Contract, Renegotiation, and Hold Up: General Results on the Technology of Trade and its Interaction with the Technology of Investment

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Abstract

This paper examines a class of contractual relationships with specific investment, a non-durable trading opportunity, and renegotiation. Furthering Watson's (2007) line of analysis, trade actions are modeled as individual and trade-action-based option contracts are explored. The paper develops simple tools for calculating the "punishment values" that determine the sets of implementable post-investment value functions, and two results are proved. The first result establishes that, with ex post renegotiation, constraining parties to use "forcing contracts" (as is implicit in public-action models) implies a strict reduction in the set of implementable value functions. The second result shows that, by using non-forcing contracts, the party without the trade action can be made residual claimant with regard to the investment action. The paper identifies an important distinction, between *divided* and *unified* investment and trade actions, that plays an important role in determining whether an efficient outcome is achieved.

The hold-up problem arises in situations in which contracting parties can renegotiate their contract between the time they make relation-specific investments and the time at which they can trade.[1] The severity of the hold-up problem depends critically on the productive technology and on the timing of renegotiation opportunities. This paper contributes to the literature by examining how the nature of the "trade action" in a contractual relationship influences the prospects for achieving an efficient outcome. We introduce a new distinction—whether the party who invests also is the one who consummates trade—that plays an important role in determining the outcome of the contractual relationship.

[1]Che and Sákovics (2008) provide a short overview of the hold-up problem, which was first described by Klein, Crawford and Alchian (1978), and Williamson (1975,1977). Analysis was provided by Grout (1984), Grossman and Hart (1986), and Hart and Moore (1988).

So that we can describe our modeling exercise more precisely, consider an example in which contracting parties "Al" and "Zoe" interact as follows. First Al and Zoe meet and write a contract that has an externally enforced element. Then one of them makes a private investment choice, which influences the *state* of the relationship. The state is commonly observed by the contracting parties but is not verifiable to the external enforcer. Al and Zoe then send individual public messages to the external enforcer. After this, they have an opportunity to renegotiate their contract.[2] Finally, the parties have a one-shot opportunity to trade and they also obtain external enforcement. Trade is verifiable to the external enforcer.

This description obviously leaves the mechanics of trade and enforcement ambiguous. In reality, the parties have individual actions that determine whether and how trade is consummated. Let us suppose that Al selects the individual trade action, which we call a. This could be a choice of whether to deliver or to install an intermediate good, for example. We then have an *individual-action model*, whereby Al chooses a and the external enforcer compels a transfer t as a function of a and the messages that the parties sent earlier. In contrast, a *public-action model* combines the trade action and the monetary transfer into a single public action (a, t) that is assumed to be taken by the external enforcer. With this modeling approach, the contract specifies how the public action is conditioned on the parties' messages.

Although the public-action model may typically be a bit unrealistic, it is simple and lends itself to elegant mechanism-design analysis (for example, as in Maskin and Moore 1999 and Segal and Whinston 2002). On the other hand, Watson (2007) demonstrates that analysis of the individual-action model can be straightforward as well. He also shows that the public-action model is equivalent to examining individual trade actions but constraining attention to "forcing contracts" in which the external enforcer forces a particular trade action as a function of messages sent by the parties (so the trade action is constant in the state). This characterization leads to the result that public-action models generally under-represent the scope of contracting and thus overstate the problem of hold up. Watson (2007) provides an example in which the restriction to forcing contracts has strict (negative) efficiency consequences.

We provide a deeper analysis for a large class of contractual relationships. We show that the properties of Watson's (2007) example are robust. Furthermore, we prove the existence of non-forcing contracts that make Al's payoff constant in the state, gross of any investment costs. This implies that Zoe can be made the "residual claimant" in the relationship. Remember that Al has the trade action in our story.[3] We thus have strong conclusions about how the technologies of trade and investment interact to determine whether the efficient outcome can be achieved. If Zoe is the party who makes the investment choice — we call this the *divided case*, because here the investment and trade actions are chosen by

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[2]This is called "ex-post" renegotiation because it occurs after messages.

[3]In general, it is possible to make the party without the trade action (player 2 in our general model) the residual claimant.

different parties — then there is a contract that induces efficient investment and trade. On the other hand, in the *unified case* in which Al makes the investment choice and also has the trade action, the efficient outcome is generally not attainable because there typically do not exist contracts that make Al the residual claimant.

Our results underscore the usefulness of modeling trade actions as individual. This is particularly salient for the setting of *cross/cooperative investment* (Che and Hausch 1999), where the investment by one party increases the benefit to the *other* party of subsequent trade. The literature has regarded cross investment settings as especially prone to the hold-up problem (and inefficient outcomes as a result). By introducing the distinction between unified and divided investment and trade actions, we thus give a basis for deeper analysis. We find that the hold-up problem can be completely solved in the case of cross investment and divided actions, whereas hold-up is more problematic in the case of cross investment and unified actions.

Our analysis utilizes mechanism-design techniques. With both the individual-action and public-action modeling approaches, analysis of the contractual problem centers on calculating the set of implementable value functions from just after the state is realized (before messages are sent). Formally, an implementable value function is the state-contingent continuation value that results in equilibrium for a given contract. Let V^{EPF} be the set of implementable value functions for the public-action model under ex-post renegotiation, and let V^{EP} be the corresponding set for the individual-action model. We also examine the case of interim renegotiation, where the parties can renegotiate only before sending messages, and let V^{I} be the set of implementable value functions for this case. Watson (2007) shows that, by their definitions, these three sets satisfy $V^{\text{EPF}} \subseteq V^{\text{EP}} \subseteq V^{\text{I}}$. In Watson's key example, the inclusion relations are strict so that $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^{\text{I}}$.

We provide simple tools to calculate the "punishment values" that determine the implementable sets for the class of relationships we analyze here. Our first theorem establishes that the inequalities $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^{\text{I}}$ always hold. In particular, in the important setting of ex post renegotiation described above, limiting attention to forcing contracts (studying V^{EPF} rather than V^{EP}) reduces the range of state-contingent continuation values. This makes it more difficult to give the investing party the incentive to invest at the beginning of the relationship. However, this does not mean that a more efficient outcome can always be achieved when actions are modeled as individual, because efficiency depends on what region of the implementable-value set is relevant for giving appropriate investment incentives. That is, in some examples we have $V^{\text{EP}} \neq V^{\text{EPF}}$ but these sets coincide where it matters to induce optimal investment. Our second theorem establishes that V^{EP} contains functions that hold fixed the value of the player with the trade action (and give the other player the full value of the relationship minus this constant). This result is the basis for our insights on the relation between the investment and trade technologies.

In the class of trade technologies that we study here, a single player (player 1, Al above) has the trade action. The key economic assumption we make is that player 1's utility

is supermodular as a function of the state and trade action. That is, this player's marginal value of the trade action is monotone in the state. Our results generalize to settings in which both players have trade actions. Our other assumptions are weak technical conditions that guarantee well-defined maxima, non-trivial settings, and the like. We argue that these conditions are likely to hold in a wide range of applications and that they are consistent with what is typically assumed in the literature. Public-action models obviously do not identify aspects of the technology of trade, although verbal accounts sometimes do.

The rest of the paper proceeds as follows. In the next section we provide the details of the model. Section 2 provides an overview of the basic analytical tools, which mostly restates material in Watson (2007). Section 3 contains our first result. Section 4 contains our second result; there we provide a detailed analysis of the interaction of the trade and investment technologies in the context of cross investment and hold up. The Conclusion includes a discussion of the case of durable trading opportunities. Some of the technical material and proofs are contained in the appendices.

1   The Theoretical Framework

We look at the same class of contracting problems and use the same notation as Watson (2007), except that we add a bit of structure on the trade technology to focus our analysis. In particular, we examine the case in which a single player has a trade action. Throughout the paper, we use the convention of labeling the player with the trade action as "player 1" and we call the other "player 2." These two players are the parties engaged in a contractual relationship with a non-durable trading opportunity and external enforcement. Their relationship has the following payoff-relevant components, occurring in the order shown:

The *state of the relationship* θ. The state represents unverifiable events that are assumed to happen early in the relationship. The state may be determined by individual investment decisions and/or by random occurrences, depending on the setting. When the state is realized, it becomes commonly known by the players; however, it cannot be verified to the external enforcer. Let Θ denote the set of possible states.

The *trade action* a. This is an individual action chosen by player 1 that determines whether and how the relationship is consummated. The trade action is commonly observed by the players and is verifiable to the external enforcer. Let A be the set of feasible trade actions.

The *monetary transfers* $t = (t_1, t_2)$. Here t_i denotes the amount given to player i, for $i = 1, 2$, where a negative value represents an amount taken from this player. Transfers are compelled by the external enforcer, who is not a strategic player but, rather, who
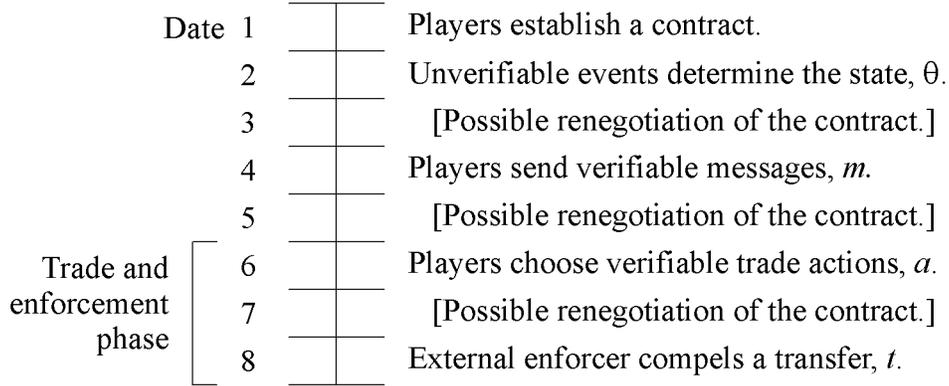
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Date	1		Players establish a contract.
	2		Unverifiable events determine the state, θ.
	3		[Possible renegotiation of the contract.]
	4		Players send verifiable messages, m.
	5		[Possible renegotiation of the contract.]
Trade and	6		Players choose verifiable trade actions, a.
enforcement	7		[Possible renegotiation of the contract.]
phase	8		External enforcer compels a transfer, t.

Figure 1: Time line of the contractual relationship.

behaves as directed by the contract of players 1 and 2.[4] Assume $t_1 + t_2 \leq 0$.

We assume that the players' payoffs are additive in money and are thus defined by a function $u : A \times \Theta \rightarrow \mathbf{R}^2$. In state θ, with trade action a and transfer t, the payoff vector is $u(a, \theta) + t$. We assume that u is bounded and that the maximal joint payoff, $\max_{a \in A}[u_1(a, \theta) + u_2(a, \theta)]$, exists for every θ.

In addition to the payoff-relevant components of their relationship, we assume that the players can communicate with the external enforcer using public, verifiable messages. Let $m = (m_1, m_2)$ denote the profile of messages that the players send and let M_1 and M_2 be the sets of feasible messages. The sets M_1 and M_2 will be endogenous in the sense that they are specified by the players in their contract.

Figure 1 shows the time line of the contractual relationship. At even-numbered dates through Date 6, the players make joint observations and they make individual decisions—jointly observing the state at Date 2, sending verifiable messages at Date 4, and selecting the trade actions at Date 6. At Date 8, the external enforcer compels transfers.

At odd-numbered dates, the players make joint contracting decisions—establishing a contract at Date 1 and possibly renegotiating it later. The contract has an externally-enforced component consisting of (i) feasible message spaces M_1 and M_2 and (ii) a *transfer function* $y : M \times A \rightarrow \mathbf{R}^2$ specifying the transfer t as a function of the verifiable items m and a. That is, having seen m and a, the external enforcer compels transfer $t = y(m, a)$. The contract also has a self-enforced component, which specifies how the players coordinate their behavior for the times at which they take individual actions. Renegotiation of the contract amounts to replacing the original transfer function y with some new function y', in which case y' is the one submitted to the external enforcer at Date 8.

The players' individual actions at Dates 4 and 6 are assumed to be consistent with sequential rationality; that is, each player maximizes his expected payoff, conditional on what

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[4]That the external enforcer's role is limited to compelling transfers is consistent with what courts do in practice.

occurred earlier and on what the other player does, and anticipating rational behavior in the future. The joint decisions (initial contracting and renegotiation at odd-numbered periods) are assumed to be consistent with a cooperative bargaining solution in which the players divide surplus according to fixed bargaining weights π_1 and π_2 for players 1 and 2, respectively. The bargaining weights are nonnegative, sum to one, and are written $\pi = (\pi_1, \pi_2)$. Surplus is defined relative to a disagreement point, which is given by an equilibrium in the continuation in which the externally enforced component of the contract has not been altered.[5] The effect of the renegotiation opportunity at Date 7 is to constrain transfers to be "balanced" — that is, satisfying

$$t \in \mathbf{R}_0^2 \equiv \{t' \in \mathbf{R}^2 \mid t_1' + t_2' = 0\}.$$

Thus, we will simply assume that transfers are balanced and then otherwise ignore Date 7.

A *(state-contingent) value function* is a function from Θ to $\mathbf{R}^2$ that gives the players' expected payoff vector from the start of a given date, as a function of the state. Such a value function represents the continuation values for a given outstanding contract and equilibrium behavior. We shall focus on continuation values from the start of Date 3, because these determine the players' incentives to invest at Date 2. Thus, our chief objective is to characterize the set of *implementable value functions* from the start of Date 3. A value function v for Date 3 is implementable if there is a contract that, if formed at Date 1, would lead to continuation value $v(\theta)$ in state θ from the start of Date 3, for every $\theta \in \Theta$.

Technology of Trade and Related Literature

Because the trade action a is assumed to be taken by player 1, we have specified here an individual-action model. A public-action model, in contrast, would abstract by treating the trade action a as something that the external enforcer directly selects. Watson (2007) shows that specifying a public-action model is equivalent to examining the individual-action model but limiting attention to "forcing contracts," as described in the next section. Also note that, so far, we are not explicitly including any specific investment technology in the model; that is, we are not describing the individual investment actions that determine the state. We leave them out for now because our first result concerns only how the trade technology is modeled. In Section 4 we investigate the interaction between the technology of trade and the technology of investment; there we add details on the investment phase of the contractual relationship.

Much of the recent contract-theory literature focuses on public-action mechanism-design models. For instance, Che and Hausch (1999), Hart and Moore (1999), Maskin and Moore (1999), Segal (1999), and Segal and Whinston (2002) have basically the same set-up as we do except that their models treat trade actions as public (collapsing together the

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[5]The rationality conditions identify a *contractual equilibrium*; see Watson (2004) for notes on the relation between "cooperative" and "noncooperative" approaches to modeling negotiation.

trade action and enforcement phase).[6] In some related papers, the verbal description of the contracting environment identifies individuals who take the trade actions, but the actions are effectively modeled as public due to an implicit restriction to forcing contracts. In some cases, such as with the contribution of Edlin and Reichelstein (1996), simple contracts (or breach remedies) are sufficient to achieve an efficient outcome and so the restriction does not have efficiency consequences.[7]

Examples of individual-action models in the literature, among others, are the articles of Hart and Moore (1988), MacLeod and Malcomson (1993), and Nöldeke and Schmidt (1995).[8] Also relevant is the work of Myerson (1982, 1991), whose mechanism-design analysis nicely distinguishes between inalienable individual and public actions (he uses the term "collective choice problem" to describe public-action models). Most closely related to our work is that of Evans (2006, 2008), who emphasizes how efficient outcomes can be achieved by conditioning external enforcement on costly individual actions. Evans (2006) examines general mechanism-design problems; Evans (2008), which we discuss more in the Conclusion, examines contracting problems with specific investment and durable trading opportunities. Related as well is the work of Lyon and Rasmussen (2004), which also shares the theme of Watson (2007), and the recent work of Boeckem and Schiller (2008) and Ellman (2006).

In classifying the related literature, another major distinction to make is between models with cross investment and models with "own investment." In the latter case, investment enhances the investing party's benefit of trade. We discuss this distinction in more detail in Section 4. Since the hold-up problem is more problematic in the case of cross investment (and there the distinction between public- and individual-action modeling is critical), Section 4 concentrates on the cross investment case.

2 Implementable Value Functions

In this section, we analyze equilibrium behavior and characterize the set of value functions from Date 3 that can be implemented by choice of contract. Characterizing this implementable set is the key to determining what can be achieved in specific applications. For example, consider a setting with specific investment, where one or both of the players makes an investment at Date 2 that influences the state. In this setting, the players' investment incentives follow directly from the value function that the parties' contract implies for Date 3. If a player can make an investment that increases the state variable, then she will have the incentive to do so only if her Date 3 continuation value increases sharply enough

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[6]Aghion, Dewatripont, and Rey (1994) is another example. The more recent entries by Roider (2004) and Guriev (2003) have the same basic public-action structure.

[7]Stremitzer (2009) elaborates on Edlin and Reichelstein (1996) by examining the informational requirements of standard breach remedies (specifically, partially verifiable investments).

[8]Hart and Moore (1988) have partial verifiability of trade.

in the state (so she is rewarded for making the investment).

Much of the analysis in this section repeats material in Watson (2007), so we keep it brief here and ask the reader to see Watson (2007) for more details. The culmination of the basic analysis here are some simple characterization results from Watson (2007), which we build upon in the subsequent sections.

The set of implementable value functions depends on whether renegotiation is possible at Dates 3 or 5.[9] We will examine two cases: *ex post renegotiation*, where the parties can renegotiate at Date 5, and *interim renegotiation*, where the parties can renegotiate at Date 3 but cannot do so at Date 5.[10] We can characterize the implementable value functions by backward induction, starting with Date 6 where player 1 selects the trade action.

State-Contingent Values from Date 6

To calculate the value functions that are supported from Date 6 (the "trade and enforce-ment phase" shown in Figure 1), we can ignore the payoff-irrelevant messages sent earlier (or equivalently, fix a message profile from Date 4) and simply write the externally en-forced transfer function as $\hat{y} : A \to \mathbf{R}^2$. That is, $\hat{y}$ gives the monetary transfer as a function of player 1's trade action.

Given the state θ, $\hat{y}$ defines a *trading game*, in which player 1 selects an action $a \in A$ and the payoff vector is then $u(a, \theta) + \hat{y}(a)$. Focusing on pure strategies, we let $\hat{a}(\theta)$ denote the action chosen by player 1 in state θ. This specification is rational for player 1 if, for every $\theta \in \Theta$, $\hat{a}$ maximizes $u_1(a, \theta) + \hat{y}_1(a)$ by choice of a. The state-contingent payoff vector from Date 6 is then given by the *outcome function* $w : \Theta \to \mathbf{R}^2$ defined by

$$w(\theta) \equiv u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta)). \tag{1}$$

Let W denote the set of supportable outcome functions. That is, $w \in W$ if and only if there are functions $\hat{y}$ and $\hat{a}$ such that $\hat{a}$ is rational for player 1 and, for every $\theta \in \Theta$, Equation 1 holds.

Public-Action Modeling and Forcing Contracts

Remember that we have specified an individual-action model, where player 1 takes the trade action at Date 6. In the public-action variant of the model, the trade action would be taken directly by the external enforcer. Since the enforcer does not observe the state, *the trade action must be constant in the state*, conditional on the verifiable items determined in earlier periods (the contract and messages). Thus, the public-action model is equivalent to the individual-action model with the restriction to *forcing contracts*, which, for any given

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[9]As noted earlier, we do not need to explicitly model the Date 2 investment technology in order to cal-culate the set of implementable value functions from Date 3. In Section 4 we analyze specific investment technologies and the hold-up problem.

[10]When ex-post renegotiation is allowed, there is no further constraint imposed by allowing renegotiation at Date 3, so we don't have to look at a separate case of renegotiation allowed at both Dates 3 and 5.

message profile, prescribe that player 1 select a particular trade action. More precisely, a forcing contract specifies a large transfer from player 1 to player 2 in the event that player 1 does not take his contractually-prescribed action; this transfer is sufficiently large to give player 1 the incentive to select the prescribed action in every state.

For example, holding the message profile fixed, the transfer function $\hat{y}$ defined as follows will force player 1 to select action a^* and impose the transfer t^* (as though the external enforcer chose these in a public-action model):

> Let L be such that $L > \sup_{a,\theta} u_1(a, \theta) - \inf_{a,\theta} u_1(a, \theta)$. Then define $\hat{y}(a^*) \equiv t^*$ and, for every $a \neq a^*$, set $\hat{y}(a) \equiv t^* + (-L, L)$.

We use the term *forcing* for any transfer function that, given the message profile, induces player 1 to select the same trade action over all of the states.[11] Let W^F be the subset of outcomes that can be supported using forcing contracts. It is easy to see that $w \in W^F$ if and only if there is a trade action a^* and a transfer vector t^* such that $w(\theta) = u(a^*, \theta) + t^*$ for all $\theta \in \Theta$. We can compare individual-action and public-action models by determining whether the restriction to forcing contracts implies a significant constraint on the set of implementable value functions.

State-Contingent Values from Date 5

So far, we have characterized the set of supportable state-contingent values from the start of Date 6, which is the outcome set W in the case of the individual-action model (unrestricted contracts) and is the set W^F in the case of the public-action model (restriction to forcing contracts). We next step back to Date 5. If there is no opportunity for ex post renegotiation, then nothing happens at Date 5 and so W and W^F are the supported state-contingent value sets from the start of Date 5 as well.

On the other hand, if ex post renegotiation is allowed, then at Date 5 the players have an opportunity to discard their originally specified contract y and replace it with another, y'. The original contract y would have led to a particular outcome w given the Date 4 message profile; by picking a new contract y', the players are effectively choosing a new outcome function w', which is freely selected from the set W or the set W^F depending on how the trade action is modeled. If the outcome w would be inefficient given the realized state and message profile, the players will renegotiate to select an efficient outcome w'. We assume that the players divide the renegotiation surplus according to fixed bargaining weights π_1 and π_2. The generalized Nash bargaining solution has this representation. Dividing the surplus in this way is feasible because W and W^F are closed under constant transfers.

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[11]One could add a public randomization device to the model for the purpose of achieving randomization over trade actions using forcing contracts. Allowing such randomization does not expand the set of implementable value functions here.

To state the bargaining solution more precisely, we let $\gamma(\theta)$ denote the maximal joint payoff that can be obtained in state θ:

$$\gamma(\theta) \equiv \max_{a \in A} [u_1(a, \theta) + u_2(a, \theta)]. \tag{2}$$

Clearly, we have $\gamma(\theta) = \max_{w \in W^F} [w_1(\theta) + w_2(\theta)]$ because the trade action that solves the maximization problem in Equation 2 can be specified in a forcing contract to yield the desired outcome. If the original contract would lead to outcome w in state θ, then the *renegotiation surplus* is

$$r(w, \theta) \equiv \gamma(\theta) - w_1(\theta) - w_2(\theta).$$

The bargaining solution implies that the players settle on a new outcome in which the payoff vector in state θ is $w(\theta) + \pi r(w, \theta)$.

An *ex post renegotiation outcome* is a state-contingent payoff vector that results when, in every state, the players renegotiate from a fixed outcome in W. That is, a value function z is an ex post renegotiation outcome if and only if there is an outcome $w \in W$ such that $z(\theta) = w(\theta) + \pi r(w, \theta)$ for every $\theta \in \Theta$. Let Z denote the set of ex post renegotiation outcomes. Note that all elements of Z are efficient in every state; also, Z and W are generally not ranked by inclusion. If trade actions are treated as public (and so attention is limited to forcing contracts) then the set of ex post renegotiation outcomes contains only the value functions of the form $z = w + \pi r(w, \cdot)$ with the constraint that $w \in w^F$. Let Z^F denote the set of ex post renegotiation outcomes under forcing contracts.

With ex post renegotiation, the set of supportable state-contingent values from the start of Date 5 is Z in the case of the individual-action model and is Z^F in the case of the public-action model. We will be a bit loose with terminology and refer to functions in Z and Z^F, in addition to functions in W and W^F, simply as outcomes.

State-Contingent Values from Dates 4 and 3

Analysis of contract selection and incentives at Date 4 can be viewed as a standard mechanism-design problem. The players' contract is equivalent to a mechanism that maps messages sent at Date 4 to outcomes induced in the trade and enforcement phase (possibly renegotiated at Date 5). The revelation principle applies in the following sense. We can restrict attention to direct-revelation mechanisms, each of which is defined by (i) a message space $M \equiv \Theta^2$ and (ii) a function that maps Θ^2 to the relevant outcome set that gives the state-contingent value functions from the start of Date 5. The outcome set is either W, W^F, Z, or Z^F, depending on whether ex post renegotiation and/or non-forcing contracts are allowed. We can concentrate on Nash equilibria of the mechanism in which the parties report truthfully in each state.[12]

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[12]The revelation principle usually requires a public randomization device to create lotteries over outcomes (or that the outcome set is a mixture space), but it is not needed here.

Let us write $\psi^{\theta_1\theta_2}$ for the outcome that the mechanism prescribes when player 1 reports the state to be θ_1 and player 2 reports the state to be θ_2. Note that, in any given state θ (the actual state that occurred), the mechanism implies a "message game" with strategy space Θ^2 and payoffs given by $\psi^{\theta_1\theta_2}(\theta)$ for each strategy profile (θ_1, θ_2). For truthful reporting to be a Nash equilibrium of this game, it must be that $\psi_1^{\theta\theta}(\theta) \geq \psi_1^{\tilde\theta\theta}(\theta)$ and $\psi_2^{\theta\theta}(\theta) \geq \psi_2^{\theta\tilde\theta}(\theta)$ for all $\tilde\theta \in \Theta$.

We proceed using standard techniques for mechanism design with transfers, following Watson (2007). The key step is observing that, for any two states θ and θ', the outcome specified for the "off-diagonal" message profile (θ', θ) must be sufficient to simultaneously (i) dissuade player 1 from declaring the state to be θ' when the state is actually θ and (ii) discourage player 2 from declaring "θ" in state θ'. Thus, we require

$$\psi_1^{\theta\theta}(\theta) \geq \psi_1^{\theta'\theta}(\theta) \quad \text{and} \quad \psi_2^{\theta'\theta'}(\theta') \geq \psi_2^{\theta'\theta}(\theta').$$

Because the outcome sets are closed under constant transfers, we can choose the outcome to effectively raise or lower $\psi_1^{\theta'\theta}$ and $\psi_2^{\theta'\theta}$ while keeping the sum constant. Thus, a sufficient condition for these two inequalities is that the sum of the two holds. Letting $\psi \equiv \psi^{\theta\theta}$ and $\psi' \equiv \psi^{\theta'\theta'}$, we thus have the following necessary condition for implementing outcome ψ in state θ and outcome ψ' in state θ':

(IC) There exists an outcome $\hat\psi$ satisfying $\psi_1(\theta) + \psi_2'(\theta') \geq \hat\psi_1(\theta) + \hat\psi_2(\theta')$.

This condition, applied to all ordered pairs (θ, θ'), is necessary and sufficient for implementation. The sum $\hat\psi_1(\theta) + \hat\psi_2(\theta')$ is called the *punishment value* corresponding to the ordered pair (θ, θ'). The punishment value plays a central role in our analysis. Lower punishment values imply a greater set of implementable outcomes.

If interim renegotiation is not allowed, then the analysis above completely determines the implementable set of value functions from Date 3. Allowing interim renegotiation has the effect of requiring each "on-diagonal" outcome to be efficient in the relevant state; that is, for each θ we need $\psi^{\theta\theta}$ to be efficient in this state. In the case of ex post renegotiation, allowing interim renegotiation entails no further constraint because every outcome in Z is efficient in every state.

It is also the case that without ex post renegotiation, W and W^{F} yield the same set of implementable value functions from Date 3. In other words, a restriction to forcing contracts does not reduce the implementable set in the case of interim renegotiation.[13] Therefore, we have three settings to compare: unrestricted contracts with ex post renegotiation, forcing contracts (public-actions) with ex post renegotiation, and forcing contracts with interim (but not ex post) renegotiation. We denote the implementable value functions for these three settings by, respectively, V^{EP}, V^{EPF}, and V^{I}.

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[13] Watson (2007), Lemma 3.

A value function $v : \Theta \to \mathbf{R}^2$ is called *efficient* if $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ for every $\theta \in \Theta$. The following theorem summarizes Watson's (2007) characterization of V^{EP}, V^{EPF}, and V^{I}:

Result 1 [Watson 2007]: *Consider any value function* $v : \Theta \to \mathbf{R}^2$.

- Implementation with Interim Renegotiation: v *is an element of* V^{I} *if and only if* v *is efficient and, for every pair of states* θ *and* θ', *there is an outcome* $\hat{w} \in W^{\mathrm{F}}$ *such that* $v_1(\theta) + v_2(\theta') \geq \hat{w}_1(\theta) + \hat{w}_2(\theta')$.

- Implementation with Ex Post Renegotiation: v *is an element of* V^{EP} *if and only if* v *is efficient and, for every pair of states* θ *and* θ', *there is an outcome* $\hat{z} \in Z$ *such that* $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$.

- Implementation with Ex Post Renegotiation and Forcing Contracts: v *is an element of* V^{EPF} *if and only if* v *is efficient and, for every pair of states* θ *and* θ', *there is an outcome* $\hat{z} \in Z^{\mathrm{F}}$ *such that* $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$.

Furthermore, the sets V^{EP}, V^{EPF}, *and* V^{I} *are closed under constant transfers.*

The following result, which follows from the characterization of the implementable sets, collects three of Watson's (2007) theorems.[14]

Result 2 [Watson 2007]: *The implementable sets are weakly nested in that* $V^{\mathrm{EPF}} \subseteq V^{\mathrm{EP}} \subseteq V^{\mathrm{I}}$. *Furthermore,* $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ *if and only if, for every pair of states* $\theta, \theta' \in \Theta$ *and every* $\hat{z} \in Z$, *there is an ex post renegotiation outcome* $\tilde{z} \in Z^{\mathrm{F}}$ *such that* $\tilde{z}_1(\theta) + \tilde{z}_2(\theta') \leq \hat{z}_1(\theta) + \hat{z}_2(\theta')$. *Likewise,* $V^{\mathrm{EP}} = V^{\mathrm{I}}$ *if and only if, for all* $\theta, \theta' \in \Theta$ *and every* $\hat{w} \in W^{\mathrm{F}}$, *there is an ex post renegotiation outcome* $\hat{z} \in Z$ *such that* $\hat{z}_1(\theta) + \hat{z}_2(\theta') \leq \hat{w}_1(\theta) + \hat{w}_2(\theta')$.

To summarize, we have thus far analyzed the players' behavior at the various dates in the contractual relationship, leading to a simple characterization of implementable value functions from Date 3. The characterization is in terms of the minimum punishment values for each pair of states, which yields a way of relating the implementable sets for the cases of interim renegotiation, ex post renegotiation, and ex post renegotiation and forcing contracts. We next turn to investigate the relation more deeply.

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[14]Watson's (2006) Lemma 1 provides some of the supporting analysis (which was not explained fully in the relevant proof in Watson 2007). This lemma establishes that, for any given ordered pair of states θ and θ' and any supportable outcome ψ, there exists an implementable value function v for which $v_1(\theta) + v_2(\theta') = \psi_1(\theta) + \psi_2(\theta')$. Because the minimum punishment values exists, in each case we can let ψ equal the outcome that attains the minimum.

3 Robustness for a Class of Trade Technologies

Watson (2007) provides an example for which $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^{\text{I}}$. The example demonstrates the importance of explicitly accounting for individual trade actions. This is because, in the realistic case that a trade action is taken by one of the contracting parties and the parties can renegotiate just before the trade action is taken, no public-action model accurately represents the scope of contracts. By not considering how trade actions can be used as options, a public-action model with ex post renegotiation understates the set of implementable value functions. On the other hand, a public-action model with interim renegotiation overstates the set of implementable value functions. Our main objective is to examine the robustness of this conclusion. We consider the wide class of contractual relationships that satisfy the following assumptions.

Assumption 1: The sets A and Θ are compact subsets of **R** and contain at least two elements, and $u_1(\cdot, \theta)$ and $u_2(\cdot, \theta)$ are continuous functions of a for every $\theta \in \Theta$.

Define $U(a, \theta) \equiv u_1(a, \theta) + u_2(a, \theta)$, which is the joint value of the contractual relationship in state θ if trade action a is selected. Define $\underline{a} \equiv \min A, \overline{a} \equiv \max A, \underline{\theta} \equiv \min \Theta,$ $\overline{\theta} \equiv \max \Theta$. Assumption 1 guarantees that these exist and that $\max_{a \in A} U(a, \theta)$ exists; that is, the efficient trade action is well-defined for each state. We make a slightly stronger assumption on $U(\cdot, \theta)$:

Assumption 2: $U(\cdot, \theta)$ is strictly quasiconcave for every $\theta \in \Theta$.

Define $a^*(\theta) \equiv \text{argmax}_{a \in A} U(a, \theta)$, so we have $U(a^*(\theta), \theta) = \gamma(\theta)$. Assumption 2 ensures that $a^*(\theta)$ is unique for each $\theta \in \Theta$.

Assumption 3: u_1 is supermodular, meaning that $u_1(a, \theta) - u_1(a', \theta) \geq u_1(a, \theta') - u_1(a', \theta')$ whenever $a \geq a'$ and $\theta \geq \theta'$.

Assumption 4: There exist states $\theta^1, \theta^2 \in \Theta$ such that $\theta^1 > \theta^2$ and either $U(\underline{a}, \theta^2) < U(\overline{a}, \theta^2)$ or $U(\underline{a}, \theta^1) > U(\overline{a}, \theta^1)$.

Assumption 5: Player 1's bargaining weight is positive: $\pi_1 > 0$.

Assumptions 1, 2, 4, and 5 are mild technical assumptions. Assumptions 1 and 2 give us a convenient and familiar technical structure to deal with. Assumption 4 basically removes a knife-edge case concerning the relative joint values of the extreme trade actions in the various states. For instance, if Θ has more than two elements and $U(\underline{a}, \theta) \neq U(\overline{a}, \theta)$ for some a strictly between $\underline{a}$ and $\overline{a}$, then Assumption 4 is satisfied. If Θ has just two elements ($\underline{\theta}$ and $\overline{\theta}$), then Assumption 4 requires that either $\underline{a}$ is the efficient trade action in the high state or $\overline{a}$ is the efficient trade action in the low state.[15]

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[15] In Watson's (2007) example, which has two states and two trade actions, $\overline{a}$ is the efficient trade action in both states.

Assumption 3 puts some structure on the payoff of player 1, the player with the trade action: Without considering transfers, player 1's marginal value of increasing his trade action weakly rises with the state. In other words, higher trade actions are weakly more attractive to him as the state increases. Note that if in a given application u_1 satisfies submodularity, one can redefine the trade action to be $-a$ and then Assumption 3 would be satisfied.

Many interesting examples in the studied in the literature satisfy these assumptions. For instance, consider a buyer/seller relationship in which a is the number of units of an intermediate good to be transferred from the seller to the buyer. The buyer's benefit of obtaining a units in state θ is $B(a, \theta)$. The seller's cost of production and delivery is $C(a, \theta)$. Suppose, as one would typically do, that B is increasing and concave in a and that C is increasing and convex in a. If a is the buyer's action (he selects how many units to install, for example), then the buyer would be player 1 and so we have $u_1 \equiv B$ and $u_2 \equiv -C$. If the seller chooses a (she decides how many units to deliver, say), then the seller is player 1 and so we have $u_1 \equiv -C$ and $u_2 \equiv B$. In either case, Assumptions 1 and 2 are satisfied. Assumption 3 adds the weak monotonicity requirement on the payoff of the player who selects a.

We have the following robustness result:

Theorem 1: *Consider any contractual relationship that satisfies Assumptions 1-5. The sets of implementable value functions in the cases of unrestricted contracts with ex post renegotiation, forcing contracts with ex post renegotiation, and interim renegotiation are all distinct. That is, $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}} \neq V^{\mathrm{I}}$.*

The analysis underlying Theorem 1 amounts to characterizing and comparing the minimum punishment values that can be supported for each of the settings of interest. Recall that the punishment value for the ordered pair (θ, θ') is the value $\psi_1(\theta) + \psi_2(\theta')$, where ψ is the outcome specified in the message game when player 1 reports the state to be θ' and player 2 reports the state to be θ. Lower punishment values serve to relax incentive conditions, so to completely characterize the sets of implementable value functions we must find the minimum punishment values. We let P^{I}, P^{EP}, and P^{EPF} denote the minimum punishment values for the settings of interim renegotiation, ex post renegotiation, and ex post renegotiation and forcing contracts, respectively:

$$
\begin{aligned}
P^{\mathrm{I}}(\theta, \theta') &\equiv \min_{w \in W^{\mathrm{F}}} w_1(\theta) + w_2(\theta'), \\
P^{\mathrm{EP}}(\theta, \theta') &\equiv \min_{\hat{z} \in Z} \hat{z}_1(\theta) + \hat{z}_2(\theta'), \\
P^{\mathrm{EPF}}(\theta, \theta') &\equiv \min_{\hat{z} \in Z^{\mathrm{F}}} \hat{z}_1(\theta) + \hat{z}_2(\theta').
\end{aligned}
$$

Our assumptions on the trade technology guarantee that these minima exist.

From Result 2, we know that Theorem 1 is equivalent to saying that there exist states $\theta, \theta' \in \Theta$ such that $P^{\mathrm{I}}(\theta, \theta') < P^{\mathrm{EP}}(\theta, \theta')$ and there exist (possibly different) states

$\theta, \theta' \in \Theta$ such that $P^{\text{EP}}(\theta, \theta') < P^{\text{EPF}}(\theta, \theta')$. Thus, to prove Theorem 1, we examine the punishment values achieved by various contractual specifications in the different settings. We develop some elements of the proof in the remainder of this section; Appendix A contains the rest of the analysis. We shall focus in this section on the relation between V^{EPF} and V^{EP}. The analysis of the relation between V^{EP} and V^{I} is considerably simpler and is wholly contained in Appendix A.

We will establish $P^{\text{EP}} < P^{\text{EPF}}$ by comparing the punishment values implied by (i) the outcome in which player 1 would be forced to take a particular trade action, and (ii) a related non-forcing specification in which player 1 would be given the incentive to select some action a in state θ and a different action a' in state θ'. We derive conditions under which a and a' can be arranged to strictly lower the punishment value for (θ, θ'), relative to the best forcing case. We then find states θ^1 and θ^2 such that the conditions must hold for at least one of the ordered pairs (θ^1, θ^2) and (θ^2, θ^1).

To explore the possible outcomes in the cases of ex post renegotiation, consider player 1's incentives at Date 6. For any given transfer function $\hat{y}$, the following are necessary conditions for player 1 to select trade action a in state θ and action a' in state θ':

$$
\begin{aligned}
u_1(a, \theta) + \hat{y}_1(a) &\geq u_1(a', \theta) + \hat{y}_1(a') \quad \text{and} \\
u_1(a', \theta') + \hat{y}_1(a') &\geq u_1(a, \theta') + \hat{y}_1(a) \quad .
\end{aligned}
\tag{3}
$$

Transfer function $\hat{y}$ can be specified so that player 1 is harshly punished for selecting any trade action other than a or a'. Then, in every state, either a or a' maximizes player 1's payoff from Date 6. Thus, we have:

Fact 1: *Consider two states $\theta, \theta' \in \Theta$ and two trade actions $a, a' \in A$. Expression 3 is necessary and sufficient for the existence of a transfer function $\hat{y}: A \to \mathbf{R}_0^2$ (defined over all trade actions) such that player 1's optimal trade action in state θ is a and player 1's optimal trade action in state θ' is a'.*

Summing the inequalities of Expression 3, we see that there are values $\hat{y}(a), \hat{y}(a') \in \mathbf{R}_0^2$ that satisfy (3) if and only if

$$
u_1(a, \theta) - u_1(a', \theta) \geq u_1(a, \theta') - u_1(a', \theta').
\tag{4}
$$

Assumption 3 then implies:

Fact 2: *If $\theta > \theta'$ then $a \geq a'$ implies Inequality 4. If $\theta < \theta'$ then $a \leq a'$ implies Inequality 4.*

Note that Fact 2 gives sufficient conditions. In the case in which $u_1(\cdot, \cdot)$ is strictly supermodular (replacing weak inequalities in Assumption 3 with strict inequalities), player 1 can only be given the incentive to choose greater trade actions in higher states.

For any two states $\theta, \theta' \in \Theta$, define

$$E(\theta, \theta') \equiv \{(a, a') \in A \times A \mid \text{Inequality 4 is satisfied.}\}.$$

Also, for states $\theta, \theta' \in \Theta$ and trade actions $a, a' \in A$ with $(a, a') \in E(\theta, \theta')$, define

$$Y(a, a', \theta, \theta') \equiv \{\hat{y} : A \to \mathbf{R}_0^2 \mid \text{Condition 3 is satisfied.}\}.$$

Condition 3, combined with the identity $\hat{y}_1 = -\hat{y}_2$, implies:

Fact 3: *For any $\theta, \theta' \in \Theta$ and $a, a' \in A$, with $(a, a') \in E(\theta, \theta')$, we have*

$$\min_{\hat{y} \in Y(a, a', \theta, \theta')} \hat{y}_1(a) + \hat{y}_2(a') = u_1(a', \theta) - u_1(a, \theta).$$

Using the definition of the set W (recall Expression 1 on page 8), any given $w \in W$ can be written in terms of the trade actions and transfers that support it. We have

$$w(\theta) = u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta))$$

and

$$w(\theta') = u(\hat{a}(\theta'), \theta') + \hat{y}(\hat{a}(\theta')),$$

where $\hat{a}$ gives player 1's choice of trade action as a function of the state and $\hat{y}$ is the transfer function that supports w.

For any state $\tilde{\theta}$ and trade action $\tilde{a}$, define $R(\tilde{a}, \tilde{\theta})$ to be the renegotiation surplus if, without renegotiation, player 1 would select $\tilde{a}$. That is, $R(\tilde{a}, \tilde{\theta}) = U(a^*(\tilde{\theta}), \tilde{\theta}) - U(\tilde{a}, \tilde{\theta})$. Combining the expressions for w in the previous paragraph with Fact 1 and the definition of ex post renegotiation outcomes, we obtain:

Fact 4: *Consider any two states $\theta, \theta' \in \Theta$ and let α be any number. There is an ex post renegotiation outcome $z \in Z$ that satisfies $z_1(\theta) + z_2(\theta') = \alpha$ if and only if there are trade actions $a, a' \in A$ and a transfer function $\hat{y}$ such that $(a, a') \in E(\theta, \theta')$, $\hat{y} \in Y(a, a', \theta, \theta')$, and*

$$\alpha = u_1(a, \theta) + \hat{y}_1(a) + \pi_1 R(a, \theta) + u_2(a', \theta') + \hat{y}_2(a') + \pi_2 R(a', \theta'). \quad (5)$$

In the last line, the first three terms are $w_1(\theta)$ plus player 1's share of the renegotiation surplus in state θ, totaling $z_1(\theta)$. The last three terms are $w_2(\theta')$ plus player 2's share of the renegotiation surplus in state θ', totaling $z_2(\theta')$.

Finding the best (minimum) punishment value for states θ and θ' means minimizing $\hat{z}_1(\theta) + \hat{z}_2(\theta')$ by choice of $\hat{z} \in Z$. For now, holding fixed the trade actions a and a' that player 1 is induced to select in states θ and θ', let us minimize the punishment value by choice of $\hat{y} \in Y(a, a', \theta, \theta')$. To this end, we can use Fact 3 to substitute for $\hat{y}_1(a) + \hat{y}_2(a')$

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in Expression 5. This yields the punishment value for trade actions a and a' in states θ and θ', respectively, written

$$\lambda(a, a', \theta, \theta') \equiv u_1(a', \theta) + \pi_1 R(a, \theta) + u_2(a', \theta') + \pi_2 R(a', \theta'). \tag{6}$$

Next, we consider the step of minimizing the punishment value by choice of the trade actions a and a', which gives us a useful characterization of $P^{\text{EP}}(\theta, \theta')$. Assumption 1 guarantees that $\lambda(a, a', \theta, \theta')$ has a minimum.

Fact 5: *The minimum punishment value in the setting of ex post renegotiation is characterized as follows:*

$$P^{\text{EP}}(\theta, \theta') = \min_{(a, a') \in E(\theta, \theta')} \lambda(a, a', \theta, \theta').$$

We obtain a similar characterization of the minimal punishment value for the setting in which attention is restricted to forcing contracts. The characterization is exactly as in Fact 5 except with the additional requirement that $a = a'$ because forcing contracts compel the same action in every state.

Fact 6: *The minimum punishment value for the setting of forcing contracts and ex post renegotiation is characterized as follows:*

$$P^{\text{EPF}}(\theta, \theta') \equiv \min_{a \in A} \lambda(a, a, \theta, \theta').$$

Recall that proof of Theorem 1 requires us to establish that $P^{\text{EP}}(\theta, \theta') > P^{\text{EPF}}(\theta, \theta')$ for some pair of states $\theta, \theta' \in \Theta$. Appendix A finishes the analysis by exploring how one can depart from the optimal forcing specification in a way that strictly reduces the value $\lambda(a, a', \theta, \theta')$.

4   Hold Up and the Technologies of Trade and Investment

Although Theorem 1 is quite general, its implications for applied settings are more nuanced. For example, we could have $V^{\text{EP}} \neq V^{\text{EPF}}$ but, still, these sets could coincide where it matters to induce optimal incentives at Date 2. To determine whether this is the case, we must add structure to the model in order to specify exactly what occurs at Date 2 — that is, specify the technology of investment — and we then must examine how the technology of investment interacts with the technology of trade.

We shall limit attention to the standard setting of "specific investment with hold up," where one of the players makes an investment choice at Date 2 and this investment influences the state. The rest of the model is unchanged; we continue to denote as "player 1"

the party who has the trade action at Date 6. Also, we focus here on the case of ex post renegotiation, so the implementable set will be either V^{EP} or V^{EPF} depending on whether there is a restriction to forcing contracts (public-action model of trade).

Our analysis here has two objectives. The first is to compare public-action and individual-action models to see if a restriction to forcing contracts implies a restriction in the level of investment that can be supported — that is, whether differences between $V^{\text{EP}} \neq V^{\text{EPF}}$ really matter for ex ante investment incentives. The second objective is to find some conditions under which efficient investment and trade can be supported using general (non-forcing) contracts. We shall provide intuition and some partial analysis toward the first objective; this leads to a general result along the lines of the second objective.

The technology of investment includes a specification of (a) which player has the investment action and (b) how the players stand to benefit from the investment. The literature has demonstrated that forcing contracts can prevent the hold-up problem in the "own-investment" case, where the investing party obtains a large share of the benefit created by the investment.[16] Therefore we will concentrate on the case of cross investment, in which the main beneficiary of the investment is the non-investing party. To simplify the discussion, we will refer to the non-investing party as the *beneficiary* even though we generally allow for the investing party to garner a small portion of the benefits of his investment. We thus have two main cases to consider:

- **Unified case** – Player 1 has both the Date 2 investment action and the Date 6 trade action, and player 2 is the beneficiary.

- **Divided case** – Player 2 has the Date 2 investment action, player 1 has the Date 6 trade action, and player 1 is the beneficiary.

Let the investment choice be denoted $x \geq 0$. We assume that the investment imposes an immediate cost of x on the investor and that investment tends to raise the state. Specifically, θ is drawn from a distribution $G(x)$ that is increasing in x in the sense of first-order stochastic dominance. We will sometimes refer to the beneficiary's payoff as $B(a, \theta)$ and the investor's payoff as $C(a, \theta)$. We take these to be gross of investment cost (that is, they do not include $-x$ for the investor). In the divided case, we thus have $u_1 \equiv B$ and $u_2 \equiv C$. In the unified case, we have $u_1 \equiv C$ and $u_2 \equiv B$. Recalling that $\gamma(\theta) = U(a^*(\theta), \theta)$ is the maximum joint value in state θ, we see that the efficient level of investment x^* solves:

$$\max_{x \geq 0} \int \gamma(\theta) dG(x) - x.$$

We also add a bit of structure to the trade technology:

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[16]See, for example, Chung (1991), Rogerson (1992), Aghion, Dewatripont, and Rey (1994), Nldeke and Schmidt (1995), and Edlin and Reichelstein (1996). An exception is the "complexity/ambivalence" setting studied by Segal (1999), Hart and Moore (1999), and Reiche (2006).

Assumption 6: The lowest trade action is $\underline{a} \equiv 0$. Further, $u_1(0, \theta) = u_2(0, \theta) = 0$ for all θ.

The first part of this assumption (that $\underline{a} = 0$) is just a normalization. We interpret $a = 0$ as "no trade." The second part is the assumption that the no-trade action always yields zero to both players, gross of the investor's cost of investment. The *null contract* in our model is the contract that forces $a = \underline{a}$ regardless of the message profile. Che and Hausch (1999) have shown that when the investor receives a sufficiently small share of the benefits of the investment, the null contract is optimal among forcing contracts. We restrict attention here to environments in which this result holds, which allows us to concentrate on evaluating what non-forcing contracts can achieve.

Typically $x^* > 0$. Thus, letting i denote the investing party, we will want to implement a value function v so that $v_i(\theta)$ is increasing in θ to some particular extent. In this way, player i will be rewarded for investing.

The rest of this section has three parts. In the first subsection, we show how non-forcing contracts improve investment incentives in both the unified and divided cases when there is *pure cross investment*. In the second subsection, we discuss the case of *near pure cross investment* and we show that the results extend in the divided case but in the unified case they depend on how investment affects the investor's benefit/cost of trade. We thus find that the hold-up problem is lessened in the divided case but can persist in the unified case. This result is strengthened and formalized in the third subsection, where we provide the result that the hold-up problem can, in fact, be completely alleviated in the divided case.

Pure Cross Investment

We begin by examining the environment in which cross investment is *pure* so that the investing party receives none of the benefit of his investment. That is, the investor's trade utility $C(a, \theta)$ is constant in the state θ.

First consider the divided case where player 1 is the beneficiary (and has the trade action) and player 2 is the investor. Player 2's motivation to invest depends on making $v_2(\theta) - v_2(\theta')$ large for $\theta > \theta'$, which requires making $v_1(\theta) + v_2(\theta')$ small. Thus it is the punishment value $\hat{z}_1(\theta) + \hat{z}_2(\theta') = \lambda(a, a', \theta, \theta')$ that we need to minimize. Using Che and Hausch's result (that the optimal forcing contract is the null contract), we see that a non-forcing contract can provide better incentives for investment if and only if there exists $a, a' \in A$ such that

$$\lambda(a, a', \theta, \theta') < \lambda(0, 0, \theta, \theta').$$

This is equivalent to the following (see Appendix B for the derivation):

$$u_1(a', \theta) + u_2(a', \theta') - \pi_2 U(a', \theta') < \pi_1 U(a, \theta). \tag{7}$$

Supermodularity of player 1's utility function implies that we must have $a' \leq a$.[17] One

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[17] That is, using the terminology of the proof of Theorem 1, we need $(a, a') \in E(\theta, \theta')$, which requires $a \geq a'$.

non-forcing contract that will ensure that Expression 7 holds is $a' = 0$ and $a = a^*(\theta)$. This makes the left hand side zero, while the right hand side is positive by Assumption 2 as long as $a^*(\theta) > 0$. Thus, we can always find a non-forcing contract that provides investment incentives for player 2 that are stronger than those under the optimal forcing contract. While the null (forcing) contract would lead player 2 to under-invest, one can find a non-forcing contract that induces a more efficient investment level.

In the case of unified decision-making, the investing party chooses the action and is therefore denoted player 1. Investment is best motivated by making $v_1(\theta) - v_1(\theta')$ large for $\theta > \theta'$, requiring $v_1(\theta') + v_2(\theta)$ to be low. Thus we want the punishment value $\hat{z}_1(\theta') + \hat{z}_2(\theta) = \lambda(a', a, \theta', \theta)$ to be small. Here a non-forcing contract can provide better investment incentives if and only if there exists a, a' such that

$$\lambda(a', a, \theta', \theta) < \lambda(0, 0, \theta', \theta).$$

This is equivalent to the following (see Appendix B for the derivation):

$$u_1(a, \theta') + u_2(a, \theta) - \pi_2 U(a, \theta) < \pi_1 U(a', \theta') \tag{8}$$

Because player 1's utility is constant in the state (and thus not strictly supermodular), we are not bound by the constraint that a must be at least as high as a'. Therefore an implementable non-forcing contract that will ensure that Expression 8 holds is $a' = a^*(\theta')$ and $a = 0$. This makes the left hand side zero, while the right hand side is positive by Assumption 2 as long as $a^*(\theta') > 0$ and so a non-forcing contract can improve on the best forcing contract just as in the divided case.

Near Pure Cross Investment

Suppose now that the setting is close to pure cross investment in the sense that the investor's trade utility $C(a, \theta)$ depends only a bit on the state θ. For example, we could have $C(a, \theta) = \varepsilon a \theta$ where ε is a constant that is close to zero. Results will depend on whether ε is positive or negative.

If the investment is such that the beneficiary receives more than the total benefit created by the investment (so $\varepsilon < 0$), then the investor's trade utility becomes submodular in θ. In the divided case, this reinforces the incentives of the beneficiary to take higher actions in higher states and the analysis from the previous subsection goes through. In the unified case, in order to satisfy Theorem 1's assumption of weak supermodularity, we simply reverse the action space: The investor's utility function is then weakly supermodular in $(-a, \theta)$. Just as in the case of pure cross investment, it is clear that there is a feasible option contract in which the investor will select $a^*(\theta')$ in state θ' and 0 in state θ.[18] This implies that, for investment incentives, a non-forcing contract can improve on a forcing contract in this case as well.

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[18]Recall that $(a^*(\theta), 0) \in E(\theta', \theta)$ is required.

Next suppose that the beneficiary receives less than the total benefit created by the investment (so $\varepsilon > 0$). For the divided case, the argument from the pure-investment setting goes through: the trade-action-based option contract improves on the investment incentives of the best forcing contract. However, in the unified case the argument is not robust. The utility function of the investing party (player 1) is strictly supermodular, and so a non-forcing contract can only induce $a \geq a'$ for two states θ, θ' with $\theta > \theta'$. Thus, it is not possible to induce player 1 to choose 0 in state θ and $a^*(\theta')$ in state θ'. As a result, Expression 8 will not hold in general, and whenever this condition fails, the incentives provided by the forcing contract cannot be improved upon using a non-forcing contract.

Supporting Efficient Investment

The analysis in the previous subsections indicates where the larger implementable set of value functions V^{EP} leads to strictly more efficient investment levels than one finds with the set V^{EPF}. We next investigate whether efficient investment levels can be supported. Since we found that non-forcing contracts sometimes do not improve on forcing contracts in the unified case, the question is whether efficient investment can be supported in the divided case, where player 2 is the investor and player 1 is the beneficiary.

In fact, we have a strong affirmative answer to this question under the following additional assumption regarding the technology of trade:

Assumption 7: γ is (weakly) increasing.

Remember that $\gamma(\theta) = U(a^*(\theta), \theta) = B(a^*(\theta), \theta) + C(a^*(\theta), \theta)$ is the maximum joint value of trade in state θ, gross of the investment cost x. Thus, Assumption 7 means that higher trade values are associated with higher states. Note that Assumptions 1-7 all have to do solely with the technology of trade; they put no constraints on the technology of investment.

We have the following general result, which shows that the implementable set V^{EP} includes value functions that hold player 1's payoff constant across the set of states:

Theorem 2: *Consider any contractual relationship that satisfies Assumptions 1-7. Let k be any real number and define value function v by $v_1(\theta) = k$ and $v_2(\theta) = \gamma(\theta) - k$ for all $\theta \in \Theta$. Then $v \in V^{\mathrm{EP}}$.*

The proof is provided in Appendix C.

Consider a value function that satisfies $v_1(\theta) = k$ and $v_2(\theta) = \gamma(\theta) - k$ for all $\theta \in \Theta$ and suppose that the players contract at Date 1 to implement this value function. Let us observe what this implies for investment in the divided case, where player 2 is the investor. Clearly player 2 selects x at Date 2 to maximize

$$\int v_2(\theta) dG(x) - x = \int \gamma(\theta) dG(x) - x - k.$$

Since k is a constant, player 2 seeks to maximize the joint value of the relationship and thus player 2's optimal investment level is x^*. Efficient investment and trade are obtained. At Date 1, the players will select such a value function to maximize the joint value of their relationship, and they will use k to divide the value between them. We formalize the conclusion by stating:

Corollary: *Under Assumptions 1-7 and in the divided case in which player 2 is the investor and player 1 has the trade action, optimal contracting induces efficient investment and trade (the first best outcome).*

In addition to proving Theorem 2, in Appendix C we provide conditions under which the conclusion holds in the unified case, where player 1 is the investor. We show that these conditions are not always satisfied.

5   Conclusion

In this paper, we have reported on the analysis of contractual relationships for a large class of trade technologies. We have provided general results on the relation between individual-action and public-action models of contractual relationships, showing that limiting attention to forcing contracts has significant implications for inefficiency. Further, we have shown that (by utilizing non-forcing contracts) the payoff of the party with the trade action can be neutralized so that the other party claims the full benefit of the investment, gross of investment costs. This result led to the key novel insight of our analysis for applications, which is to identify the distinction between the *divided* and *unified* cases of investment and trade actions. We find that, in the important setting of cross investment, the hold-up problem can be averted (and efficiency obtained) in the divided case but generally not in the unified case.

Our results reinforce the message of Watson (2007) on the usefulness of modeling trade actions as individual, particularly in settings of cross investment. They suggest that some of the conclusions of public-action models in the existing literature should be revisited. In particular, settings with cross investment are generally not as problematic as Che and Hausch's (1999) modeling exercise might suggest. In particular, efficient outcomes can be achieved in the case of divided investment and trade actions. Our results show the importance, for applied work, of differentiating between the cases of divided and unified investment and trade actions.

In our model, the trading opportunity is non-durable in that there is a single moment in time when trade can occur. One might wonder if the results differ substantially in settings with durable trading opportunities (where if trade does not occur at one time, then it can still be done at a later date). This issue has been explored by Evans (2008) and Watson and Wignall (2007), both which examine individual-action models. Evans' (2008) elegant model is very general in terms of the available times at which the players can trade and

renegotiate. He constructs equilibria in which, by having the players coordinate in different states on different equilibria in the infinite-horizon trade/negotiation game, the hold-up problem is partly or completely alleviated. Evans' strongest result (in which the efficient outcome is reached) requires the ability of the players to commit to a joint financial hostage; that is, money is deposited with a third party until trade occurs, if ever. Without the joint financial hostage, the efficient outcome may not be achieved.

Watson and Wignall (2007) examine a cross-investment setting without the possibility of joint financial hostages, and their model is more modest in other dimensions. They show that the set of implementable post-investment payoff vectors in the setting of a durable trading opportunity is essentially the same as in the setting of a non-durable trading opportunity. This suggests that, in general, the results from the current paper carry over to the durability setting. Watson and Wignall also show that, in the divided condition, there are non-stationary contracts that uniquely support the efficient outcome.

Our modeling exercise, combined with the recent literature, suggests some broad conclusions about the prospect of efficient investment and trade in contractual relationships. First, the hold-up problem is not necessarily severe, and efficient outcomes can often be achieved. Inefficiency may be unavoidable in the following problematic cases:

- when there is cross investment and unified investment and trade actions, as identified herein;

- when trade involves "complexity/ambivalence" as described by Segal (1999), Hart and Moore (1999), and Reiche (2006); and

- when the investment conveys a significant direct benefit (not requiring trade) on the non-investing party, in addition to any benefit contingent on trade.

On the last point, Ellman's (2006) model provides intuition in terms of the notion of specificity.

In each of the cases above, the hold-up problem would be reduced if the parties have some way of creating joint financial hostages, as explored by Evans (2008) and Baliga and Sjöström (2008). Bull (2009) provides a cautionary note on the ability of such financial arrangements to withstand side-contracting.

Regarding extensions of our analysis here, it may be useful to examine different classes of trade technologies, including ones with sequential actions. Perhaps more interesting would be to examine settings with partially verifiable trade actions. For example, a court may observe whether a particular trade was made but have trouble identifying which party disrupted trade (in the event that trade did not occur). Hart and Moore's (1988) model has this feature. It is straightforward to incorporate partial verifiability into the modeling framework developed here. One can represent the external enforcer's information about the trading game as a partition of the space of action profiles. One can then simply assume that the contracted transfers y must be measurable with respect to this partition.

A   Proof of Theorem 1

In this appendix, we complete the proof of Theorem 1. We start with the comparison of V^{EPF} and V^{EP} and then provide the analysis for the comparison of V^{EP} and V^{I}.

Completion of the Proof that $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}}$

We pick up from the analysis at the end of Section 3. Consider a pair of states θ^1, θ^2 that satisfies Assumption 4. That is, we have $\theta^1 > \theta^2$ and either $U(\underline{a}, \theta^2) < U(\overline{a}, \theta^2)$ or $U(\underline{a}, \theta^1) > U(\overline{a}, \theta^1)$. Let b^1 denote a solution to the forcing-contract problem

$$\min_{a \in A} \lambda(a, a, \theta^1, \theta^2)$$

and let b^2 denote a solution to the forcing-contract problem

$$\min_{a \in A} \lambda(a, a, \theta^2, \theta^1).$$

We shall demonstrate that either $P^{\mathrm{EP}}(\theta^1, \theta^2) < P^{\mathrm{EPF}}(\theta^1, \theta^2)$ or $P^{\mathrm{EP}}(\theta^2, \theta^1) < P^{\mathrm{EPF}}(\theta^2, \theta^1)$, or both, which implies that $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}}$.

Let us evaluate the minimum punishment value corresponding to the ordered pair of states (θ^1, θ^2). Specifically, compare the optimal forcing contract punishment (forcing player 1 to select b^1 in both states) with a non-forcing specification in which player 1 is induced to select b^1 in state θ^1 and $\underline{a}$ in state θ^2. This is a valid non-forcing contractual specification because, by Fact 2, $\theta^1 > \theta^2$ and $b^1 \geq \underline{a}$ imply $(b^1, \underline{a}) \in E(\theta^1, \theta^2)$.

If $V^{\mathrm{EP}} = V^{\mathrm{EPF}}$ then it must be that $\lambda(b^1, b^1, \theta^1, \theta^2) \leq \lambda(b^1, \underline{a}, \theta^1, \theta^2)$. Applying the definition of λ, this is

$$u_1(b^1, \theta^1) + \pi_1 R(b^1, \theta^1) + u_2(b^1, \theta^2) + \pi_2 R(b^1, \theta^2)$$
$$\leq u_1(\underline{a}, \theta^1) + \pi_1 R(b^1, \theta^1) + u_2(\underline{a}, \theta^2) + \pi_2 R(\underline{a}, \theta^2).$$

Canceling the second term on each side and using the definition of R, we arrive at

$$u_1(b^1, \theta^1) + u_2(b^1, \theta^2) - \pi_2 U(b^1, \theta^2) \leq u_1(\underline{a}, \theta^1) + u_2(\underline{a}, \theta^2) - \pi_2 U(\underline{a}, \theta^2).$$

Substituting $u_2(\cdot, \theta^2) = U(\cdot, \theta^2) - u_1(\cdot, \theta^2)$ on both sides, we have

$$u_1(b^1, \theta^1) + U(b^1, \theta^2) - u_1(b^1, \theta^2) - \pi_2 U(b^1, \theta^2)$$
$$\leq u_1(\underline{a}, \theta^1) + U(\underline{a}, \theta^2) - u_1(\underline{a}, \theta^2) - \pi_2 U(\underline{a}, \theta^2).$$

Finally, rearranging this expression a bit and using $\pi_1 + \pi_2 = 1$, we conclude that $\lambda(b^1, b^1, \theta^1, \theta^2) \leq \lambda(b^1, \underline{a}, \theta^1, \theta^2)$ is equivalent to

$$u_1(b^1, \theta^1) - u_1(\underline{a}, \theta^1) - [u_1(b^1, \theta^2) - u_1(\underline{a}, \theta^2)] \leq \pi_1[U(\underline{a}, \theta^2) - U(b^1, \theta^2)]. \quad (9)$$

Similarly, ordering states θ^1 and θ^2 in the opposite way, we compare the optimal forcing contract punishment (forcing player 1 to select b^2 in both states) with a non-forcing specification in which player 1 is induced to select b^2 in state θ^2 and $\overline{a}$ in state θ^1. Note that $\theta^2 < \theta^1$ and $b^2 \leq \overline{a}$ imply $(b^2, \overline{a}) \in E(\theta^2, \theta^1)$. If $V^{\mathrm{EP}} = V^{\mathrm{EPF}}$ then it must be that $\lambda(b^2, b^2, \theta^2, \theta^1) \leq \lambda(b^2, \overline{a}, \theta^2, \theta^1)$, which similar algebraic manipulation reveals to be equivalent to

$$u_1(\overline{a}, \theta^1) - u_1(b^2, \theta^1) - [u_1(\overline{a}, \theta^2) - u_1(b^2, \theta^2)] \leq \pi_1[U(\overline{a}, \theta^1) - U(b^2, \theta^1)]. \quad (10)$$

We have shown that if $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$, then Expressions 9 and 10 hold. Assumption 3 then implies that the left sides of these inequalities are non-negative, which implies

$$U(\underline{a}, \theta^2) \geq U(b^1, \theta^2) \quad \text{and} \quad U(\overline{a}, \theta^1) \geq U(b^2, \theta^1).$$

Using Assumption 2, we obtain:

Fact 7: *If $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ then $U(\underline{a}, \theta^2) \geq U(\overline{a}, \theta^2)$ and $U(\overline{a}, \theta^1) \geq U(\underline{a}, \theta^1)$.*

Assumption 4 and the contrapositive of Fact 7 provide the contradiction that proves $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}}$.

Proof that $V^{\mathrm{EP}} \neq V^{\mathrm{I}}$

We next prove the claim about the relation between V^{I} and V^{EP}. Since forcing contracts are sufficient to construct V^{I}, we have:

Fact 8: *The minimum punishment value in the setting of interim renegotiation is characterized as follows:*
$$P^{\mathrm{I}}(\theta, \theta') = \min_{a'' \in A} u_1(a'', \theta) + u_2(a'', \theta').$$

Remember that, by Result 2, $V^{\mathrm{I}} = V^{\mathrm{EP}}$ if and only if $P^{\mathrm{EP}}(\theta, \theta') = P^{\mathrm{I}}(\theta, \theta')$ for all $\theta, \theta' \in \Theta$. We can again compare the minimization problems to determine if this is the case.

Take θ^1, θ^2 satisfying Assumption 4. Consider any solution to the minimization problem that defines $P^{\mathrm{EP}}(\theta^1, \theta^2)$ and denote it (b, b'). That is, (b, b') solves

$$\min_{(a,a') \in E(\theta^1, \theta^2)} u_1(a', \theta^1) + \pi_1 R(a, \theta^1) + u_2(a', \theta^2) + \pi_2 R(a', \theta^2).$$

Then $P^{\mathrm{EP}}(\theta^1, \theta^2) = P^{\mathrm{I}}(\theta^1, \theta^2)$ is equivalent to

$$u_1(b', \theta^1) + \pi_1 R(b, \theta^1) + u_2(b', \theta^2) + \pi_2 R(b', \theta^2) = \min_{a'' \in A} u_1(a'', \theta^1) + u_2(a'', \theta^2).$$

Because $R(\cdot, \cdot) \geq 0$, we see that $P^{\mathrm{EP}}(\theta^1, \theta^2) = P^{\mathrm{I}}(\theta^1, \theta^2)$ only if b' solves the minimization problem on the right side of the above equation and also $R(b, \theta^1) = R(b', \theta^2) = 0$.

By Assumption 2, $R(b', \theta^2) = 0$ if and only if $b' = a^*(\theta^2)$. Combining this with the requirement that b' must minimize $u_1(\cdot, \theta^1) + u_2(\cdot, \theta^2)$, we derive that

$$u_1(a^*(\theta^2), \theta^1) + u_2(a^*(\theta^2), \theta^2) \leq u_1(a'', \theta^1) + u_2(a'', \theta^2)$$

for all a''. In particular, the following inequality must hold:

$$u_1(a^*(\theta^2), \theta^1) + u_2(a^*(\theta^2), \theta^2) \leq u_1(\underline{a}, \theta^1) + u_2(\underline{a}, \theta^2).$$

Using the identity $u_2 = U - u_1$ and rearranging terms, we see that this is equivalent to

$$u_1(a^*(\theta^2), \theta^1) - u_1(\underline{a}, \theta^1) - [u_1(a^*(\theta^2), \theta^2) - u_1(\underline{a}, \theta^2)]$$
$$\leq U(\underline{a}, \theta^2) - U(a^*(\theta^2), \theta^2).$$
(11)

Similarly, ordering states θ^1 and θ^2 in the opposite way, it is necessary that $a^*(\theta^1)$ must solve $P^{\mathrm{I}}(\theta^2, \theta^1)$ in order for $P^{\mathrm{EP}}(\theta^2, \theta^1) = P^{\mathrm{I}}(\theta^2, \theta^1)$. In particular, we must have

$$u_1(a^*(\theta^1), \theta^2) + u_2(a^*(\theta^1), \theta^1) \leq u_1(\overline{a}, \theta^2) + u_2(\overline{a}, \theta^1).$$

This inequality is equivalent to

$$u_1(\overline{a}, \theta^1) - u_1(a^*(\theta^1), \theta^1) - [u_1(\overline{a}, \theta^2) - u_1(a^*(\theta^1), \theta^2)]$$
$$\leq U(\overline{a}, \theta^1) - U(a^*(\theta^1), \theta^1).$$
(12)

By Assumption 3, the left sides of Expressions 11 and 12 must be non-negative, which implies both $U(\underline{a}, \theta^2) \geq U(a^*(\theta^2), \theta^2)$ and $U(\overline{a}, \theta^1) \geq U(a^*(\theta^1), \theta^1)$. From Assumption 2, we see that this is only possible if $\underline{a} = a^*(\theta^2)$ and $\overline{a} = a^*(\theta^1)$. If this is the case, Assumption 2 also implies that $U(\underline{a}, \theta^2) \geq U(\overline{a}, \theta^2)$ and $U(\overline{a}, \theta^1) \geq U(\underline{a}, \theta^1)$. Thus we obtain:

Fact 9: *If $V^{\mathrm{I}} = V^{\mathrm{EP}}$ then $U(\underline{a}, \theta^2) \geq U(\overline{a}, \theta^2)$ and $U(\overline{a}, \theta^1) \geq U(\underline{a}, \theta^1)$.*

The contrapositive of Fact 9 combined with Assumption 4 provides the contradiction that proves $V^{\mathrm{I}} \neq V^{\mathrm{EP}}$.

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B Derivations for the Setting of Pure Cross Investment

Divided Case

If we expand

$$\lambda(a, a', \theta, \theta') < \lambda(0, 0, \theta, \theta')$$

using Expression 6 (on page 17), we have

$$u_1(a', \theta) + \pi_1 R(a, \theta) + u_2(a', \theta') + \pi_2 R(a', \theta') < u_1(0, \theta) + \pi_1 R(0, \theta) + u_2(0, \theta') + \pi_2 R(0, \theta').$$

Because $u_1(0, \theta) = u_2(0, \theta')$, this reduces to

$$u_1(a', \theta) + \pi_1 R(a, \theta) + u_2(a', \theta') + \pi_2 R(a', \theta') < \pi_1 R(0, \theta) + \pi_2 R(0, \theta').$$

Using the definition of renegotiation surplus, we have

$$u_1(a', \theta) + \pi_1 U(a^*(\theta), \theta) - \pi_1 U(a, \theta) + u_2(a', \theta') + \pi_2 U(a^*(\theta'), \theta') - \pi_2 U(a', \theta')$$
$$< \pi_1 U(a^*(\theta), \theta) - \pi_1 U(0, \theta) + \pi_2 U(a^*(\theta'), \theta') - \pi_2 U(0, \theta'),$$

which simplifies to

$$u_1(a', \theta) - \pi_1 U(a, \theta) + u_2(a', \theta') - \pi_2 U(a', \theta') < -\pi_1 U(0, \theta) - \pi_2 U(0, \theta')$$

and further to

$$u_1(a', \theta) + u_2(a', \theta') - \pi_2 U(a', \theta') < \pi_1 U(a, \theta),$$

which is Equation 7 in the text.

Unified Case

We start with

$$\lambda(a', a, \theta', \theta) < \lambda(0, 0, \theta', \theta).$$

Expanding this inequality using Expression 6, we have

$$u_1(a, \theta') + \pi_1 R(a', \theta') + u_2(a, \theta) + \pi_2 R(a, \theta) < u_1(0, \theta') + \pi_1 R(0, \theta') + u_2(0, \theta) + \pi_2 R(0, \theta).$$

This reduces to

$$u_1(a, \theta') + \pi_1 R(a', \theta') + u_2(a, \theta) + \pi_2 R(a, \theta) < \pi_1 R(0, \theta') + \pi_2 R(0, \theta).$$

Using the definition of renegotiation surplus, this inequality becomes

$$u_1(a, \theta') + \pi_1 U(a^*(\theta'), \theta') - \pi_1 U(a', \theta') + u_2(a, \theta) + \pi_2 U(a^*(\theta), \theta) - \pi_2 U(a, \theta)$$
$$< \pi_1 U(a^*(\theta'), \theta') - \pi_1 U(0, \theta') + \pi_2 U(a^*(\theta), \theta) - \pi_2 U(0, \theta).$$

Simplifying yields

$$u_1(a, \theta') - \pi_1 U(a', \theta') + u_2(a, \theta) - \pi_2 U(a, \theta) < -\pi_1 U(0, \theta') - \pi_2 U(0, \theta),$$

and further

$$u_1(a, \theta') + u_2(a, \theta) - \pi_2 U(a, \theta) < \pi_1 U(a', \theta'),$$

which is Equation 8.

C   Proof of Theorem 2

Take as given a constant k. Let v be the value function defined by $v_1(\theta) = k$ and $v_2(\theta) = \gamma(\theta) - k$ for all $\theta \in \Theta$. We must show that v is implementable with ex post renegotiation. From Result 1 and the definition of the minimum punishment value P^{EP}, it is sufficient to show that, for all ordered pairs of states (θ, θ'), we have

$$v_1(\theta) + v_2(\theta') \geq P^{EP}(\theta, \theta').$$

Consider two states θ and θ' and order them so that $\theta > \theta'$. The conditions associated with these two states (for (θ, θ') and (θ', θ)) are

$$v_1(\theta) + v_2(\theta') \geq P^{EP}(\theta, \theta')$$

and

$$v_1(\theta') + v_2(\theta) \geq P^{EP}(\theta', \theta).$$

Using Fact 5 from Section 3, these conditions are equivalent to the existence of trade actions a, a', b, b' such that $(a, a') \in E(\theta, \theta')$, $(b', b) \in E(\theta', \theta)$,

$$v_1(\theta) + v_2(\theta') \geq \lambda(a, a', \theta, \theta')$$

and

$$v_1(\theta') + v_2(\theta) \geq \lambda(b', b, \theta', \theta).$$

Substituting for v_1 and v_2 using the identities $v_1(\theta) = k$ and $v_2(\theta) = \gamma(\theta) - k$, these two inequalities become:

$$\lambda(a, a', \theta, \theta') \leq \gamma(\theta') \tag{13}$$

and

$$\lambda(b', b, \theta', \theta) \leq \gamma(\theta). \tag{14}$$

Because $\theta > \theta'$, and given Assumption 3, we know that $(a, a') \in E(\theta, \theta')$ and $(b', b) \in E(\theta', \theta)$ are satisfied if we pick $a \geq a'$ and $b \geq b'$. Let $a = a^*(\theta)$ and $a' = b' = b = 0$,

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which are in the feasible set. We can use Equation 6 (from page 17) to express λ in terms of the trade values:

$$\lambda(a^*(\theta), 0, \theta, \theta') = u_1(0, \theta) + \pi_1 R(a^*(\theta), \theta) + u_2(0, \theta') + \pi_2 R(0, \theta')$$

and

$$\lambda(0, 0, \theta', \theta) = u_1(0, \theta') + \pi_1 R(0, \theta') + u_2(0, \theta) + \pi_2 R(0, \theta).$$

Noting that $R(a^*(\theta), \theta) = 0$ and the zero-trade terms are all equal to zero by Assumption 6 (which also implies that $R(0, \theta') = \gamma(\theta')$ and $R(0, \theta) = \gamma(\theta)$), these expressions simplify to $\lambda(a^*(\theta), 0, \theta, \theta') = \pi_2 \gamma(\theta')$ and $\lambda(0, 0, \theta', \theta) = \pi_1 \gamma(\theta') + \pi_2 \gamma(\theta)$. Inequalities 13 and 14 then become

$$\pi_2 \gamma(\theta') \leq \gamma(\theta')$$

and

$$\pi_1 \gamma(\theta') + \pi_2 \gamma(\theta) \leq \gamma(\theta).$$

These inequalities clearly hold, the second because of Assumption 7. This completes the proof of the theorem.

We conclude this appendix with a note about the difference between the divided and unified case. Observe that, in the unified case, the investor (player 1) would have the incentive to invest efficiently if the value function holds player 2's payoff constant; that is, we need to implement a value function v satisfying, for some constant k, $v_2(\theta) = k$ and $v_1(\theta) = \gamma(\theta) - k$ for all $\theta \in \Theta$. Calculations along the lines of those above produce the following sufficient conditions for this setting, which are the counterparts of Inequalities 13 and 14 above: There must exist trade actions a, a', b, b' such that $(a, a') \in E(\theta, \theta')$, $(b', b) \in E(\theta', \theta)$,

$$\lambda(a, a', \theta, \theta') \leq \gamma(\theta) \tag{15}$$

and

$$\lambda(b', b, \theta', \theta) \leq \gamma(\theta'). \tag{16}$$

As before, we can set $a = a^*(\theta)$ and $a' = 0$ to make Inequality 15 hold, but we are not guaranteed the existence of a pair b, b' that satisfies Inequality 16. One can use this condition to establish whether efficient investment can be obtained in specific examples with unified investment and trade actions, but a sufficient condition would be much stronger than are the assumptions we have made here.

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