The role of automatic stabilizers in the U.S. business cycle*

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Abstract

Most countries have automatic rules in their tax-and-transfer systems that are partly intended to stabilize economic fluctuations. This paper measures how effective they are at lowering the volatility of U.S. economic activity. We identify seven potential stabilizers in the data and include four theoretical channels through which they may operate in a business cycle model calibrated to the U.S. data. The model is used to compare the volatility of output in the data with counterfactuals where some, or all, of the stabilizers are shut down. Our first finding is that proportional taxes, like sales, property and corporate income taxes, contribute little to stabilization. Our second finding is that a progressive personal income tax can be effective at stabilizing fluctuations but at the same time leads to significantly lower average output. Our third finding is that safety-net transfers lower the volatility of output with little cost in terms of average output, but they significantly raise the variance of aggregate consumption. Overall, we estimate that if the automatic stabilizers were scaled back in size by 0.6% of GDP, then U.S. output would be about 7% more volatile.

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1 Introduction

Many features of the fiscal rules in most developed countries guarantee that, during recessions, tax revenues fall and transfer spending rises. The Congressional Budget Office (2011) estimates that the built-in responses of fiscal policy account for $363 of the $1294 billion U.S. deficit in 2010. These automatic stabilizers, as they are usually called, provide counter-cyclical fiscal stimulus. While there is strong disagreement on the efficacy of discretionary fiscal spending to fight recessions, there is greater consensus about the value of automatic stabilizers.¹ This consensus is especially strong among policy circles, with, for instance, the IMF (Baunsgaard and Symansky, 2009; Spilimbergo et al., 2010) recommending that countries enhance the scope of these fiscal tools as a way to reduce macroeconomic volatility, and Blanchard et al. (2010) arguing that designing better automatic stabilizers is one of the most promising routes for better macroeconomic policy. In spite of this enthusiasm, as Blanchard (2006) noted: “very little work has been done on automatic stabilization [...] in the last 20 years.”

This paper examines the efficacy of automatic stabilizers in attenuating the magnitude of the business cycle. More concretely, the goal is to answer the question: by how much do the automatic stabilizers in the U.S. tax-and-transfer system lower the volatility of aggregate activity? Our approach is to use a modern business-cycle model, calibrated to fit the U.S. data, and that captures the most important channels through which automatic stabilizers can affect the business cycle.

One of the ingredients in our model is nominal rigidities. They imply that aggregate demand plays a role in the business cycle, so that stabilizing after-tax income and the demand for consumption and investment can stabilize fluctuations. This Keynesian channel is the most often cited reason for why automatic stabilizers would be effective. Moreover, the agents in our model intertemporally optimize so that incentives and relative prices matter as well. This includes the distortions in the allocation of labor and capital induced by the tax and transfer system, which may affect behavior in a way that either attenuates or accentuates fluctuations. Households are also heterogeneous in their wealth and income in the model and there are incomplete insurance markets. Therefore, aggregate dynamics depend on the distribution of income and wealth. Because the stabilizers redistribute resources, they can potentially affect the business cycle. Furthermore, households have a precautionary demand

for savings in response to the uncertainty they face. Because some stabilizers provide social insurance, their presence changes the targets for wealth and the ability of agents to smooth out shocks.

We start in section 2 by identifying the automatic stabilizers and measuring their size in the data. We propose to use the Smyth (1966) measure of stabilization, which is the fraction by which the volatility of aggregate activity would increase if we removed some, or all, of the automatic stabilizers. This differs from the measure of “built-in flexibility” introduced by Pechman (1973), which equals the ratio of changes in taxes to changes in before-tax income, and is widely used in the public finance literature. Whereas it measures whether there are automatic stabilizers, our goal is instead to estimate whether they are effective.

Sections 3 and 4 present our quantitative business-cycle model. With complete insurance markets, the model is similar to the neoclassical-synthesis DSGE models used for business cycles, as in Christiano et al. (2005), but augmented with a series of taxes affecting every decision. With incomplete insurance markets, our model is similar to the one in Krusell and Smith (1998), but including nominal rigidities and many taxes and transfers. Methodologically, we believe this is the first model to include aggregate shocks, nominal frictions and heterogeneous agents in an analysis of aggregate fluctuations.2 A technical contribution of this paper is to use the methods developed by Reiter (2009b,a) to numerically solve for the ergodic distributions of endogenous aggregate variables so that we can compute their second moments.

Section 5 has our findings. First, under some extreme circumstances, even though the revenue and spending from the automatic stabilizers can be very cyclical, their effect on the business cycle is zero. Therefore, even if the stabilizers are present, they may not be effective. The intuition for this result leads to a lesson that persists under more general circumstances: proportional taxes, such as those on consumption, property or corporate income are ineffective stabilizers. It is well known that these taxes are distortionary, and can have a large effect on average economic activity. But their effect on the volatility of aggregate output is small, because the distortion they generate is relatively constant across time.

Second, social transfers are effective stabilizers in that they significantly lower output volatility with a negligible effect on its average level. Because they redistribute resources away from agents who choose to work longer in response and towards agents who have a

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2Guerrieri and Lorenzoni (2011) and Oh and Reis (2011) are important precursors, but they both solve only for one-time unexpected aggregate shocks, not for recurring aggregate dynamics.
higher propensity to spend them, transfers stabilize fluctuations. However, transfers greatly raise the volatility of aggregate consumption. Because they provide social insurance against idiosyncratic shocks, they induce fewer savings. Even though transfers lower the volatility of household consumption, since households have fewer assets, they are less able to smooth out aggregate shocks.

Third, the progressivity of the income tax is potentially quite stabilizing but also leads to a significantly lower average output. This progressivity ensures that marginal tax rates are procyclical, which is both stabilizing but also discouraging of work and savings on average. One common finding across these results is that social insurance and redistribution are the powerful channels through which stabilizers have their effects. Stabilizing income and cash-flow, while being the most emphasized channel in policy discussions of the stabilizers, is quantitatively weak in our calibration.

Fourth, bringing all the stabilizers together, we find that reducing their size by 0.6% of GDP would increase the volatility of output by about 7%, but also raise mean output by 6%. It is important to emphasize from the start that none of these conclusions are normative. We stay away from discussion of welfare, in part because with heterogeneous agents and fiscal redistributions, it would require controversial assumptions on how to calculate social welfare and weigh different individuals. Instead, this paper is an exercise in positive fiscal policy, in the spirit of Summers (1981) and Auerbach and Kotlikoff (1987). Like them, we propose a model that fits the US data and then change the tax-and-transfer system within the model to make positive predictions on what would happen to the business cycle.

Literature Review

There is an old literature discussing the effectiveness of automatic stabilizers (e.g., Musgrave and Miller (1948)), but very few recent papers using modern intertemporal models. Christiano (1984) uses a modern consumption model, Gali (1994) uses a simple RBC model, Andrés and Doménech (2006) use a new Keynesian model, and Hairault et al. (1997) use several models to ask a similar question to ours. However, they typically consider the effects of a single automatic stabilizer, the income tax, whereas we comprehensively evaluate several of them. Moreover, they assume representative agents, therefore missing out on the redistributive channels of the automatic stabilizers that we end up finding to play an important role. Christiano and G. Harrison (1999), Guo and Lansing (1998) and Dromel and Pintus (2008) ask whether progressive income taxes change the size of the region of determinacy of equilibrium. Instead, we use a model with a unique equilibrium, and focus on the impact of a wider set of stabilizers on the volatility of endogenous variables at this equilibrium.
Cohen and Follette (2000) are closer to our paper in their goal but their model is simple and qualitative, whereas our goal is to provide quantitative answers. van den Noord (2000) and Barrell and Pina (2004) use large macro simulation models to conduct exercises in the same spirit as ours, but their models are predominantly backward-looking and do not include most of the channels that we consider. Concurrently with our work, Veld et al. (2012) examined the role of automatic stabilizers in the response of the euro area to a shock using the European Commission’s representative-agent DSGE model. Yet, one of our main findings is that it is crucial to consider the effect of the stabilizers when there are incomplete markets and the income distribution matters for aggregate dynamics.

Huntley and Michelangeli (2011) and Kaplan and Violante (2012) are closer to us in terms of modeling, but they focus on the effect of discretionary tax rebates, whereas our attention is on the automatic features of the fiscal code. Challe and Ragot (2012) asked how precautionary savings varies over the business cycle in a model with incomplete markets and aggregate shocks in the same spirit as ours. We confirm their conclusion that precautionary savings play an important role on fluctuations, but for different reasons and asking a different question.

Empirically, Auerbach and Feenberg (2000), Auerbach (2009), and Dolls et al. (2012) use micro-simulations of tax systems to estimate the changes in taxes that follows a 1% increase in aggregate income. The OECD (Girouard and André (2005), the IMF (Fedelino et al. (2005)) and the ECB (Bouthevillain et al. (2001)), to name a few major policy institutions, instead measure automatic stabilizers using macro data, estimating which components of revenue and spending are strongly correlated with the business cycle. Blanchard and Perotti (2002), Perotti (2005) and many papers that followed use these estimates to identify the effects of fiscal policy in vector autoregressions. We take these papers’ measurement of the automatic stabilizers as inputs into our study of the effectiveness of these stabilizers.

We build on recent work by Oh and Reis (2011) and Guerrieri and Lorenzoni (2011) to try to incorporate business cycles and nominal rigidities into what Heathcote et al. (2009) call the “standard incomplete markets.” Close to our paper in emphasizing tax and transfer programs are Alonso-Ortiz and Rogerson (2010), Floden (2001), Horvath and Nolan (2011), and Berriel and Zilberman (2011), but they focus on the effects of these policies on average output, employment, and welfare. Our focus is on volatility instead.

Finally, this paper is part of a revival of interest in fiscal policy in macroeconomics.³

³For a survey, see the symposium in the Journal of Economic Literature, with contributions by Parker (2011), Ramey (2011) and Taylor (2011).
Relative to most of the literature on fiscal policy during the recession, we focus more on
taxes and government transfers, as opposed to government purchases. In the United States
in 2011, total government purchases were 2.7 trillion dollars. Government transfers amounted
to almost as much, at 2.5 trillion. Focussing on the cyclical components, during the 2007-09
recession, which saw the largest increase in total spending as a ratio of GDP since the Korean
war, 3/4 of that increase was in transfers spending (Oh and Reis (2011)), with the remaining
1/4 in government purchases. Relative to the large literature estimating purchase multipliers,
the literature on the stabilizing properties of the tax-and-transfer system is smaller. This
paper contributes to close that gap.

2 The automatic stabilizers and their role

The automatic stabilizers are sometimes presented as the fiscal rules that attenuate the busi-
ness cycle. For our purposes, this confuses defining the object of our study with measuring
its effectiveness. Before proceeding, we briefly define what are the stabilizers, discuss by
which channels they may affect the business cycle, and propose an independent measure of
their effectiveness.

2.1 What are automatic stabilizers?

An automatic stabilizer is a rule in the fiscal system that leads to significant automatic
adjustments in government revenues and outlays relative to total output in response to
business-cycle fluctuations while keeping the laws constant. In the words of Musgrave and
Miller (1948), they are the built-in-flexibility in the tax-and-transfer system that ensure that
in recessions taxes fall and spending rises. While this definition is broad, it does exclude
some government policies.

First, it focuses on fiscal stabilizers. There are many other dimensions of public policy,
notably monetary policy, that have features aimed at stabilizing real activity. Our focus is
solely on the rules in the tax code and government spending programs.

Second, it excludes discretionary changes in policy. Most recessions come with “stimulus
packages” of some form or another. There is already a tremendous amount of research on
their impact. But, as Solow (2004) put it: “The advantage of automatic stabilization is
precisely that it is automatic. It is not vulnerable to the perversities that arise when a
discretionary “stimulus package” (or “cooling-off package”) is up for grabs in a democratic
government.”

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Third, while the automatic stabilizers are a component of a fiscal policy rule, they do not include all of the systematic responses of fiscal policy. While a built-in stabilizer responds automatically, by law, to current economic conditions, a feedback rule is instead a model of the behavior of fiscal authorities in response to current and past information (Perotti (2005)). To give one example, receiving benefits when unemployed is an automatic feature of unemployment insurance, while the decision by policymakers to extend the duration of unemployment benefits in most recessions is not. There is still no consensus in the literature on what the fiscal policy rules followed by U.S. policymakers are, or even on how to best estimate them. In contrast, measuring automatic stabilizers is easier, because it requires reading and interpreting the written laws and regulations.

Fourth, we focus on the automatic fiscal rules that, either by initial design or by subsequent research, have been singled out as potentially contributing to mitigate output fluctuations. There are more government programs than a lifetime of research could study. Given these restrictions on the rules that we will consider, we turn to the components of the U.S. budget to identify the stabilizers.

### 2.2 Automatic stabilizers in the United States

The classic automatic stabilizer is the personal income tax system. Because it is progressive in the United States, its revenue fall by more than income during a recession. Because it lowers the variance of after-tax income, it is often argued that personal income taxes stabilize private spending. Figure 1 shows an estimate of the automatic component of personal income taxes due to Auerbach and Feenberg (2000). Using micro-simulations based on the TAXSIM program, they asked by how much would a 1% increase in a typical household’s income affect the amount of income taxes the household pays. The figure shows that a significant fraction of extra income goes into taxes, although this fraction has become less sensitive to the business cycle in the last decade.

While they are the most studied, personal income taxes are not the only stabilizer. Table 1 shows the main components of spending and revenue in the United States. The data comes from the National Income and Product Accounts (NIPA), so as to focus on the consolidated government flow of funds, across the different levels of government. The numbers are an average between 1988 and 2007, because earlier data would average significant changes in

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4To give another example, this one from monetary policy, the Taylor rule may be a systematic policy rule, but it is not automatic: there is no written rule that tries to enforce it on the actions of the Federal Reserve.
Figure 1: Ratio of change in taxes to change in gross income, Auerbach (2009)

The cumulative impact of changes in tax legislation is evident, as the sensitivity of taxes to income during the period 2003-7 was lower than at any time since the 1960s. Estimates for 2008 and thereafter do show an increased responsiveness, but these estimates reflect tax law as in effect midway through 2008 and therefore a stronger bite from the Alternative Minimum Tax (AMT), the encroachment of which has been continually delayed by annual legislation in recent years, and eventual repeal of essentially all of the Bush tax cuts enacted in 2001 and 2003. One might think that the growing importance of state and local taxes over this period would have partially offset the decline in federal marginal tax rates, but with essentially all states facing some form of balanced-budget requirements, the necessary tax increases and spending cuts would have undone this potential cushioning effect.

Figure 2: Automatic Responsiveness of Federal Taxes to Income

We wanted a long enough sample to capture a few business cycles, but short enough to not mix very different fiscal regimes. Appendix A describes how we aggregated the components of the government budget into the categories in the table.

Beyond personal income taxes, we consider three more stabilizers on the revenue side. Corporate income tax revenues vary by more than aggregate output because corporate profits are more volatile than national income, and it has been argued they may stabilize economic activity by lowering the volatility of corporate investment and dividends. Property taxes likewise vary with property prices and affect residential investment. Sales and excise taxes are rarely studied as automatic stabilizers, but we include them as they lower the variance of after-tax income needed to sustain a fixed real quantity of consumption. Because all of these three taxes have, approximately, a fixed statutory rate, we will refer to them as a group as proportional taxes.

On the spending side, we consider two stabilizers working through transfers. The first,

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5For instance, including the data from the start of the 1980s would imply averaging over a time with a much higher corporate income tax rate, no earned income tax credit, and significantly lower spending on health care.

6Average corporate income taxes are in fact countercyclical in the data mostly as result of recurrent changes in investment tax credits during recessions that are not automatic.
and most studied, is unemployment benefits, which greatly increase in every recession as the number of unemployed goes up. The second are safety-net programs, providing minimum support to poor households. Its main three components are food stamps, cash assistance to the very poor, and transfers to the disabled. Most of the recipients of these three programs are out of the labor force, and their numbers increase during recessions.

A seventh stabilizer is the budget deficit, or the automatic constraint imposed by the government budget constraint. The previous stabilizers do not ensure that the government budget is balanced on average across cycles. We will consider different rules for how deficits are paid and how fast this is done, in order to measure the impact of the deficit and the debt on volatility. There is no automatic rule for government purchases, so the convention in the literature measuring automatic stabilizers is to not categorize purchases as automatic stabilizers.\textsuperscript{7} We will take this as our baseline, but also consider the potential role of government purchases as an automatic stabilizer via its impact on the budget deficit.

The last rows of the table include the fiscal programs that we will exclude. Some include licenses and fines, which have no obvious stabilization role. Others include international trade, like customs taxes and transfer to the rest of the world, which we leave out because we will consider a closed-economy model. Either way, these do not account for a large share of the government budget.

\textsuperscript{7}See Perotti (2005) and Girouard and André (2005) for two of many examples.
The main omissions are retirement, both in its expenses and the payroll taxes that finance it, and health benefits, which mostly are accounted for by Medicare (for the elderly) and Medicaid (for the poor). These are large categories of the government budget, that we exclude from our study for two complementary reasons. First, in order to follow the convention. The vast literature that excludes the automatic stabilizers from measures of structural deficits almost never includes health and retirement spending as part of the stabilizers.\textsuperscript{8} Even the increase in medical assistance to the poor during recessions is questionable: for instance, in 2007-09 the proportional increase in spending with Medicaid was as high as that with Medicare. Second, we wanted our model to retain the core of conventional business-cycle models that are known to provide a satisfactory fit to the data. These models typically ignore the life-cycle considerations that dominate choices of retirement and health spending, and so do we. Exploring possible effects of public spending on health and retirement on the business cycle is a priority for future work.

\subsection*{2.3 Channels for stabilization}

The literature so far has proposed four possible channels by which automatic stabilizers can attenuate the business cycle.

First, there is the \textit{disposable income} channel, emphasized especially in Keynesian models and that dominates much of the policy discussion around stabilizers.\textsuperscript{9} The argument is that if after-tax income is less volatile than pre-tax income, then consumption and investment will also be more stable. As long as aggregate demand determines output, then this will stabilize production. All four of the tax stabilizers discussed in the previous section make after-tax income less volatile than pre-tax income. For instance, transfers provide a minimum amount of income when pre-tax income has fallen to zero as a result of losing a job or leaving the labor force. This channel requires that disposable income has an effect on aggregate demand, and in turn that aggregate demand affects the business cycle. With rational forward-looking agents, under complete markets, changes in disposable income have almost no effect on consumption, which is driven by movements in permanent income. Moreover, with flexible prices, aggregate demand affects prices but not output. We include this channel in our model by assuming that households face liquidity constraints and that firms face nominal rigidities in setting prices.

\textsuperscript{8}Darby and Melitz (2008) are an exception, arguing that health benefits have an automatic stabilizer component.

\textsuperscript{9}Brown (1955) is the classic exposition of this channel for automatic stabilization.
The second channel works through *marginal incentives*, especially on labor supply. If the previous channel focuses on aggregate demand, this one works through aggregate supply. The intertemporal response of labor supply and investment to changes in marginal returns is the key driving force behind real business cycles. We include it by having an elastic labor supply in our model. This channel works especially through the progressivity of the personal income tax. In recessions, households move to lower tax brackets, which increases the relative return to working. The progressive income tax therefore stabilizes labor supply by encouraging intertemporal substitution of labor from booms to recessions. A less studied example comes from property and corporate income taxes, which lower the variance of the after-tax return to investments.

The third channel is *redistribution*, and it interacts with the previous two. Both the progressive personal income tax and, especially, the transfer payments, imply a redistribution from higher-income to lower-income households. As discussed in Blinder (1975), this may raise aggregate demand if those that receive the funds have higher propensities to spend them than those who give the funds, and through nominal rigidities this may raise output in recessions. Redistribution may also work through labor supply, as in Oh and Reis (2011), if the recipients of transfer payments are at a corner solution with respect to their choice of hours to work, whereas those being taxed to fund the program, work more to offset the negative income effect. We include this channel by having incomplete insurance markets, so that the distribution of after-tax income affects economic aggregates.

Finally, we consider a *social insurance* channel working through precautionary savings. The automatic stabilizers provide insurance to households by lowering the taxes they pay and increasing the transfers they receive when they get hit by a bad idiosyncratic shock. On the one hand, this reduces income and wealth inequality. On the other hand, it reduces the desire for precautionary savings, lowers aggregate savings and may increase pre-tax inequality (Floden, 2001; Alonso-Ortiz and Rogerson, 2010). With incomplete markets, the wealth distribution will affect aggregate output. For instance, the social insurance provided by the stabilizers will likely lead agents to save less and become liquidity constrained more often, while at the same time making their spending choices less sensitive to hitting the liquidity constraint.

### 2.4 How to measure the effectiveness of automatic stabilizers?

At the macroeconomic level, the automatic stabilizers are effective if the variance of aggregate variables is lower in their presence. That is, letting $Y(\cdot, \tau)$ be a measure of real activity, then
each element of the vector $\tau$ measures the strength of each stabilization program. We let $\tau = 1$ correspond to the status quo, and lower elements of $\tau$ towards zero as we shrink the size of each automatic stabilizers in terms of its size in the budget. Our measure of effectiveness, following Smyth (1966), is the stabilization coefficient:

$$S = \frac{\text{Var}(Y(\cdot, \tau))}{\text{Var}(Y(\cdot, 1))} - 1.$$  

The measured $S$ is the fraction by which the volatility of aggregate activity would increase if the stabilizers were decreased to $\tau$. In the denominator is the status quo represented by our model calibrated to mimic the U.S. business cycle, while the counterfactuals in the numerator consist of shutting off different automatic stabilizers.

A line of research, of which Clement (1960) seems to have been the first and Dolls et al. (2012) is a recent example, starts from the measures of the stabilizers in figure 1 and then makes behavioral assumptions on how demand changes with income for different households and how this affects output. In the case of Dolls et al. (2012) they assume that households with certain characteristics (e.g., low financial wealth or no home) increase consumption one-to-one with income, while the marginal propensity of the other households is zero, and that aggregate demand equals output. This provides a different measure of the effectiveness of the stabilizers.\footnote{Devereux and Fuest (2009) is a recent example of the same approach but applied to corporate income taxes.}

This work measures exclusively the disposable income channel of stabilization. Moreover, it assumes extreme behavioral responses of consumption and aggregate output in the short run, while shutting off their dynamic effect especially in the long-run adjustment of prices and the wealth distribution. Finally, it does not take into account the general-equilibrium effect that changes in disposable income will have on rates of return, wages and prices in the economy. To include all of these effects and to assess how large they are, one needs a fully specified model of, not just consumers, but all agents and markets. In short, one needs a business-cycle model. The next section provides one.

3 A business-cycle model with automatic stabilizers

Following the discussion of the channels by which automatic stabilizers may matter, we need a model that includes liquidity constraints, incomplete insurance markets, nominal rigidities,
elastic labor supply, and precautionary savings. The model must also have room for the
seven stabilizers that we want to study. And finally, we would like it to be close to business-
cycle models that are known to capture the main features of the U.S. business cycle. The
model that follows is the simplest we could write—and it is already quite complicated—while
satisfying these three requirements.

Time is discrete, starting at date 0, and all agents live forever. The population has a
fixed measure of 1 + \( \nu \) households.\(^{11}\) Of these, a measure 1 refers to participants in the stock
market, or capitalists, while the remaining \( \nu \) refers to other households. The main difference
between them is that capitalists are more patient. As a result, they end up accumulating
all of the capital stock and owning all of the shares in firms. Following Krusell and Smith
(1998), having heterogeneous discount factors allows us to match the very skewed wealth
distribution that we observe in the data. Linking it to participation in financial markets
matches the well-known fact since Mankiw and Zeldes (1991) that most U.S. households do
not own any equity.

On the side of firms, there is a measure 1 of monopolistic intermediate-goods firms, a
representative final-goods firm, and another representative capital-goods firm. Some of these
agents could be centralized into a single household and a single firm without changing the
predictions of the model. We keep them separate to ease the presentation, and so that we
can introduce one automatic stabilizer with each type of agent.

The notation for the automatic stabilizers is that \( \bar{\tau} \) are taxes collected, \( \tau \) are tax rates,
and \( T \) are transfers.

### 3.1 Capitalists and the personal income tax

The stock-owners are all identical ex ante in period 0 and share risks perfectly. We assume
they have access to financial markets where all idiosyncratic risks can be insured, but this is
not a strong assumption since they enjoy significant wealth and would be close to self-insuring
even without state-contingent financial assets.

We can then talk of a representative stock-owner, whose preferences are:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 n_t^1 + \psi_2 \frac{n_t^{1+\psi_2}}{1 + \psi_2} \right]
\]

where \( c_t \) is consumption and \( n_t \) are hours worked, both non-negative. These preferences

\(^{11}\)Because we will assume balanced-growth preferences, it would be straightforward to include population
and economic growth.
ensure that there is a balanced-growth path in our economy and are can be calibrated to be consistent with the survey on the responses of labor supply to taxes in Chetty (2012).

The representative stock-owner budget constraint is:

\[ \hat{p}_t c_t + b_{t+1} - b_t = p_t [x_t - \bar{\tau}^x(x_t)] + T_t^e. \]  

(2)

The left-hand side has the uses of funds: consumption at the after-tax price \( \hat{p}_t \) plus savings in riskless bonds \( b_t \) in nominal units. The right-hand side has real after-tax income, where \( x_t \) is the pre-tax income and \( \bar{\tau}^x(x_t) \) are personal income taxes. The pre-tax price of consumption goods is \( p_t \). The \( T^e_t \) are lump-sum transfers, which we will calibrate to zero as in the data, but will be useful later to discuss counterfactuals.

The real income of the stock owner is:

\[ x_t = (i_t/p_t)b_t + w_t \bar{s}n_t + d_t. \]  

(3)

It equals the the sum of the returns on bonds at nominal rate \( i_t \), wage income, and dividends \( d_t \) from all the firms in the economy. The wage is the product of the average wage in the economy, \( w_t \), and the agent’s productivity \( \bar{s} \). This productivity could be an average of individual-specific productivities of the measure 1 of stock-owners, since these idiosyncratic draws are perfectly insured within capitalists.

The first automatic stabilizer in the model is the personal income tax system. It satisfies:

\[ \bar{\tau}^x(x) = \int_0^x \tau^x(x')dx', \]  

(4)

where \( \tau^x : \mathbb{R}^+ \rightarrow [0, 1] \) is the marginal tax rate that varies with the tax base, which equals real income. The system is progressive because \( \tau^x(\cdot) \) is weakly increasing.

### 3.2 Other households and transfers

Other households are indexed by \( i \in [0, \nu] \), so that an individual variable, say consumption, will be denoted by \( c_t(i) \). They have the same period felicity function as capitalists, but they are potentially more impatient \( \hat{\beta} \leq \beta \), as we discussed earlier. Just like capital-owners, individual households choose consumption, hours of work and bond holdings \( \{c_t(i), n_t(i), b_{t+1}(i)\} \) to maximize:

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left[ \ln(c_t(i)) - \psi_1 n_t(i)^{1+\psi_2} / (1 + \psi_2) \right]. \]  

(5)
Also like capital-owners, households can borrow using government bonds, and pay personal income taxes, so their budget constraint and real income are:

\[
\hat{p}_t c_t(i) + b_{t+1}(i) - b_t(i) = p_t \left[ x_t(i) - \tau^x(x_t(i)) \right] + p_t T_t^x(i),
\]

\[
x_t(i) = (i_t/p_t) b_t(i) + s_t(i) w_t n_t(i) + T_t^u(i).
\]

There is a further constraint on the household choices, which also applied to capitalists, but will only bind for non stock owners. It is a borrowing constraint, \( b_{t+1}(i) \geq 0 \), which is equal to the natural debt limit if households cannot borrow against future government transfers.

Unlike capital owners, households face two sources of idiosyncratic risk regarding their labor income: on their labor-force status, \( e_t(i) \), and on their skill, \( s_t(i) \). If the household is employed, then \( e_t(i) = 2 \), and she can choose how many hours to work. If \( e_t(i) \neq 2 \), then \( n_t(i) = 0 \) is an extra constraint. While working, her labor income is \( s_t(i) w_t n_t(i) \). The shocks \( s_t(i) \) captures shocks to the worker’s skill, her productivity at the job, or the wage offer she receives. They generate a cross-sectional distribution of labor income.

With some probability, the worker loses her job, in which case \( e_t(i) = 1 \) and labor income is zero. However, now the household collects unemployment benefits \( T_t^u(i) \), which are taxable in the United States. Once unemployed, the household can either find a job with some probability, or exhaust her benefits and qualify for poverty benefits. This is the last state, and for lack of better terms, we refer to their members as the needy, the poor, or the long-term unemployed. If \( e_t(i) = 0 \), labor income is zero but the household collects food stamps and other safety-net transfers, \( T_t^u(i) \), which are non-taxable. Households escape poverty with some probability at which they find a job.

There are two new automatic stabilizers at play in the household problem. First, the household can collect unemployment benefits, \( T_t^u(i) \) which equal:

\[
T_t^u(e_t(i), s_t(i)) = \min \left\{ T_t^u s_t(i), T_t^u s^u \right\} \text{ if } e_t(i) = 1 \text{ and zero otherwise.}
\]

Making the benefits depend on the current skill-level captures the link between unemployment benefits and previous earnings, and relies on the persistence of \( s_t^h \) to achieve this. As is approximately the case in the U.S. law, we keep this relation linear with slope \( T_t^u \) and a maximum cap \( s^u \).
The second stabilizer is safety-net payments $T^*_i(i)$, which equal:

$$T^*(e_t(i)) = T^* \text{ if } e_t(i) = 0 \text{ and zero otherwise.}$$

We assume that these transfers are lump-sum, providing a minimum living standard. In the data, these transfers are mean-tested, but because in our model these families only receive interest income from holding bonds, when we modified the model to put a maximum income cap to be eligible to these benefits, we found that almost no household ever hits this cap. For simplicity, we keep the transfer lump-sum.

### 3.3 Final goods’ producers and the sales tax

A competitive sector for final goods combines intermediate goods according to the production function:

$$y_t = \left( \int_0^1 y_t(j)^{1/\mu} dj \right)^{\mu}. \quad (9)$$

where $y_t(j)$ is the input of the $j^{th}$ intermediate input. The representative firm in this sector takes as given the final-goods pre-tax price $p_t$, and pays $p_t(j)$ for each of its inputs. Cost minimization together with zero profits imply that:

$$y_t(j) = \left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} y_t , \quad (10)$$

$$p_t = \left( \int_0^1 p_t(j)^{1/(1-\mu)}dj \right)^{1-\mu}. \quad (11)$$

On top of the price $p_t$, there is a sales tax $\tau^c$ so the after-tax price of the goods is:

$$\hat{p}_t = (1 + \tau^c)p_t. \quad (12)$$

This consumption tax is our next automatic stabilizer, as it makes actual consumption of goods a fraction $1/(1 + \tau^c)$ of pre-tax spending on them.

### 3.4 Intermediate goods and corporate income taxes

Each variety $j$ is produced by a monopolist firm using a production function:

$$y_t(j) = a_t k_t(j)^{\alpha} l_t(j)^{1-\alpha}. \quad (13)$$
where \( a_t \) is productivity, \( k_t(j) \) is capital used, and \( l_t(j) \) is effective labor. In the labor market, if \( l_t \) is the total amount of effective labor, then:

\[
\int_0^1 l_t(j) \, dj = \int_0^{\nu} s_t(i) n_t(i) \, di + \bar{s} n_t. \tag{14}
\]

The demand for labor on the left-hand side comes from the intermediate firms. The supply on the right-hand side comes from employed households, adjusted for their productivity, and from the labor of stock-owners.

The firm maximizes after-tax nominal profits:

\[
d_t(j) = (1 - \tau^k) \left[ p_t(j) y_t(j) / p_t - w_t l_t(j) - (r_t + \delta) k_t(j) - \xi \right], \tag{15}
\]

taking into account the demand function in (10). The firm’s costs are the wage bill to workers, the rental of capital at rate \( r_t \) plus depreciation of a share \( \delta \) of the capital used, and a fixed cost \( \xi \). The maximized profits are rebated every period to the capitalists as dividends.

Intermediate firms set prices subject to nominal rigidities a la Calvo (1983) with probability of price revision \( \theta \). Since they are owned by the capitalists, they use their discount factor \( \lambda_{t,t+s} \) to choose price \( p_t(j)^* \) at a revision date with the aim of maximizing expected future profits:

\[
E \left[ \sum_{s=0}^{\infty} (1 - \theta)^s \lambda_{t,t+s} d_{t+s}(j) \right] \quad \text{subject to: } p_{t+s}(j) = p_t(j)^* \tag{16}
\]

The new automatic stabilizer is the corporate income tax, which is a flat rate \( \tau^k \) over corporate profits. In the U.S. data, dividends and capital gains pay different taxes. While this distinction is important to understand the capital structure of firms and the choice of retaining earning, it is immaterial for the simple firms that we just described.\(^{12}\)

\(^{12}\)Another issue is the treatment of taxable losses (Devereux and Fuest (2009)). Because of carry-forward and backward rules in the U.S. tax system, these should not have a large effect on the effective tax rate faced by firms, although firms do not seem to claim most of these tax benefits. We were unable to find a satisfactory way to include these considerations into our model without greatly complicating the analysis.
3.5 Capital-goods firms and property income taxes

A representative firm owns the capital stock and rents it to the intermediate-goods firm taking $r_t$ as given. If $k_t$ denotes the capital held by this firm, then in the market for capital:

$$k_t = \int_0^1 k_t(j) dj. \quad (17)$$

This firm invests in new capital $\Delta k_{t+1} = k_{t+1} - k_t$ subject to adjustment costs to maximize after-tax profits:

$$d_t^k = (1 - \tau^k)r_t k_t - \Delta k_{t+1} - \frac{\zeta}{2} \left( \frac{\Delta k_{t+1}}{k_t} \right)^2 k_t, \quad (18)$$

The value of this firm, which owns the capital stock is then given the recursion:

$$v_t = d_t^k - \tau^p v_t + E_t[\lambda_{t+1} v_{t+1}].$$

The new automatic stabilizer, the property tax, is a fixed tax rate $\tau^p$ that applies to the value of the only property in the model, the capital stock. A few steps of algebra show the conventional results from the q-theory of investment:

$$v_t = q_t k_t, \quad (19)$$

$$q_t = 1 + \zeta \left( \frac{\Delta k_{t+1}}{k_t} \right). \quad (20)$$

Because, from the second equation, the price of the capital stock is procyclical, so will property values, making the property tax a potential automatic stabilizer.

Finally, note that total dividends sent to stock-owners, $d_t$, come from every intermediate firm and the capital-goods firm:

$$d_t = \int_0^1 d_t(j) dj + d_t^k - \tau^p q_t k_t. \quad (21)$$

We do not include investment tax credits. They are small in the data and, when used to attenuate the business cycle, they have been enacted as part of stimulus packages and not as automatic rules.
3.6 The government and budget deficits

The government budget constraint is:

\[
\tau_c \left( \int_0^\nu c_t(i)di + c_t \right) + \tau^p q_t k_t + \int_0^\nu \bar{\tau}^x(x_t(i))di + \bar{\tau}^z(x_t) + \tau^k \left[ \int_0^1 \hat{d}(j)di + r_t k_t \right] - \int_0^\nu \left[ T_t^u(i) + T_t^s(i) \right] di \\
= g_t + (i_t/p_t)B_t - (B_{t+1} - B_t)/p_t + T_t^e. \tag{22}
\]

On the left-hand side are all of the automatic stabilizers discussed so far: sales taxes, property taxes and personal income taxes in the first line, and corporate income taxes and transfers in the second line. On the right-hand side are government purchases, \(g_t\) and government bonds \(B_t\). Because government bonds are the only asset in positive net supply to the households, the market for bonds will clear when:

\[
B_t = \int_0^\nu b_t(i)di + b_t. \tag{23}
\]

In steady state, the stabilizers on the left-hand side imply a positive surplus, which is offset by steady-state government purchases \(\bar{g}\). Since we set transfers to the entrepreneurs to zero, the budget constraint then determines a steady state amount of debt \(\bar{B}\), which is consistent with the government not being able to run a Ponzi scheme.

Outside of the steady state, as outlays rise and revenues fall during recessions, the left-hand side of equation (22) increases, and so must the right-hand-side. This is the last stabilizer that we consider: the automatic increase in the budget deficit during recessions.

The debt that results must be paid over time. In our baseline, we consider a simple fiscal rule where debt is paid via a lump-sum tax on capitalists:

\[
T_t^e = -\gamma \log \left( \frac{B_t/p_t}{\bar{B}} \right) \tag{24}
\]

and purchases vary in proportion to output, so \(g_t/y_t\) is constant. The parameter \(\gamma > 0\) measures the speed at which the deficits from recessions are paid over time. If \(\gamma\) is close to infinity, then the deficits caused by recessions are paid right away the following period; if \(\gamma\) is close to zero, they take arbitrarily long to get paid. We have the tax on stock-owners adjusting because it is the fiscal tool that interferes the least with the other stabilizers, affecting neither marginal returns like the distortionary tax rates or having an important effects on the wealth and income distribution as transfers to households. In section 5, we
will consider an alternative, where it is government purchases that adjust.

### 3.7 Shocks and business cycles

Monetary policy follows a conventional Taylor rule:

\[
i_t = \bar{i} + \phi_p \Delta \log(p_t) + \phi_y \log(y_t/y) + \varepsilon_t
\]  

(25)

with \(\phi_p > 1\) and \(\phi_y \geq 0\).\(^\text{13}\)

Two aggregate shocks hit the economy: technology, \(\log(a_t)\), and monetary policy, \(\varepsilon_t\). Therefore, both aggregate-demand and aggregate-supply shocks may drive business cycles. We assume that both shocks follow independent AR(1) processes for simplicity.

The idiosyncratic shocks to households, \(e_t(i)\) and \(s_t(i)\) are first-order Markov processes. Moreover, the transition matrix of labor-force status, the three-by-three matrix \(\Pi_t\), depends on a linear combination of the two aggregate shocks. This way, we let unemployment vary with the business cycle to match Okun’s law. This approach to modeling unemployment is clearly reduced-form and subject to the Lucas critique. However, our model of the business cycle is already sufficiently complicated that endogenizing the extensive margin of labor supply is challenging. At the same time, recall that workers choose how many hours to work. Therefore, our model has an endogenous intensive margin of labor supply. The details of how we calibrate the idiosyncratic shock processes appear in Appendix D.

### 3.8 Equilibrium and volatility

An equilibrium in this economy is a collection of aggregate quantities \((y_t, k_t, d_t, v_t, c_t, n_t, b_{t+1}, x_t, d_k^t)\); aggregate prices \((p_t, \hat{p}_t, w_t, q_t)\); individual consumer decision rules \((c_t(b, s, e), n_t(b, s, e))\); a distribution of households over assets, skill levels, and employment statuses; individual firm variables \((y_t(j), p_t(j), k_t(j), l_t(j), d_t(j))\); and government choices \((B_t, i_t, g_t)\) such that:

(i) owners maximize expression (1) subject to the budget constraint in equations (2)-(3),

(ii) the household decision rules maximize expression (5) subject to their budget constraint in equations (6)-(7),

(iii) the distribution of households over assets and skill and employment levels evolves in a manner consistent with the decision rules and the exogenous idiosyncratic shocks,

(iv) final-goods firms behave optimally according to equations (10)-(12),

\(^\text{13}\)Including interest-rate smoothing had a small quantitative effect on the results (details available from the authors), so we leave it out to save on one more parameter to keep track of and calibrate.
(v) intermediate-goods firms maximize expression (16) subject to equations (10), (13), (15), (vi) capital-goods firms maximize expression (18) so their value is in equations (19)-(20), (vii) fiscal policy respects equation (22) and follows the rule in equation (24) while monetary policy follows the rule in (25), (viii) markets clear for labor in equation (14), for capital in equation (17), for dividends in equation (21) and for bonds in equation (23).

Appendix B derives the optimality conditions that we use to solve the model. We evaluate the mean and variance of aggregate endogenous variables in the ergodic distribution at the equilibrium in this economy.

4 Properties of the model

Our model is not easy to solve as one must keep track not only of the aggregate variables, but also of the distribution of assets across agents and the distribution of prices across firms. At the same time, the model has familiar foundations laid out in this section.

4.1 Optimal behavior and equilibrium wealth and capital

Figure 2 uses a simple diagram, akin to one that appears in Aiyagari (1994), to describe the stationary equilibrium of the model without aggregate shocks. For the sake of clarity, the figure depicts an environment in which there are no taxes that distort savings decisions.

The downward-sloping curve is the demand for capital, with slope determined by diminishing marginal returns. The demand of stock owners for assets is perfectly elastic at their time-preference rate just as in the neoclassical growth model. Because they are the sole holders of capital, the equilibrium capital stock in the model is determined by the intersection of these two curves. Introducing taxes on capital income, like the personal or corporate income taxes, would shift the demand curve leftwards and lower the equilibrium capital stock.

If households were also fully insured their demand for assets would be the horizontal line going through their time-preference rate, but because of the idiosyncratic risk they face, they have a precautionary demand for assets. Therefore, they are willing to hold bonds even at lower interest rates. Their asset demand is given by the upward-sloping curve. Because in the steady state without aggregate shocks, bonds and capital must yield the same return, equilibrium bond holdings by households are given by the point to the left of the equilibrium capital stock. The difference between the total amount of government bonds outstanding and those held by households gives the bond holdings of stock owners.
Figure 3 shows the optimal savings decisions of households at each of their $e_t$ states. When a household is employed, they save so the savings policy is above the 45° line, while when they do not have a job, they run down their assets. As their wealth reaches zero, they stay there until they regain employment, leading to the horizontal segment along the horizontal axis in their savings policies.

Figure 4 shows the ergodic wealth and income distributions for households. Three features of these distributions will play a role in our results. First, that households below the poverty line have essentially no assets. Given the borrowing constraint they face, they live hand to mouth. Second, that employed households are wealthier than the unemployed. When a recession comes, and more households lose their jobs, they will draw down their wealth to smooth out hard times. Third, the figure shows a counterfactual wealth distribution if the two transfer programs are significantly cut. Because not being employed now comes with higher income risk, households save more, which raises their wealth in all states. This large impact of the stabilizers on the wealth distribution will play an important role in our results.

4.2 Solution algorithm

The distribution of wealth across households is a state variable of the model, so the solution algorithm has to keep track of the dynamics of this distribution. One candidate is the
Krusell and Smith (1998) algorithm, which summarizes the distribution of wealth with a few moments of the distribution. We opt instead for the solution algorithm developed by Reiter (2009a,b) because this method can more easily be applied to models with a rich structure at the aggregate level. An appendix explain in more detail the steps we took, while we describe here the guiding principles.

The Reiter algorithm first approximates the distribution with a histogram that has a large number of bins. The mass of households in each bin then becomes a state variable of the model. The algorithm then also approximates the household decision rules with a discrete approximate (e.g. a spline). In this way, the model is converted from one that has infinite-dimensional objects to one that has a large, but finite, number of variables. It follows that, using standard techniques, one can find the stationary competitive equilibrium of this economy in which there is idiosyncratic uncertainty, but no aggregate shocks. Reiter (2009b)’s method then calls for linearizing the model with respect to aggregate shocks and solving for the dynamics of the economy as a perturbation around the stationary equilibrium without aggregate shocks using existing algorithms (e.g. Sims (2002). The resulting solution is non-linear with respect to the idiosyncratic variables, but linear with respect to the aggregate variables.
Figure 4: The ergodic wealth distribution, with and without transfers.
This approach works well for small versions of our model (e.g. with fewer than four discrete types of households). However, increasing the number of household types leads the system to grow to a size for which the application of linear rational expectation solution methods is not feasible. This is our case, since the linearized system for the full model has close to 13,000 equations. To proceed, we follow Reiter (2009a) and compress the system using model reduction techniques. As Reiter explains, this compression comes with virtually no loss of accuracy relative to the larger linearized system because many dimensions of the state space are not needed. Intuitively, this is for two reasons: because the system never varies along that dimension and/or because variation along it is not relevant for the variables of interest. We verified this claim using versions of our model for which it was possible to both solve the reduced linear system as well as the full system, and found negligible losses in accuracy. It should be noted that while the model reduction step greatly speeds up the actual solution of the model, it has its own cost, which is that the full system must be analyzed to determine how it can be reduced. As a result, the solution algorithm still takes several hours of computing time.

4.3 Calibrating the model

We calibrate as many parameters as possible to the properties of the automatic stabilizers in the data. For the government spending and revenues, we use table 1, which recall averaged over the period 1988-2007. For macroeconomic aggregates though, we average over a longer period, starting in 1960 and using quarterly data, so that we can include more recessions in the sample and periods outside the Great Moderation and do not underestimate the amplitude of the business cycle.

For the three proportional taxes, we use a parameter related to preferences or technology to match the tax base in the NIPA accounts, and choose the tax rate to match the average revenue reported in table 1, following the strategy of Mendoza et al. (1994). The top panel of table 2 shows the parameters set and the respective targets. Our average tax rates are not far from the statutory marginal tax rates for sales and corporate income.

For the personal income tax, we followed Auerbach and Feenberg (2000) and simulated TAXSIM, including federal and state taxes, for a typical household. We averaged the tax rates across states weighted by population, and across years between 1988 and 2007. We then fit a cubic function of income to the resulting scheduled, and splined it with a flat line

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14See Antoulas (2005) for a discussion of model reduction in a general context and see Reiter (2009a) for their application to forward-looking economic systems.
Table 2: Calibration of the parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Target (Source)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau^c)</td>
<td>Tax rate on consumption</td>
<td>0.054</td>
<td>Avg. revenue from sales taxes (Table 1)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Discount factor of stock owners</td>
<td>0.989</td>
<td>Consumption-income ratio = 0.689 (NIPA)</td>
</tr>
<tr>
<td>(\tau^p)</td>
<td>Tax rate on property</td>
<td>0.003</td>
<td>Avg. revenue from property taxes (Table 1)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Coefficient on labor in production</td>
<td>0.296</td>
<td>Capital income share = 0.36 (NIPA)</td>
</tr>
<tr>
<td>(\tau^k)</td>
<td>Tax rate on corporate income</td>
<td>0.282</td>
<td>Avg. revenue from corporate income tax (Table 1)</td>
</tr>
<tr>
<td>(\xi)</td>
<td>Fixed costs of production</td>
<td>1.32</td>
<td>Corporate profits / GDP = 9.13% (NIPA)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Desired gross markup</td>
<td>1.1</td>
<td>Avg. U.S. markup (Basu, Fernald, 1997)</td>
</tr>
<tr>
<td>(\bar{T}_u)</td>
<td>Unemployment benefits</td>
<td>0.185</td>
<td>Avg. outlays on unemp. benefits (Table 1)</td>
</tr>
<tr>
<td>(\bar{T}_s)</td>
<td>Safety-net transfers</td>
<td>0.169</td>
<td>Avg. outlays on safety-net benefits (Table 1)</td>
</tr>
<tr>
<td>(G/Y)</td>
<td>Steady-state purchases / output</td>
<td>0.130</td>
<td>Avg. outlays on purchases (Table 1)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Fiscal adjustment speed</td>
<td>2.2</td>
<td>Autocorr. net public savings / GDP = 0.966 (NIPA)</td>
</tr>
<tr>
<td>(B/Y)</td>
<td>Steady-state debt / output</td>
<td>1.66</td>
<td>Avg. interest expenses (Table 1)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Non-participants / stock owners</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>(\beta^h)</td>
<td>Discount factor of households</td>
<td>0.983</td>
<td>Wealth of top 20% by wealth</td>
</tr>
<tr>
<td>(\bar{s})</td>
<td>Skill level of stock owners</td>
<td>4.66</td>
<td>Income of top 20% by wealth (SCF)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Calvo price stickiness</td>
<td>0.286</td>
<td>Avg. price spell duration = 3.5 (Klenow, Malin, 2011)</td>
</tr>
<tr>
<td>(\psi_1)</td>
<td>Labor supply</td>
<td>21.6</td>
<td>Avg. hours worked = 0.31 (Cooley, Prescott, 1995)</td>
</tr>
<tr>
<td>(\psi_2)</td>
<td>Labor supply</td>
<td>2</td>
<td>Frisch elasticity = 1/2 (Chetty, 2011)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Depreciation rate</td>
<td>0.114</td>
<td>Annual depreciation expenses / GDP = 0.046 (NIPA)</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Adjustment costs for investment</td>
<td>15.0</td>
<td>Corr. of (Y) and (C) = 0.88 (NIPA)</td>
</tr>
<tr>
<td>(\rho_z)</td>
<td>Autocorrelation productivity shock</td>
<td>0.880</td>
<td>Autocorr. of log GDP = 0.864 (NIPA)</td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>St. dev. of productivity shock</td>
<td>0.004</td>
<td>St. dev. of log GDP = 1.539 (NIPA)</td>
</tr>
<tr>
<td>(\rho_m)</td>
<td>Autocorrelation monetary shock</td>
<td>0.500</td>
<td>Largest AR for inflation = 0.85 (Pivetta, Reis, 2006)</td>
</tr>
<tr>
<td>(\sigma_m)</td>
<td>St. dev. of monetary shock</td>
<td>0.005</td>
<td>Share of output variance due to shock = 0.2</td>
</tr>
<tr>
<td>(\phi_p)</td>
<td>Interest-rate rule on inflation</td>
<td>1.55</td>
<td>St. dev. of inflation = 0.638 (NIPA)</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>Interest-rate rule on output</td>
<td>0.010</td>
<td>Corr. of inflation with log (Y) = 0.198 (NIPA)</td>
</tr>
</tbody>
</table>
above a certain level of income so that the fitted function would be non-decreasing. The result is in figure 2. The cubic-linear schedule approximates the actual taxes well, and its smoothness is numerically useful. We then added an intercept to this schedule to fit the effective average tax rate. This way, we made sure we fitted both the progressivity of the tax system (via TAXSIM) and the average tax rates (via the intercept).

Panel B calibrates the parameters related to government spending. Both parameters governing transfer payments are set to equate the average outlaid from these programs. The speed at which deficits are paid comes from the autocorrelation of budget deficits in the data. Note that in the data, the budget deficit has a strong stabilizer component: increases in debt are paid for quite slowly over time.

Panel C contains parameters that relate to the distribution of income and wealth across households. We used the Survey of Consumer Finances to calculate that 0.84% of the wealth is held by the top 20% in the United States. We then picked the discount factor of the households to match this target. In addition, we use a 3-point grid for household skill levels, which we construct from data on wages in the Panel Study for Income Dynamics. Appendix D has the details.

Finally, panel D has all the remaining parameters. Most are standard, but two deserve
some explanation. First, the Frisch elasticity of labor supply plays an important role in most intertemporal business cycle models. Consistent with our focus on taxes and spending, we use the value suggested in the recent survey by Chetty (2012) on the response of hours worked to several tax and benefit changes. Second, we choose the variance of monetary shocks so that a variance decomposition of output attributes them 20\% of aggregate fluctuations. There is great uncertainty on the empirical estimates of the sources of business cycles, but this number is not out of line with at least some of the estimates reported in Christiano et al. (1999). Our results turn out to not be sensitive to this number.

4.4 Impulse responses to shocks

Before we can use our model to make counterfactual predictions, we must verify that it provides a reasonable description of U.S. business cycles. Figure 5 shows impulse responses to a positive technology shock and a contractionary monetary shock of one standard deviation.

The model generates the positive co-movement of output and consumption, as well as the persistent responses of both variables to shocks that have been emphasized in the literature. Hours have a hump-shaped response to a technology shock: at first they rise because higher TFP raises the marginal product of labor. Then, while TFP falls as the shock dissipates, the increase in the capital stock further raises the marginal product of working so hours keep rising. At a point, investment is no longer strong enough, so the fall in TFP dominates, and hours start converging back to zero. Finally, turning to inflation, as is well-known in new Keynesian models, both shocks lower inflation, but the simple Calvo model implies a fairly short-lived response.

In sum, the aggregate dynamics of our model resembles those in the standard new neoclassical synthesis model of Goodfriend and King (1997) and Woodford (2003) that has been widely used to study business cycles in the past decade.

4.5 Complete markets and the neoclassical synthesis

This similarity between our impulse response functions and those of standard business cycle models is not a coincidence. While our model is complicated, it has a familiar foundation. If prices were flexible and there were no stock owners, our model would be close to that in Krusell and Smith (1998), augmented with many taxes and transfers. This has become the standard model of incomplete markets (Heathcote et al. (2009)).

With complete markets, households can diversify idiosyncratic risks to their income. The
Figure 6: Impulse responses to the two aggregate shocks

- Output
- Technology
- Monetary

- Consumption
- Output
- Technology
- Monetary

- Hours
- Output
- Technology
- Monetary

- Inflation
- Output
- Technology
- Monetary
following assumption eliminates these risks:

**Assumption 1.** Households and capitalists trade a full set of Arrow securities, so they are fully insured. Moreover, they are equally patient, $\hat{\beta} = \beta$.

It will not come as a surprise that if this assumption holds, there is a representative agent in this economy. More interesting, the problem she solves is familiar:

**Proposition 1.** Under assumption 1, there is a representative agent with preferences:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - (1 + E_t) \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} \right\},$$

and with the following constraints:

$$\hat{p}_t c_t + b_{t+1} - b_t = p_t [x_t - \bar{\tau}(x_t)] + T^n_t$$
$$x_t = \frac{\hat{r}_t}{p_t} b_t + w_t s_t (1 + E_t)n_t + d_t + T^n_t$$
$$s_t = \left[ \frac{1}{1 + E_t} s_t^{1+1/\psi_2} + \frac{E_t}{1 + E_t} \int_0^x s_t^{1+1/\psi_2} di \right]^{1+1/\psi_2},$$

where $1 + E_t$ is total employment, including capital-owners and households and $T^n_t$ is net non-taxable transfers to the household.

The proof is in Appendix E. With the exception of the exogenous shocks to employment, the problem of this representative agent is fairly standard. Moreover, on the firm side, optimal behavior by the goods-producing firms leads to a new Keynesian Phillips curve, while optimal behavior by the capital-goods firm produces a familiar IS equation. Therefore, with complete markets, our model is of the standard neoclassical synthesis variety (Goodfriend and King (1997), Woodford (2003)) that has been intensively used to study business cycles over the past decade. We will use it to study the effectiveness of automatic stabilizers when distributional issues are set to the side.

## 5 The effectiveness of automatic stabilizers

We start with an extreme but useful case.

**Assumption 2.** The following set of conditions holds:
1. The personal income tax is proportional, so $\tau^x(\cdot)$ is constant.

2. The probability of being employed is constant over time.

3. The Calvo probability of price adjustment $\theta = 1$, so prices are flexible.

4. There are infinite adjustments costs, $\gamma \to +\infty$, and no depreciation, $\delta = 0$, so capital is fixed.

5. There are no fixed costs of production, $\xi = 0$.

These strong assumptions shut down the aggregate demand channel, since prices are flexible, and the redistribution and social insurance channels, since all households are perfectly insured against idiosyncratic shocks so the wealth distribution does not matter for aggregate dynamics. Still, there remains the effect of automatic stabilizers on marginal incentives to work, save and consume.

The appendix proves the following result:

**Proposition 2.** If assumptions 1 and 2 hold, then the variance of the log of output is equal to the variance of the log of productivity. Therefore, $S = 0$ and the automatic stabilizers are ineffective.

The steps of the proof provide some intuition for the result. With flexible prices, there is an aggregate Cobb-Douglas production function, so if the capital stock and employment are fixed, then the proposition will be true as long as the labor supply is fixed. Equating the marginal rate of substitution between consumption and leisure for households to their after-tax wage gives the standard labor supply condition:

$$n_t(i) = \left( \frac{(1 - \tau^x)s_t(i)w_t}{\psi_1c_t(i)(1 + \tau^c)} \right)^{1/\psi_2}$$

Perfect insurance implies that consumption is equated across households. But then, our balanced-growth preferences and technologies imply that $c_t/w_t$ is fixed over time, so the condition above, once aggregated over all households, gives a constant labor supply.

While this result and the assumptions supporting it are extreme, it serves a useful purpose. Note that the estimates of the size of the stabilizer following the Pechman (1973) approach would be large in this economy. Measuring the ratio of the changes in taxes over the change in output over time in a simulation of this economy would produce a plot similar
Table 3: The effect of proportional taxes on the business cycle

<table>
<thead>
<tr>
<th></th>
<th>Representative agent</th>
<th>Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance  average</td>
<td>variance  average</td>
</tr>
<tr>
<td>output</td>
<td>0.0044 0.0159</td>
<td>-0.0029 0.0164</td>
</tr>
<tr>
<td>hours</td>
<td>0.0073 0.0007</td>
<td>0.0046 -0.0002</td>
</tr>
<tr>
<td>consumption</td>
<td>-0.0290 0.0147</td>
<td>-0.0106 0.0153</td>
</tr>
</tbody>
</table>

Note: Proportional change caused by cutting the stabilizer to the one in figure 1. And yet, the stabilizers in this economy are completely ineffective using our version of the Smyth (1966) measure. An economy may have high measured built-in flexibility while not being effectively flexible at all.

5.1 The effectiveness of proportional taxes

Assumption 2 imposed no restrictions on proportional taxes, yet their effect on volatility was nil. Table 3 considers the effect on the variance and average level of output, hours, and consumption of the following experiment. We cut the tax rates $\tau^C$, $\tau^P$ and $\tau^K$ each by 10%, and replaced the lost revenue of 0.6% of GDP by a lump-sum tax on the entrepreneurs.

The first pair of columns has the effects under assumption 1, so there are complete markets, and the next pair with the full model. The effects of proportional taxes on the volatility of the business cycle in both cases are quantitatively small. At the same time, when these taxes are removed, output and consumption significantly increase on average.

Intuitively, a higher tax rate on consumption lowers the returns from working and so lowers labor supply and output on average. However, because the tax rate is the same in good and bad times, it does not induce any intertemporal substitution of hours worked, nor does it change the share of disposable income available in booms versus recessions. Likewise, the taxes on corporate and property income may discourage savings and affect the average capital stock. But they do not do so differentially across different stages of the business cycle and so they have a negligible effect on volatility.

5.2 The effectiveness of transfers

To evaluate the effectiveness of our two transfer programs, unemployment and safety-net benefits, we reduced spending on both by 0.6% of GDP, the same amount in the experiment
Table 4: The effect of transfers on the business cycle

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th>Hand-to-mouth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
</tr>
<tr>
<td>output</td>
<td>0.0719</td>
<td>0.0016</td>
</tr>
<tr>
<td>hours</td>
<td>0.1613</td>
<td>-0.0134</td>
</tr>
<tr>
<td>consumption</td>
<td>-0.1345</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

Note: Proportional change caused by cutting the stabilizer

on proportional taxes. This is a uniform 80% reduction in transfer amounts. Again, we replaced the fall in outlays with a lump-sum transfer to stock owners. The impact on the full model is in table 4.

Transfers have a very small effect on the average level of output and consumption, yet they have a large effect on their volatility. Reducing transfer payments would raise output volatility by as much as 7%. The main channel at work seems to be redistribution. In a recession, the households without a job receive higher transfers. These have no direct effect on their labor supply of hours worked, since they do not have a job in the first place. However, they are funded by higher taxes on the stock owners, who raise their hours worked in response to the reduction in their wealth. This stabilizes hours worked and output.

Transfers also provide social insurance against the major idiosyncratic shocks they face. As a result, when we cut transfers, the variance of household consumption in logs rises substantially, by 91%. Yet, because they are worse-insured without transfers, households accumulate more assets. This is visible in figure 4, with the large shift of the wealth distribution to the right when transfers are reduced. Therefore, when aggregate shocks hit, they are better able to smooth them out. Therefore, without transfers, the volatility of aggregate consumption falls is lower by 13%.

To confirm that it is the precautionary channel that is behind these changes in volatility, we performed two additional experiments. First, we lowered the households’ discount factor at the same time that we reduced transfers, so that the aggregate assets of the household did not change. This is not a valid policy experiment, since we are changing not just policy but also preferences, but it serves to highlight the role of precautionary savings. Now, when we lower transfers and the discount factor, both the volatility of output and aggregate consumption rise substantially. The higher volatility of output is no longer offset by precautionary savings.
The second experiment considered an alternative model, inspired in the savers-spenders model of Mankiw (2000). We replaced household’s optimal savings function in figure 3, with the assumption that they live hand-to-mouth, consuming all of their after-tax income at every date. Now, there are no precautionary savings. Table 4 presents the results of our experiment in this economy. As expected, eliminating the public insurance provided by transfers, raises the volatility of both household and aggregate consumption now.

Moreover, the volatility of output now only slightly goes up without transfers. The savers-spenders economy maximizes the disposable-income channel that is most often mentioned in support of automatic stabilizers. Every dollar given to households is spent, raising output because of sticky prices. Yet, we see that, quantitatively, this effect accounts for little of the stabilizing effects of transfers in our economy. Rather, is is the redistribution channel that is most at work.

5.3 The effectiveness of progressive income taxes

The next experiment replaces the progressive personal income tax with a proportional, or flat, tax that raises the same revenue in steady state. Table 5 has the results.

In the representative-agent economy, progressive income taxes have a modest effect on the volatility of output. While the increase in marginal tax rates during booms and their decline in recessions, is stabilizing in theory, the level of progressivity in the current U.S. tax system is modest, as we saw in figure 2. As a result, this effect is quantitatively small.

Progressive income taxes stabilize consumption because capital-goods firms retain dividends in expansions to avoid the higher taxes, and distribute them in recessions. This stabilizes after-tax income for stock owners, and stabilizes their consumption while making investment on capital more volatile. Moving to a flat tax would raise the average level of economic activity significantly, output by 4% and consumption by 5%.

With incomplete markets, the effect on the average level of activity is slightly larger, but the effect of progressive income taxes on volatility is much higher. Again, social insurance and precautionary savings are at work. Removing the progressivity of the personal income tax increases after-tax income risk, which on the one hand raises the variance of log household consumption by 71%, and on the other hand induces households to save more. Households therefore hold more bonds, so stock owners own a larger share of their wealth in the capital stock. They are therefore more responsive in their investment choices to changes in marginal returns on saving, and output is more volatile.

Finally, note that progressive income taxes now have a close to zero effect on aggregate
Table 5: The effect of progressive taxes on the business cycle

<table>
<thead>
<tr>
<th></th>
<th>Representative agent</th>
<th>Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
</tr>
<tr>
<td>output</td>
<td>-0.0224</td>
<td>0.0369</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0129</td>
<td>0.0369</td>
</tr>
<tr>
<td>consumption</td>
<td>0.0831</td>
<td>0.0486</td>
</tr>
</tbody>
</table>

Note: Proportional change caused by cutting the stabilizer

consumption, unlike what happened with complete markets. The reason is again redistribution and the mechanism is the same as with transfers. Removing the social insurance against idiosyncratic shocks provided by a progressive tax system again increases the volatility of household consumption, lowers assets, and so makes it easier for households to smooth out aggregate shocks. This effect works in the opposite direction to the one with complete markets, and the two happen to quantitatively cancel out.

We also repeated this experiment in the savers-spenders economy and found that removing the progressivity of personal income taxes changed output volatility by only 0.5%, similar to the representative agent economy. This further reinforces our conclusion that the disposable-income channel is not what is behind the strong stabilizing effect of progressive income taxes.

5.4 The effectiveness of budget deficits

To assess the role of the budget deficit, we conducted two final experiments. First, we increased $\gamma$ to infinity, so that the government balanced its budget every period. Table 6 shows that this had almost no effect on the volatility of any of the endogenous variables. While Ricardian equivalence does not hold in our economy, changing the time profile of the taxes on stock owners has a small quantitative effect.

In our baseline model, $g_t/y_t$ is fixed over time. In our second experiment, we replaced our fiscal rule by an alternative where deficits are paid for by cutting government purchases over GDP:

$$\log\left(\frac{g_t/y_t}{\bar{g}/\bar{y}}\right) = -\gamma \log\left(\frac{B_t/p_t}{\bar{B}}\right).$$

The last two columns in table 6 perform the counterfactual where we increase the parameter $\gamma$ in this rule from the level at which it matches the U.S. data to infinity. If budget
Table 6: The effect of budget deficits on the business cycle

<table>
<thead>
<tr>
<th></th>
<th>Balanced-budget variance</th>
<th>Balanced-budget average</th>
<th>Purchases adjust variance</th>
<th>Purchases adjust average</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>-0.0008</td>
<td>-0.0000</td>
<td>0.1471</td>
<td>-0.0000</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0001</td>
<td>0.0000</td>
<td>0.0866</td>
<td>0.0001</td>
</tr>
<tr>
<td>consumption</td>
<td>-0.0078</td>
<td>-0.0000</td>
<td>-0.5548</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

Note: Proportional change from altering the fiscal adjustment rule

deficits are paid for by government purchases, then moving to a balanced budget would significantly raise the volatility of both output and hours. The intuition is simple. After a positive aggregate shock, the economy enters a boom, and the automatic stabilizers produce a surplus. Public debt therefore falls, which, given equation (26), induces government purchases to rise. This lowers the income available for private consumption, and through this income effect, labor supply increases, amplifying the shock. A balanced budget forces the adjustment of purchases to be immediate, therefore increasing the volatility of hours and output. The same income effect works to stabilize aggregate consumption.

To conclude, changing the timing of deficits per se has little effect on the economy. But the way in which these deficits are financed can have a significant effect on volatility. In particular, not cutting government purchases in response to public deficits is an effective stabilizer.

5.5 The overall effectiveness of the automatic stabilizers

Finally, we combine all of the experiments before into a single across the board reduction in the scope of the automatic stabilizers. Table 7 shows that with complete markets, the joint role of the stabilizers is quantitatively small. Simply put, the disposable-income and marginal-incentive channels are not that important. Rather, it is redistribution and social insurance that give bite to the stabilizers.

With incomplete markets, and these channels at work, reducing the scope of the automatic stabilizers would lead to an increase in the volatility of U.S. output of 7%, of the volatility of aggregate consumption by 3% and of household consumption by 265%. On the other hand, cutting the stabilizers would also raise average output and consumption by 6% and 7% respectively.
Table 7: The effect of all stabilizers on the business cycle

<table>
<thead>
<tr>
<th></th>
<th>Representative agent</th>
<th>Full model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
</tr>
<tr>
<td>output</td>
<td>-0.0172</td>
<td>0.0556</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0052</td>
<td>0.0399</td>
</tr>
<tr>
<td>consumption</td>
<td>0.0558</td>
<td>0.0669</td>
</tr>
</tbody>
</table>

Note: Proportional change caused by cutting the stabilizer

6 Conclusion

Milton Friedman (1948) famously railed against the use of discretionary policy to stabilize the business cycle. He defended the power instead of fiscal automatic stabilizers as a preferred tool for countercyclical policy. More recently, Solow (2005) strongly argued that policy and research should both focus more on automatic stabilizers as a route through which fiscal policy could and should affect the business cycle.

We constructed a business cycle model with many of the stabilizers and calibrated it to replicate the U.S. data. The model had some interesting features in its own right. First, it nested both the standard incomplete markets model, as well as the standard new-Keynesian business cycle model. Second, it matched the first and second moments of U.S. business cycles, as well as the broad features of the U.S. wealth and income distributions. Third, solving it required using new methods that may be useful for other models that combine nominal rigidities and incomplete markets.

We found that proportional taxes, like the sales tax, the property tax, and the corporate income tax have negligible effect on the volatility of economic aggregates. The progressivity of the personal income tax and transfer payments to the unemployed and those on food stamps have been quite effective stabilizers, contributing to a lower variance of output by 7% each. However, the progressivity of the income tax also leads to significantly lower average output. Transfer payments, in turn, have a negligible effect on average output, but because they lower precautionary savings, they raise the variance of consumption substantially.

We also found that the traditional Keynesian channel used to support automatic stabilizers is quantitatively weak. While raising the disposable income of consumers during recessions increases aggregate demand and output, this has a small effect over the business cycle. We found that a more important channel for stabilization was redistributing resources...
from richer agents, that have lower marginal propensities to consume and change their labor supply as their after-tax wealth changes, towards poorer agents, with higher marginal propensities to consume and are without a job so cannot decrease hours worked any further. At the same time, because this redistribution provides social insurance against idiosyncratic shocks, households hold fewer assets to self-insure, which raises the volatility of aggregate consumption in response to aggregate shocks.

We have refrained from making welfare judgments. Our economy has many dimensions of heterogeneity so that any normative point would depend crucially on how to weight the welfare of different agents. Moreover, it was important to understand first the positive properties of the model. Future work may take up the challenge of looking at optimal policy.

A second limitation of our work is that each of the automatic stabilizers that we considered is more complex than our description and distorts behavior in more ways than the ones we modeled. We did so in part because we wanted our model to be sufficiently simple so that we could understand the channels through which the stabilizers might work, and in part because of computational limitations. To obtain sharper quantitative estimate of the role of the stabilizers, it would be desirable to include the findings form the rich micro literatures that study each of these government programs in isolation. Perhaps the main point of this paper is that to assess automatic stabilizers requires having a fully articulated business-cycle model, so that we can move beyond the disposable-income channel, and consider other channels as well as quantify their relevance. Our hope is that as computational constraints diminish, we can keep this macroeconomic approach of solving for general equilibrium, while being able to consider the richness of the micro data.
References


Appendix

A  From NIPA tables to Table 1

For each entry in Table 1, we construct a sum of one or more entries in the NIPA tables, divide by nominal GDP and average over 1988 to 2007. Here we describe the components of each entry in Table 1.

A.1  Revenues

- **Personal income taxes** are the sum of federal and state income taxes (Table 3.4) plus contributions for government social insurance less contributions to retirement programs (NIPA Table 3.6 Line 1 less Lines 4, 12, 13, 22, and 29).

- **Corporate income taxes** are from Line 5 of Table 3.1.

- **Property taxes** are the sum of business property taxes (Table 3.5) and individual property taxes (Table 3.4).

- **Sales and excise taxes** are state sales taxes (Table 3.5) plus federal excise taxes (Table 3.5).

- **Public deficit** is the residual between the two columns of the Table.

- **Customs taxes** are from Table 3.5 Line 11.

- **Licenses, fines, fees** are the residual between current tax receipts from Table 3.1 and the other revenue listed in our Table.

- **Payroll taxes** are contributions to retirement programs (Table 3.6 Lines 4, 12, 13, 22, and 29).

A.2  Outlays

- **Unemployment benefits** are from Table 3.12 Line 7.

- **Safety net programs** are the sum of the listed sub-components from Table 3.12, where “security income to the disabled” is the sum of Lines 23, 29 and 36 and “Others” is the sum of Lines 37 - 39.
• **Government purchases** are current consumption expenditure from Table 3.1.

• **Net interest income** is the difference between interest expense and interest and asset income both from Table 3.1.

• **Health benefits (non-retirement)** are spending on Medicaid (Table 3.12 Line 33). multiplied by the share of Medicaid spending that was spent on children, disabled, and non-elderly adults in 2007 plus other medical care (Table 3.12 Line 34).

• **Retirement-related transfers** are the share of Medicaid spent on the elderly plus Social Security, Medicare, pension benefit guarantees, and railroad retirement programs (all from Table 3.12).

• **Other outlays** are the difference between total outlays in Table 3.1 and those listed here.

### B Decision problems and model equations

In this section of the appendix, we go from the statement of the problem in the main text to the optimality conditions that entered our algorithm to compute the equilibrium.

#### B.1 Capital owner’s problem

The capital owner chooses \( \{c_t, n_t\} \) to maximize expression (1) subject to equations (2) and (3). Define \( \tilde{b}_t = b_t/p_t \) and \( \pi_t = p_t/p_{t-1} \) and note that \( \hat{p}_t/p_t = 1 + \tau^c \). Then we can rewrite the constraints as:

\[
(1 + \tau^c)c_t + \tilde{b}_{t+1}\pi_{t+1} - \tilde{b}_t = x_t - \tilde{\tau}^c(x_t) + T_t^e/p_t
\]

\[
x_t = i_t\tilde{b}_t + w_t\tilde{s}n_t + d_t.
\]

Setting up the Lagrangean, with \( m^1_t \) and \( m^2_t \) as the Lagrange multipliers on constraints

---

(27) and (28), respectively, the optimality conditions are:

\[ \beta^t c_{it}^{-1} = m_{it}^1(1 + \tau^e) \]
\[ m_{it}^1 \pi_{it+1} = E_t \left[ m_{it+1}^1 + i_{it+1} m_{it+1}^2 \right] \]
\[ m_{it}^2 = m_{it}^1 (1 - \tau^x(x_t)) \]
\[ \beta^t \psi_1 n_t \psi_2 = m_{it}^2 w_t s, \]

These can be rearranged to give:

\[ \psi_1 n_t \psi_2 = \left( \frac{1}{c_t} \right) \left( \frac{1 - \tau^x(x_t)}{1 + \tau^e} \right) w_t s, \tag{29} \]
\[ \frac{1}{c_t} = \beta E_t \left\{ \frac{1 + i_{it+1} (1 - \tau^x(x_{it+1}))}{c_{it+1} \pi_{it+1}} \right\}, \tag{30} \]

which are the capital-owner’s labor-supply and Euler conditions. Finally, notice that the capital owner’s stochastic discount factor is:

\[ \lambda_{t,s} = \frac{m_{t+s}^2}{m_t^2} = \frac{\beta^s c_{it+s}^{-1} (1 - \tau^x(x_t + s))}{c_t^{-1} (1 - \tau^x(x_t))}. \tag{31} \]

### B.2 Other households’ problem

The decision problem of other households can be written as

\[ \max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t(i)) - \psi_1 n_t(i)^{1+\psi_2} \frac{1 + \psi_2}{1} \right], \]

subject to

\[ c_t(i) + b_{t+1}(i) \pi_{t+1} - \bar{b}_t(i) = x_t(i) - \bar{x}^e(x_t(i)) + T^s_t(i) \]
\[ x_t(i) = i_t \bar{b}_t(i) + s_t(i) w_t n_t(i) + T^u_t(i). \]

Using the same steps as above, one can derive an Euler equation and a labor supply condition that are analogous to those for the capital owner’s problem. One difference, however, is that here the expectation operator reflects an expectation over idiosyncratic uncertainty as well as over aggregate uncertainty.
B.3 Intermediate Goods’ Firm

A firm that sets its price at date \( t \) solves

\[
\max_{p^*_t,\{y_s(j),k_s(j),l_s(j)\}} E_0 \sum_{s=t}^{\infty} (1 - \tau^K) \left[ \frac{p^*_s}{p_s} y_s(j) - w_s l_s(j) - (r_s + \delta) k_s(j) - \xi \right] \lambda_{t,s} (1 - \theta)^{s-t},
\]

subject to

\[
y_s(j) = \left( \frac{p^*_s}{p_s} \right)^{\mu/(1-\mu)} y_s
\]

\[
y_s(j) = a_s k_s(j)^{\alpha} l_j^{1-\alpha}.
\]

where the first constraint is the demand for the firm’s good and the second its production functions. Dropping the constant \( 1 - \tau^K \) and substituting in the demand curve gives the modified problem:

\[
\max_{p^*_t,\{k_s(j),l_s(j)\}} E_t \sum_{s=t}^{\infty} \left[ \left( \frac{p^*_s}{p_s} \right)^{1/(1-\mu)} y_s - w_s l_s(j) - (r_s + \delta) k_s(j) - \xi \right] \lambda_{t,s} (1 - \theta)^{s-t}
\]

subject to

\[
\left( \frac{p^*_s}{p_s} \right)^{\mu/(1-\mu)} y_s = a_s k_s(j)^{\alpha} l_j^{1-\alpha}.
\]

The first order conditions with respect to \( k_s(j) \) and \( l_s(j) \) are:

\[
\lambda_{t,s} (1 - \theta)^{s-t} (r_s + \delta) = \Omega_s a_s k_s(j)^{\alpha-1} l_s(j)^{-\alpha}.
\]

\[
\lambda_{t,s} (1 - \theta)^{s-t} w_s = \Omega_s (1 - \alpha) a_s k_s(j)^{\alpha} l_s(j)^{-\alpha},
\]

where \( \Omega_s \) is the Lagrange multiplier on the production function constraint at date \( s \).

We can derive several useful features of the solution from these two optimality conditions. First, taking their ratio:

\[
\frac{w_s}{r_s + \delta} = \frac{1 - \alpha k_s(j)}{\alpha l_s(j)},
\]

so that all firms have the same capital-labor ratio and, by market clearing, \( k_s(j)/l_s(j) = k_s/l_s \) for all firms.

Second, note that since the production function for \( y_s(j) \) can be re-written in the following
two ways:

\[ l_s(j) = \frac{y_s(j)}{a_s} \left( \frac{k_s}{l_s} \right)^{-\alpha} \]

\[ k_s(j) = \frac{y_s(j)}{a_s} \left( \frac{k_s}{l_s} \right)^{1-\alpha} , \]

therefore, total variable costs are given by

\[ w_s l_s(j) + (r_s + \delta) k_s(j) = \left[ (r_s + \delta) \left( \frac{k_s}{l_s} \right)^{1-\alpha} + w_s \left( \frac{k_s}{l_s} \right)^{-\alpha} \right] \frac{y_s(j)}{a_s} , \]

so real marginal cost are:

\[ M_s \equiv \left[ (r_s + \delta) \left( \frac{k_s}{l_s} \right)^{1-\alpha} + w_s \left( \frac{k_s}{l_s} \right)^{-\alpha} \right] \frac{1}{a_s} . \tag{34} \]

But then, rearranging equation (32)

\[ \lambda_{t,s} (1-\theta)^{s-t} (r_s + \delta) \left( \frac{k_s}{l_s} \right)^{1-\alpha} \frac{1}{a_s} = \Omega_s \alpha \tag{35} \]

and similarly for (33)

\[ \lambda_{t,s} (1-\theta)^{s-t} w_s \left( \frac{k_s}{l_s} \right)^{-\alpha} \frac{1}{a_s} = \Omega_s (1-\alpha) . \tag{36} \]

so that summing equations (35) and (36) we arrive at the relationship between \( \Omega_s \), the Lagrange multiplier, and \( M_s \), the real marginal cost:

\[ \lambda_{t,s} (1-\theta)^{s-t} M_s = \Omega_s \tag{37} \]

Third, these optimality conditions allow us already to derive the expression for dividends as a function of factor prices. To see this, note that equation (32) can be rewritten in terms of \( M_s \) as

\[ r_s + \delta = M_s a_s \left( \frac{k_s}{l_s} \right)^{\alpha-1} \]
and similarly

\[ w_s = M_s (1 - \alpha) a_s \left( \frac{k_s}{l_s} \right) \alpha. \]

Therefore we have

\[ (r_s + \delta) k_s = M_s \alpha a_s k_s^{\alpha} l_s^{1-\alpha}, \tag{38} \]
\[ w_s l_s = M_s (1 - \alpha) a_s k_s^{\alpha} l_s^{1-\alpha}. \tag{39} \]

(Of course, equation (34) is implied by equations (38) and (39).) The aggregate pre-tax dividend of the intermediate goods firms is then

\[ \int_0^1 d_i(j) dj = \int_0^1 \left[ \frac{p_t(j)}{p_t} y_t(j) - w_t(j) l_t(j) - (r_t + \delta) k_t(j) - \xi \right] dj \]

and by market clearing this becomes

\[ \int_0^1 d_i(j) dj = \int_0^1 \frac{p_t(j)}{p_t} y_t(j) dj - M_t a_s k_s^{\alpha} l_s^{1-\alpha} - \xi = y_t - M_t a_s k_s^{\alpha} l_s^{1-\alpha} - \xi. \tag{40} \]

Finally, we can turn to the final optimality condition in the problem of the firm, the one with respect to \( p_t^\ast \):

\[ \mathbb{E}_t \sum_{s=t}^{\infty} \left[ \frac{1}{1 - \mu} \left( \frac{p_t^\ast}{p_s} \right)^{1/(1-\mu)-1} \frac{y_s}{p_s} \lambda_{t,s} (1 - \theta)^{s-t} - \Omega_s \frac{\mu}{1 - \mu} \left( \frac{p_t^\ast}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \right] = 0, \tag{41} \]

Using our earlier result linking the Lagrange multiplier to real marginal costs, we can rewrite equation (41) as

\[ \mathbb{E}_t \sum_{s=t}^{\infty} \frac{1}{1 - \mu} \left( \frac{p_t^\ast}{p_s} \right)^{1/(1-\mu)-1} \frac{y_s}{p_s} \lambda_{t,s} (1 - \theta)^{s-t} = \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} M_s \frac{\mu}{1 - \mu} \left( \frac{p_t^\ast}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \]

\[ \frac{p_t^\ast}{p_t} = \frac{p_t \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} M_s \mu \left( \frac{p_t}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s}}{p_t \mathbb{E}_t \sum_{s=t}^{\infty} \left( \frac{p_t}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \lambda_{t,s} (1 - \theta)^{s-t}} \equiv \frac{\bar{p}_t^A}{\bar{p}_t^B}. \tag{42} \]

This equation gives the solution for \( p_t^\ast \). It is useful to write \( \bar{p}_t^A \) and \( \bar{p}_t^B \) recursively. To that
\[ \bar{p}_t^A = p_t \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} M_s \mu \left( \frac{p_t}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \]
\[ = M_t \mu y_t + \mathbb{E}_t p_{t+1}^{-1} \mathbb{E}_{t+1} \lambda_{t,t+1} (1 - \theta) \left( \frac{p_t}{p_{t+1}} \right)^{\mu/(1-\mu)-1} \]
\[ \times \sum_{s=t+1}^{\infty} \lambda_{t+1,s} (1 - \theta)^{s-t-1} M_s \mu \left( \frac{p_{t+1}}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \]
\[ = M_t \mu y_t + \mathbb{E}_t \left[ \lambda_{t,t+1} (1 - \theta) \pi_{t+1}^{-\mu/(1-\mu) - 1} \bar{p}_{t+1}^A \right], \quad (43) \]

where \( \pi_{t+1} \equiv P_{t+1}/p_t \). Similar logic for \( \bar{p}_t^B \) yields
\[ \bar{p}_t^B = y_t + \mathbb{E}_t \left[ \lambda_{t,t+1} (1 - \theta) \pi_{t+1}^{-\mu/(1-\mu) - 1} \bar{p}_{t+1}^B \right]. \quad (44) \]

Next, comes the relationship between \( p_t^* \) and inflation. The price index is
\[ p_t = \left( \int_0^1 p_t(j)^{1/(1-\mu)} dj \right)^{1-\mu} \]
and with Calvo pricing we have
\[ p_t = \left( (1 - \theta) \int_0^1 (p_{t-1}(j))^{1/(1-\mu)} dj + \theta (p_t^*)^{1/(1-\mu)} \right)^{1-\mu} \]
\[ = \left( (1 - \theta) p_{t-1}^{1/(1-\mu)} + \theta (p_t^*)^{1/(1-\mu)} \right)^{1-\mu}. \]

Therefore
\[ \pi_t = \left( \frac{1 - \theta}{1 - \theta \left( \frac{p_t^*}{p_t} \right)^{1/(1-\mu)} } \right)^{1-\mu}. \quad (45) \]

Finally, note that because the capital-labor ratio is constant across firms, the production of variety \( j \) follows:
\[ y_t(j) = a_t \left( \frac{k_t}{l_t} \right)^{\beta} l_t(j). \]
The demand for variety $j$ can be written in terms of the relative price to arrive at

$$
\left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} y_t = a_t \left( \frac{k_t}{\ell_t} \right)^{\alpha} \ell_t(j).
$$

Integrating both sides yields

$$
\int_0^1 \left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} dj y_t = a_t \left( \frac{k_t}{\ell_t} \right)^{\alpha} \int_0^1 \ell_t(j) dj.
$$

By market clearing we have then that:

$$
S_t y_t = a_t k_t^{\alpha} \ell_t^{1-\alpha}, \tag{46}
$$

where

$$
S_t = \int_0^1 \left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} dj.
$$

$S_t$ is inversely related to the efficiency loss due to price dispersion and it evolves according to

$$
S_t = (1 - \theta) S_{t-1} \pi_t^{\mu/(1-\mu)} + \theta \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)}. \tag{47}
$$

**B.4 Capital goods firm**

The capital goods firms chooses a sequence $\{k_{t+1}, k_{t+2}, \cdots \}$ to maximize

$$
\mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 + \tau^P)^{-(s-t+1)} \left[ (1 - \tau^k) r_s k_s - k_{s+1} + k_s - \frac{\zeta}{2} \left( \frac{k_{s+1} - k_s}{k_s} \right)^2 k_s \right].
$$

The discounting by $1/(1 + \tau^P)$ comes from the property tax since:

$$
v_t = \frac{1}{1 - \tau^P} a_t^k + \frac{1}{1 - \tau^P} \mathbb{E}_t [\lambda_{t,t+1} v_{t+1}] \tag{48}.
$$

$$
(49)
$$
This problem leads to the first-order condition

\[ 1 + \zeta \left( \frac{k_{t+1} - k_t}{k_t} \right) = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{1 - \tau^p} \left[ (1 - \tau^k) r_{t+1} + 1 - \frac{\zeta}{2} \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right)^2 + \zeta \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right) \frac{k_{t+2}}{k_{t+1}} \right] \right\}. \]  

This expression can be transformed into one that only includes variables dated \( t \) and \( t + 1 \) by writing it in terms of \( \hat{k}_t = k_{t+1} \) and introducing \( \hat{k}^{\text{lag}}_t = \hat{k}_{t-1} \). Dividends paid by the capital goods firm are the term in brackets in the objective function less \( \tau^p \) times the value of the firm, which follows equation (48).

C  Numerical solution algorithm

As the main text described, the key steps involved in solving the model are: (i) to discretize the cross-sectional distributions, (ii) to solve for the stationary equilibrium, (iii) to collect all of the many equations defining the approximate equilibrium, and (iv) reducing these to a smaller system with little loss in accuracy. We elaborate on each of these steps next.

C.1 Discretizing the model

For each discrete type of household characterized by a skill level and an employment status, we approximate the distribution of wealth by a histogram with 1000 bins. We approximate the policy rules for savings and labor supply by two piece-wise linear splines with 100 knot points each. We deal with the borrowing constraint in the approximation of the policy functions by, following Reiter (2009a), parameterize the point at which the borrowing constraint is just binding, and then constructing a grid for higher levels of assets. As a result of these approximations, there are now 1200 variables for employed workers, and 1100 variables for unemployed workers (who do not choose labor supply).

C.2 Solving for the stationary equilibrium

Solving for the steady state of the model requires two steps: first, solving for the household policy rules and distribution of wealth and second, solving for the aggregate variables including the assets and consumption of the representative capital owner. These two steps are interrelated as the equilibrium interest rate depends on the capital-owner’s marginal
tax rate, which depends on the capital-owner’s income and therefore wealth, which in turn
depends on the level of wealth held by households.

We use an iterative procedure to find the equilibrium income of the capital owners. Given a guess of the capital-owner’s income and therefore marginal tax rate, we find the equilibrium interest rate from the capital-owner’s Euler equation and then the solution of the intermediate goods firm’s problem to find the equilibrium wage. With these objects, we solve the households problem to find their consumption and asset positions. With these in hand, we use standard techniques from the analysis of representative agent models to find the rest of the aggregate variables. Finally, we check our guess of the capital-owner’s income and iterate from here.

C.3 System of equations

Keeping track of the wealth distribution We track real assets at the beginning of the period using Reiter’s procedure to allocate households to the discrete grid in a way that preserves total assets. As we have nominal bonds in the model, we account for the effect of inflation in the evolution of the household’s asset position. For each discrete type of household this provides 1000 equations.

Solving for household decision rules We use the household’s Euler equation and the household’s labor supply condition to solve for their decision rules by imposing that these equations hold with equality at the spline knot points. This provides 100 equations for unemployed households and 200 for employed households.

Aggregate equations In addition to those equations that relate to the solution of the household’s problem and the distribution of wealth across households, we have equations that correspond to the capital-owner’s savings and labor supply decisions, as well as those that correspond to the firms’ problems. These equations are discussed in more detail in appendix B. We use equations (27), (29), (30), (42), (43), (44), (45), (48), (46), (47), (49), (50), and an auxiliary variable that carries an extra lag of capital. In addition, from the main text we have equations (21), (22), (23), (24), (25), and exogenous AR(1) processes for \( \epsilon_t \) and \( a_t \). We use these equations to solve for \( c_t, n_t, b_t, M_t, p_t^s/p_t, \bar{p}_t^A, \bar{p}_t^B, S_t, \pi_t, y_t, w_t, r_t, v_t, k_t, \) lag of capital \( k_t^{\text{lag}} \), \( d_t, B_t, T_t^e \), and \( i_t \).
C.4 Linearization and model reduction

At this stage, we have a large system of non-linear equations that the discretized model must satisfy. We follow Reiter (2009a,b) in linearizing this system around the stationary equilibrium using automatic differentiation and then solving the linearized system as a linear rational expectations model. The full linear system is too large to solve directly so we use the model reduction step introduced by Reiter (2009a). The only change that we make to Reiter’s procedure is the way we select the observation matrix ($C$ in Reiter’s notation). The importance of this matrix is that it specifies those variables that the model should reproduce accurately. Reiter includes those aggregate variables that enter the household’s decision problem. In his case, that is the capital stock, which determines prices. In addition to these variables, we add those that we are interested in for our results (output, hours and aggregate consumption). Finally, we found it necessary to include the level of government debt in order to achieve an accurate solution.\footnote{We found this in initial exploration of models for which was possible to find the exact solution of the linear model (i.e. versions of the model with less heterogeneity so the model reduction step was not necessary).} We believe that the importance of government debt stems from its strong influence on the equilibrium interest rate.

D Calibration of idiosyncratic shock processes

Each household at every date has a draw of $s_t(i)$ determining the wage they receive if they are employed, and a draw of $e_t(i)$ on their employment status. This appendix describes how we calibrated the distribution and dynamics of these two random variables.

D.1 Skill shocks

We use PSID data on wages to calibrate the skill process. To do this, we start with sample C from Heathcote et al. (2010a) and work with the log wages of household heads in years 1968 to 2002. Computational considerations limit us to three skill levels and we construct a grid by splitting the sample into three groups at the 33rd and 67th percentiles and then using the median wage in each group as the three grid points. Skills are proportional to the level (not log) of these wages. Computational considerations also lead us to choose a skill
transition matrix with as few non-zero elements as possible. We impose the structure
\[
\begin{pmatrix}
1 - p & p & 0 \\
p & 1 - 2p & p \\
0 & p & 1 - p
\end{pmatrix},
\]
where \( p \) is a parameter that we calibrate as follows. From the PSID data, we compute the first, second and fourth auto-covariances of log wages. Let \( \Gamma_i \) be the \( i^{th} \) auto-covariance. We use the moments \( \Gamma_2/\Gamma_1 \) and \( \sqrt{\Gamma_4/\Gamma_2} \), each of which can be viewed as an estimate of the autoregressive parameter if the log wages follow an AR(1) process.\(^{17}\) The empirical moments are 0.9356 and 0.9496, respectively. To map these moments into a value of \( p \), we minimize the equally-weighted sum of squared deviations between these empirical moments and those implied by the three-state Markov chain. As our time period is one quarter, while the PSID data are annual, we use \( \Gamma_8/\Gamma_1 \) and \( \sqrt{\Gamma_{16}/\Gamma_8} \) from the model. This procedure results in a value of \( p \) of 0.015.

D.2 Employment shocks

Steady state In addition to differences in skill levels, households differ in their employment status. A household can be (1) employed, (2) unemployed or (3) long-term unemployed. To construct a steady state transition matrix between these three states we need six moments. First, it seems reasonable to assume that a household does not transit directly from employed to long-term unemployed or from long-term unemployed back to unemployed. Those two elements of the transition matrix are therefore set to zero.

The distribution of households across states gives us two more moments. As the focus of our work is on the level and fluctuation in the number of individuals receiving different types of transfers, we define unemployed as individuals who are receiving unemployment benefits and long-term unemployed as those receiving food stamps. In the U.S., the Supplemental Nutritional Assistance Program is the largest non-health, non-retirement social safety net program. SNAP assists low-income households in being able to purchase a minimally adequate low-cost diet. Recipients of these benefits are generally not working.\(^{18}\) One virtue of using SNAP participation as a proxy for long-term unemployment is that it avoids the subtle

\(^{17}\)The ratio \( \Gamma_1/\Gamma_0 \) is not used as this ratio is heavily influenced by measurement error, which leads to an underestimate of the persistence of wages. The moments that we use are also used by Heathcote et al. (2010b) to estimate the persistence of the wage process.

\(^{18}\)In 2009, 71% of SNAP recipient households had no earned income and only 17% had elderly individuals (Leftin et al., 2010).
distinction between unemployment and non-participation in the Current Population Survey while still focusing on those individuals who likely have poor labor market prospects. If we instead used time since last employment to identify those in long-term unemployment, we would include a number of individuals with decent opportunities to work if they chose to do so such as individuals who have retired or who choose to work in the home. Between 1971, when the data begin, and 2011, the average insured unemployment rate was 2.9%.\textsuperscript{19} Between 1974, when the SNAP program was fully implemented nationwide, and 2011, the average ratio of SNAP participation to the insured labor force was 8.7%. We refer to this as the SNAP ratio.\textsuperscript{20}

Our final two moments speak to the length of time that an individual expects to be in a particular labor market state. We use the unemployment-employment transition probability calculated by Shimer (2007) as one such moment. At a quarterly frequency, the average transition probability between 1960 and 2007 (when Shimer’s data end) was 82%. Finally, we calibrate the probability of transitioning from long-term unemployment to employment based on the finding of Mabli et al. (2011) that 3% of SNAP participants leave the program each month.

**Business cycle dynamics of employment risk**  An important component of our model is the evolution of labor market conditions over the business cycle. One effect of the fluctuations in labor market conditions is to alter the number of households receiving different types of benefits over the cycle. A second effect is to alter the amount of risk that households face, which has consequences for the consumption and work decision.

As we analyze the model with a linear approximation around the stationary equilibrium, it is sufficient to outline how the labor market risk evolves in the neighborhood of the stationary equilibrium. Let $\Pi_t$ be the matrix of transition probabilities between employment states at date $t$ and $t+1$. We impose the following structure on the evolution of $\Pi_t$

$$\Pi_t = \Pi^0 + \Pi^1 \left[ \chi \log z_t - (1 - \chi) \varepsilon_t \right],$$

where $\Pi^0$ and $\Pi^1$ are constant $3 \times 3$ matrices. $\Pi^0$ is the matrix of transition probabilities between employment states in steady state. The term in brackets is a composite of the technology and labor market shocks and the parameter $\chi$ controls how much the labor

\textsuperscript{19}The insured unemployment rate is the ratio of the number of individuals receiving unemployment insurance benefits to the number of employed workers covered by unemployment insurance.

\textsuperscript{20}This ratio is calculated as the number of SNAP participants divided by the sum of the number of workers covered by unemployment insurance and the number of individuals receiving UI benefits.
market is driven by technology shocks as opposed to monetary shocks. We set $\chi$ so that the technology shocks account for 80% of the variance of the unemployment rate in keeping with the view that they drive 80% of the variance of output. What remains is to specify the matrix $\Pi^1$. For that we need another four conditions.\footnote{The rows of $\Pi^1$ must sum to zero so that the rows of $\Pi_t$ always sum to one. We continue to assume that there are no transitions from employment to long-term unemployment or from long-term unemployment back to unemployment so those elements of $\Pi^1$ are also zero just as in $\Pi^0$.}

If the moments that determine $\Pi^0$ are viewed as first moments, we use the corresponding second moments to construct $\Pi^1$. That is, we use the variance of the insured unemployment rate ($0.9 \times 10^{-4}$), the variance of the SNAP ratio ($4.2 \times 10^{-4}$), and the variance of the unemployment-employment transition rate ($2.9 \times 10^{-3}$). That leaves us with one too few moments. What is missing is a moment that corresponds to Mabli et al.'s figure that 3% of SNAP participants exit the program each month. In their study, there is not sufficient information to discern the business cycle dynamics of this exit probability. Therefore, we assume that transition probability from long-term unemployment to employment is always proportional to the unemployment-employment transition probability.

These procedures leave us with the following:

$$
\Pi^0 = \begin{pmatrix}
0.9743 & 0.0257 & 0 \\
0.5711 & 0.1317 & 0.2972 \\
0.0873 & 0 & 0.9127 \\
\end{pmatrix}, \quad \Pi^1 = \begin{pmatrix}
1.75 & -1.75 & 0 \\
9.70 & -9.70 & 0 \\
1.48 & 0 & -1.48 \\
\end{pmatrix}, \quad \chi = 0.58,
$$

where the the $(i, j)$ element of the $\Pi$ matrices refers to the transition probability from state $i$ to state $j$ and the states are ordered as employed, unemployed, long-term unemployed.

## E Proofs for propositions

Proof of proposition 1. Under complete markets, the households will fully insure idiosyncratic risks. Therefore, we treat them as a large family that pools risks among its members. In determining the family’s tax bracket, we assume the tax collector applies the tax rate corresponding to the average income of its members.

The large family maximizes

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} + \int_0^\nu \ln c_i(i) - \psi_1 \frac{n_t(i)^{1+\psi_2}}{1 + \psi_2} \, di \right]
$$

Proof of proposition 2. Under complete markets, the households will fully insure idiosyncratic risks. Therefore, we treat them as a large family that pools risks among its members. In determining the family’s tax bracket, we assume the tax collector applies the tax rate corresponding to the average income of its members.

The large family maximizes

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} + \int_0^\nu \ln c_i(i) - \psi_1 \frac{n_t(i)^{1+\psi_2}}{1 + \psi_2} \, di \right]
$$

Proof of proposition 3. Under complete markets, the households will fully insure idiosyncratic risks. Therefore, we treat them as a large family that pools risks among its members. In determining the family’s tax bracket, we assume the tax collector applies the tax rate corresponding to the average income of its members.

The large family maximizes

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} + \int_0^\nu \ln c_i(i) - \psi_1 \frac{n_t(i)^{1+\psi_2}}{1 + \psi_2} \, di \right]
$$

Proof of proposition 4. Under complete markets, the households will fully insure idiosyncratic risks. Therefore, we treat them as a large family that pools risks among its members. In determining the family’s tax bracket, we assume the tax collector applies the tax rate corresponding to the average income of its members.

The large family maximizes

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} + \int_0^\nu \ln c_i(i) - \psi_1 \frac{n_t(i)^{1+\psi_2}}{1 + \psi_2} \, di \right]
$$

Proof of proposition 5. Under complete markets, the households will fully insure idiosyncratic risks. Therefore, we treat them as a large family that pools risks among its members. In determining the family’s tax bracket, we assume the tax collector applies the tax rate corresponding to the average income of its members.

The large family maximizes

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} + \int_0^\nu \ln c_i(i) - \psi_1 \frac{n_t(i)^{1+\psi_2}}{1 + \psi_2} \, di \right]
$$
subject to
\[ \hat{p}_t \left[ \int_0^{x_t} c_t(i) di + c_t \right] + b_{t+1} - b_t = p_t \left[ x_t - \bar{x}(x_t) \right] + T_t, \]
where \( T_t \) is net non-taxable transfers to the household and
\[ x_t = (i_t/p_t)b_t + w_t\bar{s}n_t + d_t + \int_0^{x_t} s_t(i)n_t(i) + T_t(i) di. \]
The household also faces the constraint \( n_t(i) = 0 \) if \( e_t(i) \neq 2 \). Let \( m^1_t \) be the Lagrange multiplier on the former constraint and \( m^2_t \) be the Lagrange multiplier on the latter. Then the first order conditions of this problem are
\[
\begin{align*}
\frac{\beta^t}{c_t} &= \hat{p}_t m^1_t & c_t \\
\frac{\beta^t}{c_t(i)} &= \hat{p}_t m^1_t & n_t(i) \\
m^1_t &= E_t \left\{ m^1_{t+1} + m^2_{t+1}(i_t/p_t) \right\} & b_{t+1} \\
m^1_t p_t \left[ 1 - \bar{x}(x_t) \right] &= m^2_t & x_t \\
\beta^t \psi_1 n_t^{\psi_2} &= m^2_t w_t \bar{s} & n_t \\
\beta^t \psi_1 n_t(i)^{\psi_2} &= m^2_t w_t s_t(i) & n_t(i)
\end{align*}
\]
These first order conditions can be rearranged to obtain
\[
\begin{align*}
c_t(i) &= c_t, \\
\frac{1}{c_t} &= \beta E_t \left\{ \frac{1 + i_{t+1} \left[ 1 - \tau^x(x_{t+1}) \right]}{c_{t+1} \bar{x}_{t+1}} \right\},
\end{align*}
\]
and aggregate labor input satisfies
\[
\bar{s}n_t + \int_0^{x_t} s_t(i)n_t(i) di = \left\{ \frac{1}{\psi_1 c_t} \frac{1 - \bar{x}(x_t)}{1 + \tau^c w_t} \right\}^{1/\psi_2} \left[ \bar{s}^{1+1/\psi_2} + E_t \int_0^{x_t} (s_t(i))^{1+1/\psi_2} di \right],
\]
where \( E_t \) is defined as the mass of non-capital-owner households who are employed. In this final step we should only integrate over those households that are not at a corner solution, but this is trivial as the marginal disutility of labor goes to zero as \( n_t(i) \) goes to zero so all households with positive wages are employed and it is only those who exogenously lack
employment opportunities who will set $n_t(i) = 0$.

Proceeding similarly for the representative agent decision problem stated in the proposition and defining aggregate labor input in that case to be $(1 + E_t)s_t n_t$, one reaches the conclusion that the two models will deliver the same Euler equation and condition for aggregate labor supply. Therefore, the two models will generate the same aggregate dynamics. □

**Proof of proposition 2.** Under assumption 1, we can use the representative agent formulation from proposition 1. The labor supply condition for this problem is

$$n_t = \left[\frac{(1 - \tau^x)w_t s_t}{c_t(1 + \tau^c)\psi_1}\right]^{1/\psi_2},$$

where $\tau^x$ is the (constant) marginal tax rate. Under the conditions of assumption 2, the aggregate resource constraint is: $c_t + g_t = y_t$. But, since there is a constant ratio of $g_t$ to $y_t$, the resource constraint implies that $c_t/y_t$ is constant and equal to $1 - \bar{g}/\bar{y}$. Moreover, with flexible prices, we can write $w_t = \frac{(1-\alpha)w}{\mu L_t}$, where $L_t$ is aggregate labor input. Using these two results to substitute out $c_t$ and $w_t$ we obtain

$$n_t = \left[\frac{(1 - \tau^x)(1 - \alpha)(1/\mu)y_t/n_t}{(1 - \bar{g}/\bar{y})y_t(1 + \tau^c)\psi_1}\right]^{1/\psi_2},$$

where we have used the fact that the aggregate labor input is $n_t s_t$. Using this expression, we can solve for $n_t$ as

$$n_t^{1+1/\psi_2} = \left[\frac{(1 - \tau^x)(1 - \alpha)(1/\mu)}{(1 - \bar{g}/\bar{y})(1 + \tau^c)\psi_1}\right]^{1/\psi_2}.$$

Because the right-hand-side does not depend on time, it follows that $n_t$ is constant over time.

Next, recall that capital is fixed and prices are flexible, so aggregate output is

$$y_t = a_t K s n,$$

where $K$ and $n$ are the constant inputs of capital and hours and $s$ is the skill level of the representative agent, which is also constant over time by the fact that the labor market risk is unchanging over time so the composition of the pool of workers is stable. It follows from this equation that the variance of log output is equal to the variance of log productivity, $a_t$.

That $S = 0$ follows from the fact that the productivity process is exogenous and therefore not affected by the presence or absence of automatic stabilizers. Notice that $S = 0$ holds regardless of whether one uses output or consumption as the measure of activity as $c_t/y_t$ is
constant. For hours, the ratio is not defined since there is no variation in hours worked.