

# Bidding Frictions in Ascending Auctions\*

Joachim R. Groeger      Robert A. Miller  
Carnegie Mellon University    Carnegie Mellon University

June 29, 2015

## Abstract

This paper develops a strategy for identifying and estimating the valuation distribution in ascending auctions where bidders have an unknown number of bidding opportunities. Our model leads to equilibrium jump bidding and potentially inefficient allocations. Our identification approach requires few assumptions on the types of equilibria played. In particular, we need not assume that a unique equilibrium exist or that it be separating. We apply the model to data drawn from a monthly financial market, where the treasurers of state governments purchase from local banks savings vehicles, called certificates of deposit (CD), for unallocated state funds via a procurement auction mechanism based on the offered interest rate. In this market, the state acts as a liquidity provider that allows banks to invest in opportunities that are private to the bank. The data exhibit features that are inconsistent with standard auction formats. For example, the distribution of final bids first order stochastically dominates the distribution of winning bids, which are sometimes made early in this open ascending auction. These features motivate our model of bidding frictions. We prove that the distribution of valuations, but not the individual valuations, are identified and estimation is feasible even in the presence of multiple and mixed strategy equilibria. When bidding frictions are ignored, the monotonicity property of the sealed bid framework is rejected, and the estimated valuations are orders of magnitude greater than the corresponding final bids, creating the impression that banks have a large amount of market power. The model with frictions generates more sensible valuation estimates and provides insight into a local banking market. Since our data cover the 2008 credit crisis, we can analyze the local financial effects of the recession. The estimates of frictions before and after the crisis suggest that banks face inertia in the re-organization of their activities even after large negative macroeconomic shocks have hit. We show that sealed bid mechanisms perform better than the current mechanism and that these benefits are greater in the post-2008 period.

## 1 Introduction

This paper develops a strategy for identifying and estimating valuation distributions in ascending auctions where bidders have an unknown number of submission opportunities. Our approach applies to any environment where future bidding opportunities are uncertain and therefore an incentive exists to submit jump bids. Identification only makes use of the probability of observing a bidder submitting a serious bid at a specific time in the auction to identify the probability distribution of

---

\*We would like to thank Timothy Derdenger for pointing us to this data and John Carver and Jon Smith at Grant Street for providing it. For detailed comments, we thank Dan Akerberg, Lanier Benkard, Ken Hendricks, Isabelle Perrigne, Martin Pesendorfer, John Rust and Quang Vuong. We also thank participants at the Econometric Society Meetings in Asia 2013, and at economics seminars in UT Austin, LSE, Princeton, Johns Hopkins, Vanderbilt, CEMFI, Carlos III, Autonoma, the CRES Applied Microeconomics Conference at Washington University in St. Louis and Georgetown universities.

valuations. We therefore do not rely on the monotonicity of strategies in valuations or that only one equilibrium be played in the data.

We apply our model to discriminatory auctions data that exhibit features which could not have been generated by a standard auction mechanism. For example, winning bids are not identical, yet a non-trivial proportion of them are placed before the auction ends; also the distribution of (winning and losing) final bids first order stochastically dominates the distribution of winning bids in the multiunit discriminatory auction. In the model we develop to accommodate such anomalies, frictions arise, caused by the organization of bank activities, that prevent bidders from making offers whenever they want. We prove that the distribution of equilibrium outcomes identifies the distribution of valuations and the distribution of bidding opportunities. The equations identifying the model are the basis for estimation.

The data come from a monthly financial market where the treasurers of state governments purchase from local banks savings vehicles, certificates of deposit (CD), for unallocated state funds via a procurement auction mechanism. When bidding frictions are ignored, the monotonicity property of a sealed bid framework is rejected, the estimated valuations are orders of magnitude greater than the corresponding final bids, implying that bidders have a high degree of market power. In contrast, the model of bidding frictions yields plausible estimated rates of return to bidders. The losses from bidding frictions are moderate and predominantly arise from potential bidders that fail to have an opportunity to bid at the auction. Finally our data straddle the credit crisis of 2008 and our estimates show how local financial markets became less liquid in the aftermath of the credit crisis. Our estimates also provide insight into the operations of banks. In particular, our estimates of monitoring intensities before and after the recession show little difference. This suggests that banks face inertia in re-organizing their activities even when in the midst of a financial crisis. We are also able to show that the mechanism's inefficiency are less socially costly than after the crisis where greater uncertainty generates a valuation distribution with higher dispersion.

Our study draws from both the theory of auctions and the econometric issues associated with identifying and estimating auction models. Our model is one of several ascending auction models that are not strategically equivalent to Japanese auctions (where the valuation of every losing bidder is their exit bid).<sup>1</sup> A noteworthy feature of dynamic models like ours, where bidders have multiple opportunities to place a winning bid, is that their previous bids affect the current reservation price, considerably complicating equilibrium analysis and identification. It is, however, straightforward to derive one equilibrium property of our framework, which is instrumental in proving identification and facilitating estimation: if a bidder's most recent bid is too low to win the auction, and it could profit from offering the highest bid, then it would raise its bid to at least that level if it could. This property is quite closely related to the behavioral approach of Haile and Tamer (2003). They explicitly eschew linking the primitives of the distribution of preferences to an explicit bidding mechanism through equilibrium behavior in favor of presupposing two behavioral rules: that bidders stop bidding after their valuation is reached, and that the final bid is never lower than the valuation of a losing bidder. Our model yields the first rule as an equilibrium outcome, but our data provide evidence against the second rule.<sup>2</sup>

Our model is explicitly dynamic; during the auction a bidder might discover its bid will not win

---

<sup>1</sup>Previous theoretical work on such models includes Hörner and Sahuguet (2007), and Daniel and Hirshleifer (1998). Avery (1998) analyzes jump bidding in common value settings, Birulin and Izmalkov (2011) consider English auctions with re-entry and affiliated values. Daniel and Hirshleifer (1998) and Hörner and Sahuguet (2007) consider ascending auctions with private values.

<sup>2</sup>Haile and Tamer (2003) can handle a limited amount of inefficiency. If the difference in valuations for the top two bidders is less than the difference of the bid increment then it is possible for the second highest value bidder to win. However, if the increment is very small or valuations are also defined on a grid with increments matching the bid grid the high value bidder always wins.

the auction and have an opportunity to raise it. In this respect our analysis is closer to studies of dynamic auctions in Jofre-Bonet and Pesendorfer (2003) and Groeger (2014).<sup>3</sup> Akerberg, Hirano, and Shahriar (2011) also analyzes online auctions using data on eBay auctions with a buy-it-now feature, allowing for the identification of time and risk preferences. All the aforementioned work investigates auction models in which the valuations of each bidder are point identified. In our model the distribution of valuations is identified, but not the individual valuations. Our investigation of the effects of the credit crisis is also related to the work of Cassola, Hortaçsu, and Kastl (2013), who look at bidding behavior in central bank liquidity auctions and use a structural model to identify changes in strategic behavior and shifts in the underlying primitives.

The main features of our data are described in Section 2. They comprise a sample of procurement auctions by the Texas State government for Certificates of Deposit from locally based banks, who bid on the interest rate they offer. The sample period spans the financial crash of 2008 and the liquidity crisis that followed. Summary statistics of this data provide evidence of bidding frictions, casting doubt on conventional auction models. Section 3 then describes the basic framework of the three auction mechanisms we investigate: in all mechanisms each potential bidder has a private value for an indivisible number of units, and multiple units are sold simultaneously. To justify why we introduce bidding frictions into the auction mechanism, we first propose and estimate the standard auctions in Section 3.1. We show that an auction mechanism where bidders always have the opportunity to upgrade their bids in response to rival bids is at odds with our data because successful bidders do not pay the same price, a key prediction of that setup. The data also reject modeling the procurement mechanism as a sealed bid auction (rationalized by sniping), or a sequential auction (in which bidders are asymmetrically informed about the bids of their rivals). In addition the estimated valuations from those models are wildly at variance with any reasonable notion of what alternative sources of funds might cost. Section 4 analyzes our simple model with bidding frictions. These frictions can be attributed to other activities infringing on the bidder’s time, and bidding opportunities occurring randomly throughout the auction period. The arrival rate of bidding opportunities, and the private values distribution, are identified and estimated. The current bid of each bidder affects all future reservation prices, and hence induces an intractable equilibrium selection problem when analyzing the bidder’s subsequent bids. However, we show that the distribution of valuations is identified in a straightforward manner from the first bid of each bidder and the distribution of last bids by any bidder. We also identify and estimate bounds on the efficiency of the auction. Our estimates are plausible; we find that bidding frictions do not affect the efficiency of the auction amongst bidders in a quantitatively meaningful way prior to the crisis, but do have a quantitatively significant deterrence effect on entry, especially after the financial crisis. In this way our model of bidding frictions captures illiquidity in that market. Section 6 concludes.

## 2 Auctioning Certificates of Deposit

A Certificate of Deposit (CD) is similar to a standard savings account. The key difference is that a CD has a specific, fixed term, usually three, six or twelve months. At maturity the money may be withdrawn together with the accrued interest. Thus a CD is essentially a fixed term loan with a fixed principal, repaid at maturity with the accrued interest. Our application analyzes CD auctions in Texas held by that state’s government. The Texas State government conducts multiunit procurement auctions each month.<sup>4</sup> It buys CDs issued by local qualified banks. Prior to entering

---

<sup>3</sup>These papers consider repeated first price auctions in dynamic environments and extend Guerre et al. (2000). Jofre-Bonet and Pesendorfer (2003) consider the effect of capacity constraints on bidding. Groeger (2014) studies repeated first price auctions with endogenous participation and synergies in participation.

<sup>4</sup>There are a number of other states that use the same online auction platform, for example Idaho, Iowa, Louisiana, Massachusetts, New Hampshire, Ohio, Pennsylvania and South Carolina. We focus on Texas since it is the most active

a CD auction a bank must undergo a pre-qualification process. During this process the level of collateral a bank holds is ascertained. Texas limits participation to local bank branches to ensure that tax money does not leave the state and can be used to inject liquidity into the local economy. Participating banks compete to sell these savings vehicles to the state treasurer through an on line auction. The money sold in these auctions from state funds have no immediate purpose. Banks compete in an online ascending auction that lasts 30 minutes by offering the most attractive interest rates on blocks of units.

The state operates much like a lender of last resort. During this period, the interest rate on CDs offered to retail customers is lower than the submissions observed in these auctions.<sup>5</sup> This suggests that banks use this market when retail demand has been exhausted and they have a liquidity need, for example, the arrival of a time sensitive local investment opportunity. As a result, bidding behavior in this market reflects the underlying profitability of investment opportunities banks have access to.

Each month banks bid for state funds via an open cry multiunit online auction. Entry occurs anonymously throughout the auction, and as it progresses nobody knows which bidders have dropped out. Some banks submit bids above the reserve price at the beginning of the half hour auction period and identical units are often sold at different prices. These institutional arrangements and stylized facts cannot be reconciled to a Japanese auction, where the field of bidders is announced before the auction begins, and bidders publicly withdraw as the auctioneer increases the price until the supply of units matches the demand by the bidders who are left, who all pay the same price. In an ascending electronic auction where banks can bid at the final instant, standard auction theory predicts that every bank would do that, snipe, to prevent other bidders from revising their own bids in response. While some sniping occurs in our data, it is not unusual for winning bids to be made well before the end of the auction. Bids above the reserve price are sometimes submitted near the beginning of an auction, and they tend to increase with the lowest price necessary to win, further evidence against the equilibrium outcome in sealed bid auctions, where bidders cannot react to each other. Yet our data also exhibit bids that jump in a discrete manner, in contrast to an English auction, where the auctioneer's ask price increases incrementally.

This section is an overview of our data. We describe the institutional setting and the auction mechanism; we provide several measures of the size of the auctions, and present evidence illustrating the magnitude of the difference between the highest and lowest winning bids. Then we summarize the bidding patterns that motivate some key features of our model.

## 2.1 Auction format

Before the auction begins, the state government announces a reserve interest rate, which determines the minimal acceptable bid. Bidding takes place over 30 minutes. Banks can submit a succession of increasing bids on up to five separate parcels, at different interest rates for different dollar amounts, The size of each parcel is a multiple of \$100,000, with a minimum is \$100,000, and a maximum is \$7million. The collateral of a bank determines the upper limit on the sum of its parcel sizes. A bid for a parcel can be increased as many times as the bank wants during the 30 minute period, but not decreased, and the size of the parcel cannot be reduced either. In this auction bidders do not directly observe the minimal bid that would win if no further bids were tendered, the on-the-money ONM bid.

Parcels are ranked by the interest rate on the most recent bid; size does not affect ranking. When the auction ends, an amount up to the total funds are dispersed to the banks offering the

---

state.

<sup>5</sup>The FDIC provides national average rates on CDs from May 2009 onwards. Further information can be found here <https://www.fdic.gov/regulations/resources/rates/previous.html> (accessed 22 June 2015).

highest interest rates. Banks pay the (most recent and highest) interest rate they bid on their winning parcels, and nothing on parcels that lose. In the event of a tie for the lowest winning bid, the earliest order rate at that interest rate is given precedence. Moreover, if the parcel size of the lowest winning bid exceeds the amount left for allocation, the bank is obliged to partially fill its order with the remaining funds.<sup>6</sup> Summarizing, this is a discriminatory price, multiunit ascending auction in which banks receive limited information about the status of their own bids throughout the bidding phase.

## 2.2 Auction Size

Our data set contains 78 CD auctions in Texas for the years 2006 through 2010 and in which the complete path of play for the auction is recorded. The majority of auctions involve six-month CDs. There are five auctions in our data set that are for 12 month CDs. The amount to be auctioned is on average \$76 million. The majority of auctions were for \$80 million. Eleven auctions were for \$50 million. Summary characteristics of the auction are displayed in Table 1. There is a potential pool of 53 banks, and an average of 25 banks enter each auction. A number of banks are branches of the same parent bank and find that they frequently compete against each other for funds. To compute bid level statistics, we first average across all bidders within an auction and then present data across auctions for these averages. With an average order comprising 1.6 parcels per auction, most banks only submit one parcel. Row (xi) of the table, “Number of Parcels”, shows that banks rarely increase or split parcels. For the most part, each bank bids on a fixed amount of funds at a uniform interest rate, both of which vary by bank. On average, 72 percent of participating banks win an award, and their average award size is \$4 million.

Row (iv) shows that on average 77 percent of the bids are in-the-money (INM), in other words above the reserve rate at submission. Six percent of the bids are out-of-the-money (OUTM); nothing is gained from making such a bid aside from knowing that the bank must make a more attractive offer to win any award. The third category of bids, accounting for an average of 23 percent, are on-the-money (ONM), defined as the lowest interest rate amongst those bids that would win if every bank stopped bidding at that point in the auction. In row (xix) we can see that an average of 45 percent of first bids are winning bids, suggesting that bidders jump well above the reserve rate. Row (xviii) shows the final ONM rate is on average 20 percent above the initial reserve rate, indicating that some winning first bids are large jump-bids.

We now provide details on the size of jumps. The difference between INM and ONM bids is reflected in the normalized interest rate spread, defined as the difference between the highest winning interest rate and highest losing interest rate, all divided by the highest winning rate. The average normalized interest rate spread is 10 percent and its standard deviation is 13 percent. The bottom rows of Table 1 summarize these results in terms of dollar amounts. Another way of summarizing this is the money-left-on-the-table (MLT), computed as the difference in interest payments for a bank’s submission and the highest OUTM bid. Table 1 shows the average MLT is \$1,741 and that it has a standard deviation more than twice as large, dictated in part by observations in the upper tail.

In row (xx) we also observe that nine percent of auctions did not attract enough bidders to completely allocate funds. The participating bidders still received the allocations they bid on.

---

<sup>6</sup>For example, if there is \$700,000 to be allocated, but the lowest winning bid is for \$1 million at an interest rate of 10 percent, then the bank making that bid must issue a CD for \$700,000 at the 10 percent rate.

Table 1: Summary Statistics on Auctions

		Mean	Standard Deviation	Min	Max
(i)	Number of Banks per Auction	24.50	6.50	9	41
(ii)	Number of Bids Per Player	12.81	22.17	1	276
(iii)	Number of Bids Per Parcel	4.36	10.63	0	139
(iv)	Proportion of Bids In The Money (INM)	0.77	0.18	0.29	1.00
(v)	Proportion of Bids Out of The Money (OUTM)	0.06	0.18	0.00	1.00
(vi)	Proportion of Bids On The Money (ONM)	0.23	0.37	0.00	1.00
(vii)	Proportion of Last Bids OUTM at Submission	0.10	0.30	0.00	1.00
(viii)	Proportion of Winning Bids ONM	0.07	0.05	0.00	0.27
(ix)	Proportion of Bids with Quantity Changes	0.00	0.00	0.00	0.01
(x)	Size of Parcels (millions)	1.51	0.42	0.81	2.55
(xi)	Number of Parcels	1.60	1.03	1	5
(xii)	Proportion of Banks who win	0.72	0.22	0.30	1.00
(xiii)	Annual Reserve Coupon Rate	2.19	2.11	0.06	5.30
(xiv)	Total Award Amount (millions)	75.77	10.51	50.00	80.00
(xv)	Award Amount to Winning Bank (millions)	4.29	0.62	3.20	6.31
(xvi)	Money Left on the Table (MLT)\$	1741	3831	0	65380
(xvii)	MLT Interest Rate	0.10	0.12	0.00	0.68
(xviii)	Final ONM Rate	1.18	0.24	1.00	2.10
(xix)	Proportion of Winning Bids that were First Bids	0.45	0.20	0.09	0.97
(xx)	Proportion of Auctions Not Completely Filled	0.09			

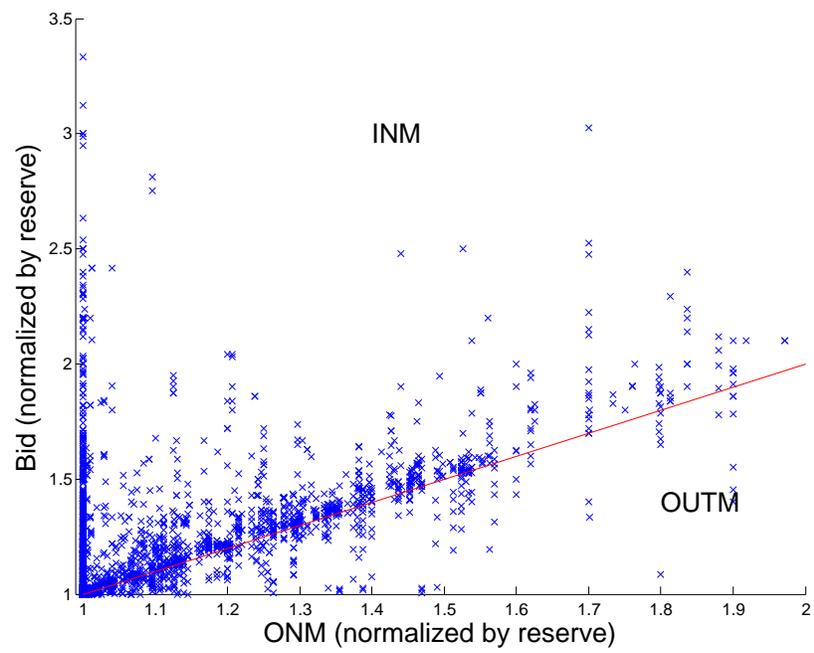
## 2.3 Bidding Patterns

In this section we describe the bidding patterns observed in the data, with a particular emphasis on understanding how bidders learn about the current ONM rate. In the data, we observe bidders re-entering the auction and “creeping” towards the ONM rate and either submitting an INM bid or remaining ONM. Occasionally bidders will stop short of the ONM rate, suggesting that the current reserve is above the bidder’s willingness to pay. As a result, we can completely observe when a bidder has reviewed their bid, except when a bidder remains INM.<sup>7</sup> Our identification results do not rely on this unique feature of our market. We will show that only initial bid data is needed to identify the primitives of interest. The extra data we have here will allow us to improve our estimates, but not substantially help identification.

Figures 1 through 3 amplify the importance of the three bidding categories by providing some simple dynamics. Figure 1 plots each bid against the current ONM (that is the lowest bid that would win an award if no more bidding took place). Thus, crosses on the 45 degree line represent ONM bids, the higher they are the further the auction progressed. Crosses to the right of the 45 degree line represent INM bids and crosses to the left represent OUTM bids. The dotted lines indicate iso-times at the minutes indicated. By inspection, Figure 1 shows that at the beginning of the auction (when ONM is at the origin), the dispersion of bids vertically arrayed is greater than at any other point during the auction. From the scatter of points arrayed in vertical lines, we can see from Figure 1 that there are many bids in all three categories throughout the auction.

<sup>7</sup>We occasionally observe bidders who are INM re-submitting the same bid. By re-submitting the same bid, bidders lose their time priority and can determine whether the ONM rate is exactly at their current bid. The number of bids we observe on average per auction that a replace INM bids is 0.49 with an average of 314 bids submitted in an auction.

Figure 1: Submitted bid against ONM bid



To help explain why we observe all three categories of bidding throughout the auction, we separated ONM bids from INM bids. Figure 2 shows that a preponderance of ONM bids were preceded by slightly lower OUTM bids (those that cannot win an award given the current state of the auction). In other words, if they had not already submitted an INM bid, banks ascertain the current ONM bid by creeping up to it with a sequence of OUTM bids.

The high number of ONM bids also suggest that bidders currently at the ONM rate cannot know whether they will win or not. In particular, bidders do not have any information whether their order will be executed first or another order has priority. Later in the model, we will provide a tie-breaking rule that reflects these strategic considerations and the institutional setting.

Taken together Figures 2 and 3 illustrate a pronounced tendency for bidders who ultimately place a bid that is ONM or INM to first determine where the boundary is by creeping to it with incremental bids, and only then deciding how much to jump into the money, if at all.

Figure 2: ONM versus Previously Submitted Bids

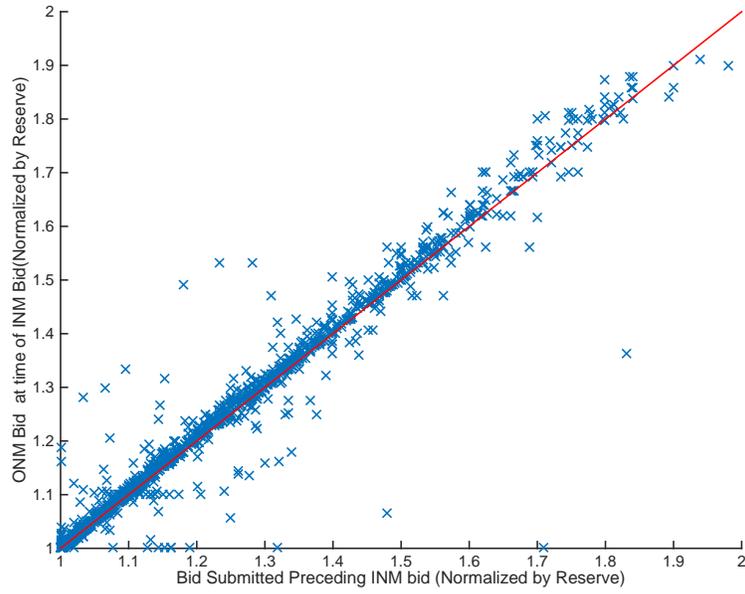
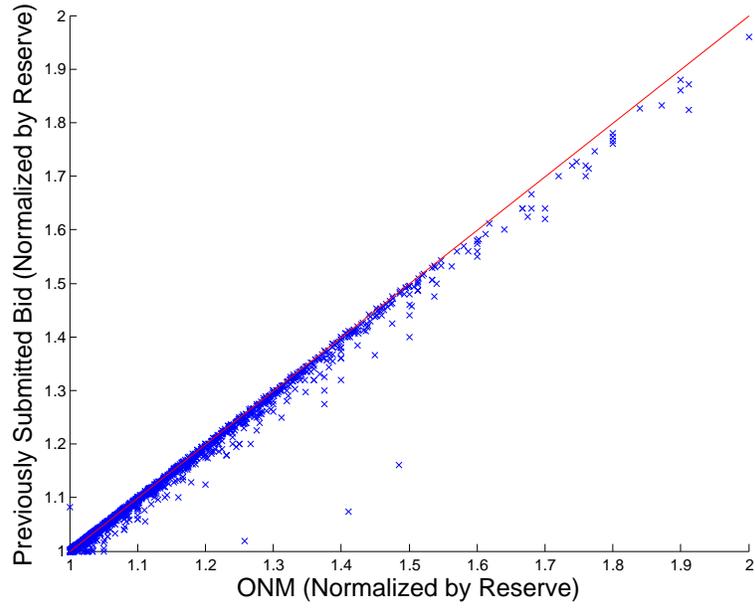
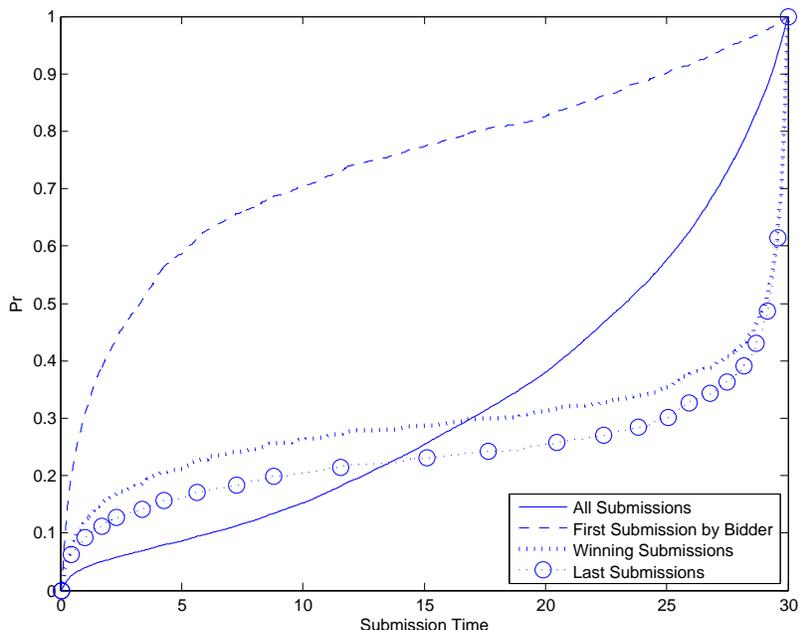


Figure 3: INM versus Previously Submitted Bids

## 2.4 Submission Times

Data on bidding submission times also provides useful information about what type of model might fit the data. In Figure 4 we provide the empirical distribution of submission times for all bids, all first bids, all last bids, and for all winning bids. (The 30-minute-auction-period is divided into 5 minute intervals in the figure.) Most first bids are made before the 10 minute mark, and the convex shape of the distribution of all bids betrays less activity in the middle of the auction than at either end. This concentration is accentuated in last bids: more than half of all final bids are submitted in the final two minutes of the auction, and almost 20 percent of them are made in the first two and half minutes.

Figure 4: Empirical Distribution of Order Submission Times

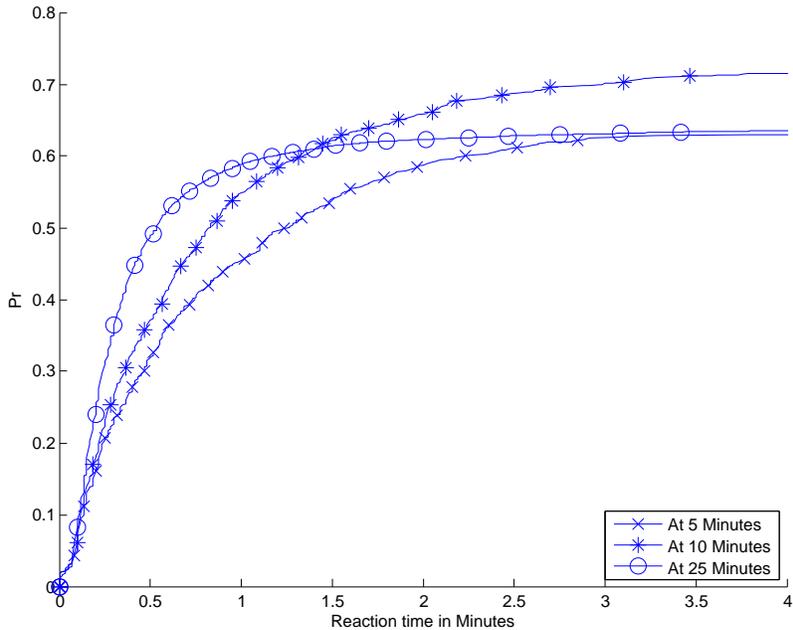


Sniping does not predominate bidding behavior of winning banks though. Figure 4 shows that most winning bids have already been submitted by the 25 minute mark, the winning bid distribution climbs to the 20th percentile within five minutes of the auction beginning but flattens between 10th and of 25th minute. The fact that a winning bid is frequently submitted in the early stages of the auction provides further evidence that some bidders do not win by incrementally increasing their bids as necessary but enter with a jump giving their rivals plenty of time to respond.

How quickly bidders respond to rivals by updating their previous bids reflects how intensively they monitor the auction. However, the average duration time between submissions by the same bidder is a biased estimator of the expected length of durations between successive monitoring opportunities, because bidders who are INM do not update their bids at every opportunity. But if a bidder was pushed out of the money, had limited bidding opportunities to revise its previous bid, and a sufficiently high valuation, a bank might increase their offer at the first available opportunity. why we display duration times between a bidder being pushed out of the money to when an updated bid restores him into the money relative to all bidders who fall out of the money.

Figure 5 shows three empirical distributions of reaction times. For every auction at 10, 15 and 25 minute marks into the bidding, we select all bidders who fall out of the money at some point within

Figure 5: Empirical Distribution of Reaction Times



the preceding five minute window. We then form percentiles measuring the proportion of those bidders who re-submit a new bid in the money within a given amount of time up to five minutes.

This measure of monitoring intensity does not account for bidders who withdraw from the auction because the lowest bids in the money have overtaken their gross value from winning since their previous bids were placed, a factor that shifts all three curves to the left. Nevertheless it is noteworthy that the distribution of reaction times at earlier points in the auction first order stochastically dominate the distribution of reaction times at later points in the auction, suggesting that bidders pay more attention to the auction in its closing minutes.

### 3 Basic Framework

We consider three auction mechanisms for selling up to  $Q$  units of a commodity or service with many common elements, two frictionless auctions that differ in the timing of how auction ends, and the amount of information each bidder has about the rival bids, and one with frictions. The second is a sequential auction in which bidders are asymmetrically informed about the final bids of the other bidders, while in the third auction we investigate, potential bidders only having limited sporadic opportunities to bid throughout the auction. The bidders in all three auction games are a set of potential bidders denoted  $\mathcal{I} \equiv \{1, \dots, I\}$ . Prior to the auction bidders observe their private values for one unit, bidder  $i \in \mathcal{I}$  drawing a private value  $v_i \in [r_0, \bar{v}]$ , independently and identically from commonly known distribution  $F_v(\cdot)$ . This valuation represents the return of a project the bank has access to, which requires cash that cannot be purchased from the retail CD market or alternative providers of liquidity.

A reserve price, denoted by  $r_0$  and set exogenously, is announced. The auction lasts one unit of continuous time  $t$ , during which bidders can submit a sequence of increasing bids. We denote a bid update by bidder  $i$  at  $t$  by  $b_{it}$ , and  $r_t$  the  $Q^{th}$  highest outstanding bid at time  $t$ . A discriminatory

pricing rule determines who wins the  $Q$  units and the price winners pay. Letting  $b_i$  denote the  $i^{th}$  bidder's final bid,  $i$  nets  $(v_i - b_i)$  if its bid exceeds the  $Q^{th}$  highest and nothing if less; precedence by time of final order submission breaks ties for the  $Q^{th}$  highest bid.

This process conforms to the institutional setting described in the previous section. Parcels in our model are one unit, yet in reality these are variable, and firms can bid on more than one. Most banks bid on just one parcel that does not vary in size.

In identifying the frictionless models, we only use data on last bids, including bids that were OUTM at time of submission. Throughout we focus on separating equilibria where bidders strategies are monotone in their private information. When we then move to our model with frictions, we avoid this restriction completely. The appendix details precisely what data we are using for each model.

### 3.1 Unlimited Opportunities to Review and Bid

Most empirical work on ascending auctions has made use of the Japanese button auction.<sup>8</sup> A multi-unit extension of this model would, in equilibrium, have bidders dropping out at their valuations until all units are allocated. The transaction price would then be equal to the valuation of the highest loser and all winners pay the same price. This is equivalent to assuming the game is played on the open interval  $t \in (0, 1)$  and bidders always have an opportunity to respond to increases in the reserve rate.

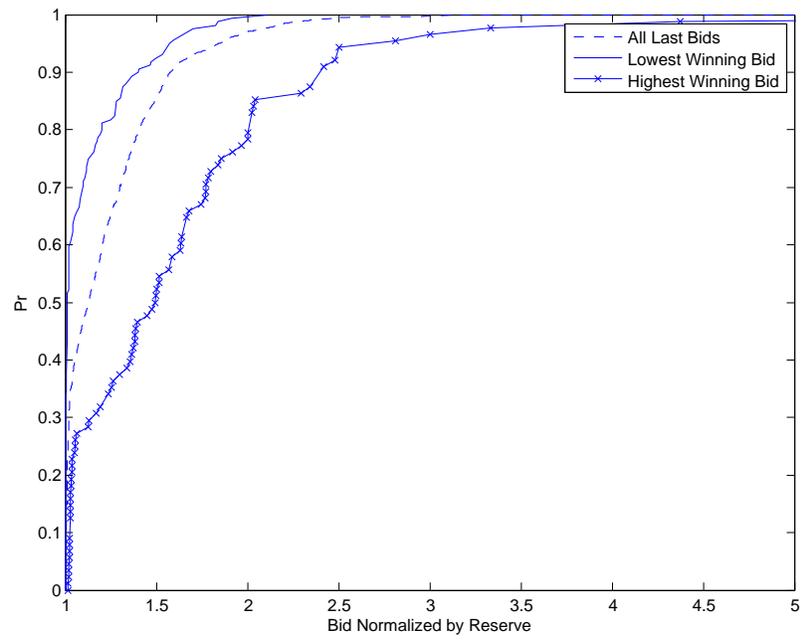
One notable alternative modeling approach is due to Haile and Tamer (2003) who bound valuations (and their distribution) on the basis of two behavioral premises. First, bidders never bid above their (private) valuations. Second, a bidder will submit another bid if its valuation exceeds the current high bid and it would lose the auction otherwise. By definition this timing rule precludes sniping. Haile and Tamer (2003) provide Monte Carlo evidence on the usefulness of their approach in obtaining bounds on valuations. All the models they consider represent departures from the standard button auction paradigm, because the empirical bid distribution does not necessarily correspond to the valuations of bidders.

We add a third behavioral assumption, that bidders do not use dominated strategies. Our agents are experienced: Figures 2 and 3 show they know how to "creep" up to the current minimal amount necessary to guarantee winning. This additional behavioral assumption restores the familiar prediction that in a private value auction, the highest valuation agent wins at the second highest valuation. Indeed the Monte Carlo results of Haile and Tamer (2003) exploit this property. Given all three assumptions, the highest and lowest losing bid distributions must coincide. However, as can be seen in Figure 6, bidders leave money on the table. Moreover, the distribution of lowest winning bids is first order stochastically dominated by the distribution of the highest winning bids and also the distribution of all last (winning and losing) bids. This constitutes evidence that neither the Japanese auction, nor the more flexible behavioral approach, obtained from imposing the three maintained behavioral rules, are applicable to this CD market.

---

<sup>8</sup>An action in this game is a dropout point at which a bidder becomes inactive. Bidders know their valuations privately and the activity status of bidders is publicly observed. A strategy maps from the number of active bidders and the valuation of the bidder to a dropout price. In the IPV paradigm, the dropout point is exactly equal to the valuation of the bidder.

Figure 6: Bid Distributions



### 3.2 Sealed Bid Auctions

The second mechanism we investigate extends the timing interval to  $[0, 1]$ , and thus facilitates “sniping”, bidders simultaneously submitting their final bid at  $t = 1$ . One equilibrium for this auction is for every bidder to snipe, so that the auction effectively degenerates to a multiunit discriminatory price sealed bid (DPSB) auctions, which generate differences in the amount paid by the winning bidders and hence differences between the distribution of high winning bids and low winning bids.

Perhaps the simplest DPSB auction that might be consistent with our institutional setting is when bidders do not take account of the ONM price when submitting their final bid at the end of the auction, and treat all previous bids as uninformative. Recall from Krishna (2009), for example, in Bayesian Nash Equilibrium bidder  $i$  chooses  $b$  to maximize  $H(b)(v_i - b)q$  where  $H(b) \equiv \Pr\{W_i = 1 | b\}$  denotes the probability a bidder wins with bid  $b$  when the other bidders make their equilibrium bid. Then  $b_i$ , the equilibrium bid of bidder  $i$ , solves a first order condition that upon rearrangement yields the equation  $v_i = b_i + H(b)/H'(b)$ . Guerre et al. (2000) prove each valuation  $v_i$  is identified from data on  $(b_i, W_i)$ ; intuitively, the regression function  $H(b)$  and its derivative can be non-parametrically estimated.<sup>9</sup>

The estimates obtained from our data suggest that, before the financial crisis markups in this market, computed as  $(v_i - b_i)/v_i$ , were very large, on average about 61 percent. Figure 8 shows that after the crisis the estimated markups are even larger, about 100 times the reserve rate. This seems implausible because in both regimes there are on average more than twenty bidders in each auction.<sup>10</sup> The estimated expected values from the winning valuation for the two regimes are unbelievably high, 32.3 times the initial reservation price in auctions preceding the financial crisis and 48.8 times the initial reservation price in auctions following the financial crisis. Since the alternative to bidding in this auction is the minimum of reducing lending for projects versus obtaining the finance from the open market, these values are simply not credible, and the fact that the value is higher in the post-crisis regime when liquidity dried offers scant reassurance.

---

<sup>9</sup>We apply a Nadaraya Watson estimator to these objects and evaluate the derivative directly as the derivative of the non-parametric estimator.

<sup>10</sup>We investigated whether a dynamic enforcement mechanism supporting an implicit agreement exists, in which bidders take turns at winning auctions. However, the omitted regression shows that the probability of winning an auction is positively related to winning a previous auction.

Figure 7: Sealed Bid Estimates pre-2008

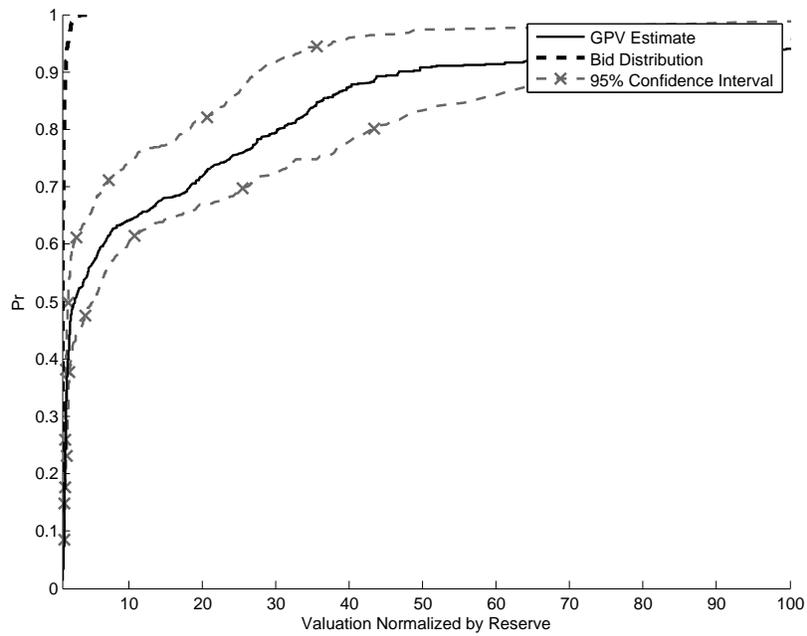
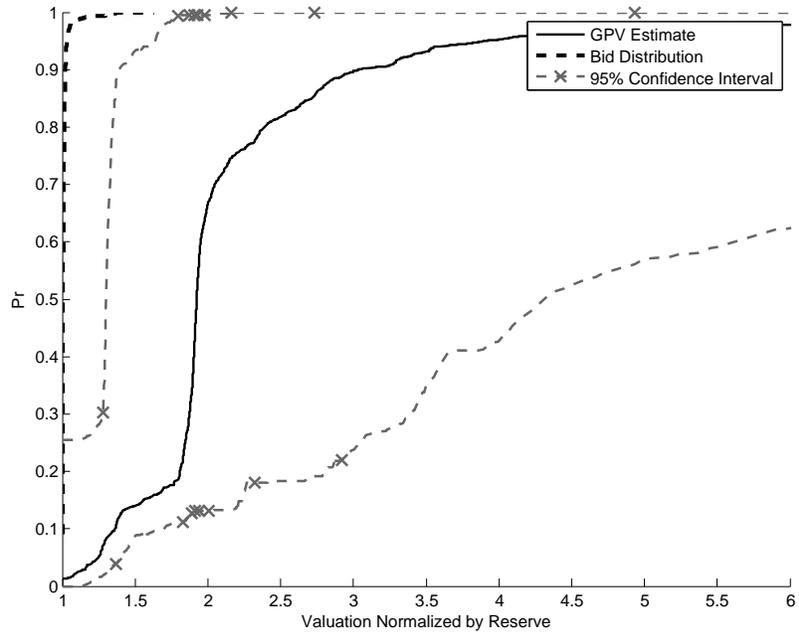


Figure 8: Sealed Bid Estimates post-2008

The equilibrium of a sealed bid first price auction exhibits monotonicity; bids increase with with valuations. This property is simply derived by differentiating the first order condition and exploiting the second order condition to obtain an expression for  $db_i/dv_i$ . Although several authors have investigated whether the markup term  $H(b)/H'(b)$ , no one to our knowledge has provided a statistic to formally test monotonicity. A variety of approaches are possible. We investigate a sample analogue to:

$$Q \equiv E \left[ \max \left\{ 0, + (H'(b_j) W_i - H'(b_i) W_j) (b_j - b_i) - H'(b_i) H'(b_j) (b_i - b_j)^2 \right\} \right] \quad (1)$$

Under the null hypothesis  $Q = 0$  because  $(v_i - v'_i) (b - b') > 0$  for all  $v_i \neq v'_i$ , and positive values of  $Q$  are attained if and only if  $(v_i - v'_i) (b - b') < 0$  with strictly positive probability.<sup>11</sup> The value of the test statistic in the pre-crisis regime is 170, the 5 percent critical value is 85 and the 1 percent critical value is 328.<sup>12</sup> In the post-crisis regime the value of the test statistic is 1833, the 5 percent critical value is 912 and the 1 percent critical value is 7619. Thus the null hypothesis of monotonicity is rejected at the 5 percent level in both the pre and post crisis regimes, but not at the 1 percent level. The rejection constitutes evidence against the discriminatory sealed bid auction.

### 3.3 Sequential Final Bids

Figures 9 and 10 below show that the empirical distribution of winning bids made in the first ten minutes of the auction first order stochastically dominate winning bids made in the last five. More generally winning bids decline as the end of the auction approaches, a finding at odds with the predictions of a sealed bid auction format, and its sniping analogue. This cannot occur in the equilibrium of a sealed bid auction where bidders have no information about the bids of rivals.

---

<sup>11</sup>This test statistic can be shown to be  $\sqrt{N}$ -consistent. See Serfling (2009).

<sup>12</sup>Critical values were estimated using a subsampling algorithm, as described in Politis, Romano, and Wolf (1999).

Figure 9: Empirical Distribution of Winning Bids Normalized by Terminal Reserve Rate by Submission Time, pre-2008

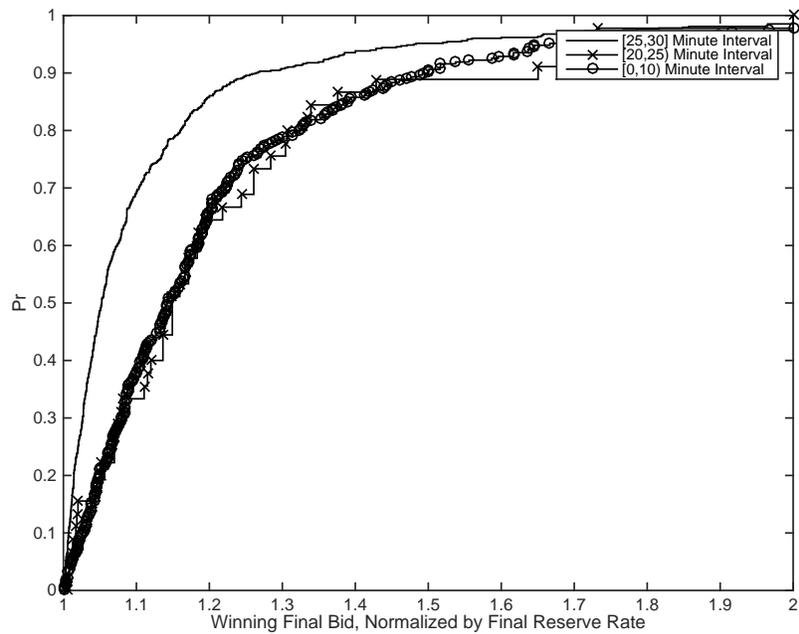
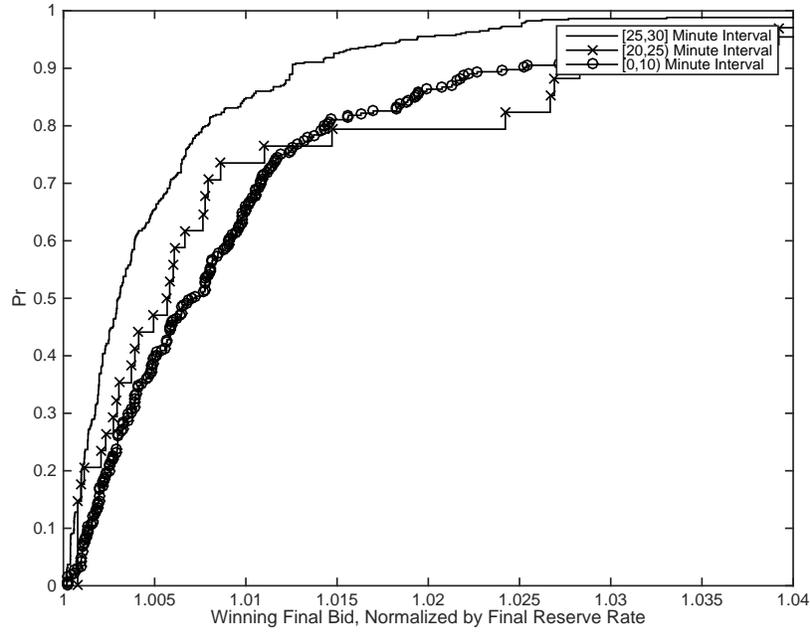


Figure 10: Empirical Distribution of Winning Bids Normalized by Terminal Reserve Rate by Submission Time, post-2008

In the spirit of Daniel and Hirshleifer (1998) we investigated a variation on the second model, where each bidder  $i \in I$  has one opportunity to bid at a predetermined random time during the auction, denoted by  $t_i < 1$ , at which point it can see the reservation price by (instantaneously) creeping up to it, as described in the previous section. This variation captures in a simple way the intuitively appealing idea that other commitments and demands on their time might prevent some bidders from bidding at the last moment the auction is open, and bid earlier instead, and that bidders who bid near the end of the auction may enjoy a strategic advantage.

This variation is a hybrid incorporating elements of ascending and DPSB auctions, which we refer to as a Sequential Discriminatory Price (SDP) auction. As in a DPSB auction, the deadlines lead bidders to solve a optimization problem that trades off probability of winning with the price a winner pays. Similar to an ascending auction, fewer bidders are willing to bid as the end of the auction approaches because the reservation price rises during the auction, but later bids are more informed about what the winners will pay. We assume the data is generated by a separating Perfect Bayesian Equilibrium equilibrium (PBE) in which each bidder  $i$  chooses  $b$  to maximize  $(v_i - b) H(b, t_i, r_i)$  where  $H(b, t_i, r_i) \equiv E[W_i | b, t_i, r_i]$  and as before  $W_i \in \{0, 1\}$  is an indicator variable signifying whether the bidder wins a unit in the auction or not.<sup>13</sup> For this auction mechanism, if  $v_i \leq r_i$  then  $b_i = v_i$  (from creeping); otherwise  $v_i > r_i$  and bidder  $i$  solves the first order condition for the problem, and we obtain an analogous solution to the DPSB auction, namely  $b_i = v_i - H(b|h)/H'(b|h)$ . Totally differentiating the first order condition with respect to  $b$  and  $t_i$  yields, upon rearrangement:

$$\frac{db}{dt} = \frac{(v_i - b) H_{bt}(b, t_i, r_i) - H_t(b, t_i, r_i)}{2H_b(b, t_i, r_i) - (v_i - b) H_{bb}(b, t_i, r_i)}$$

The second order condition ensures the denominator is positive, and  $H_t(b, t_i, r_i) > 0$  because the probability of new bids declines as the end of the auction approaches. The cross-partial is harder to sign: intuition suggests that the rate at which the probability of winning increases with higher bids declines with the number of rival bids, and hence  $H_{bt}(b, t_i, r_i) < 0$ . Formally, since  $r_i < b_i < v_i$ , for any value of  $H_{bt}(b, t_i, r_i)$ , the optimal bid strictly of bidders whose valuation is sufficiently close to  $r_i$  declines with time remaining in the auction. Consequently bidding is dampened, as Figures 3 and 4 show, and the highest valuation bidders do not necessarily win the auction.

A potential problem for estimating ascending auctions arises from bidders with valuations below the reservation price dropping out of the auction without bidding. In that case, only the selected sample of bidders who raise the reservation price record their bids in the data, which yields an upward bias in the probability distribution of the estimated valuations. This problem is averted in our particular application, because the only way bidders can ascertain whether their valuations lie above the reservation price or not is by increasing its bid to the minimum of its own valuation and the reservation price itself. It is worth mentioning that our model with frictions can deal with this selection problem without having this extra information. Thus the only essential difference in estimation between the DPSB auction and this one stems from the fact that in this case the first order condition applied when the final bid is in the money conditions on the information set of each bidder. Figures 9 and 10 show the estimated valuations are somewhat comparable to the corresponding values for the SDP estimates displayed in Figures 7 and 8. The estimated expected values from the winning valuation for the two regimes are also implausibly high, 6.9 times the initial reservation price in auctions preceding the financial crisis (lower than the estimated value obtained

---

<sup>13</sup>More precisely we assume that at least one PBE exists for this dynamic auction mechanism, and that all the auctions in the sample generate the same perfect equilibrium. The second assumption is often made in the literature on the structural estimation in dynamic games. Proving the existence of PBE in auctions with continuous bidding is complicated by the discontinuity in payoffs that arise at ties. Athey (2001), McAdams and Reny (2011) have derived sufficient conditions for the existence of a pure strategy Bayesian Nash Equilibrium in static auction games, where bidders do not have the opportunity to react to the bids of other bidders.

for a DPSB auction) and 114.4 times the initial reservation price in auctions following the financial crisis (higher than the DPSB estimates). We reject the null hypothesis of monotonicity for this auction too: the value of the test statistic in the post-crisis regime is 17,459, the 5 percent critical value is 1,962 and the 1 percent critical value is 21,052.

Figure 11: Sequential Estimates pre-2008

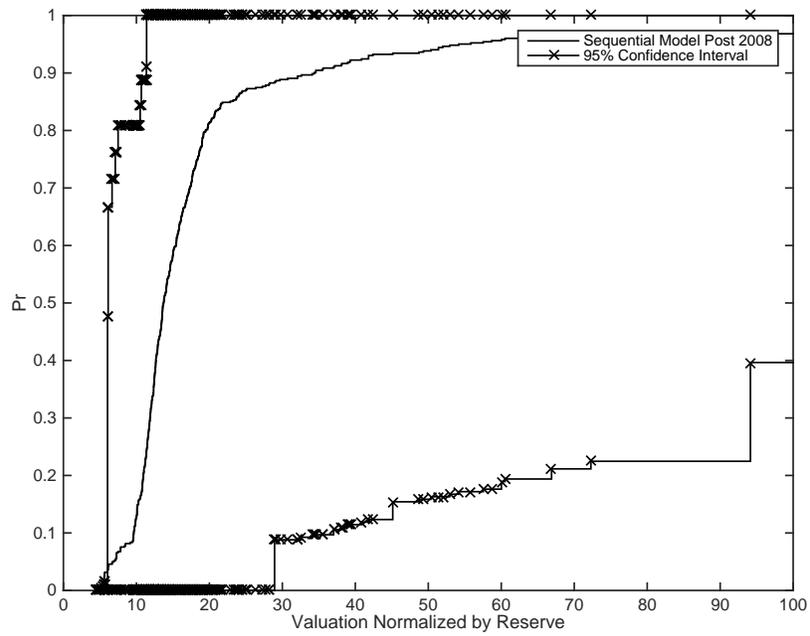
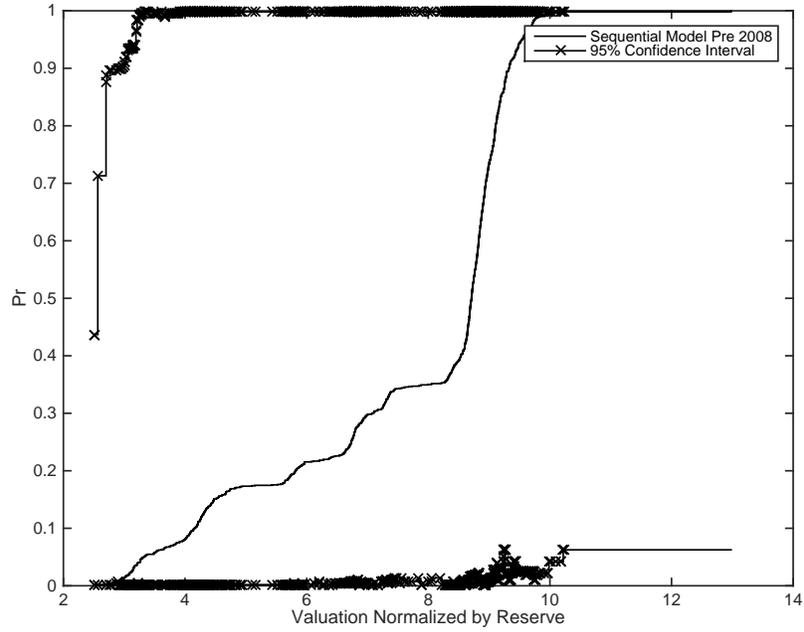


Figure 12: Sequential Estimates post-2008

## 4 Bidding Frictions

To resolve the anomalies reported in the previous section, we propose a model with bidding frictions. We provide further details in the appendix and verbally describe our model here. Bidders receive bidding opportunities at unknown random times which could occur after the auction concludes.<sup>14</sup> The technology determining the timing of opportunities is set exogenously and remains fixed across auctions and is therefore independent of bidders' private information.<sup>15</sup> The justification for this assumption is based on the idea that the technology for monitoring the progress of the auction is ultimately determined by long-term staffing arrangements and more generally competing demands for the banker's time. The process can be interpreted in two main ways: first as an implicit collusive agreement limiting the extent of bidding and deterring sniping in an attempt to lower final prices; second, as an opportunity cost of time to bidders, who could be otherwise engaged (for example, monitoring and trading in other markets, or retailing to clients). New opportunities only arrive if a previous bid has been pushed OUTM.<sup>16</sup> At each bidding opportunity the bidder learns the current reserve rate and knows the history of monitoring times, associated reserve rates as well as their own history of bids.

This leads to the following facts about behavior in equilibrium. Bidders who receive an opportunity to bid and whose valuation is below the current reservation rate or bidders with no bidding opportunity will remain inactive. In addition, bidders have an incentive to always submit a serious bid if the current reserve rate is below the bidder's valuation, since there is a probability this is the last bidding opportunity. Introducing bidding frictions in this way obliterates the sharp distinction between assuming the auction is on the open time interval versus a closed interval mentioned in the previous section. In this way our model retains some features of both frictionless models, such as bidders making multiple bids, but winners paying different prices. Our model can be interpreted as a simple extension of the sealed bid and sequential discriminatory auctions. As opportunities to bid throughout the entire auction increase without bound, the auction mechanism converges to one of the two frictionless models, the ending time of the auction determining which one. But like the sequential auction we have just estimated, the auction with bidding frictions lacks two important properties that both frictionless models share, efficiency and revenue equivalence, obvious from the fact that the bidders having the highest valuations in a model with frictions might never have the chance to bid or an opportunity to respond to a new higher reserve rate.

Identification of this model proceeds in two stages.<sup>17</sup> Since the hazard rate of bid opportunities is assumed to be independent of valuations, we can identify and estimate the hazard and its associated survivor function by focusing on the time it takes bidders to upgrade their bids amongst the set of those who (have sufficiently high valuations to) do so. Having identified (and estimated) the hazard rate of bidding opportunities, we then identify the probability distribution of valuations from the time it takes to make a first bid and the associated reserve rate. We can then compute the likelihood of observing entry by a bidder at a specific time and use this as the basis for estimation.

The ascending model we consider here is strategically complex and potentially involves multiple

---

<sup>14</sup>Song (2004) is a working paper that considers a model where bidders draw their monitoring opportunities at the beginning of the auction, independently of their valuation. The key difference to our model, is that bidders know their monitoring times before the auction begins. In our setting, bidders do not know their bidding times, and therefore have incentive to jump bid.

<sup>15</sup>There is potentially scope to allow for mixtures of monitoring functions. For example, leveraging results from Hsiu-Li, Lindsay, and Lynch (1992).

<sup>16</sup>We observe on average that 0.5 bids out of 315 bids per auction are from previously INM submissions

<sup>17</sup>The potential number of bidders is identified from the cardinality of the set of all bidders observed separately in the pre and post-2008 period. Using the FDIC's failed bank list (<https://www.fdic.gov/bank/individual/failed/banklist.html> (accessed June 22 2015)), we have determined that only one bank in our data, Washington Mutual, failed.

equilibria and pooling strategies. Like the closely related sequential auction, no one (to our knowledge) has provided a general set of sufficient conditions proving the existence of a PBE in this class of dynamic auction models, where valuations and potential bids are defined on a closed interval of the real line.<sup>18</sup> To guarantee the existence of a PBE, either a separating or pooling equilibrium, we revert to assuming that the support for the probability distribution of valuations and bids is finite with increments given by  $\Delta$ .<sup>19</sup> In addition, the tie breaking rule gives priority to the most recent bid at a given price. It is worth emphasizing that this result does not guarantee uniqueness, symmetry or even pure strategies, or monotonicity in valuations at all reservation prices. In taking this approach we also sacrifice the possibility of using the first order condition as a vehicle for identifying or estimating the model. But rather than seeking to identify the individual valuations of each bidder, we pursue the more modest objective of identifying the distribution of valuations, which for policy and welfare purposes is typically just as useful. Imposing this structure on the model allows us to relax two other assumptions that are typically made in the estimation of structural models in industrial organization. In the identification and estimation of the bidding model with frictions, we do not assume the data is generated by a (separating) PBE, nor even that they are generated by the same equilibrium.

It is also worth mentioning that order statistics methods such as those used in Athey and Haile (2002) or Haile and Tamer (2003) cannot be applied in this environment. A requirement in the identification of most ascending auctions is that the transaction price be equal to or greater than the second highest value. It is then possible to use a result from order statistics to invert the parent valuation distribution out. In our setting, we cannot guarantee that the second highest valuation bidder had an opportunity to respond to the current ONM rate.

#### 4.1 Identifying and Estimating the Hazard Rate for Bidding Opportunities

Statistically, the survivor function that prevents bidders from bidding is the vehicle used to model bidding frictions. Formally, let  $\tau(b)$  denote the last time the ONM rate does not exceed  $b$ , a bid made some time before. That is  $\tau(b) \equiv \sup \{s : r_s \leq b\}$ . We define the survivor function as:

$$G(t|\tau(b)) \equiv \Pr \{b \text{ is most recent bid at } t > \tau(b) | v \geq r_t \geq b\}$$

It is well known that  $g(t) \equiv -G'(t|\tau(b))$  is the defective probability density function for bidding at points  $t$  in the time interval  $[\tau(b), \tau(v)]$ , and that  $-G'(t|\tau(b))/G(t|\tau(b))$  is the associated hazard. We now demonstrate that  $G(t|\tau(b))$  is identified.

Suppose bidder  $i$  is active at times  $\{i_1, \dots, i_\rho\}$  and that at time  $i'_s$  the bid of bidder  $i$  at  $i_s$  is supplanted by another bid. Thus bidder  $i$  regains the high bid position at time  $i_{s+1}$  if  $i < \rho$ . After its final bid at time  $i_\rho$  it loses the auction if and when another bidder subsequently places a higher bid at  $t_s > i_\rho$ . Denote by  $G_t(s)$  the probability of a bidder surviving time length  $s \leq 1 - t$  without the opportunity to bid when its previous bid was displaced by another bidder's higher bid at time  $t$ , and let  $g_t(s) \equiv \partial \log [G_t(s)] / \partial s$  denote the associated hazard rate. We consider the contribution of the  $i^{\text{th}}$  auction to the likelihood. For the first  $\rho_i - 1$  spells it is:

$$\prod_{s=0}^{\rho_i-1} G_{i_s}(i_{s+1} - i'_s) g_{i_s}(i_{s+1} - i'_s)$$

The estimators use sample analogues to sequentially estimate  $G_t(s)$ .<sup>20</sup> Figures 13 and 14 display

<sup>18</sup>Cho, Paarsch, and Rust (2014) solve ascending auctions by focusing on “approximate equilibria”. They provide characterization of PBE in a very simplified setting with uniformly distributed valuations.

<sup>19</sup>In our setting, we observe bidders submitting bids with up to three decimal points which suggests that the grid is very fine.

<sup>20</sup>We make use of standard Nadaraya-Watson type estimators. Bandwidths are chosen using a multivariate Silverman's rule.

the estimated hazard rate for bidding opportunities. Both before and after the break point in 2008, the estimated hazard for bidding opportunities over the five minute period immediately following the time at which a bid falls OUTM becomes steeper as the end of the auction approaches. The key difference between the pre and post-crisis periods is the hazard for the last five minutes of the auction. The post-2008 hazard is steeper and reaches a higher peak than the pre-2008 regime. Overall, the hazard rates do not significantly differ before and during the crisis. This provides some evidence that the constraints on monitoring are very difficult to weaken even in the face of a large negative macroeconomic shock and that these frictions are outside of the banks' control, at least in the short run. The estimates also provide support for our assumption that these frictions are exogenous to the valuations of bidders.

Figure 13: Hazard Rate pre-2008

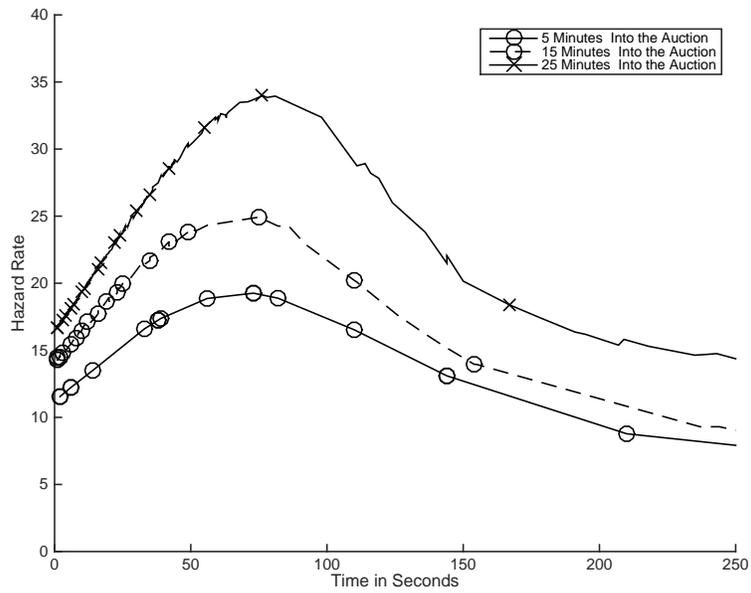
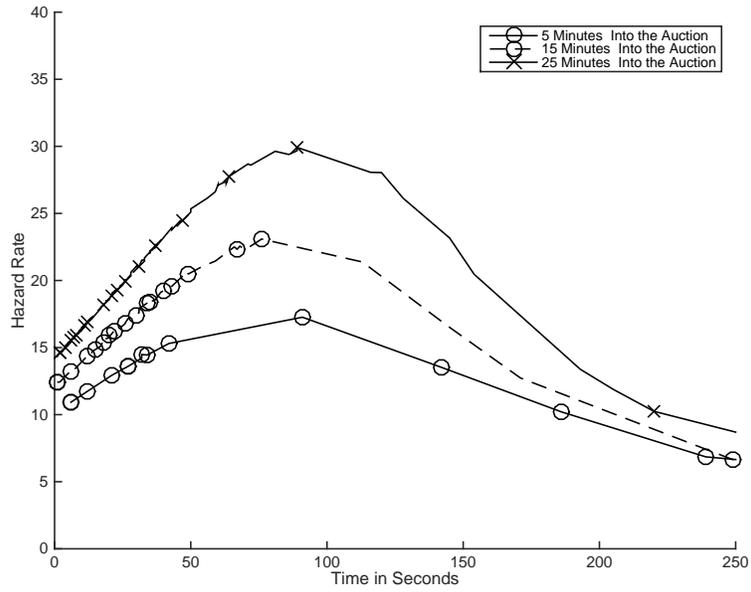


Figure 14: Hazard Rate post-2008

## 4.2 Identifying and Estimating the Distribution of Valuations

Given the survival function for bidding opportunities, the distribution function for the valuations is identified from the first bid of a bidder. Because all bids are affected by the current reservation price, each bid affects the future reservation price, and individual valuations help determine bids, a bidder's later bids are affected by its initial bid through these equilibrium considerations. Thus the later bids of bidders are used in the data only through their effect via the reservation price on the initial bids of bidders who only had their first opportunity to enter the bidding late in the auction. To model the joint distribution of the bidding sequence for any given bidder, the equilibrium selection rule must be derived.<sup>21</sup>

The distribution of bidder valuations is identified from all the spell length durations that precede the first bid of any given bidder. Key to the derivation is the fact that in equilibrium the bidding by bidder  $i$  cannot affect the bidding of the other bidders until after bidder  $i$  has made its first bid. Supposing there are  $\rho$  increments to the reservation price, the probability that bidder  $i$  does not bid during the auction is:

$$\sum_{s=0}^{\rho} G_0(t_{s+1}) [F_{r_{s+1}} - F_{r_s}]$$

where  $F_{r_s} = \Pr(v = v_s)$  and  $v_s = \arg \min_{v \in \mathbf{V}} |r_s - v|$  and where  $\mathbf{V}$  is the set of valuations. Similarly the likelihood that the  $i^{\text{th}}$  bidder has not bid before  $t_s$ , and is the first bidder to bid following  $t_s$  at time  $i_1$ , is:

$$G_0(i_1) g_0(i_1) [1 - F_{r_{i_1}}]$$

Let the indicator function  $1\{i_1 = 1\}$  denote whether or not  $i$  bids in the auction. The mixed density of the first passage time for bidder  $i$  to bid at  $i_1$  for the first time, or not at all (setting  $i_1 = 1$ ), is:

$$l(i_1 | \{t_s, r_s\}_{s=1}^{\rho_i}) = \begin{cases} G_0(i_1) g_0(i_1) [1 - F_{r_s}] & \text{for } 0 \leq t_s < i_1 < t_{s+1} \leq 1 \\ \sum_{s=0}^{\rho_i} G_0(t_{s+1}) [F_{r_{s+1}} - F_{r_s}] & \text{for } i_1 = 1 \end{cases}$$

We then maximize the likelihood using the discrete grid points  $F$  to identify the valuation distribution. Since the reduced form density of first passage times  $l(i_1 | \{t_s, r_s\}_{s=1}^{\rho_i})$  is identified and can be estimated non-parametrically for all  $i_1$ , identifying  $F_r$  reduces to proving that given  $G_0(s)$  by:

$$E \left[ \left( l(i_1 | \{t_s, r_s\}_{s=1}^{\rho_i}) - \sum_{s=0}^{\rho_i} \left( I\{t_s < i_1 < t_{s+1}\} G_0(i_1) g_0(i_1) [1 - F_{r_s}] + I\{i_1 = 1\} G_0(t_{s+1}) [F_{r_{s+1}} - F_{r_s}] \right) \right)^2 \right]$$

has a unique root at  $F_r$ . Details can be found in the appendix.

One of the problems encountered in equilibrium analysis of such auctions is that there are multiple equilibria. Typically researchers in games of incomplete information assume that all the markets exhibit the same equilibrium. Since our identification is based on a feature that is common to every equilibrium, that every bidders bids as soon as it can if its valuation exceeds the reservation price, we do not assume that all the auctions are playing the same equilibrium. Paradoxically, this speaks to an advantage of our identification and estimation procedure; multiple equilibria provide an additional source of variation in the conditioning set of existing bids for the population of first time bidders.

Our identification strategy also provides a bridge to the behavioral approach of Haile and Tamer (2003). Recall that the two behavioral assumptions of Haile and Tamer (2003) are that bidders

<sup>21</sup>See the recent work of Cho, Paarsch, and Rust (2014) on numerically solving for an ascending auction models for a given set of parameters.

not bid above their value, and if the current reserve rate is below their value they would submit a new slightly higher bid. The conditions used for identification from our equilibrium model coincide with the first behavioral assumption in Haile and Tamer (2003). The introduction of frictions and the focus on un-dominated strategies, prevents us from appealing to the second assumption. It therefore becomes necessary to provide a description of entry times, either using an equilibrium model or imposing behavioral rules. This together with the first assumption yields identification of the distribution of values. In other words, the equilibrium conditions used for identification can therefore be used as behavioral restrictions on bidders in more general environments.

A limitation of our approach is that the upper reaches of the distribution are rarely sampled by high valuation bidders bidding for the first time. Noting that there is strictly positive probability of not having the opportunity to bid until late in the auction, this empirical regularity does not affect the estimator's rate of convergence, but does affect both its estimated standard errors and its small sample properties. Our estimates of the distribution and their standard errors are shown in Figures 15 and 16.<sup>22</sup> Between one third and a half of the bidders have valuations that are barely greater than the initial reserve price, and in the pre-crisis regime only about one half have valuations more than five percent greater than the initial reserve price. In the post-crisis regime where liquidity is tighter, about half the bidders have valuations that are 50 percent higher than the initial reservation price. In the next section we show how we can take advantage of some features unique to the institution under investigation to improve our estimates.

---

<sup>22</sup>As mentioned before, we use subsampling algorithms developed by Politis et al. (1999), where the asymptotic validity of our approach is discussed.

Figure 15: Valuation Estimates pre-2008

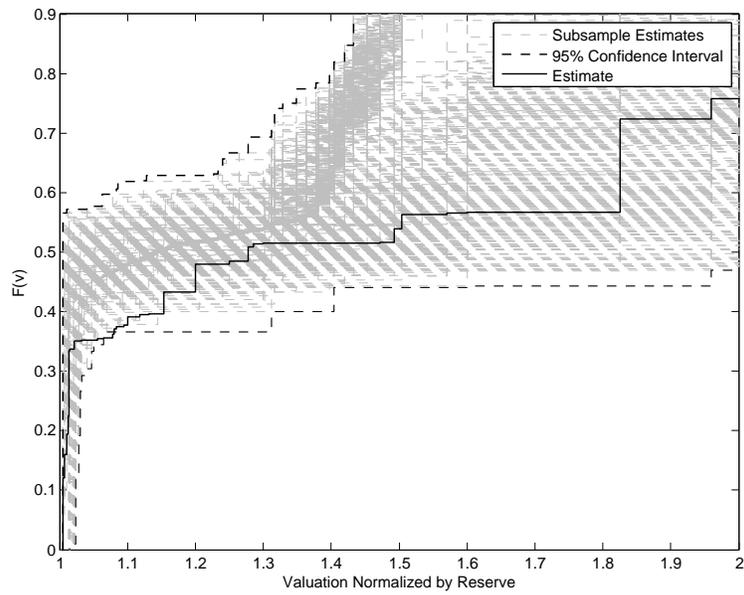
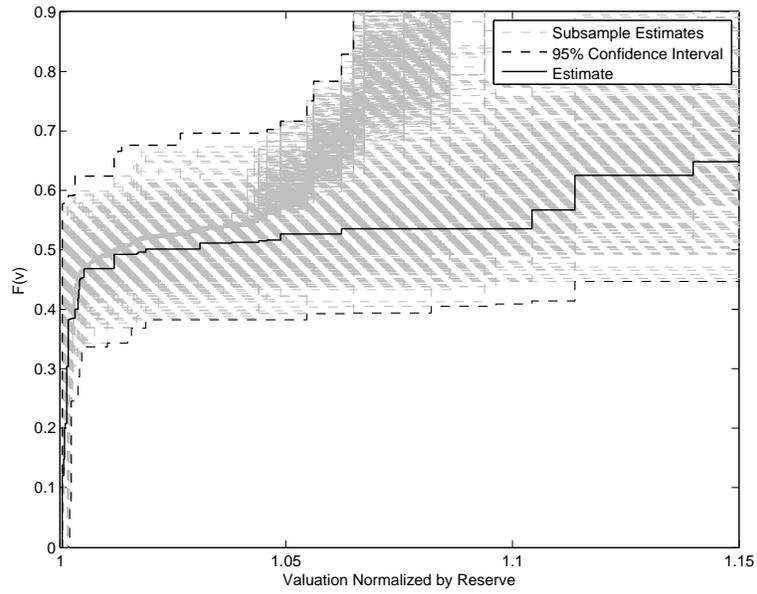


Figure 16: Valuation Estimates post-2008

### 4.3 Leveraging More Bid Information

A unique feature of this market, discussed in Section 2.3 of our paper, is that bidders cannot directly observe the reserve rate and must use a “creeping” strategy to learn it. Strategically, there is no difference between allowing bidders to know the reserve rate or using a rapid “creeping” strategy. This behavior does provide the econometrician with more information on all bidding opportunities. In particular, we can observe when bidders “pass” if the current reserve rate is above their valuation. In this case, bidders creep and drop out before hitting the reservation value. In particular, they must be stopping at close to their valuation. This re-introduces one of the features of the Japanese auction, namely the ability to equate OUTM drop-out points as observations of a bidder’s valuation.

In this setting, we are able to use the remaining bid information. We can then add to the likelihood the following terms. Each remaining bid  $k$  by bidder  $i$  that was crossed at time  $i_k$  and became active at time  $s'$  contributes to the likelihood:

$$G_{i'_k}(i_k) g_{i'_k}(i_k) [1 - F_{r_{s'}}] \tag{2}$$

This allows us to write down the likelihood,  $l(i_1, i_2, \dots, i_{\rho-1}, i_\rho | \{t_s, r_s\}_{s=1}^{\rho_i})$ . In addition, when we observe a bidder submit a final bid that is OUTM, the following replaces the final bid observation in the likelihood:<sup>23</sup>

$$G_{i'_{\rho-1}}(i_\rho) g_{i'_{\rho-1}}(i_\rho) [F_{b_\rho}] \mathbf{I}\{b_\rho < r_{i_\rho}\} \tag{3}$$

$b_\rho = v_i$ . Estimates and the corresponding confidence intervals are shown in Figures 17 and 18. Overall, the extra data yields more observations for higher reservation values. The distributions for the pre-2008 period we are able to leverage more information on valuations between 1.05 and the top-end valuations. The post-2008 valuation distribution has a similar shape to the first estimates. Again, we are able to “fill out” some of the gaps in the intermediate range of the valuation grid and the upper end as well.

---

<sup>23</sup>In implementing this estimator, we compress the creeping into an instantaneous process. As a result, we still only measure time between the time at which a previous INM bid was crossed and the next INM or OUTM final bid occurs.

Figure 17: Valuation Estimates Pre-2008

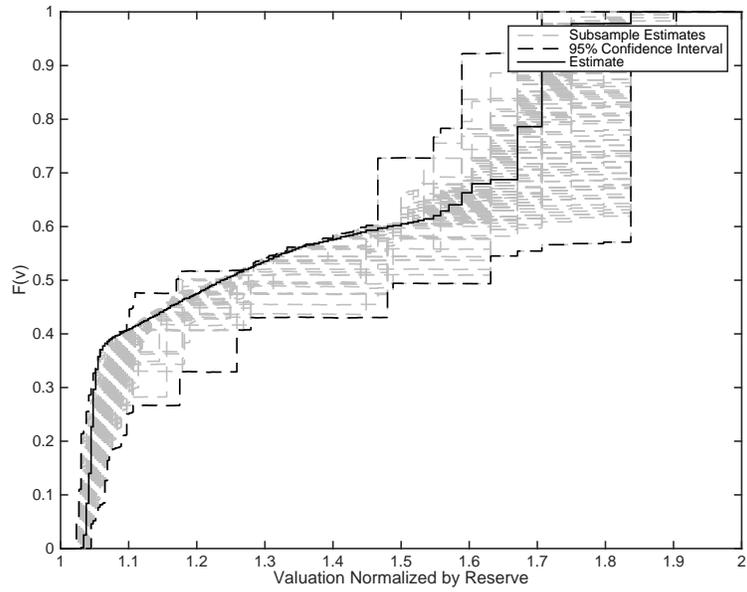
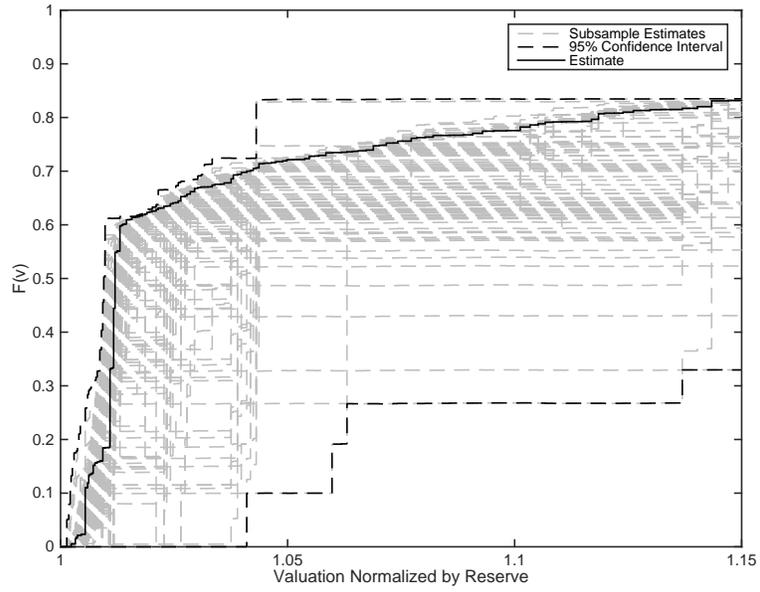


Figure 18: Valuation Estimates Post-2008

#### 4.4 What Are the Costs of Bidding Frictions?

There are two reasons bidding frictions can cause inefficiencies. The first is that some bidders never receive an opportunity to bid. The second is that high valuation bidders can be pushed OUTM if they do not receive another opportunity to respond to other bidders. Therefore, the final set of winners does not necessarily represent the highest valuation bidders. In this section we seek to determine the potential for misallocation.

In principle the set of Perfect Bayesian equilibria could be computed using the estimated structural parameters, but there are two practical objections. The first is computational: the number of equilibria is not readily determined and each equilibrium is a complicated non-stationary function in continuous time. Second the mixture of equilibria played throughout the population is unknown. Rather than pursue this route, we derived bounds on the expected value of the auction mechanism that apply regardless of which equilibrium is selected by the bidders. The bounds are based on two simple ideas: a bidder's valuation is higher than all its bids, and if its bid fell OUTM, it would raise its bid as soon as it had the opportunity if and only if its valuation was higher than the prevailing reservation bid.

Let  $W$  denote the event of placing a winning bid and  $\widetilde{W}$  its complement, losing. Appealing to the law of iterated expectations,  $E[v]$  can be expressed as a weighted sum of  $E[v|W]$  and  $E[v|\widetilde{W}]$ . Upon rearrangement we obtain:

$$E[v|W] = \left\{ E[v] - \Pr[\widetilde{W}] E[v|\widetilde{W}] \right\} / \Pr[W] \quad (4)$$

Note that  $E[v]$  is identified from  $F(v)$ . Similarly  $\Pr[W]$  is identified as the proportion of bidders who win, and  $\Pr[\widetilde{W}] \equiv 1 - \Pr[W]$ . We now prove the only remaining term  $E[v|\widetilde{W}]$  can be bounded. Denote by  $\{t_s\}_{s=1}^\rho$  the times at which the reservation price changes, and let  $t_\eta$  denote the time its final bid becomes stale, that is when the reservation price changes to  $r_\eta$ . Denoting by  $b_\eta$  its last bid, it follows that  $r_{\eta-1} < b_\eta < r_\eta$ . Since the bidder would bid at its first opportunity, after its bid falls OUTM if its valuation remains higher than the reservation price:

$$E[v|\widetilde{W}] < \left\{ \sum_{s=\eta}^\rho \frac{G_{t_\eta}(t_s) - G_{t_\eta}(t_{s+1})}{G_{t_\eta}(t_\rho)} \int_{r_\eta}^{r_s} \frac{vf(v)}{F(r_s) - F(r_\eta)} dv \right\}$$

Hence  $E[v|\widetilde{W}]$  is bounded from above. Similarly since its valuation is greater than  $b_\eta$ :

$$E[v|\widetilde{W}] > \left\{ \sum_{s=\eta}^\rho \frac{G_{t_\eta}(t_s) - G_{t_\eta}(t_{s+1})}{G_{t_\eta}(t_\rho)} \int_{b_\eta}^{r_s} \frac{vf(v)}{F(r_s) - F(b_\eta)} dv \right\}$$

Now the right side of the two inequalities above are identified because  $F(v)$  and  $G_t(t_{s+1})$  are identified. Substituting the identified expressions for  $E[v|\widetilde{W}]$  into (4) then yields identified upper and lower bounds for  $E[v|W]$ .

Rows (iii) and (iv) and rows (ix) and (x) in Table 2 compute lower and upper bounds of the expected valuation of winners, as a proportion of the initial reserve price, using initial bid data and the full bid paths, respectively. We compare them with several other metrics for valuing the auction. The bounds are quite tight across the two identification strategies. Using only initial data in auctions preceding the financial crisis, the expected gains lie between 1 and 2 percent.<sup>24</sup> In the

<sup>24</sup>Recall that the average final ONM rate was about 20 percent of the reserve. Our findings here are not at odds with this number, because some auctions clear at the reserve rate or only slightly above. The standard deviation for the final ONM rate is 0.24 and highlights the variability of the final ONM rate.

Table 2: Efficiency Measurements

		Pre 2008	Post 2008	
Initial Bid Data	(i)	Expected Valuation of Winner using Sealed Bid	32.3624	48.8139
	(ii)	Expected Valuation of Winner using Seq	6.9341	114.3642
	(iii)	Lower Bound on Expected Valuation of Winner in Data	1.0162	1.477
	(iv)	Upper Bound on Expected Valuation of Winner in Data	1.0165	1.5197
	(v)	Expected Valuation of Winner in Selected SP	1.103	1.9459
	(vi)	Expected Valuation of Winner in Unselected SP	1.13	2.0799
	(vii)	$\frac{\text{Revenue Selected SP}}{\text{Actual Revenue}}$	1.0245	1.2269
	(viii)	$\frac{\text{Revenue Unselected SP}}{\text{Actual Revenue}}$	1.0927	1.5497
All Bid Data	(ix)	Lower Bound on Expected Valuation of Winner in Data	1.062	1.5282
	(x)	Upper Bound on Expected Valuation of Winner in Data	1.0625	1.5721
	(xi)	Expected Valuation of Winner in Selected SP	1.2121	1.685
	(xii)	Expected Valuation of Winner in Unselected SP	1.288	1.7313
	(xiii)	$\frac{\text{Revenue Selected SP}}{\text{Actual Revenue}}$	1.0302	1.1623
	(xiv)	$\frac{\text{Revenue Unselected SP}}{\text{Actual Revenue}}$	1.2466	1.4148

months following the crisis the gains are 47 and 52 percent, providing evidence of a very illiquid market. Compared with the expected values of the winners as a proportion of the reserve price obtained for the other auction mechanisms displayed in first and second rows (32.3 and 48.8 for DPSB, 6.9 and 114.4 for SDP), the model with frictions yields quite plausible estimates, 1.01 and 1.4.

We find similar qualitative results for the estimates using more data. However, we can see that the estimated bounds in rows (ix) and (x) are above the initial bid data in rows (iii) and (iv), most likely due to sampling error. However, the gains we would estimate for this model are only moderately higher at about six percent. In the post-2008 period we again see large gains of the order of 52 to 57 percent. The bounds in the post regime are slightly closer to the estimates only using initial bids. This is due to the fact that a number auctions in the later post-crisis period were competitive and have more initial bid observations in the upper end of the valuation grid.

Using the estimated distribution we also calculated the potential gains from an auction where there are no bidding frictions, specifically a uniform price sealed bid (UPSB) in which winning bidders pay the bid of the highest losing bid (analogous to second price sealed bid in a multi unit auction where each bidder only demands one unit). The estimates in row (v) apply to the bidders who selected into the auction by bidding at least once. The estimates in (vi) include information for bidders who did not have the opportunity to place a bid because of bidding frictions. The most notable feature of these two rows is that excluding certain bidders from the auction reduces its overall value by about three to five percent. Comparing (iii) and (iv) with (v) and (vi), we note that since the UPSB auction allocates the units to the highest valuation bidders, the expected value in the auction exceeds gains obtained from the frictions model. In this application the bounds obtained from explicitly considering the equilibrium in the bidding frictions mechanism are less informative than the expectation of the highest valuations in the selected population of bidding bidders; however both sets of estimates are quite close.

Finally we compared the revenue that would have been collected with the UPSB. Row (viii) shows that an efficient mechanism unencumbered with bidding frictions would have generated 9 percent more gains than the bidding frictions mechanism in the pre-crisis regime and 55 percent post-crisis. Thus bidding frictions reduce revenue because some of the highest valuation bidders never have the opportunity to raise the reservation price. Using the alternative estimates we find that gains would have been slightly smaller with a pre-2008 gains being 3 percent and post-crisis about 16 percent. If we remove frictions revenue gains would be about 9 percent and 54 percent with only initial bid data and 24 to 41 percent using all bid data.

In a standard auction, expected revenue equals the expected value of the highest losing valuation amongst the bidding bidders, but in the auction with bidding frictions the prospect of being strategically disadvantaged by a latecomer to the auction leads to slightly higher bidding; each bidder must account for the possibility that any rival bidder, including one who has not bid yet, might bid after it has made its final bid. In this respect, the ascending auction mechanism allows the auctioneer to capture some of the surplus associated with the uncertainty about competition from bidders who may or may not enter the auction and bid.

To summarize, we see similar qualitative results estimates. However, our estimates using all bid data imply overall slightly smaller gains from switching to a sealed bid format. These results suggest that even with a weak equilibrium conditions and little data we are able to perform robust inference on revenues and efficiency.

## 5 Effects of the Credit Crisis

Together our estimates provide us with a window into the local market effects of the credit crisis in 2008. Our hazard rate estimates suggest that banks are unable to alter their operations even during crises, and that there is substantial rigidity in the organization of bank activities. Returns on all projects shifted down during the crisis. The valuation distribution estimates suggest that the variance of project returns is far higher post-2008, 90% of valuations lie below 1.15 of the reserve rate, compared to post-2008 where 90% lie below 1.8. Given that the hazard rates do not differ much between the pre and post-2008 period, this suggests that the variance can be completely explained by changes in the investment pool banks have access to.

This increase in the variance of valuations also means that the negative effects of frictions on allocations would be exacerbated. pre-2008 valuations varied across a narrower range. Missing out on the top end of the valuation distribution due to frictions is therefore not very costly. However, with the range of valuations moving from 1 to 1.15 the reserve rate to 1 to 2 times the reserve rate in the post-crisis period, the potential for losses is far greater. The results in Table 2 for the sealed bid auctions show that the gains from implementing these mechanisms are far greater in the post-crisis period relative to the existing mechanism. Specifically, the increase in revenues with sealed bid auctions in the post-crisis period is sometimes up to six times the pre-crisis period.

## 6 Conclusion

Motivated by patterns in the data on dynamic auctions, such as jump bids submitted in the early parts of the auction and a high intensity of activity at the beginning and at the end of the auction, we develop a dynamic model where frictions prevent agents from bidding whenever they want. Our model, which nests existing frictionless bidding models, is based on the idea of competing demands for the time of investors, and that smoothing the intensity of work effort is also costly. The existence of frictions can yield inefficient outcomes. That is, since bidders are not guaranteed a final opportunity to re-bid close to the end of the auction, the winners do not necessarily have the highest valuations. While straightforward to describe, the equilibrium analysis is complicated, because monotonicity restrictions exploited in standard auction models are not satisfied. There may be multiple equilibria and a pure strategy equilibrium might not exist.

Our identification strategy only exploits necessary equilibrium restrictions from our model that are sufficiently weak to accommodate the possibility that the data generating process produce outcomes from multiple equilibria, some or all of which may be mixed strategy equilibrium. We identify the distribution of valuations, the potential gains from efficiently allocating the auctioned items to the highest valuation demanders, and bounds on the costs of missed trading opportunities

due to friction in the existing auction mechanism. The conditions we exploit, which relate to the timing of bids conditional on the minimal threshold to win the auction, are similar but not identical to the two behavioral rules of Haile and Tamer (2003) devised for frictionless auction models. In these respects our analysis is also similar to Sutton (1991), who developed related ideas to provide robust cross-market predictions. Of course, investigators could take our equilibrium conditions and impose them as behavioral rules and bypass the construction of a complete model of behavior. In this sense our identification analysis is compatible with a behavioral approach that includes but is not limited to equilibrium, in order to accommodate complex environments.

Our data on auctions of certificates of deposit straddle the financial crisis of 2008. We show standard auction models imply unreasonably high mark-ups for this data. We derive monotonicity tests that relate valuations to final bids, and find our data reject the hypothesis that bidding strategies are monotone in valuations, a key implication of those models. In contrast, the estimated valuation distributions obtained from our model of bidding frictions are quite plausible. Finally our estimates provide useful insights into the effects of negative macroeconomic shocks to local financial markets. In particular, we show that the frictions do not change much before and after the crisis. This suggests that banks experience some inertia in the organization of their activities, even when faced with a large upheaval in the economy. As a result, the costs of frictions increase after the crisis. Their slow adaptation to this new environment, where investment returns are more widely dispersed, as evidenced by our value distribution estimates, contribute to the costs of such an upheaval through a greater misallocation away from funding to high value projects to lower value ventures.

# A Appendix

## A.1 Model with Frictions

All bidders are endowed with a set of hazard functions for all possible crossing times in the interval  $[0, T]$ ,  $\{g_\tau(x) : 0 \leq \tau \leq T\}$ .<sup>25</sup> These functions determine the arrival rate of bidding opportunities. The hazard function is only activated if a bid is crossed by the reserve rate. Bidders learn their valuations privately and receive bidding opportunities according to  $g_\tau(x)$ .

Let  $v \in \{r_0, r_0 + \Delta, r_0 + 2\Delta, \dots, r_0 + S\Delta\} \equiv \mathbf{V}$  denote the bidder's valuation from a grid of  $S$  valuations,  $W \in [0, 1]$  the fraction of the parcel the bidder wins at the end of the auction,  $s \in \{1, \dots, \rho\}$  the opportunities to bid, where  $\rho$  is its last chance (revealed only retrospectively to the bidder)  $\tau_s \in [0, 1]$  the random times at which those opportunities occur,  $b_s \in \{b_{s-1}, b_{s-1} + \Delta, \dots\} \equiv \mathbf{B}_{b_{s-1}}$  its bid at  $\tau_s$  if it chooses to bid and  $h_s \equiv \{v, \tau_s, r_s, h_s, h_s\}_{s'=1}^{s-1} \in \mathcal{H}_s$ , the history of previous opportunities reservation prices and past bids that constitutes its information set. Notice that we exclude the ‘‘creeping’’ behavior and simply assume that bidders can observe the reservation rate when they receive their bidding opportunities. A bidding strategy is a mapping:  $\beta_s : \mathbf{V} \times \mathcal{H}_s \rightarrow \mathbf{B}_{b_{s-1}}$  and a mixed strategy is a mapping  $\tilde{\beta}_s : \mathbf{V} \times \mathcal{H}_s \rightarrow \Delta \mathbf{B}_{b_{s-1}}$ . At its  $s^{\text{th}}$  opportunity to bid, the value function for the bidder is defined as:

$$V_s(h_s) = \max_{\{b_r\}_{r=s}^\rho} E[(v - b_\rho) W | h_s] \quad (5)$$

This holds if the current reserve rate is below the valuation. The value function  $V_s(h_s)$  has a recursive formulation. If  $W = 1$ , the bidder receives a payout of  $(v - b_\rho)$ , because  $b_s$  is a winning bid. We rewrite (5) recursively as:

$$V_s(h_s) = \max_b \{E_s[I\{b \geq r_T\}(v - b)W | b] + E_s[I\{s < \rho\}V_{s+1}(h_{s+1}) | b]\}$$

The decision to submit a serious bid or to pass boils down to the following comparison:

$$\max \{V_s(h_s), E_s[I\{s < \rho\}V_{s+1}(h'_{s+1}) | b_{s-1}]\}$$

where the second term in the max operator is the continuation value if the bidder does not increase its bid and receives another opportunity to bid. It is clear that  $V_s(h_s)$  is always larger than the continuation value since  $b_{s-1}$  is already OUTM. The only way a crossed bidder could have a positive probability surplus is by receiving another opportunity to bid and then submitting a competitive bid if the reserve is still below their valuation.

If the reservation price at  $\tau_s$  exceeds its valuation, the bidder effectively drops out of the auction. If not, the bidder increases its bid in return for the possibility of winning the auction. In this respect, bidding more closely resembles the Japanese auction rather than the English, since bidders with valuations higher than the ONM, when ONM is higher than their current bid are compelled to immediately update regardless of what other bidders do, as in an English auction where bidders can wait until a later point in the auction to reveal themselves.

In a separating equilibrium the optimal bid,  $b$  is characterized by the following condition

$$\begin{aligned} E_s[I\{b \geq r_T\}(v - b)W | b] + E_s[I\{s < \rho\}V_{s+1}(h_{s+1}) | b] \\ \geq E_s[I\{b' \geq r_T\}(v - b')W | b'] + E_s[I\{s < \rho\}V_{s+1}(h'_{s+1}) | b'] \end{aligned} \quad (6)$$

---

<sup>25</sup>In the main text we set  $T = 1$ .

For all  $b' \neq b$  which leads to the condition:

$$vE_s [[I\{b' \geq r_T\} - I\{b \geq r_T\}] W] + E_s [[I\{b' \geq r_T\} b' - I\{b \geq r_T\} b] W] \quad (7)$$

$$\geq$$

$$E_s [I\{s < \rho\} V_{s+1}(h_{s+1}) | b] - E_s [I\{s < \rho\} V_{s+1}(h'_{s+1}) | b']$$

The difference between a sealed bid auction equilibrium and this ascending auction is the appearance of an extra term, which is the effect of bidding on the timing and hence the value of the state variables for the continuation value  $V_{s+1}(h_{s+1})$ . In a discriminatory multiunit auction the expression on the righthand side is equal to zero. Two inequalities can be derived, one for increasing a bid from the equilibrium value and one for a downward deviation. Here increasing the bid raises the threshold for the OUTM rate, serving to postpone the next bidding opportunity, but encouraging a more aggressive response from rivals, thus accelerating the increasing reservation rate, which is captured in the change in the continuation value.

We can also see that if  $s = T - \epsilon$ , where  $\epsilon$  is small, then the probability of  $i$  being the last bidder is incredibly high, which leads to an incentive to bid at the reserve rate if  $v_i \geq r_s$ . In fact, all bidders with valuation above the reserve rate who are currently OUTM will want to bid at or close to the reserve rate. Therefore, we would expect some pooling around the reserve rate. This immediately suggests that a completely separating equilibrium need not exist and calls into question how much separation occurs at all in equilibrium during the auction.

In a mixed strategy equilibrium we would expect mixing to occur over intervals of bids defined by an upper bound equal to the valuation of the bidder and a lower bound determined by the current reserve rate. No bidder would mix over an interval that lies above its valuation.

In all of these equilibria we can be sure that a bidder will become active if the current reserve rate is below its valuation, which is the only information we leverage for identification.

## A.2 Identification and Estimation

Here we prove that the likelihood shown in the main text has a unique root at the true values for the valuation distribution. The quadratic can now be expressed as:

$$E \left[ \left( \begin{array}{c} l(n_1 | \{t_s, r_s\}_{s=1}^{\rho_n}) \\ - \sum_{s=0}^{\rho_n} \left( \begin{array}{c} I\{t_s < n_1 < t_{s+1}\} G_0(n_1) g_0(n_1) \left[ 1 - \sum_{i=1}^J \sum_{j=1}^i I\{r^{(i-1)} < r_s < r^{(i)}\} F_j \right] \\ + I\{n_1 = 1\} G_0(t_{s+1}) \sum_{i=1}^J \sum_{j=1}^i \left[ I\{r^{(i-1)} < r_{s+1} < r^{(i)}\} - I\{r^{(i-1)} < r_s < r^{(i)}\} \right] F_j \end{array} \right) \end{array} \right) \right]^2$$

or:

$$E \left[ \left( \begin{array}{c} l(n_1 | \{t_s, r_s\}_{s=1}^{\rho_n}) - \sum_{s=0}^{\rho_n} I\{t_s < n_1 < t_{s+1}\} G_0(n_1) g_0(n_1) \\ - \sum_{j=1}^J \sum_{i=j}^J \sum_{s=0}^{\rho_n} \left( \begin{array}{c} I\{t_s < n_1 < t_{s+1}\} G_0(n_1) g_0(n_1) I\{r^{(i-1)} < r_s < r^{(i)}\} \\ - I\{n_1 = 1\} G_0(t_{s+1}) \left[ I\{r^{(i-1)} < r_{s+1} < r^{(i)}\} - I\{r^{(i-1)} < r_s < r^{(i)}\} \right] \end{array} \right) F_j \end{array} \right) \right]^2$$

Defining:

$$l_n \equiv l(n_1 | \{t_s, r_s\}_{s=1}^{\rho_n}) - \sum_{s=0}^{\rho_n} I\{t_s < n_1 < t_{s+1}\} G_0(n_1) g_0(n_1)$$

$$h_n^{(j)} \equiv \sum_{i=j}^J \sum_{s=0}^{\rho_n} \left( \begin{array}{c} I\{t_s < n_1 < t_{s+1}\} G_0(n_1) g_0(n_1) I\{r^{(i-1)} < r_s < r^{(i)}\} \\ - I\{n_1 = 1\} G_0(t_{s+1}) \left[ I\{r^{(i-1)} < r_{s+1} < r^{(i)}\} - I\{r^{(i-1)} < r_s < r^{(i)}\} \right] \end{array} \right)$$

the quadratic simplifies to:

$$E \left[ \left( l_n - \sum_{j=1}^J h_n^{(j)} F_j \right)^2 \right]$$

The first order condition with respect to  $F_k$  is:

$$0 = E \left[ \left( l_n - \sum_{j=1}^J h_{nj} F_j \right) (h_{nk} - h_{nJ}) \right]$$

More directly, the structural log likelihood is thus:

$$\sum_{n=1}^N \log \sum_{j=1}^J h_n^{(j)} F_j$$

Maximizing with respect to the parameters  $\{F_j\}_{j=1}^{J-1}$  the first order condition for  $k$  is:

$$\sum_{n=1}^N \frac{h_n^{(k)} - h_n^{(J)}}{\sum_{j=0}^J h_n^{(j)} F_j} = 0$$

The objective function is concave if the negative of each additive component is convex in  $(F_1 \dots F_{J-1})$ . Differentiating twice, the cross partial is:

$$H_n^{(k,k')} \equiv \frac{- \left( h_n^{(J)} - h_n^{(k)} \right) \left( h_n^{(J)} - h_n^{(k')} \right)}{\left( \sum_{j=1}^J h_n^{(j)} F_j \right)^2}$$

The quadratic form of the square matrix formed from  $H_n^{(k,k')}$  is:

$$\frac{- \sum_{k=1}^J \sum_{k'=1}^J \Delta_k \left( h_n^{(J)} - h_n^{(k)} \right) \left( h_n^{(J)} - h_n^{(k')} \right) \Delta_{k'}}{\left( \sum_{j=1}^J h_n^{(j)} F_j \right)^2} = - \left[ \frac{\sum_{k=1}^J \Delta_k \left( h_n^{(J)} - h_n^{(k)} \right)}{\sum_{j=1}^J h_n^{(j)} F_j} \right]^2$$

The objective function is therefore concave because the quadratic form is negative.

### A.3 Subsampling Algorithm

In order to estimate confidence intervals and estimate the distribution of test statistic distributions, we follow the subsampling approach of Politis, Romano, and Wolf (1999).

We first describe how we approximate the sampling distribution of the estimator. A standard subsampling algorithm uses subsamples of size  $b \ll A$  to re-estimate the parameters of interest, where  $A$  is the number of auctions. Denote our parameter estimate by  $\hat{\theta}_A$  and the parameter estimate from subsample  $a$  of size  $b$  by  $\hat{\theta}_{A,b,a}$ . If we create  $q$  subsamples and re-estimate the parameter on these subsets we can then use the following as an approximation for the sampling distribution:

$$L_{A,b,\|\cdot\|}(x|\tau_b) \equiv q^{-1} \sum_{a=1}^q \mathbf{1} \left\{ \tau_b \|\hat{\theta}_{n,b,a} - \hat{\theta}_n\| \leq x \right\} \quad (8)$$

where  $\tau_b$  is the convergence rate of the estimator and  $\|\cdot\|$  is the norm of the normed linear space  $\Theta$ . Theorem 2.5.2 in Politis, Romano, and Wolf (1999) establishes the asymptotic validity of this approach.  $q$  represents the number of all possible combinations of the data, i.e.  $\binom{A}{b}$ . This can be

an incredibly large number. Politis et al. (1999) also prove that a stochastic approximation leads to consistent estimates by randomly selecting observations with or without replacement from the set of auctions  $\{1, \dots, A\}$  which substantially reduces the computational burden. We run 500 subsamples to estimate confidence intervals. This result is proved in Corollary 2.4.1 for a univariate parameter value but holds equally for a general parameter space.

#### A.4 Estimation Data

We now describe which data is used for the three different models. Our data set consists of  $A$  auctions with observations on bids, bid times, bid quantities for all participating bidders. For each bidder we observe the maximum number of bids submitted in the auction. We denote the maximum number of bids for bidder  $n$  in auction  $l$  by  $\rho_n^l$ . The set of active bidders in the auction is denoted by  $\mathcal{N}^l \subseteq \mathcal{N}$ . The cardinality of  $\mathcal{N}$  is estimated by taking the set of all bidders observed across  $A$  auctions. Our complete dataset is then

$$\text{data} = \left\{ (\mathbf{b}_n^l, \boldsymbol{\tau}_n^l, q_n^l)_{n \in \mathcal{N}^l} : l = 1, \dots, A \right\}$$

where  $\boldsymbol{\tau}_n = (\tau_{n,s})_{s=1}^{\rho_n}$  is the sequence of bidding times,  $\mathbf{b}_n = (b_{n,\tau_{n,s}})_{s=1}^{\rho_n}$ , is the sequence of bids. This data along with the allocation rule allow us to reconstruct the path of play each bidder observed during the auction. Specifically, we are able to construct the reserve rate path,  $r_t$  for each auction at each instant of time.

*Sealed Bid and Bound Estimates:* In estimating the “sealed bid” (SB) models we select only the last bids observed by each bidder. The data is given by:

$$\text{data}_{SB} = \left\{ (b_{n,\rho_n}^l, q_n^l)_{n \in \mathcal{N}^l} : l = 1, \dots, A \right\}$$

This is also the same data used in our discussion of Haile and Tamer (2003). When estimating the sequential sealed bid (SSB) model we include information on the reserve rate at the time of submission:

$$\text{data}_{SSB} = \left\{ (b_{n,\rho_n}^l, q_n^l, r_{\rho_n}^l)_{n \in \mathcal{N}^l} : l = 1, \dots, A \right\}$$

*Bidding Frictions:* In our model with bidding frictions, we only use entry times and reserve rates. Therefore the data is now:

$$\text{data}_{BF} = \left\{ (\tau_{n_1}^l, q_n^l, r_{\tau_{n_1}^l}^l)_{n \in \mathcal{N}^l} : l = 1, \dots, A \right\}$$

#### A.5 Post Estimation Measurements

Below is a description of how we simulate the alternative auction mechanisms. The results of these simulations are used in the measurements presented in Table 2.

1. Draw valuations from the estimated distribution.
2. Estimate the empirical distribution of parcel sizes and draw parcels for all potential bidders.
3. Use the hazard rates for the initial bid time to determine which bidders enter. If the initial bid time is in the interval  $[0, 1]$  the bidders is categorized as active. Otherwise they remain inactive.
4. A uniform price rule is used to allocate funds amongst the set of observed entrants.

5. The second simulations allocate funds for the entire set of potential bidders, essentially a mechanism where we assume away frictions.
6. We repeat this process 1000 times.

We then average across our simulations and then average across the auctions to provide the measurements in Table 2.

## References

- D. Akerberg, K. Hirano, and Q. Shahriar. Identification of time and risk preferences in buy price auctions. 2011.
- S. Athey. Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica*, 69(4):861–889, 2001.
- S. Athey and P. A. Haile. Identification of standard auction models. *Econometrica*, 70(6):2107–2140, 2002.
- C. Avery. Strategic jump bidding in english auctions. *Review of Economic Studies*, 1998.
- O. Birulin and S. Izmalkov. On efficiency of the english auction. *Journal of Economic Theory*, 2011.
- N. Cassola, A. Hortaçsu, and J. Kastl. The 2007 subprime market crisis through the lens of european central bank auctions for short-term funds. *Econometrica*, 81(4):1309–1345, 2013.
- S. Cho, H. Paarsch, and J. Rust. An empirical analysis of informationally restricted dynamic auctions of used cars. 2014.
- K.D. Daniel and D.A. Hirshleifer. A theory of costly sequential bidding. Technical report, University of Michigan Business School Working Paper No. 98028, 1998.
- J. R. Groeger. A study of participation in dynamic auctions. *International Economic Review*, 55(4):1129–1154, 2014.
- E. Guerre, I. Perrigne, and Q. Vuong. Optimal nonparametric estimation of first price auctions. *Econometrica*, 68:525–574, 2000.
- P. A. Haile and E. Tamer. Inference with an incomplete model of english auctions. *Journal of Political Economy*, 111(1):1–51, 2003.
- J. Hörner and N. Sahuguet. Costly signalling in auctions. *The Review of Economic Studies*, 2007.
- H. Hsiu-Li, B. Lindsay, and J. Lynch. On mixtures of hazards: Nonparametric maximum likelihood in certain competing risk failure models. *Journal of Nonparametric Statistics*, 2(1):89–103, 1992.
- M. Jofre-Bonet and M. Pesendorfer. Estimation of a dynamic auction game. *Econometrica*, 71:1443–1489, 2003.
- V. Krishna. *Auction Theory*. Academic Press, London, 2009.
- D.N. Politis, J.P. Romano, and M. Wolf. *Subsampling*. Springer, New York, 1999.
- P. J. Reny. On the existence of monotone pure-strategy equilibria in bayesian games. *Econometrica*, 79:499–553, 2011.
- R. J. Serfling. *Approximation theorems of mathematical statistics*, volume 162. John Wiley & Sons, 2009.
- U. Song. Nonparametric estimation of an ebay auction model with an unknown number of bidders. *University of British Columbia*, 2004.
- J. Sutton. *Sunk costs and market structure: Price competition, advertising, and the evolution of concentration*. MIT press, 1991.