Locus of Control and Business Formation*

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Abstract

This paper brings the insights from management to firm dynamics literature and introduces the notion of locus of control, according to which entrepreneurs can direct their startup efforts toward projects with different target sizes. Firm-level data from the U.S. reveal that businesses which eventually become large tend not to be started in recessions, and that financial conditions at birth are critical for the formation of such enterprises. A version of a standard firm dynamics model with financial frictions and the ability of potential entrepreneurs to choose their target size can account for the data. In the estimated model, financial frictions slow the rate at which growth-oriented businesses expand disproportionately; this discourages entrepreneurs from pursuing such ideas in recessions. From the policy perspective, I highlight the importance of improving the quality of startups rather than increasing their quantity.

Keywords: Business cycles, firm dynamics, financial frictions

JEL: E22, E32, H25

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1 Introduction

In a seminal contribution, Lucas (1978) introduced the concept of span of control which helps explain firm-size Pareto right tail in terms of the underlying distribution of managerial talent. This paper adopts the notion of locus of control (LoC); this concept goes back to at least Rotter (1966) and since then has received considerable attention in the management literature. While the span of control represents a static characteristic of a firm, LoC captures the dynamic aspect; this concept implies that entrepreneurs can direct their startup efforts toward projects of different growth potential, thereby controlling the eventual (or target) size of their enterprises. This paper makes two contributions. First, I use the data to document strong cyclical patterns in the formation of firms with different target sizes. In particular, I show that firms with large target sizes tend not to be started in downturns. Subsequently, I embed the entrepreneurial choice of the target size to the otherwise standard firm dynamics model with aggregate uncertainty, and show that this mechanism is key in accounting for the data.

It is by now well known that entrepreneurs and business managers play a central role in firm performance; numerous studies find that managerial fixed effects account for a large share of variation in investment and organizational practices across firms (e.g., Chevalier and Ellison, 1999; Bertrand and Schoar, 2003). In turn, management literature has identified several entrepreneurial characteristics, such as long-term goals and growth intentions, which are strongly correlated with firm size and growth (see, for example, Miller et al., 1982; Boone et al., 1996). Several related studies find similar characteristics of aspiring entrepreneurs to be crucial underpinnings of new ventures (Bird, 1988; Baum and Locke, 2004; Liao et al., 2009).

Importantly, the link between entrepreneurial characteristics and the subsequent performance of firms appears to be mediated by a choice of the business strategy. Starting with at least Shane and Venkataraman (2000), numerous studies looked into the so-called individual-opportunity nexus, aiming to understand how and why some entrepreneurs and not others discover and exploit particular opportunities. Chen et al. (1998) as well as Douglas (2013) find that people with high long-term goals and internal LoC pursue relatively innovative strategies or introduce differentiated products to the market, thereby achieving high organizational performance in the future. On the same line, Van den Steen (2005) and Tan et al. (2007) argue that entrepreneurial long-term goals have a material impact on the organizational structure of the firm: Enterprises with high growth aspirations invest more heavily in R&D, and are more likely to recruit qualified staff.

Critically, organizational structures are highly persistent, since constraints on change in
the core features of organizations are very strong (Hannan and Freeman, 1984). Several related papers studied the organizational inertia phenomenon in the context of different industries (e.g., Kelly and Amburgey, 1991; Amburgey et al., 1993) and found that (i) even sizable shifts in aggregate conditions are not associated with significant organizational changes, (ii) reorganizations increase the hazard rate, and (iii) probability of the organizational change declines with age. Taken at face value, these findings suggest that the choice of a business strategy upon entry can have a long-lasting impact on firm performance, and determine the eventual size of the enterprise.

Little is known about the cyclical properties of the formation of firms with different target sizes, and the existing evidence is inconclusive. Hessels et al. (2008) draw on the Global Entrepreneurship Monitor (GEM) Adult Population Survey from 2005-2006 and find GDP growth to be positively related to the probability of entrepreneurs to take up growth-oriented opportunities. By contrast, Estrin et al. (2013) use the same data but over a longer time period, and find a negative effect of aggregate conditions on individual growth aspirations controlling for a set of confounding factors. Other studies similarly find no clear effects (as reviewed by Levie and Autio, 2013), thereby suggesting a nuanced effect of business cycle fluctuations on entrepreneurial decisions.

This paper puts forward the idea that recessions particularly discourage the formation of businesses with large target sizes; throughout the paper I refer to such businesses as high-profile or high-type firms for brevity. In the context of LoC literature, this means

![Figure 1: Relative Size of Cohorts Born in Expansions vs. Recessions, by Firm Age](chart.png)
that few entrepreneurs choose ideas with high growth potential. Therefore, while it is well known that the firm entry rate falls in recessions, this paper argues that relatively few high- and relatively more low-profile new firms enter during downturns; I refer to this as a compositional effect. Figure 1 provides a basic overview of my empirical findings; it plots the relative size of the largest (those in the top decile of the firm-size distribution) and the remaining (bottom 90%) enterprises across cohorts started in expansions and recessions (which I refer to as expansionary and recessionary cohorts) by firm age. While I find that expansionary firms are on average larger than recessionary ones, the underlying microdata reveal that the difference in means can be virtually fully attributed to the differences in the right tail—the largest firms. Importantly, similar patterns arise in the majority of NAICS 2-digit industries.¹

Empirically, I draw on administrative records from the U.S. and use several metrics of the target size to show that the entry rate of businesses with large optimal size is more procyclical than that of firms with lower target size; this evidence is consistent with the view that relatively few growth-oriented business ideas get chosen during economic downturns. I also exploit cross-industry heterogeneity in the input structure as well as different measures of demand-side frictions to show that this new propagation channel of aggregate shocks cannot be accounted for by several alternative mechanisms. Furthermore, I study the link between aggregate conditions upon entry and subsequent size profiles of firms, and show that the right tail of the firm size distribution is much more procyclical than the left tail; I take this result as evidence of the compositional effect.

In starting new firms, access to financial markets can be critically important (Robb and Robinson, 2014): It typically takes time and resources to start new enterprises. During this start up phase, owners of new businesses need external resources to tide them through until the enterprise becomes viable. A widely held view is that frictions in financial markets are larger in recessions than in expansions (Jermann and Quadrini, 2012), which suggests that fewer firms will be created in downturns. I provide evidence that access to financing affects the formation of high-profile enterprises more strongly than that of small-scale businesses, which is consistent with the view that the former type of firms needs more resources to get up to scale than the latter.² Importantly, access to financing was also found to have a robust impact on entrepreneurial decisions in management literature. Using survey data on

¹With the exception of “Information” (NAICS 51), “Educational Services” (NAICS 61) and “Arts, Entertainment, and Recreation” (NAICS 71) sectors.

²This is also compatible with the idea that environments with weaker financial systems should feature fewer high-growth businesses. In a recent paper, Eslava et al. (2019) track manufacturing establishments in Colombia and the U.S. and find that slower average life cycle growth in Colombia is driven by a less enthusiastic contribution of extraordinary growth plants.
40 countries, Bowen and De Clercq (2008) document that the availability of financial capital is strongly associated with a larger share of high-growth motivated start-ups. Similarly, van der Zwan et al. (2016) find that the perception of financial conditions has a significant negative effect on opportunity entrepreneurs, and virtually no impact on necessity ones. The key role that access to financing plays in the growth of high-profile enterprises has also found support in a rich cross-country entrepreneurial survey (Albert and Caggese, forthcoming).

To evaluate the role of the compositional effect quantitatively as well as to understand which aggregate shocks account for it, I develop a general equilibrium model of firm dynamics in the spirit of Khan and Thomas (2013). The defining feature of the model is the permanent heterogeneity in firms’ spans of control; firms that have a larger span of control grow faster and eventually get larger, and in this sense have a larger target size. For simplicity, it is assumed that firms cannot change their target size after birth; this assumption captures the idea that adjusting organizational capabilities is very costly (e.g., Hannan and Freeman, 1984). Furthermore, potential entrants can direct their start-up attempts toward projects of different optimal size, thereby allowing the model to account for both the extensive (entry rate) and intensive (composition) margins of business formation. This feature, which represents a departure of my model from classic industry dynamics frameworks (i.e., Hopenhayn, 1992), reflects the idea that aggregate conditions can affect entrepreneurial decisions to pursue growth-oriented business ideas. Given the well-established link between entrepreneurial long-term goals and venture growth (Miller et al., 1982; Boone et al., 1996, among others), I exploit empirical life-cycle profiles to inform structural parameters that govern target sizes of firms in the model. In particular, when I bring the model to the data in Section 5, I target employment growth in the first 2 and 5 years of the firm’s tenure. Provided that the majority of firms are of low type, the growth in the first couple of years is informative about the eventual size of low-profile enterprises. However, it takes high-profiles businesses more time to get up to scale, thereby making firm growth over a longer horizon informative about the optimal scale of high-profile firms.

The model has three aggregate stochastic processes—financial, labor disutility and TFP shocks; while the first two processes capture the disruptions originating in capital and labor markets, the TFP shock is a reduced-form way to account for alternative, not explicitly modeled, mechanisms (Kehoe et al., 2018). The model with estimated aggregate shocks can

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3The important role ex ante characteristics of firms play in their subsequent life-cycles also finds support in administrative data (Sterk, Sedláček and Pugsley, 2021); their approach reveals that firms grow large not only because of pure luck (a sequence of high demand, low cost shocks, and alike), but also, and more importantly, because the very idea behind the business plan was good.

4This is not the first attempt to study the implications of various psychological phenomena in the context of business cycle models (e.g., Jaimovich and Rebelo, 2007)
account for the differential responsiveness of low- and high-profile firm entry to initial aggregate conditions even though this is not targeted in the estimation process. I demonstrate that the intensive margin of firm entry is an important propagation mechanism of aggregate fluctuations by way of showing that a version of the model with no entrepreneurial choice of the project type is counterfactual. Besides, I find that the financial shock is the main driver of the compositional effect over the business cycle; the formation of high-profile businesses is particularly (three times more relative to low-profile ones) sensitive to aggregate financial conditions. In the model, the cost of getting a firm with a large optimal size up to scale is higher as compared to enterprises with a smaller target size; this results in fewer high-profile businesses getting started when financial conditions deteriorate.

Finally, I briefly discuss relevant policy implications. Noting that a typical entrant operates in retail or personal services (Hurst and Lusardi, 2004) aspiring to generate only about $100,000 in sales in 5 years, a government stimulative policy of business formation does not appear as the most efficient use of public funds (Shane, 2009). In many countries local authorities are already shifting their focus from seeking to increase the entry rate to improving the quality of startups, which is reflected in the policy focus on high-profile entrepreneurship (Fischer and Reuber, 2003). In Section 7, I consider a parsimonious policy that provides additional capital to newly formed firms at the expense of the representative household. My simulations show that this policy can generate a sizable welfare improvement (about one-half of a percent of lifetime consumption) if the government targets high-profile firms. However, a more generous policy—which subsidizes all entrants—leads to welfare losses. This highlights the importance of micro-targeted government policies, as they help achieve welfare gains due to cost efficiency.

Outline The rest of the paper is organized as follows. Section 2 develops a simple model which illustrates the key mechanism of the paper. Section 3 presents empirical results. In Section 4, I lay out a structural model of firm dynamics, which I bring to the data in Section 5. Section 6 provides the results. Section 7 explores policy implications, and Section 8 concludes.

2 Simple Model

This section presents a simple model which illustrates that distortions on project growth—as compared with distortions on the eventual size of projects—disproportionately affect the value of high-profile ideas. These considerations will be further incorporated in a business cycle framework and quantified in Section 4; in that model, various aggregate shocks manifest
Figure 2: Distortions and Project Values

(A) Distortions on size

(B) Distortions on growth

Notes: Figure 2 plots values of $\mu_L$ and $\mu_H$ projects relative to the case with no distortions. See text for details.

They themselves as distortions on either eventual size of firms, or their growth.

Environment Consider a two-period economy in which firms can be one of two types indexed by Lucas (1978) span of control parameter $\mu_j \in \{\mu_L, \mu_H\}$ with $\mu_H > \mu_L$. These parameters characterize the optimal size of firms:

$$y(k) = k^{\mu_j},$$

where $k$ denotes capital, and $y$ stands for the output of type-$j$ firm. Firms are endowed with some exogenous level of capital $k_0$ at $t = 0$; capital in period $t = 1$ is determined by the investment choice made in period $t = 0$. To link the two periods across time, I assume that capital does not fully depreciate, $\delta = 0.06$.

Firms solve the following problem:

$$v_j = \max_{k'_j \geq 0} k_0^{\mu_j} - [k'_j - (1 - \delta)k_0] + \beta[k'_j]^{\mu_j},$$

where $\beta$ is the discount factor; in what follows, I set $\beta = 1$. For illustrational purposes, I set $\mu_L = 0.6$ and $\mu_H = 0.8$. Initial level of capital $k_0$ is set to match the optimal size of $\mu_L$ projects.

Distortions I consider two types of wedges. On the one hand, the eventual (or target) size can get distorted; I, therefore, introduce a wedge $\tau_S$ on a production function: $y = (1 - \tau_S)k^{\mu_j}$. According to this formulation, the optimal size of a firm declines in $\tau_S$. The second wedge
affects the ease which firms grow (invest); to this end, I introduce a wedge $\tau_G$ on the cost of investing: $i_j(k) = (1 + \tau_G)k_j' - (1 - \delta)k_0$. Thus, higher values of $\tau_G$ make investment costlier.

Results  Figure 2 illustrates the impact of various wedges on project values. In particular, panel (A) shows that when the eventual size gets distorted, both types of firms are affected similarly. Thus, from the perspective of potential entrants, relative attractiveness of $\mu_L$ and $\mu_H$ firms does not change. However, provided that it takes $\mu_H$ projects more resources to get up to scale, we see a more pronounced impact on such projects in case of growth distortions in panel (B). Thus, aspiring entrepreneurs can get discouraged from pursuing high-profile ideas when it becomes harder to expand businesses.

Next section provides empirical evidence which demonstrates strong cyclical patterns in the formation of firms with different target sizes.

3  Empirical Analysis

3.1  Data

Longitudinal Business Database  My main data source is the LBD housed by the U.S. Census Bureau. The LBD is an administrative panel dataset that covers the universe of non-farm establishments in the U.S. private sector with at least one paid employee (Jarmin and Miranda, 2002; Chow et al., 2021). The unit of observation is an establishment, which is defined as a single physical location where business is conducted. I perform my analysis at the firm-level, which requires rolling up plant-level data to the company-level. Such aggregation process is associated with several well-known issues; for example, a new firm identifier emerges in the LBD in the aftermath of a merger with another firm, which can lead to a spurious firm entry. Appendix A discusses how to aggregate establishment-level data to the firm-level in a way which is robust to these issues.

Annual Survey of Manufacturers (ASM) and Census of Manufacturers (CM)  Both the ASM and CM are mail-back surveys of U.S. manufacturing plants (NAICS 31-33); currently, these datasets span the time period 1976-2015. The CM is conducted at quinquennial frequency (years ending in 2 and 7), and covers the universe of manufacturing establishments.\footnote{Which amounts to 300-350k observations in Census years.} The ASM is conducted in non-Census years for about 50-60k establishments taken from the “mail stratum” of the manufacturing sector (Kehrig, 2015). The main
advantage of the ASM/CM is that they provide rich plant-level information on capital expenditures, value of shipments, labor input and materials. Further data details are reserved for Appendix A.2.

3.2 Entry Rates and Heterogeneity in Optimal Size

3.2.1 Measurement

In order to study the impact of business cycle fluctuation on the formation of firms with different target sizes, ideally one needs to observe decisions of potential entrepreneurs to start firms along with characteristics of their business ideas. This information is not available to me; instead, this section studies the cyclicality of entry rates of businesses with different optimal sizes by exploiting a cross-industry variation in two measures of the target size: the returns to scale (RTS) and the minimum efficient scale (MES). While the former measure is tightly linked to the model developed in Section 4, in which RTS govern the permanent heterogeneity in firm life-cycle profiles, the latter makes it possible to extend the empirical analysis to the entire U.S. private business sector.\(^6\)

This approach has two main advantages. First, industry-level analysis allows me to relate the cyclicality of entry rates to a measure of the long-run firm size. Second, the wealth of industry-level observables (e.g., input structure, startup capital requirements) makes it possible to control for several potentially confounding factors. Subsection 3.3 complements these findings with firm-level evidence.

In order to estimate returns to scale, I follow Basu and Fernald (1997) and Lee and Mukoyama (2015) and use detailed data from the ASM/CM (estimation details are in Appendix A.2.3). Subsequently, I merge the resulting RTS estimates with the LBD, thereby avoiding the ASM/CM complications with the identification of business entry arising due to the sampling practices of the Census Bureau. As per the minimum efficient scale, I follow an approach similar to Moreira (2016a) and measure it with the median industry-level employment.\(^7\) Moscarini and Postel-Vinay (2012) argue that there are concerns with the first few LBD cohorts; I, therefore, use cohorts starting from 1978.

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\(^6\)Table D7 in Appendix demonstrates that, in the data, establishments with higher returns to scale are larger over their life-cycles.

\(^7\)In particular, median employment of mature firms (aged 10 and above) within NAICS 4-digit industries is first regressed on a full set of industry, year and age dummies. Subsequently, the MES measure is obtained by standardizing the recovered industry fixed effects. Qualitatively, the results are robust to several alternative ways of the MES measure construction.
### Table 1: Target Size and Cyclicality of Entry and Exit Rates

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<tr>
<td>$\Delta GDP_t$</td>
<td>0.282***</td>
<td>0.331***</td>
<td>0.357***</td>
<td>0.367***</td>
<td>0.378***</td>
<td>0.342***</td>
<td>0.350***</td>
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<td>$\Delta GDP_t \times \text{Target Size}_i$</td>
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<td>0.073**</td>
<td>0.531***</td>
<td>0.565***</td>
<td>0.284***</td>
<td>0.087***</td>
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<td>$\Delta GDP_t \times \text{Skill Intensity}_i$</td>
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<tr>
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**Notes:** Table 1 reports the results of OLS estimation of Equation (3). The dependent variable in columns (1)-(7) is the firm entry rate, computed by dividing the number of age 0 firms in industry $i$ and year $t$ by the total count of active firms within the same industry in the preceding period. The dependent variable in column (8) is the firm exit rate, computed by dividing the number of exiting firms in industry $i$ and year $t$ by the total count of active firms within the same industry in the preceding period. The right-hand side variable $\Delta GDP_t$ is the log-deviation of real GDP from HP trend in year $t$, and Target Size$_i$ is either returns to scale or the minimum efficient scale. Observations are weighted by industry-level employment. Industry fixed effects are at NAICS 4-digit level. All industry characteristics are standardized. Entry and exit rates as well as the minimum efficient scale are winsorized at top and bottom 1%. Source of firm-level data: ASM/CM and LBD. The underlying data covers the time period 1978-2016. Standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

### 3.2.2 Empirical Specification

The baseline specification takes the form:

$$ER_{i,t} = \beta_0 \Delta GDP_t + \beta_1 (\Delta GDP_t \times \text{Target Size}_i) + \gamma X_{i,t} + \varepsilon_{i,t},$$

(3)

where the left-hand side variable $ER_{i,t}$ is the entry rate in industry $i$ at time $t$, and $X_{i,t}$ is a vector of controls that includes the intercept, industry fixed effects (at NAICS 4-digit level), and a quadratic time trend which accounts for low-frequency movements in the entry rate. Provided that the entry rate relates the number of new firms in year $t$ to the number of active firms in $t - 1$, I also control for aggregate conditions in the preceding period. The coefficient of interest $\hat{\beta}_1$ measures how the cyclicality of entry rates changes across industries with different optimal sizes.
3.2.3 Results

Table 1 reports the results. The first and second columns demonstrate that—regardless of how the target size is measured—recessions are associated with a relatively low entry of businesses with high optimal size; the entry rate in an industry with a 1 standard deviation higher RTS or MES declines by 0.07pp stronger for each extra percentage reduction in aggregate output. This is an economically significant effect, since the average entry rate was about 7-8% in recent years. Provided that the MES measure is more comprehensive, the rest of results in the table are based on that metric. Table D10 in Appendix shows that the results hold if returns to scale are used as a measure of the target size.

Adjustment Costs  A wide body of literature has documented that technological constraints in the form of adjustment costs could impede the ability of businesses to quickly adjust their size to the optimal scale (see, e.g., Hamermesh, 1989 and Cooper and Haltiwanger, 2006 for the discussion of labor and capital adjustment costs, respectively). These costs could reflect numerous factors, such as irreversibilities of projects due to the lack of liquidity in the secondary markets for capital goods, installation costs of new equipment, as well as firing/hiring costs. Thus, heterogeneity in the optimal size could reflect systematic differences in technology, whereby a large optimal size results from low adjustment costs. Provided that capital adjustment costs are typically larger than labor adjustment costs (Hall, 2004), I next include an interaction of aggregate conditions with a measure of capital adjustment costs—capital intensity.\(^8\) Column (3) demonstrates that, qualitatively, the baseline result is not affected. A larger estimate $\hat{\beta}_1$ reflects a restricted sample; several industry-level characteristics (with the exception of share of advertising and startup capital requirements) are only available for the manufacturing sector.

Innovativeness and Complexity  Another relevant characteristic of a production technology is its level of innovativeness and complexity; it was shown to be an important factor in accounting for firm entry over the business cycle (Hoffman, 2019). I adopt a notion that industries with more complex technologies require more skillful employees, and include a measure of skill intensity into Equation (3). Column (4) demonstrates that the baseline result holds.

\(^8\)NBER CES Manufacturing database was used to construct measures of skill and capital intensities. Skill intensity is measured as a ratio of non-production workers to total employment (Pierce and Schott, 2016), and capital intensity is a ratio of capital to total employment.
Demand-side Frictions  Several recent papers put forward demand-side frictions as important propagation mechanisms of initial aggregate conditions (Moreira, 2016a; Sedláček and Sterk, 2017); according to that view, mass-product firms are more likely to get started in expansions, since favorable demand conditions allow them to quickly build up their customer base and relax demand-side constraints. The next two columns of Table 1 demonstrate that the mechanism this paper focuses on is robust to these considerations. First, I exploit the notion that industries producing less durable goods are more likely to be mass-product oriented, and this could potentially account for more procyclical entry rates in sectors which happen to have large measured target sizes. Column (5) utilizes the durability index developed by Bils and Klenow (1998) and shows that the baseline qualitative result is unaffected. Furthermore, column (6) adds an interaction of aggregate conditions with the share of industry-level inputs accounted for by advertising; the idea is that entrepreneurs operating in industries where customer base accumulation is an important channel of business expansion are likely to spend relatively more of advertising.\footnote{Author's calculations based on the detailed input-output table provided by the BEA.} I find that adding this control has no material impact on $\beta_1$.

Startup Capital Requirements  One important measure for my analysis is the amount of capital needed to start a firm, since these investment requirements reflect how important access to financing is for entrepreneurs operating in a given industry. I use the Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS) to construct such a measure.\footnote{Data source: \url{https://www.census.gov/data/datasets/2007/econ/sbo/2007-sbo-pums.html}.} I focus on the “Amount of startup or acquisition capital” for each firm and group the answers to this question at the 2-digit NAICS industry level—the finest level available in the data—for active in 2007 firms. Following Hurst and Lusardi (2004) and Adelino et al. (2015), I split industries above and below the median in order to classify them into groups with high and low startup capital requirements (SCR). Column (7) indicates that the baseline result of higher procyclicality of entry rates in industries with larger operating scale is fully accounted for by a group of industries with high startup capital requirements; this points at the important role financial frictions play in the process of entry of high-profile enterprises.

Persistence of the Compositional Effect  The composition of entrants may not be persistent if high-profile businesses are more likely to exit when aggregate conditions deteriorate, thereby reducing the share of firms with large target size within a cohort. Extending this logic to the industry-level analysis, the question is whether industries with bigger enterprises
exhibit higher exit rates during recessions. Column (8) shows no evidence in favor of this view; in fact, the exit rate in industries with higher optimal size is less countercyclical.

3.3 Initial Conditions and Subsequent Life-Cycle Profiles of Firms

The results of the previous section suggest that recessionary cohorts of firms contain relatively few firms which eventually become large, since the entry rate in industries with typically large enterprises is more procyclical. This section provides direct evidence on the impact of initial aggregate conditions on subsequent firm size profiles.

3.3.1 Evidence from Probit Regressions

For the purpose of this exercise, I define a firm to be high-profile (or large) if its employment $E_{i,t}$ exceeds the 90th percentile of the within industry employment distribution $P90_{j(i)}$, where $j$ denotes the industry in which firm $i$ operates. This threshold is computed using every firm operating in a given industry, not only those from a given cohort. I then study the impact of initial aggregate conditions on the probability of exceeding this threshold using the following probit specification:

$$P(E_{i,t} \geq P90_{j(i)} | X_{i,t}) = \Phi(\gamma X_{i,t}),$$

$$\gamma X_{i,t} = \beta_0 \Delta AC_{i,0} + \lambda Y_{i,t} + \varepsilon_{i,t},$$

(4)

where the vector of controls $Y_{i,t}$ includes the intercept, as well as year, age, industry and state fixed effects; $\Delta AC_{i,0}$ measures aggregate conditions in the entry year of firm $i$.

Column (1) of Table 2 uses the log-deviation of real GDP from the HP trend as a measure of aggregate conditions, and shows that the probability of becoming large is higher if a firm gets started in expansions; the coefficient estimate $\hat{\beta}_0$ is positive and statistically significant at 1% level. Importantly, the impact of aggregate conditions at entry is not limited to the first few years of the firm’s life-cycle. Column (2) re-estimates Equation (4) on a subsample of mature firms (aged 10 and above) and finds a similar positive relationship between aggregate conditions at entry and odds of being large even 10 years into firm’s tenure.

Subsection 3.2 emphasized the role of access to financing in accounting for the formation of firms which eventually become very large. Those results imply that one should see a small difference in probabilities of being large between booms and busts under tight financial conditions. Column (3) of Table 2 adds an interaction of aggregate conditions with the financial index of Gilchrist and Zakrajšek (2012); this index is based on credit spreads and, therefore, tends to increase when financial conditions deteriorate. The data reveal that, on average, the probability of being large increases by 0.8pp when the aggregate economy
Table 2: Initial Conditions and Probability of Becoming Large

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta GDP_0$</td>
<td>0.228***</td>
<td>0.361***</td>
<td>0.476***</td>
<td>0.874***</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.060)</td>
<td>(0.074)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>$\Delta GDP_0 \times \Delta GZ_0$</td>
<td>-3.228***</td>
<td>-3.171***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.253)</td>
<td>(0.366)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta GZ_0$</td>
<td>0.026***</td>
<td>0.066***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.027</td>
<td>0.019</td>
<td>0.027</td>
<td>0.019</td>
</tr>
<tr>
<td>Ages included</td>
<td>All ≥ 10</td>
<td>All ≥ 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: Table 2 reports the estimation results of the probit model (4). The dependent variable is a binary variable, indicating whether employment of firm $i$ in year $t$ exceeds the 90th percentile of the within industry employment distribution. Right-hand side variables $\Delta GDP_0$ and $\Delta GZ_0$ are log-deviations of real GDP and Gilchrist and Zakrajšek (2012) index from their corresponding HP trends in the year of entry of firm $i$. Sample 3 is based on the LBD and includes cohorts of firms which entered between 1978 and 2006; firms are tracked until they exit or until 2016 (the last year in the LBD). See Appendix A.1.5 for additional details about samples as well as for summary statistics. Age, state, year, and industry (at NAICS 4-digit level) fixed effects are included. Standard errors are clustered at the state level. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

However, an increase in probability is only 0.2pp under unfavorable financial conditions. According to column (4), similar patterns arise if the sample is restricted to mature enterprises.

3.3.2 Initial Aggregate Conditions and Subsequent Cohort Profiles

Next, I document the impact of initial aggregate conditions on subsequent cohort profiles. The baseline specification takes the following form:

$$\ln Y_{t,a,i,s} = \beta_0 \Delta GDP_0 + \gamma X_{t,a,i,s} + \varepsilon_{t,a,i,s}. \quad (5)$$

In Equation (5), the key independent variable $\Delta GDP_0$ is the log-deviation of real GDP from its HP trend at the time of cohort entry. In order to isolate the cohort effect from aging patterns of firms, the vector of controls $X_{t,a,i,s}$ includes age fixed effects $\delta_a$. Moreover, to absorb the impact of aggregate shocks cohorts experience during their life-cycles, state-year and industry-year fixed effects ($\delta_{s,t}$ and $\delta_{i,t}$) are also included; such formulation allows me

---

11 The implied probability is 15.2% (14.4%) when GDP is 2% above (below) the trend.
12 When Gilchrist and Zakrajšek (2012) index is 20% above the trend (similar to the Great Recession episode).
### Table 3: Cohort Profiles and Initial Aggregate Conditions

<table>
<thead>
<tr>
<th></th>
<th>Employment (Log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$\tilde{\Delta}GDP_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.799</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.337</td>
</tr>
<tr>
<td>Age FE</td>
<td>✓</td>
</tr>
<tr>
<td>Industry-Year FE</td>
<td>✓</td>
</tr>
<tr>
<td>State-Year FE</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: Table 3 reports the results of OLS estimation of Equation (5). The left-hand side variable $\ln Y_{t,a,i,s}$ is the mean, 10-, 50- or 90th percentile of log employment. The independent variable is the log-deviation of real GDP from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Industry fixed effects are at NAICS 4-digit level. Sample 1 includes cohorts of firms born between 1978 and 2006, which are tracked until the age of 10. Robust standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

to control for the potentially heterogeneous impact of aggregate shocks across space and industries.

The left-hand side variable $\ln Y_{t,a,i,s}$ is the mean, 10-, 50- or 90th percentile of log employment (or log payroll) computed over observations which fall inside these four-dimensional bins. The large size of the LBD allows me to compute the required moments very accurately within narrowly defined subsets of the data. I weigh observations by the cohort count, thereby accounting for the fact that there is a strong extensive margin of the number of businesses entering in expansionary versus recessionary periods. The main coefficient of interest $\beta_0$ measures the elasticity of different moments of the cohort’s size over its lifespan with respect to economic conditions at inception. In a baseline sample, I track firms up to the age of 10, and retain businesses which exit prior to that age threshold. As a result, I consider 29 cohorts born between 1978 and 2006. Table A1 in Appendix A provides summary statistics for the underlying sample of firms.

Table 3 reports the result. The first column indicates that the average firm size is positively related to initial aggregate conditions: A 1% increase in real GDP is associated with a 0.8% increase in the cohort’s average employment over its lifespan. At the same time, elasticities of the bottom and top tails of the firm-size distribution with respect to initial conditions are remarkably different; while the 10th percentile of employment goes up by 0.13% for each 1% increase in GDP, the 50th and 90th percentiles are five and fourteen times more sensitive, respectively. This is an economically sound effect, since the top decile of the firm size distribution is nearly 20 times larger than the bottom decile (see Table A1). Overall,
the results indicate that cohorts of firms born in good aggregate times are on average larger than those which enter in recessions, and that the difference in the average size is driven by the right tail.\textsuperscript{13}

### 3.3.3 Robustness and Additional Results

The patterns documented above are pervasive. Appendix A.3 shows that similar results are present in alternative samples, at state-wide business cycle frequencies, and at the establishment-level. I also show that the effect of initial aggregate conditions is persistent, confirming findings from probit regressions.

Table D11 in Appendix demonstrates that results reported in Table 3 appear in industries with both high and low demand-side frictions as measured by the share of advertising input. In fact, the results are consistent with the demand channel being an important propagation mechanism of initial aggregate conditions (Moreira, 2016a; Sedláček and Sterk, 2017), which, however, cannot fully account for the higher procyclicality of the right tail.

Tables D12 and D13 highlight the importance of financial conditions. The former table uses the Gilchrist and Zakrajšek (2012) financial index and shows that the procyclicality of the right tail is reduced when financial conditions deteriorate. The latter table demonstrates that the higher procyclicality of the right tail disappears in industries with low startup capital requirements. Furthermore, Appendix A.4 studies the impact of disruptions in small business credit supply on subsequent cohort profiles by drawing on the Community Reinvestment Act data. All these findings are compatible with the idea that access to financing is critical for entry of growth-oriented enterprises.

### 3.4 Discussion

The first part of the empirical evidence presented in this section established that the entry rate of businesses with a large optimal size is more procyclical than that of low-profile enterprises. Critically, the data reveal that access to financing plays an important role in the process of formation of growth-oriented firms.

In the second part of this section, I studied the impact of initial aggregate conditions on subsequent firm and cohort profiles, and showed that the results are consistent with those

\textsuperscript{13}By drawing on the universe of incorporated firms in the U.K. (see Appendix A.5 for details), I provide additional support for the compositional effect. In particular, in Appendix A.5.3 I focus on cohorts of firms born in 2006-2009 and classify them into "types" by applying a grouped fixed effects estimator developed by Bonhomme and Manresa (2015). The benefit of this estimator is that it optimally assigns firms into groups so that life-cycle profiles of enterprises in each group are as similar as possible taking the effect of observables into account. I find that the share of high-profile firms was 40% smaller among businesses which entered during the Great Recession as compared with pre-recessionary cohorts.
from the first part. Next, I build a structural model of firm dynamics which is useful to interpret these empirical findings.

4 Model

4.1 Environment

Time in the model is discrete and runs forever: \( t = 0, 1, \ldots \). The economy is populated by four types of agents: potential entrepreneurs, incumbent firms, the government and a representative household.

Technology There is a finite number of projects types \( J \in \mathbb{N} \setminus \{1\} \), each of them indexed by a Lucas (1978) span of control parameter \( \mu_j \).\(^{14}\) This parameter characterizes the optimal size of each firm \( i \) in the steady-state:

\[
y_{ijt}(Z_t, z_{it}, n_{it}, k_{it}) = Z_t e^{z_{it}} [k_{it}^{\alpha} n_{it}^{\nu}]^{\mu_j},
\]

where \( k_{it} \) and \( n_{it} \) denote capital and labor inputs, and \( y_{ijt} \) stays for the output of type-\( j \) firm \( i \) at time \( t \).

Parameters \( \alpha \) and \( \nu \) are assumed to be strictly positive, with their sum being equal to 1. Span of control parameters \( \{\mu_j\} \) are less than one, which implies that every firm has a finite target size. The production function (6) is scaled by two factors: an exponent of the time-varying idiosyncratic productivity \( z_{it} \) and an aggregate TFP \( Z_t \). I postpone the description of aggregate shocks until Subsection 5.3, and at this point only specify that \( z_{it} \) follows an AR(1) process in levels:

\[
z_{i,t+1} = \rho_z z_{it} + \varepsilon_{i,t+1}, \quad \varepsilon_{i,t+1} \sim \mathcal{N}(0, \sigma_z^2),
\]

where \( \varepsilon_{i,t+1} \) are i.i.d. across time and space.

Labor The labor market is frictionless with the prevailing wage rate \( W_t \).

Investment Firms enter period \( t \) with some predetermined idiosyncratic level of capital \( k_{it} \). The capital in period \( t + 1 \) is determined by depreciation and investment made in period \( t \). Capital depreciates at a rate \( \delta \).

\(^{14}\)Hereafter, \( J \) will denote a set \( \{1, \ldots, J\} \).
**Financing** In order to finance their investment, firms can use internal and external funds, each of them subject to a friction. On one hand, firms can finance investment by way of reducing current payments to shareholders, or may even raise equity (in this case, firms bear additional costs). On the other hand, firms can also borrow external funds from a competitive financial intermediary; this channel is also subject to a friction. Following a wide body of literature (Khan and Thomas, 2013; Zetlin-Jones and Shourideh, 2017 among others), I assume that the amount of external borrowing is limited by a fraction of the firms’ installed capital—their collateral.\(^{15}\)

**Entry** Every period the economy is confronted with a large mass of ex ante identical potential entrants.\(^{16}\) In order to make an entry attempt, they have to pay a cost \(\chi\) denominated in units of the final good. They subsequently choose a project type \(\mu_j\). The entry process is subject to a coordination friction, meaning that some entry attempts will be unsuccessful. Successful entrants are endowed with exogenous amounts of capital \(k_0\) and debt \(b_0\).\(^{17}\) They draw their initial idiosyncratic productivity \(z\) from a distribution \(F^z(z)\).

**Exit** Firms can choose to leave the economy if the value of staying—the sum of operating profits plus the scrap value of capital less the debt repayment—falls short of their outside option. It is costly to operate as firms have to pay the cost \(\xi \sim F^\xi(\xi)\) denominated in units of labor in order to stay in business. Firms can also exit for exogenous reasons: At the beginning of each period \(t\) firms receive an exit shock with probability \(\pi_d\) which forces them to leave the economy in the end of that period.

**Government** The government taxes firms’ operating profits at a flat rate \(\tau\), and rebates the proceeds to the household in a lump-sum manner.

**Households** The economy is populated by a unit mass of identical households. A household consumes, supplies labor, and saves into firms’ shares and a risk-free bond.

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\(^{15}\)Both frictions are necessary in equilibrium: Without the collateral constraint, firms can obtain any level of capital by issuing debt. With zero equity issuance cost, firms can instantaneously get to the optimal size by way of financing their investment through equity issuance—even if the collateral constraint binds.

\(^{16}\)There is an extensive literature arguing that “necessity” entrepreneurship rises in recessions: People choose to become entrepreneurs because they have no better alternatives. While some papers find empirical support in favor of this view (Evans and Leighton, 1989), others offer evidence that necessity entrepreneurship does not vary substantially over the business cycle (Moreira, 2016b). In any case, businesses started out of necessity are unlikely to grow large. Thus, the quantitative results of this paper are conservative, since taking these considerations into account would make the compositional effect even more pronounced.

\(^{17}\)Letting \(k_0\) be a choice variable dramatically complicates the computation of the model. However, if \(k_0\) was a choice, there is no reason to expect it to be very different across the firm types since age 0 firms are predominantly very small in the data.
**Figure 3: Timeline of the Model**

<table>
<thead>
<tr>
<th>Firm state</th>
<th>Agg. state</th>
<th>Produce</th>
<th>Exit</th>
<th>Exit intertemp. choices</th>
<th>Payments</th>
<th>Entry</th>
<th>Div.</th>
<th>t + 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Draw z</strong></td>
<td>(ζ, X)</td>
<td>y(k, b, z; S)</td>
<td>Exog.</td>
<td>(k', b', z)</td>
<td>{W_n, R_{-1} b, \chi}</td>
<td>{C, E}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k, b, z)</td>
<td></td>
<td></td>
<td></td>
<td>(\xi \sim F^\xi)</td>
<td>(b' \leq \rho k)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Timing**  
The timing of the events within a period is as follows:

1. the aggregate state is realized;
2. each incumbent firm observes the realization of an exit shock, which is a Bernoulli random variable with parameter \(\pi_d\). Firms that receive the shock have to exit the economy in the end of the period after the production stage takes place. Other firms further learn the operating cost shock \(\xi\) and make a decision of whether to continue operations or to leave the economy in the end of the period;
3. the production stage takes place: Firms choose the optimal labor input and produce. At this point, firms which received an exit shock and those which decided to exit leave the economy. The rest of firms make intertemporal decisions \(k'\) and \(b'\);
4. the influx of new firms (successful potential entrants) enters the economy;
5. representative household consumes.

The verbal description of events is summarized in Figure 3. Next, I provide recursive formulations for individual optimization problems.

### 4.2 Incumbent Firms

The state vector of the incumbent firm contains four elements: capital \(k\), financial position \(b\), idiosyncratic productivity \(z\), and type \(j\). Since the model features aggregate uncertainty, the aggregate state vector \(S\) includes both the aggregate shocks \(X\) and the distribution of firms over the state space \(\zeta\). Therefore, the aggregate state vector is \(S = (\zeta, X)\). In order to streamline the exposition, I postpone the discussion of aggregate shocks until Subsection 5.3.

19
**Start of the Period** Let \( v_j(k, b, z; S) \) denote the value of the type-\( j \) firm at the start of the period given the idiosyncratic state \((k, b, z)\) and the aggregate state \(S\). After realization of the exogenous exit shock, the firm learns the operating cost shock \( \xi \), and decides whether to operate in the economy or to leave it. Hence the value of firms at the beginning of the period can be expressed as

\[
v_j(k, b, z; S) = \pi_d v_j^{\text{exit}}(k, b, z; S) + (1 - \pi_d) \int v_j(k, b, z, \xi; S) dF^\xi(\xi),
\]

where \( v_j^{\text{exit}}(\cdot) \) is the value of exit, and \( v_j(k, b, z, \xi; S) \) is the value of a firm with an operating cost shock \( \xi \).

After observing the cost shock \( \xi \), the firm decides whether to pay \( \xi \) and continue, or to stop its operations. Formally:

\[
v_j(k, b, z, \xi; S) = \max\{v_j^{\text{cont}}(k, b, z; S) - W(S)\xi, v_j^{\text{exit}}(k, b, z; S)\}.
\]

I assume that the operating cost shock \( \xi \) is i.i.d. across time and space, and is uniformly distributed over its support \([0, \bar{\xi}]\). The firm will choose to continue conditional on the realization of \( \xi \) if and only if

\[
v_j^{\text{cont}}(k, b, z; S) - W(S)\xi \geq v_j^{\text{exit}}(k, b, z; S).
\]

For each type-\( j \) firm indexed by its state \((k, b, z; S)\), there is a threshold value of \( \xi_j^*(k, b, z; S) \) such that the firm will always choose to continue if \( \xi \leq \xi_j^*(k, b, z; S) \), and will prefer to exit otherwise. It follows that the threshold is given by

\[
\xi_j^*(k, b, z; S) = \frac{v_j^{\text{cont}}(k, b, z; S) - v_j^{\text{exit}}(k, b, z; S)}{W(S)}.
\]

Provided that \( \xi \) has a bounded support, I reformulate the definition of the threshold to force it lie within the interval \([0, \bar{\xi}]\):

\[
\hat{\xi}_j(k, b, z; S) = \min\{\bar{\xi}, \max\{0, \xi_j^*(k, b, z; S)\}\}\}.
\]

Given that \( \xi \) is uniformly distributed, I can calculate the mathematical expectation with
respect to $\xi$ in Equation (8) analytically:

$$v_j(k, b, z; S) = \pi_d v_j^{\text{exit}}(k, b, z; S) + (1 - \pi_d) \left(1 - \frac{\xi_j(k, b, z; S)}{\xi}\right) v_j^{\text{exit}}(k, b, z; S) +$$

$$+ (1 - \pi_d) \left(\frac{\xi_j(k, b, z; S)}{\xi}\right) \left[v_j^{\text{cont}}(k, b, z; S) - W(S)\frac{\xi_j(k, b, z; S)}{2}\right]. \quad (12)$$

**Value of Operation** If the firm did not receive an exit shock and was hit by a sufficiently small operating cost shock $\xi$, it proceeds with the intertemporal choice of capital and debt. Formally, it solves the following programming problem:

$$v_j^{\text{cont}}(k, b, z; S) = \max_{k', b'} -E_j(k, b, z; S) - C(E_j(k, b, z; S)) + E[d(S', S)v_j(k', b', z'|S')] \quad (13)$$

subject to

$$i_j(k, b, z; S) = \Pi_j(k, b, z; S) + E_j(k, b, z; S) + b'_j(k, b, z; S) - b, \quad (14)$$

$$b'_j(k, b, z; S) \leq \rho(S)k, \quad (15)$$

$$S' \sim \Gamma(S), \quad (16)$$

where Equation (14) is the firm’s budget constraint, and Equation (15) is the collateral constraint. The aggregate financial shock governs the tightness of the financial constraint, $\rho(S)$.\(^\text{18}\) Firms take into account the aggregate law of motion (16).

Firms can use several sources to finance their investment expenditures. First, businesses can use retained earnings $\Pi$—funds left after they sell their output, pay the wagebill and the interest on their outstanding debt. Second, they can use debt financing and borrow $b'$ (throughout the paper it is understood that $b' > 0$ means borrowing, and $b' < 0$ corresponds to savings). Finally, firms can raise funds by issuing equity $E$. Negative values of $E$ are interpreted as dividend payments, while positive values correspond to equity issuance.

Operating profit $\Pi_j(\cdot)$ is defined as:

$$\Pi_j(k, b, z; S) = \max_{n \geq 0} (1 - \tau) \left[y(k, n, z; S) - W(S)n - (R(S - 1) - 1)b\right] + \tau \delta k, \quad (17)$$

where $\tau$ is a flat operating profit tax rate and $R(S - 1)$ is a gross interest rate from the

\(^{18}\)For simplicity, I consider tightness of the collateral constraint as the only source of financial shock in the model economy. The U.S. data show that debt repurchases are much more volatile over the business cycle than equity payout (Jermann and Quadrini, 2012; Begenau and Salomao, 2018).
preceeding period. The law of motion for physical capital is:

\[ k'_j(k, b, z; S) = (1 - \delta)k + i_j(k, b, z; S) - \frac{\varphi K}{2} \left( \frac{i_j}{k} \right)^2 k. \]  \hspace{1cm} (18)

The last term in Equation (18) denotes capital adjustment costs. Function \( C(\cdot) \) in Equation (13) captures the equity issuance cost, which will be specified below. Parameter \( \rho \) in the collateral constraint governs the tightness of financial frictions in the economy. Since all firms in this economy are owned by the household, the future stream of dividends in Equation (13) is priced according to the stochastic discount factor \( d(S', S) \).

**Value of Exit** An exiting firm produces in the current period, sells its undepreciated capital, pays the outstanding debt, and leaves the economy. Such firms do not make any intertemporal choices, their value is then given by:

\[ v^\text{exit}_j(k, b, z; S) = \Pi_j(k, b, z; S) + (1 - \delta)k - b. \] \hspace{1cm} (19)

### 4.3 Entry

The defining feature of the model is that potential entrants can direct their startup efforts toward a specific business idea. To operationalize this, I introduce a coordination friction which is reminiscent of the literature on directed search, and is formulated similarly to Sedláček and Sterk (2017).

Let \( \{\psi^j\}_{j \in J} \) be a distribution of available to potential entrants business opportunities of types \( J \). In order to enter, an aspiring potential entrant has to pay a cost \( \chi \) denominated in units of the final good. This entrance cost is designed to capture expenditures associated with market research, development of a business plan, and alike. Upon paying the cost, a potential entrant gets to choose one of projects \( j \in J \), and, subsequently, has a chance to seize one of the available ideas \( \psi^j \). Therefore, parameters \( \{\psi^j\} \) govern the relative success probability of starting different project types.

Given the coordination friction, not all business opportunities are seized, while others are seized by several aspiring entrepreneurs. This friction is modeled by a matching function which returns the mass of successful entrants of type \( j \) at time \( t \), \( m^j_t \):

\[ m^j_t = (c^j_t) ^ \phi (\psi^j)^{1 - \phi} , \] \hspace{1cm} (20)

where \( c^j_t \) is the mass of potential entrants who chose a type-\( j \) project, and \( \phi \in (0, 1) \) is an elasticity of successful entrants with respect to startup attempts.
While the equilibrium of the model will be laid out later, it is convenient to formulate one equilibrium condition here. Potential entrants are indifferent with respect to project types in equilibrium; this consideration gives rise to a set of free-entry conditions, one for each type. The free-entry condition states that the cost of starting a business has to match the expected benefit:

$$\frac{\chi}{c^j} = \frac{m^j_t}{\psi^j} \int_z v^j(k_0, b_0, z; S)dF^z(z) \quad \forall j \in J \quad z \sim F^z(z),$$

(21)

where the ratio on the right-hand side is a success probability of starting a project of type $j$. Business cycle fluctuations (shifts in elements of $S$) trigger a change in the expected benefit of starting and running projects. The free-entry condition (21) ensures that the success probability of projects adjusts correspondingly.

By way of combining the matching friction (20) with free-entry conditions (21), and defining $\tilde{v}^j(k_0, b_0; S) := \int_z v^j(k_0, b_0, z; S)dF^z(z)$, one can derive the following closed-form expression for the mass of entrants of each type:

$$m^j_t = \chi^{\phi} \left[ \tilde{v}^j(k_0, b_0; S) \right]^{\frac{1-\psi}{1-\phi}} \psi^j \quad \forall j \in J.$$

(22)

The right-hand side of Equation (22) contains a product of three terms. Given that $\phi \in (0, 1)$, a larger entry cost $\chi$ reduces the mass of entrants $m^j_t$. Furthermore, an increase in the value $\tilde{v}^j(\cdot)$ and/or project availability $\psi^j$ stimulates the entry of type-$j$ projects. Critically, Equation (22) shows how time-varying aggregate conditions affect entry decisions of potential entrepreneurs; if the value $\tilde{v}^j$ responds differently across the firm types to the same aggregate shock, the entry intensity of some firm types will change more strongly as compared with others.\(^{19}\)

### 4.4 Households

The economy is populated by a unit mass of identical households. Each household consumes $c$, supplies labor $n$, saves into a risk-free bond $\lambda'$ and firms’ shares $\omega$. The price of current

---

\(^{19}\)Recent literature explored “waiting” decisions of potential entrepreneurs; according to this view, entrepreneurs tend to postpone entry decisions during recessions (e.g., Vardishvili, 2020). I do not explicitly model that mechanism for two reasons. First, based on the evidence from the Business Formation Statistics (BFS), the waiting time to form businesses declined during the Great Recession in the U.S. (Figure C1). This implies that, if anything, the financial crisis did not generate a pronounced delayed entry effect. Moreover, the version of the model with the “waiting” mechanism will be isomorphic to the model developed in Section 4: In both cases, recessionary cohorts will contain fewer high-type enterprises.
shares is \( \rho_0(\cdot) \), and the purchase price of new shares is \( \rho_1(\cdot) \). The net risk-free interest rate is \( q_0(S)^{-1} - 1 \). The household’s dynamic programming problem is:

\[
H(S) = \max_{c,n,\lambda',\omega'} \left[ U(c, n) + \beta \mathbb{E} H(S') \right]
\]

subject to the budget constraint and the law of motion for the aggregate state:

\[
c + q_0(S)\lambda' + \int \rho_1(k', b', z', j; S)d\omega' \leq W(S)n + \lambda + \int \rho_0(k, b, z, j; S)d\omega + T,
\]

\[
S' \sim \Gamma(S).
\]

According to the budget constraint, resources available to the household consist of firms’ shares, labor income, bonds and the tax rebate from the government. A fraction of these resources is consumed, and the rest is reinvested into new firm shares as well as into a risk-free bond.

I assume that the instantaneous utility function takes the following form:

\[
U(c, n) = \frac{c^{1-\kappa}}{1-\kappa} - \frac{\theta(S)n^{1+\gamma}}{1+\gamma},
\]

where \( \theta(S) \) is an aggregate disutility from labor and \( 1/\gamma \) is the Frisch elasticity of labor supply (Jaimovich and Floetotto, 2008; Bloom et al., 2018).

4.5 Equilibrium

The recursive competitive equilibrium for this economy is a collection of decision and value functions, such that (i) household optimizes, (ii) firms solve their corresponding programming problems, (iii) labor and final good markets clear, (iv) government budget constraint is satisfied, (v) evolution of the aggregate state in consistent with individual decisions of firms. The detailed definition of equilibrium is relegated to Appendix B.1.

Solution Method

I use the perturbation method of Reiter (2009) to solve for aggregate dynamics: This approach preserves the full non-linearity of the firms’ policy functions with respect to idiosyncratic states, and perturbs these policies linearly with respect to aggregate shocks (Mongey and Williams, 2016). The method is fast, which is advantageous for estimation purposes. Further details on the solution method are in Appendix B.
**Figure 4: Decision Rules at Steady-State**

Notes: Figure 4 plots decision rules when firms’ capital is fixed at the cross-sectional mean. Panel (A) corresponds to low-type firms $\mu = \mu_L$, and Panel (B)—to high-type businesses $\mu = \mu_H$. Vertical pink dotted lines visualize the calibrated tightness of the collateral constraint (parameter $\rho$). All decision rules are normalized by the idiosyncratic level of capital $k$. Dashed lines correspond to unproductive firms $z = \bar{z}$, and solid lines—to productive firms $z = \bar{z}$.

### 4.6 Characterization of the Steady-State

In order to demonstrate the workings of the model, in Figure 4 I plot the firms’ decision rules at the calibrated steady-state. Given the dimensionality of the problem, I explore firms’ decisions by fixing their idiosyncratic capital at the cross-sectional mean $\bar{k} = \mathbb{E}[k]$.

To facilitate the visual inspection, here I focus on the lowest ($\mu_L$) and highest ($\mu_H$) types only. Furthermore, in order to capture how decisions change with respect to idiosyncratic productivity and, hence, demonstrate the full range of policy functions, I simultaneously plot decisions at the lowest and highest productivity levels, $z$ and $\bar{z}$.

There is a significant variation in financing policies across the firm types. There is nearly no difference between firms of different types at the lowest productivity level $z$: The target size of capital is below the cross-sectional mean $\bar{k}$ which is reflected in negative investment rates. At the highest productivity level $\bar{z}$, however, the investment demand of all firms rises, but more so for $\mu_H$ businesses. With an increase in the leverage ratio $b/k$, the residual capacity of debt financing decreases; the debt financing declines and the equity financing increases. Due to the equity issuance cost, firms’ investment decreases as the leverage rises: Businesses with a larger span of control reduce their investment demand by more than low-type firms. Besides, while $\mu_H$ firms issue equity (red solid line is in the positive region across all leverage ratios in the case $z = \bar{z}$), $\mu_L$ firms pay out dividends when the leverage ratio is relatively low, and start issuing equity only when they become highly leveraged.
5 Parametrization and Estimation

Strategy I split all parameters into two groups. Parameters in the first group govern the behavior of the model economy at the steady-state; I assign values to these parameters so that the model hits a set of empirical targets. The second group consists of persistence and volatility parameters for aggregate stochastic processes which I jointly estimate using Bayesian methods.

5.1 Steady-State Parameters

In this section, I provide a heuristic identification argument that justifies the choice of targeted moments. Even though every targeted moment is simultaneously affected by all the parameters, here I discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

The period in the model is one year, which matches the data frequency. I set the discount factor $\beta = 0.96$, the labor share $\nu = 0.67$ and capital share $\alpha = 0.33$. Furthermore, I set depreciation rate $\delta = 0.06$, and make the tax rate equal 0.35 (Graham, 2000; Guo, 2019). Entering firms start with the initial capital $k_0$, which is calibrated to match the relative size of entrants. Exogenous exit shock arrives with probability $\pi_d = 0.03$, which generates the 0.05 average exit rate of mature (aged 21 years and above) firms.

I let the instantaneous utility function of the household be separable between consumption and labor (Equation 24). Following the parametrizations of Khan and Thomas (2008) and Bloom et al. (2018), I set the utility curvature parameter equal 1 with the inverse Frisch elasticity of 0. Parameter $\theta_{ss}$ is the disutility from labor at the steady-state, and is set to make the household devote a third of its time endowment to market work.

The elasticity of the matching function $\phi$ is informative about volatility of firm entry over the business cycle; I, therefore, target the volatility of the number of startups relative to GDP. Parameter $\chi$ is picked so that the total amount of resources devoted to entry equals 7% of GDP (Sedláček and Sterk, 2017). Collateral constraint parameter $\rho$ directly affects the degree to which firms are financially constrained in the model; I require the model to deliver the mean leverage ratio of 0.27. Idiosyncratic productivity process parameter $\sigma_z$ along with the capital adjustment cost parameter $\varphi_K$ affect investment decisions of firms. I, therefore, include the mean investment rate as well as the frequency of investment spikes.

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20The data on the number of new firms are from the BDS (1977-2017).
21This is the average leverage ratio in the FAME dataset (see Appendix A.5 for the description of the data), which is in the ballpark of values used in the literature (e.g., Khan and Thomas, 2013; Guo, 2019).
Table 4: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target/Source</th>
<th>Data Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Labor share</td>
<td>0.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Utility curvature</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Inverse Frisch elasticity</td>
<td>$\lim \gamma \to 0$</td>
<td>Khan and Thomas (2008)</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of idiosyncratic AR(1)</td>
<td>0.86</td>
<td>Khan and Thomas (2008)</td>
<td>Graham (2000)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax rate</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\theta_{ss}$ Disutility from labor 2.04 N — 0.33
$\phi$ Elast. of match. fun. 0.81 $\sigma(\text{entry})/\sigma(Y)$ 2.98 2.79
$\chi$ Entry cost 0.35 $\sum_{j=1}^J \chi e_j/Y$ 0.07 0.05
$\rho$ Tightness of coll. constraint 0.31 $\mathbb{E}_{\{z\}} \{z\}$ 0.27 0.26
$\sigma_{\epsilon}$ Std of idiosyncratic AR(1) 0.17 $\mathbb{E}_{\{z\}} \{z\}$ 0.13 0.13
$\varphi^E$ Equity issuance cost 0.25 Dividends/GDP 0.06 0.14
$\pi_d$ Exogenous exit probability 0.03 Exit rate at age 21+ 0.05 0.05
$\xi$ Upper bound oper. cost 0.10 Exit rate at age 5 0.11 0.08
$\varphi^K$ Adjustment cost 1.32 Freq. of inv. spikes 0.19 0.24
$k_0$ Capital endowment 0.35 Relative emp. of entrants 0.32 0.32
$\mu_z$ Mean productivity of entrants -0.12 Relative prod. of entrants 0.75 0.77
$b_0$ Initial debt position 0.35 Entrants borrow $k_0$ — —
$\Delta \mu$ Medium DRS 0.82 Relative emp. ages 0 and 2 1.51 1.52
$\psi^1$ Availability of low type 0.05 Firm size (P50-P10)/(P90-P10) 0.16 0.21
$\psi^2$ Availability of med. type 0.39 Firm size (P90-P50)/(P90-P10) 0.84 0.79

in the set of empirical targets.\textsuperscript{22} I assume that the cost of issuing equity is quadratic:

\[ C(E_j) = \varphi^E \left[ \max\{0, E_j\} \right]^2. \]

Such formulation implies that firms find it increasingly hard to raise equity. Parameter $\varphi^E$ is chosen to match the dividend-to-GDP ratio.\textsuperscript{23}

New firms are assumed to draw initial idiosyncratic productivity from a normal distribution $F_z(z)$ with mean -0.12 and variance 0.17; such parametrization implies that new firms are on average 25% less productive than incumbents (Lee and Mukoyama, 2015). The upper bound of the operating cost shock $\xi$ is chosen to match the exit rate of 5-year old firms.

In my quantitative implementation, I set the number of types equal $J = 3$. Span of control parameters $\{\mu_j\}_{j=1}^J$ are chosen to hit the growth profile of young firms. In particular, I target the size of entrants relative to age 2 firms, as well as the relative size of firms aged 3 and 5. Availability parameters $\{\psi^j\}_{j=1}^J$ are picked to generate a right-skewed firm-size distribution; I target shares of the overall dispersion in firm size accounted for by the left and right tails. Table 4 reports the estimates for the structural parameters.

\textsuperscript{22} Investment spike is a situation when the investment rate exceeds 20% in absolute value.

\textsuperscript{23} The share of net corporate dividend payments in GDP over the period 2005-2020 (Federal Reserve Economic Data series B056RRC1A027NBEA).
5.2 Model Fit

Figure 5 shows the key firm demographics moments. Panel A depicts the average employment by firm age in the model and in the data. While the growth of firms in their first 5 years of tenure was targeted, the model correctly captures the growth patterns way beyond the first few years of the firm’s life-cycle without being targeted.

Panel B shows that the model delivers the downward sloping profile of exit rates. While the exit rates at ages 5 and 21 were targeted, the entire profile is close to the empirical one. Thus, the model is capable of picking up a well-documented up-or-out pattern of firm growth, according to which young firms grow fast conditional on survival (Haltiwanger et al., 2013).

The model’s success in generating an empirically relevant profile of exit rates is also reflected in panel C, which plots the distribution of firms by age. Entering businesses account for about 10% of all firms, and cohorts lose about one-half of their count due to firm exit by the age of 5. The model captures this pattern well even though it was not targeted. Appendix B.4 provides additional details about average life-cycle profiles of firms.

5.3 Estimation of Aggregate Shocks

I introduce three exogenous aggregate stochastic processes: a shock to the tightness of the collateral constraint $\rho$, a disutility of labor shock $\theta$, and an aggregate productivity shock $Z$. I assume that shocks follow AR(1) processes in logs:

$$\log X_{t+1} = \rho X \log \tilde{X}_t + \varepsilon_{X,t+1}, \quad \varepsilon_{X,t+1} \sim \mathcal{N}(0, \sigma_X^2), \quad X \in \{ \rho, \theta, Z \},$$  \hspace{1cm} (25)

where $X_t = X_{ss} \times \tilde{X}_t$. The collateral shock directly affects the ease with which firms in the model can issue debt and finance their investment. The model developed in Section 4 features frictionless labor markets; therefore, in order to account for the distressed labor markets during recessions, I introduce the disutility of labor shock $\theta$. This is a parsimonious way to model disruptions originating in the labor market. Finally, the aggregate TFP shock is a reduced-form way to account for alternative, deeper mechanisms, not captured by the benchmark model (Kehoe, Midrigan and Pastorino, 2018).\footnote{Chari, Kehoe and McGrattan (2007) develop a business cycle accounting procedure and argue that an empirically successful business cycle model should feature mechanisms which would manifest themselves as labor and efficiency wedges in a prototype neoclassical growth model. I introduce these wedges directly into my model by including labor disutility and TFP shocks.}

I use aggregate time series on GDP, hours worked and investment expenditures for the U.S. economy in order to estimate the persistence and volatility of the three stochastic
Figure 5: Model Fit

Notes: Figure 5 consists of three panels. Panel A plots the average employment by age group (with a normalization of 1 at the age of 1). Gray bars correspond to the model, black bars depict the data (BDS). Panel B plots the average exit rate by firm age. Panel C plots the firm-age distribution.

Aggregate Shocks and Business Formation

Aggregate shocks have important implications for the formation of different firm types (Figure 6).\textsuperscript{25} Panels A and C demonstrate that a negative TFP as well as a positive labor disutility shocks reduce the entry of low-, medium-, and high-type firms equally; this occurs because both shocks affect the expected

\textsuperscript{25}See Appendix B.7 for additional results.
Notes: Figure 6 plots impulse-response functions to one standard deviation innovations in TFP (panel A), financial (panel B), and labor disutility (panel C) stochastic processes. Persistence and volatility of exogenous stochastic processes are as in Table D8.

value similarly across the types according to Equation (21). The financial shock affects high-type firms stronger (panel B)—given that such businesses require more resources to get up to scale—and compress their entry more strongly as compared with lower span of control firms. Thus, it is the financial shock which primarily accounts for the compositional shift from high- to low-profile businesses in recessions.

5.4 Business Cycle Statistics

The model does a good job in picking up the key aspects of business cycle fluctuations in the U.S.: Investment is substantially more volatile than GDP, while consumption is less volatile
Table 5: Unconditional Business Cycle Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Volatility</th>
<th>Cyclicality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$\sigma(Y_t)$</td>
<td>1.21%</td>
<td>1.55%</td>
</tr>
<tr>
<td>$\sigma(C_t)/\sigma(Y_t)$</td>
<td>0.98</td>
<td>0.48</td>
</tr>
<tr>
<td>$\sigma(I_t)/\sigma(Y_t)$</td>
<td>4.68</td>
<td>3.05</td>
</tr>
<tr>
<td>$\sigma(N_t)/\sigma(Y_t)$</td>
<td>1.09</td>
<td>1.15</td>
</tr>
<tr>
<td>$\sigma(E_{xt})/\sigma(Y_t)$</td>
<td>2.98</td>
<td>2.79</td>
</tr>
<tr>
<td>$\sigma(E_{nt}(\mu_L)_{it})/\sigma(Y_t)$</td>
<td>4.16</td>
<td>1.94</td>
</tr>
<tr>
<td>$\sigma(E_{nt}(\mu_M)_{it})/\sigma(Y_t)$</td>
<td>—</td>
<td>3.08</td>
</tr>
<tr>
<td>$\sigma(E_{nt}(\mu_H)_{it})/\sigma(Y_t)$</td>
<td>—</td>
<td>3.45</td>
</tr>
</tbody>
</table>

Notes: Table 5 reports the empirical and model-generated unconditional business cycle statistics. The aggregate U.S. data have annual frequency and spans the period 1976-2018. “Model” refers to an estimated model from Section 4. Prior to computing the statistics, the data were logged and HP-filtered with the smoothing parameter of 6.25 (as suggested by Ravn and Uhlig, 2002). Data source: Federal Reserve Economic Data.

Importantly, the formation of $\mu_H$ firms is more volatile than that of $\mu_L$ and $\mu_M$ enterprises; this result is consistent with the empirical evidence reported in Section 3.2.

On the cyclicality side, consumption, investment and hours are found to be strongly positively correlated with output. I also find that the model-implied cyclicality moments are quantitatively close to empirical ones. The model correctly captures the positive correlation of entry with output. The exit margin was not the focus of analysis, and the model delivers a negative correlation of firm exit with GDP; in the data, the correlation of exit with output is positive but low.

6 Compositional Effect and the Business Cycle

I start by introducing in Subsection 6.1 a version of the model with no intensive margin of firm entry. Subsection 6.2 argues that the ability of potential entrepreneurs to target their startup efforts toward projects of different optimal size is empirically relevant. In Subsection 6.3, I quantify the contribution of each of the exogenous aggregate shocks in driving the intensive margin of business formation.

6.1 Fixed Composition Model

An alternative version of the model laid out in Section 4 with no compositional shifts can be obtained by modifying the timing of the entry problem. In particular, I now assume
Table 6: Impact of Initial Conditions: Model vs. Data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>P10</td>
<td>P50</td>
<td>P90</td>
</tr>
<tr>
<td>( \Delta GDP_0 )</td>
<td>0.80***</td>
<td>0.12***</td>
<td>0.62***</td>
<td>1.79***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>

|                  |           |               |       |       |       |
|------------------|------------|---------------|-------|-------|
|                  | Full Model |               |       |       |
| \( \Delta GDP_0 \) | 0.31       | -0.02         | 0.35  | 0.89  |

|                  |           |               |       |       |       |
|------------------|------------|---------------|-------|-------|
|                  | Fixed Composition Model |       |       |
| \( \Delta GDP_0 \) | 0.33       | 0.40          | 0.27  | 0.38  |
| Year FE          | ✓          | ✓             | ✓     | ✓     |
| Age FE           | ✓          | ✓             | ✓     | ✓     |

Notes: Table 6 reports the results of OLS estimation of Equation (5) using the LBD (“Data”) and two datasets simulated from the full and fixed composition models. For the simulation step, I feed in aggregate shocks recovered from the data using Kalman filter. See Subsection 6.1 for the description of the fixed composition model.

that potential entrants still know the distribution of type availability (parameters \( \{ \psi^j \} \)), but can no longer target their entry efforts to any particular type. Instead, each potential entrant is assigned the type randomly—according to the induced probability mass function—upon successful entry. Under this formulation, the distribution of entrants across types will mechanically be business cycle invariant.\(^{26}\)

The new free-entry condition takes the following form:

\[
\chi \text{ cost of entry} = \frac{m_t}{e_t} \int \sum_{j \in J} \sum_{i \in J} \psi_j(k_0, b_0, z; S) dF(z). \tag{26}
\]

The difference between (21) and (26) is in the integrand on the right-hand side: Now potential entrants cannot know which project they will end up operating, and so in equilibrium they balance the entry cost \( \chi \) with an average (over types) value of running a firm. Therefore, it is the extensive (overall mass of entrants)—not the intensive (distribution of firms over types)—margin of firm entry which is allowed to fluctuate over time. Thus, the difference between the two models is fully accounted for by the compositional effect.

\(^{26}\)This means that in a fixed composition model, the share of new firms of type \( j \in J \) among entrants is \( \frac{m_j}{\sum_{i=1}^{m_j} m_i} \), where \( m_j \) is the mass of entrants of type \( j \) in the steady-state of the baseline model. These shares are business-cycle invariant by construction.
6.2 Evaluating the Role of Compositional Effect

I now illustrate that the full model is able to account for the compositional effect documented in Section 3, while the fixed composition model is counterfactual. In order to operationalize it, I simulate two panels of firms using both models and fit Equation (5) on the resulting datasets.

Table 6 shows that the full model generates a pattern which is similar to U.S. data; businesses which get started in expansions are larger, and the average effect is driven by the right tail. At the same time, the fixed composition model delivers similar elasticities of the left and right tails, which is sharply at odds with the data. Importantly, the success of the full model has not been targeted; thus, I take this result as a validation for the compositional effect.

Why is the fixed composition model off? According to that model, the mass of entrants increases in expansions, which in absolute terms leads to more low-type businesses. The influx of new firms saturates the bottom of the distribution, thereby increasing the responsiveness of the left tail.

6.3 Variance Decomposition of Aggregate Shocks

Which aggregate shocks drive the intensive margin of business formation? Figure 7 reports the results of the forecast error variance decomposition for the low- and high-type firm entry (Figure C3 provides the remaining results). While the TFP shock accounts for the largest share of fluctuations in entry of all businesses, the financial shock accounts for about 30-35% of variation in the formation of the high-type (panel B) and for only 10-15% of the low-type (panel A) firms. This threefold difference indicates that the formation of firms with a large target size is much more sensitive to conditions on financial markets; this echoes the importance of access to financing for entrepreneurial decisions to pursue growth-oriented projects (see Section 1).

7 Policy Implications

In this section, I study policy implications of the model and use Lucas (1987) measure to evaluate welfare changes associated with policy interventions; this criterion calculates the percent of additional lifetime consumption that must be endowed at all future dates and states to a representative household under no policy so that the expected discounted stream of utilities is the same as in the economy where the policy is implemented. In other words, welfare benefits are measured in consumption equivalent units.
Stimulating Business Formation I consider a policy according to which the capital endowment of new firms gets increased by $d$ percent. Provided that in the model capital is in units of the final good, the higher startup capital endowment lowers consumption of the representative household.

The quantitative results of this policy are summarized in Figure 8. I find that the policy can generate a sizable welfare improvement in case it targets high-profile enterprises (about 0.4% of lifetime consumption at $d = 5.1\%$) even though the amount of capital only in the first period of firms’ operations gets increased. These gains arise because there are few high-type firms in the model, but which collectively account for a significant share of investment and employment growth. Therefore, the government can generate a welfare improvement by devoting a small fraction of resources to subsidizing the formation of such businesses.

In stark contrast with the targeted policy case, my simulations show welfare losses when the policy is applied to all firm types; this occurs because the welfare gains associated with a higher entry of low- and medium-type businesses do not outweigh the costs.

Discussion The main goal of the policy exercise described above was to illustrate the existence of sizable welfare gains associated with a more enthusiastic entry of growth-oriented businesses and to contrast it with a more generous policy, rather than to advocate for any particular policy specification. Nevertheless, the central question for the policymaker is how to stimulate the formation of high-profile businesses. One possibility is to reallocate resources
Figure 8: Assessment of Policy, % of Lifetime Consumption

Notes: Figure 8 reports welfare gains associated with the policy described in Section 7. The reported numbers are in percent of lifetime consumption.

to programs that target high growth companies; one such example is the U.S. Small Business Innovation Research Program which supports commercially viable R&D projects at small companies (Shane, 2009). Another example of a successful government initiative is the Small Business Investment Company program which uses public funds to supplement the flow of private equity capital and long-term loan funds to small businesses with high potential. The required resources can be raised by way of giving up costly transfer and subsidy schemes (e.g., home office tax deductions in the U.S.) that give marginal entrepreneurs incentives to start businesses.

8 Conclusion

Building on the insights from management literature, this paper adopts a notion of locus of control, according to which entrepreneurs have a control over the long-run (or target) size of their firms—a choice business owners make upon entry. By drawing on the administrative records of U.S. businesses, I first show that cohorts of firms entering in recessions are on average smaller than those entering in expansions, and that this average effect is primarily driven by the feature whereby firms which eventually become large tend not to be started in downturns. I also provide evidence that financial disruptions discourage the formation of high-profile firms more strongly than that of low-profile ones.

Motivated by this empirical evidence, I build and estimate a general equilibrium model of firm dynamics in which these patterns arise naturally. The primary model ingredients are the
ability of potential entrepreneurs to direct their startup efforts toward projects of different optimal size, and that firms’ growth is hampered by financial frictions. The estimated model shows that the intensive margin of firm entry is an empirically relevant mechanism, and that the financial shock accounts for the compositional shift from high- to low-profile firms in downturns. From the policy perspective, I highlight the importance of improving the quality of startups rather than increasing their quantity.

Although this paper studies business entry from the perspective of the firm dynamics literature, new insights can be obtained by examining this phenomenon through the lens of occupational choice models. For instance, such frameworks can speak to the labor market origins of the compositional effect and yield new policy prescriptions. Second, more empirical work is needed to better understand decisions of entrepreneurs to start firms with different growth potential. Individual labor market experiences, as well as educational, family, and demographic factors, can play an important role in this process. Exploration of these issues is a fruitful avenue for future research.
References


Vardishvili, Ia, “Entry Decision, the Option to Delay Entry, and Business Cycles,” *manuscript*, 2020.

SUPPLEMENTAL APPENDIX
FOR ONLINE PUBLICATION

“Locus of Control and Business Formation”

by Vladimir V. Smirnyagin
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Appendix A: Empirical Appendix

This appendix provides further details for the empirical part of the paper, including data background, sample selection and additional empirical results referenced throughout the main text.

A.1 LBD

The Longitudinal Business Database (LBD) is the comprehensive panel dataset covering the universe of U.S. businesses with at least one paid employee and spanning the years 1976-2016. The unit of observation in the LBD is an establishment, which is defined as a single physical location where business operations take place. A firm is then defined as a set of establishments that are under common ownership or control.

The LBD is based on several sources, such as the Business Register (also known as the Standard Statistical Establishment Listing—SSEL), Economic Censuses, and surveys. The LBD offers the most reliable and complete data on births, deaths, and age of establishments operating in the US. There are several data issues potentially leading to measurement errors in identification of business formation (for example, non-administratively registered establishments may not be correctly identified, or gaps in the records of establishments). See Jarmin and Miranda (2002) and Chow et al. (2021) for more details on which efforts have been undertaken to mitigate these issues in the process of the LBD construction.

A.1.1 Identification of Birth and Death

The unit of observation in the LBD is an establishment, and variable lbdnum—which is robust to mergers and acquisitions—is used to track establishments over time.

Establishments The age of an establishment is measured as the number of years elapsed since the first year the establishment appeared in the data. Age cannot be measured for establishments born prior to 1976—the first year covered by the LBD. For that reason, in my empirical exercise I exclude the cohort of 1976.

Firms Identification of firm birth and death is associated with the construction of firm linkages over time. I follow a standard approach in the literature which is robust to ownership changes and acquisitions (Haltiwanger, Jarmin and Miranda, 2013). A new firm identifier emerges in the LBD either because a new firm is born or because an existing businesses undergoes a change of ownership and control (e.g., merger and acquisition, divestitures). I register a new firm when all of its establishments are of age 0. Accordingly, when a new firm identifier arises through a merger of two preexisting firms, it is not treated as a firm birth; rather, it is assigned the age of the oldest continuing establishment of the newly combined business. The firms are then allowed to age naturally regardless of mergers and acquisitions as long as the ownership and control do not change. A firm death occurs when a firm identifier disappears and all associated establishments cease operations and exit.

27 Moscarini and Postel-Vinay (2012) and Moreira (2016a) argue that there are concerns with the first couple of LBD cohorts; I, therefore, use cohorts starting from 1978.
Table A1: Summary Statistics: LBD

<table>
<thead>
<tr>
<th></th>
<th>Sample 1</th>
<th></th>
<th>Sample 2</th>
<th></th>
<th>Sample 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Emp</td>
<td>Payroll</td>
<td>Age</td>
<td>Emp</td>
<td>Payroll</td>
</tr>
<tr>
<td>Mean</td>
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<td>2,300</td>
<td>4</td>
<td>12</td>
<td>1,300</td>
</tr>
<tr>
<td>Std</td>
<td>169</td>
<td>234,000</td>
<td>3</td>
<td>205</td>
<td>217,000</td>
</tr>
<tr>
<td>P10</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>P50</td>
<td>3</td>
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<td>4</td>
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</tr>
<tr>
<td>P90</td>
<td>17</td>
<td>514</td>
<td>9</td>
<td>18</td>
<td>596</td>
</tr>
<tr>
<td>Obs. (1000s)</td>
<td>78,540</td>
<td></td>
<td></td>
<td>47,830</td>
<td></td>
</tr>
<tr>
<td>Num. firms (1000s)</td>
<td>17,740</td>
<td></td>
<td></td>
<td>5,249</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table A1 reports the summary statistics for the three samples sourced from the LBD. For the description of samples, see Appendix A.1.5. “Emp” is the number of paid workers, “Payroll” is the total wage bill (in thousands of dollars) converted to constant 2010 dollars. “Age” is a number of years elapsed since the time of firm entry.

A.1.2 Employment

Establishments Employment (LBD variable emp) is defined as the number of full- and part-time employees as of March 12th.\(^{28}\)

Firms Firms can own a single establishment or many establishments, which may span multiple geographic areas and industries. Naturally, firm-level employment and payroll are calculated as the sum of employment and payroll across the establishments constituting that firm.

A.1.3 Industry

The LBD also includes detailed information on industry classification. The period of analysis covers the the transition from SIC to NAICS industry classification standards (which occurred in 1997), which leads to well-known classification issues. I, therefore, use a consistent NAICS 2012 industry classification variable \texttt{fk\_naics12} constructed by Fort and Klimek (2016).

Firms are frequently comprised of establishments from several industries. For that reason, to each firm I assign an industry of its largest (in terms of employment) establishment on the year-by-year basis.

A.1.4 Geographical Location

LBD also provides detailed information on the physical location of establishments. I use two pieces of information—the state and county FIPS codes—to assign establishments to counties.\(^{29}\) Similar to the case of the industry classification, I assign each firm a location of its largest establishment.

---

\(^{28}\)This measure includes employees who are on paid sick leave, holidays, and vacations. The reported number also includes salaried officers and executives of corporations, but it excludes sole proprietors and partners of unincorporated businesses.

\(^{29}\)A combination of a state and county FIPS codes gives rise to a unique identifier of U.S. counties.
A.1.5 Creation of Samples

Throughout the analysis, I consider three different samples sourced from the LBD.

**Sample 1**  The first sample consists of all firms which appeared in the LBD for the first time between 1978 and 2006, and which I track up to the age of 10.

**Sample 2**  The second sample is essentially a subset of Sample 1, where I keep only those businesses which survived through at least the age of 10.

**Sample 3**  This sample covers the same set of firms as Sample 1, but this time I track enterprises until 2016 (the last year contained in the LBD), or until they exit.\(^{30}\)

Table A1 reports summary statistics for these samples.

\(^{30}\)The 2017 and 2018 LBD snapshots became available only at late stages of the project. To avoid repeating extensive census disclosure analysis, I use data only up to 2016.
A.2 ASM and CM

The Annual Survey of Manufacturers (ASM) and the Census of Manufacturers (CM) are establishment-level datasets which cover the U.S. manufacturing sector (NAICS 31-33). As in the case of the LBD, the unit of observation is an establishment, which is defined as a single location where business is conducted. Currently, the ASM/CM are available for years 1976-2015.

A.2.1 General Information

Both the ASM and the CM are mail-back surveys; the CM covers the Census years (ending in 2 and 7), and the ASM covers the years in between them. The ASM/CM contain the information about plants in which the predominant activity is production; thus, purely administrative establishments are not included. The CM covers all the manufacturing establishments in the U.S., which amounts to 300-350 thousand observations per year. In turn, the ASM covers plants from the “mail stratum” of the manufacturing sector, which results in 50-60 thousand observations per year. The “non-mail stratum” generally consists of small establishments that collectively account for a very small fraction of aggregate activity; their chance to be selected in the ASM panel is zero. Following Kehrig (2015), in order to construct a consistent panel where the number of (weighted) observations is not driven by the sampling practices of the Census, I drop all observations from the non-mail stratum (denoted by $ET = 0$). The ASM covers all “large” establishments with certainty along with a selection of “small” establishments. The ASM is essentially a rotating panel, since every five years (years ending in 4 and 9) Census updates its small establishment sample. The Census provides frequency weights (the inverse of the sampling probability) which I use to infer the underlying population of manufacturing plants not surveyed by the Census.

A.2.2 Construction of Plant-level Variables

The ASM/CM contain a wealth of information on plants’ sizes, productivities, inputs, sales, etc. For the purposes of this project, I only need a subset of this information. In what follows, I describe how I construct different variables using the raw data from the ASM/CM data.

Measure of Real Output

Ideally, I need to obtain a measure of real production. Unfortunately, neither plant-level real output, nor prices are available. As a result, I construct a proxy for the real output following the methodology of Kehrig (2015) and Yeh (2017). In particular, I combine information on:

- total value of shipments ($TVS$),
- beginning- and end-of-year works-in-progress ($WIB$ and $WIE$),
- beginning- and end-of-year inventories ($FIB$ and $FIE$).
Provided that deflators for inventories are not publicly available (Kehrig, 2015), I use the 6-digit NAICS industry-level shipment price deflator $p_{ship}$ from the NBER-CES Manufacturing database. As a result, I construct a measure of real output of plant $p$ in year $t$ as follows:

$$Q_{p,t} = \frac{tv_{ship_{i(p),t}}}{p_{ship_{i(p),t}}} + \frac{fie_{p,t} - fib_{p,t}}{p_{ship_{i(p),t}}} + \frac{wie_{p,t} - wib_{p,t}}{p_{ship_{i(p),t}}},$$

(A.1)

where $i(p)$ denotes a 6-digit NAICS industry plant $p$ operates in.

**Labor Input** I measure labor input as a total number of hours worked. However, the ASM/CM provide the total number of hours worked for production workers only ($ph$). I follow Lee and Mukoyama (2015) and Yeh (2017), and combine two additional pieces of information to infer the total hours worked. In particular, the ASM/CM provide information on the total payroll ($sw$) and the wage bill for production workers ($ww$). I then construct the labor input as follows:

$$L_{p,t} = ph_{p,t} \times \frac{sw_{p,t}}{ww_{p,t}}.$$

(A.2)

In rare cases when either the total payroll or production workers’ wage bill is zero or negative, I use the production hours $ph_{p,t}$ as a measure of the labor input.

**Materials Input** I measure materials as the sum of expenditures on materials and parts, resales and contract work. I deflate nominal values with a 6-digit materials deflator $p_{imat}$ from the NBER-CES data. Specifically, the value of materials is then:

$$M_{p,t} = \frac{c_{p,t} + cr_{p,t} + c_{wp,t}}{p_{imat_{i(p),t}}}.$$

(A.3)

On a side note, some papers (Kehrig, 2015) treated the value of resales $cr$ as finished goods rather than materials, since resales are not used in the production process. I experimented with this alternative classification of resales and found my results to be robust:

**Energy Input** The plant-level expenditures on energy is the sum of expenditures on fuels ($cf$) and electricity ($ee$). I deflate nominal values by the 6-digit NAICS deflator $p_{ien}$ from the NBER-CES Manufacturing database. As a result, the real value of the energy input is:

$$E_{p,t} = \frac{cf_{p,t} + ee_{p,t}}{p_{ien_{i(p),t}}}.$$

(A.4)

**Capital** The construction of capital is complicated by several factors. First, the values of capital stock are reported only for years 1976-1987 (with an exception of 1986) and 1992. Second, in those years when capital stocks are reported, only the book values are available. Moreover, the imputation of capital stocks for the remaining years is complicated by the absence of information on the plant-level depreciation.

---

Fortunately, the ASM/CM report capital expenditures for all years, which makes it possible to construct a measure of capital using forward and backward inventory methods. I consider two types of capital: structures and equipment. In what follows, I describe a sequence of steps I undertake to construct a consistent over years measure of the capital input.

For plants which entered in or before 1985, I convert the reported end-of-year stocks of structures (\( \text{bae} \)) and equipment (\( \text{mae} \)) into market values using the current and historical industry-level cost of capital stocks from the BEA Fixed Asset Tables. The ASM/CM do not provide the breakdown of the end-of-year total assets (\( \text{tae} \)) into structures and equipment starting from 1988 (Census year 1992 is an exception). I, nevertheless, can recover the capital stock for the remaining years for establishments which entered in or before 1985 using the forward perpetual inventory method, since the data report capital expenditures on structures (\( \text{cbe} \)) and machinery (\( \text{cme} \)) for all years. In particular, the stock of structures and machinery for plant \( p \) in year \( t \) is constructed according to the following equations:

\[
K_{st,p,t} = (1 - \delta_{i(p),t})K_{st,p,t-1} + \frac{\text{cbe}_{p,t}}{\text{piinv}_{i(p),t}},
\]

\[
K_{eq,p,t} = (1 - \delta_{i(p),t})K_{eq,p,t-1} + \frac{\text{cme}_{p,t}}{\text{piinv}_{i(p),t}},
\]

where \( \delta_{i(p),t} \) and \( \delta_{i(p),t} \) are the 3-digit depreciation rates from the BLS Capital Tables. Deflator \( \text{piinv} \) is available at 6-digit NAICS level from the NBER-CES Manufacturing database.

For plants which first show up in the ASM/CM sample after 1987, I initialize the capital stock using the nearest Census year when the plant is still active (the total value of assets is only reported in the CM). For that purpose, I leverage information from the NBER-CES on the amounts of industry-level capital stocks of equipment and structures. In particular, I split the plant-level amount of total assets across equipment and structures according to the 6-digit industry-level distribution of capital across equipment and structures. Once the capital stock is initialized, I use the forward and backward perpetual inventory methods to impute capital in non-Census years.

**Cost Shares** In order to estimate returns to scale, I need to obtain the cost shares of production inputs. I calculate these shares for 3-digit NAICS industries as follows:

---

32 The BEA contains information at the 3-digit NAICS level with some exceptions. In particular, BEA groups industries with NAICS codes 311 and 312 into “Food and beverage and tobacco products”, 313 and 314 into “Textile mills and textile product mills” and 315 and 316 into “Apparel and leather and allied products”. I perform necessary adjustments to make these groupings consistent with the NAICS classification. The data are available at [https://www.bea.gov/national/FA2004/SelectTable.asp](https://www.bea.gov/national/FA2004/SelectTable.asp).

33 For some years, the data only report total capital expenditures (\( \text{tce} \)) along with capital expenditures on new and used machinery (\( \text{cme} \)). I calculate capital expenditures on structures as the difference between \( \text{tce} \) and \( \text{cme} \).

34 Data is available at [https://www.bls.gov/mfp/mprdload.htm](https://www.bls.gov/mfp/mprdload.htm).
\[sl_{i,t} = \frac{pay_{i,t}}{tc_{i,t}},\]
\[skeq_{i,t} = \frac{eqrkl_{i,t} \times equip_{i,t}}{tc_{i,t}},\]
\[skst_{i,t} = \frac{strkl_{i,t} \times plant_{i,t}}{tc_{i,t}},\]
\[sm_{i,t} = \frac{matcost_{i,t} - energy_{i,t}}{tc_{i,t}},\]
\[se_{i,t} = \frac{energy_{i,t}}{tc_{i,t}},\]

where \(tc_{i,t} = pay_{i,t} + eqrkl_{i,t} \times equip_{i,t} + strkl_{i,t} \times plant_{i,t} + matcost_{i,t}\). In order to reduce the noise in the industry-level estimates of cost shares (Lee and Mukoyama, 2015; Yeh, 2017), I consider divisia-based estimates of cost shares:

\[dx_{i,t} = \frac{sx_{i,t} + sx_{i,t-1}}{2},\]  

where \(x \in \{l, keq, kst, m, e\}\).

**A.2.3 Estimation of Returns to Scale**

Following Basu and Fernald (1997) and Lee and Mukoyama (2015), I assume that a firm combines labor \(L\), materials \(M\), energy input \(E\), structures \(K^{st}\) and equipment \(K^{eq}\) to produce the gross output \(Y\):

\[Y = F(L, M, E, K^{st}, K^{eq}).\]  

Cost minimization implies that returns to scale \(\gamma\) equals the ratio of average to marginal cost. Taking the log difference of the production function (A.6) yields:

\[dy = \gamma (dl \times dL + dm \times dM + de \times dE + dkst \times dK^{st} + (1 - dl - dm - de - dkst)dK^{eq}) + dt\]
\[= \gamma dx + dt,\]

where \(dy\) is the growth rate in output, and \(dL, dM, dE, dK^{st}\) and \(dK^{eq}\) are the growth rates in corresponding inputs. The growth rates are computed as log differences. The last term \(dt\) captures growth in productivity.

**Measurement Challenges** While the cost shares are in principle plant-specific, it is standard to measure them using industry-level cost shares. Moreover, a potential problem associated with a plant-level analysis is the attenuation bias caused by measurement errors. Provided that the estimation of returns to scale requires measuring changes in inputs and outputs, first-differencing may magnify the attenuation bias. As a standard response to
errors in variables, I instrument $dx$ by the cost-share-weighted growth in inputs measured over $t + 1$ and $t - 2$. As Lee and Mukoyama (2015) point out, if measurement errors are not serially correlated, an IV estimation using the instrument will yield consistent estimates of returns to scale.
A.3 Robustness Checks for Section 3.3

This section provides additional details for the robustness checks referenced in Section 3.3.

Alternative Samples I re-estimate Equation (5) on two alternative samples. In the first case, I reduce the impact of non-random attrition by way of dropping businesses which did not make it to the age of 10. I find qualitatively similar but quantitatively more pronounced results (Table D1).

Provided that a ten year horizon may be insufficient for certain firms to reach their target size, in the second alternative sample I track businesses for as long as they stay in the sample. Table D2 reports the results and confirms the differential sensitivity of the left and right tails of the firm-size distribution to aggregate conditions at inception.

Persistence of Initial Conditions To this end, I consider a version of Equation (5) augmented with an interaction term between firm age and conditions at inception. Table D3 shows that the impact of initial aggregate conditions is highly persistent, as it does not vanish as firms age.

State-wide Business Cycles I also explore the robustness of the baseline result to state-wide business cycles. To this end, I merge the LBD with County Business Patterns (CBP) data, and use state-wide fluctuations in employment to identify local business cycles. Again, I find qualitatively similar results (Tables D4 and D5).

Establishment-level Evidence Provided that establishment managers have a substantial independence in making hiring and investment decisions in the U.S. (Bloom et al., 2012), I also estimate the elasticities of left and right tails of the plant-size distribution to entry conditions and find broadly similar results (Table D6).

---

35 In this case, I am able to track some firms up to the age of 39.
A.4 Local Bank Lending Shocks

It is notoriously hard to disentangle shifts in demand from credit supply shocks. In this section, I study the impact of disruptions in small business credit supply on subsequent cohort profiles—this approach is based on the work of Greenstone et al. (2020). The idea is to exploit the variation in banks’ market shares across U.S. counties to compute each county’s exposure to commercial banks.\textsuperscript{36} Intuitively, this measure exploits the notion that some banks reduce their originations of small business loans more strongly relative to the rest of the banking sector, and that some counties are more exposed to such banks than others.\textsuperscript{37}

First, I use information on the amount of small business loans from the CRA data to fit the following equation for every pair of consecutive years:

$$g_{b,c,t} = \gamma_b + \gamma_c + \varepsilon_{b,c,t}, \quad (A.7)$$

where \(g_{b,c,t}\) is the growth rate in the amount of small business loans between years \(t\) and \(t-1\) originated by bank \(b\) in county \(c\), and \(\gamma_b, \gamma_c\) are bank and county fixed effects, respectively.\textsuperscript{38}

Next, to estimate a locally exogenous component of the growth in small business lending to county \(c\), I construct a Bartik-like measure as follows:

$$SBL_{c,t} = \sum_b \hat{\gamma}_b \times \omega_{b,c,t-1}, \quad (A.8)$$

where \(\omega_{b,c,t-1}\) is bank \(b\)’s share of small business lending in county \(c\) at \(t-1\). Thus, the constructed measure of the local credit disruption is an average computed over each bank’s change in lending (net from its exposure to unobservable local demand shocks), weighted according to the pre-existing branch footprint.\textsuperscript{39}

I then estimate the following equation:

$$\ln Y_{t,a,i,s} = \beta_0 SBL_{c,0} + \gamma X_{t,a,i,s} + \varepsilon_{t,a,i,s}. \quad (A.9)$$

where \(SBL_{c,0}\) is a county-level lending shock at the time of entry, and \(Y\) is either employment or payroll. Provided that it is hard to interpret results in case of multi-unit enterprises operating in several counties, I conduct this analysis at the establishment-level.

Table D14 reports the result. I find that county-level lending shocks are not associated with a statistically significant response of the left tail. However, the estimates suggest that a one standard deviation increase in local small business lending at the time of entry leads to a statistically significant 0.3 (0.4)\% increase in the 90\(^{th}\) percentile of employment (payroll).

\textsuperscript{36}Davis and Haltiwanger (2019) and Granja and Moreira (2020) are two closely related studies which utilize similarly constructed bank lending shocks.

\textsuperscript{37}Naturally, this approach relies on the assumptions that small business lending is inherently local and that borrowers cannot easily obtain financing from other sources. These assumptions are plausible for young firms which I am focusing on in this paper.

\textsuperscript{38}Although the CRA data explicitly specify loans to small businesses, it is reasonable to assume that there is a considerable overlap between credit supply shifts for small and young business lending (Davis and Haltiwanger, 2019).

\textsuperscript{39}I also standardize the obtained bank lending shocks for the ease of interpretation.
A.5 Evidence from the U.K.

This section provides further details for the U.K. data. First, I describe the data background. Subsequently, I lay out the details for empirical exercises mentioned in Section 3.

A.5.1 Data Background

**Firm-Level Data** The empirical results are based on a large panel dataset of firms’ financial accounts called Financial Analysis Made Easy (FAME), provided by the Bureau van Dijk (BvD). This dataset is different from commonly used Orbis and Amadeus in that it covers only U.K. registered firms. FAME is a live panel meaning that its information is accurate only at the moment of filing and not for historical reference. In order to improve the coverage, identify entry and exit of firms, multiple vintages of FAME have been combined—see Bahaj, Foulis and Pinter (2020) for a detailed explanation of this process.

The data cover the corporate universe of U.K. firms for the period 1995-2017, and encompasses approximately 1.5mln private and public firms per year. The data include both the firms’ balance sheet (assets and liabilities, debt structure, issued capital) and income statements (operating profit, turnover, cost of sales, etc.). Information on firms’ directors—a group of people (or a single person) responsible for running and promoting a firm—is also reported.

**Real Estate Price Data** Residential housing data come from the Land Registry’s Price Paid dataset (covering England and Wales) and the Registers of Scotland. These datasets cover the universe of residential property transactions since 1995. BvD provides residential addresses of firms’ directors, enabling one to measure the residential wealth of directors whose property was bought or sold at some point after 1995.\(^{40,41}\)

**Sample Selection** Standard cleaning procedures were applied to raw data. Financial and real estate sectors (FIRE), as well as firms which do not comply with Companies’ Act, were excluded. Outliers and observations for which the balance sheet identity did not hold were dropped. Summary statistics are provided in Table D9.

\(^{40}\) According to the UK law, directors must report several pieces of information (such as their name, date of birth and residential address among other things) when they register a firm. BvD contains this information.

\(^{41}\) Strictly speaking, not all directors are complete owners of their houses. Bahaj, Foulis and Pinter (2020) made use of the Product Sale Database (PSD)—an administrative data on all residential mortgages since 2005 at origination and on the stock of outstanding mortgages in 2015. The PSD is provided by the UK Financial Conduct Authority. The FCA Product Sales Data include regulated mortgage contracts only, and therefore exclude other regulated home finance products such as home purchase plans and home reversions, and unregulated products such as second charge lending and buy-to-let mortgages. PSD contains information on the borrower’s date of birth and the mortgaged property’s full postcode. Therefore, by linking directors’ home values with mortgage data, it is possible to construct a measure of *home equity*. They find that results remain largely unaffected when residential *equity* is used instead of residential real estate. This is not surprising given that approximately 90% of directors are homeowners.
A.5.2 Firm Growth and Responsiveness to Collateral Shocks

The empirical exercise consists of two steps. The first one is to classify firms into types depending on the realized growth profiles. Second, I study the responsiveness of firm-level investment to collateral shocks. In what follows, I describe these two steps in more detail. I also discuss identification issues and results.

**Assignment of Firms to Growth Groups** I follow a parsimonious approach and group firms based on how their growth rate of total assets—a uniformly reported measure of size—relates to the growth rates of their peers (same industry firms born in the same year). I assign a firm to the “fast” type if it grew faster than its median peer in at least half of the years of its tenure.\(^{42}\) I assign a firm to the “slow” type if it grew slower than its median peer in at least half of the years of its tenure. In addition to this classification, I run several robustness checks: In particular, I redefine the thresholds (40th and 60th percentiles instead of the median), increase the number of years a firm has to be above (below) its median peer in order to be classified as a “fast” (“slow”) type, and drop firms which did not survive through age 5.

**Empirical Strategy** Once I obtain the assignment of firms into groups, I study the investment response of different firm types to residential collateral shocks. Following Bahaj, Foulis and Pinter (2020), instead of measuring residential property of firm directors in every year independently, the real estate of each firm’s director is fixed at its 2002 level, and subsequently rolled forward based on the local housing price index. This approach is advantageous as it isolates fluctuations in residential wealth from potentially endogenous decisions of directors to move into bigger/smaller houses depending on the performance of their firms. Therefore, directors’ residential real estate for firm \(i\) at time \(t\) is measured as:

\[
Residential RE_{it} = \frac{\sum_{d \in \tilde{D_i}} L_{i,2002}^{d} L_{h_{d},it}^{P}}{|\tilde{D_i}|}, \tag{A.10}
\]

where \(L_{i,2002}^{d}\) is the estimated value of a house where the director \(d\) working at firm \(i\) lived in 2002, and \(L_{h_{d},it}^{P}\) is the house price index of the region \(h_{d}\) where that director lived in 2002 (with a normalization \(L_{h_{d},2002}^{P} = 1\)). According to Equation (A.10), the residential real estate for firm \(i\) is the average value of property across matched directors \(\tilde{D}_i\), multiplied by the total number of directors \(D_i\).\(^{43}\) The benchmark specification takes the following form:

\[
Investment_{it} = \alpha_i + \delta_{kt} + \mu_{it} + \sum_{j \in J} \eta_j \times 1_{\{i \in j\}} \times Residential RE_{it} + \gamma \times controls_{it} + \varepsilon_{it}, \tag{A.11}
\]

\(^{42}\)Technically, I compute median growth rates of total assets in 3 dimensional cells over age, 2-digit SIC code and year. Subsequently, for each firm, I calculate the number of years it grew faster than its peers, and divide it by the total number of years this firm was observed in the panel. I classify a firm into “fast” type if the resulting fraction exceeds 0.5. Similarly, I assign a firm to the “slow” type if it grew slower than its median peer in at least 50% of the periods in which it was observed in the sample.

\(^{43}\)As described in Bahaj, Foulis and Pinter (2020), the residential property of roughly 60% of directors was successfully valued.
where \textit{Investment} is the change in fixed assets plus depreciation, and \( J = \{\text{slow}, \text{fast}\} \). \( \alpha_i \), \( \delta_{kt} \) and \( \mu_{lt} \) are firm, region-time and industry-time fixed effects. Indicator function \( 1_{\{i \in j\}} \) takes a value of 1 if a firm \( i \) was assigned to group \( j \), and 0 otherwise. Standard errors are clustered at the level of the firm’s region. All monetary variables are scaled by lagged fixed assets, which provides a pound-to-pound interpretation of the coefficients.\textsuperscript{44} Thus, coefficients of interest \( \{\eta_j\}_{j \in J} \) show by how many pounds investment of type \( j \) firms will change if their directors’ residential wealth appreciates by £1.

**Identification** Before getting to the results, it is worth discussing some potential endogeneity concerns. The firm fixed effect \( \alpha_i \) absorbs any time-invariant omitted factors which affect firm’s behavior. The list of such factors includes the initial values of directors’ homes \( L_{d_i, 2002} \), as well as the number and composition of directors in 2002. It is also the case that \( L_{P_{ht},t} \) — the house price index for each director’s region — is typically correlated with the firm’s real estate price index \( L_{P_{j,t}} \). In turn, \( L_{P_{j,t}} \) could affect the firm’s investment opportunities; for example, by way of fueling local consumption (Mian and Sufi, 2014). I include region-time fixed effects \( \delta_{kt} \) in order to address this. Following Chaney, Sraer and Thesmar (2012), the vector of controls includes firm-level characteristics: a measure of the balance sheet strength (financial leverage) and a measure of cash flow (operating profits). As Bahaj, Foulis and Pinter (2020) point out, residential real estate does not naturally scale up with the firm’s size as, for instance, corporate real estate would; therefore, the vector of controls further includes the inverse of lagged fixed assets in order to eliminate any spurious size effects.

**Baseline Results** Table D15 reports my baseline results. It shows that the investment of rapidly growing firms is more responsive to idiosyncratic fluctuations in directors’ real estate than that of more slowly growing businesses. In particular, according to the tightest specification considered in column (8), a £1 appreciation of directors’ residential housing is associated with a 1.3p (pence) increase in fast-type firms’ investment, and with only 0.6p for the slow-type enterprises. This suggests that fast growing firms are more financially constrained than slow growing firms since the collateral shock leads to a twice bigger investment response among the former group.

**Robustness** Table D16 splits the data into young (under the age of 5) and old subsamples (columns (2) and (3)), and shows that young businesses respond stronger to collateral shocks as compared with more mature enterprises (point estimates are now 1.4p and 1p for fast- and slow-types, respectively). This is consistent with an idea that young businesses are usually below their target size, grow fast, and exhibit high investment demand. Furthermore, columns (4) and (5) in Table D16 show that larger firms (\( \geq 50 \) employees) respond stronger to fluctuations in residential wealth, potentially reflecting the higher investment expenditures such businesses need to undertake.\textsuperscript{45}

Arguably, one needs to observe a firm long enough in order to properly classify it. For that reason, Table D17 reports the estimates of the baseline specification for firms which

\textsuperscript{44} Chaney, Sraer and Thesmar (2012) also use lagged fixed assets as a scaling variable.

\textsuperscript{45} The subsamples with respect to size are constructed based on the time-average of employment within each firm. Therefore, a firm might be in the “large” subsample but be small (\(< 50\) employees) at some point.
Figure A1: Grouped Fixed Effects Estimation

(A) Controlling for business cycle (b) Controlling for business cycle and leverage

Notes: Figure A1 plots the distribution of firms across clusters based on the grouped fixed effects estimator by Bonhomme and Manresa (2015). White bars correspond to firms started in 2006-2007, and gray ones—to enterprises which entered in 2008-2009. The following linear model is considered:

\[ y_{ia} = \alpha_{g,a} + x'_{ia} \theta + \varepsilon_{ia}, \quad i = 1, \ldots, N, \quad a = 1, 8, \]

where \( y_{ia} \) is the real book value of assets of firm \( i \) of age \( a \), and \( x_{ia} \) is a vector of control variables. Panel A plots the resulting distribution of firms across clusters when \( x_{ia} \) contains the cyclical component of GDP from the HP filter with smoothing parameter 100 (annual frequency). The vector of controls in Panel B also includes the liability-based leverage of firms (ratio of total liabilities to total assets). Clustering was performed on a 20% random subsample of manufacturing firms, which I tracked for up to 8 years. The choice of age 8 as an upper bound was dictated by the panel’s duration: This is the maximal observable age of firms born in 2009. The exercise was repeated numerous times in order to ensure that the results are robust to different draws. See Appendix A.5.3 for further details. Source: BvD.

survive through age 5. I find that results are barely affected. Finally, in Table D18, I check how robust estimates are to alternative groupings of firms. I find that the estimates are largely unaffected when the requirement to spend half of tenure above (below) the median peer is increased to 75\% (column (2)), or when the threshold is shifted from the median to 40\% percentile for the slow type and the 60\% percentile for the fast type (column (3)). Finally, I re-estimate the model on the largest possible sample with no controls and total asset growth as a dependent variable (column (4)), and qualitatively confirm my baseline results.

A.5.3 Firm types and the Business Cycle

Evidence presented in Section 3 suggests that very large firms tend not to be started during economic downturns. From the model’s perspective, I interpret these findings by introducing heterogeneity in firms’ target sizes in Section 4.

In order to provide additional evidence in support of the view that fewer large firms get started in downturns, I apply a grouped fixed effects clustering algorithm developed by Bonhomme and Manresa (2015). The benefit of this approach is that it optimally assigns
firms into the pre-specified number of groups based on how similar firms’ time profiles of observables look like after the effect of controls has been taken out. By applying this estimator to the firm-level data, I can assign firms into types (small, medium and large) controlling for the sequence of aggregate shocks each firm went through.

I experiment with two different sets of controls. The first one only includes the cyclical (from HP filtering) component of GDP in order to control for the business cycle (Panel A in Figure A1). The second one adds the leverage ratio as a proxy for firms’ financial conditions (Panel B).\textsuperscript{46} In both cases, I obtain broadly similar results; the mass of high-profile firms is roughly 40\% smaller in recessionary cohorts, which is reflected in a relatively larger share of the small type.

\footnote{I use a liability-based leverage: a ratio of total liabilities to total assets.}
Appendix B: Model Appendix

The computation of the model can be broadly divided into three parts:

1. simplification of programming problems by way of combining household’s and firms’ optimization problems,
2. computation of the model at the steady-state,
3. solving for the equilibrium of the model with aggregate fluctuations using perturbation techniques.

In what follows, I lay out the key steps of the numerical algorithm. To facilitate the exposition, I assume there is only one type of firms $J = \{1\}$. I start with the formal definition of the recursive competitive equilibrium.

B.1 Definition of Equilibrium

A recursive competitive equilibrium for the economy laid out in Section 4 is a collection of functions:

$$\left\{ v_j, v_j^{\text{cont}}, v_j^{\text{exit}}, k_j', b_j', n_j, W, \rho_0, \rho_1, H, C, N, \Xi, d, m^j, e^j, \Gamma \right\}_{j \in J},$$

such that:

1. $H$ solves the household’s problem (23), and $(C, N, \Xi, \Lambda)$ are the associated policy functions,
2. $\{v_j, v_j^{\text{cont}}, v_j^{\text{exit}}\}$ solve the firms’ problem (8)-(19), and $\{k_j', b_j', n_j\}$ are the corresponding policy functions,
3. $\{m^j\}$ satisfy the free-entry conditions (21),
4. consistency condition satisfies $\forall (k, b, z, j) \in K \times B \times Z \times J$
   $$\Xi(k', b', z', j; S) = \zeta^j_j(k', b', z'),$$
5. labor market clears
   $$\left( \frac{W(S)p(S)}{\theta(S)} \right)^{1/\chi} = N(S) = \sum_{j=1}^{J} \int_{S} \left[ n_j(k, b, z; S) + \frac{\tilde{\xi}_j(k, b, z; S)^2}{2\xi} \right] d\zeta_j,$$
   where $p(S) = C(S)^{-\kappa},$
6. stochastic discount factor satisfies
   $$d(S', S) = \beta \frac{U_C'(C(S'), N(S'))}{U_C'(C(S), N(S))},$$
7. goods market clears

\[ C(S) + A(S) + E(S) = \]
\[ = \sum_{j=1}^{J} \int_{S} [Ze^z[k^{\alpha_n^\nu}a^{\mu_j} - (1 - \pi_d)(\frac{\xi_j(k, b, z; S)}{\xi})(k_j' - (1 - \delta)k_j) + \]
\[ + (\pi_d + (1 - \pi_d)\left[ 1 - \frac{\xi_j(k, b, z; S)}{\xi}\right]) (1 - \delta)k_j - m^2(S)k_0] \] \[ d\zeta_j - \sum_{j=1}^{J} \chi e^j, \]

where \( A(S) \) denotes aggregate adjustment costs defined as:

\[ A(S) = \varphi^K \sum_{j=1}^{J} \int_{S} \left( 1 - \pi_d \right) \left( \frac{\xi_j(k, b, z; S)}{\xi} \right)^2 \left( \frac{i_j(k, b, z; S)}{k} \right)^2 \] \[ d\zeta_j, \]

and \( E(S) \) captures equity issuance costs:

\[ E(S) = \varphi^E \sum_{j=1}^{J} \int_{S} \max [0, E_j]^2 d\zeta_j, \]

8. government budget constraint is satisfied:

\[ T = \tau \sum_{j=1}^{J} \int_{S} [y_j(k, b, z; S) - W(S)n - (R(S-1 - 1)b - \delta k)] d\zeta_j, \]

9. bonds market clears (by Walras law),

10. the law of motion for the aggregate state vector is consistent with firms’ policy functions.

B.2 Analysis of the Model

The model outlined in Section 4 incorporates optimization problems for three distinct types of agents: representative household, incumbent firms and potential entrants. This implies that, first, I need to solve three programming problems, then make sure that the agents’ decisions are consistent with each other, and that markets clear. Fortunately, it is possible to combine the optimality conditions for the household’s and firms’ Bellman equations, and thereby reduce the computational complexity of the problem at hand. Using \( C(S) \) and \( N(S) \) to denote the market clearing values of consumption and hours worked, it is possible to show that the market-clearing requires:

1. the real wage \( W \) be equal to the household marginal rate of substitution between leisure and consumption:

\[ W(S) = \frac{U_2'(C(S), 1 - N(S))}{U_1'(C(S), 1 - N(S))}; \]
2. the risk-free bond price $q_0^{-1}$ be equal to the expected gross real interest rate:

$$q_0(S) = \beta \mathbb{E} \left[ \frac{U'_j(C(S'), 1 - N(S'))}{U'_i(C(S), 1 - N(S))} \right];$$

3. firms’ state-contingent discount factors be consistent with the household marginal rate of substitution over time:

$$d(S', S) = \beta \frac{U'_j(C(S'), 1 - N(S'))}{U'_i(C(S), 1 - N(S))}.$$

Following Khan, Senga and Thomas (2014), I compute the recursive competitive equilibrium by effectively substituting the equilibrium implications of household optimization into the recursive problems faced by firms. Let $p(S)$ be the marginal utility of the household with respect to equilibrium consumption $C(S)$. Then equations (8), (13) and (19) can be rewritten by way of expressing each firm’s value in terms of the marginal utility of the household.

$$V_j(k, b, z; S) = \pi_d V^\text{exit}_j(k, b, z; S) + (1 - \pi_d) \int V_j(k, b, z, \xi; S)dF^\xi(\xi), \quad (B.1)$$

$$V^\text{cont}_j(k, b, z; S) = \max_{k', b'} p(S) \left[ -E_j(k, b, z; S) - C(E_j(k, b, z; S)) \right] + \beta \mathbb{E} \left[ V_j(k', b', z'|S') \right], \quad (B.2)$$

$$V^\text{exit}_j(k, b, z; S) = p(S) \left[ \Pi_j(k, b, z; S) + (1 - \delta)k - b \right]. \quad (B.3)$$

Next, I lay out the algorithm which I used to solve for the equilibrium.

### B.3 Steady-State

I use collocation methods to solve functional equations (B.1)-(B.3). In practice, I use Chebyshev polynomials to approximate value functions.

I set up a grid of collocation nodes $K \times B \times Z$, with $N_i$ nodes in each dimension, $i \in \{K, B, Z\}$. Throughout the algorithm, I compute expectations with respect to idiosyncratic productivity shocks using the Gauss-Hermite quadrature. The computation of the stationary state of the model proceeds in the following 4 steps:

1. guess the equilibrium wage rate, $W$;

2. solve for individual decision rules $(k', b')$;

3. given the decision rules, compute the stationary histogram (distribution of firms over the state space);

4. compute the excess demand on the labor market. If it exceeds some prespecified tolerance, adjust the wage guess correspondingly and go back to Step 2. Otherwise, terminate.
Once the labor market clears, the disutility parameter $\theta$ is set to be compatible with the implied aggregate consumption $C$.

### B.3.1 Approximation of Value Functions

I approximate two (normalized by the household’s marginal utility) value functions: $V(\cdot)$ and $V^{\text{cont}}(\cdot)$, which are defined in (B.1) and (B.2), respectively.\(^{47}\) In particular, I represent these value functions as weighted sums of orthogonal polynomials:

\[
\begin{align*}
V(k, b, z) &= \sum_{i, j, k=1, 1}^{N_k, N_B, N_Z} \theta^{0}_{i, j, k} T^i(k) T^j(b) T^k(z) \\
V^{\text{cont}}(k, b, z) &= \sum_{i, j, k=1, 1}^{N_k, N_B, N_Z} \theta^{1}_{i, j, k} T^i(k) T^j(b) T^k(z),
\end{align*}
\]

where $\{\theta^{0}_{i, j, k}, \theta^{1}_{i, j, k}\}$ are approximation coefficients, and $T^i(\cdot)$ is the Chebyshev polynomial of order $i$.

I use collocation method to simultaneously solve for $\{\theta^{0}_{i, j, k}, \theta^{1}_{i, j, k}\}$. Collocation method requires setting the residual equation to hold exactly at $N = N_k \times N_B \times N_Z$ points; therefore, I essentially solve for $2 \times N$ unknown coefficients. I compute the basis matrices for Chebyshev polynomials using \textit{Miranda and Fackler (2002) Comp econ toolbox}. Subsequently, I solve for a vector of unknown coefficients using Newton’s method. A much slower alternative is to iterate on the value function. Given the current guess of coefficients, I solve for the optimal policies $k'(k, b, z)$ and $b'(k, b, z)$ using nested vectorized golden search. After I solve for the policy function, I recompute decision rules on a finer grid, and, subsequently, compute the stationary distribution.

### B.3.2 Stationary Distribution

When I solve for the stationary distribution, I iterate on a mapping induced by firms’ decisions rules:

\[ L' = Q'L + L^e, \]

where $L$ is a current distribution of incumbents across the state space, and $L^e$ is a distribution of successful entrants. Matrix $Q$ is a transition matrix, which determines how mass of firms shifts in the $(k, b, z)$-space. It is a direct product of three transition matrices $Q_k$, $Q_b$ and $Q_z$:

\[
Q = Q_k \odot Q_b \odot Q_z,
\]

which govern the shift of mass along $k$-, $b$- and $z$-dimensions, respectively. While $Q_z$ is completely determined by the exogenous stochastic process (7), matrices $Q_k$ and $Q_b$ are constructed so that the model generates an unbiased distribution in terms of aggregates.\(^{48}\) More precisely, element $(i, j)$ of the transition matrix $Q_z$ with $x \in \{k, b\}$ informs which fraction of firms with the current idiosyncratic state $x_i$ will end up having $x_j$ tomorrow.

---

\(^{47}\)In practice, I approximate six functions in total—two for each type. For simplicity, here I assume that there is only one type of firms.

\(^{48}\)See Young (2010) for more details.
Therefore, this entry of the matrix is computed as:

\[ Q_x(i, j) = \left[ 1 \times \left( \frac{x' - x_j}{x_j - x_{j-1}} \right) + 1 \times \left( \frac{x_{j+1} - x'}{x_{j+1} - x_j} \right) \right]. \]

The tensor product of matrices \( Q_k, Q_b \) and \( Q_z \) is computed using the \texttt{dprod} function from the Miranda and Fackler (2002) toolkit.

B.4 Average Life Cycle Profiles

Figure B1 plots the firms’ average life-cycle profiles. Panel A plots the average idiosyncratic productivity of firms. It is evident that firms diverge in terms of average productivity, and \( \mu_H \) firms end up being more productive than firms of other types. This occurs due to the presence of endogenous exit in my model: The value of an outside option for \( \mu_H \) firms is substantially higher than that for \( \mu_M \) and \( \mu_L \) firms, thus, only sufficiently productive firms decide to continue operations.

Panel B shows how different target sizes are across firm types; while the average low-type firm ends up being around three times bigger than a one-year old business, the average high-type firm takes off pretty fast and exceed the size of entrants by a factor of 7 about 15 years into its tenure. It is also clear that it takes around 7 years for a low-type firm to reach its target size, while the high-type business is still growing even 20 years out.

Patterns observed in Panel B have important implications for debt accumulation. As panel C shows, \( \mu_H \) businesses accumulate way more debt than firms of other types. Given the borrowing constraint and the fact that firms start small and grow intensively in the first couple of years, the average debt profiles are similar across firm groups early in their tenures.

Finally, panel D plots the average investment rates of firms. The investment profiles of businesses are downward sloping, reflecting very high investment expenditures in the first several years after entry. Moreover, \( \mu_H \) firms exhibit significantly higher investment rates across all ages.

Figure B1 provides important insights for understanding the compositional effect I focus on in this paper. Provided that it takes high-type firms substantially more time—as well as resources—to get up to scale as compared with low-type businesses, tightening of the collateral constraint affects the former group of firms stronger than the latter. At the extreme, consider a firm for which its target size coincides with its initial size: Such firm will not be affected by collateral shocks at all. Therefore, the financial shock is expected to affect \( \mu_H \) firms stronger than other firm types.

On the contrary, TFP shock changes target sizes of all firm types proportionately, therefore, equally affecting the values of low, medium and high target size businesses. Thus, according to Equation (22), an aggregate productivity shock is expected to affect entry intensities of different firm types similarly.
Figure B1: Average Life Cycle Profiles

(a) Mean productivity
(b) Mean employment (relative to age 1)
(c) Mean debt (relative to age 1)
(d) Mean investment rate

Notes: Figure B1 plots the average (across idiosyncratic productivity) life-cycle profiles of firms. Panels A, B, C and D plot the mean (idiosyncratic) productivity, mean size (in terms of labor) relative to entrants, average debt, as well as the mean investment rate. Solid red, dashed green and dotted blue lines correspond to $\mu_H$, $\mu_M$ and $\mu_L$ firms, respectively.

B.5 Model with Aggregate Shocks

Once a finite representation of the system at the steady-state is obtained, I can write down equilibrium conditions as a system of difference equations, where some equations are backward-looking (e.g., the evolution of the distribution), and forward-looking (Bellman equations). Following a standard approach of using the expectation errors in forward-looking equations (denoted $\eta_{t+1}$), the equilibrium with aggregate uncertainty can be written as the finite non-linear system

$$\Gamma(\Theta_t, \Theta_{t+1}, \eta_{t+1}, \varepsilon_{t+1}) = 0,$$  \hspace{1cm} (B.4)

where the vector $\Theta_t$ contains state and jump variables (such as histogram and collocation parameters for value functions approximations), and $\varepsilon_{t+1}$ is a vector of Gaussian disturbances.
to exogenous aggregate stochastic processes. The vector \( \Theta_t \) also contains an \( M \times 1 \) vector \( g_t \), which collects observables used in the estimation step.

With this representation at hand, the computation of the steady-state boils down to solving for the value of \( \bar{\Theta} \) when aggregate shocks are turned-off; that is, it has to satisfy

\[
\Gamma(\bar{\Theta}, \bar{\Theta}, 0, 0) = 0.
\]

Subsequently, one can express (B.4) in terms of log deviations from the steady-state, \( \hat{\Theta}_t = \log(\Theta_t) - \log(\bar{\Theta}) \), and take a first-order Taylor expansion. This delivers a linear system of equations, which provides a SVAR representation of the model:

\[
\Gamma_0 \hat{\Theta}_{t+1} = \Gamma_1 \hat{\Theta}_t + \Psi \varepsilon_{t+1},
\]

(B.5)

The matrices \( \Gamma_0 \) and \( \Gamma_1 \) contain first-order partial derivatives of equilibrium conditions with respect to elements of \( \Theta_t \), which are computed numerically using automatic differentiation.\(^{49}\)

I solve the resulting system of equations (B.5) using the rational expectations solver gensys provided by Sims (2002).

B.6 Estimation

In this section, I describe the estimation procedure of \( \Omega = \{ \rho_x, \sigma_x \}_{x \in \{ x, r, n \}} \)—parameters of aggregate stochastic processes introduced in Subsection 5.3.

The solution to Equation (B.5) along with a measurement equation form the following system of equations:

\[
\begin{align*}
\hat{\Theta}_{t+1} &= A(\Omega)\hat{\Theta}_t + B(\Omega)\varepsilon_{t+1}, \\
Y_t &= C\hat{\Theta}_t + D\zeta_t,
\end{align*}
\]

where \( A(\Omega) \) describes the evolution of the state vector and \( B(\Omega) \) is an impact matrix. The second equation is a measurement equation: It relates the observable series \( \{ Y_t \}_{t=1}^T \) to a latent state \( \{ \hat{\Theta}_t \}_{t=1}^T \). With the representation above, one can compute the likelihood of any sequence of \( \{ Y_t \}_{t=1}^T \) using Kalman filter (see, for example, An and Schorfheide (2007) and Mongey and Williams (2016) for the description of that procedure).

Given a current draw of parameters \( \Omega \), let \( P(\{ Y_t \}_{t=1}^T | \Omega) \) denote the likelihood of the observed data. The posterior can be computed by combining the likelihood with the prior:

\[
P(\Omega | \{ Y_t \}_{t=1}^T) \propto P(\{ Y_t \}_{t=1}^T | \Omega) P(\Omega).
\]

In order to quantify the uncertainty about parameter estimates, I characterize the posterior by drawing from it using Markov Chain Monte Carlo; I use a Metropolis-Hastings algorithm to accomplish this step.

I estimate parameters of the three exogenous stochastic processes (6 parameters in total) using aggregate series of GDP, hours worked and investment expenditures. The data are

\(^{49}\)I use myAD toolkit written by SeHyoun Ahn, which is available at https://github.com/sehyoun/MATLABAutoDiff.
annual and spans the time period 1977-2017—see Figure C2. Table D8 reports the prior distributions used in the estimation step, and characterizes the posteriors: their medians, as well as 5% and 95% bounds.

**Figure B2: Impulse-Response Functions to Aggregate Shocks**

![Impulse-Response Functions](image)

(a) TFP shock  
(b) Fin. shock

(c) Labor shock

*Notes:* Figure B2 plots impulse-response functions to one standard deviation innovations in TFP (panel A), financial (panel B), and labor disutility (panel C) stochastic processes. Persistence and volatility of exogenous stochastic processes are as in Table D8.

### B.7 Impulse-Response Functions

Figure B2 plots the impulse-response functions for the three aggregate stochastic processes considered.\(^{50}\) For each shock, it separately plots the response of key macro aggregates—consumption, output, hours and investment. Each shock has different quantitative implications for the behavior of macroeconomic aggregates. It is clear that the financial shock affects investment way stronger than output and hours. At the same time, the aggregate

---

\(^{50}\) Aggregate processes have the estimated persistence and volatility parameters from Table D8.
productivity shock affects output stronger than hours, which is the opposite to the effect of the labor shock. Therefore, intuitively, the identification of shocks comes from the way aggregate series move relative to each other over the business cycle.
Appendix C: Additional Figures

This section contains additional figures referenced throughout the paper.

Figure C1: Business Applications and Time to Business Formation in the U.S.

(A) Business Applications

(B) Time to Business Formation

Notes: Figure C1 consists of 2 panels. Panel A plots the number of business application in the U.S., and Panel B plots the average time between application and business formation. The data are from the Business Formation Statistics and spans the period between 2004Q3 and 2013Q4 at quarterly frequency. Business application is identified as a filing of the IRS Form SS-4. Time to business formation is an average (in quarters) time between the filing of the SS-4 form and the first quarter when positive payroll is recorded. Panel B plots the detrended series (linear time trend is subtracted).

Figure C2: U.S. Aggregate Data, 1976-2017

Notes: Figure C2 plots 3 aggregate series used for Bayesian estimation: real GDP, total annual hours worked, and investment expenditures. Series were logged and HP-filtered with the smoothing parameter of 6.25. Nominal values were converted to real ones using the consumer price index. Source: Federal Reserve Economic Data.
Figure C3: Forecast Error Variance Decomposition

Notes: Figure C3 plots the forecast error variance decomposition with respect to TFP (in yellow), financial (in blue) and labor disutility (in gray) shocks. Forecast horizons up to 5 years are considered. See Subsection 6.3 for more details.
Appendix D: Additional Tables

This section contains additional tables referenced throughout the paper.

**Table D1: Firm Size and Initial Aggregate Conditions: Sample 2**

<table>
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<th>Employment (Log)</th>
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<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>( \tilde{\Delta}GDP_0 )</td>
<td></td>
</tr>
<tr>
<td>( \text{Mean} )</td>
<td>0.784***</td>
</tr>
<tr>
<td>( \text{Age FE} )</td>
<td>✓</td>
</tr>
<tr>
<td>( \text{Industry-Year FE} )</td>
<td>✓</td>
</tr>
<tr>
<td>( \text{State-Year FE} )</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
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</tr>
</tbody>
</table>

Notes: Table D1 reports the results of OLS estimation of Equation (5) using the LBD Sample 2. Depending on the specification, the left-hand side variable \( \ln Emp_{t,a,i,s} \) is the mean, 10-, 50- or 90\textsuperscript{th} percentile of log employment. The independent variable is the log-deviation of real GDP from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Sample 2 includes cohorts of firms born between 1978 and 2006, which are tracked until the age of 10. Firms which did not make it to the age of 10 are excluded. Robust standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

**Table D2: Firm Size and Initial Aggregate Conditions: Sample 3**

<table>
<thead>
<tr>
<th></th>
<th>Employment (Log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
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<tr>
<td>( \tilde{\Delta}GDP_0 )</td>
<td></td>
</tr>
<tr>
<td>( \text{Mean} )</td>
<td>1.692***</td>
</tr>
<tr>
<td>( \text{Age FE} )</td>
<td>✓</td>
</tr>
<tr>
<td>( \text{Industry-Year FE} )</td>
<td>✓</td>
</tr>
<tr>
<td>( \text{State-Year FE} )</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: Table D2 reports the results of OLS estimation of Equation (5) using the LBD Sample 3. See Appendix A.1.5 for sample construction details, as well as for summary statistics. Depending on the specification, the left-hand side variable \( \ln Emp_{t,a,i,s} \) is the mean, 10-, 50- or 90\textsuperscript{th} percentile of log employment. The independent variable is the log-deviation of real GDP from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Sample 3 includes cohorts of firms born between 1978 and 2006, which are tracked until the end of the sample, or until they exit. Firms which did not make it to the age of 10 are retained. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
### Table D3: Persistence of Initial Aggregate Conditions

<table>
<thead>
<tr>
<th></th>
<th>Employment (Log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$\tilde{\Delta}GDP_0$</td>
<td>0.392***</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
</tr>
<tr>
<td>$\tilde{\Delta}GDP_0 \times \text{Age}$</td>
<td>0.074***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.337</td>
</tr>
</tbody>
</table>

**Notes:** Table D3 reports the results of OLS estimation of Equation (5) augmented with the interaction between firm age and aggregate conditions in the year of entry. Depending on the specification, the left-hand side variable in $\ln \text{Emp}_{t,a,i,s}$ is the mean, 10th, 50th or 90th percentile of log employment. The independent variable is the log-deviation of real GDP from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Sample 1 includes cohorts of firms born between 1978 and 2006, which are tracked until the age of 10. Robust standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

### Table D4: Establishment Size and Initial Local Conditions

<table>
<thead>
<tr>
<th></th>
<th>Employment (Log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$\tilde{\Delta}Emp_0$</td>
<td>0.489***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.421</td>
</tr>
</tbody>
</table>

**Notes:** Table D4 reports the results of OLS estimation of Equation (5) using the LBD Sample 1. See Appendix A.1.5 for sample construction details, as well as for summary statistics. Depending on the specification, the left-hand side variable in $\ln \text{Emp}_{t,a,i,s}$ is the mean, 10th, 50th or 90th percentile of log employment. The independent variable is the log-deviation of state-wide employment from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Sample 1 includes cohorts of establishments born between 1978 and 2006, which are tracked until the age of 10. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
### Table D5: Establishment Size (Payroll) and Initial Local Conditions

<table>
<thead>
<tr>
<th>Payroll (Log)</th>
<th>Mean</th>
<th>P10</th>
<th>P50</th>
<th>P90</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta Emp_0)</td>
<td>0.500***</td>
<td>0.179***</td>
<td>0.475***</td>
<td>0.855***</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.063)</td>
<td>(0.051)</td>
<td>(0.063)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.443</td>
<td>0.435</td>
<td>0.427</td>
<td>0.345</td>
</tr>
<tr>
<td>Age FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Industry-Year FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>State-Year FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

*Notes:* Table D5 reports the results of OLS estimation of Equation (5) using the LBD Sample 1. See Appendix A.1.5 for sample construction details, as well as for summary statistics. Depending on the specification, the left-hand side variable in \(Pay_{t,a,i,s}\) is the mean, 10-, 50- or 90\(^{th}\) percentile of log payroll. The independent variable is the log-deviation of state-wide employment from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Sample 1 includes cohorts of establishments born between 1978 and 2006, which are tracked until the age of 10. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

### Table D6: Establishment Size and Initial Aggregate Conditions

<table>
<thead>
<tr>
<th>Employment (Log)</th>
<th>Mean</th>
<th>P10</th>
<th>P50</th>
<th>P90</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta GDP_0)</td>
<td>0.631***</td>
<td>0.208***</td>
<td>0.616***</td>
<td>1.077***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.039)</td>
<td>(0.037)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.411</td>
<td>0.432</td>
<td>0.411</td>
<td>0.315</td>
</tr>
<tr>
<td>Age FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Industry-Year FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>State-Year FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

*Notes:* Table D6 reports the results of OLS estimation of Equation (5) using the LBD Sample 1. See Appendix A.1.5 for sample construction details, as well as for summary statistics. Depending on the specification, the left-hand side variable in \(Emp_{t,a,i,s}\) is the mean, 10-, 50- or 90\(^{th}\) percentile of log employment. The independent variable is the log-deviation of real GDP from the HP trend in the year of entry. Observations are weighted by the cohort’s count. Sample 1 includes cohorts of establishments born between 1978 and 2006, which are tracked until the age of 10. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
### Table D7: Returns to Scale and Plant Size

<table>
<thead>
<tr>
<th>Labor (Log)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(Age)</td>
<td>0.362***</td>
<td>0.345***</td>
<td>0.355***</td>
<td>0.343***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Log(Age) × RTS(_j(i))</td>
<td>0.061***</td>
<td>0.023***</td>
<td>0.040***</td>
<td>0.006**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>RTS(_j(i))</td>
<td>0.071***</td>
<td>0.028***</td>
<td>0.051***</td>
<td>0.021***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>R²</td>
<td>0.095</td>
<td>0.085</td>
<td>0.089</td>
<td>0.084</td>
</tr>
</tbody>
</table>

State FE ✓ ✓ ✓ ✓
Year FE ✓ ✓ ✓ ✓
IV — ✓ — ✓
Alternative RTS — — ✓ ✓

Notes: Table D7 reports the results of OLS estimation of the following model:

\[
\log(l_{i,t}) = \alpha + \delta_i + \delta_t + \beta_0 \times \text{Age}_{i,t} + \beta_1 \times \text{RTS}_{j(i)} + \beta_2 \times \text{Age}_{i,t} \times \text{RTS}_{j(i)} + \varepsilon_{i,t},
\]

where \(l_{i,t}\) is the amount of hours worked on plant \(i\) in year \(t\), and \(\text{RTS}_{j(i)}\) is a measure of returns to scale for industry \(j\) where plant \(i\) operates. Returns to scale were computed either using OLS or IV. See Appendix A.2.3 for estimation details. The “Alternative RTS” identifies how the value of resales was handled. In columns (1)-(2), the value of resales was classified as materials. In columns (3)-(4), I follow Kehrig (2015) and do not count it as materials, but rather subtract it from the value of shipments. Observations are weighted by the sampling weights available in the ASM/CM. Returns to scale are standardized. Robust standard errors are in parentheses.

### Table D8: Estimation of Aggregate Shocks

<table>
<thead>
<tr>
<th>Param. Type</th>
<th>Prior</th>
<th>Mean</th>
<th>Std</th>
<th>Posterior (full)</th>
<th>Posterior (fixed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence</td>
<td></td>
<td></td>
<td></td>
<td>Med</td>
<td>5%</td>
</tr>
<tr>
<td>(\rho_z)</td>
<td>Beta</td>
<td>0.5</td>
<td>0.05</td>
<td>0.28</td>
<td>0.07</td>
</tr>
<tr>
<td>(\rho_r)</td>
<td>Beta</td>
<td>0.5</td>
<td>0.05</td>
<td>0.56</td>
<td>0.26</td>
</tr>
<tr>
<td>(\rho_n)</td>
<td>Beta</td>
<td>0.5</td>
<td>0.05</td>
<td>0.44</td>
<td>0.17</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>Inv. Gamma</td>
<td>0.25</td>
<td>0.06</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>Inv. Gamma</td>
<td>0.25</td>
<td>0.06</td>
<td>0.43</td>
<td>0.34</td>
</tr>
<tr>
<td>(\sigma_n)</td>
<td>Inv. Gamma</td>
<td>0.25</td>
<td>0.06</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes: Table D8 reports the results of the Bayesian estimation of aggregate exogenous stochastic processes. Each process is characterized by its persistence and standard deviation of innovations—see Equation (25). Table reports the priors for each parameter as well as the median and 90% confidence bounds based on 10000 draws from the posterior distribution. See text and Appendix B.6 for details about the estimation procedure. “Full” refers to the benchmark model, and “fixed”—to the model with a fixed composition of firm types.
Table D9: Summary Statistics: BvD Data

<table>
<thead>
<tr>
<th></th>
<th>Empl.</th>
<th>Collat.</th>
<th>ST lev.</th>
<th>LT lev.</th>
<th>Total lev.</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Private firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>103</td>
<td>0.30</td>
<td>0.19</td>
<td>0.17</td>
<td>0.29</td>
<td>12</td>
</tr>
<tr>
<td>Bottom 25%</td>
<td>10</td>
<td>0.07</td>
<td>0.04</td>
<td>0.04</td>
<td>0.13</td>
<td>3</td>
</tr>
<tr>
<td>Median</td>
<td>44</td>
<td>0.21</td>
<td>0.12</td>
<td>0.11</td>
<td>0.26</td>
<td>7</td>
</tr>
<tr>
<td>Top 25%</td>
<td>97</td>
<td>0.47</td>
<td>0.27</td>
<td>0.24</td>
<td>0.42</td>
<td>16</td>
</tr>
<tr>
<td><strong>Public firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>737</td>
<td>0.39</td>
<td>0.12</td>
<td>0.13</td>
<td>0.24</td>
<td>26</td>
</tr>
<tr>
<td>Bottom 25%</td>
<td>32</td>
<td>0.17</td>
<td>0.02</td>
<td>0.03</td>
<td>0.11</td>
<td>9</td>
</tr>
<tr>
<td>Median</td>
<td>113</td>
<td>0.36</td>
<td>0.07</td>
<td>0.09</td>
<td>0.21</td>
<td>19</td>
</tr>
<tr>
<td>Top 25%</td>
<td>628</td>
<td>0.58</td>
<td>0.16</td>
<td>0.19</td>
<td>0.34</td>
<td>37</td>
</tr>
</tbody>
</table>

Notes: Table D9 reports the descriptive statistics for the BvD data. All nominal variables were deflated using the consumer price index (2014 is a base year). Table reports summary statistics for private and public firms separately. Private firms include the “Private Limited” category. Public firms include the following categories: “Public, Quoted”, “Public, Not Quoted”, “Public, Quoted OFEX”, “Public AIM”. Collateral is the ratio of fixed assets to total assets. ST leverage is short-term debt and overdrafts divided by total assets. LT leverage is long-term debt to total assets. Total leverage is a sum of short-term debt, overdrafts and long-term debt divided by total assets. All ratios were winsorized at 1- and 99-th percentiles.

Table D10: Returns to Scale and Cyclicality of Entry and Exit Rates

<table>
<thead>
<tr>
<th></th>
<th>( \Delta GDP_t )</th>
<th>( \Delta GDP_t \times RTS_i )</th>
<th>( \Delta GDP_t \times Capital Intensity_i )</th>
<th>( \Delta GDP_t \times Skill Intensity_i )</th>
<th>( \Delta GDP_t \times Durability_i )</th>
<th>( \Delta GDP_t \times Share Advertising_i )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.282***</td>
<td>0.067**</td>
<td>0.033</td>
<td>-0.057**</td>
<td>0.029</td>
<td>-0.036*</td>
<td>0.668</td>
</tr>
<tr>
<td>(2)</td>
<td>0.285***</td>
<td>0.065**</td>
<td>(0.032)</td>
<td></td>
<td>(0.032)</td>
<td>(0.026)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>(3)</td>
<td>0.297***</td>
<td>0.093***</td>
<td>(0.032)</td>
<td></td>
<td>(0.034)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>0.326***</td>
<td>0.040</td>
<td>(0.032)</td>
<td></td>
<td>(0.060)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>0.283***</td>
<td>0.069**</td>
<td>(0.032)</td>
<td></td>
<td>(0.032)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>-0.086***</td>
<td>-0.005</td>
<td>(0.017)</td>
<td></td>
<td>(0.017)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table D10 reports the results of OLS estimation of Equation (3). The dependent variable in columns (1)-(5) is the firm entry rate, computed by dividing the number of age 0 firms in industry \( i \) and year \( t \) by the total count of active firms within the same industry in the preceding period. The dependent variable in column (6) is the firm exit rate, computed by dividing the number of exiting firms in industry \( i \) and year \( t \) by the total count of active firms within the same industry in the preceding period. The right-hand side variable \( \Delta GDP_t \) is the log-deviation of real GDP from HP trend in year \( t \), and RTS\( i \) is a measure of returns to scale. Observations are weighted by industry-level employment. Industry fixed effects are at NAICS 4-digit level. All industry characteristics are standardized. Entry and exit rates are winsorized at top and bottom 1%. Source of firm-level data: ASM/CM and LBD. The underlying data cover the time period 1978-2016. Standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
### Table D11: Adver tizing Heterogeneity and Initial Aggregate Conditions

<table>
<thead>
<tr>
<th></th>
<th>Employment (Log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$\Delta GDP_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.995***</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
</tr>
<tr>
<td>$\Delta GDP_0 \times 1_{\text{High Advertising}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.186</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.227</td>
</tr>
<tr>
<td>Age FE</td>
<td>✓</td>
</tr>
<tr>
<td>Industry-Year FE</td>
<td>✓</td>
</tr>
<tr>
<td>State-Year FE</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: Table D11 reports the results of OLS estimation of Equation (5). Depending on the specification, the left-hand side variable $\ln \hat{E}_{mp,a,i,s}$ is the mean, 10-, 50- or 90th percentile of log employment. The independent variable $\Delta GDP_0$ is the log-deviation of real GDP from the HP trend in the year of entry. The dummy variable $1_{\text{High Advertising}}$ identifies industries with high shares of the advertising input based on detailed input-output tables from the BEA. Observations are weighted by the cohort’s count. Sample 1 includes cohorts born between 1978 and 2006, which are tracked until the age of 10. Robust standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

### Table D12: Initial (Financial) Conditions

<table>
<thead>
<tr>
<th></th>
<th>Employment (Log)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$\Delta GDP_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.991***</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
</tr>
<tr>
<td>$\Delta GDP_0 \times \Delta GZ_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−3.624***</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
</tr>
<tr>
<td>$\Delta GZ_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.017***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.337</td>
</tr>
<tr>
<td>Age FE</td>
<td>✓</td>
</tr>
<tr>
<td>Industry-Year FE</td>
<td>✓</td>
</tr>
<tr>
<td>State-Year FE</td>
<td>✓</td>
</tr>
<tr>
<td>Sample</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: Table D12 reports the results of OLS estimation of Equation (5). Depending on the specification, the left-hand side variable $\ln \hat{E}_{mp,a,i,s}$ is the mean, 10-, 50- or 90th percentile of log employment. The independent variables $\Delta GDP_0$ and $\Delta GZ_0$ are log-deviations of real GDP and Gilchrist and Zakrajšek (2012) index from their corresponding HP trends in the entry year of the cohort. Observations are weighted by the cohort’s count. Sample 1 includes cohorts born between 1978 and 2006, which are tracked until the age of 10. Robust standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
Table D13: Capital Requirements and Initial Aggregate Conditions

<table>
<thead>
<tr>
<th>Employment (Log)</th>
<th>Mean</th>
<th>P10</th>
<th>P50</th>
<th>P90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta GDP_0$</td>
<td>0.169***</td>
<td>−0.036</td>
<td>0.170**</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.059)</td>
<td>(0.078)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>$\Delta GDP_0 \times 1_{\text{High SCR}}$</td>
<td>0.928***</td>
<td>0.054</td>
<td>0.531***</td>
<td>2.970***</td>
</tr>
<tr>
<td></td>
<td>(0.139)</td>
<td>(0.144)</td>
<td>(0.155)</td>
<td>(0.237)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.445</td>
<td>0.271</td>
<td>0.428</td>
<td>0.356</td>
</tr>
<tr>
<td>Age FE</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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Notes: Table D13 reports the results of OLS estimation of Equation (5). Depending on the specification, the left-hand side variable $\ln\bar{Emp}_{t,a,i,s}$ is the mean, 10th, 50th or 90th percentile of log employment. The independent variable $\Delta GDP_0$ is the log-deviation of real GDP from the HP trend in the year of entry. The dummy variable $1_{\text{High SCR}}$ identifies industries with high startup capital requirements based on SBO data. Observations are weighted by the cohort’s count. Sample 1 includes cohorts born between 1978 and 2006, which are tracked until the age of 10. Robust standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.

Table D14: Impact of Initial Local Financial Conditions

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<tr>
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<td>Payroll (Log)</td>
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<tr>
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<td>0.229</td>
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Notes: Table D14 reports the results of OLS estimation of Equation (A.9). Depending on the specification, the left-hand side variable $\ln\bar{Y}_{t,a,i,c}$ is the mean, 10th, 50th or 90th percentile of log employment or log payroll. The independent variable is a local bank lending shock constructed as in Equation (A.8). Observations are weighted by the cohort’s count. Sample 1 includes cohorts born between 1978 and 2006, which are tracked until the age of 10. Robust standard errors are in parentheses.
## Table D15: Firm Investment and Residential Wealth of Directors

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<tr>
<td>Slow</td>
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<td>0.0191***</td>
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<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
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<td>0.0082***</td>
<td>0.0261***</td>
<td>0.0269***</td>
<td>0.0271***</td>
<td>0.0270***</td>
<td>0.0148***</td>
<td>0.0126***</td>
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<td>(1.029)</td>
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<td></td>
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<td>Yes</td>
<td>Yes</td>
</tr>
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<td>Yes</td>
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</table>

Notes: Table D15 reports OLS estimates of Equation (A.11). The sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (A.10). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm “Fixed Assets”. All ratios are winsorized at the median ± 5 times interquartile range. Standard errors, clustered by firm region, are in parentheses. Column (1) reports the effect of residential real estate on firms’ investment without other controls. Column (2) adds firm fixed effects. Column (3) further adds industry-time (2-digit SIC classification) fixed effects. Column (4) adds region-year fixed effects. Column (5) adds firms’ leverage. Column (6) additionally controls for potentially spurious size effects and includes the inverse of (lagged) fixed assets. Columns (7) and (8) further include firms’ profits and a firm-region house price index. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
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<th>Size group</th>
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<td>0.0018*</td>
<td>0.0065***</td>
</tr>
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<td>(0.001)</td>
<td>(0.006)</td>
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</tr>
<tr>
<td>Fast</td>
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<td>0.0140**</td>
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Notes: Table D16 reports OLS estimates of Equation (A.11) for different sample splits. The full sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (A.10). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables are scaled by the lag of firm “Fixed Assets”. All ratios are winsorized at the median ± 5 times interquantile range. Column (1) provides baseline estimates. Columns (2) and (3) report estimates of Equation (A.11) for young (ages 1-5) and old (ages 5+) subsamples. Columns (4) and (5) report estimates for small (fewer than 50 employees) and large (more than 50 employees) subsamples. Size is determined as a time-series average of firm-level employment. Standard errors, clustered by firm region, are in parentheses. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
Table D17: Firm Investment and Residential Wealth of Directors, Conditional on Reaching Age 5

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<tr>
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<td>0.0260***</td>
<td>0.0269***</td>
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<td>0.0126***</td>
<td>0.0130***</td>
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<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
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<td>0.0776***</td>
<td>0.0769***</td>
<td>0.0776***</td>
<td>0.0769***</td>
<td>0.0776***</td>
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<td>Yes</td>
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<td>Yes</td>
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</table>

Notes: Table D17 reports OLS estimates of Equation (A.11). The sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (A.10). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm “Fixed Assets”. All ratios are winsorized at the median ± 5 times interquartile range. Standard errors, clustered by firm region, are in parentheses. Column (1) reports the effect of residential real estate on firms’ investment excluding other controls. Column (2) adds firm fixed effects. Column (3) further adds industry-time (2-digit SIC classification) fixed effects. Column (4) adds region-year fixed effects. Column (5) adds firms’ leverage. Column (6) additionally controls for potentially spurious size effects and includes the inverse of (lagged) fixed assets. Columns (7) and (8) further include firms’ profits and firm-region house price index. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
Table D18: Firm Investment and Residential Wealth of Directors: Robustness of Firms’ Assignment

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<td>(0.001)</td>
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</tr>
<tr>
<td>Region-time FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Table D18 reports OLS estimates of Equation (A.11). The sample covers reporting UK firms over the period 2002-2014. The dependent variable in columns (1) - (3) is investment (change in fixed assets plus depreciation) and change in Total Assets in column (4). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (A.10). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm “Fixed Assets”. All ratios are winsorized at the median ± 5 times interquartile range. Standard errors, clustered by firm region, are in parentheses. Column (1) is the baseline. Column(2) reports estimates for when the firm has to spend more than 75% of its tenure above the median growth rate in order to be classified as “high” type. Column (3) reports estimates when the threshold is moved from 50th percentile to 40th and 60th for “slow” and “fast” type, respectively. Column (4) corresponds to the case with no controls (other than fixed effects) and growth rate of total assets as a dependent variable. *, **, *** denote statistical significance at 10, 5 and 1 percent levels, respectively.
References for Appendix


