Vertical Integration, Exclusive Dealing, and *Ex Post* Cartelization*

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Abstract

This paper uncovers an unnoticed connection between vertical integration and exclusive dealing. A vertically integrated firm has the incentive and ability to use exclusive contracts to foreclose an equally efficient upstream competitor and to effect a cartelization of the downstream industry. Its ability to do so may be limited when downstream firms are heterogeneous and supply contracts are not contingent on uncertain market conditions. The extent of cartelization depends on the degree of downstream market concentration and on the degree to which downstream competition is localized.

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1. INTRODUCTION

Antitrust scholars have devoted much ink to the competitive effects of vertical mergers (Riordan and Salop, 1995). For the most part, the economics literature focuses on how vertical integration *per se* alters pricing incentives in relevant upstream and downstream markets. The Chicago school of antitrust, represented by Bork (1978), emphasizes that the efficiencies of vertical integration are likely to cause lower prices to final consumers, while a more recent strategic approach to the subject, represented by Ordover, Salop and Saloner (1990) and Hart and Tirole (1990), shows how vertical integration lacking any redeeming efficiencies might have the opposite purpose and effect. Choi and Yi (2000) and Church and Gandal (2000) consider richer models that feature trade-offs between anticompetitive effects and efficiencies. The debate is far from settled, in no small part because workable *indicia* of harmful vertical mergers are lacking except in special cases (Riordan, 1998).

The use of exclusive contracts by customers and suppliers in intermediate product markets is equally controversial. The courts and antitrust agencies historically have treated exclusive dealing harshly, finding in many cases such practices illegally to foreclose competition. The Chicago school disputes this approach, advising instead that exclusive contracts are presumptively efficient, because usually it is unprofitable to foreclose competition *via* exclusive contracts without good efficiency reasons (Bork, 1978). More recently, industrial organization economists have studied alternative models that demonstrate equilibrium incentives to foreclose more efficient potential entrants with exclusive contracts (Aghion and Bolton, 1987; Bernheim and Whinston, 1988; Rasmusen, Ramseyer and Wiley, 1991; Segal and Whinston, 2000).

An important institutional feature of some intermediate product markets is the coexistence of vertical integration and exclusive contracts. For instance, in *Standard Oil Co. v. U.S.* (1949), Standard Oil sold about the same amount of gasoline through its own service stations as through independent retailers with which it had exclusive dealing contracts. In *Brown Shoe Co. 62 F.T.C. 679* (1963), Brown Shoe had vertically integrated into the retailing sector while using exclusive dealing contracts with independent retailers. In *U.S.*

v. Microsoft (D.D.C. 2000), Microsoft's had license agreements with competing online service providers, requiring them to promote and distribute Microsoft's Internet Explorer to the exclusion of competitive browsers. This institutional feature is potentially important because, as we shall show, the incentive for and effects of exclusive contracts may depend on whether an upstream supplier is vertically integrated, and, conversely, the returns to vertical integration may depend on the possibility of exclusive contracting.

While the existing economics literatures on vertical integration and exclusive contracts yield important insights on the competitive effects of these practices used in isolation, the literatures generally ignore incentives for and effects of these practices in combination. The purpose of this paper is to uncover an unnoticed connection between exclusive contracts and vertical integration, and to develop a model for analyzing how these practices complement each other to achieve an anticompetitive effect. More specifically, we argue that a vertically integrated upstream firm has the ability and incentive to use exclusive contracts to exclude equally efficient upstream competitors and control downstream prices.[1] The *ex post* effect is a cartelization of the downstream industry.

The paper is organized as follows. Section 2 previews our basic ideas. We illustrate the relationship between vertical integration and exclusive dealing in a simple model of industrial organization with two identical upstream and two identical downstream firms. We then discuss potential complications that may arise if the downstream firms are heterogeneous and there are non-contractible uncertainties, providing a transition to our main model with these features. Section 3 studies the main model of the paper. We demonstrate that a vertically integrated firm can profitably employ an exclusive contract to raise input prices and to cartelize the downstream industry, but the cartelization is in general only partial when downstream monopoly prices vary with non-contractible market conditions. Employing the logic of the recent literature on private bilateral contracting (Cremer and

[1] As discussed later, the Hart and Tirole (1991) model explains the exclusion of only a less efficient competitor. While the Ordover, Salop, and Saloner (1990) model does demonstrate the equilibrium exclusion of an equally efficient competitor, the game theoretic-premises of the model limit its applicability (Hart and Tirole, 1991; Reiffen, 1992; Ordover, Salop and Saloner, 1992).

Riordan, 1987; Hart and Tirole, 1991; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Tirole, 2003), we further show that exclusive contracts may not achieve this anticompetitive effect if the industries are vertically separated. Section 4 concludes by discussing these results in the contexts of the existing economics literature and of antitrust cases. Appendices A and B relax the restrictive assumption that the downstream market is a duopoly by considering two alternative models of downstream markets with multiple independent competitors: the "spokes" model and the circle model. The results obtained earlier extend naturally to these two models, with the additional insight that the extent of upstream foreclosure and downstream cartelization depends importantly both on the nature of competition (non-localized versus localized) and on the degree of concentration in the downstream market. Proofs for some of the results in Section 3 are in Appendix C.

2. BASIC IDEAS

That vertical integration and exclusive dealing can combine to foreclose an equally efficient upstream competitor and to raise downstream prices is easy to demonstrate in a simple model of industrial organization. Suppose there are two identical upstream firms, *U1* and *U2*, and two identical downstream firms, *D1* and *D2*. The downstream firms require one unit of an intermediate good to produce one unit of the final good, for which identical consumers have a known reservation price V. Downstream costs per unit of production are equal to $C < V$ and upstream costs are normalized to zero. If the firms are independent, then Bertrand competition in the upstream market followed by Bertrand competition in the downstream market results in a final goods price equal to C. Against this backdrop, a vertically integrated *U1-D1* has an incentive to purchase an exclusive right to serve the downstream market and charge final consumers a price equal to V. For example, *U1-D1* might pay *D2* to withdraw from the market, or, alternatively, acquire *D2*. Such blatant monopolization likely would meet objections from antitrust authorities. More benign in appearance is an exclusive requirements contract that achieves the same anticompetitive effect. A contract that requires *D2* to purchase from *U1* at a price of $V - C$ fully extracts monopoly rents from the downstream market. Firm *U2* is excluded from the upstream market, and final consumers pay V to purchase from either *D1* or *D2*.

It is interesting that *D2* does not need much persuasion to agree to purchase its requirements exclusively from *U1-D1* on non-competitive terms. If *D2* were to decline an exclusive requirements contract with *U1-D1*, and instead to deal with *U2* on competitive terms, then vigorous competition from *D1* would squeeze out downstream profits to the point where *D2* would be happy to have fallen into *U1*'s exclusive arms for a small concession, e.g. a small fixed fee. The Chicago school correctly observes that a downstream firm must be compensated to agree to forgo the benefits of upstream competition (Bork, 1978), but the above simple model shows that the necessary compensation need not be large if the firm has little to lose because of vigorous downstream competition.[2] An exclusive contract effectively monopolizes the downstream industry, and the monopoly rents can be shared in some measure by all concerned firms.

It also is interesting that neither vertical integration nor exclusive dealing alone achieve these anticompetitive effects if contracts are bilateral. The vertically integrated *U1-D1* could not persuade the independent *D2* to pay a supra-competitive price for the intermediate good without an exclusive contract, because *D2* would retain an *ex post* incentive to purchase from *U2* on competitive terms and cut its retail price to steal business from *D1.* Similarly, unable to commit to a multilateral contract that binds both *D1* and *D2*, a vertically-separated *U1* is unable to pay *D1* and *D2* enough to induce them both independently to forego the competitive alternative. Thus, vertically-separated upstream firms in equilibrium maximize bilateral profits by offering each downstream firms an efficient two-part tariff that sets the unit price of the intermediate good equal to marginal cost.[3]

[2] In formalizing and qualifying Bork's argument, Bernheim and Whinston (1998) ignore downstream competition and vertical integration in their models of exclusive dealing.

[3] Hart and Tirole (1990) show that, when contracts are private, an unintegrated upstream monopolist similarly fails to achieve the monopoly outcome, and that partial forward integration (with a single downstream firm) solves the upstream monopolist's commitment problem and "restores" monopoly power (Rey and Tirole, 2003). Alternatively, the upstream monopolist could solve the commitment problem by contracting with a downstream firm exclusively. Our model shows that, when equally efficient firms compete

Matters are more complicated if downstream market conditions are uncertain and non-contractible. Suppose that C is a random variable, and that the realization of C becomes known after contracting for the intermediate good, but before setting downstream prices. Suppose further that requirements contracts take the form of uncontingent two-part tariffs. Then monopolization of the downstream industry by *U1-D1* is accomplished with an exclusive requirements contract that excludes *D2* by setting the marginal price of the intermediate good above all possible values of $V - C$. Otherwise, competition from *D2* would drive the downstream price below the monopoly level in some states of the world. Thus, under conditions of uncertainty and non-contractibility, *U1-D1* can use an exclusive contract effectively to purchase a monopoly right. The contract is hardly subtle, and such blatant exclusion likely would catch the attention of antitrust authorities.

Matters are complicated further by downstream heterogeneity. If some consumers prefer *D2*'s product, or are more cheaply served by *D2*, then a requirements contract that excludes *D2* obviously cannot fully maximize industry joint profits. Rather a fully effective *ex post* cartelization of the downstream industry would require coordinated pricing that divides the downstream market efficiently. For example, if random downstream costs have different realizations for *D1* and *D2*, then it is efficient to assign final consumers to the low cost firm. But if these uncertain downstream market conditions are non-contractible, then *U1-D1* would have the conflicting incentives both to exclude and not to exclude *D2*. *U1-D1* generally is unable both to divide the market efficiently and to fully extract rents with a two-part tariff that *D2* would accept. Thus, the combination of uncertainty, non-contractibility, and heterogeneity appear to create difficulties for *ex post* cartelization via vertical integration and exclusive dealing.

To understand fully the relationship between vertical integration and exclusive dealing, therefore, it is important to go beyond the simple case of homogeneous downstream firms and to study the relationship under conditions of downstream heterogeneity, uncertainty,

in the upstream market, either full forward integration (with both downstream firms), or a combination of partial vertical integration and exclusive dealing are needed to monopolize the downstream market (even when contracts are public).

and noncontractibility. In what follows, we analyze a game-theoretic model of an industry possessing these features. This analysis will make clear several points. First, the synergistic relationship between vertical integration and exclusive dealing is not due to the extremely vigorous nature of potential downstream competition between identical producers; rather, it holds more generally in the presence of heterogeneous downstream firms who possess some degree of market power. Second, while the vertically integrated firm has the incentive and ability to exclude upstream competition and cartelize the downstream market, its ability to do may be reduced with downstream heterogeneity and noncontractible uncertainty. In particular, the fixed payment needed to persuade *D2* to enter the exclusive contract may not be small when downstream firms are heterogeneous,[4] and only partial cartelization of the downstream industry is feasible when downstream monopoly prices vary with non-contractible market conditions. Third, extending the model to multiple independent downstream competitors, while maintaining the assumption of private bilateral contracting, reveals that the degree of *ex post* cartelization of the downstream industry depends on market concentration and on whether or not competition is localized. Fourth, the exclusive contracts that a vertically integrated firm uses to cartelize the downstream industry are not blatant antitrust violations. The vertically integrated firm subtly employs the marginal wholesale price of a two part tariff to raise the downstream price, and judicially employs the fixed fee to distribute the rents from cartelization. Because a higher wholesale price to downstream rivals also raises the opportunity cost of the vertically integrated firm itself, the elimination of double marginalization is not an efficiency of vertical integration.

3. HETEROGENEOUS DOWNSTREAM FIRMS

In this section, we study the main model of the paper. After describing the model, we consider a benchmark case in which an upstream monopolist is vertically integrated with one of the downstream duopolists. We then introduce an equally efficient non-integrated

[4] This is despite a hidden bonus to *D2*: Because the integrated firm treats foregone wholesale revenues as an opportunity cost, both of the downstream firms offer the final good at supra-competitive prices, which provides another source of compensation to *D2* for agreeing to the exclusivity.

upstream competitor, and proves that the vertically integrated firm profitably employs an exclusive contract to achieve the same market outcome as in the upstream monopoly case, except for the distribution of rents between the upstream and downstream industries. We further show that exclusive contracts are irrelevant if the industries are vertically separated. We complete this section by discussing what happens if the model is extended to allow multiple independent downstream firms.

3.1. The Model

The key properties of the model are that the costs of supplying the downstream product are uncertain, heterogeneous, and non-contractible, and requirements contracts are bilateral and private. The model is patterned roughly on markets for cement and concrete markets. Cement is a fixed proportions input into the production of concrete, and concrete producers typically procure cement supplies under requirements contracts. The demand for ready-mixed concrete is located at constructions sites that are difficult to predict or specify in contracts. Since delivered ready-mixed concrete requires a cement truck, transportation costs evidently are important and idiosyncratic to the location of the construction sites. The model captures these cost characteristics with a number of simplifying assumptions. We revisit cement and concrete markets at the end, when we discuss applications.

There is a single consumer located at $x \in [0, 1]$, who is interested in purchasing one unit of a product.[5] The consumer's uncertain reservation value V has a cumulative distribution function $F(v)$ on support $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v} < \bar{v} < \infty$. The corresponding probability density function is $f(v) > 0$ for $v \in [\underline{v}, \bar{v}]$. The consumer's uncertain reservation value gives rise to a well-behaved downward-sloping expected demand curve.[6] The corresponding expected marginal revenue function is also smooth and downward sloping under the following maintained familiar technical assumption:

[5] It is easy but cumbersome to extend the model to a finite number of consumers.

[6] We could replace the assumption of a random V with the assumption that the consumer has a conventional downward sloping demand curve.

$$A1. \qquad \frac{d\left(\frac{1-F(p)}{f(p)}\right)}{dp} \leq 0.$$

The downstream market contains two firms *D1* and *D2* with similar technologies. Each combines a component input with other inputs whose cost is normalized to zero. Additionally, to sell to the consumer *D1* incurs transportation costs τx and $D2$ incurs $\tau(1-x)$, where $\tau > 0$ is a fixed parameter, measuring the degree of *ex post* cost heterogeneity. Thus, the transportation costs of the two firms are negatively correlated. This simple spatial cost structure captures adequately the more general idea of uncertain cost heterogeneity.[7]

The downstream firms "bid" prices to the consumer, P_1 and P_2. At the time of bidding, the firms know x but do not know the realization of V. The consumer's reservation value becomes known only after the downstream firms set prices. The consumer purchases the lower priced product as long as that price is below the consumer's realized reservation value v, and nothing otherwise.

There are two upstream firms *U1* and *U2.* Each can supply the component at the same fixed cost $c \geq 0$. Suppose that *U1* and *D1* are vertically integrated. *U1* and *U2* each offer *D2* a contract requiring *D2* to purchase exclusively from *U1* or *U2.* The location of the consumer becomes known after *D2* commits to an exclusive supply relationship, but before downstream price competition. At the contract offer stage, x is uncertain and has a standard uniform distribution. Thus *D1* and *D2* are equally efficient *ex ante,* but have heterogeneous costs *ex post.*

Consumer characteristics, x and v, are not contractible. The supply contracts are assumed to take the form of a two-part tariff, specifying a fixed transfer payment from *D2* to *Ui*, t_i, and a price r_i that *D2* pays contingent on actual production.[8] The integrated

[7] The model could be extended to assume that the delivered costs of of the two products have a more general bivariate distribution. Alternatively, if the "transportation cost" is incurred directly by the consumer, as often assumed in spatial models of consumer preferences, then the parameter τ measures the degree of horizontal product differentiation.

[8] The two-part tariff allows an upstream firm to cartelize the downstream market by raisng the price

U1-D1 cannot commit to any internal transfer price that is not *ex post* jointly optimal, nor can anyone commit to a retail price through the supply contracts. The exclusive supplier produces the component only if *D2* succeeds in the downstream market.[9] The implicit assumption justifying this approach is that the transaction costs of determining the realization of x, and making the contract depend on this determination, are prohibitively high.[10] The consumer's reservation value is never observed publicly, although it is easy to write a contract contingent on production resulting from the consumer's purchase decision.

To summarize, the timing of the game is as follows:

Stage 1. *U1* and *U2* offer contracts (t_1, r_1) and (t_2, r_2).

Stage 2. *D2* chooses a contract.

Stage 3. x is realized.

Stage 4. *D1* and *D2* choose prices.

Stage 5. V is realized and the consumer makes a purchase decision.

We assume that contracting actions at Stages 1 and 2 are private.[11] This game of

of the intermediate good (r) above cost (c), while extracting rents with the fixed fee (t). If there were a large number of multiple consumers, then the fixed fee could be reinterpreted as a discount on inframarginal units of the product. Thus a cartelizing contract involves quantity premia. In practice, there are various concessions an integrated firm can make to compensate downstream firms for accepting non-competitive intermediate goods prices. For example, it is common for a manufacturer to provide fixed payments to retailers for promotional actitivities. See also the discussion of cases in the concluding section.

[9] Note that $t_i > 0$ means that *D2* pays a fee to *Ui* while $t_i < 0$ means the opposite; and that, if a contract is accepted, t_i is paid irrespective of whether any sale is made, but r_i is paid only if $D2$ actually makes a sale.

[10] A conceivable possibility, for example, is that contract terms depend on messages exchanged after x is realized, in the spirit of the Nash implementation literature (Maskin, 1985). We implicitly assume that the transactions costs associated with the necessary message game are prohibitively burdensome. Alternatively, such communication between downstream competitors might be construed to violate the antitrust laws.

[11] Our main results also hold if contracts are bilateral and public. However, if binding multilateral contracts were feasible, then vertical integration would not be a necessary ingredient of cartelization (Mathewson and Winter, 1984). The rationale for the private contracting assumption is developed by Cremer and Riordan (1987), Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994), and Rey and Tirole (2003).

imperfect information raises a subtle issue about beliefs. As will become clear, there is no (perfect Bayesian) equilibrium in which $D2$ contracts with *U2.* Accordingly, suppose in a candidate equilibrium that *D2* accepts *U1*'s contract offer. If *D2* were to deviate and reject *U1*'s offer, naturally *U1 (* and $D1$) should believe that *D2* has accepted a contract from *U2.* But then what should *D1* believe about the terms of that contract? *D1*'s belief about *D2*'s wholesale price ($\tilde{r}_2$) is important for the subgame equilibrium at Stage 4 when the downstream firms compete on price, and thus matters for what *U1* must offer at Stage 1 to gain *D2's* agreement. We assume that *D1* believes $\tilde{r}_2 = c$.[12]. We provide a rationale for this refinement later, after we have introduced more ideas and notation.

Remark 1 *The game form ignores the possibility that D2 might decline any exclusive contract and instead purchase on a spot market after learning x. A spot market is irrelevant because in equilibrium U2 offers a requirements contract on terms that are the same as would prevail in the spot market. The spot market price would be c (Hart and Tirole, 1990), providing no advantage compared to U2's contract offer.*

We further refine equilibria by requiring that *D1* and *D2* do not set prices below their costs at Stage 4, and *U2* does not offer a contract at Stage 1 that would be unprofitable if accepted by *D2.* Thus we confine our attention to equilibrium strategies with the property that a player never strictly prefers her offer to be rejected, whether in Stage 1 or in Stage 4 of the game. This property is implied by the stronger requirement that players do not use weakly dominated strategies. But that refinement is too strong for our purposes, because it would eliminate all pure strategy equilibria.[13]

[12] Hart and Tirole (1990) and Rey and Tirole (2003) do not discuss the issue, but implicitly make the same assumption in their analyses of upstream competition when one firm is vertically integrated.

[13] This is familiar from other games with infinitely many strategies, e.g. the Bertrand duopoly with cost assymetry (Kreps, 1990, p. 419, footnote d).

3.2 Upstream monopoly

We start our analysis by considering the situation where *U1* is the only supplier in the upstream market, and modify Stage 1 accordingly. In particular, if *D2* rejects *U1*'s contract offer at Stage 1, then *D1* operates as an unconstrained monopolist. This model provides a benchmark and establishes some preliminary results for our analysis of upstream duopoly. As there are only two functioning firms, *U1-D1* and *D2*, neither the exclusivity nor the privacy of contracts is an issue in the case of vertically-integrated upstream monopoly.

Suppose that *D2* accepts the contract (t_1, r_1) from *U1*. Let $p = P_1^m(x)$ maximize $\{(p - c - \tau x)[1 - F(p)]\}$ and $p = P_2^m(x, r_1)$ maximize $\{(p - r_1 - \tau(1-x))[1 - F(p)]\}$. These are monopoly prices that each downstream firm would offer consumer x in the absence of competition from the other. For any given x and r_1, $P_1^m(x)$ and $P_2^m(x, r_1)$ exist uniquely and satisfy:

$$P_1^m(x) - c - \tau x = \frac{1 - F(P_1^m(x))}{f(P_1^m(x))}, \qquad (1)$$

$$P_2^m(x, r_1) - r_1 - \tau(1-x) = \frac{1 - F(P_2^m(x, r_1))}{f(P_2^m(x, r_1))}, \qquad (2)$$

where we define $\frac{1-F(p)}{f(p)} = 0$ if $p > \bar{v}$. It is also clear that $\{(p - c - \tau x)[1 - F(p)]\}$ increases in p for $p < P_1^m(x)$ and decreases in p for $p > P_1^m(x)$. These monopoly prices are increasing, and corresponding monopoly profits are decreasing, in marginal costs. Given the regularity assumption *A1,* we then have:

Lemma 1 *(i) $P_1^m(x)$ increases in x and $P_1^m(x) - c - \tau x$ decreases in x. (ii) Assume that $P_2^m(x, r_1) < \bar{v}$. Then, $P_2^m(x, r_1)$ increases in r_1 and decreases in x, and $P_2^m(x, r_1) - r_1 - \tau(1-x)$ decreases in r_1 and increases in x.*

We will also make use of the additional technical assumption:

$$A2. \qquad P_1^m(0) \geq c + \tau.$$

A2 is satisfied if the likely values of V are not too small relative to $c + \tau$. The assumption implies that, if $r = c$, then *U2*'s willingness to supply at price equal to cost always constrains *U1*'s monopoly power. This fact is used in the proof of Proposition 1.

For any contract (t_1, r_1) that is accepted by $D2$ and for any x, there is an ensuing subgame where *D1* and D2 bid prices to the consumer, and the consumer makes a purchase decision.

Now define:

$$P_1(x, r_1) = \min \{P_1^m(x), r_1 + \tau(1-x)\}, \tag{3}$$

$$P_2(x, r_1) = \min \{P_2^m(x, r_1), \min\{P_1^m(x), r_1 + \tau x\}\}. \tag{4}$$

Lemma 2 *Suppose that $P_1^m\left(\frac{1}{2}\right) \geq r_1 + \frac{1}{2}\tau$. If U1 is the sole upstream supplier, then the following is a Nash equilibrium of the D1-D2 pricing subgame: If $x \leq \frac{1}{2}$, then D1 offers $P_1(x, r_1)$, D2 offers $r_1 + \tau(1-x)$, and the customer selects D1. If $x > \frac{1}{2}$, then D2 offers $P_2(x, r_1)$, D1 offers* $\min\{P_1^m(x), r_1 + \tau x\}$*, and the customer selects D2.*

P roof. See Appendix C.

Given r_1, $P_1(x, r_1)$ and $P_2(x, r_1)$ are the respective equilibrium prices when $x \leq \frac{1}{2}$ and $x > \frac{1}{2}$. The logic behind the construction of these two prices is as follows: *D1's* opportunity cost of making a sale (excluding τx), when the sale would have been made by *D2*, is $r_1 - c + c = r_1$. When $x < \frac{1}{2}$, *D1* is the low-cost supplier since $\tau x < \tau(1-x)$. Bertrand competition means that *D1* will set its price either at its monopoly level or at the marginal cost of *D2*, $r_1 + \tau(1-x)$, whichever is smaller. When $x > \frac{1}{2}$, *D2* becomes the low-cost supplier. *D1* is willing to lower its price to its marginal opportunity cost $r_1 + \tau x$, or, if $r_1 + \tau x > P_1^m(x)$, to its monopoly price $P_1^m(x)$ so that the probability of a sale will not be unprofitably low. Bertrand competition means that *D2* will set its price either at its monopoly price or at $\min\{P_1^m(x), r_1 + \tau x\}$, whichever is smaller.

The equilibrium prices in Lemma 2 are similar to those under Bertrand competition for a duopoly with different constant marginal costs, say $c_1 < c_2$, where the equilibrium price is c_2. Although both sellers charging a price $p \in (c_1, c_2)$ can also be supported as a Nash equilibrium, seller 2 would prefer not to be selected as the supplier at such a price. Thus, if we require that a seller should not strictly prefer to be rejected at the price it bids, the only equilibrium in our pricing game between *D1* and *D2* is the one characterized in Lemma 2. In what follows, we consider this as the unique (refined) equilibrium in the pricing subgame.[14]

[14]Notice that mixed strategy equilibria can be ruled out by standard arguments.

Returning to the entire game, we have

Lemma 3 *If U1 is the sole upstream supplier, and* (t_1, r_1) *is an equilibrium contract, then* $P_1^m\left(\frac{1}{2}\right) \geq r_1 + \frac{1}{2}\tau$.

P roof. See Appendix C.

Remark 2 *Lemma 3 also holds if D2 has some outside option for obtaining the input. This extension is relevant for the case of upstream competition considered later.*

We next define:

$$\begin{aligned}\Pi(r) &= \int_0^{\frac{1}{2}} [P_1(x,r) - \tau x - c]\,[1 - F(P_1(x,r))]\,dx \\ &\quad + \int_{\frac{1}{2}}^1 [P_2(x,r) - \tau(1-x) - c]\,[1 - F(P_2(x,r))]\,dx \end{aligned} \tag{5}$$

$$t(r) = \int_{\frac{1}{2}}^1 [P_2(x,r) - \tau(1-x) - r]\,[1 - F(P_2(x,r))]\,dx \tag{6}$$

Notice that $\Pi(r)$ is the joint upstream-downstream industry profit when *D2* contracts to purchase from *U1* at unit price r, and $t(r)$ is the transfer price that fully extracts rents from the downstream industry. We can now characterize the equilibrium of the game.

Proposition 1 *The game where U1 is the only upstream supplier has a unique equilibrium. At this equilibrium, U1 offers D2 contract* $\left(\hat{t}, \hat{r}\right)$*, which is accepted by D2, where*

$$\hat{r} = \arg\max_{c \leq r \leq v} \{\Pi(r)\}, \qquad \hat{t} = t(\hat{r}).$$

D1 is the seller with price $P_1(x, \hat{r})$ *if* $x \leq \frac{1}{2}$*, and D2 is the seller with price* $P_2(x, \hat{r})$ *if* $x > \frac{1}{2}$*. Furthermore,* $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$.

P roof. See Appendix C.

The equilibrium contract has a cartelizing effect. By charging *D2* a wholesale markup $(\hat{r} - c)$, *U1* raises *D2's* marginal cost directly, creating an incentive for *D2* to raise its prices. Thus, *D2* sells at a higher price when $x \geq 1/2$, and is less of a competitive constraint on *D1* when $x < 1/2$. The markup also raises *U1-D1's* opportunity cost, creating an incentive for *D1* to raise its prices and be less of a competitive constraint on *D2* when $x \geq 1/2$ and $P_2^m(x, \hat{r}) > \hat{r} + \tau x$. The overall effect is to lessen horizontal competition in the downstream market and to reduce consumer welfare, relative to the situation where the wholesale price for *D2* is c.[15]

The cartelization of the industry, however, is only partial, due to the assumption that x is not contractible. Full cartelization requires a monopoly price for all values of x. To see this, first consider the consumer at $x = 1$, where

$$P_2(1, \hat{r}) = \min\{P_2^m(1, \hat{r}), \min\{P_1^m(1), \hat{r} + \tau x\}\} > P_1^m(0)$$

since $P_2^m(1, \hat{r}) > P_2^m(1, c) = P_1^m(0)$, $P_1^m(1) > P_1^m(0)$, and $\hat{r} + \tau x > P_1^m(0)$. Therefore, for consumers sufficiently close to $x = 1$, we must have $P_2(x, \hat{r}) > P_1^m(1 - x)$, or the price is above the vertically-integrated industry monopoly level. Thus, there is a problem of double marginalization when cost heterogeneity is greatest. Next, consider consumers at or slightly below $x = \frac{1}{2}$. For these consumers, since $\hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$ from Proposition 1, we have $P_1(x, \hat{r}) < P_1^m(x)$, or the price is below the vertically-integrated industry monopoly level, i.e. there is a problem of excessive horizontal competition when the downstream firms have similar costs.

The obstacle to full cartelization is non-contractibility, i.e. contract terms do not vary with the location of the final consumer. This fact creates a tension between improving vertical efficiency in some circumstances and intensifying horizontal competition in others. The conflict arises in our model from the downward-sloping expected demand curve generated by the consumer's uncertain reservation price. A lower value of $\hat{r}$ causes lower downstream prices by reducing *D2*'s marginal cost as well as *U1-D1's* marginal opportunity cost. Thus, *U1* faces a trade-off in setting r_1. Reducing r_1 alleviates *D2*'s double marginalization prob-

[15] It is important for our result that *U1-D1* takes an integrated view of its operations and coordinates its upstream-downstream prices to maximize the integrated firm's expected profit. In our context, if this were not true, there would be no difference between a pair of vertically integrated or separated firms. The strategic incentives and effects can still be present, albeit to a less extent, if the interests of U1 and D1 are not completely harmonized under vertical integration.

lem at some locations, but also intensifies horizontal price competition elsewhere. If $\hat{r}$ is reduced, neither *U1-D1* nor *D2* can commit not to undercut each other for the consumer that is located closer to the rival. The problem is that downstream monopoly prices vary with the location of the consumer; and the single instrument $\hat{r}$ cannot achieve these prices in all circumstances.

3.3. Upstream Duopoly

We now return to the model where the upstream market is a duopoly. Recall that the contracts offered by *U1* and *U2* are denoted by (t_1, r_1) and (t_2, r_2), and *U1-D1* does not observe the contract offer that *U2* makes to *D2.* As we assumed earlier, if *D2* accepts *U2*'s contract off the equilibrium path, *U1-D1* believe that $\tilde{r}_2 = c$. The following lemma shows that this is implied by the belief that $U2$ and *D2* have negotiated a contract that maximizes their joint profit.

Lemma 4 *Suppose that U2 is the contracted supplier of D2. For any D1's belief $\tilde{r}_2$, U2 and D2's joint profit is maximized when $r_2 = c$.*

Proof. For any consumer $x \in [0,1]$ and any price strategy adopted by $D1$, $\tilde{P}_1(x, \tilde{r}_2)$, $D2$ will be the seller to x if

$$\tilde{P}_1(x, \tilde{r}_2) > r_2 + \tau(1-x),$$

and $D2$ will charge $\tilde{P}_1(x, \tilde{r}_2)$ for these consumers. Define

$$S_2(r_2) = \left\{x \in [0,1] : \tilde{P}_1(x, \tilde{r}_2) > r_2 + \tau(1-x)\right\},$$

then $S_2(r_2)$ is the set of consumers $D2$ sells to. ($D2$ may also sell to any consumer with x being such that $\tilde{P}_1(x, \tilde{r}_2) = r_2 + \tau(1-x)$, but including these consumers in $S_2(r_2)$ will not change our argument.) The joint profits of *U2* and *D2*, when $U2$ chooses r_2 while *D1* holds the belief $\tilde{r}_2$, are

$$\begin{aligned}\Pi_2(r_2 \mid \tilde{r}_2) &= \int_{x \in S_2(r_2)} \left(\tilde{P}_1(x, \tilde{r}_2) - (c + \tau(1-x))\right)\left[1 - F\left(\tilde{P}_1(x, \tilde{r}_2)\right)\right] dx \\ &\leq \int_{x \in S_2(c)} \left(\tilde{P}_1(x, \tilde{r}_2) - (c + \tau(1-x))\right)\left[1 - F\left(\tilde{P}_1(x, \tilde{r}_2)\right)\right] dx = \Pi_2(c \mid \tilde{r}_2),\end{aligned}$$

where the inequality is due to the fact that if $r_2 > c$, a reduction of r_2 to c potentially increases profitable sales for $D2$; and if $r_2 < c$, an increase of r_2 to c potentially reduces negative-profit sales for $D2$. ■

Thus, the only belief of $D1$ that is consistent with joint profit-maximization by *U2* and *D2* is $\tilde{r}_2 = c$. Choosing $r_2 = c$ is *U2-D2's* weakly dominant strategy*,* much like that in a second-price auction bidding her true value is each bidder's weakly dominant strategy. Here, the true marginal cost to *U2-D2* is c. For any $\tilde{P}_1(x, \tilde{r}_2)$, choosing $r_2 \neq c$ will only cause $D2$ to use the wrong marginal cost in competing with $D1$, causing $D2$ either not to make sales at prices that are above the true marginal cost or to make sales at prices that are below the true marginal cost.

Remark 3 *U1 must have correct beliefs in equilibrium. Therefore, the lemma implies that, if D2 contracts with U2 in an equilibrium, then D1's belief must be* $\tilde{r}_2 = c$*.*

If *D2* contracts with *U2,* and *U1* believes that $\tilde{r}_2 = c$, then the profits anticipated by *U1-D1* and by *U2-D2* are:

$$\int_0^{\frac{1}{2}} \tau(1-2x)\left[1 - F\left(c + \tau(1-x)\right)\right] dx = \int_{\frac{1}{2}}^1 \tau(2x-1)\left[1 - F\left(c + \tau x\right)\right] dx.$$

On the other hand, if *D2* contracts with *U1*, since $\hat{r} > c$ from Proposition 1, we have

$$\Pi(\hat{r}) > \Pi(c) = \int_0^{\frac{1}{2}} \tau(1-2x)\left[1 - F\left(c + \tau(1-x)\right)\right] dx + \int_{\frac{1}{2}}^1 \tau(2x-1)\left[1 - F\left(c + \tau x\right)\right] dx.$$

Therefore, since

$$\Pi(\hat{r}) - \int_{\frac{1}{2}}^1 \tau(2x-1)\left[1 - F\left(c + \tau x\right)\right] dx > \int_0^{\frac{1}{2}} \tau(1-2x)\left[1 - F\left(c + \tau(1-x)\right)\right] dx > 0,$$

the competition between *U1* and *U2* must mean that in equilibrium, *D2* will contract with *U1*, with *U2* offering $(0, c)$ and *U1* offering (t_1^*, r_1^*), where $r_1^* = \hat{r}$, and

$$t_1^* = \int_{\frac{1}{2}}^1 \left[P_2(x, \hat{r}) - \tau(1-x) - \hat{r}\right]\left[1 - F\left(P_2(x, \hat{r})\right)\right] dx - \int_{\frac{1}{2}}^1 \tau(2x-1)\left[1 - F\left(c + \tau x\right)\right] dx. \quad (7)$$

Notice that when r_1 increases, $P_2(x, r_1)$ is either unchanged when $P_2(x, r_1) = P_1^m(x)$, or increases otherwise; and it can be verified that there will be some interval on $(\frac{1}{2}, 1]$ on which

$P_2(x, \hat{r}) \neq P_1^m(x)$. In addition, $P_2(x, r_1) - \tau(1-x) - r_1$ weakly decreases in r_1. Thus

$$t_1^* < \int_{\frac{1}{2}}^{1} \left[P_2(x, c) - \tau(1-x) - c\right]\left[1 - F\left(P_2(x, c)\right)\right] dx - \int_{\frac{1}{2}}^{1} \tau(2x-1)\left[1 - F\left(c + \tau x\right)\right] dx = 0.$$

Furthermore, since $r_1^* = \hat{r}$, the downstream equilibrium outcome is the same as under upstream monopoly. We have thus shown:

Proposition 2 *The game where the upstream market is a duopoly has a unique equilibrium. At this equilibrium, U2 offers D2 $(0, c)$ and U1 offers D2 (t_1^*, r_1^*), where $r_1^* = \hat{r}$, D2 contracts with U1, and the downstream equilibrium outcome is the same as under upstream monopoly.*

Thus, a vertically integrated firm is able to outbid a stand-alone supplier for an exclusive relationship with a downstream competitor. When the integrated firm supplies *D2* at a price above marginal cost, the former has less incentive to undercut *D2* because of the opportunity cost of foregone input sales to *D2.* This dampening of horizontal competition explains *U1*'s advantage and ability to preempt *U2* (Gilbert and Newbery, 1982).[16] Because of downstream heterogeneity, the profitable exclusion of *U2* may nevertheless cost *U1-D1* a substantial amount. However, this cost approaches zero as the difference between $D1$ and $D2$ disappears, i.e. $t_1^* < 0$ and $\lim_{\tau \to 0} \int_{\frac{1}{2}}^{1} \tau(2x-1)\left[1 - F\left(c + \tau x\right)\right] dx = 0$ imply $\lim_{\tau \to 0} t_1^* = 0$.

Remark 4 *U1's out-of-equilibrium belief $\tilde{r}_2 = c$ matters for equilibrium value of t_1^*, but not otherwise for an equilibrium outcome. For example, if U1 believed $\tilde{r}_2 > c$ out of equilibrium, then downstream price competition would be less aggressive if U2 were to deviate and accept U2's offer, and the fixed payment t_1^* needed to gain $D2$'s compliance correspondingly would be less. Nevertheless, U1-D1 would still have an incentive to maximize joint profits by setting $r_1^* = \hat{r}$. Thus the refinement is not crucial for the equilibrium cartelization result.*

The exclusion of upstream competition leads to higher downstream prices compared to when *U2* supplies *D2.* The exclusivity of the contract clearly is important for the carteliza-

[16] While we have assumed for simplicity that *U1* and *U2* are equally efficient, the same logic would hold, and so would Proposition 2, if *U2* had a small efficiency advantage. In this case, however, *U1-D1* would have an incentive to "outsource" supplies of the input from the more efficient *U2.*

tion outcome under vertical integration. Since $\hat{r} > c$, *D2* would want to purchase from *U2* *ex post* as long as $r_2 < \hat{r}$, and *U2* would be willing to cut r_2 to as low as c to gain *D2*'s business. This implies that, if upstream firms cannot sign exclusive contracts with downstream firms, perhaps due to legal restrictions or to difficulties in contract enforcement, then the input price to *D2* must be set at $r_1^* = r_2^* = c$, with $t_1^* = t_2^* = 0$. Therefore:

Remark 5 *In the game where the upstream market is a duopoly, the cartelization of the downstream market can be achieved only if exclusive requirements contracts are feasible.*

3.4. Vertical Separation

Earlier, we showed that exclusive contracts used by a vertically integrated firm can achieve the market outcome of an upstream monopolist. To see that vertical integration is important for the cartelization effect of the exclusive contracts, we next consider a variation of our model in which *U1* and *D1* are vertically separated independent firms. We shall show that exclusive contracts are irrelevant in this case: the equilibrium input price for both downstream firms is c.

The timing of the modified game is as follows:

Stage 1. *U1* and *U2* each offer separate contracts to *D1* and *D2*.[17]

Stage 2. *D1* and *D2* choose contracts.

Stage 3. x is realized.

Stage 4. *D1* and *D2* choose prices.

Stage 5. V is realized and the consumer makes a purchase decision.

We continue to assume that contracting actions at Stages 1 and 2 are private. That is, *Dj* does not observe the contract offers made to *Di.* Unlike under the vertical integration of *U1* and *D1*, where *D1* always knows *D2*'s marginal cost when the latter contracts with *U1* and *D2* always knows the marginal cost of *D1*, under vertical separation additional

[17] To be consistent with our earlier analysis, we again assume that these are exclusive contracts requiring a downstream firm to purchase only from a certain upstream firm, although exclusive contracts are not necessary for our result that the intermediate-good price will be equal to c under vertical separation.

issues arise about beliefs when contracts are private. In particular, now when *Dj* receives an out-of-equilibrium offer, there is the issue of how it should believe about *Di's* contract terms. We shall assume that the downstream firms hold "passive beliefs". That is, *Dj* maintains the belief that *Di* has accepted an equilibrium contract offer even after receiving an out-of-equilibrium offer.[18]

We simplify our analysis from now on by requiring that prices for intermediate goods must not be below cost (i.e., $r_i \geq c$) if $c > 0$. We will explain how this strategy restriction matters later in this section. For now, we offer two justifications. First, we could dispense with the strategy restriction by assuming the existence of an outside market for the upstream product with a competitive price equal to c. In this case, c is the opportunity cost of diverting supplies from the outside market in order to supply the intermediate good to the downstream market on which our analysis focuses.[19] An implication of this interpretation is that a downstream firm could resell the intermediate good at a price of c, and this resale opportunity would make it unprofitable for an upstream firm ever to offer a contract with $r_i < c$. Second, below-cost pricing might expose an upstream firm to a predatory pricing suit, for which sufficiently high penalties would be a deterrent. Finally, we should also mention that our result will hold without the strategy restriction under an appropriate

[18] This a a standard refinement in the literature on private bilateral contracting, but it is not uncontroversial when downstream firms compete on prices (McAfee and Schwartz, 1994; Rey and Tirole, 2003; Rey and Verge 2003). The literature has studied "wary beliefs" as an alternative. Under wary beliefs a downstream firm who receives an out-of-equilibrium contract reasons that the upstream firm expects the contract to be accepted and has offered the rival downstream firm an acceptable contract that maximizes their joints profits. Wary beliefs equilibria are complicated to analyze because they implicitly involve a hierarchy of beliefs, e.g. *D1*'s belief about *D2*'s contract, *D1*'s belief about *D2*'s belief about *D1*'s contract, *et cetera*. Moreover, there are inconsistencies under wary beliefs that make the concept less than compelling. For instance, *D1* can conclude under wary beliefs that *D2* has incorrect beliefs about *D1*'s contract while continuing to believe that it has correct beliefs about *D2's* contract.

[19] To be more precise, assume that an upstream firm has an increasing marginal cost curve that determines a profit-maximizing quantity supplied to the outside market at the competitive price c. If this quantity is greater than the quantity of the intermediate good supplied to the downstream industry, then c is the opportunity cost of supplying the downstream industry.

alternative belief system, e.g. beliefs are passive except when the out-of-equilibrium contract contains $r_i < c$, in which case the downstream firm believes that its downstream rival has received the same deviation offer.[20]

Let the contract offers from *Ui* to *Dj* be denoted as (t_{ij}, r_{ij}), for $i, j = 1, 2$. Adapting our notation, let (t_j, r_j) now denote any contract that *Dj* accepts, whether offered by *U1* or *U2*. Let

$$P(x, r_1, r_2) = \min \{P^m(x, r_1), r_2 + \tau(1 - x)\}$$

with $P^m = P^m(x, r_1)$ defined implicitly by

$$P^m - r_1 - \tau x = \frac{1 - F(P^m)}{f(P^m)}.$$

P^m is the monopoly price for *D1* to serve a consumer at marginal cost $(r_1 + \tau x)$. If $[r_2 + \tau(1 - x)]$ is the marginal cost of *D2*, then equilibrium prices are $\max \{P(x, r_1, r_2), r_1 + \tau x\}$ for *D1,* and $\max \{P(1 - x, r_2, r_1), r_2 + \tau(1 - x)\}$ for *D2*. Bertrand competition implies that the downstream firm with the lowest marginal cost wins the customer. Thus, if $(r_1 + \tau x) \leq [r_2 + \tau(1 - x)]$, the equilibrium outcome is for *D1* to serve consumer x at price $P(x, r_1, r_2)$.

Market shares are determined as follows. Let $\tilde{x} = \tilde{x}(r_1, r_2)$ be defined by

$$\tilde{x} = \min \left\{ \max \left\{ \frac{r_2 - r_1}{2\tau} + \frac{1}{2}, 0 \right\}, 1 \right\}.$$

$\tilde{x}$ is the marginal consumer served by *D1,* when *D1* has marginal cost $(r_1 + \tau x)$ and *D2* has marginal cost $[r_2 + \tau(1 - x)]$.

The joint profits of an upstream-downstream pair are defined as follows. Let

$$\pi(x, r_1, r_2) = [P(x, r_1, r_2) - c - \tau x] [1 - F(P(x, r_1, r_2))].$$

[20] As is well known, there can often be multiple perfect Bayesian equilibria in games of imperfect information, supported by different beliefs. Our results are subject to the qualification that they hold under certain belief refinements. The results are strengthened, however, by the fact that they hold if contracts are bilateral and public (see Remarks 8 later), where there is no need to impose any restrictions on beliefs or strategies. Furhermore, when contracts are bilateral and private, under vertical integration the unique equilibrium outcome involves above-cost contracting and cartelization (Proposition 2); while under vertical separation marginal-cost contracting is always an equilibrium outcome (supported by at least some beliefs).

If *D1* accepts *Ui*'s contract offer, then the expected profit of the *Ui*-*D1* pair is

$$\Pi(r_1, r_2) = \int_0^{\tilde{x}(r_1, r_2)} \pi(x, r_1, r_2)\, dx.$$

We have the following result.

Proposition 3 *The game under vertical separation has a unique equilibrium outcome with* $\left(t_j^*, r_j^*\right) = (0, c)$ *for both* $j = 1, 2$.

P roof. See Appendix C.

Remark 6 *When U1 and D1 are vertically separated, exclusive contracts are irrelevant in equilibrium.*

Since in equilibrium $r_i^* = c$ for $i = 1, 2$, there is no need for exclusive contracts in equilibrium, and firms have equilibrium incentives to negotiate supply arrangements on competitive terms. The competitive contracting result accords well with previous conclusions in the literature on vertical control with private bilateral contracts (Cremer and Riordan, 1987; Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Tirole, 2003; Rey and Verge, 2003). While the details of our model and proof are different, the general logic is similar. An upstream and downstream firm cannot resist contracting efficiently to maximize their joint bilateral profit.

If both *D1* and *D2* were to contract only with *U1* at input prices above c, the downstream prices are higher and the joint profits for the upstream and downstream industries are also higher. One might then think that in equilibrium *U1* would be able to achieve this outcome, just as if *U1* and *D1* were vertically integrated. So why is this not the case? The reason is that, with *U1* and *D1* being vertically separated, each downstream firm can pair with *U2* (or with *U1*) at input price c to obtain a joint profit that is more than its joint profit with *U1* under the higher input price. This reasoning is made precise in the proof in Appendix C.

But why would *U1* be able to contract with *D2* at $r_2 > c$ when *U1* and *D1* are vertically integrated? One way to think about the intuition is the following: Since *U1* and *D1* are vertically integrated, *D1*'s pricing strategies depend on whether *D2* purchases from *U1* at $r_2 > c$. If *D2* contracts to purchase from *U1* at $r_2 > c$, *D1* would price less aggressively in the downstream market, which leads to a higher joint upstream-downstream profits. If instead *D2* contracts to purchase from *U2* at input price c, then both *D1* and *D2* will compete with marginal cost c, resulting in lower upstream-downstream joint profits. This implies that the joint profit *D2* can possibly obtain by contracting with *U2* will always be below what *U1* is willing to offer *D2* to sign it up for the exclusive contract, and hence the equilibrium obtains.

Remark 7 *Allowing $r_i < c$ would destroy the passive beliefs equilibrium if $c > 0$. U1 could profitably deviate from $[0, c]$ and offer D1 and D2 a contract $[t, r]$ with $t > 0$ and $0 < r < c$. With passive beliefs each firm would think that it alone was being offered the deviation contract and would be willing to pay for the competitive advantage. The upstream firm would profit essentially by fraudulently selling the competitive advantage twice. As explained earlier, this multilateral deviation would not be profitable if a downstream firm could resell the intermediate product at a price of c,*[21] *or if the expected antitrust penalties from below-cost pricing were sufficiently great.*[22] *Alternatively, the strategy restriction ($r_i \geq c$) could be dispensed with by postulating appropriately punishing beliefs for below cost deviation, e.g. "symmetry beliefs" that the rival has received the same below-cost offer.*

Finally, we can modify our arguments to show that our results in this and the previous section hold if contracts are bilateral and public (and the same is true for our results in the extended models in the appendices). Thus,

Remark 8 *If contracts are bilateral and public, then Proposition 3 (and our other main results) holds even without the strategy restriction. Multilateral public contracts would destroy the result. For instance, U1 could offer both D1 and D2 an r that maximizes the joint*

[21] The passive belief equilibrium always exists if $c = 0$, since a downstream firm can increase its profit by "buying" more of the intermediate good if $r_i < c$.

[22] The susceptibility of passive beliefs to multilateral deviations is discussed by McAfee and Schwartz (1995), Rey and Tirole (2003), and Rey and Verge (2003).

profits of U1-D1-D2, and if it could further stipulate in the contract that it would reduce r to c if either firm declines the contract, then the contract could be supported in equilibrium.

3.5. Extending to Multiple Downstream Firms

Our spatial model of downstream price competition is restrictive in that it only suits the case of downstream duopoly; (our simplifying assumptions of upstream duopoly and a single consumer are easily relaxed.) The logic of our results, however is more general. In Appendix A, we introduce a generalization of the model, in which n downstream competitors are located at terminal nodes of a symmetric "hub and spoke" network and consumers are distributed uniformly on the connected spokes.[23] This "spokes model" is interesting because it exhibits a strong form of non-localized competition;[24] each downstream firm possesses market power constrained by all other market competitors, who are equidistant.[25] In Appendix B, we also analyze a standard circle model of localized competition (Salop, 1979).

Our main results generalize readily to the spokes model. With $n > 2$ downstream competitors, vertical integration combines with exclusive contracts to foreclose equally efficient upstream competition and raise downstream prices, and neither of the two practices alone

[23] A key observation for the extension of our results to the spokes model of downstream oligopoly is that prices are strategic complements (Bulow, Geanakoplos, and Klemperer, 1985). Thus, an exclusive contract that raises the marginal input price to a downstream competitor has the benefit of encouraging other downstream rivals to raise their prices also. These infectious effects enable a vertically integrated cartel organizer to achieve higher downstream prices by bringing the entire downstream industry under exclusive contracts. The argument is related to Davidson and Deneckere's (1985) analysis of incentives to form coalitions.

[24] Non-localized competition means in general that a consumer may have first-choice preference over downstream products, but no strong second-choice preference, or, alternatively, a consumer has a most-efficient supplier of the downstream product, but other suppliers are equally efficient. For example, consider a case in which a consumer can buy from a single local supplier, or can buy over the Internet from more distant suppliers. Non-localized competition also applies naturally to markets with consumer switching costs.

[25] This property is reminiscent of Chamberlinian monopolistic competition; individual firms have power over price while competing against "the market". See also Hart (1985a, 1985b) and Perloff and Salop (1985).

can achieve these anticompetitive effects. There are, however, two additional results from the spokes model. First, the equilibrium upstream price under vertical integration decreases in the number of downstream competitors. This suggests that market concentration in the downstream market can be important for the evaluation of the combined effects of vertical integration and exclusive contracts. Second, under vertical integration, there may be additional equilibria that are less preferred by the industry. Thus, the most profitable equilibrium might require a measure of coordination.

Our results also extend to the circle model of localized competition. In the circle model, the vertically integrated upstream firm only brings under exclusive contract its immediate downstream neighbors, while contracting efficiently with more distant downstream firms. Thus, in the case of four or more downstream firms, upstream competitors are excluded only from supplying the portion of the downstream market that is local to the integrated firm. Nevertheless, the combination of vertical integration and exclusive dealing has an anticompetitive effect in this local market segment.

Taken together, the spokes model and the circle model indicate that the extent of upstream foreclosure and downstream cartelization depends on the nature of (localized versus non-localized) competition. We could consider a hybrid model in which the consumer locates on a spokes network with some probability and otherwise on a circle. We conjecture that *U1-D1* would contract exclusively with all downstream competitors in the hybrid case, setting intermediate goods prices that reflect the probability of non-localized competition. Thus, the extent of downstream cartelization depends on the degree to which the integrated firm is in direct competition with independent downstream competitors.

4. DISCUSSION

Our analysis has revealed a relationship between vertical integration and exclusive dealing that has gone unnoticed in the economics literature. A vertically integrated firm has the ability and incentive to use exclusive requirements contracts to effect a cartelization of the downstream industry. The ability of the vertically integrated firm to do so may be limited when downstream firms are heterogeneous and contracts cannot be contingent on uncertain market conditions. In particular a complete cartelization remains elusive when downstream monopoly prices vary with non-contractible market conditions. In such circumstances, the extent to which a vertically integrated supplier is able to cartelize the downstream industry depends on the degree of concentration in the downstream market and on the degree to which downstream competition is localized.

Hart and Tirole (1990) made an important contribution to the vertical integration literature by showing how vertical integration enables an upstream monopolist to overcome a commitment problem when contracts are private, and achieve an *ex post* monopoly outcome in the downstream market. Rey and Tirole (2003) felicitously refer to this result as "restoring" monopoly power. The essential logic is that a vertically integrated firm better internalizes the opportunity cost of cutting supply prices to downstream rivals. The same logic carries over if the upstream firm competes against inferior upstream rivals, although the ability to achieve a full monopoly outcome is constrained by potential competition from the less efficient suppliers.

The Hart-Tirole-Rey theory does not explain an incentive for partial vertical integration if the upstream rivals are equally efficient. Our analysis shows that such an incentive does exist if a vertically-integrated upstream firm has recourse to exclusive contracts. By charging a higher marginal supply price to downstream rivals, the vertically integrated supplier engineers a "more collusive" downstream outcome.[26] The resulting increase in industry profits is shared among market participants *via* lump sum transfers. In this way, an enterprising upstream firm effectively cartelizes the downstream industry.

Aghion and Bolton (1987) made an important contribution to the literature on exclusive contracting by showing how penalty contracts could exclude an equally or more efficient entrant. Our analysis complements theirs by showing how a vertically integrated firm can use exclusive contracts to exclude an equally or more efficient firm who is already in the

[26] Chen (2001) has considered the collusive effect of vertical mergers in a model that assumes linear pricing and non-exclusive contracts between upstream and downstream firms. Similar to the Hart-Tirole-Rey theory, there is no vertical merger in Chen if the upstream rivals are equally efficient.

market.[27] As suggested by the Chicago School, the exclusion of the upstream competitor is costly to the integrated firm, i.e. transfers payment are needed to gain the acquiescence of the downstream industry. But the necessary transfer payments are not so large as to make *ex post* cartelization unprofitable for the vertically integrated upstream firm. Interestingly, this cost approaches zero when the heterogeneity between downstream firms disappears: the vertically integrated firm relies on cutting its downstream prices as a (hidden) threat to persuade the independent downstream firms to accept the exclusive contract; this threat provides the most powerful incentive, and hence there is little need for explicit transfer payment, when the downstream producers become perfect substitute for each other.

If our theory is to be useful for policies concerning vertical mergers and/or exclusive contracts, it must be supported by evidence on market structure. Our analysis suggests the following relevant evidence:

- Sole source requirements contracting is a normal industry practice or at least has some industry precedent. Otherwise, the theory might be judged as too speculative about post-merger industry conduct.

- Downstream price competition is "tough" before the vertical merger or before the adoption of exclusive contracts by a vertically integrated firm, as would be the case if the firms have similar capabilities/products and were not colluding tacitly (Sutton, 1991). Otherwise, there may be little to gain from cartelization *via* exclusive contracts, or the vertically-integrated firm might be unable to exclude an equally efficient upstream competitor.

- The vertically-integrated firm is likely to have substantial excess capacity or can expand capacity easily. Otherwise, the integrated firm is unlikely to be able to supply other downstream firms on competitive terms.

[27] Clearly, there is no role for a penalty contract if upstream rivals are equally efficient. If the excluded rival, however, is more efficient, then a penalty contract could be a way for integrated firm to extract rents. It would be interesting to explore this issue in an appropriate extension of our model.

- The downstream market is concentrated, and there are barriers to entry. Otherwise, the cartelization effect is small relative to the size of the market, or would be undone by new entry.[28]

- Evidence in favor of a plausible efficiency theory should be weighed against evidence in support of an anticompetitive effect (Riordan and Salop, 1995).[29]

We close by discussing briefly two antitrust cases to illustrate the empirical relevance of our ideas. One case is Kodak v. F.T.C. (1925). Kodak had a 90% market share for raw cinematic film that it supplied to downstream picture-makers. Kodak acquired capacity to enter the downstream industry, and reached essentially an exclusive-dealing agreement with picture-makers in which it agreed not to deploy the capacity if picture-makers would refrain from purchasing imported raw film. The Court found this agreement to be an illegal restraint of trade.

Another case is TEKAL/ITALCEMENTI (A76), brought up by the Italian Antitrust Authority against Italcementi, the main cement manufacturer in Sardinia, Italy.[30] Faced with lower-priced competition from imported cement, Italcementi acquired ten concrete production facilities between April and June 1993, and began to sell its concrete at prices below variable cost, with the intention of dissuading the independent concrete producers from purchasing their cement from importers. It was then able to enter into contractual agreements with some main concrete purchasing companies that effectively excluded other concrete producers. The Italian Antitrust Authority ruled that the conduct of Italcementi was part of an overall plan to restrict access to the Sardinian cement market and constituted an abuse of dominant position, and it fined the company 3,750 billion lire.

[28] Market definition is a key issue when competition is localized. Sales to customer groups with few real alternatives may constitute a distinct product market.

[29] For example, if the upstream competition were "soft", as would be the case if the upstream firms colluded expressly or tacitly, and if uniform pricing were the normal pre-merger industry practice, then the merger arguably might increase economic efficiency by eliminating a double markup.

[30] The discussion of this case is based on Italian Antitrust Authority Annual Report 1994, published on April 30, 1995. We thank Pierluigi Sabbatini of the Italian Antitrust Authority for directing our attention to this case.

While these two cases occurred in different times, countries, and industries,[31] the strategic considerations involved in both of them are remarkably similar to those in our theory. In both cases, a vertically integrated upstream producer entered into exclusive contracts with independent downstream firms that excluded other upstream firms from market access. The independent downstream firms appeared to be willing to accept such arrangements because the integrated upstream producer used its downstream facilities to entice and discipline the independents: if the independents purchased inputs from the vertically integrated upstream producer, the vertically integrated downstream producer would compensate the independents by reducing or refraining from competition; otherwise it would aggressively cut prices. As a result, the vertically integrated firm was able to exclude an upstream competitors and likely also raised downstream prices. We also notice that the key features of our model are possibly present in the cases. In particular, for TEKAL/ITALCEMENTI (A76), the different downstream concrete producers likely had different shipping costs for consumers at different locations; downstream market condition was likely to be uncertain in that the location and the demand of a final customer might be unknown ex ante; and pricing contracts between a cement (upstream) producer and a concrete (downstream) producer did not appear to be contingent on the locations of final consumers.

Although the details of the two cases are different from our theoretical model, they do illustrate the empirical relevance of our argument that vertical integration raises heightened concerns about exclusive dealing and *vice versa*.

[31] Interestingly, there is a case similar to TEKAL/ITALCEMENTI (A76) in New Zealand, concerning a vertically integrated cement/concrete company, Fletcher Concrete and Infrastructure Limited, whose pricing behavior in the concrete market has the purpose and effect of excluding competition in the cement market and (eventually) raising concrete prices. In 2002, the New Zealand Commerce Commission investigated the case and issued a warning to the company for risking antitrust violation.

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APPENDIX A: "SPOKES" MODEL

We develop a new model of price competition by multiple downstream firms that is a natural extension of the duopoly model. In addition to extending our results, the model may also have independent interest in suggesting a new way of modeling non-localized price competition by differentiated oligopolists. To save space, we shall make our arguments mostly informally.

Suppose that the downstream has $n \geq 2$ firms, $D1, D2, ...Dn$. As before, $D1$ and $U1$ are vertically integrated. Each Di is associated with a line of length $\frac{1}{2}$, which we shall call l_i. The two ends of l_i are called origins and terminals, respectively. Firm Di is located at the origin of l_i, and the lines are so arranged that all the terminals meet at one point, which we shall call the center. This forms a network of lines connecting competing firms ("spokes"), and a firm can supply the consumer only by traveling on the lines. *Ex ante*, the consumer is located at any point of this network with equal probabilities. The realized location of the consumer is fully characterized by a vector (l_i, x_i), which means that the consumer is on l_i with distances of x_i to Di and of $\frac{1}{2} - x_i + \frac{1}{2} = 1 - x_i$ to Dj, $j \neq i$.[32] For instance, if $n = 3$ and $(l_i, x_i) = \left(l_3, \frac{1}{3}\right)$, we would know that the consumer's distance from firm 3 is $\frac{1}{3}$, and her distance from both firm 1 and firm 2 is $1 - \frac{1}{3} = \frac{2}{3}$. Obviously, the linear duopoly model is a special case of the spokes model with $n = 2$.

As in our earlier analysis, consider first the case where $U1$ is a monopolist in the upstream market. We shall assume that bilateral contracts are private (and beliefs are passive).[33] A contract offered by $U1$ to Dj, $j = 2, ...n$, can be written as (t_j, r_j). Modifying equations (1) and (2), we can define $P_1^m(x_1)$ and $P_j^m(x_j, r_j)$ as satisfying

$$P_1^m(x_1) - c - \tau x_1 = \frac{1 - F\left(P_1^m(x_1)\right)}{f\left(P_1^m(x_1)\right)}, \tag{1'}$$

[32] For the consumer located at the center, we shall denote her by $\left(l_1, \frac{1}{2}\right)$.

[33] Earlier, when *D1* and *D2* are the only two downstream firms, the vertical integration of *U1* and *D1* makes private contracting essentially the same as public contracting, since *D1* would always know *U1*'s offer to *D2* and *D2* would always know the transfer price from *U1* to *D1* is c. With several vertically independent downstream firms, private contracting potentially becomes a constraint even under the vertical integration of *U1* and *D1*.

$$P_j^m(x_j, r_j) - r_j - \tau x_j \quad = \quad \frac{1 - F\left(P_j^m(x_j, r_j)\right)}{f\left(P_j^m(x_j, r_j)\right)}, \; j = 2, ..., n. \tag{2'}$$

Let $\bar{r} \equiv \min\{r_j : j = 2, ..., n\}$. Modifying equations (3) and (4) in Section 3, for $i = 1, ..., n$ and $j = 2, ..., n$, we can define

$$P_1((l_i, x_i), \bar{r}) = \begin{cases} \min\{P_1^m(x_1), \bar{r} + \tau(1 - x_1)\} & \text{if } i = 1 \\ \min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\} & \text{if } i \neq 1 \end{cases}, \tag{3'}$$

$$P_j((l_i, x_i), r_j, \bar{r}) = \begin{cases} \min\left\{P_j^m(x_j, r_j), \max\{r_j + \tau x_j, \min\{P_1^m(1 - x_j), \bar{r} + \tau(1 - x_j)\}\}\right\} & \text{if } i = j \\ r_j + \tau(1 - x_i) & \text{if } i \neq j \end{cases}. \tag{4'}$$

Then, extending Lemma 2, in any downstream pricing game following any given $\{(t_j, r_j) : j = 2, ..., n\}$, there is a unique (refined) equilibrium outcome,[34] in which $D1$ sets $P_1((l_i, x_i), \bar{r})$ and Dj sets $P_j((l_i, x_i), r_j, \bar{r})$, with the equilibrium price for consumer (l_i, x_i) being

$$P^*((l_i, x_i), r_i, \bar{r}) = \begin{cases} \min\{P_1^m(x_1), \bar{r} + \tau(1 - x_1)\} & if \quad i = 1 \\ \min\{P_i^m(x_i, r_i), \max\{r_i + \tau x_i, \min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\}\}\} & if \quad i \neq 1 \end{cases};$$

consumer (l_i, x_i) selects *D1* if $i = 1$ or if $i \neq 1$ but $\min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\} < r_i + \tau x_i$; and consumer (l_i, x_i) selects *Di* if $i \neq 1$ and $\min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\} \geq r_i + \tau x_i$. As in Lemma 3, we require

$$P_1^m\left(\frac{1}{2}\right) \geq r_i + \frac{1}{2}\tau$$

for any equilibrium contract (t_i, r_i).

The presence of additional downstream firms introduces several issues that we must consider in extending the analysis leading to Proposition 1.

First, it is now possible that $r_j \neq r_k$ for some $j, k = 2, ..., n$ and $j \neq k$; and, should such a situation arise, consumer (l_j, x_j) may sometimes not be served by firm Dj, which creates an inefficiency since transportation costs are not minimized.

[34] As in standard Bertand competition with more than two firms, the strategy profile supporting the unique equilibrium ouctome may not be unique.

Second, suppose that $r_k = \bar{r} < r_j$ for some $j = 2, ..., n$; i.e., *Dk* has a cost advantage in supplying (l_j, x_j) when $r_k + \tau(1 - x_j) < r_j + \tau x_j$. But *Dk* cannot benefit from selling to such a consumer, since the competition from *D1* will drive the price down to $\min\{P_1^m(1-x_j), r_k + \tau(1-x_j)\} \leq r_k + \tau(1-x_j)$. This is because the perceived marginal cost for *D1* in supplying such a consumer when *Dk* is the other potential supplier and purchases from *U1* at r_k, is $c + r_k - c = r_k$.

Third, it immediately follows that to maximize joint upstream-downstream industry profits, we must have $(t_j, r_j) = (t, r)$ for $j = 2, ..., n$; because, if $r_k < r_j$ for some $j \neq k$, then slightly lowering r_j has no effect on the competition for consumer (l_i, x_i), $i \neq j$ but increases the expected industry profit from consumer (l_j, x_j). This allows us to generalize equations (5) and (6) and define

$$\begin{aligned}\Pi(r) \quad = \quad & \frac{2}{n}\int_0^{\frac{1}{2}} \left[P_1(x,r) - \tau x - c\right]\left[1 - F\left(P_1(x,r)\right)\right] dx + \\ & \frac{n-1}{n} 2 \int_0^{\frac{1}{2}} \left[P_2(x,r) - \tau x - c\right]\left[1 - F\left(P_2(x,r)\right)\right] dx, \qquad (5')\end{aligned}$$

$$t(r) = \frac{2}{n}\int_0^{\frac{1}{2}} \left[P_2(x,r) - \tau x - r\right]\left[1 - F\left(P_2(x,r)\right)\right] dx, \qquad (6')$$

where $\Pi(r)$ is the joint industry profits when $(t_j, r_j) = (t(r), r)$ for all $j = 2, ...n$. The transfer $t(r)$ fully extracts rents from the downstream industry.

Notice that an increase in r has the similar trade off here as in the downstream duopoly case: it affects positively the profit for $D1$ due to relaxed competition, but affects negatively the profits for each Dj if it worsens the double mark-up distortion. Since the second effect is more important with a higher n, we conclude that $\hat{r}$ decreases in n, where

$$\hat{r} = \arg\max_{c \leq r \leq \underline{v}} \left\{\Pi(r)\right\}.$$

As in Proposition 1, we will have $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$, and define $\hat{t} = t(\hat{r})$.

Fourth, to complete our argument that there is an equilibrium at which $U1$ offers $\left(\hat{t}, \hat{r}\right)$ to *Dj*, $j = 2, ..., n$ and these offers are accepted, we need to check that *U1* would not benefit from a deviation that privately offers different contracts to one or several *Dj*, since potentially *U1* can pair with some *Dj* to increase their profits at the expense of other independent downstream firms, as is well known in the private contracting literature.

Suppose that *U1* deviates by offering some *Dj* a contract $(t_j, r_j) \neq \left(\hat{t}, \hat{r}\right)$. It is obvious that $r_j > \hat{r}$ cannot be mutually profitable for *U1-Dj*. So suppose $r_j < \hat{r}$. This can have three possible effects: it reduces the expected profit of *U1-D1* when the consumer is located on line l_1, since *D1* will face stronger competition from *Dj* for such consumers; it reduces the joint profit of *U1* and *Dk* but does not benefit *Dj* when the consumer is located on line l_k, $k \neq j \neq 1$, since *D1* will match *Dj*'s possibly lower price for such a consumer;[35] and it increases the profit for *Dj* when the consumer is located on line l_j and hence *Dj* would be willing to make a higher transfer payment to *U1*. Since contracts are private and beliefs are passive, potentially the most desirable deviation that *U1* can make is to offer every *Dj* the reduction in r, so that every *Dj* will be willing to pay a higher t to *U1*. But of course *D1* knows the lower r for each *Dj* and each *Dj* knows that *D1* knows that.[36] The competition between *D1* and every *Dk* means that the industry profit will again be given by $\Pi(r)$ under the new r, as defined by equation (5'). Since $\hat{r}$ has already been chosen to maximize $\Pi(r)$, the new r must lead to a lower $\Pi(r)$, which means that *U1-D1* must lose more than what it gains from the increased payment of every *Dj*. Thus *U1* cannot profitably deviate from $\left(\hat{t}, \hat{r}\right)$.

Therefore, the proposed is indeed an equilibrium, and we can extend Proposition 1 to the spokes model with $n \geq 2$ downstream competitors.

Proposition 1' *The game where U1 is the only upstream supplier has a equilibrium in which U1 offers Dj contract* $\left(\hat{t}, \hat{r}\right)$, *which is accepted by Dj,* $j = 2, ..., n$. *Di is the potential seller with price* $P^*((l_i, x_i), \hat{r}, \hat{r})$ *if the consumer is located at* (l_i, x_i), $i = 1, ..., n$.

[35] Importantly, *D1* is in direct competition with *Dj* and has both the incentive and ability to constrain *Dj* whenever *Dj* attempts to sell to the consumer on l_k. This makes it irrelevant that *Dk* does not observe the contract offer to *Dj*.

[36] Thus, it is important that with the vertical integration of *U1-D1*, there is this mechanism of information exchange even when the downstream has multiple firms. This, in combination with the facts that *D1* is in direct competition with every other *D* firm and that *D1* internalizes the opportunity cost to *U1* of a lost sale at price r, allows *U1-D1* to achieve the outcome as if contracts were public.

Furthermore, $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$, and $\hat{r}$ decreases in n.

With multiple downstream competitors, however, it appears possible that there are other equilibria. For instance, it seems possible to have an equilibrium where *U1* offers (t', c) to all *Dj*, $j \neq 1$, where t' extracts *Dj*'s profit. If each *Dj* believes that other downstream firms' marginal cost is c, it may not be possible for *U1* to offer a profitable deviation to *Dj* that would be accepted. Nevertheless, the equilibrium with contract $\left(\hat{t}, \hat{r}\right)$ Pareto dominates other possible equilibria (for upstream and downstream firms, the strategic players of the game).

Thus, just as in the downstream duopoly model, the firm that is nearest to the consumer will bid the lowest price and will make the sale if this price does not exceed the consumer's valuation. The equilibrium $\hat{r}$ is above c for the same reason as in the duopoly case: it reduces downstream competition and thus raises industry profits.

Returning to upstream duopoly, we again assume that if *Dj* accepts *U2's* contract out of equilibrium, *D1* would believe that *U2* has offered *Dj* $r_j = c$, and it would indeed be optimal for *U2* to offer *Dj* $r_j = c$ if they were to contract. We continue to impose passive beliefs, and now also adopt the strategy restriction $r_j \geq c$ noting that the same justifications apply.[37] Further, when Dj contracts to purchase from $U2$ at $(0, c)$, $D1$ will charge $c + \tau(1 - x_i) < \min\{P_1^m(1 - x_i), \hat{r} + \tau(1 - x_i)\}$ if the consumer is located at (l_j, x_j) and $i \neq 1$, and thus the (expected) joint profit of *U2-Dj* is

$$\frac{2}{n}\int_0^{\frac{1}{2}} \tau(1 - 2x)\left[1 - F\left(c + \tau(1 - x)\right)\right] dx,$$

which is lower than the joint *U1-Dj* profit under $\hat{r}$.[38]

[37] The restriction is not needed in the case of upstream monopoly, because a price of $r_i < c$ similarly lowers the opportunity cost of the vertically-integrated competitor, making the out-of-equilibrium offer unattractive. In the case of upstream duopoly, however, the unintegrated $U2$ might deviate profitably by offering contracts with an intermediate goods price sufficiently below c to several independent downstream firms. Such a deviation would break the passive beliefs equilibrium if c is sufficiently greater than zero.

[38] Again, we note that, due to the vertical integration of *U1* and *D1*, *D1* knows if *Dj* deviates to contracting with *U2;* and that downstream competition is non-localized so that *D1* can effectively compete with *Dj* for consumers on any l_k, $k \neq j \neq 1$.

Since the expected profit of $D2$ when it contracts with $U1$, excluding any transfer payment, is

$$\begin{aligned} & \frac{2}{n}\int_0^{\frac{1}{2}} \left[P^*((l_2,x),\hat{r},\hat{r}) - \tau x - \hat{r}\right]\left[1 - F\left(P^*((l_2,x),\hat{r},\hat{r})\right)\right] dx \\ = & \frac{2}{n}\int_0^{\frac{1}{2}} \left[P_2\left(x,\hat{r}\right) - \tau x - \hat{r}\right]\left[1 - F\left(P_2\left(x,\hat{r}\right)\right)\right] dx, \end{aligned}$$

we can modify equation (7) to define

$$\begin{aligned} t^* = & \frac{2}{n}\int_0^{\frac{1}{2}} \left[P_2\left(x,\hat{r}\right) - \tau x - \hat{r}\right]\left[1 - F\left(P_2\left(x,\hat{r}\right)\right)\right] dx \\ & -\frac{2}{n}\int_0^{\frac{1}{2}} \tau(1-2x)\left[1 - F\left(c + \tau(1-x)\right)\right] dx, \end{aligned} \tag{7'}$$

where $t^* < 0$, and extend Proposition 2 as follows:

Proposition 2' *The game where the upstream market is a duopoly has an equilibrium in which U2 offers Dj* $(0, c)$ *and U1 offers Dj exclusive contract* $(t^*, \hat{r})$, *and Dj contracts with U1,* $j = 2, ..., n$. *The downstream outcome is the same as under upstream monopoly.*

The intuition here is the same as in the downstream duopoly case: When the integrated firm supplies *D2, ..., Dn* at a price above marginal cost, the former has less incentive to undercut the latter because of the opportunity cost of foregone input sales to *Dj*. This dampening of horizontal competition explains *U1*'s advantage and ability to preempt *U2*. The r that is optimal under upstream monopoly is again chosen to maximize the joint industry profits, and t^* is chosen so that each stand-alone firm is willing to enter the exclusive contract with $U1$. If any Dj, $j = 2, ..., n$ deviates and contracts with $U2$ at $(0, c)$, $D1$ will reduce its price to $c + \tau(1 - x_i)$ for any consumer located at (l_i, x_i), $i \neq 1$, making the expected joint profit between *U2-Dj* lower than the expected joint profit between *U1-Dj* under $\hat{r}$, which implies that no deviation would occur.[39]

Since $\hat{r} > c$, just as in the downstream duopoly case, the use of exclusive contracts is crucial for $U1$ to be able to exclude $U2$ and to raise the downstream prices.

[39] Notice that since in equilibrium *U2* offers $(0, c)$, adding additional upstream firms that are the same as *U2* will not change the results.

We now turn to the last issue: what happens if *U1* and *D1* are vertically separated? Under downstream duopoly, exclusive contracts are irrelevant when *U1* and *D1* are vertically separated, since in equilibrium any downstream firm will only accept contract at which the input price is c. This result holds when there are multiple downstream competitors as well.

First, we can argue that there can be no equilibrium where at least one of the downstream firms, say *Dj*, contracts to purchase at $r_j > c$. Suppose that an upstream firm, say *U1*, contracts with one or more downstream firms and for at least one of these downstream firms, say *Dj*, $r_j > c$. Notice that, as before, the joint profit of *U1-Dj* is maximized when $r_j = c$, given $r_i \geq c$ for $i \neq j$. If *Dj* is the only firm with which *U1* has contracted at a price above c, then clearly *U1* can benefit from a deviation of offering $r = c$ to *Dj*. If *U1* has contracted with several downstream firms, say *D1,...,Dk*, $1 < k \leq n$, at prices $r_j > c$, to prevent any one of them (*Dj*) from deviating and accepting *U2*'s contract with $r_j = c$, it is necessary that each of them receive the *U2-Dj* profit when $r_j = c$. But this implies that *U1* would earn negative profit, which cannot occur at any equilibrium.

Second, we can argue that it is an equilibrium for both *U1* and *U2* to offer $(0, c)$ to all downstream firms and *U1*'s offer is accepted by all *Di*, $i = 1, ..., n$. As before, again assume that each firm receiving a deviation offer believes that the other firms are still under the contracts of the candidate equilibrium. It suffices to consider n-step deviations by *U1* (or *U2*) that privately offer any (t, r), $r > c$, to all *Dj*, $j = 1, ..., n$. For *Di* to be willing to accept the deviation contract, it is necessary that *Di* receives a payment that compensates it for the loss in profit due to $r > c$, or

$$\begin{aligned}
-t \quad \geq \quad & \frac{2}{n}\int_0^{\frac{1}{2}} \tau(1-2x_i)\left[1 - F\left(c + \tau(1-x_i)\right)\right] dx_i \\
& -\frac{2}{n}\int_0^{\max\left\{0, \frac{1}{2} - \frac{r-c}{2\tau}\right\}} \left[\left(c + \tau\left(1 - x_i\right)\right) - \left(r + \tau x_i\right)\right]\left[1 - F\left(c + \tau(1-x_i)\right)\right] dx_i \\
> \quad & \frac{2}{n}\int_0^{\frac{1}{2}} (r-c)\left[1 - F\left(c + \tau(1-x_i)\right)\right] dx_i.
\end{aligned}$$

With an n-step deviation, *U1*'s revenue from *Di* is

$$(r-c)\,\frac{2}{n}\int_0^{\frac{1}{2}} \left[1 - F\left(\min\{P_2^m\left(x_i, r\right), c + \tau(1-x_i)\}\right)\right] dx_i.$$

Therefore, with n firms,

$$
\begin{aligned}
& n\left[(r-c)\,\frac{2}{n}\int_0^{\frac{1}{2}}\left[1-F\left(\min\{P_2^m\left(x_i,r\right),c+\tau(1-x_i)\}\right)\right]dx_i\right]-n\left(-t\right) \\
\leq\;\; & (r-c)\,2\left[\int_0^{\frac{1}{2}}\left[1-F\left(c+\tau(1-x_i)\right)\right]dx_i-\int_0^{\frac{1}{2}}(r-c)\left[1-F\left(c+\tau(1-x_i)\right)\right]dx_i\right]=0,
\end{aligned}
$$

which implies that there can be no profitable n-step deviation from the candidate equilibrium.[40] We can thus extend Proposition 3 to the case with multiple downstream competitors:

Proposition 3' *The game under vertical separation has a unique equilibrium outcome where* $\left(t_j^*, r_j^*\right) = (0, c)$ *for all* $j = 1, ..., n$.

APPENDIX B: THE CIRCLE MODEL

We now consider an alternative way of extending our model to multiple downstream firms. Instead of considering non-localized competition in the downstream market, we consider localized competition, adopting the circular city model of Salop (1979). Assume that the consumer is located with equal chance at any point of a circle with a perimeter equal to 1. Firms are located equidistant from each other on the circle. With $n > 2$ firms, $D1, D2, ..., Dn$, the distance between any two neighboring firms is simply $\frac{1}{n}$. Let *D1* be located at the bottom of the circle, followed clockwise by *D2, ..., Dn*. Thus, *D1*'s neighboring firms on the left and on the right are denoted as *D2* and *Dn*, respectively. The realized location of the consumer is denoted as $x \in [0, 1]$, where $x = 0$ if the consumer is at the bottom of the circle (the position of *D1*), and x increases clockwise (so, for instance, $x = \frac{1}{2}$ if the consumer is located at the top point of the circle). In what follows we shall only sketch our analysis.

Unlike our spokes model where each firm competes directly against the market, in the circle model each firm competes directly only against its two neighbors. If *U1* and *D1* are vertically separated, then again the only equilibrium outcome is for all downstream firms

[40] A deviation aimed at a strict subset of downstream firms is even less profitable because of sales lost to the remaining downstream competitors for whom $r_i = c$.

to purchase the input at price $c,$ same as in our basic model with rather similar reasoning. In what follows we thus assume that *U1* and *D1* are vertically integrated. For convenience, we shall focus on the case $n = 4$, and will in the end discuss the cases $n > 4$ and $n = 3$.

With $n = 4$, *D1* competes with *D2* and *D4* respectively when $x \in [0, \frac{1}{4}]$ and $x \in [\frac{3}{4}, 1]$, *D2* competes with *D3* when $x \in [\frac{1}{4}, \frac{1}{2}]$, and *D3* competes with *D4* when $x \in [\frac{1}{2}, \frac{3}{4}]$. Notice that the only firm *D1* does not compete with directly is *D3*. Denote the contract *U1* offers to *Dj* by (t_j, r_j), $j = 2, 3, 4$.

As before, we first characterize the equilibrium r_j if *U1* were the only upstream producer.

(1) We must have $r_3^* = c$ in equilibrium.

If $r_3^* > c$, *U1* can deviate by privately offering $r_3' = c$ to *D3*. This deviation has no effect on the competition between *D1* and *D2* or between *D1* and *D4*, when the consumer is located on the lower half of the circle, but it increases the joint profit of *U1* and *D3* when the consumer is located on the upper half of the circle. It would thus be profitable for *U1* to make the deviating offer and for *D3* to accept the offer, under proper transfer payment. Therefore in equilibrium we must have $r_3^* = c$.

(2) In equilibrium, *U1* is able to raise the input price of its neighbors; i.e., $r_2^* > c$ and $r_4^* > c,$ and to raise the final price for the consumer.

We shall look for r_2 and r_4 such that the joint profits of *U1-D1-D2* are maximized when the consumer is located on the left half of the circle and the joint profits of *U1-D1-D4* are maximized when the consumer is located on the right half of the circle. (Note that we already know $r_3^* = c$.) Because of symmetry, the equilibrium r_2^* and r_4^* would be equal.)

For consumer x located between *D1* and *D2* ($x \in [0, \frac{1}{4}]$), the consumer's distances from *D1* and *D2* are x and $\frac{1}{4} - x$, respectively. Since the distance of consumer x from *D3* is $\frac{1}{2} - x$, in order for the consumer to be served by either *D1* or *D2*, we need

$$r_2 + \left(\frac{1}{4} - x\right)\tau \leq c + \left(\frac{1}{2} - x\right)\tau,$$

or[41] $r_2 \leq c + \frac{1}{4}\tau$. But since $c + \frac{1}{4}\tau < c + \tau \leq P_1^m(0)$, it follows that, for any $x \in [0, \frac{1}{4}]$,

[41] If this condition is not satisfied, then $D3$ would compete with $D1$ for consumer $x \in [0, \frac{1}{4}]$. By lowering r_2 to $c + \frac{1}{4}\tau$, the price for x is not changed but the profits to $D3$ would go to $D2$. Thus, to look for the

in equilibrium *D1* and *D2* will charge prices that are below their unconstrained monopoly prices. The equilibrium prices for consumer x are thus equal to $\max\{r_2+\tau(\frac{1}{4}-x), r_2+\tau x\}$, and *D1* and *D2* each serves the consumer located between $[0,\frac{1}{8}]$ and $[\frac{1}{8},\frac{1}{4}]$, respectively.

For consumer $x \in [\frac{1}{4},\frac{1}{2}]$, for whom *D2* and *D3* compete, the marginal consumer is

$$\hat{x}_2 = \frac{c-r_2}{2\tau} + \frac{3}{8},$$

where *D2* serves if $x \in [\frac{1}{4},\hat{x}_2]$ with price $c+(\frac{1}{2}-x)\tau$ and *D3* serves if $x \in [\hat{x}_2,\frac{1}{2}]$.

Therefore, the expected joint profit of *U1-D1-D2* when the consumer is located on the left half of the circle is

$$\begin{aligned}\Pi(r_2) &= 2\int_0^{\frac{1}{8}} \left[r_2+(\frac{1}{4}-x)\tau-(c+x\tau)\right]\left[1-F\left(r_2+(\frac{1}{4}-x)\tau\right)\right]dx \\ &\quad +\int_{\frac{1}{4}}^{\hat{x}_2}\left[c+(\frac{1}{2}-x)\tau-(c+(x-\frac{1}{4})\tau)\right]\left[1-F\left(c+(\frac{1}{2}-x)\tau\right)\right]dx.\end{aligned}$$

Let

$$\hat{r}_2 \equiv \arg\max_{c\le r_2\le c+\frac{1}{4}\tau.} \Pi(r_2).$$

Then, since

$$2\int_0^{\frac{1}{8}}\left[r_2-c+(\frac{1}{4}-2x)\tau\right]\left[1-F\left(r_2+(\frac{1}{4}-x)\tau\right)\right]dx$$

is strictly increasing in r_2 at $r_2=c$, while

$$\begin{aligned}&\left.\frac{d\left[\int_{\frac{1}{4}}^{\hat{x}_2}(\frac{3}{4}-2x)\tau\left[1-F\left(c+(\frac{1}{2}-x)\tau\right)\right]dx\right]}{dr_2}\right|_{r_2=c} \\ &= \left.(\frac{3}{4}-2\hat{x}_2)\tau\left[1-F\left(c+(\frac{1}{2}-\hat{x}_2)\tau\right)\right]\left(-\frac{1}{2\tau}\right)\right|_{r_2=c}=0,\end{aligned}$$

we must have $\Pi'(r_2)|_{r_2=c}>0$, and thus $\hat{r}_2>c$. Therefore, corresponding to Proposition 1, we have:

The game where U1 is the only upstream supplier has a (refined) unique equilibrium. At this equilibrium, $r_2^*=r_4^*=\hat{r}_2>c$, *and* $r_3^*=c$. *D1 is the potential supplier when* $x\in[0,\frac{1}{8}]\smile[\frac{7}{8},1]$, *D2 is the potential supplier when* $x\in[\frac{1}{8},\hat{x}_2]$, *D3 is the potential supplier when* $x\in[\hat{x}_2,\hat{x}_3]$ *where* $\hat{x}_3=\frac{r_4^*-c}{2\tau}+\frac{1}{2}$, *and D4 is the potential supplier when* $x\in[\hat{x}_3,\frac{7}{8}]$.

optimal r_2, we need to restrict to $r_2\le c+\frac{1}{4}\tau$.

We now return to the case of upstream duopoly. If *D2* were to contract with *U2*, the contract that would maximize the joint profit of *U2-D2* and give all this profit to *D2* is $(0, c)$. The joint profit of *U1-D1-D2* when the consumer is located on the left half of the circle would then be $\Pi(c) < \Pi(\hat{r}_2)$. Notice that $D2's$ profit when it accepts $(0, c)$ from *U2* is $\frac{2}{3}\Pi(c)$, and *U1-D1*'s profit from this part of the circle is $\frac{1}{3}\Pi(c)$.

Now let t_2^* be such that $D2's$ profit when it accepts $(t_2^*, \hat{r}_2)$ from *U1* is $\frac{2}{3}\Pi(c)$. Then, $D2's$ profit when it accepts $(t_2^*, \hat{r}_2)$ from $U1$ is the same as that when it accepts $(0, c)$ from $U2$, and *U1* will indeed offer $(t_2^*, \hat{r}_2)$ to *D2* since $\Pi(\hat{r}_2) - \frac{2}{3}\Pi(c) > \frac{1}{3}\Pi(c)$. Therefore, corresponding to Proposition 2, we have:

The game where the upstream market is a duopoly has a unique equilibrium outcome, where U1 contracts with D2 and D4 at $(t_2^*, \hat{r}_2)$, *while D3 contracts with either U1 or U2 at* $(0, c)$. *The downstream equilibrium outcome is the same as under upstream monopoly.*

More generally, if $n > 4$, in equilibrium we must have $r_2^* = r_n^* > c$ and $r_j^* = c$ for $j = 3, ..., n-1$; and the downstream equilibrium outcome under upstream duopoly is the same as under upstream monopoly.

The $n = 3$ case is different because *D2* and *D3* compete directly both with *U1* and with each other. Consequently the joint profit of *U1-D1-D2* depends on r_3. By the theorem of the maximum there exists a continuous bounded function $\sigma(r_3)$ such that $r_2 = \sigma(r_3) \geq c$ maximizes the joint profit of *U1-D1-D2* given any $r_3 \geq c$, and by Brouwer's theorem there exists a fixed point $r^* = r_2(r^*)$ that defines a symmetric equilibrium $r_3^* = r_2^* = r^*$. Finally, the joint profit of *U1-D1-D2* is increasing in r_2 when $r_2 = c$, which implies $r^* > c$.

Therefore, in the circle model with multiple downstream firms, just as in our basic model and spokes model, vertical integration in combination with exclusive contracts excludes an equally (or more) efficient supplier and partially cartelizes the downstream industry. Neither of these practices alone achieves these effects. However, the extent of upstream foreclosure and downstream cartelization depends importantly on the nature of competition—whether it is localized or non-localized, in addition to on the level of concentration in the downstream market. With localized competition (the circle model), the integrated firm can only cartelize the two neighboring downstream firms and exclude an upstream competitor in supplying these two firms.

APPENDIX C: PROOFS

Proofs for Lemma 2, Lemma 3, Proposition 1, and Proposition 3 follow.

Proof of Lemma 2: First consider the cases where $x \leq \frac{1}{2}$. Notice that $\tau x \leq \tau(1-x)$. From standard arguments in Bertrand competition, $P_1(x, r_1)$ maximizes the joint profits of *U1-D1* given *D2*'s offer, *D2*'s offer is optimal for *D2* given $P_1(x, r_1)$, and the consumer will select the firm with the lower cost, which is *D1* here. The consumer will make the actual purchase if $P_1(x, r_1) \leq v$.

Next consider the cases where $x > \frac{1}{2}$. Notice that $\tau x > \tau(1-x)$ in these cases. Notice also that, since $P_1^m(x) - c - \tau x$ decreases in x from Lemma 1, we may possibly have $P_1^m(x) < r_1 + \tau x$ even though $P_1^m\left(\frac{1}{2}\right) \geq r_1 + \frac{1}{2}\tau$. We proceed with two possible situations:

(i) Suppose $P_1^m(x) > r_1 + \tau x$. At $P_2(x, r_1) = \min\left\{P_2^m(x, r_1), r_1 + \tau x\right\}$, with the customer selecting *D2*, the expected profit of *U1-D1* is $[r_1 - c]\left[1 - F\left(P_2(x, r_1)\right)\right]$.

If *D1* undercuts *D2* so that it would be selected by the customer, the expected profit of *U1-D1* is less than

$$[r_1 + \tau x - (c + \tau x)]\left[1 - F\left(r_1 + \tau x\right)\right] \leq [r_1 - c]\left[1 - F\left(P_2(x, r_1)\right)\right].$$

On the other hand, given *D1*'s offer, it is optimal for *D2* to charge $P_2(x, r_1)$ and to be selected by the customer. Thus the proposed strategies constitute a Nash equilibrium.

(ii) Suppose instead $P_1^m(x) \leq r_1 + \tau x$. We have $r_1 + \tau(1-x) < r_1 + \frac{1}{2}\tau \leq P_1^m\left(\frac{1}{2}\right) < P_1^m(x)$. With the same logic as above, competition between *D1* and *D2* must drive the price down to $P_1^m(x)$, and the consumer selects *D2*. ■

Proof of Lemma 3. Suppose that, to the contrary, there is an equilibrium contract (t_1, r_1) such that $P_1^m\left(\frac{1}{2}\right) < r_1 + \frac{1}{2}\tau$. We shall show that the expected industry profit is higher under an alternative contract (t_1', r_1'), or simply under r_1', where $P_1^m\left(\frac{1}{2}\right) = r_1' + \frac{1}{2}\tau$. Since t_1 and t_1' will be chosen such that the expected profits of *D2* are zero under the respective contracts, it follows that the expected profit for *U1-D1* must be higher under contract (t_1', r_1') than under contract (t_1, r_1), which produces a contradiction.

First consider the cases where $x \leq \frac{1}{2}$. Since $\tau x \leq \tau(1-x)$ and

$$P_1^m(x) \leq P_1^m\left(\frac{1}{2}\right) \leq r_1' + \tau(1-x) < r_1 + \tau(1-x),$$

the equilibrium price will be $P_1(x, r_1) = P_1^m(x)$, under either r_1 or r_1', and the customer will select *D1.* Therefore for $x \leq \frac{1}{2}$, both contracts produce the same expected industry profits.

Now consider the cases where $x > \frac{1}{2}$. Then $P_1^m(x) < r_1 + \tau x$ from $P_1^m\left(\frac{1}{2}\right) < r_1 + \frac{1}{2}\tau$ and from Lemma 1. Thus

$$r_1' + \tau(1-x) < r_1' + \frac{1}{2}\tau = P_1^m\left(\frac{1}{2}\right) < P_1^m(x) < r_1' + \tau x.$$

Let $\hat{x} > \frac{1}{2}$ be such that either $\hat{x}$ uniquely solves

$$P_1^m(\hat{x}) = r_1 + \tau(1-\hat{x}),$$

or $\hat{x} = 1$ if $P_1^m(1) < r_1$. Then for $\frac{1}{2} < x < \hat{x}$, $P_1^m(x) < r_1 + \tau(1-x)$.

Hence, under r_1, the equilibrium price will be $P_1^m(x)$ but *D1* will be selected by the customer for $\frac{1}{2} < x < \hat{x}$; while under r_1' the equilibrium price will also be $P_1^m(x)$ but *D2* will always be selected by the customer for $\frac{1}{2} < x \leq 1$. Therefore, for $\frac{1}{2} < x \leq 1$, industry profits will be higher under r_1' than under r_1, since $\tau(1-x) < \tau x$.

Thus expected industry profits are higher under r_1' than under r_1, contradicting that (t_1, r_1) is an equilibrium contract. ■

Proof of Proposition 1. We only need prove that $P_1^m(0) - \tau < \hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$; everything else follows directly from Lemmas 1-3 and from Assumption $A2$.

We first show that $P_1^m(0) - \tau < \hat{r}$. Suppose to the contrary $P_1^m(0) - \tau \geq \hat{r}$. Then, $P_1^m(0) > \hat{r} + \tau x$ and $P_1^m(0) > \hat{r} + \tau(1-x)$, for all $x \in (0,1)$. We thus have

$$P_1(x, \hat{r}) = \hat{r} + \tau(1-x) < P_1^m(0) < P_1^m(x) \text{ for } 0 < x \leq \frac{1}{2},$$

and

$$P_2(x, \hat{r}) = \min\{P_2^m(x, \hat{r}), \hat{r} + \tau x\}\} < P_1^m(0) < P_1^m(1-x) \text{ for } \frac{1}{2} < x < 1.$$

By raising $\hat{r}$ slightly above $P_1^m(0) - \tau$, both $P_1(x, \hat{r})$ and $P_2(x, \hat{r})$ will be closer to $P_1^m(x)$ and $P_1^m(1-x)$, respectively, for all $0 < x < 1$, which would lead to a higher expected industry profit than under $\hat{r} \leq P_1^m(0) - \tau$. This implies that it cannot be optimal for *U1* to offer $\hat{r} \leq P_1^m(0) - \tau$; and therefore $\hat{r} > P_1^m(0) - \tau$.

We next show that $\hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$. It suffices to show that $\hat{r} \neq P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$, since from Lemma 3 $\hat{r} \leq P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$. Now, from the proof of Lemma 3, if $\hat{r} = P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$, the equilibrium prices would be $P_1(x, \hat{r}) = P_1^m(x)$ for $x \leq \frac{1}{2}$ and

$$P_2(x, \hat{r}) = \min\{P_2^m(x, \hat{r}), \min\{P_1^m(x), \hat{r} + \tau x\}\} > P_1^m(1 - x) \text{ for } \frac{1}{2} < x \leq 1.$$

That is, $P_1(x, \hat{r})$ is optimal for $x \leq \frac{1}{2}$ while $P_2(x, \hat{r})$ is inefficiently too high for $x > \frac{1}{2}$. A slight reduction in $\hat{r}$ would reduce both $P_1(x, \hat{r})$ and $P_2(x, \hat{r})$ for x that is close to $\frac{1}{2}$, causing a first-order increase in industry profits for those x that are to the right of $\frac{1}{2}$ and a second-order decrease in industry profits for those x that are to the left of $\frac{1}{2}$. Therefore, in equilibrium $\hat{r} \neq P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$. ■

Proof of Proposition 3. We organize the proof in two steps. In step 1, we show that any $r_i > c$ cannot occur in equilibrium. We then show in step 2 that there exists an equilibrium where $(t_j^*, r_j^*) = (0, c)$, for $j = 1, 2$. Since step 1 implies that at any possible equilibrium $r_j^* = c$, and hence $t_j^* = 0$, combining step 1 and step 2 completes our proof.

Step 1. *There can be no equilibrium where $r_i > c$ for any i.* Suppose to the contrary that there is some equilibrium where $r_i > c$ for at least one i. Without loss of generality, suppose that $r_1 > c$, and $r_1 \geq r_2 \geq c$. There are two possible cases to consider.

Case 1: r_1 and r_2 are offered by the two different upstream firms. Without loss of generality, suppose r_1 is from *U1* and r_2 is from *U2*. The marginal consumer between *D1* and *D2* satisfies $\tilde{x} \in (0, 1)$; otherwise one of the upstream-downstream pair would have zero joint profit; and by a contract with $r_i = c$ the pair could gain a positive market share and positive expected profit. A reduction of r_1 to c would allow *D1* to make at least the same profit from consumers with $x < \tilde{x}$ and to make some additional profit from consumers with $x > \tilde{x}$; and it thus increases the joint profits of *U1-D1*. This shows that there can be no equilibrium where the downstream firms are supplied by the two separate upstream firms and at least one downstream firm contracts to receive the input at a unit price above c.

Case 2: r_1 and r_2 are offered by a single upstream firm, say, *U1*. The joint profits of the group *U1-D1-D2* equal $[\Pi(r_1, r_2) + \Pi(r_2, r_1)]$. Let α_i denote *Di*'s share of these profits. An accepted offer from *U2* to *D1* with $r_1' = c$ would generate a joint profit that is at least $\Pi(c, r_2)$, since *D2's* prices are still based on the belief that $r_1 > c$ and are higher than its prices when it knows that $r_1 = c$. (Because contracts are private, the deviating offer could also come from *U1* itself and our argument would be the same.) Therefore, in order to prevent *D1* from deviating, and accepting $(t_{21}, r_{21}) = (0, c)$, we must have

$$\alpha_1 [\Pi(r_1, r_2) + \Pi(r_2, r_1)] \geq \Pi(c, r_2) > \Pi(r_1, r_2),$$

since $r_1 > c$ and $\tilde{x}(c, r_2) = \min\left\{\frac{r_2 - c}{2\tau}, 1\right\} > 0$. Similarly in order to prevent the *U2-D2* pair from deviating, we must have

$$\alpha_2 [\Pi(r_1, r_2) + \Pi(r_2, r_1)] \geq \Pi(c, r_1) > \Pi(r_2, r_1).$$

Combining these conditions, it is necessary that

$$(\alpha_1 + \alpha_2) [\Pi(r_1, r_2) + \Pi(r_2, r_1)] > \Pi(r_1, r_2) + \Pi(r_2, r_1),$$

which means that *U1* must have a negative profit when *D1* and *D2* contract with *U1* under r_1 and r_2. Therefore there can be no equilibrium where the downstream firms are supplied by a single upstream firm and at least one downstream firm contracts to receive the input at a unit price above c.

We have thus shown that there can be no equilibrium where $r_i > c$ for any i.

Step 2. *There exists an equilibrium in which* $(t_{ij}, r_{ij}) = (0, c)$ *for* $i, j = 1, 2$, *D1 accepts the contract offered by U1, and D2 accepts the contract offered by U2.*

Consider the following two-step possible deviation from the candidate equilibrium. First, *U1* offers *D1* a new contract (t_1, r_1), with $r_1 > c$, that *D1* accepts with passive beliefs. (Passive beliefs means that *D1* continues to believe that *D2* has accepted $(0, c)$ and acts accordingly.) Second, *U1* offers *D2* a new contract (t_2, r_2), with $r_2 > c$, that *D2* accepts with passive beliefs.

For $D1$ and $D2$ to be willing to accept the contracts, *U1* needs to pay *D1* and *D2,* respectively,

$$-t_1 \quad = \quad \Pi(c, c) - \left(\Pi(r_1, c) - (r_1 - c) \int_0^{\tilde{x}(r_1, c)} [1 - F(P(x, r_1, c))]\, dx\right)$$

$$= \quad \Pi(c,c) - \int_0^{\tilde{x}(r_1,c)} \left[P\left(x,r_1,c\right) - r_1 - \tau x\right]\left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx > 0,$$

$$\begin{aligned} -t_2 &= \Pi(c,c) - \left(\Pi(r_2,c) - (r_2 - c)\int_{\tilde{x}(c,r_2)}^{1} \left[1 - F\left(P\left(1-x,r_2,c\right)\right)\right] dx\right) \\ &= \Pi(c,c) - \int_{\tilde{x}(c,r_2)}^{1} \left[P\left(1-x,r_2,c\right) - r_2 - \tau(1-x)\right]\left[1 - F\left(P\left(1-x,r_2,c\right)\right)\right] dx > 0, \end{aligned}$$

where we have used the fact that when $r_2 = c$ and *D2* believes that $r_1 = c$, from $D1's$ point of view, the marginal consumer is $\tilde{x}(r_1,c)$, *D1*'s optimal price is $P(x,r_1,c)$, and *D1*'s revenue is $\Pi(r_1,c)$, for any $r_1 > c$. We note that if *U1* only makes the first step deviation, then its payoff from the deviation would be

$$t_1 + (r_1 - c)\int_0^{\tilde{x}(r_1,c)} \left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx = -\left[\Pi(c,c) - \Pi(r_1,c)\right] < 0.$$

Thus, with the symmetry between *U1* and *U2*, the candidate equilibrium will be sustained if the two-step deviation is not profitable for *U1.* Under the new contracts for both *D1* and *D2*, *U1* will obtain from *D1* and *D2*, respectively,

$$\begin{aligned} R_1 &= (r_1 - c)\int_0^{\tilde{x}(r_1,r_2)} \left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx, \\ R_2 &= (r_2 - c)\int_{\tilde{x}(r_1,r_2)}^{1} \left[1 - F\left(P\left(1-x,r_2,c\right)\right)\right] dx, \end{aligned}$$

where we have used the fact that when D1 and D2 respectively set $P(x,r_1,c)$ and $P(1-x,r_2,c)$ under the passive beliefs, the marginal consumer is precisely $\tilde{x}(r_1,r_2)$. Thus, the candidate equilibrium will be sustained if $-(t_1+t_2) \geq R_1 + R_2$, and without loss of generality we can assume that $r_1 \leq r_2$. Now,

$$\begin{aligned} R_1 - (-t_1) &= (r_1 - c)\int_0^{\tilde{x}(r_1,r_2)} \left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx \\ &\quad + \int_0^{\tilde{x}(r_1,c)} \left[P\left(x,r_1,c\right) - r_1 - \tau x\right]\left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx - \Pi(c,c) \\ &< \int_0^{\tilde{x}(r_1,r_2)} (r_1 - c)\left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx \\ &\quad + \int_0^{\tilde{x}(r_1,r_2)} \left[P\left(x,r_1,c\right) - r_1 - \tau x\right]\left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx - \Pi(c,c) \\ &= \int_0^{\tilde{x}(r_1,r_2)} \left[P\left(x,r_1,c\right) - c - \tau x\right]\left[1 - F\left(P\left(x,r_1,c\right)\right)\right] dx - \Pi(c,c), \end{aligned}$$

where the inequality is due to $\tilde{x}(r_1, r_2) > \tilde{x}(r_1, c)$. Similarly,

$$R_2 - (-t_2) < \int_{\tilde{x}(r_1,r_2)}^{1} [P(1-x, r_2, c) - c - \tau(1-x)][1 - F(P(1-x, r_2, c))]\,dx - \Pi(c,c).$$

Notice that

$$\begin{aligned}
2\Pi(c,c) &= \int_0^{\frac{1}{2}} [P(x,c,c) - c - \tau x][1 - F(P(x,c,c))]\,dx \\
&\quad + \int_{\frac{1}{2}}^{1} [P(1-x,c,c) - c - \tau(1-x)][1 - F(P(1-x,c,c))]\,dx \\
&\geq \int_0^{\tilde{x}(r_1,r_2)} [P(x,c,c) - c - \tau x][1 - F(P(x,c,c))]\,dx + \\
&\quad \int_{\tilde{x}(r_1,r_2)}^{1} [P(1-x,c,c) - c - \tau(1-x)][1 - F(P(1-x,c,c))]\,dx,
\end{aligned}$$

since downstream costs are higher when $\tilde{x}(r_1, r_2) \neq \frac{1}{2}$. Hence:

$$\begin{aligned}
& R_1 - (-t_1) + R_2 - (-t_2) \\
< & \int_0^{\tilde{x}(r_1,r_2)} [P(x, r_1, c) - c - \tau x][1 - F(P(x, r_1, c))]\,dx + \\
& \int_{\tilde{x}(r_1,r_2)}^{1} [P(1-x, r_2, c) - c - \tau x][1 - F(P(x, r_2, c))]\,dx - 2\Pi(c,c) \\
\leq & \int_0^{\tilde{x}(r_1,r_2)} [P(x, r_1, c) - c - \tau x][1 - F(P(x, r_1, c))]\,dx \\
& - \int_0^{\tilde{x}(r_1,r_2)} [P(x,c,c) - c - \tau x][1 - F(P(x,c,c))]\,dx \\
& + \int_{\tilde{x}(r_1,r_2)}^{1} [P(1-x, c, r_1) - c - \tau x][1 - F(P(x, c, r_1))]\,dx \\
& - \int_{\tilde{x}(r_1,r_2)}^{1} [P(1-x,c,c) - c - \tau x][1 - F(P(1-x,c,c))]\,dx \\
\leq & \ 0.
\end{aligned}$$

Therefore, *U1* cannot profitably deviate from the candidate equilibrium. ■