

Life-Cycle Dynamics and the Expansion Strategies of U.S. Multinational Firms*

(Early Draft)

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Abstract

This paper examines how the activities performed by multinational firms change over their life cycle. Using a panel of U.S. multinational firms over 25 years, we classify affiliate sales as horizontal, vertical, or export platform based on their destination, and we establish two facts on the evolution of these three types of sales over the life cycle of the affiliate. First, sales to the local and other markets grow very little over the life cycle of the affiliate. Second, affiliates specialize in a core activity at birth which persists as the main activity during their life cycle; some diversification occurs later in life, particularly from horizontal to export activities. Informed by these facts, we propose a simple dynamic model of multinational activity that features entry costs to multinational activity, entry costs to export markets, and heterogenous firms. The model delivers several testable implications that are consistent with the data. Additionally, numerical exercises shed light on the nature of the costs of multinational activity, which are key to quantify the gains from multinationals' operations.

JEL Codes: F1.

Key Words: Multinational firms, Firm dynamics, Foreign direct investment.

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1 Introduction

Multinational firms are the largest players in the global economy. In 2009, 75 percent of U.S. sales to foreign customers (nearly US\$5 trillions) was accounted for the sales of foreign affiliates of U.S.-based multinationals, rather than by sales of domestically produced goods.¹ Additionally, affiliates' exports represent one third of world exports, and also one third of their total sales, according to UNCTAD (2013)'s estimates. Similar magnitudes are reported by the Bureau of Economic Analysis (BEA) for foreign affiliates of U.S. multinational enterprises (henceforth, MNEs): of the US\$4,6 billions in sales done by foreign majority-owned affiliates of U.S. MNEs, in 2009, forty percent were exports (i.e. directed to customers outside the host market of operation).² Hence, understanding the activities of multinational firms and their costs is a key factor to better understand globalization.

A frequently overlooked aspect in the analyses of the multinational firm and its affiliates is their dynamic behavior, primarily because the data requirements are large. MNEs are complex production structures, with affiliates in many locations, often engaged in different activities spanning multiple countries and sectors. Questions about the role of affiliates within the corporation and how they evolve through time have been barely addressed in the literature. Ultimately, the answers to these questions are key to dissecting the nature of the costs of multinational activity: whether these costs are country and/or activity dependent, and whether variable, fixed, or sunk costs are relatively more important. In turn, understanding the nature of the costs of multinational activity is crucial for the quantification of the gains from openness arising from MNEs operations.

This paper fills this gap in the literature by documenting salient features of the life-cycle dynamic behavior of U.S.-based MNEs and of their affiliates. Using a panel of U.S. multinational firms over 25 years from the U.S. Bureau of Economic Analysis (BEA), we examine how the organization of MNEs' activities in space evolves over time.

We classify the sales of foreign affiliates of U.S. MNEs as directed to the host market where the affiliate is located (horizontal), or to other markets (exports). We further distinguish between exports to the United States (vertical), and third markets (export platform). We trace the evolution of these three types of sales over the life cycle of the affiliate. This analysis delivers two stylized facts. First, life-cycle sales profiles are relatively flat for all types of affiliate activities, particularly when compared with new exporters' growth. Second, affiliates of U.S. multinational firms tend to

¹See Antrás and Yeaple (2014) for a detailed survey of the main facts and theories about multinational firms.

²Moreover, Ramondo et al. (2016) document that, in 2004, the median export share for a foreign manufacturing affiliate of a U.S. MNE was above 30 percent, while the average share was higher than 40 percent.

specialize in a core activity at birth, which persists as the main activity during the life cycle. Some diversification across the three types of sales is observed later in life, particularly from horizontal to export activities.

Motivated by these two facts, we present a dynamic model of the multinational firm that builds on elements from Fillat and Garetto (2015) and Fillat et al. (2015). We model a set of Home-based firms that must decide whether, how, and when to serve foreign markets through affiliate sales. Multinational activities are treated as a real option for the firm, which gets exercised once an affiliate opens abroad. Affiliate sales are associated with fixed and sunk costs of production, and the decisions of setting up an affiliate and of exporting from it are shaped by the interaction among firms' individual productivity (which we assume to be constant), a persistent aggregate productivity shock, and demand conditions in foreign markets. Starting from the observation that almost all firms in our sample have horizontal activities, we assume that firms that decide to do Foreign Direct Investment (FDI) must first set up an affiliate and sell to the local market; only then they can consider exporting from that affiliate, either back Home or to third countries. Because the problem is set up in continuous time, those two decisions – opening an affiliate and exporting from it – could be virtually made simultaneously. Additionally, writing the problem in continuous time delivers closed-form solutions for the value functions, which are simple additive functions of two terms: the firm's realized profit flow plus the option value of further expansion. Crucially, we assume that the decision of setting an affiliate and eventually exporting from it is independent across markets (e.g., whether a firm decides to export to France from an affiliate located in Germany is independent of having an affiliate in France). In this way, we avoid having to solve the complex permutational problem present in settings that model these decisions as interdependent (such as Tintelnot (2016)) and achieve tractability also in a dynamic setting.

The dynamic component of the model is built to replicate qualitatively the motivating facts described above. The cross-sectional component of the model borrows heavily from the heterogeneous firms literature.³ The model delivers several testable implications, which are confirmed by the data. First, affiliates that both serve the host market and export (“diversified” affiliates) have larger horizontal sales than affiliates devoted exclusively to serving the host market (“pure horizontal” affiliates). This fact mimics an analogous pattern about exporters that is documented in the literature.⁴ In this regard, affiliates of multinational firms are not different from standard

³See, most notably, Melitz (2003) and Helpman et al. (2004).

⁴See Bernard and Jensen (1999), among others.

domestic firms. Second, conditional on aggregate productivity growing over time, affiliates that are exporters at birth have larger horizontal sales than affiliates that become exporters later in their life cycle. Next, the model has predictions regarding the expansion strategies of MNEs worldwide. The model predicts a pecking order in the way the MNE chooses to open foreign affiliates: MNEs open first their largest affiliates, and subsequently their smaller affiliates. Also, MNEs open first affiliates in larger host countries. Finally, there is a pecking order in the way an MNE accesses markets: first affiliates are established in markets which are easier to access, and later in life the MNE opens affiliates in markets that are harder and costlier to access.

Equipped with these results, we extend the model to incorporate more realistic features that make it amenable to a quantitative analysis, namely the endogenous decision of shutting down affiliates or stop exporting from them (endogenous exit) and the choice of the destination for affiliates' exports. Thanks to the assumption whereby an MNE's affiliate location choice is independent across locations, the quantitative model can be solved as a compound option: conditional on having an affiliate in a location, firms decide whether and where to export from it. This choice determines the value of the affiliate. Knowing the potential value of an affiliate in a location (including the potential value of its export sales) firms decide whether to open the affiliate or not.

The quantitative model preserves most of the tractability of the simple framework and can be used to shed light on the nature and magnitudes of the costs of multinational activity, and to conduct welfare analysis in a dynamic setting. Simulations show, for instance, that a ten-percent change in the per-period cost of multinational activities can induce large reallocations on the activity of affiliates, and the effects of this change critically depends on which type of frictions are involved.

Most contributions in the literature have analyzed MNEs' complex choices in static settings. As evident in the models in Arkolakis et al. (2014) and Tintelnot (2016), allowing firms to set up affiliates in countries that might differ from the destinations of their sales results in a very complex combinatorial problem when fixed costs of productions are taken into account. The sharp patterns that we document, arising from the observation of affiliates over time, help to simplify this difficult problem by reducing the choice set of firms in a way that is consistent with the data. More precisely, given that most new affiliates in the data start out as entities partially or entirely specialized in horizontal FDI, and possibly diversify into other modes of operation later in life, we argue that decisions about performing different multinational activities can be separated into simple choices that happen at different points in time. This significantly simplifies the dynamic problem of the firm.

Our paper is naturally related to the literature on export dynamics, which has been mainly concerned with quantifying fixed and sunk costs of export activities and studying their welfare implications. Earlier contributions by Baldwin and Krugman (1989), Roberts and Tybout (1997), Das et al. (2007), and Alessandria and Choi (2007) find substantial sunk costs of exporting, by focusing on explaining observed patterns of export entry and exit. Subsequent analyses, such as Eaton et al. (2008) and Ruhl and Willis (2015), incorporate facts related to the life-cycle dynamics of new exporters and find that those costs are much lower. Alessandria et al. (2015) take a further step and also calculate the welfare gains from trade in a dynamic setting that matches well the life-cycle facts. Arkolakis (2016) presents rich micro evidence on firm selection and export growth that supports dynamic theories of endogenous entry costs *vis-a-vis* standard sunk costs. Finally Ghironi and Melitz (2005) study the macroeconomic dynamics of a model with heterogeneous firms based on Melitz (2003).⁵

There is also a small, but growing, literature on MNEs' dynamics, which, as mentioned above, faces as its main restriction the access to detailed data on the activity of MNEs, both through time and in space. Ramondo et al. (2013) study the implications of the proximity-concentration tradeoff under uncertainty and conclude that trade flows, relatively to FDI flows, are larger into countries that co-move less with the source country. Fillat and Garetto (2015) and Fillat et al. (2015) develop dynamic models of export and FDI that share many features with the framework that we present in this paper. The focus of these contributions is to relate firms' trade and FDI dynamic behavior to financial variables, while in this paper we characterize the location decisions of individual firms over their life cycle. Egger et al. (2014) and Conconi et al. (2016) document dynamic facts on MNEs activities, for Germany and Belgium, respectively, and claim that those facts are consistent with substantial learning. Gumpert et al. (2016), using very rich data for several countries, focus on life-cycle patterns of both MNEs and exporters, and conclude that whereas exporters have virtually zero sunk costs, MNEs face substantial entry costs.

Additionally, our paper makes contact with the literature on domestic firms' life-cycle dynamics, which goes back to the paper by Davis et al. (1996), and more recently Decker et al. (2014, 2015). In contrast to the findings for affiliates of MNEs, new domestic firms grow substantially in their first years of life. Domestic new firms also present exit rates that sharply decline with age, while foreign affiliates of U.S. MNEs present flatter exit profiles, regardless of their activity. Indeed, the

⁵As in Ghironi and Melitz (2005), we model firm-level productivity as a fixed parameter that interacts with aggregate country-level shocks.

difference between the behavior of new U.S. firms in the domestic and foreign markets is indicative of the fact that they face a different set of costs.

Finally, there is a substantial empirical literature that documents patterns of the activity of foreign affiliates of U.S. MNEs using data from the U.S. Bureau of Economic Analysis.⁶ Most of this literature focuses on static patterns, with the notable exception of Bilir and Morales (2016), who study the innovations features of affiliates of U.S. MNEs in a dynamic setting, and Kovak et al. (2015), who analyze the effect of affiliates' activities on the domestic activities of the U.S. parent. None of these papers, however, focuses on the life-cycle dynamics of the activities of new foreign affiliates of U.S. MNEs.

The rest of the paper is organized as follows. Section 2 establishes the stylized facts about affiliates' dynamics. Section 3 presents a simple and tractable version of the model and its qualitative testable implications. Section 4 shows empirical evidence in support of the model's qualitative predictions. In Section 5 we extend the model to a more realistic environment and use it for quantitative analysis. Section 6 concludes.

2 Establishing the Facts

Our descriptive empirical analysis is conducted using data from the U.S. Bureau of Economic Analysis (BEA). The BEA collects firm-level data on U.S. multinational companies' operations in its annual surveys of U.S. direct investment abroad. All U.S. located firms that have at least one foreign affiliate and that meet a minimum size threshold are required by law to respond to these surveys. The data include detailed information on the firms' operations both in the U.S. and at their foreign affiliates, for the period 1987-2011. Each foreign affiliate in the dataset is assigned an industry classification based on its primary activity according to the BEA International Surveys Industry (ISI) system, which closely follows the 3-digit Standard Industrial Classification (SIC) system.⁷ We include affiliates that list an activity in manufacturing as their primary activity and belong to a U.S. parent operating in any sector. We restrict our attention to majority-owned affiliates.

⁶Among many others, Feinberg and Keane (2006), Yeaple (2006), Hanson et al. (2001, 2005), and Ramondo et al. (2016).

⁷The BEA data use 3-digit SIC-based ISI codes for years prior to 1999. From 1999 onward, they use 4-digit NAICS-based ISI codes. For consistency, we convert the NAICS-based codes to 3-digit SIC-based ISI codes for the relevant years.

When a firm has more than one enterprise operating in the same country and industry, we group these enterprises' activities together and refer to them as a single affiliate. We do this for two reasons. First, the firms themselves are permitted to report combined data in this way, making it difficult to isolate individual plants. Based on the BEA definition, an affiliate is a business enterprise in a given industry operating in a particular host country; it thus could operate several plants in different locations within the host country. The BEA rules permit consolidated reporting for distinct enterprises located in the same country that operate in the same narrowly defined industry or otherwise are integral parts of the same business operation. Second, to the extent that the costs of opening a new affiliate are incurred at the country-industry level, this is the appropriate level of aggregation for our analysis.⁸

Additionally, we restrict our attention to affiliates that do not operate in tax haven countries. Our goal is to capture actual MNE production, and reported tax haven activity may reflect accounting practices rather than actual output. Affiliates in tax haven countries are also likely to open for different reasons and be subject to different cost structures than those in non-tax haven countries, confounding our analysis. We compile our list of tax havens using information from Gravelle (2015). We omit countries that meet some of the criteria for tax haven status but that also have a substantial amount of real FDI production from the list. If a country is in the top ten percent of U.S. FDI destinations measured by total U.S. MNE affiliate employment, we consider it to be a location for actual production rather than a strict tax haven. Based on this definition, Hong Kong and Singapore are the only two countries from the Gravelle (2015) list that we do not classify as tax havens. A full list of tax haven countries is included in Appendix A.

In most of the analysis, we focus on *new* affiliates that open during our sample period and that survive for at least ten consecutive years in the market. We further trim from the sample affiliates and parents with zero total sales, assuming that there is a reporting error. Appendix A provides more details on the BEA data and the construction of our sample.

Crucially, the BEA data allow us to classify MNEs affiliates' activities based on the destination of affiliate sales. Affiliate sales can be of three types: horizontal sales (H) are sales to the market where the affiliate is located; vertical sales (V) are sales to the home market—the United States in our data; and export-platform sales (E) are sales to third markets—outside both the local and

⁸In a robustness exercise, we use reported openings of new enterprises in a country-industry in which the firm already had existing affiliates to check whether costs are incurred at the enterprise or country-industry level. This exercise, though based on noisy data, confirms that focusing on the country-industry-firm-, rather than plant-level, is appropriate.

Table 1: Summary statistics: number of observations, by activity type.

Activity type	Horizontal	Vertical	Export-platform
Affiliates in our sample			
No. of observations	34,181	34,181	34,181
with positive sales	32,335 (94.6%)	15,869 (46.4%)	20,127 (58.9%)
<i>of which:</i>			
pure type	11,072 (32.4%)	523 (1.53%)	639 (1.89%)
pure type at birth	16,908 (52.3%)	657 (4.1%)	683 (3.4%)
Sales accounted by pure type	11.9%	0.82%	1.42%

Note: Observations are at the affiliate-year level. Affiliates in our sample, or “new” affiliates, are firms born during the sample period that survive for at least ten consecutive years. All affiliates are majority-owned in the manufacturing sector. Horizontal, vertical, and export-platform refer, respectively, to sales to: the market where the affiliate is located; the home market; and third markets—outside the local and home markets. A pure-type affiliate is an affiliate with all its sales in only one activity type.

U.S. markets.⁹

Table 1 shows how our sample of manufacturing affiliates is distributed among the three different activities. Most of our affiliate-year observations (94.6% of them) have some sales in their host country (horizontal activities), while around half of our affiliates have some vertical or export-platform activity. A third of the observations corresponding to horizontal activities are of “pure type” (i.e., affiliates with 100 percent of their sales in only one activity), while the shares of vertical and export-platform pure-type affiliates are negligible, summing up to only 3.42% of affiliates. Additionally, around 35 percent of our affiliate-year observations have all three type of activities, and a similar percentage have only one of the possible activities (not shown); the remaining observations present a mix of two out of three activities.

Table 2 shows in more detail the distribution of sales of each type, as a share of total affiliate sales. On average, almost three quarters of the sales of an affiliate are to the host market, while less than ten percent are back to the home country; the remaining 20 percent are affiliate exports to third markets. Indeed, as found by Ramondo et al. (2016), vertical sales, and to a lesser extent export-platform sales, are very concentrated in few affiliates, as indicated by the sales shares across

⁹Our classification of affiliate sales in the three groups is purely location-based. For now, we disregard the role of ownership and do not distinguish among transactions that take place at arm’s length or intrafirm.

Table 2: Affiliate sales, by activity type.

Activity type	Horizontal	Vertical	Export-platform
	(as a share of total affiliate sales)		
average	0.723	0.080	0.197
std dev	0.343	0.200	0.291
25 pc	0.540	0.000	0.000
50 pc	0.897	0.000	0.022
75 pc	1.000	0.044	0.310
90 pc	1.000	0.244	0.723
95 pc	1.000	0.529	0.905

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Horizontal, vertical, and export-platform refers, respectively, to sales to: the market where the affiliate is located; the home market; and third markets—outside the local and home markets. Percentiles are taken with respect to the variable of interest so that the X-percentile affiliates change as the sorting variable changes. Averages of the 11 firms around the indicated percentile are reported to preserve confidentiality.

the different percentiles of the distribution. While the 75th percentile for horizontal shares is one, it is less than five percent for vertical shares, and less than 30 percent for export-platform shares. Appendix A reports more detailed summary statistics.

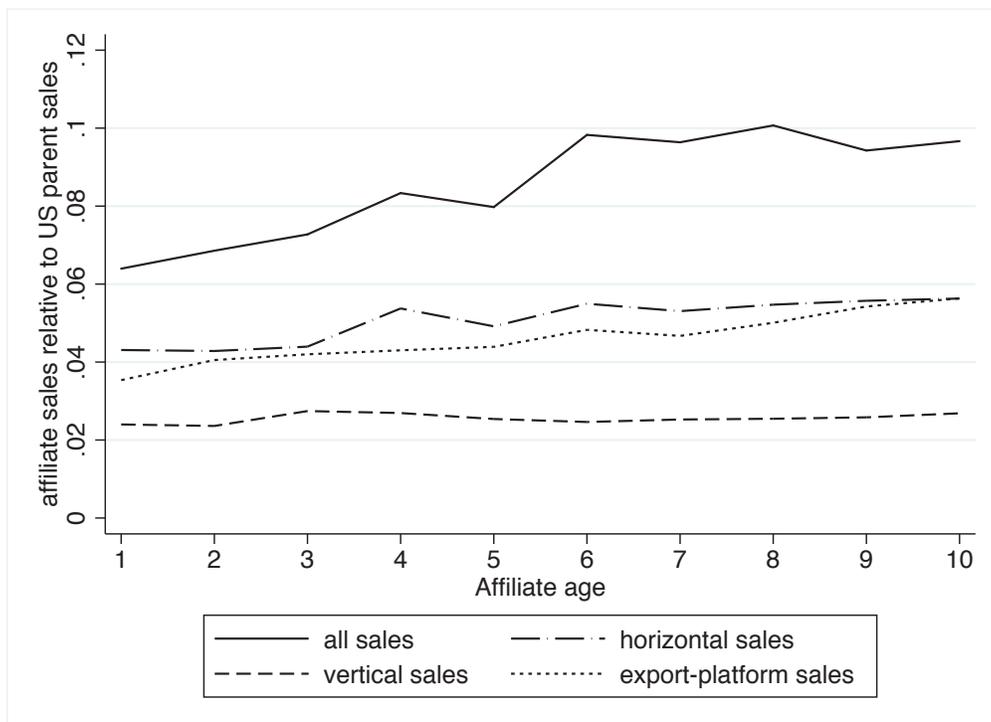
2.1 New affiliates have flat life-cycle sales profiles, regardless of their activity.

We first examine the life-cycle sales profile of new foreign affiliates of U.S. MNEs. These sales profiles are analogous to the ones documented by Ruhl and Willis (2015) for new exporters. Figure 1 shows the ratio of affiliate-to-parent sales, for each activity type, by affiliate age.

On average, new affiliates have sales volumes of about six percent of the parent’s sales. Over the initial five years of life of the affiliate, this ratio goes up to about eight percent, reaching ten percent by the 10th year of life. As a comparison, Ruhl and Willis (2015) report that export shares more than double in the first five years of exporting. Examining sales profiles by activity type, the ratio of horizontal to parent sales increases by only one percentage point from 4.6 percent. Vertical sales profiles appear flat, representing around 2.5 percent of the parent sales over the ten first years of life of the affiliate. In contrast, export platform sales, relative to the parent’s sales, increase from 3.5 to 5.5 percent during the first years of life of the affiliate.¹⁰

¹⁰The fact that affiliates sales profiles are flat does not depend on the affiliate’s position in the opening sequence

Figure 1: Affiliate to parent sales, by activity type.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Horizontal, vertical, and export-platform refers, respectively, to sales to: the market where the affiliate is located; the home market; and third markets—outside the local and home markets.

The patterns emerging from Figure 1 are captured by the following regression:

$$\text{affiliate sales}_a^i / \text{parent sales} = \beta_a \text{age} + FE + \varepsilon_a^i. \quad (1)$$

The left-hand side variable is the ratio of sales of type $i = H, V, E$, for a new affiliate of age a , to sales of the parent, age is the age of the affiliate (from 1 to 10), and ε_a^i is the error term. We include country-year and industry fixed effects, and alternatively, country-year and affiliate fixed effects.¹¹ Table 3 reports the results of estimating (1) by Ordinary Least Squares (OLS). The flat profiles observed in Figure 1 are confirmed by the regression analysis: the ratio of affiliate to parent sales is not significantly correlated with affiliate age, controlling for country-year and industry fixed effects. The result holds both with and without affiliate fixed effects

As a robustness check, we also include as controls the size of the affiliate and the size of the of the MNE. As Figure C.3 in the Appendix shows, both affiliates who are first in the opening sequence of MNEs and subsequent affiliates have fairly flat affiliate-to-parent sales ratio over their life cycle, both overall and by activity type.

¹¹We drop outliers which turn out to be five percent of observations.

Table 3: Affiliate to parent sales, by activity type. OLS.

Dep var	<i>affiliate sales</i> _a ⁱ / <i>parent sales</i>							
	<i>i = all</i>		<i>i = H</i>		<i>i = V</i>		<i>i = E</i>	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
aff. age	0.005 (0.006)	-1.40 (0.918)	0.006 (0.006)	0.397 (0.396)	-0.0007 (0.001)	-0.469 (0.414)	-0.0009 (0.003)	0.009 (0.508)
ind fe	yes	no	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes	yes	yes
aff. fe	no	yes	no	yes	no	yes	no	yes
Obs	32,470	32,470	30,718	30,718	15,076	15,076	19,121	19,121
R-sq	0.02	0.0001	0.04	0.0003	0.01	0.0001	0.01	0.0001

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variables H , V and E refer, respectively, to horizontal, vertical and export-platform sales. The dependent variable *affiliate to parent sales* refers to affiliate sales in each type, relative to the domestic sales of the U.S. parent. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

corporation, measured as employment and global sales, respectively; results are unchanged.

One could argue that the flat sales profiles observed for new affiliates may be due to the fact that firms acquire experience (and grow) in a foreign market first via exports, and only subsequently open affiliates at their optimum long-run size. Unfortunately, the BEA data do not include information about parents' exports that can inform our analysis in this respect. Few papers are able to compute export experience in a market before opening an affiliate: The data requirements are high. Using data for Belgium, Conconi et al. (2016) document that 95 percent of affiliates were preceded by the parent previously exporting to their host market. Gumpert et al. (2016), however, report that for Norway, 39 percent of new affiliates exported previously to the same market; for France, the share is 42 percent.¹²

Another concern is related to the way in which the foreign investment that led to the affiliate creation is undertaken. If MNEs establish foreign affiliates mostly through mergers and acquisitions (M&A), one could argue that "new" foreign affiliates are in reality pre-existing plants that likely grew previously and were acquired by the MNEs already at their maturity stage. This would explain the observed lack of steep sales profiles. Again, Gumpert et al. (2016) show that, for Germany, new affiliates that were previously domestic firms (i.e. were established through M&A) have flatter life-cycle sales profiles than new affiliates created through greenfield FDI. The authors attribute

¹²The finding is not sensitive to the definition of previous export experience.

the difference to partial year effects: the steeper profiles for new greenfield affiliates can just be an artifact of the data in which sales in the first year are only recorded partially.¹³ The BEA data also contains information on whether affiliates are the result of M&A or of a greenfield investment. We plan to use this information to examine separately the growth profiles of these two types of affiliates.

2.2 New affiliates are born specialized in a core activity.

We present here several pieces of evidence to show that: (1) Affiliates are born specialized and their primary specialization persists later in life, even though affiliates may incorporate secondary activities; and (2) most new affiliates are born overwhelmingly doing horizontal activities and become exporters later in their life cycle at a low intensity.

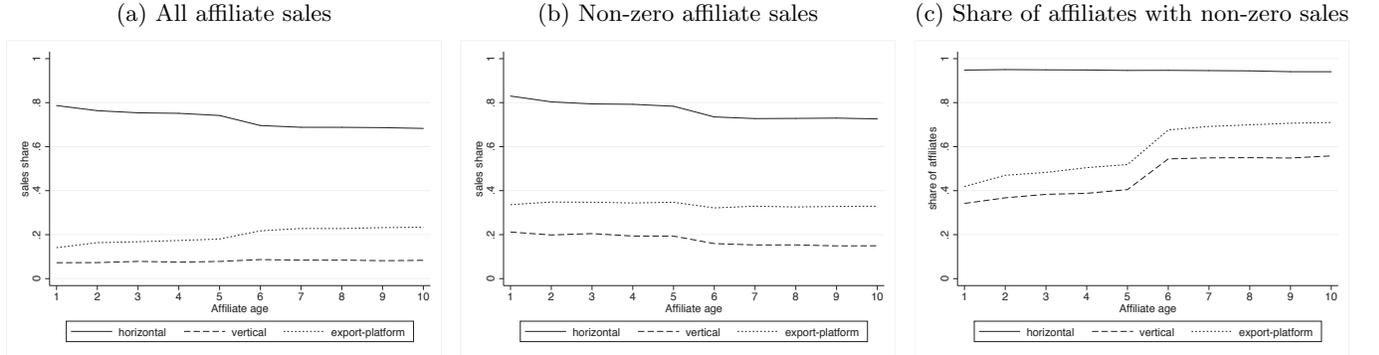
We document the evolution of sales in each activity over the life-cycle of new affiliates, and decompose it into intensive and extensive margins. Figure 2 shows the evolution of mean affiliate sales of each type, as a share of total affiliate's sales, by affiliate age. Panel (a) plots the data for all new affiliates, while panel (b) is limited to affiliates with only positive sales in a given activity (intensive margin). To capture new affiliates transitioning into different kinds of sales, panel (c) plots the percentage of affiliates with non-zero sales in a particular type of activity, so that changes in panel (c) represent the extensive margin. While the mean horizontal share decreases with affiliate age from 0.78 at birth to 0.68 in the tenth year of life of the affiliate, the average vertical sale share increases only slightly from 0.07 to 0.08, while the export platform sales share increases from 0.14 to 0.23. Changes in sales shares are mostly due to the intensive margin in the case of horizontal activities, while for vertical and export-platforms, affiliates with previously zero sales on those activities are the ones contributing to the increase in those activities.

The data suggest that, over time, affiliates born with mostly horizontal sales diversify toward vertical and export-platform sales. However, these vertical and export-platform sales are in the minority of the affiliates' total sales. In general, all affiliates move toward having at least two of the three types of activities by their tenth year of life, as also suggested by the statistics in Table 1.

Figure 2 shows the raw data without any controls, but these patterns are confirmed by re-

¹³Gumpert et al. (2016) also show that exit profiles for new affiliates are not different across FDI entry modes (greenfield vs M&A).

Figure 2: Intensive and extensive margins of sale shares, by activity type



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Horizontal, vertical, and export-platform refers, respectively, to sales to: the market where the affiliate is located; the home market; and third markets—outside the local and home markets. Average sales in each type, as a share of total affiliate sales, include all affiliates (2a) and affiliates with only positive sales in the corresponding activity (2b). Number of affiliates with positive horizontal, vertical and export-platform sales, respectively, as a share of the total number of affiliates are shown in (2c).

gressions including a battery of fixed effects. Tables 4 and 5 report the results of the following regression:

$$\text{affiliate sales}_a^i / \text{total affiliate sales}_a = \beta_a \text{age} + FE + \varepsilon_a^i \quad (2)$$

where the left-hand side variable is sales of each type—horizontal, vertical, and export-platform—for a new affiliate of age a , as a share of total sales of the affiliate, age is the age of the affiliate (from 1 to 10), and ε_a^i is the error term. We include country-year and industry fixed effects, and alternatively, country-year and affiliate fixed effects. In the robustness section, we also include the size of the affiliate and the size of the corporation measured as employment and global sales, respectively; results are unchanged.

On average, sales shares in each activity type for younger affiliates are not significantly different from the shares for older affiliates, but the share of affiliates doing vertical and export-platform activities is higher among older affiliates. Results are sharper when within-affiliate changes are considered: over time, at the intensive margin, new affiliates abandon horizontal sales (almost one-to-one) for export platform activities. At the extensive margin, affiliates add vertical and export platform activities. The patterns observed in Figure 2 are driven by within-firm variation.

The next figure and table focus on the sub-set of pure-type affiliates (i.e., affiliates with all sales in only one activity). First, it is worth noting that affiliates which are born with only horizontal sales represent half the number of observations corresponding to affiliates with some positive horizontal activity, while the ones born purely vertical or export-platform only represent less than five percent

Table 4: Sale shares and age, by activity type. OLS.

Dep var	$sales_a^i/total\ sales_a$					
	$i = H$		$i = V$		$i = E$	
	(1)	(2)	(3)	(4)	(5)	(6)
aff age	-0.002 (0.002)	-0.014*** (0.002)	0.001 (0.001)	0.001** (0.001)	0.001 (0.002)	0.01*** (0.001)
ind fe	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes
Obs	34,181	34,181	34,181	34,181	34,181	34,181
R-sq	0.095	0.025	0.067	0.056	0.001	0.026

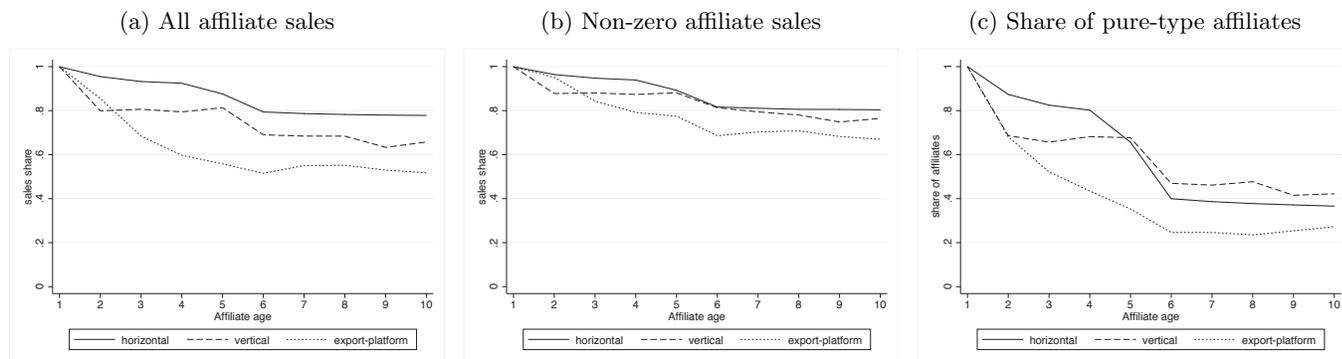
Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variables H , V and E refer, respectively, to horizontal, vertical and export-platform sales. $sales_a^i/total\ sales_a$ refers to sales of type $i = H, V, E$ for a new affiliate of age a , as a share of total affiliate's sales. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Table 5: Intensive and extensive margins of sale shares, by activity type. OLS.

Dep var	$sales_a^i(sales_a^i > 0)/total\ sales_a$						$D(sales_a^i > 0)$					
	$i = H$		$i = V$		$i = E$		$i = H$		$i = V$		$i = E$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
aff age	-0.002 (0.002)	-0.015*** (0.002)	-0.002 (0.002)	-0.003** (0.001)	-0.005** (0.002)	0.005*** (0.002)	-0.001 (0.001)	-0.000 (0.001)	0.006*** (0.002)	0.035*** (0.003)	0.014*** (0.003)	0.042*** (0.003)
ind fe	yes	no	yes	no	yes	no	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes	no	yes	no	yes	no	yes
Obs	32,335	32,335	15,869	15,869	20,127	20,127	34,181	34,181	34,181	34,181	34,181	34,181
R-sq	0.081	0.035	0.080	0.003	0.066	0.005	0.037	0.000	0.059	0.053	0.042	0.062

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variables H , V and E refer, respectively, to horizontal, vertical and export-platform sales. In columns (1)-(6), $sales_a^i(sales_a^i > 0)/total\ sales_a$ refers to sales of type $i = H, V, E$ for a new affiliate of age a , conditional on having positive sales in type i , as a share of total affiliate's sales. In columns (7)-(12), $D(sales_a^i > 0)$ is a dummy variable equal to one if sales of type $i = H, V, E$ for an affiliate of age a are positive. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Figure 3: Pure-type-born affiliates’ intensive and extensive margins of sale shares, by activity type.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Horizontal, vertical, and export-platform refers, respectively, to sales to: the market where the affiliate is located; the home market; and third markets—outside the local and home markets. Figures 3a and 3b show the average horizontal, vertical, and export-platform sales (as a share of the total affiliate sales), respectively, with and without including zero sales in the activity, for the sub-set of pure-type born affiliates (i.e. with all sales in only one activity). Figure 3c shows the number of pure-type affiliates, as a share of the total number of affiliates.

of observations in their respective activity group. That is, affiliates with all their sales in vertical or export-platform activities in their first year of life are extremely few. With this in mind, Figure 3 shows that, on average, sale shares of all pure-type born affiliates decrease with age: the initial specialization is soon followed by more diversified sales patterns. One may get the misleading impression that horizontal sale shares decrease by less than vertical and export-platform shares for affiliates born with only one type of sales, respectively. However, it is worth noting that by the tenth year of life, a pure-vertical born affiliate, for instance, is still in the 95th percentile of the vertical sale share distribution (0.52), as indicated by Table 2—and much above the average (0.08) and median (0.0001) of the distribution—while an affiliate born exclusively serving the host market, by age ten, is around the 50th percentile of the horizontal sale share distribution (0.81). A similar concentration is observed for export-platforms: affiliates born with only exports to third markets end up by age ten with an average export-platform sales share of 65 percent, which corresponds to the 75-95 percentile of this variable’s distribution. We analyze in more detail the persistence of these specialization patterns below.

Affiliates devoted exclusively to vertical and export-platform activities are a stable set of very few affiliates. The set of purely horizontal affiliates shrinks over the life cycle of affiliates indicating, once again, that new affiliates start by serving their host market almost exclusively, and then they start exporting, both back home and to other markets. This is reminiscent of the life cycle of a domestic firm: first, they serve their home market, then they become exporters (and do so at a low

Table 6: Pure-type-born affiliates' sales shares and age, by activity type. OLS.

Dep var	$sales_a^i/total\ sales_a$					
	$i = H$		$i = V$		$i = E$	
	(1)	(2)	(3)	(4)	(5)	(6)
aff age	-0.013*** (0.002)	-0.021*** (0.002)	-0.004 (0.015)	-0.042*** (0.011)	-0.030 (0.019)	-0.059*** (0.009)
ind fe	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes
Obs	16,908	16,908	657	657	683	683
R-sq	0.065	0.078	0.273	0.124	0.429	0.191

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variables H , V and E refer, respectively, to horizontal, vertical and export-platform sales. The sample is restricted to affiliates born with 100 percent of their sales in type $i = H, V, E$, with $sales_a^i/total\ sales_a$ being sales of type $i = H, V, E$, as a share of total affiliate's sales, for an affiliate of age a . Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Table 7: Pure-type-born affiliates' intensive and extensive margins of sale shares, by activity type. OLS.

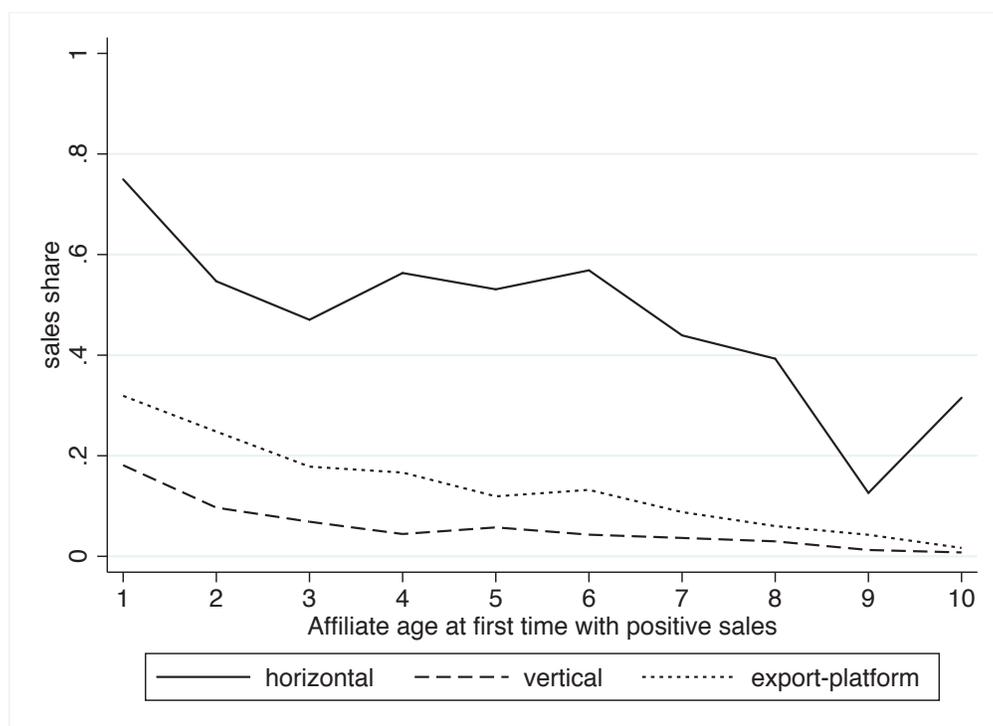
Dep var	$sales_a^i (sales_a^i > 0)/total\ sales_a$						$D(sales_a^i/total\ sales_a = 1)$					
	$i = H$		$i = V$		$i = E$		$i = H$		$i = V$		$i = E$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
aff age	-0.01*** (0.002)	-0.02*** (0.002)	0.01 (0.009)	-0.04*** (0.011)	-0.01 (0.011)	-0.06*** (0.009)	-0.04*** (0.004)	-0.06*** (0.004)	-0.01 (0.015)	-0.07*** (0.011)	-0.05** (0.020)	-0.09*** (0.010)
ind fe	yes	no	yes	no	yes	no	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes	no	yes	no	yes	no	yes
Obs	16,556	16,556	588	588	549	549	16,908	16,908	657	657	683	683
R-sq	0.060	0.078	0.226	0.125	0.564	0.192	0.081	0.160	0.282	0.198	0.386	0.239

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variables H , V and E refer, respectively, to horizontal, vertical and export-platform sales. In columns (1)-(6), the sample is restricted to pure-type born affiliates with positive sales in the respective activity type. In columns (7)-(12), $D(sales_a^i/total\ sales_a = 1)$ is a dummy variable equal to one if the affiliate at age a is of pure-type $i = H, V, E$ (i.e., the share of sales of type i in total sales is 1). Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

intensity in terms of the ratio of export to domestic sales).

Tables 6 and 7 show that the downward-sloping profile observed in Figure 3a for affiliates born with only vertical and export-platform activities is a result of within-affiliate changes. That is, on average, affiliates of different ages have undistinguishable vertical and export-platform sales shares, but a pure-type born new affiliate decreases these shares as it gets older. For affiliates born with only horizontal sales the effect is a mix of between- and within-firm effects.

Figure 4: Sales shares and entry age, by activity type.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Average sales shares in each activity for affiliates that enter that particular activity at age 1, 2, ..., 10. Horizontal, vertical, and export-platform refers, respectively, to sales to: the market where the affiliate is located; the home market; and third markets—outside the local and home markets.

Finally, Figure 4 shows the mean sales share in each activity for affiliates that start that activity at age one, two, ..., ten. This figure makes clear that affiliates which are born performing one activity will continue performing that activity for the remainder of their life. If the activity is incorporated later in life, it is not the main activity of the affiliate. Affiliates that are born doing some horizontal FDI have an average horizontal FDI sale share of above 80 percent (slightly less than the median of this variable’s distribution), while affiliates that are born doing some vertical (export-platform) activities have an average vertical (export-platform) sale share of almost 20 percent (30 percent), which correspond to almost the 90th percentile (75th percentile) of that share’s distribution. If an

affiliate were to start doing horizontal sales in its tenth year of life, on average, it would dedicate only 25 percent of its sales to the local market, while an affiliate that starts exporting in its tenth year would dedicate at most 3 percent of its sales to exports. The message from this figure complements the ones above: an affiliate is most likely to start its main life-long activity of business right away.

In the next section we propose a simple model of affiliate expansion that captures the documented facts.

3 Model

We develop here a simple dynamic model of FDI that is designed to reproduce the facts illustrated in Section 2. In this section we put forward several simplifying assumptions to present the intuition and the mechanism in a transparent way. Infact, this version of the model can be solved entirely in closed form. In Section 5 we extend the model to a more realistic environment in order to use it for quantitative analysis.

The static components of our setting follow the treatment of FDI in Helpman et al. (2004), while the dynamic choice of whether and when to enter a country with an affiliate is modeled as in Fillat et al. (2015). The model features affiliates which serve the host market as well as affiliates that export. For simplicity we do not distinguish between vertical and export platform FDI sales, or among export platform sales to different destinations, but simply treat all these sales as exports of the affiliate. We will reintroduce this distinction in the quantitative analysis in Section 5.

As in Fillat and Garetto (2015) and Fillat et al. (2015), the dynamic FDI decision is treated as a real option that the firm has for the future, and from which it derives value. The option is exercised when the MNE enters a host market with some form of FDI activity.

We assume that MNEs need to first establish an affiliate in the host country and carry out some horizontal activity before eventually engaging in export activities. Because the model is specified in continuous time, these two decisions can happen almost simultaneously: the separation in time of the two decisions is a mere artifact to gain tractability. Crucially, we assume that the choice of whether to open an affiliate in a country—and export from there—is independent for each host country (e.g., whether an affiliate in Germany exports to France is independent of having an affiliate in France).¹⁴ We can interpret this assumption as indicating that the variety that a firm produces

¹⁴We provide empirical evidence in support of this assumption in Section 3.3.

in Germany – for example – is different from the variety that the same firm produces in France (a firm-level Armington assumption). Both the independence in decisions across markets as well as the (artificial) sequential choice of the affiliate activities avoid the difficult computational problem faced by Tintelnot (2016) in a static set-up, which would be even harder to solve in a dynamic environment.

For now, we assume that exit rates are exogenous. This assumption is not entirely consistent with the data, as exit rates are negatively related to size and age (as shown in the Appendix), and will be relaxed in the quantitative analysis. Exit rates, however, are extremely similar across affiliates of different types (non-exporters and exporters) as well as across affiliates performing both activities at different intensities.

3.1 Preferences and Technology

The economy is composed by $N + 1$ countries: the Home country (the U.S. in our data) and N possibly asymmetric foreign countries. The Home country is populated by a given mass of domestic firms, who decide whether to operate only in their home market or to establish foreign affiliates in other countries.

Time is continuous. In each country, over time, consumers have linear preferences over an aggregate good Q :

$$U = \int_0^{\infty} e^{-\rho t} Q(t) dt. \quad (3)$$

$Q(t)$ aggregates a continuum of varieties, indexed by i , with constant elasticity of substitution (CES) given by $\eta > 1$.

Labor is the only factor of production. Each country is populated by a continuum of firms. Each firm in the Home country produces one variety i with a linear technology, and operates under monopolistic competition.

As in Melitz (2003), each firm is endowed with a productivity parameter φ , drawn from a distribution $G(\varphi)$. Each firm sets prices to maximize profits from sales of its own good, and prices are given by a constant mark-up over marginal cost: $p(\varphi) = \frac{\eta}{\eta-1} MC(\varphi)$, where $MC(\cdot)$ denotes the marginal cost of the firm. For domestic production, $MC_d(\varphi) = w_d/\varphi$, where w_d is the wage in the Home country. When the firm sets up an affiliate in a foreign country j ($j = 1, \dots, N$), $MC_j(\varphi) = w_j/\varphi$, as it hires labor in the country where production takes place. In order to produce,

firms also have to pay fixed continuation costs. For simplicity, we assume that there are no fixed costs associated with domestic activity, so that all firms produce in the Home market. Conversely, FDI activities entail positive fixed continuation costs and sunk entry costs.

When a firm establishes an affiliate in a foreign country, it starts by selling there, so engaging in horizontal FDI. To do so, we assume that it has to pay a one-time sunk entry cost, that we denote with $F_j^h > 0$, and a fixed cost $f_j^h > 0$. Once the affiliate is in place, the firm can expand its operations to produce volumes that are destined also to other countries, so engaging in export activities. In this way, the model can generate pure-type horizontal affiliates, but also affiliates that choose more than one mode of operation.¹⁵ We assume that affiliates' exports are associated with start-up costs $F_j^e > 0$, fixed costs $f_j^e > 0$, and variable iceberg costs $\tau_j^e > 1$.

Let $\pi_d(\varphi)$ denote a firm's domestic profits, $\pi_j^h(\varphi)$ denote the profits from local sales of an affiliate located in country j (horizontal sales), and $\pi_j^e(\varphi)$ denote the profits from exports of an affiliate located in country j , given by:

$$\pi_d(\varphi) = H \left(\frac{w_d}{\varphi} \right)^{1-\eta} P_d^\eta Q_d, \quad (4)$$

$$\pi_j^h(\varphi) = H \left(\frac{w_j}{\varphi} \right)^{1-\eta} P_j^\eta Q_j - f_j^h \equiv \bar{\pi}_j^h(\varphi) - f_j^h, \quad (5)$$

$$\pi_j^e(\varphi) = H \left(\frac{\tau_j w_j}{\varphi} \right)^{1-\eta} P_{\sim j}^\eta Q_{\sim j} - f_j^e \equiv \bar{\pi}_j^e(\varphi) - f_j^e, \quad (6)$$

where $H \equiv \eta^{-\eta}(\eta - 1)^{\eta-1}$, P_j is the aggregate price index in country j , and Q_j is the aggregate quantity demanded in country j . $P_{\sim j}^\eta Q_{\sim j}$ represents the total market size for the exports of affiliates located in j .¹⁶

Firms take decisions about whether, when and how to enter a market by computing their expected profits net of entry and continuation costs, which depend on their productivity and on market specific variables (like aggregate productivity shocks or demand shocks), which represent the aggregate state of the economy. For simplicity, we define firm-level productivity φ to be the product of a constant firm-specific component, z , and of a stochastic Home country-specific component, Z , like in Ghironi and Melitz (2005): $\varphi \equiv z \cdot Z$, where z is a firm-specific draw from a given distribution

¹⁵The model does not generate pure exporters. As we report in Section 2, pure exporters are only 3.4 percent of all affiliates in our sample, and account for only 2.2 percent of total affiliate sales.

¹⁶In this version of the model, the problem of the firm is simplified further by not modeling the choice of the destination of export platform sales (which we also cannot observe in the data). In the quantitative analysis in Section 5 we remove this simplification.

$G(z)$ (e.g., Pareto). As in Impullitti et al. (2013), we assume that $Z = e^X$, where X is a standard Brownian motion with drift:

$$dX = \mu dt + \sigma dz, \quad (7)$$

for $\mu \in \Re$ and $\sigma > 0$. This is equivalent to assume that aggregate productivity behaves like a random walk, and that productivity growth is i.i.d. This is a convenient functional form assumption, which guarantees tractability to the model.

In partial equilibrium, the state of the economy is given by the Home country-specific productivity shock X .

3.2 Value Functions

Based on this structure, we define here the Bellman equations and the value functions that characterize the dynamic problem of the firm. Let $\mathcal{V}(z, X)$ denote the expected net present value of a Home country firm with productivity z , when the state of the economy is described by X , and following optimal policy. The value of the firm is given by the value of its domestic operations (which we denote with $V_d(z, X)$), and by the value of its affiliates, which also depends on the affiliates' activities:

$$\mathcal{V}(z, X) = V_d(z, X) + \sum_{j=1}^N \max \left\{ V_j^o(z, X), V_j^h(z, X), V_j^e(z, X) \right\}, \quad (8)$$

where $V_j^o(z, X)$ denotes the option value of opening an affiliate in country j , $V_j^h(z, X)$ denotes the value of a pure type horizontal affiliate in country j , and $V_j^e(z, X)$ denotes the value of an affiliate based in country j that also exports, back home and/or to third markets.

Since all firms always operate in the domestic market, the Bellman equation for the value of domestic operations simply describes the evolution of domestic profits over time depending on the aggregate state. Over a generic time interval Δt ,

$$V_d(z, X) = \frac{1}{1 + (\rho + \delta)\Delta t} \left[\pi_d(z, X)\Delta t + E[V_d(z, X')|X] \right], \quad (9)$$

where ρ is the subjective time discount rate, δ is an exogenous death rate, and X' denotes the aggregate state in the next period.

For each foreign country $j = 1, \dots, N$, the firm may or may not have an affiliate located there. If a firm does not have an affiliate in country j , all the value from operations in j is option value,

i.e., the value of the possibility of entering j in the future. The Bellman equation describing the value of an option country is

$$V_j^o(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} E[V_j^o(z, X')|X]; V_j^h(z, X) - F_j^h \right\}, \quad (10)$$

which describes the fact that a firm may keep the option of entering market j (and get the continuation value of that option), or may enter country j by opening a horizontal affiliate there, in which case it pays the entry cost F_j^h and gets the value of a horizontal affiliate in j , $V_j^h(z, X)$.

Alternatively, a firm may have an affiliate located in country j . In this case, it gets value from horizontal sales to the local market in j . However, once the affiliate is set up, the firm may decide to produce in j to export to other markets, the Home country (vertical FDI) or third countries (export platform FDI). This option is reflected in the Bellman equation

$$V_j^h(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} \left[\pi_j^h(z, X)\Delta t + E[V_j^h(z, X')|X] \right]; V_j^e(z, X) - F_j^e \right\}, \quad (11)$$

where $V_j^e(z, X)$ is the value of sales of an affiliate in country j which is also involved in vertical and/or export platform sales, and F_j^e is the sunk cost of starting exporting from an affiliate in j . Lastly, the value of an affiliate that both sells domestically and exports is simply given by its flow profit over time,

$$V_j^e(z, X) = \frac{1}{1 + (\rho + \delta)\Delta t} \left[(\pi_j^h(z, X) + \pi_j^e(z, X))\Delta t + E[V_j^e(z, X')|X] \right]. \quad (12)$$

The structure that we imposed on the profit functions and the shock process implies that, by evaluating the Bellman Equations in their continuation regions and applying Ito's lemma, we can solve for the value functions in closed form up to multiplicative parameters.¹⁷

The value of domestic sales is simply given by the present discounted value of profits from domestic sales,

$$V_d(z, X) = \frac{\pi_d(z, X)}{\rho + \delta - \hat{\mu}}, \quad (13)$$

where $\hat{\mu} = \mu(\eta - 1) - \frac{1}{2}\sigma^2(\eta - 1)^2$ is the drift of the stochastic process for the profit flow, and the discount rate $(\rho + \delta - \hat{\mu})$ takes into account the exogenous exit rate and the effect of the evolution of aggregate productivity on profits.

¹⁷The derivation of the value functions is contained in Appendix D.

The option value of opening an affiliate is given by:

$$V_j^o(z, X) = B_j^o(z)e^{\beta X}. \quad (14)$$

$B_j^o(z) > 0$ is a firm-specific parameter to be determined and $\beta > 1$ is the positive root of: $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$. The option value is increasing in the realization of the aggregate productivity shock, indicating that there is a higher value to be obtained from opening an affiliate when aggregate productivity is high.

The value of a pure-type horizontal affiliate in country j is:

$$V_j^h(z, X) = B_j^h(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta}, \quad (15)$$

where $B_j^h(z) > 0$ is a firm-specific parameter to be determined. The value of a horizontal affiliate is the sum of discounted profits plus the option value of expanding to third markets. Also the option value of exporting is increasing in the realization of the aggregate productivity shock, indicating that there is a higher value to be obtained from exporting from an affiliate when aggregate productivity is high.

Finally, the value of an affiliate in country j who sells locally and exports is given by the present discounted value of its profits,

$$V_j^e(z, X) = \frac{\bar{\pi}_j^h(z, X) + \bar{\pi}_j^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h + f_j^e}{\rho + \delta}. \quad (16)$$

To completely characterize the problem of the firm, it remains to solve for the two parameters $B_j^o(z)$, $B_j^h(z)$ and for the thresholds in the realizations of the aggregate shocks that induce a firm to open an affiliate or to start exporting from it.

Let $X_j^h(z)$ denote the realization of X that induces a firm with productivity z to open a horizontal affiliate in j , and let $X_j^e(z)$ denote the realization of X that induces a firm with productivity z with a foreign affiliate in j to export from it. The values of $B_j^o(z)$, $B_j^h(z)$, $X_j^h(z)$ and $X_j^e(z)$ are

identified by the following system of value matching and smooth pasting conditions:

$$V_j^o(z, X_j^h) = V_j^h(z, X_j^h) - F_j^h, \quad (17)$$

$$V_j^h(z, X_j^e) = V_j^e(z, X_j^e) - F_j^e, \quad (18)$$

$$V_j^{o'}(z, X_j^h) = V_j^{h'}(z, X_j^h), \quad (19)$$

$$V_j^{h'}(z, X_j^e) = V_j^{e'}(z, X_j^e). \quad (20)$$

The above is a system of 4 equations in 4 unknowns that must be solved for each firm and for each foreign country. The simple structure of the model allows us to solve for the firms' value functions and policy functions in closed form.

The value function parameters $B_j^o(z)$ and $B_j^h(z)$ are given, respectively, by:

$$B_j^o(z) = k_B \cdot \left[\left(\frac{k_j^h(z)}{\beta(\rho + \delta - \hat{\mu})} \right)^{\frac{\beta}{(\eta-1)}} \cdot \left(\frac{f_j^h + (\rho + \delta)F_j^h}{\rho + \delta} \right)^{\frac{\eta-1-\beta}{\eta-1}} + \dots \right. \\ \left. \dots \left(\frac{k_j^e(z)}{\beta(\rho + \delta - \hat{\mu})} \right)^{\frac{\beta}{(\eta-1)}} \cdot \left(\frac{f_j^e + (\rho + \delta)F_j^e}{\rho + \delta} \right)^{\frac{\eta-1-\beta}{\eta-1}} \right], \quad (21)$$

$$B_j^h(z) = k_B \cdot \left(\frac{k_j^e(z)}{\beta(\rho + \delta - \hat{\mu})} \right)^{\frac{\beta}{(\eta-1)}} \cdot \left(\frac{f_j^e + (\rho + \delta)F_j^e}{\rho + \delta} \right)^{\frac{\eta-1-\beta}{\eta-1}}, \quad (22)$$

where k_B is a combination of parameters: $k_B \equiv \frac{(\eta-1)}{(\beta-\eta+1) \frac{(\eta-1-\beta)}{(\eta-1)}}$ and $k_j^h(z)$, $k_j^e(z)$ are firm-specific revenue terms, given by

$$k_j^h(z) = H \left(\frac{w_j}{z} \right)^{1-\eta} P_j^\eta Q_j, \\ k_j^e(z) = H \left(\frac{\tau_j w_j}{z} \right)^{1-\eta} P_{\sim j}^\eta Q_{\sim j}.$$

These parameters describe, respectively, the firm-specific components of the option value of opening an affiliate ($B_j^o(z)$) and of the option value of exporting from an existing affiliate ($B_j^h(z)$). Under the parameter restriction $\beta > \eta - 1$, equation (21) shows that the option value of opening an affiliate is decreasing in both the fixed and sunk costs of opening an affiliate and in the fixed and sunk costs of exporting from the affiliate. In other words, the less costly are an affiliate's operations in a country, the more appealing it is to open an affiliate there. Similarly, from equation (22), the option value of exporting from an affiliate is decreasing in both the fixed and sunk costs

of exporting from the affiliate. Notice that the option value of exports does not depend on the setup and operating cost of maintaining the affiliate. Finally, both option value parameters are increasing in firm-level productivity z , indicating that affiliate operations are more appealing for more productive firms.

The aggregate log-productivity thresholds that induce a firm with productivity z to open an affiliate ($X_j^h(z)$) and to start exporting from it ($X_j^e(z)$) are given by

$$X_j^h(z) = \frac{1}{\eta - 1} \log \left[\left(\frac{\beta}{\beta - \eta + 1} \right) \cdot \left(\frac{\rho + \delta - \hat{\mu}}{k_j^h(z)} \right) \cdot \left(\frac{f_j^h + (\rho + \delta)F_j^h}{\rho + \delta} \right) \right] \quad (23)$$

$$X_j^e(z) = \frac{1}{\eta - 1} \log \left[\left(\frac{\beta}{\beta - \eta + 1} \right) \cdot \left(\frac{\rho + \delta - \hat{\mu}}{k_j^e(z)} \right) \cdot \left(\frac{f_j^e + (\rho + \delta)F_j^e}{\rho + \delta} \right) \right]. \quad (24)$$

As expected, equation (23) illustrates that the aggregate productivity threshold to open an affiliate in country j is increasing in the fixed and sunk costs of opening the affiliate. Similarly, equation (24) illustrates that the aggregate productivity threshold to export from an affiliate in country j is increasing in the fixed and sunk costs of exporting from the affiliate. Moreover, both thresholds are decreasing in firm-level productivity z , indicating that more productive firms need smaller positive productivity shocks to start and expand affiliate operations compared to less productive firms.

Notice that if $\frac{f_j^h + (\rho + \delta)F_j^h}{P_j^\eta Q_j} < \frac{(f_j^e + (\rho + \delta)F_j^e)\tau^{\eta-1}}{P_{\sim j}^\eta Q_{\sim j}}$, *i.e.*, if the overall cost of a horizontal affiliate relative to its host market size is lower than the overall cost of a diversified affiliate relative to its destination market size, then $X_j^h(z) < X_j^e(z)$. We assume that this restriction holds to illustrate the predictions of the model that follow.

3.3 Qualitative Testable Implications

We argue that the simple model presented here is a useful tool to represent the patterns that we observe in the data. First, we have shown in Section 2 that MNEs' affiliates start out their operations as sizable entities, and that the affiliate-to-parent sales ratio is virtually flat in the first years after the establishment of the affiliate, regardless of the destination of its sales. Our simple shock process captures exactly this fact: as firm-specific productivity can be perfectly transferred

abroad, firm’s domestic and foreign sales perfectly co-move.¹⁸ Hence the model is consistent with the fact documented in Figure 1.

Additionally, we documented in Section 2 that more than 90 percent of all affiliates have some horizontal sales at birth and that a negligible share of affiliates are pure type vertical or export platform. The assumptions that make the model tractable are broadly consistent with these observations. In the model, firms start FDI either with purely horizontal sales or with some horizontal sales and some exports.¹⁹ There are no pure exporters in the model, but those are only 3.4 percent of affiliates in the data, and they account for only 2.24 percent of total affiliate sales during our sample period.

The specification of the aggregate productivity shock as a unit root process drives persistence in the affiliate’s type as observed in the data. Moreover, if aggregate productivity grows over time (i.e., $\mu > 0$), firms will tend to expand internationally, giving rise to the diversification pattern that we documented.

The independence assumption that allows us to keep the model tractable (i.e. the fact that the decisions of both opening an affiliate and exporting from it are independent across countries) bears support in the data. This assumption implies that it is possible for a firm to have an affiliate in a country and at the same time to have affiliates elsewhere that export to that same country. This is different from the models in Arkolakis et al. (2014) and Tintelnot (2016), in which there is only one lowest-cost location for each firm to reach consumers in a country. As we mentioned above, the BEA data contains limited information about the destination of affiliates’ exports. Some information, however, is available. Using data from the 2004 Benchmark Survey, we are able to examine the coexistence of affiliates’ exports to three countries—Canada, the United Kingdom, and Japan—and the presence of affiliates owned by the same parents in those countries. Our calculations imply that of the 20,359 affiliates that export to Canada, 13,082 (64 percent) belong to a U.S. parent that also has affiliates located in Canada. Similarly, of the 5,017 affiliates that export to the United Kingdom, 3,516 (70 percent) belong to a U.S. parent that also has affiliates located in that country. Finally, of the 5,224 affiliates that export to Japan, 2,477 (47 percent) belong to a U.S. parent that

¹⁸The ratio of affiliate to parent sales is completely flat in partial equilibrium, since the effects of aggregate shocks on both parent’s sales and affiliate’s sales cancel out. However, in an industry equilibrium, aggregate prices and quantities fluctuate and drive fluctuations in the ratio. The fluctuations induced by productivity shocks on aggregate variables are typically small in this class of models, so we expect that also in the industry equilibrium the ratio of affiliate to parent sales will be relatively stable over time.

¹⁹Setting-up out the model in continuous time implies that even if all firms technically start FDI with only horizontal sales, some firms may decide to start vertical and/or export platform instantaneously, so that observationally this is equivalent to affiliates starting FDI with a combination of modes.

Table 8: Probabilities of observing affiliates in top-ten most popular destinations.

MNEs with:	Probability of observing an affiliate in the same region					
	at least 10 affiliates		at least 5 affiliates		at least 2 affiliates	
	uncond.	conditional	unconditional	conditional	unconditional	conditional
1 Canada	0.61	0.70	0.51	0.62	0.42	0.54
2 Germany	0.71	0.72	0.60	0.62	0.40	0.51
3 France	0.70	0.71	0.57	0.59	0.36	0.45
4 China	0.66	0.68	0.48	0.54	0.29	0.44
5 Ireland	0.30	0.30	0.21	0.22	0.13	0.16
6 Mexico	0.61	0.70	0.50	0.54	0.32	0.40
7 Netherlands	0.55	0.56	0.49	0.43	0.24	0.31
8 Brazil	0.60	0.60	0.50	0.47	0.26	0.31
9 Italy	0.61	0.61	0.50	0.51	0.42	0.41
10 Singapore	0.45	0.46	0.46	0.36	0.19	0.28

Note: Countries in the list belong to four regions (Europe, Asia, Latin America, and North America). Conditional probabilities refer to the probability of observing an affiliate country i given that the parent has an affiliate in a different country of the same region. The number of parents with at least X affiliates worldwide is: 2,896 (X=10), 6,562 (X=5), and 14,974 (X=2).

also has affiliates located in Japan.

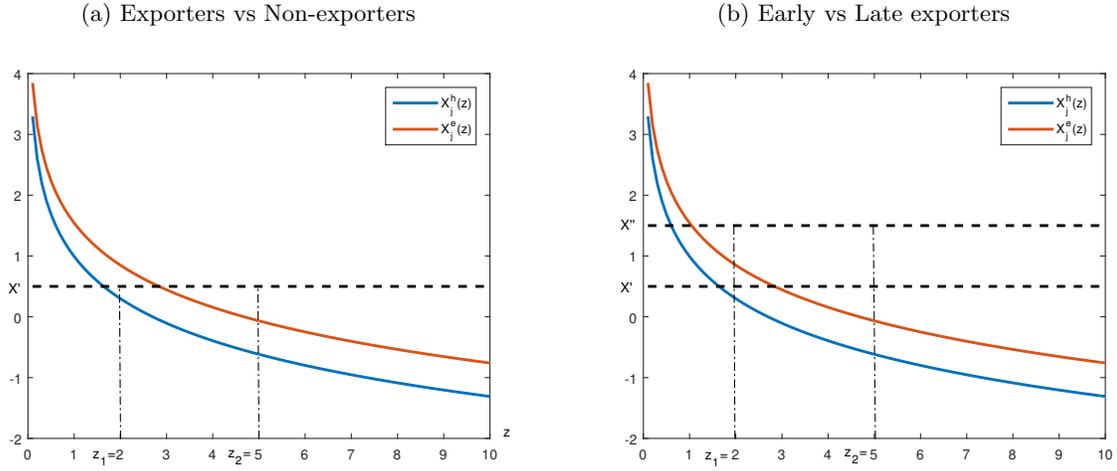
A more direct test of the independence assumption is, for a given U.S. parent, the comparison between the probability of owning an affiliate in a country and the same probability conditional on already having an affiliate in a “neighboring” country (*i.e.* in a country belonging to the same region). Of course, the comparison is only possible for U.S. parents with at least two foreign affiliates. Table 8 shows the results for MNEs with at least two, five, and ten affiliates worldwide, respectively. Countries are restricted to the ten most popular destinations for U.S. MNEs that belong to four regions: North America, Europe, Latin America, and Asia (Central, South, and East Asia, plus the Pacific).

Conditional and unconditional probabilities are strikingly similar the larger the MNE in terms of number of affiliates worldwide. The largest differences are observed when we include smaller MNEs (at least 2 affiliates) in countries such as China.

We take these pieces of evidence as supporting the assumption of independence of MNEs’ location choices. We move now to illustrate the implications of the cross-sectional, Melitz-style component of the model.

The relationship between firm-level productivity, host market characteristics, and the entry and exporting thresholds, $X_j^h(z)$, $X_j^e(z)$, has four testable implications. We start by describing predictions relating size in the host market with export status and timing of exports.

Figure 5: Affiliate size in the host market, export status, and timing of exports. Model.



Since larger and more productive firms have lower entry thresholds, we expect affiliates that are “diversified” at birth to have larger horizontal sales than affiliates born as pure horizontal affiliates. Similarly, conditional on aggregate productivity increasing over time, affiliates that start exporting later in life should have smaller horizontal sales than affiliates that start exporting at the beginning of their life cycle.

Figure 5 illustrates these predictions. Suppose that, for a hypothetical realized aggregate shock X' , we observe two firms having affiliates in the same host country j . Firm 1 (with productivity z_1) has a pure type horizontal affiliate in j , while firm 2 (with productivity z_2) has a diversified affiliate in j . Since the thresholds $X_j^h(z)$, $X_j^e(z)$ are decreasing functions of z , they are invertible. The observed selection pattern of affiliates indicates that $z_2 \geq z_j^e(X') \geq z_1$, hence $z_2 \geq z_1$ implies that the horizontal sales of the diversified affiliate of firm 2 must be larger than the horizontal sales of the pure horizontal affiliate of firm 1. To illustrate the prediction about the timing of exports, suppose now that as aggregate productivity grows on average, the realization of the aggregate shock becomes $X'' > X'$. Now, as illustrated in the right panel of the figure, $z_1 \geq z_j^e(X'')$ and also firm 1 starts exporting from its foreign affiliate. Hence early exporters are more productive and larger than affiliates that start exporting later in life, so should exhibit larger horizontal sales than late exporters. Notice that both predictions hold comparing affiliates of different firms that make the same location choices (*i.e.*, keeping the host country fixed).

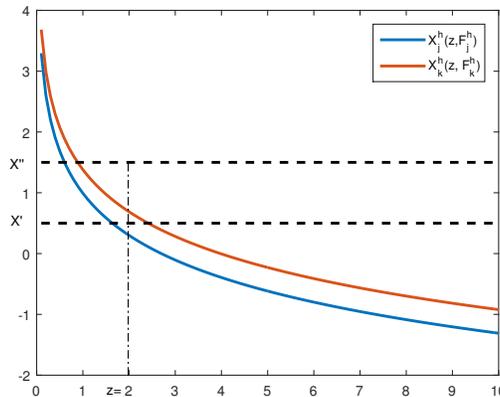
The predictions illustrated above are related to the variation in export status and timing of exports across individual affiliates of different corporations that are located in the same country.

The model also has implications about the expansion strategies of MNEs across countries, *i.e.* the extensive margin of affiliate opening.

First, the model predicts that MNEs open first their largest affiliates and subsequently their smaller affiliates. Second, MNEs open first affiliates located in larger countries and subsequently affiliates located in smaller countries. Both these predictions derive from the fact that the productivity threshold to open a horizontal affiliate is decreasing in host country size: $\frac{\partial X_j^h(z)}{\partial F_j^h} \leq 0$.

Third, the model predicts that MNEs open first affiliates in markets with lower entry costs. Since the productivity threshold to open a horizontal affiliate is increasing in entry costs ($\frac{\partial X_j^h(z)}{\partial F_j^h} \geq 0$), if a firm is evaluating whether to set up affiliates in countries j, k such that $F_j^h < F_k^h$, the corresponding thresholds will be ordered as follows: $X_j^h(z) < X_k^h(z) \Rightarrow z_j^h(X) < z_k^h(X)$. As a consequence, as aggregate productivity grows, the firm will first reach the threshold necessary to enter the cheaper country j and will possibly enter the costlier country k later in life. Figure 6 illustrates this prediction. The firm indicated in the figure only opens an affiliate in country j when the realization of the aggregate shock is X' . When aggregate productivity grows to $X'' > X'$, the firm can afford to open an affiliate also in country k . Notice that the predictions about MNEs expansion hold within corporation across affiliates located in different countries.

Figure 6: Entry costs and the timing of entry. Model.



4 Back to the Data

In this section we continue our examination of the data, now guided by the model. We present empirical evidence that strongly supports the model's predictions about affiliate size, export status,

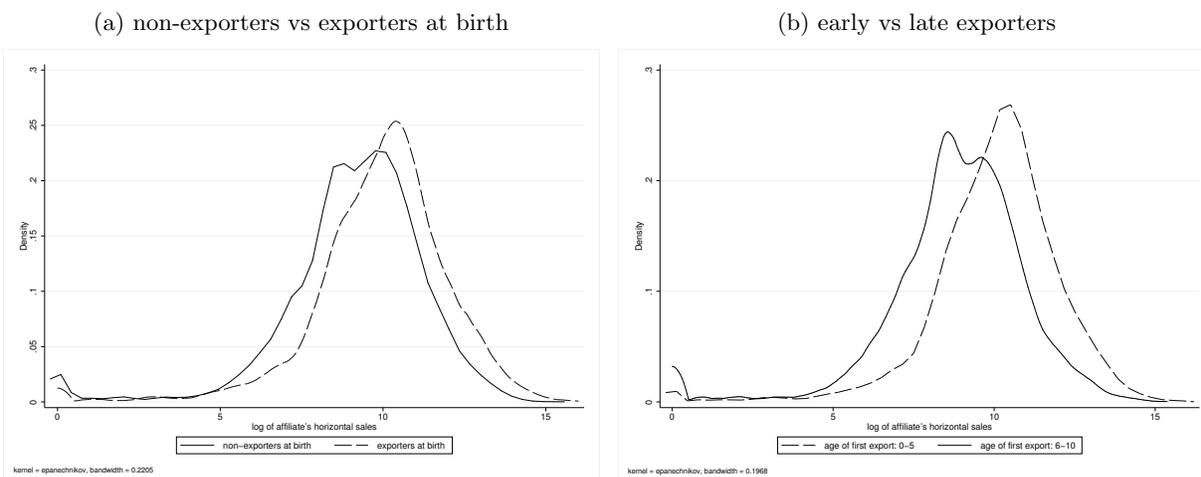
and timing of exports, as well as those about MNEs' expansion across countries.

4.1 Affiliate size, export status, and timing of exports

i. Affiliates that export have larger horizontal sales than pure horizontal affiliates at birth.

We start by illustrating graphically the relationship between the size of an affiliate in the country where it is located and the extent of its diversification across FDI modes. Figure 7a plots the kernel density of log horizontal sales for two subsets of affiliates in our sample: affiliates that are born as pure horizontal, and affiliates that are born having both horizontal sales and export sales (back Home or to third countries). The figure clearly shows that affiliates that serve their market of operation and other markets are on average larger than pure horizontal affiliates, consistent with the model's prediction.

Figure 7: Affiliate size in the host market, export status, and timing of exports.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Figure 7a shows the kernel density of log horizontal sales for affiliates that are born as pure horizontal (non-exporters), and affiliates that are born exporting (exporters). Figure 7b shows the kernel density of log horizontal sales for affiliates that start exporting in their first five years of life, and affiliates that start exporting after five years of life. Affiliates that never export are not included. Pure vertical and pure export platform affiliates are excluded from both figures.

To test the robustness of this prediction beyond the graphical illustration, we regress the affiliate's horizontal sales on a dummy variable taking value one if the affiliate is pure horizontal at birth; zero if the affiliate is also an exporter. Column 1 in Table 9 reports the results.

As expected, size in the host market is negatively correlated with being a pure horizontal

Table 9: Affiliate size in the host market, by export status. OLS.

Dep var	log of horizontal sales		
	(1)	(2)	(3)
D(pure H at birth)	-0.860*** (0.012)		-0.573*** (0.083)
age	0.071*** (0.088)	0.086*** (0.017)	0.074*** (0.013)
age at first export		-0.133*** (0.015)	-0.082*** (0.019)
Obs	31,558	27,774	27,774
R-sq	0.10	0.05	0.11

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variable $D(\text{pure } H \text{ at birth})$ is equal to one if the affiliate is born with only horizontal sales. Pure exporters (either vertical or export-platform) are not included. All specifications include country-year and industry fixed effects. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

affiliate at birth. In other words, mimicking well-documented facts on domestic exporters, affiliates that export are larger in their host country of operation than affiliates whose sales are limited to their host country. This result holds controlling for affiliate age, and is robust to the inclusion of country-year and industry fixed effects.

In a robustness exercise, we refine this statement and show that, in general, affiliates with larger shares of horizontal sales are smaller, while affiliate with larger shares of vertical sales are the largest. Moreover, within-affiliate comparisons show that affiliates grow larger as they increase their share of vertical sales.

ii. Affiliates that start exporting earlier in life have larger horizontal sales than late starters.

Figure 7b illustrates the relationship between the size of an affiliate in its host country, measured by (log of) sales in the host country, and the time in the affiliate's life cycle at which it decides to export. We split the sample between affiliates that start exporting in the first five years of life and affiliates that start exporting later in their life cycle.²⁰ As the Figure shows, affiliates that start exporting earlier in life are larger in their host country compared to affiliates that start exporting later.

Column 2 in Table 9 shows that the negative relationship between size in the host market and

²⁰This fact does not depend on the choice of the age cutoff for first exports: exporter at age one vs exporter at age older than one; and never exporter vs ever exporter.

Table 10: Affiliate size, by affiliate position in the MNE opening sequence.

Avg.	first affiliates	subsequent affiliates
affiliate employment	709	526
affiliate sales, as % of parent's	20	6.5
affiliate H-sales, as % of parent's	10	3.8
diversified affiliates (%)	65.7	59.3

Note: Observations at the affiliate level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing.

age of first export suggested by the figure is robust to the inclusion of affiliate age, country-year and industry fixed effects. Finally, column 3 in Table 9 shows that the two testable implications of our simple model jointly hold in the data.

4.2 The expansion strategies of MNEs across countries.

So far, we have looked at the data from the perspective of individual affiliates. The following facts provide support for the predictions of the model regarding the expansion strategies of MNEs as a whole.

iii. MNEs open their largest affiliates before their smaller affiliates.

Due to the monotonic relationship linking entry threshold, host market size, and affiliate sales, the model predicts that MNEs open first their larger affiliates, and subsequently their smaller affiliates.

Table 10 shows that, indeed, first affiliates are on average larger than subsequent affiliates, in terms of employment, total sales, and host market sales. Figure C.3 in the Appendix further illustrates that the three-time difference in sales between the first and subsequent affiliates is observed continuously over the affiliate's life cycle, and also regardless of the activity type (i.e., horizontal, vertical, and export-platform).²¹

This pattern is confirmed by the regressions in Table 11, which include country-year and industry fixed effects. First affiliates have affiliate-to-parent sales ratios that are significantly

²¹Figures are very similar if we consider affiliates first in a given country, as well as affiliates first in an industry, vs subsequent affiliates in that country or industry.

Table 11: Affiliate size and opening strategy sequence of MNEs. OLS.

Dep var	Affiliate-to-parent sales ratio	
	all sales	horizontal sales
age	0.004** (0.002)	0.002** (0.001)
D(1st affiliate)	0.110*** (0.035)	0.042*** (0.015)
age \times D(1st affiliate)	-0.008** (0.004)	-0.002 (0.002)
log global employment	-0.040*** (0.005)	-0.015*** (0.002)
Obs	12,203	12,203
R-sq	0.13	0.08

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. The variable $D(1st\ affiliate)$ is equal to one if the affiliate is the first in the opening sequence of the MNE. The variable "global employment" refers to the aggregate employment of the MNE, both in the United States and abroad. All specifications with country-year and industry fixed effects. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

larger than subsequent affiliates, including the aforementioned fixed effects and controlling for affiliate age and size of the corporation.

iv. MNEs open first affiliates in markets with lower entry costs.

The model predicts sorting in the order in which a MNE opens affiliates over time: affiliates located in countries with lower entry costs (F_j^h) will be opened first.

Table 12 presents the results. We follow Fillat et al. (2015) and proxy for entry costs using indicators provided by the World Bank's Doing Business Database. This database includes four measures of the costs of opening a new business in a given country: the number of administrative procedures required to open a business, the average number of days it takes to open, the cost of starting a business as a percent of GDP per capita, and the minimum capital requirement in US dollars. It is clear that, under various measures, the MNE chooses on average to open affiliates first in markets that are less costly to enter. Countries in which MNEs open their first affiliate have around a 20 percent lower number of business procedures, take 20, rather than 25, days to open a business, and have a cost of starting a business that is two thirds of the cost faced in subsequent markets. Additionally, on average, affiliates are

Table 12: Entry costs and market size, by affiliate position in the MNE opening sequence.

Avg.	first affiliates	subsequent affiliates
number of biz procedures	6.2	7.5
number of days to start biz	20.0	25.1
cost of start biz (% of GDPpc)	7.7	11.4
Min K needed to start biz (U\$)	5,139	6,311
GDP (in bill of U\$)	983	823

Note: Observations at the affiliate level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Variables related to entry costs are from the World Bank, Doing Business.

first opened in markets for which the minimum capital needed to start a business is 20 percent lower. The table also shows that MNEs, on average, open first affiliates in larger markets, where market size is proxied by GDP.

The empirical patterns we have shown in this section lend support to modeling the cross-section of MNEs’ affiliates along the lines of a Melitz model where productivity grows over time. The facts illustrate that – in the cross-section – affiliates of US MNEs broadly behave like domestic firms. On the dynamic dimension, at the contrary, MNEs’ affiliates display much flatter growth profiles, more persistence, and less extended gravity than domestic exporters.²²

5 Quantitative Analysis

In this Section we extend the model to a more realistic setting, where firms can endogenously decide to shut down affiliates, the choice of the destination of affiliate exports is endogenous and explicitly modeled, and exporting affiliates can endogenously decide to exit any export market. This richer model can be calibrated to match the three types of FDI that we observe in the data and whose dynamic behavior we described in Section 2.

Incorporating endogenous exit is important for two reasons. Empirically, exit rates are not random in the data: they decline with affiliate age and size, but are generally independent on the

²²Following Morales et al. (2015), we refer to extended gravity as the difference between the unconditional probability to export to a country and the probability to export to the same country conditional on previous exports to “neighboring” locations.

type of affiliate sales.²³ Quantitatively, incorporating endogenous exit rates in the model allows to separately identify the fixed and sunk costs of affiliate opening by matching moments on entry and exit rates.

5.1 Extended Model: Endogenous Exit and Multiple Export Destinations

Extending the model to feature endogenous exit, either from a host country or from an export market, is straightforward. The value functions include an extra term, which is the option value of exit, and additional value-matching and smooth pasting conditions deliver the equilibrium exit thresholds.

The modeling of the choice of the destination of affiliate exports is more complex. An affiliate located in any country j can in principle export to any subset of the set of potential export destinations, and the value of an exporting affiliate depends on the set of countries in which the affiliate exports. To maintain the problem tractable, we resort again to an independence assumption. Precisely, we assume that the decision of an affiliate located in country j to export to any country $k \neq j$ is independent from the decision of the same affiliate to export to any other country. Relying on this assumption, we can write the problem of the firm as a compound option and solve it backwards, as suggested by Dixit and Pindyck (1994, chap. 10). In other words, conditional on the firm having an affiliate in country j , we solve for the value of exports and of horizontal sales, and determine the thresholds that induce the affiliate to export or stop exporting to each third country $k \neq j$. Once determined the value of an affiliate in country j , we solve for the thresholds that induce the firm to open or shut down that affiliate.

In this setting, the value of a firm with productivity z when the state of the economy is described by X is denoted by $\mathcal{V}(z, X)$ and is defined as:

$$\mathcal{V}(z, X) = V_d(z, X) + \sum_{j=1}^N \max \{V_j^o(z, X), V_j^a(z, X)\}, \quad (25)$$

where $V_d(z, X)$ is the value of domestic (U.S.) sales, $V_j^o(z, X)$ is the option value of opening an affiliate in country j , and $V_j^a(z, X)$ is the value of an affiliate in country j , regardless of the

²³See Figures C.1, C.2 and Table C.1 in Appendix C.

destination of its sales. In turn, we can define $V_j^a(z, X)$ as:

$$V_j^a(z, X) = V_j^h(z, X) + \sum_{k \neq j} \max \{V_{jk}^o(z, X), V_{jk}^e(z, X)\}, \quad (26)$$

where $V_j^h(z, X)$ is the value of horizontal sales, $V_{jk}^o(z, X)$ is the option value of exporting to country k for an affiliate located in j , and $V_{jk}^e(z, X)$ is the value of exports to country k for an affiliate located in j . This formulation of the problem is analogous to a compound option because opening an affiliate in a country is equivalent to exercising an option that gives access to an another set of options: the options to export to any other country.

To solve the problem of the firm, we start by solving for $V_{jk}^o(z, X)$ and $V_{jk}^e(z, X)$, conditional on the firm having an affiliate in country j . This is a simple case of interlinked options (see Dixit and Pindyck 1994, chap. 7), that gives as solution for the value functions:²⁴

$$V_{jk}^o(z, X) = B_{jk}^o(z) e^{\beta X} \quad (27)$$

$$V_{jk}^e(z, X) = \frac{\bar{\pi}_{jk}^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_{jk}^e}{\rho + \delta} + A_{jk}^e(z) e^{\alpha X} \quad (28)$$

where $B_{jk}^o(z) > 0$, $A_{jk}^e(z) > 0$ are firm-specific parameters to be determined and $\alpha < 0$, $\beta > 1$ are the roots of: $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$. As the term $B_{jk}^o(z) e^{\beta X}$ represents the option value of exporting to country k , and is increasing in the realization of the aggregate productivity shock, similarly the term $A_{jk}^e(z) e^{\alpha X}$ is the option value of quitting the export market k , and is decreasing in the realization of the aggregate productivity shock, indicating that the option of exiting an export market has a larger value in “bad times”.

For each country pair (j, k) and for each firm with productivity z , the parameters $B_{jk}^o(z) > 0$, $A_{jk}^e(z) > 0$ and the aggregate productivity thresholds that induce the affiliate to start and stop exporting (let's denote them with X_{jk}^{OE} , X_{jk}^{EO}) can be recovered by the following system of value-matching and smooth pasting conditions:

$$V_{jk}^o(z, X_{jk}^{OE}) = V_{jk}^e(z, X_{jk}^{OE}) - F_{jk}^e \quad (29)$$

$$V_{jk}^o(z, X_{jk}^{EO}) = V_{jk}^e(z, X_{jk}^{EO}) \quad (30)$$

$$V'_{jk}{}^o(z, X_{jk}^{OE}) = V'_{jk}{}^e(z, X_{jk}^{OE}) \quad (31)$$

$$V'_{jk}{}^o(z, X_{jk}^{EO}) = V'_{jk}{}^e(z, X_{jk}^{EO}). \quad (32)$$

²⁴Details about the solution of this problem are contained in Appendix D.

Equations (27)-(28) characterize the portion of value of the affiliate coming from exports. The value of horizontal sales conditional on having an affiliate is given by:

$$V_j^h(z, X) = A_j^h(z)e^{\alpha X} + \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} \quad (33)$$

where $A_j^h(z) > 0$ is a firm-specific parameter to be determined. In other words, the value of horizontal sales is given by the present discounted value of profits from horizontal sales plus the option value of shutting down the affiliate. Hence the value of an affiliate in country j can be written as:

$$V_j^a(z, X) = A_j^h(z)e^{\alpha X} + \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} + \sum_{k \in \mathcal{A}_j(z)} \left[\frac{\bar{\pi}_{jk}^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_{jk}^e}{\rho + \delta} + A_{jk}^e(z)e^{\alpha X} \right] + \sum_{k \notin \mathcal{A}_j(z)} \left[B_{jk}^o(z)e^{\beta X} \right] \quad (34)$$

where $\mathcal{A}_j(z)$ denotes the set of export markets in which an affiliate of a firm with productivity z located in country j exports. Up to this point, the value of an affiliate $V_j^a(z, X)$ is completely characterized up to the option value parameter $A_j^h(z)$. Equation (34) is also instructive to think about the implications of the independence assumption. The value of an affiliate does not depend on the sales or on the value of the firm's other affiliates in other countries, because of the independence assumption. However, it does depend on the set of potential export destinations from the affiliate's host country.

It remains to solve for the decision of a firm to set up an affiliate in country j . The option value of opening an affiliate is:

$$V_j^o(z, X) = B_j^o(z)e^{\beta X}. \quad (35)$$

Hence for each host country j and for each firm with productivity z , the parameters $B_j^o(z) > 0$, $A_j^h(z) > 0$ and the aggregate productivity thresholds that induce the firm to open and shut down an affiliate (let's denote them with X_j^{OH} , X_j^{HO}) can be recovered by the following system of value-matching and smooth pasting conditions:

$$V_j^o(z, X_j^{OH}) = V_j^a(z, X_j^{OH}) - F_j^h \quad (36)$$

$$V_j^o(z, X_j^{HO}) = V_j^a(z, X_j^{HO}) \quad (37)$$

$$V_j^{\prime o}(z, X_j^{OH}) = V_j^{\prime a}(z, X_j^{OH}) \quad (38)$$

$$V_j^{\prime o}(z, X_j^{HO}) = V_j^{\prime a}(z, X_j^{HO}). \quad (39)$$

Lastly, the value of domestic sales is simply given by the present discounted value of profits from domestic sales,

$$V_d(z, X) = \frac{\pi_d(z, X)}{\rho + \delta - \hat{\mu}}. \quad (40)$$

5.2 Numerical Illustration: the Rise of China

We start by illustrating the mechanisms of the extended model with a simple example. Assume that there are three countries: the United States, Japan, and China. As in our data, multinational firms are headquartered in the United States. They have to decide whether and where to open affiliates and whether the affiliates are going to sell only in the market where they are located or also export.

We start with a scenario where China is identical to Japan. In this case, we set the parameters as follows: $w = P = Q = 1$, $F^h = 10$, $F^e = 8$, $\tau = 2$, and $f^e = f^h = 0.3$. A more realistic scenario is one where China is poorer, but larger, than Japan. In this case we set $w_{chn} = P_{chn} = 0.6$ and $Q_{chn} = 2$ — we will refer to this as our “baseline” scenario. Then, keeping the baseline values for wages and expenditure unchanged, we perform three comparative statics exercises, meant to depict the “rise of China” in the different ways that our model allows: a drop in iceberg trade costs from China to Japan, a drop in the sunk cost of entering China, and a drop in the sunk cost of exporting from China to Japan. To explore the effect of a reduction in affiliate entry cost, we lower the fixed cost F_{chn}^h such that the per-period cost of horizontal FDI ($f_{chn}^h + (\rho + \delta)F_{chn}^h$) drops of 10%. Similarly, to explore the differential effects of reductions in iceberg versus fixed costs of exports, we lower either $\tau_{chn,jpn}$ or $F_{chn,jpn}^e$ in such a way that the per-period cost of affiliate exports ($(f_{chn}^e + (\rho + \delta)F_{chn}^e)\tau_{chn,jpn}$) drops of 10%. Notice that in this way, a static model would generate exactly the same comparative statics following a change in F_e and a change in τ , so this exercise is well suited to illustrate the different predictions of our model compared to static ones.²⁵ The purpose of this exercise is to show that different changes in the cost structure of multinational activity across host countries generate differences in the number of affiliates, sales, and the share of affiliates that exports.

Table 13 reports the number of U.S. affiliates located in China and Japan, respectively, under

²⁵The remaining parameters of the model are: the time preference parameter, $\rho = 0.02$; the exogenous death rate, $\delta = 0.01$; the elasticity of substitution, $\eta = 2$; the drift for the productivity process, $\mu = 0.02$; the standard deviation for the productivity process $\sigma = 0.06$; the Pareto-shape parameter, $\theta = 3$; and the lower bound of the Pareto distribution, $b = 1$. We simulate 500 affiliates and 50 periods.

Table 13: Number of U.S. affiliates abroad, by country. Three-country simulation.

	symmetry	baseline	low $\tau_{chn,jpn}$	low F_{chn}^h	low $F_{chn,jpn}^e$
Japan	191	191	191	191	191
China	191	352	352	403	352

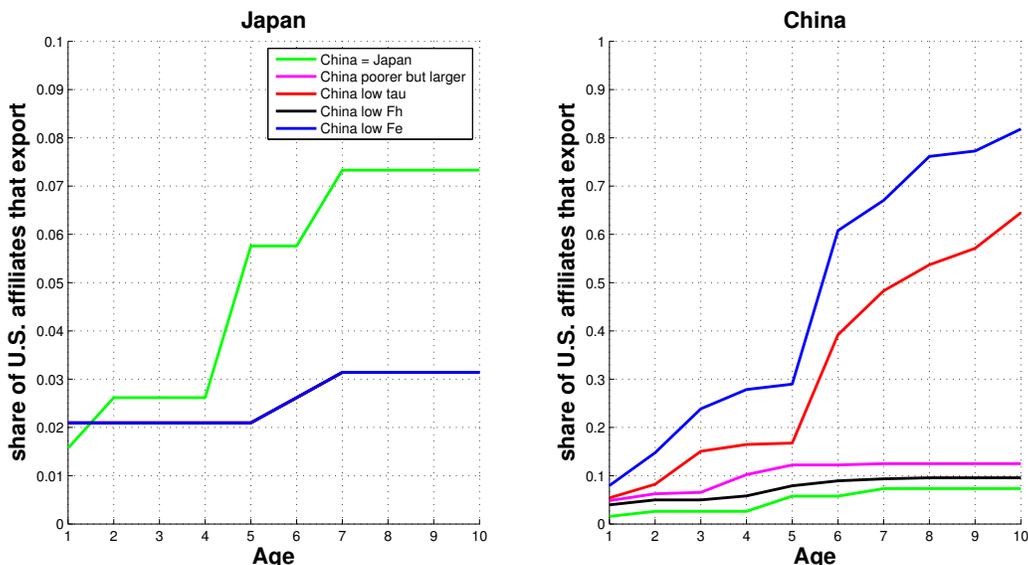
Notes: Affiliates that survive at least ten periods in the model, in each country.

each scenario. Of course in the symmetric case the U.S. opens the same number of plants in the two host countries. As expected, in the baseline scenario where China is larger and poorer than Japan, the number of affiliates located in China increases. Starting from the baseline scenario, a decrease in the sunk cost of affiliate opening increases the numbers of affiliates located in China. However, changes in the costs of affiliate exports from China to Japan do not change the number of U.S. affiliates that choose to operate in China and survive at least than ten periods. The fact that the number of U.S. affiliates located in Japan does not change with shocks to China’s MNE costs is just a consequence of our independence assumption on the MNE location choices and of the partial equilibrium nature of this example. The number of U.S. affiliates in Japan could change through general equilibrium effects (like for example changes in the price index).

Figure 8 shows the shares of exporting affiliates located in Japan (left panel) and China (right panel) in each of the described scenarios. As expected, in the symmetric case those shares are the same. In the baseline case, where China is larger and “poorer” — because of lower wages—, more U.S. firms open affiliates in China than in Japan, and those affiliates have lower costs than the ones in Japan, so they are more likely to export. Notice that quantitatively, the cost savings of U.S. affiliates in China dominate the fact that U.S. affiliates located in Japan have a larger export market.

The change in the sunk cost F_h significantly increases the number of affiliates in China, but not their likelihood to export, which actually decreases as less productive affiliates enter. On the other hand, changes in variable and sunk costs of affiliate exports don’t affect the number of affiliates, but increase their probability to export. It is interesting to notice how the reduction in the sunk cost of export generates a larger effect than the reduction in the variable cost, despite the numbers being chosen so that the change on per-period costs is the same. The reason for this difference is the following: when F_e decreases, not only the per period cost decreases, but the band of inaction

Figure 8: Share of exporters, by age. Three-country simulation

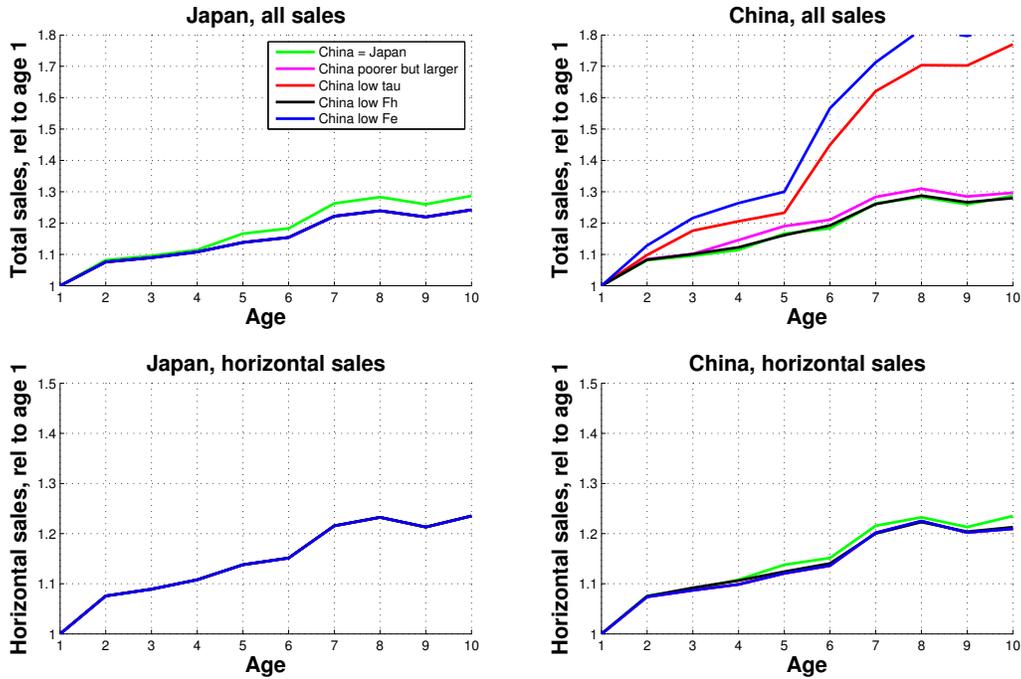


shrinks. In other words, affiliates are more likely to start exporting because now their optimal choices are more flexible. The overall effect of F_e is larger than the one of τ because it is driven by both a static and a dynamic effect.

Other numerical exercises we performed suggest that changes in different frictions may have different effects for the timing of export expansion: the expansion is likely to be faster following the decrease in τ while it takes longer following the decrease in the sunk cost of export F_e . This suggests that the nature of FDI costs also matters for the timing of MNEs expansion.

Finally, Figure 9 shows average horizontal sales and total sales of affiliates located in Japan (left panel) and China (right panel) in each of the described scenarios. The “rise of China” has interesting effects also on sales in the countries where the affiliates locate and export to. As the variable cost $\tau_{chn,jpn}$ declines, Chinese affiliates become more profitable because exporting is now less costly. This decline induces more entry in China and increases exports to Japan. Lower entry costs into China don’t have almost any effect on horizontal and total sales of Chinese affiliates, while lower sunk costs of exports for Chinese affiliates have mostly the effect of increasing total sales of Chinese affiliates via exports.

Figure 9: Affiliate sales, by age. Three-country simulation.



Notes: Affiliates that survive at least ten periods in the model. Averages across affiliates.

6 Conclusions

This paper investigates how the activities of multinational corporations evolve over their life cycle. We establish two novel facts: First, MNEs' affiliates grow little over their life cycle; and second, MNEs affiliates' are mostly specialized in a main activity, and diversify a small share of their activities over their life cycle. These facts guide us in the construction of a simple dynamic model of multinational production which, albeit stylized, delivers rich testable implications for which we find strong support in the data. Numerical simulations shed light on the implications of dynamic features of the data for the magnitudes and characteristics of the costs of FDI, an essential ingredient to study the welfare gains arising from globalization.

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Appendix

A Data Description

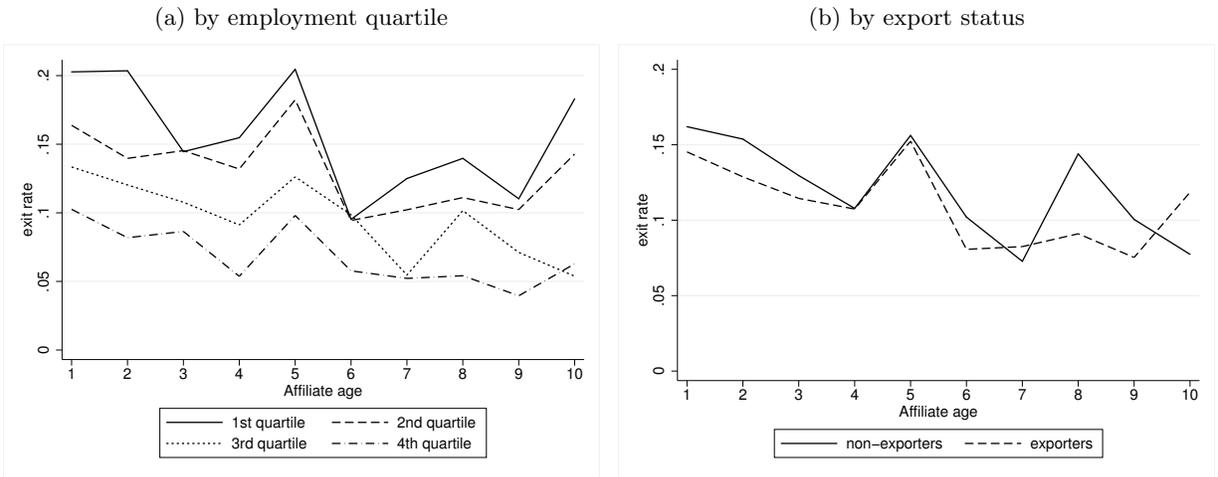
B Summary Statistics

C Additional Facts

C.1 Exit patterns.

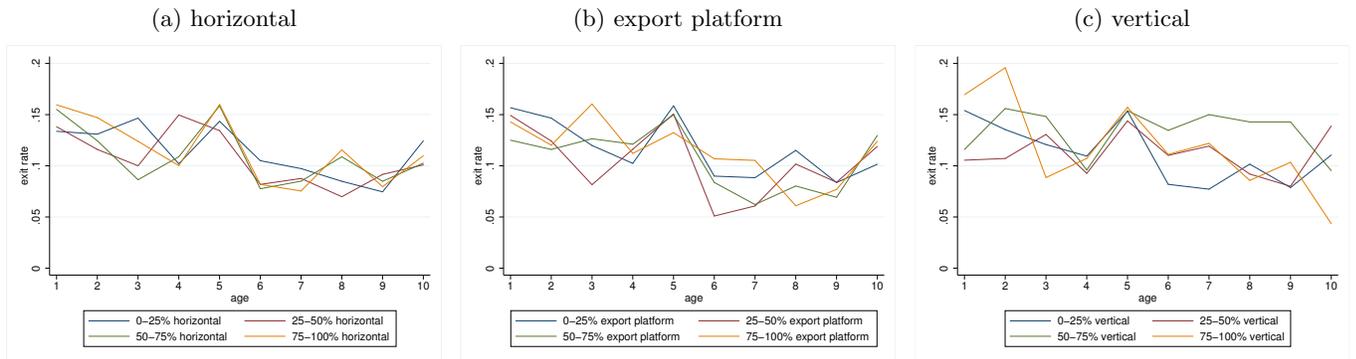
Figures C.1 and C.2 and Table C.1 present results about exit patterns. In the model, for now, we have assumed that the exit rate is exogenous and independent of firm and affiliate characteristics.

Figure C.1: Affiliate exit rates.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Pure exporters (both vertical and export-platform) are not included.

Figure C.2: Affiliate exit, by activity type intensity and age.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Pure exporters (both vertical and export-platform) are not included.

Figure C.1 shows how affiliate exit rates relate to affiliate size, age, and export status. While exit rates are broadly declining with affiliate age and employment, there do not seem to be significant differences in the exit rates of exporting affiliates compared to non-exporting ones. Figure C.2 shows that exit patterns are also very similar for firms that perform the three types of activities (horizontal, vertical, and export-platform) at different intensities.

Table C.1 confirms the patterns emerging from the figures: exit is inversely related to affiliate age and affiliate size, controlling for the size of the corporation and including country-year and industry FE. Once affiliate size is taken into account, age has a small effect on exit, which is

Table C.1: Affiliate exit, by activity type and age. OLS.

Dep var	D(affiliate exit)		
	all	pure H	exporters
	(1)	(2)	(3)
age	-0.002*** (0.0006)	-0.0016 (0.001)	-0.002*** (0.0006)
log aff employment	-0.030*** (0.003)	-0.026*** (0.005)	-0.033*** (0.003)
log global employment	-0.004* (0.002)	-0.005 (0.005)	-0.002 (0.003)
Obs	25,638	5,602	18,850
R-sq	0.035	0.041	0.039

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. Pure exporters (both vertical and export-platform) are not included. The variable $D(\text{affiliate exit})$ equals one if the affiliate exits the following year. The variable "global employment" refers to the aggregate employment, both in the United States and abroad, of the MNE. All specifications with country-year and industry fixed effects. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

significant only for exporting affiliates, not for pure horizontal affiliates.

C.2 Additional Tables and Figures

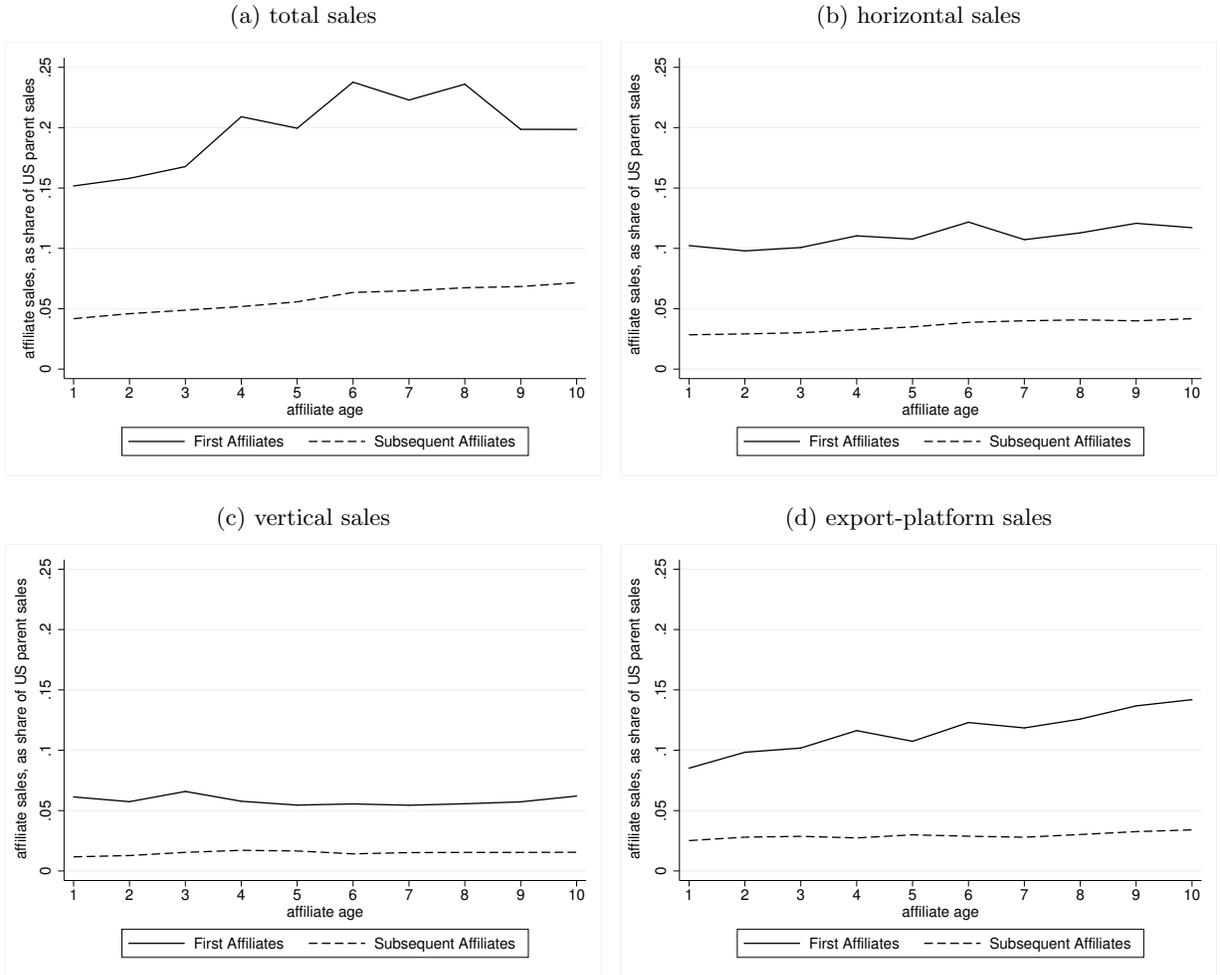
We present here a set of results that complement and deepen the descriptive empirical analysis presented in Section 2.

We documented in Section 4 that MNEs open first their largest affiliates. Figure C.3 shows that this pattern holds throughout the affiliate's life, and regardless of the affiliate's activity.

D Derivation of the Solution of the Model

In this Appendix we provide details about the solution procedure of the model in Section 3 and of its quantitative extension in Section 5.

Figure C.3: Affiliate size, by activity type, age, and position in the MNE opening sequence.



Notes: Sample of new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing. First affiliates refer to the first affiliates opened by the MNE, while subsequence affiliates refer to the rest (2nd, 3rd, ...).

D.1 Solution of the Value Functions in the Simple Model

In Section 3 we specified the problem of the firm by defining the Bellman equations. We derive here the solution of the value functions.

We start by writing equation (9) in the continuation region:

$$[1 + (\rho + \delta)\Delta t]V_d(z, X) = \pi_d(z, X)\Delta t + E[V_d(z, X')|X].$$

Taking the limit for $\Delta t \rightarrow 0$:

$$\begin{aligned}(\rho + \delta)V_d(z, X)dt &= \pi_d(z, X)dt + E[V_d(z, X')|X] - V_d(z, X) \\(\rho + \delta)V_d(z, X) &= \pi_d(z, X) + E\left[\frac{dV_d(z, X)}{dt}\right].\end{aligned}\tag{D.1}$$

From Ito's Lemma:

$$E[dV_d(z, X)] = \mu V_d'(z, X) + \frac{\sigma^2}{2} V_d''(z, X),\tag{D.2}$$

where $V_d'(z, X)$ and $V_d''(z, X)$ denote the first and second derivative of the value function with respect to X . By substituting (D.2) into (D.1), we obtain the non-arbitrage condition:

$$(\rho + \delta)V_d(z, X) = \pi_d(z, X) + \mu V_d'(z, X) + \frac{\sigma^2}{2} V_d''(z, X)\tag{D.3}$$

which states that the expected value of the asset (profit flow plus expected change) must be equal to its normal return.

We guess the following form for the value function:

$$V_d(z, X) = W_d(z, X) + e^{\xi X}$$

and substitute it into (D.3):

$$(\rho + \delta)[W_d(z, X) + e^{\xi X}] = \pi_d(z, X) + \mu[W_d'(z, X) + \xi e^{\xi X}] + \frac{\sigma^2}{2}[W_d''(z, X) + \xi^2 e^{\xi X}].$$

We solve using the method of undetermined coefficients. Collecting the homogeneous terms we obtain:

$$\begin{aligned}(\rho + \delta)e^{\xi X} &= \mu\xi e^{\xi X} + \frac{\sigma^2}{2}\xi^2 e^{\xi X} \\(\rho + \delta) &= \mu\xi + \frac{\sigma^2}{2}\xi^2.\end{aligned}$$

Hence ξ is given by the solution of the quadratic equation:

$$\xi = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2(\rho + \delta)}}{\sigma^2}.\tag{D.4}$$

Collecting the nonhomogeneous term we obtain:

$$(\rho + \delta)W_d(z, X) = \pi_d(z, X) + \mu W_d'(z, X) + \frac{\sigma^2}{2} W_d''(z, X). \quad (\text{D.5})$$

We guess the following form for the nonhomogeneous term:

$$W_d(z, X) = \frac{\pi_d(z, X)}{\kappa} = \frac{H \left(\frac{w_d}{z}\right)^{1-\eta} e^{(\eta-1)X} P_d^\eta Q_d}{\kappa}.$$

Substituting it into D.5:

$$\begin{aligned} (\rho + \delta) \frac{H \left(\frac{w_d}{z}\right)^{1-\eta} e^{(\eta-1)X} P_d^\eta Q_d}{\kappa} &= H \left(\frac{w_d}{z}\right)^{1-\eta} e^{(\eta-1)X} P_d^\eta Q_d + \mu \frac{H \left(\frac{w_d}{z}\right)^{1-\eta} (\eta-1) e^{(\eta-1)X} P_d^\eta Q_d}{\kappa} + \dots \\ &\quad \dots \frac{\sigma^2 H \left(\frac{w_d}{z}\right)^{1-\eta} (\eta-1)^2 e^{(\eta-1)X} P_d^\eta Q_d}{2 \kappa} \\ (\rho + \delta) \frac{1}{\kappa} &= 1 + \frac{\mu(\eta-1)}{\kappa} + \frac{\sigma^2}{2} \frac{(\eta-1)^2}{\kappa} \\ \kappa &= (\rho + \delta) - \mu(\eta-1) - \frac{\sigma^2}{2} (\eta-1)^2 \end{aligned}$$

so that:

$$W_d(z, X) = \frac{\pi_d(z, X)}{(\rho + \delta) - \mu(\eta-1) - \frac{\sigma^2}{2} (\eta-1)^2}.$$

Hence the general solution for the value of domestic sales is given by:

$$V_d(z, X) = \frac{\pi_d(z, X)}{(\rho + \delta) - \mu(\eta-1) - \frac{\sigma^2}{2} (\eta-1)^2} + A_d(z) e^{\alpha X} + B_d(z) e^{\beta X},$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . Since all firms always have domestic sales, there is no option value of domestic sales, so $A_d(z) = B_d(z) = 0$, and the value of domestic sales is simply given by the present discounted value of domestic profits.

We proceed in an analogous way to solve for the other value functions. By following the same steps as above, we obtain the following no-arbitrage condition for the option value of an affiliate $V_j^o(z, X)$.

$$(\rho + \delta)V_j^o(z, X) = \mu V_j^{o'}(z, X) + \frac{\sigma^2}{2} V_j^{o''}(z, X). \quad (\text{D.6})$$

We guess the following form for the value function:

$$V_j^o(z, X) = e^{\xi X}$$

and following the procedure outlined above, we obtain the general solution:

$$V_j^o(z, X) = A_j^o(z)e^{\alpha X} + B_j^o(z)e^{\beta X}$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . Notice that there are no profit flows associated with an option value function. Finally, as $X \rightarrow 0$, the option of opening an affiliate becomes worthless, so it must be that $A_j^o(z) = 0$. Conversely, the option of opening an affiliate becomes more attractive as X increases, so it must be that $B_j^o(z) > 0$.

The no-arbitrage condition for the value of a pure-type horizontal affiliate $V_j^h(z, X)$ is:

$$(\rho + \delta)V_j^h(z, X) = \pi_j^h(z, X) + \mu V_j^h(z, X) + \frac{\sigma^2}{2} V_j^{hh}(z, X). \quad (\text{D.7})$$

We guess the following form for the value function:

$$V_j^h(z, X) = W_j^h(z, X) + e^{\xi X}.$$

The homogeneous term has the same functional form as in the previous cases. We then guess the following form for the non-homogeneous term:

$$W_j^h(z, X) = \frac{\bar{\pi}_j^h(z, X)}{\kappa_1} + \frac{f_j^h}{\kappa_2}$$

and following again the method of undetermined coefficients we find:

$$\begin{aligned} \kappa_1 &= (\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2 \\ \kappa_2 &= (\rho + \delta) \end{aligned}$$

so that the general solution for the value of a pure-type horizontal affiliate is:

$$V_j^h(z, X) = A_j^h(z)e^{\alpha X} + B_j^h(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^h}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, the value of horizontal sales also goes to zero (exit is only exogenous and random in this version of the model), so it must be that $A_j^h(z) = 0$. Conversely, as X increases, the value of the affiliate also increases because of the option of starting to export, so it must be that $B_j^h(z) > 0$.

Finally, the no-arbitrage condition for the value of a diversified affiliate $V_j^e(z, X)$ is:

$$(\rho + \delta)V_j^e(z, X) = \pi_j^h(z, X) + \pi_j^e(z, X) + \mu V_j^{\prime e}(z, X) + \frac{\sigma^2}{2} V_j^{\prime\prime e}(z, X). \quad (\text{D.8})$$

Also here, we guess the following form for the value function:

$$V_j^e(z, X) = W_j^e(z, X) + e^{\xi X}.$$

The homogeneous term has the same functional form as in the previous cases. We then guess the following form for the non-homogeneous term:

$$W_j^e(z, X) = \frac{\bar{\pi}_j^h(z, X) + \bar{\pi}_j^e(z, X)}{\kappa_1} - \frac{f_j^h + f_j^e}{\kappa_2}$$

and following again the method of undetermined coefficients we find:

$$\begin{aligned} \kappa_1 &= (\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2 \\ \kappa_2 &= (\rho + \delta) \end{aligned}$$

so that the general solution for the value of a diversified affiliate is:

$$V_j^e(z, X) = A_j^e(z)e^{\alpha X} + B_j^e(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X) + \bar{\pi}_j^e(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^h + f_j^e}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, the value of the affiliate also goes to zero (exit is only exogenous and random in this version of the model), so it must be that $A_j^e(z) = 0$. Also, as X increases, the value of the affiliate converges to the discounted profit flow (there is no further expansion option), so it must be that $B_j^e(z) = 0$.

D.2 Solution of the Value Functions in the Quantitative Model

In Section 5 we outlined the solution of the model with endogenous affiliate exit, choice of affiliate export destinations, and endogenous exit from each export market. We provide here some details about the derivation of the solution of the value functions in this more general case.

Since the model has the structure of a compound option, we solve it backwards, starting from the problem of a firm that already has an affiliate in country j and has to decide whether to export to any country $k \neq j$.

The Bellman equation describing the value of the option to export to country k for a firm with an affiliate in country j is:

$$V_{jk}^o(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} E[V_{jk}^o(z, X')|X]; V_{jk}^e(z, X) - F_{jk}^e \right\}, \quad (\text{D.9})$$

which describes the fact that the affiliate may keep the option of exporting to country k (and get the continuation value of that option), or may start exporting to country k , in which case it pays the entry cost F_{jk}^e and gets the value of exporting to k from j , $V_{jk}^e(z, X)$. Writing equation (D.9) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_{jk}^o(z, X) = \mu V'_{jk}{}^o(z, X) + \frac{\sigma^2}{2} V''_{jk}{}^o(z, X). \quad (\text{D.10})$$

Following the same procedure outlined in the previous section, we can conclude that the value of the option of exporting to country k for an affiliate in country j has the following general solution:

$$V_{jk}^o(z, X) = A_{jk}^o(z)e^{\alpha X} + B_{jk}^o(z)e^{\beta X}$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . As $X \rightarrow 0$, the option of exporting becomes worthless, so it must be that $A_{jk}^o(z) = 0$. Conversely, the option of exporting becomes more attractive as X increases, so it must be that $B_{jk}^o(z) > 0$.

Similarly, the Bellman equation describing the value of exporting to country k from an affiliate in country j is:

$$V_{jk}^e(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} [\pi_{jk}^e(z, X)\Delta t + E[V_{jk}^e(z, X')|X]]; V_{jk}^o(z, X) \right\}, \quad (\text{D.11})$$

which describes the fact that the affiliate may keep exporting to country k (and get the continuation value of that option), or may stop exporting to country k , in which case it gets the value of the option of exporting to k from j , $V_{jk}^o(z, X)$. Writing equation (D.11) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_{jk}^e(z, X) = \pi_{jk}^e(z, X) + \mu V'_{jk}{}^e(z, X) + \frac{\sigma^2}{2} V''_{jk}{}^e(z, X). \quad (\text{D.12})$$

Following the same procedure outlined in the previous section, we can conclude that the value of the option of exporting to country k for an affiliate in country j has the following general solution:

$$V_{jk}^e(z, X) = A_{jk}^e(z)e^{\alpha X} + B_{jk}^e(z)e^{\beta X} + \frac{\bar{\pi}_j^e(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^e}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, there is value from the possibility of endogenously stopping to export, so it must be that $A_{jk}^e(z) > 0$. Also, as X increases, the value of exports converges to the discounted profit flow (there is no further expansion option), so it must be that $B_{jk}^e(z) = 0$.

The Bellman equation describing the value of horizontal sales for an affiliate in country j is:

$$V_j^h(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} \left[\pi_j^h(z, X)\Delta t + E[V_j^h(z, X')|X] \right]; V_j^o(z, X) \right\}, \quad (\text{D.13})$$

which describes the fact that the affiliate may keep surviving and have horizontal sales in j , or may shut down, in which case the firm gets the value of the option of opening an affiliate in j , $V_j^o(z, X)$. Writing equation (D.13) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_j^h(z, X) = \pi_j^h(z, X) + \mu V'_j{}^h(z, X) + \frac{\sigma^2}{2} V''_j{}^h(z, X). \quad (\text{D.14})$$

Following the same procedure outlined in the previous section, we can conclude that the value of horizontal sales for an affiliate in country j has the following general solution:

$$V_j^h(z, X) = A_j^h(z)e^{\alpha X} + B_j^h(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^h}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, there is value from the possibility of shutting down the affiliate, so it must be that $A_j^h(z) > 0$. Also, as X increases, the value of horizontal sales converges to the discounted profit flow (the option value of exports has been determined already above), so it must be that

$$B_j^h(z) = 0.$$

At this point, the value of an affiliate in country j , $V_j^a(z, X)$ is completely characterized up to the option value parameter $A_j^h(z)$:

$$V_j^a(z, X) = A_j^h(z)e^{\alpha X} + \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} + \sum_{k \in \mathcal{A}_j(z)} \left[\frac{\bar{\pi}_{jk}^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_{jk}^e}{\rho + \delta} + A_{jk}^e(z)e^{\alpha X} \right] + \sum_{k \notin \mathcal{A}_j(z)} \left[B_{jk}^o(z)e^{\beta X} \right] \quad (\text{D.15})$$

where $\mathcal{A}_j(z)$ denotes the set of export markets in which an affiliate of a firm with productivity z located in country j exports.

Lastly, the Bellman equation describing the value of the option to open an affiliate in country j is:

$$V_j^o(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} E[V_j^o(z, X') | X]; V_j^a(z, X) - F_j^h \right\}, \quad (\text{D.16})$$

which describes the fact that the affiliate may keep the option of opening an affiliate in country j (and get the continuation value of that option), or may open an affiliate in country k , in which case it pays the entry cost F_j^h and gets the value of an affiliate in country j , $V_j^a(z, X)$. Writing equation (D.16) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_j^o(z, X) = \mu V_j^{\prime o}(z, X) + \frac{\sigma^2}{2} V_j^{\prime\prime o}(z, X). \quad (\text{D.17})$$

Following the same procedure outlined in the previous section, we can conclude that the value of the option of exporting to country k for an affiliate in country k has the following general solution:

$$V_j^o(z, X) = A_j^o(z)e^{\alpha X} + B_j^o(z)e^{\beta X}$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . As $X \rightarrow 0$, the option of opening an affiliate becomes worthless, so it must be that $A_j^o(z) = 0$. Conversely, the option of opening an affiliate becomes more attractive as X increases, so it must be that $B_j^o(z) > 0$.

Finally, the determination of the option value parameters in the more general case needs attention: when the firm decides to open an affiliate in a country, it considers not only the value of its horizontal sales, but also the option value of potential export to any destination country. For this reason, the value-matching and smooth-pasting conditions that deliver the parameters $A_j^h(z)$,

$B_j^o(z)$ together with the aggregate productivity thresholds that induce the firm to open or shut down the affiliate (X_j^{OH} and X_j^{HO}) are computed using the option value function $V_j^o(z, X)$ and the total value of the affiliate $V_j^a(z, X)$.