tau_N(e, \theta) &= \sup_\theta \sum_i \varphi_i(\alpha_i, \widehat{e}(\theta), \theta_i) \\ &\leq \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}] = \delta [u_N - \overline{u}_N],\end{aligned}$$

where the first equality follows from (ii) and (iii); the inequality follows from (RC); and the last equality follows from (i). ∎

Corollary 1. *Any optimal stationary relational contract involves no money burning on the path of play. Moreover, any optimal relational contract σ can be replaced by an optimal stationary relational contract $\widetilde{\sigma}$ that involves no money burning following any history.*

Proof. The sufficiency proof of the above Proposition shows that, whenever (RC) holds under ownership structure α, there exists a stationary contract that implements the corresponding effort levels $\widehat{e}(\theta)$ using no-money burning following any history. Thus, the Corollary follows from the necessity direction of the above proposition and Lemma 2 (stationarity). ∎

Proof of Lemma 3. Let $\widehat{e}(\cdot)$ denote the effort schedule implemented by a second-best contract σ. Since first-best cannot be achieved, we must have $\widehat{e}(\cdot) \neq e^*(\cdot)$ over a positive measure subset of the type space. Now suppose contrary to the claim that constraint (RC) holds with slack. Also suppose we change the effort schedule to:

$$\widetilde{e}(\theta) := \lambda e^*(\theta) + (1-\lambda)\widehat{e}(\theta),$$

for some small $\lambda > 0$. Due to the continuity of $E[x \mid \cdot]$ and c_i, for all small λ constraint (RC) will still hold under schedule $\widetilde{e}(\cdot)$, which makes it feasible. But, due to the concavity of the objective (i.e., (A1)), $\widetilde{e}(\cdot)$ will deliver a strictly higher surplus than $\widehat{e}(\cdot)$, a contradiction to the fact that σ was optimal. ∎

Lemma 7. *(Based on Conditions (C1) and (C2) in the text.) Let σ be an optimal stationary contract, with ownership structure $\widehat{\alpha}$, that implements effort schedule $\widehat{e}(\cdot)$. Assume*

$$(*) \qquad \widehat{\alpha} \in \arg\min_{\alpha} \left\{ \sup_{\theta} \sum_i \varphi_i(\alpha_i, \widehat{e}(\theta), \theta_i) \right\}.$$

Then:

(a) Under (C1-Joint Effort) we must have $\sup_i \widehat{\alpha}_i < 1$, i.e., non-extreme ownership.

(b) Under (C2-Complementarity) whenever $\sup_\theta \sum_i \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta), \theta_i) > 0$ we have: $\sup_\theta \widehat{e}_i(\theta) > 0 \Rightarrow \sup_\theta \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta), \theta_i) > 0$ for all i, i.e., implicit incentives are provided for every player that (for some θ) exerts positive effort.

Proof. The proof will first deal with part (b) of the Lemma, and then part (a) will be shown as a special case. Due to the optimality of σ, under (C2) we must have that[39]

$$(i) \qquad \widehat{e}_i(\theta) \geq e_i^s(\widehat{\alpha}_i, \widehat{e}_{-i}(\theta), \theta_i) \text{ for all } i \text{ and all } \theta.$$

To see this suppose that (i) were not true for a given θ. Then, $\widehat{e}_i(\theta)$ could be increased to the static best response $e_i^s(\widehat{\alpha}_i, \widehat{e}_{-i}(\theta), \theta_i)$ simultaneously for all players i for which (i) was violated, which will weakly increase the static best responses of all players (due to (C2)), and such process

[39]More precisely, (i) could hold only over a full measure subset of the type space. But for simplicity, and without loss of optimality, this type of distinction is omitted throughout the paper.

can be repeated until convergence is achieved and (i) holds. But these changes will strictly increase surplus due to (A1) and relax (RC), a contradiction.

For notational simplicity, the proof will assume $E\left[x \mid e\right] = \sum_i e_i$. The general case follows exactly the same steps.

Suppose, towards a contradiction of (b), that WLOG $\sup_\theta \varphi_1(\widehat{\alpha}_1, \widehat{e}(\theta), \theta_1) = 0$ and $\sup_\theta \widehat{e}_1(\theta) > 0$. Then, due to (A1) we must have $\widehat{e}_1(\theta) = e_1^s(\widehat{\alpha}_1, \widehat{e}_{-1}(\theta), \theta_1)$ for all θ, and $\widehat{\alpha}_1 > 0$. It will now be shown that any small transfer $(N-1)\epsilon$ of ownership from player 1 to the rest of the players will reduce the level of $\sup_\theta \sum_i \varphi_i(\alpha_i, \widehat{e}(\theta), \theta_i)$, that is:

$$\begin{aligned} &\sup_\theta \left\{ \varphi_1(\widehat{\alpha}_1 - (N-1)\epsilon, \widehat{e}(\theta), \theta_1) + \sum_{i>1} \varphi_i(\widehat{\alpha}_i + \epsilon, \widehat{e}(\theta), \theta_i) \right\} \qquad (ii) \\ < \; &\sup_\theta \sum_i \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta), \theta_i) \text{ for all small } \epsilon > 0, \end{aligned}$$

which contradicts $(*)$.

Using a Taylor expansion we have, for all θ':

$$\begin{aligned} & \varphi_1(\widehat{\alpha}_1 - (N-1)\epsilon, \widehat{e}(\theta'), \theta_1') + \sum_{i>1} \varphi_i(\widehat{\alpha}_i + \epsilon, \widehat{e}(\theta'), \theta_i') \qquad (iii) \\ = \; & \sum_i \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta'), \theta_i') \\ & + \frac{d}{d\epsilon}\left[\varphi_1(\widehat{\alpha}_1 - (N-1)\epsilon, \widehat{e}(\theta'), \theta_1') + \sum_{i>1} \varphi_i(\widehat{\alpha}_i + \epsilon, \widehat{e}(\theta'), \theta_i') \right]_{\epsilon=0} \epsilon + o(\epsilon^2) \\ = \; & \sum_i \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta'), \theta_i') - \sum_{i>1} \left[\widehat{e}_i(\theta') - e_i^s(\widehat{\alpha}_i, \widehat{e}_{-i}(\theta'), \theta_i')\right] \epsilon + o(\epsilon^2) \\ \leq \; & \sup_\theta \left\{ \sum_i \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta), \theta_i) - \sum_{i>1} \left[\widehat{e}_i(\theta) - e_i^s(\widehat{\alpha}_i, \widehat{e}_{-i}(\theta), \theta_i)\right] \epsilon \right\} + o(\epsilon^2), \end{aligned}$$

where the second equality and the existence of the derivatives follow from Milgrom and Segal's [2002] Theorem 3 on directional derivatives, and assumption (A1). (Notice that the above derivatives are directional because $d\epsilon > 0$. Moreover, the continuity of c_i (and $E\left[x \mid \cdot\right]$) from (A1), and the fact that $\widehat{e}(\cdot)$ must be bounded, guarantee that the hypotheses of such Theorem 3 are met).

Now select a sequence θ^n such that:

$$(iv)\qquad \lim_{n\to\infty}\left\{\sum_i \varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta^n),\theta_i^n)-\sum_{i>1}\left[\widehat{e}_i(\theta^n)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta^n),\theta_i^n)\right]\epsilon\right\}$$
$$=\sup_\theta\left\{\sum_i \varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)-\sum_{i>1}\left[\widehat{e}_i(\theta)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta),\theta_i)\right]\epsilon\right\},\text{ and}$$
$$(v)\qquad \lim_{n\to\infty}\left[\widehat{e}_i(\theta^n)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta^n),\theta_i^n)\right]\text{ exists for all } i,$$

which is possible because the sequence $[\widehat{e}_i(\theta^n)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta^n),\theta_i^n)]$ must lie in a bounded space (due to (A1)).

Let $\epsilon>0$ be sufficiently small so that $\left[\widehat{e}_i(\widetilde{\theta})-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\widetilde{\theta}),\widetilde{\theta}_i)\right]\epsilon\le\varphi_i(\widehat{\alpha}_i,\widehat{e}(\widetilde{\theta}),\widetilde{\theta}_i)$ for all i and some $\widetilde{\theta}$, with strict inequality for some i, which is possible because by hypothesis we have $\sup_\theta\sum_i\varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)>0$, implying that there exist an i and a $\widetilde{\theta}$ such that $\varphi_i(\widehat{\alpha}_i,\widehat{e}(\widetilde{\theta}),\widetilde{\theta}_i)$ >0. Thus, we must have

$$(vi)\qquad \sup_\theta\left\{\sum_i \varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)-\sum_{i>1}\left[\widehat{e}_i(\theta)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta),\theta_i)\right]\epsilon\right\}>0.$$

Let $K_i:=\lim_{n\to\infty}\left[\widehat{e}_i(\theta^n)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta^n),\theta_i^n)\right]\ge 0$ (where the inequality follows from (i)), and notice that the combination of (vi) with (iv), (v), and the continuity of c_i (and $E[x\mid\cdot]$), implies that $K_j>0$ for some $j>1$. On the other hand, using (iv):

$$\begin{aligned}&\sup_\theta\left\{\sum_i \varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)-\sum_{i>1}\left[\widehat{e}_i(\theta)-e_i^s(\widehat{\alpha}_i,\widehat{e}_{-i}(\theta),\theta_i)\right]\epsilon\right\} \qquad (vii)\\ =\ &\lim_{n\to\infty}\sum_i\varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta^n),\theta_i^n)-\sum_{i>1}K_i\epsilon\\ \le\ &\sup_\theta\sum_i\varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)-K_j\epsilon.\end{aligned}$$

Thus, combining the sets of equations (iii) and (vii) we obtain:

$$\begin{aligned}&\varphi_1(\widehat{\alpha}_1-(N-1)\epsilon,\widehat{e}(\theta'),\theta_1')+\sum_{i>1}\varphi_i(\widehat{\alpha}_i+\epsilon,\widehat{e}(\theta'),\theta_i')\\ \le\ &\sup_\theta\sum_i\varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)-K_j\epsilon+o(\epsilon^2),\end{aligned}$$

and since $-K_j\epsilon+o(\epsilon^2)<0$ for all small ϵ, (ii) must hold.

Finally, for part (a) of the Lemma, assume contrary to the claim that WLOG $\widehat{\alpha}_1=1$, which from the optimality of σ implies $\widehat{e}_1(\cdot)\equiv e_1^*(\cdot)$ and thus $\varphi_1(\widehat{\alpha}_1,\widehat{e}(\theta),\theta_1)=0$ for all θ, together with $\sup_\theta\widehat{e}_1(\theta)>0$ (from (A1)). Since by hypothesis WLOG $\sup_\theta\widehat{e}_2(\theta)>0$, we must have that $\varphi_2(\widehat{\alpha}_2,\widehat{e}(\theta),\theta_2)=c_2(\widehat{e}_2(\theta),\theta_2)>0$ for some θ (where the equality follows from the fact that $\widehat{\alpha}_2=0$), and therefore $\sup_\theta\sum_i\varphi_i(\widehat{\alpha}_i,\widehat{e}(\theta),\theta_i)>0$. Thus, except for (C2), the hypotheses of part (b) of the Lemma are met and yet the conclusion is contradicted. But, since $\widehat{\alpha}_1=1$, condition (i) will be satisfied even when (C2) does not hold, and therefore the above reasoning applies (i.e., (C2) is only used to prove (i)) and the conclusion of part (b) of the Lemma must indeed hold, a contradiction to the fact that $\widehat{\alpha}_1 = 1$. ∎

Proof of Theorem 1. (Based on Lemma 7 above.) Observe that, under first-best schedule $e^*(\cdot)$, the hypothesis (C1) of part (a) in Lemma 7 is always satisfied due to (A1). Now let

$$\alpha^* \in \arg\min_{\alpha} \left\{ \sup_{\theta} \sum_i \varphi_i(\alpha_i, e^*(\theta), \theta_i) \right\}.$$

Combining Lemma 2 (stationarity) with Proposition 1, we know that first-best can be implemented using a stationary contract under ownership structure α^*. Part (a) of the Theorem then follows directly from part (a) of Lemma 7, and part (b) of the Theorem follows from (A1), i.e., $\sup_\theta \varphi_i(\alpha_i, e^*(\theta), \theta_i) > 0$ whenever $\alpha_i < 1$. ∎

Proof of Theorem 2. Suppose a second-best contract σ induces effort schedule $\widehat{e}(\cdot)$ under ownership structure $\widehat{\alpha}$. From Lemma 3 (i.e., binding (RC)) we must have

$$\widehat{\alpha} \in \arg\min_{\alpha} \left\{ \sup_{\theta} \sum_i \varphi_i(\alpha_i, \widehat{e}(\theta), \theta_i) \right\}.$$

Thus, (a) follows directly from part (a) of Lemma 7; and part (b) follows from part (b) of Lemma 7 by noticing that $\sup_\theta \sum_i \varphi_i(\widehat{\alpha}_i, \widehat{e}(\theta), \theta_i) > 0$ is implied by the hypothesis that σ delivers a surplus strictly higher than $\overline{s}$ together with the fact that (RC) must bind (Lemma 3). ∎

Sketch of proof for Lemma 4. Due to (A2), contract σ can be replaced without loss of optimality by a contract under which $\tau_i(y_i, y_{-i})$ is non-decreasing in y_i and non-increasing in y_{-i}. Moreover, by hypothesis, additively separable transfers of the form $b_i(y_i) + v_i(y_{-i})$, with

$$(i) \qquad b_i(y_i) = \tau_i(y_i, \overline{y}_{-i}) - \tau_i(\overline{y}), \text{ and } v_i(y_{-i}) = -\frac{1}{N-1} \sum_{j \neq i} b_j(y_j)$$

(where $\overline{y}$ is the highest possible value of y), will also induce efforts $\widehat{e}$ while balancing the budget. Finally, since no surplus is destroyed through money burning, transfers ω can always be selected in a way that constraints (P) and (T) hold. Indeed, the total amount of relational capital required to enforce these new transfers will not exceed the available surplus:

$$\begin{aligned} -\sum_i \inf_y \{b_i(y_i) + v_i(y_{-i})\} &= -\sum_i \inf_y \tau_i(y_i, \overline{y}_{-i}) + \sum_i \tau_i(\overline{y}) \\ &\leq -\sum_i \inf_y \tau_i(y_i, \overline{y}_{-i}) \leq -\sum_i \inf_y \tau_i(y) \\ &\leq \frac{\delta}{1-\delta} \left[s - \overline{s} \right], \end{aligned}$$

(where s is the flow surplus achieved under σ). The equality follows from (i); the first inequality follows from the (implicit) hypothesis that transfers $\tau_i(\cdot)$ are balanced; and the last inequality follows from the (implicit) hypothesis that constraint (T) is satisfied under the original contract σ. ∎

Proof of Lemma 5. Consider first the original contract σ. Let $s = (1-\delta)\sum_i u_i$ be the expected flow surplus it generates (from period 1 onward), and $u_i \geq \overline{u}_i$ the corresponding continuation payoff for player i. Also let $\tau_i(y) = \tau_i^1(y_i) + \tau_i^2(y_{-i})$ for all i. Under σ we must have:

$$(E) \qquad \widehat{e}_i \in \arg\max_{e_i} \left\{ \alpha_i E\left[x \mid e_i, \widehat{e}_{-i}\right] - c_i(e_i) + E\left[\tau_i^1(y_i) \mid e_i\right] \right\} \text{ for all } i.$$

Moreover, in order for voluntary transfers to be enforceable we need:

$$(T) \qquad -\inf_y \tau_i(y) \leq \delta\left[u_i - \overline{u}_i\right] \text{ for all } i.$$

And, finally, budget balance implies:

$$(B) \qquad \sum_i \tau_i(y) \leq 0 \text{ for all } y.$$

Now let the new contract $\widetilde{\sigma}$ satisfy:

$$\begin{aligned}
b_i(y_i) &= \tau_i^1(y_i) - \sup_{y_i} \tau_i^1(y_i) \text{ for all } i, && (i) \\
\widetilde{\tau}_i(y) &= b_i(y_i) - \sum_{j \neq i} \gamma_i^j b_j(y_j) \text{ for all } i, && (ii) \\
\widetilde{u}_i &= \overline{u}_i - \frac{1}{\delta} \inf_{y_i} b_i(y_i) \text{ for all } i < N, \text{ and} && (iii) \\
\widetilde{u}_N &= \frac{s}{1-\delta} - \sum_{i<N} \widetilde{u}_i - \frac{1}{1-\delta} E\left[\sum_i \tau_i(y) \mid \widehat{e}\right]. && (iv)
\end{aligned}$$

Observe that budget balance with no money burning occurs by construction. We must now check that the analogous conditions to (E) and (T) are satisfied under $\widetilde{\sigma}$. Denote these conditions by (E') and (T'). Notice first that (E') follows directly from (E) and (i). On the other hand, (T') for $i < N$ follows directly from (ii), (iii), and the fact that $b_i(y_i) \leq 0$; and for $i = N$ we have:

$$
\begin{aligned}
-\inf_{y}\ \widetilde{\tau}_N(y) &= -\inf_{y_N} \tau_N^1(y_N) + \sup_{y_N} \tau_N^1(y_N) \\
&\leq \delta\left[u_N - \overline{u}_N\right] + \sup_{y_N} \inf_{y_{-N}} \tau_N(y_N, y_{-N}) \\
&= \delta s - \delta \overline{u}_N - \delta \sum_{i<N} u_i + \sup_{y_N} \inf_{y_{-N}} \tau_N(y_N, y_{-N}) \\
&\leq \delta s - \delta \overline{u}_N + \sum_{i<N} \left\{ \inf_{y} \tau_i(y) - \delta \overline{u}_i \right\} + \sup_{y_N} \inf_{y_{-N}} \tau_N(y_N, y_{-N}) \\
&= \delta s - \delta \overline{u}_N + \sum_{i<N} \left\{ \inf_{y_i} b_i(y_i) - \delta \overline{u}_i \right\} + \sum_{i} \left\{ \sup_{y_i} \inf_{y_{-i}} \tau_i(y_i, y_{-i}) \right\} \\
&\leq \delta s - \delta \sum_{i<N} \widetilde{u}_i - \delta \overline{u}_N \leq \delta\left[\widetilde{u}_N - \overline{u}_N\right],
\end{aligned}
$$

where the first equality follows from (i) and (ii); the first inequality form (T); the second equality from the definition of s; the second inequality from (T); the third equality from (i); the third inequality from (iii) and (B) (i.e., $\sum_i \sup_{y_i} \inf_{y_{-i}} \tau_i(y_i, y_{-i}) \leq 0$ because $\sum_i \inf_{y_{-i}} \tau_i(y_i', y_{-i}) \leq \sum_i \tau_i(y') \leq 0$ for all y'); and the last inequality from (iv) and (B) (i.e., $E\left[\sum_i \tau_i(y) \mid \widehat{e}\right] \leq 0$).

Finally, notice that (T') and the fact that $-\inf_y \widetilde{\tau}_i(y) \geq 0$ (which follows from (i) and (ii)) imply that $\widetilde{u}_i \geq \overline{u}_i$ for all i. Thus, $\widetilde{\sigma}$ is indeed self-enforcing. ∎

Lemma 8. *Fix effort vector $\widehat{e}$, ownership structure $\widehat{\alpha}$, and bonus scheme $\widehat{b}_i(\cdot)$ such that (FOC) holds. Then, under (A2) there exist one-step bonuses $\widetilde{b}_i(\cdot)$ such that (FOC) holds for $\widehat{e}$ and $\widehat{\alpha}$, and such that:*

$$\sum_i \left\{ \sup_{y_i} \widetilde{b}_i(y_i) - \inf_{y_i} \widetilde{b}_i(y_i) \right\} \leq \sum_i \left\{ \sup_{y_i} \widehat{b}_i(y_i) - \inf_{y_i} \widehat{b}_i(y_i) \right\}.$$

Moreover, the cut-off value for each bonus $\widetilde{b}_i(\cdot)$ is given by $\widehat{y}_i$, which is uniquely defined by:

$$f_{e_i}(\widehat{y}_i \mid \widehat{e}_i) = 0.$$

Proof. It is sufficient to show that, for each player i, there exists a one-step bonus $\widetilde{b}_i(\cdot) \in \{\underline{b}_i, \overline{b}_i\}$, with $\underline{b}_i \leq \overline{b}_i$, such that:

$$
\begin{aligned}
\frac{\partial}{\partial e_i} E\left[\widetilde{b}_i(y_i) \mid \widehat{e}_i\right] &= \frac{\partial}{\partial e_i} E\left[\widehat{b}_i(y_i) \mid \widehat{e}_i\right], \text{ and} && (i) \\
\overline{b}_i - \underline{b}_i &\leq \sup_{y_i} \widehat{b}_i(y_i) - \inf_{y_i} \widehat{b}_i(y_i). && (ii)
\end{aligned}
$$

Due to (A2), there exists a $\widehat{y}_i$ such that $f_{e_i}(\widehat{y}_i \mid \widehat{e}_i) = 0$, $f_{e_i}(y_i \mid \widehat{e}_i) < 0$ over $(-\infty, \widehat{y}_i)$, and $f_{e_i}(y_i \mid \widehat{e}_i) > 0$ over $(\widehat{y}_i, \infty)$. Now suppose $\frac{\partial}{\partial e_i} E\left[\widehat{b}_i(y_i) \mid \widehat{e}_i\right] > 0$. The proof for the case in which $\frac{\partial}{\partial e_i} E\left[\widehat{b}_i(y_i) \mid \widehat{e}_i\right] < 0$ is symmetric, and trivial for the case in which $\frac{\partial}{\partial e_i} E\left[\widehat{b}_i(y_i) \mid \widehat{e}_i\right]$

$= 0$ (i.e., simply set $\underline{b}_i = \overline{b}_i$). Let $\widetilde{b}_i(\cdot)$ be such that:

$$\widetilde{b}_i(y_i) = \begin{cases} \overline{b}_i & \text{for } y_i \geq \widehat{y}_i, \\ \underline{b}_i & \text{otherwise,} \end{cases}$$

with $\underline{b}_i = \inf_{y_i} \widehat{b}_i(y_i)$, and $\overline{b}_i$ such that (i) holds, that is:

$$(iii) \qquad \int_{\widehat{y}_i}^{\infty} \overline{b}_i f_{e_i}(y_i \mid \widehat{e}_i) dy_i = \int_{-\infty}^{\widehat{y}_i} \left[\widehat{b}_i(y_i) - \underline{b}_i\right] f_{e_i}(y_i \mid \widehat{e}_i) dy_i + \int_{\widehat{y}_i}^{\infty} \widehat{b}_i(y_i) f_{e_i}(y_i \mid \widehat{e}_i) dy_i.$$

Notice that we must have $\overline{b}_i > \underline{b}_i$, because otherwise $\frac{\partial}{\partial e_i} E\left[\widetilde{b}_i(y_i) \mid \widehat{e}_i\right] \leq 0$, which is impossible due to (iii) and the assumption that $\frac{\partial}{\partial e_i} E\left[\widehat{b}_i(y_i) \mid \widehat{e}_i\right] > 0$. It now remains to show that $\overline{b}_i \leq \sup_{y_i} \widehat{b}_i(y_i)$ in order to prove (ii). Suppose on the contrary that $\overline{b}_i > \sup_{y_i} \widehat{b}_i(y_i)$, which implies:

$$\int_{\widehat{y}_i}^{\infty} \left[\overline{b}_i - \widehat{b}_i(y_i)\right] f_{e_i}(y_i \mid \widehat{e}_i) dy_i > 0 \geq \int_{-\infty}^{\widehat{y}_i} \left[\widehat{b}_i(y_i) - \underline{b}_i\right] f_{e_i}(y_i \mid \widehat{e}_i) dy_i,$$

where the second inequality follows from the fact that $\underline{b}_i = \inf_{y_i} \widehat{b}_i(y_i)$, and contradicts (iii). ∎

Lemma 9. *Suppose $\widehat{e}$, $\widehat{\alpha}$ and $\widehat{b}(\cdot)$ solve the relaxed problem with constraint (FOC) instead of (E), and each $\widehat{b}_i(\cdot)$ is a one-step bonus with cut-off value $\widehat{y}_i$ (defined in the above Lemma 8). Then, each $\widehat{b}_i(\cdot)$ must be non-decreasing.*

Proof. Let the one-step bonuses be given by

$$\widehat{b}_i(y_i) = \begin{cases} b_i^H & \text{for } y_i \geq \widehat{y}_i, \\ b_i^L & \text{otherwise.} \end{cases}$$

It must be shown that $b_i^H - b_i^L \geq 0$. Observe that the first-order condition for each i reduces to

$$(i) \qquad \widehat{\alpha}_i \frac{\partial}{\partial e_i} E\left[x \mid \widehat{e}\,\right] - \left(b_i^H - b_i^L\right) F_{e_i}\left(\widehat{y}_i \mid \widehat{e}_i\right) = c_i'(\widehat{e}_i),$$

where $F_{e_i}\left(\widehat{y}_i \mid \widehat{e}_i\right) < 0$ due to (A2).

Initially assume that first-best is achieved, i.e., $\widehat{e} = e^*$. Then, for each i, we must have

$$\widehat{\alpha}_i \frac{\partial}{\partial e_i} E\left[x \mid e^*\right] \leq \frac{\partial}{\partial e_i} E\left[x \mid e^*\right] = c_i'(e_i^*),$$

which, together with (i), establishes the claim.

Now suppose first-best is not achieved. Consider relaxing the problem further by replacing constraint $\sum_i \alpha_i = 1$ by $\sum_i \alpha_i \leq 1$ (while maintaining the same objective). It is straightforward to verify that whenever first-best can be achieved under this new problem, it can also be achieved when we require $\sum_i \alpha_i = 1$; and whenever first-best cannot be achieved under this new problem, constraints $\sum_i \alpha_i \leq 1$ and (RC) must bind. Therefore, since first-best cannot be achieved by assumption when $\sum_i \alpha_i = 1$, contract $\left\langle \widehat{e}, \widehat{\alpha}, \widehat{b}(\cdot) \right\rangle$ must also be a solution to the new problem.

Furthermore, suppose towards a contradiction that there exists a player k such that $b_k^H < b_k^L$. Then we must have $\alpha_k > 0$ (because otherwise (i) would imply that $\widehat{e}_k = 0$, which could be achieved by setting $b_i^H = b_i^L$ and thus relaxing (RC), a contradiction). Now consider modifying the contract by reducing both the level of α_k and the level of $\left(b_k^L - b_k^H\right)$ in a way that the first-order condition for player k is not altered. Notice that such a change relaxes both constraints (RC) and $\sum_i \alpha_i \leq 1$ under the new problem, without changing the objective, a contradiction. ∎

Lemma 10. *Suppose $\widehat{e}$, $\widehat{\alpha}$ and $\widehat{b}(\cdot)$ satisfy (FOC), and each $\widehat{b}_i(\cdot)$ is a non-decreasing one-step bonus with cut-off value $\widehat{y}_i$ (defined in the above Lemma 8). Then, under (A2) and (A3), $\widehat{e}$, $\widehat{\alpha}$ and $\widehat{b}(\cdot)$ also satisfy (E).*

Proof. The incentive constraint for each player i is given by

$$\widehat{e}_i = \arg\max_{e_i} \left\{ \alpha_i E\left[x \mid e_i, \widehat{e}_{-i}\right] - \left[\sup_{y_i} \widehat{b}_i(y_i) - \inf_{y_i} \widehat{b}_i(y_i)\right] F\left(\widehat{y}_i \mid \widehat{e}_i\right) - c_i(e_i) \right\},$$

which is strictly concave under (A2) and (A3). Thus, the first-order constraints (FOC) are sufficient for constraints (E). ∎

Proof of Proposition 2. For sufficiency, suppose (RC)–(ND) hold, and consider a stationary contract that uses ownership structure α and one-step bonuses $b(\cdot)$ such that, for each $i < N$,

$$b_i(y_i) = \begin{cases} p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e}) & \text{for } y_i \geq \widehat{y}_i, \\ 0 & \text{otherwise,} \end{cases}$$

and for $i = N$:

$$b_N(y_N) = \begin{cases} 0 & \text{for } y_N \geq \widehat{y}_N, \\ -p_N(\widehat{e}_N) + \alpha_N q_N(\widehat{e}) & \text{otherwise,} \end{cases}$$

where $\widehat{y}_i$ satisfies $f_{e_i}(\widehat{y}_i \mid \widehat{e}_i) = 0$. Also, let transfers $\tau_i(\cdot)$ and continuation values u_i be given by:

$$\begin{aligned} \tau_i(y) &= b_i(y_i) - \frac{1}{N-1} b_N(y_N) \text{ for } i < N, \\ \tau_N(y) &= b_N(y_N) - \sum_{i<N} b_i(y_i), \\ u_i &= \overline{u}_i \text{ for } i < N, \text{ and} \\ u_N &= \frac{\widehat{s}}{1-\delta} - \sum_{i<N} \overline{u}_i \end{aligned}$$

Notice that, due to Lemma 10 (on sufficiency of (FOC)), provided the above voluntary transfers are credible, each player will indeed select effort level $\widehat{e}_i$. Moreover, the budget will balance by assumption. It remains to verify that the voluntary transfers are credible, which follows from the fact that $\tau_i(y) \geq 0$ for all $i < N$, and for $i = N$:

$$\begin{aligned} -\inf_y \tau_N(y) &= \sum_i \{p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e})\} \\ &\leq \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}] \leq \delta [u_N - \overline{u}_N]. \end{aligned}$$

For necessity, on the other hand, suppose $\widehat{e}$ can be implemented using ownership structure α. Then, due to Lemmas 8–10, it can also be implemented using a non-decreasing one-step bonus scheme $\widetilde{b}(\cdot)$ with cutoff values $\widehat{y}_i$. Under such scheme we must have that the first-order conditions hold for all i:

$$(i) \qquad \alpha_i \frac{\partial}{\partial e_i} E[x \mid \widehat{e}\,] - \left(\sup_{y_i} \widetilde{b}_i(y_i) - \inf_{y_i} \widetilde{b}_i(y_i)\right) F_{e_i}(\widehat{y}_i \mid \widehat{e}_i) = c_i'(\widehat{e}_i),$$

and that constraint (DE) (in the text) is satisfied:

$$(ii) \qquad \sum_i \left\{\sup_{y_i} \widetilde{b}_i(y_i) - \inf_{y_i} \widetilde{b}_i(y_i)\right\} \leq \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}].$$

Conditions (RC)–(ND) follow from combining (i) and (ii). ∎

Proof of Theorem 3. Under first-best effort levels e^* we have that $c_i'(e_i^*) = \frac{\partial}{\partial e_i} E[x \mid e^*]$ for all i. Thus, condition (ND) in Proposition 2 is redundant, and (RC) becomes

$$\sum_i (1-\alpha_i)\, q_i(e^*) \leq \frac{\delta}{1-\delta}[\widehat{s} - \overline{s}],$$

which achieves a minimum when $a_k = 1$ for any $k \in \arg\max_i q_i(e^*)$. The claim then follows as a Corollary of Proposition 2. ∎

Proof of Theorem 4. Existence of a second-best contract follows from the fact that the objective is continuous in e (i.e., (A1)), and constraints (RC) and (ND) are compact (i.e., smoothness of F and (A1)). On the other hand, under any second-best contract, constraint (RC) must bind (the reasoning is analogous to that in the proof of Lemma 3). Let $\widehat{e}$ denote an optimal effort schedule. Then it follows from Proposition 2 that, given $\widehat{e}$, an ownership structure will be optimal if and only if it solves

$$\min_\alpha \sum_i \{p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e})\} \;\; s.t.$$
$$p_i(\widehat{e}_i) - \alpha_i q_i(\widehat{e}) \geq 0 \text{ for all } i.$$

Now, for the first claim, let $k \in \arg\max_i q_i(\widehat{e})$, and notice that there exists an $\widehat{\alpha}$ that solves the above problem and such that either $p_k(\widehat{e}_k) - \widehat{\alpha}_k q_k(\widehat{e}) = 0$ or $\widehat{\alpha}_k = 1$ and $p_k(\widehat{e}_k) - \widehat{\alpha}_k q_k(\widehat{e}) > 0$. However, if $\widehat{\alpha}_k = 1$, any optimal contract must have $c_k'(\widehat{e}_k) = \frac{\partial}{\partial e_k} E[x \mid \widehat{e}\,]$, and thus $p_k(\widehat{e}_k) - \widehat{\alpha}_k q_k(\widehat{e}) = 0$. Therefore, we must have $\Delta b_k = p_k(\widehat{e}_k) - \widehat{\alpha}_k q_k(\widehat{e}) = 0$.

Finally, for the second claim, assume $\arg\max_i q_i(\widehat{e}_i)$ is a singleton, and WLOG equal to k. Then, from a reasoning similar to that in the previous paragraph, any $\widehat{\alpha}$ that solves the above problem must be such that $p_k(\widehat{e}_k) - \widehat{\alpha}_k q_k(\widehat{e}_k) = \Delta b_k = 0$. ∎

Proof of Lemma 6. Consider first the original contract σ. Let $s = (1-\delta)\sum_i u_i$ be the expected flow surplus it generates (from period 1 onward), and $u_i \geq \overline{u}_i$ the corresponding continuation payoff for player i. Since $\widehat{e}(\cdot)$ is implemented in dominant strategies we must have:

$$(E) \qquad \widehat{e}_i(\theta_i) \in \arg\max_{e_i} \{\alpha_i x_i(e_i) - c_i(e_i,\theta_i) + \tau_i(e_i, e_{-i})\} \text{ for all } i,\ \theta_i \text{ and } e_{-i}.$$

Moreover, in order for voluntary transfers to be enforceable we need:

$$(T) \qquad -\inf_e \tau_i(e) \leq \delta[u_i - \overline{u}_i] \text{ for all } i.$$

And finally, budget balance implies:

$$(B) \qquad \sum_i \tau_i(e) \leq 0.$$

Now fix constants $\gamma_i^j \in [0,1]$ for all i and all $j \neq i$ such that $\sum_{i\neq j}\gamma_i^j = 1$ for all j, and let the new contract $\widetilde{\sigma}$ satisfy:

$$\begin{aligned}
b_i(e_i) &= \inf_{e_{-i}} \tau_i(e_i,e_{-i}) - \sup_{e_i}\inf_{e_{-i}} \tau_i(e_i,e_{-i}) \text{ for all } i, && (i)\\
\widetilde{\tau}_i(e) &= b_i(e_i) - \sum_{j\neq i}\gamma_i^j b_j(e_j) \text{ for all } i, && (ii)\\
\widetilde{u}_i &= \overline{u}_i - \frac{1}{\delta}\inf_{e_i} b_i(e_i) \text{ for all } i < N, \text{ and} && (iii)\\
\widetilde{u}_N &= \frac{s}{1-\delta} - \sum_{i<N}\widetilde{u}_i - \frac{1}{1-\delta}E_\theta\left\{\sum_i \tau_i(\widehat{e}(\theta))\right\}. && (iv)
\end{aligned}$$

Observe that budget balance with no money burning occurs by construction, and the functions $v_i(e_{-i})$ in part (b) of the Lemma are given by $\sum_{j\neq i}\gamma_i^j b_j(e_j)$. We must now check that the analogous conditions to (E) and (T) are satisfied under $\widetilde{\sigma}$. Denote these conditions by (E') and (T'). First notice that (E) implies:

$$\widehat{e}_i(\theta_i) \in \arg\max_{e_i}\left\{\alpha_i x_i(e_i) - c_i(e_i,\theta_i) + \inf_{e_{-i}}\tau_i(e_i,e_{-i})\right\} \text{ for all } i, \text{ and } \theta_i,$$

so that (E') follows from the fact that $\widetilde{\tau}_i(e)$ only depends on e_i through $\inf_{e_{-i}}\tau_i(e_i,e_{-i})$. On the other hand, (T') for $i < N$ follows directly from (ii), (iii), and the fact that $b_j(e_j) \leq 0$ for all j; and for $i = N$ we have:

$$\begin{aligned} -\inf_{e}\ \widetilde{\tau}_N(e) &= -\inf_{e}\tau_N(e) + \sup_{e_N}\inf_{e_{-N}}\tau_N(e_N, e_{-N}) \\ &\leq \delta\left[u_N - \overline{u}_N\right] + \sup_{e_N}\inf_{e_{-N}}\tau_N(e_N, e_{-N}) \\ &= \delta s - \delta\overline{u}_N - \delta\sum_{i<N} u_i + \sup_{e_N}\inf_{e_{-N}}\tau_N(e_N, e_{-N}) \\ &\leq \delta s - \delta\overline{u}_N + \sum_{i<N}\left\{\inf_{e}\tau_i(e) - \delta\overline{u}_i\right\} + \sup_{e_N}\inf_{e_{-N}}\tau_N(e_N, e_{-N}) \\ &= \delta s - \delta\overline{u}_N + \sum_{i<N}\left\{\inf_{e_i} b_i(e_i) - \delta\overline{u}_i\right\} + \sum_{i}\sup_{e_i}\inf_{e_{-i}}\tau_i(e_i, e_{-i}) \\ &\leq \delta s - \delta\sum_{i<N}\widetilde{u}_i - \delta\overline{u}_N \leq \delta\left[\widetilde{u}_N - \overline{u}_N\right], \end{aligned}$$

where the first equality follows from (i) and (ii); the first inequality form (T); the second equality from the definition of s; the second inequality from (T); the third equality from (i); the third inequality from (iii) and (B) (i.e., $\sum_i \sup_{e_i}\inf_{e_{-i}}\tau_i(e_i, e_{-i}) \leq 0$ because $\sum_i \inf_{e_{-i}}\tau_i(e_i', e_{-i}) \leq \sum_i \tau_i(e') \leq 0$ for all e'); and the last inequality from (iv) and (B) (i.e., $E_\theta\left\{\sum_i \tau_i\left(\widehat{e}(\theta)\right)\right\} \leq 0$).

Finally, notice that (T') and the fact that $-\inf_e \widetilde{\tau}_i(e) \leq 0$ for all i (which follows from (i) and (ii)) imply that $\widetilde{u}_i \geq \overline{u}_i$. Thus, $\widetilde{\sigma}$ is indeed self-enforcing. ∎

Proof of Proposition 3. For notational simplicity, this proof assumes that $x_i(e_i) \equiv e_i$, and $\left[\underline{\theta}_i, \overline{\theta}_i\right] = [0, 1]$ (the proof for the general case follows exactly the same steps). For sufficiency, suppose (RC) holds and let σ be a stationary contract with ownership structure α such that:

(i)
$$e(\cdot) \equiv e^*(\cdot),$$

(ii)
$$\tau_i(e) = b_i(e_i) - \frac{1}{N-1}\sum_{j\neq i} b_j(e_j) \text{ for all } i, \text{ with}$$

(ii')
$$b_i(e_i) = \begin{cases} \varphi_i(\alpha_i, e_i^*(0), 0) + (1-\alpha_i)\left[e_i - e_i^*(0)\right] & \text{if } e_i = [e_i^*(0), e_i^*(1)], \\ 0 & \text{otherwise,} \end{cases}$$

(iii)
$$u_i = \overline{u}_i - \frac{1}{\delta}\inf_{e}\tau_i(e) \text{ for } i < N,$$

(iv)
$$\text{and } u_N = \frac{s^*}{1-\delta} - \sum_{i<N} u_i.$$

Where the continuation values u_i are achieved by an appropriate selection of court enforced transfers ω. The budget will balance by construction. It will now be shown that the voluntary transfer scheme in (ii)–(ii') will induce first-best effort levels provided it is credible. First notice that no player i of type θ (the subscript is dropped for simplicity) will profit by deviating to an effort level inside $[e_i^*(0), \infty)$, other than $e_i^*(\theta)$. Moreover, by construction, if a player i is of type 0 she will not gain by deviating to an effort lower than $e_i^*(0)$. Thus, it remains to show that no player i of type $\theta > 0$ will gain by selecting an effort lower than $e_i^*(0)$, for which it will suffice to prove that

$$\begin{aligned} \alpha_i e_i^*(\theta) - c_i(e_i^*(\theta),\theta) + b_i(e_i^*(\theta)) &\geq \alpha_i e_i^*(0) - c_i(e_i^*(0),\theta) + b_i(e_i^*(0)) \\ &\geq \sup_{e_i < e_i(0)} \{\alpha_i e_i - c_i(e_i,\theta)\}. \end{aligned}$$

But the first inequality follows from the above argument, and the second inequality follows from

$$\alpha_i e_i^*(0) - c_i(e_i^*(0),0) + b_i(e_i^*(0)) \geq \sup_{e_i < e_i(0)} \{\alpha_i e_i - c_i(e_i,0)\},$$

together with the fact that $c_i(e_i,\theta)$ has decreasing differences in (e_i,θ), i.e., assumption (A1).

It must now be shown that the voluntary transfers in (ii)–(ii') are indeed credible. For every $i < N$ this follows directly form (iii), and for $i = N$ we have

$$\begin{aligned} -\inf_e \tau_N(e) &= -\sum_i \inf_e \tau_i(e) + \sum_{i<N} \inf_e \tau_i(e) \\ &= \frac{1}{N-1}\sum_i \sum_{j\neq i} \left\{\varphi_j(\alpha_j, e_j^*(0), 0) + (1-\alpha_j)\left[e_j^*(1) - e_j^*(0)\right]\right\} + \sum_{i<N} \inf_e \tau_i(e) \\ &= \sum_i \left\{\varphi_i(\alpha_i, e_i^*(0), 0) + (1-\alpha_i)\left[e_i^*(1) - e_i^*(0)\right]\right\} - \delta \sum_{i<N} [u_i - \overline{u}_i] \\ &\leq \frac{\delta}{1-\delta}\left[s^* - \overline{s}\right] - \delta \sum_{i<N} [u_i - \overline{u}_i] \\ &= \delta\left[u_N - \overline{u}_N\right], \end{aligned}$$

where the second equality follows from (ii)–(ii'); the third equality from (iii); the inequality from (RC); and the last equality from (iv).

Finally, observe that for all players $\inf_e \tau_i(e) \leq 0$. Thus, the voluntary transfer condition $-\inf_e \tau_i(e) \leq \delta\left[u_i - \overline{u}_i\right]$ implies $u_i \geq \overline{u}_i$ for all i, which provides the participation conditions and completes the proof for sufficiency.

For necessity, let the stationary relational contract σ achieve first-best surplus using ownership structure α. Due to Lemma 6, WLOG we can assume the (credible) voluntary transfers prescribed by σ are given by:

$$\tau_i(e) = b_i(e_i) - \frac{1}{N-1}\sum_{j\neq i} b_j(e_j), \text{ for all } i,$$

for some functions $b_i(\cdot)$. Now let $U_i(\theta) := \sup_{e_i}\left\{\alpha_i e_i - c_i(e_i,\theta) + b_i(e_i)\right\}$, and notice that, since $e^*(\cdot)$ is implementable by hypothesis, we must have:

(v) $$U_i(\theta) = \alpha_i e_i^*(\theta) - c_i(e_i^*(\theta),\theta) + b_i(e_i^*(\theta)) \text{ for all } \theta.$$

Application of Milgrom and Segal's [2002] envelope Theorem 2 to (v) yields:

$$U_i(\theta) = U_i(0) + \int_0^\theta \frac{\partial}{\partial\theta} c_i(e_i^*(z), z)dz,$$

which in combination with (v) implies:

$$\begin{aligned} b_i(e_i^*(\theta)) - b_i(e_i^*(0)) &= c_i(e_i^*(\theta), \theta) - c_i(e_i^*(0), 0) \\ &\quad + \int_0^\theta \frac{\partial}{\partial\theta} c_i(e_i^*(z), z)dz - \alpha_i \left[e_i^*(\theta) - e_i^*(0)\right]. \end{aligned} \qquad (vi)$$

However, since $e^*(\cdot)$ is first-best efficient we must have:

$$\frac{\partial}{\partial\theta} c_i(e_i^*(z), z) = \frac{d}{dz} \left\{e_i^*(z) - c_i(e_i^*(z), z)\right\},$$

and therefore (vi) becomes:

$$(vii) \qquad b_i(e_i^*(\theta)) - b_i(e_i^*(0)) = (1 - \alpha_i)\left[e_i^*(\theta) - e_i^*(0)\right].$$

On the other hand, in order to preclude any deviations from type $\theta = 0$, we must have:

$$\begin{aligned} b_i(e_i^*(0)) - b_i(e_i^s) &\geq \varphi_i(\alpha_i, e_i^*(0), 0) \\ \text{for } e_i^s &: = \arg\max_{e_i} \left\{\alpha_i e_i - c_i(e_i, 0)\right\}. \end{aligned} \qquad (viii)$$

Since transfers $\tau_i(e)$ are credible by hypothesis, we must have, for all i:

$$\begin{aligned} \delta\left[u_i - \overline{u}_i\right] &\geq -\tau_i(e_i^s, e_{-i}^*(1)) \\ &= -b_i(e_i^s) + \frac{1}{N-1}\sum_{j\neq i} b_j(e_j^*(1)). \end{aligned} \qquad (ix)$$

Summing up (ix) across players yields:

$$\begin{aligned} \frac{\delta}{1-\delta}\left[s^* - \overline{s}\right] &\geq \sum_i \left\{b_i(e_i^*(1)) - b_i(e_i^s)\right\} \\ &= \sum_i \left\{b_i(e_i^*(1)) - b_i(e_i^*(0)) + b_i(e_i^*(0)) - b_i(e_i^s)\right\} \\ &\geq \sum_i (1-\alpha_i)\left[e_i^*(\theta) - e_i^*(0)\right] + \sum_i \varphi_i(\alpha_i, e_i^*(0), 0), \end{aligned} \qquad (xi)$$

where the second inequality follows from (vii) and $(viii)$, and completes the proof. ∎

Proof of Theorem 5. The first claim is a consequence of Lemma 2 (stationarity) and Proposition 3: Lemma 2 implies $e^*(\cdot)$ is implementable using a stationary contract with, say, ownership structure α'. The necessity direction of Proposition 3 implies (RC) holds for α', and thus (RC) must also hold for any α such that $\sum_{i\in\Omega} \alpha_i = 1$, where $\Omega := \arg\max_i \Delta x_i^*$. The claim then follows from the sufficiency direction of Proposition 3. The second claim, on the other hand, follows from the fact that $\alpha_i = 1$ implies $\Delta b_i = 0$. ∎

Proof of Theorem 6. For notational simplicity, this proof assumes that $x_i(e_i) \equiv e_i$, and $\left[\underline{\theta}_i, \overline{\theta}_i\right] = [0,1]$ (the proof for the general case follows exactly the same steps). Consider the first claim. Existence of some optimal contract σ follows from the fact that the objective is continuous in $e(\cdot)$ (say under an L_1 metric), and the constraints are compact. For compactness, notice that under (A1) any optimal contract will involve a non decreasing and bounded schedule $e(\cdot)$, but such schedules constitute a compact set. Moreover, the subset of these schedules satisfying the constraints is closed, and hence compact. Finally, compactness regarding bonuses follows from their linearity.

The proof now proceeds by construction of a contract $\widetilde{\sigma}$ that satisfies the desired properties, and that provides a surplus at least as high as that created under σ. Throughout, let the variables with a tilde "$\widetilde{\cdot}$" correspond to contract $\widetilde{\sigma}$, and those without a tilde to the original optimal contract σ. We may assume WLOG that $\lambda_i = 0$ for all i, because such non-contingent voluntary payment can be fully replaced by court enforced transfers ω.

Claim 1: WLOG $\beta_i \leq 1 - \alpha_i$ for all i. Suppose that $\beta_j > 1 - \alpha_j$. This implies player j is selects en effort level $e_j(\cdot) \geq e_j^*(\cdot)$ (where $e_j^*(\cdot)$ is the first-best level). But if β_j was reduced to $1-\alpha_j$, player j would exert the first-best effort level for all θ, and all other incentive constraints would still hold. Thus, total surplus cannot decline, which establishes the claim.

Claim 2: WLOG $\beta_i \geq 0$ for all i. Suppose that $\beta_j < 0$. This implies player j selects an effort level $e_j(\cdot) \leq e_j^*(\cdot)$. But if β_j was increased to 0, player j would exert an effort level $e_j'(\cdot) \in [e_j(\cdot), e_j^*(\cdot)]$, and all other incentive constraints would still hold. Thus, total surplus cannot decline, establishing the claim.

Now suppose $\beta_i > 0$ for all i, otherwise the claim in the Theorem would be true. Since σ is self-enforcing we must have:

$$\delta \sum_i \left[u_i - \overline{u}_i\right] \geq \sum_i \Delta b_i \geq \sum_i \beta_i e_i(1), \text{ and} \qquad (i)$$

$$u_i \geq \overline{u}_i \text{ for all } i, \qquad (ii)$$

where (i) is the (DE) constraint required for voluntary transfers to be credible, and (ii) is required for voluntary participation. Now WLOG let $1 \in \arg\max_i e_i(1)$, and let the new contract $\widetilde{\sigma}$ be such that:

(iii) $$\widetilde{e}_i(\cdot) \equiv e_i(\cdot),$$

(iv) $$\widetilde{\alpha}_1 = \alpha_1 + \beta_1, \text{ and } \widetilde{\alpha}_i \leq \alpha_i \text{ for all } i > 1,$$

(v) $$\widetilde{\beta}_1 = 0, \text{ and } \widetilde{\beta}_i = \beta_i + \alpha_i - \widetilde{\alpha}_i \text{ for all } i > 1,$$

(vi) $$\widetilde{\tau}_i(e) = \widetilde{\beta}_i \min\{e_i, e_i(1)\} \text{ for all } i > 1, \text{ and}$$

$$\widetilde{\tau}_1(e) = -\sum_{i>1} \widetilde{\beta}_i \min\{e_i, e_i(1)\}.$$

Moreover, let the court enforced transfers $\widetilde{\omega}$ be such that:

$$(vii) \qquad \widetilde{u}_i = \overline{u}_i \text{ for all } i > 1, \text{ and } \widetilde{u}_1 = u_1 + \sum_{i>1} [u_i - \overline{u}_i],$$

Observe that if $\widetilde{\sigma}$ is self-enforcing, it will produce the same surplus as σ. We must therefore verify this is indeed the case. From (iv) and (v), the effort levels in (iii) will be optimal provided transfers are credible (i.e., total incentives remain constant: $\widetilde{\beta}_i + \widetilde{\alpha}_i = \beta_i + \alpha_i$). But since $\widetilde{\tau}_i(\cdot) \geq 0$ for all $i > 1$, in order to verify that transfers are credible it will suffice to examine the incentives for player 1, for whom we have:

$$\begin{aligned} -\inf_e \widetilde{\tau}_1(e) &= \sum_i \widetilde{\beta}_i e_i(1) \leq \sum_i \beta_i e_i(1) \\ &\leq \delta \sum_i [u_i - \overline{u}_i] = \delta [\widetilde{u}_1 - \overline{u}_1], \end{aligned} \qquad (viii)$$

where the first equality follows from (vi); the first inequality from the fact that $1 \in \arg\max_i e_i(1)$, $\widetilde{\beta}_1 < \beta_1$, $\widetilde{\beta}_i \geq \beta_i$ for all $i > 1$, and $\sum_i \widetilde{\beta}_i = \sum_i \beta_i$ (i.e., (iv) and (v)); the second inequality from (i); and the last equality from (vii).

Finally, (ii) and (vii) guarantee voluntary participation, completing the proof for the first claim.

On the other hand, suppose the second claim is false, and let σ be an optimal contract that implements $\widehat{x}(\cdot)$. Then, following exactly the same steps as above, we can build a new contract $\widetilde{\sigma}$ that achieves the same surplus as σ and for which $-\sum_i \inf_e \widetilde{\tau}_i(e) < \delta \sum_i [\widetilde{u}_i - \overline{u}_i]$ (i.e., there is slack in the voluntary transfer constraint). This is possible because the fact that $\arg\max_i \Delta\widehat{x}_i$ $(= \arg\max_i \widehat{e}_i(1))$ is a singleton implies the first inequality in $(viii)$ will be strict. Moreover, since first-best cannot be achieved, there exists an player j for whom $\widetilde{\beta}_j + \widetilde{\alpha}_j < 1$. Thus, incentives can be strengthened for j by (slightly) increasing $\widetilde{\beta}_j$ without violating any incentive constraint, and by strict concavity of the objective this will increase surplus, a contradiction. ∎

13. Appendix 2

Consider the setup of Section 8 (i.e., hidden actions), but now suppose nature's selection of θ does affect costs, i.e., $\underline{\theta}_i < \overline{\theta}_i$. Since θ is publicly observed before the selection of efforts, the reasoning in Section 8 applies following any value of θ. That is, on-step schemes will be efficient, and the power of bonuses, now a function of θ, becomes:

$$\Delta b_i(\theta) = p_i(\widehat{e}_i(\theta)) - \alpha_i q_i(\widehat{e}(\theta)),$$

Moreover, (RC) must now hold for all θ:

$$(RC) \qquad \sup_\theta \sum_i \Delta b_i(\theta) \leq \frac{\delta}{1-\delta} [\widehat{s} - \overline{s}].$$

However, since α must be selected *before* the realization of θ, the optimal ownership structure solves:

$$\min_{\alpha} \sup_{\theta} \sum_i \Delta b_i(\theta) \ s.t.$$

$$(ND) \qquad \inf_{\theta} \Delta b_i(\theta) \geq 0 \text{ for all } i.$$

Under first-best (ND) can be ignored, and extreme ownership will still arise whenever there is some player for whom the function $q_i(\widehat{e}(\theta))$ is larger than those of her opponents for all θ. This will also be the case whenever $\theta^* \in \arg\max_\theta \sum_i \Delta b_i(\theta)$[40] can be selected independent of α (e.g., this occurs if x is additively separable in a way that $E[x \mid e]$ can be expressed as $\sum_i E[x_i \mid e_i]$, for some random variables x_i that are independent of e_{-i}). The analysis for second-best will be more involved due to the presence of (ND). However, a basic principle remains: the marginal returns to ownership will be high whenever there is a combination of high marginal productivity and informational asymmetry, i.e., a high $q_i(\widehat{e}(\theta))$. Finally, if the timing was altered so that α was selected *after* observation of θ, the resulting ownership structure $\alpha(\theta)$ would be precisely that described in Section 8.

14. Appendix 3

Consider the setup of Section 9 (i.e., hidden information), but without Assumption (A4) on additive separability. Now define, for all i:

$$x_i(e_i) := E_{\theta_{-i}} E_x \left[x \mid e_i, e_{-i}(\theta_{-i}) \right].$$

That is, $x_i(e_i)$ is the expected value of output given e_i, and given that other players follow efforts $e_{-i}(\theta_{-i})$ while θ_{-i} is unknown to i. As in Section 9, let $\Delta x_i := x_i(e_i(\overline{\theta}_i)) - x_i(e_i(\underline{\theta}_i))$. It turns out that, for these functions $x_i(\cdot)$, Proposition 3 and Theorems 5–6 will remain valid.[41] However, some caveats follow:

1. Recall that a type communication stage, before the selection of e, is ruled out (e.g., footnote 16). But communication may be necessary to achieve first-best whenever the cross partials $\partial^2 E[x \mid e] / \partial e_i \partial e_j$ are not zero. As a consequence, the best outcome that can be sustained is a "limited information first-best" with efforts $e_i^{**}(\theta_i) := \arg\max_{e_i} \{x_i(e_i) - c_i(e_i, \theta_i)\}$. The generalizations of Proposition 3 and Theorem 5 refer to such limited information outcome.
2. In the absence of (A4), the equivalence between additive separability of voluntary transfers and dominant strategy implementation (Lemma 6) will no longer hold.

[40] This problem is well defined because first-best efforts guarantee that each $\Delta b_i(\theta)$ is continuous in θ.

[41] Indeed, the same proofs remain valid once the possibility that $x_i(0) > 0$ is accounted for, e.g., we can no longer assume WOLOG that $x_i(0) = 0$.

3. Under (A4), the linear bonuses in Theorem 6 had a straightforward interpretation as piece-rates. But when (A4) is relaxed, the empirical meaning of a linear function over x_i will not be as clear.

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E-mail address: `lrayo@stanford.edu`

URL: `www.stanford.edu/~lrayo`