Governing the Computers:
The London Stock Exchange, the Institute of Actuaries and the First Digital Revolution (1808-1875)

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Abstract:

Our paper proposes an economic history of scientific knowledge, focusing on financial mathematics. We show how the traditional tools of institutional and industrial economics can help make sense of the dynamic of knowledge production. We look at the history of actuarial science in the 19th century, which played an important role in the history of capitalism, as it provided the tools to calculate the returns on heteroclite financial instruments, facilitating comparison, trading and in the end, the expansion of the capital market. We explain the development of actuarial science and of the profession of actuary in England as a product of the need to resolve information asymmetries and widespread computational anxieties which was read by contemporaries as a looming tragedy of the commons. This context gave birth to an institution, the Institute of Actuaries, which rapidly imposed itself as the sole provider of legitimate financial calculus. This was done by developing a dual program that involved the production of human "computers" that were equipped with "software" in the shape of algorithms to which use the computers were trained. The final outcome was what we call a "government of calculus", in the shape of a monopoly over the production and distribution of financial knowledge. The success of the Institute and the economies of scale in knowledge production it could generate in virtue of its dominant position, eventually resulted in significant productivity gains in financial computing but also in imposing new standards such as an early statement of the rational expectations hypothesis.

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Introduction

This article explores the possibility of an economic history of scientific knowledge. The intuition which we build upon will not be surprising to economic historians knowledgeable in the modern development of the discipline. If a science is "useful" this means it is consumed. But if it is consumed, the question of the quality of science becomes paramount. This introduces what is, from the point of view of economics, a classic problem of trust. The extent to which a scientific statement, a result or a theory is received as truthful or trustworthy will depend critically on a set of institutional forces, which foster or undermine the credibility of the scientist who presents it. Seen that way, an economic history of truth and science as the one we attempt here would take us to the familiar territory of institutional economics – the world of constitutions, commitments, labels, rents, hierarchies, agencies, incentives and penalties. If the identification of truth raises problems of trust, then science (whose job it is, perhaps, to identify and distribute truth) can be studied with the conventional instruments that enable to analyze information asymmetries and the way they are being resolved. And given the importance of the usefulness of science, which authors such as Mokyr (2002) have placed at the heart of grand narratives of the Industrial Revolution, there are reasons to speculate that such questions did not only appear in *sui generis* contexts, but instead systematically.

This relevance of economic factors in science, which economic historians have ignored so far, has been in fact discussed in the works of historians of science. A key contribution in a large and growing literature is Steven Shapin's study of the world of "gentlemen scientists" in 17th century England where he emphasized the social and cultural contexts that promoted trust and facilitated persuasion (Shapin 1995). Such historians as Shapin not being economists and displaying a preference for engaging sociologists, they have cast the matter in the language of culture – the culture of trust that developed, they argue, in situated settings. Yet economic historians would have no difficulty reinterpreting the success of the gentlemen scientists in their own language. We would say for instance, that the position of the gentleman scientists (including their financial position) endowed them with credibility and that this supported their role as carriers of trustworthiness.[2] In fact, any impartial reading of the works of historians of early modern science and beyond could not miss the fact that their language is littered with economic terminology: They speak not only of trust, but of beliefs, strategies, even moral economies (Daston 1995). The term "moral" is there to ensure that we are not in the amoral world of Adam Smith, but no-one is fooled: We are right there.

In this paper, we defend the view that an economic history of science is long due by focusing on a case study. We look at the evolution of actuarial science in the 19th century, a poorly studied discipline yet one that has great importance for the history of capitalism. Actuarial science is the body of knowledge that is devoted to the design and production of financial calculations and in particular to the production of actualization formulae. Such formulae are a sine qua non in order to determine the yield of financial products with different payment streams, maturities, etc. One of the great contribution of actuarial science therefore is its ability to transform possibly complex financial contracts bonds into more transparent and unique interest rates. In so

[2] Of course, other institutions can be and have been evolved to deal with this issue, the recent work of Csiszar (2018) on the development of the scientific journal in the 19th century providing yet another example.

doing, it informs comparison across securities and helps investors with their investment decisions. As Goetzmann (2005, p. 123) has put it discussing the medieval origins of actuarial calculus and the present value model, “mathematically reducing all cash flow streams to a single point in time allows the investor to decide which is unambiguously the best”. In other words, actuarial science provides an instance of those “useful” forms of knowledge alluded to above. More precisely, the usefulness it projects is relevant to the development of credit markets.

Very little is known on the history of the actuarial profession. Historian Theodore M. Porter has emphasized that its evolution in the 19th century was shaped by the problem of establishing and maintaining trust. He suggests that the creation of this profession was a response to the need to establish credibility in the face of “flagging trust in individuals” (Porter 1994, p. 192). On the other hand, historians of finance have identified a growth in the profession’s inventive activity, taking place after the middle of the 19th century. According to Hawawini and Vora (1982) a group of English mathematicians became responsible at that point for pioneering both exact methods and approximate formulae, which enabled to calculate the yields of complex financial products.

We argue that these various phenomena were connected. Trust, itself required by the development of the financial system, the creation of a credible profession, and finally, innovative activity were separate elements in a common historical chain, which this paper will discuss. As we claim, the pivotal moment was the creation of a new institution, the Institute of Actuaries, launched in London in 1848. The main contribution of the Institute of Actuaries was to bring about what we call a “government of computations”. The Institute found its roots in widespread uncertainties pertaining to the proper calculation of yields. But the context played an important role too, as the creation occurred at a time of heightened anxieties regarding the possible manipulations by financial intermediaries in the capital market (Taylor 2013). Given that insurance companies, which were great consumers of such calculations were also large investors, their methods came naturally under scrutiny at the same time as other segments of the capital market attracted criticism.

As we shall see, it was generally felt among actuaries that the previous knowledge regime, which was open and ungoverned, had favored a proliferation of calculations which had the final effect to increase information asymmetries. These got magnified by the growing supply of diverse financial instruments, as capitalism was booming and the London Stock Exchange was becoming a global powerhouse. The situation, in other words, generated substantial transaction costs. Indeed, if the availability of an adequate technology enabling the comparison of yields is critical to the development of modern capital markets, it is easy to understand why, *a contrario*, uncertainty regarding what constitutes a correct calculation can be costly.

The creation of the Institute was meant to address the situation. It would tell what was true and what was not. It would give instructions as to how to perform computations. It would train the human “computers” in the identification and use of the right algorithm. It would govern the relations between men and machines, codifying the manner in which the computers would mobilize and implement the required algorithms. It would become, so to speak, the “server” of the financial community. One especially important innovation which the Institute brought about consisted in the creation of a degree it fully administered, and which, we will argue, was meant to reduce knowledge frictions. Granting the degree amounted to the certification that its holder would perform the assigned computations in a proper way. This was valuable for investors and the sellers of financial products who had now a way out of the thorny problem of picking the right computer. As we argue, this ensured the success of the degree, as individuals lined up to receive the Institute's valuable seal of approval. And in so doing, the Institute promoted the impression that, behind the work of actuaries, there existed a uniform and trustworthy knowledge background, in the end facilitating the operation of the capital market.

But having established itself as the relevant authority to govern the production of theorems, algorithms and other useful formulae, the Institute of Actuaries also began, by the same token, to revolutionize the process of knowledge accumulation. Rents to innovative activity were increased. "Horse races" of competing methods could be organized and their results were published. The winners received visibility and praise. The resulting set-up became a standing invitation for researchers to experiment with existing formulae and contribute new ones. The increase in status generated remunerative financial advisory work for the winners. Supporting this reading, we will demonstrate that the creation of the Institute was immediately followed by a major spurt of inventive activity which improved the quality of existing algorithms enabling to raise the power of computations that were important for mathematical finance.

Perhaps our most important message is that, by adopting the vantage point of economic history, it is possible to arrive at a novel understanding of the dynamic of ideas and the meaning of science in the development of capitalism. To show this we shall argue that the reduction of transaction costs, which the Institute had aimed to achieve, ended up shaping the contours of financial knowledge: There exists a correlation between the institutional changes brought about by the Institute, and the nature of the ideas that "emerged" subsequently in the field of financial economics. We show this in a variety of way. First, we argue that the lay-out of the theorems or formulae which the Institute promoted was not only informed but shaped by the economic forces that had underpinned the creation of the Institute. Reviewing the making of "Makeham's formula", a mode of decomposing the present value of securities which it still in use today, we show the influence of the desire to minimize the variance of calculations by putting together a template that would reduce the leeway available to computers.

A second piece of evidence we provide concerns the economic circumstances that led to the articulation, by a member of the Institute of an early statement of the rational expectations hypothesis. The hypothesis surfaced within the Institute techno-scientific machinery in 1875 through the work of a William Sutton (Sutton 1875). We argue that the background for the emergence of this hypothesis was a financial scandal that led to charges that the sellers of securities had used an existing numerical blind-spot to conceal the riskiness of the instruments they were selling to investors. As Sutton showed, adopting the rational expectation hypothesis enabled to "detect" such wrongful behavior. Following this, and despite resistance among actuaries, the method became part of the formal training, finding its way in handbooks, problem sets, and examinations.

This paper is composed of seven sections. Section I introduces an important character of our story, Francis Baily, and locates his contributions to actuarial science in the context of the history of the London Stock Exchange in the early 19th century; Section II presents the Early Modern emergence of precise algorithms aimed at solving the problem of yield extraction for a specific financial product: the annuity; Section III explains the role of intermediaries in the development financial knowledge and introduces the Institute of Actuaries as an institution whose role was to deal with the widespread computing anxieties that prevailed in the mid-19th century; Section IV presents a crucial tool that the Institute used to establish and subsequently to maintain its position as the main provider of legitimate financial figures: its certification and education system; Section V and VI discuss two significant contributions that the Institute made to financial theory in the second half of the 19th century: Makeham's formula, and a formulation of the rational expectation hypothesis, respectively; Last, Section VII provides empirical evidence for the claim that the creation of the Institute of Actuaries, by enabling to internalize R&D costs, resulted in non-negligible productivity gains in financial computing.

Section I. What's an Actuary?

At the center and at the beginning of our story are Francis Baily and the London Stock Exchange. To begin, Francis Baily: Towering over the next half century and beyond, Baily's 1808 treatise, *Doctrine of interest and annuities analytically investigated and explained*, synthesized the state of financial calculus as it had previously developed at the same time as it did lay the ground for future developments. It has sometimes been said that the technologies which the Industrial Revolution mobilized in its early phase were nothing but the application of the scientific advances of the previous two centuries: The dictum holds true for financial calculus. Indeed, Baily's treatise presented itself as a kind of encyclopedia for financial calculus. The author was concerned with providing an extensive review of what existed at his time on the subject of compound interest and the valuation of so called "annuities" – fixed income instruments that gave to the purchaser a stated annual revenue. The study of such matters went far back in time – in fact to the Commercial Revolution, when the multiplication of commercial, financial and trading occasions had created a need for financial calculus. Central to these investigations were problems connected to the calculation of interest rates. The extraction of yields from loans or securities with different characteristics was a precondition for making meaningful comparisons across alternative investments. This had generated various attempts to address such questions, and the accumulation of efforts in this direction slowly defined a field of investigation (Goetzmann (2005, p. 123)).

The late 17th and 18th centuries had been characterized by accelerating efforts in this direction and an increasingly large literature had dealt with the subject. This was as much a bounty as it was a difficulty, in that the resulting fragmented, scattered and, according to Baily, often contradictory knowledge that resulted from these achievements required ordering. For instance, Baily gave the example of Sir John Price's Short and *Easy Theorems for Finding the Differences Between the Values of Annuities* (Price 1775), which mixed up previous results by changing two assumptions at once. By Baily's own declarations, reordering knowledge had been one motivation for his endeavor, and this had required constructing a new framework – he spoke of "new-modelling" – in which the previous studies could be merged and compared. But as he further claimed, the endeavor had taken him way beyond the boundaries of the intended encyclopedia as he had found himself compelled, once two approaches were reduced to the same system of notations, to determine which one was the best. The result was a more extensive and complete treatment of the subject "than had hitherto been done" a judgment which, though it may sound self-serving, was universally endorsed by subsequent authors.

Indeed, Baily had eventually found himself extending the knowledge he had surveyed: His *Doctrine* provided novel formulae, developed “horse race” techniques for the comparison of competing approaches, and contained guidelines as to what worked best in practice.

Judging from Baily’s own declarations, there is little doubt that the economic background had played a determining role in causing the outburst of interest that had eventually led to his interventions. Previous authors have emphasized the spectacular development, during the 18th century, of the market for annuities, in the shape of life contingent contracts (Cramer 1946). Since their pricing typically involved interest rate problems, the result was an increased attention to the theory and methods used to address these, casting light on the fact that the 18th century was marked by significant research efforts, resulting in expanded publications (Bellhouse 2018). This historical phenomenon was in the background of Baily’s efforts, as reflected by the very first words of the *Preface* of his treatise. There, he alluded to his object being of “great and extensive use in the present state of society.” As he explained, the overall amount of money involved in such instruments was considerable, so that the value at risk, so to speak, was significant. As he put it, the “extent of property which is affected by circumstances involving the consideration” of the subject was giving to financial calculus “more than ordinary degree of attention” adding that when “we consider the great and extensive business which is constantly transacting in the purchase and sale of Annuities of various kinds, […] the subject assuredly acquires a degree of importance which it never before aspired to, and demands the attention of every person engaged in the public business of life.”[3]

In the last quarter of the 18th century, a major transformation in the supply of financial product was the increasing importance of the British government. It had always been a significant borrower, but with the advent of the American War of Independence followed by the French wars, the British government’s reliance on the capital market became enormous. It flooded the market with all kinds of fixed income securities that were essentially, from the point of view of the pricing techniques required, variants of the annuity. Again, successful investment strategies required being able to extract, from knowledge of the price and characteristics of these heteroclite products, the yield which they would give to the investor. Reflecting this concern, various “Tables of Equation” as Thomas Mortimer called them began to be produced. They showed in tabular format the prices at which two different securities would be equivalent thus signaling to the “discretion” of the investor, by mere comparison of these theoretical equivalences with the actual prices, which one was more attractive (Mortimer 1807).[4]

We argue that this shift of the bulk of the demand for interest calculation services from the “over the counter” market of life annuities towards the London Stock Exchange may have ensured that the encyclopedia would be produced by a stock-broker. Indeed, Baily was a member of the London Stock Exchange and in this

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[3] Baily (1808); From Preface, pp. iii-iv.

[4] He also encouraged readers to take a look at more extensive and reputable tables mentioning in particular William Fairman’s Stocks Examined and Compared. The first edition of this volume had been released in 1795 and, bearing witness of the avid demand that appear to have existed in the capital market for such tools, it was updated and reprinted several times afterwards (Fairman 1795, 1796, 1798, 1802).See *The Register of the Times: Or, Political Museum*, 1795, Vol. 7, p. 301 for a positive review of Fairman. Mortimer (1807, unnumbered page at the end of preface) also recommends “Blewere’s Tables of the Values of Stock and Annuities.”

connection we find, in Baily's own telling, the origins of the book. Not only were stock-brokers often requested by their clients to inform them on the instruments they sold (thus directing towards them the resolution of interest problems) but they were also to be benefitted themselves from the mastery of financial calculus. The jobbers, in particular, acted as market makers, that is, they managed inventories of stocks so as to be able to provide a ready counterparty to the orders they received. As a result, they could realize significant gains from arbitrage if prices differed from equilibrium relations such as captured in the tables of equation. This shows the value which such tables, or rather, such knowledge as stood behind the tables, could have for them. In other words, the flow of orders either for third parties or for themselves generated scale economies in financial calculus which encouraged brokers to internalize the costs of the expansion of financial mathematics. According to this interpretation, this was the reason why it fell upon a broker to shoulder development costs.

Indeed, as Baily explains in the Preface of his *Doctrine*, because of his trade, he was a consumer of existing treatises: The *Doctrine*, indeed, is signed "Francis Baily, of the Stock-Exchange and it was printed and sold by Richardson, the Royal Exchange's publisher. As Baily also emphasized, he was not alone with such needs. Other individuals, known to him, and thus probably connected with the market, too were "pursuing the same inquiries". But feeling frustrated by the discrepancies, differences of notations, and even contradictions that existed in previous treatises, he had felt the need for a systematic reference tool, for the "whole collectively would not form a complete treatise on the science." This consideration had led him to undertake a synthesis, because he had felt the need a a compilation "to which [he] might always refer with expedition and confidence."[5] This had been initially for his "own use". He was persuaded, however that such questions as had preoccupied him were widely shared by people interested in the subject "by inclination or duty" (thus again, many people connected with the Stock Exchange) and he was now publishing the book, presenting his action as that of the benevolent supply of a public good. By releasing the "fruits of [his] labor" he provided an "acceptable service to the public" who would be benefitted from his "rendering the path more open and easy to those who, in future, may be led […] to turn their attention to subjects of this kind."[6]

Delving further into the evidence, we find that this concern about the reliability of knowledge, may be put in relation to Baily's specific position inside the London stock exchange. Indeed, he was not merely an operator in an expanding market, but also a significant agent of the process of expansion itself. In 1801, the old coffee house arrangement that had functioned during the 18th century had been replaced by the launch of the New Stock Exchange housed in new facilities whose erection had been made possible by a subscription among supporters. As shown by the new market's Deed of Settlement, Baily was one of these so-called "proprietors" who owned up to four shares of the new market. In addition to this, Baily had emerged as one of the leaders of the market, expressing his concern over governance problems and also spearheading a legal challenge against the maintenance of the authority of the Lord Mayor over the London Stock exchange people (Baily 1806).

In other words, we suggest that Baily's pro-active role in the London stock exchange and his production of a financial encyclopedia were the two sides of the same coin. An emerging market, especially one preoccupied by setting its own rules, clearly needed to be equipped with a common understanding. One aspect of this was

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[5] Baily (1808, p. x).
[6] Baily (1808, p. xi).

the creation of a knowledge infrastructure that would be widely shared, solid and credible. Baily's preface referred to the need for *himself* to be equipped with a reference tool that would give him confidence, but there was an evident advantage for the same confidence regarding methods to be shared by more than one actor. The ability for brokers to ensure the liquidity of trading hinged on common expectations. Thus, the point was not only for the brokers to master the art of taking advantage of market fluctuations but also to contain these fluctuations through the promotion of the common understanding. Against this, stood the eclectic rules, methods, and in the end results. What if different brokers, referring to different treatises, ended up advising the same customer in different ways? Arguably, the main victim of such a situation would be the market itself. *Ergo*, it came to the stock exchange itself to develop a tool which would foster the much-needed common understanding. This was an integral part of the credibility of the market at large, of the appeal it had for customers, and of its claim indeed to self-government.

This entrenchment of knowledge inside the reputable London Stock Exchange was to have another important consequence. It contributed to laying out the foundations of the profession of actuaries. Indeed, one sector where the kind of knowledge which Baily had articulated was paramount was the business of assurance companies. In fact, this business was an ideal playing ground for the kind of reasoning Baily had sought to systematize, leading him to publish a sequel of the *Doctrine of Interest and Annuities*, called *The Doctrine of Life-annuities and Assurances* (Baily 1810a). Indeed, there were annuities both on the asset and liability side of the assurance company. Against the annual payment of a premium (essentially, an annuity), the insurance companies sold a contingent annuity. It consisted in a life insurance whose payment (a lump sum or a stream) would be triggered by the death of the subscriber. The present value of the annuities coming in would have to exceed the present value of the annuities coming out: The question at hand was to handle the inter-temporal mismatch.[7] Proper management of the insurance company required the ability to get the price of policies right, which was essentially an exercise in actualization. And the management of the money in between brought the matter back to the stock exchange where the company did acquire or sell offsetting annuities.

Thus the brokers from the Stock exchange such as Baily emerged obliquely, but for that reason perhaps even more logically, as the natural monitors of the emerging insurance companies. Indeed, Chapter 14 of Baily's second treatise dealt with insurance companies extant, which he assessed by taking them to task on the basis of their constitutions. Some were criticized, others less so. Feeling that there might be a wide market for such "rating" both because individual subscribers might wonder which policy they should purchase and because investors might ask of which joint-stock insurance company they should acquire the shares, Baily published the chapter in pamphlet form in 1810 (Baily 1810b). In its preface, he insisted that he was "uninterested in, and unconnected with, any of the societies" discussed by him. He was merely the impartial arbiter, "anxious to give a clear and unprejudiced account, deduced from their own plans and proposals, as submitted by them to the public." The pamphlet sold out in a matter of weeks, after which a second edition followed, the preface to which reminding readers that Baily had perhaps not given the "unqualified approbation" which some readers imagined to the venerable Equitable Society (Baily 1811, p. xiii). Although it was the least imperfect of all in Baily's opinion, this company was far from perfect either.

[7] It obviously applied beyond life insurance, to the other branches of the industry, whether fire or marine.

The ending was written on the wall: Baily became a kind of godfather of the insurance business. The commitment of a company to abide by Baily's best practice was taken as a hallmark of probity, enabling him to be able to sponsor a number of individuals to become actuaries in assurance companies. The appointment of someone who worked along the lines of the program delineated by Francis Baily was a signal of sound business. A case in point was the "calculator" George Barrett, a self-taught numerical genius whose remarkable skills Baily had identified and used and whom he promoted, one result being his appointment at the Hope Assurance Office (De Morgan, 1854, p. 187).

The *chef d'oeuvre* was to be the launch in 1824 of the Alliance British and Foreign Life Assurance Company. The project was indeed sponsored by several members of the London Stock Exchange. Beyond Baily, there was Benjamin Gompertz, who was later to design the famous statistical law of mortality that bears his name, as well as Moses Montefiore (a prominent member of the committee of the London Stock Exchange and Gompertz's brother in law). Gompertz, who was a personal friend of Baily, was to become the actuary and chief clerk of the company. He would also become involved in the Alliance Marine Insurance. The company was further sponsored by N.M. Rothschild, sometimes presented as the "driving force" in the project, as well as by other prominent financiers such as Francis Baring and Samuel Gurney (Schooling 1924). By lending their name to the project, these financial authorities certified the correctness of the approach. This further entrenched the role of Baily's mathematics as foundation of a knowledge system. At the end of the day, this defined what an actuary was: A group of individuals professing to operate according to the formal rules laid out by the stock-broker Francis Baily.

Section II. The Science and Art of Algorithms

A conventional view among economists and economic historians is that until the age of modern computers in the second half of the 20th century, financial mathematics remained in a backward state, notably when it came to the question of calculating the yield of fixed income securities. Some sophisticate ideas would have emerged very early on, but limitations in computing power would have held back contemporaries. Stuck as they were in what some authors have described as a kind of stone age financial economics, contemporaries would have stuck to tractable but approximate solutions to the complex problems they could not solve rigorously (Hawawini and Vora 1982). Alternatively, it has been suggested that they imagined the instruments they invested in as being essentially equivalent of others they could handle better. According to Mauro, Sussman and Yafeh (2006) for instance, late 19th century investors would have pictured each fixed income security as a perpetual bond, the reason being that computing the yield for a perpetual bond consisted in a simple mathematical formula, which consisted in dividing the coupon by the price.[8]

Not only is this picture of stone age financial economics incorrect, but it is also misleading, because it glosses over the role of actuaries as relevant financial intermediaries. We have already shown that the profession of actuaries grew out of the need to produce credible calculations, and we have emphasized the

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[8] Mauro, Sussman and Yafeh (2006, p. 41). As they further argue, this was "reasonable, given that bonds were usually of very long maturity and [the approach] probably seemed natural to a large proportion of investors who were rentiers leaving off the fixed income provided by the bonds."

role of the London Stock Exchange where clusters of expertise grew naturally out of the need to advise clients, on both the buy and the sell side of the market. The concern would not abate over time. In fact, actuaries' witness accounts from the same period when according to Mauro et al. investors behaved in a primitive fashion, suggest that they were facing sustained demand for precision, not only from the originators of financial instruments, the "financiers", but from investors who often wanted precision up to the "exact penny". As one observer declared, "it would seem that private investors are getting as much interested in these matters as financiers."[9]

As the rest of the paper will demonstrate, asking for such an extreme precision was not preposterous at all, because the methods available to experts did permit to achieve this goal. In fact, during the Early Modern period, computing techniques made headway, with tremendous productivity gains in computing being realized. One consequence of this was the ability to extract the interest rate of complicated fixed income securities in a very efficient way. The quantum leap occurred thanks to the design in the first half of the 18[th] century of the so-called "converging series method." This new technique, a canonical account of which is found in Thomas Simpson's *Treatise of Algebra* (1745/1767) emerged from the efforts of a number scientists that included on top of Simpson, Isaac Newton, Edmond Halley, Abraham de Moivre and Colin MacLaurin, whose interests straddled mathematics, physics, astronomy and indeed finance. These authors kept coming across the need to resolve equations that would not admit a closed-form solution. To such situations, as Simpson emphasized, the converging series method provided an answer that was "universal, extending to all kinds of equations".[10]

To understand the benefit which the converging series method contributed, the best way is to begin with characterizing the nature of the interest rate extraction problem which it served to solve. Going all the the way to the contributions of Fibonacci in the 14[th] century and of the Italian Nicolo Fontana and the French Jean Trenchant in the 16[th] century, the standard approach to computing yield on a given asset, was to rely on the notion, explicit or not, of present value (Sanford 1930, Poitras 2000, Goetzmann 2005). This approach posited the identity between the price paid to purchase a given financial product and its present value, namely the discounted value of the product's future income streams. In this approach, the unknown was the interest rate, thus defined as the solution of the equation just mentioned. While this was conceptually clear, the problem of actually extracting the solution from this equation was the main challenge. To see why, let's consider the case of the annuity. Adapting the notations from Baily (1808), and noting as a the price paid to purchase a unit annuity (an annuity that pays one unit of currency per year), n the maturity of the annuity and ρ the interest rate, we have the following equation:

[9] Sutton (1875, p. 96). A precision to the penny, for a 5% interest rate would mean about 0.004 or less than one basis point. He conjectured that, perhaps, this was because "the amounts dealt in [are becoming] very large." Sutton continued by stating that he was informed that "even private individuals will sometimes take £100,000 or £150,000 worth of stocks." Sutton (1875, p. 96).

[10] Simpson (1767, pp. 158 ff). Though its interest was by no means limited to finance. Interest rate problems provided one instance since they typically required the resolution of polynomial equation of a high order. It is possible therefore that the resolution of financial problems was one source of inspiration for the efforts deployed. This is consistent with the fact that several prominent contributors to the field such Edmond Halley and Thomas Simpson devoted independent studies to the problem of interest rate extraction and annuities; Such contributions include Edmond Halley's Discourse on Compound Interest (Halley 1705/1804); and Thomas Simpson's Doctrine of Annuities and Reversions (Simpson 1742).

$$a = \sum_{t=1}^{t=n} \frac{1}{(1+\rho)^t} = \frac{1-(1+\rho)^{-n}}{\rho}$$

Baily (1808, p. 172) describes this equation, which does not admit a root for n>4, “so involved that [the] true value cannot be expressed by any of the known rules of algebra […]. Such an expression is in the language of modern algebraists, called an implicit function of [the interest rate].” [11] And indeed, in the early times, similar equations had to be dealt with by “brute force” (Goetzmann 2005, p. 135). However, the converging series method could provide a dramatic simplification. At bottom, it rested on a simple intuition, the idea that even if the implicit function of the interest rate did not admit a closed form, it could be approximated near its solutions with the help of a linear function. Technically, the method posited the existence of an update function *φ(ρ)* for trial values of the unknown interest rate. If a good trial value existed, then the update function could be written as the sum of the trial value for *i* and of the error term *x* or:

$$\varphi(\rho) = \rho + x.$$

This expression could be substituted into the equation to be resolved. In the case of a polynomial equation such as generated by the annuity problem above, the error term being small, its powers higher than one could be ignored. This rendered the equation tractable: It could be solved for *x*, now a function of the trial value *ρ*, or *x*=*x*(*ρ*). In the end, the update function could be written as:

$$\varphi(\rho) = i + x(\rho)$$

Simpson’s account claimed that the method returned “the value sought, *with little trouble, to a very great degree of exactness*” (Simpson, 1767 p. 159) and subsequent commentators agreed. Baily (1808, p. 115), described this method as being the “*principal*” one to address interest rate extraction problems, referring to it being the one “by trial and error”.[12] Thanks to this efficient technique, as he asserted repeatedly in his treatise, the error term could be swiftly “reduced to nil” in fact to a precision of “sixth place of decimals, which is as great a degree of accuracy as need be desired”.[13]

These claims are important for they suggest that in fact, the amount of computing power that could be projected was much more considerable than conventionally admitted. In order to document this, we show the improvement of computing efficiency that the design of the converging series method brought about in the realm of financial calculus. We focus on the case of annuities, which is reasonable since annuities and their

[11] Baily (1808, p. 114).
[12] According to Baily (1808, p. 115): “There are several methods given, by writers on this subject, for approximating towards the roots of the higher equation; of which the principal one is by trial and error; and which consists in taking a near value for the root, and correcting the difference between this and the true root, till at last we arrive at the root required.”
[13] Baily (1808, pp. 40, 52, 115, 121, 129).

variants were a predominant type of contract in the Early Modern era according to Baily. Moreover, because of their popularity, annuities had attracted the attention of mathematicians before the converging series method was designed, resulting in the production by Michael Dary in 1674 of what is conventionally described as the first algorithm extant to solve for the annuity problem (De Morgan 1859).[14]

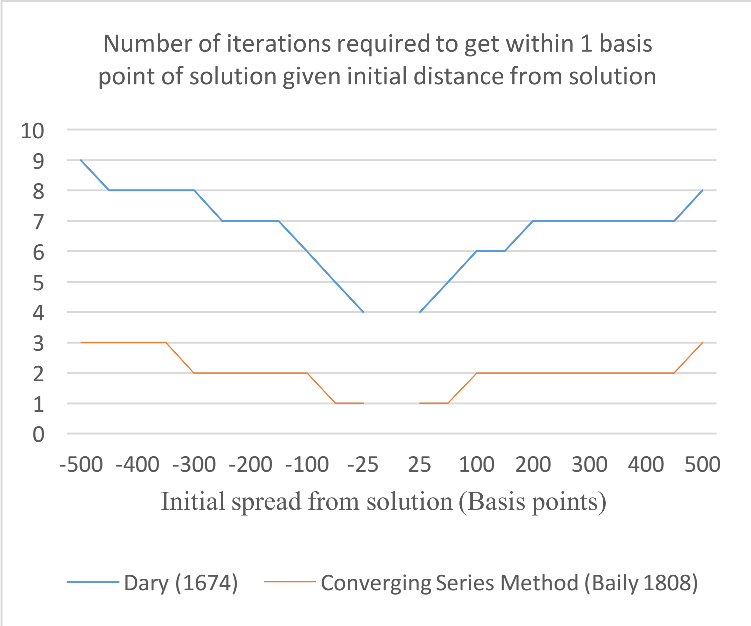
De Morgan (1859, p. 63) argues that Dary's approximation converged "but slowly" but he does not elaborate. The availability of this previous method will enable us to measure the efficiency gains realized by the converging series method. To do this, we simply compare its performance to that of an alternative technique derived form application of the converging series method to the annuity problem. This latter approach is due to Baily (1808, p. 132), who derived an explicit update function, later known as "Baily's formula". In terms of numerical complexity, it compares with Dary's, method. The question at hand, therefore, is to compare the converging power of both.[15]

The evidence we have gathered at this stage suggests strongly that contemporaries knew what they were talking about when they praised the merits of the converging series method. Figure 1 below illustrates this, by comparing the efficiency of the two methods (Dary's and Baily's adaptation of the converging series method) in the case of the "Fontana problem", a real life interest rate problem submitted in 1556 to the Italian mathematician Nicolo Fontana by a group of "gentlemen" from the city of Bari in Italy (Sanford, 1930). The chart underscores the superiority of the converging series method in relative terms: to the extent that it could be calculated, something which required to start from a point not too far from the true value, it always converged faster. But the method was remarkable too in absolute terms, as Simpson had boasted. Provided that the initial error was small – for instance in the figure above, smaller than 50 basis points in absolute value – one iteration was enough to determine the solution with an approximation of one basis point only. This is one measure of the productivity gains in financial computing as they had been realized at some point during the 18th century, thanks to the design of new, more powerful, algorithms. In other words, the period had seen the onset of a silent numerical revolution. To the extent that the matter was not yet understood widely, it is no wonder that Baily felt the need to propagate the gospel and make sure that participants in the capital market would take advantage of operating in an environment of precise numbers.

Figure 1. Dary's Algorithm vs. the Converging Series Method

[14] Note that the efficiency gains of the method were arguably even more considerable for other products because the general validity of the converging series algorithm ensured that problems that could not be solved before became tractable afterwards.

[15] The update function derived by Baily had the following form: $\varphi(\rho) = \rho + \frac{1-a\cdot\rho-(1+\rho)^{-n}}{a-n\cdot(1+\rho)^{-(n+1)}}$



Source: Authors' computations. The exact solution of the Fontana problem was 16.3560683%.

The rise of the new methods had one further consequence. The quality of the converging series method (as well as of many other tools that were to be developed subsequently) depended on the quality of the initial values which the calculator gave himself. The closer this value to the true root, the faster the convergence. Possibly, as the example above suggests, one could achieve convergence in just one iteration. This meant that the performance was highly dependent on the quality of the human computer. It became widely recognized that reliance on a trained computer would be able to speed up the process. Because of complementarities between human computing skills and the machine (the algorithm) the problem of the quality of the algorithms was bringing to the fore, with particular relevance, the question of the quality of the human calculator. It had been long observed that some individuals just had remarkable computing skills.[16] And one way to go was to rely on them. But this raised in turn the problem of how to recognize these superior "calculators", and finally, to be on the safe side, of how to manufacture them. [17]

Section III. Calculators as Intermediaries

The previous conclusion provides a starting point for our suggested interpretation of the logic that did underpin the launch of the Institute of Actuaries in 1848. We suggest to think of the human "computers" who were involved in financial calculus as financial intermediaries in their own right, that is, as agents in charge of resolving information asymmetries. For instance, the financial calculations performed by actuaries ensured that the life insurance companies in which people were investing or from which people were purchasing policies

[16] Against this backdrop, the Institute's interest in using and showcasing "extraordinary calculators" and the role that such individuals had in the development of the discipline, makes perfect sense. Famous examples included George Barrett (see De Morgan 1854); On the kind of display which this interest inspired, see "An extraordinary calculator", *Globe* July 1, 1850, on the visit of "Herr Daze".
[17] The expression "calculator" to designate number crunchers is used by De Morgan for one early 19th century representatives George Barrett (De Morgan 1854, p. 187).

were sustainable, a crucial quality for companies offering contracts spanning several decades. These financial calculations ensured that the buy-side and the sell-side of the market would read a given security in a similar fashion. In other words, as had been the case under Baily and indeed, explaining his success, actuarial calculations contributed to grease the wheels of commerce. But this was only the case to the extent that the human calculators were perceived to perform their task in a trustworthy manner. In other words, their trade was exposed to the suspicion that self-interest would lead to distort the numbers. What if they were not reliable, for instance, if they end up captured by the directors of the companies they were supposed to advise or by the sell-side of the market with which they were known to work?

The circumstances of the creation of the Institute of Actuaries, which are complex and numerous, do certainly present illustrations of these questions. The event took place against the backdrop of the rapid expansion of the life insurance industry which raised concerns over the governance in actuarial computations; Following the development of the insurance industry under the stewardship of members of the London stock exchange, the number of life insurance companies had been growing steadily. From 1830 to the Companies Act of 1844, 56 new "offices" had been created, and the Companies Act had encouraged further creations.[18] As this occurred, questionable behavior, similar to that which Baily had levelled criticism against, was reported to emerge. New entrants could undercut their competitors by offering cheaper policies, since they would typically face less out in their early years of operation. Whether this was sustainable was another matter. Instances of fraudulent practices, famously satirized by Dickens, had been exposed and this cast suspicion on the business overall.[19]

The ability of the actuaries to rein in such tendencies was being called into question. Being employed by the offices, they were easily sidelined or pressured in case they would express dissent. For instance, the actuary Henry W. Porter, remarked on the lack of consideration that the directors of companies were often showing for their own actuaries. As illustration, he gave an episode where a board had "not only acted perfectly in opposition to the most proper advice of their actuary, but also ridiculed the prominence, phrenologically speaking, of his organ of caution" (Porter 1854, p. 108).[20] Regarding the question of the extraction of yields, actuaries were known to be generally employed by "contractors", as the financiers who originated, structured, and distributed the loans were known. The financiers, it was said, wanted those figures for their "own guidance" but they rarely put them in the prospectuses, it was up to the investors to secure in their turn the correct advice as many did. And thus, there too there was a risk that the actuaries would assist the financiers in their effort to deceive the public.[21]

Additional supporting evidence for the view that the birth of the Institute resulted from the need to address governance and credibility problems can be garnered from the official discourse of the supporters of the Institute. They typically claimed that the computational context was getting awry, threatening to drift towards a state of chaos. According to this discourse, the universe of calculations lacked direction and system and as a

[18] Trebilcock (1985, p. 570).
[19] Supple (1970, p. 138).
[20] Taylor (2013) has emphasized the importance of the succession of Victorian scandals in generating calls for government intervention and parliamentary regulation.
[21] Sutton (1875, p. 95).

result, was inherently prone to errors and malpractice. They were at risk from within, owing to what they described as a lack of uniformity in the methods used by actuaries working inside offices. A much quoted statement to this effect was a pronouncement by Charles Babbage who had lamented the heterogeneous proficiency among actuaries embedded in existing offices: "The degree of knowledge possessed by persons so situated at the different institutions is exceedingly various, passing through all degrees, from the most superficial acquirements, derived merely from the routine of an office, up to the most profound knowledge of the subject" (Babbage 1826, p. xi-xii). This was in 1826 but, commenting on Babbage's remark a quarter of a century later, Henry W. Porter claimed that "if this were true in 1826, how much more so it must be at the present time it is painful to contemplate" (Porter 1854, p. 109).

But the good calculators were also at risk from without, having to assert and secure their authority continuously against what they described as the infringement of others. For instance, in the realm of yield calculations, the actuaries deplored the persistence of invalid approximations which were being used, notably by other intermediaries, such as the brokers. As advisors to investors, brokers were logically competing with actuaries. One popular approximation which drew the ire of the leaders of the Institute consisted in simplifying the yield extraction problem by assuming that securities with complex reimbursement profiles were paid back once and for all at some given future date, as opposed to gradually over time as was really the case. This artificial date was typically computed by averaging future reimbursement dates. The procedure rendered the computation of yields much more tractable and thus accessible to less sophisticate persons. This enabled them to give a quick, "back of the envelope" estimation of the yield. But as William Sutton, a future president of the Institute, deplored, for the purist, such a method was "clearly incorrect in theory, and as practically used gives results that are wide off the mark" (Sutton 1882 pp. 129-131).[22]

An illustration of the problems of the reliability of calculations both within and without the discipline is provided by a controversy that arose in the *Times*, regarding the calculation of the yield on an Austrian government loan. A "broker" had written to the money market editor of the journal to expose the loan as usurious. After calculating its cost by using one of the incorrect maturity approximations extant, he complained that the loan was costing Austria an astonishing 9.4%.[23] A few days later, he was contradicted by an anonymous and self-proclaimed "actuary" who disputed the computations, deriding the broker's faulty mathematics and arriving for his part at a more modest 7.5%.[24] However, perhaps prompted by this anonymous writer's effort at impersonating the "actuary" point of view, Peter Gray a prominent member of the Institute of Actuaries took up the matter in his own hands. In an issue of the Institute's *Journal*, he came down in detail on both calculations, finding that the alleged actuary's methods weren't any better than those of the broker. In fact, he magnanimously absolved the broker as a lucky incompetent remarking sarcastically

[22] There were two main such methods. The "probable epoch" was the date at which half of the reimbursement would have been performed. The "epoch of mean probability" was a weighted average with the weights equal to the amounts reimbursed at each date.
[23] "A broker", *Times*, November 30, 1865. The loan being reimbursed by equal instalments, the probable epoch and the epoch of mean probability coincided with one another.
[24] "An actuary", *Times*, December 4, 1865.

that, considering all the flaws, the broker's number had come "wonderfully near the truth". And the "truth", of course, was his own result, given with five digit precision: 8.92018%.[25]

Such elements shed light on the logic that underpinned the discourse of the Institute. In essence, what it claimed, was that a government of the computers was necessary, or else a tragedy of the commons would take place. With an ungoverned scientific field, the risk at hand was that the sellers of financial products would find pliable calculators at their disposal. In other words, the problem was not so much the existence of approximations but the possibility that, among these approximations, manipulations would more easily sneak in. Manipulation, in turn would produce a debasement of the profession. By weakening the market, it would also weaken those involved in ensuring its proper functioning.

In the Institute's view, the remedy consisted in developing techniques to control the computers. This required establishing a monopoly power over the subject (explaining the skirmishes against the brokers), which was itself enforced by the control which the Institute had of the certification system. This organization would reassure traders that the numbers they were fed were honestly produced. To do this, what was needed was the establishment of procedures that would ostensibly limit the ability that individual calculators would have to manipulate the figures. One was the raising of the bar of what constituted a precise calculation. As illustrated by the example of the Austrian loan above, the proper actuaries were opposing to the cruder number suggested by others a precision which, for all practical purposes sounds preposterous. What we suggest in other words, is that the reason why "truth" was associated with a greater number of decimal digits was not because such precision was needed by the consumer as such, but because it was part of he process whereby the Institute could demonstrate its ability to go to the heart of the matter. In other words, the large number of decimal figures which actuaries enjoyed flashing was a signal. It excluded from the race those who did not control the technology to produce such numbers (presumably because they would not be able to compete on this turf, or because if they tried, they would be more rapidly exposed).

A characteristic episode illustrating this logic – the need to reduce human interference to the greatest possible extent – took place in 1875 during a conversation that occurred at the Institute. One actuary participating to the meeting, Andrew Baden, challenged the use of algorithms by the presenter who had used, in Baden's description, "a number of more or less intricate formulas." He contrasted this "scientific" method with his own "unscientific" one, which was faster. It consisted in simply "guessing the proper quantity" which he had figured himself, and then plugging this number back into the equation to show that this was a indeed the solution. As he claimed "in questions of this kind we acquire, after a little practice, a sort of tact in guessing" that enabled one at "once to fix an approximate rate not very far from the mark." This triggered the strong reaction of Henry W. Porter, who chaired the meeting, and of some other actuaries. They besieged Baden with questions regarding how he had proceeded precisely and got the answer that Baden had been helped with recourse to so-called annuity tables, which were instruments that provided short-cuts to calculators as we will explain in a future section. A footnote in the transcription of the debate mentioned this and the fact that if the same tables that Baden had used were used, it was possible, by interpolation, to reach essentially the same result as the one he had allegedly divined. With relief, Porter concluded that Baden had

[25] Gray (1868b, pp. 187-8).

merely exploited “knowledge previously obtained” enabling him to derive “by intuition what was the result of his formula.”[26] Baden’s luddite approach to the algorithm had been defeated and with it the loss of face that would have followed from allegations that actuaries were just guessing the results.

Having established its position as certifier of true actuaries, who played by the rules and minimized the human element, the Institute found itself in position to provide rents to its members. This was because the government of calculus it sought to establish would raise the collective profile of the actuaries. Such was the plan, at least. As a matter of fact, the objective comes strongly out of the language used in the Institute’s “Constitution and Laws” of 1849, which equated the advancement of actuarial science with the advancement of the actuaries. As the by-laws claimed, the Institute was being "founded for the purpose of elevating the attainments and status, and promoting the general efficiency of all who engaged in occupations connected with the pursuits of an Actuary, and for the extension and improvement of the data and methods of the science."[27] It is probably no coincidence that in the enumeration of the tasks, the elevation of the actuary came first, and the data and methods of science last.

Section IV. Licensing the Calculators

The previous section has shown that the rent an intermediary (as an individual and as a member of a professional group) could expect to derive from his position depended on his credibility as a computer, that is, on his capacity to reassure his clients, actual or prospective, on the trustworthiness of his calculations. This conclusion can help make sense of some of the Institute’s regulatory features, and of some of the choices made regarding the society’s organizational form during the months leading up to its foundation. The initial plan of the insurance managers who met in London in April 1848 was not to form a society of the nature of that which would later become the Institute of Actuaries. In fact, they initially envisioned the creation of a professional association, modeled after the Scottish Association for the Managers of Life Offices, founded in 1833, and aimed at defending their common industrial interests. While this plan might have helped to curb the growth of the industry, perceived as threatening, and establish uniformity in business practice, it wouldn’t have been effective in advancing the standing and status of the actuarial profession. The plan failed to produce an agreement. [28] A few months later, after protracted discussions, an agreement finally emerged, but on an entirely new basis.

Rather than a professional association, the actuaries involved in the creation of the Institute decided to establish a scholarly society, dedicated to the development and diffusion of sound actuarial science and practices. Its ambition was not to intervene directly in the inner working of insurance companies. Rather, it was to define what constituted legitimate and proper actuarial thinking and practices, that is, to set boundaries, in order to distinguish the scientific from the arbitrary, and the rational from the confused. To put it differently, the founders of the IoA aspired to establish a property right over a label, “actuary”, to prevent outsiders from misusing it, and from tarnishing the reputation of the craft. As Samuel Brown, one of the founding members,

[26] See Sutton (1875, pp. 95-97).
[27] *Constitution and Laws of the Institute of Actuaries of Great Britain and Ireland*, p. 15, Library of the Institute and Faculty of Actuaries, London.
[28] Simmonds (1948, pp. 8-10).

stated it, uniting the members of a profession in one body would help "keep down the pretensions of ignorant or unqualified persons[29]." Like most Victorian learned societies, the IoA established its own journal, crucial vehicle to the production and diffusion of sound actuarial knowledge. And like in most Victorian learned societies, the members of the Institute gathered and met on a regular basis, to exchange ideas, read each other's papers, and discuss matters related to the society's program. Essay contests were also regularly organized on topics falling under the Institute's intellectual jurisdiction.

But in one important respect, the Institute distinguished itself from most other learned societies: its recruitment did not only proceed from cooptation. Rather, to become a Fellow of the Institute, the society's highest distinction, one had to successfully sit an examination of competency, certifying fine knowledge of mathematics and matters related to interest and probability. This requirement had the effect of erecting barriers to entry. It helped to prevent outsiders from usurping the valuable label. Those who claimed the title "actuary" for themselves without having been certified by the IoA could then be identified and denounced. The establishment of an examination system ensured that "none but those duly qualified, and who had obtained a certificate form, and admission by the council, should be entitled to practice the profession."[30] The examination system was seen as a mean to put the Institute's seal upon the business of computing in insurance and finance, and to preserve the profession's standing and prestige. "Upon this test will mainly depend the future position and character of the profession", one reads in the report on the first examination, held in June 1850. [31] In other words, such a test would contribute to create, and subsequently to protect, the brand, on which the vitality of the profession was perceived to depend.

Setting up an examination was very early on a central preoccupation of the Institute. In fact, it was consubstantial to the project, and was inscribed in the association's first Constitution.[32] But to establish itself as a legitimate legitimizer was no easy matter. A careful consideration had to be given to the content of the examination, its structure and above all its degree of difficulty: the exams could not be made too complicated, for it would have discouraged potential members to sit in and would have hampered the growth of the society. In fact, in the early years of the Institute, most candidates passed. The rate of admission was far above what it later was[33]. But the Council had to be careful to not make the exams too easy either: it would have defeated the very purpose of the plan, which was to prevent unskilled outsiders to enter the profession. The tradeoff was succinctly formulated by Henry Porter in 1854, who wrote, in an article pertaining to actuarial education,

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[29] "The Institute of Actuaries", *Journal of the Institute of Actuaries*, Vol. 13, No. 6, pp. 392, 1867.
[30] *Ibid*.
[31] "Examination Papers", *The Assurance Magazine*, Vol. 1, No. 3, p. 151, 1851.
[32] *Constitution and Laws of the Institute of Actuaries of Great Britain and Ireland*, Art. 40: "The Council, at their first meeting subsequent to the Annual General Meeting, shall appoint two Fellows, an Official Associate, and an Honorary Member, or in lieu of the last, a third Fellow, as Examiners whose duty it shall be to examine candidates for certificates of competency in accordance with a plan to be approved by the Council." Library of the Institute and Faculty of Actuaries, London.
[33] For the first two years, all candidates (11) passed. The average rate of admission was 87% between 1850 and 1859. It fell to less than 50% in the subsequent 10 years (authors' data collection, from the Journal of the Institute of Actuaries).

that, "being voluntary, [the exams] must not be made too difficult, or many will be deterred from offering themselves; nor must they be too easy, or the character of the Institute will suffer."[34]

The structure of the examination was rapidly refined. The initial plan, concretely formulated before the Council in July 1849 and consisting in a single, uniform examination for all candidates,[35] was replaced two years later by a more structured system. Aspiring English actuaries, desirous to become legitimate members of the profession, had to pass three exams, corresponding to three successive years of studies.[36] This change was expected to contribute directly "to the great object for which the Institute was organized, viz., the elevation of the status and character of the profession."[37] Success at all three levels certified fine knowledge of applied mathematics and mastery of the actuarial craft, and candidates had to pass all three to be delivered the certificate. While the first two levels focused on the technique (probablility, algrebra, geometry…), the third level tested the candidates' analytical skills, and their capacity to apply actuarial knowledge to current affairs, the working of insurance companies or political reforms. The questions posed at this latter stage were intentionally discursive. Studying formulas in books was of little help to prepare for them. This was meant as an additional barrier to entry, to prevent people without actual insurance experience from passing, and to discourage them from sitting in in the first place.[38] But for the first two levels, the Institute gave specific reading recommendations to guide students in their preparation. That way, the Institute exerted closer control to what was students were learning. Obviously, students were encouraged to frequently consult the Journal of the Institute. Third year students were often asked about their familiarity with the most recent intellectual and scientific innovations in actuarial science, as they appeared in the Institute's outlet. The production and distribution functions were thus tightly integrated.

But the Institute did not contend itself with mere certification and competency checks. The Council aspired to teach as well, to fully control the material that the candidates were to absorb. As early as 1852, a plan for the establishment of a Mathematical Professorship was proposed.[39] The plan specified that the holders of such a position would be expected, each year, to deliver twelve lectures, intended for actuaries - novice or experienced - and mathematicians in general. He would also be expected produce two articles in the Journal. More importantly, the professor would be given the freedom to set up an instruction class, that aspiring actuaries would be free to attend, to better prepare for the exams. The plan also mentioned the possibility for

[34] Porter (1854, p. 117).
[35] *Council Minute Book*, Vol. I., 1848 to 1855, ff. 79–83, Archives of the Institute of Actuaries, London.
[36] "The Institute of Actuaries", *The Assurance Magazine*, Vol. 2, No. 2, p. 195, 1852.
[37] Ibid.
[38] As Thomas Sprague argued in 1868, "There was a great deal in the [third year's examination] that could not be learnt out of books, and (…) it [is] proper that is should be so, for the professional knowledge required in an actuary could be learnt from no books. Therefore, since the Institute gave certificates that certain persons were competent to act as actuaries, they did quite right to set questions which could only be answered by persons having a practical knowledge." ("The Institute of Actuaries", *The Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 14, No. 4, p. 331–340, 1868).
[39] "Institute of Actuaries", *The Assurance Magazine, and the Journal of the Institute of Actuaries*, Vol. 3, No. 3, pp. 272 – 276, 1853.

examiners to use attendance to the class as an element to judge the motivation of the candidates, and refine their grades on this basis.[40]

The plan, as it was presented in 1852, never came to fruition. But the Institute did not give up on its desire to set up a teaching structure, and the idea was resurrected two decades later, under a different form. On June 1871, "observing the continued increase in the class of Associates and in the number of candidates presenting themselves for the examination, the Council have (*sic*) been considering whether, in addition to granting certificates of competency, the Institute could not render some assistance to the candidates in their studies, either in the form of lecture, or class instruction, or otherwise."[41] The idea was eventually implemented, and in early 1872, a fully-fledged course preparing students to the certificate of competency was established.[42] The tutorship was attributed to a promising 30 years old actuary, William Sutton (already mentioned).

The lessons were initially intended for students preparing the second-year exam, but the teaching scope was later widened to include material that first year's students were expected to master.[43] Such a diversification of the course offering was partly motivated by the observation that students sitting in the second-year examination were often deficient in their knowledge of certain facets of mathematics whose mastery was nonetheless fundamental to carry out actuarial calculations. In a privately circulated report on the classes for the second year's examination of 1872, Sutton lamented that "many of those who pass the First Year's Examination are quite ignorant of the higher part of the algebra put down among the First Year's subjects, or at any rate only acquired a faint smattering". As a result of this deficiency, "there was a considerable difficulty in taking the members thro' the preliminary subjects of the Second Year's Examination."[44] This made the teaching load of the lecturer heavier, since he then had to demonstrate what was assumed to have been assimilated at previous stages of the curriculum. In other words, the screening effect exerted by the examination was not sufficient on its own to produce standardized and efficient "computers". Setting up a proper mathematics class, covering the basics, coupled with an increase in the standard for passing the exams, were seen as means to remedy this regretted state of affair, and to facilitate human capital industrialization. [45]

In 1882, a further stone was added to the educational edifice of the Institute: its first official textbook got published.[46] This marked the completion of the triptych diplomas-lectures-textbook, which made the IoA a full-blown educational institution. By then, however, the case for an official textbook had already been made for quite some time. In 1854, for example, Henry Porter had regretted "the confusion caused by the different

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[40] *Ibid.*
[41] "The Institute of Actuaries", *Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 16, No. 4, p. 307, 1871.
[42] "The Institute of Actuaries", *The Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 17, No. 2, p. 146, 1872.
[43] "The Institute of Actuaries", *The Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 19, No. 2, pp. 148-149, 1875.
[44] "Report on the Class for the Second Year's Examination, 1872", in *Council Minute Book*, Vol. III, 1866 to 1877, pp. 1–4, Archives of the Institute of Actuaries, London.
[45] "Letter from Mr. Sprague upon the proposed revision of the Syllabus of the Examinations for the Certificate of Competency", January 23, 1873, *Miscellaneous Minute Book*, 1849–1873, f. 165, Archives of the Institute of Actuaries, London.
[46] Sutton (1882).

notations used" by the texts then in circulation, and lamented that this state of affair represented a "great obstacle in the way of the student, if not a great inconvenience even to the practiced actuary."[47] Likewise, in his inaugural lecture, delivered in October 1871, Sutton had deplored the absence of "a good text-book for students, with well selected examples such as most of the mathematical text-books of repute have, and by means of which the student can at any time easily test for himself whether his progress is sound or not."[48] In 1877, the Council of the IoA, observing the dispersed nature of the available sources, and deploring that the lack of "a homogeneous treatise […] has been severely felt by students"[49], eventually decided to arrange the production of an official text-book. William Sutton was designated as its author, and became, at that occasion, the face of actuarial teaching at the Institute, a quasi-official status he was to keep until the very end of the 19th century.

The textbook represented a key element in the educational machinery of the Institute, and a crucial vehicle for the diffusion of actuarial knowledge. Aspiring actuaries had to become familiar with its content. The material that candidates were expected to master when sitting the first two exams was presented there, in a pedagogical manner. Cases were provided so that students could try their hands at applying the concepts and theorems the textbook contained. Sutton's textbook was soon completed by an exercise handbook, containing material selected "from all available sources in Actuarial literature, and largely from the examination papers of past years"[50], meant to further facilitate practice. Another essential virtue of the official textbook was to provide unified notations, an arguably crucial condition for the practitioners of a relatively new science to discuss with one another. From the admission of the Council itself, the Journal of the Institute had failed in that regard, since the contributors had the unfortunate habit to each use their own symbols, which hampered communication.[51] With a textbook, students could draw from a common source to acquire a common language. In fact, the textbook was such a valuable educational tool that the Institute ended up producing many of them over the course of its existence, up to this day, with each instalment updated to cover the needs of the generation for which it was intended.

The Royal Charter, granted in July 1884[52], at the end of a long and arduous application process[53], strongly reinforced the Institute's claim to legitimacy. In virtue of the incorporation, the Institute became a public body, and "its members acquired a higher professional status than they formerly could claim."[54] In fact, the Council used the Institute's certification and educational endeavors as proof of the validity of its claim to legal personhood. By 1884, the Institute had already "caused to be written and published for the use of the

[47] Porter (1954, p. 115).
[48] Sutton (1872, pp. 435–436).
[49] "The Institute of Actuaries", *Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 20, No. 4, p. 307, 1877.
[50] Ackland and Hardy (1889, p. 6).
[51] *Miscellaneous Minute Book*, 1849 to 1873, f. 102, f. 142, f. 152, f. 158. Archives of the Institute of Actuaries, London. See also Simmonds (1848, p. 99).
[52] "The Institute of Actuaries", *Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 25, No. 4, p. 289, 1885.
[53] "The Royal Charter of Incorporation of the Institute of Actuaries", *Journal of the Institute of Actuaries and Assurance Magazine*, Vol. 25, No. 1, pp. 1-15, 1884.
[54] *Miscellaneous Minute Book*, 1876 to 1898, f. 122, Archives of the Institute of Actuaries, London.

profession of Actuaries and others a Text-Book of the Principles of Interest" and had "by means of the important educational functions and system of examinations […] largely contributed to the better discharge of duties affecting interests of great importance."[55] In other words, by providing a trustworthy standard of professional competency, the Institute contributed to the common good.[56] The Charter, in turn, strengthened the position of the Institute as the sole provider of computing certification in Victorian England.

Section V. The Making of Makeham's Formula

We now argue that the specific needs of the Institute shaped the manner in which it constructed financial knowledge. Securing the control of the minds of the computers was the first stage. The next was the production of algorithms that would exploit them adequately. The result was the promotion of a kind of numerical aesthetic whose main concern was to avoid overheating. A compelling example of this was the emergence of Makeham's formula. In 1874, the Institute's *Journal* featured an article on "The Solution of Problems Connected with Loans Repayable by Instalments", authored by the mathematician and Institute Fellow William Makeham, better known nowadays for his contribution to modelling human mortality.[57] In this article, Makeham proposed a transformation of the expression for the present value of a whole class of loans. His formula, which came down in financial history as "Makeham's formula", was rapidly adopted by the Institute, and became the standard framework that actuaries resorted to so as to handle the problem of interest extraction for redeemable securities.

Makeham's formula had an obvious traction: in virtue of its relative simplicity and ease of use, it could be widely deployed and applied to a large variety of cases. In other words, it generated significant economies of scale in the extraction of yield at the same time as its relative simplicity reduced the risk that individual computers go creative. Since its use only required the mobilization of computing techniques familiar to actuaries trained by the Institute, its software could easily become part of the default equipment of the new computers licensed by the Institute via the Institute's educational structures. It remains to this day, an important component of the actuarial theory of redeemable securities[58].

The origins of Makeham's formula are interesting to retrace, as they shed light on the manner in which the economic concerns of the Institute's shaped the knowledge it produced. They go back to the controversy, already mentioned, which took place in the money market section of the *Times* in late 1865, pitting against one another "a broker" and "an actuary" on the proper method to calculate the yield of a recently issued Austrian loan. The exchange had triggered the response of Peter Gray who published in 1868 a long article, released in the *Journal* in three subsequent sections under the generic title of "The Rate of Interest in Loans Repayable by

[55] "The Royal Charter", p. 7.
[56] "Copy letter addressed to the Board of Trade by the Council of the Institute of Actuaries in reply to a communication received by the Board of Trade from the Actuaries' Club in opposition to the application by the Institute of Actuaries for the Grant of a Royal Charter of Incorporation, December 1882", p. 6, *Council Minute Book*, Vol. IV, 1877 to 1887, f. 160, Archives of the Institute of Actuaries, London.
[57] On this specific aspect of actuarial science, Makeham's name is often associated with that of Benjamin Gompertz, already mentioned. Makeham is considered to have completed Gompertz's equation of human mortality (according to which the risk of mortality grows exponentially with age), by adding to it a constant term, capturing the risk of death non-related to aging (Harris 2009, pp. 36-38).
[58] Jensen (2012).

Instalments" (Gray 1868a, b and c). The article did not content itself with admonishing the numerical miscreants. It provided what Gray described as a general method capable of determining "the exact [interest] rates, at the cost of no great expenditure of time or trouble" for the whole class of securities of which the Austrian loan of 1865 provided one instance, namely bonds that were reimbursed through the payment of a constant annual sum, until they became extinguished (Gray 1868, p. 91).

From the vantage point of the borrower, such loans involved the payment of a declining cash-out, since the interest was reduced gradually as the loan was being reimbursed, while the amount amortized remained constant. In other words, such instruments were different from the standard annuity, which involved a constant annual payment, and they thus required a specific formalism. To handle this case, Gray pointed to a formula he had proposed himself some years ago in a previous issue of the *Journal* in a different context but which turned out to provide a relevant framework. It essentially consisted in a present value equation, but written in a way that made it convenient to solve. In fact, the formula could also be shown to be applicable in the case of standard annuities as well, motivating, Gray's description of his formula as "the most general annuity theorem that has yet been given[59]". Equipped with Gray's general present value equation, the actuary could in principle plug in it the parameters of the specific loan that he was dealing with, and, from there, the yield could be extracted, via a trial and error process.[60]

But Gray's method, despite the praise he gave to it, remained computationally heavy. It involved determining what he called "the law of the annuity" (the parameters to be inserted in his formula), which required for the actuary to resort to the mathematics of finite differences, a domain with which students of the Institute were far from familiar[61]. Once the law of the annuity had been established and the values of the parameters determined, the actuary had to go through a relatively strenuous trial and error process, in order to find a rate of interest such as the present value of the whole loan was equal to its market price. Each member of Gray's equation had then to be separately calculated, at each of the trial rates. There were many possibilities for mistakes along the way.

Gray's contribution played nonetheless an important function: With him, the matter was taken away from numerical outsiders, and brought on the turf of the Institute of Actuaries. Moreover, Gray was showing the possibility of a general, "industrial" solution to the problem. The question of the determination of the yield on loans redeemable by instalments became an in-house debate, and could be subject to the rigorous dissection of the Institute's mathematicians until a superior approach could be devised. Makeham's 1874 paper was, indeed, a response to Gray, and directly intended as an improvement over the latter's method. Makeham acknowledged his predecessor's "valuable labours", but his ambition was to propose of method of his own, surpassing Gray's, he explained, in ease of usefulness and tractability.

The gist of Makeham's simplification rested on an algebraic transformation of the expression of the loan's present value. More specifically, he started from the common-sense notion that the present value of any loan could be expressed as the sum of two terms: the present value of the capital repayable, and the present value

[59] Gray (1856, p. 191).
[60] See the appendix for a formal presentation of Gray's formula and discussion of its implementation.
[61] "Report on the Class for the Second Year's Examination, 1872", in *Council Minute Book*, Vol. III, 1866 to 1877, pp. 1-4, Archives of the Institute of Actuaries, London.

of the interest to be paid on this capital. He then noted that the present value of the interest was itself a function of the present value of the capital. That way, the present value of the whole loan could be expressed as a function of the capital only (See appendix for a discussion of Makeham's transformation). This came particularly handy when dealing with loans redeemable by equal instalments. Since the amortization for those loans was, by definition, constant, it meant that their present value, following Makeham's formula, was simply a function of a constant annuity. As a result, those loans, although technically different from the traditional annuities that had been familiar to actuaries for centuries, became amenable to the conventional techniques developed for these, as they were shown to be a transformation of the good old ordinary annuities.

Makeham's (1874) formula rapidly imposed itself as the main framework to deal with loans redeemable by instalments, a diffusion process set in motion and maintained by the enforcement power of the Institute's educational machinery. Sutton's textbook, published in 1882, dedicated a whole section to presenting the formula. King's (1882) and Glen's (1893) introductory textbooks drew almost entirely on Makeham's formalism when presenting the theory of redeemable loans, and so did Todhunter (1901). Todhunter's textbook, having been reedited three times between 1901 and 1937, did constitute, for half a century, the daily intellectual nourishment of aspiring actuaries, at least in the English-speaking world. And tellingly, the latest edition (as of 2019) of the Institute's official textbook on the topic of financial mathematics – one of the so-called Core Technical subjects that students need to master to become certified actuaries – devotes a generous portion of its chapter on "The Valuation of Securities" to exposing, proving, and presenting the "attractiveness" of Makeham's formula.[62] As for Gray's contribution, it rapidly fell into oblivion.

Section VI. The Coming of the Rational Expectations Hypothesis

The gradual establishment of the monopoly power of the Institute over numerical computations had the consequence of reinforcing its ability to set norms. To the extent that its machinery was primarily concerned with securing trust by reducing the scope for self-interested manipulations, it also became a natural stakeholder of "numerical confidence crises". The Institute was in position to perform a forensic role, that is, to determine after the fact the extent to which manipulations had taken place. It could also contribute the elements of a policy or regulatory response, by developing guidelines for best-practices going forward and thus, ostensibly, preventing a repetition. This was all the more so given that it had the possibility to enforce the new norm by making it part of the certification of the new generation of calculators. Thus confidence crises fed the growth of the rule of the Institute by encouraging the production of "system updates" which expanded the Institute's rule. In fact, it is possible that the causality went the other way round, with the Institute finding its advantage in dramatizing events so as to expand its territory. Given the difficulties which it encountered in expanding its influence, it benefitted from instances where it could say "I told you so." The end result was a reduction of the numerical wastelands now part of the Institute's enclosures.

We illustrate how the process took place through a case study. It consists in the response which the Institute brought to the foreign government bond debacle of 1873-1875. Following a protracted boom in foreign debt originations that had started in the 1850s, the market for foreign government debt eventually

[62] Garrett (2013, p. 134).

crashed, against charges of intermediaries' wrongdoings in a number of loans to countries now in default such as Honduras. In early 1875, this led to the gathering of a British Parliament investigation known as the Select Committee on Loans to Foreign States. The Select Committee's hearings, disclosures and "revelations" were widely advertised in the British media. As summarized in the *Report* of the Select Committee, lawmakers involved in the investigation claimed that ploys had been used to conceal the riskiness of the loans. Underwriters had charged enormous fees, but they had camouflaged the numbers from the bondholders to whom they sold the securities. They had hidden the actual amount that were being subscribed, concealing that some of the loans were really failures. Finally, they had manipulated the market through forward sales so as to puff up the securities. Such tricks, the *Report* concluded, were meant to produce the deceptive impression that the loans were less risky than they truly were.[63]

In the charges that were being brought up or discussed by the Select Committee, nothing involved directly the problem of financial calculations. Nonetheless, the Institute of Actuaries joined the chorus of attacks against contractors with an accusation of its own. According to William Sutton, who devoted, at the time when the discoveries the Select Committee were making headlines, in the Journal of the Institute, an article to the subject of the manner in which the foreign loans examined by the Select Committee had been structured had been part of the plot (Sutton 1875). As he explained, the typical method to design the loans examined by the Select Committee had consisted in issuing them at a moderately high coupon but with a very low price. For instance, in the case of the Honduras loan of 1870, it was issued at a coupon of £10, and a price of £80, when the face value was £100.

The result was that the reimbursement of the loan introduced a speculative element. Loans such as this being typically reimbursed through random drawings of the bonds' serial numbers placed in a lottery machine (See Figure 2), investors could contemplate considerable gains – if they were lucky. For instance, an investor whose security was drawn on year one would realize a revenue of £30, comprising a coupon of £10 plus a capital gain of £20, the difference between the price paid for the purchase the bond or £80 and the price at which the bond was reimbursed, or £100. For an investment of £80, a £30 revenue amounted to a hefty 37.5% interest. According to Sutton, naïve investors had been confused by this. The specific manner in which the loans had been structured had created a computational blind-spot that served to "conceal" the "actual terms on which the loan is obtained."[64] A government borrowing in such a manner had to be a high risk. As Sutton concluded, had investors been able to figure the "extravagant terms offered in some cases", this would have acted "as a caution […], and instead of serving as a bait would have [had] the contrary effect."[65]

Driving his point home, Sutton endeavored to derive what he described as the "true" approach to which we shall return below, and which, as we shall explain, consisted in adapting an early version of the rational expectations hypothesis. Next, Sutton used it to calculate the yields of a list of foreign government bonds, including several incriminated by the Select Committee and compared it to the numbers obtained under the "naïve" alternative. The output was tabulated and as the table demonstrated, the two methods exhibited the

[63] Select Committee on Loans to Foreign States, Report, pp. xxx.
[64] Sutton (1875, p. 78).
[65] Sutton (1875, p. 78).

predicted differences, with the "incorrect" method understating the "true" interest rates (Sutton, 1875, p. 87). The Honduras loan of 1870 for instance, exhibited a yield of 12.5% according to the "naïve" calculation, but 15.0263% according to Sutton's "rigorous" calculus. In other words, the bonds looked significantly riskier if one used the right method.

Figure 2. A drawing machine for the redemption of bonds

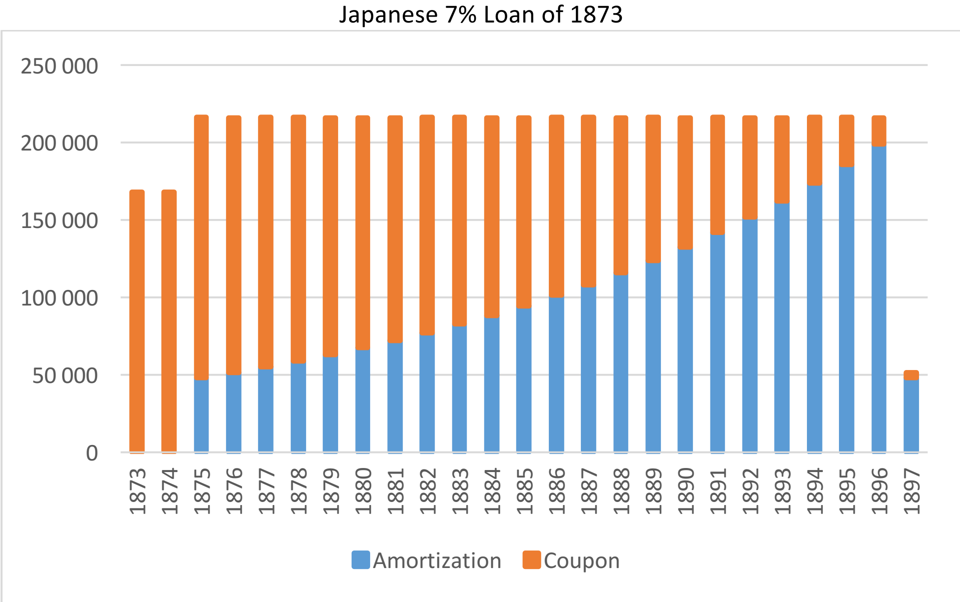
Source: Authors, from the archive of the Banque de Paris et des Pays-Bas as visited in 2011. For more details on how it operated, see letter from engineer Casanova to Directeur Comptoir d'Escompte, Arch BNP Paribas 73AH491, 1891.

However, while this difference of about 250 basis point was considerable and certainly enormous for a precision-minded individual, the conspiracy theory put together by Sutton seemed far-fetched. It is exaggerated to claim that posting a yield of 12.5%, when the true yield was 15% -- if this was really what had happened – could mislead investors *as to the speculative nature of the loan*. Underreporting the return of a high yield bond to make it look sounder feels like a rather bizarre idea even for a financier of the 1870s but the idea seems even more twisted when it is realized that the "camouflage" was imperfect at best. In fact, Sutton's explanation was in places contradicting itself. He listed, side by side, as reasons for the initial success of the foreign loans with the public, the fact that agents had been naïve (and thus missed that the loans were speculative) and the fact that agents had been "irresistibly" attracted by the loans' "gambling element" (and thus fully understood what was going on).[66]

[66] Sutton (1875, p. 87). As he continued: "The prospect of making 5 to 7 or 8 percent interest, and when your bond is drawn for repayment receiving a bonus of, say 15 to 25 or 30 percent, is undoubtedly a pleasant one, and the parties who bring out these foreign loans are well aware of the fact."

A more convincing way to understand what was at stake consists in retracing the steps that led to Sutton's crackdown. In fact, the computational blind-spot he denounced was, to a large extent, of the actuaries' own making. In the past, the calculation of the yields on bonds such as the ones examined by the Select Committee, had presented actuaries with a quandary. On the one hand, their redemption system, known as "accumulative sinking fund" facilitated the determination of yields. By this system, the amount amortized increased annually by the interest saved from already redeemed bonds (See Figure 3, which gives an illustrative example). As a result, the total annual pay-out (interests and amortization) was constant. Since the borrower paid a constant amount until maturity, this mean of borrowing was equivalent to an annuity, and the interest cost to the borrower could be calculated with absolute precision by mobilizing traditional methods. Indeed Sutton recognized in the opening lines of his 1875 paper that "it may be stated that, for all practical purposes, this [mode of borrowing] is equivalent to borrowing money by means of terminable annuities." It is likely that this had been recognized long ago and it is probable that the simplicity of the calculation had been one reason for constructing this class of bonds the way they had been.[67]

Figure 3. Breakdown of amortization and coupon in the case of an accumulative sinking fund: Japanese 7% Loan of 1873



Source: Authors' computations, various sources

What held back actuaries, however, was the problem of the point of view of the investor. The borrower knew the exact amount to be paid out as amortization from the start. But what about the investor? If she held just a few securities, the apocryphal spinster had to reckon with the fact that she might be reimbursed at an earlier or later date.[68] What should actuaries tell her? Up to the Select Committee and the publication of Sutton's piece, British actuaries felt, and said aloud, that it was not proper for them to advise on the value of a given bond being drawn. Instead, the dominant practice among members of the Institute consisted in providing

[67] Sutton (1875, p. 78). Indeed, just one page later, Sutton added that, "as is well-known, the problem is generally an extremely simple one theoretically."
[68] Gray (1868b).

customers (investors) making such inquiries with a menu of yields, according to when the bonds were drawn, because "the rate realized on each bond will depend on the period at which it is paid off" (Gray 1868a, pp. 99-100). For instance, Peter Gray's discussion of the Austrian loan of 1865 distinguished between the interest rate for the borrower (8.92018%) and the interest rate for the lender, which varied between 7.49% and 23.7%. This was expressed with a table (see Figure 4) showing the performance according to the reimbursement date (Gray, 1868b, p. 188-189). Likewise, another prominent actuary, G. F. Hardy, claimed in 1875 that he had for his part "always [given investors] two rates. I say, 'You will get so much if the bond is paid [i.e. drawn], and so much if it is not."[69]

Figure 4. Gray's Table: Interest Rate Performance for Investors in the Case of the Austrian Loan of 1865

n.	Half-yearly Rate per Cent.	Error.	Annual Rate per Cent.
5	11·221	+ ·0088	23·700
25	4·708	+ ·0008	9·635
45	4·062	– ·0022	8·288
65	3·849	– ·0019	7·845
78	3·783	– ·0055	7·709
Limit	3·678		7·490

Source, Gray (1868b), p. 189

In the language of today's economists, the reluctance that professional actuaries displayed towards synthesizing random outcomes in but one number may be stated as one of accounting for investors' attitudes toward risk. Considering that investors had different attitude towards risk, then, the proper way to advise them was to provide them with a full menu of outcome dependent performances, for them to figure out what they wanted to make of it. By contrast, aggregating all the information pertaining to the future in just one number would destroy valuable information. Indeed, the role of investors' appetite for risk was explicitly referred to when actuaries rationalized their preference for a table. Gray, for instance, suggested that risk takers would value the possibility of large financial gains.[70] To the extent that a synthetic number should be produced, actuaries hoped to get it from the market, rather than from their own calculations. Their discussions mention efforts to encourage financiers to set up an insurance system – an arrangement whereby a company would purchase from individuals the right to benefit from being drawn. Such a system would have enabled to price

[69] Sutton (1875, p. 95). Still in 1893, Ninian Glen would declare, "Of course bonds drawn within the first year or two will show a much greater return than those which are not repaid till the loan has almost expired, and it is impossible to say in the case of any individual bond what the return may be." (Glen 1893, p. 49).

[70] In his language, "it is quite conceivable that this element of uncertainty— embracing as it does the possibility of a very large return on the sum invested— may act with no small effect as a lure on minds of a certain constitution." (Gray 1868, p. 100).

this benefit. According to Hardy "if you take [the cost of insuring bonds being drawn] off, it will give you the interest rate."[71]

Such was the backdrop of Sutton's suggested solution, which consisted in assuming risk neutrality and as a result to part with the "general impression […] that in the case, say, of a purchaser of a single bond, it is impossible to say what rate of interest he will realize on his investment, as his bond may be paid off in any year of the *n* years which repayment extends" (Sutton 1875, p. 85). What Sutton suggested was that, if one used the mathematical expectations of the present value of the bond, then one could sort out the conundrum since one could show that the yield for an individual would not be affected by whether she held one bond or all the bonds extant. Sutton provided a formal proof, which consisted in the application of the law of large numbers.[72] This proof was supplemented by an intuition. Sutton likened his logic to that of an insurance company (an "office" in his terminology) that would purchase an insurance product that involved a contingent claim similar in its operation to the random reimbursement.[73] As he explained, it was conventional among insurance companies to book future cash flows, when the law of distribution was known according to their mathematical expectations. In other words, Sutton was appealing to established practices among actuaries to naturalize his suggested solution.[74] The rational expectations hypothesis thus came down as a natural assumption in a community of calculators who precisely had within their remit the affairs of insurance companies .[75]

But there was also another reason, which explains why the context created by the Select Committee mattered. The liberal approach that had prevailed before might have been suitable to the sophisticate investors who were significant clients of the actuaries. Yet, from the broader point of view of the interests of the Institute, leaving this corner of the market for computations ungoverned entailed the risk to permit the survival and proliferation of parallel, competing methods, which interfered with the government of numbers the Institute of Actuaries tried to establish. In fact, at the same time when such sophisticate methods and advice were being dispensed, the emerging middle class, which could not always afford the support of individual actuaries, had to content itself with much more rudimentary instruments. This included such instruments as were found in a number of periodicals, rough guides that converted fixed income securities into standardized products such as perpetuities. Such calculations, indeed, entailed distortions. Whether they readily served the ends of criminals lurching in computational mysteries is a disputable matter. But it did make perfect sense for a promoter of the Institute such as Sutton to exploit the widespread anxieties that arose at the time of the Select Committee and mount an attack against such methods so as to try and expand the empire of the Institute of Actuaries.

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[71] According to the actuary Cornelius Walford, just before the Overend Gurney panic of 1866, one new finance company had taken up the idea. He adds that "the scheme was a good one and was worked out with much skills and care" but the panic put a stop to the attempt (see Sutton 1875, p. 95).

[72] Sutton (1875, pp. 85-6).

[73] This was nothing but the law of large numbers or as Sutton put it, it reflected the fact that an investor might fairly "reckon" that her bonds would be "drawn for repayment year by year in the proportion the total amount of them bore to the total amount of the loan" (Sutton 1875, p. 85).

[74] As the Constitution of the Institute emphasized, their goal was the "extension and improvement […] of the methods of the science which has its origin in the application of probabilities in the affairs of life." (*Constitution and Laws*, p. 15).

[75] Sutton (1875, p. 85).

Given what we said in section IV, Sutton had the means to enforce his norm. Being responsible for the examination and later for the Institute's first textbook, he could translate the formula which he advocated into an actual software, with which all the new "calculators" produced by the Institute would be equipped in the future. Indeed, in 1882, following several years of teaching of this rational expectations approach to the determination of yields, Sutton made sure that the method would be included in the Institute's textbook (Sutton 1882, pp. 131-3).[76] From that date on, typical training material contained problem sets that involved random amortizations of loans similar to the ones dealt with in Sutton's classic paper. The candidate had to solve the problem using the new method.[77]

Reflecting the gradual success of the Institute in setting standards, the *Investor's Monthly Manual*, an influential guide for middle class investors which was owned by *The Economist*, announced in early 1883, that it was adjusting the system by which it computed the yields it reported. The *Investors' Monthly Manual*, which used to give only crude yields computation obtained by comparing the last dividend or coupon paid with the price, had taken this step following persistent criticism from its readers who typically took issue with the kind of problems to which Sutton had drawn attention. [78] As the newspaper recognized, in the leader that explained the changeover, "it has always been apparent where future redemption at par has to be taken into consideration," that neglecting it was "open to material qualification." Though the precise formulae that were to be used is lost, it appears from the description that they were informed by Sutton's approach. That the changeover occurred almost immediately after the Institute's manual had been released cannot be a mere coincidence since the content of the manual was now – by definition – "textbook stuff".[79]

Section VII. Sources of Productivity Growth in 19th Century Financial Computing

The discussions in the previous section sheds light on the possible sources of productivity gains in financial computing during the 19th century. As a result, technological progress could be advanced through any of the following, not mutually exclusive, channels: Technologies to generate superior starting values could be conceived, the power of the algorithm could be enhanced through better design, or the efficiency of the human "calculators" operating these systems could be increased. In this section we discuss these channels, focusing for simplicity on their relevance to the resolution of the annuity problem. This approach is warranted, given the centrality that this financial product has in the overall story.

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[76] Likewise, King's textbook – *The Theory of Finance* – which presented itself as consisting of "Notes of Lectures on Interest and Annuities-Certain, delivered to the Students for the Intermediate Examination of the Institute of Actuaries" declared outright that "a very frequent instance of the application" of the converging series algorithm is the "case of foreign government loans." (King 1882, pp. 217 ff.).

[77] See Glen (1893, p. 52) for such an instance.

[78] For evidence of lingering complaints by readers, see "Redemption at par", *IMM*, November 26, 1881, p. 491.

[79] *IMM*, 31 January 1883, "The Alterations Introduced in the Present Number". Compare with Mauro, Sussman and Yafeh (2006) who argue that the IMM adhered all the way to the calculation of yields with the help of the coupon/price formula. These authors appear to have overlooked the change in 1883. It is rather clear from the data that a transformation occurred at that point and that the coupon/price formula was abandoned. It would be interesting to know with greater precision what the IMM did exactly. This is an interesting subject for future research.

Let's begin with the improvement of numerical tools. With the growth in the demand of computations, the value of tabulating ready-made results, which could be used for reference, was increased. This encouraged calculators to construct and disseminate such instruments and the encouragement was especially clear if the type of calculations that were needed by the community were clearly identified. In the case of annuities, the principal tool that emerged to assist calculators consisted in the so-called "annuity tables". These provided pre-calculated results, spanning the grid of interest rates and maturities for their most common values. For instance, for an interest rate of, say, 5% and a maturity of, say, 15 years, the table gave the present value of the annuity. The tables had various uses, but an important one was that they provided a way to generate an interval for the interest rate solution. The trick consisting in reading the table backward. Knowing the present value of the annuity and the maturity, the calculator would find the two annuity values that bracketed most closely the one at hand. Then, he could simply read the interest rate bracket that corresponded to these.[80] This was a powerful instrument in view of the fact that the quality of available algorithms hinged upon the availability of high quality starting values.

Against this backdrop, the finer the granularity of the table, the better the quality of the starting values: Or, to put it in another way, improvements in the granularity of the tables translated mechanically into efficiency gains in computing. Therefore, secular trends in computing efficiency can be captured by tracking evolution of the granularity used in successive reference tables. Baily's annuity tables, printed in the appendix of the *Doctrine*, went from 2% to 5% in increments of half of a percentage points, and then from 5 to 10% in increments of one percentage point. As Baily indicated, he had lifted his tables from John Smart's *Tables of Interest, Discounts, Annuities, etc.*, dating back to 1726, because he had "neither time nor inclination to calculate anew" (Baily 1808, p. vii). Baily's table continued to serve as reference in the first half of the century.

A few years after the launch of the Institute, a new reference tool was produced that significantly increased both granularity and coverage. In 1852, George Rance's tables went from 0.25% to 10% by increments of a quarter of a percentage point (Rance 1852). This new reference instrument soon imposed itself to the profession. The first edition showed a long list of subscribers, all actuaries and members of the Institute. Rance himself was the actuary of the Sun Fire Office going. It is of course significant that the first quantum improvement of the granularity of the best available table followed on the heels of the launch of the Institute of Actuaries. It was as if the organization of the profession had helped reduce the transaction costs associated with the introduction of such a tool.[81] The appendix summarizes the main features of ten of such tables, covering a period going from 1726 to 1877.

Another aspect of the productivity gains that were realized involves increases in the power of the algorithm. The converging series method had been operationalized by Baily in the case of the annuity, but this was not the last word on the subject. Five methods were subsequently developed, due respectively to Barrett

[80] For instance, using the table provided by Baily, one could readily see that a unit annuity worth £10, and having a 15 years maturity, is bracketed by 10.3796 and 9.7122, which corresponded respectively to 5 and 6% (Baily 1808, Table VI).

[81] Rance's table dominated was later supplanted by the next generation, William Henry Oakes' *Tables of Compound Interest* (Oakes 1877). This one went from 0.75% to 10% by eighths of a percentage point. Suggestively, Oakes emphasized this finer granularity as the key superiority of his table over Rance's (Oakes 1877, p. iii).

(1811), De Morgan (1859), MacLauchlan (1874) and, finally, Hardy (1882) who developed two formulae. All these achievements had involved, at one level or another, the intervention of the Institute. The formulae of De Morgan, MacLauchlan and Hardy had been published in the Institute's *Journal*. Barrett's method, though dating back to 1811, was in fact yet another achievement of the Institute. It really consisted in three lines in the middle of a letter from Barrett to Baily, where Barrett boasted that his method "greatly improved" on Baily's. It is doubtful that it would have had any posterity, save in Barrett's own calculations, if it had not been for the fact that Baily's actuarial correspondence had been donated to the Institute in the early 1850s. This had enabled De Morgan to mine it and discover the formula, which he published in the *Journal of the Institute of Actuaries* (De Morgan 1854). After having experimented with it, its qualities were recognized by calculators and it made it into subsequent textbooks.[82]

It is possible, by comparing the "performance" of the various algorithms, to test whether the creation of the Institute was followed by a productivity shift.[83] Of course, data on financial computing costs is simply absent. But we can circumnavigate the problem by looking at technological savings (as opposed to monetary ones) brought about by an amelioration of algorithms. This enables us to generate a criterion consisting in the quantity of output (accuracy) achieved for a given amount of input (computing effort). In line with contemporary approaches, our measure of performance is the size of the error between the true yield and the yield obtained after just one iteration of the algorithm.[84]

What we do in what follows therefore, is generate randomly a large number of yield extraction problems spanning a relevant grid, and see how efficiently alternative methods handle it. Setting the horizon to 15 and 30 years respectively, we spanned the 0.5-13% interval by we generating 100 annuity problems. Next, to establish a level playing field between the different methods, we fed them with the same starting value each time, generated with the help of a theoretical interest table with a granularity arbitrarily set, without loss of generality, at .25%. This allowed us to generate 100 first iteration distances from actual value – an indicator of the quality of the algorithm.[85] Figure 5 provides the results. It shows, for each method, the average distance (over 100 trials) from the true yield, distinguishing between the 15 years and the 30 years maturity scenarios. As can be seen in the Figure, regardless of the horizon, Barrett's formula provided a significant improvement on Baily (explaining De Morgan's insistence on popularizing it). So did the other formulae developed after the creation of the Institute, although, by the third quarter of the century improvements appear to have edged off. In other words, the aftermath of the creation of the Institute was characterized by a productivity growth spurt. (The appendix shows that results have actually little variance and are robust over alternatives).

[82] Sutton (1882, p. 84) ; King (1898; p. 46).

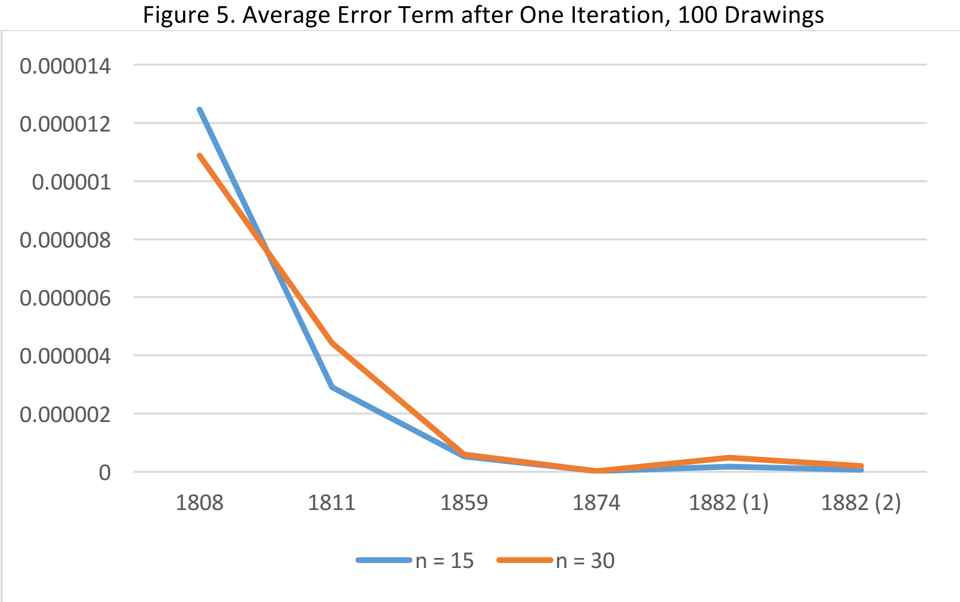
[83] For this exercise, we neglect other sources of technological progress such as the development of interest tables providing for finer granularity, the standardization of computing procedures or the improvement of the calculating workforce – all developments in which the Institute played a positive role. In other words, our evidence under-estimates the Institute's contribution.

[84] King (1898) provides a similar approach, which we industrialize. In the two subsequent editions of his handbook, he subjects a set of annuity problems to alternative algorithms, essentially, organizing a horse race between them.

[85] Consistently with the approach in King (1898) we make no effort at interpolating but simply pick the boundary of the interest interval that is closest to the actual annuity value as read from the table.

The last aspect we cover consists in the improvement of the workforce. Here we are in difficult territory because of the lack of direct evidence and the distance that may have existed between the efficiency frontier and the actual level of proficiency of individual computers. One element which comes out from the previous analysis however, is that, if we add up the improvement in the algorithms and the finer granularity of the tables, calculators had the possibility to achieve convergence "at first blush" – that, is come within a range smaller than 10^{-7} from the exact solution in but one operation. To the extent that they indeed devoted themselves to achieve this result, they were able to increase their performance. We have already seen, in the context of the debate on Baden's powers of divination the role played by familiarity with the tools. This suggests that a "grooming" of the computers was at work, that enabled to raise their productivity as they improved the mastery of their various instruments. King's textbook alludes to this process, emphasizing the importance of mastering the tools of the trade, such as the tables, and the impact this had on the productivity of the computers. According to him, enormous savings were realized from training and experience: "It will be frequently noticed that where a novice goes through an intricate calculation in answering a question, the adept produces the same result seemingly without thought or effort; *and on inquiry it will usually be discovered that tables are the tools he uses to shorten his labor and save his time*" (King 1898, p. 249).

Figure 5. Average Error Term after One Iteration, 100 Drawings



Source: authors' computations. See text.

Conclusion. Truth: A Supply Side Story

Forty years of information economics have demonstrated that information asymmetries give rise to institutional innovations. Among the "solutions" which are evolved to facilitate trade, we find in particular, in many cases, institutions that entrench hierarchies, control and monopoly power. The story of the Institute of Actuaries provides a case in point. In order to sort out anxieties pertaining to the intrinsic value of securities, a monopoly was evolved, which started to establish knowledge enclosures. The control of the members and the development and promotion of a set of methods that minimized the variance of the numerical output across individual computers served to entrench the credibility of the profession and to bolster the security of the market. This was nothing but the defense of a label. The end result was the production of a fully constituted discipline. As this happened, productivity gains were realized, the combined product of scale economies, the training of human computers and increased incentives for research and development.

One way to think of this narrative is as being the sketch of a supply side story of science, rooted in a history of the capital market. It brings to the fore the role of knowledge as part of the exchange system. In the background of the unfolding events, was the world of finance and its trading requirements – the reliability of computations reflected in a numerical precision that may be considered as preposterous, if we do not factor in the fact that precision was not about knowing the return of any single asset with extreme detail, but about knowing that the computers hummed reassuringly. The needs of the London Stock Exchange, and the dialectic relation it had with the insurance business played a crucial role in influencing the dynamics of the discipline.[86] In other words, the increased requirement for trust, and the intellectual innovations it brought, were tied to the deepening of financial capitalism. If this is granted, then we need to recognize the existence of something that we may call "capitalist science". That is to say, the fact that the capitalist regime breeds its own mode of knowing.

This leads to a final reflection, with which we would like to leave the reader. Although it has been common in grand theorizations of the role of science before, during and beyond the Industrial Revolution, to emphasize its connection with the forces of open market, a case can be made – in fact, we have just made a case – for emphasizing the hidden affinity between science and monopolistic practices. Just like it is well-known that industrial monopolies can set prices, knowledge monopolies can set truths. That they are contestable reduces the monopoly power but does not suppress it altogether. Or to put it in yet another way, the economic structures of the market place of ideas and their financial underpinnings deserve a much more careful attention than has been done heretofore.

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[86] At the end of the century, this overarching importance was acknowledged by the president of the Institute T.E. Young who, in the annual address of 1897, encouraged the aspiring actuary to develop a solid training in "Finance" (his capital) because "mastery of financial questions [had] assumed an imperative supremacy" for the profession (Young 1897).

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Appendix: Tables covering the present value of £1 for any number of years

Author	**Date**	**Years**	**Interest rate Range**	**Granularity**	**Annuity payable**	**Decimals given**
John Smart	1726	100 (half-yearly increments)	2% - 10%	50 basis points from 2% to 5%, 100 basis points afterwards	Yearly	8
Francis Baily	1808	100	2% - 10%	50 basis points from 2% to 5%, 100 basis points afterwards	Yearly	8
John Milne	1815	100	2% - 10%	50 basis points from 2% to 4%, 100 basis points from 4% to 10%	Yearly	4
Francis Corbaux	1825	100	3% - 6%	25 basis points	Yearly, Half-Yearly, Quarterly	5
Peter Hardy	1839	100	0.25% - 8%	25 basis points from 0.25% to 5%, 100 basis points from 5% to 8%	Yearly	4
David Jones	1843	100	2% - 10%	50 basis points from 2% to 5%, 100 basis points afterwards	Yearly	6
Thomas George Rance	1852	100	0.25% - 10%	25 basis points	Yearly	7
Andrew Hugh Turnbull (1)	1863	80	3% - 6%	50 basis points from 3 to 5, 100 basis points from 5 to 6	Yearly	7
Andrew Hugh Turnbull (2)	1863	40	3% - 5.5%	25 basis points from 3% to 5%, 50 basis points from 5% to 5.5%	Half-Yearly	7
Lieutenant-Colonel W. H. Oakes	1877	100	0.75% - 10%	125 basis points	Yearly	5