# A Semi-Structural Methodology for Policy Counterfactuals and an Application to Fiscal Unions*

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**Abstract**

I propose a methodology to conduct counterfactual analysis in a way that is robust to specific assumptions about primitives of linear models of dynamic stochastic economies. First, I show how to identify a set of models that yield the same counterfactual equilibrium after a policy change by imposing restrictions directly on equilibrium equations. Second, I describe how to construct this counterfactual equilibrium using data under a benchmark policy. This allows obtaining quantitative predictions with respect to policy changes with minimal *a-priori* structural assumptions while being immune to Lucas critique, enhancing credibility of the analysis. Then, I apply the methodology to quantify how fiscal unions contribute to regional stabilization. I focus on models where the federal government redistributes resources via a *transfer policy rule* in order to smooth local shocks. This rule is a function of local variables. Using US state-level data, I construct a counterfactual US economy without the rule in place. This counterfactual is identical in many fiscal union models with rich features, such as nominal rigidities and asset market incompleteness. My primary finding is that during the Great Recession fiscal integration significantly reduced cross-state employment differences by redistributing resources from states that were doing relatively well to states that were doing relatively poorly. Finally, I discuss how the methodology can further be used to falsify a set of models, provided data before and after a policy change are available.

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# 1 Introduction

During the Great Recession many US states were hit with large negative shocks that depressed their economies. How would they have fared had they not been members of a fiscal union? The most common approaches to answer this question, as well as many others in economics, are the *structural approach* and the *reduced-form approach*. The former relies on specifying primitives of a particular model and identifying its parameters whereas the later is model-free but typically subject to the critique in Lucas (1976).

In this paper, I propose a methodology to conduct counterfactual analysis in a way that is robust to particular assumptions about model primitives and parameterizations while being immune to Lucas critique. First, I show how to identify a set of linear models that yield the same counterfactual equilibrium after a policy change in dynamic stochastic economies. This is achieved by imposing restrictions directly on structural equations characterizing the equilibrium. Second, I describe how to construct this counterfactual equilibrium using data under a benchmark policy. Thus, the methodology allows researchers and policy-makers to obtain quantitative predictions with respect to changes in systematic components of policy[1] with minimal *a-priori* structural assumptions, enhancing credibility of the analysis. This is the original motivation of Sims (1980) as well as more recent literature using *sufficient statistics*.

I apply this methodology to quantify how fiscal unions contribute to regional stabilization by redistributing resources between its members through the federal tax-and-transfer system. This application provides much needed quantitative results that go beyond existing reduced-form calculations and calibration exercises in specific models,[2] and inform current discussions on European fiscal integration. In addition, I use this application to transparently illustrate the methodology. I focus on models where the federal government redistributes resources between the union members via a *transfer policy rule* that is a function of local variables. These transfers smooth the response to local, temporary shocks.[3] Using US state-level data, I construct a counterfactual US economy without a transfer policy rule. This counterfactual is identical in a set of fiscal union models whose equilibrium equations satisfy a number of simple exclusion restrictions. The set is interesting because it encompasses models with rich features, such as nominal rigidities, adjustment costs, asset market incompleteness, and correlated exogenous processes. I find that, absent the transfer policy rule, employment differences across states in the Great Recession would have been significantly larger. This is because resources were redistributed from states that were doing relatively well to states that were doing relatively poorly. I conclude that US fiscal integration contributed to stabilization of regional economies.

The paper is divided into two parts. The first part is concerned with the application to fiscal unions and the second part with the generalization of the methodology to other applications. In

[1]As opposed to analysis with respect to exogenous components of policy, i.e, policy shocks.

[2]See Section 2 for examples and notable exceptions.

[3]As an analogy, a Taylor Rule for the nominal interest rate is a function of aggregate variables and smooths response to aggregate shocks.

the first part of the paper, I start by describing several models of an economy with many islands (or states) that are hit by idiosyncratic local shocks, and where asset markets are incomplete. For instance, I present models with and without nominal rigidities, or where leisure is a durable good. The goal is to show examples where the federal transfer policy rule can be evaluated, emphasizing their differences and commonalities. Moreover, I discuss the economic intuition behind the methodology using two of these models. The two models have very different economic mechanisms (i.e., leisure durability v. wage rigidities) but are observationally equivalent and produce identical counterfactual equilibria for alternative transfer policy rules.

Next, I present the methodology to quantify the contribution of fiscal unions to regional stabilization with minimal *a-priori* structural assumptions. I focus on fiscal union models that satisfy three properties. The first is that state-level economies in log-deviations from the aggregate union behave to a first-order approximation as if they were small open economies—economies that are independent of others in the union and differ only in the realizations of purely idiosyncratic shocks that hit them.[4] The second property is that a sufficient set of variables for characterizing the state-level equilibrium in log-deviation from aggregates includes employment, nominal wages, and assets, in addition to exogenous processes for the discount rate of households, wealth of households, and productivity of firms. The third is that the federal government gives lump-sum transfers that are a function of state-level employment, wages, and assets.[5] This function is the transfer policy rule. Following this, I write a linear dynamic system of equations that characterizes equilibria in these models. It involves three equations: a log-linearized Euler equation, a sequential budget constraint, and an equation describing the labor market equilibrium. Moreover, I assume that these equilibria have a unique, stable structural vector autoregression (SVAR) representation.

Then, I present the main theoretical result that allows me to construct a counterfactual equilibrium without a transfer policy rule. I show that (i) knowledge of the SVAR representation of the equilibrium with a transfers policy rule and (ii) imposing a number exclusion restriction on the three equilibrium equations is sufficient to identify the set of fiscal union models that yield identical counterfactual SVAR representation of the equilibrium without a transfer policy rule. Furthermore, I show that the counterfactual SVAR representation can be constructed provided knowledge of the transfer policy rule. The result is useful because many rich and interesting models of fiscal unions (e.g., with nominal wage rigidities, durable leisure, asset market incompleteness, and correlated exogenous processes) are indeed consistent with the exclusion restrictions I impose.

I build the result on the following insights. Log-linearized fiscal union models can be described by the coefficients associated with state-level variables in the three equilibrium equations. These coefficients involve combinations of parameters resulting from model primitives (e.g., pref-

[4]This is an assumption about aggregation of linear economies with idiosyncratic shocks. The key conditions are that identical parameters associate with the variables in each system characterizing the equilibrium, and the idiosyncratic shocks "wash out" when aggregated. The special cases I described in the previous section satisfy these.

[5]In Section 6.7, I consider the case where taxes and transfers distort decisions at the margin.

erences and technologies). I note that models with different primitives or parameterizations but identical values for the coefficients have identical SVAR representation of the equilibrium. The standard approach to find such equilibrium is to describe the primitives, pick parameter values, and solve a *non-linear* system of equations for the SVAR given coefficient values.[6] However, the system of equations is *linear* in the coefficients given values for the SVAR. This implies that imposing a number of exclusion restrictions (i.e., zero coefficients) makes the system exactly determined, thus identifying the set of models that are observationally equivalent under a given transfer policy and yield identical counterfactual equilibrium without it. For example, I impose that assets do not enter in the Euler equation or the discount rate process does not enter in the sequential budget constraint, which is true in many models of fiscal unions. Lastly, all identified coefficients in the Euler and labor market equations are invariant to policy changes because the transfer policy rule only appears in the sequential budget constraint in an additive fashion. This implies that, given knowledge of the rule in place, it is possible to construct the counterfactual equilibrium without the transfer policy rule.

I follow by constructing a counterfactual US economy without a transfer policy rule redistributing resources between states. I use annual state-level data on employment, wages, and assets between 2006 and 2011 to estimate all inputs required to implement the semi-structural methodology. I estimate a regional SVAR where I identify the structural shocks and impulse response matrix by using a set of theoretical restrictions on the joint response of variables in the SVAR to each type of shock. These restrictions are derived from the parameterized sequential budget constraint. This identification procedure extends results in Beraja et al. (2015b) to the larger class of models considered here.[7] This is an ancillary contribution of this paper. Furthermore, I estimate the federal transfer policy rule elasticities with respect to state-level employment, wages, and assets. I use both OLS and, to deal with potential bias due to reverse causality, two-stage least squares. I instrument potentially endogenous variables using local house prices and the SVAR shocks that are orthogonal to the innovation in the policy rule but correlated with the endogenous variables.[8] Results are similar in all specifications. I find that net transfers increase between 20 and 30 percent for every 1 percent decrease in nominal total wage income, and asset variation is not significant in explaining transfers variation once wages and employment are considered.[9]

I conclude the first part of the paper by presenting primary findings of the counterfactual exercise. The employment rate's cross-state standard deviation in the US in 2010 was 2.6 percent. I find that the standard deviation would have been 3.5 percent without the transfer policy

[6]For example, by guessing a recursive equilibrium exists and then using the method of undetermined coefficients.

[7]Identification of the impulse response matrix relies on a series of theoretical linear restrictions that are implied by these equations, and link the reduced form errors to the structural shocks.

[8]OLS estimates are consistent if the error term in the policy rule has no regional component (or if the error term is measurement error). However, the issue of endogeneity might arise because of reverse causality. When the innovation in the transfer policy rule causes regional variation in employment, wages, and assets, they both cause and are caused by transfers.

[9]The estimates are similar to those in Feyrer and Sacerdote (2013), who find a 0.25 decrease, and Bayoumi and Masson (1995), who find a 0.31 decrease.

rule redistributing resources from well performing to poorly performing states. The results are similar for 2009 and 2011, and in the stationary distribution (which I construct using Monte Carlo simulations). To put these magnitudes in context, aggregate output volatility during the pre-Volcker period (1960:1 to 1979:2) was 1.8 whereas during the post-Volcker-disinflation period (1982:4 to 1996:4) it was 1.3.[10] The stabilization role of fiscal integration seems to be in the same order of magnitude than this "Great Moderation". Moreover, I assess how nominal and real rigidities interact with fiscal integration—the *rigidities channel*—by constructing a semi-structural transfer policy counterfactual in an economy without nominal or real rigidities, i.e, an economy where labor supply and demand are static. This allows me to conduct a theoretical difference-in-differences experiment to quantify the interaction between rigidities and the transfer policy rule. I find that rigidities amplified the gains from fiscal integration in poorly performing states during the Great Recession, accounting for one-third of their counterfactual employment gains, whereas it accounted for none of the counterfactual losses in the well performing states. Finally, I show that main quantitative findings are insensitive to alternative parameterizations and specifications of the policy rule. In particular, I allow for the possibility that taxes and transfers distort labor supply decisions.[11]

In the second part of the paper, I generalize the methodology to a larger class of models that are commonplace in macroeconomics. I start by describing a class of linear dynamic systems that characterize (or approximate) stable equilibria in rational expectations models. I emphasize the distinction between the part of the system that is invariant to policy, i.e., the *structure* of the economy, and the part that is not, i.e., the *policy*. Within this class of models, I define a *semi-structural policy counterfactual* as a mapping from (i) the SVAR representation of an economy under a benchmark policy, (ii) a subset of the economy's policy-invariant structure (i.e., a *semi-structure*), and (iii) the benchmark and alternative policies onto the SVAR representation of an economy under the same structure with the alternative policy. The alternative SVAR representation is identical in the set of models that are consistent with the semi-structure. I show sufficient conditions for existence and uniqueness of the mapping, as well as discuss implementation. Finally, although I do not purse this application in this paper, I discuss how to use the methodology to falsify a set of models. Provided data before and after a policy change is available, it is possible to derive a simple *Wald* $\chi^2$ *test* that compares the reduced-form VAR representation predicted by a set of models to the observed representation after the policy change. The set of models is rejected if these are "too far apart" in a statistical sense.

The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 and Section 4 introduce examples of fiscal union models. Section 5 illustrates the methodology using fiscal union models. Section 6 constructs a counterfactual US economy without fiscal

[10]See Clarida et al. (2000) for details of this calculation and analysis of interest rate policy stabilization.

[11]Findings under this alternative policy are essentially unchanged because the estimated transfer policy rule elasticities with respect to wages and employment are close to 1. What is important for determining the agents marginal decisions is the marginal effect of employment (or wages) in taxes and transfers. When coefficients are close to 1, the marginal effect is close to zero.

integration and presents primary findings. Section 7 lays out the general methodology. Section 8 shows how to falsify a set of models via the methodology. Section 9 offers concluding remarks.

## 2 Literature review

This paper relates to much literature. The original motivation of Sims (1980) seminal contribution on structural vector autoregressions (SVARs) was to do policy analysis in a way that was robust across many models. SVARs have been very successful for doing analysis with respect to the exogenous component of policy (i.e., policy shocks) in a relatively model-free way. However, because of Lucas critique, it has not been possible to use SVARs to do analysis with respect to the systematic component of policy (i.e., policy rules).[12] The semi-structural methodology allows their use by restricting the set of structural models that generate the SVAR representation of the equilibrium, while being immune to Lucas critique and retaining robustness to model selection within this restricted set.

Furthermore, since seminal research by Harberger (1964), sufficient statistics have been used to evaluate the consequences of policy changes, without requiring full knowledge of the underlying structural model. Chetty (2008) provides an excellent survey. More recent examples in trade literature include Arkolakis et al. (2012) and Blaum et al. (2015), and in macroeconomics, Alvarez et al. (2014), Auclert (2014), and Atkeson and Burstein (2015). The non-parametric counterfactuals in Adao et al. (2015) are in the same spirit. The semi-structural methodology shares their motivation and philosophy, using reduced-form statistics to construct counterfactuals that are identical in a given set of models.

Economic literature on robust control and estimation, pioneered by Hansen and Sargent (see Hansen and Sargent (2008) for a summary), is also concerned with model misspecification and its consequences for policy decision-making. Their analysis is different from the one in this paper. However, we both emphasize the study of these issues in dynamic, stochastic economies. Consequently, we use similar techniques from time series econometrics (e.g. Hamilton (1994); Stock and Watson (2001); DeJong and Dave (2011) and for solving such models (e.g. Blanchard and Kahn (1980); Uhlig (1995))

I extend the scheme proposed by Beraja et al. (2015b) for identification of structural shocks in SVARs. This identification scheme fits nicely with the semi-structural philosophy in the paper by using certain parts of the theoretical model about which we feel more confident. It is part of growing literature developing "hybrid" methods that, for example, construct optimal combinations of econometric and theoretical models (Carriero and Giacomini (2011); Del Negro and Schorfheide (2004)), or use the theoretical model to inform the econometric model's parameter (An and Schorfheide (2007); Schorfheide (2000)). Other identification schemes that use restrictions derived from theory include sign restrictions as in Uhlig (2005), long run restrictions as in

[12] In the application to fiscal unions, policy shocks would be exogenous, unanticipated transfers to a state. On the other hand, I am concerned with the systematic, anticipated component of tax-and-transfer policy as a known function of state-level variables—a transfer policy rule.

Blanchard and Quah (1989)), and short-run restrictions as in Clarida and Gertler (1997); Christiano et al. (1999).

Del Negro and Schorfheide (2009) are also concerned with doing policy analysis in misspecified models. In particular, they study changes in the nominal interest rate policy rule. They consider a benchmark DSGE model with sticky prices and a SVAR representation of the equilibrium in this model. Then, they consider an alternative "true" model that might be different from the benchmark and is unknown. They write the SVAR representation of the equilibrium in this "true" model as a sum of the benchmark SVAR and perturbation terms. They consider various assumptions about the perturbation terms concerning how they are affected by changes in policy. Taking a stance on this is necessary to do counterfactuals that are immune to Lucas critique within their framework. For example, they conduct the analysis assuming that the perturbation terms are invariant to changes in the policy rule and that they can be estimated jointly with the benchmark DSGE model parameters from past data. This approach is useful for considering the sensitivity of results to model misspecification around the benchmark model. The semi-structural methodology in this paper is rather different, although this difference is subtle. While I do not describe alternative models in terms of deviations from a benchmark model in the paper, it is possible to do so and useful for comparison with their paper. In this case, I would write the structural equations characterizing the equilibrium in the "true" model as the sum of a benchmark model and a perturbation term. Then, I would state which perturbation terms are policy invariant and which ones are not. The key difference is that my perturbation terms are placed directly in the structure of the model (e.g. Euler equations, labor demand, etc.), where it is easy to see which models are consistent with these terms being policy invariant, whereas Del Negro and Schorfheide (2009) place them directly on the SVAR representation, where most models would imply that the terms are not policy invariant.

The theory behind the stabilization benefits of fiscal integration, or more generally aggregate risk sharing arrangements, is well developed. Examples include Farhi and Werning (2014); Bucovetsky (1998)); Persson and Tabellini (1996a,b); Lockwood (1999). However, there is surprisingly little quantitative and empirical research on regional stabilization in fiscal unions. On the purely empirical side, Sala-i Martin and Sachs (1991), Feyrer and Sacerdote (2013)), and Bayoumi and Masson (1995) focus on reduced form estimates of properties of the tax-and-transfer system in fiscal unions, and in some cases, present back-of-the-envelope counterfactuals. Asdrubali et al. (1996) quantify the amount of risk sharing among states in the United States by decomposing cross-sectional variance in output. Atkeson and Bayoumi (1993) examine evidence for private insurance of regional risk in the United States and Europe.

Regarding quantitative research, Evers (2015) is closest to the application in this paper. He provides a quantitative analysis of federal fiscal rules in monetary unions by using a fully structural DSGE model, which builds on Chari and Kehoe (2007) and Kollmann (2001). The model has two countries, nominal wage, and price rigidities, and incomplete markets. It includes productivity and government spending shocks alone. He calibrates the model and finds very small differences in nearly every relevant business cycle statistic when comparing a monetary union with and without a central fiscal authority. I hypothesize that there are primarily two reasons he does not find quantitatively significant benefits from fiscal integration, while the opposite is true for this paper. First, his model does not include shifters in the Euler equation (e.g., discount rate shocks or borrowing constraints). In contrast, I find that most of the regional employment stabilization because of fiscal integration is because the tax-and-transfer system stabilizes "discount rate" shocks, and not "productivity" shocks. Second, his is a calibration exercise in a fully specified model, and so it fails to account for several statistical properties of the joint distribution of the variables in the data (e.g., the autocorrelation of consumption or cross correlation between employment and output). Oh and Reis (2012) and McKay and Reis (2013) quantify the impact of targeted transfers and automatic stabilizers in an incomplete markets model, with heterogeneous agents and nominal rigidities. This paper shares their interest in the study of real-world tax-and-transfer systems and their macroeconomic consequences. However, they are largely concerned with implications for aggregate business-cycle stabilization, whereas I am concerned with implications for regional stabilization.

Moreover, the paper contributes to the recent surge in papers that exploit regional variation to highlight mechanisms of importance for economic fluctuations. Nakamura and Steinsson (2014) use sub-national US variation to inform the size of local government spending multipliers. Hurst et al. (2015) study regional redistribution through the US mortgage market. Mian and Sufi (2014), Mian et al. (2013), and Midrigan and Philippon (2011) exploit regional variation in the United States to explore the extent to which household leverage contributed to the Great Recession. Blanchard et al. (1992), Autor et al. (2013), and Charles et al. (2014) use regional variation to measure the responsiveness of labor markets to labor demand shocks.

Finally, the paper relates to econometrics literature on "partial identification" of structural models and how to perform statistical tests in these cases. Canay and Shaikh (2015) provides an extensive summary.

## 3 A Simple Fiscal Union Model

Consider an economy comprised of many islands, inhabited by a representative household and firm. The only other agent in the economy is a federal government. Households consume, work, and save/borrow in a non-state-contingent asset—a nominal bond in zero net supply. Firms produce final consumption goods using labor and intermediate goods. By assumption, the final consumption good is non-tradable, intermediate goods are tradable, and labor is not mobile across islands. Finally, each island has an exogenous endowment of intermediate goods. The federal government sets the nominal interest rate on the nominal bond, and gives lump-sum transfers to the islands. Assume that the nominal interest rate follows an endogenous rule that is a function of only aggregate variables (together with a fixed nominal exchange rate, this implies that the islands are part of a monetary union). Also, assume that federal transfers are a function of island-level variables alone. Throughout, I assume that parameters governing preferences and production are identical across islands and the islands only differ, potentially, in the shocks that hit them—these shocks include a shifter of the households discount rate, a productivity shifter in the production function of final goods, and the exogenous endowment of tradable intermediate goods. Finally, I assume that all labor, goods and asset markets are competitive.

### 3.1 Firms and Households

Final goods producers use labor $N^y_{kt}$ and intermediates $X_{kt}$ in island $k$ at time $t$ and face prices $P_{kt}$, wages $W_{kt}$, and intermediate prices $Q_t$ (equalized across all islands because of assumed tradability). Their profits are

$$\max_{N^y_{kt}, X_{kt}} P_{kt} e^{z_{kt}} (N^y_{kt})^{\alpha} (X_{kt})^{\beta} - W_{kt} N^y_{kt} - Q_t X_{kt}$$

where $z_{kt}$ is a productivity shock and $(\alpha, \beta) : \alpha + \beta < 1$ are the labor and intermediates shares. Unlike the tradable goods prices, final good prices ($P_{kt}$) vary across islands.[13]

Households preferences are given by

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} e^{-\rho_{kt} - \delta_{kt}} \frac{(C_{kt} - \frac{\phi}{1+\phi} N_{kt}^{\frac{1+\phi}{\phi}})^{1-\sigma}}{1-\sigma} \right]$$

where $C_{kt}$ is consumption of the final good, $N_{kt}$ is labor, $\delta_{kt}$ is an exogenous processes driving the household's discount rate.

Households are able to spend their labor income $W_{kt} N_{kt}$ plus profits accruing from firms $\Pi_{kt}$ and exogenous endowment of tradable goods $Q_t e^{\eta_{kt}}$, financial income $B_{kt} i_t$ and transfers from the government $S_{kt}$, where $B_{kt}$ are nominal bond holdings at the beginning of the period and $i_t$ is the nominal interest (equalized across islands given our assumption of a monetary union where the bonds are freely traded) on consumption goods ($C_{kt}$) and savings ($B_{kt+1} - B_{kt}$). Thus, they face the period-by-period budget constraint

$$P_{kt} C_{kt} + B_{kt+1} \leq B_{kt}(1 + i_t) + W_{kt} N_{kt} + \Pi_{kt} + S_{kt} + Q_t e^{\eta_{kt}}$$

A well known issue in the international macroeconomics literature is that under market incompleteness of the type we just described there is no stationary distribution for bond holdings across islands in the log-linearized economy; and all other island variables in the model have unit roots. This is problematic for reasons both theoretical (I will like to study log-deviations from a deterministic steady state) and empirical (regional data for the US does not suggest the

[13]It is worth noting that all model shocks will generate endogenous variation in markups given assumed decreasing returns to scale. Additionally, what I call a “productivity shock" is isomorphic to any shifter of unit labor costs and, hence, labor demand schedules. I will not attempt to distinguish between the different interpretations of this shock in this paper.

presence of such unit roots). I follow Schmitt-Grohe and Uribe (2003) and let $\rho_{kt}$ be the endogenous component of the discount factor that satisfies $\rho_{kt+1} = \rho_{kt} + \Phi(.)$ for some function $\Phi(.)$ of the average per capita variables in an island. As such, agents do not internalize this dependence when making their choices. This modification induces stationarity for an appropriately chosen function $\Phi(.)$. Schmitt-Grohe and Uribe (2003) show that alternative stationary inducing modifications (a specification with internalization, a debt-elastic interest rate or convex portfolio adjustment costs) all deliver similar quantitative results in the context of a small open economy real business cycle model.

### 3.2 Federal government

The federal government budget constraint is

$$B_t^g + \sum_k S_{kt} + Q_t G = B_{t-1}^g (1 + i_t)$$

where $G$ is some exogenous level of government spending in intermediate goods. The key feature of a fiscally integrated economy is that the federal government has the ability to redistribute resources across islands via transfers $S_{kt}$. If the islands where fiscally independent such transfers would not be possible.

I assume that the federal government announces a nominal interest rate rule $i_t = i(.)$ as a function of aggregate variables in the economy alone. Moreover, it announces a transfer policy rule as a function of per-capita employment, wages and assets in an island

$$S_{kt} = \bar{S}(\tilde{W}_{kt})^{\vartheta_w} (\tilde{N}_{kt})^{\vartheta_n} (\tilde{B}_{kt-1})^{\vartheta_b}$$

Again, agents do not internalize this dependence when making their choices.

### 3.3 Exogenous shocks and processes

I assume the exogenous processes are AR(1) processes, with an identical autoregressive coefficient across islands, and that the innovations are iid, mean zero, random variables with an aggregate and island specific component. First, define $\gamma_{kt} \equiv \delta_{kt} - \delta_{kt-1}$. Then,

$$\begin{aligned} z_{kt} &= \rho_z z_{kt-1} + \tilde{\sigma}_z v_t^z + \sigma_z u_{kt}^z \\ \gamma_{kt} &= \rho_\gamma \gamma_{kt-1} + \tilde{\sigma}_\gamma v_t^\gamma + \sigma_\gamma u_{kt}^\gamma \\ \eta_{kt} &= \rho_\eta \eta_{kt-1} + \tilde{\sigma}_\eta v_t^\eta + \sigma_\eta u_{kt}^\eta \end{aligned}$$

with $\sum_k u_{kt}^z = \sum_k u_{kt}^\gamma = \sum_k u_{kt}^\eta = 0$. By assumption, I assume the average of the regional shocks sum to zero in all periods.

The "discount rate" process $\gamma_{kt}$ is a shifter of a household's discount rate, but it can be viewed as a proxy for the tightening of household borrowing limits. Such shocks have been proposed by Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2011), and Mian and Sufi (2014) as an explanation of the 2008 recession. Beraja et al. (2015b) find that these broad types of shocks explain most regional employment variation across states in the United States during the Great Recession and its aftermath. The "productivity" process $z_{kt}$ can be interpreted as actual productivity, or a shifter of firm's demand for labor or firm's mark-ups. As an example, credit supply shocks to firms such as those proposed by Gilchrist et al. (2014) fall in this category. Finally, "wealth" process $\eta_{kt}$ is modeled as an endowment of intermediate goods but can be interpreted as shifters of the budget constraint that agents face such as exogenous changes in household wealth.

### 3.4 Equilibrium

An equilibrium is a collection of prices $\{P_{kt}, W_{kt}, Q_t\}$ and quantities $\{C_{kt}, N_{kt}, B_{kt}, N^y_{kt}, X_{kt}\}$ for each island $k$ and time $t$ such that, for an interest rate rule $i_t = i(.)$ and given exogenous processes $\{z_{kt}, \eta_{kt}, \gamma_{kt}\}$, they are consistent with household utility maximization and firm profit maximization and such that the following market clearing conditions hold:

$$\begin{aligned} C_{kt} &= e^{z_{kt}}(N^y_{kt})^{\alpha}(X_{kt})^{\beta} \\ N_{kt} &= N^y_{kt} \\ G + \sum_k X_{kt} &= \sum_k e^{\eta_{kt}} \\ 0 &= \sum_k B_{kt} + B^g_t \end{aligned}$$

### 3.5 Aggregation

The first important assumption for aggregation is that all islands are identical with respect to their underlying production and utility parameters.[14] The second assumption is that the joint distribution of island-specific shocks is such that its cross-sectional summation is zero. If $K$, the number of islands, is large this holds in the limit because of the law of large numbers. I log-linearize the model around this steady state and show that it aggregates up to a representative economy where all aggregate variables are independent of any cross-sectional considerations to a first order approximation.[15] I denote with lowercase letters an island variable's log-deviation from the aggregate union equilibrium. Lowercase letters with a tilde denote deviations from

[14]Given that the broad industrial composition at the state level does not differ much across states, the assumption that productivity parameters are roughly similar across states is not dramatically at odds with the data.

[15]The model we presented has many islands subject to idiosyncratic shocks that cannot be fully hedged because asset markets are incomplete. By log-linearizing the equilibrium we gain in tractability, but ignore these considerations and the aggregate consequences of heterogeneity. As usual, the approximation will be a good one as long as the underlying volatility of the idiosyncratic shocks is not too large. If our unit of study was an individual, as for example in the precautionary savings literature with incomplete markets, the use of linear approximations would likely not be appropriate. However, since our unit of study is an island the size of a state I believe this is not too egregious of an assumption. The volatilities of key economic variables of interest at the state level are orders of magnitude smaller than the corresponding variables at the individual level.

the steady state. For example, $n_{kt} \equiv \tilde{n}_{kt} - \tilde{n}_t$ and $\tilde{n}_t \equiv \sum_k \frac{1}{K}\tilde{n}_{kt} = \sum_k \frac{1}{K} log\left(N_{kt}/\bar{N}\right)$. I assume that the monetary authority announces the nominal interest rate rule in log-linearized form: $\tilde{i}_{t+1} = \varphi_\pi \mathbb{E}_t[\tilde{\pi}_{t+1}]$ where $\tilde{\pi}_t$ is the aggregate inflation rate. Finally, I assume that the endogenous component of the discount factor is $\Phi(.) = \Phi_0 n_{kt} + \Phi_1 w_{kt} + \Phi_2 n_{kt-1}$.[16]

The following lemma present the aggregation result and shows that we can write the island level equilibrium in deviations from these aggregates.

**Lemma 1** *For given $\{z_{kt}, \gamma_{kt}, \eta_{kt}\}$, the behavior of $\{w_{kt}, n_{kt}, b_{kt}, p_{kt}, c_{kt}, x_{kt}\}$ in the log-linearized economy for each island in log-deviations from aggregates is identical to that of a small open economy where the price of intermediates and the nominal interest rate are at their steady state levels, i.e. $\tilde{q}_t = \tilde{i}_t = 0\ \forall t$.*

**Proof.** See Appendix A for a proof. ∎

# 4 Transfers Policy in Alternative Models of Fiscal Unions

The previous section described a model where many islands (I will call them states from now on) were fiscally integrated. I showed that to a first order approximation we can study each of these states separately as if they were small open economies that receive transfers from abroad. In this section, I ask: how would the equilibrium change if the transfer policy rule changed? In particular, the case when transfers are not a function of local variables $\vartheta_n = \vartheta_w = \vartheta_b = 0$ is of interest because it correspond to the case when the states are not fiscally integrated. By comparing the equilibrium of the small open economy with and without fiscal integration, it is possible to analyze the contribution of fiscal unions to regional stabilization. As a reminder, Lemma 1 implies that the following equations are sufficient to characterize the equilibrium dynamics of wages employment and assets $\{w_t, n_t, b_t\}$ of the states in log-deviations from the aggregate. For simplicity, I will drop the $k$ notation indicating a given island from now on.

$$
\begin{aligned}
0 = \mathbb{E}_t \Bigg[ & \frac{\sigma\beta}{\frac{\alpha\phi}{1+\phi} - 1}(w_{t+1} + n_{t+1} - w_t - n_t) + (\alpha + \beta - 1)(n_{t+1} - n_t) \\
& + (\beta - 1)(w_{t+1} - w_t) + \Big(1 + \frac{\sigma}{\frac{\alpha\phi}{1+\phi} - 1}\Big)(z_{t+1} - z_t) + \Phi_0 n_{kt} + \Phi_1 n_{kt-1} - \gamma_{t+1} \Bigg] \qquad \text{(Euler)}
\end{aligned}
$$

$$0 = \left(\frac{1+\phi}{\phi} - (\alpha + \beta)\right) n_t - \beta w_t - z_t \qquad \text{(Labor Market (1))}$$

$$0 = -\frac{B}{S} b_t + \frac{B}{S}(1 + r) b_{t-1} - \frac{X}{S}(w_t + n_t) + \vartheta_n n_t + \vartheta_w w_t + \vartheta_b b_{t-1} + \frac{1}{S}\eta_t \qquad \text{(SB)}$$

[16] $\Phi_0, \Phi_2, \Phi_2$ are such that they induce stationary of island level variables in log-deviations from the aggregate. At the same time, since $\Phi(.)$ depends only on these deviations, the aggregate equilibrium will feature a constant endogenous discount factor $\rho$.

Together with the exogenous processes

$$
\begin{aligned}
z_t &= \rho_z z_{t-1} + \sigma_z v_t \\
\gamma_t &= \rho_\gamma \gamma_{t-1} + \sigma_\gamma u_t^\gamma \\
\eta_t &= \rho_\eta \eta_{t-1} + \sigma_\eta u_t^\eta
\end{aligned}
$$

If parameters in this system of equations are calibrated or estimated, and the system is solved for the equilibrium, it is straightforward to construct alternative equilibria for different transfers rules simply by changing $\{\vartheta_n, \vartheta_w, \vartheta_b\}$. However, this would give an answer to the contribution of fiscal integration to regional stabilization for this particular model I described. One could imagine alternative models of fiscal unions that are reasonable a priori. For example, one could entertain the possibility that nominal wages are rigid and use a partial-adjustment model where a fraction $\lambda$ of the gap between the actual and frictionless wage is closed every period. Formally, this implies a different (Labor Market) equation

$$
\beta \frac{1-\lambda}{\lambda}(w_t - w_{t-1}) = \left(\frac{1+\phi}{\phi} - (\alpha+\beta)\right) n_t - \beta w_t - z_t \qquad \text{(Labor Market (2))}
$$

A similar specification has been used by Shimer (2010) and, more recently, by Beraja et al. (2015b) and Midrigan and Philippon (2011). Shimer (2010) argues that in labor market search models there is typically an interval of wages that both the workers are willing to accept and firms willing to pay. To resolve this wage indeterminacy he considers a wage setting rule that is a weighted average of a target wage and the past wage. The target wage in this case is the value of the marginal rate of substitution.

Alternatively, one could entertain the possibility that employment is durable as in Kydland and Prescott (1982). Formally, this implies a different (Labor Market) equation

$$
\frac{1+\phi}{\phi}\nu n_{t-1} = \left(\frac{1+\phi}{\phi} - (\alpha+\beta)\right) n_t - \beta w_t - z_t \qquad \text{(Labor Market (3))}
$$

where parameter $\nu$ governs how much one-period lagged employment choices affect current utility. Moreover, this implies a different Euler equation as well because the marginal utility of consumption is affected.

$$
\begin{aligned}
0 = \mathbb{E}_t \Bigg[ & \frac{\sigma\beta}{\frac{\alpha\phi}{1+\phi} - 1}(w_{t+1} + n_{t+1} - w_t - n_t) + (\alpha+\beta-1)(n_{t+1} - n_t) + (\beta-1)(w_{t+1} - w_t) \\
& + (1 + \frac{\sigma}{\frac{\alpha\phi}{1+\phi} - 1})(z_{t+1} - z_t) + \left(\Phi_0 - \frac{\sigma\alpha\nu}{(\alpha\frac{\phi}{1+\phi} - 1)}\right) n_{kt} + \left(\Phi_1 + \frac{\sigma\alpha\nu}{(\alpha\frac{\phi}{1+\phi} - 1)}\right) n_{kt-1} - \gamma_{t+1} \Bigg]
\end{aligned}
$$

(Euler (3))

Note that one could entertain other models with ever increasing complexity as well. To name a few, models with alternative utility functions (e.g., GHH preferences, CRRA or CARA preferences, with and without habits or durability in consumption and employment, etc.), adjustment costs for assets, debt-elastic interest rates, price and wage rigidities both forward- and backward-looking, other shocks (e.g., labor wedge shocks). The last two alternative models are particularly interesting because they both amplify the response of employment to shocks relative to response of wages. This allows to match the large employment volatility relative to wage volatility that we observe in actual economies. Hence, as a pure exercise in matching observed data generated by one of these models under a given transfer policy rule they could both do well. In this sense, some of their positive implications are observationally equivalent. I will exploit this lack of identification when constructing counterfactual equilibria for alternative transfer policy rules. I will argue that these and other models belong to a set that is indeed identified by their vector autoregression representation to the equilibrium, even if particular models are not, and that counterfactual equilibria are identical in all these models. The next section presents an example in this spirit. However, it is important to keep in mind that normative implications may be quite different, for example in models (2) and (3), because of the presence of nominal wage rigidities in the former. This cautions against the use of models in this set—that are observationally equivalent—for making normative statements, and it teaches us about when more data is needed to separately identify models in order to make such statements.

### 4.1 Different mechanisms, same counterfactual...

The purpose of this section is to show an example of fiscal union models with very different economic mechanisms that: (i) are observationally equivalent given data on employment, wages and assets alone, and (ii) produce identical counterfactual equilibrium for alternative transfer policy rule. This example is useful to gain some economic intuition before proceeding to the next section, which is more technical in nature and presents the main theoretical result for constructing semi-structural transfer counterfactuals.

I consider a model with both wage rigidities and durability in employment—combining models (2) and (3) from the previous section. For simplicity, I focus on the special case where the exogenous processes are not persistent ($\rho_\gamma = \rho_z = \rho_\eta = 0$). First, consider a benchmark parameterization $\xi^0 \equiv \{\lambda^0, \nu^0, ...\}$ and transfer policy $\{\vartheta_n^0, \vartheta_w^0, \vartheta_b^0\}$ The equilibrium $\{n_t^0, w_t^0, b_t^0\}$ can be written in recursive form as,

$$\begin{bmatrix} n_t^0 \\ w_t^0 \\ b_t^0 \end{bmatrix} = \rho^0 \begin{bmatrix} n_{t-1}^0 \\ w_{t-1}^0 \\ b_{t-1}^0 \end{bmatrix} + \Lambda^0 \begin{bmatrix} u_t^\gamma \\ u_t^z \\ u_t^\eta \end{bmatrix}$$

where $\rho^0, \Lambda^0$ are functions of structural parameters and the transfer policy rule. Second, consider an alternative parameterization $\xi^1$. Is it possible to find such parameterization that is consistent with $\{\rho^0, \Lambda^0\}$? The answer is yes, as the following claim shows, and it implies that the parameters are not identified given the recursive representation of the equilibrium. In other words, there are many models that are observationally equivalent.

**Claim 1** *Given $\{\rho^0, \Lambda^0\}$ generated by parameterization $\xi^0$, there is a parameterization $\xi^1$, with $\nu^1 > \nu^0$ and $\lambda^1 > \lambda^0$, such that $\rho^1 = \rho^0$ and $\Lambda^1 = \Lambda^0$.*

**Proof.** See Appendix B. ■

The intuition for this observational equivalence result is the following. Higher employment durability increases the elasticity of employment to shocks relative to the response of wages. Analogously, higher wage rigidity decreases the elasticity of wages to shocks relative to the response of employment. Claim 1 formalizes this intuition by showing that models with low wage rigidity (i.e., high $\lambda$) and high durability (i.e., high $\nu$) are observationally equivalent to models with high wage rigidity and low durability. However, the economic mechanisms that bring about variation in wages, employment and assets are very different.

Finally, note that the degrees of durability and rigidity only affect marginal decisions (i.e., the (Euler) and (Labor Market) equations) and not available resources (i.e., the (SB) equation). On the contrary, the transfer policy rule only affect available resources and not marginal decisions. This implies that models with different degrees of durability and rigidity will have identical (Euler), (Labor Market) and (SB) equations for any policy and any parameterization consistent with Claim 1. Hence, given a benchmark policy and parameterization that produce the "observed" benchmark equilibrium, these observationally equivalent models will have identical counterfactual equilibrium for any alternative transfer policy rule. The next section presents the methodology to describe a larger set of fiscal union models that are observationally equivalent and produce identical transfer counterfactual, instead of focusing on two particular examples with varying degrees of durability and wage rigidity.

# 5 Semi-Structural Transfer Counterfactual in Fiscal Union Models

## 5.1 A set of linear fiscal union models

I start by describing models of fiscal unions in broader terms than what I have been doing so far. I will focus on models that satisfy three properties. Examples of models with some of these properties include Farhi and Werning (2014), Nakamura and Steinsson (2014), Beraja et al. (2015b), and Evers (2015). In particular, the models in the previous sections satisfy them all.

**Assumption 1** *Models of fiscal unions satisfy the following three properties:*

1. **Transfer policy rule***: Tax-and-transfer system can be summarized as federal lump-sum transfers that are a function of state-level economic variables.*

2. **Linear aggregation:** *State-level economies in log-deviations from the aggregate union behave to a first-order approximation as if they were small open economies—independent of other states.*

3. **3 by 3:** *Employment $n_t$, nominal wages $w_t$, and assets $b_t$; and exogenous processes $\{\gamma_t, \eta_t, z_t\}$ are sufficient variables for characterizing the state-level equilibrium in log-deviation from aggregates.*

Property 1 excludes models where the tax-and-transfers system affects decisions at the margin, as it would be the case with distortionary taxation. In Section 6.7, I relax this assumption and discuss robustness of results. Property 2 excludes from the analysis models where member states are inherently different because of industrial composition or household preferences, for example, and/or exogenous processes correlation structures across states are such that idiosyncratic shocks do not average out. In property 3, assets might encompass both non-state-contingent nominal bonds and certain types of tradable physical capital. What is important for the property to hold is that no other variables that are necessary to describe the equilibrium are left out (e.g., other endogenous or exogenous state variables in the model).

Given the description of models of fiscal unions above, the system of matrix equations that characterizes the equilibrium in one island in log-deviations from the aggregate is written below. Without loss of generality, I will say that the first equation is the (Euler) equation in these models, the second is the (Labor Market) equation, and the third is the sequential budget constraint (SB) equation.

$$0 = (F + \Theta_f)\mathbb{E}_t \begin{bmatrix} n_{t+1} \\ w_{t+1} \\ b_{t+1} \end{bmatrix} + (G + \Theta_c) \begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} + (H + \Theta_p) \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} + L\mathbb{E}_t \begin{bmatrix} \gamma_{t+1} \\ z_{t+1} \\ \eta_{t+1} \end{bmatrix} + M \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix}$$

$$0 = - \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix} + N \begin{bmatrix} \gamma_{t-1} \\ z_{t-1} \\ \eta_{t-1} \end{bmatrix} + \Sigma \begin{bmatrix} u_t^{\gamma} \\ u_t^{z} \\ u_t^{\eta} \end{bmatrix} \tag{FiscalSME}$$

I will say that a particular model of fiscal unions is characterized by the *structure* $\xi$ and the *policy* $\Theta$, where $\xi \equiv \{F, G, H, L, M, N, \Sigma\}$ are policy-invariant matrices, and $\Theta \equiv \{\Theta_f, \Theta_c, \Theta_p\}$ are matrices that characterize endogenous transfer policy rule. $\Theta_f$ contains the policy parameters associated with future expected variables, $\Theta_c$ with contemporaneous variables, and $\Theta_p$ past variables. The system evaluated at $\Theta = 0$ corresponds to a benchmark economy without a transfer policy in place. Elements in $\xi$ involve combinations of subsets of structural parameters, e.g., preference and technology parameters. These combinations typically lack direct economic interpretation in terms of model primitives.

Given property 1, I will assume that the transfer policy is the same as in the models of fiscal unions from the previous section. Since the third equation is the sequential budget constraint, this implies:

**Assumption 2** *The policy $\Theta$ is:* $\Theta_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vartheta_n & \vartheta_w & 0 \end{bmatrix}$, $\Theta_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \vartheta_b \end{bmatrix}$, *and* $\Theta_f = 0_{3.3}$

## 5.2 Equilibrium representations

The description of models directly in terms of their equilibrium conditions is already a step forward toward construction of semi-structural transfers counterfactuals. The primitives or microfoundations of such models need not be specified. Two models with different primitives but that have the same structure and policy in (FiscalSME) are equivalent for my purposes.

**Assumption 3** *$\xi, \Theta$ are such that the system (FiscalSME) is stabilizable.*

Under Assumption 3, it is easy to derive, using the method of undetermined coefficients, a stable recursive solution to (FiscalSME) that can be written:

$$\begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} = P(\xi,\Theta) \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} + Q(\xi,\Theta) \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix} \tag{FiscalRR}$$

**Assumption 4** *$\xi, \Theta$ are such that $Q(\xi,\Theta)$ is a non-singular square matrix.*

**Claim 2** *If Assumptions 3 and 4 hold, there is a structural vector autoregression (SVAR) representation to the solution (RR) of the form:*

$$\begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} = \rho_1(\xi,\Theta) \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} + \rho_2(\xi,\Theta) \begin{bmatrix} n_{t-2} \\ w_{t-2} \\ b_{t-2} \end{bmatrix} + \Lambda(\xi,\Theta) \begin{bmatrix} u_t^{\gamma} \\ u_t^{z} \\ u_t^{\eta} \end{bmatrix} \tag{FiscalSVAR}$$

*where $\rho_1(\xi,\Theta) \equiv P(\xi,\Theta) + Q(\xi,\Theta)NQ(\xi,\Theta)^{-1}$; $\rho_2(\xi,\Theta) \equiv (P(\xi,\Theta) - \rho_1(\xi,\Theta))P(\xi,\Theta)$ and $V(\xi,\Theta) \equiv Var(\Lambda(\xi,\Theta)u_t) = \Lambda(\xi,\Theta)\Lambda(\xi,\Theta)'$, where $\Lambda(\xi,\Theta) \equiv Q(\xi,\Theta)\Sigma$.*

**Proof.** We can write past exogenous processes as functions of past endogenous variables by inverting $Q(\xi,\Theta)$. Next, replace this and the law of motion for the exogenous states into (FiscalRR) to obtain (FiscalSVAR) representation. ■

## 5.3 Semi-structural transfer policy counterfactual

This section describes conditions under which it is possible to construct an equilibrium with $\vartheta_n = \vartheta_w = \vartheta_b = 0$ in a set of models of fiscal unions without the need to fully specify any particular model in this set. I begin by considering models of fiscal unions that can be written as in (FiscalSME) and satisfy Assumptions 2 to 4. These models are completely described by structure $\xi$ and policy $\Theta$.

Next, I say that a set of the models is described by *semi-structure* $\xi^s \subseteq \xi$. In this application to fiscal unions, I will consider only semi-structures that take the form of exclusion restrictions, i.e., semi-structures that are described by certain elements in $\xi$ being zeros. For example, one such exclusion restriction is that assets $b_t$ do not enter in the (Labor Market) equation. Section 7 generalizes the semi-structures to linear restrictions on the elements of $\xi$.

Then, I ask: what are the semi-structures (i.e., models of fiscal unions) that are (i) consistent with a (FiscalSVAR) generated by a given $\{\xi, \Theta\}$, and (ii) deliver the same counterfactual (FiscalSVAR') if the policy changes from $\Theta$ to $\Theta'$? In particular, I am interested in $\Theta' = 0$ which corresponds to an economy without fiscal integration.

**Proposition 1** *Given $\{\rho_1, \rho_2, \Lambda\}$ and $\Theta$, if models of fiscal unions can be written as in (FiscalSME), satisfy Assumptions 2 to 4, and:*

1. *have identical (FiscalSVAR) representation $\{\rho_1, \rho_2, \Lambda\}$ when policy is $\Theta$.*

2. *have identical semi-structure $\xi^s$, specified as at least 5 zeros in each equation in (FiscalSME).*

*then counterfactual (FiscalSVAR') representation $\{\rho_1(\xi, \Theta'), \rho_2(\xi, \Theta'), \Lambda(\xi, \Theta')\}$ is identical for all these models. Moreover, this counterfactual can be constructed only with knowledge of $\{\rho_1, \rho_2, \Lambda, \Theta, \Theta'\}$.*

**Proof.** See Appendix C.■

The proposition identifies a set of models of fiscal unions that have identical counterfactual SVAR representation for alternative policies. Moreover, it shows that in order to construct a counterfactual economy without a transfers policy rule (i.e., $\Theta' = 0$) it is not necessary to fully specify and parameterize a particular model of fiscal unions that is consistent with (FiscalSME). Instead, suppose that both the structural vector autoregression representation of the equilibrium in a fiscal union, and the transfer policy $\Theta$ that generated it are known. Then, describing *sufficient semi-structure* $\xi^s$ in the proposition is enough in order to construct a counterfactual equilibrium in the set of models that are consistent with this semi-structure and are observationally equivalent in terms of their reduced-form equilibrium representation. I call this construction a *semi-structural transfer counterfactual*.[17]

The proposition is not obvious. This is because the structural vector autoregression representation is found as a solution to a highly non-linear system of equations given a structure and policy $\{\xi, \Theta\}$. Specifying a particular model, parameterizing it, and solving this system for the equilibrium under alternative policies is the standard way of evaluating them. However, fixing a SVAR representation, the mapping is linear back to the structure $\xi$ that generated it. This is the key insight that allows identification of a set of models that yield identical counterfactuals by specifying only 5 zeros per equation in (FiscalSME). The reason why 5 zeros are sufficient is because there are 14 unknown elements per equation in structure $\xi$ and there are 9 known elements per equation in structural vector autoregression representation (FiscalSVAR). Since the mapping is linear, specifying 5 of these unknown elements is enough to infer the remaining 9 unknown elements in structure $\xi$ given the 9 known elements in (FiscalSVAR) and $\Theta$. In Section 7, I show a general version of this proposition in linear dynamic stochastic models.

[17]The next section describes how to build all these inputs, in particular, how to identify the impulse response matrix in the SVAR using similar exclusion restrictions. From a methodological perspective, however, other SVAR identification schemes could be used as long as they are consistent the sufficient semi-structure.

# 6 A Counterfactual United States Economy without Fiscal Integration

I start this section by describing all inputs to construct the semi-structural transfers counterfactual using Proposition 1. I need to: (i) specify a sufficient semi-structure $\xi^s$ describing a set of fiscal union models, (ii) estimate a regional VAR using data for a fiscal union, (iii) identify the impulse response matrix, and (iv) estimate the transfer policy rule. Next, I present primary findings from the counterfactual exercise. I conclude the section by discussing the sensitivity of these findings to alternative ways of specifying the transfer policy rule.

## 6.1 The sufficient semi-structure $\xi^s$

The zeros in the matrices below describe a set of fiscal union models by specifying a semi-structure $\xi^s$ that is sufficient in the sense of Proposition 1.

**Assumption 5** *A set of fiscal union models that can be written as in (FiscalSME) is described by:*

$$F = \begin{bmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}; G = \begin{bmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix}; H = \begin{bmatrix} h_{11} & 0 & 0 \\ h_{21} & h_{22} & 0 \\ 0 & 0 & h_{33} \end{bmatrix};$$
$$L = \begin{bmatrix} 1 & l_{12} & l_{13} \\ 0 & 1 & l_{23} \\ 0 & 0 & 0 \end{bmatrix}; M = \begin{bmatrix} 0 & m_{12} & m_{13} \\ 0 & m_{22} & m_{23} \\ 0 & 0 & m_{33} \end{bmatrix}; N = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ 0 & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix}$$

The absence of expected and lagged terms beside assets makes the third equation consistent with most log-linearized, incomplete market models that include a sequential budget constraint (SB). Moreover, I assume that the only exogenous shifter in the sequential budget constraint is "wealth" process $\eta_t$. The other two exogenous processes do not appear in the sequential budget constraint, which further restricts the set models that I analyze. Finally, it is assumed that the only exogenous shifters are "wealth" shocks, and the other two exogenous processes do not appear in the sequential budget constraint. In terms of the (Euler) and (Labor Market) equations—the first and second equations—I assume that (i) assets (future, contemporaneous, or lagged) $\{b_{t+1}, b_t, b_{t-1}\}$ do not appear in them, (ii) lagged wages $w_{t-1}$ does not appear in the first equation, (iii) contemporaneous "discount rate" process $\gamma_t$ do not shift these equations, and (iv) future "discount rate" process $\gamma_{t+1}$ do not appear in the second equation. Finally, I assume that past "discount rate" process $\gamma_{t-1}$ does not cause movements in "productivity" $z_t$ and "wealth" $\eta_t$ as is evidenced by autoregressive matrix $N$. These assumptions are consistent with all models of fiscal unions described in Section 4 as well as many others. The key feature of models described by this semi-structure is that the (Labor Market) and (Euler) equations dependence on future and lagged variables is relatively unconstrained, as well as the exogenous processes and their correlation structure. This is important for the question of regional stabilization in fiscal unions because it means that this set encompasses many models with rich features.

### 6.2 Identification of $\Lambda(\xi, \Theta)$ and shocks

An important input in the construction of the semi-structural policy counterfactual is impulse response matrix $\Lambda(\xi, \Theta)$. The literature proposes myriad ways to identify it, ranging from simple ordering assumptions to more sophisticated sign and long-run restrictions. I follow Lemma 3, which extends the methodology proposed in Beraja et al. (2015b). In essence, the procedure uses elements of theory to identify underlying shocks in a VAR. These theoretical restrictions imply a series of particular linear restrictions linking the reduced form errors to the structural shocks. I use the sequential budget constraint (SB) to generate these theoretical restrictions. [18]

The first step in the procedure consists of estimating the reduced form (FiscalSVAR) to obtain the autoregressive matrices $\{\rho_1, \rho_2\}$, and the reduced form errors covariance matrix $V$. Second, derive identification restrictions that will allow us to infer $\Lambda$ and the shocks.[19] Applying the conditional expectation operator $\mathbb{E}_{t-1}(.)$ on both sides of the (SB) and constructing the reduced form expectational errors, we obtain:

$$0 = \begin{bmatrix} g_{31} + \vartheta_n & g_{32} + \vartheta_w & g_{33} \end{bmatrix} \Lambda \begin{bmatrix} u_t^\gamma \\ u_t^z \\ u_t^\eta \end{bmatrix} + m_{33}\sigma_{33}u_t^\eta \tag{Id1}$$

This equation must hold for all realizations of the shocks. Whenever there is an innovation to $u_t^\gamma$ or $u_t^z$ and $u_t^\eta = 0$, employment, wages, and debt must co-move on impact in a way that satisfies this linear relationship. Hence, it gives us two linear restrictions in the second and third columns' elements of $\Lambda$ for a given parameterization of (SB) when there are either contemporaneous $u_t^\gamma$ or $u_t^z$ shocks. These linear restrictions are the same as those implied by equation (6) in Section 7.1.

Similarly, constructing $\mathbb{E}_{t-1}(.) - \mathbb{E}_{t-2}(.)$, we obtain:

$$\begin{aligned} 0 = &\left( \begin{bmatrix} g_{31} + \vartheta_n & g_{32} + \vartheta_w & g_{33} \end{bmatrix} \rho_1 + \begin{bmatrix} 0 & 0 & h_{33} + \vartheta_b \end{bmatrix} \right) \Lambda \begin{bmatrix} u_{t-1}^\gamma \\ u_{t-1}^z \\ u_{t-1}^\eta \end{bmatrix} \\ &+ m_{33}\sigma_{33}n_{33}u_{t-1}^\eta + m_{33}\sigma_{33}n_{32}u_{t-1}^z \end{aligned} \tag{Id2}$$

This gives us one extra linear restriction in the second column's elements of $\Lambda$ for a given parameterization of (SB) when there are $u_{t-1}^\gamma$ shocks. These three linear restrictions, combined with six non-linear restrictions coming from the orthogonalization of the shocks, are sufficient to identify all nine elements in $\Lambda$. Intuitively, equation (Id1) separates the "wealth" shock from the other two shocks. If the unexpected component of employment, wages, and assets does not co-move in the linear way implied by equation (Id1), when $u_t^\eta = 0$, a "wealth" shock must have occurred. Analogously, equation (Id2) separates "discount rate" and "productivity" shocks. If

[18]In Beraja et al. (2015b), we used a different equation to identify aggregate shocks—a wage-setting equation.

[19]The shocks are identified simply by multiplying the matrix of reduced form errors from the estimated VAR by the inverse of the matrix $\Lambda$.

the unexpected component of employment, wages, and assets does not co-move in the linear way implied by equation (Id2), when $u^z_{t-1} = u^\eta_{t-1} = 0$, a "discount rate" shock occurred. For completeness, matrix $\Lambda$ solves the system:

$$\begin{bmatrix} G_{31}+\vartheta_n & G_{32}+\vartheta_w & G_{33} \end{bmatrix} \Lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\left( \begin{bmatrix} G_{31}+\vartheta_n & G_{32}+\vartheta_w & G_{33} \end{bmatrix} \rho_1 + \begin{bmatrix} 0 & 0 & H_{33}+\vartheta_b \end{bmatrix} \right) \Lambda \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0$$

$$V = \Lambda\Lambda'$$

This is the same system implied by Lemma 3 for this particular semi-structure.

### 6.3 Data description

I exclude Alaska, District of Columbia, and Hawaii from analysis, leaving 48 observations (one for each remaining state) per year, and 6 years (2006-2011) of data.

To make state-level nominal wages indices, I use data from the 2000 US Census and the 2001-2012 American Community Surveys (ACS).[20]

The 2000 Census includes 5 percent of the US population. The 2001-2012 ACS's include approximately 600,000 respondents between 2001-2004, and about 2 million after 2004. The large sample sizes allow detailed labor market information at the state level. I begin by using the data to make individual hourly nominal wages. I restrict the sample to only individuals who are employed, who report usually working at least 30 hours per week, and who worked at least 48 weeks during the prior 12 months. For each individual, I divide total labor income earned during the prior 12 months by a measure of annual hours worked during the prior 12 months.[21] The composition of workers differs across states and within a state over time, which might explain some variation in nominal wages across states over time. To account for this, I run the following regression:

$$\ln(w_{itk}) = K_t + \Gamma_t X_{itk} + u_{itk}$$

where $\ln(w_{kt})$ is log-nominal wages for household $i$ in period $t$ residing in state $k$, and $X_{kt}$ is a vector of household specific controls. The vector of controls include a series of dummy

[20] I access the data through the IPUMS-USA website https://usa.ipums.org/usa/. See Ruggles et al. (1997).

[21] Total labor income during the prior 12 months is the sum of both wage and salary earnings and business earnings. Total hours worked during the previous 12 months is a multiple of total weeks worked during the prior 12 months and the respondents' reports of their usual hours worked per week. For some years, bracketed reports are provided for weeks worked during the prior 12 months, and the usual hours per week worked. In those cases, I take the midpoint of the brackets.

variables for usual hours worked (30-39, 50-59, and 60+), a series of five-year age dummies (with 40-44 being the omitted group), 4 educational attainment dummies (with some college being the omitted group), three citizenship dummies (with native born being the omitted group), and a series of race dummies (with white being the omitted group). I run these regressions separately for each year such that both constant $K_t$ and the vector of coefficients on the controls, $\Gamma_t$, can differ for each year. I then take the residuals from these regressions for each individual, $u_{itk}$, and add back constant $K_t$. Adding back the constant from the regression preserves differences over time in average log wages. To compute average log wages within a state, holding composition fixed, I average $u_{itk} + K_t$ across all individuals in state $k$. I refer to this measure as the demographically adjusted, log-nominal wage in time $t$ in state $k$.

The measure of employment at the state level is the employment rate for each state, calculated using data from the US Bureau of Labor Statistics. The BLS reports annual employment counts and population numbers for each state and year. I divide employment counts by population to make an annual employment rate measure for each state.

Data on federal transfers net of taxes paid come from the Bureau of Economic Analysis.[22] Transfers include retirement and disability insurance benefits, medical benefits, income maintenance benefits, unemployment insurance compensation, veterans benefits, federal education and training assistance, and other transfer receipts of individuals from governments. Federal taxes are the sum of personal income taxes that are withheld, usually by employers, from wages and salaries, quarterly payments of estimated taxes on income that is usually not subject to withholding, and final settlements, which are additional tax payments made when tax returns for a year are filed, or as a result of audits by the Federal Government.[23]

Given the unavailability of official state-level data on asset positions, I construct a measure of state-level assets as the sum of physical and financial assets. From national account identities, we can derive the law of motion for assets $B_t$ in a given state as:

$$B_t = B_{t-1}(1 + r_t) + Y_t - C_t + S_t - G_t^{local} + v_t$$

where $Y_t$ is nominal gross domestic product, $C_t$ is private consumption expenditures, $S_t$ are net transfers (i.e., expenditures minus taxes) from the federal government, $G_t^{local}$ are expenditures from the local government, and $r_{t-1}$ captures the change in asset valuation between $t-1$ and $t$. Finally, error term $v_t$ includes income receipts from abroad minus income payments to foreigners, federal government expenditures not counted as federal transfers (e.g., salaries and wages), and differences in returns between physical and financial assets for which no data are available.[24] I obtain $Y_t$ and $C_t$ directly from the Bureau of Economic Analysis website. $S_t - G_t^{local}$ also comes from several variables in the BEA. I calculate it as (personal current transfers receipts) - (personal

[22] I access the data through the BEA website on regional GDP and personal income: http://www.bea.gov/iTable/index_regional.cfm

[23] Excise, Medicare and Social security federal taxes are not included in this measure.

[24] Error term $v_t$ accounts for most of the "wealth" exogenous process. The remainder is the error term $e_t$ in the difference between observed net transfers and estimated policy rule in equation Section 6.4.

current taxes paid + taxes on production and imports net of subsidies).[25] The revaluation of assets term $r_t$ is obtained residually to ensure that the growth rate of the sum of local assets across states is consistent with the growth rate of aggregate net worth in the US economy. Having all components in the law of motion for $B_t$, I calculate assets at each point in time for each state simply by iterating forward with 2006 as the initial observation. I obtain initial assets in 2006 by aggregating at the state level, the zip code total net worth data from Mian et al. (2013). In order to construct financial assets at the zip code level Mian et al. (2013) they use data on dividends and interest income from the IRS Statistics of Income (SOI). They assume that households hold identical shares of stocks and bonds (they hold the market index portfolio). Given the share of total dividends and interest income received by a zip code they can construct the share of total stocks and bonds held by that zip code. Then, they total financial assets from the Federal Reserve's Flow of Funds data to zip codes based on these shares. For the value of nominal debt owed by households they use data based on information from Equifax Predictive Services. Then they match the Federal Reserve Flow of Funds data by using the share of Equifax total debt in a zip code to allocate Flow of Funds debt. The final component of the asset measure is the value of housing wealth which they estimate using the 2000 Decennial Census data. They construct total home value as of 2000 in a zip code as the product of the number of home owners and the median home value. Then, they project it forward into later years using the CoreLogic zip code level house price index and an aggregate estimate of the change in homeownership and population growth.

### 6.4 Estimation of transfer policy rule $\Theta$

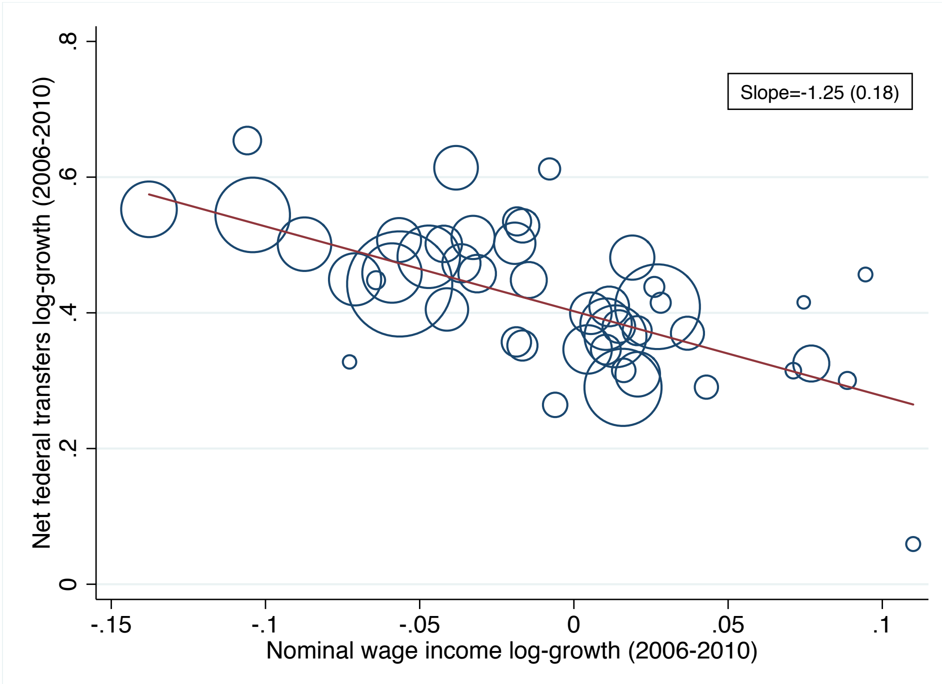
Figure 1 shows a scatter plot of net federal transfers growth (direct federal transfers minus federal taxes growth) between 2006 and 2010 against nominal wage income growth (wage plus employment growth) between 2006 and 2010 in the United States.[26]. There is a very strong, negative relationship between the two. In particular, a one percentage point increase in nominal wage income associates with a 1.25 percentage point decrease in net transfers. If the tax-and-transfer system helps stabilize regional economies, it is because a state whose economy temporarily worsens relative to the average receives some temporary relief through transfer payments or lower tax payments to the federal government—a type of insurance against temporary negative shocks. The opposite is true for states whose economies are in a relative boom. The negative relationship is a consequence of both the progressivity of the tax system in the United States and the presence of automatic stabilizers like unemployment insurance, and particularly during this period, federal emergency unemployment compensation and food stamps.[27]

[25] As long as local government expenditures plus transfers are close enough to local tax revenues (i.e., local governments have a nearly balanced budget), the calculation is accurate. If not, the difference is absorbed by error term $e_t$.

[26] See Section 6.3 for a detailed description of the construction of these data

[27] If agents understand that federal emergency unemployment compensation would always be passed in situations of economic malaise, it is correct to include them as part of the implicit net federal transfer policy rule in place in a

Figure 1: Net Transfers growth v. Wage Income growth (2006-2010)




Note: Figure shows a simple scatterplot of the log-growth of net transfers in a state between 2006 and 2010 against nominal wage income log-growth during the same period. See text for details of variable construction. The population size in 2006 of the underlying state is represented by the size of the circle in the figure. The line represents a weighted regression line from the bivariate regression, where the weights are the population in 2006.

As a reminder, the transfer policy rule is:

$$s_t = \vartheta_n n_t + \vartheta_w w_t + \vartheta_b b_{t-1} + e_t$$

For regional data to be used to estimate $\Theta$, one of the following must hold: (1) the innovations to the policy rule have no regional component ($e_t = 0$)—in which case, a simple OLS regression produces consistent estimates—or (2) valid instruments can be found that isolate movements in $n_t, w_t, b_{t-1}$ that are orthogonal to $e_t$. The issue of endogeneity arises because of reverse causality. When the innovation in the policy rule is part of the "wealth" shock $u_t^\eta$, employment and wages both cause and are caused by net transfers in the equation above. To deal with the endogeneity of $n_t, w_t, b_{t-1}$, I proceed variously. First, I estimate a regression of net transfers onto nominal wage income alone (assuming $\theta_w = \theta_n$) using house price growth between 2006 and 2010 as an instrument. This accords with many recent papers, including Mian and Sufi (2014). Contemporaneous housing price growth strongly predicts contemporaneous nominal wage income growth. The instrument is valid as long as local housing prices are orthogonal to the transfer policy rule shock, which appears plausible. In the second approach, I use "discount rate" and "productivity"

fiscal union.

shocks in 2008, estimated from (FiscalSVAR), as instruments for wages and employment. They are linear combinations of wages, employment, and assets in 2008 that are orthogonal to the "wealth" shock, and hence $e_t$, by construction.

Table 3 in Appendix G presents results for several specifications. The dependent variable is the log-growth rate of transfers minus the growth rate of taxes between 2006 and 2010 for each state. The independent variables are the log-growth rate of nominal wages between 2006 and 2010 and the log-growth rate of employment between 2006 and 2010 in the first two columns, and the log-growth rate of assets between 2006 and 2009 in the third column. In the fourth column, the independent variable is the sum of wage and employment growth. The first line is a simple OLS regression. The second presents two-stage, least-squares results using the "discount rate" and "productivity" shocks in 2008 $u^{\gamma}_{2008}, u^{z}_{2008}$. The third uses house price log-growth between 2006 and 2008 as an instrument instead. The fourth uses all three instruments. For all specifications, and when possible, I consider case (1) when $b_{t-1}$ is not endogenous, and case (2) when $b_{t-1}$ might be endogenous.

I find that the policy rule estimates have the expected sign and are significant in all specifications. They are also similar in magnitude, ranging from -1.3 to -1.6 for $\vartheta_n$ and -0.9 to -1.4 for $\vartheta_w$. Lagged assets have nearly no independent explanatory power for net transfers across all specifications. To give a sense of the magnitudes involved, when net transfers increase by 30 percent for every 1 percent decrease in nominal wage income, and the average income tax rate is 0.17, for every 1 dollar decrease in nominal wage income, a state receives 0.22 dollars in federal transfers. This result is similar to findings by Feyrer and Sacerdote (2013), who find a 0.25 decrease, and Bayoumi and Masson (1995), who find a 0.31 decrease.

## 6.5 Main Findings

I estimate the vector autoregression (FiscalSVAR) via weighted OLS where the weights are the 2006 population in the state, using data described in subsection 6.3. For each variable and year, I take the cumulative log-growth between 2006 and 2011 and express it in log-deviations from the average across states. I pool all data between 2006 and 2011, leaving 240 observations (5 years*48 states), and estimate common autoregressive coefficients $\rho_1, \rho_2$ and reduced form errors $U$ covariance matrix for all states $V = \frac{UU'}{240-3*2}$. I follow 6.2 to identify $\Lambda$, setting $g_{33} \equiv \frac{\bar{B}}{\bar{S}} = 2.25$ to match the median net worth to revenues ratio across states in the United States in 2006 and $\vartheta_n = -1.6, \vartheta_w = -0.9, \vartheta_b = -0.03$, which correspond to the OLS policy rule estimates in Table 3. Following Proposition 1, I construct a semi-structural transfers counterfactual for $\vartheta_n = \vartheta_w = \vartheta_b = 0$ which corresponds to an economy without a federal transfer policy rule.

Figure 2 in Appendix G shows the impulse response functions of employment, wages, and assets to a one-standard-deviation "discount rate" shock $\gamma$, both in the actual and the counterfactual economy without transfers.[28] I find that employment and wages both decrease on impact,

[28] Mian and Sufi (2014) and Beraja et al. (2015b), among others, argue that this type of shocks were key drivers of regional business cycles during the Great Recession in the United States.

whereas assets increase in response to a "discount rate" shock. This accords with the theoretical impulse response functions in models in Section 4. As for the effects of fiscal integration, I find that amplification and persistence of "discount rate" shocks are mitigated by the transfer policy rule—the employment response (after two years) is -1.2 percent in the actual economy, whereas it is -2.1 percent in the counterfactual economy without transfers.

Table 4 presents moments of the employment distribution in the actual and counterfactual economies without transfers. The cross-state employment standard deviation in the US data in 2010, $\sigma_n^{2010}$, was 2.6 percent (this corresponds to the first line and third column in the table, when all shocks are present). I then consider the following thought experiment. At the end of 2007, it is announced that from 2008 onwards the United States federal government will cease to give transfers and collect taxes to/from the states in the union. How would employment, wages, and assets have evolved were regional economies hit by the same sequence of shocks? I find that, absent a federal transfer policy rule, the standard deviation of employment in 2010 would have been 3.5 percent. The skewness of the distribution, $sk_n^{2010}$, would have been -0.3 instead of -0.2. To give some context to these numbers, aggregate output volatility during the pre-Volcker period (1960:1 to 1979:2) was 2.7, whereas during the post-Volcker-disinflation period (1982:4 to 1996:4) volatility was 2.06.[29] Much literature examines causes of this "Great Moderation". The consequences of the US federal tax-and-transfer system are in the same order of magnitude.

Table 4: Employment statistics: Fiscal Integration v. Fiscal Autonomy

| | | $\gamma$ | $(\gamma, z)$ | $(\gamma, z, \eta)$ |
|---|---|---|---|---|
| $\sigma_n^{2010}$ | $\Theta \neq 0$ | 2.3 | 2.5 | 2.6 |
| | $\Theta = 0$ | 3 | 3.4 | 3.5 |
| $sk_n^{2010}$ | $\Theta \neq 0$ | -0.26 | -0.29 | -0.2 |
| | $\Theta = 0$ | -0.39 | -0.37 | -0.3 |
| $\overline{\sigma}_n$ | $\Theta \neq 0$ | 2.3 | 2.7 | 3.5 |
| | $\Theta = 0$ | 3.9 | 4.5 | 4.9 |
| $\overline{sk}_n$ | $\Theta \neq 0$ | -0.03 | -0.09 | 0.02 |
| | $\Theta = 0$ | -0.02 | -0.07 | -0.01 |
| $\sqrt{s_n(0)}$ | $\Theta \neq 0$ | 1.5 | 6.3 | 7.8 |
| | $\Theta = 0$ | 5.5 | 9.7 | 10.7 |

Note: $\sigma_n^{2010}$ is the standard deviation of the distribution of employment $n_t$ across states in 2010 in percentages. $sk_n^{2010}$ is the skewness of the distribution in 2010. $\overline{\sigma}_n$ and $\overline{sk}_n$ are the standard deviation and skewness in the stationary distribution. $s_n(0)$ is the spectrum at zero frequency (the long-run variance of $n_t$). Line $\Theta \neq 0$ corresponds to the fiscal union economy, and line $\Theta = 0$ to results from the semi-structural counterfactual. Column $\gamma$ corresponds to the case with only "discount rate" shocks, and column $(\gamma, z)$ to the case with "discount rate" and "supply shocks. Column $(\gamma, z, \eta)$ corresponds to the case with all shocks.

Results above imply that the federal tax-and-transfer system helped stabilize regional economies

[29]These numbers come from Clarida et al. (2000).

by redistributing resources from regions that were doing relatively well to regions that were doing relatively poorly. Figure 3 in Appendix G elaborates on this point, showing the employment gain (or loss) from fiscal integration for each state in 2010, where states are sorted according to their employment in 2010 from lowest to highest. We observe that fiscal integration increased employment in states with the worst employment outcomes, and the opposite is true for states with the best employment outcomes.

In the first and second columns of the table, I calculate the same statistics if regional economies had been hit by only "discount rate" shocks or both "discount rate" and "productivity" shocks. Comparing the first and last columns in the first line, I find that most of the employment variation across states in 2010 is accounted for by "discount rate" shocks (approximately 90 percent). This accords with findings in Beraja et al. (2015b), in which data, and particularly the identification strategy, are different. Similarly, the federal transfer policy rule reduced employment dispersion primarily by stabilizing regional "discount rate" shocks. This is evidenced by comparing the first column in the second line of the table to the other columns. Of the 0.9 volatility reduction, 0.7 is achieved because of stabilization of "discount rate" shocks, and only 0.2 because of "productivity" shocks.[30]

In the lower half of Table 4, I present Monte Carlo estimates of the standard deviation $\overline{\sigma}_n$ and skewness $\overline{sk}_n$ of employment in the stationary distribution. I construct them by sampling with replacement 1,000,000 observations from the empirical distribution of shocks, feeding them to the (FiscalSVAR) and calculating the corresponding statistic. In the last line of the table, I present an estimate of the square root of the long-run variance of employment. The long-run variance is constructed as the diagonal element corresponding to employment in the spectrum at frequency zero of the multivariate (FiscalSVAR). Its square root is a common measure of the persistence in the series, as proposed by Pesaran et al. (1993). It associates with the speed at which the impulse response function to a given set of shocks decays, and hence their persistence. Results for the reduction in stationary employment volatility are qualitatively similar to the ones during the Great Recession. Finally, I find that the persistence of employment increases from 7.8 to 10.9 in the counterfactual economy without a federal transfer policy rule. Interestingly, most of the persistence in employment in the fiscally integrated economy is due to "productivity" shocks. In the counterfactual economy, "discount rate" shocks generate much larger persistence in employment.

### 6.6 Channels of regional stabilization

To elucidate channels behind measured benefits from fiscal integration, I examine the contribution of what I define as a *rigidity channel*, which includes nominal and real rigidities in comparison to other broad frictions like asset market incompleteness and/or labor mobility costs. The idea is to construct an economy without nominal and/or real rigidities using the semi-structural

[30]The volatility reduction of 0.9 is the difference between the actual and counterfactual volatilities of 3.5 and 2.6. Analogously, 0.7 is the difference between 3 and 2.3, and 0.2 is the difference between 3.4 minus 2.5 and 0.7.

methodology, and conduct the transfer policy counterfactual in this alternative economy. Then, follow a diff-in-diff approach to evaluate contributions of nominal/real rigidities to the benefits of fiscal integration by using differences across all four combinations; that is, an economy with rigidities and transfers $(r, s)$, rigidities and no transfers $(r, ns)$, no rigidities and transfers $(nr, s)$, and no rigidities and no transfers $(nr, ns)$. Any observation $\omega$ can be written as:

$$\omega = \beta_{nr,ns} + \beta_r \mathbb{I}_r + \beta_s \mathbb{I}_s + \beta_{r,s} \mathbb{I}_r \mathbb{I}_s$$

where $\mathbb{I}_r$ is an indicator variable that is turned on when rigidities are present, and $\mathbb{I}_s$ is an indicator variable turned on when transfers are present. For example, in the previous section, I identify $\beta_s + \beta_{r,s}$ for the employment distribution. This corresponds to the total difference between economies with and without transfers (the transfers counterfactual). The purpose of this section is to identify $\beta_{r,s}$, which measures the contribution of nominal/real rigidities to the total difference in the transfers counterfactual $\beta_s + \beta_{r,s}$ (i.e., the *rigidity channel*). Following diff-in-diff logic, we can identify $\beta_{r,s}$ by comparing the transfers counterfactual in economies with and without rigidities. That is:

$$\beta_{r,s} = (\omega_{r,s} - \omega_{r,ns}) - (\omega_{nr,s} - \omega_{nr,ns})$$

To construct counterfactual economies without nominal or real rigidities, I follow the semi-structural methodology in this paper. Given the semi-structure specified in Section 6.1, the second equation in (FiscalSME) is a combination of labor demand, labor supply, wage-setting, and/or price-setting schedules. This point was made in the aforementioned section. For the purpose of this counterfactual exercise, I further assume that parameters related to the degree of forward- and backward-lookingness in labor demand, price-setting, labor supply, or wage-setting schedules in the presence of nominal and real rigidities affect only the second equation in (FiscalSME). Then, the semi-structural methodology is also useful for defining a counterfactual economy in which there are no nominal and real rigidities. This counterfactual economy is consistent with models in which labor supply and demand are static. The way to construct such counterfactual economy is by imposing counterfactual policy matrices that cancel the forward- and backward-looking terms in the equation. Given the way we specified the semi-structure $\xi^s$, this is achieved by setting $\Theta'_{f,21} = -f_{21}; \Theta'_{f,22} = -f_{22}; \Theta'_{p,22} = -h_{22}; \Theta'_{p,21} = -h_{21}$.

Table 5 presents results for the $20^{th}$, $50^{th}$, and $80^{th}$ percentiles of the distribution of employment growth between 2008 and 2010. Figure 4 in Appendix G presents kernel density estimates for all four distributions used in the construction of Table 5, and summary statistics.

Findings for the *transfers counterfactual* are as mentioned in the previous section. The increase in employment (1.3 percent) because of fiscal integration in the bottom part of the employment distribution (below the 20th percentile) is about the same as the decrease (-1.1 percent) in the upper part (above the 80th percentile). The degree to which nominal and real rigidities amplify (or attenuate) the consequences of fiscal integration attenuates, the *rigidity channel*, is not the same

Table 5: Employment in the Great Recession: Channels Decomposition

| Percentiles | $\mathbf{20^{th}}$ | $\mathbf{50^{th}}$ | $\mathbf{80^{th}}$ |
|---|---|---|---|
| *Rigidity channel* ($\boldsymbol{\beta_{rs}}$) | 0.4 | -0.2 | 0 |
| *Transfers counterfactual* ($\boldsymbol{\beta_s} + \boldsymbol{\beta_{rs}}$) | 1.3 | 0 | -1.1 |

across the employment distribution. I find that about one-third of the 1.3 percent employment increase because of fiscal integration in states with the worst employment outcomes during the Great Recession was due to this channel. On the other hand, the channel was irrelevant for states with the best employment outcomes, meaning that at least for these states, the employment reduction because of fiscal integration involved frictions other than nominal and real rigidities.

### 6.7 Robustness to alternative policy specifications

Results from the previous section are based on a particular transfer policy specification. As a reminder, the benchmark policy matrices were:

$$\Theta_f = 0; \Theta_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vartheta_n & \vartheta_w & 0 \end{bmatrix}; \Theta_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \vartheta_b \end{bmatrix}$$

where $\vartheta_n = -1.6, \vartheta_w = -0.9, \vartheta_b = 0.03$, which corresponds to the OLS estimates of the transfer policy rule in Table 3 in Appendix G.

First, we would like to evaluate the sensitivity of results to estimates of the transfer policy rule other than the benchmark OLS estimates. Second, the specification is consistent with households in each state receiving lump-sum transfers from the federal government because the only affected equation that characterizes the equilibrium is the sequential budget constraint (the third equation in this case). In practice, lump-sum transfers are rare, and instead, the federal government uses distortionary taxes. This section explores the sensitivity of the counterfactual exercise to both of these issues.

Table 6 in Appendix G shows the counterfactual employment standard deviation in an economy without transfers, both in 2010 and in the stationary distribution, for alternative initial policy parameterizations corresponding to the instrumental variable estimates in Table 3. Results are qualitatively similar to those reported in the previous section for the benchmark policy rule. Although quantitatively reduced somewhat for some of the parameterizations, gains from fiscal integration in terms of the reduction of the dispersion in employment across states remain large. The largest quantitative difference is for the case in which I restrict coefficients on employment and wages in the policy rule to be identical ($\vartheta_n = \vartheta_w = -1.1$). As the last column in the table shows, the counterfactual employment standard deviation in 2010 would have been 3.1 percent (instead of 2.6 percent in the data), and the counterfactual standard deviation in the stationary distribution is 4.5 percent (instead of 3.5). The counterfactual using the benchmark policy estimates instead resulted in counterfactual standard deviations of 3.5 and 4.9 percent, respectively.

Evaluating sensitivity to an alternative policy that accounts for the potentially distortionary effects of taxes is less straightforward. For simplicity, consider the transfer policy without lagged assets in log-deviations from the aggregate.[31]

$$s_t = \vartheta_n n_t + \vartheta_w w_t$$

This implies that tax rate $\tau_t$ per unit of nominal labor income $w_t + n_t$ in log-deviations from the aggregate can be written as:

$$\tau_t = -(1+\vartheta_n)n_t - (1+\vartheta_w)w_t$$

The potential labor supply (or wage-setting) tax distortion relates to $\tau_t$, not total transfers $s_t$ for which we estimated elasticities $\vartheta_w, \vartheta_n$. If the federal tax-and-transfer system affects equilibrium equations beyond the sequential budget constraint, $(1+\theta_w), (1+\theta_n)$ would appear in these equations, not $\theta_w, \theta_n$.

I consider the case in which the second equation in (FiscalSME) is a wage-setting equation, as in Section 4. If the federal tax-and-transfer system is distortionary, and given semi-structure $\xi_s$ from Section 6.1, we can write it as:

$$\begin{aligned} 0 =& f_{21}\mathbb{E}_t[n_{t+1}] + f_{22}\mathbb{E}_t[w_{t+1}] + \left(g_{21} + \frac{\bar{\tau}\vartheta_n}{1-\bar{\tau}\vartheta_n}(1+\vartheta_n)\right) n_t + \left(g_{22} + \frac{\bar{\tau}\vartheta_n}{1-\bar{\tau}\vartheta_n}(1+\vartheta_w)\right) w_t \\ &+ h_{21}n_{t-1} + h_{22}w_{t-1} + \mathbb{E}_t[z_{t+1}] + l_{23}\mathbb{E}_t[\eta_{t+1}] + m_{22}z_t + m_{23}\eta_t \end{aligned}$$

The equation above accords with the tax rate that affects the target wage in the wage-setting equation by distorting the marginal rate of substitution. The distortion is given by terms $1+\theta_n$ and $1+\theta_w$. For example, the case $\theta_w = \theta_n = -1$ is such that the tax schedule is flat (i.e., a proportional labor income tax). Due to the lack of curvature, it would not affect the island's log-deviations of the marginal rate of substitution from the aggregate.

I construct a semi-structural policy counterfactual using this alternative policy specification, in which I set $\bar{\tau} = 0.17$ to match the average tax rate in the US economy. Results from the previous section are unchanged because estimates of transfer policy rule $\vartheta_n, \vartheta_w$ are close to $-1$. Thus, policy-related terms that distort this equation are very small in absolute magnitude, in comparison with terms in the policy-invariant structure. To see this, consider the case in which the second equation is interpreted as a static labor supply equation, and the policy rule depends on employment alone: $w_t = \left(-g_{21} - \frac{\bar{\tau}\vartheta_n}{1-\bar{\tau}\vartheta_n}(1+\vartheta_n)\right) n_t$. Plausible calibrations of labor supply Frisch elasticity $-g_{21}$ are in the range 0.5 to 4.[32] The policy-related term for the case when $\bar{\tau} = 0.17, \vartheta_n = -1.6$ is -0.08, which is an order of magnitude smaller than the Frisch elasticity.

[31]Since the estimated coefficient on lagged assets is so small, results are identical whether we include it.

[32]See Chetty et al. (2011).

# 7 Semi-Structural Policy Counterfactuals: A General Methodology

This section describes the construction of semi-structural policy counterfactuals in a large class of linear models of dynamic stochastic economies. First, I define the class of models I consider, and develop notation and language that I use throughout the section. Second, I offer a formal definition for a *semi-structural policy counterfactual* in this class of models. I conclude by showing how to construct it and what is needed to do so depending on what we are willing to assume about the underlying structure of the economy.

The class of linear models I consider are such that equilibrium is characterized by the system of matrix equations,

$$\begin{aligned}
0 &= (F+\Theta_f)\mathbb{E}_t[x_{t+1}] + (G+\Theta_c)x_t + (H+\Theta_p)x_{t-1} + L\mathbb{E}_t[z_{t+1}] + Mz_t \\
0 &= -z_t + Nz_{t-1} + \Sigma u_t; \quad \text{iid } u_t \text{ with } \mathbb{E}[u_t]=0, Var(u_t)=1 \qquad \text{(SME1)} \\
0 &= y_t + R_1 x_t + R_2 x_{t-1} + R_3 z_t \\
&\ (H+\Theta_p)x_0; Mz_0 \text{ given}
\end{aligned}$$

where $x_t$ is a column vector that includes all endogenous state variables, and could include endogenous control variables too; $z_t$ is a column vector of exogenous state variables; and $y_t$ a column vector of other endogenous variables not included in $x_t$. In the application of the methodology to fiscal unions $x_t$ included log-deviations from the aggregate union of employment, wages and assets at the state-level, and $z_t$ included the discount rate, productivity and wealth processes. Alternatively, in the context of Hansen (1985) real business cycles model, $x_t$ includes log-deviations from the steady state of capital and potentially employment, $z_t$ includes productivity, and $y_t$ consumption.

For initial conditions, I say that a particular economy is characterized by the *structure* $\{\xi, \xi^a\}$ and the *policy* $\Theta$, where $\xi \equiv \{F, G, H, L, M, N, \Sigma\}$, and $\xi^a \equiv \{R_1, R_2, R_3\}$ are policy-invariant matrices, and $\Theta \equiv \{\Theta_f, \Theta_c, \Theta_p\}$ are matrices that characterize endogenous policy rules. $\Theta_f$ contains the policy parameters associated with future expected variables, $\Theta_c$ with contemporaneous variables, and $\Theta_p$ past variables. The system (SME1) evaluated at $\Theta = 0$ corresponds to a benchmark economy without a policy in place.[33] Elements in $\{\xi, \xi^a\}$ typically involve combinations of subsets of parameters derived from a fully specified model. They typically lack direct economic interpretation in terms of the primitives of such a model. Elements in $\Theta$ generally have an economic interpretation. For example, the former could include non-linear combinations of labor supply and demand elasticities (as in the application to fiscal unions), and the latter, the transfers rule elasticities or the elasticity of a Taylor rule for nominal interest rates with respect to inflation in New Keynesian models. The description of models directly, in terms of its equilibrium conditions, is already a step forward toward construction of semi-structural policy counterfactuals. The primitives or micro-foundations of such models need not be specified. Two models with dif-

[33] Uhlig (1995), from whom I borrow some notation, studies a very similar system of equations, and provides a computational toolkit for finding recursive solutions.

ferent primitives but that have the same representation (SME1) are equivalent for the purposes of this section.

**Assumption 6** *$\xi, \Theta$ are such that the system (SME1) is stabilizable.*

Under Assumption 6, it is easy to derive, using the method of undetermined coefficients, a stable recursive solution to (SME1) that can be written:

$$\begin{aligned} x_t &= P(\xi,\Theta)x_{t-1} + Q(\xi,\Theta)z_t \\ z_t &= Nz_{t-1} + \Sigma u_t \end{aligned} \tag{RR}$$

**Assumption 7** *$\xi, \Theta$ are such that $Q(\xi,\Theta)$ is a non-singular square matrix.*

**Claim 3** *If Assumptions 6 and 7 hold, there is a structural vector autoregression (SVAR) representation to the solution (RR) of the form:*

$$x_t = \rho_1(\xi,\Theta)x_{t-1} + \rho_2(\xi,\Theta)x_{t-2} + \Lambda(\xi,\Theta)u_t \tag{SVAR}$$

*where $\rho_1(\xi,\Theta) \equiv P(\xi,\Theta) + Q(\xi,\Theta)NQ(\xi,\Theta)^{-1}$; $\rho_2(\xi,\Theta) \equiv (P(\xi,\Theta) - \rho_1(\xi,\Theta))P(\xi,\Theta)$ and $V(\xi,\Theta) \equiv Var(\Lambda(\xi,\Theta)u_t) = \Lambda(\xi,\Theta)\Lambda(\xi,\Theta)'$, where $\Lambda(\xi,\Theta) \equiv Q(\xi,\Theta)\Sigma$.*

**Proof.** We can write $z_{t-1} = Q(\xi,\Theta)^{-1}(x_{t-1} - P(\xi,\Theta)x_{t-2})$ and replace it and the law of motion for the exogenous states into the law of motion for the endogenous variables to obtain the VAR(2) representation ■.

To summarize, linear models that can be written as in (SME1), where $\xi, \Theta$ satisfy Assumptions 6 and 7, formalize the general class of models on which semi-structural policy counterfactuals can be constructed.

The counterfactual answers how the structural vector autoregression representation of equilibrium (SVAR) changes when the policy changes from $\Theta$ to $\Theta'$.

**Definition 1** *For $\xi^s \subseteq \xi$, a semi-structural policy counterfactual is a mapping $\Omega$ such that:*
$\Omega : \{\rho_1(\xi,\Theta), \rho_2(\xi,\Theta), \Lambda(\xi,\Theta), \Theta, \Theta', \xi^s\} \to \{\rho_1(\xi,\Theta'), \rho_2(\xi,\Theta'), \Lambda(\xi,\Theta')\}$

The mapping $\Omega$ requires knowledge of the matrices in (SVAR), the policy $\Theta$ that generated the (SVAR) and counterfactual policy $\Theta'$. It also requires knowledge of a subset $\xi^s$ regarding the structure of economy $\xi$. I refer to this subset as a *semi-structure*. The definition nests the fully structural counterfactual when $\xi^s = \xi$, where one could directly solve the model (SME1) given the counterfactual policy. The fact that only a semi-structure $\xi^s$ is required is what gives substance to the definition.

**Assumption 8** *Given $\{\rho_1(\xi,\Theta), \rho_2(\xi,\Theta)\}$, the equation $\rho_2(\xi,\Theta) = (P - \rho_1(\xi,\Theta))P$ has a unique solution $P$ with all eigenvalues inside the unit circle.*

**Proposition 2** *If Assumption 8 holds, a semi-structural policy counterfactual $\Omega$ exists and is unique, and knowledge of $\{F, G\}$ is sufficient to construct $\Omega$, i.e. $\xi^s \subset \{F, G\}$.*

**Proof.** See Appendix D.

The key to proving the proposition is to notice that $\{\rho_1(\xi, \Theta), \rho_2(\xi, \Theta), \Lambda(\xi, \Theta)\}$ are the solution to a linear system of matrix equations for all $\xi, \Theta$, and that two such systems corresponding to $\Theta$ and $\Theta'$ are linear translations of each other. Since these systems are linear, we can construct the semi-structural policy counterfactual simply by subtracting one from the other. Knowledge of the matrices $\{F, G\}$ is sufficient because only these elements in $\xi$ enter in a particular non-linear fashion in the systems (i.e., multiplying future and contemporaneous endogenous variable).

Construction of the semi-structural policy counterfactual in Proposition 2 might be difficult to implement in practice. In many applications, we might not want to, or be able to, specify $\{F, G\}$ entirely because, for example, the elements they contain might be the most controversial ones. In the the application to fiscal unions, for example, this includes parameters in the wage-setting equation governing how forward-looking wages are. Moreover, the construction requires knowledge of the impulse response matrix $\Lambda(\xi, \Theta)$. The next section demonstrates that both of these issues can be resolved by specifying an alternative, sufficient semi-structure $\xi^s$.

## 7.1 Implementation: How to describe the semi-structure $\xi^s$

This section discusses specification of the semi-structure $\xi^s$ in a way that is useful to applications of the semi-structural methodology—one that allows identification of $\{F, G, \Lambda(\xi, \Theta)\}$ given $\{\rho_1(\xi, \Theta), \rho_2(\xi, \Theta), V(\xi, \Theta), \Theta\}$. I proceed in two steps. First, I describe identification of $\{F, G\}$ for a fixed matrix $\Lambda(\xi, \Theta)$. Second, I describe identification of $\Lambda(\xi, \Theta)$ by extending the methodology in Beraja et al. (2015b) to a more general class of linear models considered in this paper.[34]

I note that the matrices in the vector autoregression representation (SVAR) are found as a solution to a non-linear system of equations, given a structure and policy. The system of equations (1)-(4) is found in the proof of Proposition 2 in Appendix D.[35] This is the typical way to proceed when solving fully specified, explicit models of the economy. Notice, however, that we can think of this same system as being linear in the structural matrices, given a policy and the vector autoregression representation matrices. In general, the number of unknown elements in the full structure is larger than the number of equations in the system. Thus, the linear system is underdetermined, and it is not possible to infer the full structure with knowledge of the policy and vector autoregression representation matrices alone. The sufficient semi-structure is described by a minimum number of the elements in the full structure that, once specified, make the linear system in the remaining unspecified elements of the structure, exactly determined. I proceed to formalize this logic in what follows.

Given matrices $\{\rho_1(\xi, \Theta), \rho_2(\xi, \Theta), \Lambda(\xi, \Theta), \Theta\}$, the structure $\xi$ satisfies the linear system of equations,

[34]Beraja et al. (2015b) discuss identification in a special case of this class of models.

[35]See Uhlig (1995) and Blanchard and Kahn (1980) for a formal treatment.

$$(FP + G + \Theta_c)P + H + \Theta_p = 0 \quad (5)$$
$$(LN + M)\Sigma + (F\rho_1 + G + \Theta_c)\Lambda = 0 \quad (6)$$
$$(LN + M)\Sigma\Sigma^{-1}N\Sigma + ((F\rho_1 + G + \Theta_c)\rho_1 + F\rho_2 + H + \Theta_p)\Lambda = 0 \quad (7)$$
$$(P - \rho_1)P - \rho_2 = 0 \quad (8)$$

This new system can be obtained from the system of equations (1)-(4). Per Assumption 7, all matrices in the system are $k \times k$ square matrices, where $k$ is the number of variables in $x_t$. Suppose we are interested in identifying the first line in matrices $\{F, G\}$ (identification of other lines follows identical logic). We can count the number of equations involving the elements in the first line of $\{F, G\}$, and the number of unknown elements in the structure in this line. Then, we specify enough elements in the structure $\xi$ such that the elements in the first line of $\{F, G\}$ are exactly determined (i.e., there are the same number of equations and unspecified unknown elements). The following lemma formalizes this.

**Lemma 2** *Given matrices $\{\rho_1(\xi, \Theta), \rho_2(\xi, \Theta), \Lambda(\xi, \Theta), \Theta\}$, the elements in line 'l' of the $k \times k$ matrices $\{F, G\}$ are uniquely identified if the following are specified:*

1. *$j_l$ elements in line 'l' of $(LN + M)\Sigma[\ I \quad \Sigma^{-1}N\Sigma\ ]$.*

2. *$n_l$ elements in line 'l' of $\{F, G, H\}$, where $n_l \equiv 2k - j_l - (1 - rank(\Theta_l))$.*

**Proof.** See Appendix E.■

The description above is not unique but useful since it separates required knowledge into two dimensions. First is knowledge about what exogenous variables enter in what equations, and their autoregressive matrix $N$. Second is knowledge about what endogenous variables enter in what equations, and how to parameterize certain elements in $\{F, G, H\}$. In the application to fiscal unions these were exclusion restrictions implying certain elements are zeros in the structure. In general, however, specifying a number of linear restrictions on the elements in the structure would suffice.

If $\Lambda(\xi, \Theta)$ were known, we could follow Lemma 2 for all lines 'l' from 1 to $k$. This would specify a sufficient semi-structure $\xi^s$ to identify all lines of $\{F, G\}$. But what if $\Lambda(\xi, \Theta)$ is unknown? Identification of the impulse response matrix in vector autoregression models requires assumptions—ordering of shocks, sign restrictions, long-run restrictions, etc. These represent several routes that could be followed. Alternatively, the primary methodological insight in Beraja et al. (2015b) is that the impulse response matrix can be identified through specification of one or more structural equations, or in the terminology used in this paper, a semi-structure.

The following lemma describes this semi-structural identification scheme. It is an extension of the methodology in Beraja et al. (2015b) to the general class of linear models described in the previous section.

**Lemma 3** *Take as given the matrices $\{\rho_1(\xi,\Theta), \rho_2(\xi,\Theta), V(\xi,\Theta)\}$ and all the elements in 's' lines of $\{F, G, H, \Theta\}$, where $s < k$. Let $\mathbb{I}_s$ be a selection matrix for the 's' lines, and $A(s) \equiv \begin{bmatrix} \mathbb{I}_s(LN+M)\Sigma \\ \mathbb{I}_s(LN+M)N\Sigma \end{bmatrix}$ and $B(s) \equiv \begin{bmatrix} \mathbb{I}_s(F\rho_1 + G + \Theta_c) \\ \mathbb{I}_s((F\rho_1 + G + \Theta_c)\rho_1 + F\rho_2 + H + \Theta_p) \end{bmatrix}$. The impulse response matrix $\Lambda(\xi,\Theta)$ is identified uniquely if the following are satisfied:*

1. *$B(s)$ has full rank.*
2. *There are $r(k) = k(k-1)/2$ elements specified in $A(s)$.*
3. *There is at least one specified element in $r_k$ in each of '$k-1$' columns of $A(s)$.*
4. *$\Lambda(\xi,\Theta)\Lambda(\xi,\Theta)' = V(\xi,\Theta)$.*

**Proof.** See Appendix F.■

Conditions 1 through 3 in the lemma imply a set of linear restrictions linking the reduced form error from (SVAR) to the structural shocks. They impose a linear co-movement that the unexpected components in the variables in (SVAR) (i.e., the columns in the impulse response matrix) must satisfy. These restrictions, combined with orthogonalization conditions in 4, are sufficient to identify all elements in the matrix $\Lambda(\xi,\Theta)$. The application to fiscal unions in this paper is a special case, with three endogenous variables and one specified structural equation (i.e., $k = 3$ and $s = 1$). Section 6.2 describes construction of the linear restrictions implied by Lemma 3 corresponding to this application, and provides intuition. Beraja et al. (2015b) is another example.

I conclude this section by noting that $r(k)$ elements in the '$s$' lines from Lemma 3, combined with $\{j_l, n_l\}$ elements in lines $l \notin s$ from Lemma 2, describe a sufficient semi-structure $\xi^s$ for identifying $\{F, G, \Lambda(\xi,\Theta)\}$ uniquely.

# 8 Falsifying a Set of Models via the Semi-Structural Methodology

So far I have discussed how to construct a counterfactual economy after a policy change given observations of the economy before the policy change. There are many instances, however, where observations before and after the policy change are available. For instance, in the context of the application to stabilization in fiscal unions, we could entertain the possibility that in the future Catalunya decided to secede from the fiscal union that is Spain or that Europe would fiscally integrate. Or that only country-level monetary policy rules changed before and after the European Monetary Union, going from being country-specific to union-wide determined. In cases like this, I argue that we could compare the predicted VAR representation from a set of models and the actual VAR representation that we observe after the policy change. If these are "too far apart" in a statistical sense, we can reject the null hypothesis that this set of models generated the observed data. Moreover, under normality of the shocks, the OLS estimates of the VAR are the maximum likelihood estimators. Hence, the statistical definition of "too far part" is given by a simple Wald test. Thus, the methodology can further be used to falsify a set of models in a way that is reminiscent to relatively new literature in econometrics on “partially identified” models.

Formally, we wish to test the null hypothesis

$$\mathbf{H_0} : h\left(\rho_1, \rho_2, V, \rho_1', \rho_2', V', \Theta, \Theta'\right) \equiv \{\rho_1(\xi, \Theta'), \rho_2(\xi, \Theta'), V(\xi, \Theta')\} - \{\rho_1', \rho_2', V'\} = 0$$

where $\{\rho_1(\xi, \Theta'), \rho_2(\xi, \Theta'), V(\xi, \Theta')\}$ is the predicted VAR representation under the new policy $\Theta'$ by a set of models with structure $\xi$, and $\{\rho_1', \rho_2', V'\}$ is the observed VAR representation. The predicted representation is constructed via the semi-structural policy counterfactual mapping as a function of the observed VAR representation $\{\rho_1, \rho_2, V\}$ under the original policy $\Theta$, the new policy $\Theta'$ and the semi-structure $\xi^s$. Hence, the test is about the null hypothesis that semi-structure $\xi^s$ generated observed VAR representation after the change in policy.

**Assumption 9** *The function* $h\left(\rho_1, \rho_2, V, \rho_1', \rho_2', V', \Theta, \Theta'\right)$*:*

1. *is continuously differentiable on a neighborhood of* $\{\tilde{\rho}_1, \tilde{\rho}_2, \tilde{V}, \tilde{\rho}_1', \tilde{\rho}_2', \tilde{V}', \tilde{\Theta}, \tilde{\Theta}'\}$.

2. *has full rank first derivative matrix.*

3. *has positive definite second derivative matrix.*

Finally, given OLS consistent estimators $\{\tilde{\rho}_1, \tilde{\rho}_2, \tilde{V}, \tilde{\rho}_1', \tilde{\rho}_2', \tilde{V}', \tilde{\Theta}, \tilde{\Theta}'\}$, we can construct the Wald statistic that has asymptotic $\chi^2$ distribution under the null hypothesis $H_0$. The set of models described by semi-structure $\xi^s$ is falsified if the null hypothesis is rejected.

# 9 Conclusions

In this paper, I propose a methodology to conduct quantitative counterfactual analysis in widely used models in macroeconomics with minimal *a-priori* structural assumptions. This semi-structural methodology allows the use of vector autoregression models to construct counterfactuals with respect to changes in policy rules. This analysis is robust across many models while being immune to Lucas critique, in the spirit of Sims (1980). While I have emphasized benefits and feasibility of the semi-structural methodology, it also has some limitations when compared to fully structural or “sufficient statistics” approaches. The most important is that it is an inherently *linear* methodology. Counterfactual questions that involve non-linearities are not easily, or at all, accommodated. [36] A further limitation is that the methodology does not directly lend itself to welfare comparisons or optimal policy analysis.

[36] For example, in Beraja et al. (2015a), we study monetary policy in an environment where the pass-through from posted interest rates to effective interest rates is influenced by collateralized lending, a type of lending that naturally introduces non-linearities.

I apply the methodology to quantify how fiscal unions, through their federal tax-and-transfer system, contribute to regional stabilization. This quantitative question has received surprisingly little attention in the literature beyond reduced-form calculations and simple calibration exercises in specific models, despite existing theoretical work on fiscal unions and its relevance for current discussions about European fiscal integration. My primary quantitative finding is that during the Great Recession fiscal integration significantly reduced cross-state employment differences by redistributing resources from states that were doing relatively well to states that were doing relatively poorly. On the one hand, this finding may overstate regional stabilization benefits because fiscal integration could partially displace existing private risk-sharing arrangements. On the other hand, measured gains from fiscal integrations would be larger if the reduction in state-level volatility reduces within-state individual risk exposure by more than it reduces per-capita (or average) risk exposure of the state's "representative agent".

In future research, the semi-structural methodology could help answering other questions in macroeconomics where stochasticity and dynamics are important and linear models provide a good approximation—for example, questions related to stabilization via monetary policy, financial integration, tax reform, exchange rate dynamics. Moreover, the methodology could be used in applications in other fields, like industrial organization and labor economics, where models typically involve simultaneous systems of equations of first order conditions that need to be estimated. Finally, I discussed how to use the methodology to falsify a set of models provided data before and after a policy change are available.

## References


Adao, R., A. Costinot, and D. Donaldson (2015): "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade," Tech. rep., National Bureau of Economic Research.

Alvarez, F., H. L. Bihan, and F. Lippi (2014): "Small and large price changes and the propagation of monetary shocks," Tech. rep., National Bureau of Economic Research.

An, S. and F. Schorfheide (2007): "Bayesian analysis of DSGE models," *Econometric reviews*, 26, 113–172.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

Asdrubali, P., B. E. Sorensen, and O. Yosha (1996): "Channels of interstate risk sharing: United States 1963-1990," *The Quarterly Journal of Economics*, 1081–1110.

Atkeson, A. and T. Bayoumi (1993): "Do private capital markets insure regional risk? Evidence from the United States and Europe," *Open Economies Review*, 4, 303–324.

Atkeson, A. and A. Burstein (2015): "Aggregate Implications of Innovation Policy," .

Auclert, A. (2014): "Monetary policy and the redistribution channel," *Unpublished Manuscript, Massachusetts Institute of Technology*.

Autor, D. H., D. Dorn, and G. H. Hanson (2013): "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *The American Economic Review*, 103, 2121–2168.

Bayoumi, T. and P. R. Masson (1995): "Fiscal flows in the United States and Canada: lessons for monetary union in Europe," *European Economic Review*, 39, 253–274.

Beraja, M., A. Fuster, E. Hurst, and J. Vavra (2015a): "Regional heterogeneity and monetary policy," Tech. rep., Staff Report Federal Reserve Bank of New York.

Beraja, M., E. Hurst, and J. Ospina (2015b): "The Aggregate Implications of Regional Business Cycles," *Unpublished Manuscript, University of Chicago*.

Blanchard, O. J. and C. M. Kahn (1980): "The solution of linear difference models under rational expectations," *Econometrica: Journal of the Econometric Society*, 1305–1311.

Blanchard, O. J., L. F. Katz, R. E. Hall, and B. Eichengreen (1992): "Regional evolutions," *Brookings papers on economic activity*, 1–75.

Blanchard, O. J. and D. Quah (1989): "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *The American Economic Review*, 79, 655–673.

Blaum, J., C. Lelarge, and M. Peters (2015): "The Gains From Trade in Firm-Based Models of Importing," .

Bucovetsky, S. (1998): "Federalism, equalization and risk aversion," *Journal of Public Economics*, 67, 301–328.

Canay, I. and A. Shaikh (2015): "Practical and Theoretical Advances for Inference in Partially Identified Models," .

Carriero, A. and R. Giacomini (2011): "How useful are no-arbitrage restrictions for forecasting the term structure of interest rates?" *Journal of Econometrics*, 164, 21–34.

Chari, V. V. and P. J. Kehoe (2007): "On the need for fiscal constraints in a monetary union," *Journal of Monetary Economics*, 54, 2399–2408.

Charles, K. K., E. Hurst, and M. J. Notowidigdo (2014): "Housing Booms, Labor Market Outcomes, and Educational Attainment," *Manuscript, University of Chicago*.

Chetty, R. (2008): "Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods," Tech. rep., National Bureau of Economic Research.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins," *The American Economic Review*, 101, 471–475.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1999): "Monetary policy shocks: What have we learned and to what end?" *Handbook of macroeconomics*, 1, 65–148.

Clarida, R., J. Gali, and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 147–180.

Clarida, R. H. and M. Gertler (1997): "How the Bundesbank conducts monetary policy," in *Reducing inflation: Motivation and strategy*, University of Chicago Press, 363–412.

DeJong, D. N. and C. Dave (2011): *Structural macroeconometrics*, Princeton University Press.

Del Negro, M. and F. Schorfheide (2004): "Priors from general equilibrium models for vars*," *International Economic Review*, 45, 643–673.

——— (2009): "Monetary Policy Analysis with Potentially Misspecified Models," *The American Economic Review*, 99, 1415.

Eggertsson, G. B. and P. Krugman (2012): "Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach*," *The Quarterly Journal of Economics*, qjs023.

Evers, M. P. (2015): "Fiscal Federalism and Monetary Unions: A Quantitative Assessment," *Journal of International Economics*.

Farhi, E. and I. Werning (2014): "Fiscal unions," *Unpublished Manuscript, Massachusetts Institute of Technology*.

Feyrer, J. and B. Sacerdote (2013): "How Much Would US Style Fiscal Integration Buffer European Unemployment and Income Shocks?(A Comparative Empirical Analysis)," *The American Economic Review*, 103, 125–128.

Gilchrist, S., J. W. Sim, and E. Zakrajšek (2014): "Uncertainty, financial frictions, and investment dynamics," Tech. rep., National Bureau of Economic Research.

Guerrieri, V. and G. Lorenzoni (2011): "Credit crises, precautionary savings, and the liquidity trap," Tech. rep., National Bureau of Economic Research.

Hamilton, J. D. (1994): *Time series analysis*, vol. 2, Princeton university press Princeton.

Hansen, G. D. (1985): "Indivisible labor and the business cycle," *Journal of monetary Economics*, 16, 309–327.

Hansen, L. P. and T. J. Sargent (2008): *Robustness*, Princeton university press.

Harberger, A. C. (1964): "The measurement of waste," *The American Economic Review*, 58–76.

Hurst, E., B. J. Keys, A. Seru, and J. S. Vavra (2015): "Regional Redistribution Through the US Mortgage Market," Tech. rep., National Bureau of Economic Research.

Kollmann, R. (2001): "Explaining international comovements of output and asset returns: the role of money and nominal rigidities," *Journal of Economic Dynamics and Control*, 25, 1547–1583.

Kydland, F. E. and E. C. Prescott (1982): "Time to build and aggregate fluctuations," *Econometrica: Journal of the Econometric Society*, 1345–1370.

Lockwood, B. (1999): "Inter-regional insurance," *Journal of Public Economics*, 72, 1–37.

Lucas, R. E. (1976): "Econometric policy evaluation: A critique," in *Carnegie-Rochester conference series on public policy*, Elsevier, vol. 1, 19–46.

McKay, A. and R. Reis (2013): "The role of automatic stabilizers in the US business cycle," Tech. rep., CEPR Discussion Papers.

Mian, A., K. Rao, A. Sufi, et al. (2013): "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics*, 128, 1687–1726.

Mian, A. and A. Sufi (2014): "What Explains the 2007–2009 Drop in Employment?" *Econometrica*, 82, 2197–2223.

Midrigan, V. and T. Philippon (2011): "Household leverage and the recession," Tech. rep., National Bureau of Economic Research.

Nakamura, E. and J. Steinsson (2014): "Fiscal Stimulus in a Monetary Union: Evidence from US Regions," *The American Economic Review*, 104, 753–792.

Oh, H. and R. Reis (2012): "Targeted transfers and the fiscal response to the great recession," *Journal of Monetary Economics*, 59, S50–S64.

Persson, T. and G. Tabellini (1996a): "Federal fiscal constitutions: Risk sharing and moral hazard," *Econometrica: Journal of the Econometric Society*, 623–646.

——— (1996b): "Federal fiscal constitutions: risk sharing and redistribution," *Journal of Political Economy*, 979–1009.

Pesaran, M. H., R. G. Pierse, and K. C. Lee (1993): "Persistence, cointegration, and aggregation: A disaggregated analysis of output fluctuations in the US economy," *Journal of Econometrics*, 56, 57–88.

Ruggles, S., M. Sobek, C. A. Fitch, P. K. Hall, and C. Ronnander (1997): *Integrated public use microdata series: Version 2.0*, Historical Census Projects, Department of History, University of Minnesota.

Sala-i Martin, X. and J. D. Sachs (1991): "Fiscal Federalism and Optimum Currency Areas: Evidence for Europe from the United States," *NBER Working Paper*.

Schorfheide, F. (2000): "Loss function-based evaluation of DSGE models," *Journal of Applied Econometrics*, 15, 645–670.

Shimer, R. (2010): *Labor Markets and Business Cycles*, Princeton University Press.

Sims, C. A. (1980): "Macroeconomics and reality," *Econometrica: Journal of the Econometric Society*, 1–48.

Stock, J. H. and M. W. Watson (2001): "Vector autoregressions," *Journal of Economic perspectives*, 101–115.

Uhlig, H. (1995): "A toolkit for analyzing nonlinear dynamic stochastic models easily," .

——— (2005): "What are the effects of monetary policy on output? Results from an agnostic identification procedure," *Journal of Monetary Economics*, 52, 381–419.

# A Proof of Lemma 1

The following equations characterize the log-linearized equilibrium

$$
\begin{aligned}
\tilde{w}_{kt} - \tilde{p}_{kt} &= \frac{1}{\phi}\tilde{n}_{kt} \\
\tilde{w}_{kt} - \tilde{p}_{kt} &= (\alpha - 1)\tilde{n}_{kt} + \beta\tilde{x}_{kt} + \tilde{z}_{kt} \\
\tilde{q}_t - \tilde{p}_{kt} &= \alpha\tilde{n}_{kt} + (\beta - 1)\tilde{x}_{kt} + \tilde{z}_{kt} \\
0 &= \mathbb{E}_t\left(\tilde{m}u_{kt+1} - \tilde{m}u_{kt+1} - (\tilde{p}_{kt+1} - \tilde{p}_{kt}) - \gamma_{kt+1} - \Phi_0(\tilde{c}_{kt} - \tilde{c}_t) + \tilde{i}_{t+1}\right) \\
\tilde{m}u_{kt+1} &= -\frac{\sigma}{C - \frac{\phi}{1+\phi}N^{\frac{1+\phi}{\phi}}}\left(C\tilde{c}_{kt+1} - N^{\frac{1+\phi}{\phi}}\tilde{n}_{kt+1}\right) \\
\tilde{c}_{kt} &= \tilde{w}_{kt} - \tilde{p}_{kt} + \tilde{n}_{kt} \\
B\tilde{b}_{kt} &= B(1+r)(\tilde{b}_{kt-1} + \tilde{i}_t) + \eta_{kt} - X(\tilde{q}_t + \tilde{x}_{kt}) + S\tilde{s}_{kt} \\
\sum_k \tilde{x}_{kt} &= \sum_k \tilde{\eta}_{kt} \\
B^g\tilde{b}^g_t + S\sum_k \tilde{s}_{kt} + G\tilde{q}_t &= B^g(1+r)(\tilde{b}^g_{t-1} + \tilde{i}_t) \\
\tilde{s}_{kt} &= \vartheta_w\tilde{w}_{kt} + \vartheta_n\tilde{n}_{kt} + \vartheta_b\tilde{b}_{kt-1} \\
\tilde{i}_{t+1} &= \phi_p\mathbb{E}_t[\tilde{p}_{t+1} - \tilde{p}_t]
\end{aligned}
$$

After adding up, the aggregate log-linearized equilibrium evolution of $\{\tilde{w}_t - \tilde{p}_t, \tilde{n}_t\}$ is characterized by

$$
\begin{aligned}
0 &= \mathbb{E}_t(\tilde{m}u_{t+1} - \tilde{m}u_t + (\phi_p - 1)(\tilde{p}_{t+1} - \tilde{p}_t) + \tilde{\gamma}_{t+1}) \\
0 &= \frac{1}{\phi}\tilde{n}_t - (\tilde{w}_t - \tilde{p}_t) \\
\tilde{w}_t - \tilde{p}_t &= (\alpha - 1)\tilde{n}_t + \tilde{z}_t + \beta\tilde{\eta}_t \\
\tilde{m}u_{t+1} &\equiv -\frac{\sigma}{C - \frac{\phi}{1+\phi}N^{\frac{1+\phi}{\phi}}}\left(C(\tilde{w}_{t+1} - \tilde{p}_{t+1} + \tilde{n}_{t+1}) - N^{\frac{1+\phi}{\phi}}(\tilde{n}_{t+1})\right)
\end{aligned}
$$

which is equivalent to the system of equations characterizing the log-linearized equilibrium in a representative agent economy with a production technology that utilizes labor alone with an elasticity of $\alpha$, no endogenous discounting and only 2 exogenous processes $\{\tilde{z}_t + \beta\tilde{\eta}_t, \tilde{\gamma}_t\}$.

Next, take log-deviations from the aggregate in the original system. This results in the system characterizing the evolution of $\{p_{kt}, w_{kt}, n_{kt}, b_{kt}, c_{kt}\}$ for given $\{z_{kt}, \eta_{kt}, \gamma_{kt}\}$

$$
\begin{aligned}
0 = \mathbb{E}_t \Bigg[ & \frac{\sigma\beta}{\frac{\alpha\phi}{1+\phi} - 1}(w_{kt+1} + n_{t+1} - w_{kt} - n_{kt}) + (\alpha + \beta - 1)(n_{kt+1} - n_{kt}) \\
& +(\beta - 1)(w_{kt+1} - w_{kt}) + (1 + \frac{\sigma}{\frac{\alpha\phi}{1+\phi} - 1})(z_{kt+1} - z_{kt}) + \Phi_0 c_{kt} - \gamma_{kt+1} \Bigg] \\
\beta w_{kt} &= \left( \frac{1+\phi}{\phi} - (\alpha + \beta) \right) n_{kt} - z_{kt} \\
\frac{B}{S} b_{kt} &= \frac{B}{S}(1+r) b_{kt-1} - \frac{X}{S}(w_{kt} + n_{kt}) + \vartheta_n n_{kt} + \vartheta_w w_{kt} + \vartheta_b b_{kt-1} + \frac{1}{S}\eta_{kt} \\
c_{kt} &= w_{kt} - p_{kt} + n_{kt} \\
p_{kt} &= (1-\beta) w_{kt} - (z_{kt} + (\alpha + \beta - 1) n_{kt})
\end{aligned}
$$

This system is independent of all aggregate variables and is analogous to the system characterizing the equilibrium in a small open economy without movements in the terms of trade and nominal interest rate, proving the Lemma.

# B Proof of Claim 1

To ease notation let $\tilde{\Phi}_0 \equiv \Phi_0 - \nu\alpha\frac{\sigma}{\alpha - \frac{1+\phi}{\phi}}$, $\tilde{\Phi}_2 \equiv \Phi_2 + \nu\alpha\frac{\sigma}{\alpha - \frac{1+\phi}{\phi}}$, and $\tilde{\sigma} \equiv 1 + \frac{\sigma}{(\alpha - \frac{1+\phi}{\phi})\frac{\phi}{1+\phi}}$. Guess that $\beta(\lambda) = \frac{\beta_0}{\frac{1-\lambda_0}{\lambda_0}}\frac{1-\lambda}{\lambda}, \nu(\phi, \lambda) = \frac{\nu_0\frac{1+\phi_0}{\phi_0}}{\frac{1-\lambda_0}{\lambda_0}}\frac{1-\lambda}{\lambda}\frac{\phi}{1+\phi} - 1, \alpha(\phi, \lambda) = \frac{1+\phi}{\phi} + \frac{(\alpha_0 - \frac{1+\phi_0}{\phi_0})}{\frac{1-\lambda_0}{\lambda_0}}\frac{1-\lambda}{\lambda}, \sigma_z(\lambda) = \frac{\sigma_z^0}{\frac{1-\lambda_0}{\lambda_0}}\frac{1-\lambda}{\lambda}, \tilde{\Phi}_2(\lambda) = \frac{\tilde{\Phi}_2^0}{\frac{1-\lambda_0}{\lambda_0}}\frac{1-\lambda}{\lambda}$ and $\frac{X}{S} = \frac{X^0}{S^0}, \frac{B}{S} = \frac{B^0}{S^0}, r = r^0$. Also, let $\tilde{P}^0 \equiv (P^0)^{-1}$ and let $\tilde{\sigma}, \Phi_1, \tilde{\Phi}_0$ solve the system below

$$
\begin{aligned}
-\beta(P^0_{13} + P^0_{23}) + (1 - \alpha P^0_{13})\frac{1}{\tilde{\sigma}} + \tilde{\phi}_2 \tilde{P}^0{}_{13} &= 0 \\
-\frac{\alpha}{\tilde{\sigma}} P^0_{12} - (\beta - \frac{1}{\tilde{\sigma}})(P^0_{12} + P^0_{21} - 1) + \Phi_1 + \tilde{\phi}_2 \tilde{P}^0{}_{12} &= 0 \\
-(\beta - \frac{1}{\tilde{\sigma}})(P^0_{11} + P^0_{21} - 1) + \frac{\alpha}{\tilde{\sigma}}(1 - P^0_{11}) + \tilde{\Phi}_0 + \tilde{\phi}_2 \tilde{P}^0{}_{11} &= 0
\end{aligned}
$$

So we can construct a parameterization $\lambda^1 > \lambda^0$ and $\nu^1 > \nu^0$ if $\phi^1 : \nu^1 = \frac{\nu_0\frac{1+\phi_0}{\phi_0}}{\frac{1-\lambda_0}{\lambda_0}}\frac{1-\lambda^1}{\lambda^1}\frac{\phi^1}{1+\phi^1}$.

To verify the guess, replace it in the system of equations characterizing the equilibrium. We obtain the exact same system of equations corresponding to $\xi^0$ and hence identical $P^0, \Lambda^0$, which completes the proof.

# C Proof of Proposition 1

The proof is by construction. Applying the method of undetermined coefficients to (FiscalSME) implies that $\{P, \Lambda\}$ solve

$$\begin{aligned}(FP + G + \Theta_c)P + H + \Theta_p &= 0 \\ (LN + M)\Sigma + F\Lambda\Sigma^{-1}N\Sigma + (FP + G + \Theta_c)\Lambda &= 0\end{aligned}$$

Also from Claim 3 we have

$$\begin{aligned}\Lambda\Sigma^{-1}N\Sigma + (\rho_1 - P)\Lambda &= 0 \\ (P - \rho_1)P - \rho_2 &= 0\end{aligned}$$

Replacing in the first system, we obtain a new system

$$\begin{aligned}(FP + G + \Theta_c)P + H + \Theta_p &= 0 \\ (LN + M)\Sigma + (F\rho_1 + G + \Theta_c)\Lambda &= 0\end{aligned}$$

Given $\{\rho_1, \rho_2\}$, let $P$ be a solution with all eigenvalues inside the unite circle to the matrix quadratic equation $0 = (P - \rho_1)P - \rho_2$. Without loss of generality, take equations involving the first line of structure $\xi$, i.e. the Euler equation. Because the first line of $\Theta$ is all zeros by Assumption 2, this system can be written as

$$\begin{bmatrix} (P')^2 & P' & I & 0 \\ \Lambda'\rho_1' & \Lambda' & 0 & I \end{bmatrix}_{6\times 12} \begin{bmatrix} F_{1.}' \\ G_{1.}' \\ H_{1.}' \\ ((LN+M)\Sigma)_{1.}' \end{bmatrix}_{12\times 1} = \mathbf{0}_{6\times 1}$$

There are 12 unknowns in the first line of structure $\xi$. There are 6 equations in this system. One of the unknown elements can be normalized to 1 because the system is equalized to zero. This implies that by specifying at least 5 zeros in the first line of structure $\xi$ we obtain a linear system of equations that is exactly determined and can be solved for the remaining unspecified elements in the structure (as long as the matrix containing $\{P, \Lambda, \rho_1\}$ has full rank).

An identical proof holds for identifying the structure $\xi$ for in the other two equations: the sequential budget constraint (SB) and (Labor market).

Finally, we solve the following system for $\{P(\xi, \Theta'), \Lambda(\xi, \Theta'\})$ given $\{P(\xi, \Theta), \rho_1(\xi, \Theta), \Lambda(\xi, \Theta)\}$

and $\{F, G, \Theta\}$. The system results from subtracting (SME Fiscal) for policy $\Theta$ and $\Theta' = 0$.

$$
\begin{aligned}
&0 = F(P(\xi,\Theta')^2 - P(\xi,\Theta)^2) + G(P(\xi,\Theta') - P(\xi,\Theta)) - \Theta_c P(\xi,\Theta) - \Theta_p \\
&0 = \left(FP(\xi,\Theta') + G\right)\left(\Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1} - I\right) + F\left(\Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1} - I\right)(\rho_1(\xi,\Theta) - P(\xi,\Theta)) \\
&\quad + F(P(\xi,\Theta') - P(\xi,\Theta)) - \Theta_c
\end{aligned}
$$

And obtain $\{\rho_1(\xi,\Theta'), \rho_2(\xi,\Theta')\}$ as:

$$
\begin{aligned}
\rho_1(\xi,\Theta') &= P(\xi,\Theta') + \Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1}(\rho_1(\xi,\Theta) - P(\xi,\Theta))\left(\Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1}\right)^{-1} \\
\rho_2(\xi,\Theta') &= (P(\xi,\Theta') - \rho_1(\xi,\Theta'))P(\xi,\Theta')
\end{aligned}
$$

which concludes the proof. ■

# D Proof of Proposition 2

The proof is by construction. We have that $P(\xi,\Theta), \Lambda(\xi,\Theta), \rho_1(\xi,\Theta), \rho_2(\xi,\Theta)$ satisfy the following system of matrix equations for all $\{\xi,\Theta\}$,

$$
((F + \Theta_f)P + G + \Theta_c)P + H + \Theta_p = 0 \tag{1}
$$

$$
(LN + M)\Sigma + (F + \Theta_f)\Lambda\Sigma^{-1}N\Sigma + ((F + \Theta_f)P + G + \Theta_c)\Lambda = 0 \tag{2}
$$

$$
\Lambda\Sigma^{-1}N\Sigma + (\rho_1 - P)\Lambda = 0 \tag{3}
$$

$$
(P - \rho_1)P - \rho_2 = 0 \tag{4}
$$

Under Assumption 8, equation (4) has a unique solution $P(\xi,\Theta)$ with all eigenvalues inside the unit circle for given $\rho_1(\xi,\Theta), \rho_2(\xi,\Theta)$. Subtracting equation (1) evaluated at $\Theta$ and $\Theta'$ from each other, and doing the same for equation 2 (where we replace $N$ using 3) we obtain a system of matrix equations to solve for $\{P(\xi,\Theta'), \Lambda(\xi,\Theta')\}$ given $\{P(\xi,\Theta), \Lambda(\xi,\Theta), F, G\}$:

$$
\begin{aligned}
0 =& (F + \Theta'_f)(P(\xi,\Theta')^2 - P(\xi,\Theta)^2) + (G + \Theta'_c)(P(\xi,\Theta') - P(\xi,\Theta)) \\
&+ (\Theta'_f - \Theta_f)P(\xi,\Theta)^2 + (\Theta'_c - \Theta_c)P(\xi,\Theta) + \Theta'_p - \Theta_p \\
0 =& \left((F + \Theta'_f)P(\xi,\Theta') + G + \Theta'_c\right)\left(\Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1} - I\right) \\
&+ (F + \Theta'_f)\left(\Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1} - I\right)(\rho_1(\xi,\Theta) - P(\xi,\Theta)) \\
&+ (F + \Theta'_f)(P(\xi,\Theta') - P(\xi,\Theta)) + (\Theta'_f - \Theta_f)\rho_1(\xi,\Theta) + \Theta'_c - \Theta_c
\end{aligned} \tag{SME2}
$$

Finally, we can obtain $\{\rho_1(\xi,\Theta'),\rho_2(\xi,\Theta')\}$ as:

$$\rho_1(\xi,\Theta') = P(\xi,\Theta') + \Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1}(\rho_1(\xi,\Theta) - P(\xi,\Theta))\left(\Lambda(\xi,\Theta')\Lambda(\xi,\Theta)^{-1}\right)^{-1}$$
$$\rho_2(\xi,\Theta') = (P(\xi,\Theta') - \rho_1(\xi,\Theta'))P(\xi,\Theta')$$

which concludes the proof. ■

# E Proof of Lemma 2

From system (5)-(8), we can write the equations involving the first line of $\{F,G\}$ as:

$$\begin{bmatrix} P'P' & P' \\ \Lambda'\rho_1' & \Lambda' \\ \Lambda'(\rho_1'\rho_1' + \rho_2') & \Lambda'\rho_1' \end{bmatrix}\begin{bmatrix} F' \\ G' \end{bmatrix}\begin{bmatrix} 1 \\ 0_{k.1} \end{bmatrix} + \begin{bmatrix} I_{k.k} \\ 0_{k.k} \\ \Lambda' \end{bmatrix} H' \begin{bmatrix} 1 \\ 0_{k.1} \end{bmatrix}$$
$$+ \begin{bmatrix} P' & I \\ \Lambda' & 0 \\ \Lambda'\rho_1' & \Lambda' \end{bmatrix}\begin{bmatrix} \Theta_c' \\ \Theta_p' \end{bmatrix}\begin{bmatrix} 1 \\ 0_{k.1} \end{bmatrix} + \begin{bmatrix} 0_{k.k} \\ \Sigma \\ \Sigma N' \end{bmatrix}(LN+M)'\begin{bmatrix} 1 \\ 0_{k.1} \end{bmatrix} = 0_{3k.1}$$

There are $3k$ elements in the first line of $\{F,G,H\}$. Let $j$ be the number of specified elements in $[\ 1 \quad 0_{1.k}\ ](LN+M)\Sigma[\ I \quad N\ ]$. Then there are $k+j$ zero elements in $[\ 1 \quad 0_{1.k}\ ](LN+M)\Sigma[\ 0_{k.k} \quad I \quad N\ ]$. Also if $rank([\ 1 \quad 0_{1.k}\ ][\ \Theta_c \quad \Theta_p\ ]) = 0$ we can normalize one of the elements in the first line of $\{F,G,H\}$ to 1. This implies that it is necessary to specify $n \equiv 3k - k - j - (1 - rank([\ 1 \quad 0_{1.k}\ ][\ \Theta_c \quad \Theta_p\ ])$ elements in the first line of $\{F,G,H\}$ alone.

# F Proof Lemma 3

There are $k^2$ elements in $\Lambda(\xi,\Theta)$ that we would like to identify. The proof of the lemma consist in deriving $k^2$ independent equations to generate a system that is exactly determined.

From the orthogonalization conditions in 4., we obtain $k(k+1)/2$ equations. Since $V(\xi,\Theta)$ is symmetric we do not obtain $k^2$ equations, but only a number of equations equal to the lower (or upper) triangle. Conditions 1. to 3. imply that the system derived from picking $'s'$ lines in equations (6) and (7) and specifying $r(k) = k(k-1)/2$ elements in $A(s)$

$$A(s) + B(s)\Lambda(\xi,\Theta) = 0$$

generates $r(k)$ independent equations that includes all the elements in all the columns of $\Lambda(\xi,\Theta)$.

Finally, we have described a total of $k(k+1)/2 + r(k) = k^2$ equations to identify the $k^2$ parameters in $\Lambda(\xi,\Theta)$.

# G Appendix Tables and Figures

Table 2: Employment Impact Elasticity to $\gamma$ in (%)

| | | $\boldsymbol{\lambda}$ | | | |
|---|---|---|---|---|---|
| | $\kappa$ | **1** | **0.7** | **0.5** | **0.3** |
| $\vartheta_n = -1.6$ | **0** | 0.7 | 1.6 | 2 | 2.4 |
| $\vartheta_n = 0$ | | 0.7 | 2.1 | 2.9 | 3.6 |
| $\vartheta_n = -1.6$ | **0.5** | 0.7 | 1.8 | 2.1 | 2.4 |
| $\vartheta_n = 0$ | | 0.7 | 2.3 | 3.1 | 3.8 |
| $\vartheta_n = -1.6$ | **0.9** | 0.7 | 1.9 | 2.2 | 2.5 |
| $\vartheta_n = 0$ | | 0.7 | 2.5 | 3.3 | 3.9 |

Note: Table shows the employment impact elasticity to a discount rate shock, in percentage deviations from the steady state, for alternative calibrations of $\kappa, \lambda$ in the model in Section **??**.

Table 3: Policy rule baseline estimates

| | $\vartheta_n$ | $\vartheta_w$ | $\vartheta_b$ | $\vartheta_{w+n}$ | $R^2$ |
|---|---|---|---|---|---|
| **OLS** | $-1.6^{**}$ (0.5) | $-0.9^{*}$ (0.7) | $-0.03$ (0.02) | . | 0.41 |
| **IV w/ shocks (1)** | $-1.3^{*}$ (0.7) | $-1.4^{*}$ (0.8) | $-0.02$ (0.02) | . | 0.41 |
| **IV w/ house prices (1)** | . | . | $-0.03$ (0.02) | $-1.1^{**}$ (0.4) | 0.42 |
| **IV w/ house prices and shocks (1)** | $-1.4^{*}$ (0.6) | $-1.2^{*}$ (0.6) | $-0.02$ (0.02) | . | 0.43 |
| **(2)** | $-1.3^{*}$ (0.7) | $-1.4^{*}$ (0.7) | 0.01 (0.08) | . | 0.38 |

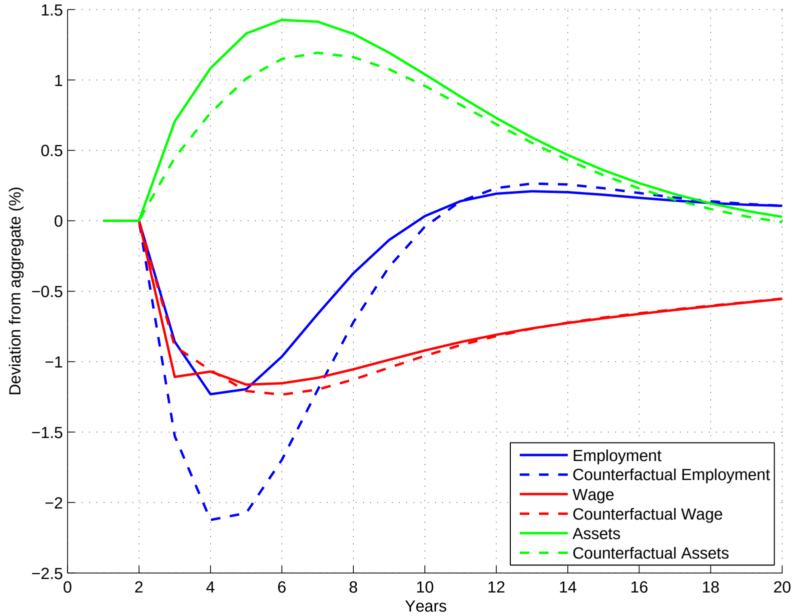
Note: Numbers in parenthesis are OLS (or second stage) standard errors. Variables with '$*$' are significant at a 5% level. Variables with '$**$' are significant at 1%. All variables are state log-growth rates between 2006 and 2010. $b_{t-1}$ is exogenous in (1) and endogenous in (2)

Table 6: Counterfactual Employment Dispersion without Fiscal Integration

| | $(\vartheta_n, \vartheta_w, \vartheta_b)$ | | | |
|---|---|---|---|---|
| | **(-1.6,-0.9,0.03)** | **(-1.4,-1.2,0)** | **(-1.2,-1.4,0)** | **(-1.1,-1.1,0)** |
| $\sigma_n^{2010}$ | 3.5 | 3.4 | 3.3 | 3.1 |
| $\overline{\sigma}_n$ | 4.9 | 4.9 | 4.7 | 4.5 |

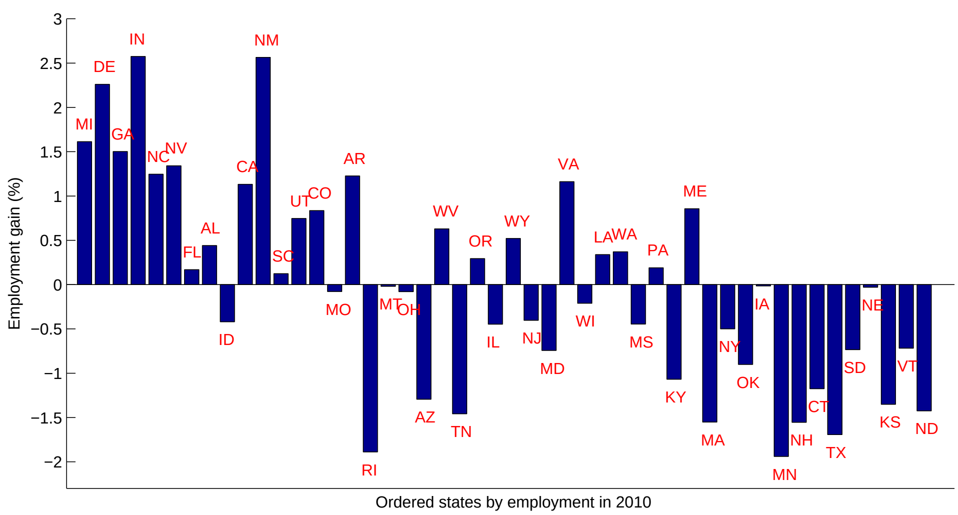
Note: Table shows selected results of the counterfactual exercise for alternative parameterizations of the prevalent transfer policy rule. $\sigma_n^{2010}$ is the standard deviation of the counterfactual distribution of employment across states in the year 2010 in percents in an economy without transfers. $\overline{\sigma}_n$ is the counterfactual standard deviation in the stationary distribution.

Figure 2: Impulse Response Function to a Demand shock



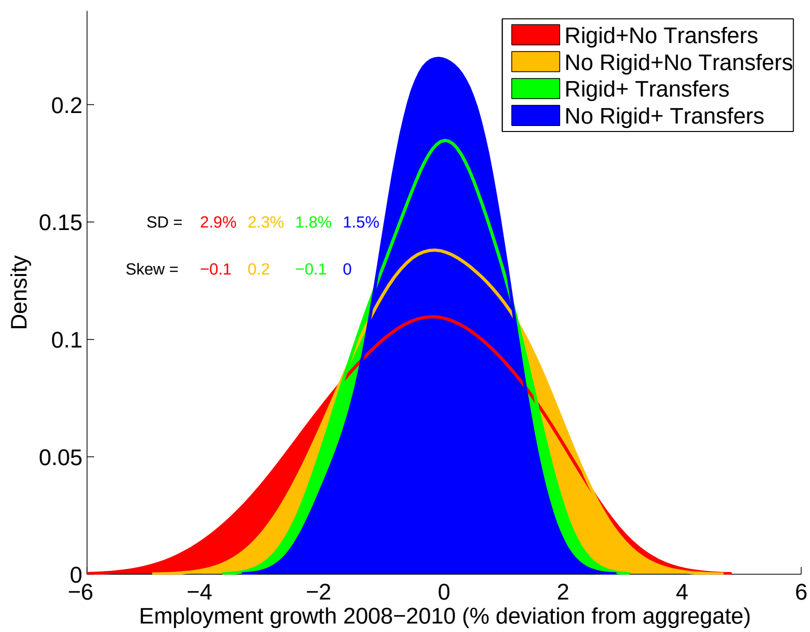

Note: Figure shows the response of employment, wages and assets to a one-standard-deviation "discount rate" shock; both in the actual and counterfactual economy without transfers. The units are a state's percentage deviation from the aggregate.

Figure 3: Employment Gains from Fiscal Integration by State in 2010




Note: For each state, the figure shows the employment difference between the counterfactual economy without a federal transfer policy rule, constructed using the semi-structural methodology, and the actual economy in 2010. The states were sorted according to their actual employment in 2010 in ascending order. To the left, the states with the worst employment realizations; and, to the right, the states with best.

Figure 4: Employment in the Great Recession: Channels Decomposition




Note: Figure shows kernel density estimates of the distribution of employment growth between 2008 and 2010 across states. This is done for the actual economy with rigidities and transfers, and for all other counterfactual economies constructed in Section 6.6.