

Inefficient Automation*

Martin Beraja

MIT and NBER

Nathan Zorzi

Dartmouth

September 6, 2021

Abstract

How should the government respond to automation? We study this question in a heterogeneous agent model that takes worker displacement seriously. We recognize that displaced workers face two frictions in practice: reallocation is slow and borrowing is limited. We first show that these frictions result in inefficient automation. Firms are effectively too patient when they automate, and (partly) overlook the time it takes for workers to reallocate and for the benefits of automation to materialize. We then analyze a second best problem where the government can tax automation but lacks the tools to fully overcome borrowing frictions. The equilibrium is (constrained) inefficient — automation and reallocation impose pecuniary externalities on workers. The government finds it optimal to tax automation while labor reallocates, even when it has no preference for redistribution. Using a quantitative version of our model, we find significant welfare gains from slowing down automation.

*Martin Beraja: maberaja@mit.edu. Nathan Zorzi: nathan.g.zorzi@dartmouth.edu. We thank Daron Acemoglu, Adrien Bilal, Ricardo Caballero, Arnaud Costinot, Mariacristina De Nardi, John Grigsby, Jonathon Hazell, Roozbeh Hosseini, Anders Humlum, Pablo Kurlat, Monica Morlacco, Jeremy Pearce, Elisa Rubbo, Gustavo Ventura, Jesús Fernández-Villaverde, Conor Walsh, Iván Werning and Christian Wolf for helpful discussions. All errors are our own.

1 Introduction

Automation technologies — like AI and robots — raise productivity but disrupt labor markets, displacing workers and lowering their earnings (Graetz and Michaels, 2018; Acemoglu and Restrepo, 2018a). The increasing adoption of automation has fueled an active debate about appropriate policy interventions (Atkinson, 2019; Acemoglu et al., 2020). Despite the growing public interest in this question, the literature has yet to produce optimal policy results that take into account the frictions that workers face in practice when they are displaced by automation.

The existing literature that justifies taxing automation assumes that worker reallocation is frictionless or absent altogether. First, recent work shows that a government that has a preference for redistribution should tax automation to mitigate its distributional consequences (Guerreiro et al., 2017; Costinot and Werning, 2018; Thue mmel, 2018; Korinek and Stiglitz, 2020). This literature assumes that automation and labor reallocation are intrinsically efficient, and that the government is willing to sacrifice efficiency for equity. Second, an extensive literature finds that a government should tax capital — and automation, by extension — to finance government spending (Chari and Kehoe, 1999 for a survey), prevent dynamic inefficiency (Diamond, 1965; Aguiar et al., 2021), or improve insurance when markets are incomplete (Heathcote et al., 2009 for a survey). This literature abstracts from worker displacement and labor reallocation.

In this paper, we take worker displacement seriously and study how a government should respond to automation. In particular, we recognize that workers face two important frictions when they reallocate or experience earnings losses. First, reallocation is slow: workers face barriers to mobility and may go through unemployment or retraining spells before finding a new job (Davis and Haltiwanger, 1999; Jacobson et al., 2005; Lee and Wolpin, 2006). Second, credit markets are imperfect: workers have a limited ability to borrow against future incomes (Jappelli and Pistaferri, 2017), especially when moving between jobs (Chetty, 2008). We show that these frictions result in *inefficient* automation. A government finds it optimal to tax automation — even if it has no preference for redistribution — when it lacks the instruments to fully alleviate the frictions. Quantitatively, we find large welfare gains from slowing down automation.

We incorporate reallocation and borrowing frictions in a dynamic model with endogenous automation and heterogeneous agents. There is a continuum of occupations, and workers come in overlapping generations. Firms invest in automation to expand their productive capacity. Automated occupations become less labor intensive, which displaces workers. These workers face reallocation frictions: they receive random opportunities to

move between occupations, experience a temporary period of unemployment or retraining when they do so (Alvarez and Shimer, 2011), and incur a permanent productivity loss when reallocating due to the specificity of their skills (Violante, 2002; Adão et al., 2020). Workers also face financial frictions: they are not insured against the risk that their occupation is automated and face borrowing constraints (Huggett, 1993; Aiyagari, 1994) which can prevent consumption smoothing.

We have two main theoretical results. Our first result shows that the interaction between slow reallocation and borrowing constraints results in inefficient automation. Displaced workers experience earnings losses when their occupation is automated, but expect their income to increase as they slowly reallocate. This creates a motive for borrowing to smooth consumption during this transition. When the borrowing and reallocation frictions are sufficiently severe, displaced workers are pushed against their borrowing constraints.¹ This drives a wedge between the (intertemporal) marginal rate of substitution of displaced workers and the equilibrium interest rate that firms face when they automate. Effectively, firms are excessively patient when they automate. They (partly) overlook the time it takes for labor to reallocate and for the benefits of automation to materialize.

Our second result characterizes optimal policy. In principle, the government could restore efficiency using redistributive taxes and transfers that fully relax borrowing constraints.² In practice, the government is unlikely to have access to this rich set of instruments.³ This motivates us to study second best interventions, where the government can tax automation and (potentially) implement active labor market interventions but is unable to fully alleviate the borrowing constraints of displaced workers by redistributing income directly.

We find that the equilibrium is generically (constrained) inefficient, as defined by Geanakoplos and Polemarchakis (1985) and Farhi and Werning (2016). Automation and reallocation choices impose *pecuniary externalities* on workers. Firms do not internalize that automation displaces workers and lowers their earnings, and workers do not internalize how their reallocation affects the wage of their peers. These pecuniary externalities do not net out when displaced workers are pushed against their borrowing constraints. We show that the government should tax automation on efficiency grounds — even when

¹ This is consistent with the empirical evidence. Displaced workers increase their borrowing to smooth consumption (Sullivan, 2008; Collins et al., 2015) when they are able to. Many workers are constrained and are either unable to borrow or forced to delever their existing debt (Bethune, 2017; Braxton et al., 2020). Job displacement also increases defaults (Gerardi et al., 2018; Keys, 2018), which further limits access to credit.

² In fact, the government can decentralize any first best allocation using targeted lump sum transfers — a version of the Second Welfare Theorem holds in our model.

³ The absence of (targeted) lump sum transfers is precisely what motivates the existing literature on the taxation of automation. We allow for various sources of social insurance in our quantitative model.

it has no preference for redistribution. The government should intervene while labor reallocation takes place, but has no reason to tax automation in the long-run. By *slowing down* automation, this intervention prevents excessive investment in automation and raises consumption early on during the adjustment process, precisely when displaced workers are borrowing-constrained.

Overall, our results show that optimal policies can potentially improve *both* efficiency and equity — while there is necessarily a trade-off in efficient economies (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020). The rationale we propose for curbing automation also complements a large literature on long-run capital taxation, although it is distinct from the ones encountered there. In particular, the rationale relies neither on equity considerations (Judd, 1985; Chamley, 1986), nor on the presence of uninsured idiosyncratic risk (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012), nor on dynamic inefficiencies (Phelps, 1965; Diamond, 1965; Aguiar et al., 2021). Instead, the rationale that we propose applies during the *transition* to the long-run while displaced workers are borrowing constrained.

We conclude the paper with a quantitative exploration. We extend our baseline model along various dimensions that are important for welfare analysis. In particular, we introduce gross flows across occupations (Kambourov and Manovskii, 2008; Moscarini and Vella, 2008) and uninsured earnings risk (Floden and Lindé, 2001). These two features are important in the data and affect the ability of displaced workers to self-insurance against automation through mobility and savings. We also introduce progressive income taxation (Heathcote et al., 2017) and unemployment benefits (Krueger et al., 2016) to account for existing sources of insurance that can benefit displaced workers. Slowing down automation generates large welfare gains in our numerical simulations — even absent any equity considerations.

Our paper relates to several additional strands of the literature. We contribute to an important literature studying pecuniary externalities in economies with incomplete markets (Geanakoplos and Polemarchakis, 1985; Lorenzoni, 2008; Farhi and Werning, 2016; Dávila and Korinek, 2018). In our model, the pecuniary externalities imposed on workers do not net out when those initially employed in automated occupations are borrowing constrained. These externalities occur when firms and workers make *technological choices* — such as automation or reallocation — and borrowing constraints distort the (shadow) prices that these agents face.⁴ As such, this type of pecuniary externalities is conceptually

⁴ Labor reallocation — just like automation — is isomorphic to a technological choice in the Arrow-Debreu construct. Each worker owns a firm that chooses the type of labor services to provide. We show that this choice is efficient *when* workers' borrowing constraints do not bind.

distinct from the type commonly encountered in the incomplete markets literature.⁵ In particular, it relies neither on the presence of uncertainty and incomplete markets, nor on endogenous borrowing constraints.

Methodologically, our quantitative model borrows from three literatures: (i) the one studying reallocation and wage dispersion using island models in the tradition of [Lucas and Prescott \(1974\)](#) and [Alvarez and Shimer \(2011\)](#); (ii) the one concerned with the impact of technological innovations and trade using dynamic discrete choice models with mobility shocks ([Heckman et al., 1998](#); [Artuç et al., 2010](#); [Caliendo et al., 2019](#)); and (iii) the one interested in consumption and insurance using heterogeneous-agent models with idiosyncratic income shocks, life-cycle features, and borrowing constraints ([Heathcote et al., 2010](#); [Low et al., 2010](#)). Finally, our policy analysis contributes to the public finance literature studying optimal taxation ([Conesa and Krueger, 2006](#); [Ales et al., 2015](#); [Heathcote et al., 2017](#)) and social insurance ([Imrohoroglu et al., 1995](#); [Golosov and Tsyvinski, 2006](#); [Braxton and Taska, 2020](#)) in dynamic models with heterogeneous agents.

Layout. We introduce our baseline environment in Section 2. Section 3 characterizes the set of first best allocations. Section 4 describes the laissez-faire equilibrium. Section 5 presents our inefficiency result. Section 6 characterizes optimal policy. Section 7 introduces our quantitative model. Finally, Section 8 describes our calibration strategy and quantitative exercise.

2 Model

Time is continuous, and there is no aggregate uncertainty. Periods are indexed by $t \geq 0$. The economy consists of a continuum of workers, a continuum of occupations indexed by $h \equiv [0, 1]$, and a final goods producer. In this section, we specify the technologies, preferences, reallocation frictions, and resource constraints of this economy. We will describe assets, incomes, and borrowing frictions in Section 4 when discussing the decentralized equilibrium.

⁵ This literature distinguishes between “distributional externalities” and “collateral externalities” ([Dávila and Korinek \(2018\)](#)). Distributional externalities occur when financial markets are incomplete, and households’ marginal rates of substitution do not co-move across states — either due to borrowing constraints (e.g. across time) or non-homothetic preferences (e.g. across goods). Collateral externalities occur when households’ borrowing constraints are indexed by an aggregate state.

2.1 Technology

Occupations use labor as an input and another factor that we assume is fixed.⁶ Final goods are produced by aggregating the output of all occupations.

Occupations. Occupations are indexed by $h \equiv [0, 1]$. They use a decreasing returns to scale technology

$$y_t^h = F^h(\mu_t^h), \quad (2.1)$$

where μ_t^h denotes effective labor.

Technology adoption. Each occupation becomes partially automated in period $t = 0$ with probability ϕ . The degree of automation is α . The occupations' technology is given by

$$F^h(\cdot) = \begin{cases} \hat{F}(\cdot; \alpha) & \text{if automated} \\ F(\cdot) & \text{otherwise} \end{cases}, \quad (2.2)$$

where $F(\cdot)$ and $\hat{F}(\cdot)$ are neoclassical technologies. By definition, $\hat{F}(\cdot; 0) \equiv F(\cdot)$ absent automation. Automated occupations are less labor intensive than non-automated occupations — i.e., $\hat{F}_\mu(1; \alpha)$ decreases with α . Automation can raise output directly, but it also comes at a cost. The technology has to be maintained: it requires some continued investment which diverts resources away from production. The technology $\hat{F}(\cdot; \alpha)$ implicitly captures these two effects. We will impose some regularity assumptions in the next section.

Final good. Aggregate output is produced by combining the output y^h of all occupations with a neoclassical technology

$$Y_t = G\left(\{y_t^h\}\right), \quad (2.3)$$

In the following, we suppose that these inputs are complements. Moreover, we impose some *symmetry* across occupations.

Assumption 1 (Symmetry). *The technology of the final good producer $G(\cdot)$ is continuously differentiable, additively separable and symmetric in its arguments.*

This assumption insures that economy behaves as if there were only automated ($h = A$) and non-automated ($h = N$) occupations.⁷ This allows us to define the aggregate

⁶ The fixed factor plays the same role as capital in our quantitative model (Section 7). In this richer model, capital reallocation is slow (Ramey and Shapiro (2001)) due to adjustment costs at the occupation-level.

⁷ Formally, this follows from Assumption 1 together with the (strict) concavity of $G(\cdot)$.

production function

$$G^*(\boldsymbol{\mu}; \alpha) \equiv G\left(\left\{F^h\left(\mu^h\right)\right\}\right) \quad (2.4)$$

where $\boldsymbol{\mu} \equiv (\mu^A, \mu^N)$ are the flow of workers employed in each automated and non-automated occupations, with the degree of automation α being implicit in $\{F^h(\mu^h)\}$.⁸ The technology $G^*(\cdot)$ is total production net of automation costs. For concision, we refer to it as *output* in the following.

2.2 Workers

There are overlapping generations of workers who are born and die at a constant rate $\chi \in [0, +\infty)$.⁹ A worker is indexed by four idiosyncratic states: their initial occupation of employment (h); their age (s); their productivity (ξ); and their employment status (e). In the following, we let $\mathbf{x} \equiv (h, s, \xi, e)$ denote the workers' idiosyncratic states and π denote the associated measure.

Preferences. Workers consume, and supply inelastically one unit of labor when employed. Workers' preferences are represented by¹⁰

$$U_0 = \mathbb{E}_0 \left[\int \exp(-\rho t) u(c_t) dt \right] \quad (2.5)$$

for some discount rate $\rho > 0$ and some isoelastic utility $u(c) \equiv \frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$.

Reallocation frictions. We assume that the process of labor reallocation is *slow* and *costly*. Reallocation is slow for three reasons. First, existing generations of workers are given the opportunity to reallocate to another occupation with intensity $\lambda > \chi$.¹¹ Second, workers who reallocate across occupations enter a temporary state of unemployment or retraining

⁸ For illustration, suppose that the technology belongs to the [Kimball \(1995\)](#) class. Then, the aggregate production function solves

$$1 = \phi \Gamma \left(\frac{\hat{F}(\mu^A; \alpha)}{G^*(\mu^A, \mu^N; \alpha)} \right) + (1 - \phi) \Gamma \left(\frac{F(\mu^N)}{G^*(\mu^A, \mu^N; \alpha)} \right),$$

for some $\Gamma(\cdot)$ increasing and concave.

⁹ We suppose throughout the paper that $1/\chi$ is bounded away from 0 — each generation has a positive (expected) life span.

¹⁰ The expectation is over the realization of the aggregate shock and the worker's survival.

¹¹ That is, we assume that the workers' mobility decision is purely time-dependent, which delivers tractable expressions. We also suppose that existing generations disappear at a sufficiently low rate for labor mobility to occur at equilibrium. We allow for state-dependent mobility in our quantitative model (Section 7), by assuming that workers face a discrete choice problem *à la* [McFadden \(1973\)](#) between occupations.

which they exit at rate $\kappa > 0$.¹² Third, new generations of workers enter the labor market gradually at rate χ — at which point they can choose any occupation.¹³ In turn, reallocation is costly for two reasons. First, workers do not produce while unemployed. Second, they incur a permanent productivity loss $\theta \in (0, 1]$ after they have reallocated to a new occupation, i.e. $h'_t \neq h$. This productivity loss captures the workers' skill specificity, i.e. the lack of transferability of their skills across occupations.¹⁴ Thus, workers' productivity evolves as

$$\tilde{\zeta}_t = \mathbf{1}_{\{e_t=1\}} \zeta_t \quad \text{with} \quad \zeta_t = \begin{cases} \lim_{\tau \uparrow t} \zeta_\tau & \text{if } h'_t(\mathbf{x}) = h \\ (1 - \theta) \times \lim_{\tau \uparrow t} \zeta_\tau & \text{otherwise} \end{cases} \quad (2.6)$$

with $\zeta_t \equiv e_t \equiv 1$ at birth. In turn, the employment status is $e_t = 1$ at birth, switches to $e_t = 0$ upon reallocation, and reverts $e_t = 1$ upon exiting unemployment at rate κ .

Occupational choices. Occupational decisions for existing or new generations consist of choosing the occupation with the highest value

$$\max_{h' \in \{A, N\}} V_t^{h'}(\mathbf{x}'(h'; \mathbf{x})) \quad (2.7)$$

where $V_t^h(\cdot)$ denotes the continuation value associated to automated and non-automated occupations. For existing generations, the state $\mathbf{x}'(h'; \mathbf{x})$ captures the unemployment or retraining spells that displaced workers go through and the permanent productivity loss they experience. Newborns are subject to neither of those.

2.3 Resource Constraint

The occupation outputs are given by

¹² This assumption follows [Alvarez and Shimer \(2011\)](#). The temporary state could be interpreted either as involuntary employment due to search frictions or a temporary exit of the labor force during which workers retrain for their new occupation. Empirically, labor reallocation across occupations/sectors raises unemployment ([Lilien, 1982](#); [Chodorow-Reich and Wieland, 2020](#)).

¹³ We explicitly introduce overlapping generations to allow for a realistic source of labor reallocation across occupations. New cohorts achieve a substantial share of labor reallocation ([Adão et al., 2020](#); [Hobijn et al., 2020](#); [Porzio et al., 2020](#)).

¹⁴ We are interested in the effect of *permanent*, occupation-specific shocks. In this version of the model, workers never return to the occupation they have previously left. Therefore, they incur the productivity loss θ at most once. For this reason, we do not distinguish in (2.6) between a worker who moves to a new occupation she never worked in (and incurs a productivity loss) and one that returns to the occupation she was previously employed in (and recovers part of her high productivity).

$$y_t^h = F^h \left(\int \mathbf{1}_{\{h(\mathbf{x})=h\}} \zeta d\pi_t \right) \quad (2.8)$$

for each occupation h . Finally, the aggregate resource constraint is

$$G^* \left(\left\{ \mu_t^h \right\} \right) = \int c_t(\mathbf{x}) d\pi_t \quad (2.9)$$

where $c_t(\mathbf{x})$ is the consumption of a worker with idiosyncratic state \mathbf{x} .

3 Efficient Allocation

We now characterize the set of efficient allocations. We state the planner's problem in Section 3.1 and characterize its solution in Section 3.2.

3.1 First Best Problem

The planner faces two choices: how to reallocate labor after a shock (ex post); and what type of innovation to pursue (ex ante). We start by discussing our choice of Pareto weights. We then state the ex post problem, before turning to the ex ante counterpart. In the following, we let $\phi^A \equiv \phi$ and $\phi^N \equiv 1 - \phi$ denote the mass of automated and unautomated occupations, respectively.

Pareto weights. We allow the planner to place different weights $\eta \equiv \{\eta_s^h\}$ on workers based on their birth period and their initial occupation of employment.¹⁵ The dispersion of these weights *within* a generation determines the strength of the equity motive. Without loss of generality, we suppose that these weights add up to 1. In the following, we let $\bar{\eta}_s > 0$ denote the *average* weight within a given generation with $\bar{\eta}_s^{\frac{1}{\sigma}} \equiv \sum_h \phi^h (\eta_s^h)^{\frac{1}{\sigma}}$.

Ex post. We start with the efficient reallocation of labor after automation has occurred. The planner solves

$$V^{FB}(\alpha; \eta) \equiv \max_{\{c_t, m_t, \hat{m}_t, \mu_t\}_{t \geq 0}} \sum_h \phi^h \int_{-\infty}^{+\infty} \eta_s^h \int_0^{+\infty} \exp(-(\rho + \chi)t) u(c_{s,t}^h) dt ds \quad (3.1)$$

¹⁵ By convention, we treat symmetrically all workers who are not born yet in period $t = 0$, i.e. $\eta_s^A = \eta_s^N = \eta_s$ for all periods $s > 0$. Therefore, we can effectively treat the mass of workers indexed by (h, s) as constant over time — conditional on survival.

subject to the constraints (3.2)–(3.7) below. Here, $c_{s,t}^h$ denotes the consumption in period t of the generation born in period s and initially located in an occupation $h \in \{A, N\}$ (if born yet). First, an allocation must satisfy the resource constraint

$$C_t = G^*(\boldsymbol{\mu}_t; \alpha) \quad (3.2)$$

where

$$C_t \equiv \sum_h \phi^h \int_0^{+\infty} \chi \exp(-\chi s) c_{s,t}^h ds \quad (3.3)$$

denotes aggregate consumption. The laws of motion for *effective* labor supplies $\boldsymbol{\mu}_t \equiv \{\mu_t^A, \mu_t^N\}$ — accounting for productivity losses upon reallocation — are

$$d\mu_t^A = -(\lambda m_t + \chi \hat{m}_t) \mu_t^A dt \quad \text{with} \quad \mu_0^A = 1 \quad (3.4)$$

in automated occupations $h = A$, for some $m_t, \hat{m}_t \in [0, 1]$, and

$$d\mu_t^N \equiv -\frac{\phi}{1-\phi} d\mu_t^A - d\tilde{\mu}_t - \theta d\hat{\mu}_t \quad \text{with} \quad \mu_0^N \equiv 1 \quad (3.5)$$

in the non-automated occupations $h = N$, with

$$d\tilde{\mu}_t = \left(\lambda \frac{\phi}{1-\phi} m_t \mu_t^A - (\kappa + \chi) \tilde{\mu}_t \right) dt \quad \text{with} \quad \tilde{\mu}_0 = 0 \quad (3.6)$$

$$d\hat{\mu}_t = (\kappa \tilde{\mu}_t - \chi \hat{\mu}_t) dt \quad \text{with} \quad \hat{\mu}_0 = 0 \quad (3.7)$$

for each $t \geq 0$. From (3.4), we see that labor reallocation happens through two margins: the reallocation of existing generations (at rate λ) or the entry of new generations (at rate χ). The planner chooses the shares m_t and \hat{m}_t of each of these workers to reallocate. From (3.6), we see that existing generations who reallocate enter a temporary pool of unemployed which they leave gradually – either their unemployment ends (at rate κ) or they are replaced by a new generation (at rate χ). At that point, they become active in their new occupation (3.7) but incur a productivity loss ($\theta \leq 1$) and are replaced gradually (at rate χ). From (3.5), we see how the effective labor supply in non-automated occupations evolves. The term Θ_t captures the productivity distortion due to unemployment and productivity losses.

The planner increases output by closing the wedge between the marginal productivities of labor across occupations $\left(\frac{\partial}{\partial \mu^A} G^*(\cdot) \text{ and } \frac{\phi}{1-\phi} \frac{\partial}{\partial \mu^N} G^*(\cdot) \right)$. It does so by reallocating workers. But using each margin of labor reallocation has a different cost. When reallocating existing generations, there is a productivity loss ($\theta > 0$), the foregone production

while unemployed ($1/\kappa > 0$), and the delay in closing the wedge by waiting for workers to slowly reallocate ($1/\lambda > 0$) and exit unemployment ($1/\kappa > 0$). When reallocating new generations, there is the delay in closing the wedge by waiting for them to slowly enter the labor market ($1/\chi > 0$). The condition in Assumption 2 guarantees that these costs are such that reallocation happens through both margins at the first best.

Ex ante. We now turn to the efficient automation decision. The planner solves

$$\max_{\alpha \geq 0} V^{FB}(\alpha; \boldsymbol{\eta}) \quad (3.8)$$

Automation creates a wedge between the marginal productivities of labor across occupations. The production possibility frontier expands as labor gets reallocated between those. The efficient level of automation maximizes the present discounted value of the additional output that this reallocation allows, given reallocation frictions $(\lambda, \kappa, \chi, \theta)$ and the cost of automation.

Finally, we impose regularity assumptions to rule out corner solutions. These are needed for a meaningful discussion of automation and labor reallocation. First, we assume that the cost of automation is such that there is positive but partial automation at the first best.¹⁶ Second, we suppose that parameters governing reallocation costs — i.e. the average unemployment or retraining duration ($1/\kappa$) and the productivity loss (θ) — are small enough that reallocation takes place at the first best.

Assumption 2 (Interior solutions). *The direct effect of automation $G^*(\mu, \mu'; \alpha)$ is concave in α and satisfies $\partial_\alpha G^*(\mu, \mu'; \alpha)|_{\alpha=0} > 0$ and $\lim_{\alpha \rightarrow +\infty} \partial_\alpha G^*(\mu, \mu'; \alpha) = -\infty$ for any $0 \leq \mu < 1$ and $\mu' > 1$. Finally, the average unemployment duration ($1/\kappa$) and the productivity loss (θ) are sufficiently small*

$$\theta \leq 1 - \frac{Z^A}{\int_0^{+\infty} (1 - \exp(-\kappa t)) Z_t^N dt}$$

that labor reallocation takes place at the first best. The coefficients Z_A and $\{Z_{N,t}\}$ are defined in Appendix A.1, and are positive, exogenous, and independent of (θ, κ) .

¹⁶ The first part of Assumption 2 is satisfied whenever the cost of automation $F(1) - \hat{F}(1; \alpha)$ is sufficiently convex — i.e. more and more workers are diverted from production as the level of automation increases.

3.2 Efficient Automation and Reallocation

We now characterize labor reallocation and automation at the first best. We define the following felicity function

$$U_t(C_t; \boldsymbol{\eta}) \equiv \beta_t(\boldsymbol{\eta}) u(C_t) \quad (3.9)$$

with

$$\beta_t(\boldsymbol{\eta}) \equiv \chi \int_0^{+\infty} \bar{\eta}_{t-s} \exp(-\rho(s-t) - \chi s) u \left(\frac{(\bar{\eta}_{t-s} \exp(-\rho s))^{\frac{1}{\sigma}}}{\chi \int_0^{+\infty} \exp(-\chi \tau) (\bar{\eta}_{t-\tau} \exp(-\rho \tau))^{\frac{1}{\sigma}} d\tau} \right)^{1-\sigma} ds$$

We focus on aggregate, first best allocations $X_t \equiv \{C_t, \mu_t, \Theta_t\}$. It is understood that the planner can choose any set of individual allocations that satisfy (3.3).

Reallocation. We begin by characterizing the solution to the ex post problem — the efficient reallocation of labor *fixing* the degree of automation α .

Proposition 1 (Efficient labor reallocation). *An aggregate allocation $\{X_t\}$ is part of an efficient allocation if and only if there exist two stopping times (T_0^{FB}, T_1^{FB}) with $0 < T_0^{FB} < T_1^{FB} < +\infty$ such that: (i) existing generations reallocate to non-automated occupations until T_0^{FB} and new generations until T_1^{FB} ; (ii) after T_1^{FB} , new generations are allocated so that marginal productivities are equalized across occupations; and (iii) these stopping times satisfy the smooth pasting conditions*

$$\int_{T_0^{FB}}^{+\infty} \exp(-\rho t) U_t'(C_t; \boldsymbol{\eta}) \Delta_t dt = 0 \quad \text{and} \quad \mathcal{Y}_{T_1^{FB}}^N = \mathcal{Y}_{T_1^{FB}}^A \quad (3.10)$$

where

$$\Delta_t \equiv \underbrace{\exp(-\chi t)}_{\text{OLG}} \left\{ \underbrace{(1-\theta)}_{\text{Productivity cost}} \underbrace{(1 - \exp(-\kappa(t-T_0)))}_{\text{Unemployment spell}} \mathcal{Y}_t^N - \mathcal{Y}_t^A \right\} \quad (3.11)$$

for all $t \geq T_0^{FB}$ denotes the response of output to labor reallocation, $C_t = G^*(\boldsymbol{\mu}_t; \alpha)$ denotes aggregate consumption, and $\mathcal{Y}_t^h \equiv 1/\phi^h \partial_h G^*(\boldsymbol{\mu}_t; \alpha)$ denotes the marginal productivity of labor.

Proof. See Appendix A.1. □

Automation drives a wedge between the marginal productivities of labor in the two occupations, i.e. $\mathcal{Y}_0^A < \mathcal{Y}_0^N$. The planner reallocates workers between those to increase

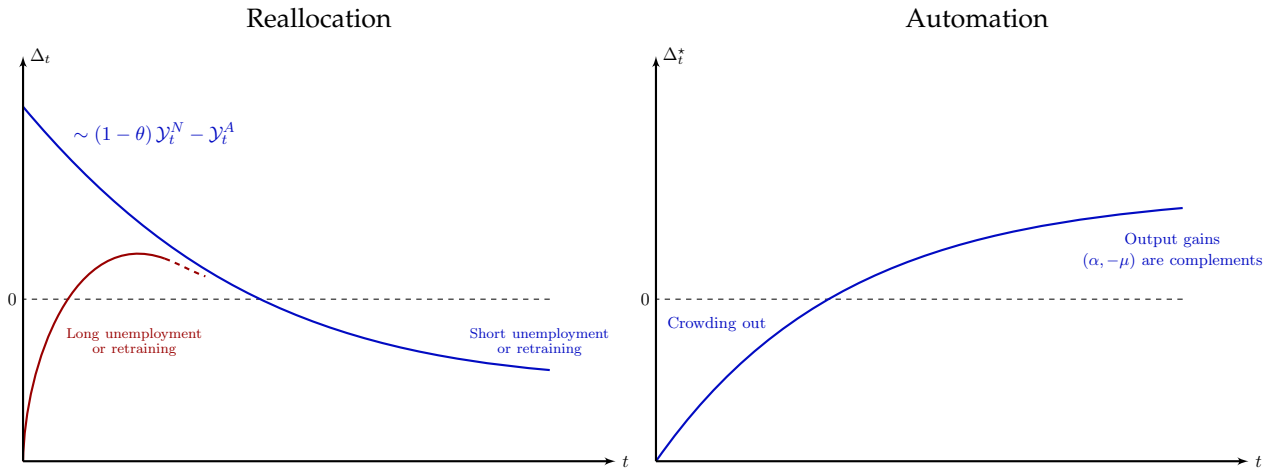
output. Effective labor supplies evolve as

$$\mu_t^A = \exp\left(-\lambda \min\{t, T_0^{\text{FB}}\} - \chi t\right) \quad (3.12)$$

$$\mu_t^N = 1 - \mu_t^A + (\theta - 1) \frac{\phi}{1 - \phi} \exp(-\chi t) \left\{1 - \exp\left(-\lambda \min\{t, T_0^{\text{FB}}\}\right)\right\} \quad (3.13)$$

for all $t \in [0, T_1^{\text{FB}})$ with no unemployment or retraining, or as (A.7)–(A.10) in the general case. Initially, both existing generations (at rate λ) and new generations (at rate χ) are assigned to non-automated occupations. This adjustment process is slow and labor misallocation declines gradually, i.e. $\mathcal{Y}_0^A / \mathcal{Y}_0^N < \mathcal{Y}_t^A / \mathcal{Y}_t^N$ for $t > 0$. In period $t = T_0^{\text{FB}}$, the planner finds it optimal to stop reallocating existing generations since they experience a productivity loss when moving. New generations continue to be assigned to non-automated occupations until period $t = T_1^{\text{FB}}$. After that, the planner allocates new generations across occupations so as to keep the marginal productivities equal across those.

Figure 3.1: Impulse responses of output to reallocation and automation



The left panel of Figure 3.1 illustrates the dynamics of the output response to reallocation Δ_t for the case of no overlapping generations ($\chi \rightarrow 0$). This response governs the incentives to reallocate workers displaced by automation, as seen in equation (3.10). When unemployment / retraining spells are short ($1/\kappa \rightarrow 0$), the flows Δ_t are *front-loaded*. The response is initially positive and then gradually declines as more workers enter non-automated occupations. At longer horizons the flows become negative ($\lim_{t \rightarrow +\infty} \Delta_t < 0$), as the productivity of non-automated occupations is depressed by the productivity loss due to skill specificity ($\theta > 0$).¹⁷ On the contrary, the flows Δ_t are *back-loaded* when un-

¹⁷ In the general case with overlapping generations, this distortion decreases and vanishes asymptotically as these workers are replaced by new, more productive generations.

employment spells are sufficiently long. The response is negative at short horizons since displaced workers who reallocate do not produce while unemployed. It becomes positive later on, as workers exit unemployment and produce in non-automated occupations, and then negative again at long horizons due to the productivity loss.

Automation. We now turn to the ex ante problem — the efficient degree of automation α^{FB} . The next proposition characterizes this choice.

Proposition 2 (Efficient automation). *The degree of automation α^{FB} is unique and interior. A necessary and sufficient condition is*

$$\int_0^{+\infty} \exp(-\rho t) U_t'(C_t; \boldsymbol{\eta}) \Delta_t^* dt = 0 \quad (3.14)$$

where

$$\Delta_t^* \equiv \frac{\partial}{\partial \alpha} G^*(\boldsymbol{\mu}_t; \alpha^{FB}) \quad (3.15)$$

for all $t \geq 0$ denotes the response of output to automation, with consumption $\{C_t\}$ and the labor supplies $\{\boldsymbol{\mu}_t\}$ are given by Proposition 1 when evaluated at α^{FB} .

Proof. See Appendix A.2 □

An increase in the degree of automation α has two effects on output (net of automation costs). First, automation increases output as labor gradually reallocates to non-automated occupations with a higher labor productivity.¹⁸ Second, automation comes at cost as it diverts some labor away from production. The exact time profile of these benefits and costs depends on the degree of complementarity between automation and reallocation. We maintain the following assumption throughout.

Assumption 3 (Complementarity). *Automation and labor reallocation are complements. That is,*

$$\hat{G}(\mu, \alpha) \equiv G^*(\mu, 1 + \Theta(1 - \mu); \alpha)$$

has decreasing differences in (μ, α) for all $\mu \in (0, 1)$ and $\Theta \in (0, 1]$.

This restriction ensures that the gains from automation get realized gradually, as more workers reallocate to occupations where the marginal productivity of labor is higher. This assumption is mostly innocuous. For instance, it is satisfied in our quantitative model (Section 7) where we adopt standard functional forms. Figure 3.1 depicts the returns on automation Δ_t^* in this case. Automation crowds out consumption early on, but eventually

¹⁸ The formulation (2.4) also allows for direct productivity gains through automation.

expands the production possibility frontier as labor reallocates. In other words, the returns on automation are *back-loaded*.

4 Decentralized Equilibrium

We now turn to the decentralized equilibrium. We first describe the problem of a representative firm which chooses automation and labor demands. We next describe the workers' problem, including the assets they trade, the frictions they face and their sources of incomes. Finally, we define a competitive equilibrium.

4.1 Firms

The representative firm chooses the degree of automation α and labor demands μ to maximize the value of its equity

$$\max_{\alpha \geq 0} \hat{V}(\alpha) \quad \text{with} \quad \hat{V}(\alpha) \equiv \int_0^{+\infty} Q_t \hat{\Pi}_t(\alpha) dt \quad (4.1)$$

where $\{Q_t\}$ is the appropriate stochastic discount factor,¹⁹ and

$$\hat{\Pi}_t(\alpha) \equiv \max_{\mu \geq 0} G^*(\mu; \alpha) - \phi \mu^A w_t^A - (1 - \phi) \mu^N w_t^N \quad (4.2)$$

are optimal profits given equilibrium wages $\{w_t^h\}$.

4.2 Workers

We now specify the assets that workers trade, and the constraints they face beyond reallocation frictions.

Assets and states. Workers save in bonds available in zero net supply. We suppose that financial markets are incomplete: workers are unable to trade contingent securities against the risk that their occupation becomes automated.²⁰ We suppose that workers initially

¹⁹ Equity is implicitly priced by workers who are marginally unconstrained.

²⁰ We rule out complete markets for two reasons: financial markets participations is limited in practice (Mankiw and Zeldes, 1991; Heaton and Lucas, 2000); and workers' equity holdings are typically not hedged against their employment risk (Poterba, 2003; Massa and Simonov, 2006; Huberman, 2001). The absence of contingent securities (or lump sum transfers) is precisely what motivates the existing literature on the regulation of automation (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018). The equilibrium would be efficient if workers could trade contingent securities before the automation

employed in automated occupations form a large household. This allows them to achieve full risk sharing against the risk of being allowed to move across occupations (at rate λ) or not.²¹ Workers trade annuities (Blanchard, 1985; Yaari, 1965) against the risk of their death. Workers are now indexed by five idiosyncratic states: their holdings of bonds (a); their initial occupation of employment (h); their age (s); their idiosyncratic productivity (ξ); and their employment status (e). We let $\mathbf{x} \equiv (a, h, s, \xi, e)$ denote the vector of idiosyncratic states and π denote the associated measure.

Budget constraint. Worker's flow budget constraint is

$$da_t(\mathbf{x}) = [\mathcal{Y}_t^*(\mathbf{x}) + (r_t + \chi) a_t(\mathbf{x}) - c_t(\mathbf{x})] dt \quad (4.3)$$

where \mathcal{Y}_t^* denotes total income consisting of labor income, profits and taxes, $r_t \geq 0$ denotes the return on savings, and c_t denotes individual consumption. The initial condition is $a_s(\mathbf{x}) = a^{\text{birth}}(\mathbf{x})$ at birth, where $a^{\text{birth}}(\mathbf{x})$ is the stock of assets inherited by a given generation.

Borrowing frictions. Workers are subject to borrowing constraints

$$a_t(\mathbf{x}) \geq \underline{a} \quad (4.4)$$

where the borrowing parameter $\underline{a} \leq 0$ determines the level of self-insurance that a worker can achieve.

Income and occupational choice. Total income consists of effective labor income $\hat{\mathcal{Y}}_{s,t}^h$, profits Π_t and lump sum taxes $T_t(\mathbf{x})$. That is,

$$\mathcal{Y}_t^*(\mathbf{x}) = \hat{\mathcal{Y}}_{s,t}^h + \Pi_t - T_t(\mathbf{x}) \quad (4.5)$$

We suppose that workers initially employed in automated occupations form a large family. They act in their interest, but insure each other against idiosyncratic reallocation opportunities.²² As a result, their effective labor income is not indexed by their reallocation

risk is realized.

²¹ This restriction allow us to retain tractability by preventing an artificial dispersion in the distribution of asset holdings. We relax this assumption in our quantitative model (Section 7).

²² This assumption allows us to abstract from insurance considerations at this point. We relax this assumption in our quantitative model.

history. Profits are

$$\Pi_t \equiv G^*(\boldsymbol{\mu}_t; \alpha) - \int \mathbf{1}_{\{h(\mathbf{x})=h\}} \tilde{\zeta} w_t^h d\pi_t \quad \text{with} \quad \mu_t^{h'} = \frac{\int \mathbf{1}_{\{h=h'\}} \tilde{\zeta} d\pi_t}{\phi^h} \quad (4.6)$$

For simplicity, we suppose that profits are claimed symmetrically — all our results carry through if we assume that workers displaced by automation claim no profits.²³ To retain tractability and prevent a dispersion in the wealth distribution, we suppose that the returns on annuities are taxed lump sum by the government $T_t(\mathbf{x}) \equiv \chi a_t(\mathbf{x})$ and rebated to new generations.²⁴ Finally, workers still face the occupational choice (2.7) with the values now indexed by the new idiosyncratic state \mathbf{x} .

4.3 Equilibrium

The rest of the model is unchanged. The resource constraint is still given by (2.8)–(2.9). The wages that insure labor market clearing in each occupation are

$$w_t^h \equiv 1/\phi^h G_h^*(\boldsymbol{\mu}_t; \alpha) \quad (4.7)$$

All agents act competitively. We choose the price of the final good as *numéraire*. A competitive equilibrium is defined as follows.

Definition 1 (Competitive equilibrium). A competitive equilibrium consists of a degree of automation α , and sequences for effective labor supplies $\{\mu_t^h\}$, consumption and savings functions $\{c_t(\mathbf{x}), a_t(\mathbf{x})\}$, interest rate, wages, profits and incomes $\{r_t, \{w_t^h\}, \Pi_t, \{\mathcal{Y}^*(\mathbf{x})\}\}$, and distributions of states $\{\pi_t(\mathbf{x})\}$ such that: (i) automation and labor demands are consistent with the firm's optimization (4.1)–(4.2); (ii) consumption and savings are consistent with workers' optimization; (iii) interest rates insure that the resource constraint is satisfied

$$\int c_t(\mathbf{x}) d\pi_t = G^*(\boldsymbol{\mu}_t; \alpha); \quad (4.8)$$

(iv) wages, profits and incomes are given by (4.5)–(4.7); and (v) the distribution of states evolves consistently with workers' optimal choices and the law of motion of productivity (2.6).

²³ We implicitly assume that all workers are endowed with one unit of equity, which is fully illiquid. That is, we effectively impose two restrictions. First, workers are unable to hedge ex ante against the risk that their occupation becomes automated. Second, workers are limited in their ability to self-insure ex post by selling their equity after being affected by automation. These two restrictions are in-line with the evidence discussed in footnote 20.

²⁴ We relax this last assumption in our quantitative model (Section 7).

5 Inefficient Automation

We now show that automation is inefficient when reallocation and borrowing frictions are sufficiently severe. Section 5.1 proves that the equilibrium is inefficient and discusses the role of the frictions. Section 5.2 explains why automation and labor reallocation are inefficient. Section 5.3 contrasts our results to the existing literature.

5.1 Inefficient Equilibrium

We now state the first main result of this paper. The decentralized equilibrium is inefficient when reallocation is slow and borrowing constraints are tight.²⁵

Proposition 3 (Failure of First Welfare Theorem). *The laissez-faire equilibrium is inefficient if and only if reallocation frictions (λ, κ, χ) and borrowing frictions (\underline{a}) are such that $a^*(\lambda, \kappa, \chi) < \underline{a} \leq 0$ for some threshold $a^*(\cdot)$ defined in Appendix A.4. The threshold satisfies $a^*(\lambda, \kappa, \chi) < 0$ — i.e., inefficiency can occur — if and only if labor reallocation is slow $1/\lambda > 0$ or $1/\kappa > 0$ or $1/\chi < +\infty$.*

Proof. See Appendix A.4. □

Figure 5.1 illustrates this result in the space of reallocation frictions $(1/\lambda)$ and borrowing frictions (\underline{a}) .²⁶ This space is partitioned in two main regions. The equilibrium is efficient as long as the frictions fall in the white region — that is $\underline{a} \leq a^*(\cdot)$. This occurs when either reallocation frictions are sufficiently small or borrowing constraints are sufficiently loose. In contrast, the equilibrium is inefficient when the frictions fall into either one of the colored regions — that is $\underline{a} > a^*(\cdot)$. Inefficiency is more likely to occur when reallocation is slow, and borrowing constraints are tight.²⁷

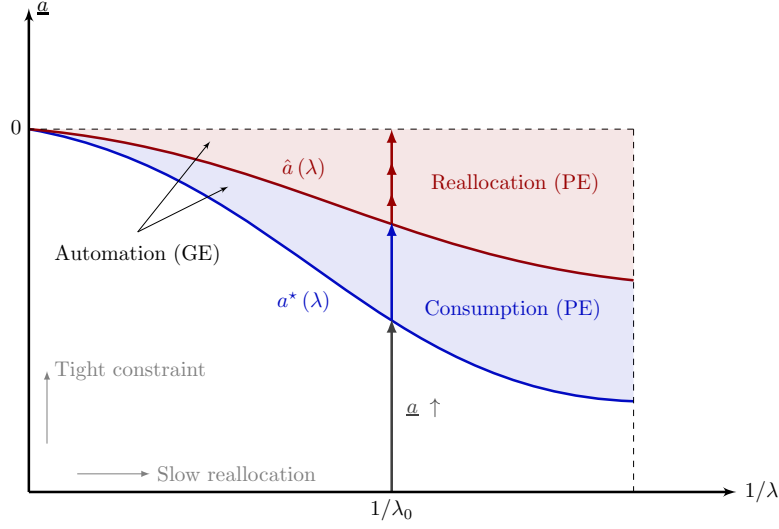
To understand the nature of this inefficiency, we momentarily adopt a partial equilibrium (PE) approach and fix the sequence of prices that prevail in an efficient economy. This allows us to focus on how reallocation and borrowing frictions directly affect workers' consumption, savings and reallocation decisions. The discussion remains informal at this point. We formalize these insights in Appendix A.10.

²⁵ Throughout, we continue to assume that workers incur a productivity loss $\theta > 0$ when moving across occupations (Section 2.2).

²⁶ For exposition, we focus on the effect of the slow arrival of mobility opportunities $(1/\lambda)$ and abstract from the other forms of slow reallocation $1/\kappa \rightarrow 0$ and $1/\chi \rightarrow +\infty$.

²⁷ This is the case in Figure 5.1. More generally, the threshold $a^*(\lambda)$ is non-monotonic in its arguments. In particular, $\lim_{1/\lambda \rightarrow +\infty} a^*(\lambda) = 0$ when existing workers never reallocate (as in (Guerreiro et al., 2017)). The case of interest (.

Figure 5.1: Distorsions at the laissez-faire



Labor incomes for workers born in period s are²⁸

$$\hat{\mathcal{Y}}_{s,t}^h = w_t^A + \overbrace{(1 - \exp(-\lambda \min\{t, T_0\}))}^{\text{Mass of workers who reallocate}} \overbrace{\left[\hat{\Theta}_t(\lambda, \kappa) \times (1 - \theta) w_t^N - w_t^A \right]}^{\text{Self-insurance through reallocation}} \quad (5.1)$$

when these workers are initially employed in automated occupations, i.e. $h = A, s < 0$, and $\hat{\mathcal{Y}}_{s,t}^h = w_t^N$ otherwise.²⁹ Here, the term $\hat{\Theta}_t(\lambda, \kappa)$ captures the share of workers who exited unemployment or retraining among those who reallocated (Appendix A.3). Figure 5.2a depicts the paths of the average labor incomes in each of the occupations. When reallocation is *slow*, automation decreases the income of workers initially employed in automated occupations. This decrease is not fully persistent though. Their income slowly rises after they reallocate — it increases from \mathcal{Y}_t^A to $(1 - \theta)\mathcal{Y}_t^N$ — or their peers do — \mathcal{Y}_t^A increases over time. This makes workers displaced by automation want to borrow while they reallocate. The following remark states this insight.

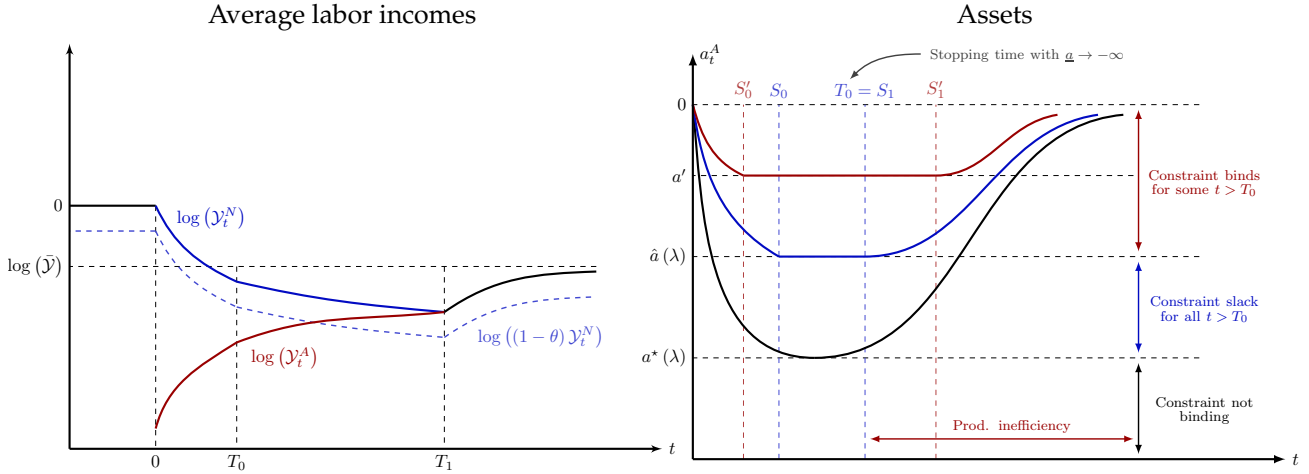
Remark 1. *Workers displaced by automation expect their income to increase as they slowly reallocate. This creates a motive for borrowing.*

²⁸ The large family of workers initially employed in automated occupations earns w_t^A for each worker who has not relocated, and $(1 - \theta) w_t^A$ for each worker who reallocated *and* exited unemployment at rate κ — which is captured by the term $\hat{\Theta}_t(\lambda, \kappa)$ (Appendix A.3).

²⁹ Except for workers initially employed in automated occupations, all workers effectively earn the wage that prevails in non-automated occupations. The reason is that they are either initially employed in non-automated occupations and remain there, or are born in period $s \geq 0$ in which case they either choose to locate in a non-automated occupation ($t < T_1$) or are indifferent between automated and non-automated occupations ($t \geq T_1$).

When reallocation and borrowing frictions are sufficiently mild, workers are never borrowing constrained — the black curve in Figure 5.2b — and the equilibrium is efficient — the white region in Figure 5.1.

Figure 5.2: Incomes and assets



As the frictions become more severe, borrowing constraints eventually bind $\underline{a} > a^*(\cdot)$. In this case, workers initially employed in automated occupations are unable to achieve consumption smoothing over $t \geq 0$. So consumption choices are distorted— the blue region in Figure 5.1. When borrowing constraints are still relatively loose, workers are constrained at some point (S_0) but they still stop being constrained (S_1) *before* they would have stopped reallocating absent borrowing frictions (T_0) — the blue curve in Figure 5.2b. In this case, reallocation decisions remain undistorted since they are forward-looking. As the frictions become even more severe $\underline{a} > \hat{a}(\cdot)$, workers remain constrained in the period when they were supposed to stop reallocating (T_0) — the red curve in Figure 5.2b. In this case, the reallocation decision becomes distorted too — the red region in Figure 5.1.

Turning to the general equilibrium (GE), the distortions in consumption and labor allocations alter the path of interest rates and wages. This causes automation to become distorted and adds to the partial equilibrium distortion of labor reallocation – the colored regions in Figure 5.1. We elaborate on this point in the next section.

5.2 Why Is Automation Inefficient?

To understand why automation is inefficient, we compare the private and social incentives to automate³⁰

$$(LF) \quad \int_0^{+\infty} \exp(-\rho t) u'(c_{0,t}^N) \Delta_t^* dt = 0 \quad (5.2)$$

$$(FB) \quad \int_0^{+\infty} \exp(-\rho t) u'(c_{0,t}^A) \Delta_t^* dt = 0 \quad (5.3)$$

respectively, where

$$\Delta_t^* \equiv \frac{\partial}{\partial \alpha} G^*(\mu_t; \alpha) \quad (5.4)$$

is the response of output to automation — an increase in α . Firms — just like the government — increase automation until the returns Δ_t^* are zero in present discounted value. The (intertemporal) *marginal rate of substitution* (MRS) that they internalize are potentially different, however. Absent borrowing constraints, all workers share the same MRS — which is inversely related to the gross interest rate. In this case, the private and social incentives to automate coincide, and the equilibrium is efficient. When workers in automated occupations become borrowing constrained, their planning horizon is effectively shorter (Woodford, 1990) than their peers' in non-automated occupations. Thus, binding borrowing constraints drive a wedge between the interest rate that firms face when they automate and the MRS of workers displaced by automation. In this case, private and social incentives to automate differ. The following remark states this insight. In Section 6, we further show that automation is *excessive* when automation and labor reallocation are *complements* — the gains from automation are realized gradually as more workers reallocate.

Remark 2. *The degree of automation is inefficient. Firms are effectively too patient and (partly) overlook the time it takes for labor to reallocate and for the benefits of automation to materialize.*

It is worth noting that the economy can be *inefficient* while still achieving *production efficiency* (Diamond and Mirrlees, 1971). This is the case when borrowing and reallocation frictions are sufficiently severe to prevent consumption smoothing, but not sufficiently so to distort the reallocation choices — i.e., the blue region in Figure 5.1. In this case, displaced workers are still pushed against their borrowing constraints, and the private (5.2) and social (5.3) incentives to automate still differ. However, reallocation is efficient

³⁰ To obtain (5.3), we use Proposition 2, and the envelope condition (A.39) in Appendix A.4. This expression holds for *any* weights η that the planner assigns to workers.

conditional on the equilibrium degree of automation. The distortion in automation simply affects the timing of the output and consumption stream $\{C_t\}$ and the the economy moves *along* its production possibility frontier (as opposed to *inside*).³¹ Production efficiency also fails whenever the borrowing and reallocation frictions are particularly severe — i.e., the red region in Figure 5.1.

5.3 Relation to the Literature

To conclude this section, we draw a connection to two strands of the literature.

Regulation of automation. A burgeoning literature argues that a government should tax automation when it has a preference for redistribution. This literature has worked with *efficient* economics. These efficient benchmarks obtain in our economy in two limit cases. Suppose first that the process of labor reallocation is *instantaneous* — as in Costinot and Werning (2018).³² In our model, this occurs when workers receive immediately reallocation opportunities $1/\lambda \rightarrow 0$, do not go through unemployment or retraining spells when moving between occupations $1/\kappa \rightarrow 0$, and no reallocation takes place through new generations $1/\chi \rightarrow +\infty$. In this case, the model is static $T_0, T_1 \rightarrow 0$ — the economy jumps to its new steady state. To understand why the equilibrium is efficient in this case, it is useful to inspect workers' labor incomes (5.1). When reallocation is instantaneous, workers initially employed in automated occupations flow immediately across occupations so as to insure $(1 - \theta) w_t^N = w_t^A$. Workers initially employed in non-automated occupations earn w_t^A . In other words, automation has *distributional* effects. However, there is no motive for borrowing since income changes are fully permanent. As a result, borrowing frictions are irrelevant and the equilibrium is efficient. This explain why slow reallocation is *necessary* for inefficiency.

In turn, suppose that reallocation is slow but there are no borrowing frictions — as in Guerreiro et al. (2017).³³ In our model, this occurs when the borrowing constraints are sufficiently loose $\underline{a} \rightarrow -\infty$. In this case, wages remain persistently higher in non-automated occupations, until the gap closes in period $t = T_1$. Therefore, automation has distributional effects and creates a motive, but there is no wedge between the MRS the firms and

³¹ In particular, this consumption stream $\{C_t\}$ is supported as a first best for *some* Pareto weights η .

³² The general production technology in Costinot and Werning (2018) effectively allows for labor reallocation between occupations.

³³ In Guerreiro et al. (2017), reallocation taking place (entirely) through new generations $1/\chi < +\infty$. That is, existing generations are not allowed to move in their model. In our model, this corresponds to the case where workers never receive reallocation opportunities $1/\lambda \rightarrow +\infty$ or unemployment spells $1/\kappa$ are prohibitively long (Assumption 2).

the planner face.

Dynamic inefficiency. An extensive literature argues that capital investment can be dynamically inefficient.³⁴ This can occur in economies with overlapping generations (Samuelson, 1958; Phelps, 1965; Diamond, 1965), or when precautionary saving depresses the interest rate (Aiyagari, 1995; Aguiar et al., 2021). In these environments, the stock of capital is excessively high in the *long-run* and a planner can achieve a Pareto improvement by redistributing resources across generations. The source of inefficiency is different in our model. In Section 6.4, we extend our baseline environment to allow for gradual investment in automation (as opposed to a one-time choice). We find that the equilibrium is inefficient during the *transition* — while labor reallocates and displaced workers are borrowing constrained — but converges to an efficient allocation in the long-run. The inefficiency that we document relies on the presence of multiple occupations and slow reallocation — two features that the literature on dynamic inefficiency and capital taxation abstracts from.

6 Optimal Policy Interventions

We now discuss optimal policy. We state the Ramsey problem and discuss our choice of policy instruments in Sections 6.1. In Section 6.2, we show that the equilibrium is generically *constrained inefficient* due to pecuniary externalities. In Section 6.3, we show that the government should tax automation on efficiency grounds — even when it does not value equity directly. Section 6.4 draws connections to the literature.

6.1 Ramsey Problem

We start by stating the government’s problem in its general form, allowing for a rich set of policy instruments. We then discuss the types of interventions that would implement first best outcomes. Even if these tools are unrealistic in practice, this discussion clarifies and motivates the restrictions that we impose on the set of policy instruments. Finally, we state the constrained Ramsey problem that we work with in the remaining of this section. For tractability and to obtain more compact expressions, we assume in the following that workers cannot borrow $\underline{a} \rightarrow 0$ and abstract from overlapping generations $1/\chi \rightarrow +\infty$.³⁵

³⁴ A related literature finds that capital investment is not only inefficient but also *constrained* inefficient too — even when limiting the tools that the government has access to. We discuss this literature in Section 6.4, as part of our second best analysis.

³⁵ All the insights carry through in the general case with $1/\chi < +\infty$.

6.1.1 General Problem

We suppose at this point that the government has access to an arbitrary set of taxes $\{\tau_t\}$. This set can include complex lump sum transfers $\{T_t^h\}$, a distortionary tax on automation $\{\tau^x\}$, severance payments $\{\zeta_t\}$, non-linear income taxes $\{\mathcal{T}_t(\cdot)\}$, etc. The government chooses these taxes to solve

$$\max \sum_h \phi^h \eta^h \int_0^{+\infty} \exp(-\rho t) u(c_t^h) dt \quad (6.1)$$

for a given set of Pareto weights η , subject to the following implementability constraints. First, consumption and reallocation choices are consistent with workers' optimization, i.e., equations (A.18)–(A.22), (A.26) and (A.28) in Appendix A.3 augmented with taxes. Second, effective labor supplies are given by equations (A.7)–(A.10). Third, automation is consistent with firms' optimality condition (A.36) given taxes. Finally, wages and profits are given by (A.29)–(A.30) and labor incomes are given by (5.1).

6.1.2 Implementing the First Best

The type of inefficiency that we document operates when displaced workers are pushed against their borrowing constraints. Any intervention that fully alleviates these borrowing constraints would restore efficiency. In particular, *targeted* lump sum transfers $\{T_t^h\}$ could implement any efficient allocation — a version of the Second Welfare Theorem holds in our economy (Proposition 10 in Appendix B.5). However, this type of transfers are unlikely to be feasible in practice. They would require the government to know which occupations are automated and discriminate between workers who are displaced and those who are temporarily unemployed. These informational constraints motivate a large literature on optimal income taxation (Piketty and Saez, 2013 for a review) and the existing literature on the regulation of automation (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020).

Alternatively, the government could undo workers' borrowing constraints by borrowing on their behalf and balancing its budget with *symmetric* transfers $\{T_t\}$ to all workers (Araujo et al., 2015). In theory, this intervention would implement a first best and could be achieved through a form of negative income tax or Universal Basic Income (Friedman, 1962; Moffitt, 2003). In practice, the fiscal cost is likely to be prohibitive. The payments would have to be given to *all* workers and be particularly generous to ensure that no workers are unconstrained — a scenario that the literature on heterogeneous agents has not seriously considered. Moreover, these programs would have to be financed with dis-

torsionary taxation with potentially large welfare costs (Daruich and Fernández, 2020; Conesa et al., 2021; Guner et al., 2021; Luduvic, 2021).

Finally, it is worth noting that a non-linear income tax $\mathcal{T}_t(\cdot)$ (Mirrlees, 1971; Atkinson and Stiglitz, 1976) or unemployment insurance could benefit displaced workers and help relax their borrowing constraints. However, these interventions would typically not implement first best allocations in practice — e.g., they would reduce labor supply and distort the incentives to reallocate between occupations. In addition, non-linear income taxation is a particularly blunt and costly tool to redistribute resources *across* occupations when there is a relatively large dispersion in incomes *within* occupations due to idiosyncratic shocks (as in our quantitative model).

6.1.3 Constrained Ramsey Problem

In the following, we assume that the government is not able to *fully* alleviate the borrowing constraints of displaced workers. In this case, the equilibrium is inefficient (Section 5.1). For simplicity, we abstract from social insurance programs altogether and reintroduce them later in our quantitative analysis. Instead, we assume that the government has access to a simple set of taxes that depend on calendar time alone: a linear tax on automation τ^x ; and active labor market interventions (Card et al., 2010 for a survey) that subsidize labor reallocation $\{\zeta_t\}$.³⁶ It is understood that these taxes and subsidies can be positive or negative. These instruments are effectively already used in the U.S. and other advanced economies, and do not require the government to know which occupations become automated or which workers are displaced.³⁷

The government effectively controls two choices with its instruments: the degree of automation α ; and the reallocation of displaced workers T_0 . All other choices must be consistent with workers' and firm's optimality. The government's constrained Ramsey problem reduces to the following primal problem (Lucas and Stokey, 1983).

Lemma 1 (Primal problem). *The government's problem reduces to*

$$\max_{\{\alpha, T_0, \mu_t, c_t\}} \sum_h \phi^h \eta^h \int_0^{+\infty} \exp(-\rho t) u(c_t^h) dt$$

³⁶ Because we abstract from social insurance at this point, we suppose that the government requires the large family (Section 4.2) to reimburse any reallocation subsidies received by its members. These subsidies can take the form of credits for retraining programs or unemployment insurance (when positive), or penalties such as imperfect vesting of retirement funds (when negative).

³⁷ The tax code already treats various forms of capital differently in many countries. In particular, Acemoglu et al. (2020) show that the U.S. tax code already favors investment in automation over investment in labor-augmenting capital.

subject to the laws of motion for effective labor

$$\begin{aligned}\mu_t^A &= \exp(-\lambda \min\{t, T_0\} - \chi t) \\ \mu_t^N &= \left\{ 1 + (\theta - 1) \frac{\phi}{1 - \phi} \exp(-\chi t) \frac{1 - \exp(-\lambda \min\{t, T_0\})}{1 - \mu_t} \right\} (1 - \mu_t^A)\end{aligned}$$

and the consumption allocations

$$c_t^h = \underbrace{1/\phi^h \partial_h G^*(\boldsymbol{\mu}_t; \alpha)}_{\text{Initial wage}} + \underbrace{(1 - \exp(-\lambda \min\{t, T_0\})) \Gamma_t^h}_{\text{Reallocation gains}} + \underbrace{G^*(\boldsymbol{\mu}_t; \alpha) - \sum_h \mu_t^A \partial_h G^*(\boldsymbol{\mu}_t; \alpha)}_{\text{Profits}},$$

where reallocation gains are

$$\Gamma_t^h \equiv (1 - \theta) 1/\phi^h \partial_h G^*(\boldsymbol{\mu}_t; \alpha) - 1/\phi^A \partial_A G^*(\boldsymbol{\mu}_t; \alpha)$$

for each occupation $h \in \{A, N\}$, in the particular case without unemployment / retraining spells ($1/\kappa \rightarrow 0$). The general case is similar but involves the richer laws of motion for effective labor (A.7)–(A.10) and reallocation gains (A.24)–(A.25) in Appendices A.1 and A.3.

6.2 Constrained Inefficiency

We now show that the government should intervene even when its instruments are limited — the equilibrium is *constrained inefficient*.³⁸ To understand why this is the case, it is useful to compare the private and social incentives to automate and reallocate

$$\begin{aligned}\int_0^{+\infty} \exp(-\rho t) u'(\mathcal{Y}_{0,t}^N + \Pi_t) \Delta_t^* dt &= -\Phi^*(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}) \\ \underbrace{\int_{T_0^{\text{SB}}}^{+\infty} \exp(-\rho t) u'(\mathcal{Y}_{0,t}^A + \Pi_t) \Delta_t dt}_{\text{laissez-faire}} &= \underbrace{-\Phi(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta})}_{\text{pecuniary externalities}}\end{aligned}$$

where the terms $\Phi^*(\cdot)$ and $\Phi(\cdot)$ capture the *pecuniary externalities* that automation and reallocation impose on workers — which we define in Appendix A.5. The government takes into account that an increase in automation (α) reduces wages in automated occupations, but increases profits that benefit all workers (or *some* workers when profits are

³⁸ By definition, an allocation is constrained efficient whenever there exists *some* set of Pareto weights $\boldsymbol{\eta}$ such that the allocation coincides with the solution to (6.1) given these weights $\boldsymbol{\eta}$.

not claimed symmetrically).³⁹ Similarly, the government takes into account that an increase in reallocation (T_0) reduces wages in non-automated occupations, but lifts wages in automated occupations. Firms and workers do not internalize these effects.

We show in the following that these pecuniary externalities typically do not net out in presence of reallocation and borrowing frictions. Formally, we establish that the laissez-faire is *generically* constrained inefficient in the sense of [Geanakoplos and Polemarchakis \(1985\)](#) and [Farhi and Werning \(2016\)](#). That is, whenever the laissez-faire and the second best coincide for some weights η , there exists a perturbation of the production function $G^{*,\prime} = \mathcal{G}(G^*, \epsilon)$ such that: (i) $G^{*,\prime}(G^*, \epsilon) \rightarrow G^*$ uniformly as $\epsilon \rightarrow 0$; and (ii) the resulting laissez-faire and the second best do not coincide when $\epsilon > 0$ is sufficiently small.

Proposition 4 (Constrained inefficiency). *Fix the production function G^* . Suppose that the laissez-faire is constrained efficient for some Pareto weights η . Then, there exists a perturbation of the production function $G^{*,\prime} = \mathcal{G}(G^*, \epsilon)$ and a threshold $\bar{\epsilon} > 0$ such that the resulting second best and laissez-faire do not coincide for all $0 < \epsilon \leq \bar{\epsilon}$.*

Proof. See Appendix [A.5](#). □

In other words, the government should intervene *regardless* of its preference for redistribution η . This finding echoes the constrained-inefficiency results in the incomplete markets literature ([Lorenzoni, 2008](#); [Farhi and Werning, 2016](#); [Dávila and Korinek, 2018](#)). The nature of the inefficiency is different, however. Constrained inefficiency occurs in our economy despite the absence of uncertainty and incomplete markets, or endogenous borrowing constraints. Instead, it occurs when firms and workers make *technological choices*, and borrowing constraints distort the (shadow) prices that these agents face.⁴⁰ It is well-known that technological choices can result in inefficiencies by *themselves* ([Acemoglu and Zilibotti, 1997](#); [Acemoglu, 2009](#)). However, our model is set up so that they are a source of inefficiency *only* when borrowing constraints bind, as we have shown in Section [5](#).

6.3 Taxing Automation on Efficiency Grounds

We now present the second main set of results of this paper, which characterizes and signs optimal policy interventions. We show that the government should tax automation on efficiency grounds — even when it does not have a preference for redistribution. We start

³⁹ Again, all our results carry through in the case where displaced workers do not claim profits. Assuming that profits are claimed symmetrically is conservative, if anything, since the increase in profits partly compensates for the decline in labor income experienced by displaced workers.

⁴⁰ Labor reallocation — just like automation — is isomorphic to a technological choice in the Arrow-Debreu construct. Each worker owns a firm that chooses the type of labor services to provide.

by discussing our choice of Pareto weights.

Pareto weights. Taxing automation has two effects. The first effect is *aggregate*: it generates an intertemporal substitution between current resources (the automation cost, or investments more generally) and future output. The importance of this first effect for the government depends on the distribution of marginal utilities over time — the intertemporal marginal rate of substitution. The second effect is *distributional*: some workers benefit more from this intervention than others through the pecuniary externalities we discussed above. The importance of this second effect depends on the distribution of marginal utilities across workers and the Pareto weights that the government places on each worker. To highlight the new rationale for policy intervention that we propose, we initially abstract from equity altogether — the second effect. We suppose that the government intervenes *exclusively* on efficiency grounds — the first effect. This is achieved by choosing Pareto weights η^{effic} such that the distributional effects net out when taking into account the worker’s marginal utilities and the weights η^{effic} .⁴¹ In particular, these weights are such that the government values constrained workers relatively *less* compared to a utilitarian government that values equity. We reintroduce equity considerations in Section 6.4.

6.3.1 With Active Labor Market Interventions

We are now ready to sign optimal policy interventions. At this point, we continue to assume that the government has the necessary tools to intervene ex post in the labor reallocation process. The following result shows that the government should *curb* (or *tax*) automation on efficiency grounds.

Proposition 5 (Second best). *Suppose that the government controls automation, as well as labor reallocation. Then, curbing automation is optimal.*

Proof. See Appendix A.6. □

To understand this result, it is useful to inspect the private and social benefits to automate

$$(LF) \quad \int_0^{+\infty} \exp(-\rho t) u' \left(\mathcal{Y}_{0,t}^N + \Pi_t \right) \Delta_t^* dt = 0 \quad (6.2)$$

$$(SB) \quad \int_0^{+\infty} \exp(-\rho t) \left\{ \sum_h \phi^h \eta^{h,\text{effic}} u' \left(\mathcal{Y}_{0,t}^h + \Pi_t \right) \right\} \Delta_t^* dt = 0 \quad (6.3)$$

⁴¹ The details are provided in Appendices A.6–A.7. These weights are inversely related to the workers’ marginal utilities. Absent borrowing constraints, these weights take the familiar form $1/\eta^{\text{effic},h} \propto u' \left(c_0^h \right)$.

where Δ_t^* is the response of output to automation and is given by (5.4). Firms — just like the government — increase automation until the returns Δ_t^* are zero in present discounted value. Their effective (intertemporal) *marginal rate of substitution* (MRS) are different, however. The reason is that the government takes into account the welfare of *all* workers. In contrast, the firm’s decisions are based on the equilibrium interest rate which reflects the welfare of unconstrained workers who were initially employed in non-automated occupations. In other words, firms are excessively patient compared to the government.

The direction of the intervention depends on the time profile of $\{\Delta_t^*\}$. By assumption, automation and reallocation are complements (Assumption 3). Therefore, the flows Δ_t^* are *back-loaded* — as depicted in the left panel of Figure 3.1. The firm initially incurs a productivity cost when adopting automation technologies ($\Delta_t^* < 0$ for small t), and the benefits get realized gradually as labor reallocates between occupations ($\Delta_t^* > 0$ for large t). Comparing (6.2) and (6.3), it follows that the government prefers a flatter time profile of $\{\Delta_t^*\}$. Curbing automation achieves so by reducing the cost of automation in the short-run at the cost of smaller productivity gains in the long-run. The following remark states this insight.

Remark 3. *Taxing automation prevents excessive investment and raises consumption early on in the transition, precisely when displaced workers are borrowing-constrained.*

6.3.2 Without Active Labor Market Interventions

In practice, ex post policies can be difficult to implement. Active labor market interventions often produce mixed results (Heckman et al., 1999; Card et al., 2010; Doerr and Novella, 2020), or have unintended consequences for untargeted workers (Crépon and van den Berg, 2016).⁴² For this reason, we now suppose that the government controls automation (ex ante) but is unable to control directly labor reallocation (ex post).

Proposition 6 (Second best — ex ante only). *Suppose that the government only controls automation — but labor reallocation T_0 must be consistent with workers’ optimization. This reinforces the government’s desire to curb automation when unemployment / retraining spells are short ($1/\kappa \rightarrow 0$). On the contrary, this reduces the government’s desire to curb automation unemployment / retraining spells are long ($1/\kappa > 1/\kappa^*$) for some threshold $1/\kappa^* > 0$.⁴³*

⁴² This would be the case with *gross* labor flows between occupations — as in our quantitative model (Section 7).

⁴³ The average duration of unemployment spells $1/\kappa$ is bounded above by Assumption 2. In theory, the case where the government curbs automation *less* might not present itself. This is an empirical question that we address with our quantitative model (Section 7).

Again, it is useful to inspect the social incentives to automate

$$(SB) \quad \int_0^{+\infty} \exp(-\rho t) \times \left\{ \sum_h \phi^h \eta^{h, \text{effic}} u' \left(\mathcal{Y}_{0,t}^h + \Pi_t \right) \right\} \left\{ \Delta_t^* + T_0' \left(\alpha^{\text{SB}} \right) \Delta_t \right\} dt = 0, \quad (6.4)$$

and compare them to the private incentives (6.2). Here, Δ_t denotes the response of output to reallocation, and is defined by (3.11). Missing active labor market interventions provide an additional motive for policy intervention. The government internalizes the indirect effect of automation on output Δ_t due to the reallocation it induces $T_0'(\cdot) > 0$, in addition to the direct effect. Workers's reallocation at the laissez-faire satisfies

$$(LF) \quad \int_{T_0^{\text{LF}}}^{+\infty} \exp(-\rho t) u' \left(\mathcal{Y}_{0,t}^A + \Pi_t \right) \Delta_t dt = 0 \quad (6.5)$$

Absent borrowing constraints, all workers share the same MRS. In this case, the indirect effect of automation Δ_t is no cause for intervention either, given (6.5). When borrowing constraints bind, the private and social incentives to automate differ due to *both* the direct effect Δ_t^* and the indirect effect Δ_t . The government should curb automation based on the direct effect (Section 6.3.1). The sign of the indirect effect depends on the duration of unemployment / retraining spells.

When unemployment spells are short $1/\kappa \rightarrow 0$, the flows Δ_t are *front-loaded* (see Figure 3.1). Workers enjoy a higher wage after they reallocate $\Delta_0 > 0$, but their new wage declines gradually as more workers enter non-automated occupations $\lim_{t \rightarrow +\infty} \Delta_t < 0$. Constrained workers put an excessive weight on early, positive payoffs: binding borrowing constraints incentivize them to rely excessively on mobility as a source of self-insurance. This indirect effect reinforces the government's desire to *curb* automation.

When unemployment spells are sufficiently long, the flows Δ_t are *back-loaded* instead. Workers' earnings decrease during unemployment $\Delta_0 < 0$, before they are paid the wage in their new occupation.^{44,45} Constrained workers put an excessive weight on early, negative payoffs: binding borrowing constraints limit their ability to use mobility as a source of self-insurance. The indirect effect dampens the government's desire to curb automation, and could in principle lead the government to *stimulate* automation.

⁴⁴ See footnote 43. In the medium term, $\Delta_t > 0$. Eventually, $\lim_{t \rightarrow +\infty} \Delta_t < 0$ since workers experience a permanent productivity loss.

⁴⁵ The result generalizes immediately when allowing a replacement rate during unemployment that is less than 1. We allow for such a replacement rate in our quantitative model (Section 7).

6.4 Relation to the Literature

To conclude this section, we clarify our contribution relative to the literatures on the taxation of automation on equity grounds and on long-run capital taxation.

Equity concerns. A growing literature argues that a government should curb automation when it has a preference for redistribution (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020). To draw a connection to this literature, we now introduce equity concerns in our model. We denote by η^* the weights that support the decentralized allocation with no borrowing frictions.⁴⁶ In turn, we denote by $(\eta^{\text{utilit}})_s^{\frac{1}{\sigma}} \equiv \sum_h (\eta_s^{h,*})^{\frac{1}{\sigma}}$ the symmetric weights that a *utilitarian* government would use *within* each generation s .⁴⁷ We show below that the government curbs automation in an efficient economy with no borrowing frictions when it has a preference for redistribution.

Proposition 7 (Second best with equity concerns). *Consider the special case of our model with no borrowing frictions — so that the laissez-faire is efficient. Suppose that the government is utilitarian, i.e., uses symmetric weights η^{utilit} within generations. Suppose that the government can either control automation and reallocation, or only automation (with reallocation consistent with workers' optimization). Then, the optimal policy curbs automation.*

Proof. See Appendix A.8. □

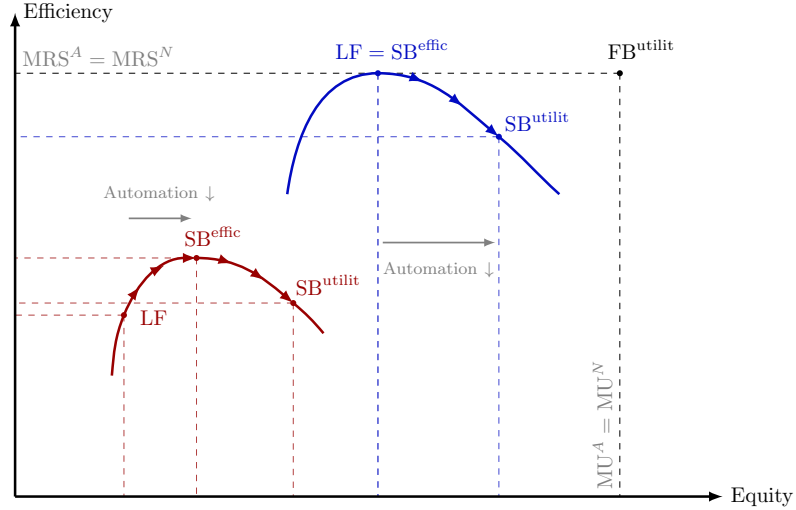
Figure 6.1 illustrates this result schematically. Automation has distributional effects: it reduces equity at the laissez-faire (LF) compared to the first best of a utilitarian planner (FB^{utilit}). Displaced workers are worse off and their marginal utility is (persistently) higher than other workers' $MU^A > MU^N$. We consider two economies. The first one is efficient (in blue), which occurs when borrowing constraints are sufficiently loose (Section 5.1). In this case, the (intertemporal) marginal rates of substitutions of displaced workers coincides with the equilibrium interest rate faced by firms who automate $MRS^A = MRS^N$. The government does not intervene (LF = SB^{effic}) unless it has a preference for redistribution (SB^{utilit}), in which case it taxes automation and sacrifices efficiency to improve equity. Equity gains can be achieved at a relatively small efficiency cost in this case — an envelope condition applies. This is the canonical trade-off emphasized in the existing literature on the taxation of automation. The second economy is inefficient (in red), which occurs when borrowing constraints are relatively tight. In this case, displaced workers are

⁴⁶ These weights exist since the equilibrium is efficient in this case (Proposition 3).

⁴⁷ This particular averaging was introduced in Section 3.1, and ensures that the equity motives does not affect the aggregate allocation at the first best — i.e. (3.9).

pushed against their borrowing constraints. This drives a wedge between the (intertemporal) marginal rate of substitution of displaced workers and the equilibrium interest rate faced by firms $MRS^A > MRS^N$. Firms are effectively too patient: automation is inefficient. The government can improve *both* efficiency and equity by taxing automation — there is no trade-off.

Figure 6.1: Second best with **efficiency** ($\underline{a} \rightarrow -\infty$) and **inefficiency** ($\underline{a} \rightarrow 0$)



Capital taxation. An extensive literature on capital taxation with incomplete markets argues that capital should be taxed in the long-run. This literature has proposed two main arguments for taxing capital. First, it can improve insurance against earnings risk by affecting the relative price between labor and capital services (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012). Second, it can improve dynamic efficiency by reducing capital accumulation in economies where the interest rate is reduced by precautionary savings (Chamley, 2001; Aguiar et al., 2021). These two rationales share two features: they rely on the presence on uninsured idiosyncratic risk; and optimal policies affect investment in the long-run.

The rationale that we propose is conceptually distinct. First, we find that taxing automation is optimal even absent idiosyncratic uncertainty. Second, the government should tax automation during the *transition* to the long-run, but has no reason to intervene once labor reallocation is complete. To clarify this point, we now extend our model and allow automation to take place gradually due to convex adjustment costs. Output (net of investment costs) is

$$Y_t = G^*(\mu_t; \alpha_t) - x_t \alpha_t - \Omega(x_t / \alpha_t) \alpha_t, \quad (6.6)$$

where x_t is the gross investment rate in automation and $\Omega(\cdot)$ is a convex function. The

law of motion of automation is $d\alpha_t = (x_t - \delta\alpha_t) dt$ for some depreciation rate $\delta > 0$. Automation is costly as installing (x_t) or maintaining (δ) these technologies diverts resources away from production activities.⁴⁸ To obtain realistic long-run dynamics, we reintroduce overlapping generations $1/\chi < +\infty$.

Proposition 8 (No intervention in the long-run). *A (utilitarian) government does not intervene in the long-run $\alpha_t^{LF} / \alpha_t^{FB} \rightarrow 1$ as $t \rightarrow +\infty$.*

Proof. See Appendix A.9. □

The government should *slow down* automation while labor reallocation takes place and workers are borrowing constrained, but has no reason to tax automation in the long-run. This rationale for intervention is specific to automation — as opposed to capital accumulation — since it displaces workers and pushes them against their borrowing constraints.^{49,50} It should be noted that a positive long-run tax on automation could still be optimal for reasons that are not captured in this baseline model — e.g., for insurance purposes or to prevent dynamic inefficiency. This partly motivates our quantitative analysis, where we allow for uninsured idiosyncratic risk.

7 Quantitative Model

In the previous sections, we established that the government should slow down automation when reallocation and borrowing frictions are sufficiently severe. In the remaining of this paper, we quantify the welfare gains from optimal policy interventions. To this end, we enrich our baseline model along several dimensions that are potentially important for optimal policy. First, workers are subject to uninsured idiosyncratic earnings risk (Aiyagari, 1994; Huggett, 1993). It is well-known that incomplete markets play an important role for capital taxation, since they create an insurance motive and can contribute to dynamic inefficiency (Section 5.3). Second, workers now face idiosyncratic preference shocks

⁴⁸ This specification provides a micro-foundation for the cost of automation in our baseline model. In this baseline model, automation takes place once and for all and entails a productivity cost as labor is diverted away from production. In our quantitative model, automation requires maintenance to offset depreciation, which effectively diverts labor away from production. The effective production function for automated occupations — net of investment — is $\hat{F}(\mu; \alpha) \equiv A(\alpha + \mu)^{1-\eta} - (\delta + \Omega(\delta))\alpha$ at the new steady state.

⁴⁹ Empirically, displaced workers have a higher risk of being borrowing constrained (footnote 1).

⁵⁰ In theory, workers could also become borrowing constrained after an occupation-specific TFP shock — without ever experiencing an earnings loss. The reason is that expect their earnings to go up in the future after they reallocate to this occupation. In practice, this anticipatory effect is likely to be quantitatively small (Poterba, 1988). The effect might actually be the opposite: workers borrowing constraints are relaxed as their future earnings increase (Jappelli, 1990; Carman et al., 2003).

for mobility across occupations (Artuç et al., 2010; Caliendo et al., 2019). This generates gross flows between occupations, which is an important feature of the data (Kambourov and Manovskii, 2008; Moscarini and Vella, 2008) and can create an additional insurance motive to stimulate wages in automated occupations. We also introduce progressive income taxation (Heathcote et al., 2017) and unemployment benefits (Krueger et al., 2016) to account for existing sources of insurance that can benefit displaced workers (Section 6.1.2). Finally, automation now takes place gradually due to convex adjustment costs. As in our baseline model, time is continuous, and there is no aggregate uncertainty. Periods are still indexed by $t \geq 0$.

7.1 Firms

Production. Occupations are still indexed by $h \in \{A, N\}$. These occupations produce intermediate goods by combining labor, automation and a fixed factor

$$y_t^h = F(\mu_t^h; \alpha_t^h) = A^h (\psi^h \alpha_t^h + \mu_t^h)^{1-\eta} \quad (7.1)$$

for some elasticity $\eta \in (0, 1)$ and productivities $A^h, \psi^h > 0$, with μ_t^h denoting effective labor and $\{\alpha_t^h\}$ denoting automation.⁵¹ We set $\psi^A \equiv 1$ in automated occupations — so that automation displaces labor *gradually* — and $\psi^N \equiv 0$ in non-automated occupations. The aggregate technology has a constant elasticity of substitution

$$G(\{y_t^h\}) \equiv \left(\sum_h \phi^h (y_t^h)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} \quad (7.2)$$

for some elasticity $\nu > 1$, where $\phi^A \equiv \phi$ is the mass of automated occupations and $\phi^N \equiv 1 - \phi$ is the mass of non-automated occupations. Automated occupations rent the stock of automation on spot markets (Guerreiro et al., 2017; Costinot and Werning, 2018) at rate $\{r_t^*\}$ from a mutual fund.

Investment. A mutual fund invests workers' savings in government's bonds and automation. The initial stock of automation is $\alpha_0 = 0$. The law of motion of automation is

$$d\alpha_t = (x_t - \delta\alpha_t) dt, \quad (7.3)$$

⁵¹ That is, labor and automation are perfect substitutes. Each occupation can be interpreted as a task in the frameworks of Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018b). The only difference is that automation is *as* productive as labor in our model, so that labor is not displaced immediately. Instead, labor is crowded out progressively as the stock of automation increases over time.

where δ is the rate at which the automation depreciates, and x_t is the investment rate. Investment is subject to quadratic adjustment cost $\Omega(x_t; \alpha_t) = \omega(x_t/\alpha_t)^2 \alpha_t$ as in (6.6). The government taxes automation linearly at rate $\{\tau_t^x\}$ and rebates the proceedings to the mutual fund. The mutual fund is competitive so that its return is the return on bonds $\{r_t\}$.

7.2 Workers

There are still overlapping generations of workers that are replaced at rate $\chi \in [0, +\infty)$. A worker is indexed by five idiosyncratic states: their holdings of liquid assets (a); their current occupation of employment (h); their employment status (e); a binary variable that indicates whether they ever switched between occupations (ξ); and the mean-reverting component of their productivity (z). In the following, we let $\mathbf{x} \equiv (a, h, e, \xi, z)$ denote the workers' idiosyncratic states, and π denote the corresponding measure.

Assets and constraints. Workers now save in two financial assets: a liquid asset (mutual fund) with return $\{r_t\}$, and an illiquid asset (equity). Following Auclert et al. (2018), we suppose that each worker is endowed with one unit of the illiquid asset $\hat{a} \equiv 1$ and is unable to trade it away. This asset pays a random dividend, which is proportional to a worker's mean-reverting productivity (z). In addition, workers have access to annuities (Blanchard, 1985; Yaari, 1965) which allows them to self-insure against survival risk. Financial markets are otherwise incomplete: workers are unable to trade contingent securities against the risk that their occupation becomes automated, against the risk that they are not able to relocate across occupations, against unemployment risk, and against their idiosyncratic productivity risk. Workers now face the flow budget constraint

$$da_t(\mathbf{x}) = [\mathcal{Y}_t^{\text{net}}(\mathbf{x}) + (r_t + \chi) a_t(\mathbf{x}) - c_t(\mathbf{x})] dt \quad (7.4)$$

where $\mathcal{Y}_t^{\text{net}}(\mathbf{x})$ denotes net income and r_t is the return on the liquid asset.⁵² Workers still face the borrowing constraint (4.4). They hold $a^{\text{birth}}(\mathbf{x}) = 0$ assets at birth.

Occupational choice. Workers choose their first occupation of employment at birth. During their life, workers supply labor and are given the opportunity to move between automated and non-automated occupations with intensity λ .⁵³ Workers are subject to linearly

⁵² For simplicity, we suppose that the capital income earned through bonds is not taxed.

⁵³ Our model reduces to a standard income fluctuations problem when $\lambda = 0$ and all workers are employed initially.

additive taste shocks when choosing between occupations. These taste shocks are independent over time and distributed according to an Extreme Value Type-I distribution with mean 0 and variance $\gamma > 0$.⁵⁴ In particular, workers choose a *non-automated* occupation with hazard⁵⁵

$$\mathcal{S}_t(\mathbf{x}) = \frac{(1 - \phi) \exp\left(\frac{V_t^N(\mathbf{x}'(N;\mathbf{x}))}{\gamma}\right)}{\sum_{h'} \phi^{h'} \exp\left(\frac{V_t^{h'}(\mathbf{x}'(h';\mathbf{x}))}{\gamma}\right)}, \quad (7.5)$$

where $V_t^h(\cdot)$ denotes the continuation value associated to automated ($h = A$) and non-automated ($h = N$) occupations, and the parameter $\gamma > 0$ governs the elasticity of labor supply between those. If workers reallocate between occupations, they enter a state of unemployment or retraining which they exit at rate $\kappa > 0$. Upon exiting unemployment, workers enter their new occupation and experience a permanent productivity loss $\theta \in (0, 1)$.⁵⁶ Newborns are subject to neither unemployment nor a productivity loss when they choose their first occupation.

Income. Employed workers ($e = E$) earn a gross labor income

$$\mathcal{Y}_t^{\text{labor}}(\mathbf{x}) = \xi \exp(z) w_t^h, \quad (7.6)$$

with the productivity consisting of a permanent component (ξ) and a mean-reverting component (z). The permanent component captures the productivity cost (2.6) that workers incur when reallocating between occupations. The mean-reverting component evolves as

$$dz_t = -\rho_z z_t dt + \sigma_z dW_t \quad (7.7)$$

with persistence $\rho_z^{-1} > 0$ and volatility $\sigma_z > 0$. Following [Krueger et al. \(2016\)](#), we suppose that unemployed workers ($e = U$) receive benefits that are proportional to the gross labor income they would have earned if they had remained employed in their previous occupation. The replacement rate is $b \in [0, 1]$, and we assume that these earnings take the form of home production ([Alvarez and Shimer \(2011\)](#)).⁵⁷ Workers net income is

$$\mathcal{Y}_t^{\text{net}}(\mathbf{x}) = \mathcal{T}_t \left(\mathcal{Y}_t^{\text{labor}}(\mathbf{x}) + \exp(z) \Pi_t \right)$$

⁵⁴ This specification is standard in the literature — e.g. [Artuç et al. \(2010\)](#) and [Caliendo et al. \(2019\)](#).

⁵⁵ Our focus is on the reallocation process *between* automated occupations or non-automated occupations. Therefore, we abstract from gross flows *within* those.

⁵⁶ Workers experience this productivity loss at most once during their lifetime.

⁵⁷ This last assumption is mostly innocuous. Its only purpose is to avoid introducing an additional motive for distortionary taxation to finance unemployment insurance.

where $\mathcal{T}_t(y) = y - \tau_t y - \psi_0 y^{1-\psi_1}$ with τ_t capturing linear taxation and $\psi_0, \psi_1 > 0$ captures progressive taxation (Heathcote et al., 2017).

7.3 Policy and Equilibrium

The government's flow budget constraint is⁵⁸

$$dB_t = (T_t + r_t B_t - G_t) dt \quad (7.8)$$

where B_t is the government's asset holdings, T_t is tax revenues and G_t is government spending. The government maintains a constant ratio of spending to GDP G_t/Y_t . It adjusts its stock of assets B_t to: (i) maintain a constant ratio of liquidity to GDP $(-B_t + \alpha_t)/Y_t$ in the long-run; and (ii) ensure that liquidity converges to its long-run level with a half-life of 14 years (Guerrieri and Lorenzoni, 2017). The government chooses the linear taxes τ_t so that its budget clears (7.8). The rest of the model is unchanged. The resource constraint is now

$$\int a_t(\mathbf{x}) d\pi_t = -B_t + E_t \quad (7.9)$$

where E_t is the equity of the mutual fund. The wages that insure labor market clearing in each occupation are still given by (4.7). The rental rate of the stock of automation adjusts so that $\alpha_t^A = \alpha_t$. All agents act competitively. We normalize the price of the final good to 1 (*numéraire*). A competitive equilibrium is defined as before.

8 Quantitative Evaluation

We now calibrate the quantitative model and we use it to evaluate the quantitative importance of our mechanisms and perform various optimal policy experiments. Section 8.1 discusses the calibration. Section 8.3 describes our numerical experiments. Finally, Appendix B provides details about our numerical implementation.

8.1 Calibration

We parameterize the model using a mix of external and internal calibration. We first describe the external calibration, before discussing the targeted moments and the parameters we calibrate internally. Table 8.1 shows the parametrization.

⁵⁸ The government also taxes investment in automation. The proceedings are rebated lump sum to occupations.

External calibration. External parameters are set to standard values in the literature. We interpret our initial stationary equilibrium, i.e. before automation, as the year 1970. We set the initial labor share η to 0.64 based on BLS data. We choose a depreciation rate of automation δ of 10%, as in [Graetz and Michaels \(2018\)](#) and [Artuç et al. \(2020\)](#). We set the elasticity of substitution across occupations ν to 0.75, in between the values in [Buera and Kaboski \(2009\)](#) and [Buera et al. \(2011\)](#).⁵⁹ We choose an inverse elasticity of intertemporal substitution parameter σ to 2. We set the replacement rate χ to obtain an average active life of 45 years. We pick the unemployment exit hazard parameter κ to match the average unemployment duration in the U.S., as measured by [Alvarez and Shimer \(2011\)](#). The productivity loss θ when moving between occupations is set to match the earnings losses estimated by [Kambourov and Manovskii \(2009\)](#). As in [Auclert et al. \(2018\)](#), we rule out borrowing $\underline{a} = 0$. We use the annual income process estimated by [Floden and Lindé \(2001\)](#) using PSID data and choose the mean reversion ρ_z and volatility σ_z in our continuous time model accordingly. We choose a replacement rate when unemployed b of 0.4, following [Shimer \(2005\)](#) and [Ganong et al., 2020](#). We suppose that the profits are rebated to workers in proportion to their idiosyncratic productivity z , following [Auclert et al. \(2018\)](#).⁶⁰ The parameter that governs the elasticity of the progressive tax schedule ψ_1 is 0.181 as in [Heathcote et al. \(2017\)](#). We choose the level ψ_0 to obtain an average income-weighted marginal tax rate of 34% at the initial steady state. Government spending over GDP G_t/Y_t is 20%. Finally, we choose the long-run ratio of liquidity to GDP $(-B_t + \alpha_t)/Y_t$ to one-fourth, following [Kaplan et al. \(2018\)](#).

Internal calibration. We calibrate seven parameters internally: the discount rate (ρ); the mobility hazard (λ); the Fréchet parameter (γ); the occupations' productivities (A^h); the adjustment cost for automation (ω), and the share of automated occupations (ϕ). We pick these parameters to jointly match seven moments. We use the discount rate to target an annualized real interest rate of 2 percent. We adjust the mobility hazard to match an annual occupational mobility rate of 10% per year, which corresponds to the U.S. level in 1970 in [Kambourov and Manovskii \(2008\)](#). We pick the Fréchet parameter so as to obtain an elasticity of labor supply of 0.5 for the stock of workers (i.e. all generations). This

⁵⁹ We interpret automated occupations as routine-intensive occupations which are well represented in manufacturing. So we set the elasticity of substitution between the groups of automated and non-automated occupations as broadly corresponding to that between manufacturing and other sectors. Evidence from the structural change literature strongly suggest gross complementarity as the empirically relevant case.

⁶⁰ In a one-sector model, this assumption implies that workers claim *both* the wage bill and profits in proportion to their productivity. This assumption is the most neutral possible, in that it insures that the government has no incentives to regulate automation to improve insurance by affecting the profit share of aggregate output.

is between the 0.26 Hicksian labor supply elasticity in [Chetty et al. \(2011\)](#) and the 0.79 occupational choice elasticity to expected earning changes in [Arcidiacono et al. \(2020\)](#).⁶¹ We obtain a Fréchet parameter of roughly 0.06, which is an order of magnitude smaller than conventional estimates in the literature ([Artuç et al., 2010](#); [Hsieh et al., 2019](#)). The reason is twofold: our model features gross flows, so that wage changes are perceived as non-permanent; and old generations have accumulated financial wealth and therefore respond less to perceived changes in human wealth. We choose the occupations' productivity $\{A^h\}$ to normalize output in the initial stationary equilibrium to 1 and insure that wages at the initial stationary equilibrium are symmetric across automated and non-automated occupations. We set the mass of automated occupations ϕ to target the employment share in routine occupations in 1970 ([Bharadwaj and Dvorkin, 2019](#)). Finally, we choose the adjustment cost for automation ω to match the employment share in routine occupations in 2015 ([Bharadwaj and Dvorkin, 2019](#)), along the laissez-faire transition.

8.2 Automation, Reallocation and Inequality

We start by simulating the transition of our economy to its new stationary equilibrium with automation. The economy is initially at its (unstable) steady state without automation. At that point, investment is prohibitive and no automation takes place.⁶² In period $t = 0$, an exogenous increase in automation ($\alpha_0 > 0$) initiates a convergence to the final (stable) steady state with positive automation. We interpret the first period in our simulation as the year 1990 — when automation was still in its infancy. We choose the initial stock α_0 so that it is roughly 1/4 of its level in 2015 ([Acemoglu and Restrepo, 2020](#)) along the laissez-faire transition.⁶³

Aggregate dynamics. Figures 8.1–8.2 illustrate the transition dynamics for key aggregate variables. Automation converges to its steady state with a half-life of roughly 40 years. The rise in automation expands the economy's productive capacity over time, but displaces labor since automation and labor are substitutes. The mass of workers employed in automated occupations declines from 56% to 42% over 25 years — as targeted by our calibration — and to 32% in the long-run. The profit share (the complement of the labor share) initially falls below its steady state value (36%) as the firm invests in automation. It

⁶¹ We compute this elasticity in our model by simulating a 10% wage increase in one of the occupations and leaving the others unchanged.

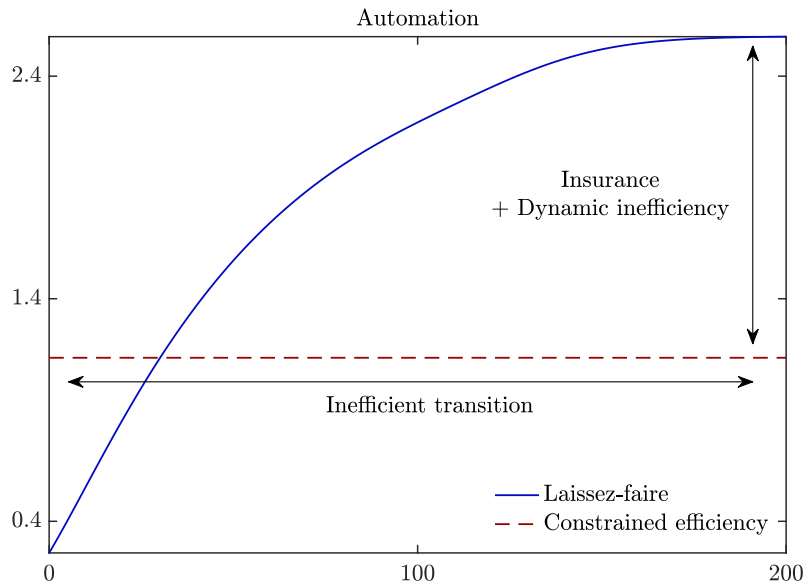
⁶² The reason is that we have assumed quadratic adjustment costs of the form $\Omega(x_t; \alpha_t) = \omega (x_t/\alpha_t)^2 \alpha_t$. This specification is standard in the investment literature ([Cooper and Haltiwanger, 2006](#)).

⁶³ We solve the model non-linearly, so that the rate of convergence over the first decades depends on the initial condition.

Table 8.1: Calibration

Parameter	Description	Calibration	Target / Source
<i>Workers</i>			
ρ	Discount rate	0.13	2% real interest rate
σ	EIS (inverse)	2	-
χ	Death rate	1/45	Average working life of 45 years
\underline{a}	Borrowing limit	0	Auclert et al. (2018)
<i>Technology</i>			
A^A, A^N	Productivities	(0.89, 1.26)	Initial output (1)
η	Initial labor share	0.36	1970 labor share (BLS)
δ	Depreciation rate	0.1	Graetz and Michaels (2018)
ϕ	Fraction of automated occupations	0.53	Routine occs. share in 1970
ω	Adjustment cost	16	Routine occs. share in 2015
ν	Elasticity of subst. across occs.	0.75	(Buera and Kaboski, 2009; Buera et al., 2011)
<i>Mobility frictions</i>			
λ	Mobility hazard	0.49	Occupational mobility rate in 1970
$1/\kappa$	Average unemployment duration	1/3.2	Alvarez and Shimer (2011)
$1 - \kappa^*$	Probability of return move	0.44	Carrillo-Tudela and Visschers (2020)
θ	Productivity loss from relocation	0.18	Kambourov and Manovskii (2009)
γ	Fréchet parameter	0.06	Elasticity of labor supply
<i>Government</i>			
G/Y	Spending / GDP	0.20	BEA
ψ_1	Progressive taxation	0.181	Heathcote et al. (2017)
$(-B + \alpha) / Y$	Liquidity / GDP	0.26	Liquid assets / GDP (Kaplan et al., 2018)
<i>Income process</i>			
ρ_z	Mean reversion	0.0228	Floden and Lindé (2001)
σ_z	Volatility	0.1025	Floden and Lindé (2001)
b	Replacement rate	0.4	(Shimer, 2005; Ganong et al., 2020)

Figure 8.1: Automation



then rises gradually to 39% as labor gets reallocated to non-automated occupations where it is more productive.⁶⁴

Individual allocations. Figure 8.3 describes the distributional effects of automation for workers initially employed in automated and non-automated occupations. Investment in automation crowds out consumption initially. The decline in consumption is larger for workers initially employed in automated occupations. The reason is that automation decreases the marginal productivity of labor in these occupations, increases the supply of these goods — and hence depresses their price — and forces workers into unemployment. As time passes, workers reallocate across occupations and average consumption converges across these two groups. Importantly, the consumption profile is steeper for workers initially employed in automated occupations. This reflects the higher share of hand-to-mouth (HtM) workers in these occupations, as they borrow to self-insure against their income drop and anticipate that their future reallocation across occupations will improve their earnings in the medium-run.⁶⁵ As a result, workers initially employed in automated occupations are more *impatient* than their peers in non-automated occupations. This wedge in intertemporal marginal rates of substitution (MRS) is a source of inefficiency that distorts automation choices.

⁶⁴ In comparison, the labor share (BLS) decreased from 64% in 1970 to 58% in 2019 in the U.S.

⁶⁵ The share of HtM workers is roughly 31% at the initial steady state. This figure is in line with the estimated found in the literature on heterogeneous agent models (Kaplan et al., 2018; Aguiar et al., 2020).

Figure 8.2: Aggregate dynamics

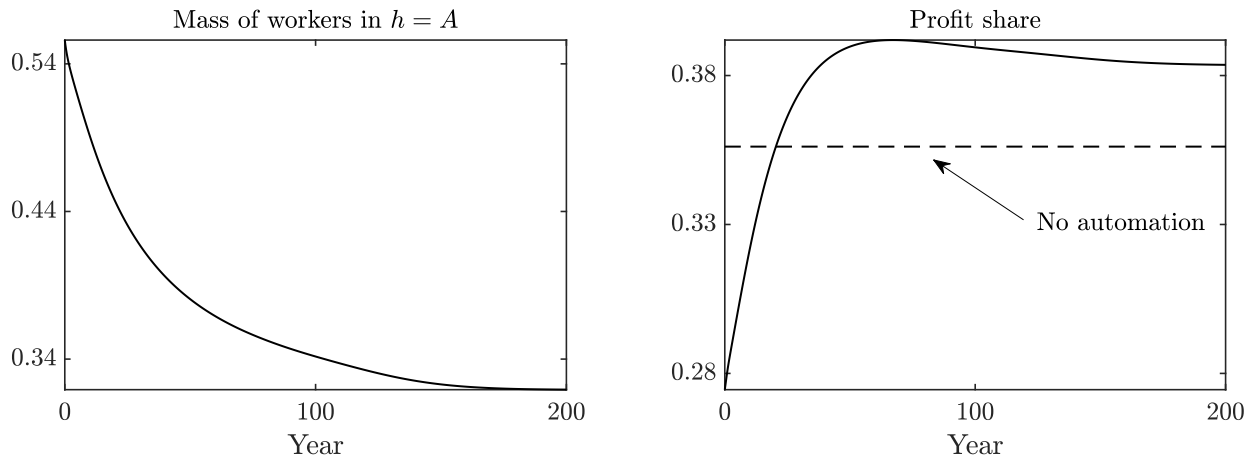
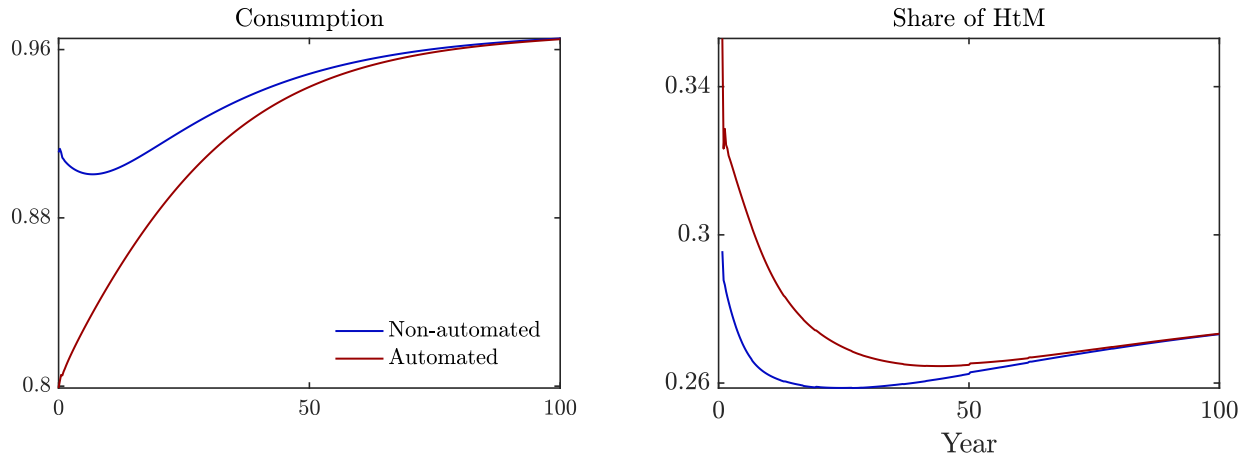


Figure 8.3: Individual allocations



Long run taxation. The rationale for intervention that we propose in this paper concerns the *transition* to the long-run steady state, while labor reallocates gradually and automated workers are borrowing constrained. In particular, this motive does not justify by *itself* to tax automation in the long run (Section 6.4). However, our quantitative model also features uninsured idiosyncratic risk which introduces an additional motive for intervention. It is well-known that a long-run tax (or subsidy) on *capital* can be optimal when markets are incomplete — it can improve insurance and / or prevent dynamic inefficiency (Section 5.3). We find that this is the case too in the context of automation. Figure 8.1 plots the constrained efficient level of automation that maximizes long-run utilitarian welfare (Figure B.1 in Appendix B.5). The optimal level of automation is roughly 45% that at the *laissez-faire*.

8.3 Slowing Down Automation [In Progress]

We showed in Section 5 that the government should *slow down* automation, even when it has no preference for redistribution. We now quantify the welfare gains from taxing automation. The government maximizes

$$\mathcal{W}(\boldsymbol{\eta}) \equiv \int_{-\infty}^{+\infty} \int \eta_t(\mathbf{x}) V_t^{\text{birth}}(\mathbf{x}) d\pi_t(\mathbf{x}) dt \quad (8.1)$$

by choosing the sequence of taxes on investment $\{\tau_t^x\}$ and rebating the proceedings lump sum to the mutual fund. Here, $V_t^{\text{birth}}(\mathbf{x})$ denotes the value function of a worker born in period t that draws a state \mathbf{x} , and $\eta_t(\mathbf{x})$ denotes some Pareto weights.⁶⁶

As in our benchmark model, we work with the primal problem since the government can implement any sequence of automation $\{\alpha_t\}$ by choosing taxes appropriately. We focus on relatively simple (or arguably more realistic) interventions, where the government chooses the *speed* at which automation converges to its constrained-efficient level in the long-run (Section 8.2). That is,

$$\alpha_t = \alpha_0 + (1 - e^{-\varphi t}) (\alpha^{\text{CE}} - \alpha_0)$$

where $\varphi > 0$ is the speed of convergence. We consider two half-lives that are longer than the one at laissez-faire (40 years). For each of those, we compute the transition dynamics, evaluate welfare (8.1) and express the gains in consumption-equivalent terms.

Table 8.2: Welfare Gains

Half-life (years)	40	70	100
$\mathcal{W}(\boldsymbol{\eta}^{\text{effic}}; r_t)$	0%	2.3%	3.2%
$\mathcal{W}(\boldsymbol{\eta}^{\text{utilit}}; r_t)$	0%	2.8%	3.9%

Table 8.2 reports the welfare gains from slowing down automation. We use two sets of Pareto weights $\{\eta_t(\mathbf{x})\}$, which we describe in details in Appendix B.3. The first set of weights $\boldsymbol{\eta}^{\text{effic}}$ focuses on *efficiency*. In this case, the government has no preference for redistribution. The welfare gains are substantial. In other words, slowing down automation improves efficiency. The second set of weights $\boldsymbol{\eta}^{\text{utilit}}$ is *utilitarian*. In this case, the

⁶⁶ By assumption, the government treats all *existing* generations (in period $t = 0$) symmetrically. That is, $\eta_t(\mathbf{x}) = \eta_s(\mathbf{x})$ for all $s, t < 0$.

government does has a preference for redistribution. We find even larger welfare gains, since slowing down automation improves not only efficiency but also equity.

References

- Acemoglu, Daron**, *Introduction to Modern Economic Growth*, Princeton University Press, 2009.
- **and David Autor**, “Skills, Tasks and Technologies: Implications for Employment and Earnings,” in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Vol. 4 of *Handbook of Labor Economics*, Elsevier, 2011, chapter 12, pp. 1043–1171.
 - **and Fabrizio Zilibotti**, “Was Prometheus Unbound by Chance? Risk, Diversification, and Growth,” *Journal of Political Economy*, August 1997, 105 (4), 709–751.
 - **and Pascual Restrepo**, “Artificial intelligence, automation and work,” Technical Report, National Bureau of Economic Research 2018.
 - **and —**, “The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment,” *American Economic Review*, June 2018, 108 (6), 1488–1542.
 - **and —**, “Robots and jobs: Evidence from US labor markets,” *Journal of Political Economy*, 2020, 128 (6), 2188–2244.
 - **, Andrea Manera, and Pascual Restrepo**, “Does the US Tax Code Favor Automation?,” *Brookings Papers on Economic Activity*, Spring 2020.
- Adão, Rodrigo, Martin Beraja, and Nitya Pandalai-Nayar**, “Technological transitions with skill heterogeneity across generations,” Technical Report, National Bureau of Economic Research 2020.
- Aguiar, Mark A, Manuel Amador, and Cristina Arellano**, “Micro Risks and Pareto Improving Policies with Low Interest Rates,” Working Paper 28996, National Bureau of Economic Research July 2021.
- Aguiar, Mark A., Mark Bilts, and Corina Boar**, “Who Are the Hand-to-Mouth?,” NBER Working Papers 26643, National Bureau of Economic Research, Inc January 2020.
- Aiyagari, S. Rao**, “Uninsured Idiosyncratic Risk and Aggregate Saving,” *The Quarterly Journal of Economics*, 1994, 109 (3), 659–684.
- Aiyagari, S Rao**, “Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting,” *Journal of Political Economy*, December 1995, 103 (6), 1158–1175.

- Ales, Laurence, Musab Kurnaz, and Christopher Sleet**, “Technical change, wage inequality, and taxes,” *American Economic Review*, 2015, 105 (10), 3061–3101.
- Alvarez, Fernando and Robert Shimer**, “Search and Rest Unemployment,” *Econometrica*, January 2011, 79 (1), 75–122.
- Araujo, Alosio, Susan Schommer, and Michael Woodford**, “Conventional and Unconventional Monetary Policy with Endogenous Collateral Constraints,” *American Economic Journal: Macroeconomics*, January 2015, 7 (1), 1–43.
- Arcidiacono, Peter, V. Joseph Hotz, Arnaud Maurel, and Teresa Romano**, “Ex Ante Returns and Occupational Choice,” *Journal of Political Economy*, 2020, 128 (12), 4475–4522.
- Artuç, Erhan, Paulo Bastos, and Bob Rijkers**, “Robots, Tasks, and Trade,” CEPR Discussion Papers 14487, C.E.P.R. Discussion Papers March 2020.
- , **Shubham Chaudhuri, and John McLaren**, “Trade Shocks and Labor Adjustment: A Structural Empirical Approach,” *American Economic Review*, June 2010, 100 (3), 1008–1045.
- Atkinson, A. B. and J. E. Stiglitz**, “The design of tax structure: Direct versus indirect taxation,” *Journal of Public Economics*, 1976, 6 (1-2), 55–75.
- Atkinson, Robert D**, “The Case Against Taxing Robots,” Technical Report, Information Technology and Innovation Foundation April 2019.
- Auclert, Adrien, Matthew Rognlie, and Ludwig Straub**, “The Intertemporal Keynesian Cross,” NBER Working Papers 25020, National Bureau of Economic Research, Inc September 2018.
- Bethune, Zachary**, “Aggregate Labor Market Dynamics,” Mimeo, University of Virginia 2017.
- Bharadwaj, Asha and Maximiliano A. Dvorkin**, “The Rise of Automation: How Robots May Impact the U.S. Labor Market,” Regionaal Economist Second Quarter 2019, Federal Reserve Bank of St. Louis 2019.
- Blanchard, Olivier J**, “Debt, Deficits, and Finite Horizons,” *Journal of Political Economy*, April 1985, 93 (2), 223–247.
- Braxton, J Carter and Bledi Taska**, “Technological Change and the Consequences of Job Loss,” *Wisconsin mimeo*, 2020.

Braxton, J. Carter, Kyle F. Herkenhoff, and Gordon M. Phillips, “Can the Unemployed Borrow? Implications for Public Insurance,” NBER Working Papers 27026, National Bureau of Economic Research, Inc April 2020.

Buera, Francisco J. and Joseph P. Kaboski, “Can Traditional Theories of Structural Change Fit The Data?,” *Journal of the European Economic Association*, 04-05 2009, 7 (2-3), 469–477.

—, —, and **Yongseok Shin**, “Finance and Development: A Tale of Two Sectors,” *American Economic Review*, August 2011, 101 (5), 1964–2002.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro, “Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock,” *Econometrica*, May 2019, 87 (3), 741–835.

Card, David, Jochen Kluge, and Andrea Weber, “Active Labour Market Policy Evaluations: A Meta-Analysis,” *Economic Journal*, November 2010, 120 (548), 452–477.

Carman, Katherine Grace, Jagadeesh Gokhale, and Laurence J Kotlikoff, “The Impact on Consumption and Saving of Current and Future Fiscal Policies,” Working Paper 10085, National Bureau of Economic Research November 2003.

Carrillo-Tudela, Carlos and Ludo Visschers, “Unemployment and Endogenous Reallocation over the Business Cycle,” CEPR Discussion Papers 14697, C.E.P.R. Discussion Papers May 2020.

Carroll, Christopher D., “The method of endogenous gridpoints for solving dynamic stochastic optimization problems,” *Economics Letters*, June 2006, 91 (3), 312–320.

Chamley, Christophe, “Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives,” *Econometrica*, May 1986, 54 (3), 607–622.

—, “Capital income taxation, wealth distribution and borrowing constraints,” *Journal of Public Economics*, 2001, 79 (1), 55–69.

Chari, V.V. and Patrick J. Kehoe, “Optimal fiscal and monetary policy,” in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, December 1999, chapter 26, pp. 1671–1745.

Chetty, Raj, “Moral Hazard versus Liquidity and Optimal Unemployment Insurance,” *Journal of Political Economy*, April 2008, 116 (2), 173–234.

- , **Adam Guren, Day Manoli, and Andrea Weber**, “Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins,” *American Economic Review*, May 2011, 101 (3), 471–475.
- Chodorow-Reich, Gabriel and Johannes Wieland**, “Secular Labor Reallocation and Business Cycles,” *Journal of Political Economy*, 2020, 128 (6), 2245–2287.
- Collins, J. Michael, Kathryn Edwards, and Maximilian Schmeiser**, “The Role of Credit Cards for Unemployed Households in the Great Recession,” FDIC Consumer Research Symposium 2015.
- Conesa, Juan Carlos and Dirk Krueger**, “On the optimal progressivity of the income tax code,” *Journal of Monetary Economics*, October 2006, 53 (7), 1425–1450.
- , **Bo Li, and Qian Li**, “A Quantitative Evaluation of Universal Basic Income,” Mimeo, Stony Brook University 2021.
- , **Sagiri Kitao, and Dirk Krueger**, “Taxing Capital? Not a Bad Idea after All!,” *American Economic Review*, March 2009, 99 (1), 25–48.
- Cooper, Russell W. and John C. Haltiwanger**, “On the Nature of Capital Adjustment Costs,” *Review of Economic Studies*, 2006, 73 (3), 611–633.
- Costinot, Arnaud and Iván Werning**, “Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation,” NBER Working Papers 25103, National Bureau of Economic Research, Inc September 2018.
- Crépon, Bruno and Gerard J. van den Berg**, “Active Labor Market Policies,” *Annual Review of Economics*, October 2016, 8 (1), 521–546.
- Daruich, Diego and Raquel Fernández**, “Universal Basic Income: A Dynamic Assessment,” Mimeo, USC Marshall 2020.
- Dávila, Eduardo and Anton Korinek**, “Pecuniary Externalities in Economies with Financial Frictions,” *Review of Economic Studies*, 2018, 85 (1), 352–395.
- Dávila, Julio, Jay H. Hong, Per Krusell, and José-Victor Ríos-Rull**, “Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks,” *Econometrica*, November 2012, 80 (6), 2431–2467.
- Davis, Steven J. and John Haltiwanger**, “Gross job flows,” in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Vol. 3 of *Handbook of Labor Economics*, Elsevier, 1999, chapter 41, pp. 2711–2805.

- Diamond, Peter A.**, “National Debt in a Neoclassical Growth Model,” *The American Economic Review*, 1965, 55 (5), 1126–1150.
- Diamond, Peter A and James A Mirrlees**, “Optimal Taxation and Public Production: I–Production Efficiency,” *American Economic Review*, March 1971, 61 (1), 8–27.
- Doerr, Annabelle and Rafael Novella**, “The Long-Term Effects of Job Training on Labor Market and Skills Outcomes in Chile,” IDB Publications (Working Papers) 10767, Inter-American Development Bank October 2020.
- Druedahl, Jeppe and Thomas Høgholm Jørgensen**, “A general endogenous grid method for multi-dimensional models with non-convexities and constraints,” *Journal of Economic Dynamics and Control*, 2017, 74 (C), 87–107.
- Farhi, Emmanuel and Iván Werning**, “A Theory of Macroprudential Policies in the Presence of Nominal Rigidities,” *Econometrica*, September 2016, 84, 1645–1704.
- Floden, Martin and Jesper Lindé**, “Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?,” *Review of Economic Dynamics*, July 2001, 4 (2), 406–437.
- Friedman, Milton**, *Capitalism and freedom*, University of Chicago press, 1962.
- Ganong, Peter, Pascal Noel, and Joseph Vavra**, “US unemployment insurance replacement rates during the pandemic,” *Journal of Public Economics*, 2020, 191 (C).
- Geanakoplos, John and Heracles M. Polemarchakis**, “Existence, Regularity, and Constrained Suboptimality of Competitive Allocations When the Asset Market Is Incomplete,” Cowles Foundation Discussion Papers 764, Cowles Foundation for Research in Economics, Yale University August 1985.
- Gerardi, Kristopher, Kyle F. Herkenhoff, Lee E. Ohanian, and Paul S. Willen**, “Can’t Pay or Won’t Pay? Unemployment, Negative Equity, and Strategic Default,” *Review of Financial Studies*, 2018, 31 (3), 1098–1131.
- Golosov, Mikhail and Aleh Tsyvinski**, “Designing Optimal Disability Insurance: A Case for Asset Testing,” *Journal of Political Economy*, April 2006, 114 (2), 257–279.
- Graetz, Georg and Guy Michaels**, “Robots at Work,” *The Review of Economics and Statistics*, December 2018, 100 (5), 753–768.

- Guerreiro, Joao, Sergio Rebelo, and Pedro Teles**, “Should Robots be Taxed?,” NBER Working Papers 23806, National Bureau of Economic Research, Inc September 2017.
- Guerrieri, Veronica and Guido Lorenzoni**, “Credit Crises, Precautionary Savings, and the Liquidity Trap,” *The Quarterly Journal of Economics*, 2017, 132 (3), 1427–1467.
- Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura**, “Rethinking the Welfare State,” CEPR Discussion Papers 16275, C.E.P.R. Discussion Papers June 2021.
- Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante**, “Quantitative Macroeconomics with Heterogeneous Households,” *Annual Review of Economics*, May 2009, 1 (1), 319–354.
- , —, and —, “The Macroeconomic Implications of Rising Wage Inequality in the United States,” *Journal of Political Economy*, August 2010, 118 (4), 681–722.
- , —, and **Giovanni L Violante**, “Optimal Tax Progressivity: An Analytical Framework,” *The Quarterly Journal of Economics*, 2017, 132 (4), 1693–1754.
- Heaton, John and Deborah Lucas**, “Portfolio Choice in the Presence of Background Risk,” *Economic Journal*, January 2000, 110 (460), 1–26.
- Heckman, James J., Robert J. Lalonde, and Jeffrey A. Smith**, “The economics and econometrics of active labor market programs,” in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Vol. 3 of *Handbook of Labor Economics*, Elsevier, 1999, chapter 31, pp. 1865–2097.
- Heckman, James, Lance Lochner, and Christopher Taber**, “Explaining Rising Wage Inequality: Explanations With A Dynamic General Equilibrium Model of Labor Earnings With Heterogeneous Agents,” *Review of Economic Dynamics*, January 1998, 1 (1), 1–58.
- Hobijn, Bart, Todd Schoellman, and Alberto Vindas**, “Structural Transformation by Cohort,” Technical Report, Technical Report, Arizona State University 2020.
- Hsieh, Chang-Tai, Erik Hurst, Charles I. Jones, and Peter J. Klenow**, “The Allocation of Talent and U.S. Economic Growth,” *Econometrica*, September 2019, 87 (5), 1439–1474.
- Huberman, Gur**, “Familiarity Breeds Investment,” *Review of Financial Studies*, 2001, 14 (3), 659–680.
- Huggett, Mark**, “The risk-free rate in heterogeneous-agent incomplete-insurance economies,” *Journal of Economic Dynamics and Control*, 1993, 17 (5-6), 953–969.

- Imrohoroglu, Ayse, Selahattin Imrohoroglu, and Douglas H Joines**, “A Life Cycle Analysis of Social Security,” *Economic Theory*, June 1995, 6 (1), 83–114.
- Iskhakov, Fedor, Thomas H. Jørgensen, John Rust, and Bertel Schjerning**, “The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks,” *Quantitative Economics*, July 2017, 8 (2), 317–365.
- Jacobson, Louis S., Robert J. LaLonde, and Daniel G. Sullivan**, “Is retraining displaced workers a good investment?,” *Economic Perspectives*, 2005, 29 (Q II), 47–66.
- Jappelli, Tullio**, “Who is Credit Constrained in the U. S. Economy?,” *The Quarterly Journal of Economics*, 1990, 105 (1), 219–234.
- **and Luigi Pistaferri**, *The Economics of Consumption: Theory and Evidence* number 9780199383153. In ‘OUP Catalogue.’, Oxford University Press, 2017.
- Judd, Kenneth L.**, “Redistributive taxation in a simple perfect foresight model,” *Journal of Public Economics*, October 1985, 28 (1), 59–83.
- Kambourov, Gueorgui and Iourii Manovskii**, “Rising Occupational And Industry Mobility In The United States: 1968-97,” *International Economic Review*, 2008, 49 (1), 41–79.
- **and —**, “Occupational Specificity Of Human Capital,” *International Economic Review*, February 2009, 50 (1), 63–115.
- Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante**, “Monetary Policy According to HANK,” *American Economic Review*, March 2018, 108 (3), 697–743.
- Keys, Benjamin J.**, “The Credit Market Consequences of Job Displacement,” *The Review of Economics and Statistics*, July 2018, 100 (3), 405–415.
- Kimball, Miles S.**, “The Quantitative Analytics of the Basic Neomonetarist Model,” *Journal of Money, Credit and Banking*, November 1995, 27 (4), 1241–1277.
- Korinek, Anton and Joseph E. Stiglitz**, “Steering Technological Progress,” Mimeo, University of Virginia November 2020.
- Krueger, D., K. Mitman, and F. Perri**, “Macroeconomics and Household Heterogeneity,” in J. B. Taylor and Harald Uhlig, eds., *Handbook of Macroeconomics*, Vol. 2 of *Handbook of Macroeconomics*, Elsevier, December 2016, chapter 0, pp. 843–921.
- Lee, Donghoon and Kenneth I Wolpin**, “Intersectoral labor mobility and the growth of the service sector,” *Econometrica*, 2006, 74 (1), 1–46.

- Lilien, David M**, "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, August 1982, 90 (4), 777–793.
- Lorenzoni, Guido**, "Inefficient Credit Booms," *Review of Economic Studies*, 2008, 75 (3), 809–833.
- Low, Hamish, Costas Meghir, and Luigi Pistaferri**, "Wage Risk and Employment Risk over the Life Cycle," *American Economic Review*, September 2010, 100 (4), 1432–1467.
- Lucas, Robert Jr. and Edward C. Prescott**, "Equilibrium search and unemployment," *Journal of Economic Theory*, February 1974, 7 (2), 188–209.
- **and Nancy L. Stokey**, "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 1983, 12 (1), 55–93.
- Ludvige, André Victor Doherty**, "The Macroeconomic Effects of Universal Basic Income Programs," Mimeo, Federal Reserve Bank of Cleveland 2021.
- Mankiw, N. Gregory and Stephen Zeldes**, "The consumption of stockholders and non-stockholders," *Journal of Financial Economics*, 1991, 29 (1), 97–112.
- Massa, Massimo and Andrei Simonov**, "Hedging, Familiarity and Portfolio Choice," *The Review of Financial Studies*, 01 2006, 19 (2), 633–685.
- McFadden, Daniel**, *Conditional Logit Analysis of Qualitative Choice Behavior*, Institute of Urban and Regional Development, University of California, 1973.
- Mirrlees, J. A.**, "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 1971, 38 (2), 175–208.
- Moffitt, Robert A.**, "The Negative Income Tax and the Evolution of U.S. Welfare Policy," *Journal of Economic Perspectives*, Summer 2003, 17 (3), 119–140.
- Moscarini, Giuseppe and Francis G. Vella**, "Occupational Mobility and the Business Cycle," NBER Working Papers 13819, National Bureau of Economic Research, Inc February 2008.
- Phelps, Edmund S.**, "Second Essay on the Golden Rule of Accumulation," *The American Economic Review*, 1965, 55 (4), 793–814.
- Piketty, Thomas and Emmanuel Saez**, "Chapter 7 - Optimal Labor Income Taxation," in Alan J. Auerbach, Raj Chetty, Martin Feldstein, and Emmanuel Saez, eds., *Handbook of Public Economics*, vol. 5, Vol. 5 of *Handbook of Public Economics*, Elsevier, 2013, pp. 391–474.

- Porzio, Tommaso, Federico Rossi, and Gabriella Santangelo**, “The Human Side of Structural Transformation,” CEPR Discussion Papers 15110, C.E.P.R. Discussion Papers July 2020.
- Poterba, James M.**, “Are Consumers Forward Looking? Evidence from Fiscal Experiments,” *American Economic Review*, May 1988, 78 (2), 413–418.
- Poterba, James M.**, “Employer Stock and 401(k) Plans,” *American Economic Review*, May 2003, 93 (2), 398–404.
- Ramey, Valerie A. and Matthew D. Shapiro**, “Displaced Capital: A Study of Aerospace Plant Closings,” *Journal of Political Economy*, October 2001, 109 (5), 958–992.
- Rouwenhorst, Geert**, *Asset Pricing Implications of Equilibrium Business Cycle Models*, Vol. Chapter 10, Frontiers of Business Cycle Research, Princeton University Press, 1995.
- Samuelson, Paul A.**, “An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money,” *Journal of Political Economy*, 1958, 66, 467–467.
- Shimer, Robert**, “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, March 2005, 95 (1), 25–49.
- Sullivan, James X.**, “Borrowing During Unemployment: Unsecured Debt as a Safety Net,” *Journal of Human Resources*, 2008, 43 (2), 383–412.
- Thuemmel, Uwe**, “Optimal Taxation of Robots,” CESifo Working Paper Series 7317, CE-Sifo 2018.
- Violante, Giovanni L.**, “Technological Acceleration, Skill Transferability, and the Rise in Residual Inequality,” *The Quarterly Journal of Economics*, 2002, 117 (1), 297–338.
- Woodford, Michael**, “Public Debt as Private Liquidity,” *American Economic Review*, May 1990, 80 (2), 382–388.
- Yaari, Menahem E.**, “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer,” *Review of Economic Studies*, 1965, 32 (2), 137–150.
- Young, Eric R.**, “Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm and non-stochastic simulations,” *Journal of Economic Dynamics and Control*, January 2010, 34 (1), 36–41.

A Proofs and Derivations

A.1 Proof of Proposition 1

The proof consists of two steps. In the first step, we decompose the problem (3.1)–(3.7) into a dynamic problem and a sequence of statics ones. In the second step, we characterize the efficient allocation of labor across occupations and the associated level of consumption.

1. **Objective.** The planner's problem is equivalent to

$$\begin{aligned} \max_{\{C_t, Q_t, m_t, \hat{m}_t, \mu_t, \Theta_t\}_{t \geq 0}} \int_0^{+\infty} \exp(-\rho t) U_t(C_t) dt \\ \text{s.t. (3.2) – (3.7)} \end{aligned} \quad (\text{A.1})$$

with the felicity function⁶⁷

$$\begin{aligned} \exp(-\rho t) U_t(C_t) \equiv \max_{\{c_s^h\}} \chi \sum_h \phi^h \int_0^{+\infty} \eta_{t-s}^h \exp(-(\rho + \chi)s) u(c_s^h) ds \\ \text{s.t. } C_t = \chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi s) c_s^h ds \end{aligned} \quad (\text{A.2})$$

Here, $c_s^h \equiv c_{t-s,t}^h$ denotes the consumption in period t of the generation born in period $t - s$ and initially located in occupation h . Solving the static problem,

$$c_s^h = \frac{(\eta_{t-s}^h \exp(-\rho s))^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi \tau) (\eta_{t-\tau}^h \exp(-\rho \tau))^{\frac{1}{\sigma}} d\tau} C_t \quad (\text{A.3})$$

and the felicity function $U_t(\cdot)$ is given by (3.9) in the text.

2. **Labor allocation.** Fix some period $T \geq 0$. Consider the planner's decision to reallocate workers employed in automated occupations, i.e. the choice of $\{m_t\}$ and $\{\hat{m}_t\}$. Using a standard variational argument, it is optimal for the planner to reallocate all members of existing generations ($m_t = 1$) if and only if the present discounted values of the marginal labor productivities is higher in non-automated occupations⁶⁸

$$\int_T^{+\infty} \exp(-(\rho + \chi)(t - T)) U_t'(C_t) \Delta_t dt > 0, \quad (\text{A.4})$$

⁶⁷ The normalizing factor χ in (A.2) is for convenience. It insures that $U_t(C) = u(C)$ when the planner discounts each generation with its own discount factor (ρ), i.e. $\eta_{t-s} = \exp(-\rho(t-s))$.

⁶⁸ A share $\exp(-\chi t)$ of the marginal workers survive at any given horizon $t \geq 0$. Hence the term in χ in (A.4).

where $\Delta_t \equiv (1 - \theta) (1 - \exp(-\kappa(t - T))) \mathcal{Y}_t^N - \mathcal{Y}_t^A$ captures the marginal increase in output when the large family reallocates additional workers. This term reflects the difference in marginal productivities $\{\mathcal{Y}_t^h\}$ (Proposition 3.1), the productivity loss θ , and the average duration of unemployment spells $1/\kappa$. The planner reallocates none of these workers ($m_t = 0$) if and only if the inequality (A.4) is reversed.⁶⁹ Here, similarly, the planner reallocates all members of entering generations ($\hat{m}_t = 1$) if and only if

$$\int_T^{+\infty} \exp(-(\rho + \chi)(t - T)) U'_t(C_t) [\mathcal{Y}_t^N - \mathcal{Y}_t^A] dt > 0, \quad (\text{A.5})$$

reallocates none of them ($\hat{m}_t = 0$) if and only if the inequality is reversed. The planner chooses an interior solution ($\hat{m}_t \in (0, 1)$) otherwise. By Assumptions 1–2, there exists some $T_0^{\text{FB}} > 0$ such that the planner reallocates all members of existing generations ($m_t = 1$) for all $t \in [0, T_0^{\text{FB}})$. In period $T = T_0^{\text{FB}}$, the left-hand side of (A.4) is zero. Inspecting (A.4)–(A.5), the planner continues to reallocate entering generations. That is, there exists some T_1^{FB} with $0 < T_0^{\text{FB}} < T_1^{\text{FB}}$ such that the planner reallocates all members of new generations ($\hat{m}_t = 1$) for all $t \in [0, T_1^{\text{FB}})$. Furthermore, $T_1^{\text{FB}} < +\infty$ since the technologies $F(\cdot)$ and $\hat{F}(\cdot)$ satisfy Inada conditions. From $t = T_1^{\text{FB}}$ onward, the left-hand side of (A.5) holds with equality and the planner chooses $\hat{m}_t \in (0, 1)$ to insure that the marginal productivities are equalized $\mathcal{Y}_t^N = \mathcal{Y}_t^A$ for all $t \in [T_1^{\text{FB}}, +\infty)$. The planner does not reallocate existing generations ($m_t = 0$) for all $t \geq T_0^{\text{FB}}$. Summing up,

$$m_t = \begin{cases} 1 & \text{if } t \in [0, T_0^{\text{FB}}) \\ 0 & \text{if } t \in [T_0^{\text{FB}}, +\infty) \end{cases} \quad \text{and} \quad \hat{m}_t = \begin{cases} 1 & \text{if } t \in [0, T_1^{\text{FB}}) \\ \in (0, 1) & \text{if } t \in [T_1^{\text{FB}}, +\infty) \end{cases} \quad (\text{A.6})$$

with $\{\hat{m}_t\}$ chosen for $t \geq T_1^{\text{FB}}$ such that the effective labor supplies in the two occupations remain constant over time. The two stopping times satisfy (3.10) in the text. Solving the differential equation (3.4) and evaluating using (A.6) gives

$$\mu_t^A = \exp(-\lambda \min\{t, T_0\} - \chi t) \quad (\text{A.7})$$

for all $t \in [0, T_1)$, evaluated at $T_0 = T_0^{\text{FB}}$ and $T_1 = T_1^{\text{FB}}$. Solving (3.5)–(3.7) gives

$$\mu_t^N \equiv 1 - \mu_t^A - \tilde{\mu}_t - \theta \hat{\mu}_t \quad (\text{A.8})$$

⁶⁹ As shown below, $\hat{m}_t \geq m_t$ for all $t \geq 0$ so that the left-hand side of (A.4) holds with equality in a single period.

and

$$\tilde{\mu}_t = \frac{\lambda}{\lambda - \kappa} \frac{\phi}{1 - \phi} \exp(-(\kappa + \chi)t) (1 - \exp(-(\lambda - \kappa) \min\{t, T_0\})) \quad (\text{A.9})$$

$$\begin{aligned} \hat{\mu}_t = & \frac{\lambda}{\lambda - \kappa} \frac{\phi}{1 - \phi} \exp(-\chi t) \times \quad (\text{A.10}) \\ & \left[(1 - \exp(-(\kappa - \chi) \min\{t, T_0\})) - \frac{\kappa}{\lambda} (1 - \exp(-\lambda \min\{t, T_0\})) \right] \\ & + \frac{\lambda}{\lambda - \kappa} \frac{\phi}{1 - \phi} (1 - \exp(-(\lambda - \kappa) T_0)) \exp(-\chi t) [\exp(-\kappa T_0) - \exp(-\kappa \max\{t, T_0\})] ds \end{aligned}$$

for all $t \in [0, T_1)$, evaluated at $T_0 = T_0^{\text{FB}}$ and $T_1 = T_1^{\text{FB}}$. For $t \geq T_1^{\text{FB}}$, the effective labor supplies μ_t adjust so that the marginal productivities are equalized across occupations $\mathcal{Y}_t^N = \mathcal{Y}_t^A$. The expression (3.13) in the text is obtained by taking the limit with no unemployment ($1/\kappa \rightarrow 0$). Finally, consumption is given by aggregate output $C_t = G^*(\mu_t; \alpha)$.

We have supposed so far that the average unemployment duration and the productivity loss are sufficiently small that labor mobility takes place at the first best, i.e. $T_0^{\text{FB}} > 0$. This occurs whenever the productivity cost associated to reallocation is sufficiently small

$$\theta \leq 1 - \frac{\int_0^{+\infty} \exp(-(\rho + \chi)t) U'_t(\tilde{C}_t) \tilde{\mathcal{Y}}_t^A dt}{\int_0^{+\infty} (1 - \exp(-\kappa t)) \exp(-(\rho + \chi)t) U'_t(\tilde{C}_t) \tilde{\mathcal{Y}}_t^N dt} \quad (\text{A.11})$$

where the terms on the right-hand side are defined as above, but evaluated with an alternative technology and a counterfactual sequence of (effective) labor supplies. These labor supplies are still given by (A.7)–(A.10) but are now evaluated at $T_0 \equiv 0$ and T_1 given by (3.10). The technology $G^*(\mu, \mu'; \alpha)$ is evaluated at some automation level $\bar{\alpha} > 0$ such that $\partial_\alpha G^*(\mu, \mu'; \bar{\alpha}) > 0$.^{70,71} By definition, the sequences of consumption and the marginal productivities in (A.11) are not indexed by any of the mobility parameters (θ, κ). Therefore, the restriction (A.11) effectively puts an upper bound (jointly) on the average unemployment duration $1/\kappa$ and the productivity loss θ . The coefficients $\{Z^A, Z_t^N\}$ in Assumption 2 can be read from the numerator and denominator in (A.11).

⁷⁰ Such a threshold $\bar{\alpha} > 0$ exists by Assumptions 1–2.

⁷¹ Whenever the level of automation satisfies $\alpha \geq \bar{\alpha}$ — which turns out to be the case at the first best (Proposition 2) — the right-hand side in (A.11) remains larger than θ so labor reallocation still takes place.

A.2 Proof of Proposition 2

We first consider a perturbation of the planner's ex post problem as the level of automation changes and we derive an envelope condition. We then state the optimality condition for the planner's ex ante problem.

1. **Envelope.** By Proposition 1, the planner's ex post problem (3.1)–(3.7) can be equivalently formulated as

$$V^{\text{FB}}(\alpha; \boldsymbol{\eta}) = \max_{\{T_0, T_1\}} \int_0^{+\infty} \exp(-\rho t) U_t(C_t) dt \quad (\text{A.12})$$

subject to the resource constraint $C_t = G^*(\boldsymbol{\mu}_t; \alpha)$, the effective labor supplies given by (A.7)–(A.10) and the restriction $0 < T_0 < T_1 < +\infty$. Note that the problem is differentiable in α and is Lipschitz continuous in $\{T_0, T_1\}$. Therefore, the following envelope condition applies

$$\frac{\partial}{\partial \alpha} V^{\text{FB}}(\alpha; \boldsymbol{\eta}) = \int_0^{+\infty} \exp(-\rho t) U'_t(C_t) \times \underbrace{\frac{\partial}{\partial \alpha} G^*(\boldsymbol{\mu}_t; \alpha)}_{\equiv \Psi_t(\alpha)} dt = 0, \quad (\text{A.13})$$

where consumption $\{C_t\}$, the labor supplies $\{\boldsymbol{\mu}_t\}$ and the terms $\{\tilde{\mu}_t, \hat{\mu}_t\}$ are those characterized in Appendix A.1 when evaluated at α .

2. **Optimality.** The solution to the planner's ex ante problem (3.8) is unique and interior. We first show that the solution is interior. First, note that $\alpha^{\text{FB}} > \bar{\alpha}$ where $\bar{\alpha}$ is the exogenous level of automation implicit in (A.11) — i.e. Assumption 2. The reason is that $\Psi_t(\alpha) > 0$ for all $\alpha \in [0, \bar{\alpha}]$ and all $t > 0$. This follows by Assumptions 1–2 and the fact that $\mu_t < 1$ and $\Theta_t(1 - \mu_t) > 1$ since (at least) some members of existing generations reallocate. Therefore,

$$\int_0^{+\infty} \exp(-\rho t) U'_t(C_t) \times \Psi_t(\alpha) dt > 0 \quad (\text{A.14})$$

for all $\alpha \in [0, \bar{\alpha}]$, so $\alpha^{\text{FB}} > \bar{\alpha}$. Furthermore, $\alpha^{\text{FB}} < 1$ since

$$\lim_{\alpha \rightarrow 1} \int_0^{+\infty} \exp(-\rho t) U'_t(C_t) \times \Psi_t(\alpha) dt = -\infty \quad (\text{A.15})$$

by Assumption 2. Therefore, the solution is interior. Uniqueness follows from the concavity of the value (A.12) in α . To see that, consider two automation levels (α_0, α_1) and let $\mu_t(\alpha)$ and $\mu'_t(\alpha)$ denote the associated effective labor supplies in $h \in \{A, N\}$ at

the first best. Now, consider a convex combination $\hat{\alpha} \equiv c\alpha_0 + (1-c)\alpha_1$ of the automation levels, for some $c \in (0,1)$. Note that the effective labor supplies $\hat{\mu}_t^{(t)} = c\mu_t^{(t)}(\alpha_0) + (1-c)\mu_t^{(t)}(\alpha_1)$ are feasible under the laws of motions (3.4)–(3.7). Therefore,

$$\begin{aligned} G^*(\mu_t(\hat{\alpha}), \mu'_t(\hat{\alpha}); \hat{\alpha}) &\geq G^*(\hat{\mu}_t, \hat{\mu}'_t; \hat{\alpha}) \\ &\geq cG^*(\mu_t(\alpha_0), \mu'_t(\alpha_0); \alpha_0) \\ &\quad + (1-c)G^*(\mu_t(\alpha_1), \mu'_t(\alpha_1); \alpha_1) \end{aligned} \tag{A.16}$$

for all periods $t \geq 0$. The second inequality follows by concavity of the aggregate technology with respect to labor supplies and α (Assumption 2).⁷² The concavity of the value (A.12) then follows immediately by concavity of the felicity function (3.9). Therefore, a necessary and sufficient condition for an optimum is

$$\frac{\partial}{\partial \alpha} V^{\text{FB}}(\alpha^{\text{FB}}; \eta) = 0 \tag{A.17}$$

since $V^{\text{FB}}(\cdot; \eta)$ is differentiable everywhere. The result follows immediately from (A.13) and (A.17).

A.3 Characterization of Equilibrium

We now characterize the competitive equilibrium in our baseline model (Section 4). We omit potential distortionary and lump sum taxes (Section 6) for notation.

Ex post. We start by characterizing the equilibrium conditional on an automation level α . The presence of borrowing frictions implies that some workers are potentially borrowing constrained. As a result, the decentralized equilibrium is characterized by three of stopping times: the times until which existing and new generations reallocate to non-automated occupations (T_0, T_1); and the times between which workers initially employed in automated occupations are borrowing constrained (S_0, S_1).⁷³ We start by characterizing the latter, before turning to the former and solving for equilibrium prices.

1. **Consumption-savings.** The time at which workers initially employed in automated

⁷² The aggregate technology G^* inherits the concavity of neoclassical technologies used by the final good producer (G) and in each occupation (F, \hat{F}).

⁷³ In principle, workers initially employed in non-automated occupations and new generations could be borrowing constrained *after* wages have converged across occupation (T_1). We abstract from this possibility, since it does not occur for the range of parameters we are interested in.

occupations become borrowing constrained (S_0) is such that workers deplete their savings⁷⁴

$$u'(\hat{c}_{0,S_0}^A) = \exp\left(\int_0^{S_0} \hat{r}_t dt - (\rho + \chi) S_0\right) u'(\hat{Y}_{0,S_0}^A + \Pi_{S_0} + r_{S_0} \underline{a}) \quad (\text{A.18})$$

given the budget restriction

$$\hat{c}_{0,S_0}^A = \frac{\int_0^{S_0} \exp\left(-\int_0^t \hat{r}_s ds\right) \left(\hat{Y}_{0,t}^A + \Pi_t\right) dt + a_0^A - \exp\left(-\int_0^{S_0} \hat{r}_s ds\right) \underline{a}}{\int_0^{S_0} \exp\left(-\int_0^t \hat{r}_s ds\right) \hat{u}\left(\exp\left(\int_0^t r_s ds - \rho t\right)\right)^{-1} dt} \quad (\text{A.19})$$

where $\hat{u} \equiv 1/(u')^{-1}$, $\hat{Y}_{0,t}^A$ is labor income, Π_t are profits, $\hat{r}_t \equiv r_t + \chi$ is the effective return on bonds, and $a_0^A \equiv 0$. The time at which these workers stop being borrowing constrained (S_1) is the one where their savings flow equals the change in their borrowing constraint⁷⁵

$$\hat{c}_{S_1,+\infty}^A = \hat{Y}_{0,S_1}^A + \Pi_{S_1} + r_{S_1} \underline{a} \quad (\text{A.20})$$

with $\hat{c}_{t,t'}^h$ defined by analogy with (A.19) and $a_{S_1}^A \equiv \underline{a}$. The same workers are unconstrained for all $t \geq S_1$. That is, the consumption of workers initially employed in automated occupations is given by $c_t^A = \hat{u}\left(\exp\left(\int_0^t r_s ds - \rho s\right)\right) \hat{c}_{0,S_0}^A$ before the borrowing constraint binds $t \in [0, S_0)$, $c_t^A = \hat{Y}_t^A + \Pi_t$ when the borrowing constraint binds $t \in [S_0, S_1)$ and

$$c_t^A = \hat{u}\left(\exp\left(\int_{S_1}^t r_s ds - \rho(t - S_1)\right)\right) \times \hat{c}_{S_1,+\infty}^A \quad (\text{A.21})$$

afterwards. In turn, workers initially employed in non-automated occupations and members of generations born at $s > 0$ are unconstrained for all $t \geq 0$. Their consumption is given by

$$\hat{c}_t^N = \hat{u}\left(\exp\left(\int_0^t r_s ds - \rho t\right)\right) \hat{c}_{0,+\infty}^N \quad (\text{A.22})$$

⁷⁴We have $S_0 = S_1 \rightarrow +\infty$ when these workers never become borrowing constrained, since all workers effectively become hand-to-mouth as the economy converges to its new stationary equilibrium. Without loss of generality, we can also set $S_0 = S_1 \equiv 0$. For notational convenience, we choose to do so in the following.

⁷⁵In theory, workers could be constrained over multiple, separate intervals of time. We rule this case out since it does not occur for the parametrizations of interest. This explains why (A.20) implicitly assumes that workers are unconstrained for all periods $t \geq S_1$.

Finally, aggregate consumption is given by

$$C_t = \phi \exp(-\chi t) c_t^A + (1 - \phi \exp(-\chi t)) \hat{c}_t^N \quad (\text{A.23})$$

2. **Labor reallocation.** Labor income $\hat{y}_{s,t}^h$ in period t for a generation born in s and initially located in occupation h is

$$\hat{y}_{s,t}^h = w_t^A + (1 - \exp(-\lambda \min\{t, T_0\})) \left(\Theta_t(\lambda, \kappa) (1 - \theta) w_t^N - w_t^A \right) \quad (\text{A.24})$$

if $h = A, s < 0$ and $\hat{y}_{s,t}^h = w_t^N$ otherwise, where

$$\Theta_t(\lambda, \kappa) \equiv \frac{1 - \phi}{\phi} \frac{\hat{\mu}_t}{1 - \exp(-\lambda \min\{t, T_0\})} \quad (\text{A.25})$$

is the share of workers who exited their unemployment spell after changing occupation, with $\{\hat{\mu}_t\}$ given by (A.10) evaluated at the equilibrium stopping times.

In any period $t = T$, workers initially employed in automated occupations — i.e. $h = A, s \leq 0$ — decide as a large household whether to reallocate to non-automated occupations or not. It is never optimal to postpone mobility. Thus, these workers effectively choose a stopping time T_0 . When making this choice, they internalize the effect of this stopping time on labor income, taking prices as given — i.e. the direct effect of T_0 in (A.24)–(A.25) as well as the impulse response of $\{\hat{\mu}_t\}$. Therefore, the optimal stopping time satisfies⁷⁶

$$\int_{T_0}^{+\infty} \exp(-\rho t) u'(\hat{c}_t^A) \Delta_t dt = 0 \quad (\text{A.26})$$

where $\Delta_t \equiv \exp(-\chi t) \{(1 - \theta) (1 - \exp(-\kappa(t - T_0))) w_t^N - w_t^A\}$ captures the marginal increase in labor incomes when the large family reallocates additional workers.⁷⁷ This condition becomes

$$\int_{T_0}^{+\infty} \exp\left(-\int_{T_0}^t r_\tau d\tau\right) \Delta_t dt = 0 \quad (\text{A.27})$$

in the case where existing workers are unconstrained after they stop reallocating $t \geq T_0$.

⁷⁶ Condition (A.26) applies whether workers are constrained or not after they stop reallocating $t \geq T_0$.

⁷⁷ A worker who reallocates between occupations internalizes the risk that she will die through her discount factor $\exp(-(\rho + \chi)t)$, not through the flows Δ_t — contrary to the planner. We chose the formulation (A.26) to preserve the symmetry with the first best (3.10)–(3.15).

The second stopping time T_1 is such that wages are equalized across occupations

$$w_{T_1}^A = w_{T_1}^N \quad (\text{A.28})$$

Fixing a sequence of interest rates $\{r_t\}$, the conditions (A.18)–(A.20), (A.24)–(A.25) and (A.27)–(A.28) pin down the equilibrium stopping times (T_0, T_1) and (S_0, S_1) .⁷⁸ Effective labor supplies μ_t are given by (A.7)–(A.10) evaluated at the stopping times (T_0, T_1) .

3. **Equilibrium prices.** Equilibrium wages and profits are

$$w_t^h = 1/\phi^h \partial_h G^*(\mu_t; \alpha) \quad \forall h \quad (\text{A.29})$$

$$\Pi_t \equiv Y_t - \phi \mu_t^A w_t^A - (1 - \phi) \mu_t^N w_t^N \quad (\text{A.30})$$

where $Y_t \equiv G^*(\mu_t; \alpha)$ is equilibrium output. Finally, the interest rate that insures that $C_t = Y_t$ at equilibrium is

$$r_t = \rho + \frac{\sigma}{Y_t} \left(\phi w_t^A \partial_t \mu_t^A + (1 - \phi) w_t^N \partial_t \mu_t^N \right) \quad (\text{A.31})$$

when the borrowing constraint does not bind $t \in [0, S_0) \cup [S_1, +\infty)$. The expression for the interest rate when the borrowing constraint binds $t \in [S_0, S_1)$ involves additional terms, so we omit it for concision since we do not use it in the following. Finally,

$$\partial_t \mu_t^A = \left(\mathbf{1}_{\{t < T_1\}} \chi + \mathbf{1}_{\{t < T_0\}} \lambda \right) \mu_t^A \quad (\text{A.32})$$

$$\begin{aligned} \partial_t \mu_t^N &= -\frac{\phi}{1 - \phi} \partial_t \mu_t^A - \left(\lambda \frac{\phi}{1 - \phi} \mathbf{1}_{\{t < T_0\}} \mu_t^A - (\kappa + \chi) \tilde{\mu}_t \right) \\ &\quad + (\theta - 1) (\kappa \tilde{\mu}_t - \chi \hat{\mu}_t) \end{aligned} \quad (\text{A.33})$$

using (3.4)–(3.7) and the definition of the stopping times.

Ex ante. We now characterize the equilibrium choice of automation. A necessary condition for an interior optimum is^{79,80}

$$\int_0^{+\infty} \exp\left(-\int_0^t r_s ds\right) \frac{\partial}{\partial \alpha} \Pi_t(\alpha) dt = 0 \quad (\text{A.34})$$

⁷⁸ We can actually show that (T_0, T_1) and (S_0, S_1) are unique, given $\{r_t\}$.

⁷⁹ The static profit function (4.2) is differentiable in the level of automation by Assumption 1

⁸⁰ We suppose that equity — which is fully illiquid in our model — is priced using the stochastic discount factor of unconstrained workers. By no arbitrage with bonds, the return on equity (pre-annuities) is $\{r_t\}$.

Furthermore, the following envelope condition applies

$$\frac{d}{d\alpha}\Pi_t(\alpha) = \frac{\partial}{\partial\alpha}G^*(\boldsymbol{\mu}_t; \alpha) \quad (\text{A.35})$$

Therefore,

$$\int_0^{+\infty} \exp\left(-\int_0^t r_s ds\right) \frac{\partial}{\partial\alpha}G^*(\boldsymbol{\mu}_t; \alpha) = 0 \quad (\text{A.36})$$

This condition is both necessary and sufficient, by Assumption 2.

A.4 Proof of Proposition 3

The result states that laissez-faire equilibrium is inefficient if and only if the borrowing constraints are sufficiently important $\underline{a} > a^*$ for some $a^* \leq 1$. For our purpose, it is sufficient to show that the laissez-faire either satisfies all the restrictions that characterizes first best allocations (Section 3) or violates *at least* one of those. At this point, we do not elaborate on the nature of the inefficiency. Throughout, we define the aggregate and individual allocation $\{\bar{X}_t\}$ with $\bar{X}_t \equiv (\{\bar{c}_{s,t}^h, \bar{a}_{s,t}^h\}, \{\bar{y}_t^h\}, \bar{Y}_t)$ to be the one that occurs in the laissez-faire equilibrium without borrowing frictions ($\underline{a} \rightarrow -\infty$). We let (\bar{T}_0, \bar{T}_1) denote the associated stopping times. Prices are defined similarly. For notation, the dependence on the reallocation parameters $(\lambda, \eta, \theta, \chi)$ is implicit when there is no ambiguity. We show sufficiency first, then necessity.

Sufficiency. Define the threshold $a^* \equiv \inf_t \bar{a}_{s,t}^0$ for any existing generations $s < 0$. Then, the laissez-faire allocation coincides with $\{\bar{X}_t\}$ whenever $\underline{a} \leq a^*$.⁸¹ It suffices to show that $\{\bar{X}_t\}$ is efficient — i.e. there exist some weights $\{\eta_s^h\}$ that implement this allocation as a first best. When workers are unconstrained,

$$\bar{c}_{s,\tau}^h / \bar{c}_{s,t}^h = \hat{u} \left(-\exp\left(\int_t^\tau r_k dk - \rho(\tau - t)\right) \right) \quad \text{for all } (h, s) \text{ and } t, \tau \geq s \quad (\text{A.37})$$

using Assumption 3. This quantity does not depend on the initial occupation of employment (h) nor the birth date (s). Therefore, there exists a set of weights $\{\eta_s^h\}$ and coefficients $\{b_t\}$ such that $\bar{c}_{s,t}^h = b_t (\eta_{t-s}^h \exp(-\rho s))^{\frac{1}{\sigma}} C_t$ for all initial occupations (h), generations (s) and periods $t \geq s$. The sequence $\{b_t\}$ is chosen to satisfy the definition of aggregate consumption (3.2). As a result, the equilibrium consumption allocation coincides with its first best counterpart (A.3) when the planner uses the weights $\{\eta_s^h\}$. It remains

⁸¹ All other agents are net savers at equilibrium $\bar{a}_{s,t}^h \geq 0$ for any occupation h and generation $s \geq 0$.

to show that the equilibrium stopping times (\bar{T}_0, \bar{T}_1) also coincide with their first best counterparts. When workers are unconstrained, the first stopping time is characterized by (A.26). Then,

$$\int_{T_0}^{+\infty} \exp(-(\rho + \chi)t) u'(\bar{c}_{s,t}^h) \Delta_t dt = 0 \quad \text{for all } (h, s) \quad (\text{A.38})$$

using the workers' optimality conditions (A.27) and (A.37). Furthermore, the following envelope condition applies

$$U'_t(C_t) = \eta_s^h \exp(-\rho s) u'(\bar{c}_{s,t}^h) \quad \text{for all } (h, s) \text{ and } t \geq s \quad (\text{A.39})$$

using the planner's intratemporal problem (A.2) with the proportionality factor independent of the period t . It follows that the first stopping time (\bar{T}_0) coincides with its first best counterpart (3.10), using (A.38)–(A.39). Finally, so does the second stopping time (\bar{T}_1) since effective labor supplies still evolve as (A.7)–(A.10) in both cases. To complete the proof of sufficiency, note that $-\infty < a^* \leq 0$. In the limit where reallocation is fast $1/\lambda, 1/\kappa \rightarrow 0$ and $1/\chi \rightarrow +\infty$, we have $\bar{y}_t^A = \bar{y}_t^N$ for all $t \geq 0$ by Proposition 1. Therefore no borrowing takes place and $a^* \rightarrow 0$ in this limit.

Necessity. Define a^* as above. Let $\underline{a} > a^*$. Then, there exist some periods $0 \leq t < \tau$ such that

$$c_{s,\tau}^A / c_{s,t}^A > \hat{u} \left(-\exp \left(\int_t^\tau r_k dk - \rho(\tau - t) \right) \right) \quad \text{for all } s < 0 \quad (\text{A.40})$$

at the laissez-faire for workers initially employed in automated occupations, using Assumption 3. In contrast, the relation above holds with equality for all other workers h and s since they are unconstrained at equilibrium. It follows that there exist occupation h , generations $s < 0$ and s' and periods $s' \leq t < \tau$, such that $c_{s,\tau}^A / c_{s,t}^A \neq c_{s',\tau}^h / c_{s',t}^h$. Therefore, the equilibrium allocation does not satisfy the first best restriction (A.3). We conclude that this equilibrium is inefficient.

A.5 Proof of Proposition 4

The government's optimality conditions to reallocate and automate are

$$\int_{T_0^{\text{SB}}}^{+\infty} \exp(-(\rho + \chi)t) u'(c_{0,t}^A) \Delta_t dt = -\exp(\lambda T_0^{\text{SB}}) \frac{\Phi(\alpha^{\text{SB}}, T_0^{\text{SB}}; \eta)}{\phi^A \eta^A} \quad (\text{A.41})$$

and

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) u'(c_{0,t}^N) \Delta_t^* dt = -\frac{\Phi^*(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta})}{\phi^N \eta^N} \quad (\text{A.42})$$

respectively. The terms on the left-hand side of (A.41)–(A.42) correspond to the private incentives to automate and reallocate, respectively. The terms on the right-hand capture pecuniary externalities that affect workers through wages and profits — that firms and do not internalize. These pecuniary externalities are given by

$$\Phi(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}) \equiv \int_{T_0^{\text{LF}}}^{+\infty} \exp(-(\rho + \chi)t) \hat{\Phi}_t(\cdot) dt \quad (\text{A.43})$$

$$\Phi^*(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}) \equiv \int_0^{+\infty} \exp(-(\rho + \chi)t) \hat{\Phi}_t^*(\cdot) dt \quad (\text{A.44})$$

where

$$\begin{aligned} \hat{\Phi}_t(\cdot) \equiv & \phi^A \eta^A u'(c_{0,t}^A) \left[\exp(-\lambda T_0^{\text{SB}}) \hat{w}_t^A + (1 - \theta) \hat{\mu}_t(T_0^{\text{SB}}) \hat{w}_t^N - \sum_h \phi^h \mu^h \hat{w}_t^h \right] \\ & + \phi^N \eta^N u'(c_{0,t}^N) \left[\hat{w}_t^N - \sum_h \phi^h \mu^h \hat{w}_t^h \right] \end{aligned} \quad (\text{A.45})$$

$$\begin{aligned} \hat{\Phi}_t^*(\cdot) \equiv & \phi^A \eta^A u'(c_{0,t}^A) \left[\Delta_t^* + \exp(-\lambda T_0^{\text{SB}}) \hat{w}_t^{A,*} + (1 - \theta) \hat{\mu}_t(T_0^{\text{SB}}) \hat{w}_t^{N,*} - \sum_h \phi^h \mu^h \hat{w}_t^{h,*} \right] \\ & + \phi^N \eta^N u'(c_{0,t}^N) \left[\hat{w}_t^{N,*} - \sum_h \phi^h \mu^h \hat{w}_t^{h,*} \right] \end{aligned} \quad (\text{A.46})$$

for the reallocation and automation decisions, respectively.⁸² In turn, $\hat{\mu}_t(T_0^{\text{LF}})$ denotes the mass of workers (A.10) who have reallocated and completed their unemployment spell, while the sequences $\{\hat{w}_t^h\}$ and $\{\hat{w}_t^{h,*}\}$ denote the perturbation of wages $w_t^h \equiv \partial_h G(\mu_t, \Theta_t(1 - \mu_t); \alpha)$ with respect to a change in T_0 and α , respectively.⁸³

The equilibrium is *constrained efficient* if and only if

$$\Phi(\alpha^{\text{LF}}, T_0^{\text{LF}}; \boldsymbol{\eta}) = \Phi^*(\alpha^{\text{LF}}, T_0^{\text{LF}}; \boldsymbol{\eta}) = 0 \quad (\text{A.47})$$

for *some* weights $\boldsymbol{\eta}$. We now show that whenever these conditions hold, there exists a small perturbation of the production function such that the resulting second best and

⁸² The last term in each of the brackets in (A.45)–(A.46) corresponds to the change in profits. This is obtained using the definition of profits $\Pi_t = G^*(\cdot) - \phi w_t^A \mu_t - (1 - \phi) w_t^N \Theta_t(1 - \mu_t)$ and equilibrium wages $w_t^h = 1/\phi^h G_h^*(\cdot)$.

⁸³ Effective labor supplies $\{\mu_t, \Theta_t\}$ are effectively indexed by T_0 , as is apparent from (A.7)–(A.10). These quantities are evaluated at the degree of automation α^{SB} and the stopping time T_0^{SB} .

laissez-faire do not coincide. To see this, suppose that (A.47) holds for some weights η . Consider the variation

$$\mathcal{G}(G^*, \epsilon) = G^* + \epsilon g(\boldsymbol{\mu}; \alpha) \quad (\text{A.48})$$

where g is any function that satisfies

$$g(\boldsymbol{\mu}_t^{\text{LF}}; \alpha^{\text{LF}}) = 0 \quad (\text{A.49})$$

for all $t \geq 0$,

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) u'(c_{0,t}^A) \partial_h g(\boldsymbol{\mu}_t^{\text{LF}}; \alpha^{\text{LF}}) dt = 0 \quad (\text{A.50})$$

for each occupation $h \in \{A, N\}$, and

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) u'(c_{0,t}^N) \partial_\alpha g(\boldsymbol{\mu}_t^{\text{LF}}; \alpha^{\text{LF}}) dt = 0 \quad (\text{A.51})$$

along the initial equilibrium. For instance,

$$g(\boldsymbol{\mu}_t; \alpha) \equiv \left\{ \mu_t^{A,\text{LF}} + \varrho \mu_t^{N,\text{LF}} \right\} (\alpha^{\text{LF}} - \alpha) \quad (\text{A.52})$$

satisfies (A.49)–(A.51) when choosing $\varrho < 0$ appropriately.

Then, the allocation $(\boldsymbol{\mu}_t^{\text{LF}}; \alpha^{\text{LF}})$ still satisfies all equilibrium conditions — workers' reallocation (??), firms' automation (5.2), and the resource constraint (4.8) — after a variation $\epsilon > 0$. That is, the laissez-faire is unchanged. It follows that the pecuniary externality that concerns labor reallocation (A.45) still nets out $\Phi(\alpha^{\text{LF}}, T_0^{\text{LF}}; \boldsymbol{\eta}) = 0$ after this variation. The reason is that this pecuniary externality involves exclusively terms in $D_\mu^2 G^*$, while the perturbation (A.52) is linear in $\boldsymbol{\mu}$ and cannot affect these terms.

Now, note that $\partial_\alpha g(\boldsymbol{\mu}_t; \alpha)$ is *increasing* over time when evaluated at $\alpha = \alpha^{\text{LF}}$, and $\partial_\alpha g(\boldsymbol{\mu}_0; \alpha) < 0$ and $\lim_{t \rightarrow +\infty} \partial_\alpha g(\boldsymbol{\mu}_t; \alpha) > 0$.⁸⁴ Furthermore, note that the sequence of relative marginal utilities $\left\{ u'(\mathcal{Y}_{0,t}^A + \Pi_t) / u'(\mathcal{Y}_{0,t}^N + \Pi_t) \right\}$ is *decreasing* over time given (5.1). It follows that

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) \left\{ \sum_h \phi^h \eta^{h,\text{effic}} u'(\mathcal{Y}_{0,t}^h + \Pi_t) \right\} \partial_\alpha g(\boldsymbol{\mu}_t^{\text{LF}}; \alpha^{\text{LF}}) dt < 0 \quad (\text{A.53})$$

given (A.51). Therefore, we have constructed a variation $\mathcal{G}(G^*, \epsilon)$ such that $\Phi^*(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}) \neq$

⁸⁴ The first property follows from the definition (A.52) and the law of motions (A.7)–(A.10) for effective labor supplies $\{\boldsymbol{\mu}_t\}$. The second and third properties follow immediately from (A.51). It is worth noting that the sequence of labor supplies — and hence $\partial_\alpha g(\boldsymbol{\mu}_t; \alpha)$ — could actually be non-monotonic after the second stopping time T_1 when there are overlapping generations — which we abstract from here (Section 6.1).

0. It follows that

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) u'(c_{0,t}^N) \Delta_t^* dt \neq 0, \quad (\text{A.54})$$

which is inconsistent with firms' automation (5.2). That is, the second best and the laissez-faire do not coincide after the perturbation $\varepsilon > 0$. Finally, $G^{*'}(G^*, \varepsilon) \rightarrow G^*$ uniformly as $\varepsilon \rightarrow 0$ given (A.52), as claimed.

A.6 Proof of Proposition 5

We first derive the optimality conditions associated to the problem (6.1). We then sign the wedge at the laissez-faire. We abstract from overlapping generations ($1/\chi \rightarrow +\infty$) to streamline the exposition and obtain more compact expressions.⁸⁵

The equilibrium level of automation α^{LF} satisfies

$$\int_0^{+\infty} \exp(-\rho t) \times u'(\hat{\mathcal{Y}}_{0,t}^A + \Pi_t) \Delta_t^* dt = 0, \quad (\text{A.55})$$

where Δ_t^* is defined by (5.4) and denotes the response of aggregate output to automation. The income and profits streams in (A.55) are pinned down by the implementability conditions of the problem (6.1) and T_0^{LF} . In turn, the second-best level of automation $\alpha^{\text{SB}}(\eta)$ satisfies

$$\int_{T_0^{\text{SB}}(\eta)}^{+\infty} \exp(-\rho t) \times \sum_h \phi^h \eta^h u'(\hat{\mathcal{Y}}_{0,t}^h + \Pi_t) (\Delta_t^* + \Phi_t^{*,h}) dt = 0 \quad (\text{A.56})$$

where $\{\Phi_t^{*,h}\}$ capture distributional effects between workers employed in different occupations, with $\sum_h \phi^h \Phi_t^{h,*} \equiv 0$ for all periods t . Similarly, the income and profits streams in (A.56) are pinned down by the implementability conditions of the problem (6.1) and $T_0^{\text{SB}}(\eta)$.⁸⁶ By assumption, the government chooses weights $\eta \equiv \eta^{\text{effic}}$ that insure that the distributional terms net out. Therefore,

$$\int_{T_0^{\text{SB}}(\eta)}^{+\infty} \exp(-\rho t) \times \sum_h \phi^h \eta^h u'(\hat{\mathcal{Y}}_{0,t}^h + \Pi_t) \Delta_t^* dt = 0 \quad (\text{A.57})$$

⁸⁵ The result applies to the general case with $1/\chi \leq +\infty$ — up to the caveat discussed in footnote 84. The same remark applies to Appendices A.7–A.8.

⁸⁶ The income and profits streams in (A.55) and (A.56) are implicitly indexed by the stopping times T_0^{LF} and $T_0^{\text{SB}}(\eta)$, respectively. We account for that in the following. We omit this dependence to keep the notation concise.

In the following, we let $\lambda_t^h \equiv u'(\hat{\mathcal{Y}}_{0,t}^h + \Pi_t)$. The sequence $\{\lambda_t^A\}$ is more *front-loaded* than $\{\lambda_t^N\}$ since $\hat{\mathcal{Y}}_{0,t}^A \leq \hat{\mathcal{Y}}_{0,t}^N$ and the two converge eventually (Appendix A.3). For the reasons outlined in Section 6.3.1, the sequence $\{\Delta_t^*\}$ is itself *back-loaded* by Assumption 3. Thus, the left-hand side of (A.57) is negative at α^{LF} since the government's values relatively less flows which are more distant in the future. Therefore, the government *curbs* automation.

A.7 Proof of Proposition 6

We proceed as in Appendix A.6. The equilibrium stopping time T_0^{LF} satisfies

$$\int_{T_0^{\text{LF}}}^{+\infty} \exp(-\rho t) \times u'(\hat{\mathcal{Y}}_{0,t}^A + \Pi_t) \Delta_t = 0, \quad (\text{A.58})$$

where Δ_t is defined by (3.11) and denotes the response of aggregate output to labor reallocation. In turn, the second-best level of automation $\alpha^{\text{SB}}(\boldsymbol{\eta})$ satisfies

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) \times \left\{ \sum_h \phi^h \eta^h u'(\mathcal{Y}_{0,t}^h + \Pi_t) \right\} \left\{ \Delta_t^* + T_0^{\text{LF}}(\alpha^{\text{SB}}) \Delta_t + \hat{\Phi}_t^h \right\} dt = 0, \quad (\text{A.59})$$

where $T_0^{\text{LF}}(\cdot) > 0$ denotes the response of reallocation at the laissez-faire and $\{\hat{\Phi}_t^h\}$ capture distributional effects between workers employed in different occupations, with $\sum_h \phi^h \hat{\Phi}_t^h \equiv 0$ for all periods t . By assumption, the government chooses weights $\boldsymbol{\eta} \equiv \boldsymbol{\eta}^{\text{effic}}$ that insure that the distributional terms net out. Therefore,

$$\int_0^{+\infty} \exp(-(\rho + \chi)t) \times \left\{ \sum_h \phi^h \eta^h u'(\mathcal{Y}_{0,t}^h + \Pi_t) \right\} \left\{ \Delta_t^* + T_0^{\text{LF}}(\alpha^{\text{SB}}) \Delta_t \right\} dt = 0, \quad (\text{A.60})$$

Again, let $\lambda_t^h \equiv u'(\hat{\mathcal{Y}}_{0,t}^h + \Pi_t)$. Note that the sequence $\{\lambda_t^A\}$ is more *front-loaded* than $\{\lambda_t^N\}$. For the reasons outlined in Section 6.3.2, the sequence $\{\Delta_t\}$ can itself be front- or back-loaded depending on the average duration of unemployment / retraining spells. When this reallocation is fast, i.e. $1/\kappa$ small, the sequence $\{\Delta_t\}$ is *front-loaded*.⁸⁷ In this case, the term involving $\{\Delta_t\}$ in (A.60) is negative at α_0^{LF} since the government's values relatively more flows which are more distant in the future. This reinforces the government's desire to *curbs* labor reallocation. When this reallocation is slow, i.e. $1/\kappa > 1/\kappa^*$

⁸⁷ The sequence is initially positive as $(1 - \theta) w_t^N > w_t^A$ at the equilibrium stopping time (Appendix A.3 and the left panel of Figure 5.1). It declines over time as wages converge, and eventually becomes negative.

large, the sequence $\{\Delta_t\}$ is *back-loaded*.⁸⁸ Therefore, term involving $\{\Delta_t\}$ in (A.60) is α_0^{LF} is positive. This reduces the government's desire to curb automation. In theory, this case might not present itself. The reason is that workers might decide not to reallocate altogether if the average duration of unemployment is too long (Assumption 2). In this case, we set $1/\kappa^* \equiv +\infty$.

A.8 Proof of Proposition 7

The proof is very similar in the cases where the government intervenes ex ante (automation) or ex post (labor reallocation). We focus on the latter to streamline the exposition. Suppose that there are no borrowing frictions ($\underline{a} \rightarrow -\infty$). Then, the decentralized equilibrium is efficient (Proposition 3). As a result, there exist some weights η^* that support this allocation as a first best (Section 3). This allocation is necessarily second best as well. Abstracting again from overlapping generations ($\chi \rightarrow 0$), the equilibrium level of automation α^{LF} satisfies

$$\int_0^{+\infty} \exp(-\rho t) \times \sum_h \phi^h \eta^{h,*} u' \left(\hat{\mathcal{Y}}_{0,t}^h + \Pi_t \right) \left\{ \Delta_t^* + \Phi_t^{*,h} + T'_0 \left(\alpha^{\text{LF}} \right) \left(\Delta_t + \Phi_t^h \right) \right\} = 0 \quad (\text{A.61})$$

where Δ_t and Δ_t^* are defined by (3.15) and (3.15) evaluated at the relevant allocation, and Φ_t^h and $\Phi_t^{*,h}$ are distributional effects associated to more automation and more reallocation, respectively. By definition, these pecuniary effects satisfy $\sum_h \phi^h \Phi_t^h = \sum_h \phi^h \Phi_t^{h,*} = 0$ for all periods $t \geq 0$. Now, note that

$$\exp(-\rho t) \eta^{h,*} u' \left(\hat{\mathcal{Y}}_{0,t}^h + \Pi_t \right) = \exp(-(\rho + \chi)t) U' \left(C_t^{\text{LF}} \right) \quad (\text{A.62})$$

using (A.39), where C_t^{LF} denotes aggregate output at the laissez-faire. Therefore, the pecuniary effects net out

$$\int_0^{+\infty} \exp(-\rho t) u' \left(C_t^{\text{LF}} \right) \left\{ \Delta_t^* + T'_0 \left(\alpha^{\text{LF}} \right) \Delta_t \right\} = 0 \quad (\text{A.63})$$

Now, consider the second best problem for a government which values equity and uses symmetric weights $\bar{\eta}^{\frac{1}{\sigma}} \equiv \sum_h \phi^h \left(\eta^{h,*} \right)^{\frac{1}{\sigma}}$. The second best degree of automation with equity

⁸⁸ In the limit with infinitely long unemployment spells, $1/\kappa \rightarrow +\infty$, the sequence is entirely back-loaded since workers are unemployed for a long-time. The sequence thus increases over time. However, $(1 - \theta) w_t^N < w_t^A$ when workers exit unemployment (Appendix A.3 and the left panel of Figure 5.1), so workers would choose not to reallocate in the first place.

$\alpha^{\text{SB}}(\bar{\eta})$ satisfies (A.61) with these new weights.

Following the same approach as in Appendices A.7–A.6, we evaluate the government’s optimality condition at the laissez-faire level of automation. The second best level of automation remains unchanged if

$$\int_0^{+\infty} \exp(-\rho t) \times u' \left(C_t^{\text{LF}} \right) \sum_h \left(\frac{(\eta^{h,*})^{\frac{1}{\sigma}}}{\sum_{h'} \phi^{h'} (\eta^{h',*})^{\frac{1}{\sigma}}} \right)^{-\sigma} \left\{ \Delta_t^* + \Phi_t^{*,h} + T_0' \left(\alpha^{\text{LF}} \right) \left(\Delta_t + \Phi_t^h \right) \right\} = 0 \quad (\text{A.64})$$

Equivalently,

$$\int_0^{+\infty} \exp(-\rho t) u' \left(C_t^{\text{LF}} \right) \sum_h \left(\frac{(\eta^{h,*})^{\frac{1}{\sigma}}}{\sum_{h'} \phi^{h'} (\eta^{h',*})^{\frac{1}{\sigma}}} \right)^{-\sigma} \left\{ \Phi_t^{*,h} + T_0' \left(\alpha^{\text{LF}} \right) \Phi_t^h \right\} = 0, \quad (\text{A.65})$$

using (A.63). Furthermore, note that $\eta^{A,*} < \eta^{N,*}$. The reason is that workers initially employed in automated occupations claim a lower human wealth (Appendix A.3), and the weights are inversely proportional to consumption (A.39). In addition, note that

$$\Phi_t^{*,A} + T_0' \left(\alpha^{\text{LF}} \right) \Phi_t^A < 0 \quad \text{and} \quad \Phi_t^{*,N} + T_0' \left(\alpha^{\text{LF}} \right) \Phi_t^N < 0, \quad (\text{A.66})$$

since these terms capture the distributional effects of automation in general equilibrium. As automation increases, workers initially employed in these occupations are worse off. They relocate more as a result of this change, but still earn no more than those initially employed in automated occupations (Appendix A.3). Putting this together,

$$\int_0^{+\infty} \exp(-\rho t) u' \left(C_t^{\text{LF}} \right) \sum_h \left(\frac{(\eta^{h,*})^{\frac{1}{\sigma}}}{\sum_{h'} (\eta^{h',*})^{\frac{1}{\sigma}}} \right)^{-\sigma} \left\{ \Phi_t^{*,h} + T_0' \left(\alpha^{\text{LF}} \right) \Phi_t^h \right\} < 0, \quad (\text{A.67})$$

since the left-hand side of (A.67) puts a higher weight on negative payoffs. Therefore, automation is *excessive*, regardless of the average duration of unemployment / retraining spells.

A.9 Proof of Proposition 8

We show that the laissez-faire allocation converges to its *first best* counterpart in the long-run. It follows that it also converges to its *second best* counterpart, regardless of whether the government has commitment or not.⁸⁹

We now guess and verify that the laissez-faire converges to the first best with utilitarian weights $\eta_s^h \propto \exp(-\rho s)$. It suffices to verify that the equilibrium sequence of interest rates $r_t \rightarrow \rho$ as $t \rightarrow +\infty$. The reason is twofold. First, other aggregates allocations are continuous in $\{r_t\}$ (Section A.3) so that the guess that $\{\alpha_t, x_t, \mu_t, \Pi_t\}$ converges to its first best steady states counterparts is verified too — this part is very similar to the proof of Proposition 10 in Appendix A.10 so we omit it. Second, individual allocations \mathbf{c} are symmetric across workers $c_{s,t}^h = C_t$ both at the laissez-faire and the first best with weights η .⁹⁰ As a result, individual allocations necessarily coincide too in the long-run.

To show that $r_t \rightarrow \rho$ as $t \rightarrow +\infty$, note that all workers are unconstrained at equilibrium except for the surviving mass $\exp(-\chi t)$ of workers born in $s < 0$ and initially employed in automated occupations $h = A$. Furthermore, note that all these other workers earn the same income

$$\hat{y}_t^{\text{unconstr}} \equiv \frac{1}{1 - \phi \exp(-\chi t)} \left\{ G^*(\cdot) - x_t \alpha_t - \omega x_t^2 \alpha_t - \phi \exp(-\chi t) \hat{y}_t^{\text{constr}} \right\} \quad (\text{A.68})$$

where $\hat{y}_t^{\text{constr}} < +\infty$ is the income of constrained workers. Therefore, the income of unconstrained workers converges to the long-run aggregate consumption at the first best $\hat{y}_t^{\text{unconstr}} \rightarrow C^{\text{FB}}$ as $t \rightarrow +\infty$, using the fact that all other aggregates converge to their first best counterpart and the aggregate resource constraint. It follows that individual consumption $c_t^{\text{unconstr}} \rightarrow C^{\text{FB}}$ as $t \rightarrow +\infty$ by market clearing. As a result, the interest rate converges to the subjective discount factor $r_t \rightarrow \rho$ as $t \rightarrow +\infty$, using (A.18).

A.10 Additional Results

Proposition 9 (Distorsions in PE and GE). *Fix prices and profits at the level that prevails in an efficiency economy without borrowing constraints $\underline{a} \rightarrow -\infty$. Then, the consumption choices are distorted if and only if $\underline{a} > a^*(\lambda, \kappa, \theta, \chi)$ where $a^*(\cdot)$ is defined in Proposition 3. Furthermore, the labor supply choices are distorted if and only if $\underline{a} > \hat{a}(\lambda, \kappa, \theta, \chi)$ for some threshold $\hat{a}(\cdot) \geq a^*(\cdot)$.*

⁸⁹ The reason is that workers are hand-to-mouth $\underline{a} \rightarrow 0$ so that all the state variables in the government's problem are under its control.

⁹⁰ At the laissez-faire, workers are hand-to-mouth so that $c_{s,t}^h = \hat{y}_{s,t}^h$ and $\hat{y}_{s,t}^h / \hat{y}_{s',t}^{h'} \rightarrow 1$ as $t \rightarrow +\infty$ for all occupations (h, h') and generations (s, s') , using (5.1) and the fact that wages are equalized in the long-run. At the first best, symmetry follows directly from (A.3).

Turning to general equilibrium, automation α and reallocation $\{\mu_t\}$ are distorted if and only if $\underline{a} > a^*(\lambda, \kappa, \theta, \chi)$.

Proof. This result consists of two parts: a partial equilibrium one, and a general equilibrium one. We consider the former first, and return to the latter at the end of the proof. We have already shown that consumption choices are if and only if $\underline{a} > a^*(\lambda, \kappa, \theta)$ as part of Proposition 3. We now show that labor supply choices are distorted if and only if $\underline{a} > \hat{a}(\lambda, \kappa, \theta, \chi)$ for some threshold $\hat{a}(\cdot) \geq a^*(\cdot)$. Throughout, we denote by \bar{T}_0 the stopping time that prevails at the efficiency equilibrium without borrowing constraints. All prices are understood to be the ones at this particular equilibrium.

Sufficiency. We proceed in three steps. First, we show that the labor supply choices are distorted only if borrowing constraints are binding at equilibrium at $t = \bar{T}_0$, where \bar{T}_0 is the stopping time in the frictionless economy ($\underline{a} \rightarrow -\infty$). Second, we show that these borrowing constraints are binding in period $t = \bar{T}_0$ if $\underline{a} = 0$. Third, we show that there exists some \hat{a} with $a^* \leq \hat{a} \leq 0$ such that the borrowing constraints are binding at equilibrium in period $t = \bar{T}_0$ if $\hat{a} < \underline{a} \leq 0$. Finally, we show that the borrowing constraints are *not* binding at equilibrium in period $t = \bar{T}_0$ if $\underline{a} \leq \hat{a}$. The desired result follows immediately.

Step 1. We show that labor supply choices are distorted only if borrowing bind at equilibrium at $t = \bar{T}_0$. To see this, note that the reallocation decisions (A.27) and (A.27) when unconstrained are purely forward-looking. In particular, they are not indexed by workers' asset holdings, and whether they were constrained in any period $t < \bar{T}_0$. Therefore, the labor supply choices are distorted only if borrowing constraints bind in period $t = \bar{T}_0$.⁹¹

Step 2. We now show that the borrowing constraints are binding in period $t = \bar{T}_0$ if $\underline{a} = 0$. To derive a contradiction, suppose that this is not the case. Then, all workers are hand-to-mouth since none of them can save at equilibrium. Furthermore, their Euler equations hold with equality since they are unconstrained. Therefore,

$$\frac{\hat{y}_{s,t}^A + \Pi_t}{\hat{y}_{s,\bar{T}_0}^A + \Pi_{\bar{T}_0}} = \frac{\hat{y}_{s,t}^N + \Pi_t}{\hat{y}_{s,\bar{T}_0}^N + \Pi_{\bar{T}_0}} \quad (\text{A.69})$$

for all $s < 0$ and $t \geq \bar{T}_0$ since preferences are isoelastic. However, this restriction cannot hold since \hat{y}_t^A increases over time while \hat{y}_t^N decreases, using labor incomes (A.24)–(A.25). This leads to the desired contradiction.

⁹¹ We have assumed continue to assume that borrowing constraints either bind in period $t = T_0$ or never bind afterwards (Appendix A.3).

Step 3. By continuity of the equilibrium with respect to \underline{a} , there exists some \hat{a} with $a^* \leq \hat{a} \leq 1$ such that the borrowing constraints are binding at equilibrium in period $t = \bar{T}_0$ if $\hat{a} < \underline{a} \leq 0$. This threshold satisfies

$$\hat{a} = \begin{cases} a' & \text{if } a' \leq 0 \\ -\infty & \text{otherwise} \end{cases}, \quad (\text{A.70})$$

where a' insures that workers initially employed in automated occupations do not want to save or dissave in period $t = \bar{T}_0$

$$\frac{\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^t \hat{r}_s ds\right) (\hat{Y}_t^0 + \Pi_t) dt + a'}{\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^t \hat{r}_s ds\right) \hat{u}\left(\exp\left(\int_{\bar{T}_0}^t r_s ds - \rho(t - \bar{T}_0)\right)\right)^{-1} dt} = \hat{Y}_{\bar{T}_0}^A + \Pi_{\bar{T}_0} + \hat{r}_{\bar{T}_0} a', \quad (\text{A.71})$$

where incomes and prices are those that prevail at the equilibrium with no borrowing frictions.

Step 4. It remains to show that the borrowing constraints are *not* binding at equilibrium in period $t = \bar{T}_0$ if $\underline{a} \leq \hat{a}$. This is the case when

$$\hat{r}_{\bar{T}_0}^{-1} \geq \int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^t \hat{r}_s ds\right) \hat{u}\left(\exp\left(\int_{\bar{T}_0}^t r_s ds - \rho(t - \bar{T}_0)\right)\right) dt \quad (\text{A.72})$$

as total income exceeds consumption when $\underline{a} \leq \hat{a}$. Otherwise, $a' > 0$ using (A.71), since the sequence $\{\hat{Y}_t^0 + \Pi_t\}$ is increasing at the original equilibrium, and $r_t \geq \rho$ for all t if the economy without borrowing constraints grows over time. Therefore, $\hat{a} = -\infty$ — i.e. borrowing constraints are always binding — and $\underline{a} \leq \hat{a}$ is never satisfied.

Necessity. Let $\underline{a} > \hat{a}$ where the threshold is given by (A.70). Then, workers initially employed in automated occupations are constrained in period $t = \bar{T}_0$. We now show that the labor reallocation and automation decisions are distorted in this case. To see this, fix the continuation sequence of interest rates $\{\bar{r}_t\}_{t \geq \bar{T}_0}$ and wages $\{\bar{w}_t^h\}_{t \geq \bar{T}_0}$ that prevail in the frictionless equilibrium. By definition of the stopping time \bar{T}_0 ,

$$\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^t r_\tau d\tau\right) \bar{\Delta}_t dt = 0 \quad (\text{A.73})$$

using (A.27), where Δ_t was defined in Appendix A.3. However, these workers are constrained in period $t = \bar{T}_0$, by assumption. That is, they are hand-to-mouth over a choice-specific interval $t \in [\bar{T}_0, S_1)$. Note that the optimality condition (A.26) is *generically* not

satisfied when (A.73) holds.⁹² We show in Section 6 that reallocation can be excessive ($T_0 > \bar{T}_0$) or insufficient ($T_0 < \bar{T}_0$) depending on the average duration of unemployment / retraining spells $1/\kappa$.

Turning to the general equilibrium part of the result, automation and reallocation are distorted when $\underline{a} > a^*(\lambda, \kappa, \theta, \chi)$. The reason is that borrowing constraint bind in this case, by definition of $a^*(\cdot)$. Therefore, the equilibrium sequence of interest rates $\{r_t\}$ differs from the one $\{\bar{r}_t\}$ in the economy without borrowing constraints. As a result, labor supply choices (A.27) are distorted — even if workers remain unconstrained in period $t = \bar{T}_0$ — and so is the automation choice (A.36). \square

Proposition 10 (Second Welfare Theorem). *A first best allocation supported by some Pareto weights η can be decentralized with lump sum transfers*

$$\tau_{s,t}^h = \frac{(\eta_s^h \exp(-\rho(t-s)))^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi\tau) (\eta_{t-\tau}^h \exp(-\rho\tau))^{\frac{1}{\sigma}} d\tau} C_t - \left\{ \hat{y}_{s,t}^h + C_t - \sum_k \phi^k \hat{y}_{s,t}^k \right\}$$

for each initial occupation h , all ages s and calendar time t , where the quantities on the right-hand side are given by Proposition 1 and (5.1).

Proof. We first show that the first best allocation $\{X_t\}$ associated to the weights η is part of an equilibrium (ex post), given the transfers

$$\tau_{s,t}^h = \frac{(\eta_s^h \exp(-\rho(t-s)))^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi\tau) (\eta_{t-\tau}^h \exp(-\rho\tau))^{\frac{1}{\sigma}} d\tau} C_t - \left\{ \hat{y}_{s,t}^h + C_t - \sum_k \phi^k \hat{y}_{s,t}^k \right\} \quad (\text{A.74})$$

and the level of automation α^{FB} . Then, we show that the equilibrium level of automation is α^{FB} (ex ante) when anticipating $\{X_t\}$.

Ex post. Fix the level A.3 of automation α^{FB} and the transfers (A.74). We conjecture and verify that the following sequence of interest rate $\{r_t\}$, wages $\{w_t^h\}$, profits $\{\Pi_t\}$ are part of an equilibrium⁹³,

$$\exp\left(-\int_0^t r_s ds\right) = \exp(-\rho t) \frac{U_t'(C_t)}{U_0'(C_0)} \quad \text{for all } t \geq 0, \quad (\text{A.75})$$

⁹² Whenever this condition happens to be satisfied, there exists a small perturbation of the average duration of unemployment spells $1/\kappa$ that insures it does not.

⁹³ The sequence $\{C_t\}$ characterized by (3.10)–(3.13) is continuous but not differentiable at $t \in \{T_0, T_1\}$.

and

$$w_t^h = \mathcal{Y}_t^h \quad \text{for each } h \in \{0, 1\} \text{ and all } t \geq 0, \quad (\text{A.76})$$

and

$$\Pi_t \equiv C_t - \phi \mu_t^A \mathcal{Y}_t^A - (1 - \phi) \mu_t^N \mathcal{Y}_t^N, \quad (\text{A.77})$$

and that the associated allocation coincides with the first best. The quantities on the right-hand side of (A.75)–(A.77) correspond to the first best. It suffices to show that the planner's allocations of consumption (A.3) and labor (A.7)–(A.10) are consistent with workers' optimality given these prices. By construction, the remaining equilibrium conditions are satisfied: labor markets clear given wages (A.76) and the resource constraint (4.8) is satisfied.

Focusing on the consumption allocation first, we now show that: (i) workers can afford these consumption allocations with a balanced budget given the sequence of wages (A.76), profits (A.77) and the optimal mobility decision implicit in (A.7)–(A.10); and (ii) these consumption allocations insure that workers' Euler equations hold with equality given the interest rate (A.75). Pre-transfer labor incomes $\hat{\mathcal{Y}}_{s,t}^h$ are given by (A.24)–(A.25). By construction, transfers (A.74) insure that the first best consumption allocations allocations of consumption (A.3) are affordable for workers and insure that they have a balanced budget

$$\frac{(\eta_s^h \exp(-\rho(t-s)))^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi\tau) (\eta_{t-\tau}^h \exp(-\rho\tau))^{\frac{1}{\sigma}} d\tau} C_t = \hat{\mathcal{Y}}_{s,t}^h + \Pi_t + \tau_{s,t}^h \quad (\text{A.78})$$

since wages $\{w_t^h\}$ and profits $\{\Pi_t\}$ are given by (A.76)–(A.77) and equilibrium output satisfies $Y_t = C_t$. We still have to show that the consumption allocations (A.78) are optimal. Consider the planner's intratemporal problem (A.2). At the optimum of this problem, consumption allocations satisfy

$$\eta_{t-s}^h \exp(-\rho s) u' \left(c_{t-s,t}^h \right) = \exp(-\rho t) U'_t(C_t) \quad (\text{A.79})$$

It follows that workers' Euler equations hold with equality, by definition of the sequence of interest rates (A.75) and using restriction (A.79).

Turning to the labor allocation, we now show that the effective labor supplies coincide with the first best ones (A.7)–(A.10) given the sequence of wages (A.76), profits (4.6) and interest rate (A.75). Occupational choices are still characterized for two equilibrium stopping times (T_0^{LF}, T_1^{LF}) . We now show that those coincide with their first best counter-

parts $(T_0^{\text{FB}}, T_1^{\text{FB}})$. The first stopping time (T_0^{LF}) is characterized by

$$\int_{T_0^{\text{LF}}}^{+\infty} \exp\left(-\int_{T_0^{\text{LF}}}^t r_\tau d\tau\right) \exp(-\chi t) \times \left((1-\theta)\left(1-\exp\left(-\kappa\left(t-T_0^{\text{LF}}\right)\right)\right)\right) w_t^N - w_t^A dt = 0 \quad (\text{A.80})$$

since transfers (A.74) insure that workers are unconstrained, and using (A.27). It follows that the equilibrium stopping time coincides with the first best $T_0^{\text{LF}} = T_0^{\text{FB}}$, using the definition of the first best stopping time (3.10), wages (A.76) and the stochastic discount factor (A.75). The proof for the second stopping time (T_1^{LF}) is very similar, so we omit it for concision.

Ex ante. Finally, we show that the first best degree of automation α^{FB} solves the firm's problem (4.1) when it anticipates the equilibrium sequence $\{X_t\}$. Using (A.36), the interest rates (A.75) and the fact that labor reallocation is unchanged when the degree of automation is α^{FB} ,

$$\int_0^{+\infty} \exp(-\rho t) U_t'(C_t) \frac{\partial}{\partial \alpha} G^*(\mu_t; \alpha^{\text{FB}}) = 0 \quad (\text{A.81})$$

with $C_t = G^*(\mu_t; \alpha^{\text{FB}})$, so the degree of automation is efficient $\alpha^{\text{LF}} = \alpha^{\text{FB}}$. \square

Lemma 2. *Suppose that either: there are no reallocation frictions $1/\lambda, 1/\kappa, \theta \rightarrow 0$; or there are no borrowing frictions $\underline{a} \rightarrow -\infty$. Then, $\tau_{s,t}^h = \mathbf{0}$ implements a first best allocation.*

Proof. Consider first the case without mobility frictions $1/\lambda, 1/\kappa, \theta \rightarrow 0$. Then, the marginal productivities are equalized across occupations $\mathcal{Y}_t^A = \mathcal{Y}_t^N$. Fix the set of weights $\eta_t^A = \eta_t^N \propto \exp(\rho t)$. Using (A.74), $\{T_{s,t}^h\} = \mathbf{0}$ with these particular weights.

Now, consider the case without borrowing frictions $\underline{a} \rightarrow -\infty$. Automation has distributional consequences in this case, but the economy is otherwise efficient since borrowing is frictionless. Fix the set of weights

$$\left(\eta_t^h\right)^{\frac{1}{\sigma}} \propto \int \exp(-(\rho + \chi)t) U_\tau'(C_\tau) \left(1 + \sum_k (1 - \phi^k) \mathcal{Y}_{t,\tau}^k / C_\tau\right) C_\tau d\tau \quad (\text{A.82})$$

Using (A.74),

$$\int \exp(-(\rho + \chi)\tau) U_\tau'(C_\tau) T_{t,\tau}^h d\tau = 0 \quad (\text{A.83})$$

Therefore, $\{T_{s,t}^h\} = \mathbf{0}$ implements the same allocation since workers are unconstrained. \square

B Quantitative Appendix

In this appendix, we describe our quantitative model in more details and we discuss the approach used to simulate and calibrate the model. Section B.1 provides a recursive formulation of the workers' problem. Section B.2 states and characterizes the solution to the occupations' problem. Section B.3 discusses our choice of Pareto weights for our normative exercise. Finally, Section B.4 provides details about our numerical implementation.

B.1 Workers' Problem

We discretize time into periods of constant length $\Delta \equiv 1/N > 0$, and solve the workers' problem in discrete time.⁹⁴ The workers' problem can be formulated recursively

$$\begin{aligned} V_t^h(a, e, \xi, z) &= \max_{c, a'} u(c) \Delta + \exp(-(\rho + \chi)\Delta) V_{t+\Delta}^{h,*}(a', e, \xi, z) \\ \text{s.t. } a' &= (\mathcal{Y}_t(\mathbf{x}) - c) \Delta + \frac{1}{1 - \chi\Delta} (1 + r_t\Delta) a \\ a' &\geq 0 \end{aligned} \quad (\text{B.1})$$

for employed workers ($e = E$) and unemployed workers ($e = U$) The continuation value V^* before workers observe the mean-reverting component of their income is given by

$$V_t^{h,*}(a', e, \xi, z) = \int \hat{V}_t^h(a', 1, \xi, z') P(dz', z), \quad (\text{B.2})$$

where $\hat{V}_t^h(\cdot)$ is the continuation value associated to the discrete occupational choice. The continuation value for employed workers ($e = E$) associated to this discrete choice problem is⁹⁵

$$\begin{aligned} \hat{V}_t^h(a, e, \xi, z) &= (1 - \lambda\Delta) V_t^h(a, e, \xi, z) + \\ &\quad \lambda\Delta\gamma \log \left(\sum_{h'} \phi^{h'} \exp \left(\frac{V_t^{h'}(a, e'(h', \mathbf{x}), \xi, z)}{\gamma} \right) \right) \end{aligned} \quad (\text{B.3})$$

⁹⁴ Alternatively, we could have formulated the workers' problem in continuous time and solved the associated partial differential equation using standard finite difference methods. However, (semi-)implicit schemes are non-linear in our setting due to the discrete occupational choice. This requires iterating on (B.1)–(B.5) to compute policy functions — which limits the efficiency of these schemes. We found that explicit schemes were unstable unless we use a particularly small time step Δ — which again proves relatively inefficient. In contrast, formulating and solving the workers' problem in discrete time proves to be relatively fast.

⁹⁵ See Artuç et al. (2010) or Caliendo et al. (2019) for the derivation.

with $e'(\cdot) = E$ if $h' = h$ and $e'(\cdot) = U$ otherwise. The associated mobility hazard across occupations is

$$\mathcal{S}_t(h'; \mathbf{x}) = \frac{\phi^{h'} \exp\left(\frac{V_t^{h'}(\mathbf{x}'(h'; \mathbf{x}))}{\gamma}\right)}{\sum_{h''} \phi^{h''} \exp\left(\frac{V_t^{h''}(\mathbf{x}'(h''; \mathbf{x}))}{\gamma}\right)} \quad (\text{B.4})$$

In turn, the continuation value for unemployed workers ($e = U$) is \tilde{n}

$$\hat{V}^h(a, e, \zeta, z) = (1 - \kappa\Delta) V^h(a, e, \zeta, z) + \kappa\Delta \sum_{h'} \mathcal{S}(h', \mathbf{x}) V^{h'}(a, 1, \zeta'(h', \mathbf{x}), z) \quad (\text{B.5})$$

where $\mathcal{S}(\cdot)$ is the mobility hazard, and $\zeta'(\cdot) = \zeta$ if $h' \neq h$ and $\zeta'(\cdot) = (1 - \theta)\zeta$ otherwise. The associated mobility hazard across occupations is

$$\mathcal{S}_t(h'; \mathbf{x}) = \begin{cases} 1 - \kappa^* & \text{if } h' \neq h \\ \kappa^* & \text{otherwise} \end{cases} \quad (\text{B.6})$$

New generations who enter the labor market draw a random productivity z from its stationary distribution and then choose their occupation with a hazard similar to the employed workers'. The only difference is that they experience neither an unemployment spell nor a productivity loss. Worker's labor income is

$$\mathcal{Y}_t(\mathbf{x}) = \begin{cases} (1 - \tau_t) \zeta \exp(z) w_t^h & \text{if } e = E \\ b\mathcal{Y}_t^{h'}(a, E, \zeta, z) / (1 - \tau_t) & \text{otherwise} \end{cases} \quad (\text{B.7})$$

with $h' \neq h$ denoting the previous occupation of employment. The permanent component of workers' income (ζ) is reduced by a factor $(1 - \theta)$ whenever a worker who exits unemployment chooses to enter her new occupation in (B.6). Finally, the mean-reverting component income (z) evolves as

$$z' = (1 + (\rho_z - 1)\Delta)z + \sigma_z \sqrt{\Delta} W' \quad \text{with } W' \sim \text{i.i.d. } \mathcal{N}(0, 1) \quad (\text{B.8})$$

B.2 Firms' Problem

The occupations, the final good producer and the mutual fund can be consolidated into a single representative firm. We solve the problem of this firm in continuous time. This problem can be formulated recursively

$$\begin{aligned}
r_t W_t^h(\alpha) = \max_{\{\mu, x, \alpha'\}} & \left\{ A^h p_t^h (\alpha + \mu)^{1-\eta} - w_t^h \mu - (1 + \tau_t^x) x - \omega \left(\frac{x}{\alpha}\right)^2 \alpha \right. \\
& \left. + (x - \delta \alpha) W_t^{h'}(\alpha) \right\} + \frac{\partial}{\partial t} W_t^h(\alpha) \\
\text{s.t. } x \in \Sigma^h &
\end{aligned} \tag{B.9}$$

where x denotes gross investment, i.e. $(d\alpha_t = (x_t - \delta\alpha_t) dt)$. The constraint set Σ^h captures the non-negativity of x and the constraint $\alpha^{(N)} = 0$ for non-automated occupations. For notation, we set $\tau_t^x = 0$ in the following. At optimum, the labor demand is characterized by

$$(1 - \eta) A^h p_t^h (\alpha_t^h + \mu_t^h)^{-\eta} = w_t^h, \tag{B.10}$$

where $\{\mu^h\}$ denotes effective labor (demand). In turn, the automation choice for automated occupation ($h = A$) is characterized by two differential equations. First, investment satisfies

$$(r_t + \delta) (1 + 2\omega x_t^*) = \left\{ (1 - \eta) A p_t^h (\mu_t^h + \alpha_t)^{-\eta} + \omega (x_t^*)^2 \right\} + 2\omega \partial_t x_t^* \tag{B.11}$$

together with a standard transversality condition. Second, automation satisfies the law of motion

$$d\alpha_t = (x_t^* - \delta) \alpha_t dt \tag{B.12}$$

with the initial condition $\alpha_0 = 0$. The price of the good produced in each occupation is given by

$$p_t^h = \left(\sum_g \phi^g \left\{ A^g (\alpha_t^g + \mu_t^g)^{(1-\eta)} \right\}^{\frac{v-1}{v}} \right)^{\frac{v}{v-1}-1} \left\{ A^h (\alpha_t^h + \mu_t^h)^{(1-\eta)} \right\}^{-\frac{1}{v}} \tag{B.13}$$

in the two occupations. The equity of the mutual fund is

$$E_t \equiv \int_0^{+\infty} \exp\left(-\int_0^s r_{t+s}\right) ds$$

where is the rental rate of automation. The firms' profits are

$$\Pi_t \equiv$$

and are rebated in proportion to each worker's mean-reverting productivity (z).

B.3 Pareto Weights

The government's objective is

$$\mathcal{W}(\mathbf{c}; \boldsymbol{\eta}) \equiv \int_0^{+\infty} \chi \sum_h \phi^h \int_0^{+\infty} \eta_{t-s}^h \exp(-(\rho + \chi)s) u(c_{s,t}^h) ds dt \quad (\text{B.14})$$

using (A.1)–(A.2). We consider two sets of weights $\boldsymbol{\eta}$. The first weights are *utilitarian*. In this case, the government weights agents symmetrically regardless of their initial occupation of employment. We suppose that the government values different generations using the subjective discount rate, so that consumption is symmetric for all workers $c_{s,t}^h = C_t$ at the first best, using (A.3). Therefore, $\eta_s^h \propto \exp(-\rho s)$ for all occupations h and generations s . The objective (B.14) can be decomposed into two terms for old generations (born at $t < 0$) and new generations (born at $t \geq 0$) so that

$$\mathcal{W}^{\text{utilit}}(\mathbf{c}; \boldsymbol{\eta}) \equiv \int V_0(\mathbf{x}) d\pi_0 + \chi \int_0^{+\infty} \exp(-\rho s) V_s^{\text{new}} dt \quad (\text{B.15})$$

where V_s^{new} is the expected value at birth of new generations, given by (B.2). The second set of weights we consider focuses on the *efficiency* motive. Our approach is similar to the one we adopted with our benchmark model (Section 6.3). The weights that the government puts on a given worker are inversely related to this worker's marginal utility at birth. This insures that the government has no incentive to redistribute resources to improve equity. In particular, the government weights (constrained) workers with a higher marginal utility *less* compared to a utilitarian government that values equity. Therefore, we choose $\eta_s^h \propto 1/V_s'(\cdot) \exp(-\rho s)$ where $V_s'(\cdot)$ is marginal utility at birth. The objective (B.14) becomes

$$\begin{aligned} \mathcal{W}^{\text{effic}}(\mathbf{c}; \boldsymbol{\eta}) \equiv & \int 1/V_0'(\mathbf{x}) \times V_0^h(\mathbf{x}) d\pi_0 \\ & + \chi \int_0^{+\infty} \exp(-\rho s) 1/V_s^{\text{new},'}(\cdot) \times V_s^{\text{new}} dt \end{aligned} \quad (\text{B.16})$$

where

$$V_t^{\text{new},'}(\cdot) = \int \sum_h \tilde{\mathcal{S}}_{t-1}(h, z) \tilde{V}_t^{h,'}(h, z) P^*(dz') \quad (\text{B.17})$$

is the (expected) marginal utility at birth for new generations, with $\tilde{V}_t^h(\cdot)$ and $\tilde{\mathcal{S}}_t(\cdot)$ denoting the value (B.1) and mobility hazard (B.4) of a worker with no initial assets $a = 0$ and

who has never moved between occupations before $\xi = 0$, and P^* denoting the ergodic distribution associated to the income process P .

B.4 Numerical Implementation

We now describe the numerical approach we adopt to solve for the stationary equilibrium and the transition dynamics.

Workers' problem. The problem (B.1) is potentially non-convex since it involves a discrete choice across occupations.⁹⁶ To address this issue, we follow an approach similar to [Druehl and Jørgensen \(2017\)](#) and [Iskhakov et al. \(2017\)](#). Specifically, we recover a map $a' \mapsto a$ using the standard endogenous grid method ([Carroll, 2006](#)). This map is not necessarily monotonic since the problem is non-convex. In other words, this map defines a correspondence $a \rightrightarrows a'$ — this map contains the optimal policy $a \mapsto a'$. We recover this optimal function as follows. We first partition the map $a' \mapsto a$ into monotone segments. For each a , we interpolate linearly each of these segments to obtain candidates for a' . We then compare the values (B.1) obtained using these candidates. Optimal policies are those who achieve the highest value. We use a generalization of [Young \(2010\)](#)'s non-stochastic simulation method with multiple assets to iterate on the distribution. Finally, we discretize the income process on a 11-point grid using the method of [Rouwenhorst \(1995\)](#).

Firm's problem. The firm's optimal choice of investment and automation is characterized by the non-linear system of differential equations (B.11)–(B.12). We solve this system as follows when computing the transition paths from the initial to the final stationary equilibrium. First, we fix $\alpha_t = 0$ for all t and $x_T = \delta\alpha^*$ for some terminal period T , where α^* denotes the level of automation in $h = A$ at the final stationary equilibrium. We then solve for the sequence $\{x_t\}$ using (B.11) and a standard Runge-Kutta method. Next, we solve for the sequence $\{\alpha_t\}$ using (B.12). We repeat the previous two steps until the sequence $\{\alpha_t\}$ converges.

B.5 Additional Results

In Section 8.3, we discussed why the government finds it optimal to tax automation in the long-run. Figure B.1 plots utilitarian welfare as a function of the steady state level of

⁹⁶ A finite elasticity of labor supply ($\gamma < +\infty$) insures that V^* does not feature kinks. However, V^* might remain convex on part of its domain. An Euler equation is still necessary but not sufficient since it can admit multiple candidates for consumption c and savings $s(c, \mathbf{x})$.

automation α as a share of its laissez-faire counterpart α^{LF} .

Figure B.1: Constrained efficiency (steady state)

