THE ECONOMICS OF RELATIONSHIPS*

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1 Introduction

1.1 Relationships: Two Illustrations

Each year, about $60 billion dollars worth of diamond jewelry is sold worldwide. Over the course of its journey from mine to warbrobe, a diamond typically passes through numerous intermediaries in search of just the right buyer. Because diamonds are easy to conceal, difficult to distinguish, portable and valuable, the opportunity to cheat on diamond deals are many. One would accordingly expect them to be handled with the utmost care. To the contrary, virtually no care at all is taken:[1]

> Once gems leave the vault-like workshops, they do so in folded sheets of tissue paper, in the pockets of messengers, dealers and traders. They are not logged in and out ... or marked to prevent substitution. They are protected from embezzling only by the character of those who transport. ... On that slender record, gems worth thousands of dollars traverse the street and are distributed among buyers from Bombay to Buenos Aries, Pawtucket and Dubuque.

In Puccini's opera *Gianni Schicchi*, the deceased Buoso Donati has left his estate to a monastery, much to the consternation of his family.[2] Before anyone outside the family learns of the death, Donati's relatives engage the services of the actor Gianni Schicchi, who is to impersonate Buoso Donati, write a new will leaving the fortune to the family, and then feign death. Anxious that Schicchi do nothing to risk exposing the plot, the family explains that there are severe penalties for tampering with a will and that any misstep puts Schicchi at risk. All goes well until the time arrives for Schicchi to write the new will, at which point he instructs that the entire estate be left to the great actor, Gianni Schicchi. The relatives watch in horror, afraid to object lest their plot be exposed and they pay the penalties with which they had threatened Schicchi.

[1] This account of diamond transactions is taken from Richman (2005) (the sales figure from page 10 (footnote 29) and the quotation from page 14 (noting that it is originally from an article by Roger Starr, "The Real Treasure of 47th Street," in the *New York Times* (March 26, 1984, Section A, page 18))).

[2] The use of *Gianni Schicchi* to illustrate the (lack of) incentives in an isolated interaction is due to Hamermesh (2004, p. 164).

The outcomes in these situations are strikingly different. Those involved in the diamond trade face constant opportunities to return one less diamond than they received, with the recipient unable to prove they had been shortchanged, but refrain from doing so. Gianni Schicchi sees a chance to steal a fortune and grabs the money. The diamond handlers are involved in a relationship. They know they will deal with one another again, and that opportunistic behavior could have adverse future consequences, even if currently unexposed. Gianni Schicchi was not involved in a relationship with the family of Buoso Donati. Nothing came into play beyond the current interaction, which he turned to his advantage.

Throughout our daily lives we similarly react to incentives created by relationships (or their absence). This paper examines work on the economics of relationships.

1.2 What is a Relationship?

A relationship is an interaction featuring

(1) agents who are tied together with identified partners over the course of a number of periods,

(2) incentives that potentially spill across periods, and

(3) future outcomes that are tailored to current actions (so as to create current incentives) not by contracts, but by the appropriate provision of future incentives.

If one prefers a more specific term than "relationship" for such interactions, likely alternatives are "relational contract" or "relational incentive contract" (e.g., Baker, Gibbons, and Murphy (2002), Levin (2003)).

The first building block in the study of relationships is an information, incentive, property right or contracting problem that pushes us away from competitive markets (Debreu (1959)) or contracting (Coase (1960)) as means for effectively allocating resources. In the diamond market, for example, the formal remedies for reneging on a deal are ineffective. In the words of one dealer, "The truth is that if someone owes you money, there's no real way to get it from him if he doesn't want to pay you." [3] The legal difficulties that prevented Buoso Donati's family from writing a contract with Gianni Schicchi recur in a variety of relationships, perhaps most notably those that are governed by antitrust legislation. Throughout, we simply take such constraints as given.[4]

[3] Richman (2005) (page 18, reporting a personal interview).

[4] The questions of which markets function effectively in an economy and which contracts can be written merit further study. There were once no insurance markets; they are now commonplace. There were once no mutual funds and no futures markets.

In the face of such incentive problems, it can make a great deal of difference to Alice whether she anticipates dealing with Bob once or whether she has a continuing relationship with Bob.[5] Forming a relationship potentially allows the two of them to create more effective current incentives by appropriately configuring their future interactions.[6]

The phrase "relationship-specific investment" is sufficiently familiar and sufficiently similar as to warrant comment.[7] Work in this area has focussed on the inefficiencies that can arise when two two trading partners are locked together but cannot write complete contracts.[8] We thus have the first characteristic of a relationship, but the remaining two aspects, repeated interactions with incentives that potentially spill over from one interaction to the next, are missing. A repeated relationship-specific-investment problem would bring us into the realm of relationships.

1.3 Why Study Relationships?

Economics is often (perhaps somewhat narrowly) defined as the study of resource allocation. Within this study, attention is tyically focussed on the role of prices and markets. We touch here on a few points of

The markets for buying and selling people in many countries are not as active as they once were, and many legal systems put limits on the penalties that can be written into (especially employment) contracts.

[5] Alice is interested in whether she will trade again with Bob rather than someone else not because they trade different goods—our standard models of markets allow some people to prefer buying musical performances from Itzhak Perlman while others prefer Pearl Jam—but because they are characterized by different past behavior and information.

[6] The mere fact that current exchanges might have implications for the future does not suffice to create a relationship. People constantly trade claims on future resources, often in the form of money or savings or via a variety of other contracts, but the contractural nature of these claims prompts us to stop short of calling them a relationship.

[7] Early references, including Grossman and Hart (1986), Grout (1984), and Hart and Moore (1988), directed attention to the inefficiency caused by the inability to contract simultaneously on current investments and future exchange. Subsequent papers have explored the circumstances under which such inefficiency is inevitable and under which it can be avoided. See, for example, Aghion, Dewatripont, and Rey (1994), Che and Hausch (1999), Chung (1991), Meza and Lockwood (1998), Edlin and Reichelstein (1996), Felli and Roberts (2001), Hart (1990), Hart and Moore (1990), Hart and Moore (1999), Hermalin and Katz (1993), MacLeod and Malcomson (1993), Maskin and Tirole (1999), and Nöldeke and Schmidt (1995). Malcomson (1997), Moore (1992), Palfrey (1992), Palfrey (2002), and Salanie (1997) provide surveys.

[8] It is tempting to view the ex post market power that arises in such interactions as the cause of the inefficiency. However, Mailath, Postlewaite, and Samuelson (2004) examine a model in which buyers and sellers again make investments that enhance the value of a subsequent trade, but in which the ex post market is competitive. Each buyer in the ex post market is indifferent as to the seller from which he purchases, as are sellers indifferent over buyers. Nonetheless, in the absence of the ability to contract on the ex post price when making investments, the equilibrium outcome is inefficient. Behind this inefficiency is the fact that investments, while having no implications for the question of with whom one trades, are important for the question of *whether* one trades.

entry into the large literature, much of it from outside economics, describing how relationships provide an alternative mechanism that also plays in important role an allocating resources.

Greif and his coauthors (Greif (1997, 2005), Grief, Milgrom, and Weingast (1994)) highlight the role of appropriate institutions in making possible the trade upon which modern economies are built. In some cases, these institutions provided the foundations for markets and contracts. In other cases, they "thickened" the information flows, allowing relationships to come to the fore.[9]

Relationships continue to play an important role in our contemporary economy. Macauley (1963) (an early and often-echoed classic) argues that business-to-business relations typically rely on the prospect of future interactions rather than contracts or legal recourse to shape deals and to mediate subsequent disputes. Ellickson (1991) suggests that such a reliance on relationships is pervasive, to the extent that relationships are the primary means of allocating many resources. Putnam's (2000) concerns about deteriorating social capital sound very much like a lament that our relationships are deteriorating.

Evolutionary psychology (e.g., Cosmides and Tooby (1992a,b)) suggests that our evolutionary past may have equipped us with an ability to sustain relationships as basic as our propensity for language (Chomsky (1980), Pinker (1994)). The argument here is that monitoring relationships was at one point so crucial to our evolutionary success that our brains have developed specialized resources for doing so.

The common theme is that understanding relationships can help us understand how our economy allocates resources. More importantly, understanding relationships can help us design our economy to better allocate resources.

1.4 An Example

An example will be helpful in putting relationships into context. Consider an economy with two goods, x and y, and an even number N of agents with utility functions $u(x,y) = \ln(1+x) + \ln(1+y)$. In each period $t = 0,1,\ldots$, each agent is endowed with one unit of good y. Each agent is also endowed with either zero or two units of good x in each period, with each endowment being equally likely and with precisely half of the agents receiving each endowment. An agent maximizes the expected value of the normalized (by $1-\delta$) discounted sum of payoffs, given by

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t u(a^t),$$

where a^t is the agent's period-t consumption bundle.

[9] Similarly, Richman (2005) attributes the preponderance of Jewish merchants in the diamond trade to the resulting ability to strengthen information flows so that relationships can be effective.

In the absence of trade, we have the autarkic equilibrium in which the consumption bundles $(0,1)$ and $(2,1)$ are equally likely in each period. Expected utility is given by

$$\frac{1}{2}[\ln 1 + \ln 2] + \frac{1}{2}[\ln 3 + \ln 2] = \frac{1}{2}\ln 12 = 0.54.$$

At the other end of the spectrum, we have an economy with complete markets. A state ω in this economy now identifies, in every period, which agents are endowed with two units of good x. Trades can be made contingent on the state.[10] The symmetric efficient allocation is also the unique competitive equilibrium allocation of the economy, in which each agent consumes one unit of x and one unit of y in each period, removing all individual uncertainty from the outcome, for a utility of:

$$\frac{1}{2}[\ln 2 + \ln 2] + \frac{1}{2}[\ln 2 + \ln 2] = \frac{1}{2}\ln 16 = 0.60.$$

Now suppose that in each period t, no trade can occur involving a commodity dated in some subsequent period (the contracting difficulty that potentially gives rise to relationships). We then have a countable number of separate markets, one for each period, each of which must clear independently. Each such market again has a unique competitive equilibrium in which goods x and y trade on a one-to-one basis. An agent endowed with no units of good x consumes half a unit of each good, while an agent endowed with two units of good x consumes three-halves of each good. Expected utility is

$$\frac{1}{2}\left[\ln\left(\frac{3}{2}\right) + \ln\left(\frac{3}{2}\right)\right] + \frac{1}{2}\left[\ln\left(\frac{5}{2}\right) + \ln\left(\frac{5}{2}\right)\right] = \ln\left(\frac{15}{4}\right) = 0.57.$$

Can the agents do better, given their inability to contract? Suppose that in each period, agents are randomly (and independently across periods) sorted into pairs, fortuitously arranged so that one agent has two units of x and the other has none (an assumption to which we return). Let each agent who finds himself endowed with two units of good x give one unit to his x-less partner in that period, as long as no agent has yet failed to deliver on this implicit promise. Should any agent ever fail to make this transfer, the autarkic equilibrium appears in each subsequent period. The resulting outcome duplicates that of the complete-markets outcome. Moreover, this behavior is an equilibrium if the agents can make the required observations and are sufficiently patient.[11]

[10] Agents can thus trade units of good x, in period t and given a state in which they are endowed with two such units in t and none in t' (and so on), for units of good x in period τ and given a state in which they are endowed with two units in t and also two in t'.

[11] The incentive constraint for an agent to carry on with the prescribed behavior, rather than pocketing his two units of good x when so endowed (at the cost of subsequent autarky), is given by $(1-\delta)\ln 6 + \delta\frac{1}{2}\ln 12 \leq \frac{1}{2}\ln 16$, or $\delta \geq 0.74$.

This arrangement brings us two-thirds of the way to a relationship. The incentives for current behavior involve the dependence of future behavior on current outcomes, and this dependence is enforced not by contractural arrangements but by future incentives. However, the agents are still anonymous—there is no need for agents to be tied together in relationships.

It is important for this arrangement that every agent observes every aspect of the history of play, so that any missed transfer triggers a switch to the autarkic equilibrium. Suppose instead that the agents can observe only whether a transfer is made *in their pair*. At first glance, it appears as if this information limitation is devastating. An agent who consumes her two units of good x rather than sharing with her hapless partner is on to a new partner before any retribution can be extracted. However, suppose that an agent follows the practice of making a transfer whenever endowed with two units of good x, unless a previous opponent failed to deliver on such a transfer, in which case the current agent makes no transfer. A defection thus sets off a contagion of defections that eventually returns to haunt the original transgressor, even when encountering partners for the first time. If the population is not too large and the players are sufficiently patient, then we have an equilibrium duplicating the complete-markets outcome.[12]

If the population is too large, then the previous scheme will be unable to support risk sharing. As an alternative, suppose the agents can arrange to meet the *same* opponent in every period. Suppose now, even more fortuitously (and more deserving of future commentary), that only one agent in a pair is endowed with two units of x in each period, with that agent's identity randomly drawn each period. Let the equilibrium call for the agent with two units of good x to offer one unit to the agent with none, as long as such behavior has prevailed in the past, and to retain his endowment otherwise. The incentive constraint for this to be an equilibrium is that

$$(1-\delta)\ln 6 + \delta \frac{1}{2} \ln 12 \leq \frac{1}{2} \ln 16,$$

which we can solve solve for $\delta \geq 0.74$ (cf. footnote 11), regardless of population size. The agents are exploiting their relationships to achieve the complete-markets outcome. A relationship concentrates the flow of information across periods, with each agent entering the current period knowing the history of their opponent's play, allowing more effective incentives to be created.

[12]Ellison (1994) and Kandori (1992) (see also Harrington, Jr. (1995)) establish this result in a model based on the prisoners' dilemma, with Ellison (1994) distinguished by allowing a public correlation device that turns out to be quite valuable. Okuno-Fujiwara and Postlewaite (1995) examine a related model in which each player is characterized by a history-dependent status that allows partial transfer of information across encounters. Ahn and Suominen (2001) examine a model with indirect information transmission.

Could we generate the information flow required to sustain incentives without sorting agents into relationships, while stopping short of assuming that all information is publicly available? One of the purposes of money is convey information (Kocherlakota (1998), Kocherlakota and Wallace (1998)). Suppose we introduce money into the economy by endowing the agents with certificates. An agent endowed with no x could exchange a certificate for a unit of the good, while agents called upon to relinquish a unit of x could exchange it for a certificate, confident that the latter will elicit the required reciprocation when needed. An agent need no longer worry whether future partners can observe that he has contributed when endowed with x, instead showing the resulting certificate when the need arises.

Unfortunately, there will inevitably be some agents who encounter extraordinarily long strings of bad luck, in the form of zero endowments of good x, and others who would run into similar strings of good luck. Eventually, the former will run out certificates, while the latter will have accumulated so many that they prefer to buy more than one unit of good x in a period. We may then be able to achieve some risk sharing, but cannot do so perfectly.

An analogous problem arises in relationships, buried beneath the assumption that only one of the agents in a relationship is endowed with x at a time. We have no reason to expect such coordination. Instead, there will inevitably be periods in which neither agent is endowed with x, as well as periods when both are. In these cases, the relationship is stuck. One could respond by creating larger relationships, matching pools of people in each period instead of just two. Presumably, however, the information flows required to make the relationship work deteriorate as the group grows larger. Otherwise, we would simply mass the entire economy into a single relationship, returning us to a case in which risk sharing poses no difficulties.

If forced to rely exclusively on either relationships or money, we thus face a tradeoff. The relationship must remain small, in order to capture its informational advantages, but at the cost of unavoidable idiosyncracy in endowments within a period. Money is vulnerable to idiosyncratic draws across periods. Relationships thus have a useful role to play, but are not a panacea for inadequate formal arrangements. As David Levine noted in his comments, there are many desperately poor countries who have lots of relationships. At the same time, there is every reason to believe that there are tremendous gains to be had from designing appropriate relationships.

1.5 Preview

The study of relationships begins with the study of repeated games. Repeated games have been featured at two preceding World Congresses, in presentations by Fudenberg (1992), Mertens (1982), Pearce (1992), and Rubinstein (1992). What has changed, and what is new?

We stress developments in five areas:

1. **Payoffs for patient players.** The basic tool for modeling relationships is the theory of repeated games. The primary results here are folk theorems, characterizing the set of equilibrium payoffs for the limiting case of (arbitrarily) patient players. Section 2 describes recent work.

2. **Characterizing payoffs.** It may not be easy to determine whether a folk theorem holds, and we may be interested in situations in which the folk theorems do not apply, perhaps because players are not sufficiently patient or the information flows are not sufficiently rich. Section 3 describes methods for characterizing the set of equilibrium payoffs in a repeated game.

3. **Characterizing behavior.** Much of the initial work in repeated games concentrated on characterizing the set of equilibrium payoffs. Attention has increasingly turned to the study of the behavior behind these payoffs. This work bridges the gap between the theory of repeated games and the economics of relationships. Section 4 presents examples.

4. **Reputations.** The concept of a reputation is a familiar one. We readily speak of people having reputations for being diligent or trustworthy, or of institutions as having reputations for providing high quality or being free of corruption. Recent work has made great gains in the study of reputations while also opening new questions. Section 5 considers reputations.

5. **Modelling relationships.** Perhaps the most important remaining questions concern how we are to interpret and use models of a relationship. When are relationships important in allocating resources, and why? Which of the many equilibria should we expect to see in a relationship? Why should we expect the people involved to come to an equilibrium at all? Section 6 considers these and similar questions.

2 Payoffs for Patient Players

We begin with the best-known results in repeated games, the folk theorems. It is helpful to place these results in the context of a familiar example, the prisoners' dilemma, shown in Figure 1. As is well known,

	C	D
C	$2,2$	$-1,3$
D	$3,-1$	$0,0$

Figure 1: The prisoner' dilemma

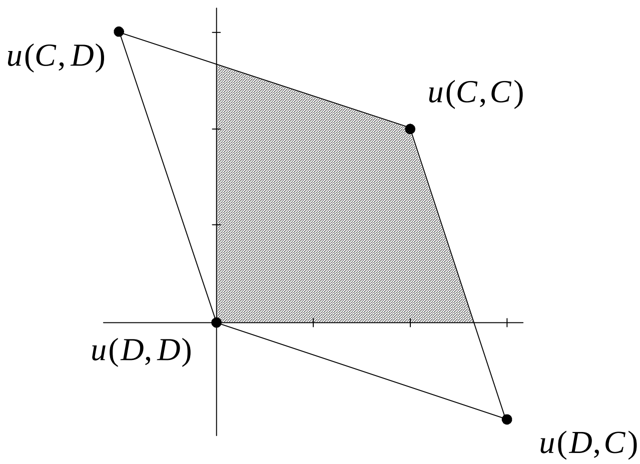

Figure 2: Feasible payoffs (the polygon and its interior) and folk-theorem outcomes (the shaded area) for the infinitely-repeated prisoners' dilemma.

this game has a unique but inefficient Nash equilibrium, in which both players defect.

2.1 Perfect Monitoring

Suppose the prisoners' dilemma is (infinitely) repeated, played in each of periods $0,1,\ldots$.[13] The two players share a common discount factor δ and maximize the discounted sum of their average payoffs,

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t u_i(a^t),$$

where $u_i(a^t)$ is player i's payoff from the period-t action profile a^t. The normalization factor $(1-\delta)$ ensures that the repeated game and stage game have the same sets of feasible payoffs.

There is a subgame perfect equilibrium of this repeated game in which both players defect at every opportunity. However, there may also be other equilibria. Consider a "grim trigger" strategy profile

[13] We focus throughout on infinitely-repeated games. Benoit and Krishna (1985) and Friedman (1985) show that finitely-repeated games whose stage games have multiple Nash equilibria are qualitatively similar to infinitely repeated games.

in which the agents cooperate, and continue to do so as long as there has been no defection, defecting otherwise. Suppose player 1 reaches a period, possibly the first, in which no one has yet defected. What should player 1 do? One possibility is to continue to cooperate, and in fact to do so forever. Given the strategy of player 2, this yields a (normalized, discounted) payoff of 2. The only other candidate for an optimal strategy is to defect (if one is going to defect, one might as well do it now), after which one can do no better than to defect in all subsequent periods (since player 2 will do so), for a payoff of $(1-\delta)[3+\sum_{t=\tau+1}^{\infty}\delta^{t-\tau}0] = 3(1-\delta)$. Cooperation is optimal if $2 \geq 3(1-\delta)$, or

$$\delta \geq \frac{1}{3}.$$

Sufficiently patient players can thus support mutual cooperation.

The players can do more than this. Figure 2 shows the set of payoffs for the repeated prisoners' dilemma that are feasible and individually rational, in the sense that each player earns at least the zero payoff that can be ensured by relentless defection. For any payoff in this set and for a sufficiently large discount factor, there is a subgame perfect equilibrium of the repeated game with that payoff.

This result is quite general. With Aumann and Shapley (1976), Friedman (1971), and Rubinstein (1977, 1979), as predecessors, and subsequently pursued by Abreu, Dutta, and Smith (1994) and Wen (1994), the basic folk theorem result for subgame perfect equilibria in discounted repeated games is due to Fudenberg and Maskin (1986).[14] Let A be a finite set of pure strategy profiles for a game with n (possibly more than two) players. Let $\underline{v}_i$ identify the *minmax* value for player i, given by $\max_{\alpha_i \in \mathscr{A}_i} \min_{\alpha_{-i} \in \mathscr{A}_{-i}} u_i(\alpha_i, \alpha_{-i})$ (where $\mathscr{A}_i$ and $\mathscr{A}_{-i}$ are the sets of mixed strategies for player i and the remaining players, respectively). This is the smallest payoff to which the other players can constrain player i, if they are maniacally determined to reduce his payoff. Let $\mathscr{F}^\dagger$ be the convex hull of the payoff profiles that can be produced by the action profiles in A. This is the set of feasible payoffs.

Proposition 1 (The Perfect-Monitoring Folk Theorem) *Suppose $\mathscr{F}^\dagger$ has nonempty interior. Then for every $v \in \operatorname{int}\mathscr{F}^\dagger$ with the property that $v_i > v > \underline{v}_i$ for some $v \in \mathscr{F}^\dagger$ and for all i, there exists $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, there exists a subgame perfect equilibrium of the perfect-monitoring repeated game with value v.*

Intuitively, this result indicates that every feasible, individually rational payoff can be obtained in the repeated game, as long as the players are sufficiently patient.

[14] The statement here is taken from Mailath and Samuelson (2006, Proposition 3.8.1).

2.2 Imperfect Public Monitoring

An important ingredient in the folk theorem for perfect monitoring games is that the players can observe each others' behavior.[15] For example, this allows them to punish defection in the prisoners' dilemma. Moreover, if the threat of punishment creates the proper incentives, then the agents fortuitously never have to actually do the punishing.

We might expect the players to have quite good information about others' play, but perhaps not perfect information. If so, we are interested in games of *imperfect* monitoring. Green and Porter (1984) and Porter (1983b,a) popularized games of imperfect *public* monitoring, meaning that the agents observe noisy signals of play, but all agents observe the same signals.

We first illustrate with the prisoners' dilemma of Figure 1. We now assume that players cannot observe their opponent's actions, instead in each period observing either signal $\underline{y}$ or signal $\overline{y}$, generated according to a probability distribution that depends upon the action profile a taken in that period according to:

$$\Pr\{\overline{y} \mid a\} = \begin{cases} p, & \text{if } a = CC, \\ q, & \text{if } a = DC \text{ or } CD \\ r, & \text{if } a = DD, \end{cases} \tag{1}$$

where $0 < r < q < p < 1$. For example, we might interpret the prisoners' dilemma as a partnership game whose random outcome is either a success ($\overline{y}$) or failure ($\underline{y}$).[16]

Let us first examine the counterpart of the grim trigger strategy for this game of imperfect monitoring. Since defection makes the signal $\underline{y}$ more likely, we examine a strategy in which players initially cooperate, do so as long as the signal $\overline{y}$ is received, but switch to permanent defection once $\underline{y}$ is received.

For these strategies to constitute an equilibrium, it is again necessary and sufficient that an agent be

[15]We encountered the importance of good information about previous actions in Section 1.4.

[16]Can't player 1 figure out what 2 has chosen by looking at the payoffs 1 receives? We assume that player 1's payoffs are determined as a function of 1's actions and the public signal as follows:

	$\overline{y}$	$\underline{y}$
C	$\frac{(3-p-2q)}{(p-q)}$	$-\frac{(p+2q)}{(p-q)}$
D	$\frac{3(1-r)}{(q-r)}$	$-\frac{3r}{(q-r)}$

The same is true for player 2. This ensures that the distribution of 2's action, conditional on the public signal and 1's payoff, is the same as the distribution conditional on just the public signal, ensuring that payoffs contain no additional information. It also ensures that the payoffs given in Figure 1 are the expected payoffs as a function of the agents' actions, so that the players face a prisoners' dilemma.

willing to cooperate when called upon to do so, or

$$(1-\delta)2+\delta pV \geq (1-\delta)3+\delta qV, \tag{2}$$

where V is the expected value of playing the game, given that no one has yet defected. The calculation recognizes that with probability p (if the agent cooperates) or q (if the agent defects), the signal $\overline{y}$ appears and the game enters the next period with expected payoff V, while with the complementary probability, signal $\underline{y}$ appears and subsequent defection brings a zero payoff. We can solve $V=(1-\delta)2+\delta pV$ for

$$V=\frac{2(1-\delta)}{1-\delta p},$$

and then insert in (2) to calculate that the proposed strategies are an equilibrium if

$$\delta(3p-2q)\geq 1.$$

Hence, we have an equilibrium if the players are sufficiently patient and the signals are sufficiently informative, in the sense that p must be large enough relative to q. Impatient players or signals that provide insufficiently clear indications of defection will disrupt the equilibrium.

We thus have some equilibrium cooperation, but with payoffs that are less attractive than those of the perfect monitoring case. Eventually, the signal $\underline{y}$ will appear, no matter how diligent the agents are about cooperating, after which these strategies doom them to defection. Indeed, as the players become increasing patient ($\delta \to 0$), the expected payoff from this equilibrium converges to zero, as less and less importance is attached to the transient string of initial cooperation.

Perhaps we have simply chosen our strategy poorly. Could we do better? Since the difficulty with grim trigger is that the players eventually end up in a permanent punishment, let us make the punishment temporary. Suppose that the players initially cooperate and do so after every instance of the signal $\overline{y}$. If they observe signal $\underline{y}$ in a period in which they are supposed to cooperate, they defect for a single period and then return to cooperation.

It is obvious that the players have no incentive to do anything other than defect when they are supposed to, since the opponent is then also defecting and nothing can speed the return to cooperation. These strategies will be an equilibrium if it is optimal to cooperate when called upon to do so. We can calculate that this incentive constraint is given by

$$2\delta(p-q)\geq \frac{1-\delta p-\delta^2(1-p)}{1-\delta}.$$

In the limit, as $\delta \to 1$, this becomes $3p \geq 2(1+q)$. Hence, the proposed strategies are again an equilibrium if the players are sufficiently patient and the signals sufficiently informative.

As the players become increasing patient, the expected payoff from this strategy profile approaches

$$\frac{2}{2-p}.$$

This is better than the zero payoff of our grim trigger adaptation, but still falls short of the payoff 2 to be had from persistent mutual cooperation.

These two examples reflect a basic property of equilibria in games of imperfect monitoring: punishments happen. The only way to create incentives for the players to do anything other than defect is to ensure that some signals bring lucrative continuation play and others bring bleak continuation play. But if this is to be the case, unlucky signal realizations will sometimes bring punishments. Moreover, the players will inflict such punishments even though they know that no one did anything to warrant such a response. In equilibrium, players who have observed $\underline{y}$ know that everyone has cooperated and that they were unlucky to have drawn signal $\underline{y}$, but nonetheless they punish (and indeed would be punished for not doing so). In essence, players are not punished because they are guilty, but are guilty (or deserving of punishment) because they are punished. Why would anyone participate in such a crazy equilibrium? It is not clear that player can choose whether to participate, or can choose an equilibrium if they do participate, but it is worth noting that this equilibrium can bring higher expected payoffs than an equilibrium in which no one is ever punished.

Given the inevitability of punishment, it is natural to conjecture that inefficiency is a general property of imperfect monitoring. Against this background, Fudenberg, Levine, and Maskin (1994) produced a startling result, in the form of a folk theorem for games of imperfect public monitoring. The following version of the result is taken from from Mailath and Samuelson (2006, Proposition 9.2.1), with the understanding that the terms "pairwise full rank" and "individual full rank" are yet undefined:

Proposition 2 (The Public-Monitoring Folk Theorem) *Suppose $\mathscr{F}^\dagger$ has nonempty interior, and all the pure action profiles yielding the extreme points of $\mathscr{F}^\dagger$ have pairwise full rank for all pairs of players. If the vector of minmax payoffs $\underline{v} = (\underline{v}_1, \ldots, \underline{v}_n)$ is Pareto inefficient and the profile $\hat{\alpha}^i$ that minmaxes player i has individual full rank for all i, then for all $v \in \operatorname{int}\mathscr{F}^\dagger$ with $v_i > \underline{v}_i$ for all i, there exists $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, v is a subgame-perfect equilibrium payoff.*

There are two keys to this result. First, we need the set of feasible payoffs to have an interior. This ensures that, in response to a signal that is relatively likely when player i deviates, we can push continuation payoffs toward payoffs that are worse for i but better for the other players. This allows us to create incentives without sacrificing efficiency. The second requirement is that the signals be sufficiently informative to give us information not only about whether a deviation has occurred, but also about who has deviated. This is reflected in the "individual full rank" and "pairwise full rank" conditions in the theorem. We will leave the statement and discussion of these conditions to Fudenberg, Levine, and Maskin (1994) and Mailath and Samuelson (2006, Chapter 9)). Intuitively, they ensure that a deviation from equilibrium play by each player i has a distinctive effect on the public signals, so that there exists a signal that is "relatively likely when player i deviates." For example, the prisoners' dilemma with which we introduced imperfect monitoring in this section *fails* these conditions. Given action profile CC. the signals can provide information about whether there has been a deviation (i.e., $\underline{y}$ is more likely if someone played D), but no information about who might have deviated. As a result, we are constrained to inefficiency.

2.3 Private Monitoring

Just as players may not always have precise information about previous play, so may they often not have precisely the same information. We are then in the realm of *private* monitoring.

Suppose we again have the prisoners' dilemma of Figure 1. Given a choice of actions, let a hypothetical or "latent" signal be drawn from the set $\{\underline{y}, \overline{y}\}$ according to the distribution given by (1). However, instead of assuming that the agents both observe this signal, we use it as a basis for constructing a pair of private signals.

2.3.1 Conditionally Independent Signals: A First Result

Suppose first that, when signal $\overline{y}$ is drawn, each player i independently observes signal $\overline{y_i}$ with probability $1-\zeta$ and signal $\underline{y_i}$ with probability ζ. Things are reversed when signal $\underline{y}$ is drawn. We refer to this as the case of conditionally independent monitoring, since, conditioning on the action profile, agent i's signal provides no information about agent j's signal.

When ζ is very small, the two players almost certainly receive the same signals. How much difference could it make that they don't have exactly the same information? Consider a strategy profile in which the agents player CC in the first period, and in which signal $\underline{y_i}$ causes agent i to switch to a punishment phase beginning with D. This seems a promising start on an equilibrium strategy. An agent makes the signal $\underline{y_i}$ more likely by deviating D, so that attaching punishments to $\underline{y_i}$ should provide the required incentives to cooperate.

Suppose now that each player adopts such a strategy, that player 1 dutifully chooses C in the first period, and then unluckily draws signal $\underline{y}_1$. Player 1 can then reason, "player 2 has certainly chosen her equilibrium action of C (since that is how the equilibrium hypothesis asks me to reason), and has almost certainly observed $\bar{y}_2$ (since I played C, and there is not much noise in the signals), and hence is prepared to continue with cooperation next period. If I choose D, I make it very likely that she sees $\underline{y}_2$ and switches to her punishment phase, to my detriment. If I choose C again, then there is a good chance we can avoid the punishment phase altogether, at least for a while." As a result, player 1 will not enter the punishment phase, precluding the optimality of the strategy profile. Even the tiniest amount of privateness disrupts the proposed equilibrium.[17]

On the strength of this reasoning, initial expectations were that equilibria in repeated games with private monitoring must be inefficient, and perhaps presented very little prospect for effectively using intertemporal incentives. This in turn raises the fear that repeated games of public or perfect monitoring might be a hopelessly special case. As was the case with imperfect public monitoring, these expectations were displaced by a surprising result, this time from Sekiguchi (1997).

Say that private monitoring is *ε-perfect* if, for each player i and each action profile a, there is a signal that player i receives with probability at least $1-\varepsilon$ when action profile a is played.[18] Working with the prisoners' dilemma, Sekiguchi (1997) showed the following:

Proposition 3 *For all $\eta > 0$, there exists $\varepsilon > 0$ and $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}, 1)$, if the private monitoring is ε-perfect, then there is a sequential equilibrium in which each player i's average payoffs are within at least η of $u_i(C,C)$.*

It is not surprising that the monitoring technology is required to be sufficiently informative (ε small), for much the same reason that we need the players to be patient. Otherwise, we have no hope of creating intertemporal incentives.[19] The surprise here is the ability to achieve efficiency with private signals, no matter how close to perfect.

We can provide an indication of the basic technique involved in the equilibrium construction. Suppose that each player mixes in period 1, placing probability $1-\xi$ on C and probability ξ on D. Suppose

[17] See Bagwell (1995) for a precursor of this argument and Bhaskar (2005), Güth, Kirchsteiger, and Ritzberger (1998), and Hurkens (1997) for subsequent discussion.

[18] Notice that for the signals discussed in the opening paragraphs of this section to be ε-perfect, not only must ζ be small, so that each player almost certainly observes the public signal without error, but the distribution given by (1) must approach perfect monitoring. Sekiguchi's result requires the monitoring to be ε-perfect, but not conditionally independent

[19] For an extreme illustration, consider the completely uninformative case in which each of player i's signals appears with equal probability, independently of j's signals and no matter what the action profile.

further that ξ is large relative to ε, the measure of noise in the private monitoring. Let player i continue with action C in the second period if i happened to choose C in the first period and observed signal $\bar{y}_i$, and otherwise let i switch to action D. Now consider again our previously problematic case, that in which i played C and observed signal $\underline{y}_i$. Player i can now reason, "I've seen signal $\underline{y}_i$. Either player 2 chose C and I happened to see the unlikely signal $\underline{y}_i$, or 2 chose D and I received the (then relatively more likely) signal $\underline{y}_i$. Because ε is small relative to ξ, the latter is more likely. Hence, 2 will enter the punishment phase next period, and so should I."

This at least allows the prospect of coordinating punishments. There are many details to be taken care of in converting this intuition into an equilibrium. In particular, we must ensure that 1 indeed finds it a best response to enter the punishment, given that 1 thinks 2 is likely but not certain to do so. We must make sure that we have the indifference conditions required for mixing in the first period. Finally, we have the problem that this mixing itself introduces some inefficiency. Fortunately, this inefficiency can be made small as ε becomes small, opening the door to an efficiency result.

2.3.2 Belief-Free Equilibria

The equilibrium constructed by Sekiguchi (1997) is a *belief-based* equilibrium, in the sense that each player keeps track of beliefs about the signals the other player has observed. The difficulty is that such beliefs quickly become quite complicated. We describe here a more recent but all the more surprising development, *belief-free* equilibria, introduced by Piccione (2002), simplified and extended by Ely and Välimäki (2002)), and characterized by Ely, Hörner, and Olszewski (2005).

We continue with our prisoners' dilemma example, allowing arbitrary private monitoring technologies. We consider an equilibrium in which each player i's strategy is built from four mixtures, that we refer to as $\alpha^{C\bar{y}_i}$, $\alpha^{C\underline{y}_i}$, $\alpha^{D\bar{y}_i}$, $\alpha^{D\underline{y}_i}$. In each period, player i chooses C with probability $\alpha^{C\bar{y}_i}$ if i chose C and saw $\bar{y}$ in the previous period (choosing D with complementary probability); chooses C with probability $\alpha^{C\underline{y}_i}$ if he chose C and saw $\underline{y}$; and so on. It is then useful to think of player i's strategy as consisting of four states, one corresponding to each of the mixtures i might choose, and as player i being in one of these states in each period, depending upon his experience in the previous period.

The potential difficulty in showing that these strategies are an equilibrium is that each time player i is called upon to mix, i must be indifferent between the actions C and D. The payoffs to these actions depend upon what player j is doing, again raising the potentially very difficult problem of player i having to keep track of beliefs about what player j has observed and hence is playing. The surprising result is that this is unnecessary. One can choose the various mixtures so that player i is indifferent between C and D no matter what state player j is in, and hence no matter what i believes about player j. Hence, i can dispense with the need to keep track of beliefs at all, prompting the name "belief-free" equilibrium.

One's first thought is that the conditions required to support such indifference must be hopelessly special, often failing and allowing very little control over the payoffs they produce when they are satisfied. To the contrary, it turns out that there are many such equilibria. Indeed, we have a partial folk theorem. In our prisoners' dilemma example, any payoff profile v with $v_i \in (0,2)$ can be achieved as an equilibrium outcome if the players are sufficiently patient and the monitoring sufficiently close to perfect.[20] Private monitoring thus poses no obstacle to a prisoners'-dilemma folk theorem.

Ely, Hörner, and Olszewski (2005) provide a general characterization of the set of belief-free payoffs in games with patient players. They find that the prisoners' dilemma is rather special. In most games they are not sufficient to prove a folk theorem, even for vanishing noise. However, in the course of this analysis, Ely, Hörner, and Olszewski (2005) show that the basic techniques for working with games of perfect or public monitoring extend to games of private monitoring.[21] Moreover, belief-free behavior can serve as a point of departure for constructing folk theorems. Matsushima (2004) uses review strategies, familiar from Radner's (1985) work on repeated principal-agent problems, to extend the belief-free folk theorem for the prisoners' dilemma with almost perfect private monitoring to cases in which the monitoring is quite noisy. Hörner and Olszewski (2005) prove a general folk theorem for almost-perfect private monitoring using profiles that have some of the essential features of belief-free equilibria.

2.3.3 Almost Public Monitoring

Interest in repeated games centers around the ability to use future play to create current incentives. We thus think of the players as using the history of play to coordinate on a continuation equilibrium. In the

[20] More complicated strategies allow equilibria to be constructed in which one player receives a payoff larger than 2.

[21] The hallmark of games with perfect monitoring is their recursive structure—each period marks the beginning of a continuation subgame that is identical to the original game. Games of public monitoring have a similar recursive structure, as long as the players use *public* strategies—strategies in which actions depend only upon the public history of signals, and not players' private information about their own past actions. (For many purposes, this restriction causes no difficulties—for example, the public-monitoring folk theorem requires only public strategies—though Kandori and Obara (2003) explore circumstances under which it can be limiting to restrict attention to public strategies). It appears as if the recursive structure has been lost forever in games of private monitoring. In each period past the first, the players have different information, both about their signals and their actions, ensuring that the repeated game has no proper subgames at all. However, Ely, Hörner, and Olszewski (2005) show that the recursive techniques of Abreu, Pearce, and Stacchetti (1990) extend to private-monitoring games.

prisoners' dilemma, for example, the players support equilibrium cooperation by using histories featuring defection as a signal to coordinate on a continuation equilibrium featuring relentless defection.

In the belief-free equilibria of private-monitoring games, this sense of using histories to coordinate continuation play is lost. Instead of coordinating future play with player j, player i gives no thought to what j might do. Is there any prospect of constructing equilibria in games of private monitoring that have more of the coordination flavor of equilibria from perfect or public monitoring games?

Mailath and Morris (2002, 2005) examine games of *almost-public* monitoring. Return to our prisoners' dilemma example. Once again, we think of an intermediate signal being drawn according to the the distribution given by (1). Now suppose that if signal $\bar{y}$ is drawn, with probability $1-\zeta$ players i and j observe $\bar{y}_i$ and $\bar{y}_j$, while with probability $\frac{\zeta}{2}$ player i observes $\bar{y}_i$ and j observes $\underline{y}_j$ (with the reverse pattern also having probability $\frac{\zeta}{2}$). Similarly, if $\underline{y}$ is drawn, with probability $1-\zeta$ player i observes $\underline{y}_i$ and j observes $\underline{y}_j$. Notice that i's signal now provides considerable information about j's, unlike the case of conditionally independent signals. As $\zeta \to 0$, we approach the case of public monitoring.[22]

Now consider strategies that play C in the first period and switch to D upon observing the signal $\underline{y}$. Suppose player 1 cooperates and observes the signal $\underline{y}$. Unlike the case of conditionally independent private monitoring, player 1's inference is now that player 2 has almost certainly (when ζ is small) also observed $\underline{y}$, and is also switching to D. We thus avoid the difficulties that immediately scuttled such strategies in the case of conditionally independent monitoring.

Ensuring that we have an equilibrium hinges upon showing that players can always be reasonably confident of where their opponent is in their strategy. Consider first a strategy in which player i initially cooperates and does so after any signal $\bar{y}_i$. After any signal $\underline{y}_i$, player i defects (switching back to cooperation at the next $\bar{y}_i$ signal). If the signals are sufficiently informative and the players sufficiently patient, then this strategy profile is a *strict equilibrium* under the imperfect public monitoring scheme given by (1). The strategies will then also be an equilibrium for private monitoring that is sufficiently close to being public. The key to this result is that the strategies in question have bounded (in this case, 1-period) recall, meaning that only a finite string of signals is required to identify a player's action. This ensures that player i's estimate of player j's action depends only on a limited number of signals and accordingly can never be too far away from j's actual action (given monitoring sufficiently close to public).

This result is more general. For any strategy profile that has bounded recall and that is a strict equi-

[22]Unlike the case of ε-perfect monitoring for small ε, there is no presumption here that the limiting public monitoring distribution be close to perfect monitoring.

librium an a game with public monitoring, there is a corresponding strategy profile that is an equilibrium in the associated private monitoring game, if the monitoring in the latter is sufficiently close to the public monitoring of the former.

The same result does not hold for strategies with unbounded recall. For example, let player i initially cooperate and continue to do so until the first signal $\underline{y}_j$, at which point i switches to defecting. Defection continues until the first signal $\underline{y}_j$, at which point i switches to cooperating, and so on. Hence, we can think of player i as having a cooperate state and a defect state, switching whenever $\underline{y}_j$ is observed. There are again conditions under which this strategy is an equilibrium of the public-monitoring game. However, no matter how close to public monitoring is the private-monitoring game, it is not an equilibrium for the two players to each choose the counterpart of this strategy under private monitoring. The difficulty is that the strategy has infinite recall, in the sense that one must know the entire history of signals in order to identify the strategy's current state. This ensures that eventually player i will have virtually no clue as to player j's state, disrupting the equilibrium conditions.

2.3.4 Working with Private Monitoring

Recent years have witnessed surprising progress in working with repeated games of private monitoring, making it clear that the equilibrium possibilities in such games are richer than initially suspected. Much now depends upon the interpretation of belief-free equilibria, and in particular upon how one views the pervasive randomization upon which they are constructed. While mixed strategies are used routinely in economic models, many economists persist in viewing them uneasily (e.g., Rubinstein (1992)), an unease that is likely to be heightened by the central role they play in belief-free equilibria. Moreover, it is not clear whether such equilibria can be purified (Harsanyi (1973)), possibly foreclosing one of the most popular interpretations of mixtures.[23] One the one hand, belief-free equilibria appear to miss the connection between histories of play and continuation equilibria that is commonly the centerpiece of work in repeated games. However, the strongest and most complete results for private-monitoring games have been obtained with belief-free equilibria. It remains to be seen whether belief-free equilibria will become the standard tool for working with such games, or whether interest will turn to other techniques.

[23] More specifically, it is not clear if, in general, a belief-free equilibrium can be approximated by any strict equilibrium in nearby games of incomplete information, where the incomplete information is generated by independently distributed (over time and players) payoff shocks.

2.4 Interpreting The Folk Theorem

The folk theorem asserts that “anything can be an equilibrium.” The only payoffs for which equilibrium behavior in a repeated game cannot account are obviously uninteresting, and can be so classified without the help of an elaborate theory, being either infeasible or offering some player less than his minmax payoff. This result is sometimes viewed as an indictment of the repeated games literature, implying that game theory has no empirical content. The common way of expressing this is that “a theory that predicts everything predicts nothing.”

The first point to note in response is that multiple equilibria are common in settings that range far beyond repeated games. Coordination games, bargaining problems, auctions, Arrow-Debreu economies, mechanism design problems, and signaling games (among many others) are notorious for multiple equilibria. Moreover, a model with a unique equilibrium would be quite useless for many purposes. Behavioral conventions differ across societies and cities, firms and families. Only a theory with multiple equilibria can capture this richness. How else can we conveniently study a world in which people drive on the right side of the street in some countries, and on the left in others? If there is a problem with repeated games, it cannot be that there are multiple equilibria, but that there are “too many” multiple equilibria.

An argument that repeated games have too many multiple equliabria is unconvincing on four counts. First, a theory need not make unique predictions to be useful. Even when the folk theorem applies, the game-theoretic study of long-run relationships deepens our understanding of the incentives for opportunistic behavior and the institutional responses that might discourage such behavior. For example, repeated games help us understand why efficiency might require the ability to punish some players while rewarding others (cf. Section 2.2) or why we might see nonstationary behavior in stationary settings (Section 4.2). Without such an understanding, a useful model of behavior is beyond our grasp.

Second, we are often interested in cases in which the conditions for the folk theorem fail. The players may be insufficiently patient or the monitoring technology may be insufficiently informative. There is much to be learned from studying the set of equilibrium payoffs in such cases. The techniques developed in for working with repeated games allow us characterize equilibrium payoffs when the folk theorem holds *and* when it fails.

Third, the folk theorem places bounds on payoffs but says nothing about *behavior*. The strategy profiles used in proving folk theorems are chosen for analytical ease rather than descriptive relevance. As repeated games find increasing application in economics, interests increasingly focusses on its behavioral implications. We may want to know not just whether the firms in a market can use their repeated interactions to support collusion, but what behavioral evidence this collusion might leave if they do. Whether the folk theorem holds or fails, it is then only the point of departure for the study of behavior.

Finally, there is a classical view of game theory in which constructing one's model consists of specifying the game, at which point the calculation of an equilibrium is part of the analysis of the model. If one takes this view and is interested in the theory as a tool of economic analysis, then a natural response to the great multitude of equilibria is to look for some other tool. However, an alternative view treats both the construction of the model and the selection of an equilibrium as part of the modelling exercise. Equilibrium behavior that might be quite plausible in some contexts may be uninteresting in others. Depending upon the nature of the interaction to be studied, one might be interested in equilibria that are efficient or satisfy the stronger efficiency notion of renegotiation proofness, that make use of only certain types of (perhaps "payoff relevant") information, that are in some sense simple, or that have some other properties. We would then be concerned if the game did *not* exhibit multiple equilibria, and hence the flexibility to be applied to the wide variety of contexts that can be modelled as repeated games. We can lament the multiplicity of the folk theorem only if we had hoped to have the model do the work that properly falls to the modeler. We return to this issue in Section 6.3.

3 Characterizing Payoffs

Section 2 describes progress in understanding the set of equilibrium payoffs for arbitrarily patient players with a sufficiently informative monitoring structure, culminating in the folk theorems. Suppose, however, we were interested in agents who are not perfectly patient, or in games in which the monitoring structure is not sufficiently informative as to allow a folk theorem. Can we characterize the set of payoffs?

Less is known about these questions. For example, we do not have a complete understanding of how the set of equilibrium payoffs for the prisoners' dilemma varies in the discount factor.[24] At the same time, great progress has been made in developing techniques for characterizing equilibrium payoffs.

[24] Stahl (1991) provides a complete characterization for the case in which players can use a public random variable to correlate their actions. Mailath, Obara, and Sekiguchi (2002) illustrate the complexities that can arise when they cannot do so. See Sorin (1986) and van Damme (1991, Section 8.4) for earlier results.

3.1 Self Generation

The difficulty in working with repeated games is that the strategy sets are vast, making it cumbersome to construct an equilibrium or verify that a candidate is an equilibrium. A first step in simplifying this procedure is Abreu, Pearce, and Stacchetti (1986, 1990)'s characterization of equilibrium payoffs as self-generating sets.

We illustrate with pure strategies in games of perfect monitoring (see Mailath and Samuelson (2006, Chapters 2 and 7) for a presentation of the general case). We say that a pure action profile $a^* \in A$ is *enforceable* on the set of payoff *profiles* $\mathscr{W}$ if there exists some specification of continuation promises $\gamma : A \to \mathscr{W}$ such that, for each player i and action $a_i \in A_i$,

$$(1-\delta)u_i(a^*) + \delta\gamma_i(a^*) \geq (1-\delta)u_i(a_i, a^*_{-i}) + \delta\gamma_i(a_i, a^*_{-i}).$$

A payoff $v \in \mathscr{F}^\dagger$ is *pure-action decomposable on* $\mathscr{W}$ if there exists an profile a^* enforceable on $\mathscr{W}$ such that

$$v_i = (1-\delta)u_i(a^*) + \delta\gamma_i(a^*),$$

where γ is a function that enforces a^*.

In other words, an action profile is enforceable in the current period if we can arrange continuation payoffs, as a function of the current actions, so as to make it a best response for each agent to play their part of the action profile in the current period. A payoff is decomposable if it can be produced by an enforceable action.

This initially appears to be of little help, since we have made no assumptions about the set of possible continuation payoff profiles $\mathscr{W}$. The next step is to notice that subgame perfection requires every agent to choose a best response at every opportunity. This suggests that we should be interested in the case where the set of continuation payoff profiles is the set of subgame perfect equlibrium payoffs. Letting $\mathscr{E}$ be the set of subgame-perfect equilibrium payoff profiles, we have:

Proposition 4 *If the payoff profile v is pure-action decomposable on $\mathscr{E}$, then v is a subgame-perfect equilibrium payoff. Any set of payoff profiles $\mathscr{W}$ with the property that every payoff in $\mathscr{W}$ can be pure-action decomposed on $\mathscr{W}$ is a set of subgame-perfect equilibrium payoffs. The set $\mathscr{E}$ of pure subgame-perfect equilibrium payoffs is the largest such set.*

The first statement simply repeats that subgame perfect equilibria couple choices that are currently optimal with equilibrium continuation payoffs. The second statement gives us a method for identifying

	C	D
C	a, a	$-c, b$
D	$b, -c$	$0, 0$

Figure 3: Prisoners' dilemma, where $b > a$, $c > 0$, and $b - c < 2a$.

subgame-perfect equilibrium payoffs as "self-generating" sets of payoffs. The third identifies the set of subgame-perfect equilibrium payoffs as the largest such set. These results generalize to mixed strategies and to public monitoring.

We now have two possibilities for identifying subgame-perfect equilibrium payoffs. One is to identify a set of payoffs as likely to be self generating, and then confirm that it indeed has this property. Some clever guesswork may be required here, but there are often clues as to what such a set might be. The other is to start with the entire feasible set of payoffs and successively calculate the payoffs that can be decomposed, proceeding until hitting a fixed point that will be the set of subgame-perfect equilibrium payoffs.

To illustrate, let us again consider the prisoners' dilemma, this time with the payoffs shown in Figure 3, chosen to make clear what role the various payoffs play in the result. We ask when there exists a subgame-perfect equilibrium in which both players cooperate in every period. In light of Proposition 4, this is equivalent to identifying the discount factors for which there is a self-generating set of payoff profiles $\mathscr{W}$ containing (a, a). If such a set $\mathscr{W}$ is to exist, then the action profile CC must be enforceable on $\mathscr{W}$, or,

$$(1-\delta)a + \delta\gamma_1(CC) \geq (1-\delta)b + \delta\gamma_1(DC)$$

$$\text{and} \qquad (1-\delta)a + \delta\gamma_2(CC) \geq (1-\delta)b + \delta\gamma_2(CD),$$

for $\gamma(CC)$, $\gamma(DC)$ and $\gamma(CD)$ in $\mathscr{W}$. These inequalities are least restrictive when $\gamma_1(DC) = \gamma_2(CD) = 0$. In addition, the singleton set of payoff profiles $\{(0,0)\}$ is itself self-generating (since there are no incentive issues in asking players to defect, and hence a future of defection suffices to make defection optimal in the first period). We thus sacrifice no generality by assuming the self-generating set contains $(0,0)$, and can then set $\gamma_1(DC) = \gamma_2(CD) = 0$. Similarly, the pair of inequalities is least restrictive when $\gamma_i(CC) = a$ for $i = 1, 2$. We can thus take $\mathscr{W}$ to be the set $\{(0,0), (a,a)\}$. Inserting the continuation values in the incentive constraints and simplifying, the conditions for decomposability hold when

$$\delta \geq \frac{b-a}{b}. \tag{3}$$

The inequality given by (3) is thus necessary for the existence of a subgame-perfect equilibrium giving payoff (a,a), and is also sufficient, since it implies that the set $\{(0,0),(a,a)\}$ is self-generating. For the prisoners' dilemma of Figure 1, we have the familiar result that $\delta \geq \frac{1}{3}$.

3.2 Bounding Payoffs

The techniques for identifying sets of equilibria described in Section 3.1 are general and powerful. They allow the calculation of equilibrium payoffs to be reduced to what looks like a dynamic programming problem. There is just one hitch—the crucial constraint in calculating equilibrium payoffs is that continuation payoffs come from the set $\mathscr{E}$, the set of equilibrium payoffs that one is trying to calculate.

Remarkably, Fudenberg and Levine (1994) (see also Mailath and Samuelson (2006, Chapter 8)), with a refinement by Kandori and Matsushima (1998) and with Matsushima (1989) as a precursor, allow us to avoid this self reference, leading to what is now the standard tool for identifying equilibria.

We illustrate with the prisoners' dilemma of Figure 1, under imperfect public monitoring, with the monitoring technology given by (1). Suppose we are interested in the equilibrium that maximizes the weighted sum $\lambda_1 v_1 + \lambda_2 v_2$ of the players' payoffs, for any values λ_1 and λ_2 that are not both zero (and that may be negative), and suppose that we knew that this equilibrium began with the play of CC. Then the equilibrium resulting equilibrium payoff profile v^* must be decomposed, meaning that there must exist a function γ, associating equilibrium payoff profiles with signals, that allows us to characterize v^* as

$$\begin{aligned} \lambda_1 v_1^* + \lambda_2 v_2^* &= \max_{v,\gamma} \lambda_1 v_1 + \lambda_2 v_2 \\ \text{subject to} \quad & v_i = (1-\delta)2 + \delta[p\gamma_i(\bar{y}) + (1-p)\gamma_i(\underline{y})], \quad i = 1,2 \\ & v_i \geq (1-\delta)3 + \delta[q\gamma_i(\bar{y}) + (1-q)\gamma_i(\underline{y})], \quad i = 1,2 \\ & \gamma(\cdot) \in \mathscr{E}. \end{aligned}$$

The first equation ensures that we are maximizing the sum of the players' payoffs, the second that we hit the desired payoff, the third imposes the incentive constraints, and the fourth ensures that continuation payoffs are equilibrium payoffs. Now consider the set of payoffs

$$H = \{\gamma : \lambda_1 \gamma_1 + \lambda_2 \gamma_2 \leq \lambda_1 v_1^* + \lambda_2 v_1^*\}.$$

The set H must contain $\mathscr{E}$, the set of subgame-perfect equilibrium payoffs, since by assumption no equilibrium gives a weighted sum of payoffs higher than $\lambda_1 v_1^* + \lambda_2 v_1^*$ and H contains any payoff profile with a lower sum. As a result, we can replace our characterization of v^* with an upper bound:

$$\begin{aligned}
\lambda_1 v_1^* + \lambda_2 v_2^* \quad &\leq \quad \max_{v,\gamma} \lambda_1 v_1 + \lambda_2 v_2 \\
\text{subject to} \qquad & v_i \ = \ (1-\delta)2 + \delta[p\gamma_i(\overline{y}) + (1-p)\gamma_i(\underline{y})], \quad i=1,2 \\
& v_i \ \geq \ (1-\delta)3 + \delta[q\gamma_i(\overline{y}) + (1-q)\gamma_i(\underline{y})], \quad i=1,2 \\
& \lambda_1\gamma_1(\cdot) + \lambda_2\gamma_2(\cdot) \ \leq \ \lambda_1 v_1^* + \lambda_2 v_2^*.
\end{aligned}$$

Now redefining the variables to let $\tilde{\gamma} = \frac{\delta}{1-\delta}[\gamma_i - v_i]$, it suffices for this problem that

$$\begin{aligned}
\lambda_1 v_1^* + \lambda_2 v_2^* \quad &\leq \quad \max_{v,\tilde{\gamma}} \lambda_1 v_1 + \lambda_2 v_2 \\
\text{subject to} \qquad & v_i \ = \ 2 + p\tilde{\gamma}_i(\overline{y}) + (1-p)\tilde{\gamma}_i(\underline{y})], \quad i=1,2 \\
& v_i \ \geq \ 3 + [q\tilde{\gamma}_i(\overline{y}) + (1-q)\tilde{\gamma}_i(\underline{y})], \quad i=1,2 \\
& \tilde{\gamma}_i(\cdot) \ \leq \ 0, \quad i=1,2.
\end{aligned}$$

In the course of making these transformations, we have replaced a characterization with a bound. In return, we have made considerable improvement in simplifying this characterization of v^*. The first step eliminated the set of equilibrium payoffs from the problem, getting us around having to speculate about the object we hope to calculate, while the second step eliminates the discount factor.

We have assumed that the equilibrium maximizing the weighted sum of the players' payoffs begins with the play of CC. This need not be the case. For example, if $\lambda_1 = \lambda_2 = -1$ (so that we our maximization is actually minimizing payoffs), then the solution is an equilibrium in which both players defect in every period. But whatever weighted sum we are maximizing, the equilibrium begin with *something*, and so we can represent v^* as

$$\lambda_1 v_1^* + \lambda_2 v_2^* \quad \leq \quad \max_{\alpha,v,\tilde{\gamma}} \lambda_1 v_1 + \lambda_2 v_2 \tag{4}$$

$$\text{subject to} \qquad v_i \ = \ \left\{u_i(\alpha) + [\text{prob}(\overline{y} \mid \alpha)\tilde{\gamma}_i(\overline{y}) + \text{prob}(\underline{y} \mid \alpha)\tilde{\gamma}_i(\underline{y})]\right\} \tag{5}$$

$$v_i \ \geq \ u_i(\alpha_i', \alpha_{-i}) + [\text{prob}(\overline{y} \mid (\alpha_i', \alpha_{-i}))\tilde{\gamma}_i(\overline{y}) + \text{prob}(\underline{y} \mid (\alpha_i', \alpha_{-i}))\tilde{\gamma}_i(\underline{y})] \tag{6}$$

$$\tilde{\gamma}_i(\cdot) \ \leq \ 0, \quad i=1,2. \tag{7}$$

This gives us a straightforward problem we can solve for bounds on equilibrium payoffs.

For example, this bound on equilibrium payoffs allows us to show that, no matter what the discount factor, the symmetric equilibrium payoff in our prisoners' dilemma (based on Figure 1 and the noisy monitoring technology (1)) is inefficient. Hence, it is not simply that we chose our strategies unwisely in Section 2.2, but that we are doomed to inefficiency.[25] It suffices to show that in any equilibrium beginning with CC, the sum of the two players' payoffs is bounded away from 4.[26] Setting $\lambda_1 = \lambda_2 = 1$, the incentive constraint given by (6) implies

$$\tilde{\gamma}_i(\overline{y}) \geq \tilde{\gamma}_i(\underline{y}) + \frac{1}{p-q}.$$

Hence, we must have (inserting this result in (5) and recalling from (7) that that $\tilde{\gamma} \leq 0$)

$$\begin{aligned} v_1^* + v_2^* &\leq 4 + p(\tilde{\gamma}_1(\overline{y}) + \tilde{\gamma}_2(\overline{y})) + (1-p)(\tilde{\gamma}_1(\underline{y}) + \tilde{\gamma}_2(\underline{y})) \\ &= 4 + p(\tilde{\gamma}_1(\overline{y}) + \tilde{\gamma}_2(\overline{y})) + (\tilde{\gamma}_1(\overline{y}) + \tilde{\gamma}_2(\overline{y})) - (1-p)(\tilde{\gamma}_1(\overline{y}) + \tilde{\gamma}_2(\overline{y}) - \tilde{\gamma}_1(\underline{y}) - \tilde{\gamma}_2(\underline{y})) \\ &\leq 4 + p(\tilde{\gamma}_1(\overline{y}) + \tilde{\gamma}_2(\overline{y})) - (1-p)(\tilde{\gamma}_1(\overline{y}) + \tilde{\gamma}_2(\overline{y}) - \tilde{\gamma}_1(\underline{y}) - \tilde{\gamma}_2(\underline{y})) \\ &\leq 4 - 2\frac{1-p}{p-q}. \end{aligned}$$

We are thus bounded away from an efficiency by an amount that approaches zero as the monitoring becomes increasingly perfect, i.e., as $p \to 1$.

We can go further. For any pair of weights $(\lambda_1, \lambda_2) \equiv \lambda$, let $\alpha(\lambda)$ be the action profile that solves (4)–(7). Then the set of equilibrium values must be contained in the set

$$H^*(\lambda) = \{\gamma : \lambda_1\gamma_1 + \lambda_2\gamma_2 \leq \lambda_1 v_1(\alpha(\lambda)) + \lambda_2 v_1(\alpha(\lambda))\},$$

since this is the set containing the set of subgame-perfect equilibrium payoffs that got us started on the simplification of the characterization. This must hold for every λ. Hence, the set of subgame perfect equilibria must be contained in the intersection of the sets $H^*(\lambda)$, for every nonzero $\lambda \in \mathbb{R}^2$.

This provides a tool for studying subgame-perfect equilibrium payoffs, imposing bounds whose usefulness we've seen in examining the public-monitoring prisoners' dilemma. The more remarkable result is that, as $\delta \to 1$, the set of equilibrium payoffs converges to this intersection ($\cap_\lambda H^*(\lambda)$). This is a straightforward recipe for an exact characterization of the set of equilibrium payoffs for patient players.

4 Characterizing Behavior

Theoretical models based on repeated games have been used to examine a variety of economic relationships. This section briefly presents three examples.

[25]This is consistent with our observation, at the end of Section 2.2, that this game and monitoring technology fail the sufficient conditions of Proposition 2.

[26]This leaves open the possibility that mixed equilibria may allow symmetric payoffs that are arbitrarily close to efficient, but a similar argument excludes this possibility as well.

4.1 Time Consistency

The idea of time (in)consistency appears regularly in discussion of government policy (e.g., Chari and Kehoe (1990), Kydland and Prescott (1977), and Ljungqvist and Sargent (2004, Chapter 22)). To fit this into our repeated-games context, let us think of player 1 is a government. The role of player 2 is filled by a continuum of consumers. Each player 2 is a negligible portion of the economy, and can hence expect her current actions to have no effect on future play. Each consumer accordingly chooses a myopic best response in each period.

In each period, each player 2 is endowed with one unit of a consumption good. The consumer divides this unit between consumption c and capital $1-c$. Capital earns a gross return of R, so that the consumer amasses $R(1-c)$ units of capital. The government sets a tax rate t on capital, collecting revenue $tR(1-c)$ with which it produces a public good. One unit of revenue produces $\gamma > 1$ of the public good. Untaxed capital is consumed. Notice that all of this happens within a period. There are no savings across periods and the problem of time consistency will arise completely within a period.

The consumer's utility is given by

$$c+(1-t)R(1-c)+2\sqrt{G}, \tag{8}$$

where $R-1<\gamma<R$ and G is the quantity of public good. The government chooses its tax rates so as to maximize the consumer's utility. There is thus no conflict of interest.

Each individual consumer makes a negligible contribution to the government's tax revenues, and accordingly treats G as fixed. The consumer thus chooses c to maximize $c+(1-t)R(1-c)$. The consumer's optimal behavior, as a function of the government's tax rate t, is then given by:

$$c(t)=\begin{cases} 0 & \text{if } t<\frac{R-1}{R} \\ 1 & \text{if } t>\frac{R-1}{R}. \end{cases} \tag{9}$$

If every consumer chooses c, the government's best response is to choose the tax rate so as to maximize the consumers' utility, or

$$c+(1-t)R(1-c)+2\sqrt{\gamma tR(1-c)},$$

where the government recognizes that the quantity of the public good depends upon its tax rate. We can take a derivative of (8) in t and solve to obtain the government's optimal tax rate as a function of c, given by

$$t(c)=\frac{\gamma}{R(1-c)}. \tag{10}$$

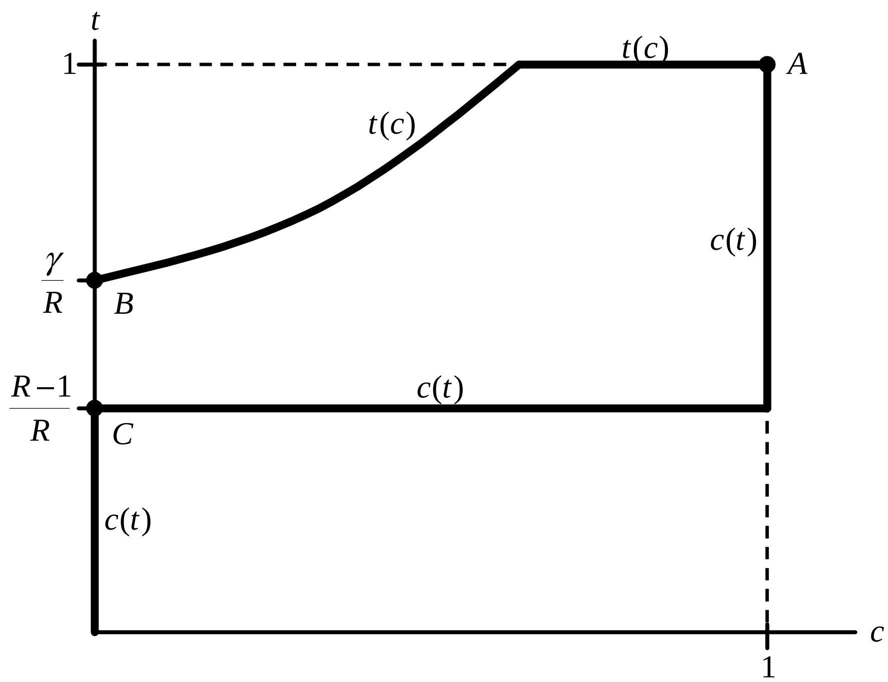

Figure 4: Consumer best response $c(t)$ as a function of the tax rate t, and government best response $t(c)$ as a function of the consumption c.

Figure 4 illustrates the best responses of consumers and the government.[27]

The efficient outcome calls for consumers to set $c = 0$. Since $R > 1$, investing in capital is productive, and since the option remains of using the accumulated capital for either consumption or the public good, this ensures that it is efficient to invest all of the endowment. The optimal tax rate (from (10)) is $t = \frac{\gamma}{R}$. This gives the allocation B in Figure 4.

It is apparent from Figure 4 that the stage game has a unique Nash equilibrium outcome in which consumers invest none of their investment ($c = 1$) and the government tax rate is set sufficiently high as to make investments suboptimal. Outcome A in Figure 4 is an example.[28] This equilibrium minmaxes both the consumer and the government. Remarkably, this result arises in a setting where the government and the consumers have identical utility functions.

There are no circumstances in the repeated game under which we can hope to obtain the efficient

[27] We omit in Figure 4 the fact that if consumers set $c = 1$, investing none of their endowment, then the government is indifferent over all tax rates, since all raise a revenue of zero.

[28] There are other Nash equilibria in which the government sets a tax rate less than one, since the government is indifferent over all tax rates when $c = 1$, but they all involve $c = 1$.

allocation (B in Figure 4) as an equilibrium outcome, since the consumers are not choosing myopic best responses. Respecting the constraint that consumers must choose such best responses in any equilibrium, the allocation that maximizes the government's (and hence also the consumers') payoffs is C in Figure 4. The government sets the highest tax rate consistent with consumers' investing, given by $\frac{R-1}{R}$, and the latter invest all of their endowment. Let $\overline{v_1}$ denote the resulting payoff for the government.

The government could obtain payoff $\overline{v_1}$ if it could choose its tax rate first, with this choice observed by consumers before making their investment decision. In the absence of the ability to do so, we can say that the government has a commitment problem—its payoff could be increased by the ability to commit to a tax rate before consumers make their choices. Alternatively, this is often described as a time consistency problem, or the government is described as having a tax rate ($\frac{R-1}{R}$) that is optimal but "time inconsistent." The notion of time inconsistency has arisen in a variety of contexts, but in each case is ultimately the observation that the government's payoff could be increased by the ability to commit to a policy.

Could repeated play allow the government to commit to more moderate tax rates?[29] The result is expected. Suppose the government begins the repeated game with tax rate $\frac{R-1}{R}$ and consumers begin by investing all of their endowment. These actions are repeated, in the absence of deviations, while any deviation prompts a reversion to the permanent play of the (minmaxing) stage-game equilibrium. Using these strategies, the proof of the following is a straightforward calculation.

Proposition 5 *There exists $\underline{\delta}$ such that, for all $\delta \in [\underline{\delta}, 1)$, there exists a subgame perfect equilibrium of the repeated game in which the constrained efficient allocation (C in Figure 4) is obtained in every period.*

4.2 Adverse Selection

This section examines a problem of repeated adverse selection.[30] There are two firms, denoted 1 and 2. In each period of the repeated game, Nature first independently draws, for each firm, a constant marginal

[29] Kydland and Prescott's (1977) introduction of the issue of consistency has been followed by a collection papers examining, in various contexts, the ability of history-dependent strategies in repeated games to implicitly provide the ability to commit. The example presented in this section raises issues similar to those examined by Chari and Kehoe (1990). Chari and Kehoe (1993a) and Chari and Kehoe (1993b) examine the question of why governments repay their debts, and why governments are able to issue debt in the first place, given the risk that they will not repay.

[30] We work with a simplified version of a model examined by Athey and Bagwell (2001) and Athey, Bagwell, and Sanchirico (2004).

cost equal to either $\underline{\theta}$ or $\overline{\theta} > \underline{\theta}$, with the two values being equally likely. The firms then simultaneously choose prices, drawn from $\mathbb{R}_+$. There is a unit mass of consumers, each potentially buying a single unit of the good, with a reservation price of $r > \overline{\theta}$. A consumer purchases from the firm setting the lower price if it does not exceed r. Consumers are indifferent between the two firms if the latter set identical prices, in which case we specify consumer decisions as part of the equilibrium. A firm from whom the consumers all purchase at price p, with cost θ, earns payoff $p-\theta$.

The stage game has a unique symmetric Nash equilibrium. A firm whose cost level is $\overline{\theta}$ sets price $\overline{\theta}$ and earns a zero expected profit. A low-cost firm chooses a price according to a distribution $F(p)$ with support on $[\frac{\underline{\theta}+\overline{\theta}}{2}, \overline{\theta}]$.[31] The expected payoff to each firm from this equilibrium is given by $\frac{1}{4}[\overline{\theta}-\underline{\theta}]$. If r is much larger than $\overline{\theta}$, the firms are falling far short of the monopoly profit. An upper bound on the payoffs in a symmetric-payoff equilibrium arises if both firms set price r, but with only low-cost firms (if there is such a firm) selling output, for an expected payoff to each firm of

$$\frac{1}{8}(r-\overline{\theta}) + \frac{3}{8}(r-\underline{\theta}) \equiv v^*.$$

The repeated game is one of imperfect public monitoring, in the sense that, given a strategy that attaches different prices to different cost levels, the stage-game outcome reveals only one of these prices. We are interested in an equilibrium of the repeated game that maximizes the firms' payoffs, subject to the constraint that they receive the same payoff.

Proposition 6 *For any $\eta > 0$, here exists a $\overline{\delta} < 1$ such that for all $\delta \in (\overline{\delta}, 1)$, there exists a pure perfect equilibrium with payoff at least $v^* - \varepsilon$ for each player.*

We present an equilibrium with the desired property. Our candidate strategies for the firms specify that a high cost firm choose price r and a low cost firm price $r-\varepsilon$ for some small $\varepsilon > 0$, after any history featuring no other prices, and that any history featuring any other price prompts play of the stage-game Nash equilibrium. We also specify that if an out-of-equilibrium price has ever been set, consumers thereafter split equally between the two firms whenever the latter set identical prices.

To describe the behavior of consumers in response to equilibrium prices, define three market share "regimes," B, I and II, each specifying how consumers behave when the firms both set price r or both

[31] It is straightforward that prices above $\overline{\theta}$ are vulnerable to being undercut by one's rival and hence will not appear in equilibrium, so that high-cost firms must set price $\overline{\theta}$. The lower bound on the support of the low-cost firm's price distribution must make the firm indifferent between selling with probability 1 at that price and selling with probability $\frac{1}{2}$ at price $\overline{\theta}$, or $p-\underline{\theta} = \frac{1}{2}(\overline{\theta}-\underline{\theta})$, giving $p = \frac{\underline{\theta}+\overline{\theta}}{2}$.

State	Prices $r-\varepsilon, r-\varepsilon$	$r-\varepsilon, r$	$r, r-\varepsilon$	r, r
B	split	1	2	split
I	1	1	2	1
II	2	1	2	2

Figure 5: Market share regimes B, I, and II, each identifying how the market is split between the two firms, as a function of their prices.

set price $r-\varepsilon$. These regimes are shown in Figure 5, where "split" indicates that the market is to be split equally, and otherwise the indicated firm takes the entire market. Play begins in regime B, which treats the firms identically and splits the market whenever they set the same price. Regime I rewards firm 1 and Regime II rewards firm 2. The regime shifts to I whenever firm 1 sets price r and firm 2 sets price $r-\varepsilon$. The regime shifts to II whenever firm 2 sets price r and firm 1 sets price $r-\varepsilon$. Hence, a firm is rewarded for choosing price r (while the opponent reports chose price $r-\varepsilon$) by a presumption that the firm receives the lion's share of the market if the two firms set equal prices.

The prescribed actions always allocate the entire market to the low-cost producer, ensuring that the proposed equilibrium outcome is efficient. The three market share regimes differ in how the market is to be allocated when the two firms have the same cost level. The payoffs thus shift along a frontier passing through the equilibrium payoff profile, with a slope of -1. Transitions between states thus correspond to transfers from one agent to the other. As we have noted in Section 2.2, these are precisely the types of punishments we should expect if we are to achieve efficient outcomes under imperfect monitoring.

It is a straightforward calculation that expected payoffs from this strategy profile approach v^* for each firm (as we make ε small and the firms patient), and that if firms are sufficiently patient, neither will ever prefer to abandon equilibrium play, triggering permanent play of the stage-game Nash equilibrium, by setting a price other than r or $r-\varepsilon$. To complete the argument, we must verify that each firm prefers to "identify its cost level truthfully," in the sense that it prefers to make the appropriate choice from the set $\{r-\varepsilon, r\}$, given the history of play and its realized cost. We examine the incentive constraints for the limiting case of $\varepsilon = 0$, establishing that they hold with strict inequality for sufficiently patient firms. They will continue to hold if ε is sufficiently small.

Let $\bar{V}$ be the payoff to firm 1 from a continuation game that begins in regime I (or, equivalently, the value of firm 2 of regime II). Conversely, let $\underline{V}$ be the value of firm 2 when beginning in regime I or firm 1 in regime II. The requirement that a low-cost firm 1 optimally set price $r-\varepsilon$ rather than r in regime I is

$$(1-\delta)(r-\underline{\theta})+\delta(\frac{1}{2}\underline{V}+\frac{1}{2}\overline{V}) \geq (1-\delta\frac{1}{2}(r-\underline{\theta})+\delta\overline{V}.$$

The requirement that a high cost firm 1 optimally choose price r in regime I is:

$$(1-\delta)\frac{1}{2}(r-\overline{\theta})+\delta\overline{V} \geq (1-\delta)(r-\overline{\theta})+\delta(\frac{1}{2}\underline{V}+\frac{1}{2}\overline{V}).$$

The requirement that a low-cost firm 2 set price $r-\varepsilon$ in regime I is

$$(1-\delta)\frac{1}{2}(r-\underline{\theta})+\delta\underline{V} \geq \delta(\frac{1}{2}\underline{V}+\frac{1}{2}\overline{V}).$$

The requirement that a high-cost firm 2 optimally choose price r in regime I is

$$\delta(\frac{1}{2}\underline{V}+\frac{1}{2}\overline{V}) \geq (1-\delta)\frac{1}{2}(r-\overline{\theta})+\delta\underline{V}.$$

Regime II yields equivalent incentive constraints. Let V be the expected value of a continuation game beginning in regime B, which is identical to the two firms. For a low-cost firm to optimally choose price $r-\varepsilon$, we have

$$(1-\delta)\frac{3}{4}(r-\underline{\theta})+\delta(\frac{1}{2}V+\frac{1}{2}\underline{V}) \geq (1-\delta)\frac{1}{4}(r-\underline{\theta})+\delta(\frac{1}{2}V+\frac{1}{2}\overline{V}).$$

For a high-cost firm to optimally choose price r, we need

$$(1-\delta)\frac{1}{4}(r-\overline{\theta})+\delta(\frac{1}{2}V+\frac{1}{2}\overline{V}) \geq \frac{3}{4}(r-\overline{\theta})+\delta(\frac{1}{2}V+\frac{1}{2}\underline{V}).$$

It is then a matter of calculation to show that these constraints hold, and hence that we have an equilibrium, for sufficiently large δ.

This calculation raises three points. First, the further we progressed through the presentation, the more the language sounded like that of a mechanism design problem, culminating in a collection of "truth-telling" incentive constraints. This is indicative of the *mechanism design approach* to repeated games with private-information stage games.[32] The mechanism design approach begins by dividing the prices in this market (or more generally, the actions in a game) into two sets, *equilibrium* prices and *out-of-equilibrium* prices. Out-of-equilibrium prices unambiguously reveal a deviation from equilibrium

[32]The mechanism design approach is introduced by Athey and Bagwell (2001) and Athey, Bagwell, and Sanchirico (2004), developed and extended by Miller (2005a,b), and applied by Athey and Miller (2004).

play. We can then attach the worst available punishment to these actions, knowing that this punishment will have no implications for equilibrium payoffs and will deter the deviations (for sufficiently large discount factors). This allows us to concentrate on equilibrium prices. By viewing continuation payoffs in the game as transfers, this part of the analysis can be treated as a mechanism design problem, allowing us to apply the tools of mechanism design theory.

Second, the incentive for firm 1 to set a high price when drawing cost $\bar{\theta}$ is that a low price is punished by a shift to regime II. The distinguishing feature of regime II is that indifferent consumers purchase from firm 2. Firms thus set high prices because consumers punish them for low prices. How crazy can a model be in which firms collude because their customers punish them for not doing so?

Upon reflection, perhaps not so crazy, because we actually see such arrangements. Firms routinely advertise that they will "never knowingly be undersold" and that they will "meet any competitor's price," schemes that appear to be popular with consumers. These pricing policies are commonly interpreted as devices to facilitate collusion by making it less profitable to undercut a collusive price. Consumers who march into store 1 to demand the lower price they found at store 2 are in fact punishing store 2 for its low price rather than store 1 for its high price, in the process potentially allowing the firms to collude.

More generally, we return to Section 2.4's point that we cannot evaluate an equilibrium within the confines of the model. Instead, we must select an equilibrium as part of constructing the model of the strategic interaction in question. Depending upon the nature of this interaction, consumers may well behave in such a way as to support collusion on the part of the firm. This behavior may appear counterintuitive in the stark confines of the model, while appearing perfectly natural in its actual context.

Third, the firms are ex ante symmetric in our model, and we have focussed attention on maximizing their payoffs given that they earn the same expected payoffs. It is then natural to suspect that the resulting equilibrium would feature symmetric and stationary outcomes—that along the equilibrium path, we would see the same (symmetric) outcome in each period.[33] Instead, we find an equilibrium that makes important use of nonstationarity and asymmetry along the equilibrium path.[34] This is not simply an artifact of the particular equilibrium we have eamined. Efficiency requires that the firms sometimes set price

[33]We cannot expect the stronger version of stationarity that would require the same actions after every history, both in and out of equilibrium. Instead, we expect that deviations from equilibrium will trigger punishments. For example, the only equilibrium in the repeated prisoners' dilemma satisfying this stronger stationarity property is one in which players defect in every period, while an equilibrium in which the cooperate in every period, with deviations punished by subsequent defection, features outcomes that are stationary along the equilibrium path.

[34]The asymmetry is not simply ex post, in the sense that firms with different cost realizations are treated differently, but ex ante, in the sense that the firms fare differently conditioned on cost realizations, depending upon the history of play.

r and sometimes price $r - \varepsilon$ for small ε, all without a high-cost firm having an incentive to sweep up all the consumers by setting price $r - \varepsilon$. This can be done only if future payoffs following the equilibrium prices r and $r - \varepsilon$ differ, giving rise to nonstationary equilibrium outcomes. If these in turn are to create effective incentives without inefficiency, they must be asymmetric. Nor is this an artifact of the particular game we have examined, instead being quite common.[35]

4.3 Consumption Dynamics

The dynamics of consumption behavior have attracted attention because individual consumption is commonly observed to be positively correlated with current and lagged values of individual income. People consume more when they earn more, and people consume more when they have earned more in the past (controlling for a variety of factors such as aggregate consumption, so that we are not simply making the observation that everyone consumes more when there is more to be consumed). If a risk averse agent's income varies, there are gains to be had from smoothing the resulting consumption stream by insuring against the income fluctuations. Why aren't consumption fluctuations perfectly insured?

This section illustrates how such behavior naturally emerges as part of efficient behavior in a relationship. We work with a model in which agents are subject to perfectly-observed income shocks. In the absence of any impediments to contracting on these shocks, the agents should enter into insurance contracts with one another, with each agent i making transfers to others when i's income is relatively high and receiving transfers when i's income is relatively low. In the simple examples we consider here, featuring no fluctuations in aggregate income, each agent's equilibrium consumption would be constant across states.

The conventional wisdom is that consumption fluctuates more than is warranted under a full insurance outcome. This excess consumption sensitivity must represent some difficulties in conditioning consumption on income. We focus here on one such difficulty, an inability to commit.[36] In particular,

[35] For example, Bond and Park (2002) suggest that gradualism in trade agreements between countries may reflect the fact that some efficient payoffs may be achievable only in nonstationary strategies.

[36] An alternative model of consumption dynamics arises out of the assumption that income shocks are private information. See Hertel (2004) for a model in which agents again cannot make commitments. Thomas and Worrall (1990) examine a case in which a risk-averse agent with a private income shock faces a risk-neutral insurer. Thomas and Worrall assume that the agents can commit to contracts, in the sense that participation constraints must be satisfied only at the beginning of the game, in terms of ex ante payoffs. The inability to completely smooth the risk-averse agent's consumption within a period prompts some risk to be optimally postponed until future periods. As a result, the risk-averse agent's equilibrium consumption shrinks to zero as time proceeds and her continuation value becomes arbitrarily negative, both with probability 1, clearly eventually violating

in each period, and after observing the current state, each agent is free to abandon the current insurance contract.[37] As a result, any dependence of current behavior on current income must satisfy incentive constraints. The dynamics of consumption and income will arise out of this restriction.

The stage game features two consumers, 1 and 2.[38] There is a single consumption good. A random draw first determines the players' endowments of the consumption good to be given by one of three possibilities:

	Endowment	Probability
Endowment $e(1)$	$(\bar{y}, \underline{y})$	$\frac{1}{3}$
Endowment $e(m)$	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$\frac{1}{3}$
Endowment $e(2)$	$(\underline{y}, \bar{y})$	$\frac{1}{3}$,

where $\bar{y} \in (\frac{1}{2}, 1]$. After observing the endowment, players 1 and 2 simultaneously transfer nonnegative quantities of the consumption good to one another, and then consume the resulting net quantities, evaluated according to the utility function $u(\cdot)$. The function u is strictly increasing and strictly concave.

This stage game obviously has a unique Nash equilibrium outcome in which no transfers are made. Because the consumers are risk averse, this outcome is inefficient. Each agent earns his minmax value under this equilibrium.

Suppose now that the consumers are infinitely-lived, playing the game in each period $t = 0, 1, \ldots$. The endowment draws are independent across periods. Players discount at the common rate δ. If the discount factor is high enough, then there exists a full-insurance equilibrium in which each player's

participation constraints in the continuation game. Extensions of this model are examined by Atkeson and Lucas, Jr. (1992) (to a general equilibrium setting) and Atkeson and Lucas, Jr. (1995) (to study unemployment insurance). Wang (1995) shows that if utility is bounded below, then (obviously) continuation utilities cannot be come arbitrarily negative, and (more importantly) consumption does not shrink to zero with probability one, since then the accumulation of utility at the lower bound would preclude the creation of incentives. Ljungqvist and Sargent (2004, Chapter 19) provide a useful discussion.

[37] Ljungqvist and Sargent (2004, Chapter 19) examine a variation on this model in which one side of an insurance contract can be bound to the contract, while the other is free to abandon the contract at any time. We might interpret this as a case in which an insurance company can commit to insurance policies with its customers, who can terminate their policies at will. Bond (2003), Ray (2002) and Thomas and Worrall (1994) similarly examine models in which a principal can commit to a contract but the agent cannot. We follow Kocherlakota (1996), Ljungqvist and Sargent (2004, Chapter 20) and Thomas and Worrall (1988) in examining a model in which either party to an insurance contract has the ability to abandon the contract at will. This might be interpreted as a case in which agents must insure one another.

[38] We draw here on a model of Kocherlakota (1996), discussed by Ljungqvist and Sargent (2004, Chapters 19–20). Ligon, Thomas, and Worrall (2002) and Thomas and Worrall (1988) present related models. See Koeppl (2003) for a qualification of Kocherlakota (1996).

consumption in each period is independent of the endowment and the history of play. The transfers required to achieve such consumption are enforced by a switch to the stage-game Nash equilibrium (i.e., to mutual minmaxing) should they fail to be made.

Suppose that δ falls short of $\underline{\delta}$, so that full insurance is impossible, but is large enough that some nontrivial equilibria exist. We will examine the efficient, symmetric-payoff equilibrium for this case. We can construct a first candidate for such an equilibrium by assuming that whenever endowment $e(i)$ is received, agent i transfers ε to agent j. Any failure to do so triggers permanent play of the stage-game Nash equilibrium. We choose ε as large as possible, namely to satisfy

$$(1-\delta)u(\bar{y}-\varepsilon)+\delta\frac{1}{3}\left(u(\bar{y}-\varepsilon)+u\left(\frac{1}{2}\right)+u(\underline{y}+\varepsilon)\right)=(1-\delta)u(\bar{y})+\delta\underline{v}.$$

This is the incentive constraint that an agent having drawn a high endowment be willing to transfer ε and continue with equilibrium play rather than pocketing the relatively favorable endowment and switching to the stage-game Nash equilibrium. Given our assumption that the discount factor is large enough to support more than the autarky equilibrium, but too small to support full insurance, this equation is solved by some $\varepsilon \in (0, \bar{y}-\frac{1}{2})$. This equilibrium leaves consumption untouched in endowment $e(m)$, while smoothing consumption i endowments $e(1)$ and $e(2)$. We refer to this equilibrium as σ.

This equilibrium provides some insurance, but we can provide more. Let us separate ex ante histories into two categories, category 1 and category 2. A history is in category i if agent i is the most recent one to have drawn a high endowment. Now fix $\zeta > 0$, and let the strategy profile prescribe consumption $(\frac{1}{2}+\zeta, \frac{1}{2}-\zeta)$ whenever the agents find themselves facing endowment $e(m)$ after a category-1 history and consumption $(\frac{1}{2}-\zeta, \frac{1}{2}+\zeta)$ when facing endowment $e(m)$ after a category-2 history. In essence, we are using consumption in endowment $e(m)$ to reward the last agent who has had a large endowment and transferred part of it to the other agent.

This modification of profile σ has two effects. We are introducing risk in endowment $e(m)$, but with a second-order effect on total expected payoffs (when ζ is small). However, because we now allocate endowment-$e(m)$ consumption in order to reward the last agent to make a transfer, this adjustment gives a first-order increase in the expected continuation payoff to agent i after a history of category i. This relaxes the incentive constraints facing agents when drawing endowments $e(1)$ and $e(2)$. We can thus couple the increase in ζ with an increase in ε, where the latter is calculated to restore equality in the incentive constraints in endowments $e(1)$ and $e(2)$, thereby allowing more insurance in endowments $e(1)$ and $e(2)$. The increased volatility of consumption in endowment $e(m)$ thus buys reduced volatility in endowments $e(1)$ and $e(2)$, allowing a first-order gain on the latter at a second-order cost on the former.

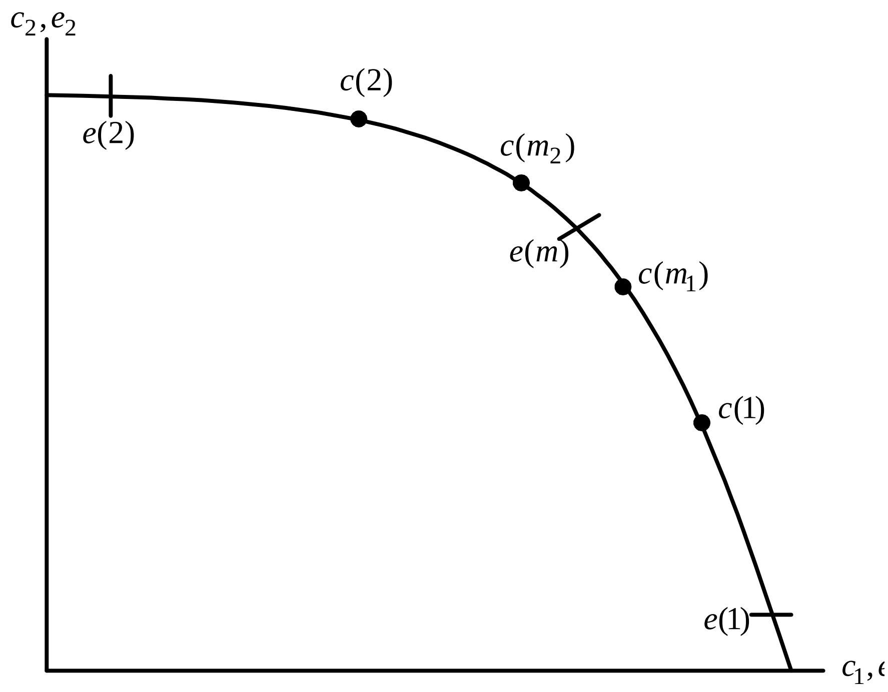

Figure 6: Illustration of the three endowments $e(1)$, $e(m)$ and $e(2)$, as well as four consumption profiles $c(1)$, $c(m_1)$, $c(m_2)$ and $c(2)$, for the efficient, symmetric equilibrium. Consumption bundle $c(m_i)$ follows a history in which endowment $e(m)$ has been drawn, and the last endowment other than $e(m)$ to be drawn was endowment $e(i)$.

Figure 6 illustrates the resulting consumption pattern. Because profile σ was an equilibrium, this new profile is also an equilibrium for sufficiently small ζ.[39] Choosing ζ to be either the largest value for which this profile is an equilibrium or the value for which $\bar{y} - \varepsilon - \frac{1}{2} + \zeta$ (in which case $c(i) = c(m_i)$ in Figure 6), whichever is smaller, gives us the equilibrium we seek. As in Section 4.2, we have a symmetric-payoff equilibrium that is not stationary. Current consumption depends upon whether the history is in category 1 or category 2, in addition to the realization of the current endowment. Intuitively, we are now spreading risk across time as well as states within a period, exchanging a relatively large transfer from a high-endowment agent for a relatively lucrative continuation payoff. In terms of consumption dynamics, agents with high endowments in their history are now more likely to have high

[39]The essential observation here is that, under σ, the expected continuation payoff conditional on state m exceeds that of the repeated play of the Nash equilibrium, introducing slack into the incentive constraints for making transfers in state m that allows the new equilibrium to support small transfers.

	h	ℓ
H	$2,3$	$0,2$
L	$3,0$	$1,1$

Figure 7: The product choice game

current consumption.

5 Reputations

It is common to speak of people, firms, governments, and other institutions as having reputations. The idea of a reputation typically carries some connotation of foregoing an opportunity for short-terms gains—perhaps by providing high quality instead of shoddier but cheaper merchandize, exerting effort instead of shirking, behaving honestly rather than opportunistically—even though the immediate incentives for doing so are not compelling. The rewards for doing so are viewed as arising from links between current actions and future expectations about one's behavior. The concept of a reputation thus fits naturally into a study of relationships.

We will discuss reputations in the context of the "product choice" game shown in Figure 7. We think of player 1 as a firm who can produce high quality (H) or low quality (L). Player 2 is a consumer who simultaneously purchases either a high-cost product from the firm (h) or a low-cost product (ℓ). The consumer prefers to buy the expensive product if the firm is providing high quality, and the low-cost product otherwise. For example, the consumer may prefer dinner with all the trimmings at a fine restaurant but a sandwich at the local diner. A patient may prefer heart surgery from a competent physician but a herbal remedy from a quack. The firm finds high quality expensive and so always earns a higher payoff from low quality, but prefers that the consumer choose the high cost option. The stage game has a unique Nash equilibrium in which $L\ell$ is played, for payoffs $(1,1)$.

There are two approaches to thinking about reputations. The "equilibrium interpretation" approach selects an equilibrium of the repeated game and then interprets various of its features in reputation terms. For example, when the players are sufficiently patient, there is an equilibrium in the repeated product-choice game in which Hh is played in every period, with any deviations from such play prompting a switch to (permanent) play of the stage-game equilibrium $L\ell$. We might then refer to the firm as maintaining a reputation for high quality along the equilibrium path, and any deviation as having destroyed this reputation.[40] However, there is no necessary link between past behavior and expectations about future behavior in such models. For example, there remains an equilibrium in which $L\ell$ is played in every period, and it remains an equilibrium to continue with such behavior even if player 1 desperately tries to build a reputation by playing H in the first thousand periods. The idea of a reputation is helpful in keeping track of the behavior in the selected equilibrium, but adds nothing formal to the analysis.

The "adverse selection" approach to reputations, considered here, rests upon the introduction of incomplete information concerning players' characteristics. For example, suppose there is some small probability that player 1 in the product choice game is a commitment type who always plays H, otherwise being the normal type specified by the stage game. This incomplete information is a mechanism for creating a necessary link between past play and expected future behavior. In particular, there is no longer an equilibrium of the repeated product-choice game in which the normal type of player 1 and player 2 invariably choose $L\ell$. Given such a candidate equilibrium, the normal type would choose H in the first period, leading player 2 to conclude she was facing the commitment type and hence leading to subsequent play of Hh. Results in the adverse selection approach consist of restrictions on the set of equilibria rather than statements about a particular equilibrium.

5.1 Short-Run Opponents

Our examination of the adverse selection approach to reputations begins with a setting in which player 1 is a long-run player, who appears in every period of the infinitely-repeated game. The role of player 2 is filled by a succession of short-run players, each of whom participates in the market for only a single period. For example, a new customer may come to the firm every period. As a result, each player 2 chooses a myopic best response to player 1's behavior.

5.1.1 Perfect Monitoring

Suppose first that actions are perfectly monitored.[41] There are two possible types of player 1, a "normal" type whose payoffs are given by Figure 7 and a "Stackelberg" type who always plays H. The game begins with a draw of player 1's type, revealed only to player 1, and perhaps attaching very high probability to player 1 being normal.

[40] For examples of this approach, see Barro and Gordon (1983), Canzoneri (1985), Ljungqvist and Sargent (2004, Chapter 22), and Rob and Fishman (2005).

[41] The basic result is from Fudenberg and Levine (1989). Celentani and Pesendorfer (1996) present an analogous result for dynamic games. Fudenberg and Levine (1992) extend the argument to imperfect monitoring.

In general, let

$$v_1^* = \max_{a_1} \min_{\alpha_2 \in B(a_1)} u_1(a_1, \alpha_2),$$

where $B(a_1)$ is the set of player-2 myopic best replies to a_1. This is player 1's pure Stackelberg payoff, identifying the payoff player 1 could earn in the stage game if 1 could publicly choose a pure action before 2 chooses, with 2 then choosing a best response. Let a_1^* denote the accompanying pure "Stackelberg" action. In the product choice game, this payoff is 2, secured by choosing action H. Let μ be the prior probability attached to the Stackelberg type. We have:

Proposition 7 *For any probability $\mu > 0$ and for any $\varepsilon > 0$, there exists a $\underline{\delta} < 1$ such that for any $\delta \in (\underline{\delta}, 1)$, the payoff to the normal type of player 1 in any Nash equilibrium of the repeated game is at least $v_1^* - \varepsilon$.*

To see what lies behind this result, fix an equilibrium of the game and consider player 1's options. One possibility is to play a_1^* in every period. The key step to the reputation result describes the consequences of such play:

Lemma 1 *For any $\eta > 0$, there is a number $n(\eta)$ such that if the normal type of player 1 chooses a_1^* in every period, the number of periods in which player 2 can attach probability less than $1-\eta$ to player 1 choosing a_1^* is less than $n(\eta)$.*

Thus, if player 2 observes ever-longer strings of action a_1^*, then eventually player 2 must come to expect action a_1^* to be played with high probability. In particular, suppose we reach a period t in which player 1 has hitherto played the Stackelberg action and player 2's current expectation is that the Stackelberg action need *not* appear. This can only happen if equilibrium play attaches some probability to the event that player 1 is the normal type and will not play the Stackelberg action. But then Bayes' rule ensures that observing the Stackelberg action in period t results in a posterior that must put increased weight on the Stackelberg type. As a result, the probability that player 2 attaches to seeing action a_1^* marches upward, and player 2 must eventually believe either that she is almost certainly facing the Stackelberg type or facing a normal type who plays like the Stackelberg type. Either way, player 2 must expect to see the Stackelberg action.

This characterization of beliefs allows us to establish the lower bound on player 1's equilibrium payoffs. Let the normal type of player 1 choose a_1^* in every period. After some finite number of periods, player 2 must believe that a_1^* is sufficiently likely as to play a best response, giving player 1 the payoff

v_1^*. Player 1 may earn lower payoffs in the early periods before 2 expects a_1^*, but these early periods are insignificant if player 1 is sufficiently patient. This ensures a repeated-game payoff arbitrarily close to v_1^* for a patient player 1. Notice that continually playing a_1^* may *not* be optimal for player 1, but 1's optimal strategy must give a payoff at least this large. In the case of the product choice game, continually playing H suffices for player 2 to eventually always play h, for a payoff close to 2.

5.1.2 Imperfect Monitoring

With some additional technical complication, a stronger result applies to games of imperfect monitoring (cf. FudenbergLevine92). For example, suppose that when the product choice game is played, player 2 observes not player 1's actions, but a signal from the set $\{\underline{y}, \bar{y}\}$, with probabilities

$$\rho(\bar{y} \mid a) = \begin{cases} p, & \text{if } a_1 = H, \\ q, & \text{if } a_1 = L, \end{cases} \tag{11}$$

where $0 < q < p < 1$. Player 2's inferences about player 1's behavior are now clouded by the imperfect monitoring. The arguments behind Lemma 1 are accordingly replaced by conceptually similar probabilistic arguments (e.g., Sorin (1999)) reflecting the randomness in the monitoring process. If player 1 invariably plays a_1, player 2's expectation of a_1 must increase on average, and she eventually must play a best response to a_1^* with very high probability. This suffices to ensure a patient player 1 a payoff arbitrarily close to 2 in the product choice game.

The same techniques that allow us to work with imperfect monitoring allow us to extend the results to commitment types who play mixed actions, in games of either perfect or imperfect monitoring, in the process strengthening the reputation result.[42] We illustrate with the product choice game, though the result is general. Player 1 would like player 2 to choose h. It suffices for this for player 1 be (known to be) committed to H, but there is a sense in which this is more of an investment in high quality than is required. It would suffice for 2 to find h a best response that player 1 choose H with probability just over $\frac{1}{2}$. The reputation argument ensures that, if the commitment type of player 1 chooses such a mixture, the (patient, normal) player 1 must receive a payoff in any equilibrium of the repeated game that is arbitrarily close to that from being known to be committed to such a mixture, in this case $\frac{5}{2}$. This is all the more remarkable in light of the fact that, in the case of imperfect monitoring (as in (11)) and no uncertainty

[42]Because player 2 observes only the realized actions of player 1 and not the underlying mixture, perfectly-monitored opponents who play mixed actions present inference problems similar to those of imperfectly monitored opponents.

about player 1's type, an argument analogous to that applied to the prisoners' dilemma in Section 3.2 allows us to conclude that player 1's payoff is bounded above by

$$2 - \frac{1-p}{p-q},$$

while in the case of perfect monitoring player 1's type is bounded above by 2 (Mailath and Samuelson (2006, Section 2.7.2)). The adverse-selection-based reputation thus not only pushes player 1 to the top of the set of equilibrium payoffs, but expands the set itself.

This discussion may leave the impression that reputation arguments depend critically upon having just the right commitment type in the model, and perhaps on having only that type, or having that type be sufficiently likely. None of these is the case. The arguments extend to the case in which player 1 may be one of an infinite number of possible types, which may not include the pure Stackelberg type. The bound on player 1's payoff is then given by allowing him to "choose" from the set of possible types the one to whose behavior it would be most profitable to commit, regardless of the how likely are the various types. The arguments further extend to the case in which there is (sufficiently small) uncertainty about player 2's type.

5.1.3 Temporary Reputations

Cripps, Mailath, and Samuelson (2004b) establish conditions under which player 1 will eventually spend his reputation. Under fairly general conditions, player 2 must eventually learn player 1's type, with play converging to an equilibrium of the complete-information game defined by player 1's type.[43] In the product-choice game, for example, reputation effects may constrain player 1's ex ante payoff to be very close to $\frac{5}{2}$, but his continuation payoff is eventually less 2.

To build intuition for this result, suppose that player 1 may be either a normal or Stackelberg type, and we have a candidate equilibrium in which player 2 does not learn player 1's type. Then player 2 must expect Stackelberg-type behavior from both types of player 1. Otherwise, she would eventually get enough information, even under imperfect monitoring, to sort out which type of player she faced.[44] Player 2, being a short-run player, will then play a best response to the Stackelberg type. But then the normal type of player 1 has an incentive to deviate from the Stackelberg behavior, potentially contradicting player 2's belief that player 1 will exhibit Stackelberg behavior.[45]

[43] Jackson and Kalai (1999) examine another sense in which reputations are temporary, in finitely-repeated games.

[44] An assumption plays a role here, namely that the signals are sufficiently informative that player 2 can distinguish different actions of player 1, given sufficient observations.

[45] Three assumptions are embedded here. First, that the structure of the game and signals is such that player 1 knows player

This is a limiting result, describing beliefs and behavior in the possibly very distant future. While the short-run properties of equilibria are interesting, we believe that the long-run equilibrium properties are also relevant in many situations. For example, an analyst may not know the age of the relationship to which the model is to be applied. We sometimes observe strategic interactions from a well-defined beginning, but we also often encounter on-going interactions whose beginnings are difficult to identify. Long-run equilibrium properties may be an important guide to behavior in the latter cases. Alternatively, one might take the view of a social planner who is concerned with the continuation payoffs of the long-run player and with the fate of all short-run players, even those in the distant future. Our analysis also suggests that the short-run players may have definite preferences as to where they appear in the queue of short-run players, offering a new perspective on the incentives created by repeated games. Finally, interest often centers on the *steady states* of models with incomplete information, again directing attention to long-run properties.

We view our results as suggesting that a model of *long-run* reputations should incorporate some mechanism by which the uncertainty about types is continually replenished. For example, Holmström (1982), Cole, Dow, and English (1995), Mailath and Samuelson (2001), and Phelan (2005) assume that the type of the long-run player is governed by a stochastic process rather than being determined once and for all at the beginning of the game. In such a situation, reputations can indeed have long-run implications. We return to this in Section 5.4.

How do we reconcile the finding that reputations are temporary with the fact that reputations impose nontrivial bounds on ex ante payoffs? The answer lies in the "eventually" aspect of our result. There may well be a long period of time during which player 2 is uncertain of player 1's type, and in which play does not resemble an equilibrium of the complete-information game. The length of this period will depend upon the discount factor, being longer for larger discount factors, and in general being long enough to have a significant effect on player 1's payoffs. Eventually, however, such behavior must give way to a regime in which player 2 is (correctly) convinced of player 1's type.

We thus have an order of limits calculation. For any prior probability μ that the long run player

2's belief, and hence knows that there is a profitable deviation. Cripps, Mailath, and Samuelson (2004a) relax this assumption. Second, that the normal player 1 does not find Stackelberg behavior a best response to player 2's (best response to Stackelberg) behavior. For example, there would be no difficulty in player 1 maintaining a reputation in the product choice game for being a commitment type who always played L—let the equilibrium simply specify (L,ℓ) in every period, regardless of history. Third, that the monitoring is imperfect, ensuring that the normal player 1's deviation cannot be deterred by a sufficiently draconian punishment.

is the commitment type and for any ε, there is a discount factor $\underline{\delta}$ sufficiently large that player 1's expected payoff is within ε of his Stackelberg payoff. This holds no matter how small μ. As a result, it is tempting to think that, even as the game is played and the posterior probability of the commitment type falls, we should be able to choose a period, think of it as the beginning of the game, and apply the standard reputation argument to conclude that uncertainty about player 1's type still has a significant effect. However, for any fixed $\underline{\delta}$ and in any equilibrium, there is a time at which the posterior probability attached to the commitment type has dropped below μ, becoming too small (relative to $\underline{\delta}$) for reputation effects to operate. We are then on the path to completely revealing player 1's type.

5.2 Two (Asymmetric) Long-Run Players

Section 5.1 examined a model in which player 1, the reputation builder, faced short-run opponents. In many reputation settings, such as a firm facing a succession of consumers, this seems quite natural. But what if both players are long-run?

Let player 1 and 2 both be long run players, with (possibly different) discount factors δ_1 and δ_2. We assume that player 1 may be either a normal or a Stackelberg player and ask whether the normal player 1 can exploit this uncertainty to impose a lower bound on his payoff in the repeated game.

Consider the product choice game. We can again proceed by arguing that, if player 2 always observes action H, player 2 must eventually belief the continued play of H is quite likely. This suffices to ensure a *short-run* player 2 plays h, the second important piece in the argument. However, the same is not necessarily the case for a long-run player 2. In particular, 2's expectation that H will very likely be played in the future is consistent with 2 attaching probability to player 1 being an normal type whose strategy calls for action H, a Stackelberg type committed to H, and a "punishment" type who plays H until the first time 2 plays h, after which the punishment type plays L. The latter type is of no concern to a short-run player 2, but if a long-run player 2 thinks this latter type sufficiently likely (a belief that continued observations of H will do nothing to dispel), 2 will play ℓ. We thus cannot simply transfer the previous argument to the case of two long-run players without imposing some additional structure on the problem.[46]

[46]The simplicity of the product choice game requires that we work with a punishment type whose behavior may appear counterintuitive or contrived. There is in general no obvious way of avoiding this difficulty by restricting attention to a set of "plausible" commitment types.

5.2.1 Conflicting Interests

We begin with a result from Schmidt (1993). Let v_1^* and a_1^* again be the Stackelberg payoff and action for player 1. We say that the stage game has *conflicting interests* if

$$\max_{\alpha_2 \in \mathscr{A}_2} u_2(a_1^*, \alpha_2) = \min_{\alpha_1 \in \mathscr{A}_1} \max_{\alpha_2 \in \mathscr{A}_2} u_2(\alpha_1, \alpha_2).$$

Hence, player 2's best response to player 1's Stackelberg action minmaxes player 2.

We then have:

Proposition 8 *Let the stage game be one of conflicting interests. Then for any $\delta_2 < 1$ and $\varepsilon > 0$, there exists a $\underline{\delta} < 1$ such that for all $\delta \in (\underline{\delta}_1, 1)$, the expected payoff of the normal type of player 1 is at least $v_1^* - \varepsilon$.*

As expected, the basic tool in the argument is to note that when facing a steady stream of a_1^*, player 2 must eventually come to expect the Stackelberg action. The argument that a long-run player 2 will then play a best response to a_1^* exploits the fact that this best response minmaxes player 2 (given conflicting interests), which in turn must be better than 2's payoff from any other response. We then note that there are no punishments worse for player 2 than being minmaxed, and hence nothing that can induce player 2 to consistently *not* play a best response to a_1^*. This allows a sufficiently patient player 1 to be assured a payoff close to the Stackelberg payoff.

Notice that the argument requires an asymmetry in discount factors, with player 2's discount factor being fixed and player 1's allowed to approach 1. Player 2's discount factor plays a role in imposing a bound on the number of times 2 can fail to play a best response to a_1^*. We must then have the freedom to let player 1's discount factor increase, to ensure that these periods have an insignificant effect on 1's payoff.

What sorts of games exhibit conflicting interests? The prisoners' dilemma does, but the reputation result is of no interest here, since player 1's Stackelberg type always defects and the result ensures player 1 a payoff of at least 0. The product choice game does not exhibit conflicting interests, and so is not covered by the result. The chain-store game (Figure 8) satisfies conflicting interests, and the result implies that player 1 (the incumbent) can earn a payoff arbitrarily close to that to be had from being committed to fighting entry.

Is there anything we can do to expand the result beyond games of conflicting interests? Cripps, Schmidt, and Thomas (1996) show that in the absence of conflicting interests, a sufficiently patient player

	Enter	*Out*
A	2,2	5,0
F	−1,−1	5,0

	L	*R*
T	0,0	3,1
B	1,3	0,0

Figure 8: Chain store (left) and battle of the sexes game.

1 can be assured a payoff arbitrarily close to that which arises if player 1 commits to an action and player 2 chooses the response that is worst for player 1, conditional on 2 earning at least her minmax payoff. In the battle of the sexes game, shown in Figure 8 (and which does not have conflicting interests), the minmax utilities for the two players are $(\frac{3}{4}, \frac{3}{4})$. Suppose that with positive probability, player 1 is thought to be the Stackelberg type who always plays T. The set of responses to T in which player 2 receives at least her minmax utility is the set of actions that place at least probability $\frac{3}{4}$ on R, ensuring player 1 a payoff of $\frac{9}{4}$. Hence, the normal player 1, if sufficiently patient and for any player-2 discount factor, must receive a payoff arbitrarily close to $\frac{9}{4}$.[47]

5.2.2 Imperfect Monitoring

Celentani, Fudenberg, Levine, and Pesendorfer (1996) show that reputations can be more effective in games of imperfect monitoring. As we have seen, the difficulty in establishing a reputation when facing a long-run player 2 is that consistently playing like the commitment type will lead player 2 to expect such behavior on the equilibrium path, but imposes few restrictions on what 2 can believe about play off the equilibrium path.[48] Celentani, Fudenberg, Levine, and Pesendorfer (1996) note that the sharp distinction between being on and off the equilibrium path disappears in games of imperfect monitoring. Player 2 should then have ample opportunity to become well acquainted with all of player 1's behavior, including any punishment possibilities. They exploit this insight to show that, without requiring conflicting interests, for any player-1 discount factor δ_2 and $\varepsilon > 0$, there is a $\underline{\delta}_1$ such that for all $\delta_1 \in (\underline{\delta}_1, 1)$, the normal type of player 1 earns at least $v_1^* - \varepsilon$ in every Nash equilibrium of the repeated game. Indeed, by allowing more sophisticated commitment types, they show that player 1 can be assured a payoff arbitrarily close to the largest stage-game payoff consistent with individual rationality for player 2, providing the strongest

[47]In the product choice game, ℓ ensures 2 her minmax payoff no matter what 1 chooses, and is always the worst action for player 1. The reputation result thus places a lower bound on 1's payoff that is no higher than can be earned when 2 always chooses ℓ, and hence imposes no restrictions in this case.

[48]Recall that this is not a difficulty in games of conflicting interests because player 2's payoff along the equilibrium path is sufficiently low that she has nothing to fear in being pushed off the path.

possible reputation result.

5.2.3 Commitment Types who Punish

Evans and Thomas (1997) (see also Evans and Thomas (2001)) take an alternative approach to player 2's concerns about out-of-equilibrium punishments. Suppose we have a commitment type who punishes player 2 for *not* playing a best response to the Stackelberg action. In the product-choice game, for example, suppose that a commitment type plays L once, the first time player 2 fails to choose h; plays L twice, the next time; and so on. Eventually, player 2 faces arbitrarily long punishments for not playing h. Now let the normal player 1 mimic this behavior. For familiar reasons, player 2 must come to expect such play, and must choose a best response. In the product choice game, this ensures that 2 will eventually play h, ensuring a patient (normal) player 1 a payoff close to 2. In general, by choosing the commitment type carefully, we again have the result that player 1 can be assured a payoff arbitrarily close to the largest stage-game payoff consistent with individual rationality for player 2. This is once more a strong result, one that does not require imperfect monitoring. This argument does require somewhat more active commitment types than those appearing in many reputation results. No longer does it suffice for a commitment type to simply play the action for which player 1 would like to develop a reputation, such as H in the product choice game. Instead, the commitment type must play an active role in "teaching" player 2 to play a best response to this action.

5.2.4 Reputations with Long-Run Opponents

The message that emerges from these results is that reputations can be effective against long-run opponents, but that the conditions for reputation building become more stringent than required for short-run opponents. Once again, the arguments do not require that *only* the right commitment types be present, or that the various commitment types appear in the right proportions, or that uncertainty be limited to only one player. However, if the results are to apply to a general class of games, then we must have either imperfect monitoring or relatively sophisticated commitment types. One of missing elements in the study of reputations is a theory of commitment types that would provide some guidance as to what sort of uncertainty about types captures the intuitive links between current behavior and future expectations that lie at the heart of a reputation.

Cripps, Mailath, and Samuelson (2004a) extend the "temporary reputations" result of Cripps, Mailath, and Samuelson (2004b) to games with two long-run players, so that the ex ante and asymptotic implications of reputation models can be quite different. We again see that an assessment of reputation results must depend upon the setting for which the model is intended.

5.3 Two (Symmetric) Long-Run Players

Sections 5.1 and 5.2 have examined reputations in games where player 2 is a short-run player, and then in which player 2 is a long-run player but not as patient as player 1. What if both are long run players and equally patient, or $\delta_1 = \delta_2 > 1$?[49]

Notice first that some asymmetry must be involved in a reputation result. The essence of a reputation result is a lower bound on the reputation builder's payoff, with particular emphasis on the cases in which this lower bound is close to a suitably defined "Stackelberg" payoff. Both players cannot earn nearly their Stackelberg payoff in the battle of the sexes (Figure 8), for example, since payoffs near $(3,3)$ are not feasible. We can then hope only for sufficient conditions for one player to develop such a reputation. But then we have no hope for such conditions in a perfectly symmetric game, with nothing to distinguish one of the players as the potential reputation builder.[50]

In moving from a short-run player 2 through a long-run but less patient player 2 to two long-run and equally patient players, we have wrung all of the asymmetry out of the players' discounting. We must accordingly look for asymmetries in the structure of the game. Two results are available.[51]

Suppose first that player 1's Stackelberg action is a strictly dominant action for the normal type in the stage game, and that player 2's best response to this action produces the highest stage-game payoff available to player 1. Figure 9 presents an example. We say that such a game is a *strictly dominant action game*. Notice that player 2's best response to 1's Stackelberg action need not minmax player 2,

[49]The symmetric case of *perfectly* patient players on both sides is studied by Cripps and Thomas (1995), Shalev (1994) and Israeli (1999). Cripps and Thomas (1995) show that when there is some prior probability that player 1 (only) may be a Stackelberg type, the normal player 1's payoff in any Nash equilibrium must be close to the bound established by Cripps, Schmidt, and Thomas (1996) for discounted games with player 1 arbitrarily more patient than player 2. Cripps and Thomas (2003) study games of incomplete information with payoff types (i.e., types defined in terms of preferences rather than actions), with positive probability attached to a (payoff) Stackelberg type. They show that as the common discount factor of player 1 and 2 approaches unity, there exist equilibria in which player 1's payoff falls short of the bound established in Cripps, Schmidt, and Thomas (1996).

[50]There may exist equilibria in which one of the players receives their Stackelberg payoff, but this does not reflect the limitation on the set of equilibrium payoffs characteristic of a reputation result.

[51]Cripps and Thomas (1997) and Celentani, Fudenberg, Levine, and Pesendorfer (1996) present examples showing that incomplete information can be remarkably ineffective in limiting equilibrium payoffs when the players are long-run and equally patient, ensuring that positive results cannot extend far beyond the ones presented here.

	L	R
T	$2,1$	$0,0$
B	$0,0$	$-1,2$

Figure 9: Game in which player 1's Stackelberg action, T, is a dominant action in the stage game for the normal type.

	L	R
T	$2,0$	$0,-1$
B	$1,0$	$0,1$

Figure 10: Game of strictly conflicting interests.

and hence that conflicting interests are not required. Chan (2000) shows that if such a game is perturbed to add a single possible commitment type for player 1, in the form of a type who always plays the Stackelberg action, then the normal player 1 receives the Stackelberg payoff v_1^* in any sequential equilibrium. Interestingly, this result holds regardless of the discount factors of the two agents.

Second, a game of *strictly conflicting interests* is a game of conflicting interests in which (*i*) the combination of player 1's Stackelberg action and any of player 2's best responses yields the highest stage-game payoff possible for player 1 and the minmax playoff $\underline{v}_2$ to player 2, and (*ii*) every other action profile giving player 1 this maximum payoff also minmaxes player 2. Figure 10 presents an example. The Stackelberg action is T. Player 2's best response yields a payoff to player 2 of 0, the minmax level. Cripps, Dekel, and Pesendorfer (2004) show that a reputation result obtains for this class of games. For sufficiently patient players, player 1's payoff is arbitrarily close to the Stackelberg payoff.

5.4 An Alternative Reputation Model

This section present an alternative reputation model, a simplified version of models examined by Mailath and Samuelson (2001) and Mailath and Samuelson (2006, Chapter 18). Our presentation begins with a model taken from the latter.

5.4.1 Motivation

Our motivation for this model comes in three parts. First, existing models do not readily capture the full spectrum of issues encompassed in the popular use of the word "reputation." It is common to think of reputations as assets—things of value that require costly investments to build and maintain, that can be enhanced or allowed to deteriorate, that gradually run down if neglected, and that can be bought and sold. We would like a model that captures this richness.

The repeated games of adverse selection that form the heart of existing work on reputations may well have equilibria capturing many of these features. The argument that player 2 must eventually come to expect the Stackelberg action if player 1 invariably plays it is suggestive of a reputation-building phase, while we have also seen that ultimately, reputations are optimally depleted. However, these models do not provide the explicit links between the structure of the interaction and equilibrium behavior that would be especially useful in studying reputations.

Second, reputations in standard models are built by mimicking behavior to which one would like to be committed. We refer to these as "pooling" reputations, since the payoff bounds arise out of pooling one's actions with those of the commitment type. In contrast, this section focusses on "separating" reputations, in which players strive to distinguish themselves from types for whom they would like to not be mistaken. Stackelberg types may not always be conveniently available. Consumers may approach the market not in terms of finding a firm who necessarily provides good service, but with avoiding the one who is incapable of doing so. The normal firm may then find that there are effectively no Stackelberg types with whom to pool, but that providing good service is essential in distinguishing himself from inept types.

Third, many equilibria in repeated games require what often appears to be an implausible degree of coordination among the players. We will work in this chapter with models deliberately designed to limit such coordination. This in turn will provide a natural setting for reputations based on separation.

5.4.2 The Model

The model is based on a variant of the product choice game (cf. Figure 7). Player 1, a long-run firm, can choose either high quality (H) or low quality (L) in each period. Low quality is costless, while high quality imposes a cost of $c > 0$. We assume throughout that c is sufficiently small as to make high quality the Stackelberg action for the firm.

We now interpret player 2 as a continuum of consumers. Each consumer recognizes that their actions have a negligible effect on the market outcome and hence no effect on future play (as in Section 4.1), and hence behave myopically.

In each period, each consumer buys one unit of the good from the firm. The good generates two possible utility levels for the consumer, which we take to be 0 and 1. The realized utility is random and depends upon the quality chosen by the firm, with a good outcome (utility 1) appearing with probability ρ_H if the firm chose high quality and ρ_L if the firm chose low quality, where

$$0 < \rho_L < \rho_H < 1.$$

Each consumer pays a price equal to the expected utility of the good.

We view the various player 2s as receiving idiosyncratic signals. If the firm chooses high quality, then each consumer receives a good utility with probability ρ_H, and precisely ρ_H of the consumers receive good utilities. This idiosyncracy in signals disrupts the coordination that typically plays a central role in creating intertemporal incentives. Suppose we attempted to construct an equilibrium in which player 1 chooses H, deterred from choosing L by the fact that bad outcomes trigger punishments. A consumer who has just received a bad outcome has no way of knowing whether this is simply an unlucky draw from the firm's choice of high effort or whether it is a signal that the firm chose low effort. By itself, this inference problem is not particularly problematic. In a standard public-monitoring game, bad signals trigger punishments even though players know they are *not* an indication of shirking (in equilibrium) (Section 2.2). However, for this behavior to be consistent with equilibrium, it is important that there be coordination in the punishment, not only among the small anonymous players but also with player 1. This is possible because an agent receiving a bad signal knows that (every else knows that...) everyone else also received a bad signal. The idiosyncratic signals disrupt these inferences, robbing the players of the ability to coordinate. As a result, this game has a unique equilibrium in which the firm always exerts low effort.

The next step is to add incomplete information about player 1's type. There are two types of firm, *normal* and *inept*. An inept firm can only choose low effort. Notice that the extra type of player 1 is not a Stackelberg type, but a type with whom player 1 would like to not be confused. Reputation concerns in this model will arise out of player 1's efforts to separate from a bad type rather than pool with a good type.[52]

We have seen that something must be done if player 1 is to choose H in equilibrium, since the game of incomplete information features a unique equilibrium in which L is always chosen. How do inept types help? Consider a candidate equilibrium in which the normal player 1 always chooses H. When there

[52]What difference would it make if the commitment type always played H, rather than being inept? Our separating model always features an equilibrium in which both the inept and normal type of player 1 exert low effort, with player 2 never drawing any inferences about player 1 and with both players receiving their minmax payoffs. The incomplete information in our model thus does not impose a lower bound on player 1's payoff, but raises the *possibility* that player 1 can achieve a higher payoff.

is uncertainty about player 1's type, each short-run player's signals provide information about player 1. Given the proposed behavior, the more likely is the firm to be normal, the more likely is high effort and hence the higher the price paid by the consumer. The firm may then find it optimal to exert high effort because doing so increases the consumer posterior that the firm is normal and hence leads to higher future prices. Unfortunately, this behavior contains the seeds of its own destruction. Eventually, the consumers' posteriors will come arbitrarily close to attaching probability one to the firm's being normal. At this point, further experience has virtually no effect on consumer posteriors and hence on prices. But then the firm has an irresistible incentive to deviate to low effort, unravelling the putative equilibrium. Increased patience might allow this unravelling to be postponed, but it cannot be avoided. Hence, even with the possibility that the firm is inept, there is a unique equilibrium in which the firm always exerts low effort.

The difficulty is that a firm who builds a reputation does too good a job of it. Eventually, almost all consumers become almost certain that the firm is normal, in the sense that the posterior probability attached to a normal firm gets arbitrarily close to one for an arbitrarily large subset of consumers. At some point, the current outcome will then have such a small effect on the current belief that the cost c of high effort overwhelms the very small difference in beliefs caused by a good rather than a bad outcome, and the normal firm will find it optimal to revert to low effort.

To obtain an equilibrium with consistent high effort, consumers' posteriors about the firm must be bounded away from certainty. Such a bound might appear for a number of reasons. For example, the consumers may have bounded memory, using only some finite number of their most recent observations in drawing inferences about the firm's type. Overwhelming evidence that the firm is normal could then never be amassed.

We adopt a different approach here, assuming that in every period there is some possibility that the firm is replaced by a new firm whose type is randomly drawn from a prior distribution over types. Consumers understand the possibility of such replacements, but cannot observe them. Intuitively, the possibility of changing types plays a role whenever one's response to a disappointing experience with a firm known for good outcomes is not simply "I've been unlucky" but also "I wonder if something has changed?" This again ensures that consumer can never be too certain about the firm, and hence that the firm always faces incentives to choose high effort.[53]

In introducing the prospect that a firm's characteristics or even identity are constantly subject to

[53]See Holmström (1982) for an early application of this idea, Cole, Dow, and English (1995) and Phelan (2005) for more recent examples, and Hörner (2002) for an alternative approach.

revision, we place an upper bound on the effective discount factors, no matter how large patient the firm happens to be. As a result, appealing to the limit as the discount factor gets arbitrarily close to one is no longer an effective way to create incentives. However, if the cost of effort c is sufficiently small, then we have a "high effort" equilibrium in which the normal firm always chooses high effort. Let λ be the probability that the firm is replaced in each period. We have:

Proposition 9 *Suppose $\lambda \in (0,1)$. If the prior probability that the firm is normal is not too close to 0 or 1, then there exists $\overline{c} > 0$ such that a high effort equilibrium exists for all $0 \leq c < \overline{c}$.*

We thus have the seemingly paradoxical result that it can be good news for the firm to have consumers constantly fearing that the firm might "go bad." The purpose of a reputation is to convince consumers that the firm is normal and hence will produce high quality. As we have seen, the problem with maintaining a reputation in the absence of replacements is that the firm essentially succeeds in convincing consumers it is normal. If replacements continually introduce the possibility that the firm has turned bad, then the firm can never do "too good" a job of convincing consumers it is normal. But then the incentive for the firm to continually reinforce consumer beliefs induces the firm to consistently choose high quality.

Ely, Fudenberg, and Levine (2002) and Ely and Välimäki (2002) present an alternative model in which a reputation builder's attempts to separate himself from a bad type has strikingly counterproductive effects, consigning the reputation building to his lowest possible payoff. In the current model, the action H that player 1 takes to separate from the inept type is an action that consumers value. Player 1 has a richer action space in the "bad reputation" games of Ely, Fudenberg, and Levine (2002) and Ely and Välimäki (2002), with the action that most effectively distinguishes the normal player from his inept counterpart being bad for consumers. In addition, consumers in the bad reputation model have the option of not purchasing from the firm, shutting off the information flow that is essential to building a reputation. As a result, the circumstances in which player 1 is most anxious to separate from the inept type are precisely those in which he cannot do so, because consumers anticipate an unfavorable action from the firm and hence refrain from purchasing, unleashing an unravelling that precludes any chance of building a reputation.

5.4.3 Markets for Reputations

If reputations can be sold, who buys them? This section considers this question, based on Mailath and Samuelson (2001).[54]

In each period, there is again a probability λ that a current firm leaves the market, to be replaced by a new entrant. Now, instead of drawing the type of this entrant from an exogenous distribution, we assume that that a collection of normal and inept potential entrants, each having drawn an opportunity cost of entering the market, bid for the right to replaced the existing firm. The replacement will be a normal firm if the net (of opportunity costs) value of a normal firm's participating in the market exceeds the corresponding difference for a inept firm. These values will depend upon the firm's current reputation. Let us assume that this can be represented by a a single posterior probability $\phi \in [0, 1]$ that the firm is normal (as is the case in Mailath and Samuelson (2001)). We are thus prompted to ask, for whet values of of ϕ does is a normal firm especially relatively profitable in the market? We have:

Proposition 10 *The difference between the continuation payoffs of a normal and inept firm are first increasing in ϕ and subsequently decreasing in ϕ.*

This result implies that replacements are more likely to be normal firms for intermediate values of ϕ and less likely to be normal firms for extreme values of ϕ. Hence, firms with low reputations are relatively likely to be replaced by inept firms. Normal firms find it too expensive to build up the reputation of such a name. On the other hand, firms with very good reputations are also relatively likely to be replaced by inept firms. These names are attractive to normal firms, who would prefer to inherit a good reputation to having to build up a reputation, and who would maintain the existing, good reputation. However, these names are even more attractive to inept entrants, who will enjoy the fruits of running down the existing high reputation (recall that if consumers believe that the firm is almost certainly normal, then bad outcomes do not change consumer beliefs by a large amount)

Replacements are more likely to be normal firms for intermediate reputations. These are attractive to normal firms because less expenditure is then required to build a reputation than is the case when the exiting firm has a low reputation. At the same time, these reputations are less attractive than higher reputations to inept entrants, because the intermediate reputation offers a smaller stock that can be profitably depleted. We can thus expect reputations to exhibit two features. Low reputations are likely to remain

[54] Tadelis Tadelis (1999, 2002, 2003) presents an argument that reputations will in general be traded when consumers cannot observe firm turnover.

low. Further up the market, there will be churning: high reputations will be depleted while intermediate reputations will be enhanced.

5.5 Group Reputations

Reputations often appear to be attached to groups rather than individuals. We may feel more comfortable trading with a business who belongs to the local chamber of commerce than one that does not. When travelling, we may be more apt to frequent hotels or restaurants with names we recognize, though we have no experience with them. Some New Yorkers prefer Mormons as nannies (Frank (1988, pp. 111–112)) while the diamond merchants of New York City capitalize on their common Jewish heritage (Richman (2005)). Membership in an organization can make a difference, even with someone with whom we will interact only once.

Tirole (1988) offers one model of collective reputations, based on moral hazard. A group may be able to monitor its members and sanction irresponsible behavior. The recognizable restaurant is a good bet because its parent company monitors its quality. The chamber of commerce is a useful seal of approval because it monitors business practices.

Frank (1988, Chapter 6) offers an alternative, based on adverse selection. People may come in different types, some of whom are more valuable in interactions than others, and those who are more valuable may also also be better suited to be members of a particular organization. The same tastes that make one a likely candidate to be Mormon may make one well suited to be a nanny. Again, we have the foundations for a group reputation.

A third alternative arises out of the statistical discrimination literature.[55] Here, otherwise irrelevant group markers endogenously acquire meaning, once again leading to collective reputations.

Each of these models captures a piece of the process by which groups acquire reputations. People spend great energy seeking to belong to the right groups—the right schools, clubs, political organizations, and social circles. Some of the value of such membership is contained in obvious benefits such as access to important people or the ability to "be in the right place at the right time." Some of the value appears to be more elusive, consisting of being known for being a member. The resulting reputation may reflect the forces identified in the preceding paragraphs, but may reflect more. This is again an important area for further work.

[55] See Cain (1986) for a survey of the original work and Mailath, Samuelson, and Shaked (2000) for a recent contribution.

6 Modelling Relationships

There are many technical questions still open in the study of relationships—results to be generalized, assumptions to be relaxed, proofs to be refined, necessary and sufficient conditions to be pushed closer together, and so on. However, the more challenging questions involving how we apply and interpret these techniques.

6.1 The Boundaries of Relationships

Though it is convenient to think of relationships as alternative devices for allocating resources, and to examine them in isolation, we can in practice expect relationships and contracts, prices and favors, markets and games to be complements rather than substitutes. How do they interact, and what are the implications? For example, given the ability to contract on some aspects of an interaction on not on others, how do the parties determine which arrangements to make specific and which to implicit? One might think that contracts would be used whenever possible, with relationships left to fill in the holes, but the interaction between the two makes it far from obvious that such an arrangement is optimal (see Baker, Gibbons, and Murphy (1994) and Schmidt and Schnitzer (1995) for examples). In addition, Ellickson (1991) argues that people widely shun the use of prices and markets, even ones that could work perfectly well, in order to rely on relationships. Rather than balance current exchanges by making transfers, people prefer to adjust continuation payoffs.[56] In somewhat more common terms, people prefer to trade and bank "favors" rather than use transfers to settle accounts.

As a result, the dividing line between prices and markets on the one hand and relationships on the other is not so obvious as the statement that relationships are used to allocate resources when markets fail. What are the advantages of relationships and what determines when people rely on them? Why is the response to being invited to dinner that "we owe them a dinner," rather then to estimate the appropriate transfer and make it at the end of the evening?

Some insightful steps along these lines have been taken by recent papers on trading favors (Abdulkadiroğlu and Bagwell (2005), Hauser and Hopenhayn (2002), and Möbius (2001)), each based on the presence of private information that stymies the use of conventional means of exchange. The next steps will require a clearer understanding of the advantages of relationships. Perhaps an offer to make a transfer rather than rely on future payoffs is taken as an indication that one does not plan to be around in

[56] For example, "Brennan declined this offer of compensation, ... because he would rather have Ellis in debt to him...." (Ellickson (1991, p. 56))

the future, to the detriment of the interaction. Perhaps transfers make information common that is currently privately held, again with adverse consequences. Perhaps we need to think carefully about which aspects of our interactions are private and which public. Overall, this is an important area for further work, with the stakes being an understanding of basic issues in the theory of resource allocation.

6.2 Designing Relationships

If relationships are important in allocating resources, then the theory of relationships should provide guidance as to how we might design relationships to do so more effectively. It is now taken for granted that economists can offer useful advice on how to design markets or structure contracts. What do we have to say about relationships?

This section offers one elementary example, taken from Andreoni and Samuelson (2005) (see Baker, Gibbons, and Murphy (2002) for an alternative approach). We begin with the observation that experiments with the prisoners' dilemma consistently find considerable heterogeneity in behavior. Some agents always defect while others cooperate, even in the one-shot prisoners' dilemma, though with the incidence of cooperation falling over the course of a finitely-repeated prisoners' dilemma (but not to zero).[57] Moving beyond the experimental laboratory and the prisoners' dilemma, we have the impression that people often act "cooperatively"—they donate to charity, they vote, they provide public goods, they come to the aid of those in need—in circumstances under which many economic models would suggest that they not do so.

How do we explain such behavior? One possibility is to posit that they are involved in a repeated game, and have settled on an equilibrium that calls for such behavior. This is the analogue of the equilibrium interpretation approach to reputations (cf. Section 5). We pursue here a model more closely related to the adverse selection approach to reputations, for much the same reasons, namely that such a model will more effectively identify links between the structure of the interaction and its equilibrium behavior. We accordingly posit that some people have preferences that lead them to sometimes prefer cooperation in the prisoners' dilemma. However, we stop well short of suggesting that everyone has such preferences, or that such preferences always lead to cooperation, and hence short of suggesting that such preferences complete our model of behavior. Instead, we are interested in how an appropriately designed relationship can leverage such preferences to enhance the extent of cooperation. The argument will provide an indication of why it might be effective to have a relationship "start small," beginning with

[57] See, for example, Andreoni and Miller (1993), Rabin (1993), Roth and Murnighan (1978) and their references.

	C	D
C	$3x_1, 3x_1$	$0, 4x_1$
D	$4x_1, 0$	x_1, x_1

Period one

	C	D
C	$3x_2, 3x_2$	$0, 4x_2$
D	$4x_2, 0$	x_2, x_2

Period two

Figure 11: *Stage games for the twice-played prisoners' dilemma, where* $x_1, x_2 \geq 0$.

relatively small stakes and building up to more important interactions.[58]

We consider two-period games whose stage games are the prisoners' dilemmas shown in Figure 11. Let

$$\lambda = \frac{x_2}{x_1 + x_2}.$$

We consider a class of such twice-played prisoners' dilemma games in which $x_1 + x_2$ is fixed, but λ ranges from zero to one. When $\lambda = 0$, all of the payoffs are concentrated in the first of the two prisoners' dilemmas. As λ increases, the second period becomes relatively more important, with $\lambda = \frac{1}{2}$ corresponding to equal payoffs in the two periods and $\lambda = 1$ corresponding to all payoffs being concentrated in the second period.

We study a model in which (*i*) players prefer that their opponents cooperate in the prisoners' dilemma, (*ii*) players sometimes prefer to cooperate themselves, (*iii*) players are more likely to cooperate when their opponent is more likely to cooperate, and (*iv*) players differ in the strength of this taste for cooperation. While the various models of preferences for cooperation that have been offered in the literature (e.g., Bolton and Ockenfels (2000), Falk and Fischbacher (2001), Fehr and Schmidt (1999), Levine (1998), and Rabin (1993)) differ in many details, these features provide a concise summary of their common ground.[59]

We can then solve for the equilibrium of the two-period game of incomplete information. With the help of some additional (primarily technical) structure, the equilibrium has the following features:

[58] Binmore, Proulx, Samuelson, and Swierzbinski (1998) present experimental results in which players are more likely to trust a randomly chosen opponent if they must first risk relatively small amounts to do so, than if the high-stakes trust opportunities come first. Theoretical models in which relationships optimally start small are examined by Diamond (1989) and Watson (1999, 2002).

[59] For example, Rabin (1993), citing evidence from psychology for these assumptions, designed a model of fairness to capture them. The one-period version of our model will have many predictions in common with Rabin's, but without specifying the intentions or motives of our players.

- Cooperation will be more prevalent in the first than in the second period of play.
- First-period play for $\lambda = 0$ will match second-period play for $\lambda = 1$.
- The incidence of first-period cooperation increases as λ does.
- Certain outcomes of the game become more likely, and others less likely, as λ grows. For example, when λ is small, we predict that an outcome of mutual cooperation in the first period should be followed by mutual cooperation in the second. However, as λ increases above a threshold, the incidence of mutual cooperation followed by one defection (denoted by CC,DC) increases, and the incidence of mutual cooperation followed by mutual defection (CC,DD) becomes positive, but has an ambiguous comparative static in λ.

The behavioral patterns outlined in the previous point give rise to conflicting effects on payoffs that, with some functional-form assumptions, combine to produce what we regard as an intuitive effect:

- The expected monetary payoff from the two-period game initially increases in λ, achieves a interior maximum at a value of $\lambda > \frac{1}{2}$, and then decreases.

Cooperation in the first period, by enhancing an opponent's estimate of one's unobserved taste for cooperation, leads to more opponent cooperation in the second period. This enhances the value of first-period cooperation. As a result, the model shares the common prediction that players are more likely to cooperate at the beginning of a sequence of prisoners' dilemmas. The model becomes more interesting when we consider the effects of varying the relative payoffs between the two periods. First, one of the two periods is trivial whenever $\lambda = 0$ or $\lambda = 1$, suggesting that we should observe identical behavior and payoffs from the nontrivial period in each case. More importantly, second-period cooperation is more valuable the higher is λ. As a result, higher values of λ induce agents to cooperate more in the first period as an investment in second-period cooperation, as well as inducing a number of more specific behavioral shifts, including those described in the fourth point above. Finally, as λ increases, we trade off increased first-period cooperation for decreased first-period payoffs, as payoffs are shifted to the second period. The combined effects suggest that monetary payoffs will be minimized when $\lambda = 0$ or $\lambda = 1$, and will achieve an interior maximum.

We present two figures containing partial results of an experimental implementation of the model. Figure 12 shows that the incidence of cooperation indeed increases is λ increases, shifting the stakes to the second period. Figure 13 provides results for some the paths of play CC,DC and CC,DD. Both

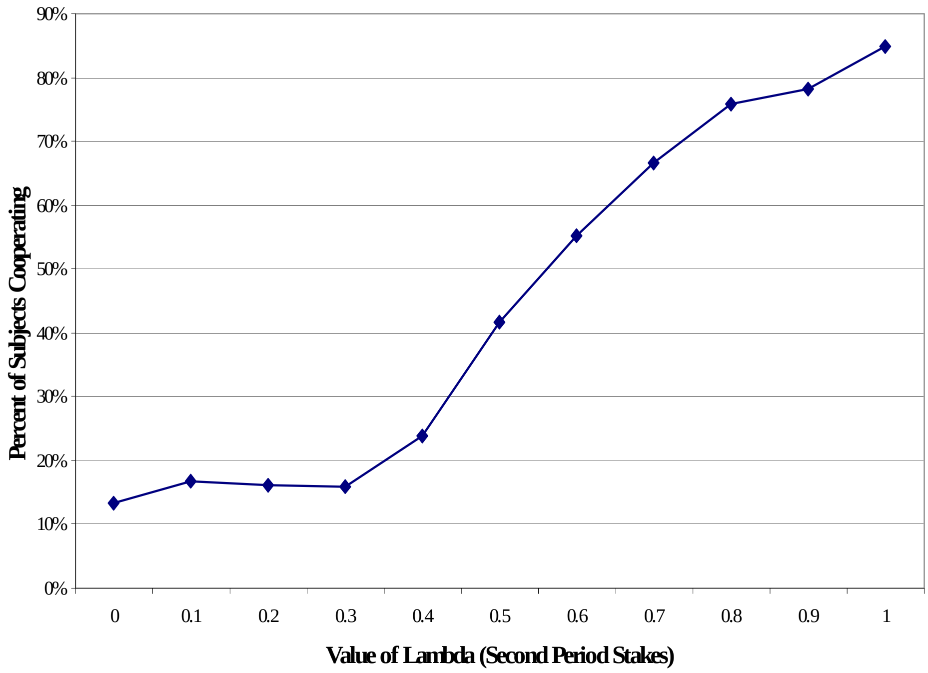

Figure 12: Percentage of experimental subjects cooperating in the first period of the two-period prisoners' dilemma, as a function of λ.

figures report expected patterns of play.

How does these results relate to the study of reputations? First, we have some insight into modelling. An alternative implementation of the adverse-selection approach to reputations would stick closer to the models of Kreps, Milgrom, Roberts, and Wilson (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982) by assuming that most players are rational and have defection as a dominant strategy, but that there is some possibility that one's opponent is an "irrational" or "altruistic" type who sometimes cooperates.[60] Our analysis can be viewed as an extension of such models. A standard "gang-of-four" model would include two types of agents, committed defectors (rational types) and "irrational" types who play a strategy such as TIT-FOR-TAT, while we have a continuum of irrational types, differing in their taste for cooperation. What do we gain by such an extensions? In a two-period version of the gang-of-four model, there are two possibilities. For small values of λ, the only sequential equilibrium calls for every rational agent to defect at every opportunity, with variations in λ or the period having no

[60] See Andreoni and Miller (1993), Camerer, Ho, and Chong (2002), Camerer and Weigelt (1988), and McKelvey and Palfrey (1992) for experimental studies of such models.

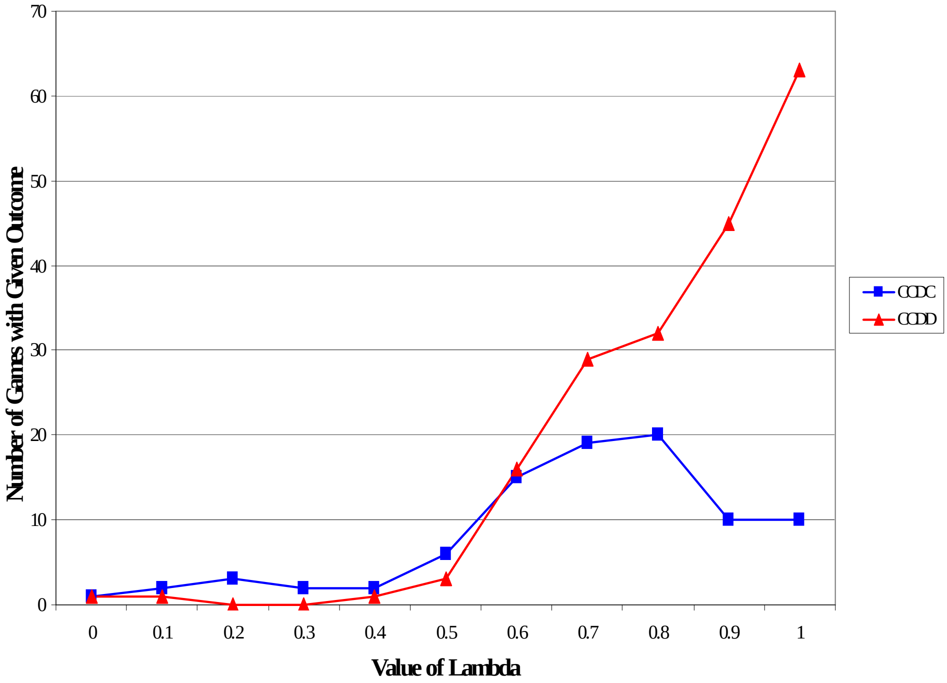

Figure 13: *Frequency of outcomes* (CC, DD)*, and* (CC, DC) *as a function of* λ*. The model predicts the incidence of* (CC, DD) *and* (CC, DC) *will be approximately zero over a common range of relatively small values of* λ*, above which* (CC, DD) *is increasing and* (CC, DC) *is positive but with an ambiguous comparative static.*

effect on behavior. For larger values of λ, rational agents will all cooperate in the first period and defect in the second period, with variations in λ again having no effect.[61] Hence, we should either see universal defection in the first period or universal defection in the second, with the probability of cooperation in the other period in each case equaling the prior probability of a TIT-FOR-TAT agent. These differences remind us that much can depend upon the "types" that appear in adverse-selection reputation models, emphasizing again that this is a feature of our models that warrants more attention.

Second, in our experimental results, the total payoffs from the interaction are maximized when the second-period stakes are one-and-a-half to two times as large as those of the first period. Results such as these can form the beginnings of an understanding of how relationships might be designed to more effectively allocate resources.

[61] To see this, note that rational agents must defect in the final period of the model. A rational agent can optimally cooperate in the first period of play only in order to induce TIT-FOR-TAT opponents to also cooperate in the final period. This will be optimal only if λ is sufficiently large (relative to the prior probability of a TIT-FOR-TAT opponent).

6.3 Equilibrium

If we have learned anything about repeated games, it is that they have lots of equilibria. Which ones should command our attention? Behind this simple question lurks a collection of more difficult ones, culminating in questions about how we interpret an equilibrium and even how we interpret a repeated game.

6.3.1 Renegotiation

It is common to be especially interested in equilibria that are efficient in the repeated game. The reasoning appears to proceed as follows. "An equilibrium of the repeated game involving nontrivial intertemporal incentives requires coordination on the part of the players. Something must lie behind this coordination, perhaps some explicit process in which the players could communicate and agree on an equilibrium. But then it seems as if this process should direct the players to an efficient equilibrium. Why would the players settle for an inefficient equilibrium when a better one is available?" Alternatively, the selection of an equilibrium with nontrivial intertemporal incentives is often viewed as something that the players do actively, whereas simply repeating the Nash equilibrium of the stage game in each period is viewed as something more passive. This view implicitly appears when we speak of firms as "colluding" when setting prices above those of the stage-game Nash equilibrium, with no such word required for repetition of the stage-game equilibrium. Why would the players actively strive for anything less than efficiency? This view suggests that at least any equilibrium of interest that features nontrivial intertemporal incentives should be efficient.

These arguments lead to two puzzles. First, if there is some process that directs attention to efficient equilibria, why does it occur just once, at the beginning of the game? Why doesn't the same process come into play every time a new period appears, and hence a new subgame with a new continuation equilibrium?

Pursuing this question leads to the idea of a renegotiation-proof equilibrium, in which the players are assumed to never settle for an inferior continuation equilibrium. A body of work has grown around this idea.[62] Interestingly, there is also a large body of research in economics centered on the premise

[62] It is not obvious how one should make this criterion precise. One might first think of rejecting an equilibrium σ if, after any history of play, it prescribes continuation play that is strictly dominated by some other equilibrium σ'. However, it is then natural to limit the set of possible blocking equilibria σ' to those that are not similarly blocked, building a self-reference into the definition that opens the door can lead to all sorts of difficulties. The result has been a large literature, with some useful examples being Abreu, Pearce, and Stacchetti (1993), Asheim (1991), Baliga and Evans (2000), Benoit and Krishna (1993),

that economic systems do *not* always yield efficient outcomes, to the extent that events such as bank failures (Diamond and Dybvig (1983)), discrimination (Coate and Loury (1993)) or economic depressions (Cooper and John (1988)) are explained as events in which players have coordinated on an inefficient equilibrium. One cannot help but be struck by these contrasting views of efficiency.

Second, it is not clear what to make of the view that an equilibrium with nontrivial intertemporal incentives somehow involves more active coordination than does the repetition of a stage-game Nash equilibrium. In all but the simplest of games, the assertion that *any* equilibrium is played involves a belief that the players can somehow coordinate their behavior. Even in games with unique Nash equilibria, it is not clear that we can obviously expect equilibrium play. Simply asserting the players are rational, and that this rationality is common knowledge, does not suffice for equilibrium play. The standard story in game theory is now an evolutionary one, that either the players' own history or a collective history that they share with others brings them to equilibrium.[63] We can apply similar ideas to repeated games, but there appears to be no reason to believe that the outcome will be efficient, nor that equilibria with nontrivial intertemporal incentives will be treated differently than those without.[64] People live lives full of intertemporal tradeoffs. Why should we expect the default position to be that they ignore these tradeoffs in a repeated game, at any point, other than for the fact that stage games came first in the development of game theory?

What can we conclude? Our theory of repeated games and relationships needs a more carefully considered account of what we are doing when we select an equilibrium, addressing both the question of which equilibrium might be selected and why we might expect an equilibrium at all. Harping on a recurring theme, this equilibrium selection is properly viewed as part of the modelling process. We need methods for dealing with this part of the modelling that will move us beyond our current "I know it when I see it."

6.3.2 Punishments

The question of how players coordinate on an equilibrium reappears when thinking about how to use punishments in the construction of equilibria. It is common to assume that deviations from equilibrium

Bernheim and Ray (1989), Evans and Maskin (1989), Farrell and Maskin (1989), and Wen (1996).

[63]Kalai and Lehrer (1993) provide one perspective on this process, while the large bodies of work on learning (Fudenberg and Levine (1998)) and evolutionary games (Samuelson (1997), Weibull (1995)) provide others.

[64]For two evolutionary models that do show a penchant for efficiency in repeated games, see Binmore and Samuelson (1992) and Fudenberg and Maskin (1990, 1993).

play trigger the subsequent play of a stage-game Nash equilibrium. There are often more severe punishments available, and the use of such punishments could make a difference.[65] Why work with Nash reversion?

In some applications, nothing is lost by focussing on Nash reversion. In the prisoners' dilemma and the product choice game, the (unique) stage-game Nash equilibrium gives each player the minimum level consistent with individual rationality, ensuring that there are no more severe punishments. In other cases, Nash reversion is adopted as an analytical convenience. It frees us from having to worry about incentives off the equilibrium path. As long as one ensures that the results do not depend upon the restrictions inherent in this analytical convenience, the restriction raises no difficulties. However, the preference for Nash reversion is sometimes fuelled by the view that it may be reasonable to think of players as coordinating on repeated-game equilibria that bring them high payoffs, but absurd to think of them as actively coordinating on low payoffs.

The foundations of this last view are less apparent. The industrial organization literature contains no lack of models and stories about price wars. What is a price war other than a coordinated punishment, looking very much like the combination of a temporary penalty coupled with a return to higher payoffs that characterizes the work of Abreu (1986) or Fudenberg and Maskin (1986)? In this setting, actively coordinated punishments look quite reasonable. In paraticular, it is not at all clear that a price war requires more coordination or is less "natural" than Nash reversion. If one is to advance the view that "active coordination," whatever the phrase might mean, is more reasonable in some circumstances than others, the argument must be more nuanced than simply the claim that such behavior is unlikely to characterize punishments. Again, we return to the need for better insight into how we view the equilibria of the repeated games with which we work.

6.3.3 Markov Equilibrium

It is common, especially in applications, to restrict attention to Markov equilibria. The most common characterization of Markov equilibrium is that it restricts attention to only 'payoff relevant" information about the history of play. The motivation is that the players in the game should find some types of information more salient than others, and the term payoff relevance is designed both to identify the

[65]For example, these more severe punishments can allow a particular outcome to be supported as equilibrium behavior for a broader range of discount factors, an idea illustrated in a familiar setting by Abreu (1986). They can also expand the set of equilibrium payoffs, including efficient equilibrium payoffs, for a given discount factor or in the limiting case as the discount factor approaches one, as Fudenberg and Maskin (1986) make clear.

	In	*Out*
In	$-1,-1$	$1,0$
Out	$0,1$	$0,0$

Figure 14: Entry game.

salient information and provide the motivation for its being salient.

Markov equilibria are also sometimes motivated as being simple, an argument somewhat in tension with the observation that a precise specification of "payoff relevant" (see Maskin and Tirole (2001)) is neither obvious nor simple. A second source of tension arises out of the fact that, in repeated games (i.e., games involving precisely the same stage game in every period, as opposed to dynamic games, where the stage game evolves over the course of play), Markov equilibria must feature a stage-game Nash equilibrium in each period.[66] For example, the only Markov equilibrium of the repeated prisoners' dilemma features defection in every period. This clashes with the view that equilibria featuring nontrivial intertemporal incentives, perhaps allowing cooperation in the prisoners' dilemma, are often of interest in such games.

The first step on the road to Markov equilibrium seems straightforward enough. The players in a repeated game, as in any other game, are surrounded by all sorts of information. Their histories of play already contain a wealth of information. If we take seriously that the model is a tool for examining real strategic interactions, then we must recognize that the environment in which the interaction takes place contains another wealth of information that we have excluded from the model. It also seems quite plausible that the players cannot make use of all of the available information, so that some criterion for which information is to be salient must play a role, thought it is not yet clear that the criterion is likely to be payoff relevance.

To go further, observe that we conventionally do *not* restrict players to conditioning their behavior only on information that is contained in the model. To focus on this point, put repeated games aside and consider a one-shot interaction, the game shown in Figure 14. Interpret this as a game in which two firms must both decide whether to enter a market that is large enough to hold just one of them. The game is symmetric, with nothing to distinguish the two players. A theory of equilibrium selection that strictly confines itself to information contained within the game must respect this symmetry, choosing game's unique symmetric equilibrium, in which each firm mixes between entering the market and staying

[66]Markov equilibria allow more latitude in dynamic games.

	L	C	R
T	$4,4$	$0,5$	$0,5$
M	$5,0$	$3,3$	$0,0$
B	$5,0$	$0,0$	$1,1$

Figure 15:

out (e.g., Harsanyi and Selten (1988)). However, many analysts would choose an asymmetric pure-strategy equilibrium, in which one firm enters and the other does not, when working with the game. The interpretation of the asymmetry in the equilibrium would be that it reflects some asymmetry present in the actual interaction—one firm got there first, or was larger, or had lower costs—that can show up in the model only in the form of an equilibrium selection that it not grounded in any information contained in the model.

More generally, we return to the point that the construction of the game and the selection of an equilibrium jointly comprise our model of the strategic interaction. But there is then no particular reason to expect the equilibrium to respect informational criteria that can be defined only in terms of the game.

Now let us more closer to repeated games by considering the game shown in Figure 15. Suppose this game is played once. There are two pure Nash equilibria, (M,C) and (B,L). Though some might have a preference for the efficient equilibrium, it would be common to admit either as equilibrium candidates, with the relevant one depending upon details of the context in which the game is played. For example, the way the game has been played in the past might be cited as a particularly relevant such detail.

Now suppose that the game is played twice, with the players maximizing the sum of their payoffs over the two interactions. An application of Maskin and Tirole's (2001) payoff relevance concludes that nothing about first-period play is payoff relevant, and hence that second-period play must be independent of the first-period history. This precludes the efficient equilibrium in which (T,L) is played in the first period followed by (M,C) (with first-period deviations prompting play of (B,R)). But if either (M,C) or (B,R) are considered reasonable equilibria when the game is played once, with the choice between the two resting upon environmental details not captured by the game, why cannot first-period play serve the same function when reaching the second period of the repeated game?

This suggests that in the course of thinking about equilibria in repeated games, we need an account of which aspects of their environments and history the players come to think of as relevant and which they ignore. One of the difficulties here is that which information is relevant is itself likely to be an equilibrium phenomenon, raising the specter of an infinite regress. One possibility is an evolutionary approach. Some quite limited beginnings along these lines, in one-shot games, are taken by Binmore and Samuelson (2000, 2001, 2005).

6.3.4 Complexity

One cannot work with repeated games without coming to the view that some strategies and some equilibria are more complex than others. In addition, it appears to be quite compelling that simple strategies should have their attractions. Identifying precisely what one means by simple is somewhat more challenging.

An early formulation of complexity in repeated games captured the idea that strategies should not contain contingencies that are never used (Abreu and Rubinstein (1988), Rubinstein (1986)). Subsequent work pursued the notion of complexity in a number of directions, showing in the process that the results could be quite sensitive to precisely how one models complexity.

This work took the view that the complexity of a strategy was properly assessed within the confines of the game in which the strategy is played. For example, an argument about complexity might proceed as follows. Consider the strategy grim trigger in the prisoners' dilemma. It includes the specification that the player defect in response to previous defection. In equilibrium this capability is never used. Hence, the strategy can be simplified by deleting this capability, with no effect on how the game is played, and players who are concerned about complexity should surely do so. Once the ability to punish has been deleted, however, we no longer have an equilibrium. Complexity considerations thus suggest that that cooperation in the repeated prisoners' dilemma is suspect.

There are several ways of responding to this point while keeping the analysis within the confines of a single game.[67] However, let us return to an observation made in Section 1.3, that an ability to monitor interactions and punish transgressions may be part of our evolutionary heritage (Cosmides and Tooby (1992a,b)). If evolution has equipped us with a propensity to monitor and punish, then perhaps we should not remain within the confines of the game. Instead, we might think of players as having a "punishment" phase in their thinking, flexible enough that it can be applied whenever needed, and that survives precisely because it is sometimes needed (perhaps in games where the discount factors and

[67] For example, we can ensure that all possibilities are on the equilibrium path, and hence that the equilibrium is not threatened by "free" simplifications, by working with imperfect public monitoring. Alternatively, we could look for a strategy that cooperates most of the time, but incorporates defections into its equilibrium play that could provide tools for punishing that are not completely redundant and hence are not candidates for deletion.

incentives are such that punishment is the only possible equilibrium outcome). If so, then attaching a punishment capability to a strategy should not be viewed as making the strategy more complex. This in turn suggests that viewing the repeated play of a stage-game Nash equilibria as being particularly simple may be misleading. Cooperating in the prisoners' dilemma, with deviations punished, may be effectively just as simple as always defecting.

6.3.5 Modelling Games

How do we pull these ideas together? Once clear theme is that we need to think of the structure of the repeated game and the accompanying equilibrium as jointly determined as part of the model of the strategic interaction. Second, in constructing this model, we must ask ourselves how the agents themselves model the interaction. What information do they view as relevant, and what relevant information do they ignore? Do they view it in isolation, or as part of a larger interaction? Do they view it as repeated, or do they ignore the prospect of the future? For the same reasons that economists work with models to examine a reality that is too complicated to study in full detail, so should we expect the players in the game to rely on models. There is then no reason to expect their view of "payoff relevant" or "simple" or "requiring coordination," or even of the boundary of the model, to match ours.

We thus need a theory of relationships that begins with a model of how the agents perceive their environment. For example, Jehiel (2005) examines a model in which agents fail to distinguish histories that a conventional model would necessarily identify as distinct, including histories of different lengths. The agents thus effectively play a different extensive form that than posited by the modeler. Samuelson (2001) examines a model in which evolution determines such features as whether agents treat distinct games differently or as being identical.

The difficulty is that the modelling choices embedded in such an analysis inevitably seem arbitrary, while often themselves being an equilibrium phenomenon that simply pushes the problems back one step. My inclination is to think that an evolutionary approach provides the best hope for making progress in the fact of such obstacles. The result would not be another exercise in evolutionary game theory, with the players' behavior adjusting shaped by an adaptive process, but would be a study of how the actual process of evolution has shaped the way people analyze their strategic interactions. Such an approach faces great challenges, but also promises great rewards in our understanding of relationships.

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