Dynamic Auction Environment with Subcontracting

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Abstract

We study a dynamic procurement environment with capacity constraints where contractors may re-sell part of the work in the subcontracting market. In our model projects are allocated to contractors in the primary market through first price sealed bid auctions. Project costs are contractors' private information. They vary stochastically across projects and independently across contractors. Capacity constraints impact costs through previous commitments (backlogs). We develop a novel numerical strategy for solving this dynamic game. This methodology is used to characterize the equilibrium and to isolate and quantitatively assess the magnitude of mechanisms through which the availability of subcontracting shapes this market.

Keywords: dynamic games, capacity constraints, generalized all-pay auction, subcontracting, procurement

JEL Classification: C73, C63, D44, H57, L20

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1 Introduction

Recent developments in the auctions literature are characterized by the enhanced appreciation of the impact that capacity constraints have on the performance of many markets. For example, about 30% of funds spent on government procurement are allocated to construction and maintenance, environments that are traditionally associated with capacity constraints. In addition to raising the cost of work in periods of high capacity utilization, the presence of capacity constraints is thought to promote firms' cost asymmetry, which leads to relaxation of competition and results in government paying high prices. Recent empirical studies by Jofre-Bonet and Pesendorfer (2003), Groeger (2012) and Balat (2012) documented significant cost increases associated with capacity constraints in the markets for highway maintenance. For example, Jofre-Bonet and Pesendorfer (2003) estimate that an increase in capacity utilization from one standard deviation below the average to one standard deviation above the average results in a 12% increase in the cost. These studies, however, do not account for the key prevalent feature of these environments: the ability of firms to outsource part of their work in the subcontracting market. In this paper we argue that this omission has an important impact on the quantitative findings in the literature and on their interpretation.

The presence of capacity constraints dynamically links firms' performance and bidding strategies across time periods. In such an environment the key effects of the ability to outsource part of the work also arise from modifying intertemporal incentives. To study these effects we consider an infinite-horizon procurement game where a service for a single project is sought each period. The project is allocated to contractors in the primary market through a first price sealed bid auction. We assume that the capacities of contractors are small relative to the size of the project. This implies that work on the project is carried over several periods. The amount of work carried over is summarized by a backlog. We assume that contractors' costs vary stochastically across projects and independently across contractors. These are standard assumptions in the procurement auction literature, and we find it essential to preserve them in this study as the firms rely on subcontracting precisely to modify their cost distributions over time. The cost persistence is captured by the firm's backlog that works to increase the firm's costs in the manner of first-order stochastic dominance. Firms' backlogs are observable to all market participants. The realization of the project's costs is the private information of the firm. To focus on the dynamic incentive effects provided by the ability to subcontract in the primary market, our modeling of the subcontracting market is deliberately simple. We assume that the contractors can outsource part of the work in the secondary market, which is composed of a large number of small firms that undertake only the amount of work they can complete in one period. Such a market can then be summarized by a static subcontracting supply curve. We consider an environment where contractors have to commit to the subcontracting policy at the time when they submit their bids. This is consistent with the rules adopted in many procurement markets but alternative rules may also be of interest.

While this model is a natural and a relatively simple extension of existing models, it has never been studied before in part because of the associated computational challenges.[1] The first contribution of this paper is methodological. In particular, we develop a numerical method to solve the dynamic game with subcontracting and capacity constraints where the work is allocated through first price sealed bid auctions. We exploit this methodology to understand the equilibrium properties of our model and of the specific role played by the ability of firms to subcontract part of their work. More generally, the numerical methodology we develop is important for future empirical analysis of such markets since it could be used to re-solve the game while evaluating counterfactual scenarios. Specifically, our numerical approach extends that of Saini (2013), who solves the dynamic game with capacity constraints but without subcontracting. He shows that pricing strategies in such a game can be obtained as a solution to a standard auction game with asymmetric bidders and modified distributions of costs. He then draws on the numerical methodologies developed for simulating solutions to such auction games and exploits a specification where a closed-form solution of the auction game exists. Analysis of the dynamic game with subcontracting is more challenging since it involves deriving two interrelated policy functions (subcontracting and bidding). In addition, the payoff from losing under subcontracting depends on the contractor's bid, which means that bidding functions are determined by a generalized version of the "all-pay" auction game with asymmetric bidders that does not have a closed-form solution for any of the known distributions. This means that bidding strategies have to be obtained as a numeric solution to the system of differential equations with boundary conditions. In contrast to Saini (2013), we compute equilibrium in our game as a limit of Markov Perfect Equilibria of finite horizon games. This alleviates concerns about the multiplicity of equilibria by providing a consistent and robust equilibrium selection rule that enables us to compare equilibrium outcomes for different models and parameter values.

The second contribution of the paper is substantive. We use this newly developed numerical technique to derive equilibrium properties of the game with subcontracting and to compare these properties to those of the game without subcontracting. We find that the availability of subcontracting has profound and multi-layered effects on the environment. First, conditional on the vector of backlogs, subcontracting results in modification of project cost distributions. Second, it gives greater control over backlog accumulation. These cost effects have direct implications for pricing strategies. Last, the subcontracting changes the dynamic incentives in pricing. These three sets of effects are interrelated, and we loosely separate them in the discussion below for expositional purposes only.

For a given vector of backlogs, which fixes the set of cost distributions for the participating bidders, the availability of subcontracting results in the modification of these distributions. As might be expected, bidders with high cost realizations are more likely to subcontract. Moreover, if the price of subcontracting is increasing in the amount of work outsourced, the subcontracting strategies are increasing in own cost realizations. Thus, subcontracting helps bidders mitigate their high cost realizations and results in project cost distributions that have lower means, variances and upper

[1] In Section 9 we will argue that this model captures the key trade-offs in the procurement markets with subcontracting and discuss how various additional empirical features of such markets map into objects in our model and affect our insights.

bounds than the cost distributions without subcontracting for the same vector of backlogs. Among other things, this reduces the importance of private information since it lowers the variance of private costs. This cost mitigation leads to lower prices because: (a) costs are lower on average, (b) cost distributions become more similar, and thus, the competition intensifies, and (c) the variance of the cost distribution decreases, which reduces informational rents associated with private information.

In addition to changing the conditional cost distributions, the availability of subcontracting modifies equilibrium backlog accumulation. It tends to reduce the backlog levels on average and makes backlog levels more similar among market participants. This lowers the average costs and further intensifies the competition as the cost distributions become even more similar. Hence, these dynamic equilibrium effects also lead to lower prices.

The most subtle effects of subcontracting relate to its impact on dynamic pricing considerations. In the environments with capacity constraints, winning an auction has a dual effect: on one hand it entails collecting profit for a given project; on the other hand, it leads to an increase in future costs (due to an increase in backlog), and thus a weakening of the contractor's position in future auctions. Conversely, losing could be strategically valuable, since it results in an increase in the competitor's backlog. Thus, losing today implies a higher probability of winning in the future and with higher prices. These trade-offs are summarized by the option value of losing relative to winning. These option values are heterogeneous across bidders with different backlog levels.[2]

The option value of losing is lower in the environment with subcontracting because subcontracting ameliorates the impact of winning on backlog accumulation. Specifically, the fact that inefficient players subcontract a substantial part of the project reduces the option value of losing for the lowest backlog competitors. Thus, the probability that the project is allocated to a bidder with a lower backlog increases and prices decline. Since the low backlog bidders have lower costs on average, this improves allocative efficiency.

Without subcontracting, the option value of losing does not depend on the bidder's cost realization because continuation values conditional on winning or losing do not depend on current cost realization. With subcontracting, the option value of losing depends on the bidder's costs. More specifically, since subcontracted amounts are increasing in own costs, the impact of winning on the backlog is higher for bidders with low cost realizations relative to bidders with high cost realizations, which makes the option value decreasing in own costs. Thus, under subcontracting, option value considerations flatten bidding strategies as a function of costs. This tends to lower the allocative efficiency.

Further, the option value of losing relative to winning depends on the distribution of future costs. Since the availability of subcontracting reduces the variance of these distributions and eliminates

[2]Note that this reflects an additional dynamic bidder asymmetry that exists in addition to cost asymmetry due to the differences in cost distributions among contractors with different backlog levels. Both types of asymmetry work to raise bidders' prices and to lessen the efficiency of the auction environment.

very high cost realizations, it essentially serves as an insurance device. It thus enhances dynamic option values and in that it works against the dynamic symmetrization induced by subcontracting.

To assess the relative quantitative importance of these effects we explore the properties of subcontracting equilibria across a range of subcontracting supply functions. The steepness of the baseline subcontracting supply function is chosen so that the model matches the average level of subcontracting observed in highway procurement data used in the empirical studies cited above. We also consider steeper and flatter functions representing different levels of subcontracting availability. The other parameters of the model are chosen to make primitives of the model comparable to those estimated in the empirical studies.

First, consider an equilibrium with a steep subcontracting supply function, implying that outsourcing is expensive. In this case, contractors subcontract a small part of the work for most cost realizations. They do so mostly for the dynamic reason of limiting the backlog accumulation, which, in turn, limits the probability of very high cost draws in the future. Even if they all subcontract little, the effect on eliminating extreme cost realizations is large, resulting in a substantial decline in the average costs relative to an equilibrium with no subcontracting. As the subcontracting supply function becomes progressively flatter, this effect persists; however, it stops being the dominant effect for costs. Instead, when subcontracting is cheap, firms use it aggressively in the case of high cost realizations. In other words, as subcontracting becomes cheap, the reduction in the expected costs is driven more by the elimination of high costs through subcontracting than by the reduction in the average level of backlogs.

The importance of the dynamic pricing effects also varies systematically with the steepness of the subcontracting supply curve. When subcontracting is expensive, the option value component in pricing is relatively large because winning is associated with a substantial increase in backlog and consequently future costs. There is a substantial option value associated with all backlog levels, and it is increasing in backlog. This option value is passed through into prices and leads to an increase in markups charged by bidders (as compensation for the negative future cost consequences of winning). When subcontracting becomes cheaper, firms rely less on this pricing strategy to control backlog accumulation, since now the winning bidder can always mitigate large future cost realizations by subcontracting more of the work. Thus, the importance of the option value component in prices declines as the cost of subcontracting goes down.

Interestingly, regardless of the slope of the subcontracting supply curve, we always find that the variation in subcontracting strategies across cost levels while holding the backlog levels fixed by far dominates the variation in subcontracting strategies across backlog levels while holding the cost realization fixed. This happens because when subcontracting is cheap, firms are less concerned with backlog and rely on subcontracting to minimize the impact of high cost realizations. When subcontracting is expensive, firms continue to use subcontracting to modify costs but primarily rely on dynamic pricing to limit backlog accumulation.

As subcontracting becomes cheaper, the costs become smaller, more symmetric, and the role of the option value in prices declines. The importance of private information in the market declines as well. Thus, the average profits under subcontracting are always lower relative to the profits without subcontracting and a greater availability of subcontracting results in a larger deterioration of contractors' profits. In our simulations, the industry as a whole would prefer to commit to not using subcontracting if it could. In contrast, the auctioneer gains from subcontracting, since project costs as well as mark-ups under subcontracting are substantially lower than the costs and mark-ups in the environment without subcontracting.

The third contribution of the paper is to understand the implications of using a mis-specified model without subcontracting to empirically study an environment with subcontracting. As mentioned above, cost modification as well as option value considerations works to lower and flatten bidding strategies as a function of unmodified cost realizations under subcontracting. This implies that the shape of the bid distribution conditional on backlog levels under subcontracting differs substantially from the corresponding distribution of bids in the environment without subcontracting. Existing estimation methodologies rely on these bid distributions and participation patterns to identify the primitives of this environment. Consequently, estimating a model with no subcontracting on the data generated by an environment with subcontracting will yield biased estimates. We find that the bias is likely to be large. In our baseline parameterization, the mean of the cost distribution estimated using a mis-specified model without subcontracting would have a downward bias of around 30%, the estimated variance of the cost distribution would have a downward bias of approximately 50%, and the importance of capacity constraints would be underestimated by around 70%. These biases are determined, to a large extent, by imputing wrong dynamic option effects relative to those present in an environment with subcontracting. The recovered distribution of costs is not close even to the distribution of costs modified through the use of subcontracting. Even if it were, the modified distribution is not a primitive of the model and thus cannot be used to perform counterfactual experiments.

Finally, we study the implications of the capacity constraints in the presence of subcontracting for the optimal procurement policy in terms of size versus frequency trade-off, and compare them to predictions of the model without subcontracting. We find that in the environment with subcontracting the burden of a larger project size is reduced substantially. For example, consider a change in procurement policy in our benchmark model so that projects that are twice as large are auctioned half as frequently. In response to this change, procurement cost which is already 30% lower in our benchmark model relative to the no subcontracting environment, increases by only 2% in the presence of subcontracting as opposed to 12% without subcontracting. This occurs, of course, because an increase in project size results in higher levels of subcontracting, which (a) ameliorates the backlog accumulation that arises in the absence of subcontracting, and (b) more strongly modifies cost distribution. This leads to lower costs but also (c) intensifies competition in the market, and (d) reduces informational rents. The last two effects lead to lower mark-ups. Effects (a) – (d) result in lower prices relative to the case without subcontracting and thus lower procurement costs. The policy prescription developed in the capacity constraints literature without subcontracting (Saini, 2013; Groeger, 2012) is to sub-divide projects and auction them more frequently, since it was estimated that this would result in substantial savings. We find that in the environment with subcontracting, this effect is unlikely to be very large. In fact, if sub-dividing projects is costly, an auctioneer may consider improving the availability of subcontracting as a substitute for such a policy. Our results underscore the importance of taking subcontracting into account when performing policy experiments. Indeed, any policy that exogenously affects contractors' cost is likely to result in the adjustment of subcontracting and pricing schedules. This, in turn, would change backlog accumulation and winning patterns as well as firms' profits and procurement costs.

The rest of the paper is organized as follows. In Section 2 we summarize the related literature. Section 3 describes the model. In Section 4, we characterize the equilibrium with subcontracting. In Section 5, we develop a numerical simulation algorithm. We analyze the properties of computed equilibrium in Section 6, study the implication of using the mis-specified model without subcontracting in estimation in Section 7 and focus on the policy implications of subcontracting in Section 8. Section 9 discusses empirically relevant extensions. Section 10 concludes.

2 Related Literature

Our paper is related to an older literature that studies the boundaries of firms and is represented by Coase (1937), Coase (1988), Williamson (1975), Jensen and Mechling (1976), Alchian and Demsetz (1972) and other studies. This literature analyzes the factors that determine what components of firms' production should be outsourced rather than performed in-house. Some of the factors they mention are dynamic (capacity) constraints, quality control, and the difficulty of creating appropriate incentives for outside workers. Our analysis abstracts from most of these issues and focuses only on the gains from subcontracting in the presence of asymmetric stochastic costs as well as capacity constraints.

We are more closely related to the literature that studies the effect of subcontracting on the performance of static auctions. For example, Wambach (2009) investigates the benefits of committing to subcontracting strategy at the time of bidding. Gale, Hausch, and Stegeman (2000) investigate subcontracting in sequential auctions. They are interested in questions similar to the ones we pose in this paper. However, they focus on the environment with perfect information, where projects are allocated through a second price auction. As we do, they find, among other things, that firms subcontract higher amounts subsequent to recent winning. This literature also includes a considerable number of empirical papers such as Miller (2012), De Silva, Kosmopoulou, and Lamarche (2011), Moretti and Valbonesi (2011), and an experimental analysis by Nakabuyashi and Watanabe (2010). Empirical research focuses on the effect of long-term relationships on subcontracting, and preferential treatment in the subcontracting market as well as the effect of uncertainty on the amount of subcontracting.

Finally, we build on the empirical literature, represented by Jofre-Bonet and Pesendorfer (2003), Groeger (2012), and Balat (2012), that measures the importance of capacity constraints in the procurement markets organized as a sequence of first price sealed bid auctions. Our paper extends the models used by these studies. We also rely on their estimates when choosing the set of parameters we use in our study. As we mentioned already, our numerical strategy is an extension of the method proposed in Saini (2013), who studies a dynamic procurement environment with capacity constraints but without subcontracting.

3 Model

This section describes a model of a dynamic procurement auction with endogenous subcontracting. The model is developed in the context of construction procurement but could be adjusted to describe other similar markets.

3.1 Setting

We consider an infinite horizon environment where a buyer (for example, a government) seeks to allocate a project of size x to a contractor every period. We assume that projects consists of providing a certain amount of homogeneous service. Projects are allocated one at a time among two infinitely lived firms via first-price sealed-bid auctions. The contractors, upon winning a project, may engage to do all the work in-house or may decide to re-sell a part of the project to subcontractors operating in the secondary market.

Subcontracting Market To simplify the exposition we assume that the subcontracting market is summarized by a (possibly) increasing supply curve, $P(.)$, which is constant over time. In this we abstract from any possible contractor-subcontractor alliances, contractor-specific bargaining, or the possibility of capacity constraints arising in the subcontracting market. All of these issues are important on their own but they might obscure the main issue that we would like to study in this paper. In the setting we have in mind, the subcontracting market consists of a large number of small firms that could be very heterogeneous in their costs. In addition, the project can be sub-divided into small tasks that could be completed by a subcontractor in one period. Under these circumstances the subcontracting supply curve would remain nearly static. Several subcontracting firms may need to be hired to fulfill subcontracting demand on a given project. This accounts for the possibility that the subcontracting price may be increasing in quantity. We believe that these features characterize the majority of real-life subcontracting markets. Our setting accounts for the fact (in a degenerate way) that the contractors are likely to draw correlated subcontracting costs since they are shopping in the same market. We further assume that while the supply schedule in the subcontracting market may be flat over a large range of job sizes, it eventually starts sloping upward so that it is impossible to subcontract the whole project at a price that is below the costs drawn in the primary market. We use this assumption to capture the effect of an upper limit on subcontracting imposed in most primary markets.

Productivity and Backlog Contractors that operate in this market are endowed with capacity, K_i, $i = 1, 2$. The amount of work a contractor completes within a given period of time, his productivity, may depend on several factors (such as weather) that are outside his control. Following the literature, we model contractor i's within-period productivity as a random variable, ϵ_{it}, that takes it values from an interval $[0, K_i]$ and is distributed according to $F_{\epsilon,i}$. In this market, project size x is usually large relative to contractors' capacities. This regularity re-enforced by stochastic productivity implies that a certain amount of outstanding obligations may be carried from period to period. The work that contractor i has undertaken to complete but which remains unfinished at the beginning of period t is summarized by contractor i's backlog in period t, $\omega_{i,t}$. We assume that the contractor's backlog levels are known to all market participants. Further, in our environment the issue of sequencing jobs does not arise since projects are homogeneous. The contractor works on them in the same order in which they arrive.

Backlog and Contractors' Costs We assume that project costs are given by $c_{i,t}x$ where the marginal cost $c_{i,t}$ is the private information of contractor i. Marginal cost is drawn from the distribution $F_c(.|R_{it})$, which depends on the contractor's current capacity utilization defined as $R_{i,t} = \frac{\omega_{i,t}}{K_i}$. Capacity utilization essentially is equal to the number of periods before the contractor would be able to start work on any new load under the best possible scenario. We assume that higher capacity utilization has an adverse effect on the project costs distribution in the sense of first-order stochastic dominance. It is easy to get a sense for the effect of this variable and its relationship to so-called capacity constraints if one imagines that the contractor has to complete the project within a certain number of periods. Then the closer capacity utilization is to the allocated number of periods the less likely it is that the project will be completed on time. If the cost of missing the deadline is positive (and proportional to the project's size), then the marginal cost of the project will be increasing in capacity utilization. We do not explicitly assume any restrictions on the duration of the project to avoid unnecessary complications in solving the model. The dependence of the cost distribution on capacity utilization captures the possible effects of deadlines or any other potential cost effects associated with working at full capacity over a substantial amount of time. In the simulation part of the paper we consider a number of ways in which capacity utilization may impact the cost distribution, such as a shift in the support of the cost distribution, or the re-allocation of the mass toward the upper end of the cost distribution.

3.2 Timeline

Each period in the game is divided into two stages. In the first stage the new projects are allocated and in the second stage the work on the projects is performed. At the beginning of the period the state of the world is characterized by a vector of contractors' backlogs, $\omega_t = (\omega_{1,t},\, \omega_{2,t})$. This vector determines contractors' capacity utilizations and hence the distribution of their costs.

Project Allocation In the first stage contractors draw their marginal project costs from respective distributions and simultaneously decide on their bids, b, and subcontracting amounts, h, for the new project. For simplicity we assume that participation in the auction is costless. Therefore, all contractors submit a bid. We will address the issue of effective participation later in the paper. We assume that the contractor is required to commit to a subcontracting strategy prior to the auction and cannot renege on his commitment later. This assumption is based on the rules followed in most real-life procurement markets, though alternative specification could be of interest as well. The contractor with the lowest bid wins the auction and the new project is added to his backlog. In the analysis that follows we assume that total payment is paid and that all of the cost is incurred right after the auction. This simplifying assumption is made for convenience of exposition. However, it is not very far from reality. In real markets contractors are usually paid at the end of the job, whereas they are required to post a bond that is used to pay their suppliers and subcontractors before the auction. This implies that the problem of sub-dividing the costs or the payments over periods when work on the project lasts does not arise.

Backlog Depreciation After the auction stage is concluded, the contractors observe their productivity draw, ϵ, and reduce their backlogs by corresponding amounts.

State Transition Denote the winner of period t's auction by w. Then, contractor i's backlog evolves according to the transition function $\sigma(\omega_{i,t}, \epsilon_{i,t}, c_{i,t}, w)$ such that:

$$\omega_{i,t+1} = \sigma(\omega_{i,t}, \epsilon_{i,t}, c_{i,t}, w) \;\; = \;\; \max\{\omega_{i,t} - \epsilon_{i,t} + 1(i \; = \; w_t)(1 - h_{i,t}(c; \omega_t))x, 0\} \tag{1}$$

Similar to the previous literature our specification of the transition of the states ensures that the evolution of contractors' backlogs is stochastic, and the rate of transition is more favorable to large firms. In simulations, we assume that the backlog amount is limited from above by some large positive constant M. Therefore, the state space in our game is given by $\Omega = [0,\, M] \times [0,\, M]$.

3.3 Markov Perfect Equilibrium

The contractors in our model are forward-looking: as in the environment without subcontracting they take into account how winning a project today impacts their competitiveness and profitability in the future. Winning has a dichotomous effect: on the one hand, the winner collects a profit in the current period; on the other hand, winning increases backlog and, due to capacity constraints, implies higher costs in the near future. Similarly, losing increases the competitor's backlog and, therefore, provides a competitive edge in the next few periods. The contractor chooses his optimal strategy by weighting current profit against the difference between the continuation values of losing and winning. In the environment with subcontracting these considerations become even more subtle. First, in the model with subcontracting the amount of work that the contractor commits to complete himself and which is added to his backlog may and does depend on his cost realization. In addition, contractor i's bid, which is a function of current costs, determines the competitor's cost levels to which contractor i loses, and thus more flexibly controls the competitor's backlog accumulation. In short, dynamic incentives in the environment with subcontracting depend on current costs realization, and thus they impact optimal strategies differentially across cost levels. This is in contrast to the environment without subcontracting, where dynamic considerations affect contractors' behavior (their pricing) uniformly across cost levels.

In the paper we analyze Markov Perfect Equilibria of the dynamic auction game as defined in Maskin and Tirole (1988). In particular, we consider strategies that depend only on payoff-relevant histories. In our case, payoff-relevant information is summarized by a vector of contractors' backlogs. Indeed, own backlog fully determines the distributions of the contractor's cost and productivity in period t. Thus, current backlog variables determine the contractor's profitability in the current period as well as his backlogs in future periods. Hence, we can summarize the state of the market at time t by the vector of contractors' backlogs at the beginning of period t. Contractor i decides on state-dependent optimal action consisting of bidding, $\mathbf{b}_i(\cdot;\omega)$, and subcontracting, $\mathbf{h}_i(\cdot;\omega)$, functions that for every realization of his private costs determine the bid he submits in the auction and the portion of work he commits to completing in-house upon winning. We define a stage payoff of our game as the expected value from participating in the stage auction. Note that this stage payoff is stationary, that is, it does not change over time conditional on state (backlogs) and actions. This fact enables us to restrict our attention to stationary strategies.

Let $\mathbf{b}_i(\cdot;\omega_i,\,\omega_{-i})$ and $\mathbf{h}_i(\cdot;\omega_i,\,\omega_{-i})$ denote contractor i's stationary bidding and subcontracting strategies respectively, where we use a standard convention to denote the competitor of contractor i and the competitor's backlog by $-i$ and ω_{-i} correspondingly. As standard theory suggests, for each strategy profile $\mathbf{g} = \{(\mathbf{h}_1(\cdot\,;\omega), \mathbf{h}_2(\cdot\,;\omega), \mathbf{b}_1(\cdot\,;\omega), \mathbf{b}_2(\cdot\,;\omega))\}_{\omega\in\Omega}$, and a starting state ω^0 there exists an (almost) unique Markov process that determines the joint distribution of private costs $c_{i,t}$, states ω_t, and actions $b_{i,t} = \mathbf{b}_i(c_{i,t}, \omega_t)$, $h_{i,t} = \mathbf{h}_i(c_{i,t}, \omega_t)$ for each $t = 0, \ldots, \infty$. For a given strategy profile we define a value function of contractor i as a sum of discounted future expected profits where the expectation is taken with respect to the stochastic process. Formally,

$$V_i(\omega_0; \mathbf{g_i}, \mathbf{g_{-i}}) = E_{\{c_t,\epsilon_t\}_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \delta^t \mathbf{1}_{\{b_{i,t}<b_{-i,t}\}} (b_{i,t} - (1-h_{i,t})c_{i,t}x - P(h_{i,t}x)h_{i,t}x) \middle| \omega_0 \right], \qquad (2)$$

where δ denotes the discount rate common to all contractors, and the expression in the brackets

$$\mathbf{1}_{\{b_{i,t}<b_{-i,t}\}}(b_{i,t}-(1-h_{i,t})c_{i,t}x-P(h_{i,t}x)h_{i,t}x)$$

summarizes the period t profit of contractor i. We refer to this value function as the *ex-ante value function*, since it describes the value to the contractor before he acquires private information about his costs in the current period.

We consider Markov Perfect Equilibria $\mathbf{g}^* = (\mathbf{g}_\mathbf{1}^*, \mathbf{g}_\mathbf{2}^*)$, such that $V_i(\omega_0|\mathbf{g_i}^*, \mathbf{g_{-i}}^*) \geq V_i(\omega_0|\mathbf{g_i}, \mathbf{g_{-i}}^*)$ for all $\mathbf{g_i}$, $i = 1, 2$, for all $\omega_0 \in \Omega$, and given that contractors have correct beliefs about the distribution of their competitors' private costs.

4 Equilibrium Characterization

4.1 Bellman Equation

Under standard assumptions contractors' optimal behavior in this environment can be summarized by a Bellman equation. To simplify the presentation we develop the relevant Bellman equation in steps.

Conditional on the realization of private costs c_i, and submitting bid b_i and subcontracting action h_i, contractor i's dynamic payoff from winning is given by

$$(b_i-(1-h_i)c_ix-P(h_ix)h_ix)+\delta E_\epsilon V_i(\omega_i-\epsilon_i+(1-h_i)x,\omega_{-i}-\epsilon_{-i}) \tag{3}$$

where the expectation is taken with respect to the distribution of within period realizations of productivity ϵ. In order to simplify the notation we drop the maximum term that keeps the backlog positive in the state transition. For the full expression see equation (1).

Contractor i's dynamic payoff from losing is given by

$$\begin{gathered}\delta E_{\epsilon,c_{-i}|\mathbf{b}_{-i}<b_i}V_i(\omega_i-\epsilon_i,\omega_{-i}-\epsilon_{-i}+(1-\mathbf{h}_{-i}(c_{-i}))x)= \\ \delta\int_{\underline{c}_{-i}}^{\mathbf{b}_{-i}^{-1}(b_i)}E_\epsilon[V_i(\omega_i-\epsilon_i,\omega_{-i}-\epsilon_{-i}+(1-\mathbf{h}_{-i}(c_{-i}))x)]dF_{-i,c}(c_{-i}|c_{-i}<\mathbf{b}_{-i}^{-1}(b_i)).\end{gathered} \tag{4}$$

In the remainder of the paper we frequently drop the dependence of bidding and subcontracting strategies as well as cost distribution on the state ω to keep the notation simple. Notice that if contractor i loses the auction, his competitor's backlog increases. However, as opposed to an environment without subcontracting, the amount by which the competitor's backlog increases depends on the competitor's costs c_{-i} through the subcontracting strategy. In turn, contractor i's bid determines the set of the competitor's cost to which he may lose. Thus, contractor i's payoff from losing depends on his own bid.

We put together the above pieces to obtain a Bellman equation

$$V_i(\omega) = \int_{c_i} \Bigg\{ \max_{b_i,h_i} W_i(b_i,\omega;\mathbf{g_{-i}}) \Big[b_i - (1-h_i)c_i x - P(h_i x)h_i x + \delta E_\epsilon V_i(\omega_i - \epsilon_i + (1-h_i)x, \omega_{-i} - \epsilon_{-i}) \Big]$$

$$+(1 - W_i(b_i,\omega;\mathbf{g_{-i}}))\delta \int_{\underline{c}_{-i}}^{\mathbf{b}_{-i}^{-1}(b_i)} E_\epsilon V_i(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + (1 - \mathbf{h}_{-i}(c_{-i}))x) dF_{-i}(c_{-i}|c_{-i} < \mathbf{b}_{-i}^{-1}(b_i)) \Bigg\} dF_i(c_i). \tag{5}$$

Here, $W_i(b_i,\omega;\mathbf{g_{-i}}) = (1 - F_{-i}(\mathbf{b}_{-i}^{-1}(b_i)))$ denotes the probability that bidder i wins the auction given that he submits the bid b_i and his competitor uses strategy $\mathbf{g_{-i}}$.

4.2 Optimal Subcontracting Policy

We solve the problem sequentially. Notice that the payoff from losing and the probability of winning do not depend on bidder i's subcontracting strategy. We, therefore, first, solve for contractor i's optimal subcontracting level conditional on winning; that is,

$$\mathbf{h_i}(c_i;\omega) = \arg\max_{h_i}(b_i - (1-h_i)c_i x - P(h_i x)h_i x + \delta E_\epsilon V_i(\omega_i - \epsilon_i + (1-h_i)x, \omega_{-i} - \epsilon_{-i})).$$

Notice that the second derivative of the payoff with respect to the subcontracting strategy is negative

$$-P''(h_i x)h_i x^2 - 2P'(h_i x)x + x\delta E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x(1-h_i), \omega_{-i} - \epsilon_{-i}) < 0$$

if one of the following conditions is satisfied

(A1) The subcontracting supply schedule, $P(.)$, is convex in quantity supplied and the expected future value function, $E_\epsilon V_i(.,.)$, is concave in own state.

(A1') $E_{c_i',\epsilon} V_i(.,.)$ is not concave but $E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x(1-h_i), \omega_{-i} - \epsilon_{-i})$ is small relative to $-P''(h_i x)h_i x^2 - 2P'(h_i x)x$ and $P(.)$ is convex or vice versa if $P(.)$ is not convex but $-P''(h_i x)h_i x^2 - 2P'(h_i x)x$ is small relative to $x\delta E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x(1-h_i), \omega_{-i} - \epsilon_{-i})$ and $E_\epsilon V_i(.,.)$ is concave in own state.

It would be quite challenging to establish the concavity of the expected value function in this very general setting. We, therefore, verify this property in simulations.

Proposition 1 *If condition (A1) or (A1') holds, then the optimal subcontracting action h_i^*, exists,* *is unique, and is determined by the following equations:*

$$
\begin{aligned}
&h_i^* = 1 \;\; \textit{if}\; c_i - P'(x)x - P(x) - \delta E_{c_i',\epsilon} V_{i,1}'(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i}) > 0 \\
&h_i^* = 0 \;\; \textit{if}\; c_i - P(0) - \delta E_{c_i',\epsilon} V_{i,1}'(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i}) < 0 \\
&0 < h_i^* < 1 \;\; \textit{if}\; c_i - P'(h_i^* x) h_i^* x - P(h_i^* x) - \delta E_\epsilon V_{i,1}'(\omega_i - \epsilon_i + x(1 - h_i^*), \omega_{-i} - \epsilon_{-i}) = 0.
\end{aligned} \tag{6}
$$

As can be seen from the above, an optimal h_i^* does indeed depend on the state and on contractor i's current cost realization, i.e., $h_i^* = \mathbf{h}_i(c_i;\omega)$. Notice that the contractor subcontracts only at costs levels that are sufficiently high relative to $P(0) + \delta E_\epsilon V_{i,1}'(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i})$, which is the marginal cost of the first unit purchased in the subcontracting market net of the dynamic cost of completing the whole project. In general, when deciding on the level of subcontracting the bidder weights the marginal cost (both static and dynamic) of completing this unit in-house, $c_i - \delta E_\epsilon V_{i,1}'(\omega_i - \epsilon_i + x(1 - h_i^*), \omega_{-i} - \epsilon_{-i})$ against the cost of purchasing this unit in the subcontracting market, $P'(h_i^* x) h_i^* x - P(h_i^* x)$, which accounts for the fact that the price in the subcontracting market may potentially grow as he attempts to purchase more units.

Corollary 1 *If condition (A1) or (A1') holds, the optimal subcontracting policy is weakly increasing in the realization of static marginal cost.*

This property obtains by differentiating the first-order condition for $0 < h_i < 1$ with respect to c_i:

$$
\begin{gathered}
1 - P''(\mathbf{h}_i(c_i)x)\mathbf{h_i}'(c_i)\mathbf{h_i}(c_i)x^2 - 2P'(\mathbf{h_i}(c_i)x)\mathbf{h_i}'(c_i)x - \\
\delta x E_\epsilon V_{i,1}''(\omega_i - \epsilon_i + x(1 - \mathbf{h_i}(c_i)), \omega_{-i} - \epsilon_{-i}))(-\mathbf{h_i}'(c_i)) = 0
\end{gathered} \tag{7}
$$

and

$$
\mathbf{h_i}'(c_i) = (P''(\mathbf{h_i}(c_i)x)\mathbf{h_i}(c_i)x^2 + 2P'(\mathbf{h_i}(c_i)x)x - \delta x E_\epsilon V_{i,1}''(\omega_i - \epsilon_i + x(1 - \mathbf{h_i}(c_i)),\ \omega_{-i} - \epsilon_{-i}))^{-1},
$$

where $\mathbf{h_i}'$ denotes the derivative of the subcontracting function with respect to the current realization of per unit project costs.

4.2.1 Dependence on state variables

Let us define $c^L = \max\{\underline{c}_i,\ \min\{\bar{c}_i,\ P(0) + \delta E_\epsilon V_{i,1}'(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i})\}\}$, then c^L is the highest cost realization at which contractor i decides not to use subcontracting services. Similarly, define $c^U = \min\{\bar{c}_i,\ \max\{\underline{c}_i,\ P'(x)x + P(x) + \delta E_\epsilon V_{i,1}'(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i})\}\}$ to be the lowest cost level at which bidder i subcontracts everything.

Corollary 2 *If condition (A1) or (A1') holds, then the interval $[c^L, c^U]$ shifts to the left as ω_i increases. In general, the optimal subcontracting policy is weakly increasing in own state.*

If the expected value function is concave in own state then c^L and c^U decrease at the rates $\delta E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i})$ and $\delta E_\epsilon V''_{i,11}(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i})$, respectively, when ω_i increases holding ω_{-i} fixed. In addition, by differentiating the first order condition for $0 < \mathbf{h_i} < 1$ with respect to ω_i obtains

$$\frac{\partial \mathbf{h_i}(c_i;\omega)}{\partial \omega_i} = \frac{\delta E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x(1-\mathbf{h_i}), \omega_{-i} - \epsilon_{-i})}{\delta x E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x(1-\mathbf{h_i}), \omega_{-i} - \epsilon_{-i}) - P''(\mathbf{h_i}x)\mathbf{h_i}x^2 - 2P'(\mathbf{h_i}x)x} > 0$$

Similar results can be obtained with respect to dependency on ω_{-i}. That is, if in addition to (A1) or (A1'), the cross-partial derivative of the expected value function is positive, then as ω_{-i} increases while holding ω_i fixed, the interval $[c^L, c^U]$ shifts to the right and the optimal policy is weakly decreasing in the competitor's state. This follows by differentiating the first-order condition for $0 < \mathbf{h_i} < 1$ with respect to ω_{-i}:

$$\frac{\partial \mathbf{h_i}(c_i;\omega)}{\partial \omega_{-i}} = \frac{\delta E_\epsilon V''_{i,12}(\omega_i - \epsilon_i + x(1-\mathbf{h_i}), \omega_{-i} - \epsilon_{-i})}{\delta x E_\epsilon V''_{i,11}(\omega_i - \epsilon_i + x(1-\mathbf{h_i}), \omega_{-i} - \epsilon_{-i}) - P''(\mathbf{h_i}x)\mathbf{h_i}x^2 - 2P'(\mathbf{h_i}x)x}.$$

To summarize, several useful properties of subcontracting functions arise in the areas of the state space where the expected value function is concave in own state and has positive cross-partial derivatives. However, theoretically there are no guarantees that these properties of the expected value function hold everywhere or even at some subset of the state space. We address these issues in simulations.

4.2.2 Flat supply curve

It is instructive to consider the case of a flat subcontracting supply curve, $P(z) = p_0$. In a static game, contractors choose not to subcontract if $p_0 > c_i$ and to subcontract everything if $p_0 < c_i$. Subcontracting is only used to "improve" high cost realizations. In contrast, in a dynamic game, non-zero amounts of subcontracting are optimal as long as

$$c_i > \delta E_\epsilon V'_{i,1}(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i}) + p_0.$$

Notice that $p_0 > \delta E_\epsilon V'_{i,1}(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i}) + p_0$ if $E_\epsilon V'_{i,1} < 0$, a condition we would expect to hold in equilibrium, is satisfied. Contractors outsource more since they additionally use subcontracting to alleviate future capacity constraints, sometimes at the expense of short-term efficiency. It is still optimal to subcontract to the limit if $p_0 < c_i$. However, full subcontracting remains optimal if $p_0 > c_i > \delta E_\epsilon V'_{i,1}(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i}) + p_0$. Also, in contrast to the static game, intermediate levels of subcontracting occur on a non-degenerate interval

$$[\delta x E_\epsilon V'_{i,1}(\omega_i - \epsilon_i + x, \omega_{-i} - \epsilon_{-i}) + p_0,\ \delta x E_\epsilon V'_{i,1}(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i}) + p_0].$$

The width of this interval depends on the curvature of the value function, the discount factor and the size of the contract.

4.3 Bidding Strategies

In this section we show that after optimal subcontracting strategies are determined and given a vector of value functions, the contractors' optimization problem can be re-arranged to resemble a static asymmetric procurement auction with an "all-pay" feature. An "all-pay" feature arises since in the environment with subcontracting bidder i's continuation value of losing an auction depends on his bid. This is so because the competitor's subcontracting levels (and therefore the impact of winning on the competitor's backlog) depend on his cost realization, whereas contractor i's bid determines the set of the competitor's costs to which he may lose. This is in contrast to the model without subcontracting, where both the continuation value of winning and the continuation value of losing do not depend on the contractor's costs.

For the purpose of this section we assume that the subcontracting supply schedule is such that contractors choose to subcontract at all cost realizations and it is never optimal to subcontract the whole project. We believe that such an assumption is without loss of generality for the purpose of our analysis: indeed the subcontracting supply schedule could be always chosen in such a way that a very small portion of the project is subcontracted at the low cost realization, whereas the subcontracted share approaches one (without being equal to one) at the high cost realizations. We make this assumption in order to maintain the smoothness of the bidding problem, which in turn facilitates the convergence of the simulation algorithm. We further presume that the ex-ante value function is decreasing in own state and is increasing in the state of the competitor, i.e., $EV'_{i,1}(\omega_i,\, \omega_{-i}) < 0$, and $EV'_{i,2}(\omega_i,\, \omega_{-i}) > 0$, and either condition (A1) or condition (A1') holds (and therefore the optimal subcontracting strategy is increasing in the bidder's own cost, $\mathbf{h_{i,1}}'(c_i; \omega) > 0$). We expect the first two properties to arise due to the presence of capacity constraints and the limited availability of subcontracting. Indeed, as in the game without subcontracting, the high level of own backlog increases the risk of the high current cost realization. In addition, the relative sizes of projects and the contractor's capacity ensure that high backlogs persist into the future. Limited subcontracting means that these concerns could not be completely eliminated. Similarly, the high levels of the competitor's backlog implies a higher chance of the competitor having high costs both in the current and in the next few future periods. While these properties are intuitively justified, it would be difficult to establish them formally. We verify these properties later numerically.

Denote $E_\epsilon[V_i(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + (1 - \mathbf{h_i}(\bar{c}_{-i})))$ by $\underline{V}_i(\omega)$. This is the lowest possible payoff from losing given ω since $h_{-i}(.)$ is at its highest possible level. We re-arrange the right-hand side of the Bellman equation, which represents the contractor's objective function conditional on ω, in the following way[3]

$$\max_{b_i} W_i(b_i,\omega;\mathbf{g_{-i}})\Big[b_i - ((1-h_i)c_i x + P(h_i x)h_i x - \delta(E_\epsilon V_i(\omega_i - \epsilon_i + (1-h_i)x, \omega_{-i} - \epsilon_{-i}) - \underline{V}_i(\omega')))\Big]$$
$$+\delta \int_{\underline{c}_{-i}(\omega)}^{\mathbf{b}_{-i}^{-1}(b_i,\omega)} (E_\epsilon V_i(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + (1-\mathbf{h_{-i}})x) - \underline{V}_i(\omega)) f_{-i}(c_{-i};\omega) dc_{-i} - \delta \underline{V}_i(\omega). \quad (8)$$

For the purpose of further exposition we introduce a new object, $\phi_i(c_i,\omega)$, which we refer to as *effective costs* as opposed to *original* (or *current*) costs c_i

$$\phi_i(c_i;\omega) = (1-\mathbf{h_i})c_i x + P(\mathbf{h_i}x)\mathbf{h_i}x - \delta(E_\epsilon V_i(\omega_i - \epsilon_i + (1-\mathbf{h_i})x, \omega_{-i} - \epsilon_{-i}) - \underline{V}_i(\omega')). \quad (9)$$

The first part of ϕ is a *static effective cost*, which captures the immediate impact of subcontracting on markups. The second part represents a dynamic opportunity cost of winning the auction against the least efficient opponent. Since we presumed that V_i is decreasing in the own backlog, and increasing in the backlog of the opponent, we know that

$$-\delta(E_\epsilon V_i(\omega_i - \epsilon_i + (1-\mathbf{h_i})x, \omega_{-i} - \epsilon_{-i}) - \underline{V}_i(\omega')) > 0,$$

i.e., the dynamic cost component is always positive and therefore raises the cost of the project. Further, notice that

$$\begin{aligned} \phi'_{i,1}(c_i;\omega) &= (1-\mathbf{h_i})x - c_i \mathbf{h_i}' x + P'(\mathbf{h_i}x)\mathbf{h_i}' x \mathbf{h_i} x + P(\mathbf{h_i}x)\mathbf{h_i}' x - \delta E_\epsilon V'_{i,1}(-\mathbf{h_i}' x) \quad (10) \\ &= (1-\mathbf{h_i})x + \{P'(\mathbf{h_i}x)\mathbf{h_i}x + P(\mathbf{h_i}x) + \delta E_\epsilon V'_{i,1} - c_i\}\mathbf{h_i}' x > 0, \end{aligned}$$

where the non-negativity of the term in the brackets follows from the necessary first-order conditions for the optimality of the subcontracting function and from our assumption that $\mathbf{h_i}' > 0$. We, therefore, can re-write the contractor's optimization problem in terms of ϕ_i. More specifically, through the change of variables we will view the bidding function as a function of effective costs, $\phi_i(c)$, rather than c, i.e.,

$$\mathbf{b}_i(\cdot;\omega): \ [\underline{\phi}_i(\omega), \overline{\phi}_i(\omega)] \to [\underline{b}(\omega), \overline{b}(\omega)].$$

Similarly, we define the inverse bid function, which maps bids into effective costs rather than into real costs

$$\xi_i(\cdot;\omega): \ [\underline{b}(\omega), \overline{b}(\omega)] \to [\underline{\phi}_i(\omega), \overline{\phi}_i(\omega)].$$

Then, the objective function of bidder i becomes

$$\max_{b_i} \ (1 - F_{-i,\phi}(\mathbf{b}_{-i}^{-1}(b_i,\omega)))(b_i - \phi_i) + \quad (11)$$
$$\delta \int_{\underline{\phi}_{-i}(\omega)}^{\mathbf{b}_{-i}^{-1}(b_i,\omega)} (E_\epsilon V_i(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + (1-\mathbf{h_{-i}})x) - \underline{V}_i(\omega)) f_{-i,\phi}(\phi_{-i};\omega) d\phi_{-i} - \delta \underline{V}_i(\omega).$$

Notice that the last term, $\delta\underline{V}_i(\omega)$, does not depend on the bid and therefore does not affect the

[3] The normalization is introduced for the purpose of deriving the boundary conditions of the optimal bid problem. See the derivation in the Appendix.

optimal bidding. On the other hand, the term before the last is strictly increasing in the bid and represents an analogue of the "all-pay" component of the payoff. In general, the equilibrium of the bidding game is characterized by a system of first-order differential equations in ξ_i and ξ_{-i}, which represent the necessary first-order conditions associated with the bidding problem for $i = 1, 2$,

$$(1 - F_{\phi,-i}(\xi_{-i}(b,\omega))) - \big(b - \xi_i(b,\omega) - \tag{12}$$

$$\delta[E_\epsilon V(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + x(1 - \mathbf{h_{-i}}(\phi_{-i}^{-1}(\xi_{-i}(b,\omega)),\omega)) - \underline{V}_i]\big) f_{\phi,-i}(\xi_{-i}(b,\omega))\xi'_{-i}(b,\omega) = 0, \tag{13}$$

and boundary conditions. Notice that, in our setting, the supports of effective cost distributions are naturally different for contractors with different backlog levels. We adjust the standard argument (see Kaplan and Zamir (2012)) accounting for the all-pay component in order to obtain the boundary condition for our optimization program. More specifically, without loss of generality, assume that $\bar{\phi}_1 \leq \bar{\phi}_2$.[4] Then,

$$\begin{aligned} \xi_1(\overline{b},\omega) &= \overline{\phi}_1 \\ \xi_2(\overline{b},\omega) &= \overline{b} \\ \xi_1(\underline{b},\omega) &= \underline{\phi}_1 \\ \xi_2(\underline{b},\omega) &= \underline{\phi}_2, \end{aligned} \tag{14}$$

where $\overline{b}$ is the highest equilibrium bid. The proposition below summarizes conditions that determine the value of $\overline{b}$.

Proposition 2 *Let* $\tilde{b} = \min\{\tilde{b}_0, \bar{\phi}_2\}$ *where* $\tilde{b}_0$ *is such that* $\big[-(\tilde{b}_0 - \bar{\phi}_1) + \delta E_\epsilon V_1(\omega'_1, \omega'_2 + x(1 - \mathbf{h_2}(\phi_2^{-1}(\tilde{b}_0);\omega))\big] f_{\phi,2}(\tilde{b}_0) = 0$, *and* b_0 *is indirectly defined by* $(1 - F_{\phi,2}(b_0)) - \big[(b_0 - \overline{\phi}_1) - [E_\epsilon V_1(\omega_1 - \epsilon_1, \omega_2 - \epsilon_2 + (1 - \mathbf{h_2})x) - \underline{V}_2]\big] f_{\phi,2}(b_0) = 0$ *then either (a)* $\overline{b} = \tilde{b}$ *if* $\tilde{b} = \bar{\phi}_2$ *or (b)* $\overline{b} = min\{b_0, \bar{\phi}_2\} \geq \tilde{b}$.

The proof is in the Appendix.

The problem in (12) with boundary conditions defined in (14) and Proposition 2 satisfies all of the usual conditions sufficient to guarantee the existence and uniqueness of the pair of equilibrium bidding functions (see Reny and Zamir (2004) and Athey (2001)).

4.4 Dynamic Option Effect

The optimal pricing behavior in the environment with capacity constraints is driven in part by option value considerations since winning or losing the auction has implications beyond collecting expected

[4]Notice that the ranking of $\bar{\phi}_1$ and $\bar{\phi}_2$ does not necessarily reflect the ranking of ω_1 and ω_2 due to the dynamic cost component.

within-period profit. Notice that in the environment without subcontracting, the contractor's optimization problem can be represented as

$$\max_{b_i}(1-F_{-i}(\mathbf{b}_{-i}^{-1}(b_i)))\Big(b_i-c_ix+\delta\Big[E_\epsilon V_i(\omega_i-\epsilon_i+x,\omega_{-i}-\epsilon_{-i})-E_\epsilon V_i(\omega_i-\epsilon_i,\omega_{-i}-\epsilon_{-i}+x)\Big]\Big)$$
$$+\delta E_\epsilon V_i(\omega_i-\epsilon_i,\omega_{-i}-\epsilon_{-i}+x). \quad (15)$$

Therefore, the option value impact on optimal pricing could be conveniently summarized by

$$\delta\Big[E_\epsilon V_i(\omega_i-\epsilon_i+x,\omega_{-i}-\epsilon_{-i})-E_\epsilon V_i(\omega_i-\epsilon_i,\omega_{-i}-\epsilon_{-i}+x)\Big],$$

the difference between the continuation value conditional on losing and the continuation value conditional on winning. Notice that this term enters the contractor's optimal bidding problem in such a way that it can be treated as a constant shift in the support of the cost distribution. That is why the option value effect basically translates into a uniform upward shift of static bidding strategies.

The situation is more complex in the environment where subcontracting is available. The optimal pricing behavior in (12) is affected by dynamic option value considerations through the terms $E_\epsilon V(\omega_i-\epsilon_i,\omega_{-i}-\epsilon_{-i}+x(1-\mathbf{h_{-i}}(\phi_{-i}^{-1}(\xi_{-i}(b))))$ and $E_\epsilon V(\omega_i-\epsilon_i+x(1-\mathbf{h_i}(\phi_i^{-1}(\xi_i(b)))),\omega_{-i}-\epsilon_{-i})$. The first term enters the necessary first-order conditions directly and represents bidder i's marginal continuation value conditional on losing, whereas the second term enters optimality conditions through the effective costs term, ϕ_i, and represents bidder i's continuation value conditional on winning. These terms enter the optimality conditions in several places and cannot be conveniently localized in contrast to the model without subcontracting. In addition, the dynamic considerations in the model with subcontracting depend on the current cost realization through the subcontracting function and therefore affect the shape (slope and curvature) as well as the level of the bidding function. In an auction environment with asymmetric bidders this impact could not be easily derived since no closed-form solution exists.

We can obtain an insight into the effect of option value considerations on pricing by analyzing bidding behavior in symmetric states, i.e., states such that $\omega_i=\omega_{-i}$, where the closed-form expression for bidding strategies exists. We suppress the reference to ω wherever possible for brevity. Using the first order conditions from (12) and imposing symmetry obtains:

$$\tilde{\beta}(c)=\frac{1}{(1-F_c(c))}\int_c^{\bar{c}}\big((1-\mathbf{h})c'x+P(\mathbf{h}x)\mathbf{h}x\big)f_c(c')dc'-\frac{\delta}{(1-F_c(c))}\times$$
$$\times\int_c^{\bar{c}}[E_\epsilon V_i(\omega_0-\epsilon_i+(1-\mathbf{h}(c'))x,\omega_0-\epsilon_{-i})-E_\epsilon V_i(\omega_0-\epsilon_i,\omega_0-\epsilon_{-i}+x(1-\mathbf{h}(c'))]\big)f_c(c')dc'$$

Notice that the first term in the sum on the right represents bidding schedule in a static procurement auction where the costs are given just by the static part of effective costs. The second term accounts for the option values.[5] Since the ex-ante value function is decreasing in the bidder's own state and

[5]Notice that $EV_i(\omega_0+\mathbf{h},\omega_0)\neq EV_i(\omega_0,\omega_0+\mathbf{h})$ since the value function is not exchangeable in own and competitor's

is increasing in the competitors state, the second summand is always positive; hence, the bidding function from a dynamic model lies above the bidding function based on the static part of effective costs. In addition, the derivative of

$$\int_{c}^{\bar{c}} [E_\epsilon V_i(\omega_0 - \epsilon_i + (1 - \mathbf{h}(c'))x, \omega_0 - \epsilon_{-i}) - E_\epsilon V_i(\omega_0 - \epsilon_i, \omega_0 - \epsilon_{-i} + x(1 - \mathbf{h}(c')))]\big) f_c(c')dc'$$

is always negative, so the contribution of this term to the bidding function diminishes with c. In general, adjustment by $\frac{\delta}{(1-F_c(c))}$ may break the monotonicity of this effect on bids. However, in most cases the effect indeed declines with c as we demonstrate in the "Computed MPE" section. This implies that the dynamic option value under subcontracting flattens bidding strategies in symmetric states.

Motivated by the insight from a symmetric states analysis, we measure the effect of the option value consideration on prices in asymmetric states by computing a difference between the bidding function from the full model and the bidding function based on the static part of the effective cost. We discuss the numerical results related to the dynamic option effects in the "Computed MPE" section.

5 Numerical Implementation

In this section we provide details of the numerical algorithm that computes the equilibrium of the dynamic game with subcontracting.

5.1 Computational Algorithm

Our algorithm is an extension of the method used in Chen, Doraszelski, and Harrington Jr (2009) to dynamic auctions. It involves computing a limit on Markov Perfect Equilibria in the finite horizon games, which alleviates multiplicity of equilibria by providing a consistent and robust equilibrium selection rule. The algorithm is composed of two parts: (i) an inner-loop computing optimal subcontracting and bidding strategies, as well as the value function of the game with n periods, and (ii) an outer-loop computing an equilibrium of an infinite horizon game.

In the remainder of this section the value function of the n-stage game is denoted as $V^{(n)}$. The value function of the game with n+1 stages can be obtained from $V^{(n)}$ using the following Bellman

states.

equation:

$$
\begin{aligned}
V_i^{(n+1)}(\omega) = \int_{c_i} \Big[& \max_{b_i,h_i} W_i(b_i,\omega;\mathbf{g_{-i}}) \times \\
& \times (b_i - (1-h_i)c_i x - P(h_i x)h_i x + \delta E_\epsilon V_i^{(n)}(\omega_i - \epsilon_i + (1-h_i)x, \omega_{-i} - \epsilon_{-i})) \\
& + \delta \int_{\underline{c}_{-i}(\omega)}^{\mathbf{b}_{-i}^{-1}(b_i,\omega)} E_\epsilon V_i^{(n)}(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + (1-\mathbf{h_{-i}}(c_{-i}))x) dF_{-i}(c_{-i}) \Big] dF_i(c_i).
\end{aligned} \tag{16}
$$

We follow the *parametric value function iteration* procedure suggested in Judd (1998) to parametrically approximate $V^{(n)}(\cdot)$, $\hat{V}^{(n)}(\cdot|\theta^n) \approx V^{(n)}(\cdot)$. More specifically, we define a 2-dimensional grid on the state space, $\mathbf{\Omega^D} = \{(\omega_1^d, \omega_2^d) : d = 1, \ldots, D\}$. We use 7 grid points on each dimension, which, taking into account the symmetry of the environment, results in 28 grid points. We use the parametric approximation of the value function from the n-stage game, $\hat{V}^{(n)}$, to obtain data pairs (ω^d, v^d) for $\hat{V}^{(n+1)}$ by evaluating the right-hand side of equation (16) on the grid. The parametric approximation of $\hat{V}^{(n+1)}$ is then obtained through a 4th-order Chebyshev regression. We stop when $\|\hat{V}^{(n)} - \hat{V}^{(n+1)}\|$ is small.

The most computationally intensive part of the algorithm involves solving the right hand-side of the Bellman equation to produce the interpolation data (ω^d, v^d) – an inner-loop of our algorithm. The procedure involves multiple steps. First, we note that we can precompute the expected future value function

$$
\hat{\hat{V}}_i^{(n)}(\omega) = E_\epsilon \hat{V}_i^{(n)}(\bar{\omega}_i - \epsilon_i, \bar{\omega}_{-i} - \epsilon_{-i}),
$$

where ω is an interim backlog after adding the current auction results but before subtracting the utilization. An interim backlog is given by $\bar{\omega}_i = \omega_i - (1-h_i)x$ in case player i won the auction and is given by $\bar{\omega}_i = \omega_i$ in case player i loses the auction. This expectation is numerically computed on the grid $\mathbf{\Omega^D}$ using an adaptive Simpson quadrature, and interpolated using a 4^{th}-order Chebyshev regression. Without precomputing this expectation the algorithm would be numerically infeasible. The value of the Bellman equation v^d at the grid point ω^d can now be obtained by using

$$
\begin{aligned}
v_i^d = \int_{c_i} \Bigg\{ & \max_{b_i,h_i} W_i(b_i,\omega^d;\mathbf{g_{-i}}) \Big[b_i - (1-h_i)c_i x - P(h_i x)h_i x + \delta \hat{\hat{V}}_i(\omega_i^d + (1-h_i)x, \omega_{-i}^d)^{(n)} \Big] \\
& + \delta \int_{\underline{c}_{-i}(\omega^d)}^{\mathbf{b}_{-i}^{-1}(b_i,\omega^d)} \hat{\hat{V}}_i^{(n)}(\omega_i^d, \omega_{-i}^d + (1-\mathbf{h}_{-i}(c_{-i}))x) dF_{-i}(c_{-i},\omega^d) \Bigg\} dF_i(c_i).
\end{aligned} \tag{17}
$$

Next, we compute the optimal subcontracting functions and then we use the subcontracting functions to compute the optimal bidding functions. This is the most challenging part of the process. For every state grid point $\omega^d \in \mathbf{\Omega^D}$ we define a grid on the support of the distribution of original costs, $\mathbf{C}_i(\omega) = \big(\underline{c}(\omega_i), \ldots, c_i^r, \ldots, \overline{c}(\omega_i)\big)$. Having solved for the optimal subcontracting level at each cost grid point, we then obtain the subcontracting strategy, and the effective-cost functions as well as the distribution functions of the effective costs through cubic spline interpolation.

Having computed all of the components, namely $V\hat{(}\cdot), h(\cdot), \phi(\cdot)$ (as well as $F_\phi(\cdot)$ and $f_\phi(\cdot)$), we proceed to solve for inverse bid strategies $(\xi_1(\cdot, \omega^d), \xi_2(\cdot, \omega^d))$ using the system of differential equations (12) with boundary conditions given by Proposition 2. We use a shape-preserving projection method with the Chebyshev basis to guarantee the monotonicity of the inverse bid functions. Our basis consists of 4^{th}-order complete Chebyshev polynomials and is defined on a Chebyshev grid.[6] We reduce our task to the following constrained optimization problem:

$$\begin{aligned}
\min_{\underline{b}, \theta_1^\xi, \theta_2^\xi} \quad & \sum_{b^k \in \mathbf{B}} \left[R_1(b^k|\theta_1^\xi)\right]^2 + \left[R_2(b^k|\theta_2^\xi)\right]^2 \\
\text{s.t.} \quad & \forall k; \hat{\xi}_1'(b^k|\theta_1^\xi) > 0, \quad \hat{\xi}_2'(b^k|\theta_2^\xi) > 0 \\
& \forall k; \hat{\xi}_1(b^k|\theta_1^\xi) < b^k, \quad \hat{\xi}_2(b^k|\theta_2^\xi) < b^k \\
& \underline{b} < \bar{b} + \tau \\
& \underline{\phi}_1 = \hat{\xi}_1(\underline{b}|\theta_1^\xi), \quad \underline{\phi}_2 = \hat{\xi}_2(\underline{b}|\theta_2^\xi) \\
& \bar{\phi}_2 = \hat{\xi}_1(\bar{b}|\theta_1^\xi), \quad \bar{\phi}_2 = \hat{\xi}_2(\bar{b}|\theta_2^\xi)
\end{aligned} \tag{18}$$

where R_i is the residual from evaluating first-order conditions (12) using the approximation of the inverse bid function $\hat{\xi}$ instead of the true inverse bid function ξ. Note that one inequality contains a bandwidth parameter τ, which controls the flatness of the bid strategies. It is set to a very small non-binding number and is used to improve the stability of the numerical iterations.

To summarize we provide a flow description of the algorithm:

(I) Fix the terminal value $V_i^{(0)} \equiv 0$. Fix a D-point grid of the state space $\mathbf{\Omega^D} = \{(\omega_1^d, \omega_2^d) : d = 1, \dots, D\}$.

(1) For every point ω^d and given n

  (a) For both players, given the value functions in the n-stage game $V_i^{(n)}$, solve for an optimal subcontracting strategy $h_i^{(n)}(\cdot, \omega^d)$, effective-cost functions $\phi_i^{(n)}(\cdot, \omega^d)$ and determine the CDF and PDF of the pseudo cost $\phi_i^{(n)}$.

  (b) Solve the Boundary Value Problem for inverse bidding strategies $\xi_i^{(n)}(\cdot, \omega^d)$

  (c) Perform an iteration on the Bellman equation (16) to compute $V_i^{(n+1)}(\omega^d)$

(2) Use a projection method to fit a parametric approximation $\hat{V}_i^{(n+1)}(\omega|\theta^{(n+1)})$ outside of the grid $\mathbf{\Omega^D}$

(3) For each point on the grid $\mathbf{\Omega^D}$, perform an integration of ϵ to obtain $E_\epsilon \hat{V}_i^{(n+1)}(\omega^d - \epsilon)$, where ω^d is an interim backlog

[6] A Chebyshev grid is composed of the roots of a Chebyshev polynomial of the first kind. Using the roots of the Chebyshev polynomial instead of an equidistant grid makes the numerical procedure more stable for ill-conditioned problems. For more discussion see Judd (1998).

(4) Use a projection method to fit a parametric approximation $\hat{\hat{V}}_i^{(n+1)}(\omega|\theta_2^{(n+1)})$ outside of the grid Ω^D

(S) Stop if $\|\hat{V}^{(n)} - \hat{V}^{(n+1)}\| < \epsilon$ or goto (1).

5.2 Simulation Details

The computational algorithm described above provides an approximation of the equilibrium value function $\hat{V}^{(n+1)}$ as well as the equilibrium bidding and subcontracting strategies on the cost and backlog grid, $\{(c_i^r, \omega^d) : c_i^r \in \mathbf{C}_i(\omega^d), \omega^d \in \mathbf{\Omega^D}\}$. To simulate the equilibrium path we need to know the strategies outside of the cost and backlog grid. One option is to re-solve for optimal strategies as needed for a given ω on the path (see Saini (2013)). However, this option is computationally infeasible because the strategies do not have closed-form solutions. We solve this issue by interpolating $b_i(c_i, \omega)$, and $h_i(c_i, \omega)$ outside the cost and backlog grid.

Note that our 3-dimensional grid is non-rectangular since the support of $C_i(\omega^d)$ depends on ω^d. That is why we perform the interpolation in several steps. First, a cubic spline interpolation for the upper bound $\bar{b}$ is constructed. It is later used to determine if the contractor is priced out of the market. Next, ω^d-specific linear transformation is used to project a uniform grid $C_i(\omega^d)$ onto a $[0, 1]$ interval. This procedure converts our grid into a rectangle one and enables fitting the 3-dimensional cubic splines to obtain $\hat{h}_i$ and $\hat{b}_i$.

The strategies can now be evaluated at an arbitrary point (c_i, ω) in the following way. First, c_i is projected onto a $[0, 1]$ interval using a correct ω-dependent linear transformation, then $\hat{h}_i$ or $\hat{b}_i$ is evaluated at a corresponding point. Once the bids are known, the winner is determined and backlogs are adjusted. We record that player i lost the auction if c_i is greater than the cut-off point $\bar{b}(\omega)$.

We use the interpolated strategies to simulate the stationary distribution and long-run industry path. The stationary distribution is used to obtain average industry statistics, while the discounted procurement cost is computed along the equilibrium path. In order to obtain a stationary distribution we perform 10^4 warm-up draws and average subsequent 10^5 draws. To obtain the long-run industry path we simulate 10^3 draws of 80 consecutive periods and assume that the 81^{st} state persists forever. Note that the contribution of the 81^{st} period is equal to $\delta^{81} = 0.002$.

5.3 Parameterization

In this section we describe the parameterization of our dynamic game.

We constrain the backlogs to be within a finite interval, that is, $\omega \in [0, M]$. M is chosen to be high enough so that it is reached along the equilibrium path with negligible probability. In our benchmark specification we use truncated normal[7] cost distributions $F_i(\cdot, \omega_i)$ with the mean given by $\mu_i = \theta_1 + \theta_2\omega_i + \theta_3\omega_i$. The truncations are chosen to be $\underline{c}_i(\omega_i) = \underline{C} + \theta_2\omega_i$ and $\bar{c}_i(\omega_i) = \bar{C} + \theta_2\omega_i$. Parameter θ_2 represents the uniform shift in the cost distribution and θ_3 represents the shift in the mass within the support caused by the backlog ω_i. The distribution of productivity $f_\epsilon(\epsilon_i)$ is assumed to be uniform with the support $[0, K]$. Parameter K summarizes the contractor's capacity.

Note that in our environment robustly computing equilibria with (partially) flat subcontracting schedules is very labor intensive. This is because such strategies involve "corner" solutions for many backlog configurations. That is why we restrict our attention to the subcontracting supply schedules that do not result in full subcontracting. We believe that doing so does not result in a loss of generality since the subcontracting supply curve can always be chosen in such a way that the subcontracting policy approaches one within an arbitrary small margin. We consider the subcontracting supply schedules of the form

$$p(q) = \alpha_1 + \alpha_2\frac{q}{1-q} + \alpha_3 q.$$

In subsequent sections we compare equilibria across five subcontracting supply schedules described by this functional form. These schedules range from quite steep to relatively flat curves. We depict them in Figure 1 and summarize their parameters in Table 1. The baseline subcontracting supply curve is chosen in such a manner that average subcontracting levels in the associated equilibrium coincide with those observed in the data.

6 Computed MPBE

In this section we describe and compare computed Markov Perfect Equilibria for the dynamic games with and without subcontracting that are based on the baseline values of the parameters. We begin by discussing the simulated ex-ante value function. After this we discuss policy functions and the steady-state properties of the environment.

6.1 Ex-ante Value Function

Figure 3 depicts the computed ex-ante value functions for the games with and without subcontracting. It shows both the three-dimensional graphs of the ex-ante value function over a grid of state values and the cross-sections of the value functions for different values of the contractor's own state. The value functions of both games are decreasing in the contractor's own state and are increasing in the competitor's state. We relied on these properties in the theory section when we described an optimal bidding and subcontracting strategies and now we have an opportunity to verify them. We

[7] To investigate the sensitivity of our results to this assumption, we recomputed the equilibrium using uniform cost distributions. The results stay qualitatively the same.

observe the monotonicity in the own and the competitor's states for the majority of parameter values considered in this study. It appears to break down for very high levels of the discount rate.[8]

These properties are more sharply pronounced in the game without subcontracting. In contrast, the value function of the game with subcontracting is much flatter both in the own and the competitor's state as we would expect, since subcontracting serves to mitigate the effects of backlog accumulation and, thus, diminishes differences across states. The results of our analysis show that the slope of the value function is positively related to the slope of the subcontracting supply curve. Thus, the variation in the levels of the own or the competitor's backlog becomes less important, as greater subcontracting availability helps to insure more fully against the risk of a high current cost and backlog persistence.

The value function of the game with subcontracting is concave at the states with high levels of competitor's backlog and convex at the states with low levels of competitor's backlog, a property it shares with the value function of the game without subcontracting. However, at the baseline subcontracting supply function, the change in curvature is very small and the value function appears almost linear in contrast to the model without subcontracting, which exhibits strong concavity for high values of the competitor's state. Notice that similar to the model without subcontracting, the effect of an increase in the own backlog on the contractor's ex-ante value function is stronger than the effect of an increase in competitor's backlog. This is because the own backlog impacts both the distribution of the project's costs and the probability of winning, whereas the competitor's backlog impacts the probability of winning only.

The value function of the game with subcontracting is substantially lower than the value function of the game without subcontracting at all states. In broad terms this effect arises because the availability of subcontracting results in a higher degree of contractors' symmetrization both conditional on the state and in the increased probability of visiting more symmetric states. Such symmetrization leads to more intense competition, which in turn lowers firms' mark-ups. In addition, the subcontracting reduces the variation in private costs (or the importance of the private information), which additionally lowers informational rents that contractors are able to collect in this environment. We discuss these issues when we describe the properties of the optimal bidding and subcontracting policies. The findings of this section are summarized in the statement below.

Result 1. *The ex-ante value function of the dynamic game with subcontracting is lower and flatter than the ex-ante value function of the dynamic game without subcontracting. In addition, the change in the curvature of the value function is less pronounced in the case of the game with subcontracting.*

[8] When the discount rate is close to one and contractors' states are at zero, the value of an individual contractor is quite high since his cost is low and the probability of winning in a given period is high. However, as the opponent accumulates some backlog the contractor's own probability of winning declines since he prefers to wait for his opponent to accumulate a much higher level of backlog, which in turn would allow the contractor to win the next project at a very high mark-up. As a result the value function assumes u-shaped form.

6.2 Subcontracting Policy Function

As we noted in the previous section the ex-ante value function of our game is not concave for some states. Nevertheless, it either concave or sufficiently flat (see Conditions A1 and A1') for the subcontracting strategy to exist and to be monotone in own costs for the majority of parameter values we have considered.

Figure 5 shows subcontracting policy as a function of the contractor's private cost, c, while Table 3 reports expected subcontracting levels across different states. The subcontracted portion of the project is close to zero for very low values of contractor's private costs and monotonically increases over the range of costs while becoming relatively flat at the high cost realizations. The panels demonstrate how the subcontracting function changes in own as well as the competitor's state. Own-state effect is mostly confined to extending the subcontracting policy to accommodate shifting support of the cost distribution. One should not underestimate this effect. As Table 3 shows, under the baseline subcontracting supply function the expected subcontracting levels increase from 0.43 to 0.61 to 0.73 as the own state moves from 0 to 1 and to 2 and the competitor's state is fixed at 0. The level of subcontracting conditional on cost realization increases very slightly as the contractor's own backlog increases. In fact, the variations in subcontracting levels across a range of own states holding the realization of private costs and the competitor's state fixed or across competitor's states holding the realization of private costs and own state fixed are small relative to the variation in subcontracting levels across cost realizations and for a fixed vector of states.

We explore the sensitivity of the subcontracting policy to the availability of subcontracting opportunities as summarized by the slope of the subcontracting supply schedule. As the slope of the subcontracting supply schedule becomes flatter and thus subcontracting becomes more available, contractors choose to subcontract weakly more at all cost levels and across all states. In particular, the slope of the subcontracting policy increases as the subcontracting supply schedule becomes flatter.

Result 2 *The subcontracting strategy is increasing in own costs. Variation in subcontracting levels across states for a given value of private costs is small relative to the variation in subcontracting levels across private cost values for a given state. Contractors choose to subcontract weakly more at all cost levels and across all states as subcontracting becomes more available. The slope of the subcontracting policy increases as the subcontracting supply schedule becomes flatter.*

6.3 The Effect of Subcontracting on Project Costs

The availability of subcontracting directly impacts the distributions of the project's costs. First, subcontracting allows bidders to lower their costs for a range of high cost realizations. The costs are reduced when a part of the project is completed by subcontractors who on average have lower costs. This is demonstrated in Figures 6 and 7, which plot the static part of effective costs against the original costs under the various states and the subcontracting supply schedules. The graphs show that the static part of effective costs is monotone in original cost and is flatter than the 45-degree line, especially at high cost values. This translates into the static effective cost distribution with a lower upper bound of the support, high realizations "bunched" together, and characterized by mean and variance that are lower than those of the original cost distribution. These properties of the model have two important implications. First, subcontracting reduces the importance of private information (variation in private costs), which translates into lower informational rents or mark-ups that bidders charge in equilibrium. In addition, the effect of a backlog on the distribution of project costs is mitigated by eliminating the high cost realizations that may occur due to the high levels of capacity utilization. Thus, the capacity constraints could only be measured correctly if equilibrium subcontracting strategies are taken into account when recovering the distribution of project costs.

The full effective cost also includes a dynamic component that is equal to the expected continuation value of winning net of the expected continuation value of losing to the highest cost competitor. This component depends on the original cost realization, since the expected continuation value of winning depends on the amount the bidder decides to subcontract. Therefore, inclusion of this component means that the overall variance of the effective cost distribution might be larger than the variance of the static part of the effective costs. We find that this potential increase in variance is largely canceled out by a covariance term which is negative. However, the impact of dynamic subcontracting considerations on bidding extends beyond this variance effect. We discuss this in more detail in the next section.

The panels of Figures 6 and 7 also show that the "bunching" effect becomes stronger as own state increases. The subcontracting works to symmetrize the effective cost distributions in asymmetric states. This forces contractors to bid more aggressively and, thus, results in lower mark-ups of all bidders. This symmetrizing effect becomes stronger as the slope of the subcontracting supply schedule decreases.

Table 4 reports the means and variances of the original and effective cost distributions across different state levels and for different slopes of the subcontracting supply schedules. The second and third columns report the mean and variance of the original cost distribution. In our main specification the mean of the original cost distribution increases in the contractor's own state, while the variance remains unchanged. However, the effects we document remain unchanged under alternative specifications that allow for the variance of the cost distribution to be affected by the backlog accumulation. The sixth and the seventh columns show the mean and variance of the static part of the effective costs distribution. Under the baseline subcontracting supply schedule both the mean and the variance are substantially reduced. The reduction increases in the contractor's own state: it is equal to 13%, 20%, and 27% in the case of means, and 39%, 54% and 69% in the case of variances for backlog levels 0, 1, and 2, respectively. These reductions are even stronger in the case of a flatter subcontracting supply schedule: 17%, 23% and 30% for means, and 46%, 62% and 77% for variances. On the other hand, the reductions are smaller in the case of a steeper subcontracting schedule: 8%, 12% and 17% for means, and 30%, 38% and 50% for variances. These results show that subcontracting substantially reduces means and variances of the cost distributions over a wide range of the subcontracting supply schedules. They underscore that one of the main consequences of subcontracting is the reduction in the variability of private costs. This result is summarized below.

Result 3 *In the presence of subcontracting, contractors' bids are based on the cost distributions with means, variances and upper bounds of the supports that are lower than the means, variances and upper bounds of the supports of the original cost distributions. Thus, subcontracting results in the diminished importance of private information and, thus, a reduction in informational rents. The magnitudes of these effects increase in the contractor's own state and the availability of subcontracting (decreases as the slope of the subcontracting schedule increases).*

6.4 Analysis of Pricing (Bidding) Strategies

We now turn to the analysis of pricing behavior under capacity constraints when subcontracting is available. Recall from section 4.3 that equilibrium bidding strategies in our environment coincide with the equilibrium bidding strategies for the first price auction with an "all-pay" component where the distributions of private costs are given by the distributions of the effective costs. Therefore, two features distinguish our setting from the regular first price auction based on the distributions of original costs: (1) the distributions of project costs are reshaped by subcontracting (compressed toward lower bound of the support of the original cost distribution); (2) the optimization problem incorporates dynamic option value considerations through the "all-pay" component and part of the effective costs. Figures 8 and 9 as well as Table 5 summarize the bidding strategies in the environment with subcontracting and compare them to the bidding strategies that arise in the environment without subcontracting.

6.4.1 Important features of bidding strategies

Participation The graphs show important differences in the supports of private costs. They arise due to capacity constraints that induce the distribution of original costs to shift to the right as backlog levels increase. For the effective costs, the supports are further shifted by dynamic option value considerations that differ across states and across own backlog levels. As a result, the competing contractors may be characterized by the effective cost distributions with different supports.[9] In equilibrium, the least efficient (in terms of the effective costs) contractor may be priced out of the market at the upper end of the support. This feature can be seen in the graphs where thhe least

[9] Notice that the contractor with a lower backlog is not necessarily more efficient in terms of effective costs, i.e., his support may be further to the right relative to the contractor with higher backlog. This is because the option value for the contractor with lower backlog may be higher than that of the contractor with higher backlog and this effect may, in principle, dominate the direct impact of the backlog on the support of original costs. However, in all of the figures shown in the paper, the contractor with a lower backlog is also more efficient in terms of effective costs.

efficient contractor is shown not to submit a bid for high cost realizations. In the framework of our model this is equivalent to the contractor participating in the market with probability less than one in a given period.

Two important observations can be made in relation to this effect. First, in this environment a single bid submitted in the auction is not motivated by the regular monopoly considerations. The entering contractor's bidding strategy "deters" the participation of the non-entering contractor, which imposes an upper limit on the entering contractor's bid schedule. Thus, the environment with a single bidder resembles a contestable rather than a regular monopoly. Second, this participation effect excludes bidders (labeled by their effective cost realizations) that are most affected by the capacity constraints either through the original cost realization or through the option value considerations. This means that if we judge the magnitude of capacity constraints on the basis of bids only while disregarding participation behavior, we would underestimate the importance of the technological capacity constraints. Table 8 investigates the effect of subcontracting on the participation decision. We find that the availability of subcontracting produces a small reduction in the probability of not participating.

The Effect of Cost Assymetries on Pricing The graphs in Figures 8 and 9 confirm that as usual in the environments with private information the more efficient contractor prices less aggressively at every cost realization, exploiting his advantage in terms of the distribution of effective costs. On the other hand, Table 5 shows that the less efficient contractor submits higher bids on average relative to the more efficient contractor. This feature arises because the less efficient cost distribution allocates a larger mass to the higher cost realizations relative to the more efficient cost distribution. The difference in bidding strategies between more and less efficient contractors increases with the difference in their backlogs. On the other hand, the difference in the bidding strategies decreases as subcontracting becomes more available (the subcontracting supply curve becomes flatter). In fact, for the baseline and flatter subcontracting supply curves, the bidding strategies used by contractors with different backlog levels are practically indistinguishable for a large part of the support. This regularity arises due to the symmetrization effect of subcontracting (both in terms of the distribution of project costs and option value considerations).

The Effect of Subcontracting on Levels and Shapes of Pricing Schedules The graphs also indicate that bidding strategies in the environment with subcontracting are lower and flatter than the bidding functions in the environment without subcontracting. Table 5 quantifies this effect. It shows that under the baseline subcontracting supply function, contractor 1 submits bids that are on average 25%, 32% and 35% lower than those he would submit in the environment without subcontracting for $(0,0)$, $(1,0)$ and $(2,0)$ backlog configurations, respectively. Unsurprisingly, the effect of subcontracting on prices is more substantial in the states where the backlog differences and therefore the scope for symmetrization are large. This effect also becomes more important as subcontracting availability increases. For example, the difference between average bids under no subcontracting and under subcontracting is somewhat smaller under the "even steeper" subcontracting supply function:

it is only 6%, 9% and 10% correspondingly for contractor 1 and $(0,0)$, $(1,0)$ and $(2,0)$ backlog configurations, respectively. On the other hand the reduction is substantially larger under the "even flatter" subcontracting supply curve: it is 33%, 40% and 43% correspondingly for contractor 1 and $(0,0)$, $(1,0)$ and $(2,0)$ backlog configurations, respectively. We investigate the mechanisms that shape the bidding strategies in the dynamic environment with subcontracting next.

6.4.2 Cost mitigation versus option value considerations

The results of this analysis are recorded in Figure 13. We focus on two mechanisms: (a) the mitigation of bidders' private project costs through subcontracting (as we have discussed in section 6.3), and (b) dynamic considerations in the form of the option value of losing instead of winning that in the environment with subcontracting depends on the realization of the bidder's original costs. To understand how these features impact pricing (bidding), we perform the following comparison. We begin by deriving and plotting the bidding strategies that arise in a regular static first-price auction where the distributions of private costs coincide with the distributions of the original costs at a given state. We compare these strategies to the bidding strategies for a regular static first-price auction where the distribution of private costs coincides with the distribution of the static part of the effective costs. In this exercise we use the effective costs (and their static part) that arise in the dynamic equilibrium with subcontracting. The bidding strategies computed in such a manner reflect the cost reduction effect of subcontracting but ignore the option considerations.

As we discussed in the previous section, under subcontracting the distribution of private costs is compressed toward the lower bound of the support. The bidding strategies that arise in this environment charge lower mark-ups over the appropriate costs (in this case the static part of the effective costs). The mark-ups are lower because private information is less important (the variance is reduced) and because the cost distributions are now less asymmetric. In addition, the bidding functions based on the static part of the effective costs as plotted against the original costs lie under the 45-degree line for a large part of the support and, therefore, below the bidding strategies based on the original distribution of the private costs. They are also flatter than the bidding strategies based on original costs.

Last, we plot the optimal bidding strategies that arise in the dynamic environment with subcontracting. Relative to the bidding strategies discussed in the previous paragraph, these strategies also incorporate an option value consideration in addition to cost reduction effect. To gain a better understanding of this mechanism we first plot the portion of the bid related to option considerations[10] for different states, ω, and under different subcontracting supply functions in Figure 12. The graphs confirm the intuition developed in section 4.3 that the option value function is positive for all cost realizations, is lower than the option value function from the environment without subcontracting,

[10] We generate it as the difference between the full bid submitted in the dynamic environment with subcontracting and the bid from the static model based on the static part of the effective costs.

and is decreasing in the original costs.

Table 7 summarizes the average price effect associated with option values across states and for various subcontracting price schedules. It shows that while the option value effect is non-monotone in the backlog difference under the “no subcontracting” regime, it increases with differences in backlogs and then flattens out under subcontracting. This feature has important implications for the probability of winning, which we address later in the paper.

The table also indicates that the option value considerations become less and less important as subcontracting becomes more available. It still accounts for 3% - 7% of the bid under the “even steeper” subcontracting supply curve (as compared to 4%-11% under “no subcontracting”) but it declines to 0.7% - 2.1% under the baseline schedule and 0.6% -1.5% under the “even flatter” subcontracting supply schedule. This effect can also be seen in Figure 13, which shows that under the “even steeper” and “steeper” subcontracting supply functions, the option value’s impact on bidding strategies is comparable in magnitude to the cost reduction effect. However, under flatter supply schedules, the cost reduction effect clearly dominates.

To summarize: (1) the option considerations increase bids (the contribution to the bid associated with option values is always positive); (2) the impact of option considerations on the bid is decreasing in the cost realization and thus works to flatten the bid curve; (3) the magnitude of option value’s effect on the bid under subcontracting is lower than under the “no subcontracting” regime, which combined with the cost reduction effect of subcontracting, results in bidding strategies that are lower and flatter than those under “no subcontracting.”

6.4.3 Probability of winning

Figure 10 investigates the impact of subcontracting on the probability of winning across different states and for various subcontracting supply functions. We begin by considering graphs that describe the probability of winning in the absence of subcontracting. As the graphs show, the project is more likely to be allocated to the contractor with the higher backlog level in the states when one contractor is empty (zero backlog) and the other has an intermediate backlog level. This pattern is partially reversed when the difference in backlog levels increases with the low-backlog competitor winning more often for a large range of cost realizations. This regularity is consistent with the option value properties we have documented in Table 7. More specifically, the dynamic option value is very high in the states where the contractor with low backlog faces a competitor with somewhat higher backlog. This is because by losing today, the efficient contractor guarantees himself high prices tomorrow since he faces a very weak competitor at that time. The option value decreases as the competitor becomes very full, since the participation effect does not allow full internalization of the competitor’s capacity constraints under very high capacity utilization.

In the presence of subcontracting the option value of losing rather than winning is much smaller in magnitude at all states, since subcontracting mitigates both the backlog accumulation and the very high cost realizations that may arise under high backlog levels. As a result, the low-backlog contractor is less motivated to lose to the high-backlog competitor. Thus, in equilibrium the project is more likely to be allocated to a contractor with a lower backlog.

Result 4 *The least effective contractors (in terms of effective costs) may participate in the market with probability less than one. Cost mitigation and option value considerations work to make the bidding strategies in the environment with subcontracting lower and flatter than those used in the environment without subcontracting. Under subcontracting the project is more likely to be allocated to the contractor with lower backlog. This is in contrast to the environment without subcontracting, where the option value dictates that the low-backlog contractor should lose to the high-backlog contractor in some states in order to enjoy monopoly power in the future.*

6.5 Steady-State Properties

Figure 14 shows the stationary distribution of the individual contractor's backlog and the distribution of differences in backlogs for the game without subcontracting as well as for the game with subcontracting under different subcontracting supply schedules. This figure confirms that the availability of subcontracting reduces backlog accumulation in equilibrium as well as reduces the differences in backlogs between subcontractors. This effect becomes more pronounced as the availability of subcontracting improves.

Table 9 reports several steady-state statistics under different degrees of subcontracting availability starting with the "no subcontracting" case. It confirms the effect of subcontracting availability on the expected backlog of an individual contractor and the expected difference in backlogs (columns 7 and 8). Columns 4 and 5 document the effect of the reduction in backlog accumulation on the expected cost of the winner. We find that it results in a 4.7% to 8.7% reduction in the original costs of the winner and an 8.1% reduction under the baseline specification. The reduction in the effective costs is even more drastic: it ranges from 6.3% to 19.4% and is equal to 14.4% for the baseline specification. Notice that at low levels of subcontracting availability, the dynamic effect on costs (through the reduction in backlog accumulation) dominates the static effect (from removing the very high cost realization) (0.03-0.04 reduction in original costs relative to additional 0.01 - 0.03 in effective costs measured in absolute units).[11] However, as subcontracting becomes more accessible, the static effect on costs dominates (it is a 0.09 - 0.17 reduction in effective costs in addition to a 0.05-0.06 reduction in original costs). The expected amount subcontracted by the winner documented in column 6 is quite low (7-17%) under low subcontracting availability but it increases substantially (27% - 40%) as subcontracting becomes more easily accessible.

The reduction in costs is combined with the mark-up compression (expected mark-up of the

[11] We ignore the continuation value component inside the effective costs for this calculation.

winner = expected bid of the winner - expected effective costs of the winner). The latter result arises due to the dynamic and static symmetrization induced by subcontracting. In the equilibrium with subcontracting the mark-ups of the winner are reduced by 20% to 65% (55% under the baseline specification) relative to the mark-ups charged in the equilibrium without subcontracting. In terms of the effect on the cost of procurement (expected bid), the mark-up compression is almost twice as important as the total cost reduction induced by subcontracting (mark-ups are reduced by 0.08 - 0.26 in absolute units as opposed to a 0.04 - 0.13 reduction in the effective costs).[12] Therefore, subcontracting leads to a drastic reduction in contractors' profitability.

Result 5 *Subcontracting results in the compression of the stationary distribution of backlogs toward zero. The reduction in the expected static part of the effective costs dominates the reduction in the expected original costs, i.e., the static cost reduction effect is stronger than the dynamic cost reduction effect under high subcontracting availability. The mark-up compression induced by the symmetrization effect of subcontracting dominates the cost reduction effect.*

6.6 Cost Mitigation versus Reduction in Backlog Accumulation

As we remarked in the previous section subcontracting affects the market through two channels. It reduces backlog accumulation and allows contractors to mitigate their project cost realizations for a given state and own backlog level. The first channel unequivocally leads to dynamic symmetrization. However, the role of the second channel is more complex.

In order to understand the mechanism through which project cost mitigation works in equilibrium, we conduct the following experiment. First, we compute an equilibrium with subcontracting. Next, we re-compute the bidding strategies and the value function while holding the equilibrium subcontracting strategies fixed but using them only in state transitions. That is, in this experiment the adjustment in the static part of the effective costs due to subcontracting (cost mitigation) is eliminated. It is worth noting that this exercise does not compute the equilibrium without cost mitigation effect of subcontracting, since the optimality considerations that drive the subcontracting strategies take this effect into account. Rather, the results of this analysis should be viewed as a measurement of a partial effect, i.e., an isolation of the backlog accumulation channel through which dynamic symmetrization works.

Figure 15 reports the results of this analysis for different subcontracting supply schedules. All the graphs show a cross-section of the value function for the equilibrium without subcontracting and the full equilibrium with subcontracting and of the value function computed under the partial effect analysis. An interesting pattern emerges. In all the graphs the value function of the model with subcontracting lies below the value function of the model without subcontracting, an effect induced

[12]The expected mark-up of the winner is equal to twice the expected profit of the individual contractor in our model.

by symmetrization. However, under the steep subcontracting supply schedule, the partial effect value is the lowest, whereas under the baseline specification, the partial effect value almost coincides with the full equilibrium value function and is only slightly higher than the full equilibrium value function under higher competitor's states. Finally, the partial effect value function is noticeably higher than the full equilibrium value function under the flat subcontracting schedule.

The regularities that we observe in the case of the "steeper" subcontracting supply function should not arise if the role of the cost mitigation channel is only to symmetrize the static part of the effective cost distribution. Indeed, as we have seen in section 6.4 such symmetrization invariably results in lower mark-ups and should induce the value function from the full model to be lower than the partial effect value function. Therefore, cost mitigation must have dynamic implications that are most significant under steep subcontracting supply functions. Figure 16 confirms this intuition. It graphs the outside option for the dynamic equilibrium without subcontracting, the full dynamic equilibrium with subcontracting, and the partial effect analysis in the case of "steeper" and "flatter" subcontracting supply curves. This graph confirms that under the "steeper" subcontracting supply curve, the option value in the full subcontracting equilibrium is higher than the option value in the case of partial analysis. The effect is reversed and the difference between (as well as magnitudes of) option values is almost negligible in the case of the "flatter" subcontracting supply curve.

The option value reflects the difference between the continuation value conditional on losing this period and the continuation value conditional on winning. This effectively summarizes the value of continuing with a lower backlog level against a weaker competitor versus continuing with an increased backlog level against a strengthened competitor or, in other words, the relative profitability of competing under changed backlog configurations. This measure is stochastic (and therefore is associated with risk), since it depends on future cost realizations as well as future productivity. The availability of cost mitigation reduces this risk and in this way increases the option value. Therefore, this insurance feature works in the opposite direction of dynamic symmetrization. On the other hand, cost mitigation also results in static symmetrization, which leads to a reduction in mark-up (profitability). It appears that under the "steeper" subcontracting supply curve, the backlog asymmetry remains quite important, and therefore, the magnitude of the option value is quite large relative to the static cost symmetrization (we have seen this in Figure 13 as well). Hence, the reduction in risk dominates the static symmetrization effect. In contrast, under the "flatter" subcontracting supply curve, the dynamic asymmetry is largely eliminated. As a result the static symmetrization effect appears to be more important in magnitude than the risk reduction effect. Hence, as we move from partial to full equilibrium with subcontracting, the value (and the option value) is reduced. This result is summarized below.

Result 6 *The cost mitigation feature of subcontracting partially insures contractors against bad cost draws in the future and thus works to increase the option value. The insurance effect dominates the static symmetrization at low subcontracting availability. This relationship is reversed when subcontracting availability is high.*

7 Implication for Estimation Bias

The bidding strategies that arise in the dynamic environment with subcontracting are lower and flatter than the bidding strategies in the environment without subcontracting. As Jofre-Bonet and Pesendorfer (2003) and Balat (2012) show, the estimation methodology that recovers primitives of this environment exploits the observed distributions of bids conditional on backlog configuration and the contractor's participation behavior. Therefore, the estimation procedure based on the model without subcontracting and applied to the data generated by the model with subcontracting is likely to produce erroneous estimates of the distribution of the bidders' private costs as well as of the technological capacity constraints. We explore this surmise below.

In particular, we use the distributions of bids conditional on state that we obtained for the model with subcontracting to recover the value function and the distribution of private costs that would be consistent with these bid distributions under the dynamic model without subcontracting. To do this we modify the methodology proposed in Jofre-Bonet and Pesendorfer (2003) to allow for stochastic backlog depreciation. We then compare the distribution of private costs recovered under the misspecified model to the underlying distribution of the private costs. To fix a benchmark for comparison purposes, we also show the recovered cost distribution and its moments for the case when the methodology is correctly applied to the bid distributions generated by the model without subcontracting. The details of the methodology are summarized in the Appendix.

The results of the analysis are depicted in Figure 17. Recall that the distribution of original costs is specified to be truncated normal. The graphs show that our procedure indeed recovers the cost distribution that is reasonably close to the primitive when it is applied to the data from the model without subcontracting. In particular, the recovered distribution is symmetric around the mean and the first two moments of the recovered distribution are very close to the first two moments of the primitive (means: estimated 0.73 compared to true mean of 0.75; and standard deviations: 0.27 compared to 0.26). Notice that we report no standard errors since our methodology is based on exact bid distribution rather than the distribution estimated on the basis of the sample. All of the discrepancies between the primitive and the recovered distribution of costs are due to numerical error.

In contrast, the methodology recovers the cost distribution with the mean and variance that are lower than those of the primitive distribution when applied to the bid distributions generated by the model with subcontracting. In general, the cost distribution recovered from the misspecified model allocates most of the mass near the lower end of the support of the primitive distribution. The downward bias is substantial and it becomes more pronounced as the availability of subcontracting increases. Thus, this analysis confirms that failing to account for subcontracting in estimation is likely to result in biased estimates for the distribution of private costs.

Further, the difference in the means of the cost distributions that correspond to different backlog values inform us of the strength of capacity constraints. We recover this parameter using the data without subcontracting as well as the data with subcontracting under varying degrees of subcontracting availability. We find that when this methodology is applied to the correct model, it produces reasonably accurate estimates of the capacity constraint parameter (estimated 0.17 versus the true value of 0.15). On the other hand the estimation under the misspecified model produces estimates with a significant downward bias.

It is worth noting that these calculations abstract from backlog mismeasurement issues that are likely to arise in many realistic settings. The mismeasurement occurs when the backlog is computed as the sum of loads awarded to a given contractor without accounting for subcontracted portions. Such mismeasurement would additionally induce us to underestimate technological capacity constraints, and by impacting the option value, measurement is likely to further bias our estimates of the distribution of costs.

Result 7 *The estimation procedure based on the misspecified model without subcontracting underestimates the mean and the variance of the project cost distribution as well as the magnitude of capacity constraints when applied to the data generated by the environment with subcontracting.*

8 Optimal Procurement Policy

The existence of capacity constraints has an important implication for procurement policy. Previous studies have investigated market performance under a variety of procurement policies that reflect project size versus frequency trade-off. Formally, the procurement policy can be summarized by a pair (x, q) where q reflects the probability that a project is allocated in any given period and the project size is given by x. To ensure that the same amount of work, $\bar{x}$, is completed on average, the comparison is across the policies that satisfy $xq = \bar{x}$.

In the environment where subcontracting is not available it was shown previously (and our results in Table 11 confirm this) that lumpier policies, i.e., policies that allocate large projects but less frequently, result in higher profits, higher costs of procurement, higher average backlog accumulation and larger differences in backlogs. It is a natural consequence of a lumpier policy that higher states as well as more asymmetric states are reached in equilibrium than under a less lumpy policy. Due to the limited and stochastic productivity as well as the lower frequency of the award, both high states and high asymmetry are persistent. Asymmetry naturally relaxes competition and therefore results in higher mark-ups. This feature, in combination with higher costs associated with higher backlog levels, leads to a higher cost of procurement under a lumpier policy. Our results also indicate that the total surplus (=- procurement cost + profit) is higher when projects are smaller and more frequent. Thus, in the environment without subcontracting the smaller projects are preferred by the auctioneer and society whereas, the industry prefers larger projects.

We document that in the presence of subcontracting (under the baseline subcontracting schedule) the lumpier policy induces larger levels of subcontracting: the average portion subcontracted increases from 27% under our benchmark size to 49% under the policy that auctions projects that are three times larger but three times less frequently. Nevertheless, the expected backlog accumulation increases substantially (by 204% relative to benchmark case). This, of course, leads to the increase in costs due to capacity constraints even though both the cost levels (0.63 versus 1.19) and the rate of increase (4.16% versus 15.6%) are lower than in the environment without subcontracting. The difference is even more striking if we consider the effective costs. They increase at an even lower rate (3.3% versus 4.16% reduction in the original costs) even though the incremental reduction is not very large. Finally, the increase in subcontracting does increase symmetrization at the static level, while the backlog accumulation works to increase asymmetry at the dynamic level. The net effect is a slight mark-up compression (0.14 under the benchmark policy versus 0.13 under the $(3\bar{x}, 1/3)$ policy, and 0.12 under the $(6\bar{x}, 1/6)$ policy). The cost increase and mark-up compression result in lower profits. Since mark-up compression is very small, the cost effect dominates and the procurement cost increases as projects become lumpier. However, the increase in the procurement costs is much smaller than in the environment without subcontracting (an increase of 2% from 0.75 to 0.76 compared to an increase of 12.3% from 1.06 to 1.19 if subcontracting is not available).

Result 8 *The procurer benefits from subcontracting under all procurement policies, while a less lumpy policy is more advantageous. Contractors strongly benefit from a less lumpy policy, since a lumpier policy increases costs, forces them to subcontract more and results in more aggressive competition, mark-up compression, and lower profits.*

9 Extensions and Additional Empirical Considerations

In this analysis we focused on a very simple model of a procurement market with subcontracting that emphasized the features we believe to be of first-order importance. We now discuss some other factors that typically arise in real-life procurement markets with subcontracting.

Heterogeneous Units In some markets, such as construction or maintenance procurement, the projects do not consist of a homogeneous amount of work; rather, they are composed of heterogeneous units such that each unit represents a homogeneous amount of work. The implication of this is that the costs of work may not be evenly allocated across these units, and in fact, a contractor may have a relatively low cost of completing some units and high costs of completing some others. In addition, the work from different units may have to be subcontracted in different markets. Under these circumstances a separate backlog may have to be considered for each type of work that could be included in the project. A contractor would have to decide on a separate subcontracting function for each submarket and then aggregate these decisions in a measure of effective costs reminiscent of the one we use in this paper. The analysis of such an environment may be challenging due to the dimensionality issue but is unlikely to provide any new insights.

Contractor-Specific Price of Subcontracting In our analysis we assume that all contractors have access to subcontracting at the same price. In reality subcontracting prices are often negotiated and as a result are contractor-specific. If negotiated prices are public information, this feature does not substantially complicate the analysis except that otherwise symmetric contractors choose different subcontracting and bidding strategies in this environment.

If subcontracting prices are the private information of the contractor-subcontractor pair, then the model changes. Formally, in this environment contractors will have to form expectations about the subcontracting price and, therefore, about subcontracted levels of their competitors conditional on the cost draw. As a result the competitive effect of subcontracting will be softened. If subcontracting prices are correlated as they are likely to be if contractors are working with the same set of subcontractors, then subcontracting will induce correlation in the effective costs of bidders. This will result in the reduction of the variance of competitor's costs conditional on the contractor's own draw and will result in more aggressive bidding. However, this effect will be softer relative to the model we present in this paper.

Horizontal Subcontracting In some markets the primary and subcontracting markets may not be clearly separated. More specifically, firms participating in the subcontracting market may occasionally submit bids in the primary market. If this is the case, the subcontracting market cannot be summarized by the subcontracting supply curve. It would be important to model the decision to subcontract work to a specific firm as well as to account for the correlation in bids by the company that participates in both markets and the companies that use this subcontractor. Such an environment is potentially much more complicated than the one we consider in this paper. Fortunately, horizontal subcontracting does not occur very often in the markets that we had in mind when writing this paper. Marion (2011) investigates the issue of horizontal subcontracting using auction data from the California highway procurement market. He finds that during the period between 1996 and 2005, about 10% of projects received a bid from a company that was also listed as a subcontractor on another bid. This feature affected 7% of the bids submitted during this time period. These figures, while not negligible, indicate that horizontal subcontracting is not a main concern that needs to be addressed when investigating the effect of subcontracting availability on the functioning of such markets.

Timing of Subcontracting We assume that the decision to subcontract is made at the time of bidding. In some markets these decision could be made as part of the on going work on the project. Formally, it means that the contractor can adjust the subcontracting levels in every period in which he works on the project if we keep subcontracting project-specific. The subcontracting strategy will depend on productivity realization in addition to state. Such a contingent strategy would have to be "integrated back" to compute the expected costs of the project at the time of bidding.

10 Conclusion

This paper studies the role of subcontracting in the dynamic procurement environment with capacity constraints and private costs. We develop a numerical strategy of solving for the equilibrium of this game and use it to study properties of the procurement environment with subcontracting.

We demonstrate that subcontracting leads to dynamic symmetrization and as lowers overall backlog accumulation: the stationary distribution of states allocates a larger mass toward the states with lower and more similar backlog levels. On the other hand, in a given state the subcontracting works to mitigate high cost realizations, including those that arise due to capacity constraints. This feature reduces the means and variances of project cost distributions and thus renders private information less important in this environment as well as reduces the differences in project cost distributions across backlog levels within the same state (static symmetrization).

Capacity constraints make losing in a given period preferable to winning under some state configurations, a feature summarized by the option value considerations in the contractor's pricing problem. Under subcontracting, the option value depends on the contractor's current cost realization. It is always positive and is decreasing in costs. Cost mitigation and option values shape bidding strategies used by contractors in this environment. Both of these features work to make bid functions flatter relative to the ones used in the environment without subcontracting. Cost mitigation lowers bids, whereas the option value pushes bids up. However, the magnitude of the option value under subcontracting is lower than the option value in the environment without subcontracting. As a result the pricing (or bidding) strategies used by contractors in this market differ substantially (are lower and flatter) than those that arise in the environments without subcontracting. This finding has important implications for empirical research. We show that the omission to account for subcontracting introduces a substantial bias in the estimates of the distribution of private costs and of the measure of technological capacity constraints if they are based on the data from the market where subcontracting is available.

The availability of subcontracting has important implications for public policy. Previous research has shown that in the presence of capacity constraints, the procurer incurs higher costs if he auctions large projects infrequently. Reducing the size of the projects while increasing their number allows to obtain the same amount of service but under substantially lower costs. We show that the effect is not very large if subcontracting is available. Subcontracting works through two channels to contain the increase in the cost of procurement when larger projects are auctioned. Under these circumstances the higher fraction of the project is subcontracted. This reduces backlog accumulation and thus reduces the effect of capacity constraints on costs. It also puts downward pressure on prices through the static symmetrization channels. Thus, policy makers may substitute the reduction in the size of the project for the greater subcontracting availability when appropriate. Of course, this is a complex decision that needs to take into account the impact of greater subcontracting availability on the overall quality work. We leave the full analysis of these trade-offs to future research.

We abstract from many details of the subcontracting market when we summarize it by an upward-sloping subcontracting supply schedule. Several dimensions in which real subcontracting markets might be richer than our environment are considered in section 9. While each of these features would change the equilibrium of our model to some degree, the main lessons of our paper remain unchanged.

One other important feature deserves a separate comment. In our analysis, the subcontracting supply function is assumed to be exogenous. In some settings where the subcontracting market exclusively serves the main contracting market, this assumption will not hold. Luckily, this concern does not arise in government procurement because the subcontracting industry it draws on serves many other markets. Studing the endogenous effect of changes in the primary market on the subcontracting market for those environments where such concern arises is a direction for future research.

References

Alchian, A., and H. Demsetz (1972): "Production, Information Costs, and Economic Organization," *American Economic Review*, 62(6), 777–795.

Athey, S. (2001): "Single Crossing Properties and the Existense of Pure Strategy Equilibria in Games of Incomplete Information," *Rand Journal of Economics*, 69(4), 861–890.

Balat, J. (2012): "Highway Procurement and the Stimulus Package: Identification and Estimation of Dynamic Auctions with Unobserved Heterogeneity," Working paper, Johns Hopkins University.

Chen, J., U. Doraszelski, and J. Harrington Jr (2009): "Avoiding market dominance: Product compatibility in markets with network effects," *The RAND Journal of Economics*, 40(3), 455–485.

Coase, R. H. (1937): "The nature of the Firm," *Economica*, 4(16), 386–405.

——— (1988): "The nature of the firm: Influence," *Journal of Law, Economics and Organization*, 4(1), 33–47.

De Silva, D., G. Kosmopoulou, and C. Lamarche (2011): "Survival of Contractors with Previous Subcontracting Experience," *Economic Letters*, 117(1), 1–9.

Gale, I., D. Hausch, and M. Stegeman (2000): "Sequential Auctions with Subcontracting," *International Economic Review*, 41(4), 12–35.

Groeger, J. (2012): "A Study of Participation in Dynamic Auctions," Working paper, Tepper School of Business.

Jensen, M., and W. Mechling (1976): "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3(4), 305–360.

Jofre-Bonet, M., and M. Pesendorfer (2003): "Estimation of a Dynamic Auction Game," *Econometrica*, 71(5), 1443–1489.

Judd, K. (1998): *Numerical methods in economics*. MIT press.

Kaplan, T. R., and S. Zamir (2012): "Asymmetric first-price auctions with uniform distributions: analytic solution to the general case," *Economic Theory*, –(–), 1–34.

Marion, J. (2011): "Sourcing from the Enemy: Horizontal Subcontracting in Highway Procurement," Working paper, University of California, Santa Cruz.

Maskin, E., and J. Tirole (1988): "A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs," *Econometrica*, pp. 549–569.

Miller, D. P. (2012): "Subcontracting and Competitive Bidding on Incomplete Contracts," Working paper, Clemson University.

Moretti, L., and P. Valbonesi (2011): "Subcontracting in Public Procurement," Working paper, University of Padova.

Nakabuyashi, J., and N. Watanabe (2010): "Experimental Behavior in Subcontract Auctions," Working paper, University of Tsukuba.

Reny, P. J., and S. Zamir (2004): "On the Existence of Pure Strategy Monotone Equilibria in Asymmetric First-Price Auctions," *Econometrica*, 72(4), 1105–1125.

Saini, V. (2013): "Endogenous Asymmetry in a Dynamic Procurement Auction," *Rand Journal of Economics*, –, –, Forthcoming.

Wambach, A. (2009): "How to Subcontract," *Economics Letters*, 102(2), 152–155.

Williamson, O. E. (1975): *Markets and Hierarchies: Analysis and Antitrust Implications*. New York: The Free Press.

Appendix

Proof of Proposition 2. The expected profit of bidder 1 with effective cost realization $\bar{\phi}_1$ is maximized at $\bar{b}$, i.e.

$$(\bar{b}-\bar{\phi}_1)(1-F_{\phi,2}(\beta_2^{-1}(\bar{b};\omega)))+\delta\int_{\underline{\phi}_2}^{\beta_2^{-1}(\bar{b};\omega)}[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]f_{\phi,2}(\phi)d\phi \geq$$

$$(b-\bar{\phi}_1)(1-F_{\phi,2}(\beta_2^{-1}(b;\omega)))+\delta\int_{\underline{\phi}_2}^{\beta_2^{-1}(b;\omega)}[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]f_{\phi,2}(\phi)d\phi$$

or

$$(\bar{b}-\bar{\phi}_1)(1-F_{\phi,2}(\xi_2(\bar{b};\omega)))+\delta\int_{\underline{\phi}_2}^{\xi_2(\bar{b};\omega)}[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]f_{\phi,2}(\phi)d\phi \geq \quad (19)$$

$$(b-\bar{\phi}_1)(1-F_{\phi,2}(\xi_2(b;\omega)))+\delta\int_{\underline{\phi}_2}^{\xi_2(b;\omega)}[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]f_{\phi,2}(\phi)d\phi$$

Next, the derivative of $\bar{\pi}_1(b,\overline{\phi}_1)$ with respect to ξ_2 is given by

$$\bar{\pi}'_{1,\xi_2}=\big[-(b-\bar{\phi}_1)+\delta[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2(\phi_2^{-1}(\xi_2(b)))x)-\underline{V}_2]\big]f_{\phi,2}(\xi_2(b))$$

notice that $\bar{\pi}'_{1,\xi_2}(b=\bar{\phi}_1)>0$ and is decreasing in b, so there must exist $\tilde{b}\in[\bar{\phi}_1,\bar{\phi}_2]$ such that $\bar{\pi}'_{1,\xi_2}\geq 0$ for $b\leq\tilde{b}$ and $\bar{\pi}'_{1,\xi_2}\leq 0$ for $b\geq\tilde{b}$ (i.e. $\bar{\pi}'_{1,\xi_2}(\tilde{b})=0$ or $\tilde{b}=\bar{\phi}_2$).

Then $\bar{\pi}_1(b,\overline{\phi}_1)$ is maximized at $\tilde{b}$ for $b\leq\tilde{b}$ since

$$\bar{\pi}'_{1,b}=(1-F_{\phi,2}(\xi_2(b;\omega)))+\bar{\pi}'_{1,\xi_2}\xi'_2(b)\geq 0.$$

Notice that $\bar{\pi}'_{1,b}\geq 0$ for some range of b such that $b\geq\tilde{b}$.

Further, $\bar{\pi}_1(b,\overline{\phi}_1)$ is decreasing in ξ_2 for $b\geq\tilde{b}$. Recall that in equilibrium $\xi_2(b)\leq b$ then

$$(b-\bar{\phi}_1)(1-F_{\phi,2}(\xi_2(b;\omega)))+\delta\int_{\underline{\phi}_2}^{\xi_2(b;\omega)}[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]f_{\phi,2}(\phi)d\phi\geq$$

$$(b-\bar{\phi}_1)(1-F_{\phi,2}(b))+\delta\int_{\underline{\phi}_2}^{b}[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]f_{\phi,2}(\phi)d\phi$$

The right-hand side of the inequality above is maximized at b_0 such that

$$(1-F_{\phi,2}(b_0))-\big[(b_0-\overline{\phi}_1)-[E_{c',\epsilon}V_1(\omega_1-\epsilon_1,\omega_2-\epsilon_2+(1-\hat{h}_2)x)-\underline{V}_2]\big]f_{\phi,2}(b_0)=0.$$

Thus combining (19) with the discussion above we obtain that two cases are possible

(a) $\bar{b}=\tilde{b}$ if $\tilde{b}=\bar{\phi}_2$

(b) $\bar{b}=\min\{b_0,\bar{\phi}_2\}\geq\tilde{b}$ where b_0 is defined as above.

Methodological Details from the "Estimation Bias" Section

We use the bid distributions computed for the model with subcontracting to recover the cost distribution that would be consistent with these distributions under the misspecified model without subcontracting. We extend the method proposed by Jofre-Bonet and Pesendorfer (2003) to allow for stochastic backlog depreciation.

Following Jofre-Bonet and Pesendorfer (2003) we use the necessary first order conditions from the contractor's bidding problem to recover the inverse bid function consistent with the observed bid distribution and the model without subcontracting:

$$cx = b - \delta[E_\epsilon V_i(\omega_i - \epsilon_i, \omega_j + x - \epsilon_j) - E_\epsilon V(\omega_i - \epsilon_i + x, \omega_j - \epsilon_j)] + \frac{1}{h_i(b;\omega)},$$

where $h_i(b;\omega) = \frac{f_{b,i}(b;\omega)}{1-F_{b,i}(b;\omega)}$ is a hazard rate, while $f_{b,i}$ and $F_{b,i}$ are the density and cumulative distribution function of the equilibrium bid distribution for contractor i under state ω respectively. Jofre-Bonet and Pesendorfer (2003) show that the value function used in the equation above can be inferred from the distribution of bids. We modify their argument to obtain:

$$V_i(\omega) = \int_{\underline{b}(\omega)}^{\bar{b}(\omega)} \frac{f_{b,i}(b;\omega)}{h_{-i}(b;\omega)} db + \delta E_\epsilon V(\omega_i - \epsilon_i, \omega_j + x - \epsilon_j).$$

Jofre-Bonet and Pesendorfer (2003) solve the corresponding equation by an approximate interpolation that assumes that the backlog process never leaves the grid. It makes the integration with respect to the productivity shock infeasible. Instead, we solve this equation by a projection method on the grid $\mathbf{\Omega^D}$, which uses 2-dimensional, second degree, complete Chebyshev polynomial basis functions. The residual function for a given set of projection coefficients θ is given by

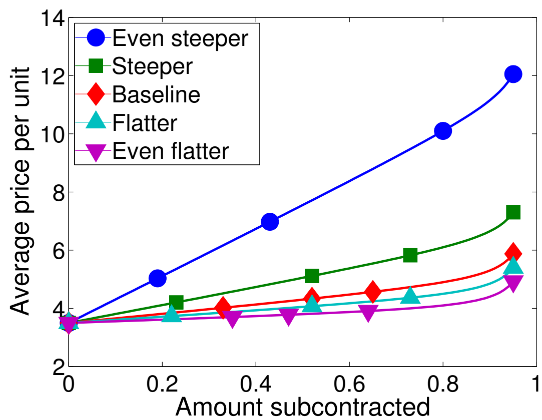
$$R_i(\omega^d;\theta) = \int_{\underline{b}(\omega^d)}^{\bar{b}(\omega^d)} \frac{\hat{f}_{b,i}(b;\omega^d)}{\hat{h}_{-i}(b;\omega^d)} db + \delta E_\epsilon \hat{V}(\omega_i^d - \epsilon_i, \omega_j^d + x - \epsilon_j;\theta) - \hat{V}(\omega^d;\theta)$$

This residual function is minimized jointly for both contractors to determine θ^*.

11 Figures and Tables

11.1 Specification Details

Figure 1: Subcontracting Price Schedules



This figure shows the various subcontracting supply functions we use in simulations. It plots the per unit price of subcontracting services (vertical axis) versus the subcontracted amount (horizontal axis).

Table 1: Parameters of Subcontracting Schedules

	Subcontracting schedule				
	Even steeper	Steep	Baseline	Flatter	Even flatter
Intercept (α_1)	3.5	3.5	3.5	3.5	3.5
Linear part (α_2)	8	3	1.5	1	0.5
Hyperbolic part (α_3)	0.05	0.05	0.05	0.05	0.05

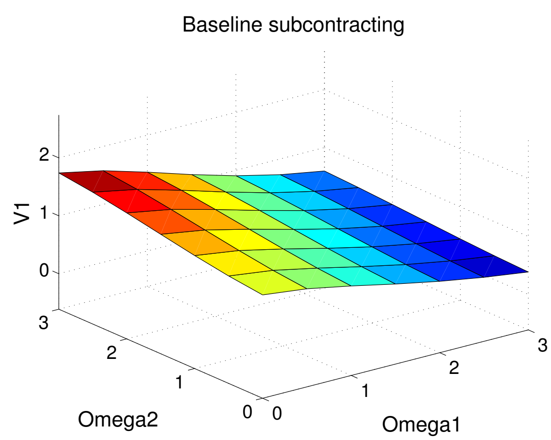
The simulations are based on the subcontracting supply schedules given by equation $P(q) = \alpha_1 + \alpha_2 q + \alpha_3 \frac{q}{1-q}$. The various parameter combinations used in the paper are summarized in the table above.

Table 2: Parameters of Benchmark Model

Cost distribution	Baseline lower bound ($\underline{C}$)	1
	Baseline upper bound ($\bar{C}$)	9
	Mean intercept (θ_1)	5
	Backlog support shift (θ_2)	1
	Backlog mean shift (θ_3)	0
	Standard deviation (σ)	2
Backlog process	Project size ($\bar{x}$)	0.15
	Maximum productivity (K)	0.16
	Maximum backlog (M)	3
Discount factor	(δ)	0.925

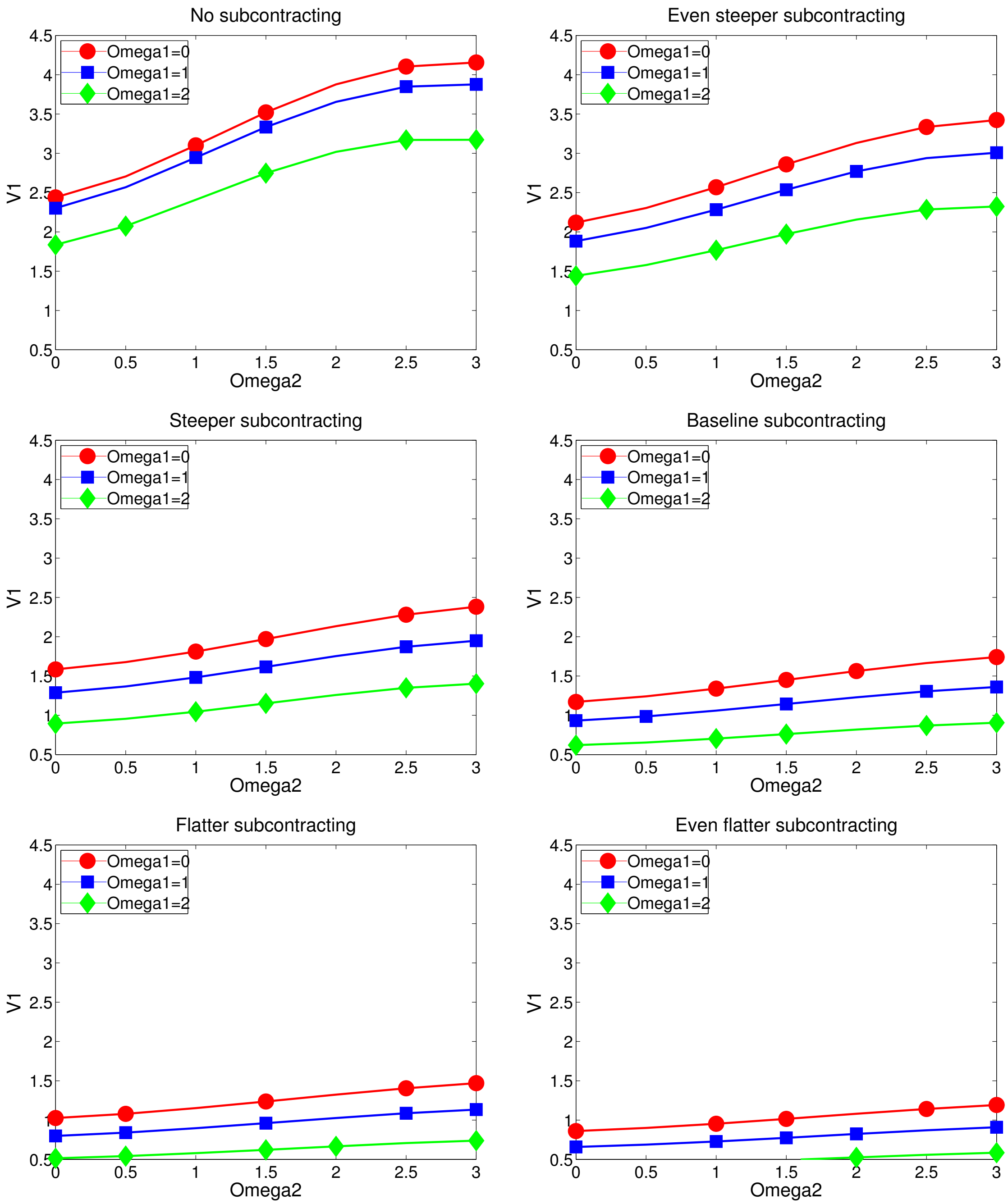
11.2 Computed MPE

Figure 2: Value Function 3D



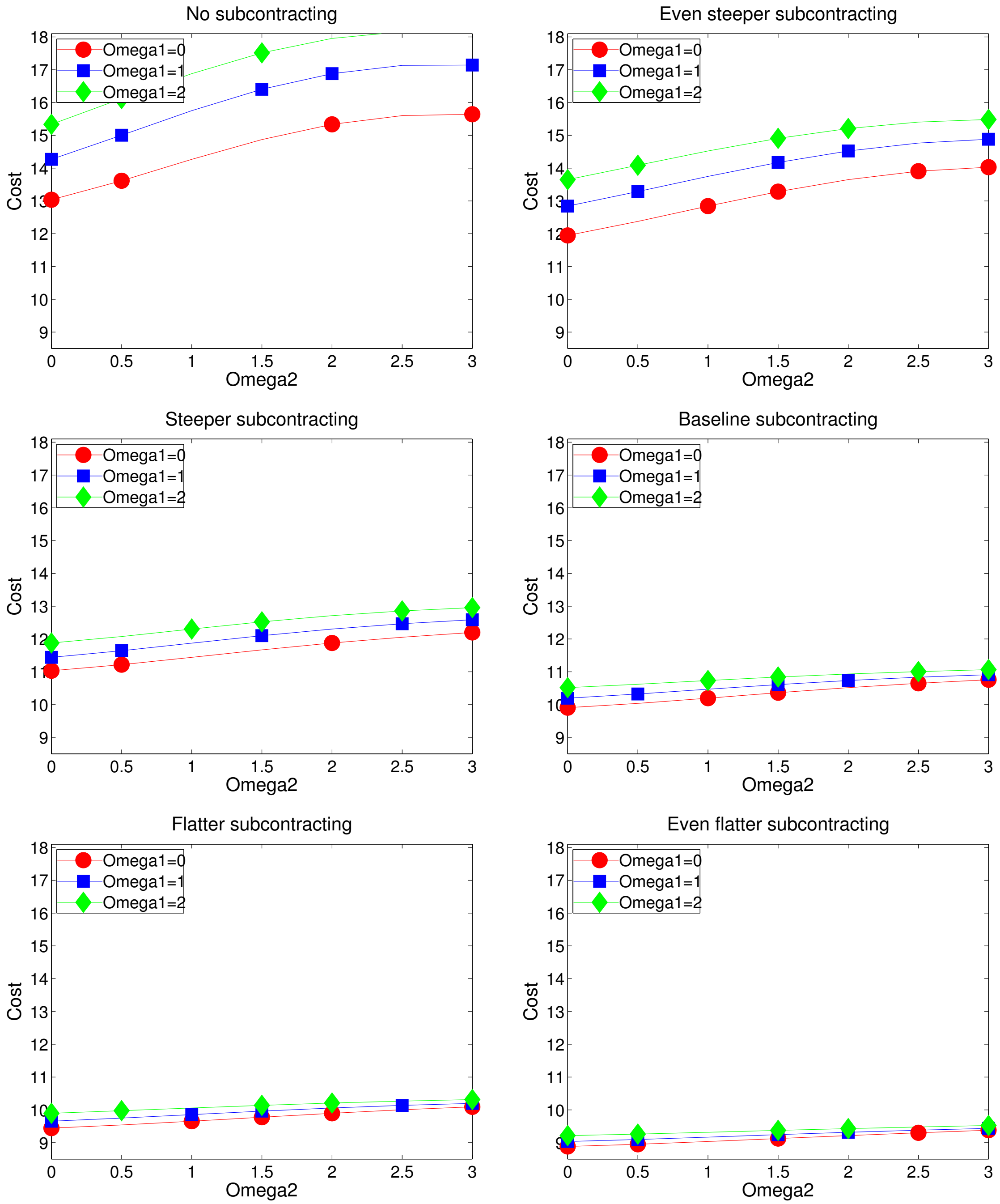
This figure shows the value function of contractor 1 as a function of own and competitor's backlogs.

Figure 3: Value Function of Contractor 1



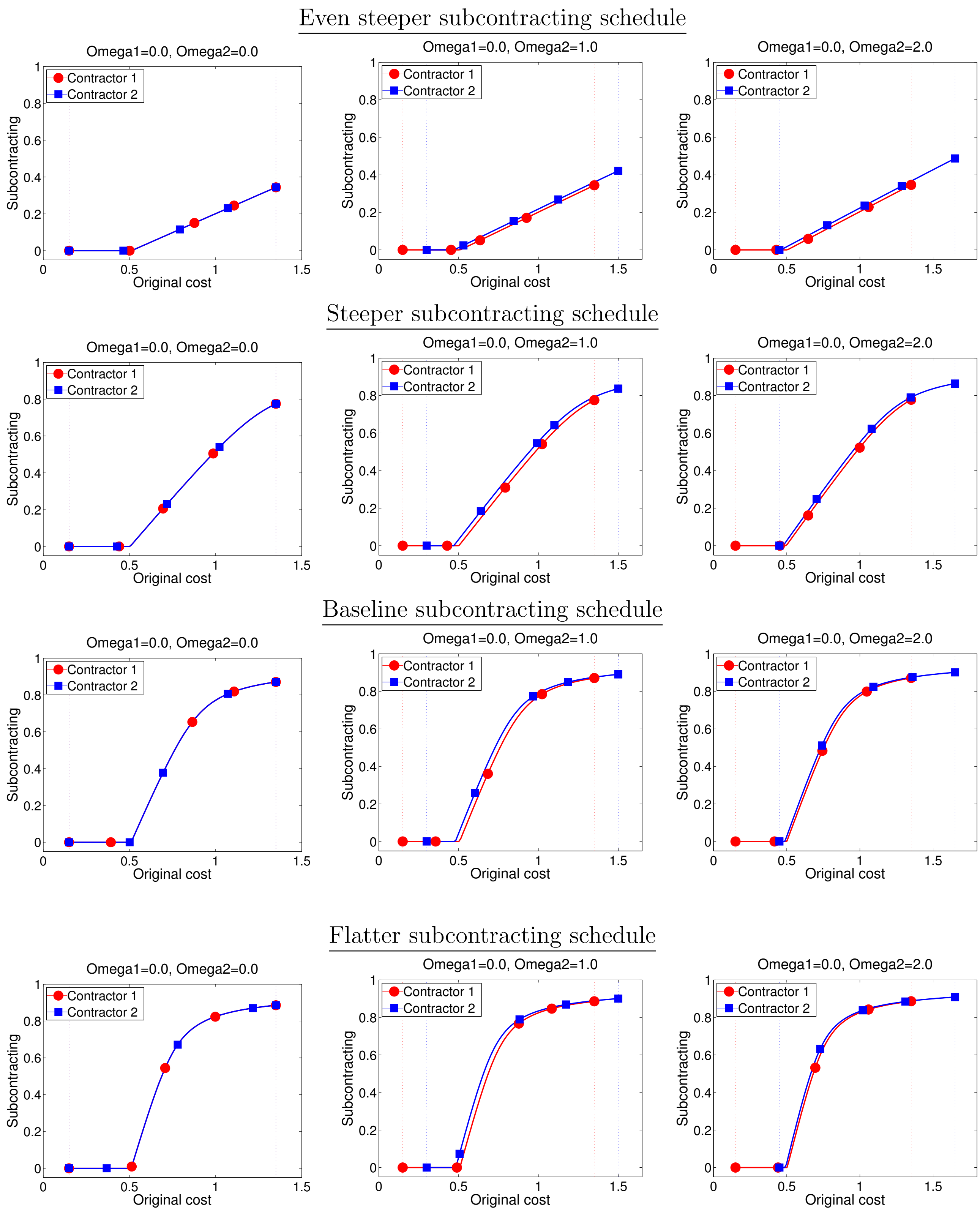
This figure shows the sections of the value function that correspond to different levels of own backlog. It graphs the value function against the level of the competitor's backlog.

Figure 4: The Cost of Procurement to Auctioneer



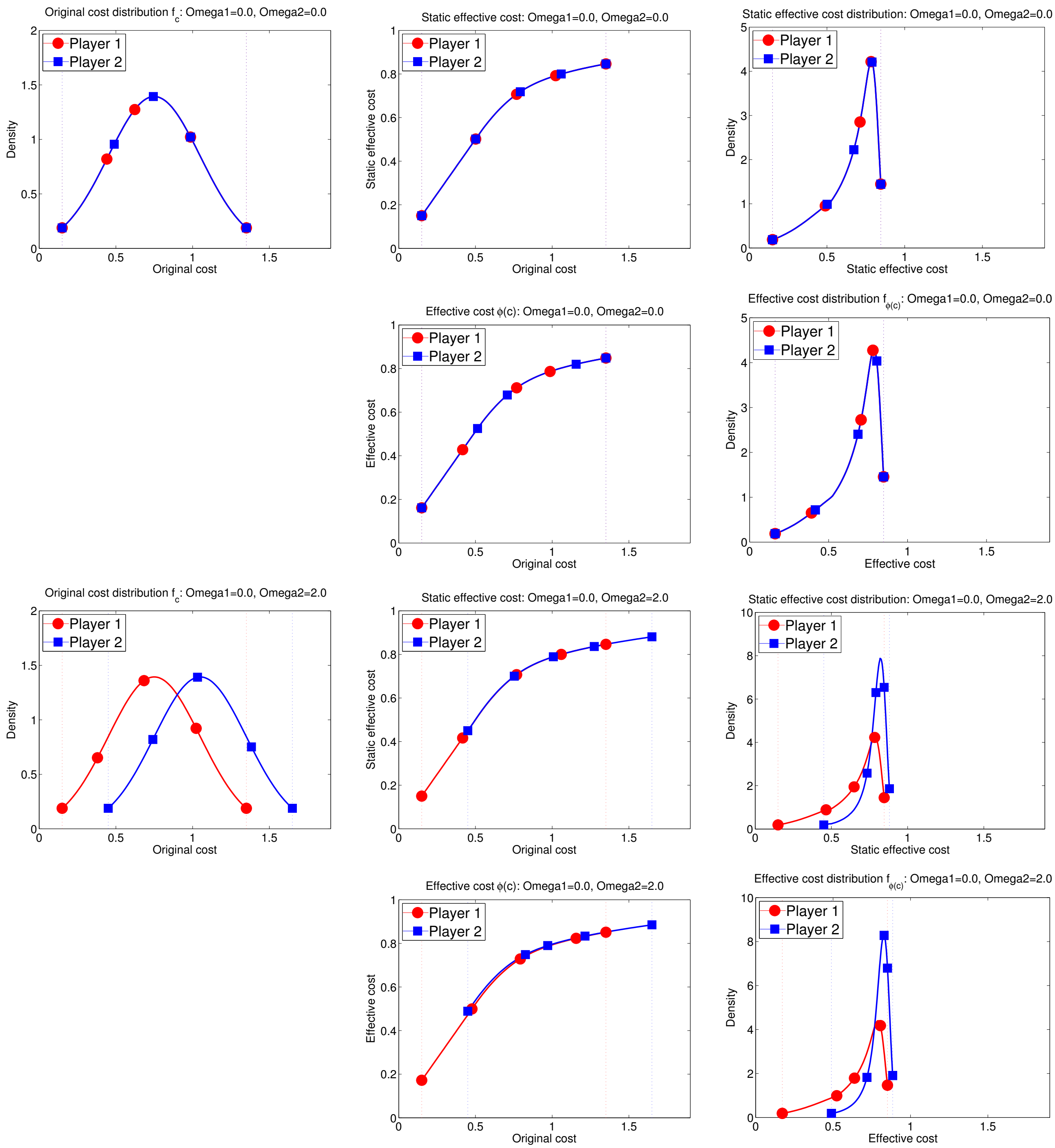
This figure shows the sections of the cost of procurement surface for various backlog levels of contractor 1 and as a function of the backlog level of contractor 2.

Figure 5: Subcontracting Strategy



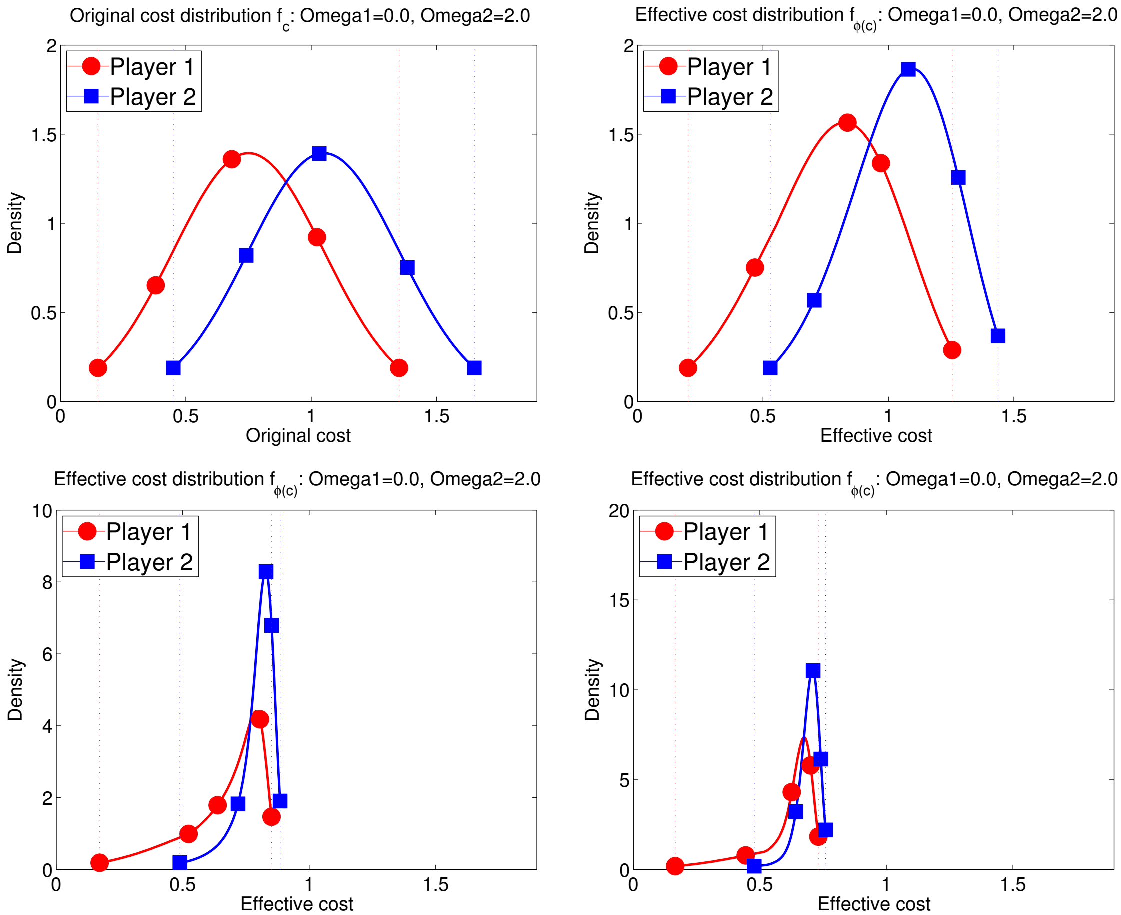
This figure shows the subcontracting strategy as a function of original costs under various backlog configurations and for different subcontracting supply schedules.

Figure 6: Distribution Density Functions of Original and Effective Costs



This figure compares the distributions of original and effective costs given a baseline subcontracting supply schedule and across different backlog configurations.

Figure 7: Distribution Density Functions of Original and Effective Cost



This figure compares the distributions of effective costs across different subcontracting supply schedules and for (0, 2) backlog configuration.

Table 3: Expected Subcontracting Levels

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 2$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 1$	$\omega_1 = 0$ $\omega_2 = 2$
No	0.00	0.00	0.00	0.00	0.00
Even steeper	0.11	0.18	0.25	0.11	0.11
Steeper	0.28	0.44	0.56	0.28	0.29
Baseline	0.43	0.61	0.73	0.44	0.44
Flatter	0.50	0.67	0.78	0.51	0.51
Even flatter	0.57	0.73	0.82	0.58	0.58

This table summarizes the expected share of the project that is subcontracted under various backlog configurations and for different subcontracting supply schedules.

Table 4: Moments of the Distributions of Private Costs

Even steeper subcontracting schedule

ω_1	$E(c\bar{x})$	std($c\bar{x}$)	$E(\phi(c))$	std($\phi(c)$)	$E(\phi(c)_S)$	$std(\phi(c)_S)$	$E(\phi(c)_D)$	$std(\phi(c)_D)$
0.00	0.75	0.26	0.75	0.24	0.73	0.24	0.025	0.001
1.00	0.90	0.26	0.92	0.22	0.86	0.22	0.059	0.006
2.00	1.05	0.26	1.04	0.20	0.98	0.21	0.063	0.007

Steeper subcontracting schedule

ω_1	$E(c\bar{x})$	std($c\bar{x}$)	$E(\phi(c))$	std($\phi(c)$)	$E(\phi(c)_S)$	$std(\phi(c)_S)$	$E(\phi(c)_D)$	$std(\phi(c)_D)$
0.00	0.75	0.26	0.70	0.19	0.69	0.20	0.013	0.004
1.00	0.90	0.26	0.82	0.15	0.79	0.16	0.033	0.013
2.00	1.05	0.26	0.90	0.12	0.87	0.13	0.024	0.011

Baseline subcontracting schedule

ω_1	$E(c\bar{x})$	std($c\bar{x}$)	$E(\phi(c))$	std($\phi(c)$)	$E(\phi(c)_S)$	$std(\phi(c)_S)$	$E(\phi(c)_D)$	$std(\phi(c)_D)$
0.00	0.75	0.26	0.65	0.16	0.65	0.16	0.006	0.003
1.00	0.90	0.26	0.74	0.11	0.72	0.12	0.018	0.012
2.00	1.05	0.26	0.79	0.07	0.78	0.08	0.011	0.008

Flatter subcontracting schedule

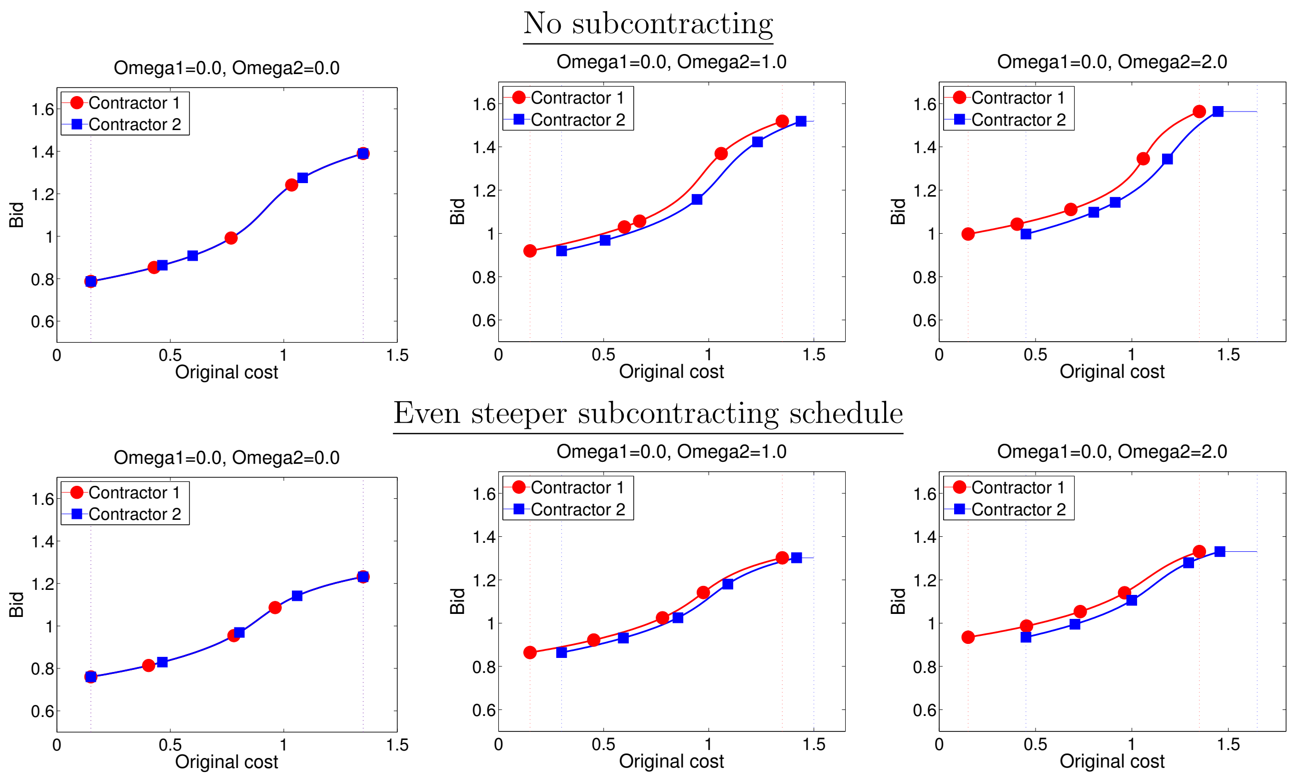
ω_1	$E(c\bar{x})$	std($c\bar{x}$)	$E(\phi(c))$	std($\phi(c)$)	$E(\phi(c)_S)$	$std(\phi(c)_S)$	$E(\phi(c)_D)$	$std(\phi(c)_D)$
0.00	0.75	0.26	0.63	0.14	0.62	0.14	0.005	0.003
1.00	0.90	0.26	0.70	0.09	0.69	0.10	0.014	0.011
2.00	1.05	0.26	0.74	0.06	0.73	0.06	0.008	0.006

Even flatter subcontracting schedule

ω_1	$E(c\bar{x})$	std($c\bar{x}$)	$E(\phi(c))$	std($\phi(c)$)	$E(\phi(c)_S)$	$std(\phi(c)_S)$	$E(\phi(c)_D)$	$std(\phi(c)_D)$
0.00	0.75	0.26	0.60	0.12	0.59	0.12	0.004	0.003
1.00	0.90	0.26	0.66	0.07	0.65	0.08	0.010	0.009
2.00	1.05	0.26	0.69	0.05	0.68	0.05	0.005	0.004

This table summarizes moments of the distribution of private costs as a function of own backlog while holding the competitor's backlog at 0. The second and third columns show the mean and the variance of the original costs (i.e., costs before subcontracting is taken into account), whereas the fourth and the fifth columns present the mean and the variance of the effective costs (i.e., costs after subcontracting is taken into account plus the continuation value of winning). Columns 6-7 and 8-9 show the moments of the static and dynamic components of effective costs separately.

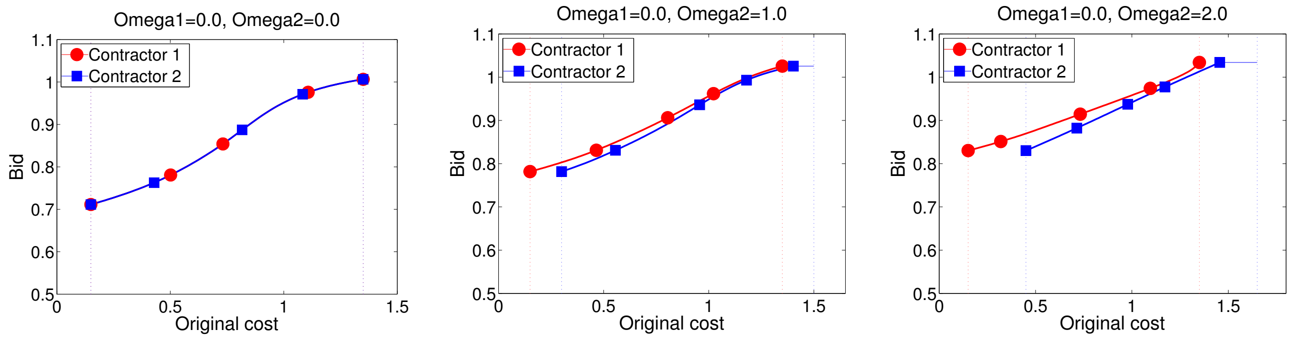
Figure 8: Bidding Strategy



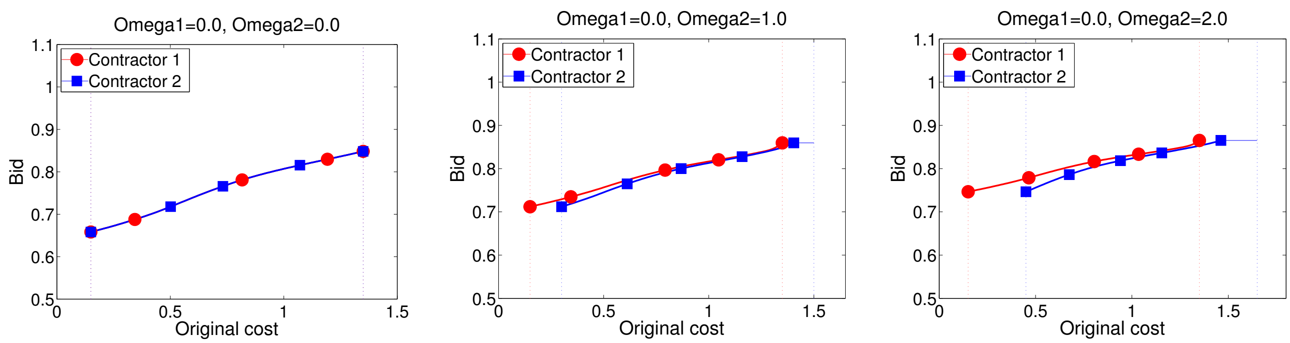
This figure shows the equilibrium bidding strategies as functions of respective effective costs for the model without subcontracting and the model with subcontracting under the "even steeper" subcontracting supply curve over various backlog configurations.

Figure 9: Bidding Strategy

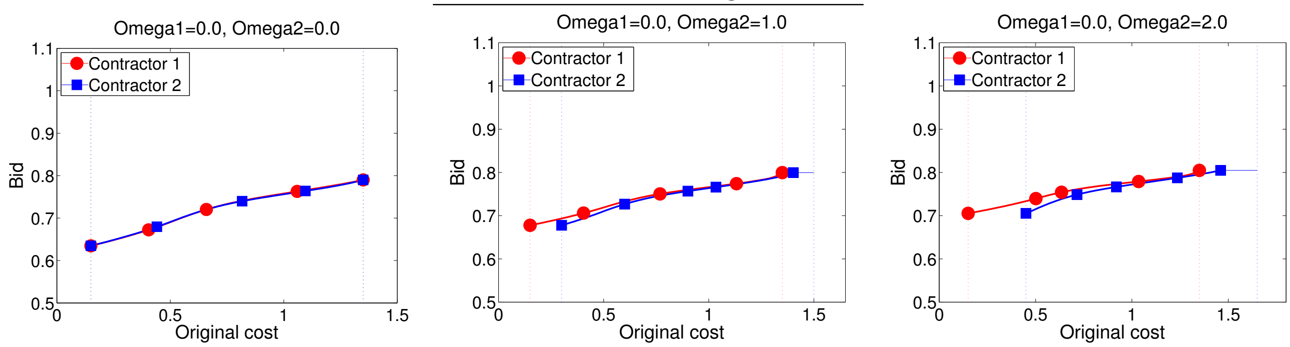
Steeper subcontracting schedule



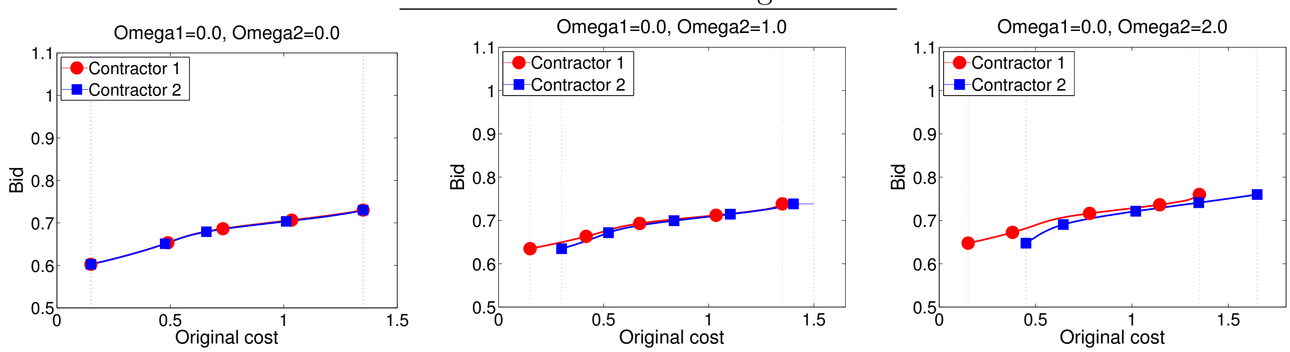
Baseline



Flatter subcontracting schedule



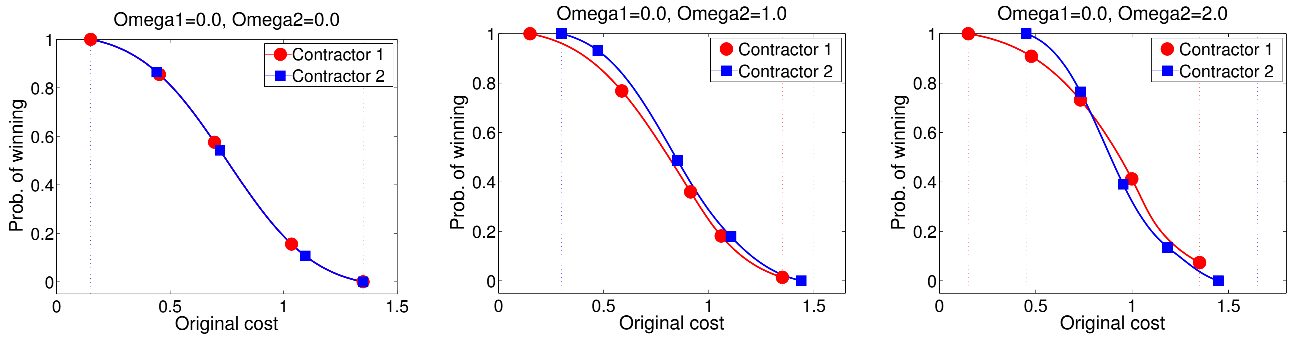
Even flatter subcontracting schedule



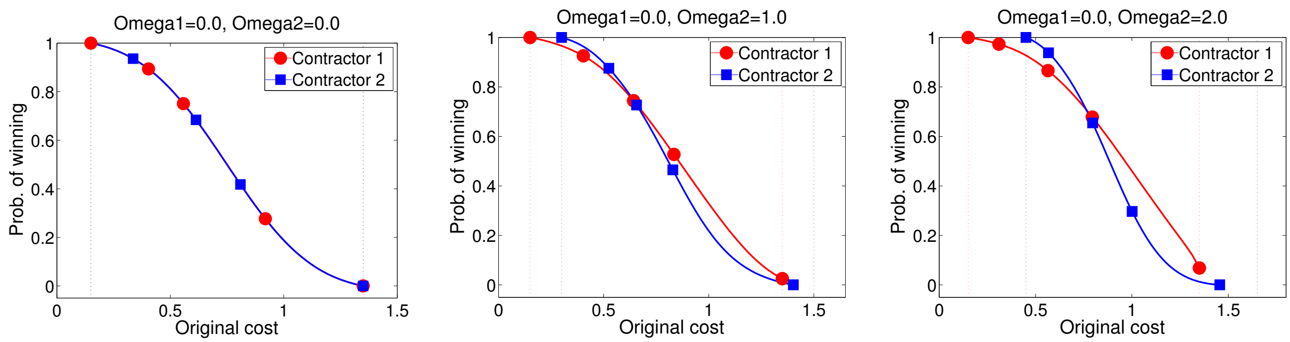
This figure shows the equilibrium bidding strategies as functions of respective effective costs for the models with subcontracting under various subcontracting supply schedules and for several different backlog configurations.

Figure 10: Probability of winning

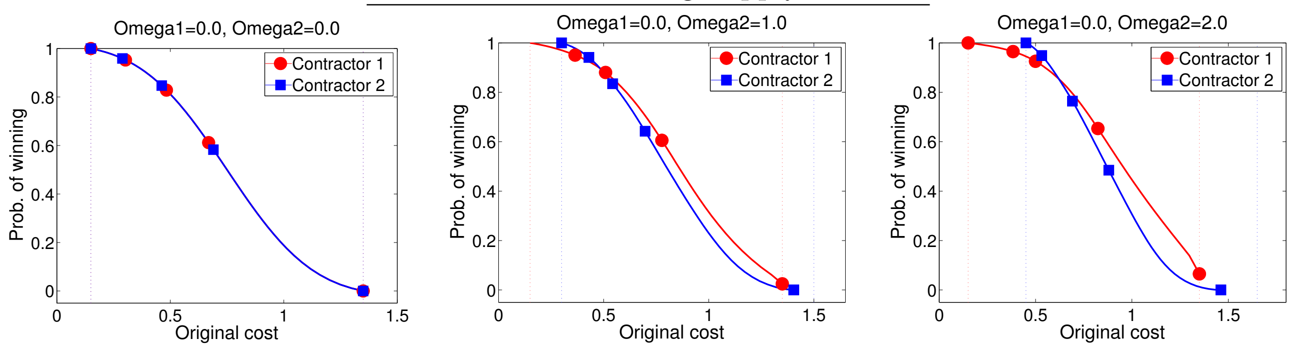
No subcontracting



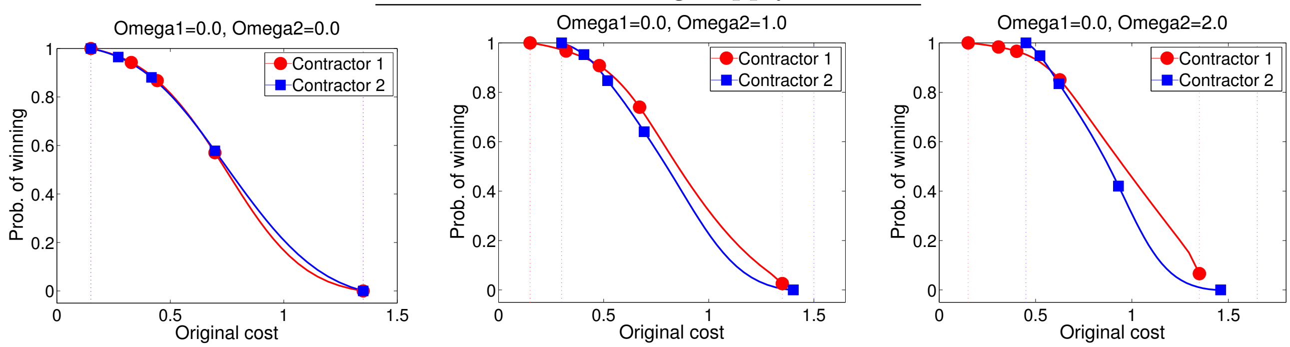
Steeper Subcontracting Supply Schedule



Baseline Subcontracting Supply Schedule



Flatter Subcontracting Supply Schedule



This figure shows the probability of winning under various backlog configurations and for different subcontracting supply schedules.

Table 5: Expected Bid

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 2$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 1$	$\omega_1 = 0$ $\omega_2 = 2$
No	1.02	1.17	1.27	1.14	1.18
Even steeper	0.96	1.07	1.14	1.04	1.08
Steeper	0.87	0.91	0.95	0.89	0.92
Baseline	0.76	0.80	0.83	0.79	0.81
Flatter	0.72	0.75	0.77	0.74	0.76
Even flatter	0.68	0.70	0.72	0.69	0.71

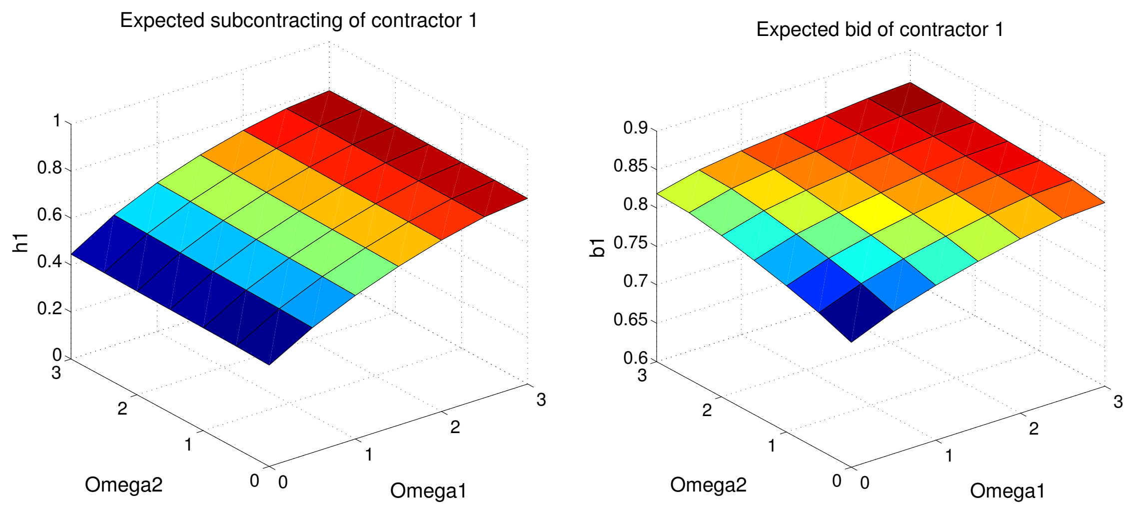
This table shows contractor's 1 expected bid under various backlog configurations and for different subcontracting environments.

Table 6: Expected Probability of Winning

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 2$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 1$	$\omega_1 = 0$ $\omega_2 = 2$
No	0.50	0.45	0.34	0.55	0.66
Even steeper	0.50	0.43	0.34	0.57	0.66
Steeper	0.50	0.41	0.32	0.59	0.68
Baseline	0.50	0.40	0.32	0.60	0.68
Flatter	0.50	0.40	0.31	0.60	0.69
Even flatter	0.50	0.40	0.35	0.60	0.65

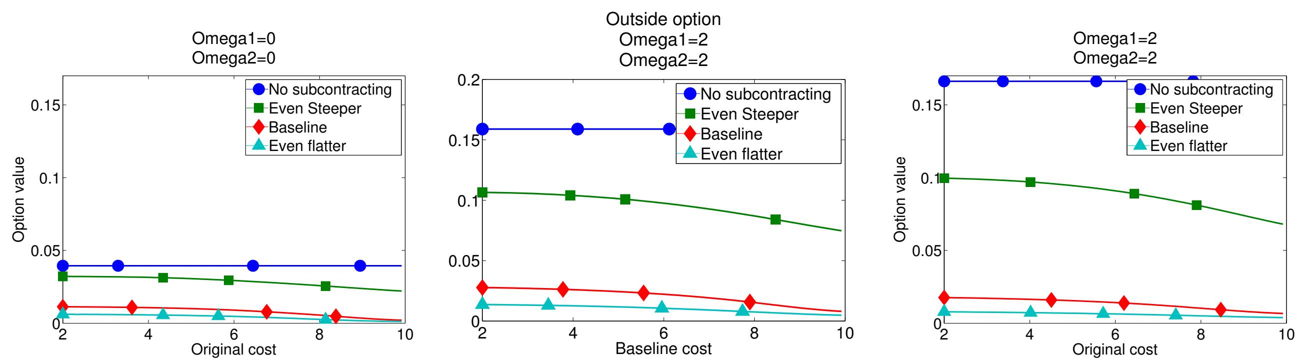
This table shows contractor's 1 expected probability of winning under various backlog configurations and for different subcontracting environments.

Figure 11: Expected Strategies



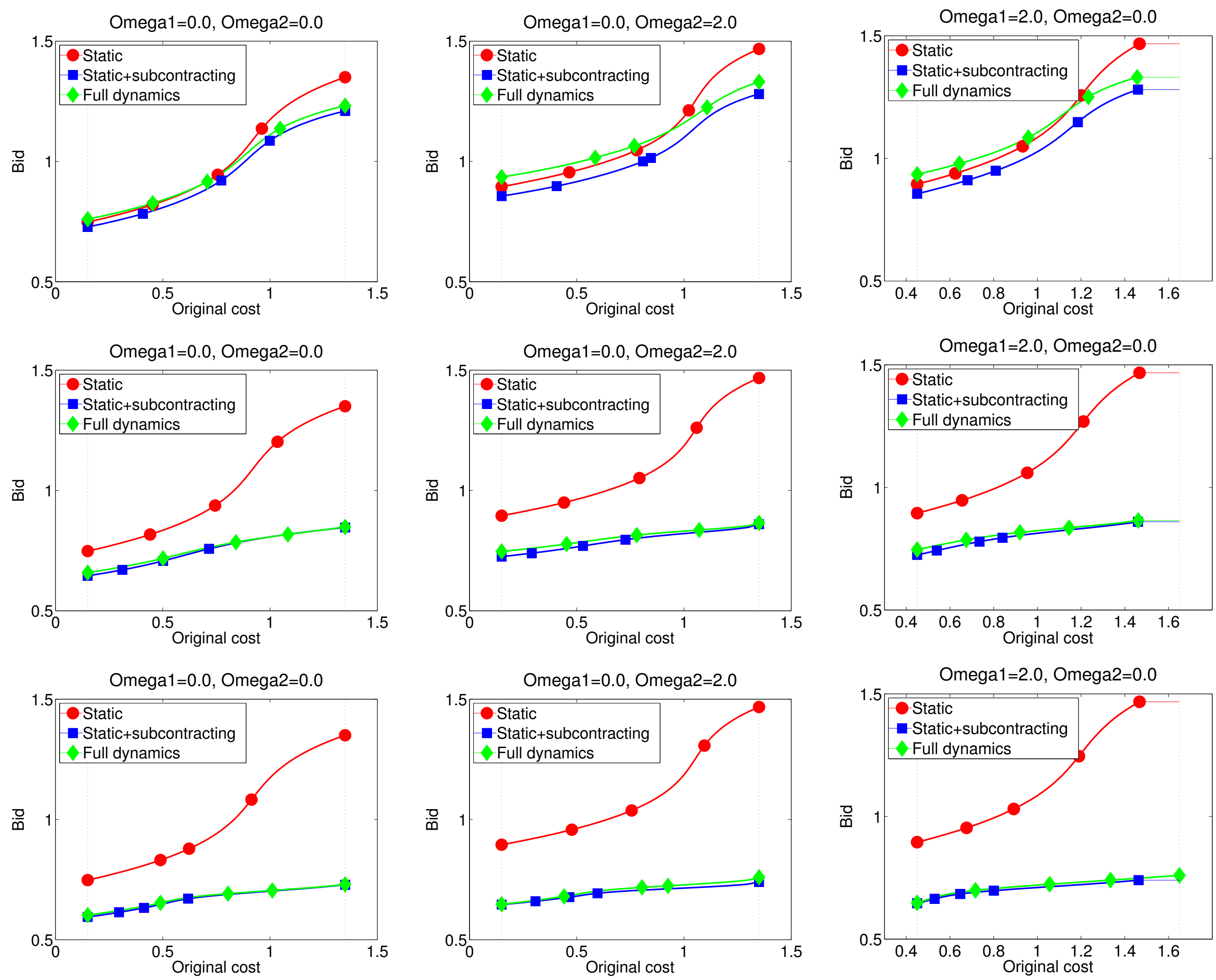
This figure shows expected subcontracting and bidding strategies of contractor 1 conditional on backlog configuration.

Figure 12: Option Value



This figure plots the effect of the option value on bids as a function of the contractor's private costs for symmetric states and across different subcontracting supply functions. This effect is computed using equation 16 from section 3.

Figure 13: Deconstruction of the Bidding Function



This figure isolates several different effects that characterize bidding behavior in the dynamic model with subcontracting. All the bidding functions are computed on the basis of the equilibrium subcontracting strategy from the fully dynamic model. The red line corresponds to the bidding strategy that would be optiomal in a static auction given the cost distributions implied by the corresponding backlog configuration. The blue line corresponds to the bidding strategy that would be optimal in a static auction given the cost distributions that coincide with the distributions of the static parts of the effective costs (implied by the subcontracting function from the dynamic model). The green line corresponds to the bidding strategy from a fully dynamic model. It adds option value considerations to the bidding behavior described by the blue line.

Table 7: Expected Bid Effect Associated with Option Value

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 2$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 1$	$\omega_1 = 0$ $\omega_2 = 2$
No	0.04	0.11	0.10	0.10	0.10
	4.0%	10.2%	8.9%	9.0%	8.8%
Even steeper	0.03	0.07	0.07	0.06	0.07
	3.0%	6.8%	7.1%	6.9%	7.0%
Steeper	0.01	0.03	0.03	0.03	0.02
	1.7%	3.3%	3.0%	3.0%	2.3%
Baseline	0.01	0.02	0.01	0.01	0.01
	0.7%	2.1%	1.7%	1.8%	1.4%
Flatter	0.01	0.01	0.01	0.01	0.00
	0.8%	1.6%	0.8%	0.9%	0.2%
Even flatter	0.00	0.01	0.01	0.01	0.01
	0.6%	1.2%	1.5%	1.1%	1.6%

The expected bid effect associated with the option value is computed as the expected difference between the optimal bid in a dynamic environment with subcontracting and the optimal bid in a static environment where the distribution of costs coincides with the static part of the effective costs.

Table 8: The Probability of Non-Participation

	$P_{non-part}$	$\bar{\phi}_2 - \bar{\phi}_1$	$\tilde{\phi}_2 - \bar{\phi}_1$	ΔV	$f_{\phi,2}(\bar{\phi}_2)$	$f_{\phi,2}(\tilde{\phi}_2)$
No	**0.07**	0.33	**0.13**	**0.00**	0.19	**0.58**
Even steeper	**0.06**	1.20	**0.49**	**-0.01**	0.05	**0.13**
Steeper	**0.07**	0.36	**0.15**	**-0.00**	0.20	**0.44**
Baseline	**0.06**	0.23	**0.09**	**-0.00**	0.28	**0.70**
Flatter	**0.06**	0.21	**0.08**	**-0.00**	0.30	**0.77**
Even flatter	**0.06**	0.19	**0.08**	**-0.00**	0.33	**0.84**

This table documents the probability of non-participation that arises due to the difference in supports of private costs across contractors participating in the same auction. The table further breaks this probability into several components using the formula for boundary conditions derived in Proposition 2:

$$(1 - F_{\phi,2}(\tilde{\phi}_2)) - ((\tilde{\phi}_2 - \bar{\phi}_1) - \delta E[V_1(\omega_1', \omega_2' + x(1 - h_2(\phi_2^{-1}(\tilde{\phi}_2), \omega)))] + \delta \underline{V}_1) f_{\phi,2}(\tilde{\phi}_2)) = 0$$

or

$$P_{non_part} = [\tilde{\phi}_2 - \bar{\phi}_1 + \Delta V] f_{\phi,2}(\tilde{\phi}_2)$$

. These three components are

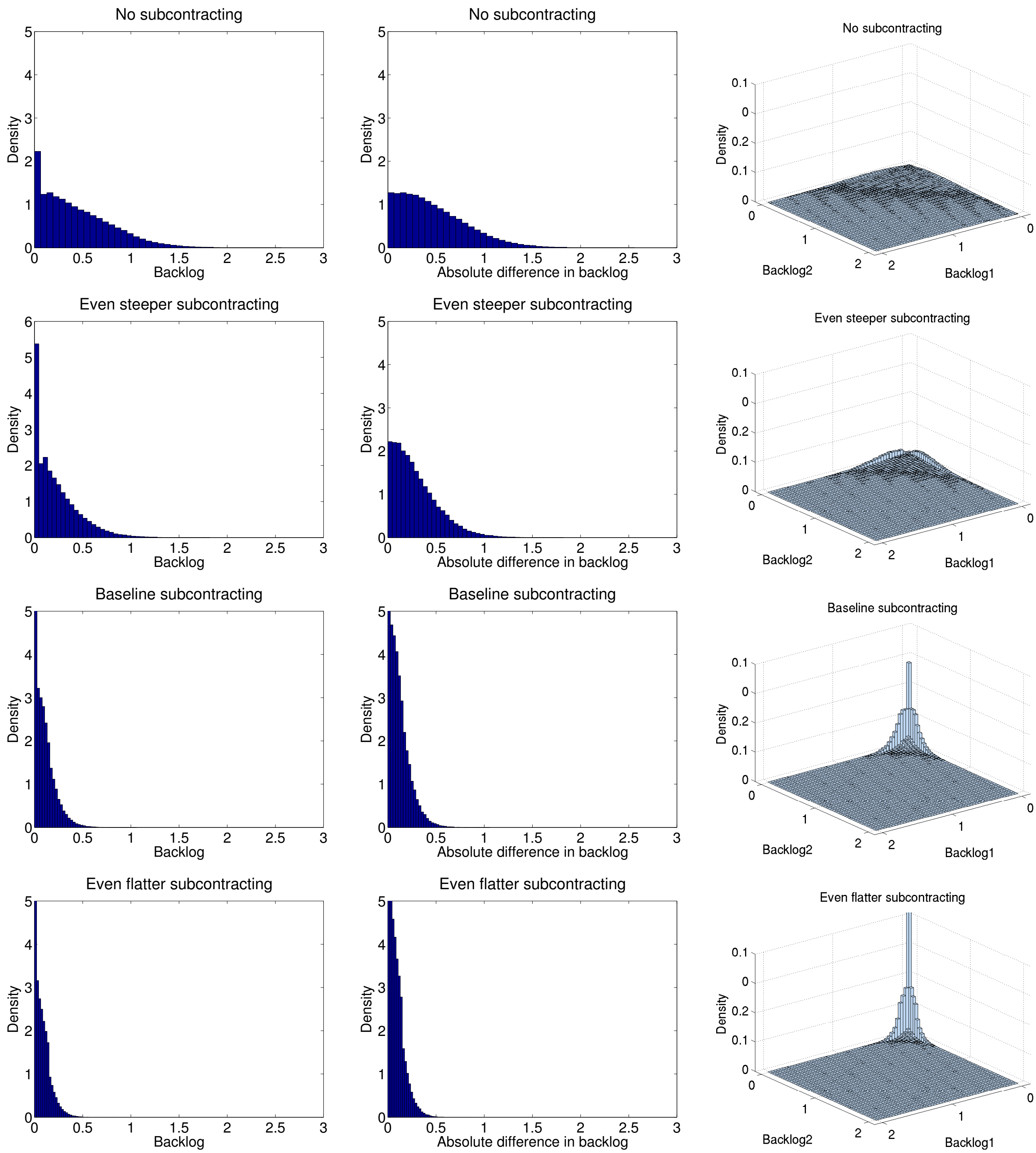
(i) The difference in the upper support bounds for participating contractors $\tilde{\phi}_2 - \bar{\phi}_1$

(ii) The differences in continuation values of losing: $\Delta V = \delta\Delta(E[V_1(\omega_1', \omega_2' + x(1 - h_2(\phi_2^{-1}(\tilde{\phi}_2), \omega))) - \underline{V}_1)$

(iii) The value of the effective cost density at the participation threshold: $f_{\phi,2}(\widetilde{\phi_2})$.

The results are shown for different subcontracting supply schedules and $(0, 2)$ backlog configuration.

Figure 14: Stationary Distribution of Backlogs



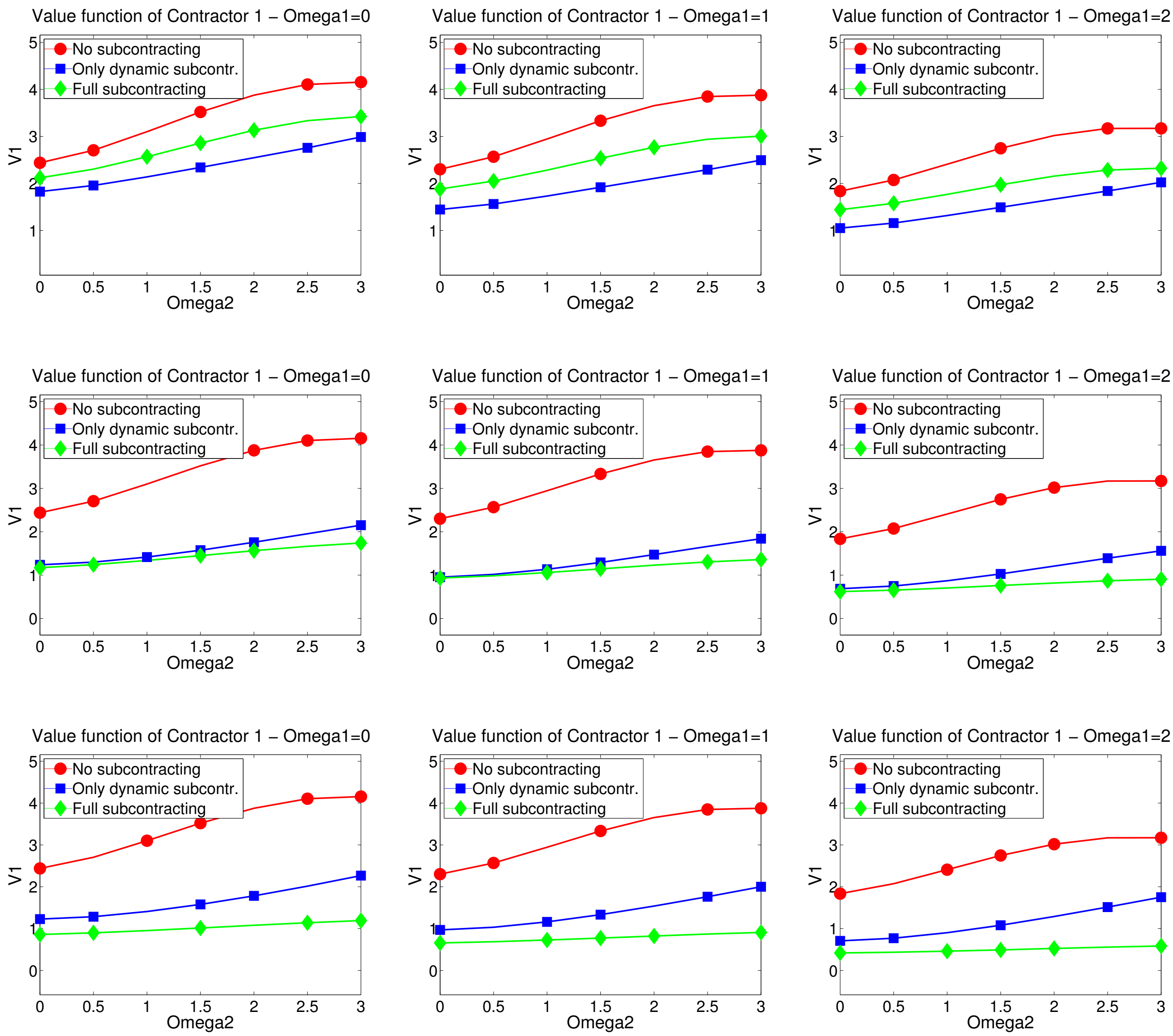
The graphs in the left column of this figure show the stationary distribution of the individual contractor's backlog, whereas the graphs in the middle column depict the distribution of differences in contractor's backlogs across different subcontracting supply functions. The right column shows the stationary joint distribution of competitors' backlogs.

Table 9: Summary of Equilibrium Variables

Subcontracting price schedule	Firm's profit	Procurement cost	Original cost	Effective cost	Work done by the firm	Backlog	Difference in backlog
No	0.20	1.06	0.66	0.66	1.00	0.44	0.47
Even steeper	0.16	0.94	0.63	0.62	0.93	0.23	0.29
	-19.27%	-11.01%	-4.75%	-6.29%	-6.90%	-48.67%	-39.26%
Steeper	0.12	0.83	0.62	0.59	0.83	0.12	0.17
	-39.24%	-21.25%	-7.13%	-10.75%	-16.81%	-73.32%	-64.18%
Baseline	0.09	0.75	0.61	0.57	0.73	0.07	0.11
	-55.14%	-29.50%	-8.09%	-14.44%	-27.20%	-83.51%	-75.89%
Flatter	0.08	0.71	0.61	0.55	0.67	0.06	0.09
	-60.63%	-32.88%	-8.45%	-16.56%	-33.14%	-86.99%	-80.28%
Even flatter	0.07	0.67	0.60	0.53	0.60	0.04	0.07
	-66.60%	-36.88%	-8.73%	-19.39%	-40.45%	-90.03%	-84.31%

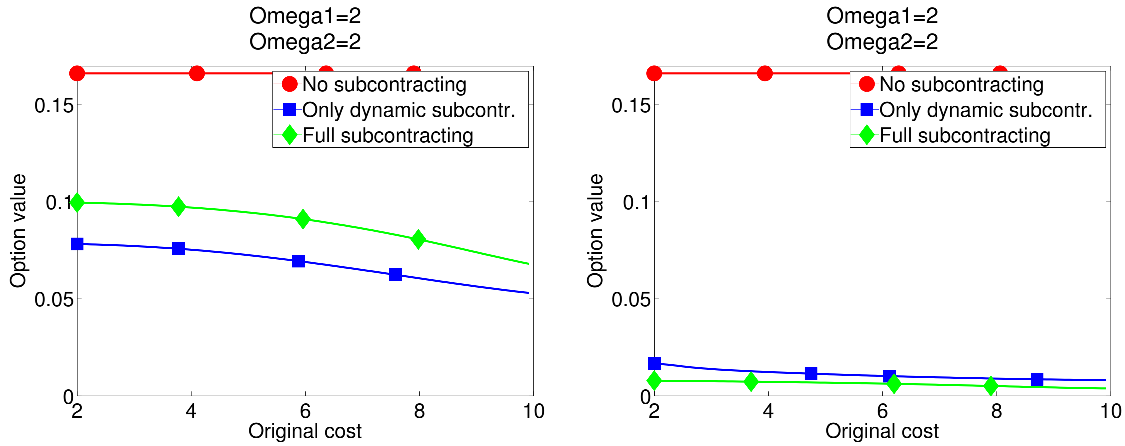
This table shows the expected values of various variables computed using the stationary distribution of states.

Figure 15: Isolating the Dynamic Effect of Subcontracting



This figure isolates the dynamic effect of subcontracting while holding the subcontracting strategies from the fully dynamic model fixed. More specifically, the green line is obtained in the following way: the equilibrium with subcontracting is computed in the first step; then the bidding strategies and the value function are re-computed while holding the equilibrium subcontracting strategies fixed but using them only in state transitions. Thus, in this experiment the adjustment in the static part of the effective costs due to subcontracting is eliminated. It is worth noting that this exercise does not compute the equilibrium without the static effect of subcontracting since subcontracting strategies take this effect into account. The figure plots the values to contractor 1 as functions of competitor's backlog for various subcontracting supply schedules.

Figure 16: Isolating the Dynamic Effect of Subcontracting: Option Value



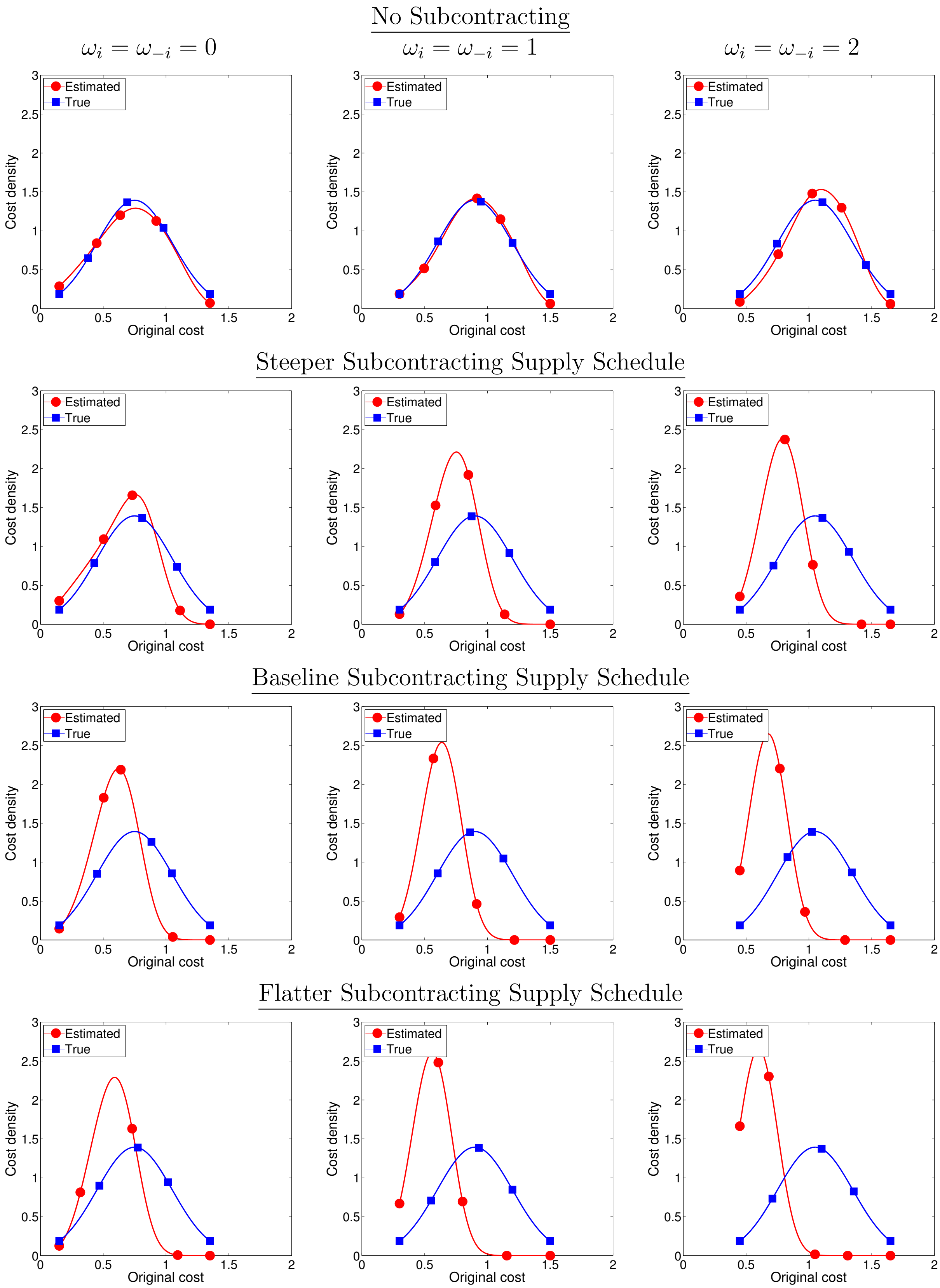
This figures depicts the option value effect on prices for the model without subcontracting, the partial effect model (see explanation of Figure 15), and the model with subcontracting.

Table 10: Quantifying Estimation Bias

	Subcontracting	No	Even steeper	Steeper	Baseline	Flatter	Even Flatter
$\omega_i = 0$	True mean	0.75					
	Estimated mean	0.73	0.71	0.67	0.59	0.57	0.54
$\omega_i = 1$	True mean	0.9					
	Estimated mean	0.90	0.86	0.74	0.63	0.62	0.57
$\omega_i = 2$	True mean	1.05					
	Estimated mean	1.08	0.97	0.79	0.69	0.64	0.61
$\Delta\omega_i = 1$	θ_2	0.15					
	Estimated mean	0.17	0.15	0.07	0.04	0.05	0.03
$\omega_i = 0$	True std. dev.	0.26					
	Estimated std. dev.	0.27	0.26	0.22	0.17	0.17	0.16
$\omega_i = 1$	True std. dev.	0.26					
	Estimated std. dev.	0.27	0.25	0.17	0.15	0.14	0.14
$\omega_i = 2$	True std. dev.	0.26					
	Estimated std. dev.	0.27	0.23	0.15	0.13	0.12	0.11

This table reports the estimated means and standard deviations of the distributions of private project costs recovered under the assumption of no subcontracting from the data generated by the model without subcontracting as well as the model with subcontracting and subcontracting supply functions respectively given by schedules listed in the columns of the table.

Figure 17: Estimation Bias



11.3 Optimal Procurement Policy

This exercise compares procurement policies that differ in the size of projects as well as in the frequency with which the projects are auctioned. More specifically, the policies we consider can be summarized as (x_0, q_0) where x_0 and q_0 denote the size and the frequency, respectively. In our benchmark model, projects are auctioned every period ($q_0 = 1$) and are of size $x_0 = \bar{x}$. We study a set of policies that auction the same amount of work on average, i.e., $x_0 q_0 = \bar{x}$.

Table 11: Optimal Procurement Policy (no subcontracting)

	Firm's profit	Procurment cost	Firm's cost	Backlog	Difference in backlog
$(\bar{x}, 1)$	0.20	1.06	0.66	0.44	0.47
$(3\bar{x}, \frac{1}{3})$	0.21	1.19	0.77	1.13	0.74
Change relative to $(\bar{x}, 1)$	6.50%	12.27%	15.57%	157.18%	55.56%
$(6\bar{x}, \frac{1}{6})$	0.20	1.17	0.77	1.22	0.90
Change relative to $(\bar{x}, 1)$	1.10%	10.87%	16.42%	176.25%	90.76%
$(9\bar{x}, \frac{1}{9})$	0.19	1.15	0.76	1.20	0.95
Change relative to $(\bar{x}, 1)$	-3.10%	8.65%	15.21%	172.82%	100.89%

Table 12: Optimal Procurement Policy (subcontracting)

	Firm's profit	Procurement cost	Original cost	Effective cost	Work done by the firm	Backlog	Difference in backlog
$(\bar{x}, 1)$	0.09	0.75	0.61	0.57	0.73	0.07	0.11
$(3\bar{x}, \frac{1}{3})$	0.09	0.76	0.63	0.59	0.51	0.22	0.29
Change relative to $(\bar{x}, 1)$	-1.21%	1.99%	4.16%	3.27%	-29.27%	204.31%	154.99%
$(6\bar{x}, \frac{1}{6})$	0.09	0.77	0.65	0.60	0.48	0.38	0.48
Change relative to $(\bar{x}, 1)$	-4.10%	3.17%	7.08%	5.32%	-33.60%	418.95%	318.88%
$(9\bar{x}, \frac{1}{9})$	0.08	0.77	0.66	0.60	0.47	0.49	0.61
Change relative to $(\bar{x}, 1)$	-5.73%	3.57%	8.90%	6.44%	-36.06%	575.65%	435.53%