

An Evolutionary Approach Towards Time Preferences*

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Abstract

A gene is interpreted as a choice behavior in an environment where choices have intertemporal effects. A gene survives the evolutionary process if the population corresponding to the gene grows at the largest possible rate. Our goal is to represent the choice behavior of the surviving genes by preference relations. We show that if choices only affect the number of offspring, but not the reproductive ability of descendants, this representation has the following attributes. It is always time-consistent, the discount factor is the inverse of the population growth factor, and the felicity utility function is the reproduction function.

We also show that if newborn offspring are heterogeneous, for example because of altruistic acts, then the utility representation is more subtle. The discount factor is still the inverse of the population growth factor, but the felicity utility function is essentially the sum of the discounted reproductive values of those descendants who are affected by the choices.

Key words: evolution, time preferences.

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1 Introduction

Most models in economics take preferences as given and derive choices induced by these preferences. In this paper we do just the opposite. We attempt to characterize those choice behaviors that could possibly survive the evolutionary process. Then we represent these choice behaviors with preference relations. That is, we identify preferences that would induce the same choice behaviors that could have survived evolution.

We interpret a gene as a choice behavior. Hence, the choices an individual makes along her life time is determined solely by her gene. Since children inherit their genes from their parents, they make the same choices as their parents would have if they were in the same situation. A population is defined as a group of individuals having the same genes. Populations with different genes potentially grow at different rates. The premise of this paper is that only those genes can survive the evolutionary process that induce the highest possible population growth rate given the physical environment.

Our focus is on time preferences. Hence, in our model individuals face intertemporal trade-offs: their current choices potentially affect their lives in the future. In particular, current choices might not only affect current reproductivity, but also reproductivity in the future. We show that, in general, the behaviors of those genes that survive evolution are impossible to characterize. However, they all have very simple utility–representations. The main result of the paper is that the surviving gene has a time–consistent utility representation where the discount factor is the inverse of the population growth factor and the felicity utility function is the reproduction function. Our approach and some of our results can be best illustrated with the following simple example.

Example. Suppose that an individual lives for two periods. She has one unit of endowment and can transform it into offspring according to a concave, and increasing function f . A newborn offspring is facing the same environment as her predecessor. A gene is defined as a decision rule $x \in [0, 1]$, the use of endowment in the first period. (The use of endowment in the second period is $1 - x$.) Since offspring have the same gene as their predecessor, they are using the same rule.

Fix a gene, x , and denote the number of individuals who is one-year-old at time t by y_t . The measure of two-year-old individuals at time t is y_{t-1} . The following equation recursively defines the law of motion of y_t :

$$y_{t+1} = f(x) y_t + f(1 - x) y_{t-1}.$$

Dividing both sides by y_t yields

$$\frac{y_{t+1}}{y_t} = f(x) + f(1-x) \frac{y_{t-1}}{y_t}.$$

It can be shown that $y_{t+1}/y_t = g$ asymptotically. That is, no matter what the initial proportions of one-year-old and two-year-old individuals in the population are, the gene x determines a constant asymptotic growth factor g . Therefore, the previous equation can be rewritten as

$$g^2 = gf(x) + f(1-x).$$

Let $g(x)$ denote the positive solution of the previous equation for g . Then

$$g^2(x) - f(x)g(x) - f(1-x) \equiv 0 \text{ for all } x \in [0, 1].$$

After differentiating this equation:

$$2g(x)g'(x) - f(x)g'(x) - f'(x)g(x) + f'(1-x) = 0.$$

Let x^* denote the gene generating the largest possible growth rate and $g^* = g(x^*)$. Then $g'(x^*) = 0$, and the previous equation becomes

$$f'(x^*) = f'(1-x^*)/g^*.$$

This equation implies that the optimal decision x^* is the solution for the following maximization problem¹:

$$\max_x f(x) + \frac{f(1-x)}{g^*}. \tag{1}$$

Equation (1) means that when an individual makes choices, she behaves as if she would maximize the expected discounted present value of her offspring, when the discounting is done according to the inverse of the population growth factor. The intuition behind this observation is the following. Suppose that the expected number of descendents from an individual t periods later is z_t . If t is large, then z_{t+1} is approximately gz_t . Hence the value of an offspring today, measured as her asymptotic contribution to the gene pool, is g times as high as that of an offspring tomorrow. This explains why offspring born a period later are discounted by g . In order to complete the intuition, one must argue that an individual of a population corresponding to the surviving gene indeed should maximize

¹Of course, this example is too simple to talk about utility representation because the domain of choices is not large enough.

her asymptotic contribution to the gene pool. The reason why this is non-trivial is because she must take into account that if she changes her behavior, each of her descendants will do the same.

The result described in the above example is substantially generalized in the paper. In particular, the reproduction function as well as the choice set of an individual are allowed to depend on previous decisions, the age of the individual, and random variables.

Notice that in the example offspring are homogenous. The decision of an individual affects only her number of offspring, but has no impact on the *lives* of her offspring. We consider a model where offspring are heterogenous and individuals' choices might affect the reproductivity of their descendants. For example, an individual might save some of her resources and donate it to her offspring. Offspring with *rich* parents are likely to be more reproductive than offspring with *poor* parents. In the presence of such altruistic acts the utility-representation of surviving genes is more subtle. The reason is that the *reproductive value* of an individual is a more complicated object than the discounted present value of the expected number of offspring. We show that the surviving gene still has a time-consistent utility representation. The discount factor is again the inverse of the population growth factor. However, the felicity utility function at age t is roughly the sum of the reproductive values of all descendants affected by the choices made by the individual at age t .

The premise of the paper, according to which only those populations survive evolution that generate the highest possible growth factor, deserves some explanation. The implicit assumption justifying this premise is that populations compete for a limited amount of resources. The larger is a population the larger its share of total resources. If a population grows slower than other ones, then its share of resources tends to zero, and will eventually die out. We formalize this argument by introducing *carrying capacity constraint* into our basic model. This means that per capita reproduction decreases in total population, and converges to zero as the total population converges to infinity. Such a constraint then drives down the long run growth factor to be one. Nonetheless the choice behavior surviving evolution still maximizes the growth factor. In this case, our model still predicts that the felicity utilities are identical to the reproduction functions, but the discount factor is forced to be one. Hence, time preferences are merely determined by the variations in fertility.

Literature Review

The idea that preferences can be, at least partially, explained by evolution is present

in numerous papers. The first one was probably Becker (1976). The author uses an evolutionary argument to explain altruistic behavior. Most of the papers on the evolution of preferences focused on attitude towards risk, altruism, and information processing. Excellent overviews of the theories on the relationship between biology and economic behavior can be found in Robson (2001 and 2002). Below we review only some of those papers which are concerned with the evolution of time preferences.

The paper closest related to ours is Rogers (1994). As a matter of fact the author has the exact same goal as us: to explain time preferences by natural selection. The author's objective is to characterize indifference curve maps shaped by evolution. Rogers (1994) does not set up a rigorous mathematical model, his arguments are inconsistent, and many of his formulas are incorrect. Most importantly, we shall show that given the framework of Rogers (1994), it is not possible to identify indifference curves representing choice behaviors. In order to make these statements precise, we introduce a model in Section 4, which can be viewed as a version of the one in Rogers (1994). At the end of this section we provide an excessive discussion of Rogers (1994), and compare our analysis with his one.

Hansson and Stuart (1990) consider a neoclassical growth model, where *clans* compete for shares of resources. The production of a clan depends on the stock of clan-specific capital and on the total population of all clans. The clans face a carrying capacity constraint, meaning that per capita production decreases in total population. An individual is active only for one period, and the behavior of a clan is described by a consumption-saving decision. Higher consumption results a higher immediate population growth, but a smaller capital stock, and hence output, for the next generation. Clans must grow at a rate zero in the long-run equilibrium, because of the carrying capacity constraint. As a result, evolution selects clans with zero rate of time preferences. That is, agents behave as if they maximize the sum of per capita felicities of current and future generations.

Robson and Wooders (1997) also derives zero rate of type preferences in a growth model where total output depends on capital and labor. Furthermore, both labor and capital can be of many types. The growth rate of labor of a certain type is determined by its per capita income. The authors show that when the balanced growth rate is maximized, the income must be distributed across individuals in accordance with marginal product pricing.

Although, both Hansson and Stuart (1990) and Robson and Wooders (1997) analyze intertemporal trade-offs based in evolutionary models, the choice problems investigated in

these papers are not rich enough to establish a tight relationship between utility functions and fertilities. As we mentioned earlier, we also introduce carrying capacity constraint into our model and derive that the discount factor is one in the long run equilibrium. Notice however that as long as the felicity utilities are age-dependent, a discount factor of one does not imply zero rate of time preferences. To see this, compare the following two cases. In case one, the reproduction function, f , does not vary with age and the growth factor is $g > 1$. In case two, the population grows at a rate zero, but the fertility is exponentially declining in age, say $f_t = f/g^t$ where f_t is the reproduction function at age t . The preferences in both cases are identical, although in case one the discounting is due to the population growth rate, while in case 2 it is due to the declining fertility.

There are two recent papers on evolutionary economics, in which the conclusion is that time-inconsistent preferences might arise. Samuelson and Swinkels (2005) assume that agents are unable to process information perfectly. In particular, agents have to make decisions equipped with incorrect priors. As a result, time-inconsistent preferences can survive evolution. The reason is that time-inconsistent preferences compensate for faulty information processing.

Maskin and Dasgupta (2005) argue that during the evolutionary process individuals could have faced the following type of choice problems. There are two random options A and B . At time zero, the expected return of option A is larger than that of option B . However, after some time, if A still has not yielded fruit, option B becomes more attractive. As a result, an individual rationally chooses option A at time zero, and switches to option B after some time. Although this behavior is dynamically consistent, it is seemingly time-inconsistent. The authors argue that in modern times, individuals face *atypical* situations (which they did not face during the evolutionary process) and behave in a time-inconsistent manner inherited from the optimal behavior developed during evolution. That is, Maskin and Dasgupta (2005) take the view that although individuals' behavior are shaped optimally by the evolution, they behave suboptimally in the civilized society. Unlike these papers, we assume that neither the information structure, nor the choice problems of the individuals change over time.

2 The Model

Time is discrete and an individual lives for T periods. In every period an individual has to make a choice from a subset of \mathbb{R} . The choice set is determined by the history of

previous decisions and some stochastic variables. Let $\alpha_t (\in A_t \subset \mathbb{R}^{k_t})$ denote the vector determining the choice set of an individual and c_t her choice at age t . For example, α_t can be $(c_1, \dots, c_{t-1}, s_1, \dots, s_t)$ where s_i is a realization of some random variable at time i . The random variables can be correlated and can be also influenced by previous choices. Let $[0, b_t(\alpha_t)]$ be the set from which an individual has to make a choice at age t . One can think of b_t as the resources available for the individual at age t . The expected number of an individual's offspring is determined by her choices and other random variables. More precisely, the expected number of offspring at age t is $f_t(c_t, \beta_t)$, where $\beta_t (\in B_t \subset \mathbb{R}^{m_t})$ denotes the vector of variables determining fertility at age t . For example, β_t can be $(c_1, \dots, c_{t-1}, s'_1, \dots, s'_t)$ where s'_i is a realization of some random variable at time i . The random variables can be again correlated and influenced by previous decisions. We also allow that s_i and s'_i are realizations of correlated, possibly the same, random variables. It turns out to be useful to distinguish between those variables that influence only choice sets, from those which influence fertility.

A gene is defined as a collection of choices $\{c_t\}_{t=1}^T$, where $c_t : A_t \times B_t \rightarrow \mathbb{R}$, such that $c_t(\alpha_t, \beta_t) \in [0, b_t(\alpha_t)]$ for all $\alpha_t \in A_t$. That is, if an individual with a gene $\{c_t\}_{t=1}^T$ faces a choice set S at age t , and the realization of the random variable is (α, β) , she chooses $c_t(\alpha, \beta) \in S$. Since offspring have the same gene as their predecessors, they make the same choices.

Let C_t denote $A_t \times B_t$, and $\gamma_t = (\alpha_t, \beta_t)$. Let $G_t(\cdot, \gamma_{t-1}, c_{t-1})$ denote the distribution function of γ_t , and $g_t(\cdot, \gamma_{t-1}, c_{t-1})$ the corresponding density.²

Assumptions. — The functions $\{f_t\}_1^T$ are totally differentiable. The function f_t is strictly increasing in c_t and strictly concave for all $t \in \{1, \dots, T\}$. Furthermore, $\partial f_t / \partial c_t|_{c_t=0} = \infty$. Finally, $f_t(0, \beta_t) = 0$, and if $c_t(\alpha_t, \beta_t) = b_t(\alpha_t)$ then $b_{t+1}(\alpha_{t+1}) = 0$.

Most of these assumption are made for technical convenience. More precisely, we shall use the first-order approach in analysis. The assumptions above guarantee that this approach is valid, and the solution is interior. The only assumption which has a substantive consequence is the concavity of the reproduction functions. This *decreasing-marginal-return* assumption results a form of *patience*.

Our objective is to analyze the genes inducing the highest population growth rate. More precisely, assume that at time zero there is a measure of $\lambda_t (\in (0, 1))$ individuals at age t , all of them having the same gene. Furthermore, in order to apply the Law of

²Notice that this notation does not imply that the distribution of ϖ_t is not influenced by events occurred more than one period before t . This is because those events can be summarized in ϖ_{t-1} .

Large Numbers, we assume that the random variables described above are independent across individuals. This means that although individuals are facing uncertainty, the population is not affected by aggregate shocks. We shall show that the population growth rate corresponding to a gene is constant asymptotically and does not depend on the initial conditions, $\{\lambda_t\}_{t=1}^T$. Therefore, our objective to search for the gene generating the largest growth rate is well-defined.

Population growth. — Fix a gene $\{c_t\}_1^T$. The expected number of offspring of a one-year-old individual is $a_1 = \int f_1(c_1(\gamma_1), \beta_1) dG_1(\gamma_1)$. The age zero expected number of offspring of a two-years-old is $a_2 = \int \int f_2(c_2(\gamma_2), \beta_2) G_2(d\gamma_2, \gamma_1, c_1(\gamma_1)) dG_1(\gamma_1)$. In general, the expected number of offspring of a t -year-old individual is

$$a_t = \int \dots \int f_t(c_t(\gamma_t), \beta_t) G(d\gamma_t, \gamma_{t-1}, c_{t-1}(\gamma_{t-1})) \dots G(\gamma_1). \quad (2)$$

Let y_τ denote the measure of one-year-olds at time τ in the population. The following equation describes the law of motion of y_τ :

$$y_\tau = \sum_{i=1}^T y_{\tau-i} a_i. \quad (3)$$

Pollard (1973), in Chapter 4.6., shows that if the evolution of generations can be described by an equation like (3), then the population grows according to a fixed rate asymptotically. That is $y_\tau/y_{\tau-1} \equiv g$ asymptotically. Hence (3) can be rewritten as

$$g = \frac{y_\tau}{y_{\tau-1}} = \sum_{i=1}^T \frac{y_{\tau-i}}{y_{\tau-1}} a_i = \sum_{i=1}^T \prod_{j=0}^{i-2} \frac{y_{\tau+j}}{y_{\tau+j+1}} a_i = \sum_{i=1}^T \frac{a_i}{g^{i-1}}. \quad (4)$$

Given the previous equation, we can restate our objective formally. We seek to identify the gene $\{c_t\}_1^T$, that maximizes g defined by (4) and (2).

3 Results

Our goal in this section is to find a utility-representation of the surviving genes. That is, we want to find a function, mapping from choices to the real numbers, such that the choice of the surviving gene is the solution of the maximization problem if the maximand is the function.

Suppose that the gene $\{c_t^*\}_1^T$ maximizes the growth rate of the corresponding population. Choose any set of functions $\{V_{t,\gamma_t}\}_{t,\gamma_t}$, where V_{t,γ_t} is a mapping from $[0, b_t(\alpha_t)]$ to \mathbb{R} ,

such that $c_t^*(\gamma_t) = \arg \max_{c \in [0, b_t(\alpha_t)]} V_{t, \gamma_t}(c)$. Then the behavior induced by maximizing V_{t, γ_t} at age t would obviously induce the same choice behavior as $\{c_t^*\}_1^T$. We would not want to call such a collection of functions as utility representation, since it does not reflect to the dynamic nature of the decision problem of individuals. We require a stronger form of representation laid out formally below.

Recursive Utility-Representation. — The collection of choices, $\{c_t\}_{t=1}^T$, has a recursive utility representation by the collection of felicity utility functions, $\{U_t\}_{t=1}^T$, if

$$c_T(\alpha_T, \beta_T) = \arg \max_{c \in [0, b_T(\alpha_T)]} U_T(c, \beta_T) \quad \forall (\alpha_T, \beta_T) \in A_T \times B_T,$$

and for all $t < T$, and $(\alpha_t, \beta_t) \in A_t \times B_t$:

$$c_t(\alpha_t, \beta_t) = \arg \max_{c \in [0, b_t(\alpha_t)]} U_t(c, \beta_t) + E_t \left(\sum_{i=t+1}^T U_i(c_i, \beta_i) \mid c_{t+1}, \dots, c_T \right).$$

Requiring recursive utility-representation already implies a certain form of time-consistency. Solving an individual's maximization problem at age t , involves her expectations about her future decisions at ages $t+1, \dots, T$. These expectations are correct. Since the vector α_t influences only the choice sets we do not allow the instantaneous utility to depend on this vector.

Theorem 1 *The surviving gene has the following utility-representation:*

$$U_t(c_t, \beta_t) = \frac{f_t(c_t, \beta_t)}{(g^*)^{t-T}},$$

where g^* is the largest possible growth factor satisfying (4).

Theorem 1 implies that an individual's choice at age t , who has a surviving gene, maximizes

$$E \sum_{i=t}^T \frac{f_i(c_i, \beta_i)}{(g^*)^i}. \quad (5)$$

That is the discounted present value of expected number of offspring, where the discounting is done according to the population growth factor. Fisher (1958) refers to the maximand in (5) as the reproductive value of an individual.

Proof. The proof of the theorem consists of two steps. First, we show that the first-order conditions corresponding to the maximization of g (defined by (4) and to maximizing (5) are identical. Second, we show that these first-order conditions are sufficient as well as necessary in both maximization problems.

Fix c_i $i \neq j$, and view a_i as a function of the choice rule c_j using (2). Notice that a_i does not depend on c_j if $i < j$. Hence, after multiplying both sides by g^{T-1} , (4) can be rewritten as

$$g^T(a_1(c_j), \dots, a_T(c_j)) = \sum_{i=1}^T a_i(c_j) g^{T-i}(c_j).$$

We shall use an argument of small variations. Fix an arbitrary continuous function h , and consider the following identity:

$$g^T(a_1(c_j + \varepsilon h), \dots, a_T(c_j + \varepsilon h)) \equiv \sum_{i=1}^T a_i(c_j + \varepsilon h) g^{T-i}(c_j + \varepsilon h).$$

Differentiating the previous identity according to ε we get

$$T g^{T-1} \sum_{k=1}^T g_k \frac{da_k}{d\varepsilon} = \sum_{i=1}^T \left[\frac{da_i}{d\varepsilon} g^{T-i} + a_i (T-i) g^{T-i-1} \sum_{k=1}^T g_k \frac{da_k}{d\varepsilon} \right], \quad (6)$$

where $g_k = \partial g / \partial a_k$. Notice however, that if (c_1, \dots, c_T) defines a surviving gene, then

$$\left. \sum_{k=1}^T g_k \frac{da_k}{d\varepsilon} \right|_{\varepsilon=0} = 0.$$

For otherwise given $\{c_i\}_{i \neq j}$, c_j would not maximize the growth factor g . Hence (6) can be rewritten as

$$0 = \left. \sum_{i=1}^T \frac{da_i}{d\varepsilon} g^{T-i} \right|_{\varepsilon=0}. \quad (7)$$

Recall, that

$$\begin{aligned} a_t &= \int \dots \int f_t(c_t(\gamma_t), \beta_t) dG_t(\gamma_t, \gamma_{t-1}, c_{t-1}(\gamma_{t-1})) \dots dG_1(\gamma_1) \\ &= \int \dots \int f_t(c_t(\gamma_t), \beta_t) g_t(\gamma_t, \gamma_{t-1}, c_{t-1}(\gamma_{t-1})) \dots g_1(\gamma_1) d\gamma_t \dots d\gamma_1. \end{aligned}$$

For all $t \geq j$, let $\vartheta_t(c_j(\gamma_t))$ denote

$$\int \dots \int f_t(c_t(\gamma_t), \beta_t) g_t(\gamma_t, \gamma_{t-1}, c_{t-1}(\gamma_{t-1})) \dots g_{j+1}(\gamma_{j+1}, \gamma_t, c_j(\gamma_t)) d\gamma_t \dots d\gamma_{j+1}.$$

Hence for all $t \geq j$:

$$a_t(c_j(\gamma_j) + \varepsilon h(\gamma)) = \int \dots \int \vartheta_t(c_j(\gamma_j) + \varepsilon h(\gamma)) dG_j(\gamma_j, \gamma_{j-1}, c_{j-1}(\gamma_{j-1})) \dots dG_1(\gamma_1).$$

and

$$\frac{da_i}{d\varepsilon} = \int \frac{d\vartheta_i(c_j(\gamma) + \varepsilon h(\gamma))}{d\varepsilon} h(\gamma_j) dG_j(\gamma_j, \gamma_{j-1}, c_{j-1}(\gamma_{j-1})) \dots dG_1(\gamma_1).$$

Since (7) is true for all h , it follows that

$$0 = \sum_{i=j}^T \frac{d\vartheta_i(c_j(\gamma))}{d\varepsilon} g^{T-i} \text{ for all } \gamma.$$

But these are exactly the first-order conditions defining the choice functions generated by the recursive utility representation if $U_i(c_t, \beta_t) = f_t(c_t, \beta_t) / g^{t-1}$.

The first-order conditions are obviously sufficient as well as necessary in both maximization problems. ■

Uniqueness of the Representation

A natural question to ask is: Under what circumstances is the utility-representation unique? Recall from standard consumer theory, the preferences of a consumer can only be deduced from her behavior if she faces sufficiently many choice problems. For example, she has to make consumption decisions with different prices and different incomes. In other words, in order to identify a consumer's preferences, one must be able to observe her choices made from various budget sets. Similarly, we shall show that our utility-representation of the surviving gene is unique if and only if there is enough variations in the resource constraints relative to the variations in their fertilities of the individuals. Instead of developing a general theorem of uniqueness we show by example that the representation is unique if there is enough variation in the resources an individual potentially has.

Notice that if $\{U_t\}_{t=1}^T$ is a utility-representation of the surviving gene, and $V_t = aU_t + b_t$ where $a_1, \dots, a_T, b \in \mathbb{R}_+$, then $\{V_t\}_{t=1}^T$ is also a utility-representation. Hence, in what follows we only show uniqueness up to these linear transformations.

Example 1. We modify the example described in the introduction as follows. An individual still lives for two periods, and has access to the reproduction function f . However, a new-born individual has endowment $b \in (0, \infty)$ to allocate between the two periods, and faces a saving technology characterized by $R \in (0, \infty)$. The numbers b and R are realizations of independent random variables, and they are also independent across individuals. If an individual saves $b - x$ of her resources, she receives $R(b - x)$ a period later. In this example, a gene is a function $c : (0, \infty)^2 \rightarrow [0, 1]$. That is, if an individual has endowment b and can save with interest R , she uses $c(b, R)$ of her endowment in the first period and $R(b - c(R))$ in the second one.

The surviving gene is characterized by the following first-order conditions:

$$f'(c(R)) = \frac{Rf'(R(b - c(R)))}{g^*} \text{ for all } (b, R) \in (0, \infty)^2, \quad (8)$$

where g^* is the largest possible growth factor.

Notice that for all $x, y \in (0, \infty)$ there exist $R, b \in (0, \infty)$ such that $c(b, R) = x$ and $R(b - c(b, R)) = y$.³ Let $R(x, y)$ denote the interest corresponding to x and y . The previous first-order conditions can be rewritten as

$$f'(x) = \frac{R(x, y) f'(y)}{g^*} \quad \text{for all } x, y \in (0, \infty).$$

Suppose now, that $\{U_1, U_2\}$ is the set of felicities in the recursive utility-representation. Then, they also must satisfy the following first-order condition for all $x, y \in (0, \infty)$

$$U_1'(x) = \frac{R(x, y) U_2'(y)}{g^*}. \quad (9)$$

Fix $x_0 \in (0, \infty)$. Since the felicities can be determined only up-to linear transformation, normalize $U_1'(x_0)$ to be $f'(x_0)$. In order to establish uniqueness, we shall show that $U_1' = U_2' = f'$. Notice that for all $y \in (0, \infty)$

$$U_2'(y) = \frac{g^* U_1'(x_0)}{R(x_0, y)} = \frac{g^* f'(x_0)}{R(x_0, y)} = f'(y),$$

where the first equality follows from (9), the second one from the normalization of $U_1'(x_0)$, and the third one from (8). From (9), for all $x \in (0, \infty)$

$$U_1'(x) = \frac{g^* U_2'(y)}{R(x, y)} = \frac{g^* f'(y)}{R(x, y)} = f'(x).$$

Commonly Used Preferences

Next, we investigate under what conditions does Theorem 1 imply preferences often used by economists. A desired and often required property of preferences in dynamic models is time-separability.

Remark 1 *The utility representation of the surviving gene is time-separable if and only if the reproduction functions, $\{f_t\}_{t=1}^T$, do not depend on previous decisions. That is, for all $t \in \{2, \dots, T\}$,*

$$\frac{\partial f_t}{\partial c_p} = 0 \quad \text{for all } p \in \{1, \dots, t-1\}.$$

³To see this notice that if $R = f'(x) / [g^* f'(y)]$ and $b = x + y/R$ the previous first-order condition is satisfied.

Notice that although felicities depend on past choices in general, they never depend on future choices. This is because the reproductive ability of an individual cannot be influenced by choices made in the future.

In addition to time-separability, most model assumes that the felicity utility function of an agent is age-invariant.

Remark 2 *The utility representation of the surviving gene is time-separable and the felicity utility function is the same if and only if the reproduction functions depend on neither previous decisions nor on age.*

Most economic model also assumes that the realization of states of the world has no impact on felicities. Our model predicts that this is only true as long as the state of the world has no impact on fertility.

Remark 3 *The felicities in the utility representation of the surviving gene are state-independent if and only if the reproduction function does not depend on these states.*

The Impossibility of Characterizing the Optimal Behavior

Consider the example described in the introduction, except that an individual lives for five periods, that is $T = 5$. Then, the equation defining the growth factor in terms of the choices would be

$$g^5 = \sum_{i=1}^5 g^{i-1} f(x_i).$$

That is, g is the solution for a polynomial of degree five, which, in general, cannot be determined analytically.

Carrying Capacity Constraint

As we have mentioned in the introduction, one reason why one might expect only genes generating the highest growth rate to survive evolution is that world resources are limited. If a gene grows slower than some others then its share of resources which are necessary for reproduction becomes asymptotically zero, and hence the gene dies out.

A simple way to model the competition for resources among individuals is to introduce carrying capacity constraint.⁴ Assume that the reproduction function depends on an index of total world population, that is, the expected number of offspring at age t is

⁴The same constraint is introduced for example in Hansson and Stuart (1990) and Robson and Wooders (1997).

$f_t(c_t, \beta_t, N)$ where N is an aggregate index of total population. The number N can be thought of, for example, as the sum of the sizes of various populations facing the same physical environment, but also as the sum of the sizes of populations facing different environment.⁵ What is important is that if the size of at least one population goes to infinity, N also converges to infinity. Further assume that $\partial f_t / \partial N < 0$, and $\lim_{N \rightarrow \infty} f_t = 0$.

Notice that if for a fixed N a choice behavior induces a growth factor larger than one, the size of the population corresponding to the choice behavior as well as the size of total population increases, and makes individuals less reproductive. Hence the growth rate of the population declines, and eventually is forced to be zero. Therefore, in a steady state equilibrium the aggregate index of the total population must be constant, say N^* . Then, only those populations survive evolution that maximize the growth rate for N^* . Hence, the solution of the model is characterized by the same equations as before and the additional constraint guaranteeing that the growth factor is one. More precisely, the choice behavior surviving the evolution still maximizes g subject to (4). And there is an additional constraint saying that the g defined by (4) is one.

The recursive utility-representation in this model is identical to the one described in Theorem 1 with $g^* = 1$. Hence any attribute of time preferences is due to the age-dependence of fertility and the shape of the reproduction functions. Individuals might prefer immediate rewards because their fertility is declining with age, and hence they can transform resources into offspring more efficiently today than tomorrow. They might prefer to transfer some of their resources to the future because of the concavity of their reproduction function.

Pairwise Mating

What if, as often in the nature, two individuals are necessary for reproduction? An offspring inherit a gene from one of her parent with probability half. What if individuals with different genes can produce offspring? In this case, a population should be defined as the group of those individuals who can mate. In the long run evolutionary equilibrium, there can be many genes represented in the population. In steady state, the relative proportions of genes are constant. Suppose that the steady state proportion of a certain gene is μ . Then, the probability of an offspring having the same gene as her parent is $(1 + \mu) / 2$. (This is because the offspring inherit her gene from one of her parent with

⁵Instead of a single number, one could also introduce a vector determining reproductivity. A coordinate of a vector corresponds to the size of a certain population.

probability half, and the other parent has the same gene with probability μ .) Then the number of offspring at age t having the same gene as the individual is $(1 + \mu) f_t/2$. This implies that in the utility-representation of a surviving gene, the felicity is $(1 + \mu) f_t/2$ which is just a linear transformation of the representation in Theorem 1.

Coexisting Populations

Recall, the premise of this paper is that a population growing at a rate slower than another one cannot survive evolution. If one is willing to accept this premise then one can take it a step further and apply it to populations facing different physical environments. For example: What if two populations have access to different reproduction functions? Our logic implies that two populations can only coexist if they grow at the same rate.

A consequence of Theorem 1 is that the discount factor in the preferences representing surviving genes is the inverse of the growth factor. Since coexisting populations must grow at the same rate one can make the following

Remark 4 *Individuals corresponding to different coexisting populations might have different felicity utility functions but they must have the same discount factor.*⁶

This remark implies that if one observes heterogeneity in preferences among individuals, this must be due to the heterogeneity in felicity utility functions but not in discount factors.

If one specifies two different physical environments, the surviving genes corresponding to the two environments will grow at different rates generically. We argue however that if there are interactions between different populations, their growth rates can be forced to be the same in steady state equilibria.⁷ To see this, suppose that there are two different physical environments. One is described by the reproduction functions $\{f_t\}_1^T$ and the other is by $\{h_t\}_1^{T'}$. Further assume, that these populations have positive externalities on each other. Formally, let ρ denote the relative proportion of population one to population two. Assume that f_t and h_t depend on ρ and satisfy $\partial f_t/\partial \rho < 0$, and $\partial h_t/\partial \rho > 0$. That is, an individual corresponding to one of the populations is more fertile if the size of the other population is large relative to her own. Further assume that $\lim_{\rho \rightarrow \infty} f_t = 0$, $\lim_{\rho \rightarrow 0} f_t = \infty$, and similarly $\lim_{\rho \rightarrow \infty} h_t = \infty$, $\lim_{\rho \rightarrow 0} h_t = 0$. In steady state equilibria ρ must be constant and equivalently the two populations must grow at the same rates for

⁶We are grateful to Phil Reny for pointing out this consequence.

⁷We are grateful to Kevin Murphy for making this observation.

the following reason. Suppose that population one grows faster than the other one. This implies that ρ increases, and in turn individuals of population one become less and less fertile while individuals of the other population become more fertile. This decreases the growth rate of population one and increases the growth rate of population two.

4 Altruism

In the previous sections we maintained the assumption that choices made by an individual affect only her number of offspring but have no impact on other individuals otherwise. In this section we analyze a simple model where an individual might transfer some of her endowment to her offspring in order to increase the reproductivity of the offspring at a cost of reducing her own.

We emphasize that the reason why the results, in particular the utility representations, are different in this section from those in the previous one, is due to the fact that newborn individuals are no longer identical. As a consequence of the ability to transfer resources to future generations, individuals with wealthy parents are likely to receive more transfers and become more reproductive. This introduces heterogeneity among newborn individuals. As a result, we shall show, the surviving gene does not behave as if it maximized the discounted present value of expected number of descendants. This is because even if two descendants are in the same generation, they might have different reproductive abilities and their asymptotic contributions to the gene pool are different.

Suppose that an individual lives for at most T periods. In each period an individual produces exactly one offspring if she is still alive. Furthermore, at age t , an individual receives endowment I_t . The endowment, I_t , is a random variable distributed on $[0, 1]$ according to a cdf G_t . Assume that there is a corresponding density, g_t , on $(0, 1]$. The individual can save part of the endowment and transfer it to her new-born offspring. The new-born offspring will receive this transfer (and potentially some interest) one period later, at the age of one. Let $s_t(I_t)$ denote the saving of the individual at age t if her endowment is I_t . Whatever is left from the saving is consumed by the individual. The new born offspring receives $Rs(I_t)$ a period later, where $R \in \mathbb{R}_+$. The individual of age t survives until the next period with probability $p_t(c_t)$, where c_t is the consumption of the individual at age t . A newborn offspring surely survives until the age of one. The functions $\{p_t\}_{t=1}^{T-1}$ are assumed to be increasing, concave, and satisfy the Inada conditions. We assume that when a one-year-old individual makes her saving-consumption decision,

she observes neither the age, nor the transfer of her parent. For simplicity we assume that $I_T \equiv 0$.⁸

A gene is a collection of saving decisions, $\{s_t\}_1^{T-1}$, where $s_t : [0, 1] \rightarrow [0, 1]$.

The trade-off every individual is facing in this model is the following. Consuming more increases the individual's own survival probability, but decreases the survival probability of her new-born child from age one to age two.

We could have specified a more complicated model, where individuals can have more than one offspring in each period, they can save for their own future, parents can transfer resources to offspring older than one, and the transfers can affect not only survival probabilities but also reproductive ability. However, this simple model already serves our purpose to demonstrate the subtleties associated to the external effects of choices. In particular, we show that the reproductive value of an individual is a more complicated object than the one derived in the model without externalities. In addition, we specified the setup so that it is easy to compare our results with that of Rogers (1999).

In what follows, we again use the first-order approach to analyze the model. This is valid, for example, if I_t has an atom at 0 for all $t = 1, \dots, T$.

Population Growth. — Notice that the expected survival probability of an individual at age one depends on the age of the parent. Let μ_t denote the probability that an individual's parent is of age t . Introduce the following notations:

$$P_1^t = \int_0^1 \int_0^1 p_1(I_1 - s_1(I_1) + Rs_t(I_t)) dG_1(I_1) dG_t(I_t).$$

That is, P_1^t is the ex-ante probability that a new-born offspring survives until the age of two, if she had a parent of age t . Then the probability of surviving until the age of two unconditional on the age of the parent is

$$P_1 = \sum_{t=1}^T \mu_t P_1^t.$$

Furthermore, let

$$P_t = \int_0^1 p_t(I_t - s_t(I_t)) dG_t(I_t) \quad t = 2, \dots, T - 1,$$

the expected surviving probability of an individual of age t until age $t + 1$.

⁸Since the individuals die at age T , they would transfer all their endowments to their new-born offspring anyway.

Let y_τ denote the number of new-born offspring at time τ . The following equation recursively defines the motion of y_τ :

$$y_\tau = y_{\tau-1} + y_{\tau-2}P_1 + y_{\tau-3}P_1P_2 + \dots + y_{\tau-T}P_1\dots P_{T-1}. \quad (10)$$

In steady state, the population grows at a fixed factor g , and μ_t ($t = 1, \dots, T$) does not change. Notice that

$$\mu_1 = \frac{y_{\tau-1}}{y_\tau} = \frac{1}{g}.$$

Furthermore,

$$\mu_t = \frac{\mu_2 \prod_{i=2}^{t-1} P_i}{g^{t-2}} \quad t = 3, \dots, T.$$

Since $\mu_1 + \dots + \mu_T = 1$, it must follow that

$$\mu_2 \left(1 + \sum_{t=3}^T \frac{\prod_{i=2}^{t-1} P_i}{g^{t-2}} \right) = 1 - \frac{1}{g} = \frac{g-1}{g}.$$

Let Q denote the coefficient of μ_2 in the previous equation. Then, (10) can be rewritten as follows

$$\begin{aligned} g &= 1 + \left(\frac{P_1^1}{g} + \frac{P_1^2(g-1)}{gQ} + \dots + \frac{P_1^T(g-1)P_2\dots P_{T-1}}{g^{T-1}Q} \right) \left(\frac{1}{g} + \frac{P_2}{g^2} + \dots + \frac{P_2\dots P_T}{g^{T-1}} \right) \\ &= 1 + \left(\frac{P_1^1}{g} + \frac{P_1^2(g-1)}{gQ} + \dots + \frac{P_1^T(g-1)P_2\dots P_{T-1}}{g^{T-1}Q} \right) \frac{Q}{g} \\ &= 1 + \frac{P_1^1Q}{g^2} + \frac{P_1^2(g-1)}{g^2} + \dots + \frac{P_1^T(g-1)P_2\dots P_{T-1}}{g^T}. \end{aligned}$$

After multiplying both sides by g^T :

$$g^{T+1} = g^T + P_1^1Qg^{T-2} + (g-1)P_1^2g^{T-2} + \dots + (g-1)P_1^T P_2\dots P_{T-1}.$$

Finally, using the definition of Q :

$$g^{T+1} = g^T + P_1^1 \left(g^{T-2} + \sum_{t=3}^T g^{T-t} \prod_{i=2}^{t-1} P_i \right) + (g-1) \sum_{t=2}^T P_1^t g^{T-t} \prod_{i=2}^{t-1} P_i. \quad (11)$$

In what follows, using the previous equation, we characterize the gene that maximizes its corresponding growth rate.

The Surviving Gene. — What is the first-order-condition with respect to $s_t(I_t)$? Notice, that $s_t(I_t)$ affect only P_1^t and P_t . Differentiating (11) according to $s_t(I_t)$, and using that $\partial g/\partial s_t(I_t) = 0$,

$$0 = P_1^1 \left(\frac{\partial P_t}{\partial s_t(I_t)} \left(g^{T-t-1} \prod_{\iota=2}^{t-1} P_i + \sum_{j=t+2}^T g^{T-j} \prod_{\substack{\iota=2 \\ \iota \neq t}}^{j-1} P_i \right) \right) + (g-1) \frac{\partial P_1^t}{\partial s_t(I_t)} g^{T-t} \prod_{\iota=2}^{t-1} P_i \\ + (g-1) \frac{\partial P_t}{\partial s_t(I_t)} \left(P_1^{t+1} g^{T-t-1} \prod_{\iota=2}^{t-1} P_i + \sum_{j=t+2}^T P_1^j g^{T-j} \prod_{\substack{\iota=2 \\ \iota \neq t}}^{j-1} P_i \right).$$

That is

$$-\frac{\partial P_t}{\partial s_t(I_t)} \left(P_1^1 \sum_{j=t+1}^T g^{T-j} \prod_{\substack{\iota=2 \\ \iota \neq t}}^{j-1} P_i + (g-1) \sum_{j=t+1}^T P_1^j g^{T-j} \prod_{\substack{\iota=2 \\ \iota \neq t}}^{j-1} P_i \right) = (g-1) \frac{\partial P_1^t}{\partial s_t(I_t)} g^{T-t} \prod_{\iota=2}^{t-1} P_i.$$

After multiplying both sides by $g^{t-T} \prod_{\iota=2}^{t-1} P_i / (g-1)$ we get

$$-\frac{\partial P_t}{\partial s_t(I_t)} \left(\frac{P_1^1}{g-1} \sum_{j=t+1}^T g^{t-j} \prod_{\iota=t+1}^{j-1} P_i + \sum_{j=t+1}^T P_1^j g^{t-j} \prod_{\iota=t+1}^{j-1} P_i \right) = \frac{\partial P_1^t}{\partial s_t(I_t)}. \quad (12)$$

This first-order condition looks substantially more complicated than the one in the previous section. A natural question to ask is: Does it have a natural interpretation? Next, we show that it does.

Intuition. — Notice, that in this model it is not clear how one should define the reproductive value. The reproductive values of one-year-old individuals are different. A newborn individual with a wealthy parent receives a high transfer and more likely to survive from the age of one to the age of two. How can one compare the reproductive values of different individuals? Notice, that at the age two, before the realization of I_2 , all individuals are identical. Hence, a candidate for measuring the reproductive value of an individual is the discounted present value the expected number of two-years-old descendents. The discounting is again done according to the inverse of the growth factor g .

Let V_t denote the reproductive value of an individual at age t . Then

$$V_T = \frac{P_1^T}{g^2} + \frac{P_1^1}{g^3} \frac{g}{g-1}. \quad (13)$$

The reason is the following. At age T , an individual has one offspring who will survive until the age of two with probability P_1^T . Since the offspring is going to be two-years old in two years, she must be discounted by g^2 . The offspring also produces an offspring at age one, who survives until the age of two with probability P_1^1 . As a matter of fact, each offspring produces one offspring at the age of one, and each offspring survives with probability P_1^1 . This explains the second term. Suppose now that $t < T$. Then

$$V_t = \frac{P_1^t}{g^2} + \frac{P_1^1}{g^3} \frac{g}{g-1} + \frac{P_t}{g} V_{t+1}. \quad (14)$$

The first and second terms correspond to the discounted present value of the two-year-old descendants of the newborn offspring of the individual. The last term is the individual's own discounted reproductive value. Equations (13) and (14) enables one to express V_t explicitly. It turns out that

$$V_t = \frac{P_1}{g^3} \frac{g}{g-1} \left(1 + \sum_{j=t}^{T-1} \frac{\prod_{k=t}^j P_k}{g^{j-t+1}} \right) + \sum_{j=t+1}^T P_1^j \frac{\prod_{k=t}^{j-1} P_k}{g^{j-t+2}}.$$

Suppose now, that an individual at age t , has endowment I_t . Intuitively, when making choices, she should maximize the properly discounted reproductive value of those individuals who are affected by her choices. That is, she solves the following maximization problem:

$$\max_{s_t(I_t) \in [0, I_t]} p_t(I_t - s_t(I_t)) \frac{V_{t+1}}{g} + \int_0^1 p_1(I_1 - s_1(I_1) + R s_t(I_t)) dG_1(I_1) \frac{V_2}{g^2}.$$

The first-order condition for the problem is

$$p'_t(I_t - s_t(I_t)) \frac{V_{t+1}}{g} = R \int_0^1 p'_1(I_1 - s_1(I_1) + R s_t(I_t)) dG_1(I_1) \frac{V_2}{g^2}. \quad (15)$$

Notice that

$$\begin{aligned} p'_t(I_t - s_t(I_t)) &= \partial P_t / \partial s_t(I_t) / g_t(I_t), \\ R \int_0^1 p'_1(I_1 - s_1(I_1) + R s_t(I_t)) dG_1(I_1) &= \partial P_1^t / \partial s_t(I_t) / g_t(I_t). \end{aligned}$$

Since $V_2 = 1^9$, plugging the previous equations into (15) yields

$$-\partial P_t / \partial s_t(I_t) g V_{t+1} = \partial P_1^t / \partial s_t(I_t).$$

⁹Recall, the reproductive value is defined as the discounted present value of expected number of two-years-old descendants of an individual. This explains $V_2 = 1$, since the individual is exactly at age two.

Plugging in V_{t+1} from (14)

$$-\partial P_t / \partial s_t(I_t) g \left(\frac{P_1}{g^3} \frac{g}{g-1} \left(1 + \sum_{j=t}^{T-1} \frac{\prod_{k=t}^j P_k}{g^{j-t+1}} \right) + \sum_{j=t+1}^T P_1^j \frac{\prod_{k=t}^{j-1} P_k}{g^{j-t+2}} \right) = \partial P_1^t / \partial s_t(I_t).$$

This equation is identical to (12).

Recursive Utility Representation

We can find again a utility-representation for the surviving gene, that is for the behavior defined by the first-order conditions in (12).

Proposition 1 *The surviving gene has the following utility-representation:*

$$U_t(I_t - s_t(I_t)) = \frac{P_1 \dots P_{t-1}}{g^t} \left[\frac{P_1^1}{g^3} \frac{g}{g-1} + \frac{P_1^t}{g^2} \right].$$

Observe that the choice $I_t - s_t(I_t)$ affects only the probability P_1^t . Notice that this representation implies that at age i , the individual maximizes

$$E \sum_{t=i+1}^T \frac{P_{i+1} \dots P_{t-1}}{g^t} \left[\frac{P_1^1}{g^3} \frac{g}{g-1} + \frac{P_1^t}{g^2} \right], \quad (16)$$

that is the discounted present value of the expected number of two-year-old descendants, where the discounting is again done according to the inverse of the population growth.

It is worth comparing the felicity utility functions in (5) and in (16). Recall, the felicity utility at age t , in the case where external effects are not present, is the number of offspring reproduced at age t . Since offspring are identical, their reproductive values are the same, and can be normalized to be one. Hence, the number of offspring is also the sum of the reproductive values of newborn offspring. Since the choices of an individual at age t directly influences only the number of offspring who are born when her age is t , one can give the following interpretation to the felicity utility function. The felicity utility at age t is the sum of the reproductive values of those individuals, who are directly affected by the age- t choices of the individual. Below, we argue that this is exactly the interpretation of the felicity utility in (5). In this respect, (16) can be viewed as a generalization of (5).

The expression $P_1^t/g^2 + P_1^1/g^2(g-1)$ is exactly the discounted present value of the expected reproductive value¹⁰ of the offspring of an individual who gave birth to the

¹⁰With probability P_1^t the offspring lives until the age of two, in which case her reproductive value is one. With probability $1 - P_1^t$, she is dead by the age of two in which case her reproductive value is zero.

offspring at age t . In the maximization problem (16), this term is multiplied by $P_{i+1} \dots P_{t-1}$, because this is the probability that the offspring mentioned above will actually be born. Hence, the age- t felicity utility of an individual is again the sum of the expected discounted reproductive values of all those individuals who are directly affected by the choices made by the individual at age t .

Measuring Reproductive Value. — In the model of this section, all two-year-old individuals are identical. That's why a natural measure of reproductive value is the discounted present value of expected number of two-year old descendants. Of course, in a more general setting individuals are always heterogeneous. A natural question to ask is: Is there a way to measure reproductive value in such models? We think that the reproductive value should be measured as the discounted present value of expected number of dead descendants, as opposed to newborn ones. This is because at the moment of death all individuals become identical.

4.1 Discussion of Rogers (1994)

As we have pointed out in the introduction, Rogers (1994) also has the objective to explain time preferences by natural selection. The model in Rogers (1994) is similar to the one in this section. The author is not explicit about the choices available for individuals, but as far as we understood, the author had in mind similar consumption-saving decisions as the ones discussed here. The main difference is that Rogers (1994) assumes that a gene can occur in two different forms: altruistic and non-altruistic. The author refers to a form of the gene as *allele*¹¹. An altruistic allele survives evolution, according to Rogers (1994), if it is selectively neutral. That is, the properly discounted sum of the changes of the reproductive values of the donor and recipient is zero¹². Below, we point out several problems of the analysis of Rogers (1994) and argue that his conclusions are not valid.

Domain of Choice-Problems. — Rogers (1994) analyzes a deterministic model, where an individual faces a single choice problem.¹³ This means that any two individuals at the same age face the exact same decision problem. This makes it impossible to represent the choice behavior of an allele by indifference curves. To make this point clear, suppose

¹¹See Footnote 3 on pg. 462 in Rogers (1994).

¹²This is roughly the first-order condition corresponding to the maximization of the discounted reproductive values.

¹³In Section III, Rogers (1994) introduces uncertainty, but even in that model individuals face only a single choice problem.

that an individual faces a choice set consisting of the following three elements: x, y , and z . Choice x means murdering a new-born offspring and producing another one at the same time. Choice y is being inactive, and choice z is producing a new offspring. An allele surviving evolution obviously chooses z . However, one cannot conclude anything about the preferences over the choices x , and y . The choice of the allele is consistent with $x \sim y \prec z$, $x \prec y \prec z$, and $y \prec x \prec z$. As a matter of fact, the choice x is consistent even with $x \sim y \sim z$.¹⁴ In order to conclude anything about the preference over choices x and y , one must observe the individual making a choice from the set $\{x, y\}$. Hence, Rogers's (1994) goal to identify indifference curves representing choices is impossible to achieve given his framework.

Reproductive Value. — We emphasize that the notion of reproductive value must endogenously be determined in evolutionary models. Whether a gene survives evolution or not is determined by the choices it induces. One cannot assume that the choices satisfy a predefined maximization problem. Once the surviving choice behavior is identified, one might be able to find maximization problems for which the choices are solutions. The maximand of this problem is the reproductive value in general.

Instead of deriving endogenously, Rogers (1994) assumes that the reproductive value of an individual with an altruistic allele is the discounted present value of expected number of descendants, that is, the maximand in (5).¹⁵ In other words, Rogers (1994) assumes in his model of altruism what we have derived in our model without altruism. As we have proved in this section, in the presence of altruistic acts, the reproductive value is no longer the discounted present value of expected number of offspring. As a result, Rogers (1994)'s analysis is built on the wrong notion of reproductive value. The author mistakenly assumes that the choices generated by the surviving gene are the solutions of maximizing an ill-defined reproductive value.

Pairwise mating. — In many models on evolutionary theory, and indeed in the nature too, two individuals are necessary for reproduction. Offspring inherit half of the genes of each of their parents. Many authors assume, including Rogers (1994), that as a consequence, an individual *should care* only half as much about her child as about herself. To illustrate this point, assume that an altruistic act instantaneously increases the number

¹⁴In addition, choices x and y are selectively neutral by the definition of Rogers (1994). Hence, according to the author they should survive evolution. This shows that investigating first-order conditions instead of maximization problems might be misleading.

¹⁵See equation (5) on pg. 464 in Rogers (1994).

of grandchildren by a , but instantaneously decreases the number of children by b . Then this altruistic act, according to Rogers (1994), can only survive evolution if $a/2 \geq b$. This is because children have half of the genes of the donor, while grandchildren have only a quarter of them.

This argument implies that the probability of an offspring having the altruistic gene, conditional on one of her parent having it, is one half. This is true if and only if the probability of the two parents having the altruistic gene is zero. That is, the frequency of the altruistic gene in the population is zero. Maximizing the growth rate of the altruistic gene in a population where its frequency is zero is a valid exercise when one is checking whether a population can be invaded by altruistic genes. This is because in that case the question is whether altruistic genes can grow faster than other genes if other genes dominate the population.

Notice however, that if one represents the surviving choice behavior by preferences, weighting children by a half and grandchildren by a quarter becomes unjustified. This is because the zero frequency of a gene in evolutionary equilibrium implies that the gene has not survived the evolution. Rogers (1994) is aware of this problem, and he claims that his theory only holds when the frequency of the altruistic gene is near zero. As a matter of fact, Rogers (1994)'s formulas are only valid if the frequency of the altruistic gene is literally zero. Hence, what Rogers (1994) analyzed in his paper is the best-response of an altruistic gene against a population without the altruistic gene, instead of characterizing the choice behavior in the long-run evolutionary equilibrium.

Yet another problem with the argument above is that an altruistic act might increase the growth rate of genes which were not provided by the donor. As a result of altruistic acts, other genes might grow faster than the one corresponding to the donor. In other words, although it is possible that the altruistic act maximizes the absolute growth rate of the altruist's gene, the growth rate of this gene relative to other genes might actually decrease. Assuming that the altruistic act survives the evolution even though it might increase the growth rate of other genes is controversial at best.

In our altruism model, we have avoided these problems by assuming that only one individual is necessary for reproduction. We could have assumed that offspring have two parents, as long as they have the same genes. Since a gene in this paper is interpreted as a choice behavior, this would merely mean that the parents make similar choices. As a result, in our model the relative and absolute growth rate of a gene are identical.

We do recognize the importance of analyzing the consequences of the possibility of

mating between individuals with different choice behaviors. It seems to be, however, a particularly difficult situation to model. The difficulty stems from the fact that whether a choice behavior generates high or low growth rates relative to other choice behaviors heavily depends on the distribution of choice behaviors in the population.

Long Term Interest Rate. — The (verbal) argument about the market interest rate in Rogers (1994) is the following. An individual can transfer some of her income to her offspring, who will receive it one generation later. That is, the recipient's age is the same when she receives the transfer as the donor's age when she gives it away, hence the reproductive values of the donor and the recipient are the same. The marginal effect on the survival probability of the donor is the derivative of the survival probability. The marginal effect on the survival probability of the recipient is the derivative of the survival probability times R^T , where R is the interest, and T is the length of a generation. Since the transfer affects the offspring T periods later, this effect must be multiplied by the inverse of $2g^T$. (The factor 2 is due to the assumption that an offspring inherits only half of her parent's genes.) The donor and the recipient are identical and hence their survival probabilities are the same. This implies, that the altruistic act is selectively neutral if $R^T/2g^T = 1$. Since in the long run the population growth rate is approximately zero, Rogers (1994) concludes that $R^T = 2$.

In this section, we have demonstrated that evolutionary equilibria can be consistent with any interest rate. In our model we imposed no restriction on the interest R . Notice that the interest rate is an exogenous part of the physical environment in these models. It describes the saving technology. Of course, the behavior induced by the surviving gene does depend on the interest rate. But we emphasize that it cannot happen vice versa: the behavior of the individuals have no impact on the saving technology unless one explicitly model an interaction between choice behaviors and saving technologies.¹⁶

What is wrong with Roger (1994)'s argument? Recall that Rogers (1994) analyzes a deterministic model, where individuals have to solve only a single choice problem. Therefore, if the interest rate is large enough individuals save all of their endowments for their offspring, and if it is small they save nothing. There is no particular reason to assume that the solution to the problem is interior. The correct statement of Rogers (1994) should be the following. If the environment is deterministic and the solution happens to be interior then the interest satisfies $R^T = 2$.

¹⁶One possibility of modelling such an interaction is introducing capital into the model, and assuming decreasing return to scale. The larger is the aggregate saving the smaller the interest rate becomes.

5 Concluding Remarks

The goal of this paper was to understand what kind of restrictions evolution imposes on time preferences. We have shown that one can always find a utility representation for those choice behaviors that survive evolution. We have established strong relationships between felicities and fertilities, and between discount factors and population growth rates. In particular, we have shown that in the utility representation, the instantaneous utility functions are identical to the reproduction values, and the discount factor is the inverse of the growth factor of the population. In our representation there is a natural cardinal interpretation of utilities. The instantaneous utility is simply the expected number of offspring whenever newborn individuals are identical. More generally, the felicity of an individual is the sum of the reproductive values of those who are affected by the choices of the individual.

Extrapolation. — Our theory implies that time preferences surviving evolution must be time-consistent. We think that one must be careful when using evolutionary arguments to analyze consumption-saving decisions in modern times. Preferences, as well as indifference curves, simply represent choice behaviors. If one aims at understanding the evolution of preferences by natural selection, one should restrict attention to those choice problems which were faced by individuals during evolution. It is not straightforward to draw conclusions based on evolutionary arguments about individuals' choices unless similar choices were offered during the evolutionary process.¹⁷ Nonetheless, there is a strong evolutionary force to make individuals time consistent. Time inconsistent behaviors are likely to be associated to choices which are new to populations and individuals had no time to adopt the optimal behavior.

In order to extrapolate results based on evolutionary arguments to choice problems offered only in modern civilizations, more structure is required on the biological part of the model. One must go beyond interpreting a gene merely as a choice behavior. One way to extrapolate is to assume a principal-agent relationship between gene and individual. The gene is interpreted as a principle rewarding and punishing the individual (agent) for her choices. The individual's objective is to maximize rewards. One can think of

¹⁷Rogers (1994) claims that “[...] one need not assume that credit markets have been important throughout evolution. The hypothesis of evolutionary equilibrium merely specifies the form of indifference curves.” We find this argument to be conceptually wrong and is based on misunderstanding preferences. It seems unlikely that during the evolution individuals had access to saving technologies which are even remotely similar to those used in modern civilizations.

rewards as agronomical pleasure, sexual satisfaction etc. Then the felicities identified in this paper can be interpreted as the reward-scheme corresponding to the surviving gene. If one is willing to interpret felicities as rewards, then extrapolation becomes less controversial. To illustrate this point, suppose that during the evolution individuals could only save at zero interest rate, and the optimal reward for consumption turned out to be the reproduction function. In modern times individuals might have access to better saving technology. However, if the reward scheme is hardwired in individuals, they still maximize the discounted present value of the same felicities.

Social Discounting. — Another important implication of our model is related to social discounting. An interesting yet unsettled question in economic theory is: How should a Social Planner discount future generations? In other words, what should be the relative weights of current and future generations in social welfare functions? In our model, the choices made by individuals having the surviving gene maximize population growth rate. Individuals behave as if their goal was to maximize the asymptotic size of their own population. Therefore, the preferences of individuals belonging to the same population, whether or not they are in the same generation, are completely in line. When making choices, individuals appropriately take into account the impact of their own decisions on future generations. This implies, that the weight on future generations should be zero, since they are already represented in the preferences of individuals of the current generation. Of course, one must be careful to draw such conclusions. For example, if societies consists of several populations, then the argument above becomes invalid.

Social Norms. — We assumed that choice behaviors are hard-wired in individuals. Their choices are merely determined by their genes. One could also apply similar arguments to situations where behavior of individuals are shaped by their upbringing, and social environments. Then, our theory can be used to predict what kind of social norms could survive and what kinds will extinguish. This interpretation also helps to extrapolate our results.

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