When Tariffs Disrupt Global Supply Chains*

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Abstract

We study unanticipated tariffs on imports of intermediate goods in a setting with firm-to-firm supply relationships. Firms that produce differentiated products conduct costly searches for potential input suppliers and negotiate bilateral prices with those that pass a reservation level of match productivity. Global supply chains are formed in anticipation of free trade. Once they are in place, the home government surprises with an input tariff. This can lead to renegotiation with initial suppliers or new search for replacements. We identify circumstances in which renegotiation generates improvement or deterioration in the terms of trade. The welfare implications of a tariff are ambiguous in this second-best setting, but plausible parameter values suggest a welfare loss that rises rapidly at high tariff rates.

Keywords: global supply chains, global value chains, input tariffs, imported intermediate goods

JEL Classification: F13, F12

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1 Introduction

Intermediate inputs now comprise as much as two thirds of world trade (Johnson and Noguera, 2012). Although firms purchase some of these inputs on anonymous international markets, many transactions take place within global supply chains. The 2020 World Development Report highlights the distinctive features of such supply chains. They derive from technological advances that make feasible the *fragmentation* of production processes. They impose non-trivial *search costs* on participants, as downstream firms hunt for suitable suppliers and upstream firms seek customers. They require *matching* of compatible partners to ensure productive exchanges. They often are governed by *incomplete contracts* that give rise to frequent *renegotiation*. And yet they typically involve *durable* relationships, because the sunk nature of search and customization costs impart "stickiness" to the pairings.[1]

A burgeoning literature examines firms' participation in global supply chains, the geography of international sourcing, the implications of these arrangements for productivity and market structure, and the persistence and economic significance of firm-to-firm networks.[2] Yet with just a few exceptions (that we discuss below), little attention has been paid to how trade policies might disrupt supply chains and with what implications for consumer prices and welfare. A prominent feature of the trade policy landscape pre-2018 readily explains this lacuna: in almost all countries, tariffs and non-tariff barriers were notably escalated, with much greater protection afforded to final goods than to intermediate inputs. Bown and Crowley (2018, p.20) report, for example, that MFN tariffs applied by G20 countries to imports of final goods are 70-75% higher than those levied on imports of intermediate goods. Using the tariff schedules published by the U.S. International Trade Commission and import data from the U.S. Census Bureau, we calculate that tariff protection applied to consumption goods in the United States during the period from 2010 to 2017 was more than four times as high as that applied to intermediate inputs. The average U.S. tariff applied to imports of intermediate goods was a miniscule 0.9% percent in 2017.[3]

But history changed course with the policies introduced by the Trump administration beginning in 2018, especially those imposed as "special protection" against imports from China. By September 2018, new tariffs levied by the United States covered 82% of intermediate goods imported from China, but only 29% of final consumer goods and 38% of capital equipment (Bown, 2019a). Under the phase one trade deal between the two countries that went into effect in early 2020, 93% of intermediate goods from China continue to be subject to special tariffs (Bown, 2019b).

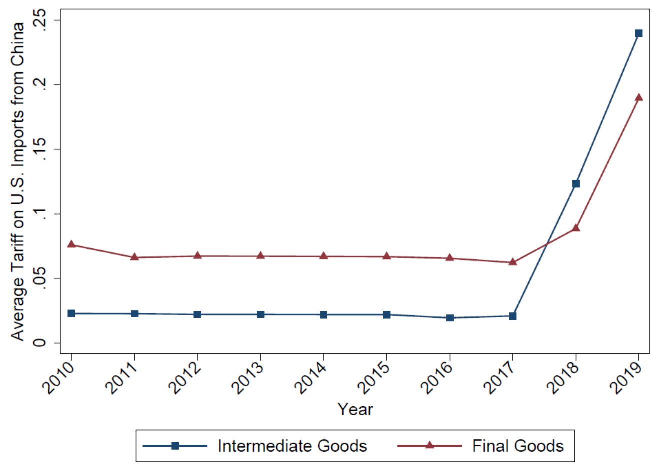
Figure 1 plots the average tariff rates applied by the United States to imports from China of intermediate goods and of final goods for the period from 2010 to 2019. To calculate these averages, we used the MFN tariff schedules reported annually by the U.S. International Trade Commission,

[1] See also Antàs (2020), upon which parts of the World Development Report are based.

[2] See, for example, Antràs and Helpman (2004), Grossman and Rossi-Hansberg (2008), Antràs and Chor (2013), Baldwin and Venables (2013), Halpern et al. (2015), Antràs et al. (2017), Bernard and Moxnes (2018), and many others.

[3] Excluding oil and petroleum products, the weighted average U.S. tariff on intermediate goods was still only 1.05% in 2017. See the appendix for details about the data and the averaging.

the tariff data collected by Fajgelbaum et al. (2020) for the early rounds of Trump tariffs, and the data assembled by Chad Bown for subsequent tariff hikes. We weighted HTS10 tariff rates by the value share of each category in total U.S. imports from China for the year. As the figure shows, tariffs on imports of intermediate goods from China rose dramatically with the introduction of the Trump tariffs from an average of less than three percent in 2017 to almost 24% by the end of 2019. Those on consumer goods increased as well, but by considerably less. The disruption of supply chains and the decoupling of integrated production processes were very much a part of the administration's intention with these aggressive policies. In fact, in August 2019, President Trump advised U.S. firms to "immediately start looking for an alternative to China" (Breuninger, 2019).



Note: Classification into end-use categories based on the United Nation's Classification by Broad Economic Categories (4th Revision). Weighted average tariffs use the value of imports from China in each HTS10 category in 2015 as weights. Further details in the appendix.

Figure 1: Average U.S. Tariffs on Imports from China

Anecdotes abound that reorganization of supply chains indeed took place in response to the large and unanticipated U.S. tariffs. The business press reported shifts in sourcing away from China toward Vietnam, Thailand, Indonesia, Malaysia, Cambodia, and others. Relocation of import supply allegedly was undertaken by companies such as Samsonite, Cisco Systems, Macy's, Ingersoll-Rand, and the Fossil Group, and in diverse industries such as electronics, furniture, hand luggage, and auto parts.[4]

Slightly less anecdotally, we can adapt the difference-in-difference methodology proposed by Amiti et al. (2019, 2020) in their investigations of the price and volume effects of the Trump tariffs to hunt for evidence of supply-chain reorganization in the U.S. trade data. To this end, we use monthly U.S. customs data for imports of intermediate goods at the HTS10-country-of-origin level for the period from January 2016 through October 2019. We regress the log of the value of imports from China and the log of the value of imports from a group of 13 Asian low-cost countries (LCC)

[4]See Master et al. (2018), Bloomberg News (2019), Huang (2019), Hufford and Tita (2019), Kawanami and Shiraishi (2019), Reed (2019), and Soon (2019).

on product fixed effects, month fixed effects, and the log difference between one plus the ad valorem tariff rate on imports from China and one plus the weighted-average tariff rate on imports from these other sources.[5] The results presented in Table 1 provide suggestive evidence of the relocation of supply chains from China to the other LCCs. Imports of intermediate inputs from China were significantly lower for goods that experienced large tariff hikes, and imports from the other Asian countries were correspondingly higher.

	Imports from China (1)	Imports from 13 LCCs (2)
Log Difference in Tariffs	-1.609**	0.441*
	(0.212)	(0.224)
R Squared	0.85	0.84
Obs	110132	110132

Note: Observations are at the HTS10-source-month level for the period January 2016 to October 2019 and include all products categorized as intermediate goods in the U.S. Census Bureau's 5-digit End-Use Code. Regressions include only products with positive imports from both sources and incorporate product and month fixed effects. We drop oil imports from our regressions due to the sensitivity of import values to fluctuations in oil prices. The dependent variable in column 1 is the log of the value of U.S. imports from China. The dependent variable in column 2 is the log of the value of combined U.S. imports from the 13 LCCs. The independent variable is the log difference between one plus the ad valorem tariff rate on imports from China and one plus the weighted-average ad valorem tariff rate on imports from the LCCs. The weighted average tariffs use the annual import values in 2017 as weights. Standard errors are clustered at the HTS8 level. Further details in the appendix.

* and ** indicate significance levels of $p < 0.05$ and $p < 0.01$, respectively.

Table 1: Imports of Intermediate Goods from China and 13 Low-Cost Countries

Motivated by these observations, we aim to study the effects of unanticipated input tariffs on sourcing and pricing in global supply chains. In Section 3, we develop a model of trade in intermediate inputs that captures many of the defining characteristics of supply chains mentioned in the 2020 World Development Report. Firms search for partners to form their chains. Search is costly. Matches vary in productivity. Relationships are governed by short-term contracts that can be renegotiated at any time. Sunk costs generate stickiness in relationships, but renewed search occurs in response to some shocks.

The model builds on Venables (1987). There are two sectors, one that produces a homogeneous good with labor alone and another that produces differentiated products. Firms enter the latter sector in anticipation of some initial trade policy, which we take to be one of free trade. Entrants produce unique varieties by combining labor and a composite intermediate input. The latter comprises a unit continuum of differentiated inputs in fixed proportions. Each producer can manufacture the set of inputs it needs using a backstop technology, but we focus on circumstances in which they prefer to engage input suppliers in a low-wage country. The firms pay search costs that deliver draws from a known distribution of productivities for each of the inputs they require.

[5] The thirteen LCCs include Bangladesh, Cambodia, Hong Kong, India, Indonesia, Malaysia, Pakistan, Philippines, Singapore, Sri Lanka, Taiwan, Thailand, and Vietnam. These are the countries identified by Kearney (2020), in addition to China, as "traditional offshoring trade partners," when calculating their annual Reshoring Index. See the appendix for more detail on the data sources and regression procedures that underlie Table 1.

Once they identify a potential supplier of an input, they learn the productivity of the pairing and decide whether to negotiate a renewable short-term contract or resume their search for a better match. When a match is acceptable, the buyer and supplier conduct Nash-in-Nash bargaining (i.e., pairwise Nash bargaining that takes other bargaining outcomes as given) that determines the set of input prices and thus the perceived marginal cost of the composite intermediate good. This and the wage rate govern the optimal production technique, which yields the minimum unit cost. Consumers demand the differentiated products with a love of variety and producers engage in markup pricing, as usual, under monopolistic competition with a constant elasticity of substitution between brands. The model determines the mass of varieties and the prices and quantities of each, along with the optimal search strategy and the negotiated input prices that reflect the extant trade policy and the match-specific productivities.

In Section 3, we consider the introduction of small tariffs that were not anticipated at the time of entry. By "small", we mean two things. First, the tariffs do not induce exit, considering that entry and search costs have already been sunk and firms need only cover their operating costs in order to remain active. Second, the tariffs do not alter the ideal destination for search should firms contemplate replacing any of their initial suppliers. For example, if the supply chains initially were located in China, then China remains the optimal place for search despite the added cost of the tariff. We show that the effects of the tariff depend on the elasticity of demand for the group of differentiated products. If demand for differentiated products is elastic, all initial supply relationships survive. However, suppliers renegotiate their short-term contracts in the shadow of the tariff. The renegotiations generate higher ex-factory prices for all inputs and an increase in the cost of intermediates that is proportionately greater than the tariff rate. The input price hikes reflect the deterioration of the bargaining position of downstream producers, who find their outside option of renewed search to be less attractive with the tariff in place. The rise in input prices represents a deterioration in the home country's terms of trade.

In contrast, when demand for differentiated products is inelastic, the tariffs create opportunities for profitable entry by new final producers. The tariffs raise input costs for potential entrants, but they also dampen competition from existing producers who face similarly higher costs, and the latter effect dominates when demand is inelastic. The new entrants seek and find suppliers of the inputs they need using the same search strategy that was optimal pre-tariffs. The original suppliers preserve their supply chains and find no pressures to renegotiate their contracts. Consumer prices rise, but the terms of trade are not affected.

With elastic demand, tariffs reduce output of differentiated products from the sub-optimally low levels caused by markup pricing. For this familiar reason, they impose a welfare cost on the home country. We also identify two new elements in the welfare calculus that are unique to a setting with global supply chains. First, the deterioration in the terms of trade resulting from renegotiation of supply contracts harms welfare. As we have just noted, the terms of trade respond to changes in the attractiveness of search, which determines the outside option for the downstream buyers. These terms-of-trade effects are present even though no new searches take place. Second, a distortion in the mix of factors used to produce differentiated products results from the bargaining process, and tariffs alleviate this distortion. All told, we cannot rule out the possibility of welfare-enhancing tariffs, but we provide sufficient conditions and plausible parameter values under which welfare falls. Importantly, our analysis implies that the usual welfare analysis of tariffs is bound to miss important channels when imports take place within supply chains.

A different welfare calculus applies when demand for differentiated products is inelastic. Then, the impact on the operating profits of initial producers is neutralized by entry, and new producers earn zero profits. Moreover, the terms of trade do not change. The welfare effect of input tariffs combines the adverse implications for consumer surplus with the positive tariff revenue collected by the government. We find that tariffs harm welfare when the buyers have most of the bargaining power in their procurement relationship, when differentiated products are relatively poor substitutes, and when match productivities are not widely dispersed.

Section 4 addresses larger tariffs, those that alter the ideal location for search. The new optimal search destination might be another low-wage country, if the tariff discriminates against the original suppliers but exempts other potential sources. The evidence suggests that this has happened in response to the recent U.S. tariffs on China, which apparently led companies to consider moving parts of their supply chains to other Asian countries. Alternatively, the new optimal search destination might be the home country, in which case final producers might choose to "reshore" some inputs initially sourced from abroad. Some effects of these larger tariffs are analogous to those for smaller tariffs, but new forces come into play. First, the implications for renegotiation with ongoing suppliers may be quite different. Whereas with small tariffs and elastic demand there is a positive relationship between the size of the tariff and the ex-factory price of inputs, the relationship is negative for a range of tariffs that are large enough to alter the optimal destination for search. With small tariffs, the incentive for renewed search in the original location diminishes with the size of the tariff, thereby tilting bargaining power in favor of suppliers. But with larger tariffs, the threat to search in a new location becomes ever more credible, tilting the renegotiation in the opposite direction. Consequently, we establish a non-monotonic relationship between the size of the tariff and the terms of trade. Second, tariffs that induce partial relocation of supply chains generate switches from low-cost to higher-cost sources. When the new destination for search is a different foreign country, this relocation amounts to Vinerian trade diversion, with the usual adverse implications for the terms of trade and welfare. The welfare implications are less severe when tariffs induce reshoring, because the profit losses suffered by downstream producers from higher input prices are offset by higher profits for upstream firms, which are also a component of home welfare in this case.

In a setting with global supply chains, the welfare analysis of large tariffs can be quite complex. Changes in output levels, factor mix, negotiated prices, and search costs all must be taken into account. Analytical results remain elusive. But numerical calculations with plausible parameter values suggest that 25 percent tariffs on inputs imported through global supply chains impose sizable welfare losses on the country that levies them and that the losses escalate rapidly at high tariff rates.

As we noted at the outset, our paper contributes to a small literature on the effects of tariffs that are applied to intermediate inputs and an even smaller literature that considers trade policy in the context of global supply chains. The earliest papers on input tariffs focused on effective rates of protection; see, for example, the various papers collected in Grubel and Johnson (1971). The effective rate of protection adjusts the nominal tariff on a final good for the cost of tariffs levied on the imported inputs used to produce that good. Ruffin (1969) and Casas (1973) study second-best tariffs on intermediate goods in small countries that protect their final producers, while Das (1983) considered optimal tariffs on intermediate and final goods in a large country, all in neoclassical settings with perfect competition and constant returns to scale. Blanchard et al. (2017) represents a more recent effort in this same vein. Using an approach that emphasizes the national origin of the value-added content of traded goods, they relate the structure of optimal protection to the sources of value added. Caliendo and Parro (2015) is a well-known paper that brings input tariffs and input-output linkages to quantitative modeling of multi-country trade so as to conduct welfare analysis of trade liberalization.

The papers most closely related to ours are by Ornelas and Turner (2008, 2012) and Antràs and Staiger (2012). These authors focus on the hold-up problems that arise when relationship-specific investments occur with incomplete contracts. Ornelas and Turner (2008) study bilateral relationships in which a foreign supplier must make a relationship-specific investment to sell an input to a downstream, home producer. Tariffs dampen the foreign firm's incentive to do so, thereby exacerbating the underinvestment problem that results from the incomplete contracting. The endogenous investment responses make trade flows more sensitive to trade policy than they would be with conventional, anonymous trade. In Ornelas and Turner (2012), in contrast, specialized inputs are provided by domestic suppliers, whereas imports offer a more generic alternative. In such a setting, tariffs reduce the attractiveness of the outside option to the downstream firm and thereby enhance incentives for relationship-specific investment by the domestic upstream firm. Tariffs on cheap but generic inputs can improve home welfare by mitigating the hold-up problem.

Antràs and Staiger (2012) study a setting with two small countries and a single, homogeneous good sold at a fixed world price. The producer of the final good is located in the home country, whereas the input supplier is located abroad. The input must be customized for the buyer, so that it has no value outside the relationship. Due to incomplete contracting, the terms of exchange are negotiated after the inputs have been customized and produced. In this setting, the authors identify the optimal input and output taxes and subsidies and the policies that result from non-cooperative policy setting in the two countries. Efficiency can be achieved by an input subsidy that resolves the hold-up problem together with free trade in the final good. But the governments have unilateral incentives to invoke sub-optimal policies, because the benefits of any subsidy paid by the home country are shared by firms in the foreign country. As in our model below, trade policy influences the bilateral negotiations between suppliers and buyers, and thereby impacts the terms of trade. But the focus on relationship-specific investments, as opposed to search, and the very different market environment, make the two papers complements rather than substitutes.[6]

A recent paper by Ornelas et. al (2020) examines the reorganization of supply chains induced by preferential trading arrangements. As in their earlier work, they focus on relationship-specific investment in a world of incomplete contracts. Like us, they consider discriminatory trade policies that can divert trade away from the lowest-cost sources. They allow for matching of buyers with heterogeneous suppliers, albeit in a frictionless setting that yields globally-efficient pairings and lacks any stickiness from sunk costs. Their welfare analysis has a second-best flavor similar to ours, although their inefficiencies arise from a different source, namely the insufficiency of investment owing to the hold-up problem. Interestingly, a preferential trade agreement might generate welfare gains in their setting even in the absence of any trade creation.

The remainder of our paper is organized as follows. Section 2 develops our model of global supply chains with costly search and negotiated input prices. Section 3 analyzes unanticipated tariffs that do not change the ideal local for supply chains. In Section 4 we study larger tariffs that render some exempted foreign country or the home market a better place for new searches. Section 5 concludes.

2 Foreign Sourcing with Search and Bargaining

In this section, we develop a simple model of global supply chains. Firms in a monopolistically competitive industry combine labor and a composite intermediate good to produce differentiated products. The intermediate good requires a continuum of inputs in fixed proportions. Each firm can produce any input it needs using a "backstop" technology or it can search for an external supplier of that input at home or in its choice of foreign markets. When a firm locates a supplier, it learns the productivity of the potential match. Then it can bargain with the supplier over a short-term (but renewable) contract, or it can choose to resume its search. Time is continuous and the interest rate is equal to the subjective discount rate.

In this section, we characterize an initial, long-run equilibrium. We assume that entry takes place in anticipation of free trade, although we could just as easily use any fixed tariff rate as the starting point. In succeeding sections, we introduce small and large tariff shocks and study how they impact the supply-chain relationships.

2.1 Preferences and Demands

We adapt a familiar model of monopolistic competition from Venables (1987). Consumers demand a homogeneous good and an array of differentiated products. Preferences are characterized by

$$\Omega(X, Y) = Y + U(X),$$

[6] In an appendix, Antràs and Staiger (2012) introduce search costs. But they focus on whether search yields a match or not, and optimal search determines how many buyers search in each of several foreign markets, not the intensity of search or the productivity of the resulting matches.

where $\Omega(X,Y)$ is the quasi-linear utility of the representative individual, Y is her consumption of the homogeneous good, and X is an index of consumption of differentiated varieties. We take the mass of consumers to measure one and the subutility $U(\cdot)$ to have constant elasticity, i.e.,

$$U(X) = \begin{cases} \frac{\varepsilon}{\varepsilon-1}\left(X^{\frac{\varepsilon-1}{\varepsilon}}-1\right) & \text{for } \varepsilon \neq 1 \\ \log X & \text{for } \varepsilon = 1 \end{cases}. \tag{1}$$

The consumption index takes the familiar form,

$$X = \left[\int_0^n x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \sigma > 1,$$

where $x(\omega)$ is consumption of variety ω, n is the measure of varieties available in the home country, and σ is the constant elasticity of substitution between any pair of brands. The corresponding real price index is

$$P = \left[\int_0^n p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}, \tag{2}$$

where $p(\omega)$ denotes the per-unit price of brand ω.

In order to focus most sharply on the supply chains, we assume that the differentiated final goods are not tradable; this allows us to ignore the determinants of foreign demand for home brands.[7] The representative home consumer purchases differentiated products up to the point where $U'(X) = P$ or $X = \mathcal{X}(P) = P^{-\varepsilon}$. Each individual demands variety ω as a function of its price and the aggregate price index according to

$$x[p(\omega), P] = \mathcal{X}(P)\left[\frac{p(\omega)}{P}\right]^{-\sigma}. \tag{3}$$

This is also the aggregate demand for variety ω, in view of the unit mass of consumers.

The demand for brand ω declines, of course, with own price. We want the demand for an individual brand to be increasing in the price index for competitor brands, so we henceforth assume that $\sigma > \varepsilon$.

2.2 Production

The homogeneous good is produced competitively with labor alone and is freely tradable. By choices of units and numeraire, one unit of good Y requires one unit of labor and bears a normalized price of one. This fixes the home wage rate at one in units of the homogeneous good.

Firms in the imperfectly-competitive sector produce unique varieties of the differentiated final good using labor, ℓ, and bundles of a composite intermediate good, m, subject to a constant-

[7] We could, alternatively, consider a home country that is small in the market for differentiated products, as in, for example, Demidova and Rodriguez-Clare (2009). They assume that the prices and variety of home products have no effect on either foreign expenditures on these products nor on the foreign price index. Introducing such fixed export demand would have little effect on our analysis.

returns-to scale production function $z(\ell, m)$. The composite intermediate good comprises a unit continuum of inputs indexed by j in fixed proportions, with one unit of each input needed for each unit of the composite.[8]

In the main text, we will often invoke a Cobb-Douglas form for the technology for producing final goods. Then we will refer to

Assumption 1 *The marginal cost of any differentiated product takes the form $c(\phi) = \phi^{\alpha}$, with $0 < \alpha < 1$.*

Here, ϕ represents the cost to the producer of a marginal unit of m. Clearly, $c(\phi) = \phi^{\alpha}$ is dual to a Cobb-Douglas production function with exponents $1 - \alpha$ and α on ℓ and m, respectively, when the wage rate is one.

In addition to variable costs, a firm producing any variety ω bears a one-time entry cost of F_e units of home labor, as well as a recurring fixed operating cost of f_o. Moreover, it bears a cost of finding partners for its global supply chain, which we describe in the next section.

2.3 Search

The creation of supply chains requires that producers locate suppliers. The cost of search can be an important component in the response to changes in trade policy. We suppose that firms can search for potential suppliers in one or more of several countries, $i \in \{1, \ldots, I\}$. One value of i represents the home country, so that producers of differentiated products might seek out domestic outsourcing relationships. With the symmetry that we impose across inputs, it is always optimal for a firm to search for all of its suppliers in a single country, although that target country might change following the imposition of a tariff. With free trade and the other assumptions described below, the optimal location for any supply chain is the country that has the lowest (efficiency-adjusted) wage. For now, we take the foreign country A to have the lowest wage, i.e., $w_A = \min\{w_1 \ldots, w_I\}$. All home producers conduct their searches in country A, so we describe the search process without reference to the i index and write w instead of w_A. However, once the home country introduces a tariff on inputs imported from country A, producers might seek out new suppliers at home or in some other country (if there is any) that is exempt from the tariff.

Search requires home labor. A firm ω seeking a supplier for input j can take a draw from a cumulative distribution $G(\cdot)$ at a capital cost of F. The realization of this draw, a, reveals the quality of the match between the producer and the particular supplier. Specifically, a potential supplier with match-specific (inverse) productivity a can produce a unit of input j for brand ω at a cost of aw. The firm producing ω decides whether to negotiate a short-term but renewable contract to buy input j from the potential supplier or whether to continue its search by taking another, independent draw from $G(\cdot)$ at an additional cost of F. For simplicity, we abstract from

[8] Inasmuch as the input suppliers must be identified through search and they provide match-specific productivity at a negotiated price, it is immaterial whether the inputs used by different final producers are physically the same or not, so long as all aspects of the search, matching and bargaining are symmetric across producers.

the time that may elapse between draws and assume, instead, that all search takes place in an instant. We assume that $g(a) \equiv G'(a) > 0$ for all $a \in (0,1]$ and $g(a) = 0$ for all $a > 1$. The firm producing brand ω also has access to an inferior but viable backstop technology for producing every input j that requires one unit of labor per unit of output. As we shall see, this option—that might be a fallback in case of a sequence of failed negotiations—proves to be irrelevant to the equilibrium outcome whenever supply chains form.

The optimal search strategy, as usual, involves a reservation stopping rule.[9] Let $\bar{a}$ be the reservation level, which the firms choose optimally. Then a firm takes another draw for the input j if and only if all of its prior draws for that input had inverse match productivities that exceed $\bar{a}$. Ultimately, all of a firm's suppliers will have inverse productivities in the range $[0, \bar{a}]$, with densities given by $g(a)/G(\bar{a})$. Given the continuum of inputs and the independence across them, the search process (plus bargaining) leads to a deterministic cost for a given quantity of the composite intermediate.

We can readily calculate the total cost of a firm's search effort, $S(\bar{a})$, as a function of the stringency of its stopping rule. When a firm takes its first draw, it pays F. Then, with probability $G(\bar{a})$ it achieves at least its reservation level of match productivity, in which case there are no further search costs. With the remaining probability, $1 - G(\bar{a})$, it encounters a supplier with $a > \bar{a}$, in which case it finds itself facing again a search cost of $S(\bar{a})$. It follows that $S(\bar{a}) = F + [1 - G(\bar{a})] S(\bar{a})$, or

$$S(\bar{a}) = \frac{F}{G(\bar{a})}.$$

This is the expected capital cost of search for any one input as well as the aggregate cost of search for the measure one of inputs in the bundle.

As with the cost function, it will prove useful to posit a convenient functional form for $G(\cdot)$. In the main text, we shall often make use of

Assumption 2 *The distribution function $G(a)$ takes the form $G(a) = a^{\theta}$, $\theta > 1$,*

where θ captures (inversely) the spread of productivities in this Pareto distribution.

2.4 Bargaining

In principle, a downstream firm might bargain with its suppliers over both prices and quantities. However, full efficiency would require a joint negotiation of quantities with all suppliers and this would be quite impractical with many of them. Instead, we invoke simultaneous but separate ("Nash-in-Nash") bargaining; i.e., each negotiation between a buyer and a potential supplier takes all other bargaining outcomes as given.[10] In our setting with a Leontief technology, this takes bargaining over quantities off the table; once a firm has decided to purchase m units of every input

[9] See, for example, Benkert et al. (2018) for proof that a reservation stopping rule is optimal in this environment.

[10] Note that a Stole-Zwiebel (1996) protocol would not yield different results in our setting, because with every input j essential to production, a failed negotiation would result in a potential supplier being replaced by another, with negligible impact on the other bargains.

from its many other suppliers, it has no use for any more than this amount from the individual supplier with whom it is bargaining, nor can it manage with less without wasting the purchase of other inputs. Inasmuch as the price of a single input has a negligible effect on the cost of the bundle, the buyer and each of its suppliers have no conflict over quantity given the outcome of the other negotiations. Instead, each pair takes m as given and the parties haggle only over price. We assume Nash bargaining with exogenous weights β for the buyer and $1-\beta$ for the seller and denote the agreed price per unit of an input produced with inverse productivity a by $\rho(a)$.[11]

The seller has no outside option. Therefore a seller with match productivity a earns a surplus from the relationship equal to the difference between its revenues $\rho(a)m$ and its production costs, wam, considering that the m units of the composite require m units of each of its components. The buyer, in contrast, has two options should the negotiation break down. It can produce input j using its backstop technology, with a labor coefficient of one and a wage of one. Or it can resume its search for an alternative supplier. Clearly, the latter option dominates, or else it would not have begun to search in the first place. Therefore, the outside option for the buyer is the expected cost of finding a new supplier plus the payment it would expect to make to that supplier. Continued search engenders an expected capital cost of $S(\bar{a})$, or a flow cost of $rS(\bar{a})$, where r is the constant interest rate, equal to the representative individual's subjective discount rate. The expected payment to an alternative supplier is $\mu_\rho(\bar{a})m$, where

$$\mu_\rho(\bar{a}) = \frac{1}{G(\bar{a})}\int_0^{\bar{a}} \rho(a)\,g(a)\,da$$

is the expected price of an input drawn randomly from the truncated distribution with domain $[0, \bar{a}]$. Thus,

$$\rho(a) = \arg\max_q (qm - wam)^{1-\beta}\left[\mu_\rho(\bar{a})m + rS(\bar{a}) - qm\right]^\beta.$$

The Nash bargaining solution implies

$$\rho(a) = \beta wa + (1-\beta)\,w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})} \tag{4}$$

and that

$$\mu_\rho(\bar{a}) = w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})}\ ,$$

where $\mu_a(\bar{a})$ is the conditional mean of a for $a \leq \bar{a}$ and $f \equiv rF$ is the debt service on the capital expenditure F. When the producer follows the same search strategy and bargaining process for all of its inputs, it pays $\mu_\rho(\bar{a})$ per unit for its composite intermediate good plus the fixed cost of search, $f/G(\bar{a})$. Thus, the total cost of m units of the intermediate good runs to $\left[w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})}\right]m+$

[11] Technically speaking, there exist many Nash-in-Nash equilibria, because once all other negotiations have generated a quantity of some $\tilde{m}$, an individual pair of buyer and supplier has every incentive to agree to this same quantity. Among the Nash-in-Nash equilibria, we focus on the one most preferred by the buyer, who is the only party engaged in multiple negotiations. This amounts to the same as allowing the buyer to specify the quantity of each input in advance of the individal, bilateral negotiations.

$f/G(\bar{a}) = w\mu_a(\bar{a})\, m + f/\beta G(\bar{a})$.[12] Note that each firm perceives a constant marginal cost of $\phi = w\mu_a(\bar{a})$ for each unit of the composite intermediate good.

2.5 Cost Minimization

To minimize cost, the firm chooses the optimal search strategy $\bar{a}$ for producing m units of the intermediate, and the optimal factor mix, m and ℓ, for producing x units of its brand. The factor mix minimizes $\ell + w\mu_a(\bar{a})\, m + f/\beta G(\bar{a})$, subject to $z(\ell, m) \geq x$. Notice that the third term in the minimand is independent of ℓ and m. Evidently, the firm perceives a fixed search cost (including the fact that the search costs weaken the buyer's bargaining position) of $f/\beta G(\bar{a})$ and a constant marginal cost of $c[1, w\mu_a(\bar{a})]$, where $c(\cdot)$ is the unit cost function dual to $z(\cdot)$. We shall henceforth suppress the first argument in $c(\cdot)$—which is the constant, unitary home wage—and write the unit cost more compactly as $c(\phi)$, where $\phi = w\mu_a(\bar{a})$ is the perceived marginal cost of a unit of m. Shephard's Lemma then gives us the factor demands, so that $m = xc'$ and $\ell = x(c - w\mu_a c')$.

Turning to the optimal search strategy, the total (flow) cost of m units of the composite intermediate comprises the aggregate payment to suppliers, $m\mu_\rho(\bar{a}) = mw\mu_a(\bar{a}) + (1-\beta) f/[\beta G(\bar{a})]$, and the debt service on the capital cost of search, $f/G(\bar{a})$. The tradeoff facing each firm is clear. On the one hand, a more exacting strategy generates a better average match productivity and thus a lower variable component in the payment to suppliers. On the other hand, a more stringent search strategy spells higher fixed costs of search and a larger fixed component in the payment to suppliers. Each firm chooses $\bar{a}$ to minimize the sum, i.e., $\bar{a} = \arg\min_a [mw\mu_a(a) + f/\beta G(a)]$. Then, if an interior solution exists, the first-order condition implies

$$mw\mu_a'(\bar{a}) = \frac{fg(\bar{a})}{\beta G(\bar{a})^2}. \tag{5}$$

Noting that $\mu_a'(\bar{a}) = g(\bar{a})[\bar{a} - \mu_a(\bar{a})]/G(\bar{a})$, and substituting (5) into (4), we can write the negotiated price of an input with inverse productivity a as

$$\rho(a) = \beta wa + (1-\beta)\, w\bar{a}\ , \tag{6}$$

a weighted average of the supplier's production cost and the cost of producing the input with the reservation match productivity.

We can gain further insight into the optimal search strategy by applying Assumption 2. In this case, $\mu_a(\bar{a}) = \frac{\theta}{\theta+1}\bar{a}$ and $g(\bar{a})/G(\bar{a})^2 = \theta/\bar{a}^{\theta+1}$. Then, the first-order condition can be written as

$$\bar{a}^{\theta+1} = \frac{f(\theta+1)}{\beta mw}.$$

Intuitively, the stopping rule is more tolerant (higher $\bar{a}$) when search draws are more costly or

[12] Inasmuch as the firm can produce the inputs in-house at a cost of m, outsourcing proceeds if and only if there exists an $\bar{a}$ for which $w\mu_a(\bar{a}) + \frac{1}{\beta}\frac{f}{mG(\bar{a})} < 1$.

the distribution of productivities is tighter. Search effort is greater (lower $\bar{a}$) when the foreign wage is higher, the scale of production is larger, or the buyers have more bargaining power; in these situations, the producers have more at stake in the search process. The greater is the search effort, the lower are the resulting transaction prices of all inputs, per (6). Of course, the scale of production and the demand for intermediates are endogenous in the full equilibrium, so the total effect of the parameters f, θ, β, and w must include the indirect effects that operate through m.

2.6 Profit Maximization and Monopolistically-Competitive Equilibrium

The firms in the differentiated-products sector face a constant elasticity of demand, per (3). They maximize profits, as usual, by charging a proportional markup over marginal cost,

$$p = \frac{\sigma}{\sigma - 1} c(\phi). \tag{7}$$

These prices yield operating profits of

$$\pi_o = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \mathcal{X}(P) P^\sigma c(\phi)^{1-\sigma} - \frac{(1-\beta) f}{\beta G(\bar{a})} - f_o. \tag{8}$$

The first term in (8) is the difference between revenues and variable costs when the marginal cost of production is $c(\phi)$, $\phi = w\mu_a(\bar{a})$ and firms practice the pricing rule in (7) subject to the demands in (3). The second term represents the sum of ongoing fixed payments to suppliers that result from the Nash bargains prescribed by (4). The last term in (8) is the recurring, fixed operating cost.

In a symmetric equilibrium, all firms charge the same price, p. Then (2) implies

$$P = n^{-\frac{1}{\sigma-1}} p.$$

As usual, the index increases linearly with the price of a typical brand, but decreases with the number of brands. This reflects the "love of variety" inherent in the Dixit-Stiglitz formulation.

Finally, in a monopolistically-competitive equilibrium with free entry, the present value of operating profits matches the fixed costs of entry and of search, or

$$\pi_o = f_e + \frac{f}{G(\bar{a})} \ ,$$

where $f_e = rF_e$ denotes the debt service on the one-time entry cost and $f/G(\bar{a})$ represents the debt service on the sunk search costs. Together, the model determines n, x, and p, as in the original Venables (1987) setting, as well as the demand for intermediates per brand, m, and the search intensity, $\bar{a}$, that result from the formation of supply chains. The equilibrium described in this section will serve as the initial condition when we study unanticipated tariffs in Sections 3 and 4 below.

2.7 Properties of the Initial Equilibrium

To elucidate some of the properties of the free-trade equilibrium, we invoke Assumptions 1 and 2 that posit common and convenient functional forms for the production function and the distribution of match productivities.

First, we examine the conditions for an interior optimal stopping rule in the low-wage country; i.e., when is $0 < \bar{a} < 1$? For this, we need the second-order condition also to be satisfied at the $\bar{a}$ that satisfies (5) and we need the solution for $\bar{a}$ to be less than one when m takes on its equilibrium value. In the appendix, we prove that the second-order condition is satisfied at $\bar{a}$ under Assumptions 1 and 2 if and only if $\theta > \alpha(\sigma - 1)$. This condition is more likely to be satisfied if the dispersion of productivities is relatively low (θ high), if output is relatively unresponsive to the volume of intermediates (α low) and if the differentiated varieties are relatively poor substitutes for one another. Otherwise, costs may be monotonically increasing with $\bar{a}$ and it may be optimal to search indefinitely despite the prohibitive fixed cost of doing so, because operating profits rise even faster than fixed costs as production costs go to zero. To abstract from such an unrealistic situation, we label for future reference

Assumption 3 *When the production function satisfies Assumption 1 and the productivity distribution satisfies Assumption 2,* $\theta > \alpha(\sigma - 1)$.

However, as we also show in the appendix, when the second-order condition is satisfied, a higher value of θ generates a greater cutoff, $\bar{a}$. This makes intuitive sense, inasmuch as a less dispersed distribution of productivities implies a smaller return to search. For θ sufficiently large, firms take only a single draw from $G(a)$ and accept any outcome; i.e., $\bar{a} = 1$. An interior value for $\bar{a}$ thus requires that θ should be neither too small nor too large.

Under Assumptions 1 and 2, we can solve explicitly for $\bar{a}$. We find

$$\bar{a}^{\theta} = \frac{f}{f_o + f_e} \frac{\theta - \alpha(\sigma - 1)}{\beta\alpha(\sigma - 1)}. \tag{9}$$

The right-hand side of (9) is positive under Assumption 3. It is less than one if the cost of search is not too large compared to the one-time cost of entry and the fixed cost of operation and if the buyers' bargaining power is not too low. We henceforth assume parameter values that ensure $\bar{a} < 1$.

Using the value of $\bar{a}$ in (9), we can solve for the price index P and the number of varieties n from a system of two equations; see the appendix. As in other models of monopolistic competition, variety is greater and the price index of differentiated products is lower when the one-time cost of entry and the fixed cost of operation are small. A lower value of the price index P corresponds to a higher level of welfare. As for the search costs, a lower value of f also implies a lower equilibrium price index and greater welfare. The equilibrium number of firms may increase or decrease with f, according to whether $\varepsilon < 1$ or $\varepsilon > 1$.

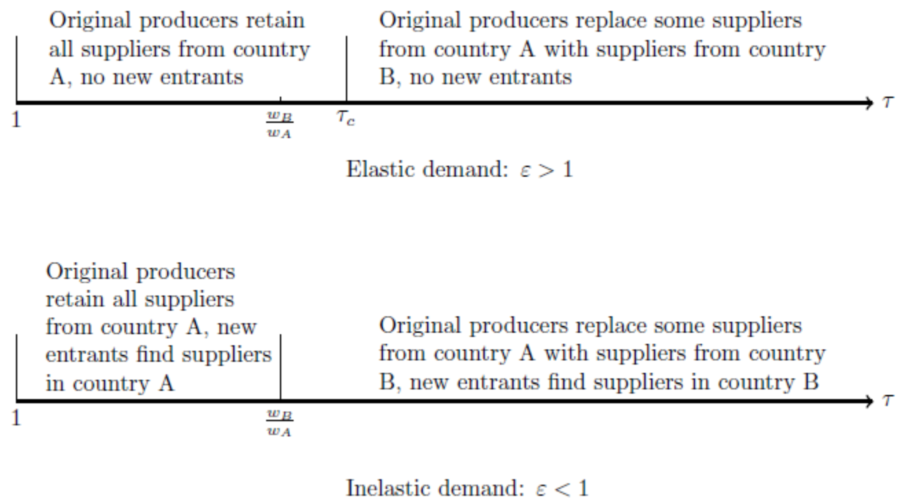


Figure 2: Sourcing Patterns

2.8 Departing from Free Trade

We are now ready to introduce tariffs on imported inputs. We will study tariffs that come as a surprise to downstream producers who have already formed their supply chains. Once the tariffs have been implemented, firms expect them to persist indefinitely. Let τ denote one plus the *ad valorem* tariff rate. We assume that τ is not so large as to induce exit by any of the original producers. These firms have already borne the sunk costs of entry and search, so they need only cover their fixed and variable operating costs to remain active. Since $\pi_o = f_e + f/G(\bar{a}) > 0$ in the initial equilibrium, there is room for input costs to rise without their causing exit.

We distinguish two sizes of tariffs. In the next section, we consider "small" tariffs that do not alter the preferred location for search by any suppliers, while in Section 4 we study "large" tariffs. Under free trade, the price distribution of inputs sourced from country A dominates that for any other country. However, a tariff raises the potential cost of importing from country A. If there happens to exist another country B that is exempt from the tariff (including the home country) such that $w_B < \tau w_A$, then country B takes over as the destination for all prospective searches once the tariff is put in place. Thus, small tariffs are those that satisfy $\tau < w_B/w_A$ whereas large tariffs are those such that $\tau > w_B/w_A$, for some country B that is not subject to the tariff.

As a guide to what follows, it is helpful to consider Figure 2, in which we show various sizes of the tariff and further distinguish final goods facing elastic demand ($\varepsilon > 1$) from those facing inelastic demand ($\varepsilon < 1$). In either case, the demarcation between small and large tariffs comes at $\tau = w_B/w_A$. When demand is elastic, we will find that a small tariff reduces operating profits for all actual and potential final-good producers, so that no new entry occurs. Moreover, the initial producers see a diminished incentive for search relative to free trade, so none resume their searching. For large tariffs, the ideal destination for potential new searches shifts from country

A to some country B. When the tariff rate is only slightly greater than w_B/w_A, no new searches actually take place. For still larger tariffs in excess of some τ_c, renegotiation takes place in enduring relationships, but producers also replace their least-productive initial suppliers.

The case of inelastic demand is rather different. Then incentives for search intensify given n, and potential profits rise. The rise in potential profits induces entry by new producers of differentiated products. The stiffer competition chokes off incentives for more stringent search. In the equilibrium with a small tariff, the original producers maintain their relationships with all of their initial suppliers at the initial terms of trade. The new entrants find their suppliers in country A and negotiate prices similar to those paid by the original producers. Large tariffs also induce entry. Then the original producers replace their least productive suppliers in country A with new suppliers in country B. New entrants form their supply chains entirely in country B. We will elaborate all of these claims in what follows.

3 Small, Unanticipated Tariffs

We begin by examining the renegotiation that takes place in enduring relationships under the shadow of a small tariff. We then address the decision by original producers to engage in renewed search to replace relatively unproductive suppliers and the decision by potential entrants to bear the fixed cost of entry and of establishing a supply chain. Later in this section, we study the effects of small tariffs on input prices, output prices, and welfare.

3.1 Renegotiation in Enduring Relationships

A small tariff alters the parties' bargaining positions in two ways. First, it imposes a direct burden that must be borne by one or both parties. Second, it alters a producer's optimal search strategy and thereby its outside options. If the initially agreed price does not exactly balance these new considerations, then renegotiation generates new terms.

Let $\rho(a,\tau)$ denote the renegotiated price that a producer pays to its supplier of some input j when the inverse match productivity is a and the *ad valorem* tariff rate is $\tau-1$. Upon importing the input, the producer incurs a customs charge of $(\tau-1)\,\rho(a,\tau)$. The outside option for the producer is to conduct a new search in country A—with optimal stopping rule $\bar{a}(\tau)$—and to pay an expected tariff-inclusive price to a new supplier of $\tau\mu_\rho[\bar{a}(\tau),\tau]$, where $\mu_\rho[\bar{a}(\tau),\tau]$ is the mean of $\rho(a,\tau)$ conditional on $a\le\bar{a}(\tau)$. The producer's net benefit from remaining with its original supplier amounts to $\tau\mu_\rho[\bar{a}(\tau),\tau]\,m(\tau)+f/G[\bar{a}(\tau)]-\tau\rho(a,\tau)\,m(\tau)$, where $m(\tau)$ is the quantity of the composite intermediate good that the firm assembles with the tariff in place. For the supplier, the surplus is simply the difference between revenue and production cost, or $[\rho(a,\tau)-wa]\,m(\tau)$, as before. Therefore, renewed Nash bargaining yields

$$\rho(a,\tau)=\arg\max_q\left[\tau\mu_\rho[\bar{a}(\tau),\tau]+\frac{f}{m(\tau)\,G[\bar{a}(\tau)]}-\tau q\right]^\beta(q-wa)^{1-\beta}$$

which implies that

$$\rho(a,\tau) = \beta wa + (1-\beta)\, w\mu_a[\bar{a}(\tau)] + \frac{1-\beta}{\beta}\frac{f}{\tau m(\tau)\, G[\bar{a}(\tau)]} \tag{10}$$

and

$$\mu_\rho[\bar{a}(\tau),\tau] = w\mu_a[\bar{a}(\tau)] + \frac{1-\beta}{\beta}\frac{f}{\tau m(\tau)\, G[\bar{a}(\tau)]}.$$

We can find the optimal search strategy as before. A firm that conducts new searches after the small tariff has been introduced will choose $\bar{a}(\tau)$ to minimize $\tau m(\tau)\, \mu_\rho[\bar{a}(\tau),\tau] + f/G[\bar{a}(\tau)]$, the sum of procurement costs and the debt burden imposed by search costs. The new first-order condition becomes

$$\tau m(\tau)\, w\mu_a'[\bar{a}(\tau)] = \frac{fg[\bar{a}(\tau)]}{\beta G[\bar{a}(\tau)]^2} \tag{11}$$

which, after rearranging terms, can be written as

$$w\{\bar{a}(\tau) - \mu_a[\bar{a}(\tau)]\}\, G[\bar{a}(\tau)] = \frac{f}{\beta\tau m(\tau)}. \tag{12}$$

Note that left-hand side of (12) is increasing in $\bar{a}(\tau)$; the derivative is $G[\bar{a}(t)] > 0$. It follows that $\bar{a}(\tau) > \bar{a}$ if and only if $\tau m(\tau) < m$; more on the conditions for this below.

Now we can substitute (12) into (10) to derive

$$\rho(a,\tau) = \beta wa + (1-\beta)\, w\bar{a}(\tau). \tag{13}$$

Evidently, if $\beta < 1$, all input prices rise in enduring relationships if $\bar{a}(\tau) > \bar{a}$ and all prices fall if $\bar{a}(\tau) < \bar{a}$. Only if bargaining power rests entirely with the buyer are the negotiated prices immune to changes in the outside option. Adjustments in the negotiated prices amount to changes in the terms of trade, much as in Antràs and Staiger (2010) and Ornelas and Turner (2012).

3.2 Replacing Unproductive Suppliers

In response to a tariff, producers might choose to end some of their supply relationships and recommence search for better matches. If so, they will terminate the relationships that had the worst initial match productivities. With this strategy in mind, we denote by a_c the inverse productivity of the marginal match, so that producers retain their supply relationships for all inputs with $a \in [0, a_c]$, while replacing suppliers with $a \in (a_c, \bar{a}]$. Of course, if $a_c = \bar{a}$, firms preserve their original global supply chains in their entirety.

As we noted above, there are two possibilities for the new, optimal search strategy should a firm choose to re-engage in search. First, $\bar{a}(\tau)$ might be (weakly) greater than $\bar{a}$, as it will be if $\tau m(\tau) \leq m$. Alternatively, $\bar{a}(\tau)$ might be smaller than $\bar{a}$, as it will be if $\tau m(\tau) > m$. In the first scenario, all existing supply relationships already meet or surpass the reservation level of match productivity; there is nothing to be gained by resuming search for any of them. In the second scenario, there exists a set of inputs for which $a \in (a_c, \bar{a}]$. For all of these, the firms opt to renew searches until they achieve match productivities at least as good as $\bar{a}(\tau)$. In short, each producer minimizes the cost of procuring $m(\tau)$ units of every input by setting $a_c = \min\{\bar{a}(\tau), \bar{a}\}$.

To identify circumstances in which supply chains are disturbed by the introduction of a small tariff, we must examine whether $\bar{a}(\tau)$ is ever strictly less than $\bar{a}$. To this end, we consider the marginal cost of a composite intermediate good in the tariff equilibrium. For the fraction of inputs $G(a_c)/G(\bar{a})$, the producers retain their initial suppliers. For these inputs, they perceive an average marginal cost of $\beta\tau w\mu_a(a_c) + (1-\beta)\tau w\mu_a[\bar{a}(\tau)]$, according to (10). For the remaining inputs (if any), they perceive an average marginal cost of $\tau w\mu_a[\bar{a}(\tau)]$. The weighted average gives the marginal cost of m that firms use in making their decisions about production techniques and consumer prices, which we denote by $\phi(\tau)$. After collecting terms, we have[13]

$$\phi^\tau = \beta\frac{G(a_c)}{G(\bar{a})}\tau w\mu_a(a_c) + \left[1-\beta\frac{G(a_c)}{G(\bar{a})}\right]\tau w\mu_a(\bar{a}^\tau) \tag{14}$$

and then optimal pricing implies

$$p^\tau = \frac{\sigma}{\sigma-1}c(\phi^\tau). \tag{15}$$

In Figure 3, the kinked curve labeled MM depicts the relationship between ϕ^τ and $\bar{a}^\tau$ implied by (14) for a particular value of τ, when $a_c = \min\{\bar{a}^\tau, \bar{a}\}$ and demand for differentiated products is elastic ($\varepsilon > 1$). We illustrate for the case of a Pareto distribution, namely

$$\phi^\tau = \begin{cases} \frac{\theta}{\theta+1}\tau w\bar{a}^\tau & \text{for } \bar{a}^\tau < \bar{a} \\ \beta\frac{\theta}{\theta+1}\tau w\bar{a} + (1-\beta)\frac{\theta}{\theta+1}\tau w\bar{a}^\tau & \text{for } \bar{a}^\tau \geq \bar{a} \end{cases}. \tag{16}$$

Here, we have drawn the curve associated with $\tau = 1$ (i.e., a tariff rate of zero). Evidently, the MM curve is piecewise linear with a kink at $\bar{a}$.

We can derive a second relationship between ϕ^τ and $\bar{a}^\tau$ by using the first-order condition for $\bar{a}^\tau$ in (12), the first-order condition for $m^\tau = x^\tau c'(\phi^\tau)$, the expression for demand for variety ω in (3), and the expression for the price index, $P^\tau = p^\tau(n^\tau)^{-1/(\sigma-1)}$. Combining these equations, using $c'(\tau) = \alpha(\phi^\tau)^{\alpha-1}$ and $p^\tau = \frac{\sigma}{\sigma-1}(\phi^\tau)^\alpha$, and hypothesizing that there is no induced entry of final producers (i.e., $n^\tau = n$), we have under Assumptions 1 and 2,

$$\frac{(\theta+1)f}{w\beta(\bar{a}^\tau)^{\theta+1}} = \tau n^{-\frac{\sigma-\varepsilon}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon}\alpha(\phi^\tau)^{\alpha(1-\varepsilon)-1}, \tag{17}$$

which we have depicted by the curve NN in Figure 1. The left-hand side of (17) is a decreasing function of $\bar{a}^\tau$, while the right-hand side is a decreasing function of ϕ^τ. Thus, the NN curve is upward sloping. Under Assumptions 1 and 2, it has a constant elasticity of $(\theta+1)/[1-\alpha(1-\varepsilon)]$. For $\tau = 1$, the two curves intersect at $\bar{a}(1) = \bar{a}$ and $\phi(1) = [\theta/(\theta+1)]w\bar{a}$. When the second-order

[13] To reduce notational clutter, we will sometimes write the value of a variable y in the tariff equilibrium as y^τ. For example, $\phi^\tau = \phi(\tau)$ and $\bar{a}^\tau = \bar{a}(\tau)$.

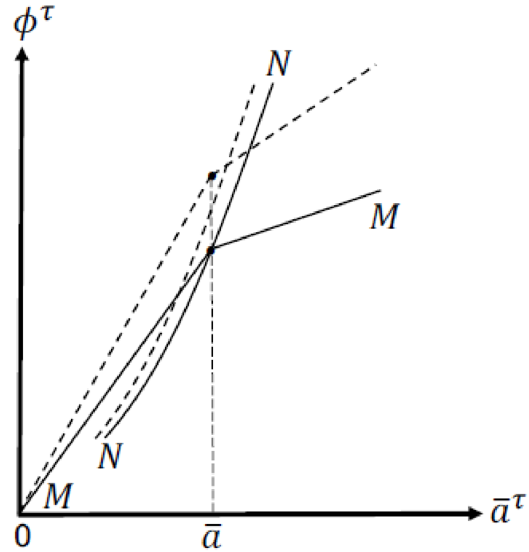


Figure 3: Small-Tariff Equilibrium with Elastic Demand for Differentiated Products

condition for $\bar{a}^\tau$ is satisfied, the slope of NN must be steeper than that of MM at the point of intersection, as drawn.[14]

Now suppose that a positive tariff is introduced, so that τ rises proportionately by $d\tau/\tau = \hat{\tau} > 0$ from an initial value of $\tau = 1$. The figure illustrates the resulting shift in the curves. The MM curve shifts upward at every point in proportion to $\hat{\tau}$, with a kink still at $\bar{a}$. The NN curve also shifts upward, but in proportion to $[1 + \alpha(\varepsilon - 1)]^{-1}\hat{\tau} < \hat{\tau}$. Therefore, the intersection of the new MM curve and the new NN curve must come to the right of the kink in the former, which implies that $\bar{a}^\tau > \bar{a}$. The stopping rule becomes less stringent in this case, because, as we have seen, the benefit from search is proportional to τm^τ, the stake that firms have in finding more productive matches. When demand for differentiated products is elastic, the derived demand for the intermediate good declines more than in proportion to τ, so τm^τ falls. The search effort follows the change in the marginal benefit from search. Operating profits fall, but remain positive for small enough τ. The fall in profits validates our hypothesis of no induced entry.

Now consider Figure 4, which depicts the case of inelastic demand ($\varepsilon < 1$). The solid MM and NN curves again intersect at $\bar{a}$, because they apply to $\tau = 1$. Once the tariff is introduced, the MM curve again shifts up in proportion to $\hat{\tau}$. Now, however, the upward shift in the NN curve is $[1 - \alpha(1 - \varepsilon)]^{-1}\hat{\tau} > \hat{\tau}$. So the new intersection of the dashed MM curve and the dashed NN curve falls to the left of the kink in the former, which suggests an enhanced incentive to search and a more stringent stopping rule than $\bar{a}$.

However, this is not the end of the story. If the new search strategy were to invoke a stopping rule more stringent than $\bar{a}$—as at the intersection of the two dashed curves—the balance of power

[14] The elasticity of the NN curve at $\bar{a}$ is $(\theta + 1)/[1 - \alpha(1 - \varepsilon)]$, while that of the steeper branch of the MM curve is 1. But $(\theta + 1)/[1 - \alpha(1 - \varepsilon)] > 1$ when $\sigma > \varepsilon$ and Assumption 3 holds.

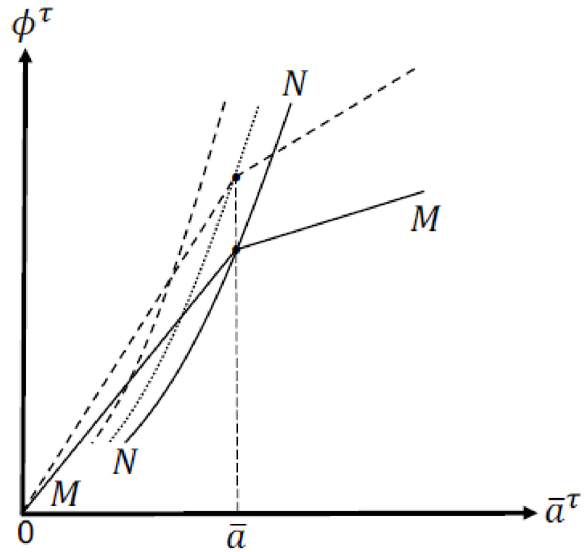


Figure 4: Small-Tariff Equilibrium with Inelastic Demand for Differentiated Products

in the price negotiations would tilt in favor of the buyers. The marginal cost of the composite intermediate would rise, but by proportionally less than $\hat{\tau}$, thanks to a fall in the ex-factory price.[15] Meanwhile, the price index P would rise, because marginal costs would be higher for all firms and they would pass along their increased costs to consumers. A potential entrant would face a higher cost of intermediates than ϕ, but would also experience greater demand for its product as a function of price than in free trade. As we show in the appendix, with $\varepsilon < 1$ and when $\theta > \alpha(\sigma - 1)$ per Assumption 3, the latter effect dominates.[16] In other words, the tariff equilibrium cannot remain at the intersection of the dashed MM and NN curves in Figure 4, because that point implies positive profit opportunities for potential entrants.

Entry pushes the NN curve downward (see (17)) until it reaches the place of the dotted curve. At the intersection of this curve and the dashed MM curve, $\bar{a}^\tau = \bar{a}$, $\phi^\tau = \tau\phi$, and $\tau m^\tau = m$.[17] The original producers see their stake in search restored to its free-trade level, so their outside option is to use the same search strategy as before. They do not replace any of their suppliers, nor do they re-negotiate any input prices. Their operating profits remain unchanged, as their higher costs are

[15] The less than proportionate rise in the marginal cost of inputs can be seen in Figure 2 from the fact that the MM curve shifts up in proportion to $\hat{\tau}$ and the NN curve shifts up by proportionally more. The fact that $\hat{\phi}^\tau > 0$ in this case follows from the fact that the leftward shift in MM is in proportion to $\hat{\tau}$, whereas the leftward shift in NN is in proportion to $\hat{\tau}/(\theta + 1) < \hat{\tau}$.

[16] A new entrant foresees potential profits of

$$\max_a \frac{(\sigma-1)^{(\sigma-1)}}{\sigma^\sigma} P(\tau)^{\sigma-\varepsilon} \left[\tau w \mu_a(a)\right]^{\alpha(1-\sigma)} - \frac{f}{\beta G(a)} - f_o - f_{e,}$$

while taking $P(\tau)$ as given. By the envelope theorem, potential profits are a rising function of τ if and only if $P(\tau)^{\sigma-\varepsilon}\tau^{\alpha(1-\sigma)}$ is a rising function of τ. In the appendix, we show that the second-order condition, $\theta > \alpha(\sigma-1)$, ensures that this is so.

[17] Note that entry proceeds until $(P^\tau)^{\sigma-\varepsilon}\tau^{-\alpha(\sigma-1)} = P^{\sigma-\varepsilon}$, which implies $n^\tau = n\tau^{\frac{\alpha(1-\varepsilon)(\sigma-1)}{\sigma-\varepsilon}}$.

offset by a higher, post-entry price index, P^{τ}, that boosts demand at any price. Meanwhile, the new entrants adopt the same search strategy as that previously used by the original producers. They achieve the same distribution of input prices, the same marginal cost of the composite intermediate, and an operating profit that just covers their fixed entry-plus-search costs. The greater is the tariff rate, the larger is the number of new entrants.

3.3 Effect of Small Tariffs on Input Prices, Output Prices, and Search

We have just seen that a tariff reduces the stringency of optimal search when demand for differentiated products is elastic, but ultimately leaves the search strategy unchanged when demand is inelastic. In either case, $a_c = \bar{a}$, which means that the original producers do not replace any of their initial suppliers. With elastic demand, the extant suppliers insist on renegotiating prices, which, according to (13), generates a price hike for all inputs in the new Nash bargains. With inelastic demand, by contrast, the optimal stopping rule post entry remains at $\bar{a}$, leaving outside options and negotiated prices as before. Entrants introduce new varieties in the latter case, but not the former.

We can use (11), (12), and (17) to calculate the effect of a small tariff on the marginal cost of the composite intermediate good. We find

$$\hat{\phi}^{\tau} = \left[\frac{\theta + 1 - \gamma^{\tau}}{\theta + 1 - \gamma^{\tau} - \gamma^{\tau}\alpha\left(\varepsilon - 1\right)}\right]\hat{\tau} \geq \hat{\tau}, \tag{18}$$

where $\gamma^{\tau} = \frac{(1-\beta)\bar{a}^{\tau}}{\beta\bar{a}+(1-\beta)\bar{a}^{\tau}}$ and thus $0 \leq \gamma^{\tau} \leq 1$. The average price paid to foreign suppliers can be computed using (10) and the fact that a is distributed on $[0, \bar{a}]$ according to the truncated distribution, $G(a)/G(\bar{a})$. This gives $\rho^{\tau} = \beta w \mu_a(\bar{a}) + (1-\beta) w \bar{a}^{\tau}$ or

$$d\rho^{\tau} = (1-\beta) w d\bar{a}^{\tau}.$$

Finally, markup pricing according to (15), the expression for the price index (2), and a fixed number of producers imply

$$\hat{p}^{\tau} = \alpha\hat{\phi}^{\tau} = \hat{P}^{\tau} = \left[\frac{\theta + 1 - \gamma^{\tau}}{\theta + 1 - \gamma^{\tau} - \gamma^{\tau}\alpha\left(\varepsilon - 1\right)}\right]\alpha\hat{\tau}\ . \tag{19}$$

In contrast, when demand for differentiated products is inelastic and entry takes place until profit opportunities are eliminated,

$$\hat{\phi}^{\tau} = \hat{\tau},$$

$$d\rho^{\tau} = 0,$$

$$\hat{p}^{\tau} = \alpha\hat{\tau},$$

and[18]

$$\hat{P} = \frac{\alpha(\sigma-1)}{\sigma-\varepsilon}\hat{\tau}. \tag{20}$$

We summarize in

Proposition 1 *Suppose Assumptions 1-3 hold. (i) If $\varepsilon > 1$, a small tariff generates no new searches and no entry or exit, but renegotiation with suppliers leads to higher input prices. Consumer prices rise and the price index rises. (ii) If $\varepsilon < 1$, a small tariff generates no new searches by the original producers and no changes in the f.o.b. prices they pay to their suppliers. Entry occurs and new producers adopt the same search strategies as the original producers. Consumer prices rise and the price index rises despite the increase in product variety.*

3.4 Welfare Effects of Small Tariffs

Welfare comprises total income (the sum of labor income, dividends paid by firms from their operating profits net of interest payments, and rebated tariff revenue) plus consumer surplus. We let $V(\tau) = \Pi(\tau) + T(\tau) + \Gamma(\tau)$ represent the sum of the three components of aggregate welfare that might vary with a small tariff, where $\Pi(\tau)$ denotes aggregate variable profits net of debt service on any new capital costs induced by the tariff τ, $T(\tau)$ denotes tariff revenue, and $\Gamma(\tau)$ represents the aggregate consumer surplus from purchases of differentiated products. In this section, we invoke Assumptions 1 and 2 to derive explicit expressions for each component of $V(\tau)$ and then calculate how aggregate welfare responds to a small but positive tariff in the presence of global supply chains. We consider separately the cases of elastic and inelastic demand for differentiated products inasmuch as the welfare calculus differs in these alternative scenarios.

3.4.1 Elastic Demand for Differentiated Products: $\varepsilon > 1$

When demand for differentiated products is elastic, operating profits for existing producers fall, firms undertake no novel searches and so bear no new capital costs, and no costly entry takes place. Aggregate variable profits net of debt service on new capital costs amount to $\Pi(\tau) = n(p^\tau x^\tau - \tau\rho^\tau m^\tau - \ell^\tau)$, the difference between revenues and input costs of active firms. The government collects and rebates tariff revenue of $T(\tau) = n(\tau-1)\rho^\tau m^\tau$ on the nm^τ units of imports by downstream producers at an average price of ρ^τ. Consumer surplus can be written as $\Gamma(\tau) = U(X^\tau) - np^\tau x^\tau$. Summing these components, we have

$$V(\tau) = U(X^\tau) - n\rho^\tau m^\tau - n\ell^\tau, \tag{21}$$

[18] The fact that $d\pi_o/d\tau = 0$ implies

$$(\sigma-\varepsilon)\hat{P}^\tau + \alpha(1-\sigma)\hat{\phi}^\tau = 0$$

or

$$\hat{P}^\tau = \frac{a(\sigma-1)}{\sigma-\varepsilon}\hat{\phi}^\tau.$$

the difference between aggregate utility from consuming differentiated products and the real resource cost of producing them.

Differentiating (21), we find

$$\frac{1}{n}\frac{dV^{\tau}}{d\tau} = \left(\frac{\sigma}{\sigma-1}-1\right)\frac{d\ell^{\tau}}{d\tau} + \left(\frac{\sigma}{\sigma-1}\phi^{\tau}-\rho^{\tau}\right)\frac{dm^{\tau}}{d\tau} - m^{\tau}\frac{d\rho^{\tau}}{d\tau}, \tag{22}$$

where we have used the fact that firms hire labor and purchase intermediate goods up to the point at which marginal revenue product of each factor equals its marginal cost. The first term on the right-hand side of (22) represents the net social benefit that results from a change in labor input in the differentiated-products sector. Since $\sigma/(\sigma-1)>1$, an increase in employment raises welfare, all else the same; the monopoly pricing of differentiated varieties drives a positive wedge between the marginal social product of labor and the market wage. In the Cobb-Douglas case, employment is proportional to aggregate spending on differentiated products, which falls when demand is elastic. The induced drop in employment contributes to a decline in aggregate welfare, much as in other settings with markup pricing.[19]

The second term represents the welfare effect of reduced purchases of intermediate goods. Here, there are offsetting considerations at work. On the one hand, $\sigma/(\sigma-1)>1$ suggests underutilization of intermediate goods, for much the same reason that market-generated employment is suboptimally low with markup pricing. This adverse effect of a tariff would be present even if inputs were purchased on anonymous markets. On the other hand, firms base their input demands on ϕ^{τ}, the perceived marginal cost of a unit of the composite intermediate good. But for τ close to one, $\phi^{\tau}<\rho^{\tau}$, where ρ^{τ} is the average amount actually paid to foreign suppliers for the inputs that comprise m. The excess of resource cost over perceived marginal cost suggests that firms might *overutilize* intermediate goods. A tariff that discourages input usage could actually contribute to higher welfare in this context, if all else remains constant. This novel effect of the tariff is specific to settings in which prices are negotiated bilaterally within multi-input supply chains.

How can we understand this potential benefit of a tariff on inputs purchased from supply chains? We recognize that, if buyers could negotiate collectively with all their suppliers at the same time, they would agree on a jointly-optimal choice of m and would share the gains from productive efficiency. But joint negotiations are impractical with large numbers of suppliers. Instead, we have assumed "Nash-in-Nash" bargaining whereby firms negotiate individually with each of their suppliers, taking the outcome of their other negotiations as given.[20] Buyers cannot discuss separately with each supplier the choice of m, because the technology requires that all inputs be used in fixed proportions. Instead, the buyer chooses m unilaterally and negotiates prices for this quantity of each input. In such circumstances, the downstream firm has an incentive to "overuse" intermediates in order to enhance its bargaining position vis-à-vis each of its suppliers. From (13) we see that the price falls with m; therefore, each buyer recognizes that it enjoys monopsony power through

[19] See, for example, Helpman and Krugman (1989, pp. 137-145) or Campolmi et al. (2018).

[20] For a discussion of the game-theoretic foundations of Nash-in-Nash bargaining, see Collard-Wexler et al. (2019).

bargaining and sets its input demands accordingly.

Note, however, that the government also has a tool to influence the negotiated prices. From (13), we see that the bargaining outcomes respond to τm^{τ}, the total stake that buyers have in their negotiations. By introducing a tariff, the government can tilt the bargaining in favor of home firms and alleviate their incentive to demand extra intermediate goods for that purpose. In other words, a tariff allows the home country to achieve a given ex-factory price at lesser resource cost. This positive effect of a small tariff is strongest when the home firms' bargaining position is weak (β small), as that creates the largest gap between ϕ^{τ} and ρ^{τ}. In fact, the potential efficiency-enhancing role of the tariff disappears entirely as β approaches one, because ϕ^{τ} and ρ^{τ} are approximately the same when $\tau = 1$ and β is close to one.

Finally, the third term on the right-hand side of (22) manifests yet another consideration that arises in supply chain relationships but is absent with arms-length purchase of intermediate goods. As in other settings with imperfect competition, trade policy redistributes profits from one party to the other.[21] Here, this works through the bilateral negotiations. As we have seen, any tariff that reduces τm^{τ} also dampens the incentives for search. But a less stringent stopping rule $\bar{a}^{\tau}$ carries with it a less imposing threat if a negotiation collapses, so a tariff tilts the table in favor of the suppliers. In short, any positive tariff delivers higher ex-factory prices for all inputs than under free trade, which imposes a terms-of-trade loss on the home country.

We can combine the three terms on the right-hand side of (22) to derive a necessary and sufficient condition for welfare to be declining in τ at $\tau = 1$. This requires some algebra, which we relegate to the appendix.[22] There, we prove

Proposition 2 *Suppose Assumptions 1-3 hold. If $\varepsilon > 1$, $dV/d\tau < 0$ locally at $t = 1$ if and only if*

$$\frac{\theta\varepsilon\left(\theta+\beta\right)}{\theta+\beta-\alpha\left(\varepsilon-1\right)\left(1-\beta\right)} > \left(1-\beta\right)\left(\sigma-1\right). \tag{23}$$

Clearly, (23) is satisfied if $\beta = 1$; indeed, if all bargaining power resides with the home producers, then any positive tariff reduces home welfare. The condition also is satisfied if $\theta/\left(\sigma-1\right) > \left(1-\beta\right)$, which is equivalent to $\left[\sigma/\left(\sigma-1\right)\right]\phi\left(1\right) > \rho\left(1\right)$; i.e., the middle term in (22) is negative when evaluated at $\tau = 1$. Another sufficient condition is $\alpha\varepsilon > \left(1-\beta\right)$.[23] Moreover, for parameter values typically found in the literature, the inequality is satisfied with slack.[24]

A point worth emphasizing, however, is that the usual welfare cost of an input tariff that reflects the underproduction of differentiated varieties in a setting of monopolistic competition is augmented

[21]See the seminal papers on the use of tariffs to extract monopoly rents by Katrak (1977) and Svedberg (1979), and subsequent work by Brander and Spencer (1984), Helpman and Krugman (1989), and many others.

[22]In the appendix, we also provide sufficient conditions for welfare to be declining in τ for all $\tau \geq 1$.

[23]Inequality (23) is equivalent to

$$\frac{\theta\varepsilon}{\sigma-1} > \left(1-\beta\right) - \frac{\alpha\left(\varepsilon-1\right)\left(1-\beta\right)^{2}}{\theta+\beta}$$

and Assumption 3 ensures that $\theta\varepsilon/\left(\sigma-1\right) > \alpha\varepsilon$.

[24]For example, if $\sigma = 5$, $\theta = 4$, $\varepsilon = 1.5$, and $\alpha = \beta = 1/2$, the left-hand side of (23) is equal to $216/35 \approx 6.17$, whereas the right-hand side is equal to 2.

by two additional considerations when producers create supply chains via costly search. First, a tariff alleviates misallocation associated with inefficient overuse of intermediates relative to labor in the production of final goods. This inefficiency results from a process of piecemeal negotiations with multiple suppliers. Second, a tariff worsens the terms of trade when producers negotiate with suppliers over input prices and resuming search becomes less attractive. The overall welfare cost may be larger or smaller than with competitive input markets and, under some unlikely conditions, a small tariff might even increase home welfare.

3.4.2 Inelastic Demand for Differentiated Products: $\varepsilon < 1$

When demand for differentiated products is inelastic, a small tariff induces entry by producers of new varieties. Then

$$V(\tau) = U(X^\tau) - n^\tau \rho^\tau m^\tau - n^\tau \ell^\tau - (n^\tau - n)\left[f_e + \frac{f}{G(\bar{a})} + f_o\right], \tag{24}$$

where (24) is just like (21), except that we have added a term capturing the debt service on fixed costs paid by new entrants. Differentiating (24), and noting that $d\rho^\tau = 0$ in this case, we find

$$\begin{aligned}\frac{1}{n^\tau}\frac{dV^\tau}{d\tau} = {} & \frac{1}{\sigma-1}\frac{d\ell^\tau}{d\tau} + \left(\frac{\sigma}{\sigma-1}\phi^\tau - \rho\right)\frac{dm^\tau}{d\tau} + \\ & \frac{1}{n^\tau}\left(\frac{\sigma}{\sigma-1}p^\tau x^\tau - \rho m^\tau - \ell^\tau - f_e - \frac{f}{G(\bar{a})} - f_o\right)\frac{dn^\tau}{d\tau}.\end{aligned} \tag{25}$$

Now, the fact that $\phi^\tau m^\tau$ is constant and $\ell^\tau = \alpha\phi^\tau m^\tau/(1-\alpha)$ imply that $d\ell^\tau/d\tau = 0$, so the first term in (25) vanishes; with no change in employment per firm, there is no welfare gain or loss from this source. The second term has the same interpretation as before, and since $dm^\tau/d\tau = -\left(1/\tau^2\right) < 0$, it contributes to a welfare gain or loss according to whether $\frac{\sigma}{\sigma-1}\phi^\tau$ exceeds or falls short of ρ. Again, this requires a comparison of the socially suboptimal use of intermediate inputs due to markup pricing versus the socially excessive use of intermediates due to the difference between the perceived marginal cost and the true social cost. Finally, the third term represents the welfare change generated by new entry. The zero-profit condition for new entrants implies $p^\tau x^\tau - \tau\rho m^\tau - \ell^\tau = f_e - f/G(\bar{a}) - f_o$, so the term in square brackets is equal to $\frac{1}{\sigma-1}p^\tau x^\tau + (\tau-1)\rho m^\tau$. This is always positive; the entry induced by a positive tariff contributes to aggregate welfare, because entrants do not capture all of the social gain from expanded variety.

We can compare the terms in (25) by expressing $V(\tau)$ in an equivalent, but somewhat different manner. The operating profits generated by entrants just cover the debt service on their entry and search costs and thus contribute nothing to national income. Meanwhile, the original producers see their operating profits restored to the initial levels, so there is no change in this component of income either. What remains is revenue generated by the tariff, $T(\tau) = (\tau-1)n^\tau\rho m^\tau$, and consumer surplus, $\Gamma(\tau) = U(X^\tau) - P^\tau X^\tau$. Using (1) and $X^\tau = (P^\tau)^{-\varepsilon}$, we can express $\Gamma(\tau) = \frac{\varepsilon}{1-\varepsilon} - \frac{1}{1-\varepsilon}(P^\tau)^{1-\varepsilon}$. Also, $n^\tau\phi^\tau m^\tau = \alpha\frac{\sigma-1}{\sigma}P^\tau X^\tau$ and $\phi^\tau m^\tau = \phi m$, so $T(\tau) = (\tau-1)\alpha\frac{\rho}{\phi}\frac{\sigma-1}{\sigma}(P^\tau)^{1-\varepsilon}$.

Thus, we can write

$$V(\tau) = \frac{\varepsilon}{1-\varepsilon} + \left[(\tau - 1)\,\alpha \frac{\rho}{\phi}\frac{\sigma-1}{\sigma} - \frac{1}{1-\varepsilon}\right](P^{\tau})^{1-\varepsilon} \quad . \tag{26}$$

Differentiating (26), using $\rho/\phi = (\theta + 1 - \beta)/\theta$, and evaluating at $\tau = 1$, we find

$$(P^{\tau})^{\varepsilon-1} \left.\frac{dV^{\tau}}{d\tau}\right|_{\tau=1} = \frac{\theta+1-\beta}{\theta}\frac{\alpha(\sigma-1)}{\sigma}\hat{\tau} - \hat{P}^{\tau}.$$

But (20) relates the increase in the price index generated by a tariff hike to the increase in the tariff rate, namely $\hat{P}^{\tau} = \frac{\alpha(\sigma-1)}{\sigma-\varepsilon}\hat{\tau}$. It follows that the introduction of an infinitessimal tariff reduces welfare if and only if $\theta > (1-\beta)\left(\frac{\sigma}{\varepsilon} - 1\right)$. In the appendix, we show that if welfare declines at $\tau = 1$, it also declines with the tariff rate for all $\tau > 1$. We then have

Proposition 3 *Suppose Assumptions 1-3 hold. If* $\varepsilon < 1$*,* $dV^{\tau}/d\tau < 0$ *for all* $\tau \geq 1$ *if*

$$\theta > (1-\beta)\left(\frac{\sigma}{\varepsilon} - 1\right).$$

Evidently, tariffs harm welfare when the downstream buyers hold most of the bargaining power in their procurement relationships, when differentiated products are relatively poor substitutes for one another, and when match productivities are not widely dispersed.

4 Larger, Unanticipated Tariffs

In the last section, we studied small tariffs. By small, we meant both that the tariff does not displace country A as the ideal location for producers' supply chains and that it does not induce exit from the industry. In this section, we consider larger tariffs, ones that make some other market the preferred place to search for suppliers. If the tariff is discriminatory and some other low-wage source is exempt, firms might relocate part of their supply chains to a different country. Or, if $\tau w_A > 1$ and there are no better foreign alternatives, firms might bring parts of their supply chains home. We attach the label B to the country that becomes the optimal destination for search once the tariff is introduced. We will consider both situations where B identifies a foreign country that is exempt from the tariff and where it represents the home country. In any case, we shall continue to assume that, despite the higher cost of inputs, all firms are able to cover their fixed operating costs and the debt service on new searches. Thus, the number of firms does not fall below that in the free-trade equilibrium.

We now must distinguish wages in the location of the original supply chains from those where new searches may take place. The fact that country A was the ideal destination for search before the tariff but country B becomes so afterward implies $w_A < w_B < w_A\tau$. Firms that conduct searches in country B draw match-specific (inverse) productivities from the distribution $G(\cdot)$, which is the same as for country A. We let b denote the realization of such a draw and $\bar{b}^{\tau} = \bar{b}(\tau)$ denote the optimal stopping rule in the tariff equilibrium, analogous to a and $\bar{a}^\tau$, respectively.[25]

When a large tariff is introduced, producers might nonetheless retain some of their most productive suppliers in country A, while replacing others that are less productive. All new search takes place in country B and bargaining occurs in the shadow of potential searches there. Let a_B be the inverse productivity of the marginal supplier that is retained after the tariff comes into effect, so that firms renegotiate with suppliers in country A that have $a \in (0, a_B]$ and replace their original suppliers that have match productivities $a \in (a_B, \bar{a}]$ with new partners in country B. Of course, it may be that $a_B = \bar{a}$, in which case there are no new searches.

We can calculate the optimal stopping rule as we have done before, to derive an equation that relates $\bar{b}^\tau$ to the derived demand for the composite intermediate good, analogous to that for $\bar{a}$ in (5); see the appendix for details. Then we substitute this first-order condition for $\bar{b}^\tau$ into the Nash bargaining solution to obtain negotiated prices for inputs imported from countries A and B, respectively, as functions of the inverse match productivities, a and b.[26] This gives

$$\rho_A(a,\tau) = \beta w_A a + (1-\beta)\frac{w_B \bar{b}^\tau}{\tau} \tag{27}$$

and

$$\rho_B(b,\tau) = \beta w_B b + (1-\beta) w_B \bar{b}^\tau. \tag{28}$$

These bargaining outcomes imply that tariff-inclusive prices, $\tau\rho_A(a,\tau)$ and $\rho_B(b,\tau)$, are weighted averages of the unit cost of production-cum-delivery and the unit cost of an input that could be produced by a supplier in country B with the reservation level of productivity. In this sense, (27) and (28) are analogous to (13). Moreover, these price equations imply that two inputs with the same unit cost of production-cum-delivery but different countries of origin carry the same delivered price. Notice that, if $w_B\bar{b}^\tau/\tau < w_A\bar{a}$, suppliers in country A bear some of the burden of the tariff.

Facing these potential input prices, producers can make their optimal sourcing decisions. By definition, the stopping rule identifies the worst match that a buyer would accept conditional on searching in country B and recognizing the costliness of further search. This worst match yields an opportunity to purchase an input at delivered price $\rho_B\left(\bar{b}^\tau,\tau\right) = w_B\bar{b}^\tau$. However, even before commencing a new search, the buyer has access to a supplier from whom it can buy at delivered price $\tau\rho_A(a,\tau) = \beta\tau w_A a + (1-\beta) w_B\bar{b}^\tau$ for a match with productivity a. If $\tau w_A a < w_B\bar{b}^\tau$, the

[25] We use $\bar{b}(\tau)$ to express the reservation level as a function of the tariff rate, and $\bar{b}^\tau$ to denote the value of $\bar{b}(\tau)$ in the cum-tariff equilibrium.

[26] The Nash bargain with a supplier in country A with inverse match productivity a yields a price

$$\rho(a,\tau) = \arg\max_q \left[w_B\mu_b\left(\bar{b}^\tau\right) + \frac{f}{\beta m(\tau) G\left(\bar{b}^\tau\right)} - \tau q\right]^\beta (q - w_A a)^{1-\beta}.$$

The Nash bargain with a supplier in country B with inverse match productivity b yields a price

$$\rho(b,\tau) = \arg\max_q \left[w_B\mu_b\left(\bar{b}^\tau\right) + \frac{f}{\beta m(\tau) G\left(\bar{b}^\tau\right)} - q\right]^\beta (q - w_B b)^{1-\beta}.$$

original supplier offers a better deal than the reservation match. Conversely, if $\tau w_A a > w_B \bar{b}^\tau$, search in country B yields a cost saving even if the firm realizes the worst possible match among those it will accept. It follows that $a_B = \min\left\{w_B \bar{b}^\tau / \tau w_A, \bar{a}\right\}$ and that producers retain suppliers with $a \leq \frac{w_B}{\tau w_A}\bar{b}^\tau$ while replacing those (if any) with $a > \frac{w_B}{\tau w_A}\bar{b}^\tau$.

We are ready to examine the equilibrium effects of larger tariffs, i.e., those with $\tau \geq w_B/w_A$. Again, we invoke Assumptions 1 and 2 and distinguish cases of elastic and inelastic demand. We use ϕ^τ, as before, to denote the tariff-inclusive marginal cost of the composite intermediate good for the original producers of final goods. Recall that these producers perceive a lower marginal cost of inputs than the average price that they pay for them, because they recognize that price per unit falls with the volume m^τ. For a fraction $G(a_B)/G(\bar{a})$ of inputs, the original producers continue to buy from their existing suppliers in country A and perceive an average marginal cost of $\beta \tau w \mu_a(a_B) + (1-\beta) w_B \mu_b\left(\bar{b}^\tau\right)$. For the remaining fraction $1 - G(a_B)/G(\bar{a})$ of inputs (if any), they source from country B and perceive an average marginal cost of $w_B \mu_b\left(\bar{b}^\tau\right)$. After collecting terms, the weighted average becomes

$$\phi^\tau = \beta \frac{G(a_B)}{G(\bar{a})} \tau w_A \mu_a(a_B) + \left[1 - \beta \frac{G(a_B)}{G(\bar{a})}\right] w_B \mu_b\left(\bar{b}^\tau\right) .$$

In Figures 5 and 6, the solid curve MM depicts the relationship between ϕ^τ and $\bar{b}^\tau$ for $\tau = w_B/w_A$. Under Assumption 2 of a Pareto distribution for match productivities, the curve is piecewise linear, with

$$\phi^\tau = \begin{cases} \frac{\theta}{\theta+1} w_B \bar{b}^\tau & \text{for } \bar{b}^\tau < \tau w_A \bar{a}/w_B \\ \frac{\theta}{\theta+1}\left[\beta \tau w_A \bar{a} + (1-\beta) w_B \bar{b}^\tau\right] & \text{for } \bar{b}^\tau > \tau w_A \bar{a}/w_B \end{cases} . \tag{29}$$

For $\bar{b}^\tau < \tau w_A \bar{a}/w_B$, it has a slope of $\frac{\theta}{\theta+1} w_B$, whereas for $\bar{b}^\tau > \tau w_A \bar{a}/w_B$, it has the shallower slope of $(1-\beta)\frac{\theta}{\theta+1} w_B$. With $\tau = w_B/w_A$, the curve kinks at $\bar{b}^\tau = \bar{a}$.

As before, we need a second relationship between ϕ^τ and $\bar{b}^\tau$ to locate the equilibrium. We begin with the case of elastic demand, as depicted in Figure 5. Recall that $n(w_B/w_A) = n$, because operating profits per firm are smaller when $\tau = w_B/w_A$ than when $\tau = 1$, and thus there is no entry beyond the free-entry level. We use the first-order condition for $m^\tau = x^\tau c'(\phi^\tau)$, the expression for the demand for variety ω in (3), and the expression for the price index, $P^\tau = p^\tau n^{-1/(\sigma-1)}$, much as we did in constructing the NN curve in Figure 3. Combining these equations, and applying Assumption 1 of a Cobb-Douglas technology and Assumption 2 of a Pareto distribution of match productivities, we find the new NN curve,

$$\frac{(\theta+1) f}{w_B \beta \left(\bar{b}^\tau\right)^{\theta+1}} = n^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha \left(\phi^\tau\right)^{\alpha(1-\varepsilon)-1} . \tag{30}$$

We have seen that the stopping rule with a large tariff $\tau = w_B/w_A$ is the same as the stopping rule with a small tariff of this size, and that both are less stringent than under free trade; i.e.,

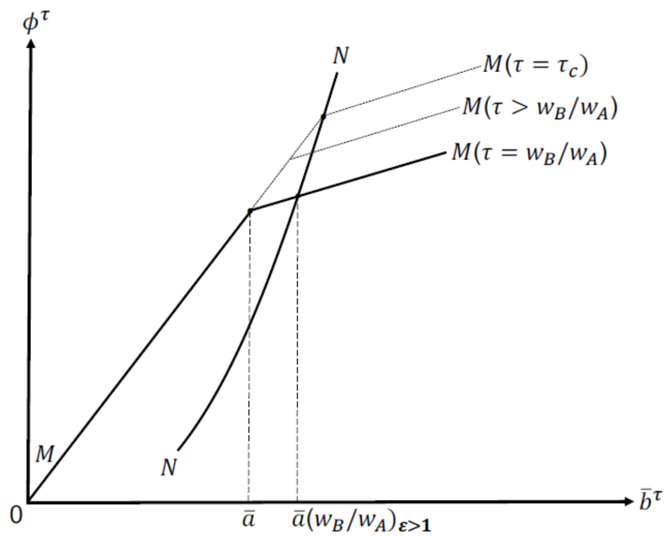


Figure 5: Large-Tariff Equilibrium with Elastic Demand

$\bar{b}(w_B/w_A) = \bar{a}(w_B/w_A) > \bar{a}$. It follows that the intersection of the MM curve and the new NN curve in Figure 5 takes place to the right of the kink in the former curve, as drawn. Now let τ be something larger than w_B/w_A. The tariff rate does not appear in (30), except insofar as it influences the variables on the axes or the number of active firms. But as we raise τ above w_B/w_A, the portion of the MM curve to the right of the kink shifts upward, as can be seen from (29). For τ somewhat greater than w_B/w_A, the equilibrium occurs at the intersection of NN and the lowermost dashed curve in the figure. Here, $\bar{b}^\tau > \bar{a}$, but $\tau w_A \bar{a} < w_B \bar{b}^\tau$, so the original producers preserve the entirety of their supply chains. The parties renegotiate the terms of their exchange against the new outside option of search in country B. Moreover, since operating profits are a declining function of τ in this range, no entry takes place.

For some still-higher tariff rate, the original producers of differentiated products are indifferent between relocating their worst matches to country B and continuing on with their original suppliers. This tariff, which we denote by τ_c in the figure, is defined implicitly by $\tau_c w_A \bar{a} = w_B \bar{b}(\tau_c)$. Tariffs larger than τ_c disrupt the supply chains. For $\tau \geq \tau_c$, $a_B = \frac{w_B}{\tau w_A}\bar{b}(\tau_c) = \tau_c \bar{a}/\tau$ and so $\phi^\tau = \frac{\theta}{\theta+1}\tau_c w_A \bar{a}$. Further tariff hikes do not generate any further shifts in the MM curve at the equilibrium point. Rather, the stopping rule remains $\bar{b}^\tau = \bar{b}(\tau_c)$ and a_B declines with the size of the tariff. In other words, the higher the tariff for $\tau > \tau_c$, the more extensive is the reorganization of the supply chain. In this range, operating profits remain constant but profits net of additional search costs fall.[27]

Figure 6 depicts the equilibrium for a large tariff when demand for differentiated products is inelastic. The curves are drawn for $\tau = w_B/w_A$. The MM curve is the same as in Figure 5, but the NN curve is somewhat different. Recall that input tariffs induce entry of new final producers when demand is inelastic. When $\tau = w_B/w_A$, all producers face the same distribution of prices in country

[27] In the appendix, we derive an explicit expression for τ_c, namely $\tau_c = (w_B/w_A)^{\frac{\theta}{\theta-\alpha(\varepsilon-1)}}$

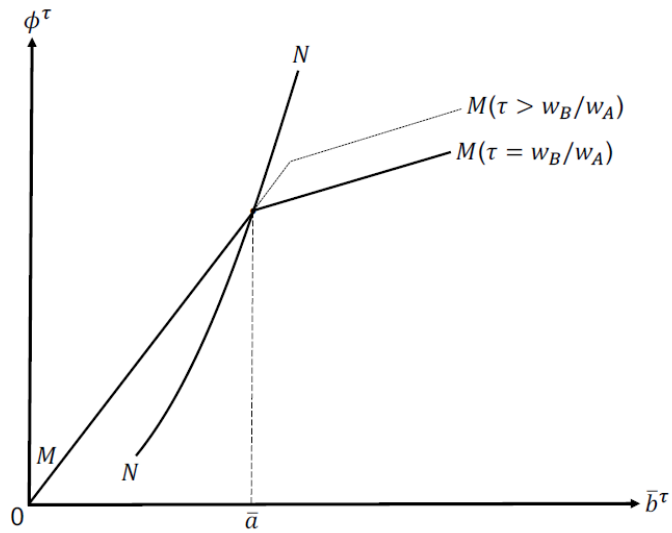


Figure 6: Large-Tariff Equilibrium with Inelastic Demand

A as in country B. It follows that they use the same stopping rule, i.e., $\bar{b}\,(w_B/w_A) = \bar{a}\,(w_B/w_A)$, but $\bar{a}\,(w_B/w_A) = \bar{a}$, so the new NN curve must intersect the MM curve at this point, as drawn. To find the shape of this curve, we first combine the zero-profit condition for new entrants and the first-order condition for their optimal search strategy to derive an expression for the price index; see the appendix. Then we use this value of P^τ for $\tau = w_B/w_A$ together with the first-order conditions for the choice of $\bar{b}^\tau$ and $\bar{m}^\tau$ by the original producers to derive the NN curve for this case,

$$\frac{(\theta+1)\,f}{w_B\beta\left(\bar{b}^\tau\right)^{\theta+1}} = \left(P^\tau\right)^{\sigma-\varepsilon}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\alpha\left(\phi^\tau\right)^{\alpha(1-\sigma)-1}\ . \tag{31}$$

Notice that the elasticity of this curve is $(\theta+1)\,/\,[1+\alpha\,(\sigma-1)] > 1$, so NN intersects the lower branch of MM from below.

Now suppose that τ exceeds w_B/w_A. The tariff rate does not appear separately in (31), because the price index P^τ that is consistent with zero profits for new entrants does not depend on the tariff. This in turn reflects the fact that the new entrants search only in country B, so their input costs are independent of the tariff rate. But with P^τ fixed for all values of $\tau > w_B/w_A$, so too is the location of the NN curve. Similarly, the left-most branch of MM is independent of τ. The right-most branch of MM shifts up, as in Figure 5, but this is irrelevant because the intersection of the two curves stays put at the point vertically above $\bar{a}$. In other words, the marginal costs for the original producers and their optimal search strategies are independent of the tariff rate for all values of $\tau > w_B/w_A$.

With inelastic demand, larger tariffs greater than $w_B/w_A - 1$ induce the original suppliers to replace ever larger portions of their supply chains. The inverse productivity of their marginal supplier in country A is $a_B = \frac{w_B}{\tau w_A}\bar{b}^\tau = \frac{w_B}{\tau w_A}\bar{a}$. The new entrants search for all their suppliers in country B, using the reservation inverse-productivity level $\bar{b}^\tau_{new} = \bar{a}$. Since the costs and demands facing the entrants are the same for all tariff levels, so too are their operating profits, and $n^\tau = n(w_B/w_A)$ for all $\tau \geq w_B/w_A$.

We recap the effects of larger tariffs on the number and organization of supply chains in

Proposition 4 *Suppose Assumptions 1-3 hold and that $\tau > w_B/w_A$ for some country B that is exempt from the tariff (possibly the home country). (i) For $\varepsilon > 1$, there is no new entry and the original producers preserve their entire supply chains in country A for all $\tau < \tau_c$ defined by $\tau_c w_A \bar{a} = w_B \bar{b}(\tau_c)$; for $\tau > \tau_c$, these producers retain their initial suppliers in country A for $a \leq \frac{\tau_c}{\tau}\bar{a}$, while replacing those with $\bar{a} \geq a > \frac{\tau_c}{\tau}\bar{a}$. The number of active firms is $n^\tau = n(1)$ for all $\tau > w_B/w_A$. (ii) For $\varepsilon < 1$, the original producers of final goods retain their suppliers in country A for $a \leq \frac{w_B}{\tau w_A}\bar{a}$, while replacing those with $\bar{a} \geq a > \frac{w_B}{\tau w_A}\bar{a}$ with suppliers in country B. The number of active firms is $n^\tau = n(w_B/w_A) > n(1)$ for all $\tau > w_B/w_A$ and the entrants source all of their inputs in country B.*

4.1 Effect of Larger Tariffs on Input Prices, Output Prices, and the Terms of Trade

In this section, we discuss the implications of larger tariffs for input prices, output prices, and the terms of trade. We begin with the case of elastic demand.

4.1.1 Larger Tariffs with Elastic Demand

For tariffs in the range $\tau \in [w_B/w_A, \tau_c]$, there is no entry of new brands. The original producers continue to procure all of their inputs in country A, paying the prices recorded in (27). We see here the offsetting forces at work on the negotiated price. On the one hand, a higher tariff directly raises the value of a buyer's outside option to search in a tariff-free location. On the other hand, a higher tariff means that buyers would have less incentive to search intensely in country B, were they to undertake such searches. In the appendix we show that $\bar{b}^\tau$ rises less than in proportion to τ, so $\bar{b}^\tau/\tau$ declines with τ. It follows that higher tariffs improve the buyers' bargaining position vis-à-vis all of their suppliers and so reduce net-of-tariff input prices. The average price becomes

$$\rho^\tau = \beta w_A \mu_a(\bar{a}) + (1-\beta)\frac{w_B \bar{b}^\tau}{\tau},$$

which is a declining function of τ. Inasmuch as all inputs continue to be sourced in country A, the fall in ρ^τ represents an improvement in the home country's terms of trade.

Next consider tariffs large enough to induce partial relocation of supply chains to country B. We have seen that search intensity is not affected by the size of the tariff in such circumstances; rather $\bar{b}^\tau = \bar{b}(\tau_c)$ for all $\tau > \tau_c$. Nonetheless, the terms of trade respond to two offsetting forces. From (27), we see that the prices of all inputs that continue to be imported from country A fall with the tariff, as the option to shift production to a tariff-free source strengthens the buyers' bargaining position. Meanwhile, parts of the supply chain move from a relatively low-cost source to one with

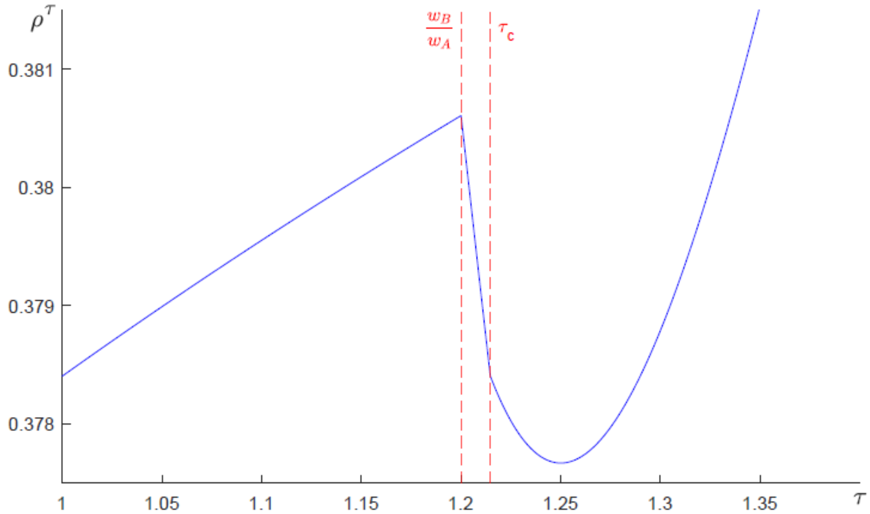


Figure 7: Terms of Trade for Different Tariff Rates

higher wages. When the best alternative to the original source is another foreign country, this amounts to Vinerian trade diversion, and it contributes to a deterioration in the overall terms of trade. We write the weighted average of inputs from the alternative sources as

$$\rho^{\tau} = \frac{G(a_B)}{G(\bar{a})}\left[\beta w_A \mu_a(a_B) + (1-\beta)\frac{w_B \bar{b}^{\tau}}{\tau}\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right]\left[\beta w_B \mu_b\left(\bar{b}^{\tau}\right) + (1-\beta) w_B \bar{b}^{\tau}\right] , \quad (32)$$

where $a_B = \frac{w_B}{\tau w_A}\bar{b}(\tau_c)$ in these circumstances. In the appendix we show that the fall in prices from country A outweighs the shift in production to the higher-cost country B if and only if $\tau < (\theta+1)/\theta$. If $\tau_c < (\theta+1)/\theta$, then there exists a range of tariffs above τ_c in which higher tariffs imply better terms of trade. Moreover, $\rho(t_c) = \rho$; i.e., at τ_c the terms of trade are the same as when $\tau = 1$.[28] So, when $\tau_c < (\theta+1)/\theta$, there also exists a range of tariffs for which the home country enjoys better terms of trade than with zero tariffs. For sufficiently high tariffs, however, most imports are sourced from country B, where ex-factory prices are higher than those in country A, so the terms of trade must be worse than those under free trade.

Figure 7 highlights the non-monotonic relationship between the size of the tariff and the home country's terms of trade when the best alternative to searching in the original location of supply chains is another foreign country and $\varepsilon > 1$.[29] The figure shows the entire range of positive tariffs, including those we have termed small and large. For $\tau < w_B/w_A$, country A remains the preferred location for search, and larger tariffs result in higher import prices as the buyers' outside option

[28] At τ_c, $a_B = \bar{a}$ and $\bar{b}^{\tau} = \tau_c w_A \bar{a}/w_B$. Therefore, (32) implies

$$\rho(\tau_c) = \beta w_A \mu_a(\bar{a}) + (1-\beta) w_A \bar{a} = \rho.$$

[29] The figure uses the same, "plausible" parameter values described in footnote 24, along with $w_A = 0.5$, $w_B = 0.6$, $f = 5$, and $f_e = f_o = 10$. However, the qualitative features of Figure 7 apply more generally.

deteriorates. For $w_B/w_A < \tau < \tau_c$, the best search option switches to country B. Although no new searches actually take place, the threat to do so becomes more credible for higher tariff rates, which shifts the bargaining outcomes in favor of the buyers. Finally, for $\tau > \tau_c$, higher tariffs further enhance the buyers' bargaining power vis-à-vis their original suppliers, but they also generate costly trade diversion that raises real input costs. The latter force must eventually dominate, although it need not do so for tariffs just above τ_c, as illustrated in the figure.

What if the label B refers to the home country, rather than to some foreign country that is exempt from the tariff? The identity of country B makes no difference to firms' optimal sourcing decisions nor to their bargaining position vis-à-vis their original and new suppliers. Input prices are the same no matter whether country B is a foreign country or not. The only difference is that higher prices paid to home suppliers are not generally considered a deterioration in the terms of trade, nor do they have the same adverse implications for home welfare (as we discuss below). In fact, when firms reshore portions of their supply chains, it becomes difficult to define a meaningful measure of changes in the terms of trade. Firms negotiate better prices for those inputs they continue to import, but other inputs—for which they pay higher prices than before—disappear from the import basket. Thus, the terms of trade apply to a changing bundle of goods, which poses the usual challenge for defining an appropriate price index.

Finally, we turn to output prices. Producers of differentiated varieties set these prices, as before, at a fixed markup over their perceived marginal costs. As we have seen in Figure 5, when demand for final goods is elastic, ϕ^τ is an increasing function of τ for all $\tau \in (w_B/w_A, \tau_c)$. So, higher input tariffs give rise to higher output prices throughout this range. For still higher tariffs such that $\tau > \tau_c$, firms' perceive marginal costs of the composite intermediate to be independent of the tariff rate. Since consumer prices are a fixed markup over perceived marginal costs, higher tariffs do not generate higher consumer prices when $\tau > \tau_c$, although the level of these prices must be higher than under free trade.

4.1.2 Larger Tariffs with Inelastic Demand

When demand is inelastic, downstream producers' optimal stopping rule is $\bar{b}^\tau = \bar{b}\,(w_B/w_A) = \bar{a}$ for all tariffs with $\tau > w_B/w_A$. The original producers retain their suppliers with $a \leq \frac{w_B}{\tau w_A}\bar{a}$, while replacing the rest. The negotiated prices obey (27) for the former group and (28) for the latter. Meanwhile, new producers enter and form supply chains in country B. For them, input prices are given by (28) for all relevant realizations of b.

As with the case of elastic demand and $\tau > \tau_c$, there are two offsetting influences of higher tariffs on the terms of trade. The higher is the tariff, the better is the price that the original producers negotiate with their retained suppliers in country A, as the outside option to search in country B is more attractive for greater τ. But higher tariffs induce greater reorganization of the supply chains by the original producers and the switch in sourcing represents diversion to a higher-cost supplier. On net, the former effect dominates for $\tau < (\theta + 1)/\theta$ and the latter for $\tau > (\theta + 1)/\theta$.[30] The real

[30] Note to EH: Is this correct? I'm not sure that we looked at the correct weighted average. On the other hand,

cost of inputs may fall and then rise as a function of the tariff rate in the range of large tariffs, or it may rise monotonically.

With a larger tariff in place and $\varepsilon < 1$, firms perceive a marginal cost of intermediate goods of $\phi^{\tau} = \frac{\theta}{\theta+1} w_B \bar{b}(w_B/w_A) = \frac{\theta}{\theta+1} w_B \bar{a}$. Under free trade, the perceived marginal cost is $\phi = \frac{\theta}{\theta+1} w_A \bar{a}$. Since wages are higher in country B than in country A, $\phi^{\tau} > \phi$ for all $\tau > w_B/w_A$. The tariff raises the perceived (tariff-inclusive) marginal costs of intermediate goods relative to that under free trade, so prices paid by final consumers are correspondingly higher.

4.2 Welfare Effects of Larger Tariffs

We begin our welfare analysis by identifying again the components of aggregate utility that vary with the tariff rate. Recall that $V(\tau) = \Pi(\tau) + T(\tau) + \Gamma(\tau)$, the sum of variable profits net of debt service on new capital costs, tariff revenues and consumer surplus. We can evaluate $V(\tau)$ for $\tau > w_B/w_A$ using $V(\tau) = \Pi(1) + \Gamma(1) + \int_{w_B/w_A}^{\tau} V'(t)\,dt$. In Section 3.4, we examined $V'(\tau)$ for $\tau < w_B/w_A$. In this section, we consider $V'(\tau)$ for $\tau > w_B/w_A$, making use once again of Assumptions 1- 3. As before, we distinguish the cases of elastic and inelastic demand for differentiated products.

4.2.1 Elastic Demand for Differentiated Products

For $\tau \in (w_B/w_A, \tau_c)$, there are no new searches and no entry. Recognizing that supply chains remain in country A and thus tariffs are applied to all imports in this case, we can write $V(\tau) = U(X^{\tau}) - \rho^{\tau} n m^{\tau} - n\ell^{\tau}$, as in (21). Then, differentiating this expression, we have

$$\frac{1}{n}\frac{dV^{\tau}}{d\tau} = \frac{1}{\sigma-1}\frac{d\ell^{\tau}}{d\tau} + \left(\frac{\sigma}{\sigma-1}\phi^{\tau} - \rho^{\tau}\right)\frac{dm^{\tau}}{d\tau} - m^{\tau}\frac{d\rho^{\tau}}{d\tau}, \text{ for } w_B/w_A < \tau < \tau_c,$$

which has the same form as (22). It is not necessary to repeat the arguments from Section 3.4.1, except to note that the first term again is negative, the second can be negative or positive according to the sign of the expression in parenthesis, and the last term is positive now, because higher tariffs in this range *improve* the terms of trade.

Turning to still larger tariffs with $\tau > \tau_c$, we have several new considerations in the welfare analysis. First, tariffs apply only to imports from country A and thus only for inputs with $a \in (0, a_B]$. Second, ϕ^{τ} is independent of τ in this range, so that $d\ell^{\tau}/d\tau = dm^{\tau}/d\tau = 0$ and $dX^{\tau}/d\tau = dP^{\tau}/d\tau = 0$. Third, if the label B identifies the home country, then the final producers' payments to suppliers net of production costs contribute to home welfare. Finally, fresh searches in country B forge new debts that must be serviced.

Suppose first that B denotes a foreign country. New searches are conducted by all n original producers for a fraction $1 - G(a_B)/G(\bar{a})$ of their inputs. These searches each have an expected flow cost of $f/G\left[\bar{b}(\tau_c)\right]$. Tariff revenues collected by the home government exactly offset the tariffs

it is only the original suppliers that see their ex-factory prices change as τ rises in the range of large tariffs; the new producers source only in B, their stopping rule doesn't change and so their average price doesn't change with τ.

payments made by home producers. So, using Assumption 2, we can write

$$V(\tau) = U(X^\tau) - n\rho^\tau m^\tau - n\ell^\tau - nf\left[\left(\frac{\tau w_A}{w_B}\right)^\theta - \frac{1}{\bar{a}^\theta}\right].$$

Since the tariff revenues collected by the home government exactly offset the tariffs payments made by home producers, we can write $\Pi(\tau) + T(\tau) = P^\tau X^\tau - n\rho^\tau m^\tau - n\ell^\tau$. With $\bar{b}^\tau = \bar{b}(\tau_c)$ for all $\tau > \tau_c$, perceived marginal costs, prices and factor demands are independent of τ. Only the terms of trade and the search costs vary with the tariff rate. Substituting $m^\tau = m/\tau$, we have

$$\frac{\tau}{n}\frac{dV^\tau}{d\tau} = -m\frac{d\rho^\tau}{d\tau} - \theta f\left(\frac{w_A}{w_B}\right)^\theta \tau^\theta, \text{ for } \tau > \tau_c.$$

We have already observed that the terms of trade might improve or deteriorate with the size of the tariff, according to whether $\tau < (\theta+1)/\theta$ or $\tau > (\theta+1)/\theta$. Of course, the search costs only grow with higher tariffs, as they induce more new searches. In the appendix, we show that aggregate welfare increases with the tariff rate for $\tau > \tau_c$ if and only if

$$\tau < \frac{\theta + 1 - \beta}{\theta}. \tag{33}$$

Note that a higher tariff might improve the terms of trade and nonetheless reduce welfare, because the searches for new suppliers impose additional costs. The greater is the buyers' bargaining weight, the smaller is the terms of trade effect, although the direction of the price movement does not depend on β. Meanwhile, the responsiveness of search costs to the tariff rate increases with β. From (33) we see that higher tariffs in this range are more likely to harm welfare when the distribution of match productivities is less dispersed (θ is large). The same is true when the buyers secure a greater share of the bargaining surplus; indeed a larger tariff must result in lower welfare when $\beta = 1$.

Figure 8 plots the change in welfare (expressed as a fraction of free-trade spending on differentiated products) as a function of the tariff rate, using the plausible parameter values that we have described before. Notably, $\sigma = 5$, $\theta = 4$, $\varepsilon = 1.5$, $\alpha = \beta = 0.5$, and wages in country B are 20 percent higher than those in country A. We see that welfare falls with the tariff over the range of small tariffs, with a welfare loss that reaches approximately 2.7 percent of initial spending for $\tau = 1.2$. (Note that $\tau = 1.2$ implies an *ad valorem* tariff of 20% on inputs that comprise 40% of the value of output.) There is a slight rebound in aggregate welfare, thanks to the terms of trade improvement, for large tariffs up to τ_c. Then welfare falls again as a function of the tariff rate, reaching losses of 3.06 percent of initial spending for $\tau = 1.25$. If wages in country B are only 10 percent higher than those in country A, the welfare loss from a 25% tariff is only 1.86 percent; see the appendix for the corresponding figure. In either case, the marginal efficiency cost of a higher tariff expands as the tariff rate increases.

Now suppose that B denotes the home country, so that the reorganization of the supply chain

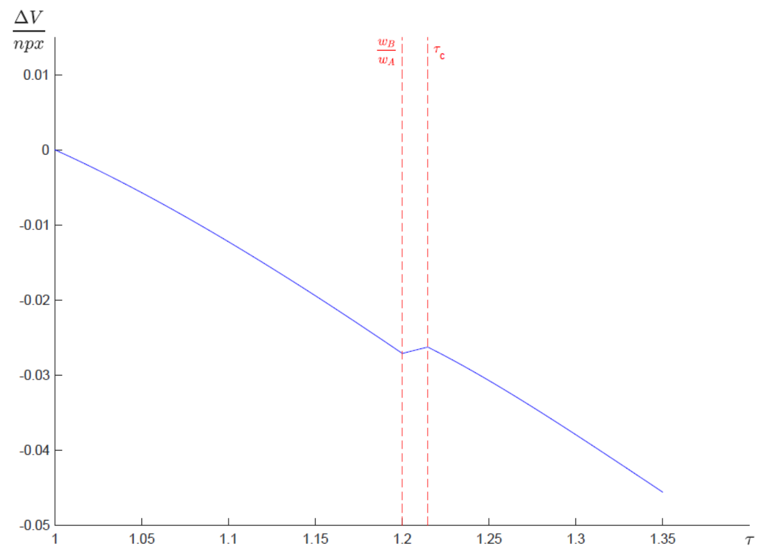


Figure 8: Welfare Effects of Unanticipated Tariffs: Elastic Demand

involves the reshoring of some inputs. In such circumstances, home welfare should include the profits earned by home input suppliers. The social cost of inputs then becomes

$$\rho^\tau = \frac{G(a_B)}{G(\bar{a})}\left[\beta w_A \mu_a(a_B) + (1-\beta)\frac{\bar{b}^\tau}{\tau}\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right] w_B \mu_b\left(\bar{b}^\tau\right),$$

where the second term now represents the cost of producing inputs at home rather than the prices that buyers pay for them. Using this expression for ρ^τ, we find that $d\rho^\tau/d\tau > 0$ if and only if $\tau > \left(\frac{\theta+1}{\theta}\right)\left(\frac{\theta+1-\beta}{\theta}\right)$. Since $(\theta+1-\beta)/\theta > 1$, this condition leaves more room for the real cost of inputs to fall when profits are shared domestically rather than with foreign suppliers. The calculations in the appendix prove that aggregate welfare increases with the tariff rate in this case if and only if

$$\tau < \left(\frac{\theta+1}{\theta}\right)\frac{\theta+1-\beta}{\theta+\beta}.$$

Comparing this inequality to (33), we see that welfare increases for a wider range of tariffs when the disruption of supply chains induces reshoring than when it encourages relocation abroad. Still, even with reshoring, a larger tariff results in lower welfare when $\beta = 1$.

4.2.2 Inelastic Demand for Differentiated Products

When demand for differentiated products is inelastic, a tariff greater than $\tau = w_B/w_A$ always disrupts the supply chains. Moreover, the optimal stopping rule for searches in country B is given by $\bar{b}^\tau = \bar{b}(w_B/w_A)$, which is independent of the tariff rate. Then $\phi^\tau = \frac{\theta}{\theta+1}\bar{b}^\tau$ for both original and new producers of differentiated varieties, which also is independent of the tariff rate. Output prices and factor demands are linked to perceived marginal costs. With no variation in ϕ^τ, there is no variation in ℓ^τ, m^τ, P^τ, or X^τ. With no change in P^τ, there is no room for entry by firms

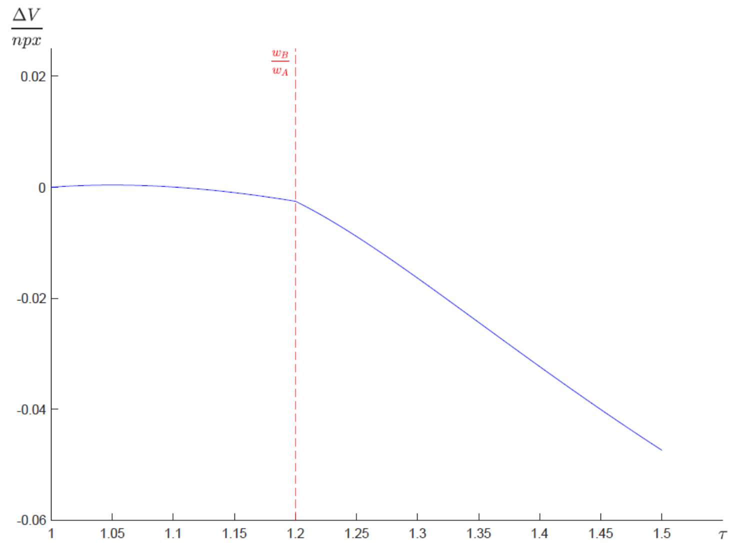


Figure 9: Welfare Effects of Unanticipated Tariffs: Inelastic Demand

that would search for suppliers in country B beyond the entry that occurs for $\tau = w_B/w_A$. Higher tariff rates affect welfare for $\tau > w_B/w_A$ through two channels: they influence the terms of trade via renegotiation and trade diversion and they generate additional search costs.

Indeed, there is no need for further analysis. The comparative statics with respect to changes in τ for $\tau > w_B/w_A$ and $\varepsilon < 1$ are identical to those for $\tau > \tau_c$ and $\varepsilon > 1$ that we studied in the last section. If the new searches take place in a foreign country B, higher tariffs result in better home terms of trade if and only if $\tau < (\theta + 1)/\theta$ and they generate greater home welfare on the margin if and only if $\tau < \frac{\theta+1-\beta}{\theta}$. If new searches instead take place in the home country, higher tariffs reduce the real cost of inputs if and only if $\tau < \left(\frac{\theta+1}{\theta}\right)\left(\frac{\theta+1-\beta}{\theta}\right)$ and they boost welfare if and only if $\tau < \left(\frac{\theta+1}{\theta}\right)\left(\frac{\theta+1-\beta}{\theta+\beta}\right)$.

Figure 9 depicts the relationship between social welfare and the tariff rate for the same parameter values used in Figure 8, except that $\varepsilon = 0.5$. In this example, welfare rises imperceptibly above the free-trade level for a range of small tariffs up to about 10.3 percent. Here, the social benefit from added variety nearly perfectly offsets the net social loss from reduced output by the original producers; the net welfare gain is less than 0.1% of initial spending at the peak. Once supply chains begin to relocate to country B, welfare falls precipitously with the tariff rate due to the socially-wasteful added search costs and the induced Vinerian trade diversion. Moreover, the marginal harm from the tariff grows larger as the tariff rate increases.

Finally, Figure 10 in the appendix illustrates a case where protection is clearly beneficial, especially if protection induces reshoring of input supply to the home country. There, $w_A = 0.9, w_B = 1.0$, $\beta = 0.3$, and the other parameters are the same as in Figure 9. This is a case where the wage gap between the cheapest and second cheapest suppliers is only ten percent and suppliers enjoy more of the bargaining power in their bilateral relationships with downstream producers. If country B is a foreign country (which requires w_B a bit below one), a large tariff can be used to extract some

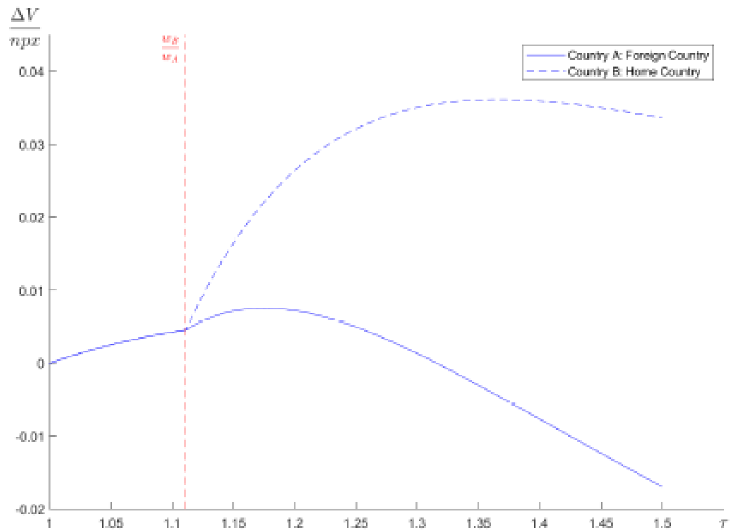


Figure 10: Welfare Effects of Unanticipated Tariffs with Inelastic Demand and Weak Buyer Bargaining Power

of the rents that the downstream firms concede to suppliers in their price negotiations. A tariff of 20% generates a modest welfare gain of about 0.46% of initial spending. If country B instead is the home country, then the optimal tariff is approximately 36.7% and it generates a welfare gain of more than 3.33% of initial spending. These welfare gains reflect the substantial profit shifting from foreign suppliers to domestic suppliers that occurs in this case.

5 Conclusions

Traditional tariff analysis focuses on supply and demand elasticities and Harberger triangles. Of course, subsequent literature has addressed many types of market imperfections, including those arising from monopoly power and from factor-market distortions. Yet, the rise of global supply chains introduces some novel considerations to the evaluation of trade barriers, especially when tariffs are applied to imports of intermediate goods.

In this paper, we have stressed the relational aspects of supply chains, as highlighted in the 2020 World Development Report. The formation of supply chains often requires costly search. Partnerships may vary in productivity. Supply relationships might be governed by imperfectly-enforceable contracts that can be renegotiated when circumstances change. Bargaining might take place separately with many, independent suppliers.

We have identified several new mechanisms by which unanticipated tariffs on intermediate inputs impact prices and welfare. First, negotiations with suppliers may be conducted in the shadow of renewed search. When the outside option for a buyer is to find an alternative supplier, the negotiated price depends upon the factors that govern the intensity of search and its eventual prospects. If a tariff weakens the incentives for search, the bargaining table tilts in favor of suppliers.

In contrast, if a tariff makes search in some different destination relatively more attractive, the negotiations may result in shared incidence of the levy.

Second, bargaining can drive a wedge between the marginal cost of inputs as perceived by final-good producers and their true social cost. When a downstream firm bargains independently with many suppliers, it becomes impractical to negotiate levels of input demands that are jointly efficient. If, instead, the downstream firm decides its factor demands unilaterally, it will recognize a connection between that choice and the eventual per-unit price. The firm will perceive a marginal cost of inputs different from their average cost, which generates an inefficient (but privately profitable) choice of production technique.

Third, large tariffs can induce firms to replace their least efficient suppliers with alternatives at home or in countries that are exempt from the tariff. In the latter case, the relocation of portions of the supply chain amounts to Vinerian trade diversion. In both cases, the additional search costs become a hidden component of the welfare calculus.

We have analyzed tariffs that are introduced after global supply chains are already in place. With original search and entry costs sunk, firms remain active as long as they can cover their operating costs and supply relationships endure in the face of shocks. We consider tariffs that are small enough to leave the location of the supply chain as originally situated and larger tariffs that make a new destination more attractive. We identify the elasticity of demand for differentiated products as an important parameter in determining the impacts of an input tariff, so we analyze separately cases with elastic and inelastic demand.

In our second-best setting, input tariffs generate positive and negative effects on home welfare. Measurement requires attention to numerous details, including some that leave no visible trail in the trade data. Although the theoretical analysis leaves open the possibility of welfare-improving tariffs, this does not seem to be the likely outcome for plausible parameter values. In fact, we find that the marginal welfare cost of protection grows with the size of the tariff, so large tariffs such as those recently implemented by the United States.

More generally, our paper contributes a tractable analytic framework for studying the complex adjustments that occur when various unanticipated shocks disrupt global supply chains. Our framework can be extended to allow for heterogeneous suppliers who enjoy comparative advantage in different parts of the production process. Comparative advantage would provide a ready explanation for multi-country sourcing, as in Blaum et al. (2017) and Antràs et al. (2017). And whereas we have set aside the holdup problems emphasized by Ornelas and Turner (2008) and Antràs and Staiger (2012) in order to focus on costly search, it should be possible to combine these features in a fuller analysis.

References

[1] Amiti, Mary, Redding, Stephen J., and David E. Weinstein, 2019. “The Impact of the 2018 Tariffs on Prices and Welfare,” *Journal of Economic Perspectives 33:4*, 187-210.

[2] Amiti, Mary, Redding, Stephen J., and David E. Weinstein, 2020. “Who’s Paying for the US Tariffs? A Longer-Term Perspective,” *AEA Papers and Proceedings 110*, 541-546..

[3] Antràs, Pol, 2020, “Conceptual Aspects of Global Value Chains,” *The World Bank Economic Review 34*, forthcoming.

[4] Antràs, Pol and Davin Chor, 2013. "Organizing the Global Value Chain,” *Econometrica 81:6*, 2127-2204.

[5] Antràs, Pol, Fort, Theresa, and C. and Felix Tintelnot, 2017. “The Margins of Global Sourcing: Theory and Evidence,” *American Economic Review 107:9*, 2514-2564.

[6] Antràs, Pol and Elhanan Helpman, 2004. “Global Sourcing,” *Journal of Political Economy 112:3*, 552-580.

[7] Antràs, Pol and Robert W. Staiger, 2012. “Offshoring and the Role of Trade Agreements,” *American Economic Review 102:7*, 3140-3183.

[8] Baldwin, Richard E. and Anthony J. Venables, 2011. “Spiders and Snakes: Offshoring and Agglomeration in the Global Economy,” *Journal of International Economics 90:2*, 245-254.

[9] Benkert, Jean-Michel, Letina, Igor, and Georg Nödeke, 2018. “Optimal Search from Multiple Distributions with Infinite Horizon,” *Economic Letters 164*, 15-18.

[10] Bernard, Andrew B. and Andreas Moxnes, 2018. “Networks and Trade,” *Annual Review of Economics 10*, 65-85.

[11] Blanchard, Emily J., Bown, Chad P and Robert C. Johnson, 2017. “Global Value Chains and Trade Policy,” Dartmouth College, mimeo.

[12] Blaum, Joaquin, Lelarge, Clair and Michael Peters, 2018. “The Gains from Input Trade with Heterogeneous Importers,” *American Economic Journal: Macroeconomics 10:4*, 77-127.

[13] Bloomberg News, 2019.“Trump Tariffs Seal the Deal for Companies Looking to Quit China,” *Bloomberg News* May 16, 2019, https://www.bloomberg.com/news/articles/2019-05-16/trump-tariffs-seal-the-deal-for-companies-looking-to-quit-china.

[14] Bown, Chad P., 2019a. “The 2018 US-China Trade Conflict After 40 Years of Special Protection,” Peterson Institute for International Economics Working Paper 19-7.

[15] Bown, Chad P., 2019b. “Phase One China Deal: Steep Tariffs are the New Normal,” *Trade and Investment Policy Watch*, Peterson Institute of International Economics, December 19, 2019.

[16] Bown, Chad P. and Meredith A. Crowley, 2016. “The Empirical Landscape of Trade Policy,” in K. Bagwell and R.W. Staiger eds., *Handbook of Commercial Policy Volume 1A*, Amsterdam: North Holland.

[17] Brander, James A. and Barbara Spencer, J. “Trade Warfare: Tariffs and Cartels,” *Journal of International Economics 16:3*, 227-242.

[18] Breuninger, Kevin, 2019. “Trump says He’s Ordering American Companies to Immediately Start Looking for an Alternative to China,” *CNBC*, August 23, 2019, https://www.cnbc.com/2019/08/23/trump-says-hes-ordering-american-companies-to-immediately-start-looking-for-an-alternative-to-china.html.

[19] Caliendo, Lorenzo and Fernando Parro, 2015. "Estimates of the Trade and Welfare Effects of NAFTA,” *Review of Economic Studies 82:1*, 1-44.

[20] Campolmi, Alessia, Fadinger, Harald, and Chiata Forlati, 2018. “Trade and Domestic Policies in Models with Monopolistic Competition,” http://fadinger.vwl.uni-mannheim.de/Research_. . . les/CFF_2018_1004.pdf.

[21] Casas, Francisco R., 1973. “Optimal Effective Protection in General Equilibrium,” *American Economic Review 64:3*, 714-716.

[22] Collard-Wexler, Allan, Gowrisankaran, Gautam, and Robin S. Lee, 2019. “‘Nash-in-Nash’ Bargaining: A Microfoundation for Applied Work,” *Journal of Political Economy 127:1*, 163-195.

[23] Das, Satya, 1983. “Optimal Tariffs on Final and Intermediate Goods,” *International Economic Review 24:2*, 493-508.

[24] Demidova and Andrés Rodríguez-Clare, 2013. “The Simple Analytics of the Melitz Model in a Small Open Economy,” *Journal of International Economics 90:2*, 266-272.

[25] Fajgelbaum, Pablo D., Goldbeg, Pinelopi K., Kennedy, Patrick J. and Amit K. Khandelwal, 2020. “The Return to Protectionism,” *Quarterly Journal of Economics 135:1*, 1-55.

[26] Grossman, Gene M. and Esteban Rossi-Hansberg, 2008. “Trading Tasks: A Simple Theory of Offshoring,” *American Economic Review 98:5*, 1978-1997.

[27] Halpern, László, Koren, László, and Adam Szeidl, 2015. “Imported Inputs and Productivity,” *American Economic Review 105:12*, 3660-3703.

[28] Helpman, Elhanan and Paul R. Krugman, 1989. *Trade Policy and Market Structure*, Cambridge, MA: The MIT Press.

[29] Huang, Eustance, 2019. "Thailand is a 'Leading Beneficiary' of the Trade War, says Standard Chartered", *CNBC* Nov 4, 2019, https://www.cnbc.com/2019/11/05/thailand-is-a-leading-beneficiary-of-the-us-china-trade-war-stanchart.html.

[30] Hufford, Austen and Bob Tita, 2019. "Manufacturers Move Supply Chains Out of China," *The Wall Street Journal* July 14, 2019, https://www.wsj.com/articles/manufacturers-move-supply-chains-out-of-china-11563096601.

[31] Katrak, Homi, 1977. "Multi-National Monopolies and Commercial Policy," *Oxford Economic Papers 29:2*, 283-291.

[32] Kawanami, Takeshi and Takeshi Shiraishi, 2019. "Trump's Latest China Tariffs to Shock Global Supply Chains," *Nikei Asian Review* May 12, 2019, https://asia.nikkei.com/Economy/Trade-war/Trump-s-latest-China-tariffs-to-shock-global-supply-chains.

[33] Kearney, 2020. *Trade War Spurs Sharp Reversal in 2019 Reshoring Index, Foreshadowing COVID-19 Test of Supply Chain Resilience*, Washington DC: Kearney.

[34] Master, Farah, Sriring, Orathai, and Anne Marie Roantree, 2018. "Supply Chain Reaction: Trade War Refugees Race to Relocate to Vietnam, Thailand," *Reuters* November 28, 2018, https://www.reuters.com/article/us-usa-trade-china-vietnam/supply-chain-reaction-trade-war-refugees-race-to-relocate-to-vietnam-thailand-idUSKCN1NX2XZ.

[35] Ornelas, Emanuel and John L. Turner, 2008. "Trade Liberalization, Outsourcing, and the Hold-up Problem," *Journal of International Economics 74:1*, 225-241.

[36] Ornelas, Emanuel and John L. Turner, 2012. "Protection and International Sourcing," *The Economic Journal 122:559*, 26-63.

[37] Ornelas, Emanuel, Turner, John L. and Grant Bickwit, 2020. "Preferential Trade Agreements and Global Sourcing," mimeo.

[38] Reed, J.R., 2019. "President Trump Order Firms to Ditch China, but Many Already Have and More are on the Way," *CNBC* September 1, 2019, https://www.cnbc.com/2019/09/01/trump-ordered-us-firms-to-ditch-china-but-many-already-have.html.

[39] Ruffin, Roy J., 1969. "Tariffs, Intermediate Goods and Domestic Protection," *American Economic Review 59:3*, 261-269.

[40] Soon, Stella, 2019. "Investors Have Set Their Eyes on Thailand as the Trade War Deepens, says CEO," *CNBC* August 27, 2019, https://www.cnbc.com/2019/08/27/us-china-trade-war-companies-looking-to-invest-in-thailand-vietnam.html.

[41] Stole, Lars A. and Jeffrey Zwiebel, 1996. "Infor-firm Bargaining under Non-Binding Contracts," *Review of Economic Studies 63,* 375-410.

[42] Svedberg, Peter, 1979. "Optimal Tariff Policy on Imports from Multinationals," *Economic Record 55,148*, 64-67.

[43] Venables, Anthony J., 1987. "Trade and Trade Policy with Differentiated Products: A Chamberlinian-Ricardian Model," *The Economic Journal 97:387*, 700-717.

[44] World Bank, 2020. *World Development Report 2020: Trading for Development in the Age of Global Value Chains*, Washington DC: The World Bank.

Appendix

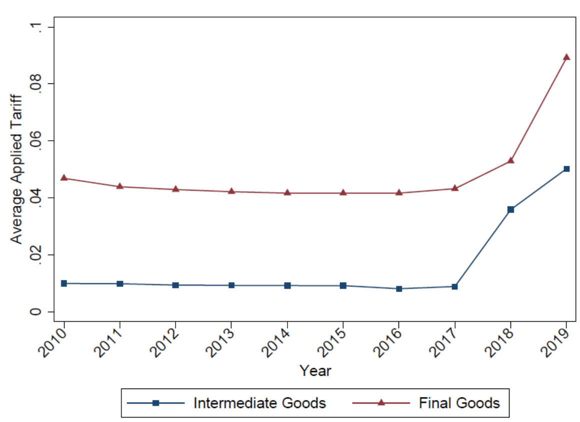
When Tariffs Disturb Global Supply Chains
by
Gene M. Grossman and Elhanan Helpman

Section 1 Introduction

For applied tariffs, we used the Harmonized Tariff Schedule (HTS) of the United States prepared by the U.S. International Trade Commission and available at https://dataweb.usitc.gov/tariff/annual. Since these tariff rates are reported at the HTS8 level, we assumed that all HTS10 items in the same HTS8 category were subject to the average tariff for that category. We calculated the ad valorem equilibrium of specific tariffs using reported unit values and we accounted for "special rates" due to preferential trade agreements and administered protection. For the first wave of U.S. tariff increases in February 2018 and the second wave of tariff increases in March 2018, we used the tariff data from Fajgelbaum et al. (2020). For subsequent tariff hikes that began in July 2018, we used data kindly provided to us by Chad Bown.

Import data are from the U.S. Census Bureau, available at https://usatrade.census.gov/.

To compute average applied tariffs on intermediate goods and final goods, we categorized HTS10 products according to the Broad Economic Categories provided by the United Nations. The crosswalk between HTS10 codes and end-use categories is available at https://unstats.un.org/unsd/tradekb/Knowledgebase/50090/Intermediate-Goods-in-Trade-Statistics. We weighted the applied tariffs by the annual import shares in 2015 at the HTS10 level. Figure A1 presents the results of these calculations.



Note: Classification into end-use categories based on the United Nation's Classification by Broad Economic Categories (4th Revision). Weighted average tariffs use the value of imports from each country in each HTS10 category in 2015 as weights. Applied tariffs include MFN *ad valorem* tariffs, *ad valorem* equivalents of MFN specific tariffs, and "special tariffs" due to preferential agreements and administered protection.

Figure A1: Average U.S. Applied Tariffs on Intermediate Goods and Final Goods

Figure 1 in the main text uses the tariff rates applicable to China, weighted by the annual imports shares in 2015 at the HTS10 level.

To construct Table 1, we estimated

$$\ln M_{ijt} = \zeta_i + \psi_t + \delta_j \ln \frac{\tau_{i,China,t}}{\tau_{i,LCC,t}} + \nu_{ijt}$$

where M_{ijt} is the value of imports of intermediate good i from country j in month t, ζ_i is an HTS10-product fixed effect, ψ_t is a month fixed effect, and $\tau_{i,j,t}$ is one plus the ad valorem tariff on good i from country j in month t. In the case of the low-cost countries ($j = LCC$), we aggregated the imports from Bangladesh, Cambodia, Hong Kong, India, Indonesia, Malaysia, Pakistan, Philippines, Singapore, Sri Lanka, Taiwan, Thailand, and Vietnam and we weighted the applicable tariff rates by the HTS10 import values shares in 2017. We used only product-month pairs with positive imports from both China and the LCC countries. Following Amiti et al. (2019), we excluded petroleum products due the sensitive response to volatile oil products. To allow comparability with Amiti et al. (2019), we classified goods according to the end-use codes provided by the U.S. Census Bureau at https://www.census.gov/foreign-trade/schedules/b/2015/imp-code.txt. However, using the slightly different categorization provided by the United Nation yields qualitatively similar results. Standard errors are clustered at the HTS8 level.

Section 2 Foreign Sourcing with Search and Bargaining

We start from the bargaining game, which determines the payment to a supplier with inverse match productivity a for one unit of the intermediate input. The Nash bargaining solution solves

$$\rho(a) = \arg\max_q (qm - wam)^{1-\beta} \left[\mu_\rho(\bar{a})\, m + \frac{f}{G(\bar{a})} - qm\right]^\beta .$$

The first-order condition for the maximization on the right-hand side yields

$$\frac{1-\beta}{\rho(a) - wa} = \frac{\beta}{\mu_\rho(\bar{a}) + \frac{f}{mG(\bar{a})} - \rho(a)}$$

and therefore

$$\rho(a) = \beta wa + (1-\beta)\,\mu_\rho(\bar{a}) + (1-\beta)\frac{f}{mG(\bar{a})}.$$

Taking the conditional mean of both sides of this equation for $a \leq \bar{a}$, we have

$$\mu_\rho(\bar{a}) = w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})}. \tag{34}$$

Substituting this result back into the $\rho(a)$ function then gives

$$\rho(a) = \beta wa + (1-\beta)\, w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})}, \tag{35}$$

which is equation (4) in the main text. Next we use (5), the first-order condition for $\bar{a}$. This states

$$mw\mu_a'(\bar{a}) = \frac{fg(\bar{a})}{\beta G(\bar{a})^2}. \tag{36}$$

Note, however, that

$$\mu_a(\bar{a}) = \frac{1}{G(\bar{a})}\int_0^{\bar{a}} ag(a)da$$

and therefore

$$\mu_a'(\bar{a})\, G(\bar{a}) = g(\bar{a})\left[\bar{a} - \mu_a(\bar{a})\right]. \tag{37}$$

Substituting this into (36), we obtain

$$w\left[\bar{a} - \mu_a(\bar{a})\right] = \frac{f}{\beta m G(\bar{a})}. \tag{38}$$

Substituting (38) into (35) then yields equation (6),

$$\begin{aligned}\rho(a) &= \beta w\left[a - \mu_a(\bar{a})\right] + \beta w \mu_a(\bar{a}) + (1-\beta)\, w\bar{a} \\ &= \beta wa + (1-\beta)\, w\bar{a}.\end{aligned}$$

We next use the demand equation (3), the pricing equation (7), and (34) to compute operating profits. These profits are

$$\pi_o = x(p-c) - \frac{1-\beta}{\beta}\frac{f}{G(\bar{a})} - f_o,$$

where

$$p = \frac{\sigma}{\sigma-1}c,$$

$$x = X\left(\frac{p}{P}\right)^{-\sigma} = XP^{\sigma}\left(\frac{\sigma}{\sigma-1}c\right)^{-\sigma}, \tag{39}$$

and the aggregate cost of m units of the intermediate input is

$$w\mu_a(\bar{a})\, m + \frac{1-\beta}{\beta}\frac{f}{G(\bar{a})}.$$

Therefore,

$$\pi_o = XP^{\sigma}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}c^{1-\sigma} - \frac{1-\beta}{\beta}\frac{f}{G(\bar{a})} - f_o, \tag{40}$$

where

$$c = c\left[w\mu_a(\bar{a})\right],$$

as stated in equation (8). By Shephard's Lemma, m is given by

$$m = XP^{\sigma}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}c^{-\sigma}c'. \tag{41}$$

A firm chooses $\bar{a}$ to maximize profits net of search costs, taking P and X as given. That is,

$$\begin{aligned}\bar{a} &= \arg\max_a XP^{\sigma}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}c\left[w\mu_a(a)\right]^{1-\sigma} - \frac{1-\beta}{\beta}\frac{f}{G(a)} - \frac{f}{G(a)} - f_o \\ &= \arg\max_a XP^{\sigma}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}c\left[w\mu_a(a)\right]^{1-\sigma} - \frac{f}{\beta G(a)} - f_o.\end{aligned}$$

For an interior solution, the first-order condition is

$$-XP^{\sigma}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}c\left[w\mu_a(\bar{a})\right]^{-\sigma}c'\left[w\mu_a(\bar{a})\right]w\mu_a'(\bar{a}) + \frac{fg(\bar{a})}{\beta G(\bar{a})^2} = 0,$$

which is the same as (5) in view of (41). Using Assumptions 1 and 2, this condition can be written as

$$-\alpha XP^{\sigma}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}\left(w\frac{\theta}{\theta+1}\bar{a}\right)^{-\alpha(\sigma-1)-1}\left(w\frac{\theta}{\theta+1}\right) + \theta\frac{f}{\beta\bar{a}^{\theta+1}} = 0.$$

Therefore the second-order condition for profit maximization is satisfied at the optimal choice of $\bar{a}$ if and only if $\theta > \alpha(\sigma-1)$, as stipulated in Assumption 3. This first-order condition can be expressed as

$$\bar{a}^{\theta-\alpha(\sigma-1)}XP^{\sigma} = \frac{\theta f}{\alpha\beta}\left(\frac{w\theta}{\theta+1}\right)^{\alpha(\sigma-1)}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}. \tag{42}$$

Substituting this expression into (40) yields

$$\pi_o - \frac{f}{G(\bar{a})} = \frac{\theta-\alpha(\sigma-1)}{\beta\alpha(\sigma-1)}f\bar{a}^{-\theta} - f_o.$$

The free entry condition is

$$\pi_o - \frac{f}{G(\bar{a})} = f_e,$$

which, together with the previous equation, yields equation (9):

$$\bar{a}^{\theta} = \frac{f}{f_o+f_e}\frac{\theta-\alpha(\sigma-1)}{\beta\alpha(\sigma-1)}. \tag{43}$$

The solution to this cutoff is interior if and only if

$$\frac{f}{f_o+f_e}\frac{\theta-\alpha(\sigma-1)}{\beta\alpha(\sigma-1)} < 1.$$

Substituting (43) and $XP^{\sigma} = P^{\sigma-\varepsilon}$ into (42) provides a solution for P. And substituting this equation into

$$P = \frac{\sigma}{\sigma-1}\left(w\frac{\theta}{\theta+1}\bar{a}\right)^{\alpha}n^{-\frac{1}{\sigma-1}}$$

provides a solution for n. Note that

$$\hat{n} = (\sigma - 1)\left(\alpha\widehat{\bar{a}} - \hat{P}\right),$$

where a hat over a variable represents a proportional rate of change, e.g., $\hat{y} = dy/y$. For an increase in the search cost f we have, from (42),

$$\hat{P} = \frac{\hat{f} - [\theta - \alpha(\sigma - 1)]\,\widehat{\bar{a}}}{\sigma - \varepsilon}$$

and from (43),

$$\widehat{\bar{a}} = \frac{1}{\theta}\hat{f}.$$

Therefore,

$$\hat{P} = \frac{\alpha(\sigma - 1)}{\theta(\sigma - \varepsilon)}\hat{f},$$

$$\hat{n} = \frac{\alpha(\sigma - 1)}{\theta}\frac{1 - \varepsilon}{\sigma - \varepsilon}\hat{f}.$$

These results are summarized in

Lemma 1 *Suppose Assumptions 1-3 hold and*

$$\frac{f}{f_o + f_e}\frac{\theta - \alpha(\sigma - 1)}{\beta\alpha(\sigma - 1)} < 1.$$

Then lower search costs f lead to a lower cutoff $\bar{a}$ and a lower price index P. They also generate more variety n if and only if $\varepsilon > 1$.

Section 3 A Small, Unanticipated Tariff

In this case, the ex-factory price paid to a foreign supplier with inverse match productivity a is $\rho(a, \tau)$, which is the solution to

$$\rho(a, \tau) = \arg\max_q \left[\tau\mu_\rho[\bar{a}(\tau), \tau] + \frac{f}{m(\tau)\,G[\bar{a}(\tau)]} - \tau q\right]^\beta (q - wa)^{1-\beta}.$$

This f.o.b. price excludes the tariff levy. The first-order condition for this maximization problem is

$$\frac{1 - \beta}{\rho(a, \tau) - wa} = \frac{\beta}{\mu_\rho[\bar{a}(\tau), \tau] + \frac{f}{\tau m(\tau) G[\bar{a}(\tau)]} - \rho(a, \tau)},$$

which yields

$$\rho(a, \tau) = \beta wa + (1 - \beta)\,\mu_\rho[\bar{a}(\tau), \tau] + (1 - \beta)\frac{f}{\tau m(\tau)\,G[\bar{a}(\tau)]}. \tag{44}$$

Taking conditional expectations on both sides of this equation for $a \leq \bar{a}(\tau)$, we find

$$\mu_\rho [\bar{a}(\tau), \tau] = w\mu_a [\bar{a}(\tau)] + \frac{1-\beta}{\beta} \frac{f}{\tau m(\tau) G[\bar{a}(\tau)]}. \tag{45}$$

Next, substituting this expression into (44), we obtain

$$\rho(a,\tau) = \beta w a + (1-\beta) w \mu_a [\bar{a}(\tau)] + \frac{1-\beta}{\beta} \frac{f}{\tau m(\tau) G[\bar{a}(\tau)]}, \tag{46}$$

which is equation (10) in the main text. As explained in the text, using the optimal search cutoff $\bar{a}(\tau)$ yields

$$w\{\bar{a}(\tau) - \mu_a [\bar{a}(\tau)]\} = \frac{f}{\beta \tau m(\tau) G[\bar{a}(\tau)]}. \tag{47}$$

Now substitute this equation into (46) to obtain

$$\rho(a,\tau) = \beta w a + (1-\beta) w \bar{a}(\tau). \tag{48}$$

Next note that it is cheaper to sources inputs from the original supplier a whenever

$$\tau \rho(a,\tau) \leq \tau \mu_\rho [\bar{a}(\tau), \tau] + \frac{f}{m(\tau) G[\bar{a}(\tau)]}.$$

Using (45) and (47), the right-hand side of this inequality equals $\tau w_A \bar{a}(\tau)$. Therefore this inequality can be expressed as

$$a \leq \bar{a}(\tau).$$

From this result, we have

Lemma 2 *For a given $\bar{a}(\tau)$ the cost minimizing cutoff a_c is*

$$a_c = \min\{\bar{a}(\tau), \bar{a}\}.$$

As explained in the main text, the marginal cost of m is given by equation (14),

$$\phi^\tau = \beta \frac{G(a_c)}{G(\bar{a})} \tau w \mu_a (a_c) + \left[1 - \beta \frac{G(a_c)}{G(\bar{a})}\right] \tau w \mu_a (\bar{a}^\tau)$$

and then optimal (mark-up) pricing implies

$$p^\tau = \frac{\sigma}{\sigma - 1} c(\phi^\tau).$$

Using Assumption 2 and Lemma 2, the marginal cost can be expressed as

$$\phi^\tau = \begin{cases} \frac{\theta}{\theta+1} \tau w \bar{a}^\tau & \text{for } \bar{a}^\tau < \bar{a} \\ \beta \frac{\theta}{\theta+1} \tau w \bar{a} + (1-\beta) \frac{\theta}{\theta+1} \tau w \bar{a}^\tau & \text{for } \bar{a}^\tau > \bar{a} \end{cases}. \tag{49}$$

This is the MM curve in Figures 3 and 4.

We next derive the NN curve, using the first-order condition for $\bar{a}^{\tau}$ in (47), Shephard's Lemma $m^{\tau} = x^{\tau} c'(\phi^{\tau})$, the expression for the demand for variety ω in (39), and the expression for the price index, $P^{\tau} = p^{\tau}(n^{\tau})^{-1/(\sigma-1)}$. This expression of the price index assumes that all firms, new and old, charge the same price p^{τ}, which we verify below. First, in the Pareto case (47) becomes

$$w\bar{a}(\tau)^{\theta+1} = \frac{f(\theta+1)}{\beta\tau m(\tau)}. \tag{50}$$

Second,

$$\begin{aligned} m^{\tau} &= X^{\tau}\left(\frac{p^{\tau}}{P^{\tau}}\right)^{-\sigma} c'(\phi^{\tau}) \\ &= X^{\tau}(n^{\tau})^{-\frac{\sigma}{\sigma-1}} c'(\phi^{\tau}) = (P^{\tau})^{-\varepsilon}(n^{\tau})^{-\frac{\sigma}{\sigma-1}} c'(\phi^{\tau}) \\ &= (p^{\tau})^{-\varepsilon}(n^{\tau})^{-\frac{\sigma-\varepsilon}{\sigma-1}} c'(\phi^{\tau}) \end{aligned} \tag{51}$$

Combining these equations, we obtain

$$\frac{(\theta+1)f}{w\beta(\bar{a}^{\tau})^{\theta+1}} = \tau(n^{\tau})^{-\frac{\sigma-\varepsilon}{\sigma-1}}(p^{\tau})^{-\varepsilon} c'(\phi^{\tau}) \ ,$$

which is equation (17) in the main text. Using $p^{\tau} = c(\phi^{\tau})\sigma/(\sigma-1)$ and $c(\phi^{\tau}) = (\phi^{\tau})^{\alpha}$, this equation becomes

$$\frac{(\theta+1)f}{w\beta(\bar{a}^{\tau})^{\theta+1}} = \tau(n^{\tau})^{-\frac{\sigma-\varepsilon}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon}\alpha(\phi^{\tau})^{\alpha(1-\varepsilon)-1} \ . \tag{52}$$

This implies that the NN curve is higher the greater is the tariff rate and that all along this curve,

$$\hat{\phi}^{\tau} = \frac{\theta+1}{1-\alpha(1-\varepsilon)}\hat{\bar{a}}^{\tau}.$$

The denominator is positive for all $\varepsilon > 0$, and since $\varepsilon < \sigma$ and $\theta > \alpha(\sigma-1)$, $\theta+1 > 1+\alpha(\varepsilon-1)$. Therefore the elasticity of the NN curve is larger than one. The upward shift of the curve in response to a rise in τ satisfies

$$\hat{\phi}^{\tau} = \frac{1}{1-\alpha(1-\varepsilon)}\hat{\tau}.$$

Therefore, ϕ^{τ} rises proportionately more than τ if $\varepsilon < 1$ and proportionately less if $\varepsilon > 1$. As a result, the marginal cost ϕ^{τ} declines in the inelastic case and rises in the inelastic case, holding constant the number of firms.

Now consider the incentives for entry by new firms in response to the tariff. We begin with the inelastic case, $\varepsilon < 1$. In this case,

$$\phi^{\tau} = w\frac{\theta}{\theta+1}\tau\bar{a}^{\tau}, \tag{53}$$

$$\rho^{\tau} = w\frac{\theta+1-\beta}{\theta+1}\bar{a}^{\tau}. \tag{54}$$

Therefore, holding constant the number of firms, (16)-(17) imply

$$\hat{\phi}^{\tau} = \frac{\theta}{\theta - \alpha(\varepsilon - 1)} \hat{\tau}, \tag{55}$$

$$\widehat{\bar{a}}^{\tau} = \frac{\alpha(\varepsilon - 1)}{\theta - \alpha(\varepsilon - 1)} \hat{\tau}. \tag{56}$$

It follows that ϕ^{τ} rises less than proportionately to τ, while $\bar{a}^{\tau}$ declines. A new entrant seeks to maximize operating profits minus search costs and entry costs. Using (40) and (49), the highest profit such an entrant can achieve as a function of the tariff is

$$\pi(\tau) = \max_a P(\tau)^{\sigma - \varepsilon} \frac{(\sigma - 1)^{\sigma - 1}}{\sigma^{\sigma}} [\tau w \mu_a(a)]^{\alpha(1-\sigma)} - \frac{f}{\beta G(a)} - f_o - f_e. \tag{57}$$

For $\tau = 1$, the solution to the problem on the right-hand side of this equation is $a = \bar{a}$ and $\pi(1) = 0$. Therefore there is new entry as long as $\pi'(\tau) > 0$. Using the Envelope Theorem, $\pi'(\tau) > 0$ if and only if $P(\tau)^{\sigma-\varepsilon} \tau^{\alpha(1-\sigma)}$ is increasing in τ. Using (55) and $\theta > \alpha(\sigma - 1)$, we obtain

$$\begin{aligned}
\frac{(\sigma - \varepsilon)\hat{P}^{\tau} - \alpha(\sigma - 1)\hat{\tau}}{\alpha\hat{\tau}} &= \frac{(\sigma - \varepsilon)\hat{\phi}^{\tau} - (\sigma - 1)\hat{\tau}}{\hat{\tau}} \\
&= \frac{\theta(\sigma - \varepsilon)}{\theta + \alpha(1 - \varepsilon)} - (\sigma - 1) \\
&> \frac{\alpha(\sigma - 1)(\sigma - \varepsilon)}{\alpha(\sigma - 1) + \alpha(1 - \varepsilon)} - (\sigma - 1) = 0.
\end{aligned}$$

It follows that there is entry of new firms as long as $\bar{a}^{\tau} < \bar{a}$. The entry of new firms reduces the price index and entry proceeds until

$$\begin{aligned}
P(\tau)^{\sigma-\varepsilon} \tau^{-\alpha(\sigma-1)} &= P^{\sigma-\varepsilon}, \\
P(\tau) &= \tau^{\frac{\alpha(\sigma-1)}{\sigma-\varepsilon}} P,
\end{aligned}$$

at which point

$$\bar{a}(\tau) = \bar{a}.$$

Using the equation for the price index,

$$P(\tau) = \frac{\sigma}{\sigma - 1} c\{\tau w \mu_a[\bar{a}(\tau)]\} n(\tau)^{-\frac{1}{\sigma-1}},$$

we obtain

$$n(\tau) = n\tau^{\frac{\alpha(1-\varepsilon)(\sigma-1)}{\sigma-\varepsilon}}.$$

Hence, there is more entry the higher the tariff. Moreover, with the number of firms n^{τ},

$$\phi(\tau) = \tau\phi,$$
$$\rho(\tau) = \rho.$$

The optimal choice of $\bar{a}(\tau)$,

$$w\{\bar{a}(\tau) - \mu_a[\bar{a}(\tau)]\} = \frac{f}{\beta\tau m(\tau) G[\bar{a}(\tau)]},$$

implies

$$\tau m(\tau) = m.$$

In summary, we have

Lemma 3 *Suppose Assumptions 1-3 hold and $\varepsilon < 1$. Then for small tariffs new firms enter and: (i) $P(\tau) = \tau^{\frac{\alpha(\sigma-1)}{\sigma-\varepsilon}} P$; (ii) $n(\tau) = \tau^{\frac{\alpha(1-\varepsilon)(\sigma-1)}{\sigma-\varepsilon}} n$; (iii) $\bar{a}(\tau) = \bar{a}$; (iv) $\phi(\tau) = \tau\phi$; (v) $\rho(\tau) = \rho$; (vi) $\tau m(\tau) = m$.*

In this equilibrium, entrants and the original producers share the same marginal cost, $\tau\phi$, and they charge the same price p^{τ} for their final goods, as conjectured above.

In the elastic case, $\varepsilon > 1$, equations (15), (16) and (17) imply

$$\hat{\phi}^{\tau} = \frac{\theta + 1 - \gamma^{\tau}}{\theta + 1 - \gamma^{\tau} - \gamma^{\tau}\alpha(\varepsilon - 1)}\hat{\tau} \tag{58}$$

and

$$\hat{\bar{a}}^{\tau} = \frac{\alpha(\varepsilon - 1)}{\theta + 1 - \gamma^{\tau} - \gamma^{\tau}\alpha(\varepsilon - 1)}\hat{\tau}, \tag{59}$$

where

$$\gamma^{\tau} = \frac{(1-\beta)\bar{a}^{\tau}}{\beta\bar{a} + (1-\beta)\bar{a}^{\tau}}.$$

The objective function of a potential entrant is (57). Therefore $\pi'(\tau) > 0$ if and only if $P(\tau)^{\sigma-\varepsilon}\tau^{\alpha(1-\sigma)}$ is rising in τ. However, this time we use (58) and $\theta > \alpha(\sigma - 1)$ to obtain

$$\begin{aligned}
\frac{(\sigma-\varepsilon)\hat{P}^{\tau} - \alpha(\sigma-1)\hat{\tau}}{\alpha\hat{\tau}} &= \frac{(\sigma-\varepsilon)\hat{\phi}^{\tau} - (\sigma-1)\hat{\tau}}{\hat{\tau}} \\
&= \frac{(\theta+1-\gamma^{\tau})(\sigma-\varepsilon)}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha(\varepsilon-1)} - (\sigma-1) \\
&< \frac{[\alpha(\sigma-1)+1-\gamma^{\tau}](\sigma-\varepsilon)}{\alpha(\sigma-1)+1-\gamma^{\tau}-\gamma^{\tau}\alpha(\varepsilon-1)} - (\sigma-1) \\
&= -\frac{(1-\gamma^{\tau})(\varepsilon-1)[\alpha(\sigma-1)+1]}{\alpha(\sigma-1)+1-\gamma^{\tau}-\gamma^{\tau}\alpha(\varepsilon-1)} < 0.
\end{aligned}$$

It follows that potential entrants face negative profits for all small tariff levels. Therefore, we have

Lemma 4 *Suppose Assumptions 1-3 hold and $\varepsilon > 1$. Then for small tariffs there is no entry of new final-good producers and prospective profits of potential entrants decline with the tariff rate.*

Next consider the welfare effects of small tariffs. We showed in the main text that, apart from a constant, welfare can be expressed as

$$V(\tau) = U(X^\tau) - n^\tau \rho^\tau m^\tau - n^\tau \ell^\tau - n^\tau f \left[\frac{1}{G(a_c)} - \frac{1}{G(\bar{a})} \right].$$

In the elastic case, i.e., $\varepsilon > 1$, $a_c = \bar{a}$ and there are no additional search costs. Moreover, there is no entry, so that $n^\tau = n$. Therefore

$$V(\tau) = U(X^\tau) - n \rho^\tau m^\tau - n \ell^\tau$$

and

$$\frac{dV}{d\tau} = P^\tau \frac{dX^\tau}{d\tau} - n \frac{d\ell^\tau}{d\tau} - n\rho^\tau \frac{dm^\tau}{d\tau} - n m^\tau \frac{d\rho^\tau}{d\tau}.$$

The CES aggregator implies that

$$X^\tau = n^{\frac{\sigma}{\sigma-1}} z(\ell^\tau, m^\tau)$$

and therefore

$$\begin{aligned} P^\tau \frac{dX^\tau}{d\tau} &= n^{\frac{\sigma}{\sigma-1}} P^\tau \left(z_\ell \frac{d\ell^\tau}{d\tau} + z_m \frac{dm^\tau}{d\tau} \right) \\ &= n^{\frac{\sigma}{\sigma-1}} \frac{\sigma}{\sigma-1} \frac{P^\tau}{p^\tau} \left(\frac{d\ell^\tau}{d\tau} + \phi^\tau \frac{dm^\tau}{d\tau} \right) \\ &= n \frac{\sigma}{\sigma-1} \left(\frac{d\ell^\tau}{d\tau} + \phi^\tau \frac{dm^\tau}{d\tau} \right). \end{aligned}$$

The second line is obtained from the first by noting that the marginal revenue generated by an increase in an input equals the input's marginal cost, which is one for labor and ϕ^τ for intermediate inputs. The third line is obtained from $P = pn^{-\frac{1}{\sigma-1}}$. Using this result, we obtain

$$\frac{dV}{d\tau} = n \frac{1}{\sigma-1} \frac{d\ell^\tau}{d\tau} + n \left(\frac{\sigma}{\sigma-1} \phi^\tau - \rho^\tau \right) \frac{dm^\tau}{d\tau} - n m^\tau \frac{d\rho^\tau}{d\tau}, \tag{60}$$

which is equation (22) in the main text.

Next, the assumption of a Cobb-Douglas technology implies

$$\ell^\tau = \frac{1-\alpha}{\alpha} \phi^\tau m^\tau$$

and therefore

$$\frac{1}{\sigma-1} \frac{d\ell^\tau}{d\tau} = \frac{1}{\sigma-1} \frac{1-\alpha}{\alpha} \frac{d(\phi^\tau m^\tau)}{d\tau}.$$

However, spending on intermediate inputs is a fraction α of spending on all inputs,

$$n\phi^{\tau}m^{\tau} = \alpha\frac{\sigma-1}{\sigma}P^{\tau}X^{\tau}, \tag{61}$$

and therefore

$$\begin{aligned} n\frac{1}{\sigma-1}\frac{d\ell^{\tau}}{d\tau} &= n\frac{1}{\sigma-1}\frac{1-\alpha}{\alpha}\frac{d\left(\phi^{\tau}m^{\tau}\right)}{d\tau} = \frac{1-\alpha}{\sigma}\frac{d\left(P^{\tau}X^{\tau}\right)}{d\tau} \\ &= -\frac{1-\alpha}{\tau\sigma}\left(\varepsilon-1\right)\alpha\left(\frac{d\phi^{\tau}}{d\tau}\frac{\tau}{\phi^{\tau}}\right)P^{\tau}X^{\tau}. \end{aligned} \tag{62}$$

Using $P^{\tau} = \left(\phi^{\tau}\right)^{\alpha} n^{-\frac{1}{\sigma-1}}\sigma/\left(\sigma-1\right)$, the last equality is obtained from

$$\frac{d\left(P^{\tau}X^{\tau}\right)}{d\tau} = \frac{d\left(P^{\tau}\right)^{1-\varepsilon}}{d\tau} = -\left(\varepsilon-1\right)\alpha\left(\frac{d\phi^{\tau}}{d\tau}\frac{\tau}{\phi^{\tau}}\right)\frac{1}{\tau}P^{\tau}X^{\tau}.$$

Therefore, using (58),

$$n\frac{1}{\sigma-1}\frac{d\ell^{\tau}}{d\tau} = -\frac{1-\alpha}{\tau\sigma}\left(\varepsilon-1\right)\alpha\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau}.$$

This gives us the first term in (60). Since $\varepsilon > 1$, the tariff reduces employment and this has a negative (partial) effect on welfare.

To obtain the second term in (60), we again use (61) and (58), which gives

$$\begin{aligned} n\phi^{\tau}\frac{dm^{\tau}}{d\tau} &= \frac{\sigma-1}{\sigma}\alpha\frac{d\left(P^{\tau}X^{\tau}\right)}{d\tau} - nm^{\tau}\frac{d\phi^{\tau}}{d\tau} \\ &= -\frac{1-\alpha}{\tau\sigma}\left(\varepsilon-1\right)\alpha\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau} - \frac{1}{\tau}nm^{\tau}\phi^{\tau}\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)} \\ &= -\frac{1}{\tau}\frac{\sigma-1}{\sigma}\alpha\left[\left(\varepsilon-1\right)\alpha+1\right]\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau}. \end{aligned}$$

Now, (13) and (16) imply

$$\phi^{\tau} = \tau w\frac{\theta}{\theta+1}\left[\beta\bar{a}+\left(1-\beta\right)\bar{a}^{\tau}\right] \tag{63}$$

and

$$\rho^{\tau} = \beta w\frac{\theta}{\theta+1}\bar{a}+\left(1-\beta\right)w\bar{a}^{\tau}. \tag{64}$$

Therefore,

$$\begin{aligned} n\left(\frac{\sigma}{\sigma-1}\phi^{\tau}-\rho^{\tau}\right)\frac{dm^{\tau}}{d\tau} &= \left(\rho^{\tau}-\frac{\sigma}{\sigma-1}\phi^{\tau}\right)\frac{1}{\tau\phi^{\tau}}\frac{\sigma-1}{\sigma}\alpha\left[\left(\varepsilon-1\right)\alpha+1\right]\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau} \\ &= \left(\frac{\theta+\gamma^{\tau}}{\theta}-\frac{\sigma}{\sigma-1}\tau\right)\frac{1}{\tau^{2}}\frac{\sigma-1}{\sigma}\alpha\left[\left(\varepsilon-1\right)\alpha+1\right]\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau}. \end{aligned}$$

While the tariff reduces demand for the composite intermediate good, the welfare effect is ambiguous for the reasons discussed in the main text. This component of the welfare effect is positive if and only if

$$\frac{\theta + \gamma^\tau}{\theta} > \frac{\sigma}{\sigma - 1}\tau.$$

This is the second term in (60).

To obtain the third term in the welfare formula, we use (64) and (59) to obtain

$$\begin{aligned} nm^\tau \frac{d\rho^\tau}{d\tau} &= wnm^\tau (1-\beta) \frac{d\bar{a}^\tau}{d\tau} \\ &= \frac{1}{\tau} nm^\tau (1-\beta) \frac{\alpha (\varepsilon - 1) w\bar{a}^\tau}{\theta + 1 - \gamma^\tau - \gamma^\tau \alpha (\varepsilon - 1)}. \end{aligned}$$

Next, (61) and (63) imply

$$nm^\tau = \frac{1}{\tau w \frac{\theta}{\theta+1} [\beta \bar{a} + (1-\beta) \bar{a}^\tau]} \frac{\sigma - 1}{\sigma} \alpha P^\tau X^\tau.$$

Therefore,

$$nm^\tau \frac{d\rho^\tau}{d\tau} = \frac{1}{\tau^2} \frac{\theta + 1}{\theta} \gamma^\tau \frac{\sigma - 1}{\sigma} \alpha \frac{\alpha (\varepsilon - 1)}{\theta + 1 - \gamma^\tau - \gamma^\tau \alpha (\varepsilon - 1)} P^\tau X^\tau.$$

So, in this case, $d\rho^\tau / d\tau > 0$; i.e., the terms of trade deteriorate.

Combining the three terms in the expression for the change in welfare, we have

$$\begin{aligned} &\frac{\theta + 1 - \gamma^\tau - \gamma^\tau \alpha (\varepsilon - 1)}{\theta + 1 - \gamma^\tau} \frac{\sigma \tau^2}{\alpha P^\tau X^\tau} \frac{dV}{d\tau} = \\ &- \tau (1-\alpha)(\varepsilon - 1) \\ &+ \left(\frac{\theta + \gamma^\tau}{\theta} - \frac{\sigma}{\sigma - 1}\tau \right) (\sigma - 1) [(\varepsilon - 1)\alpha + 1] \\ &- \frac{\theta + 1}{\theta} (\sigma - 1) \frac{\alpha (\varepsilon - 1) \gamma^\tau}{\theta + 1 - \gamma^\tau}. \end{aligned} \tag{65}$$

A marginal tariff raises welfare if and only if the right-hand side of this equation is positive. Since at free trade $\gamma(1) = 1 - \beta$, it follows that, starting with free trade, a very small tariff reduces welfare if and only if

$$\frac{\theta \varepsilon (\theta + \beta)}{\theta + \beta - (\varepsilon - 1)\alpha (1 - \beta)} > (\sigma - 1)(1 - \beta).$$

Next, note that, holding γ^τ constant, the right-hand side of (65) is declining in τ. Hence, any positive tariff must reduce welfare if

$$\frac{\theta \varepsilon (\theta + 1 - \gamma^\tau)}{\theta + 1 - \gamma^\tau - (\varepsilon - 1)\alpha \gamma^\tau} > (\sigma - 1)\gamma^\tau \quad \text{for all } \tau \geq 1.$$

In the inelastic case, i.e., $\varepsilon < 1$, Lemma 3 describes the equilibrium outcomes. The difference in welfare between this tariff-ridden equilibrium and an equilibrium without the tariff results only from the differences in consumer surplus and tariff revenue, because profits of new entrants are zero and operating profits of incumbents do not change. We therefore have

$$V(\tau) - V(1) = \frac{\varepsilon}{\varepsilon - 1}\left[X(\tau)^{\frac{\varepsilon-1}{\varepsilon}} - X^{\frac{\varepsilon-1}{\varepsilon}}\right] - \left[P(\tau)X(\tau) - PX\right] + (\tau - 1)\rho n(\tau) m(\tau).$$

However, in view of Lemma 3, (61) implies

$$\tau\phi n(\tau) m(\tau) = \alpha\frac{\sigma - 1}{\sigma} P(\tau) X(\tau),$$

and therefore

$$V(\tau) - V(1) = \frac{\varepsilon}{\varepsilon - 1}\left[X(\tau)^{\frac{\varepsilon-1}{\varepsilon}} - X^{\frac{\varepsilon-1}{\varepsilon}}\right] - \left[P(\tau)X(\tau) - PX\right] + \frac{\tau - 1}{\tau}\frac{\rho}{\phi}\alpha\frac{\sigma - 1}{\sigma} P(\tau) X(\tau).$$

Since, in this case,

$$\begin{aligned}
\rho &= \frac{\theta + 1 - \beta}{\theta + 1} w\bar{a}, \\
\phi &= \frac{\theta}{\theta + 1} w\bar{a}, \\
\frac{\rho}{\phi} &= \frac{\theta + 1 - \beta}{\theta},
\end{aligned}$$

we can use $X(\tau) = P(\tau)^{-\varepsilon}$ and Lemma 3(i) to obtain

$$\begin{aligned}
\frac{V(\tau) - V(1)}{P^{1-\varepsilon}} &= -\frac{\varepsilon}{1 - \varepsilon}\left[\tau^{\frac{\alpha(\sigma-1)(1-\varepsilon)}{\sigma-\varepsilon}} - 1\right] - \left[\tau^{\frac{\alpha(\sigma-1)(1-\varepsilon)}{\sigma-\varepsilon}} - 1\right] + \frac{\tau - 1}{\tau}\frac{\theta + 1 - \beta}{\theta}\alpha\frac{\sigma - 1}{\sigma}\tau^{\frac{\alpha(\sigma-1)(1-\varepsilon)}{\sigma-\varepsilon}} \\
&= \frac{1}{1 - \varepsilon} + \left(\frac{\tau - 1}{\tau}\frac{\theta + 1 - \beta}{\theta}\alpha\frac{\sigma - 1}{\sigma} - \frac{1}{1 - \varepsilon}\right)\tau^{\frac{\alpha(\sigma-1)(1-\varepsilon)}{\sigma-\varepsilon}}.
\end{aligned}$$

This implies

$$\frac{\tau^{-\frac{\alpha(\sigma-1)(1-\varepsilon)}{\sigma-\varepsilon}+1}}{\alpha(\sigma - 1) P^{1-\varepsilon}} V'(\tau) = \frac{1}{\tau}\frac{\theta + 1 - \beta}{\theta\sigma} + \frac{1 - \varepsilon}{\sigma - \varepsilon}\left[\left(1 - \frac{1}{\tau}\right)\frac{\theta + 1 - \beta}{\theta\sigma}\alpha(\sigma - 1) - \frac{1}{1 - \varepsilon}\right]. \tag{66}$$

Since $\sigma > \varepsilon + (1 - \varepsilon)\alpha(\sigma - 1)$, the right-hand side of (66) is declining in τ. Welfare declines in τ for all $\tau > 1$ if it declines at $\tau = 1$. Note that

$$\begin{aligned}
\operatorname{sign}\left[V'(1)\right] &= \operatorname{sign}\left[\frac{\theta + 1 - \beta}{\theta\sigma} - \frac{1}{\sigma - \varepsilon}\right] \\
&= \operatorname{sign}\left[(1 - \beta)(\sigma - \varepsilon) - \theta\varepsilon\right].
\end{aligned}$$

Therefore,

$$V'(1) < 0 \;\text{ if and only if }\; \theta > (1 - \beta)\left(\frac{\sigma}{\varepsilon} - 1\right).$$

This is more likely to be satisfied the larger is the bargaining power of the buyers, the lower is the elasticity of substitution, and the larger is the elasticity of demand for differentiated products as a whole.

Section 4 Large, Unanticipated Tariffs

In this section, the outside option for buyers is to search for new suppliers in country B. The outside option is the same when a buyer bargains with a supplier in country A as when it bargains with one in country B. Since there are no tariffs on inputs purchased in country B, the bargaining game with a supplier in country B yields

$$\rho_B(b,\tau) = \arg\max_q \left[qm(\tau) - w_B bm(\tau)\right]^{1-\beta} \left[w_B \mu_b\left[\bar{b}(\tau)\right]m(\tau) + \frac{f}{\beta G\left[\bar{b}(\tau)\right]} - qm(\tau)\right]^{\beta}.$$

The first-order condition for this problem is

$$\frac{1-\beta}{\rho_B(b,\tau) - w_B b} = \frac{\beta}{w_B\mu_b\left[\bar{b}(\tau)\right] + \frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]} - \rho_B(b,\tau)},$$

and therefore

$$\rho_B(b,\tau) = \beta w_B b + (1-\beta) w_B \mu_b\left[\bar{b}(\tau)\right] + (1-\beta)\frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]}. \tag{67}$$

Taking the conditional mean of both sides of this equation for $b \leq \bar{b}(\tau)$, yields

$$\mu_{\rho_B}\left[\bar{b}(\tau)\right] = w_B\mu_b\left[\bar{b}(\tau)\right] + \frac{1-\beta}{\beta}\frac{f}{m(\tau) G\left[\bar{b}(\tau)\right]}. \tag{68}$$

Now use the first-order condition for $\bar{b}(\tau)$ that minimizes costs,

$$w_B\left\{\bar{b}(\tau) - \mu_b\left[\bar{b}(\tau)\right]\right\} = \frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]}, \tag{69}$$

to obtain

$$\rho_B(b,\tau) = \beta w_B b + (1-\beta) w_B \bar{b}(\tau). \tag{70}$$

Note that this cost of inputs depends on the tariff only through $\bar{b}(\tau)$ and it is the same for the original producers and new entrants.

Bargaining with suppliers in country A yields

$$\rho_A(a,\tau) = \arg\max_q \left[qm(\tau) - w_A am(\tau)\right]^{1-\beta} \left[w_B \mu_b\left[\bar{b}(\tau)\right]m(\tau) + \frac{f}{\beta G\left[\bar{b}(\tau)\right]} - \tau qm(\tau)\right]^{\beta}.$$

The first-order condition for this problem is

$$\frac{1-\beta}{\rho_A(a,\tau)-w_A a}=\frac{\beta\tau}{w_B\mu_b\left[\bar{b}(\tau)\right]+\frac{f}{m(\tau)G\left[\bar{b}(\tau)\right]}-\tau\rho_A(a,\tau)}$$

and therefore

$$\tau\rho_A(a,\tau)=\beta\tau w_A a+(1-\beta)w_B\mu_b\left[\bar{b}(\tau)\right]+(1-\beta)\frac{f}{\beta m(\tau)G\left[\bar{b}(\tau)\right]}. \tag{71}$$

Substituting (68) and (69) into this equation we obtain

$$\rho_A(a,\tau)=\beta w_A a+(1-\beta)w_B\frac{\bar{b}(\tau)}{\tau}. \tag{72}$$

This negotiated price depends on τ through the ratio $\bar{b}(\tau)/\tau$. In these circumstances, it is cheaper to source an input a from country A if

$$\tau\rho_A(a,\tau)\leq\mu_{\rho_B}\left[\bar{b}(\tau)\right]+\frac{f}{m(\tau)G\left[\bar{b}(\tau)\right]}.$$

Using (68) and (69), the right-hand side of this inequality equals $w_B\bar{b}(\tau)$. Therefore this inequality can be expressed as

$$\tau w_A a\leq w_B\bar{b}(\tau).$$

From this result we have

Lemma 5 *For given $\bar{b}(\tau)$, the cost minimizing cutoff a_B is*

$$a_B=\min\left\{\frac{w_B\bar{b}(\tau)}{\tau w_A},\bar{a}\right\}. \tag{73}$$

Now consider the perceived marginal cost of the composite intermediate good for one of the original producers. From (67), we see that the average marginal cost of sourcing from country B is $w_B\mu_b\left[\bar{b}(\tau)\right]$, while from (71) we see that the average marginal cost of sourcing from country A is $\beta\tau w_A\mu_a(a_B)+(1-\beta)w_B\mu_b\left[\bar{b}(\tau)\right]$. Since an incumbent firm sources a fraction $G(a_B)/G(\bar{a})$ of its inputs from country A and the remaining fraction $1-G(a_B)/G(\bar{a})$ from country B, its marginal cost of the intermediate input is

$$\begin{aligned}\phi^\tau&=\frac{G(a_B)}{G(\bar{a})}\left[\beta\tau w_A\mu_a(a_B)+(1-\beta)w_B\mu_b\left(\bar{b}^\tau\right)\right]+\left[1-\frac{G(a_B)}{G(\bar{a})}\right]w_B\mu_b\left(\bar{b}^\tau\right)\\&=\beta\frac{G(a_B)}{G(\bar{a})}\tau w_A\mu_a(a_B)+\left[1-\beta\frac{G(a_B)}{G(\bar{a})}\right]w_B\mu_b\left(\bar{b}^\tau\right),\end{aligned}$$

where we have replace the function $\bar{b}(\tau)$ with the value of $\bar{b}$ at the tariff level τ, $\bar{b}^\tau$. Using (73) and properties of the Pareto distribution yields the equation for the MM curve,

$$\phi^{\tau} = \begin{cases} \frac{\theta}{\theta+1} w_B \bar{b}^{\tau} & \text{for } \bar{b}^{\tau} < \tau w_A \bar{a}/w_B \\ \frac{\theta}{\theta+1} \left[\beta \tau w_A \bar{a} + (1-\beta) w_B \bar{b}^{\tau}\right] & \text{for } \bar{b}^{\tau} > \tau w_A \bar{a}/w_B \end{cases} . \tag{74}$$

New entrants (if any exist) search for suppliers only in country B. Equation (68) implies that an entrant's marginal cost is

$$\phi^{\tau}_{new} = w_B \mu_b \left(\bar{b}^{\tau}\right) = \frac{\theta}{\theta+1} w_B \bar{b}^{\tau}. \tag{75}$$

For the tariff level $\tau = w_B/w_A$, the equilibrium values are $\bar{b}^{\tau} = \bar{a}$ and $\phi^{\tau}_{new} = \phi^{\tau} = \tau\phi = \frac{\theta}{\theta+1} w_B \bar{a}$.

We next derive the equation for the NN curve, first for the case of $\varepsilon > 1$ and then for $\varepsilon < 1$. In either case, we have (69). As we explained in the previous section, when all the firms are identical, m^{τ}, the volume of imported intermediate goods, is given by (see (51))

$$\begin{aligned} m^{\tau} &= \left(p^{\tau}\right)^{-\varepsilon} \left(n^{\tau}\right)^{-\frac{\sigma-\varepsilon}{\sigma-1}} c'\left(\phi^{\tau}\right) \\ &= \left[\frac{\sigma}{\sigma-1} c\left(\phi^{\tau}\right)\right]^{-\varepsilon} \left(n^{\tau}\right)^{-\frac{\sigma-\varepsilon}{\sigma-1}} c'\left(\phi^{\tau}\right) \\ &= \alpha \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \left(n^{\tau}\right)^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\phi^{\tau}\right)^{\alpha(1-\varepsilon)-1}, \end{aligned} \tag{76}$$

where $n^{\tau} = n$ in the elastic case. Since higher tariffs do not raise profits when $\varepsilon > 1$, there is no entry of new firms. Substituting the expression for m^{τ} into (69) yields

$$\frac{(\theta+1) f}{w_B \beta \left(\bar{b}^{\tau}\right)^{\theta+1}} = n^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha \left(\phi^{\tau}\right)^{\alpha(1-\varepsilon)-1} , \tag{77}$$

which is the NN curve in the elastic case. It follows that the elasticity of the NN curve in this case is $(\theta+1)/[1-\alpha(1-\varepsilon)]$, which is larger than one under Assumption 3 for all $\varepsilon < \sigma$. From (74), the slope of the MM curve is smaller than one and therefore NN is steeper at the intersection point of the two curves, as drawn in Figure 5.

In the inelastic case ($\varepsilon < 1$) we derive the NN curve as follows. A new entrant who searches for suppliers in country B, has zero profits, so

$$0 = \max_b \left(P^{\tau}\right)^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} c\left[w_B \mu_b(b)\right]^{1-\sigma} - \frac{f}{\beta G(b)} - f_o - f_e ,$$

where

$$\begin{aligned} c\left[w_B \mu_b(b)\right] &= \left(w_B \frac{\theta}{\theta+1} b\right)^{\alpha}, \\ G(b) &= b^{\theta}. \end{aligned}$$

Let $\bar{b}^{\tau}_{new}$ be the optimal search strategy for an entrant. The first-order condition for an interior solution for $\bar{b}^{\tau}_{new}$ is

$$-\alpha \left(P^{\tau}\right)^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}} \left(w_B \frac{\theta}{\theta+1} \bar{b}^{\tau}_{new}\right)^{-\alpha(\sigma-1)-1} \left(w_B \frac{\theta}{\theta+1}\right) + \theta \frac{f}{\beta \left(\bar{b}^{\tau}_{new}\right)^{\theta+1}} = 0, \tag{78}$$

and the zero profit condition becomes

$$\left(P^{\tau}\right)^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} \left(w_B \frac{\theta}{\theta+1} \bar{b}^{\tau}_{new}\right)^{-\alpha(\sigma-1)} - \frac{f}{\beta \left(\bar{b}^{\tau}_{new}\right)^{\theta}} = f_o + f_e. \tag{79}$$

The last two equations provide a solution to the cutoff $\bar{b}^{\tau}_{new}$ and the price index P^{τ}. These are the same conditions that lead to (43):

$$\left(\bar{b}^{\tau}_{new}\right)^{\theta} = \frac{f}{f_o + f_e} \frac{\theta - \alpha(\sigma-1)}{\beta\alpha(\sigma-1)}. \tag{80}$$

Note that this solution does not depend on the wage rate or the tariff rate. It follows that new entrants choose the same cutoff that the original entrants chose in country A when they entered the industry. That is,

$$\bar{b}^{\tau}_{new} = \bar{a}.$$

And the solution to the price index is,

$$\left(P^{\tau}\right)^{\sigma-\varepsilon} = \frac{\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}} \left(f_o + f_e + \frac{f}{\beta \bar{a}^{\theta}}\right) \left(w_B \frac{\theta}{\theta+1} \bar{a}\right)^{\alpha(\sigma-1)},$$

which is independent of the size of the tariff for $\tau > w_B/w_A$. The price index under free trade is

$$(P)^{\sigma-\varepsilon} = \frac{\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}} \left(f_o + f_e + \frac{f}{\beta \bar{a}^{\theta}}\right) \left(w_A \frac{\theta}{\theta+1} \bar{a}\right)^{\alpha(\sigma-1)},$$

so we have

$$\left(P^{\tau}\right)^{\sigma-\varepsilon} = P^{\sigma-\varepsilon} \left(\frac{w_B}{w_A}\right)^{\alpha(\sigma-1)} > P^{\sigma-\varepsilon}. \tag{81}$$

Moreover, P^{τ} for τ slightly above w_B/w_A is the same as the price index in the small-tariff setting when τ approaches w_B/w_A from below.

Although P^{τ} does not depend on the tariff rate, changes in the tariff rate might nonetheless affect the scale of entry by new firms if the tariff impacts the prices charged by the original producers, because (2) implies

$$\left(P^{\tau}\right)^{1-\sigma} = \frac{\sigma}{\sigma-1} \left[n \left(\phi^{\tau}\right)^{\alpha(1-\sigma)} + n^{\tau}_{new} \left(w_B \frac{\theta}{\theta+1} \bar{a}\right)^{\alpha(1-\sigma)}\right], \tag{82}$$

where n is the number of original entrants, ϕ^{τ} is their marginal cost and n^{τ}_{new} is the number of new entrants. If higher tariffs were to lead to higher marginal costs ϕ^τ, they would also lead to fewer entrants. However, we will soon see that ϕ^τ does not change with the tariff level as long as $\tau \geq w_B/w_A$ so that, in fact, n^τ_{new} is independent of the tariff rate for all $\tau > w_B/w_A$.

To derive the equilibrium marginal cost of the incumbents, ϕ^τ, we first note that the MM curve defined by (74) applies to this case too. Next use the first-order condition for $\bar{b}^\tau$ that minimizes costs of m^τ (for an original producer),

$$w_B\left[\bar{b}^\tau - \mu_b\left(\bar{b}^\tau\right)\right] = \frac{f}{\beta m^\tau G\left(\bar{b}^\tau\right)},$$

to obtain

$$w_B\left(\bar{b}^\tau\right)^{\theta+1} = \frac{f\left(\theta+1\right)}{\beta m^\tau}.$$

However,

$$\begin{aligned} m^\tau = x^\tau c'\left(\phi^\tau\right) &= \left(P^\tau\right)^{\sigma-\varepsilon}\left[\frac{\sigma}{\sigma-1}c\left(\phi^\tau\right)\right]^{-\sigma} c'\left(\phi^\tau\right) \\ &= \alpha\left(P^\tau\right)^{\sigma-\varepsilon}\left[\frac{\sigma}{\sigma-1}\left(\phi^\tau\right)^\alpha\right]^{-\sigma}\left(\phi^\tau\right)^{\alpha-1}. \end{aligned}$$

Substituting this expression for m^τ into the previous equation yields the upward sloping NN curve when demand is inelastic, namely

$$w_B\left(\bar{b}^\tau\right)^{\theta+1} = \left(\frac{\sigma}{\sigma-1}\right)^\sigma \frac{f\left(\theta+1\right)}{\beta\alpha\left(P^\tau\right)^{\sigma-\varepsilon}}\left(\phi^\tau\right)^{1+\alpha(\sigma-1)}. \tag{83}$$

In this equation P^τ is a constant, given by (81). The elasticity of the NN curve now is

$$\frac{\theta+1}{1+\alpha\left(\sigma-1\right)} > 1.$$

Equations (74) and (83) yield the solution $\bar{b}^\tau = \bar{a}$ and $\phi^\tau = \frac{\theta}{\theta+1}w_B\bar{a}$ for $\tau = w_B/w_A$, as depicted in Figure 6. For higher tariff rates, $\tau > w_B/w_A$, the price index P^τ remains the same and therefore only the MM curve shifts, as explained in the main text. As a result, in the inelastic case, $\bar{b}^\tau = \bar{a}$ and $\phi^\tau = \frac{\theta}{\theta+1}w_B\bar{a}$ for all $\tau \geq w_B/w_A$.

Under these circumstances (82) implies that the number of new entrants also is constant and equal to $n^\tau_{new} = n\left(w_B/w_A\right) - n$. Moreover, m^τ is constant and $m^\tau_{new} = m^\tau$, and all firms face the same marginal cost, be they incumbents or new entrants. It follows that all employ the same amount of labor. As a result, the difference between revenue and production costs do not change and profits net of entry costs are zero for all entrants. It follows that changes in welfare result from changes in tariff revenue, which is generated from the remaining imports from country A by the original producers and by changes in the search costs in country B by these firms. We summarize these findings in:

Lemma 6 *Suppose Assumptions 1-3 hold and* $\varepsilon < 1$. *Then for all* $\tau \geq w_B/w_A$, $\bar{b}^\tau = \bar{b}^\tau_{new} = \bar{a}$ *and* $\phi^\tau = \phi^\tau_{new} = \frac{\theta}{\theta+1} w_B \bar{a}$ *and* $n^\tau_{new} = n(w_B/w_A) - n$.

Now consider the response of ϕ^τ and $\bar{b}^\tau$ to tariff changes, beginning with the elastic case. First suppose that τ is such that $\bar{b}^\tau < \tau w_A \bar{a}/w_B$. In this case, there is sourcing from both countries and (74) and (77) imply that neither ϕ^τ nor $\bar{b}^\tau$ change as long as tariffs remain in the region with $\bar{b}^\tau < \tau w_A \bar{a}/w_B$. In contrast, consider an increase in the tariff when $\bar{b}^\tau > \tau w_A \bar{a}/w_B$. Then (74) and (77) imply

$$\hat{\phi}^\tau = \gamma_B \hat{\bar{b}}^\tau + (1-\gamma_B)\hat{\tau},$$
$$(\theta+1)\hat{\bar{b}}^\tau = [1+\alpha(\varepsilon-1)]\hat{\phi}^\tau,$$

where

$$\gamma_B = \frac{(1-\beta) w_B \bar{b}^\tau}{\beta \tau w_A \bar{a} + (1-\beta) w_B \bar{b}^\tau}.$$

Therefore,

$$\hat{\phi}^\tau = \frac{(\theta+1)(1-\gamma_B)}{\theta+1-\gamma_B-\gamma_B\alpha(\varepsilon-1)}\hat{\tau}, \quad (84)$$

$$\hat{\bar{b}}^\tau = \frac{[1-\alpha(1-\varepsilon)](1-\gamma_B)}{\theta+1-\gamma_B-\gamma_B\alpha(\varepsilon-1)}\hat{\tau}. \quad (85)$$

The numerators and the denominators of both equations are positive, implying that higher tariffs raise the cutoff and the marginal costs of intermediate inputs. Moreover, note from (85) that

$$\hat{\bar{b}}^\tau - \hat{\tau} = -\frac{(1-\gamma_B)[\theta-\alpha(\varepsilon-1)]}{\theta+1-\gamma_B-\gamma_B\alpha(\varepsilon-1)}\hat{\tau}.$$

The denominator on the right-hand side of this equation is positive. The numerator is negative under Assumption 3, because $\sigma > \varepsilon$. We conclude that the ratio $\bar{b}^\tau/\tau$ is declining with the tariff level.

As shown in the text, for $\tau \in (w_B/w_A, \tau_c)$ we have $\bar{b}^\tau > \tau w_A \bar{a}/w_B$, where τ_c is the tariff level at which $\tau_c w_A \bar{a} = w_B \bar{b}(\tau_c)$. For tariffs in this range, a higher tariff raises both ϕ^τ and $\bar{b}^\tau$ according to (84) and (85). In contrast, ϕ^τ and $\bar{b}^\tau$ are invariant to the tariff rate for all $\tau > \tau_c$. In this range, $a_B = w_B \bar{b}(\tau_c)/\tau w_A$ and $\bar{b}^\tau = \bar{b}(\tau_c)$, so we can express the weighted average of the foreign cost of the inputs using (70) and (72) as

$$\begin{aligned}\rho^\tau &= \frac{G(a_B)}{G(\bar{a})}\left[\beta w_A \mu_a(a_B) + (1-\beta) w_B \frac{\bar{b}^\tau}{\tau}\right] + \left[1-\frac{G(a_B)}{G(\bar{a})}\right]\left[\beta w_B \mu_b\left(\bar{b}^\tau\right) + (1-\beta) w_B \bar{b}^\tau\right] \\ &= \left(\frac{\tau_c}{\tau}\right)^\theta \frac{\theta+1-\beta}{\theta+1}\frac{w_B \bar{b}^\tau}{\tau} + \left[\left\{1-\left(\frac{\tau_c}{\tau}\right)^\theta\right\}\right]\frac{\theta+1-\beta}{\theta+1} w_B \bar{b}^\tau.\end{aligned}$$

The second line reveals the offsetting effects on the terms of trade: ρ^τ declines as a result of the decline in prices paid to suppliers in country A, but it rises with reallocation of supply from country A to country B, because net-of-tariff costs are higher in country B. The combined impact can be seen by rewriting the equation for ρ^{τ} as

$$\rho^{\tau} = \left\{1 - \frac{\tau - 1}{\tau}\left(\frac{\tau_c}{\tau}\right)^{\theta}\right\}\frac{\theta + 1 - \beta}{\theta + 1}w_B\bar{b}^{\tau}.$$

From this, we obtain

Lemma 7 *Suppose $\varepsilon > 1$. Then for $\tau > \tau_c$, higher tariffs generate better terms of trade if and only if*

$$\tau < \frac{\theta + 1}{\theta}.$$

We now examine the welfare effects of tariffs for $\tau > w_B/w_A$, beginning with the case of elastic demand, i.e., $\varepsilon > 1$. First, consider tariffs in the range $\tau \in (w_B/w_A, \tau_c)$. In this range, there are no new searches by any of the incumbent producers and country A continues to supply all intermediate inputs. As a result, tariffs are imposed on all imports, generating a revenue of $(\tau - 1)\,\rho^{\tau}m^{\tau}$. Tariff revenue plus variable profits plus consumer surplus sum to

$$\begin{aligned} V(\tau) &= T(\tau) + \Pi(\tau) + \Gamma(\tau) \\ &= (\tau - 1)\,\rho^{\tau}m^{\tau} + [P^{\tau}X^{\tau} - \tau\rho^{\tau}nm^{\tau} - n\ell^{\tau}] + [U(X^{\tau}) - P^{\tau}X^{\tau}] \\ &= U(X^{\tau}) - \rho^{\tau}nm^{\tau} - n\ell^{\tau}. \end{aligned}$$

Differentiating this equation gives

$$\begin{aligned} \frac{1}{n}\frac{dV}{d\tau} &= \frac{1}{n}P^{\tau}\frac{dX^{\tau}}{d\tau} - \frac{d\ell^{\tau}}{d\tau} - \rho^{\tau}\frac{dm^{\tau}}{d\tau} - m^{\tau}\frac{d\rho^{\tau}}{d\tau} \\ &= \left(\frac{\sigma}{\sigma - 1} - 1\right)\frac{d\ell^{\tau}}{d\tau} + \left(\frac{\sigma}{\sigma - 1}\phi^{\tau} - \rho^{\tau}\right)\frac{dm^{\tau}}{d\tau} - m^{\tau}\frac{d\rho^{\tau}}{d\tau}. \end{aligned}$$

We have shown that, in this range, $\bar{b}^{\tau}$ is larger for larger tariffs whereas $\bar{b}^{\tau}/\tau$ is smaller for larger tariffs. The optimal choice of $\bar{b}^{\tau}$ for a given m^{τ}, equation (69), therefore implies that m^{τ} declines with the tariff, while (72) implies that ρ^{τ} declines. For these reasons, the change in the sourcing of intermediate inputs raises welfare if and only if

$$\frac{\sigma}{\sigma - 1}\frac{\phi^{\tau}}{\rho^{\tau}} = \tau\frac{\sigma}{\sigma - 1}\frac{\frac{\theta}{\theta+1}\left[\beta\tau w_A\bar{a} + (1 - \beta)\,w_B\bar{b}^{\tau}\right]}{\frac{\theta}{\theta+1}\beta\tau w_A\bar{a} + (1 - \beta)\,w_B\bar{b}^{\tau}} = \frac{\sigma}{\sigma - 1}\frac{\theta\tau}{\theta + \gamma_B} < 1.$$

Meanwhile, better terms of trade always contribute to higher welfare. Finally, since

$$n\ell^{\tau} = (1 - \alpha)\,\frac{\sigma - 1}{\sigma}P^{\tau}X^{\tau}$$

and ϕ^{τ} rises with the tariff level, it follows that $P^{\tau}X^{\tau}$ declines with the size of the tariff in the elastic case. As a result, ℓ^{τ} declines, which reduces welfare, all else the same. Clearly, in this case, a marginal increase in the tariff rate may increase or reduce welfare.

We next consider $\tau > \tau_c$ for the elastic case. In this range, $d\ell^\tau/d\tau = dm^\tau/d\tau = dX^\tau/d\tau = dP^\tau/d\tau = 0$, because neither ϕ^τ nor $\bar{b}^\tau$ vary with the size of the tariff. As a result,

$$\frac{dV}{d\tau} = -nm^\tau \frac{d\rho^\tau}{d\tau} - \frac{d\Sigma}{d\tau},$$

where $\Sigma(\tau)$ is the cost of the new searches that take place by incumbent producers. Using (69) and $a_B = \frac{w_B \bar{b}(\tau_c)}{\tau w_A}$, the cost of new searches amounts to

$$\begin{aligned}\Sigma &= n\left[1 - \frac{G(a_B)}{G(\bar{a})}\right]\frac{f}{G\left[\bar{b}(\tau_c)\right]} \\ &= nm^t\left[1 - \left(\frac{\tau_c}{\tau}\right)^\theta\right]\frac{\beta}{\theta+1} w_B \bar{b}(\tau_c).\end{aligned}$$

Therefore, the variation in the search cost that results from a slightly higher tariff is

$$\frac{d\Sigma}{d\tau} = nm^\tau \frac{\theta}{\tau^{\theta+1}}(\tau_c)^\theta \frac{\beta}{\theta+1} w_B \bar{b}(\tau_c).$$

The terms of trade now are a weighted average of the cost of sourcing from country A and the cost of sourcing from country B,

$$\rho^\tau = \frac{G(a_B)}{G(\bar{a})}\left[\beta w_A \mu_a(a_B) + (1-\beta) w_B \frac{\bar{b}^\tau}{\tau}\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right] w_B \left[\beta \mu_b\left(\bar{b}^\tau\right) + (1-\beta)\bar{b}^\tau\right].$$

The first term on the right-hand side represents the fraction of goods sourced from country A, $G(a_B)/G(\bar{a})$, times the average cost of goods sourced from that country, while the second term represents the fraction of goods sourced from country B times the average cost of those inputs. Using $a_B = \frac{w_B \bar{b}(\tau_c)}{\tau w_A}$ and properties of the Pareto distribution, this equation becomes

$$\begin{aligned}\rho^\tau &= \left(\frac{\tau_c}{\tau}\right)^\theta \frac{\theta+1-\beta}{\theta+1}\frac{w_B \bar{b}(\tau_c)}{\tau} + \left[1 - \left(\frac{\tau_c}{\tau}\right)^\theta\right]\frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c) \\ &= \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c)\left[1 - \frac{\tau-1}{\tau^{\theta+1}}(\tau_c)^\theta\right],\end{aligned}$$

$$\frac{d\rho^\tau}{d\tau} = \frac{\theta(\tau-1)-1}{\tau^{\theta+2}}(\tau_c)^\theta \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c)$$

Since the right-hand side of the last equation is negative if and only if

$$\tau < \frac{\theta+1}{\theta},$$

it follows that the terms of trade improve if $\tau < (\theta+1)/\theta$ and deteriorate if $\tau > (\theta+1)/\theta$.

Combining terms, we now have

$$
\begin{aligned}
\frac{1}{nm^{\tau}}\frac{dV}{d\tau} &= -\frac{d\rho^{\tau}}{d\tau} - \frac{1}{n^{\tau}m^{\tau}}\frac{d\Sigma}{d\tau} \\
&= w_B\bar{b}\left(\tau_c\right)\frac{\theta+1-\beta-\theta\tau}{\tau^{\theta+2}}\left(\tau_c\right)^{\theta}.
\end{aligned}
$$

Therefore, welfare rises with the tariff for $\tau > \tau_c$ if and only if

$$
\tau < \frac{\theta+1-\beta}{\theta}.
$$

In the main text, we displayed in Figure 8 the relationship between V and τ for $\sigma = 5$, $\theta = 4, \varepsilon = 1$, $\alpha = \beta = 0,5$, $f_e = f_o = 10$, $f = 5$, $w_A = 0.5$ and $w_B = 0.6$. The following figures shows the relationship when $w_B = 0.55$, which implies a smaller gap between wages in country A and country B and thus a relocation of a greater portion of the supply chain for any $\tau > 1.1$.

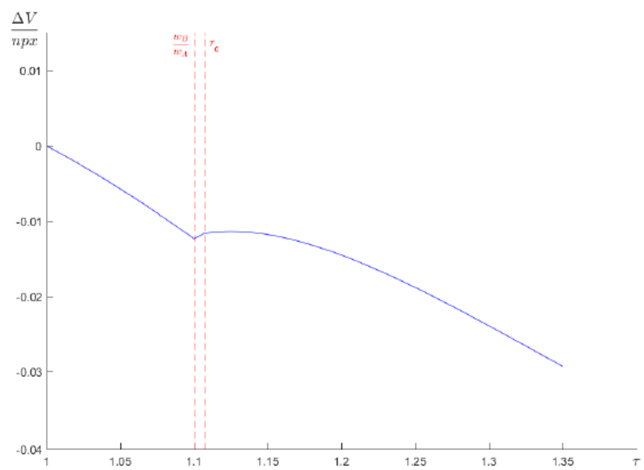


Figure A2: Welfare Effects of Unanticipated Tariffs: Small Wage Gap

Evidently, the overall welfare effects of a tariff of any given size are quite similar in these alternative scenarios.

When the label B denotes the home country, the social cost of inputs is

$$
\rho^{\tau} = \frac{G\left(a_B\right)}{G\left(\bar{a}\right)}\left[\beta w_A\mu_a\left(a_B\right) + (1-\beta)\frac{\bar{b}^{\tau}}{\tau}\right] + \left[1 - \frac{G\left(a_B\right)}{G\left(\bar{a}\right)}\right] w_B\mu_b\left(\bar{b}^{\tau}\right),
$$

where the second term now represents the cost of producing inputs at home. Using properties of the Pareto distribution and $a_B = \frac{w_B\bar{b}(\tau_c)}{\tau w_A}$, we have

$$
\rho^{\tau} = \left(\frac{\tau_c}{\tau}\right)^{\theta}\frac{\theta+1-\beta}{\theta+1}\frac{w_B\bar{b}\left(\tau_c\right)}{\tau} + \left[1 - \left(\frac{\tau_c}{\tau}\right)^{\theta}\right]\frac{\theta}{\theta+1}w_B\bar{b}\left(\tau_c\right),
$$

$$\begin{aligned}
\frac{d\rho^{\tau}}{d\tau} &= -\frac{\theta+1}{\tau^{\theta+2}}\left(\tau_c\right)^{\theta}\frac{\theta+1-\beta}{\theta+1}w_B\bar{b}\left(\tau_c\right)+\frac{\theta}{\tau^{\theta+1}}\left(\tau_c\right)^{\theta}\frac{\theta}{\theta+1}w_B\bar{b}\left(\tau_c\right)\\
&= \frac{1}{\left(\theta+1\right)\tau^{\theta+2}}\left(\tau_c\right)^{\theta}\left[\tau\theta^2-\left(\theta+1\right)\left(\theta+1-\beta\right)\right]w_B\bar{b}\left(\tau_c\right).
\end{aligned}$$

In this case, the resource cost of inputs declines with the tariff if and only if

$$\tau<\frac{\left(\theta+1\right)\left(\theta+1-\beta\right)}{\theta^2}.$$

The effect of a higher tariff on social welfare can now be expressed as

$$\begin{aligned}
\frac{1}{n^{\tau}m^{\tau}}\frac{dV}{d\tau} &= -\frac{d\rho^{\tau}}{d\tau}-\frac{1}{n^{\tau}m^{\tau}}\frac{d\Sigma}{d\tau}\\
&= -\frac{1}{\left(\theta+1\right)\tau^{\theta+2}}\left(\tau_c\right)^{\theta}\left[\tau\theta^2-\left(\theta+1\right)\left(\theta+1-\beta\right)\right]w_B\bar{b}\left(\tau_c\right)\\
&\quad -\frac{\theta}{\tau^{\theta+1}}\left(\tau_c\right)^{\theta}\frac{\beta}{\theta+1}w_B\bar{b}\left(\tau_c\right)\\
&= w_B\bar{b}\left(\tau_c\right)\frac{-\tau\theta^2+\left(\theta+1\right)\left(\theta+1-\beta\right)-\beta\theta\tau}{\left(\theta+1\right)\tau^{\theta+2}}\left(\tau_c\right)^{\theta}.
\end{aligned}$$

Therefore, welfare rises with the tariff if and only if

$$\tau<\frac{\left(\theta+1\right)\left(\theta+1-\beta\right)}{\theta\left(\theta+\beta\right)}.$$

Next, we derive an equation for τ_c. From (38) we have

$$\frac{1}{\theta+1}w_A\bar{a}=\frac{f}{\beta m\bar{a}^{\theta}},$$

where m is the volume of intermediates in the free-trade equilibrium, before any tariff is imposed. From (69) we have

$$\frac{1}{\theta+1}w_B\bar{b}\left(\tau_c\right)=\frac{f}{\beta m\left(\tau_c\right)\bar{b}\left(\tau_c\right)^{\theta}}$$

when the tariff is τ_c. Therefore,

$$\frac{w_B\bar{b}\left(\tau_c\right)^{\theta+1}}{w_A\bar{a}^{\theta+1}}=\frac{m}{m\left(\tau_c\right)}.$$

However, from (76),

$$\begin{aligned}
m &= \alpha\left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon}n^{-\frac{\sigma-\varepsilon}{\sigma-1}}\left(\frac{\theta}{\theta+1}w_A\bar{a}\right)^{\alpha(1-\varepsilon)-1},\\
m\left(\tau_c\right) &= \alpha\left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon}n^{-\frac{\sigma-\varepsilon}{\sigma-1}}\phi\left(\tau_c\right)^{\alpha(1-\varepsilon)-1}.
\end{aligned}$$

However, (74) implies that,

$$\phi\left(\tau_c\right)=\frac{\theta}{\theta+1}w_B\bar{b}\left(\tau_c\right)=\frac{\theta}{\theta+1}\tau_c w_A\bar{a}$$

and therefore,

$$\frac{w_B \bar{b}(\tau_c)^{\theta+1}}{w_A \bar{a}^{\theta+1}} = \left(\frac{w_A}{w_B}\right)^{\theta} (\tau_c)^{\theta+1} = \frac{m}{m(\tau_c)} = \frac{1}{(\tau_c)^{\alpha(1-\varepsilon)-1}}.$$

It follows that,

$$\tau_c = \left(\frac{w_B}{w_A}\right)^{\frac{\theta}{\theta-\alpha(\varepsilon-1)}}.$$

Since $\tau_c w_A \bar{a} = w_B \bar{b}(\tau_c)$, this implies

$$\bar{b}(\tau_c) = \left(\frac{w_B}{w_A}\right)^{\frac{\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} \bar{a}.$$

It remains to consider welfare in the inelastic case, i.e., $\varepsilon < 1$. But, as is evident from the analysis in the main text, in this case welfare changes are the same as in the elastic case with $\tau > \tau_c$, although the welfare *levels* differ between these two scenarios. The reason for the level difference is that, in the inelastic case, the number of firms is larger, i.e., $n(\tau) = \left(\frac{w_B}{w_A}\right)^{\frac{\alpha(1-\varepsilon)(\sigma-1)}{\sigma-\varepsilon}} n > n$, and the search cutoff is smaller, i.e., $\bar{a} < \bar{b}(\tau_c) = \left(\frac{w_B}{w_A}\right)^{\frac{\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} \bar{a}$. But the conditions for welfare changes are similar in both cases; i.e., in both cases the variations in tariff revenue and the search costs, which are the only sources of welfare changes, are produced by the n original producers.

Finally, note that in the inelastic case and with $\tau \geq w_B/w_A$, the average ex-factory cost of a bundle of inputs for an incumbent firm is (using $a_B = \frac{w_B \bar{a}}{\tau w_A}$)

$$\begin{aligned}
\rho^{\tau} &= \frac{G(a_B)}{G(\bar{a})}\left[\beta w_A \mu_a(a_B) + (1-\beta) w_B \frac{\bar{a}}{\tau}\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right] w_B \left[\beta \mu_b(\bar{a}) + (1-\beta)\bar{a}\right] \\
&= \left(\frac{w_B}{\tau w_A}\right)^{\theta} w_B \left[\beta \frac{\theta}{\theta+1} + (1-\beta)\right] \frac{\bar{a}}{\tau} + \left[1 - \left(\frac{w_B}{\tau w_A}\right)^{\theta}\right] w_B \left[\beta \frac{\theta}{\theta+1} + (1-\beta)\right] \bar{a}
\end{aligned}$$

while for a new entrant this cost is

$$\rho^{\tau}_{new} = w_B \mu_b(\bar{a}) = \frac{\theta}{\theta+1} w_B \bar{a}.$$

The average ex-factory cost of a bundle of m units of the composite intermediate good is

$$\frac{n}{n(w_B/w_A)} \rho^{\tau} + \frac{n(w_B/w_A) - n}{n(w_B/w_A)} \rho^{\tau}_{new}.$$

Since ρ^{τ}_{new} does not depend on the tariff level, the home country's terms of trade are an increasing function of the size of the tariff if and only if ρ^{τ} is declining in τ. Note, however, that

$$\mathtt{sign} \frac{\partial \rho^{\tau}}{\partial \tau} = \mathtt{sign} \frac{d}{d\tau}\left(\tau^{-\theta-1} - \tau^{-\theta}\right).$$

It follows that the terms of trade are increasing in the size of the tariff if and only if

$$\tau < \frac{\theta + 1}{\theta}.$$