Selection Bias in a Controlled Experiment: The Case of Moving to Opportunity.

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Web Appendix Included.

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Abstract

The Moving to Opportunity (MTO) is a social experiment designed to evaluate the effects of neighborhoods on the economic and social outcomes of disadvantaged families in the United States. It targeted over 4,000 families living in high poverty housing projects during the years of 1994–1997 across five U.S. cities. MTO randomly assigned voucher subsidies that incentivize families to relocate from high poverty housing projects to better neighborhoods. Nearly half of the families assigned to vouchers moved. Although the MTO randomization is well suited to evaluate the causal effects of offering vouchers to families, it is less clear how randomized vouchers might be used to assess the causal effects of neighborhoods on outcomes. I exploit the experimental design of the MTO to nonparametrically identify the causal effects of neighborhood relocation on socioeconomic outcomes. My identification strategy employs revealed preference analysis and combines it with tools of causal inference developed in the literature on Causal Bayesian Networks. I find that neighborhood relocation has statistically significant causal effects on labor market outcomes. I decompose the widely reported treatment-on-the-treated parameter - the voucher's effect divided by the compliance rate for the voucher – into components that are unambiguously interpreted in terms of neighborhood effects. The method that I develop is general and applies broadly to unordered choice models with categorical instrumental variables and multiple treatments.

Keywords: Moving to Opportunity, Randomization, Selection Bias, Social Experiment; Causal Inference.

JEL codes: H43, I18, I38. J38.

1 Introduction

Willian J. Wilson's influential book (1987) studies the power of neighborhoods in shaping the life outcomes of individuals in the United States. His work has spawned a large literature that relates the decline of inner city neighborhoods to the life outcomes of their residents (Sampson et al., 2002). According to this literature, the strong correlation between a neighborhood's quality and the well-being of residents can be attributed in part to the effects of neighborhood characteristics on family outcomes.

In a seminal work, Durlauf (2012) investigates the theoretical and empirical difficulties of evaluating of neighborhood effects. He explains that residential sorting poses a fundamental problem for assessing the causal effects of neighborhood quality. The characteristics that foster economic prosperity of the residents of affluent neighborhoods also affect their choice of residential location. This residential sorting is a source of selection bias that impairs causal inference about a neighborhood's characteristics and the socioeconomic outcomes of its residents.

The potential social benefits of neighborhood effects stimulated a variety of housing policies that operate by relocating poor families living in distressed neighborhoods to better ones. Although a large body of literature examines these housing programs (e.g., van Ham et al. (2012)), the causal link between the neighborhood characteristics and resident's outcomes has seldom been assessed (Bergstrom and van Ham, 2012; Curley, 2005; Sampson et al., 2002). This paper contributes to this literature by defining and quantifying neighborhood causal effects that account for residential sorting. I solve the econometric problems generated by neighborhood self-selection using a novel method that explores the economic incentives built into the Moving to Opportunity (MTO) Project.

MTO is a housing experiment that used the method of randomized controlled trials to investigate the consequences of relocating families from America's most distressed neighborhoods to low poverty communities (Orr et al., 2003). The project targeted over 4,000 households living in high poverty housing projects during the years of 1994 to 1997 across five U.S. cities (Baltimore, Boston, Chicago, Los Angeles, and New York).

The MTO project randomly assigned tenant-based vouchers that could be used to subsidize housing costs if the family agreed to relocate. Eligible families who volunteered to participate in the project were placed in one of three assignment groups: control (30% of the sample), experimental

(40% of the sample), or Section 8 (30% of the sample). Families assigned to the control group were offered no voucher, while those assigned to the experimental group could use their vouchers to lease a unit in a low poverty neighborhood[1]. Families assigned to the Section 8 group could use their vouchers to lease a unit in either low or high poverty neighborhoods.[2] MTO vouchers did not force neighborhood relocation but rather created incentives to move. Nearly 50% of experimental families and 60% of Section 8 families used the voucher to relocate.

Figure 1 describes the relocation decisions faced by families according to their assignment groups. Families assigned to the control group decided between not moving, moving to a low poverty neighborhood, or moving to a high poverty neighborhood without the incentive of a voucher. Those assigned to the experimental group who used the voucher had to relocate to a low poverty neighborhood. The experimental families that did not use the voucher faced the same choices as the control families. Families assigned to the Section 8 group that used the voucher could relocate to either low or high poverty neighborhoods. The Section 8 families that did not use the voucher faced the same choices as the control families.

An influential literature exploits the experimental design of MTO to evaluate the intention-to-treat (ITT) and the treatment-on-the-treated (TOT) effects.[3] The ITT evaluates the causal effect of being offered a voucher. The ITT effect for the experimental (or Section 8) voucher is identified by the difference between the average outcome of experimental (or Section 8) families and the average outcome of control families. Kling et al. (2005) explain that the TOT is a Bloom (1984) estimator that evaluates the causal effect of being offered a voucher for the families that relocate using the voucher (i.e., the voucher compliers).[4] The TOT effect for each type of voucher is obtained from the ratio of its ITT divided by the voucher compliance rate.

The ITT and TOT are important parameters to evaluate the housing policy of the MTO program. These are the most useful parameters to investigate the effects of offering rent subsidizing

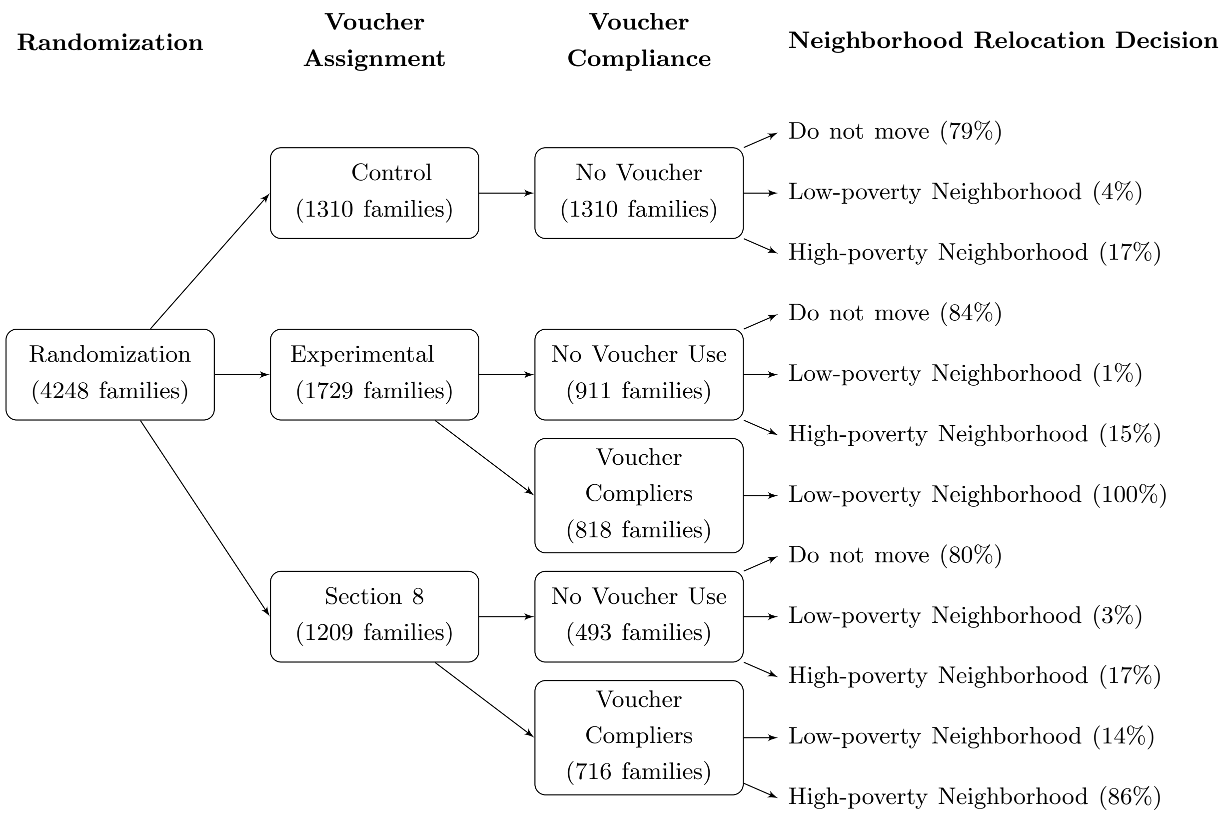
[1] A neighborhood is classified as low poverty if less than 10% of its households are considered poor according to the income poverty threshold of the US 1990 Census. See Section 2 for details.

[2] Section 8 is a well-known public housing program created by the Housing Act of 1937 (Title 42 of US Code, subchapter 1437f). It allows low-income families to rent dwellings in the private housing market by subsidizing a fraction of the families' rent. Section 8 is financed by the federal funds of the US Department of Housing and Urban Development (HUD) and is administered locally by the public housing agencies (PHAs). Section 8 benefits approximately 3 million low-income households nationwide.

[3] Examples of this literature are Gennetian et al. (2012); Hanratty et al. (2003); Katz et al. (2001, 2003); Kling et al. (2007, 2005); Ladd and Ludwig (2003); Leventhal and Brooks-Gunn (2003); Ludwig et al. (2005, 2001).

[4] Under the weak assumption of no average effect of being *offered* the MTO voucher on those who do not use the voucher.

Figure 1: **Neighborhood Relocation by Voucher Assignment and Compliance**



Notes: This figure describes the possible decision patterns of families in MTO that resulted from voucher assignment and family compliance. Families assigned to the control group could decide to not move, to move to a low poverty neighborhood or to move to a high poverty neighborhood. Families assigned to experimental or Section 8 groups that did not use the voucher to relocate faced the same choices. These families could still move to low or high poverty neighborhood without using the subsidy rendered by the voucher. Experimental compliers could only move to low poverty neighborhoods. Section 8 compliers could move to either high or low poverty neighborhoods.

vouchers to families. Yet it is not clear how to interpret the these parameters in terms of the causal effects of neighborhood relocation. Clampet-Lundquist and Massey (2008), for instance, caution that the TOT is not well suited for the evaluation of neighborhood effects because it does not account for the selection bias generated by compliance with the vouchers.[5] Aliprantis (2014) agrees with Clampet-Lundquist and Massey (2008). According to his work, the TOT parameter requires strong model restrictions to render a clear causal interpretation of neighborhood effects.

The goal of this research is to exploit the exogenous variation of the MTO randomized vouchers to nonparametrically identify the causal effects of *neighborhoods* on labor market outcomes. To achieve this goal, I consider a stylized version of the MTO intervention in which the vouchers play the role of instrumental variables for the choice of neighborhood at the intervention onset, and families decide among three neighborhoods alternatives: (1) housing projects targeted by MTO; (2) low poverty neighborhoods; and (3) high poverty neighborhoods. These neighborhood alternatives correspond respectively to three relocation decisions: (1) not to relocate; (2) to relocate to a low poverty neighborhood; or (3) to relocate to a high poverty neighborhood. Counterfactual outcomes are defined as the potential outcomes generated by fixing relocation decisions for population members. Neighborhood causal effects are defined by differences in counterfactual outcomes among the three neighborhoods categories listed above.[6] The average neighborhood effect associated with low poverty relocation compares the counterfactual outcome in which all MTO families relocate to low poverty neighborhoods and the counterfactual outcome wherein no family relocates. Similar reasoning applies to the average neighborhood effect associated with the high poverty relocation arm of the experiment.

Those who attempt to nonparametrically identify neighborhood effects in MTO face a major challenge: MTO vouchers are insufficient to identify the expected outcomes for all possible counterfactual relocation decisions. This challenge is an instance of a more familiar identification problem of choice models in which the variation of the instrumental variable is insufficient to render iden-

[5] Clampet-Lundquist and Massey (2008) state that:

> "*... compliance with the terms of the program was highly selective... a variable measuring assignment to the treatment group (the ITT estimate) or the use of the non-experimental TOT estimate can successfully measure the effects of the policy initiative, but is not well suited to capturing neighborhood effects.*"

[6] Counterfactual outcomes are defined as potential outcomes generated by the causal operation of *fixing* the neighborhood choice at some value among the possible alternative (see Heckman and Pinto (2014b) for a discussion on causality). Each neighborhood choice correspond to a relocation decision. I use the terms neighborhood effects and relocation effects interchangeably.

tification of treatment effects. I address the MTO identification problem by combining economic theory and tools of causal inference to exploit the experimental variation of MTO. Nevertheless the method that I develop is general and applies broadly to unordered choice models with categorical instrumental variables and multiple treatments.

I employ economic reasoning such as the Strong Axiom of Revealed Preferences to identify a set of counterfactual relocation choices that are economically justifiable. This approach allows me identify (1) the distribution of patterns of unobserved counterfactual of a range of counterfactual outcomes and bounds for the causal effects of neighborhood relocation. Then I extend the analysis of Kling et al. (2007) to achieve point identification of neighborhood causal effects. Kling et al. (2007) use data on a neighborhood's poverty level as a metric that summarizes a bundle of unobserved variables that are associated with neighborhood quality. Their study evaluates the impact of a neighborhood's poverty level on the outcomes of residents using a parametric two-stage least squares procedure that employs MTO vouchers as instrumental variables. I demonstrate that a weaker version their assumptions can be used to *nonparametrically* point identify (1) the neighborhood causal effects conditioned on counterfactual relocation decisions and (2) the average causal effect of neighborhood relocation.

My identification strategy explores concepts from Causal Bayesian Networks (Lauritzen, 1996; Pearl, 2009) not commonly used in economics. Specifically, I exploit the assumption that the overall quality of a neighborhood is not directly caused by the unobserved family variables even though neighborhood quality correlates with these unobserved family variables due to neighborhood sorting. I show that this assumption is testable and can be used to achieve point identification. I test and no not reject this assumption.

My analysis produces fresh insights into the MTO project. I contribute to analyses of the MTO by identifying parameters not previously estimated in MTO studies that lead to clear causal interpretations. I partition the sample of MTO families into unobserved subsets associated with economically justified counterfactual relocation choices and estimate the causal effects of neighborhood relocation conditioned on these partition sets.

My analysis focuses on labor market outcomes and concurs with the previous studies that show no statistically significant TOT effects. That said, by focusing on relocation effects rather than voucher effects I obtain sharper results than my predecessors. I conclude that the causal effect of relocating from housing projects to low poverty neighborhood generates statistically significant results on labor market outcomes. The causal effect of relocation is 65% higher than the TOT effect for adult earnings. Both parameters are estimated conditioned on the sites of the intervention. Elsewhere (Pinto, 2014) I examine a wider selection of outcomes plus additional aspects of the MTO study.

This paper proceeds as follows. Section 1.1 explains the identification problem faced in the MTO program. I motivate the discussion using the familiar choice model in which the treatment and the instrumental variable are binary indicators. Section 2 presents a description of the MTO project. In Section 3, I develop my methodology. In Section 3.1, I describe the economic model that frames the relocation decision as a utility maximization problem. Section 3.2 examines the necessary and sufficient conditions to nonparametrically identify the treatment effects generated by a general unordered choice model with categorical instrumental variables and multiple treatments. Section 3.3 identifies the neighborhoods causal effects conditioned on counterfactual relocation decisions. Sections 3.4 and 3.5 investigate properties of the choice model being used such as separability and monotonicity. Section 3.6 decomposes the TOT into a weighted average of interpretable causal effects of neighborhood relocation. Section 3.7 presents the point identification of the causal effects of relocation by response-types. Section 4 presents empirical results and Section 5 summarizes my findings.

1.1 Investigating the Identification Problem of MTO

I use a familiar binary choice model to clarify the nature of the identification problem in the MTO project. Consider a simplified housing experiment that randomly assigns families to a voucher group and to a no voucher group. Let $Z_\omega \in \{0,1\}$ denote a voucher indicator such that $Z_\omega = 0$ if family ω does not receives a voucher and $Z_\omega = 1$ if family ω receives it. Let T_ω denote the relocation decision of family ω where $T_\omega = 0$ if family ω does not relocate and $T_\omega = 1$ if family ω relocates. Define $T_\omega(z)$ as the indicator for the counterfactual relocation decision that family ω would choose had it been assigned to voucher $z \in \{0,1\}$. Let $(Y_\omega(0), Y_\omega(1))$ denote the potential counterfactual outcomes when relocation decision T_ω is *fixed* at zero (no relocation) and at one (relocation occurs). The observed outcome for family ω is given by $Y_\omega = Y_\omega(0)(1 - T_\omega) + Y_\omega(1)T_\omega$. The model is completed by invoking the standard assumption that the instrumental variable Z_ω is independent of counterfactual variables, i.e. $(Y_\omega(0), Y_\omega(1), T_\omega(0), T_\omega(1)) \perp\!\!\!\perp Z_\omega$, where $\perp\!\!\!\perp$ denotes independence.

A key concept in my identification analysis is the response variable S_ω, which I define as the unobserved vector of potential relocation decisions that family ω would choose if voucher assignments were set to zero and to one, i.e., $S_\omega = [T_\omega(0), T_\omega(1)]'$.[7] $S_\omega = [0,1]'$ means that family ω does not relocate if assigned no voucher ($T_\omega(0) = 0$) but would relocate if assigned a voucher ($T_\omega(1) = 1$) and $S_\omega = [1,0]'$ means that family ω relocates if assigned no voucher ($T_\omega(0) = 1$) but would not relocate if assigned a voucher ($T_\omega(1) = 0$). Table 1 describes the four vectors of potential response-types that S_ω can take. Angrist et al. (1996) term these response-types as: never takers ($S_\omega = [0,0]'$), compliers ($S_\omega = [0,1]'$), always takers ($S_\omega = [1,1]'$), and defiers ($S_\omega = [1,0]'$). I later show that the response variable S_ω can be interpreted as a coarse partition of the variables generating unobserved heterogeneity across families in an unordered choice model with multiple treatments.

Table 1: Possible Response-types for the Binary Relocation Choice with Binary Voucher

Voucher Types	Voucher Assignment	Relocation Counterfactuals	**Response-types** Never Takers	Compliers	Always Takers	Defiers
No Voucher	$Z_\omega = 0$	$T_\omega(0)$	0	0	1	1
Voucher Recipient	$Z_\omega = 1$	$T_\omega(1)$	0	1	1	0
		Response variable	$S_\omega = [0,0]'$	$S_\omega = [0,1]'$	$S_\omega = [1,1]'$	$S_\omega = [1,0]'$

In this simplified model, the ITT, i.e. $E(Y_\omega|Z_\omega = 1) - E(Y_\omega|Z_\omega = 0)$, can be expressed as the weighted sum of the causal effect of relocation compared to no relocation for compliers ($S_\omega = [0,1]$) and the causal effect of not relocating compared to relocation for defiers ($S_\omega = [1,0]$), that is: [8]

$$ITT = \underbrace{E(Y_\omega(1) - Y_\omega(0)|S_\omega = [0,1]')}_{\text{causal effect for compliers}} \underbrace{P(S_\omega = [0,1]')}_{\text{compliers probability}} + \underbrace{E(Y_\omega(0) - Y_\omega(1)|S_\omega = [1,0]')}_{\text{causal effect for defiers}} \underbrace{P(S_\omega = [1,0]')}_{\text{defiers probability}}. \quad (1)$$

The observed propensity score difference between voucher assignments identifies the difference

[7] Different concepts of the response-type variable have been used in the literature. Robins and Greenland (1992) explored the concept of a vector of counterfactual choices, Frangakis and Rubin (2002) coined the term "principal stratification." Both use the language of counterfactuals to express the concept of response-types. Balke and Pearl (1994) used "response variable" while Heckerman and Shachter (1995) used "mapping variable" and Heckman and Pinto (2014b,c) used the term strata variable for S_ω. In these works, the response variable is generated by a causal model. This is the approach adopted here.

[8] See Appendix B for proofs of these claims.

between compliers and defiers probabilities:

$$\underbrace{P(T_\omega = 1|Z_\omega = 1) - P(T_\omega = 1|Z_\omega = 0)}_{\text{difference of propensity scores between vouchers}} = \underbrace{P(S_\omega = [0,1]') - P(S_\omega = [1,0]')}_{\text{probability difference between compliers and defiers}}. \quad (2)$$

The causal interpretation of the ratio between ITT (1) and the propensity score difference (2) hinges on assumptions that reduce the number of response-types. For example, the Bloom (1984) approach assumes that only voucher recipients can relocate, i.e. $P(T_\omega = 1|Z_\omega = 0) = 0$, which implies no defiers or always takers. Under this assumption, TOT identifies the causal effect of neighborhood relocation for compliers:

$$TOT = \frac{ITT}{P(T_\omega = 1|Z_\omega = 1)} = \frac{E(Y_\omega(1) - Y_\omega(0)|S_\omega = [0,1]')\, P(S_\omega = [0,1]')}{P(S_\omega = [0,1]')} = E(Y_\omega(1) - Y_\omega(0)|S_\omega = [0,1]'),$$

where the first equality defines TOT and the second equality comes from Equations (1)–(2) under the assumption that $P(T_\omega = 1|Z_\omega = 0)$ and $P(S_\omega = [1,0]')$ are equal to zero.

Imbens and Angrist (1994) define the Local Average Treatment Effect ($LATE$), defined as ITT (1) divided by the difference of propensity scores (2). $LATE$ assumes no defiers which generates the identification of the causal effect of neighborhood relocation for compliers:

$$LATE = \frac{ITT}{P(T_\omega = 1|Z_\omega = 1) - P(T_\omega = 1|Z_\omega = 0)} = E(Y_\omega(1) - Y_\omega(0)|S_\omega = [0,1]'), \text{ if } P(S_\omega = [1,0]') = 0.$$

MTO differs from the simplified model just discussed. It assigns families to three randomized groups (control, experimental, and Section 8) and allows for three relocation choices (no relocation, low poverty neighborhoods, and high poverty neighborhoods). Notationally, I use $Z_\omega = z_1$ to denote no voucher (control group), $Z_\omega = z_2$ to denote the experimental voucher and $Z_\omega = z_3$ to denote the Section 8 voucher. I use $T_\omega = 1$ to denote no relocation, $T_\omega = 2$ for low poverty neighborhood relocation, $T_\omega = 3$ for high poverty neighborhood relocation. Let $T_\omega(z)$ denote the relocation decision that family ω would choose if assigned voucher $z \in \{z_1, z_2, z_3\}$.

The response-type of the MTO family ω is represented by the unobserved three-dimensional vector $S_\omega = [T_\omega(z_1), T_\omega(z_2), T_\omega(z_3)]'$ whose elements denote the counterfactual relocation decision that family ω would take if assigned to the control group z_1, the experimental group z_2, and the Section 8 group z_3. For example, if family ω is of response-type $S_\omega = [3,2,3]'$, the family relocates to a high poverty neighborhood if assigned to the control group ($T_\omega(z_1) = 3$), relocates to a low poverty neighborhood if assigned to the experimental group ($T_\omega(z_2) = 2$), and relocates to a high poverty neighborhood ($T_\omega(z_3) = 3$) if assigned a Section 8 voucher.

The support of S_ω is given by the combination of all of the possible values that each element $T_\omega(z)$ takes for $z \in \{z_1, z_2, z_3\}$. For instance, $T_\omega(z_1)$ can take three possible values: one, two, or three. For each value of $T_\omega(z_1)$, $T_\omega(z_2)$ can also take the same three values. This generates nine possible relocation patterns by voucher assignment. Further, for each value of $T_\omega(z_1)$ and $T_\omega(z_2)$, $T_\omega(z_3)$ can also take the same three values, thus generating the 27 possible response-types. These response-types summarize the unobserved heterogeneity across families and are depicted in Table 2 in lexicographic order. Thus suppose a family ω is of response-type $s_2 = [1, 1, 2]$, then this family would chose not to relocate if assigned to either control group or the experimental group, i.e. $T_\omega(z_1) = 1$ and $T_\omega(z_2) = 1$, but would relocate to a low poverty neighborhood if assigned to the Section 8 group, i.e. $T_\omega(z_3) = 2$.

Table 2: MTO Possible Response-types

		Possible Response-types (lexicographic ordering)													
Voucher	Z Assignments	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	$\cdots$	s_{24}	s_{25}	s_{26}	s_{27}
Control	$Z = z_1$	1	1	1	1	1	1	1	1	1	2	$\cdots$	3	3	3	3
Experimental	$Z = z_2$	1	1	1	2	2	2	3	3	3	1	$\cdots$	2	3	3	3
Section 8	$Z = z_3$	1	2	3	1	2	3	1	2	3	1	$\cdots$	3	1	2	3

Like in the simplified model, the ITT evaluates a weighted sum of the causal effects of neighborhood relocation across a subset of response-types. For instance the ITT that compares the experimental voucher (z_2) with the no voucher assignment (z_1), i.e., $E(Y_\omega|Z_\omega = z_2) - E(Y_\omega|Z_\omega = z_1)$ can be expressed as a weighted sum of relocation causal effects across all response-types S_ω whose first element ($T_\omega(z_1)$) and second element ($T_\omega(z_2)$) differ. Table 3 extracts the 18 response-types of Table 2 that fall into this category. The large number of response-types not only prevents the identification of relocation effects but also impairs interpretation of the ITT parameter in terms of relocation effects. I classify the these 18 response-types of Table 3 into six blocks according to the counterfactual relocation choice for control assignment $T_\omega(z_1)$ and experimental group assignment

Table 3: Relevant Response-types for the *ITT* of Experimental Voucher (z_2) vs. No Voucher (z_1)

		Block 1			Block 2			Block 3			Block 4			Block 5			Block 6		
		$T_\omega(z_1)=1$ $T_\omega(z_2)=2$			$T_\omega(z_1)=1$ $T_\omega(z_2)=3$			$T_\omega(z_1)=2$ $T_\omega(z_2)=1$			$T_\omega(z_1)=2$ $T_\omega(z_2)=3$			$T_\omega(z_1)=3$ $T_\omega(z_2)=1$			$T_\omega(z_1)=3$ $T_\omega(z_2)=2$		
Voucher Assignment	Z	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}
Control	$Z=z_1$	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3
Experimental	$Z=z_2$	2	2	2	3	3	3	1	1	1	3	3	3	1	1	1	2	2	2
Section 8	$Z=z_3$	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3

$T_\omega(z_2)$. Block 1 compares low poverty relocation $T_\omega(z_2)=2$ and no relocation $T_\omega(z_1)=1$, while Block 3 makes the opposite comparison. The same contrast occurs between Blocks 2 and 5 and between Blocks 4 and 6.

The econometric model underlying MTO is an unordered choice model with a categorical instrumental variable and multiple treatments. This paper examines the necessary and sufficient conditions for nonparametrically identifying the treatment effects for this class of models. I show that the identification of relocation effects in MTO relates to the identification strategy of the simplified model previously discussed. Specifically, the identification of relocation effects in both models hinges on assumptions that reduce the number of possible response-types.

Some response-types of MTO are unlikely to occur. For example, if $S_\omega=[2,1,1]'$, family ω chooses to relocate to a low poverty neighborhood with no voucher ($T_\omega(z_1)=2$) but does not relocate if assigned an experimental voucher ($T_\omega(z_2)=1$). This is an implausible decision pattern as the experimental voucher subsidizes relocation to low poverty neighborhoods. If $S_\omega=[1,2,1]'$, then family ω chooses to relocate to a low poverty neighborhood under the experimental voucher ($T_\omega(z_2)=2$), but chooses not to relocate under the Section 8 voucher ($T_\omega(z_3)=1$). However, both the Section 8 and the experimental vouchers subsidize relocation to low poverty neighborhoods, which makes $T_\omega(z_3)=1$ unlikely.

A natural way to examine the identification of neighborhood effects in the MTO is to extend the monotonicity condition of Imbens and Angrist (1994) to the case of three unordered treatment choices. Web Appendix H examines this idea. An extended monotonicity condition that accounts for the relocation incentives generated by MTO vouchers would generate 17 response-types and does not render the identification of relocation causal effects.

My approach uses revealed preference analysis to systematically reduce the number of response-

Table 4: Economically Justifiable MTO Response-types

Voucher	Z Assignment	Response-types s_1	s_2	s_3	s_4	s_5	s_6	s_7
Control	$Z = z_1$	1	2	3	1	1	3	1
Experimental	$Z = z_2$	1	2	3	2	2	2	1
Section 8	$Z = z_3$	1	2	3	3	2	3	3

types. I model the family's choice of relocation as a utility maximization problem and use the Strong Axiom of Revealed Preferences to reduce the 27 possible response-types of the MTO into the 7 response-types shown in Table 4. This allows for the identification of: (a) the response-type distribution, (b) the distribution of pre-intervention variables conditional on each response-type, (c) a range of counterfactual outcomes and (d) bounds for the relocation causal effects. I also express the TOT parameter that compares the experimental and control groups as a weighted average of two causal effects: the effect of relocating to a low poverty neighborhood versus not relocating, and the effect of relocating to a low poverty neighborhood versus relocating to a high poverty neighborhood.

2 MTO: Experimental Design and Background

The MTO is a housing experiment implemented by the Department of Housing and Urban Development (HUD) between June 1994 and July 1998. It was designed to investigate the social and economic consequences of relocating poor families from America's most distressed urban neighborhoods to low poverty communities.

The experiment targeted low-income households living in public housing or Section 8 project-based housing located in disadvantaged inner city neighborhoods in five US cities – Baltimore, Boston, Chicago, Los Angeles, and New York. The eligible households consisted of families with children under 18 years of age that lived in areas with very high poverty rates (40% or more). The final sample consisted of $4,248$ families, two-thirds of whom were African-American. The remaining were mostly Hispanic.[9] Three-quarters of the families were on welfare and less than half of the

[9] The initial sample consist of $4,608$ families, but it was restricted to $4,248$ families in order to assure that at least four years had passed for all of the families surveyed by the interim study.

household heads had graduated from high school. Nearly all of the households (92%) were headed by a female and had an average of three children (Orr et al., 2003).

The MTO experiment used the RCT method to assign vouchers that could be used as rent subsidies for families who sought to relocate. Each family was assigned by lottery to one of the three groups:

1. **Control group :** members were not offered vouchers but continued to live in public housing or received some previous project-based housing assistance.

2. **Experimental group :** members were offered housing vouchers that could be used to lease a unit in a low poverty neighborhood.

3. **Section 8 group :** members were offered Section 8 vouchers with no geographical restriction. Families could use their vouchers to move to low or high poverty neighborhoods of their own choosing.

The low poverty neighborhoods were those whose fraction of poor households was below 10% according to the 1990 US Census. [10] By the time of the relocation, about half of the neighborhood destinations of the participants who used the experimental voucher had poverty rates below 10%, although most had poverty rates were below 20% percent (Orr et al., 2003).

The experimental families who complied with the voucher requirements were requested to live for a period of one year in low poverty areas in order to retain their vouchers. After this period, the families could use the voucher to relocate without geographical constraints. Less than two percent of the families that move using the vouchers returned to their original neighborhood.

The vouchers consisted of a tenant-based subsidy, wherein HUD paid rent of an eligible dwelling directly to the landlord. The voucher beneficiary was requested to pay 30% of the household's monthly adjusted gross income for rent and utilities.[11]

[10] The poverty levels were computed as the ratio of the number of poor residents calculated as the sum of the 1990 US Census variables P1170013–P1170024 (which account for 12 age groups) divided by the total number of residents calculated as the sum of the 1990 US Census variables P1170001– P1170012 and P1170013-P1170024.

[11] The subsidy amounts were calculated based on the Applicable Payment Standard (APS) criteria set by HUD. The rental subsidy differed depending on the number of bedrooms in the dwelling and family size. The eligible units comprised all of the houses and apartments available for rent that complied with the APS criteria. The landlords of an eligible dwelling could not discriminate against a voucher recipient who met the same requirements as a renter without a voucher. The lease was renewed automatically unless the owner (or the voucher recipient) stated otherwise in a written notice.

Table 5: Compliance Rates by Site

Site	All Sites	Baltimore	Boston	Chicago	Los Angeles	New York
Experimental Compliance Rate	47 %	58 %	46 %	34 %	67 %	45%
Section 8 Compliance Rate	59 %	72 %	48 %	66 %	77 %	49%

This tables presents the fraction of voucher recipients that actually used the voucher (compliance rate) for relocation by site.

The experimental group's average compliance rate was 47%, while the compliance rate for Section 8 vouchers was 59%. Table 5 shows that the compliance rates differ by site for both experimental and Section 8 vouchers.

A baseline survey was conducted at the onset of the intervention, after which families were re-contacted in 1997 and 2000. An impact interim evaluation conducted in 2002 (four to seven years after enrollment) assessed six study domains: (1) mobility, housing, and neighborhood; (2) physical and mental health; (3) child educational achievement; (4) youth delinquency; (5) employment and earnings; and (6) household income and public assistance.[12] The MTO Long Term Evaluation consists of data collected between 2008 and 2010.

Table 6 presents a statistical description of the MTO baseline variables surveyed before any relocation decisions. Columns 2–6 of Table 6 show that, apart from sampling variations, the variables are reasonably balanced across the voucher assignments. However, Columns 7–9 and 10–12 show that the means of the baseline variables differ significantly depending on the relocation choices. Table 6 shows evidence that families with fewer social connections are more likely to move using the voucher. Families whose household head is not married, do not have teenage siblings, and have fewer friends in the neighborhood are more likely to use the voucher to move. Living in an unsafe neighborhood is an incentive for relocating using vouchers. In contrast, families that had lived in the same neighborhood for more than five years were less likely to use the vouchers. Web Appendix C estimates the distribution of the poverty of the neighborhoods for the MTO families by voucher assignment and their relocation decision.

My goal is to solve the statistical problems that impede the nonparametric identification of neighborhood effects. To this end I consider a stylized version of the MTO intervention that allows exploitation of the exogenous variation in the MTO voucher assignments to identify the effects of

[12] See Gennetian et al. (2012); Orr et al. (2003) for detailed descriptions of the intervention and the available data.

Table 6: Baseline Variables of MTO by Voucher Assignment and Compliance

Variable	Full Sample					ExperimentalGroup			Section 8 Group		
	Control Group	Experimental minus Control		Section 8 minus Control		Used Voucher	Did not Use minus Voucher Use		Used Voucher	Did not Use minus Voucher Use	
	Mean	Diff	p-val	Diff	p-val	Mean	Diff	p-val	Mean	Diff	p-val
	2	3	4	5	6	7	8	9	10	11	12
Family											
Disable Household Member	0.15	0.01	0.31	0.00	0.82	0.15	0.04	0.34	0.13	0.06	0.23
No teens (ages 13-17) at baseline	0.63	-0.03	0.12	-0.01	0.55	0.65	-0.10	**0.00**	0.66	-0.11	**0.00**
Household size is 2 or smaller	0.21	0.01	0.48	0.01	0.39	0.26	-0.08	**0.04**	0.23	-0.03	0.56
Neighborhood											
Victim last 6 months (baseline)	0.41	0.01	0.41	0.01	0.45	0.45	-0.05	**0.08**	0.45	-0.06	**0.09**
Living in neighborhood > 5 yrs.	0.60	0.00	0.97	0.02	0.28	0.59	0.03	0.17	0.59	0.08	**0.00**
Chat with neighbor	0.53	-0.01	0.60	-0.03	0.19	0.50	0.05	**0.04**	0.51	-0.01	0.77
Watch for neighbor children	0.57	-0.02	0.31	-0.03	0.16	0.51	0.07	**0.00**	0.55	-0.03	0.36
Unsafe at night (baseline)	0.50	-0.02	0.27	-0.00	1.00	0.52	-0.08	**0.00**	0.54	-0.10	**0.00**
Moved due to gangs	0.78	-0.01	0.52	-0.02	0.24	0.79	-0.04	**0.00**	0.78	-0.04	**0.00**
Schooling											
Has a GED (baseline)	0.20	-0.03	**0.04**	0.00	0.81	0.18	-0.03	0.46	0.20	-0.00	0.98
Completed high school	0.35	0.04	**0.01**	0.01	0.47	0.41	-0.02	0.57	0.39	-0.06	0.10
Enrolled in school (baseline)	0.16	0.00	0.95	0.02	0.22	0.19	-0.07	0.10	0.19	-0.04	0.45
Never married (baseline)	0.62	-0.00	0.97	-0.02	0.36	0.66	-0.06	**0.00**	0.63	-0.05	**0.02**
Teen pregnancy	0.25	0.01	0.41	0.01	0.69	0.27	-0.02	0.49	0.29	-0.09	**0.05**
Missing GED and H.S. diploma	0.07	-0.01	0.12	-0.01	0.54	0.04	0.03	0.50	0.06	0.01	0.80
Sociability											
No family in the neighborhood	0.65	-0.02	0.35	0.00	1.00	0.65	-0.03	**0.06**	0.65	-0.01	0.57
Respondent reported no friends	0.41	-0.00	0.78	-0.01	0.56	0.44	-0.06	**0.02**	0.41	-0.02	0.48
Welfare/economics											
AFDC/TANF Recipient	0.74	0.02	0.34	0.00	0.85	0.78	-0.04	**0.00**	0.78	-0.08	**0.00**
Car Owner	0.17	-0.01	0.65	-0.01	0.43	0.19	-0.04	0.26	0.17	-0.04	0.48
Adult Employed (baseline)	0.25	0.02	0.28	0.01	0.75	0.26	0.01	0.84	0.27	-0.03	0.51

This table presents a statistical description of MTO baseline variables by group assignment and compliance decision. By baseline variables I mean pre-program variables surveyed at the onset of the intervention before neighborhood relocation. Columns 2–6 present the arithmetic means for selected baseline variables conditional on Voucher assignments. Column 2 presents the control mean. Columns 3 presents the difference in means between the Experimental and Control groups. Columns 4 shows the double-sided single-hypothesis p-value associated with the equality in means test. Inference is based on the bootstrap method. Columns 5–6 compare the Section 8 group with the control group in the same fashion as columns 3–4. Columns 7–9 examine baseline variables for the experimental group conditional on the choice of voucher compliance. Column 7 presents the variable mean conditioned on voucher compliance. Column 8 gives the difference in means between the families assigned to the Experimental voucher that did not use the voucher and the ones that used the voucher for relocation. Columns 9 shows the double sided p-value associated with the equality in means test. Columns 10–12 analyze the families assigned to the Section 8 group in the same fashion as columns 7–9.

neighborhood relocations.

At the onset of the intervention, a family has three location options: (1) to not relocate, (2) to relocate to a low poverty neighborhood, or (3) to relocate to a high poverty neighborhood. A relocation choice can be interpreted as a bundle that consists of the relocation choice at the onset of the intervention plus the neighborhood mobility pattern associated with this relocation choice.

MTO vouchers play the role of instrumental variables because of their impact on neighborhood choice. That is, vouchers impact family outcomes by affecting the family's choice of neighborhood relocation. Voucher assignment is assumed to be independent of the counterfactual outcomes generated by fixing the relocation decisions, even though voucher assignments are not independent of observed outcomes conditioned on relocation choice. Thus, voucher income effects cannot explain the difference in the outcome distribution of the families who relocate to a low poverty neighborhood whether they use their vouchers or not. This difference is explained by the confounding effects of the unobserved family variables that affect both the choice of neighborhood relocation.

To exploit the exogenous variation of randomized vouchers, it is necessary to summarize the patterns of neighborhood relocation of the MTO families into the three relocation alternatives just described. The MTO data from the interim evaluation classifies participating families into three categories: families that do not move, families that move using the voucher, and families that move without using the voucher.[13] I use this classification to allocate the choice of no relocation to families that do not move, and the choice of low poverty relocation to the families that move using the experimental voucher. I use poverty levels of the 1990 U.S. Census to classify the relocation choice of families that move using the Section 8 voucher into low or high poverty neighborhood.

It remains to classify the relocation choice of control families that move plus the experimental and Section 8 families that move without using the voucher. To this end, I investigate the cumulative distribution of the numbers of days from the onset of the intervention until the first move for families that relocate. This distribution is presented in Table 7.

[13]This classification refers to the variable "m–movepatt1" of the MTO data documentation for the interim evaluation.

Table 7: Cumulative Distribution of Days from Onset until First Move

	Move using the Voucher		No Voucher Use		
Days until	Experimental	Section 8	Control	Experimental	Section 8
First Move	2	3	4	5	6
50	0.04	0.06	0.03	0.03	0.03
70	0.11	0.14	0.04	0.04	0.05
90	0.20	0.27	0.05	0.06	0.08
110	0.30	0.39	0.07	0.07	0.09
130	0.40	0.54	0.08	0.08	0.11
150	0.49	0.64	0.11	0.10	0.13
170	0.59	0.76	0.12	0.11	0.14
200	0.71	0.88	0.14	0.13	0.18
250	0.80	0.95	0.18	0.15	0.24
375	0.90	0.98	0.27	0.23	0.34
525	0.95	0.99	0.41	0.40	0.45

This table shows the cumulative distribution of days from the intervention onset until first move for MTO participating families that move conditional on voucher assignment and voucher usage. The first column gives days until first move. Columns 2–3 provide the cumulative distribution for the families that move using the experimental and Section 8 vouchers. Next column gives the cumulative distribution of days until first move for families assigned to the control group that move. The remaining two columns give the cumulative distribution for families assigned to the experimental group and Section 8 group that move without using the vouchers.

Columns 2–3 provide the cumulative distribution for the families that move using the experimental and Section 8 vouchers. Their relocation decision has already been defined. My quest is to ascribe relocation choices for the families that move without using vouchers. Those families are represented in Columns 4–6 of Table 7.

A simple approach is to estimate a threshold for the number of days until first move such that the cumulative distribution of days until first move of the families that do not use the voucher matches the respective distribution for the families that move using the vouchers.[14] The families that move before this threshold are considered to have relocated. Next, I use the poverty level in the 1990 US Census to classify classify the relocation choice of families that move using the Section 8 voucher into low or high poverty neighborhood according to the 10% criteria in the MTO design.

[14] Pinto (2014) models the relocation classification as an unobserved categorical variable and estimates the probability of relocation alternatives for the families that relocate without using vouchers.

3 An Economic Model for the MTO Program

Randomized controlled trials (RCTs) are often called the gold standard for policy evaluation. In the case of the MTO, the randomization of the vouchers allows for the evaluation of the causal effects of voucher assignment. Yet this randomization does not identify the causal effects of the neighborhood relocations. To assess those, I need to address the potential selection bias generated by the family's relocation decision.

3.1 An Economic Model for Neighborhood Relocation

I express a family's choice of neighborhood relocation as a utility maximization problem. I use the real valued function $u_\omega(k,t)$ to represent the rational preferences for family ω over its consumption goods $k \in \text{supp}(K)$ (including dwelling characteristics) and the relocation decision t. The argument t of the utility function accounts for the nonpecuniary preferences of the relocation such that $t = 1$ stands for not relocating, $t = 2$ for relocating to a low poverty neighborhood, and $t = 3$ for relocating to a high poverty neighborhood.

The impact of the MTO vouchers is captured by allowing the family's budget set to vary according to the voucher assignment and the relocation decision. Namely, $W_\omega(z,t) \subset \text{supp}(K)$ is the budget set of family ω under relocation decision $t \in \{1,2,3\}$ and MTO voucher $z \in \{z_1, z_2, z_3\}$ where z_1 stands for no voucher (control group), z_2 stands for the experimental voucher, and z_3 for the Section 8 voucher.

The choice of relocation for family ω under MTO voucher z is given by:

$$C_\omega(z) = \underset{t\in\{1,2,3\}}{\arg\max} \left(\max_{k \in W_\omega(z,t)} u_\omega(k,t) \right) \tag{3}$$

where the bundle of optimal consumption goods and the choice of relocation $t \in \{1,2,3\}$ given a voucher assignment $z \in \text{supp}(Z)$ for family ω is $[k_\omega(z,t), t]$. Equation (3) implies that if $C_\omega(z) = t \in \{1,2,3\}$, then the bundle $[k_\omega(z,t), t]$ is preferred to $[k_\omega(z,t'), t']$ where t' is a relocation choice in $\{1,2,3\}$ other than t.

S_ω is termed a response variable and denotes the unobserved three-dimensional vector of decisions to relocate that occur if family ω is assigned respectively to the control group (z_1), experimental group (z_2), or the Section 8 (z_3) group:

$$S_\omega = [C_\omega(z_1)\ ,\ C_\omega(z_2)\ ,\ C_\omega(z_3)]' \tag{4}$$

where $\text{supp}(S) = \{s_1, s_2, \ldots, s_{N_S}\}$ denote support of S_ω and a value $s \in \text{supp}(S)$ is termed a response-type. Each element $C_\omega(z); z \in \{z_1, z_2, z_3\}$ of S_ω can take a value in $\{1, 2, 3\}$. There are a total of $3 \cdot 3 \cdot 3 = 27$ possible response-types in $\text{supp}(S)$ as explained in the Introduction 1. The remainder of this section uses economic reasoning such as the Strong Axiom of Revealed Preference to investigates which of the possible response-types are economically justified.

The MTO vouchers subsidize the rent of an eligible dwelling when that rent is in excess of 30% of the family's monthly income. Housing subsidies allow families to afford consumption bundles that would exceed the family's available income if the subsidy were not available. Furthermore, the Section 8 subsidy can be used in both low and high poverty neighborhoods. The experimental voucher subsidy is restricted to low poverty neighborhood relocations. These statements can be translated into the following budget restrictions:

Assumption A-1. Budget Restrictions:

$$W_\omega(z_1, 2) \ \subset \ W_\omega(z_2, 2) \ = \ W_\omega(z_3, 2), \tag{5}$$

$$W_\omega(z_1, 3) \ = \ W_\omega(z_2, 3) \ \subset \ W_\omega(z_3, 3). \tag{6}$$

Equation (5) states the budget set relations for families that relocate to low poverty neighborhoods ($t = 2$). The family's budget set under the experimental and Section 8 vouchers is bigger than under no voucher. Equation (6) characterizes the budget sets of families that move to high poverty neighborhoods ($t = 3$). A Section 8 voucher increases the family's budget set compared to no voucher or an experimental voucher.

I assume that the family's budget set does not change across relocation choices if the family is not offered a voucher (control group). I also assume that the family's budget set is the same for the relocation choices for which the experimental or Section 8 vouchers do not apply. Formally:

Assumption A-2. Budget Equalities:

$$W_\omega(z_1,1) \;=\; W_\omega(z_1,2) \;=\; W_\omega(z_1,3) \;=\; W_\omega(z_2,1) \;=\; W_\omega(z_2,3) \;=\; W_\omega(z_3,1).$$

The following lemma applies the strong axiom of revealed preferences (SARP) to translate the budget restrictions in Assumptions **A-1**–**A-2** into constraints on the Choice Rule (3):

Lemma L-1. If preferences are rational and if the agent is not indifferent to relocation choices, then, under Assumptions **A-1**–**A-2**, the family Choice Rules C_ω must satisfy the following assertions:

1. $C_\omega(z_1) = 2 \Rightarrow C_\omega(z_2) = 2 \text{ and } C_\omega(z_3) \neq 1,$
2. $C_\omega(z_1) = 3 \Rightarrow C_\omega(z_2) \neq 1 \text{ and } C_\omega(z_3) \neq 1,$
3. $C_\omega(z_2) = 1 \Rightarrow C_\omega(z_1) = 1 \text{ and } C_\omega(z_3) \neq 2,$
4. $C_\omega(z_2) = 3 \Rightarrow C_\omega(z_1) = 3 \text{ and } C_\omega(z_3) = 3,$
5. $C_\omega(z_3) = 1 \Rightarrow C_\omega(z_1) = 1 \text{ and } C_\omega(z_2) = 1,$
6. $C_\omega(z_3) = 2 \Rightarrow C_\omega(z_2) = 2.$

Proof. See Mathematical Appendix. □

I further assume that a neighborhood is a normal good. This assumption means that if a family decides to relocate to a low or high poverty neighborhood under no subsidy, then the family does not change its decision if a subsidy is offered for the chosen relocation. Notationally this assumption translates to:

Assumption A-3. The neighborhood is a normal good, that is, for each family ω, and for $z, z', t \in \{z_1, z_2, z_3\}$, if $C_\omega(z) = t$ and $W_\omega(z,t)$ is a proper subset of $W_\omega(z',t)$ then $C_\omega(z') = t$.

Theorem **T-1** uses Lemma **L-1**, and Assumption **A-3** to reduce the number of potential response-types from 27 to 7:

Theorem T-1. Under Assumptions **A-1**–**A-3**, the set of possible response-types is given by:

Voucher	Z Assignment	**Possible Response-types** s_1	s_2	s_3	s_4	s_5	s_6	s_7
Control	$Z = z_1$	1	2	3	1	1	3	1
Experimental	$Z = z_2$	1	2	3	2	2	2	1
Section 8	$Z = z_3$	1	2	3	3	2	3	3

This table shows the possible values that the response variable S_ω can take under the restrictions of Choice Rules C_ω presented in Lemma **L-1** and Assumption **A-3**.

Proof. See Mathematical Appendix. □

Next I use the concept of response-types to develop a general instrumental variable model (IV) that describes the MTO intervention.

3.2 A General Instrumental Variable Model for MTO Experiment

The results described here apply to any unordered choice model with categorical instrumental variables and multiple treatments.

A general IV model is described by four main variables, $V_\omega, Z_\omega, T_\omega$ and Y_ω, defined in the common probability space $(\Omega, \mathscr{F}, P)$ in which $Y_\omega \in \Omega$ denotes a measurement of the random variable Y for family ω. These variables are described as follows:

1. Z_ω denotes an observed categorical instrumental variable.

2. T_ω denotes an observed categorical treatment.

3. Y_ω denotes an observed post-treatment outcome.

4. V_ω denotes an unobserved random vector affecting both the treatment and the outcome.

In the case of the MTO, Z_ω denotes the voucher assigned to family ω that takes its value from the support $\text{supp}(Z) = \{z_1, z_2, z_3\}$, where z_1 stands for the control group, z_2 for the experimental group, and z_3 for the Section 8 group. I use the term instrumental variable or assigned voucher interchangeably. Treatment T_ω denotes the relocation decision whose support is $\text{supp}(T) = \{1, 2, 3\}$, where $T_\omega = 1$ stands for not relocating, $T_\omega = 2$ for relocating to a low poverty neighborhood, and $T_\omega = 3$ for relocating to a high poverty neighborhood. I use the terms treatment and relocation choices interchangeably. The random vector V_ω represents all of the unobserved characteristics of family ω that affect the outcome Y_ω and the relocation choice T_ω. Therefore, the distribution of the outcome conditioned on the relocation choices might differ due to the differences in the conditional distribution of V_ω instead of the relocation itself. The V_ω is often called a confounding variable, and it is the source of the selection bias in the IV model.

I denote the observed pre-program variables by X_ω. For the sake of notational simplicity, they are not explicitly included in the model. All of the analyses described in this subsection can be understood as conditional on any pre-program variables that I need to control for.

The following structural equations govern the causal relations among the variables of this model:[15]

$$Y_\omega = f_Y(T_\omega, V_\omega, \epsilon_\omega). \tag{7}$$

$$T_\omega = f_T(Z_\omega, V_\omega). \tag{8}$$

The arguments of Equations (7)–(8) are said to cause Y_ω and T_ω respectively. Equation (7) states that outcome Y_ω is caused by relocation decision T_ω, the unobserved variable V_ω, and by an error term ϵ_ω that is independent of any variable other than Y_ω; that is, $\epsilon_\omega \perp\!\!\!\perp (V_\omega, Z_\omega, T_\omega)$. Equation (8) states that the relocation decision T_ω is caused by the voucher Z_ω and the vector of the family's unobserved characteristics V_ω. I suppress the error term in Equation (8) to simplify the notation. The lack of an error term does not constitute a model restriction because V_ω has an arbitrary dimension and can subsume the supposed error term.

A key property generated by the MTO randomization is that the vouchers are independent of the family's unobserved characteristics:

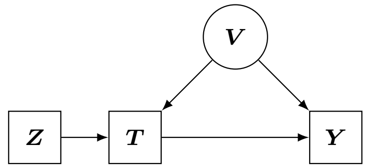
$$Z_\omega \perp\!\!\!\perp V_\omega. \tag{9}$$

The model is completed by the following regularity conditions:

Assumption A-4. The expectation of Y_ω exists, that is, $E(|Y_\omega|) < \infty$.

[15] By structural I mean that the equations that have the autonomy property of Frisch (1938), i.e., a stable mechanism that is represented by a deterministic function that remains invariant under the external manipulation of its arguments.

Figure 2: **General IV Model**



This figure represents the MTO Model as a DAG. Arrows represent direct causal relations. Circles represent unobserved variables. Squares represent observed variables. Y is the observed outcome. T an observed neighborhood decision that causes outcome Y. V is an unobserved confounding variable generating selection bias and causing T and Y. Z is the MTO voucher assignment that plays the role of instrumental variable that causes the relocation decision T.

Assumption A-5. $P(Z_\omega = z | X_\omega) > 0 \, \forall \, z \in \text{supp}(Z)$ and $P(T_\omega = t | X_\omega) > 0 \, \forall \, t \in \text{supp}(T)$.

Assumption **A-4** assures that the mean treatment parameters are well defined. Assumption **A-5** assures that the families are randomized to each MTO voucher with a positive probability and that some families pick each relocation choice.

Figure 8 uses the nomenclature of Bayesian networks (Lauritzen, 1996) to represent the IV model as a Directed Acyclic Graph (DAG). In the figure, causal relations are indicated by directed arrows, unobserved variables are represented by circles, and squares represent observed variables. In a DAG, error disturbances typically are implied rather than depicted.

The IV model defined by Equations (7)–(9) is more general than standard representation of the well-known Generalized Roy Model for unordered choices described in Heckman et al. (2006, 2008); Heckman and Vytlacil (2007) and Heckman and Urzúa (2010). Both the general IV model and the Roy model allow for a categorical treatment, and both impose no restriction on the random *vector* of the unobserved variables that impact the outcome. In contrast to Equation (8), the standard form of the Generalized Roy model assumes that the treatment choice T_ω is governed by a function that is separable from the instrumental variables Z_ω and the unobserved variables V_ω. Notationally, the standard form of the Generalized Roy model distinguishes the random vector of the unobserved variables that cause the outcome Y_ω from the unobserved variables that cause T_ω. The General IV model assumes no specific functional form for Equations (7)–(8) nor does it impose any restriction on the dimension of V_ω. Thus, for sake of notational simplicity, I use unobserved random vector V_ω for both the outcome and the treatment equations.

A counterfactual outcome is generated by *fixing* the treatment variable T_ω to $t \in \text{supp}(T)$ (Haavelmo, 1944; Heckman and Pinto, 2014b). By fixing I mean setting the argument T_ω of

Equation (7) to a value $t \in \text{supp}(T)$, that is:

$$Y_\omega(t) = f_Y(t, V_\omega, \epsilon_\omega);\ t \in \text{supp}(T). \tag{10}$$

The average causal effect comparing treatment choices t against t' on outcome Y_ω is defined as $E(Y_\omega(t) - Y_\omega(t')); t, t' \in \text{supp}(T)$ and the observed outcome Y_ω can be expressed as:

$$Y_\omega = \sum_{t \in \text{supp}(T)} Y_\omega(t) \cdot \mathbf{1}[T_\omega = t] \tag{11}$$

where $\mathbf{1}[\psi]$ is an indicator function that equals one if ψ is true and zero otherwise. Under this notation, the potential treatment choice T_ω when instrument Z_ω is fixed at $z \in \text{supp}(Z)$ is given by $T_\omega(z) = f_T(z, V_\omega); z \in \text{supp}(Z)$. I use Equation (10), Relation (9) and the independence properties of error term ϵ_ω to state the following counterfactual independence relations:

$$Z_\omega \perp\!\!\!\perp Y_\omega(t) \text{ and } Y_\omega(t) \perp\!\!\!\perp T_\omega | V_\omega. \tag{12}$$

A consequence of the first relation of (12) is that the vouchers can only cause outcome Y_ω through its impact on the relocation decision T_ω. This property characterizes Z_ω as an instrumental variable for the treatment T_ω. The second relation of (12) assigns a *matching property* to the variable V_ω. That is, if V_ω were known, then the outcome's counterfactual expectation could be evaluated by:

$$E(Y_\omega|T_\omega = t, V_\omega) = E\left(\sum_{t \in \text{supp}(T)} Y_\omega(t) \cdot \mathbf{1}[T_\omega = t] \big| T_\omega = t, V_\omega \right) = E(Y_\omega(t)|T_\omega = t, V_\omega) = E(Y_\omega(t)|V_\omega)$$

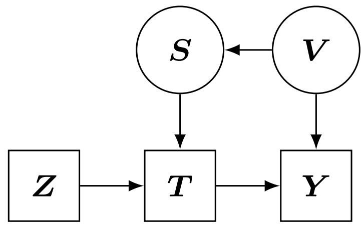
where the first equality comes from (11) and the last one from (12).

The response variable S_ω in subsection 3.1 denotes the unobserved vector of the counterfactual treatments T_ω when the instrument Z_ω is fixed at z_1, z_2, or z_3. Notationally, S_ω is expressed by:

$$S_\omega = [T_\omega(z_1)\ ,\ T_\omega(z_2)\ ,\ T_\omega(z_3)]' = [f_T(z_1, V_\omega)\ ,\ f_T(z_2, V_\omega)\ ,\ f_T(z_3, V_\omega)]' = f_S(V_\omega). \tag{13}$$

Equation (13) shows that S_ω is a function of the unobserved variables V_ω and therefore does not add new information to the model. The relocation decision indicator T_ω can be written in terms

Figure 3: **General IV Model with Response Variable S**



This figure shows the MTO model with the response variable S as a DAG. Arrows represent direct causal relations. Circles represent unobserved variables. Squares represent observed variables. Y is the observed outcome. T an observed neighborhood decision that causes outcome Y. V is an unobserved confounding variable that generates the selection bias and causes the response variable S and Y. Z is the MTO voucher assignment that plays the role of the instrumental variable that causes the relocation decision T.

of the response variable S_ω as:

$$T_\omega = [\mathbf{1}[Z_\omega = z_1], \mathbf{1}[Z_\omega = z_2], \mathbf{1}[Z_\omega = z_3]] \cdot S_\omega. \tag{14}$$

A useful consequence of (80) is that the treatment choice T_ω is deterministically conditioned on the instrumental variable Z_ω and the unobserved response variable S_ω. The IV model with response variable S_ω is represented as a DAG in Figure 10.

The next lemma states three useful relations of the response variable S_ω :

Lemma L-2. The following relations hold for the IV model of Equations (7)–(8):

$$S_\omega \perp\!\!\!\perp Z_\omega, \quad Y_\omega \perp\!\!\!\perp Z_\omega | (S_\omega, T_\omega) \text{ and } Y_\omega(t) \perp\!\!\!\perp T_\omega | S_\omega.$$

Proof. See Mathematical Appendix. □

The first relation in **L-2** states that the voucher Z_ω is independent of the potential choice of relocation in S_ω. The second relation states that the outcomes and the assigned vouchers are independent when they are conditioned on the neighborhood decision T_ω and the unobserved response variable S_ω. The last relation states that S_ω shares the same matching property of V_ω. Indeed, relocation choice T_ω only depends on Z_ω when conditioned on the response variable S_ω. And Z_ω is independent of unobserved variables V_ω generating bias. Conceptually, S_ω solves the problem of the confounding effects of the unobserved variables V_ω by generating a coarse partition of the sample space such that the distribution of V_ω is the same across the relocation choices T_ω

within the partitions generated by S_ω. Most important, the matching property of S_ω allows the evaluation of the counterfactual outcomes by conditioning on S_ω :

$$E(Y_\omega|T_\omega = t, S_\omega) = E\left(\sum_{t\in\text{supp}(T)} Y_\omega(t)\cdot\mathbf{1}[T_\omega = t]|S_\omega, T_\omega = t\right) = E(Y_\omega(t)|S_\omega, T_\omega = t) = E(Y_\omega(t)|S_\omega = s)$$

where the first equality comes from (11) and the last one from **L-2**.

The matching property of S_ω motivates the definition of the average treatment effect of the responses ($RATE$), that is, the causal effect of T_ω on Y_ω when T_ω is fixed at t compared to t' that is conditioned on response-type $S_\omega = s \in \text{supp}(S)$:

$$RATE_s(t,t') = E(Y_\omega(t) - Y_\omega(t')|S_\omega = s) = E(Y_\omega|T_\omega = t, S_\omega = s) - E(Y_\omega|T_\omega = t', S_\omega = s). \quad (15)$$

The subscript s in Equation (15) denotes an element $s \in \text{supp}(S)$. I also use set $\tau \subset \text{supp}(S)$ as subscript to denote $RATE_\tau(t,t') = E(Y_\omega(t) - Y_\omega(t')|S_\omega \in \tau)$. If $P(T_\omega = t|S_\omega = s) > 0$ and $P(T_\omega = t'|S_\omega = s) > 0$ for all $s \in \text{supp}(S), t, t' \in \text{supp}(T)$, then the average treatment effect (ATE) can be expressed as a weighted average of $RATE$s:

$$ATE(t,t') = E(Y_\omega(t) - Y_\omega(t')) = \sum_{s\in\text{supp}(S)} RATE_s(t,t')\, P(S_\omega = s). \quad (16)$$

From Equations (15)–(16), the identification of the causal effects of T_ω on Y_ω relies on the evaluation of the unobserved quantities $E(Y_\omega|T_\omega = t, S_\omega = s)$, $P(S_\omega = s)\, \forall s \in \text{supp}(S), t \in \text{supp}(T)$ based on the observed quantities $E(Y_\omega|T_\omega = t, Z_\omega = z)$, $P(T_\omega = t, Z_\omega = z)$; $t \in \text{supp}(T), z \in \text{supp}(Z)$. The next theorem uses the relations of Lemma **L-2** to express these unobserved quantities in terms of the observed ones:

Theorem T-2. The following equation holds for the IV model of Equations (7)–(8):

$$E(Y_\omega \cdot \mathbf{1}[T_\omega = t]|Z_\omega) = \sum_{s\in\text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega]\, E(Y|T_\omega = t, S_\omega = s)\, P(S_\omega = s). \quad (17)$$

Proof. See Mathematical Appendix. □

Equation (17) expresses the unobserved outcome expectations conditioned on the response-types in terms of the observed expectations of the outcomes and the voucher assignments. If I set outcome Y_ω as a constant, then Equation (17) generates the following equality:

$$P(T_\omega = t|Z_\omega = z) = \sum_{s \in \text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega = z]\, P(S_\omega = s). \tag{18}$$

Equation (18) expresses the response-type probabilities $P(S_\omega = s); s \in \text{supp}(S)$ in terms of the propensity scores $P(T_\omega = t|Z_\omega = z); t \in \{1,2,3\}, z \in \{z_1, z_2, z_3\}$.

The left-hand sides of Equations (18)–(17) are observed, the right-hand sides are not. It is helpful to transcribe the linear equations of (18)–(17) into matrix algebra in order to investigate conditions for identifying treatment effects. The support of S_ω is a matrix $\boldsymbol{A}$ that is denoted by $\boldsymbol{A} = [s_1, \ldots, s_{N_S}]; \text{supp}(S) = \{s_1, \ldots s_{N_S}\}$. I express the element in the i-th row and j-th column of a matrix $\boldsymbol{A}$ as $\boldsymbol{A}[i,j] = (T_\omega|Z_\omega = z_i, S_\omega = s_j); i \in \{1,2,3\}, j \in \{1,\ldots,N_S\}$. I use $\boldsymbol{A}_t$ for the $|\boldsymbol{A}|$-dimensional binary matrix that is defined by $\boldsymbol{A}_t[i,j] = \mathbf{1}[\boldsymbol{A}[i,j] = t]$. As a short hand notation, I use $\boldsymbol{A}[i,\cdot]$ for the i-th row, $\boldsymbol{A}[\cdot,j]$ for the j-th column and $\boldsymbol{A}_t = \mathbf{1}[\boldsymbol{A} = t]$ to denote $\boldsymbol{A}_t$. The matrix $\boldsymbol{A_S}$ denotes the binary matrix that is generated by stacking the matrices $\boldsymbol{A}_t$ as t takes the values of 1,2, and 3. The matrix $\boldsymbol{A_D}$ denotes the binary matrix generated by setting the matrices $\boldsymbol{A}_t$ as t takes the values of 1,2, and 3 as a diagonal block matrix, that is:

$$\boldsymbol{A_S} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \boldsymbol{A}_3 \end{bmatrix}, \quad \boldsymbol{A_D} = \begin{bmatrix} \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{A}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{A}_3 \end{bmatrix} \tag{19}$$

where $\mathbf{0}$ is a matrix of element zeros with the same dimension of $\boldsymbol{A}_t$. The matrix $\boldsymbol{A_S}$ has $|\text{supp}(Z)| \cdot |\text{supp}(T)|$ rows and N_S columns. The matrix $\boldsymbol{A_D}$ also has $|\text{supp}(Z)| \cdot |\text{supp}(T)|$ rows but $N_S \cdot |\text{supp}(T)|$ columns.

The following matrix notation stacks the observed and the unobserved parameters into vectors. The $\boldsymbol{P}_Z(t)$ denotes the vector of observed propensity scores $P(T_\omega = t|Z_\omega)$ when Z_ω ranges in

$\{z_1, z_2, z_3\}$, and $\boldsymbol{P}_Z$ stacks vectors $\boldsymbol{P}_Z(t)$ as t ranges in $\{1,2,3\}$:

$$\boldsymbol{P}_Z(t) = [P(T_\omega = t|Z_\omega = z_1), P(T_\omega = t|Z_\omega = z_2), P(T_\omega = t|Z_\omega = z_3)]',$$
$$\boldsymbol{P}_Z = [\boldsymbol{P}_Z(1)' \,,\ \boldsymbol{P}_Z(2)' \,,\ \boldsymbol{P}_Z(3)']. \tag{20}$$

Vectors $\boldsymbol{Q}_Z(t)$ and $\boldsymbol{Q}_Z$ focus on the outcome expectations in the same manner as $\boldsymbol{P}_Z(t)$, and $P(T_\omega = t|Z_\omega)$ focuses on the propensity scores:

$$\boldsymbol{Q}_Z(t) = [E(Y_\omega|T_\omega = t, Z_\omega = z_1), E(Y_\omega|T_\omega = t, Z_\omega = z_2), E(Y_\omega|T_\omega = t, Z_\omega = z_3)]' \odot \boldsymbol{P}_{Z_\omega}(t),$$
$$\boldsymbol{Q}_Z = [\boldsymbol{Q}_Z(1)' \,,\ \boldsymbol{Q}_Z(2)' \,,\ \boldsymbol{Q}_Z(3)'], \tag{21}$$

where $\odot$ denotes the Hadamard product.[16] The vector of the unobserved response-type probabilities is denoted by $\boldsymbol{P}_S$. The vectors of the outcome expectations that are conditional on the response-types are given by $\boldsymbol{Q}_S(t)$ and $\boldsymbol{Q}_S$:

$$\boldsymbol{P}_S = [P(S_\omega = s_1), \ldots, P(S_\omega = s_{N_S})]',$$
$$\boldsymbol{Q}_S(t) = [E(Y_\omega|T_\omega = t, S_\omega = s_1), \ldots, E(Y_\omega|T_\omega = t, S_\omega = s_{N_S})]' \odot \boldsymbol{P}_S,$$
$$\boldsymbol{Q}_S = [\boldsymbol{Q}_S(1)' \,,\ \boldsymbol{Q}_S(2)' \,,\ \boldsymbol{Q}_S(3)'].$$

Under this notation, Equations (18)–(17) can be rewritten as:

$$\boldsymbol{P}_Z = \boldsymbol{A_S}\boldsymbol{P}_S, \tag{22}$$
$$\boldsymbol{Q}_Z = \boldsymbol{A_D}\boldsymbol{Q}_S. \tag{23}$$

Equations (22)–(23) can be used to generate bounds and to identify counterfactual estimates. To this end, $\boldsymbol{C}^+$ denotes the Moore-Penrose pseudoinverse[17] of the matrix $\boldsymbol{C}$.

[16] Element-wise multiplication.

[17] A $n \times m$ matrix $\boldsymbol{C}^+$ is the Moore-Penrose inverse of a matrix $\boldsymbol{C}$ if:

$$\boldsymbol{C}\boldsymbol{C}^+\boldsymbol{C} = \boldsymbol{C}$$
$$\boldsymbol{C}^+\boldsymbol{C}\boldsymbol{C}^+ = \boldsymbol{C}^+$$
$$(\boldsymbol{C}\boldsymbol{C}^+)' = \boldsymbol{C}\boldsymbol{C}^+$$
$$(\boldsymbol{C}^+\boldsymbol{C})' = \boldsymbol{C}^+\boldsymbol{C}$$

Theorem T-3. Consider the IV model of Equations (7)–(8). If λ is a real-valued vector of dimension N_S such that $\left(\boldsymbol{I}_{N_S} - \boldsymbol{A}_{\boldsymbol{S}}^{+}\boldsymbol{A}_{\boldsymbol{S}}\right)'\lambda = \boldsymbol{0}$, then $\lambda'\boldsymbol{P}_S$ is identified. And, if λ is a real-valued vector of dimension $\kappa = N_S \cdot |\operatorname{supp}(T)|$ such that $\left(\boldsymbol{I}_{\kappa} - \boldsymbol{A}_{\boldsymbol{D}}^{+}\boldsymbol{A}_{\boldsymbol{D}}\right)'\lambda = \boldsymbol{0}$, then $\lambda'\boldsymbol{Q}_S$ is identified.

Proof. See Mathematical Appendix. □

Corollary **T-3** is not restricted to the case of the MTO but holds for unordered choice models with categorical instruments and multiple treatments. It simplifies the identification of causal effects as it only depends on the binary properties of the matrices $A_{\boldsymbol{S}}$ and $A_{\boldsymbol{D}}$.

For instance, suppose $\boldsymbol{A}_{\boldsymbol{S}}$ has a full column-rank. Then $\left(\boldsymbol{I}_{N_S} - \boldsymbol{A}_{\boldsymbol{S}}^{+}\boldsymbol{A}_{\boldsymbol{S}}\right)'$ is equal to a matrix with all its entries being zero, and $\lambda'\boldsymbol{P}_S$ is identified for any vector of dimension N_S. In particular, $\lambda'\boldsymbol{P}_S$ is identified when λ is set to each vector of the identity matrix. Therefore all of the response-type probabilities are identified if $\boldsymbol{A}_{\boldsymbol{S}}$ has a full rank. A consequence of corollary **T-3** is that the identification of the response-type probabilities does not render the identification of the causal effects, because the full-rank of $\boldsymbol{A}_{\boldsymbol{S}}$ does not imply that $\boldsymbol{A}_{\boldsymbol{D}}$ has a full rank. On the other hand, if $\boldsymbol{A}_{\boldsymbol{D}}$ has a full rank, then $\boldsymbol{A}_{\boldsymbol{S}}$ also has full rank.

The binary-treatment, binary-instrument model mentioned in this paper's introduction exemplifies the concepts discussed here. This binary model generates four response-types: never takers ($s_\omega = [0,0]'$), compliers ($s_\omega = [0,1]'$), always takers ($s_\omega = [1,1]'$), and defiers ($s_\omega = [1,0]'$). The monotonicity assumption of Imbens and Angrist (1994) eliminates the defiers from this set of possible response-types. Under this assumption, the response-type matrices $\boldsymbol{A}, \boldsymbol{A}_{\boldsymbol{S}}$ and $\boldsymbol{A}_{\boldsymbol{D}}$ for the binary-treatment model are given by:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \ \boldsymbol{A}_{\boldsymbol{S}} = \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \ \boldsymbol{A}_{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{A}_0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}. \tag{24}$$

The column-rank of $\boldsymbol{A}_{\boldsymbol{S}}$ in (24) is $\operatorname{rank}(\boldsymbol{A}_{\boldsymbol{S}}) = 3$. Thus, $\boldsymbol{A}_{\boldsymbol{S}}$ has a full rank, and all of the response-types probabilities (never takers, compliers, and always takers) are identified. The matrix $\boldsymbol{A}_{\boldsymbol{D}}$ has six columns and $\operatorname{rank}(\boldsymbol{A}_{\boldsymbol{D}}) = 4$. Therefore, it does not have a full rank. However, the

The Moore-Penrose matrix always exists and is unique. Moreover, if $\boldsymbol{C}$ is square and $det(\boldsymbol{C}) \neq 0$, then $\boldsymbol{C}^{+} = \boldsymbol{C}^{-1}$.

matrix $\boldsymbol{A_D}$ in (24) is such that $\left(\boldsymbol{I}_6 - \boldsymbol{A}^{+}_{\boldsymbol{D}}\boldsymbol{A_D}\right)'\lambda = \boldsymbol{0}$ for $\lambda = [0, -1, 0, 1, 0]'$. Therefore, according to **T-3**, $\lambda'\boldsymbol{Q}_S$ is identified for $\lambda = [0, -1, 0, 1, 0]'$. For this value of λ, I have:

$$\lambda'\boldsymbol{Q}_S = \bigg(E(Y_\omega | T_\omega = 1, S_\omega = [0, 1]') - E(Y_\omega | T_\omega = 0, S_\omega = [0, 1]') \bigg) P(S_\omega = [0, 1]'),$$

and therefore the causal effect for the compliers, i.e. $S_\omega = [0, 1]'$, is identified.

Corollary **T-3** translates the identification requirements of the IV model's causal effects into a search for the assumptions under which the matrices $\boldsymbol{A_S}$ and $\boldsymbol{A_D}$ have full ranks. A challenge with this approach is that while the number of possible values that the instrument or treatment takes grows linearly, the number of possible response-types grows exponentially. Specifically, while the number of rows in $\boldsymbol{A_S}$ is given by $|\,\text{supp}(Z)|$ times $|\,\text{supp}(T)|$, the number of possible columns, that is, the number of possible response-types, is given by $|\,\text{supp}(Z)|$ raised by $|\,\text{supp}(T)|$. Thus, matrix $\boldsymbol{A_S}$ is typically a matrix whose column dimension far exceeds its row dimension. But the rank of a matrix is less or equal to its row dimension. As a consequence, a necessary condition for achieving identification is a reduction in the number of columns in $\boldsymbol{A_S}$, i.e. the response-types, in order to decrease the gap between the row and column dimensions of $\boldsymbol{A_S}$.

The next subsection uses corollary **T-3** and the response-types of **T-1** to identify the causal effects of the relocation.

3.3 Identifying Counterfactual Expectations

The response matrix of **T-1** consists of seven economically justifiable response-types generated by the MTO design. The response-types s_1, s_2, and s_3 of **T-1** refer to the families whose relocation choices do not vary across the voucher assignments: s_1 stands for families that never relocate, s_2 for families that always relocate to low poverty neighborhoods and, s_3 for families that always relocate to high poverty neighborhoods. The variation in the voucher assignments cannot be used to evaluate the causal effects of the relocation of those response-types because the choice does not change across the voucher assignments.

The response-type s_4 refers to compliers in the sense that a family does not move if no voucher is assigned, moves to a low poverty neighborhood under the experimental voucher, or moves to a high poverty neighborhood under the Section 8 voucher. The remaining response-types also refer to the families whose relocation choices vary according to the voucher assignments.

The causal effect of moving to a low poverty neighborhood ($T_\omega = 2$) versus not moving ($T_\omega = 1$) can be evaluated only for response-types that can make these relocation choices, that is, s_4 and s_5 :

$$\begin{aligned} RATE_{\{s_4,s_5\}}(2,1) &= E(Y_\omega|T_\omega = 2, S_\omega \in \{s_4,s_5\}) - E(Y_\omega|T_\omega = 1, S_\omega \in \{s_4,s_5\}) \\ &= \frac{RATE_{s_4}(2,1)\,P(S_\omega = s_4) + RATE_{s_5}(2,1)\,P(S_\omega = s_5)}{P(S_\omega = s_4) + P(S_\omega = s_5)} \end{aligned} \tag{25}$$

But, the causal effect of moving to a high poverty neighborhood versus not moving can only be evaluated for response-types s_4 and s_7 :

$$\begin{aligned} RATE_{\{s_4,s_7\}}(3,1) &= E(Y_\omega|T_\omega = 3, S_\omega \in \{s_4,s_7\}) - E(Y|T_\omega = 1, S_\omega \in \{s_4,s_7\}) \\ &= \frac{RATE_{s_4}(3,1)\,P(S_\omega = s_4) + RATE_{s_7}(3,1)\,P(S_\omega = s_7)}{P(S_\omega = s_4) + P(S_\omega = s_7)} \end{aligned} \tag{26}$$

The next theorem describes the parameters that can be identified by the response-types of matrix **T-1**:

Theorem T-4. Under Assumptions **A-1**–**A-3**:

1. Response-type Probabilities $\boldsymbol{P}_S$, that is $P(S_\omega = s)$; $\forall\, s \in \text{supp}(S)$, are identified.

2. The following outcome expectations that are conditioned on the response-types and the relocation choice are identified:

No Relocation	Low Poverty Neighborhood	High Poverty Neighborhood
$E(Y_\omega(1)\|S_\omega = s_1)$	$E(Y_\omega(2)\|S_\omega = s_2)$	$E(Y_\omega(3)\|S_\omega = s_3)$
$E(Y_\omega(1)\|S_\omega = s_7)$	$E(Y_\omega(2)\|S_\omega = s_5)$	$E(Y_\omega(3)\|S_\omega = s_6)$
$E(Y_\omega(1)\|S_\omega \in \{4,5\})$	$E(Y_\omega(2)\|S_\omega \in \{4,6\})$	$E(Y_\omega(3)\|S_\omega \in \{4,7\})$

3. For any variable X_ω such that $X_\omega \perp\!\!\!\perp T_\omega|S_\omega$ holds, $E(X_\omega|S_\omega = s)$ is identified for all $s \in \text{supp}(S)$.

Proof. See Mathematical Appendix. □

Item (1) of Theorem **T-4** shows that the response-type probabilities $\boldsymbol{P}_S$ are identified. This identification result is consequence of the response-types in **T-1** for which response matrix $\boldsymbol{A_S}$ has full column rank. Response-types probabilities can be evaluated through Equation (22) by using the Moore-Penrose pseudo-inverse of matrix $\boldsymbol{A_S}$:

$$\boldsymbol{P}_S = \boldsymbol{A}_{\boldsymbol{S}}^{+}\boldsymbol{P}_Z = \begin{bmatrix} 1 & 1 & 7 & 1 & 1 & -2 & 1 & 1 & -2 \\ -2 & 1 & 1 & 7 & 1 & 1 & -2 & 1 & 1 \\ 1 & -2 & 1 & 1 & -2 & 1 & 1 & 7 & 1 \\ 3 & -6 & 3 & 3 & 3 & -6 & -6 & 3 & 3 \\ 3 & 0 & -3 & -6 & 0 & 6 & 3 & 0 & -3 \\ -3 & 3 & 0 & -3 & 3 & 0 & 6 & -6 & 0 \\ 0 & 6 & -6 & 0 & -3 & 3 & 0 & -3 & 3 \end{bmatrix} \cdot \frac{\boldsymbol{P}_Z}{9}; \tag{27}$$

Furthermore, if a matrix $\boldsymbol{A_S}$ has full rank, then its Moore-Penrose pseudo-inverse can be computed as $\boldsymbol{A}_{\boldsymbol{S}}^{+} = (\boldsymbol{A}'_{\boldsymbol{S}}\boldsymbol{A}_{\boldsymbol{S}})^{-1}\boldsymbol{A}'_{\boldsymbol{S}}$. In other words, $\boldsymbol{A}_{\boldsymbol{S}}^{+}$ is equal to the closed-form expression of an Ordinary Least Square that uses the vectors of $\boldsymbol{A_S}$ for covariates. Hence the estimated values of the response-type probabilities can be interpreted as the coefficients of a linear regression that uses the propensity scores as dependent variables and the response-types indicators of $\boldsymbol{A_S}$ of as regressors.

Item (2) of Theorem **T-4** lists the counterfactual outcome expectations identified according to the response matrix of **T-1**.

Item (3) states that the conditional expectation of the pre-intervention variables are identified for all of the response-types. Table A.8 of Web Appendix G shows the estimates for the pre-intervention variables variables described in subsection 2 by response-types.

The response matrix of **T-1** also allows the mapping of the content of the TOT parameter in terms of the causal effects of the relocation. This feature is discussed in subsection 3.6.

The identification results of Theorem **T-4** are pivoted in the economic behavior of MTO families. This analysis can be placed in the economic literature that examines the impact of individual rational behavior on observed data (Blundell et al., 2003, 2008, 2014; Kline and Tartari, 2014; McFadden, 2005; McFadden and Richter, 1991). Web Appendix E presents a brief discussion of this literature. Kitamura et al. (2014) suggest a nonparametric test that verifies whether the observed empirical data on prices and consumption are consistent with rational agents. Web Appendix E maps the identification presented here into their framework.

3.4 Separability

Aliprantis and Richter (2014) examine MTO through an ordered choice model in which families decide among neighborhoods that differ on a quality index. They assume that the family choice is governed by additive-separable functions of unobserved family characteristics.[18] Their work is inline with Heckman et al. (2006, 2008), who also use separability conditions to achieve identification.

In this paper, I assume that family choices are economically rational,[19] which, in turn, generates the following separability condition:

Theorem T-5. Under Assumptions **A-1**–**A-3**, the relocation decision T_ω is separable in V_ω and Z_ω, that is, there are functions $\varphi : \text{supp}(V) \times \text{supp}(T) \to \mathbb{R}$ and $\zeta : \text{supp}(Z) \times \text{supp}(T) \to \mathbb{R}$ such that $P\left(T_\omega = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}[\varphi(V_\omega, t) + \zeta(Z_\omega, t) \geq 0]\right) = 1$.

Proof. See Mathematical Appendix. □

Vytlacil (2002) investigates the duality between a separability condition and response-types in the binary choice model. He shows that the monotonicity condition of Angrist et al. (1996) is a sufficient and necessary condition to obtain a separable function of treatment assignments in the case of a binary treatment. Intuitively stated, the response matrix of a choice model with binary treatment in which monotonicity holds contains the same information content generated by a separable function of treatment assignments.

Vytlacil's equivalence is not applicable for the case of multiple choices of MTO. The response matrix **T-1** is a sufficient but not a necessary condition to obtain the separability condition **T-5**. Otherwise stated, the response matrix of **T-1** contains more information than the separability condition **T-5**. Indeed, the number of response matrices generated by combinations of 7 response-types taken from the 27 possible ones totals 888.030. Among those, 66 response matrices satisfy the separability condition **T-5** – including the economically justified response matrix of **T-1**.[20] All these matrices share a lonesum property, which is explained in the next section.

3.5 Useful Property of Response-types – An Equivalence Result

In this section I explain that the Separability **T-5** can be traced to a useful property of the response matrix in **T-1**. Namely, each binary matrix $\mathbf{A}_1, \mathbf{A}_2$ and $\mathbf{A}_3$ associated with **T-1** is uniquely recovered by its row and column sums. Binary matrices of this type are termed lonesum (Brualdi,

[18] They also assume that the unobserved characteristics of families can be modeled by a three-dimensional vector of normally distributed random variables. They identify an outcome expectation that is conditioned on one of the these three unobserved variables.

[19] By rational I mean preferences that generate an ordering of goods that is both transitive and complete.

[20] Web Appendix D presents these 66 response matrices.

1980; Ryser, 1957). I show that this lonesum property renders the identification of counterfactual outcome expectations. I also state an equivalence result that links this lonesum property to generalized concepts of separability and monotonicity.

Some notation is necessary. Let $r_{i,t} = \sum_{j=1}^{N_S} \mathbf{A}_t[i,j]$ be the sum of the i-th row and $c_{j,t} = \sum_{i=1}^{N_Z} \mathbf{A}_t[i,j]$ be the sum of the j-th line of the binary matrix $\mathbf{A}_t$. Let $r'_{1,t} \leq \cdots \leq r'_{N_S,t}$ be the ordered values of $r_{1,t}, \cdots, r_{N_S,t}$, then the maximal of a matrix $\mathbf{A}_t$ is the $|\mathbf{A}_t|$-dimensional matrix whose i-th row is given by $r'_{i,t}$ elements one followed by elements zero. Two matrices are said to be *equivalent* if one can be transformed into another by a row or column permutations.

Table 8 presents the row and column sums of matrix $\mathbf{A}_2$. To show that matrix $\mathbf{A}_2$ is lonesum, consider reordering the columns and rows of matrix $\mathbf{A}_2$ based on increasing values of column and decreasing values of row sums. The reordered matrix is shown in Table 9 and it is equal to the maximal of $\mathbf{A}_2$. Namely, the matrix rows are described by elements one followed by elements zero and thereby fully characterized by the row sum. Thus $\mathbf{A}_2$ is lonesum as the matrix reordering and matrix characterisation were based only on rows and columns sums. The same feature occurs for matrices $\mathbf{A}_1$ and $\mathbf{A}_3$.

Table 8: Rows and Columns Sums of Matrix $\mathbf{A}_2$

Row Sum	Row Index	**Matrix $\mathbf{A}_2 = \mathbf{1}[\mathbf{A} = 2]$**						
1	$r_{1,2}$	0	1	0	0	0	0	0
4	$r_{2,2}$	0	1	0	1	1	1	0
2	$r_{3,2}$	0	1	0	0	1	0	0
	Column Index	$c_{1,2}$	$c_{2,2}$	$c_{3,2}$	$c_{4,2}$	$c_{5,2}$	$c_{6,2}$	$c_{7,2}$
	Column Sum	0	3	0	1	2	1	0

Table 9: Reordered Matrix $\mathbf{A}_2$ According to Increasing Values of Rows and Columns Sums

Row Sum	Row Index	**Reordered Rows and Columns by Sums**						
1	$r_{1,2}$	1	0	0	0	0	0	0
2	$r_{3,2}$	1	1	0	0	0	0	0
4	$r_{2,2}$	1	1	1	1	0	0	0
	Column Index	$c_{2,2}$	$c_{5,2}$	$c_{4,2}$	$c_{6,2}$	$c_{1,2}$	$c_{3,2}$	$c_{7,2}$
	Column Sum	3	2	1	1	0	0	0

Theorem **T-6** translates the lonesum property into monotonicity and separability properties of the underlying choice model with multiple treatments being examined.

Theorem T-6. Let the Model (7)–(8) and where T_ω and Z_ω are categorical variables, and $\text{supp}(Z) = \{z_1, \ldots, z_{N_Z}\}$ and the response-type S_ω is given by $S_\omega = [f_T(z_1, V_\omega), \ldots, f_T(z_{N_Z}, V_\omega)]$. Then following statements are equivalent:

(i) Each response matrix $\mathbf{A}_t$; $t \in \text{supp}(T)$ is lonesum.

(ii) Each response matrix $\mathbf{A}_t$; $t \in \text{supp}(T)$ is equivalent to its maximal.

(iii) **Monotonicity:** given $t \in \text{supp}(T)$, for all $v \in \text{supp}(V)$ and for any $z, z' \in \text{supp}(Z)$, we must have that:

$$\big(\mathbf{1}[T_\omega = t] | Z_\omega = z, V_\omega = v)\big) \geq \big(\mathbf{1}[T_\omega = t] | Z_\omega = z', V_\omega = v)\big),$$
$$\text{or } \big(\mathbf{1}[T_\omega = t] | Z_\omega = z, V_\omega = v) \leq \big(\mathbf{1}[T_\omega = t] | Z_\omega = z', V_\omega = v)\big).$$

(iv) **Separability:** treatment T is separable on V and Z, that is, there exist functions, $\varphi : \text{supp}(V) \times \text{supp}(T) \to \mathbb{R}$ and $\zeta : \text{supp}(Z) \times \text{supp}(T) \to \mathbb{R}$ such that

$$\mathbf{P}\left(T_\omega = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}[\varphi(V_\omega, t) + \zeta(Z_\omega, t) \geq 0]\right) = 1.$$

(v) Let the utility function of an agent $\omega \in \Omega$ over Z_ω, T_ω be represented by a function $u(T_\omega, Z_\omega, V_\omega)$ in $\mathbb{R}$ where unobserved variables V_ω accounts for the agent's preferences. Let the choice of treatment T_ω of an agent ω whose unobserved variables V_ω takes value $v \in \text{supp}(V)$ and the instrument Z_ω takes value $z \in \text{supp}(Z)$ be $P(T_\omega = f_T(v, z) | V_\omega = v, Z_\omega = z) = 1$ where $f_T(v, z) = \arg\max_{t \in \text{supp}(T)} u(t, z, v)$ and $u(t, z, v)$ takes the additive form $u(t, z, v) = \varphi(v, t) + \zeta(z, t) + \psi(v, z)$.

Proof. See Mathematical Appendix. ☐

In the binary case, Item (iv) of Theorem **T-6** translates to the latent index representation of Vytlacil (2002). Item (iii) renders the monotonicity condition of Imbens and Angrist (1994), which generates a binary response matrix that is lower triangular (as in Equation (24)). But a binary lower triangular matrix is equivalent to a maximal matrix (by reverting the order of columns) and complies with the lonesum property.

3.6 Interpreting Treatment-on-the-Treated

This subsection investigates the causal interpretation of the TOT parameter, which is defined here as the ratio of the causal effect on outcome Y from being assigned voucher z versus z' (ITT) divided by the difference in the propensity score of a relocation choice $\tau \subset \text{supp}(T)$ that is induced by the voucher change:

$$TOT_{\tau}(z, z') = \frac{E(Y_{\omega}|Z_{\omega} = z) - E(Y_{\omega}|Z_{\omega} = z')}{P(T_{\omega} \in \tau|Z_{\omega} = z) - P(T_{\omega} \in \tau|Z_{\omega} = z')}; \; \tau \subset \text{supp}(T), z, z' \in \text{supp}(Z). \tag{28}$$

The denominator of Equation (28) differs from the Bloom estimator used in the MTO literature. In Equation (28), I use the difference in the propensity of the relocation choice induced by the voucher change instead of the voucher's compliance rate. Empirically, the propensity difference and the compliance rate generate similar values and hence similar estimates.[21] Theoretically, those denominators yield distinct interpretations. Kling et al. (2007) explain that the Bloom estimator differs conceptually from the $LATE$ parameter of Angrist et al. (1996) because the endogenous variable examined is the voucher itself rather than the relocation choice. Equation (28) generates the $LATE$ parameter of Imbens and Angrist (1994) in the case of a binary treatment and is more suitable to examine the causal effects of the relocation. Henceforth, I use TOT for the parameter defined in (28) and refer to the vouchers' effects divided by the compliance rates as Bloom estimators.

Theorem T-7 disentangles the TOT into components associated with the causal effects of the relocation by response-types:

Theorem T-7. If $z, z' \in \text{supp}(Z)$ and $\tau \subset \text{supp}(T)$, then $TOT_{\tau}(z, z')$ of (28) can be expressed as

[21] The experimental group's voucher induces the relocation to a low poverty neighborhood. Thus, the denominator of Equation (28) stands for the difference in the fraction of people who relocate to low poverty neighborhoods between the experimental and the control groups. This difference accounts for two types of families that are not assessed by the experimental group's compliance rate: (1) the control families that relocate to low poverty neighborhoods without a voucher, and (2) the few experimental families that relocate to low poverty neighborhoods without using the voucher.

the weighted average of causal effects of the response-types $RATE_s; s \in \text{supp}(S)$, that is:

$$TOT_\tau(z,z') = \sum_{j=1}^{N_S} \left(RATE_{s_j}(\boldsymbol{A}[i,j], \boldsymbol{A}[i',j])\right) H_{s_j}$$
$$\text{such that } H_{s_j} = \frac{\left(\sum_{t\in\tau}(\boldsymbol{A}_t[i,j] - \boldsymbol{A}_t[i',j])\right) P(S_\omega = s_j)}{\sum_{j=1}^{N_S}\left(\sum_{t\in\tau}(\boldsymbol{A}_t[i,j] - \boldsymbol{A}_t[i',j])\, P(S_\omega = s_j)\right)}$$

where $RATE_s$ is given by (15), $\boldsymbol{A}_t$ as defined in subsection 3.2, $\tau \subset \text{supp}(T)$ denotes the set of choices induced by the change in values of the instrumental variable, and $H_s; s \in \text{supp}(S)$ are positive weights that total one.

Proof. See Mathematical Appendix. □

The TOT that compares the experimental to the control group is denoted by $TOT_2(z_2, z_1)$ and is defined by the ITT parameter of being assigned the experimental group versus control group, that is, $E(Y_\omega|Z_\omega = z_2) - E(Y_\omega|Z_\omega = z_1)$, divided by the difference in the propensity to relocate to the low poverty neighborhood across these voucher assignments $P(T_\omega = 2|Z_\omega = z_2) - P(T_\omega = 2|Z_\omega = z_1)$. According to (**T-7**) and the response matrix of **T-1**, $TOT_2(z_2, z_1)$ is given by:

$$TOT_2(z_2,z_1) = \frac{RATE_{\{s_4,s_5\}}(2,1)\, P(S_\omega \in \{s_4,s_5\}) + RATE_{s_6}(2,3)\, P(S_\omega = s_6)}{P(S_\omega \in \{s_4,s_5\}) + P(S_\omega = s_6)}. \tag{29}$$

Equation (29) shows that $TOT_2(z_2, z_1)$ is a weighted average of two causal effects that differ on the relocation choices. Specifically, $TOT_2(z_2, z_1)$ is a weighted average of: (1) the effect of relocating to a low poverty neighborhood ($T_\omega = 2$) versus no relocation ($T_\omega = 1$) for response-types s_4, s_5; and (2) the effect of relocating to the low poverty neighborhood ($T_\omega = 2$) versus the high poverty neighborhood ($T_\omega = 3$) for the response-type s_6.

The TOT parameter associated with the Section 8 group is defined as the fraction of the ITT that compares the Section 8 voucher with no voucher divided by the difference in the propensity to relocate to either a low or a high poverty neighborhood:

$$TOT_{\{2,3\}}(z_3,z_1) = \frac{RATE_{\{s_4,s_7\}}(3,1)\, P(S_\omega \in \{s_4,s_7\}) + RATE_{s_5}(2,1)\, P(S_\omega = s_5)}{P(S_\omega \in \{s_4,s_7\}) + P(S_\omega = s_5)} \tag{30}$$

In this case, $TOT_{\{2,3\}}(z_3, z_1)$, compares relocating to the high poverty neighborhood ($T_\omega = 3$)

and no relocation ($T_\omega = 1$) for response-types $S_\omega \in \{s_4, s_7\}$, but relocating to the low poverty neighborhood ($T_\omega = 2$) versus no-relocation ($T_\omega = 1$) for response-type $S_\omega = s_5$.

3.7 Point Identification

Theorem **T-4** states that economically justified response-types allow for the identification of all of the response-type probabilities (Item 1) and a range of causal parameters (Item 2). In Section 3.6, I use these response-types to map the causal content of the TOT parameter in terms of the causal effects of the relocation.

Nevertheless, these economically justified response-types do not guarantee the point identification of the causal effects of relocation. According to **T-4**, $RATE_{\{s_4,s_5\}}(2,1)$ of Equation (25) is not identified. While $E(Y_\omega|T_\omega = 1, S_\omega \in \{4,5\})$ is identified, $E(Y_\omega|T_\omega = 2, S_\omega \in \{4,5\})$ is not. I cannot disentangle $E(Y_\omega|T_\omega = 2, S_\omega = 4)$ from the identified parameter $E(Y_\omega|T_\omega = 2, S_\omega \in \{4,6\})$. The causal effect $RATE_{\{s_4,s_7\}}(3,1)$ of Equation (26) is not identified either. While $E(Y_\omega|T_\omega = 1, S_\omega = s_7)$ and $E(Y_\omega|T_\omega = 3, S_\omega \in \{4,7\})$ are identified, $E(Y_\omega|T_\omega = 1, S_\omega = s_4)$ is not. I cannot disentangle $E(Y_\omega|T_\omega = 1, S_\omega = s_4)$ from the identified expectation $E(Y_\omega|T_\omega = 1, S_\omega \in \{4,5\})$.

In this subsection, I investigate additional assumptions that yield the point identification of the causal effects for the response-types. My identification strategy is a product of ideas drawn from the literature on causality and Bayesian networks. I use the available data on the post-intervention characteristics of a neighborhood and the causal relations between these characteristics and the family's unobserved variables. Specifically, I exploit the assumption that the overall quality of the neighborhood is not directly caused by the unobserved variables of a family. Even though the neighborhood quality correlates with the family unobserved variables because of neighborhood sorting.

Formally, let G_ω denote the post-intervention characteristics of the neighborhood faced by family ω that cause the outcome Y_ω. I account for G_ω in the MTO framework by recasting Model (7)–(8)

and (13) into the following equations:

$$Y_\omega = f_Y(G_\omega, V_\omega, \epsilon_\omega), \tag{31}$$

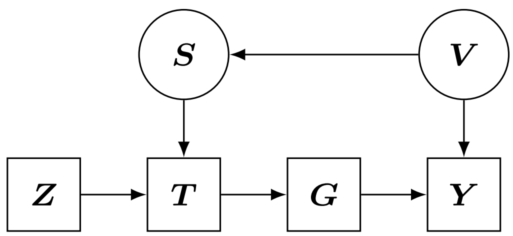
$$S_\omega = [f_T(z_1, V_\omega)\ ,\ f_T(z_2, V_\omega)\ ,\ f_T(z_3, V_\omega)] = f_S(V_\omega), \tag{32}$$

$$G_\omega = f_G(T_\omega, \xi_\omega), \tag{33}$$

$$T_\omega = [\mathbf{1}[Z_\omega = z_1], \mathbf{1}[Z_\omega = z_2], \mathbf{1}[Z_\omega = z_3]] \cdot S_\omega \tag{34}$$

where f_T is the same as in (8) and ξ_ω stands for an error term statistically independent of $Z_\omega, V_\omega, S_\omega, T_\omega$, and ϵ_ω. Figure 4 represents Model (31)–(34) as a DAG.

Figure 4: **MTO model with Post-intervention Neighborhood Characteristics**



This figure represents Model (31)–(34) as a DAG. Arrows represent direct causal relations. Circles represent unobserved variables. Squares represent observed variables. Y is the observed outcome. T is an observed relocation choice. V is an unobserved confounding variable that generates a selection bias and causes the response variable S and Y. G denotes the post-intervention neighborhood characteristics. Z stands for the MTO vouchers that play the role of the instrumental variable for relocation choice T.

Variable G_ω in the MTO model (31)–(34) represents neighborhoods intrinsic characteristics that affect the family outcomes. Examples of these characteristics are the quality of public schools, the supply of local jobs or the level of public safety generated by police patrols. Family unobserved variables V_ω correlate with these neighborhood characteristics G_ω as they are linked through the relocation choice T_ω via the causal path $V_\omega \rightarrow S_\omega \rightarrow T_\omega \rightarrow G_\omega$. In other words, neighborhood sorting induces a correlation between neighborhood characteristics G_ω and the family unobserved variables V_ω. But family unobserved variables V_ω also cause outcomes Y_ω. Therefore V_ω constitutes a confounding variable generating selection bias that impairs causal inference between observed G_ω and Y_ω in the same fashion that V_ω impairs causal inference between relocation choice T_ω and outcomes Y_ω.

The fact that selection bias plagues the relation between G_ω and Y_ω as well as between T_ω

and Y_ω raises the following question: How does the inclusion of G_ω in the MTO model help to identify the causal effects of T_ω on Y_ω if the relation between G_ω and Y_ω is impaired by the same problem we ought to solve? The answer lies in a key property of model (31)–(34), namely, the family unobserved variables V_ω do not *directly* cause neighborhood characteristics G_ω.

It is helpful to summarize the main identification results of Sections 3.2–3.3 in order to gain intuition on the identification result of this section. Equation (22) shows that the relation between observed propensity scores $P(T_\omega = t|Z_\omega = z)$ and unobserved response-type probabilities $P(S_\omega = s)$ is governed by matrix $\boldsymbol{A_S}$, which has full rank and renders the identification of the response-type probabilities. On the other hand, Equation (23) shows that the relation between the observed outcome expectations $E(Y_\omega|T_\omega = t, Z_\omega = z)$ and the unobserved outcome expectations conditioned on response-types $E(Y_\omega|T_\omega = t, S_\omega = s)$ is governed by matrix $\boldsymbol{A_D}$, which does not have full rank and does not generate the point identification of outcome expectations by response-types.

Now, if V_ω does not *directly* cause neighborhood characteristics G_ω, then the independence property $Y_\omega \perp\!\!\!\perp (Z_\omega, T_\omega)|(S_\omega, G_\omega)$ holds (proved in the following Lemma **L-3**). I use this property to show that the relation between the observed outcome expectations $E(Y_\omega|G_\omega = g, T_\omega = t, Z_\omega = z)$ and the unobserved outcome expectations conditioned on response-types $E(Y_\omega|G_\omega = g, S_\omega = s)$ is governed by matrix $\boldsymbol{A_S}$ (instead of $\boldsymbol{A_D}$), which renders the identification of the outcome expectation conditioned on response-types $S_\omega = s$ and $G_\omega = g$.

Another identification insight is given by the analysis of Pearl (1995), who studies a similar version of Model (31)–(34) termed the "Front-door model". His insight can be summarized as follows. First, if V_ω does not *directly* cause G_ω, then the relation between relocationship choice T_ω and observed neighborhood characteristics G_ω is causal as there is no confounding effect of V_ω. Moreover, T_ω is a matching variable for the impact of G_ω on Y_ω, in the same fashion that S_ω is a *matching variable* for the impact of T_ω on Y_ω. By matching variable I mean that T_ω solves the confounding effect generated by V_ω on the relation between G_ω on Y_ω. As a consequence, the causal effect of G_ω on Y_ω can be evaluated by conditioning on T_ω, which is observed. Thus the causal effect of T_ω on Y_ω can be computed as the weighted average of the causal effect of G_ω on Y_ω weighted by the distribution of G_ω conditioned on T_ω.[22]

[22] See Heckman and Pinto (2014b) for a discussion of this model and its relation to more standard approaches in economics.

The response variable S_ω also shares the same matching property of T_ω for the impact of G_ω on Y_ω. Theorem **T-10** explores this property to identify the average causal effect of T_ω on Y_ω as a weighted average of the the expectation of Y_ω conditioned on S_ω and G_ω weighted by the distribution of G_ω conditioned on T_ω.

A familiar example of the labor economics literature can clarify how the logic of the Front-door identification differs from the standard matching approach. Consider the quest for identifying the causal effect of going to college on income. Let Y_ω denote the observed income for an individual ω and let T_ω be a binary indicator that takes the value $T_\omega = 1$ if individual ω goes to college and $T_\omega = 0$ otherwise. Let V_ω represent the unobserved variables, e.g. cognition, that cause both college attendance and income. Therefore V_ω is the source of selection bias that prevents the evaluation of the causal effects of college on income based on observed data. A standard matching approach assumes a proxy for V_ω. For instance, the Stanford-binet IQ score. In this example, it is difficult to conceive a variable that share the properties that G_ω has in the MTO model (31)–(34). For this model of college return, it is necessary that a variable that *is caused* by college attendance *causes* income but *is not caused* by cognition. This example illustrates that the causal relations of the MTO model (31)–(34) are more an exception than a rule in microeconomic models.

The potential identifying power of variable G_ω in the MTO model (31)–(34) derives from the possibility of using available data on the post-intervention characteristics of the neighborhood as a good proxy for variable G_ω. The identifying assumption requires that G_ω is observed.

Kling et al. (2007) postulate that neighborhood characteristics (poverty, in their case) can be used as a good proxy for the unobserved neighborhood characteristics that affect the outcomes. They evaluate the impact of the poverty levels on the outcomes through a two-stage least squares (2SLS) model that uses the MTO vouchers by intervention site as the instrumental variables. Clampet-Lundquist and Massey (2008) also assume that the poverty levels are among the main driving forces that generate the neighborhood effects. In contrast with the 2SLS method employed by Kling et al. (2007), the identification analysis presented here is nonparametric. It is valid when structural equations for the outcome are nonlinear and nonseparable. Moreover, the assumption that generates the point identification of the causal effects of neighborhood relocation is testable.

Lemma **L-3** shows the independence relations of Model (31)–(34) that are used to identify the unobserved expectations $E(Y_\omega|T_\omega = t, S_\omega = s)$:

Lemma L-3. The following relations hold Model (31)–(34):

$$S_\omega \perp\!\!\!\perp Z_\omega, \quad Y_\omega \perp\!\!\!\perp (Z_\omega, T_\omega)|(S_\omega, G_\omega), \quad G_\omega \perp\!\!\!\perp (S_\omega, Z_\omega)|T_\omega.$$

Proof. See Mathematical Appendix. □

The next theorem uses Lemma **L-3** to state two equations that allow the identification of the outcome counterfactual expectations by response-types.

Theorem T-8. The following equation holds for Model (31)–(34):

$$E(Y_\omega|G_\omega = g, Z_\omega = z_j, T_\omega = t)\, P(T_\omega = t|Z_\omega = z_j) = \sum_{s_i \in \text{supp}(S)} \boldsymbol{A}_t[j,i]\, E(Y|G_\omega = g, S_\omega = s_i)\, P(S_\omega = s_i).$$

$$E(Y_\omega|T_\omega = t, S_\omega = s) = \int_{g \in \text{supp}(G)} E(Y|G_\omega = g, S_\omega = s) dF_{G_\omega|T_\omega = t}(g), \text{ where}$$

$$P(T_\omega = t|S_\omega = s) > 0 \text{ and } F_{G_\omega|T_\omega = t}(g) = P(G_\omega \leq g|T_\omega = t).$$

Proof. See Mathematical Appendix. □

The equations of Theorem **T-8** are useful in identifying $E(Y|T_\omega, S_\omega)$. Let $\boldsymbol{Q}_Z(g)$ denote the vector of the stacked expectations $E(Y_\omega|G_\omega = g, Z_\omega = z, T_\omega = t)\, P(T_\omega = t|Z_\omega = z)$ in the same fashion that $\boldsymbol{Q}_Z$ in Equation (20) stacks the expectations $E(Y_\omega|T_\omega = t, Z_\omega = z)\, P(T_\omega = t|Z_\omega = z)$ across the values that T_ω and Z_ω take. The $\boldsymbol{Q}_S(g)$ denotes the vector of the stacked expectations $E(Y_\omega|G_\omega = g, S_\omega = s)\, P(S_\omega = s)$ across the values $s \in \text{supp}(S)$. In this notation, I can express the first equation of **T-8** using the following matrix notation:

$$\boldsymbol{Q}_Z(g) = \boldsymbol{A}_{\boldsymbol{S}} \boldsymbol{Q}_S(g), \tag{35}$$

where $\boldsymbol{A}_{\boldsymbol{S}}$ is defined by (19). According to the response matrix of **T-1**, the rank of $\boldsymbol{A}_{\boldsymbol{S}}$ is equal to seven, which is equal to its column dimension. By the same reasoning of **T-3**, $E(Y|G_\omega = g, S_\omega = s)$ is identified for all $s \in \text{supp}(S)$ because $\boldsymbol{Q}_S(g)$ can be obtained by $\boldsymbol{A}_{\boldsymbol{S}}^{+} \boldsymbol{Q}_Z(g)$. The second equation of Theorem **T-8** shows that $E(Y_\omega|T_\omega = t, S_\omega = s)$ is a function of the identified parameters $E(Y|G_\omega = g, S_\omega = s)$ and the observed probabilities $P(G_\omega \leq g|T_\omega = t)$. Therefore, $E(Y_\omega|T_\omega = t, S_\omega = s)$ are identified for all $s \in \text{supp}(S)$ and $t \in \text{supp}(T)$ such that $P(T_\omega = t|S_\omega = s) > 0$. The next theorem formalizes this identification result.

Theorem T-9. Under Assumptions **A-1**–**A-3** and Model (31)–(34), the outcome expectations $E(Y_\omega|T_\omega = t, S_\omega = s)$ are identified for $s \in \text{supp}(S)$ and $t \in \text{supp}(T)$ such that $P(T_\omega = t|S_\omega = s) > 0$.

Proof. See Mathematical Appendix. □

Theorem **T-9** can be used to determine whether the G_ω, which represents the available data, are good proxies for the impact of the neighborhood characteristics on outcomes. The specification test of the model consists of comparing the identified causal parameters of **T-4** with the ones computed using G_ω. Another testable restriction is given by $G_\omega \perp\!\!\!\perp Z_\omega|T_\omega$ of Lemma **L-3**. In Web Appendix F, I present the model's specification tests for the labor market outcomes of the interim evaluation. The hypothesis that the observed neighborhood data are good proxies for the impact of the neighborhood characteristics on the outcomes cannot be rejected.

The next theorem an identification result for the average counterfactual outcome of the neighborhood relocation for Model (31)–(34):

Theorem T-10. Under Assumptions **A-1**–**A-3**, the counterfactual expectations of the outcomes $E(Y_\omega(t)); t \in \text{supp}(T)$ are identified in Model (31)–(34) by the following Equation:

$$E(Y_\omega(t)) = \int_{g \in \text{supp}(G)} \left(\sum_{s \in \text{supp}(S)} E(Y|G_\omega = g, S_\omega = s)\, P(S_\omega = s) \right) \frac{P(T_\omega = t|G_\omega = g)}{P(T_\omega = t)} dF_G(g), \tag{36}$$

where $F_G(g) = P(G_\omega \leq g)$.

Proof. See Mathematical Appendix. □

The Equation (36) shows that the counterfactual expectation of the outcome is a function of the unobserved expectations $E(Y|G_\omega = g, S_\omega = s)$ and the unobserved probabilities of the response-types $P(S_\omega = s)$. Those quantities are identified according to Theorems **T-9** and **T-4** respectively. The remaining quantities of Equation (36), that is, $P(T_\omega = t|G_\omega = g)$, $P(T_\omega = t)$ and $F_G(g)$, are observed.

Web Appendix G investigates another identification strategy that relies on the work of Altonji et al. (2005). Namely, $RATE_{\{s_4,s_5\}}(2,1)$ can be identified by assuming that $E(Y_\omega|T_\omega = 2, S_\omega =$

$s_4) = E(Y_\omega | T_\omega = 2, S_\omega \in \{s_4, s_6\})$ and $RATE_{\{s_4,s_7\}}(3,1)$ is identified by assuming that $E(Y_\omega | T_\omega = 3, S_\omega \in \{s_4, s_7\})$. Web Appendix G explains how to use Item (3) of Theorem **T-4** to make a causal inference under those assumptions.

The causal analysis using in this paper follows tools commonly used in economics. It is often unnoticed that the causal concept of fixing is an ill-defined concept in statistical theory.[23] This generates some confusion in realms of statistics and economics. For instance, fixing depends on the direction of causal relations while statistical theory lacks directionality. Fixing differs from statistical conditioning in that fixing a variable does not affect the distribution of variables not caused by the variable being fixed. Conditioning, on the other hand, affects the dependency structure of all random variables. Most important, fixing does not comply to range of statistical manipulations such as the law of iterated expectation. In Web Appendix I, I develop a causal framework that harmonizes causal concepts with standard statistical tools. Namely, I develop a causal framework that is fully developed within standard statistical theory and does not require additional mathematical tools to assess causal operations.

4 Empirical Analysis

The novel method employed here (see Section 3) sheds new light on the impacts of the MTO project. I evaluate new parameters that have a clear interpretation in terms of the causal effects of neighborhood relocation and that have not previously been estimated in the MTO literature. I also investigate the pre-program variables and the outcomes of MTO families according the seven economically justified response-types described in Table 10.

The response-types s_1, s_2, and s_3 consist of families whose relocation decisions are not affected by the voucher assignment. The remaining response-types consist of families whose relocation decision vary according to voucher assignments.

Table 10 presents the response-type probabilities. It shows that the response-types s_1, s_2, and s_3 account for 43% of the MTO families. The exogenous variation in the MTO vouchers cannot be used to assess the causal effects of relocating for those families because their relocation choice does not vary by the voucher assignment. In spite of this lack of variation, the comparison of outcome

[23] See Heckman and Pinto (2014b) for a recent discussion on fixing.

counterfactual expectations among s_1,s_2, and s_3 is of interest. Suppose that the distribution of family unobserved characteristics that affect outcomes were similar across these response-types. Then large differences in outcome expectations conditioned on these response-types, say s_2 and s_1, would suggest possible effects of neighborhood relocation.

Table 10: Response-type Probabilities

	Response-types						
Voucher	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Control (z_1)	1	2	3	1	1	3	1
Experimental (z_2)	1	2	3	2	2	2	1
Section 8 (z_3)	1	2	3	3	2	3	3
	Response-type Probabilities						
All Sites	0.31	0.04	0.08	0.31	0.05	0.09	0.12
	Response-type Probabilities by Site						
Baltimore	0.21	0.05	0.10	0.33	0.08	0.13	0.10
Boston	0.39	0.05	0.08	0.28	0.04	0.11	0.05
Chicago	0.24	0.03	0.10	0.22	0.01	0.08	0.31
Los Angeles	0.15	0.03	0.06	0.45	0.09	0.11	0.12
New York	0.45	0.04	0.05	0.31	0.06	0.05	0.04

This table presents the estimated probabilities of the response-types by site according to Equation (27).

The response-types s_4, s_5, s_6, and s_7 consist of families that change their relocation decisions as the voucher assignment varies. Those response-types represent 57% of the MTO sample and comprise the families that generate the policy conclusions of the MTO vouchers.

Families of type s_4 are the most responsive to MTO voucher policy. Those families do not relocate if no voucher is offered, relocate to a low poverty neighborhood if given the experimental voucher and relocate to high poverty neighborhood if given the Section 8 voucher.

Families of type s_5 can be comprehended as families that intend to relocate to a low poverty neighborhood but will do so only if a subsidizing voucher is offered. On the other hand, families of s_6 prefer to relocate to a high poverty neighborhood but would relocate to a low poverty neighborhood if the experimental voucher is offered. Families of type s_7 would relocate only if they could use a voucher to lease a unit in a high poverty neighborhood.

According to Item (3) of Theorem **T-4**, the expected values of pre-program variables conditioned on response-types are identified. These estimates are presented in Table 11.

Families that never move (response-type s_1) are more likely to have a disable family member. Families that always move to a low poverty neighborhood (response-type s_2) are more likely to be victims of crimes in their original neighborhoods. These s_2-type families have more schooling, are more likely to employed, less likely to be on welfare and fare better economically than the families of any other response-type. For example, low poverty movers of response-type s_2 are twice as likely to have a car or be employed than the never movers of response-type s_1. The families that are most responsive to the MTO vouchers (response-type s_4) are less likely to have teenage family members. Families most dependent on vouchers to relocate to low poverty neighborhoods (response-type s_5) are also the families that most depend on welfare.

The causal effect of low poverty neighborhood relocation can only be assessed for the response-types whose decisions include relocate to low poverty neighborhoods and not to relocate. These response-types, s_4 and s_5, account for a third of the MTO families. I use $RATE_{\{s_4,s_5\}}(2,1)$ to denote the causal effect for these response-types of relocating to a low poverty neighborhood. In the same fashion, the causal effects of high poverty neighborhood relocation can only be determined for response-types that access the choices of no relocation and high poverty neighborhoods as the voucher assignment changes. These families belong to response-types s_4 and s_7 and represent 43% of the sample. The causal effect of relocating to a high poverty neighborhood for these response-types is denoted by $RATE_{\{s_4,s_7\}}(3,1)$.

I evaluate the causal effects of relocation for labor market outcomes surveyed in the interim evaluation. These outcomes are divided into five domains: (1) adult earnings, (2) total income, (3) poverty, (4) self-sufficiency, and (5) employment.

The estimates presented below are conditioned on the sites of the intervention. By this I mean that each estimate is a weighted average of the parameters computed by site. [24] Table 12 presents a

[24] I follow a parsimonious criteria to select the site indicators as conditioning variables. This selection allows for

Table 11: Expected Value of Pre-program Variables by Response-type

	Response-type s_1		Response-type s_2		Response-type s_3		Response-type s_4		Response-type s_5		Response-type s_6		Response-type s_7	
Variable Name	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
Response-type														
Probabilities	0.31	0.01	0.04	0.01	0.08	0.01	0.31	0.02	0.05	0.01	0.09	0.01	0.12	0.02
Family														
Disable Household Member	**0.21**	0.02	0.14	0.08	0.13	0.04	0.13	0.02	0.14	0.10	0.16	0.06	0.12	0.06
No teens (ages 13-17) at baseline	0.55	0.02	0.63	0.11	0.70	0.05	**0.72**	0.03	0.54	0.14	0.55	0.08	0.54	0.08
Household size is 2 or smaller	0.19	0.02	**0.44**	0.10	0.14	0.04	0.22	0.03	0.24	0.12	0.32	0.07	0.16	0.07
Neighborhood														
Baseline Neighborhood Poverty	52.24	0.64	56.06	3.70	54.84	2.01	53.35	1.12	58.83	4.53	60.55	2.74	67.12	3.40
Victim last 6 months (baseline)	0.39	0.02	**0.56**	0.11	0.38	0.06	0.43	0.03	0.47	0.13	0.45	0.08	0.42	0.08
Living in neighborhood > 5 yrs.	0.66	0.02	0.75	0.11	0.59	0.05	0.61	0.03	0.50	0.14	0.52	0.08	0.55	0.08
Chat with neighbor	0.51	0.02	**0.36**	0.11	0.51	0.06	0.46	0.03	**0.70**	0.14	0.56	0.08	0.66	0.08
Watch for neighbor children	0.55	0.02	0.52	0.11	0.61	0.06	0.51	0.03	0.55	0.14	0.55	0.08	0.65	0.08
Unsafe at night (baseline)	0.43	0.02	0.41	0.11	0.52	0.06	0.57	0.03	0.51	0.14	0.49	0.08	0.51	0.08
Moved due to gangs	0.73	0.02	0.77	0.09	0.72	0.05	0.78	0.03	0.76	0.12	0.87	0.07	0.82	0.07
Schooling														
Has a GED (baseline)	0.17	0.02	**0.13**	0.09	0.20	0.04	0.22	0.03	0.24	0.11	0.13	0.06	0.19	0.06
Completed high school	0.35	0.02	**0.58**	0.11	0.38	0.05	0.35	0.03	0.35	0.14	0.46	0.08	0.38	0.08
Enrolled in school (baseline)	0.13	0.02	0.22	0.08	0.14	0.04	0.21	0.02	0.07	0.11	0.22	0.06	0.14	0.06
Never married (baseline)	0.60	0.02	0.60	0.11	0.64	0.05	0.65	0.03	0.65	0.14	0.66	0.08	0.52	0.08
Teen pregnancy	0.21	0.02	0.30	0.10	0.20	0.05	0.24	0.03	0.25	0.12	0.36	0.07	0.35	0.07
Missing GED and H.S. diploma	0.08	0.01	0.06	0.06	0.07	0.02	0.05	0.02	0.04	0.07	0.05	0.04	0.11	0.04
Sociability														
No family in the neigborhood	0.64	0.02	0.51	0.11	0.70	0.05	0.68	0.03	**0.79**	0.14	0.57	0.08	0.55	0.08
Respondent reported no friends	0.39	0.02	0.42	0.11	0.37	0.05	0.43	0.03	0.46	0.14	0.47	0.08	0.36	0.08
Welfare/economics														
AFDC/TANF Recepient	0.71	0.02	**0.64**	0.10	0.70	0.05	0.78	0.03	**0.82**	0.12	0.74	0.07	0.79	0.07
Car Owner	**0.13**	0.01	**0.28**	0.09	0.20	0.04	0.21	0.03	0.25	0.11	0.05	0.06	0.14	0.06
Adult Employed (baseline)	0.23	0.02	**0.46**	0.10	0.22	0.05	0.20	0.03	0.27	0.13	0.37	0.07	0.34	0.07

This table shows expected value of the MTO pre-program variables by response-type. All estimates are weighted according to the weighting index suggested by the MTO Interim evaluation. Standard deviations of the estimates are computed using the boostrap method. Some values of interest are in bold type.

statistical description of outcome expectations for labor market outcomes conditioned on relocation choices and voucher assignments. For every outcome, the expected values for the families who chose to relocate to a low poverty neighborhood are greater than the values for the families that do not relocate. There are substantial differences across the treatment cells.

Consider the comparison between control families that relocate to a low-poverty and families that decide not to relocate (columns 10 and 5 of Table 12). The outcome expectation differences for the three variables in Total Income domain (second set of rows in Table 12) are \$ 4,812, \$ 7,300 and \$ 6,775 respectively. The corresponding values for control families that relocate to a high-poverty neighborhood are \$ 1,360, \$ 1,776 and \$ 2,214 respectively. The outcome differences associated with low-poverty relocation account for less than a third of the differences associated with low-poverty relocation. These differences do not have a causal interpretation as they do not account for the selection bias generated by the choice of neighborhood relocation.

The Bloom estimator – the voucher's effect divided by the compliance rate for the voucher – is a useful parameter for evaluating the causal effects of the voucher for families that use the voucher. The parameter is of special importance for those interested in examining the policy implications of MTO voucher assignments. Table 13 presents (1) the Bloom estimator for the experimental group versus control group and (2) the treatment-on-the-treated estimator suggested by Equation (28). Section 3.6 explained that although these parameters yield different interpretations, they are likely to generate similar results in terms of estimation and inference. Table 13 supports this claim.

The first column of Table 13 presents the name of the variables and the second column indicates if the variable is reversed so that greater values of a variable are in accord with the expected direction of the effects. Bloom estimates are presented in columns 3–6. Column 3 presents the outcome mean for the control group and column 4 the Bloom estimator.[25] Columns 5 and 6 show the single-hypothesis and multiple-hypothesis single-sided p-values for the null hypothesis of no effect.[26] Columns 7–10 provide the same analysis of columns 3–6 to the treatment-on-the-treated estimator of Equation (28), that is, $TOT_2(z_2, z_1)$.

nonparametric conditioning and avoids potential model misspecification arising from imposing linearity assumptions. In Pinto (2014), I examine the question of covariate selection in greater detail. I use a within-site Bayesian Model Averaging following the methods suggested in Hansen (2007, 2008).

[25] Estimates are conditioned on site and weighted according to the weighting index recommended by the MTO interim evaluation.

[26] The p-values are computed using the Bootstrap method (Efron, 1981; Romano, 1989) and the multiple hypothesis inference uses the stepdown algorithm of Romano and Wolf (2005).

Table 12: Outcome Expectations by Neighborhood Relocation and Voucher Assignments

			No Relocation				Low-poverty Neighborhood				High-poverty Neighborhood			
			All	Conditioned on Vouchers			All	Conditioned on Vouchers			All	Conditioned on Vouchers		
		Sample	Vouchers	Cntl.	Exper.	Sec. 8	Vouchers	Cntl.	Exper.	Sec. 8	Vouchers	Cntl.	Exper.	Sec. 8
Variable	Rev.	Size	3	4	5	6	7	8	9	10	11	12	13	14
Adult Earnings														
Earnings (2001)	No	3313	8440.1	8649.3	8584.5	7472.4	9684.8	11779.2	9568.0	9994.3	9392.0	9235.5	9579.2	9220.4
Current Weekly earnings	No	3311	173.2	178.8	173.0	157.7	189.2	199.9	189.1	189.4	184.1	178.4	211.1	178.9
Earnings Range (1 to 6)	No	3313	2.35	2.38	2.39	2.22	2.54	2.94	2.53	2.56	2.53	2.54	2.55	2.50
Total Income														
Total income (head)	No	3526	11512.6	11365.1	11689.6	11398.0	12665.9	16177.1	12636.2	12571.6	12361.4	12725.0	13211.0	11999.5
Total household income	No	3526	12871.9	13022.1	12909.3	12450.3	14364.2	20322.1	14415.4	13194.4	13614.5	14798.1	14770.2	12911.9
Sum of all income	No	3526	15196.4	14957.4	15284.5	15454.5	16527.9	21732.7	16709.0	15679.3	15413.3	17171.6	17325.0	14375.9
Poverty Line														
Income $<$ 50% poverty line	Yes	3526	-0.34	-0.35	-0.35	-0.31	-0.33	-0.25	-0.31	-0.45	-0.37	-0.35	-0.32	-0.38
Income $\geq$ 150 % poverty line	No	3526	0.15	0.14	0.15	0.18	0.18	0.17	0.18	0.20	0.15	0.19	0.12	0.13
Income $>$ poverty line	No	3526	0.27	0.26	0.29	0.26	0.33	0.60	0.33	0.30	0.31	0.39	0.33	0.28
Self-sufficiency														
Economic self-sufficiency	No	3499	0.17	0.17	0.18	0.18	0.20	0.30	0.20	0.18	0.20	0.20	0.19	0.20
Employed (no welfare)	No	3472	0.45	0.45	0.46	0.47	0.49	0.40	0.49	0.51	0.47	0.49	0.52	0.45
Not in the labor force	Yes	3508	-0.39	-0.37	-0.38	-0.42	-0.31	-0.39	-0.30	-0.38	-0.33	-0.41	-0.30	-0.31
Self-reported Employment														
Employed	No	3517	0.51	0.52	0.51	0.49	0.57	0.52	0.57	0.55	0.55	0.53	0.61	0.55
Employed with health insurance	No	3483	0.30	0.30	0.32	0.24	0.33	0.30	0.33	0.32	0.31	0.29	0.34	0.31
Employed full-time	No	3488	0.39	0.40	0.39	0.36	0.40	0.30	0.40	0.39	0.40	0.39	0.45	0.40
Employed above poverty	No	3311	0.31	0.32	0.31	0.30	0.34	0.29	0.34	0.35	0.35	0.35	0.36	0.35
Job for more than a year	No	3475	0.38	0.37	0.39	0.38	0.39	0.23	0.39	0.38	0.38	0.36	0.44	0.38

This table shows the expected values of labor market variables by relocation choice and voucher assignment. Expectations are conditioned on MTO sites and weighted according to the index suggested by the MTO Interim Evaluation. Variables are divided into five domains separated by horizontal lines: (1) Adult Earnings; (2) Total Income; (3) Poverty; (4) Self-sufficiency; and (5) Employment. The first column gives the variable name. The second column states if the variable is reversed or not. By reversed I mean that the variable is multiplied by -1 such that greater values of the variable denotes a desired score. The third column denotes the sample size. The remaining columns show the expected value of labor market outcomes conditioned on the neighborhood relocation choice and voucher assignment. Column 3 presents the outcome expectation for families that do not relocate. Columns 4–6 present the outcome expectation for families that do not relocate conditioned on voucher assignment: Control group, Experimental group and Section 8 group respectively. Columns 7–10 present the same conditional outcome expectations for families that relocate to low-poverty neighborhood. Columns 11–14 focus on the hight-poverty neighborhood relocation.

Table 13 shows that both methods generate very similar results and no effect survives the multiple hypothesis inference aside from the labor force indicator. A few measures of employment are statistically significant when considering single-hypothesis inference.

Theorem **T-7** shows that the TOT parameter defined by Equation (28) is a weighted average of the causal effects of relocation across the response-types whose decision changes as the voucher assignment varies. In the case of the experimental group versus the control group, the $TOT_2(z_2, z_1)$ parameter captures the relocation effects associated with response-types s_4, s_5, and s_6. The $TOT_2(z_2, z_1)$ assesses half of the MTO sample and is a mixture of the causal effects associated with relocating to a low poverty neighborhood for response-types s_4 and s_5, that is, $RATE_{\{s_4,s_5\}}(2, 1)$, and relocating to a high poverty neighborhood for response-type s_6, that is, $RATE_{s_6}(2, 3)$.

Section 3.7 provides the expressions to compute counterfactual outcomes conditioned on response-types as well as the average causal effect of neighborhood relocation. The identification of the causal components of the $TOT_2(z_2, z_1)$ uses post-intervention data on neighborhood poverty levels. Specifically, I rely on available data on the proportion of time that MTO participants lived in neighborhoods that have different poverty levels.

Table 14 presents three analyses that compare counterfactual outcomes of low-poverty neighborhood relocation and no relocation. The first analysis (columns 3–6) compares families that always relocate to low-poverty neighborhood (response-type s_2) with families that never relocate (response-type s_1). Notationally, columns 3–6 report the estimates for $E(Y_\omega|S_\omega = s_2) - E(Y_\omega|S_\omega = s_1)$. As mentioned, this comparison is not causal nor is it captured by $TOT_2(z_2, z_1)$. Theorem **T-4** states that the identification of $E(Y_\omega|S_\omega = s_2) - E(Y_\omega|S_\omega = s_1)$ results from the assumptions that generate the seven economically justified response-types of Table 10.

The second analysis (columns 7–10) shows the estimates for the causal effect of low-poverty relocation compared to no relocation evaluated for the response-types that access these two relocation choices as voucher assignments vary. Notationally, columns 7–10 report the $RATE_{\{s_4,s_5\}}(2, 1)$ which can also be written in terms of counterfactual expectations: $E(Y_\omega(2) - Y_\omega(1)|S_\omega \in \{s_4, s_5\})$.

The identification of $RATE_{\{s_4,s_5\}}(2, 1)$ is discussed in Section 3.7 and is based on the assumption that the overall neighborhood quality is not directly caused by the family's unobserved variables. I use available data on post-intervention neighborhood poverty as a proxy for neighborhood qual-

Table 13: Treatment-on-the-Treated Comparison for No Move versus Low-poverty Relocation

		Voucher Effects divided by Compliance Rates				Voucher Effects divided by Difference in Relocation Propensities			
		Baseline	**Bloom**	**Inference**		**Baseline**	**TOT**	**Inference**	
		Mean	**Estimator**	**Single**	**SD**	**Mean**	**Estimator**	**Single**	**SD**
Variable	**Rev.**	3	4	5	6	7	8	9	10
Adult Earnings									
Earnings (2001)	No	8878.1	677.8	0.27	0.37	8878.1	706.5	0.28	0.39
Current Weekly earnings	No	179.0	9.96	0.33	0.33	179.0	11.02	0.33	0.33
Earnings Range (1 to 6)	No	2.42	0.11	0.27	0.39	2.42	0.12	0.27	0.38
Rank Average	No	0.50	0.02	0.24	–	0.50	0.02	0.24	–
Total Income									
Total income (head)	No	11803.1	1031.4	0.16	0.24	11803.1	1105.0	0.15	0.23
Total household income	No	13597.7	291.2	0.41	0.41	13597.7	313.8	0.41	0.41
Sum of all income	No	15621.2	823.5	0.30	0.33	15621.2	882.4	0.29	0.33
Rank Average	No	0.50	0.03	0.15	–	0.50	0.03	0.15	–
Poverty Line									
Income $<$ 50% poverty line	Yes	-0.35	0.04	0.23	0.46	-0.35	0.04	0.23	0.46
Income $\geq$ 150 % poverty line	No	0.15	0.02	0.25	0.38	0.15	0.03	0.25	0.38
Income $>$ poverty line	No	0.30	0.02	0.33	0.33	0.30	0.02	0.33	0.33
Rank Average	No	0.50	0.01	0.22	–	0.50	0.01	0.22	–
Self-sufficiency									
Economic self-sufficiency	No	0.18	0.02	0.28	0.28	0.18	0.02	0.28	0.28
Employed (no welfare)	No	0.45	0.07	**0.10**	0.17	0.45	0.07	**0.10**	0.17
Not in the labor force	Yes	-0.38	0.10	**0.03**	**0.06**	-0.38	0.10	**0.03**	**0.06**
Rank Average	No	0.49	0.03	**0.06**	–	0.49	0.03	**0.06**	–
Self-reported Employment									
Employed	No	0.52	0.05	0.18	0.31	0.52	0.05	0.18	0.30
Employed with health insurance	No	0.29	0.07	**0.08**	0.17	0.29	0.08	**0.08**	0.18
Employed full-time	No	0.39	0.02	0.33	0.44	0.39	0.03	0.32	0.43
Employed above poverty	No	0.32	0.01	0.43	0.43	0.32	0.01	0.42	0.42
Job for more than a year	No	0.36	0.08	**0.05**	0.15	0.36	0.09	**0.05**	0.15
Rank Average	No	0.50	0.03	0.12	–	0.50	0.03	0.12	–

This table compares the labor market outcomes of the Experimental group versus the Control group. It presents the Bloom estimator – voucher effect divided by voucher compliance rate – and the treatment-on-the-treated estimator suggested by Equation (28). Outcomes grouped in blocks separated by horizontal lines. The last line of each block of outcomes examines the average of the participant rank across the outcomes within each block. It represent a summary index for the selected variables within each block. First column states the variable name. Second column indicates if the variable is reversed, i.e., multiplied by -1 so that greater values of a variable are inline with the expected direction of the effects. Bloom estimates are presented in columns 3–6. Column 3 gives the outcome mean for the control group and column 4 gives the Bloom estimator. Columns 5 and 6 present the single-hypothesis and multiple-hypothesis single-sided p-values for the null hypothesis of no effect. Columns 7–10 provide the same analysis of columns 3–6 to the treatment-on-the-treated estimator of Equation (28). All estimates are weighted by the weighing index recommended by the MTO Interim evaluation. The p-values are computed using the Bootstrap method (Efron, 1981; Romano, 1989) and the multiple hypothesis inference uses the stepdown algorithm of Romano and Wolf (2005). Estimates are nonparametrically conditioned on the site of intervention.

ity and I compute $RATE_{\{s_4,s_5\}}(2,1)$ according to Equation (35). In Section 3.7, I explain that the assumptions that render the identification of $RATE_{\{s_4,s_5\}}(2,1)$ are testable. I test these assumption in Tables A.5–A.7 of Web Appendix F and do not reject the assumptions that identify $RATE_{\{s_4,s_5\}}(2,1)$.

The third analysis (columns 11–14) shows the estimates for the average causal effect of low-poverty relocation versus no relocation. Notationally, columns 11–14 report $E(Y_\omega(2)-Y_\omega(1))$ which is estimated according to Equation (36). The identification of this average causal effect, too, relies on the assumptions that identify $RATE_{\{s_4,s_5\}}(2,1)$.

Each analysis consists of four columns: (1) a baseline mean; (2) the expected difference of the outcome counterfactuals; (3) the one-sided p-values for single-hypothesis inference that the estimated effect is equal to zero; (4) the one-sided multiple-hypothesis p-values. All estimates are weighted by the weighing index recommended by the MTO Interim evaluation. The p-values are computed using the Bootstrap method (Efron, 1981; Romano, 1989) and the multiple hypothesis inference employs the stepdown algorithm of Romano and Wolf (2005). The reported parameters are conditioned on the site of intervention.

I find statistically significant effects on the labor market outcomes associated with Adult Earning, Total Income and Poverty-line (first three blocks of variables) while the effects for the outcomes associated with Self-sufficiency and Self-reported Employment (last two block of variables) are not statistically significant. Those results are in contrast with the TOT estimates of Table 13.

In summary, the biggest counterfactual differences are derived from comparisons of response-types s_2 and s_1, followed by $RATE_{\{s_4,s_5\}}(2,1)$ and then by the average relocation effect $E(Y_\omega(2)-Y_\omega(1))$. Columns 5–6 of Table 14 reveals that the outcome difference between response-types s_2 and s_1 is statistically significant for Adult Earnings and total income. The inference survives the multiple hypothesis correction. The table also documents statistically significant results for the causal effect on neighborhood relocation (Columns 7–10) on labor market outcomes and average causal effect of relocation (Columns 11–14) for total income outcomes. The results for self-sufficiency and employment displayed in Table 14 are not statistically significant.

Table 14: Causal Effects and Response-type Comparison for Low-poverty Relocation versus No Relocation

		Response-type Comparison $S = s_2$ versus $S = s_1$				Causal Effects for Response-types s_4 and s_5				Average Treatment Effects All Response-types			
		Base	**Diff.**	**Inference**		**Base**	**Diff.**	**Inference**		**Base**	**Diff.**	**Inference**	
		Mean	**Means**	**Single**	**SD**	**Mean**	**Means**	**Single**	**SD**	**Mean**	**Means**	**Single**	**SD**
Variable Name	**Rev.**	3	4	5	6	7	8	9	10	11	12	13	14
Adult Earnings													
Earnings (2001)	No	7363.9	4447.8	**0.03**	**0.05**	8652.5	1156.1	**0.09**	0.13	8576.3	1059.1	**0.10**	0.13
Current Weekly earnings	No	154.2	22.830	0.27	0.27	198.6	8.171	0.28	0.28	180.0	10.740	0.23	0.23
Earnings Range (1 to 6)	No	2.208	0.731	**0.04**	**0.05**	2.329	0.176	0.14	0.17	2.364	0.153	0.16	0.20
Rank Average	No	0.466	0.073	**0.09**	–	0.504	0.015	0.26	–	0.498	0.015	0.26	–
Total Income													
Total income (head)	No	11460.9	4433.9	**0.01**	**0.02**	10892.3	1047.7	**0.10**	**0.10**	11514.9	1019.3	**0.09**	**0.09**
Total household income	No	12545.8	7416.0	**0.01**	**0.01**	12596.5	1676.9	**0.07**	**0.07**	12745.1	1522.8	**0.07**	**0.07**
Sum of all income	No	15565.5	5733.3	**0.03**	**0.03**	14240.4	2264.7	**0.02**	**0.05**	14788.2	1933.4	**0.03**	**0.07**
Rank Average	No	0.489	0.152	**0.01**	–	0.464	0.026	0.13	–	0.486	0.023	0.16	–
Poverty Line													
Income < 50% poverty line	Yes	-0.302	0.038	0.36	0.53	-0.337	0.032	**0.08**	0.21	-0.347	0.025	0.12	0.30
Income ≥ 150 % poverty line	No	0.179	-0.006	0.53	0.53	0.138	0.065	**0.02**	**0.03**	0.150	0.049	**0.03**	**0.06**
Income > poverty line	No	0.253	0.341	**0.00**	**0.00**	0.219	0.030	0.19	0.19	0.279	0.023	0.23	0.23
Rank Average	No	0.505	0.062	**0.05**	–	0.485	0.021	0.11	–	0.496	0.016	0.16	–
Self-sufficiency													
Economic self-sufficiency	No	0.179	0.109	0.17	0.29	0.155	0.035	0.15	0.29	0.168	0.034	0.13	0.24
Employed (no welfare)	No	0.468	-0.090	0.81	0.81	0.471	-0.009	0.52	0.52	0.461	0.010	0.35	0.35
Not in the labor force	Yes	-0.430	0.040	0.30	0.43	-0.300	0.022	0.24	0.34	-0.348	0.019	0.22	0.35
Rank Average	No	0.487	0.012	0.33	–	0.506	0.008	0.32	–	0.499	0.010	0.28	–
Self-reported Employment													
Employed	No	0.489	-0.008	0.51	0.79	0.578	0.008	0.38	0.56	0.536	0.012	0.33	0.51
Employed with health insurance	No	0.240	0.041	0.29	0.61	0.316	-0.001	0.46	0.54	0.304	0.007	0.39	0.46
Employed full-time	No	0.357	-0.088	0.86	0.93	0.429	0.005	0.37	0.56	0.392	0.015	0.29	0.49
Employed above poverty	No	0.288	-0.029	0.62	0.84	0.347	-0.006	0.47	0.47	0.329	0.006	0.37	0.37
Job for more than a year	No	0.373	-0.154	0.95	0.95	0.399	0.003	0.39	0.55	0.390	0.008	0.35	0.52
Rank Average	No	0.487	-0.027	0.76	–	0.515	-0.000	0.44	–	0.505	0.004	0.39	–

This table examines causal parameters associated with the choice of relocation to low-poverty neighborhood versus no relocation for labor market outcomes. MTO Outcomes on this table are grouped in blocks separated by horizontal lines. The last line of each block of outcomes examines the average of the participant rank across the outcomes within block. First column states the variable name. Second column indicates if the variable is reversed, i.e., multiplied by -1 so that greater values of a variable are inline with the expected direction of the effects. The remaining columns refer to three analysis of counterfactual outcome expectations. The first analysis compares families that always move to low-poverty neighborhoods (response-type s_2) and families that never move (response-type s_1). The second analysis shows the causal effects of low-poverty neighborhood relocation that are defined for response-types s_4 and s_5., i.e., $RATE_{\{s_4,s_5\}}(2,1)$. The third analysis evaluates the average causal effect for low-poverty neighborhood relocation. Results of each analysis are displayed in four columns. The first column shows the baseline outcome expectation. The second column shows the difference in the outcome expectations. The third and fourth columns show the single-hypothesis and multiple-hypothesis single-sided p-values for the null hypothesis of no effect respectively. All estimates are weighted by the weighing index recommended by the MTO Interim evaluation. The p-values are computed using the Bootstrap method (Efron, 1981; Romano, 1989) and the multiple hypothesis inference uses the stepdown algorithm of Romano and Wolf (2005). Estimates are nonparametrically conditioned on the site of intervention.

5 Summary and Conclusions

This paper examines the causal effects of neighborhood relocation on socioeconomic outcomes. A fundamental challenge in neighborhood-level research is accounting for the selection bias generated by residential sorting. I address this challenge by exploiting the features of the Moving to Opportunity (MTO) project, a prominent social experiment that devised the method of randomized controlled trials to investigate neighborhood effects.

MTO is a housing experiment designed to investigate the social and economic consequences of relocating poor families from America's most distressed urban neighborhoods to low-poverty communities. MTO randomly assigns vouchers that can be used to subsidize the rent of a housing unit if the family agrees to relocate to a better neighborhood.

The intervention assigned participating families to three groups: a control group, an experimental group and the Section 8 group. The families assigned to the control group were offered no voucher. Experimental families were offered a voucher that could be used to lease a unit in a low poverty neighborhood if the family agreed to relocate. The Section 8 recipients were offered a voucher that could be used to lease a unit in either low or high poverty neighborhoods. The MTO program did not force voucher compliance but rather created incentives for neighborhood relocation.

An influential literature on MTO used randomized vouchers to evaluate the intention-to-treat ITT effect – the voucher's causal effect – and the treatment-on-the-treated TOT effect – the voucher's effect divided by the voucher's compliance rate. These effects are the most useful and appropriate parameters to examine the policy implications of the MTO voucher assignments. Randomized vouchers, however, do not render the identification of the causal effects of *neighborhood relocation* on socioeconomic outcomes. To assess those, it is necessary to account for the selection bias generated by the family relocation decision.

I contribute to the MTO literature by quantifying a new set of parameters that have a clear interpretation in terms of the causal effects of neighborhood relocation. The interpretation of these effects is based on the seven economically justified response-types described in Table 10. I develop a novel method that explores the economic and causal features of the project's design to solve the problem of selection bias generated by neighborhood sorting. To this end, I consider an unordered choice model in which vouchers play the role of instrumental variables for neighborhood relocation and families decide among three relocation alternatives: (1) no relocation, (2) relocation to a low-poverty neighborhood; and (3) relocation to a high-poverty neighborhood.

Those who intend to nonparametric identify the causal effects of neighborhood relocation face a major challenge: the experimental variation of MTO vouchers is insufficient to identify all possible counterfactual relocation decisions. My identification strategy overcomes this challenge by combining economic theory, the tools of causal inference and the experimental variation from data.

I employ economic reasoning such as the Strong Axiom of Revealed Preferences to reduce the gap between the number of counterfactual relocation decisions and the values that the instrumental variable takes. Upon this approach, I identify a range of counterfactual outcomes and latent probabilities associated economically justifiable relocation patterns of MTO participants. This method allows to decompose the TOT parameter into interpretable components associated with the causal effects of neighborhood relocation. For instance, the TOT parameter that compares the experimental group with the control group can be expressed as a mixture of two effects: (1) the causal effect of relocating to a low poverty neighborhood versus not relating; and (2) the causal effect of relocating to a low poverty neighborhood versus relocating to a high poverty neighborhood.

I achieve point identification by using tools of causal inference from the literature on causality and Bayesian networks (Lauritzen, 1996; Pearl, 2009). I exploit the assumption that the overall quality of a neighborhood is not directly caused by the unobserved family variables. Even though the neighborhood quality correlates with the family unobserved variables because of neighborhood sorting. I use available data on post-intervention neighborhood poverty as a proxy for neighborhood quality. Under these assumptions, I show that it is possible to nonparametrically identify the causal effects of neighborhood relocation.

My empirical analysis agrees with the previous literature that shows no statistically significant TOT effects on economic outcomes. However, I obtain sharper results than my predecessors by focusing on the causal effects of neighborhood relocation rather than voucher effects. I find statistically significant effects of neighborhood relocation for identifiable subpopulations of MTO. Moreover, I find statistically significant results for the average causal effects of neighborhood relocation.

The identification challenge posed by the MTO intervention is an instance of a more general class of econometric problems wherein variation in instrumental variables is insufficient to identify the treatment effects of interest. The methodology proposed here is general and applies widely to the case of unordered choice models with categorical instrumental variables and multiple treatments.

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A Mathematical Appendix - Proofs

This appendix presents the proofs of lemmas and theorems stated in the main paper. **Proof of Lemma L-1:**

Proof. Let $x_\omega(z,t) = \max_{(k)\in W_\omega(z,t)} u_\omega(x,t)$ be choice of consumption goods when family is faced with instrument $z \in \{1,2,3\}$ and neighborhood relocation $t \in \{1,2,3\}$. Also let $[x_\omega(z,t),t]$ is a bundle of goods and neighborhood choices. Then, by SARP:

$$\begin{aligned} &\text{if } [x_\omega(z,t),t] \succ [x_\omega(z,t'),t'] \\ &\text{and } W_\omega(z,t) \subseteq W_\omega(z',t),\ W_\omega(z,t') \supseteq W_\omega(z',t') \\ &\text{then } [x_\omega(z',t),t] \succ [x_\omega(z',t'),t']. \end{aligned} \tag{37}$$

Also, we have that:

$$C_\omega(z) = t \Leftrightarrow [x_\omega(z,t),t] \succ [x_\omega(z,t'),t'];\ \forall\, t' \in \{1,2,3\} \setminus \{t\}, \tag{38}$$

We now proof the Theorem choice restrictions:

1. $C_\omega(1) = 2 \Rightarrow [x_\omega(1,2),2] \succ [x_\omega(1,1),1]$ and $[x_\omega(1,2),2] \succ [x_\omega(1,3),3]$.

    - By **A-1**, $W_\omega(1,2) \subset W_\omega(2,2)$, and by **A-2**, $W_\omega(1,1) = W_\omega(2,1)$ thus by (37) and $[x_\omega(1,2),2] \succ [x_\omega(1,1),1]$ we have that $[x_\omega(2,2),2] \succ [x_\omega(2,1),1]$. Again, by **A-1**,$W_\omega(1,2) \subset W_\omega(2,2)$, and by **A-2**, $W_\omega(1,3) = W_\omega(2,3)$ thus by (37) and $[x_\omega(1,2),2] \succ [x_\omega(1,3),3]$ we have that $[x_\omega(2,2),2] \succ [x_\omega(2,3),3]$. Therefore, by (38), $C_\omega(2) = 2$.

    - By **A-1**, $W_\omega(1,2) \subset W_\omega(3,2)$, and by **A-2**, $W_\omega(1,1) = W_\omega(3,1)$ thus by (37) and $[x_\omega(1,2),2] \succ [x_\omega(1,1),1]$ we have that $[x_\omega(3,2),2] \succ [x_\omega(3,1),1]$. Therefore $C_\omega(3) \in \{2,3\}$.

2. $C_\omega(1) = 3 \Rightarrow [x_\omega(1,3),3] \succ [x_\omega(1,1),1]$ and $[x_\omega(1,3),3] \succ [x_\omega(1,2),2]$.

    - By **A-1**, $W_\omega(1,3) = W_\omega(2,3)$, and by **A-2**, $W_\omega(1,1) = W_\omega(2,1)$ thus by (37) and $[x_\omega(1,3),3] \succ [x_\omega(1,1),1]$ we have that $[x_\omega(2,3),3] \succ [x_\omega(2,1),1]$. Therefore, by (38), $C_\omega(2) \in \{2,3\}$.

    - By **A-1**, $W_\omega(1,3) \subset W_\omega(3,3)$, and by **A-2**, $W_\omega(1,1) = W_\omega(3,1)$ thus by (37) and $[x_\omega(1,3),3] \succ [x_\omega(1,1),1]$ we have that $[x_\omega(3,3),3] \succ [x_\omega(3,1),1]$. Therefore $C_\omega(3) \in \{2,3\}$.

3. $C_\omega(2) = 1 \Rightarrow [x_\omega(2,1),1] \succ [x_\omega(2,2),2]$ and $[x_\omega(2,1),1] \succ [x_\omega(2,3),3]$.

    - By **A-2**, $W_\omega(2,1) = W_\omega(1,1)$ and by **A-1**, $W_\omega(2,2) \supset W_\omega(1,2)$ thus by (37) and $[x_\omega(2,1),1] \succ [x_\omega(2,2),2]$ we have that $[x_\omega(1,1),1] \succ [x_\omega(1,2),2]$. Again, by **A-2**,$W_\omega(2,1) = W_\omega(1,1)$ and $W_\omega(2,3) = W_\omega(1,3)$ thus by (37) and $[x_\omega(2,1),1] \succ [x_\omega(2,3),3]$ we have that $[x_\omega(1,1),1] \succ [x_\omega(1,3),3]$. Therefore, by (38), $C_\omega(1) = 1$.

    - By **A-2**, $W_\omega(2,1) = W_\omega(3,1)$, and by **A-1**, $W_\omega(2,2) = W_\omega(3,2)$ thus by (37) and $[x_\omega(2,1),1] \succ [x_\omega(2,2),2]$ we have that $[x_\omega(3,1),1] \succ [x_\omega(3,2),2]$. Therefore $C_\omega(3) \in \{1,3\}$.

4. $C_\omega(2) = 3 \Rightarrow [x_\omega(2,3), 3] \succ [x_\omega(2,2), 2]$ and $[x_\omega(2,3), 3] \succ [x_\omega(2,1), 1]$.

   - By **A-2**, $W_\omega(2,3) = W_\omega(1,3)$ and $W_\omega(2,1) = W_\omega(1,1)$ thus by (37) and $[x_\omega(2,3), 3] \succ [x_\omega(2,1), 1]$ we have that $[x_\omega(1,3), 3] \succ [x_\omega(1,1), 1]$. Again, by **A-2**,$W_\omega(2,3) = W_\omega(1,3)$, and by **A-1**, $W_\omega(2,2) \supset W_\omega(1,2)$ thus by (37) and $[x_\omega(2,3), 3] \succ [x_\omega(2,2), 2]$ we have that $[x_\omega(1,3), 3] \succ [x_\omega(1,2), 2]$. Therefore, by (38), $C_\omega(1) = 3$.

   - By **A-1**, $W_\omega(2,3) \subset W_\omega(3,3)$, and by **A-2**, $W_\omega(2,1) = W_\omega(3,1)$ thus by (37) and $[x_\omega(2,3), 3] \succ [x_\omega(2,1), 1]$ we have that $[x_\omega(3,3), 3] \succ [x_\omega(3,1), 1]$. Again, by **A-1**,$W_\omega(2,3) \subset W_\omega(3,3)$ and $W_\omega(2,2) = W_\omega(3,2)$ thus by (37) and $[x_\omega(2,3), 3] \succ [x_\omega(2,2), 2]$ we have that $[x_\omega(3,3), 3] \succ [x_\omega(3,2), 2]$. Therefore, by (38), $C_\omega(3) = 3$.

5. $C_\omega(3) = 1 \Rightarrow [x_\omega(3,1), 1] \succ [x_\omega(3,2), 2]$ and $[x_\omega(3,1), 1] \succ [x_\omega(3,3), 3]$.

   - By **A-2**, $W_\omega(3,1) = W_\omega(1,1)$, and by **A-1**, $W_\omega(3,2) \supset W_\omega(1,2)$ thus by (37) and $[x_\omega(3,1), 1] \succ [x_\omega(3,2), 2]$ we have that $[x_\omega(1,1), 1] \succ [x_\omega(1,2), 2]$. Again, by **A-2**,$W_\omega(3,1) = W_\omega(1,1)$, and by **A-1**, $W_\omega(3,3) \supset W_\omega(1,3)$ thus by (37) and $[x_\omega(3,1), 1] \succ [x_\omega(3,3), 3]$ we have that $[x_\omega(1,1), 1] \succ [x_\omega(1,3), 3]$. Therefore, by (38), $C_\omega(1) = 1$.

   - By **A-2**, $W_\omega(3,1) = W_\omega(2,1)$, and by **A-1**, $W_\omega(3,2) = W_\omega(2,2)$ thus by (37) and $[x_\omega(3,1), 1] \succ [x_\omega(3,2), 2]$ we have that $[x_\omega(2,1), 1] \succ [x_\omega(2,2), 2]$. Again, by **A-1**,$W_\omega(3,1) = W_\omega(2,1)$ and $W_\omega(3,3) = W_\omega(2,3)$ thus by (37) and $[x_\omega(3,1), 1] \succ [x_\omega(3,3), 3]$ we have that $[x_\omega(2,1), 1] \succ [x_\omega(2,3), 3]$. Therefore, by (38), $C_\omega(2) = 1$.

6. $C_\omega(3) = 2 \Rightarrow [x_\omega(3,2), 2] \succ [x_\omega(3,1), 1]$ and $[x_\omega(3,2), 2] \succ [x_\omega(3,3), 3]$.

   - By **A-1**, $W_\omega(3,2) = W_\omega(2,2)$, and by **A-2**, $W_\omega(3,1) = W_\omega(2,1)$ thus by (37) and $[x_\omega(3,2), 2] \succ [x_\omega(3,1), 1]$ we have that $[x_\omega(2,2), 2] \succ [x_\omega(2,1), 1]$. Again, by **A-2**,$W_\omega(3,2) = W_\omega(2,2)$, and by **A-1**, $W_\omega(3,3) \supset W_\omega(2,3)$ thus by (37) and $[x_\omega(3,2), 2] \succ [x_\omega(3,3), 3]$ we have that $[x_\omega(2,2), 2] \succ [x_\omega(2,3), 3]$. Therefore, by (38), $C_\omega(2) = 2$.

□

Proof of Theorem T-1:

Proof. The theorem comes as a consequence of applying the rules described in Lemma **L-1** and Assumption **A-3** to the possible values values, i.e. response-types, variable S can take. Table 15 presents a matrix with all possible values of S and indicates whether those values violate the choice rules stated in the items of Lemma **L-1** and the Assumption **A-3**. □

Proof of Lemma L-2:

Proof. The independence relation $S_\omega \perp\!\!\!\perp Z_\omega$ comes from the fact that $V_\omega \perp\!\!\!\perp Z_\omega$ and that S_ω is a function of only V_ω. Let $t \in \text{supp}(T)$, then $Y(t) \perp\!\!\!\perp T|S$ is a consequence of the independence of error term and the Structural Equations

Table 15: Restrictions on the Possible Values that Response Variable S takes

Response-types	Values Instrumental Variable Z takes			Restriction Analysis		
	No Voucher $Z=1$	Experimental $Z=2$	Section 8 $Z=3$	No Voucher $Z=1$	Experimental $Z=2$	Section 8 $Z=3$
1	1	1	1	✓	✓	✓
2	1	1	2	✓	Item 6 of **L-1**	Item 3 of **L-1**
3	1	1	3	✓	✓	✓
4	1	2	1	✓	Item 5 of **L-1**	✓
5	1	2	2	✓	✓	✓
6	1	2	3	✓	✓	✓
7	1	3	1	Item 4 of **L-1**	Item 5 of **L-1**	✓
8	1	3	2	Item 4 of **L-1**	Item 6 of **L-1**	✓
9	1	3	3	Item 4 of **L-1**	✓	✓
10	2	1	1	Item 3 of **L-1**	Item 1 of **L-1**	Item 1 of **L-1**
11	2	1	2	Item 3 of **L-1**	Item 1 of **L-1**	Item 3 of **L-1**
12	2	1	3	Item 3 of **L-1**	Item 1 of **L-1**	✓
13	2	2	1	Item 5 of **L-1**	Item 5 of **L-1**	Item 1 of **L-1**
14	2	2	2	✓	✓	✓
15	2	2	3	Ass. **A-3**	Ass. **A-3**	Ass. **A-3**
16	2	3	1	Item 4 of **L-1**	Item 1 of **L-1**	Item 1 of **L-1**
17	2	3	2	Item 4 of **L-1**	Item 1 of **L-1**	✓
18	2	3	3	Item 4 of **L-1**	Item 1 of **L-1**	✓
19	3	1	1	Item 3 of **L-1**	Item 2 of **L-1**	Item 2 of **L-1**
20	3	1	2	Item 3 of **L-1**	Item 2 of **L-1**	Item 3 of **L-1**
21	3	1	3	Item 3 of **L-1**	Item 2 of **L-1**	✓
22	3	2	1	Item 5 of **L-1**	Item 5 of **L-1**	Item 2 of **L-1**
23	3	2	2	Ass. **A-3**	Ass. **A-3**	Ass. **A-3**
24	3	2	3	✓	✓	✓
25	3	3	1	Item 5 of **L-1**	Item 5 of **L-1**	Item 2 of **L-1**
26	3	3	2	✓	Item 6 of **L-1**	Item 4 of **L-1**
27	3	3	3	✓	✓	✓

This table presents all possible values that the response variable S can possibly take when instrumental variable Z and treatment status T range over $\text{supp}(Z) = \text{supp}(T) = \{1, 2, 3\}$. The first column enumerates the 27 possible response-types. Columns 2 to 4 presents the response-types according to the vector of the values the instrumental values Z takes. The remaining three columns examine are associated with the three previous ones. Columns 5 to 7 indicate whether the response-type violates any of the restrictions imposed by the items of Lemma **L-1** and the Assumption **A-3**. A check mark sign means that the associate response-type does not violate any rule. Otherwise, the table declares the rule being violated.

of the general IV model:

$$\Big((V_\omega, \epsilon_\omega) \perp\!\!\!\perp Z_\omega\Big) \Rightarrow \Big(f_Y(t, V_\omega, \epsilon_\omega) \perp\!\!\!\perp g_T(Z_\omega, f_S(V_\omega)) | f_S(V_\omega)\Big) \Rightarrow \Big(Y_\omega(t) \perp\!\!\!\perp T_\omega | S_\omega\Big)$$

Also,

$$\Big((V_\omega, \epsilon_\omega) \perp\!\!\!\perp Z_\omega\Big) \Rightarrow \Big((f_Y(t, V_\omega, \epsilon_\omega), f_S(V_\omega)) \perp\!\!\!\perp Z_\omega\Big) \Rightarrow \Big((Y_\omega(t), S_\omega) \perp\!\!\!\perp Z_\omega\Big).$$

We now apply the Weak Union Property of conditional independence relations of Dawid (1976) [27] to obtain $Y_\omega(t) \perp\!\!\!\perp Z_\omega|S_\omega$. But T_ω is a linear function of Z_ω when conditioned on S_ω (see (80)), thus we have that $Y_\omega(t) \perp\!\!\!\perp (T_\omega, Z_\omega)|S_\omega$. Again, by Weak Decomposition we have that $Y_\omega(t) \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)$. We according to Representation 11:

$$\Big(Y_\omega(t) \perp\!\!\!\perp Z_\omega|(T_\omega, S_\omega)\Big) \Rightarrow \Big(\sum_{t\in \text{supp}(T)} Y_\omega(t)\cdot \mathbf{1}[T_\omega = t] \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)\Big) \Rightarrow \Big(Y_\omega \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)\Big).$$

□

Proof of Lemma T-12:

Proof.

$$\begin{aligned}
&E(Y_\omega|T_\omega = t, Z_\omega = z) = \sum_{s\in \text{supp}(S)} E(Y_\omega|T_\omega = t, S_\omega = s, Z = z)\, P(S_\omega = s|T_\omega = t, Z_\omega = z)\\
&= \sum_{s\in \text{supp}(S)} E(Y_\omega|T_\omega = t, S_\omega = s, Z_\omega = z)\frac{P(T_\omega = t|S_\omega = s, Z_\omega = z)\, P(S_\omega = s|Z_\omega = z)}{P(T_\omega = t|Z_\omega = z)}\\
&\therefore E(Y_\omega|T_\omega = t, Z_\omega = z)\, P(T_\omega = t|Z_\omega = z) =\\
&= \sum_{s\in \text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega = z]\, E(Y_\omega|T = t, S_\omega = s)\, P(S_\omega = s).
\end{aligned} \tag{39}$$

The second equality comes from Bayes Rule. The first term of Equation (94) comes from the fact that T_ω is deterministic conditional on Z_ω and S_ω. The second and third terms come from $Y_\omega \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)$ and $S_\omega \perp\!\!\!\perp Z_\omega$ of Lemma **L-2**. □

Proof of Theorem ??:

Proof. A general solution for the matrix form of a system of linear equations is obtained by (Magnus and Neudecker, 1999):

$$\boldsymbol{b} = \boldsymbol{B}\boldsymbol{x} \Rightarrow \boldsymbol{x} = \boldsymbol{B}^+(\boldsymbol{I} - \boldsymbol{B}^+\boldsymbol{B})\lambda \tag{40}$$

such that λ is an arbitrary real-valued $|\boldsymbol{b}|$-dimension vector, $\boldsymbol{I}$ is an identity matrix of the same line dimension and $\boldsymbol{B}^+$ is the Moore-Penrose Pseudoinverse of matrix $\boldsymbol{B}$. Theorem comes as a direct consequence of applying the general solution for the matrix form of a system of linear equations of Equation (40) to $\boldsymbol{P}_Z = \boldsymbol{A_S}\boldsymbol{P}_S$ (Equation (22)) and $\boldsymbol{Q}_Z = \boldsymbol{A_D}\boldsymbol{Q}_S$ (Equation (23)). □

[27] The Graphoid axioms are a set of conditional independence relations first presented by Dawid (1976):

$$\begin{aligned}
\text{Symmetry: } & X \perp\!\!\!\perp Y|Z \Rightarrow Y \perp\!\!\!\perp X|Z.\\
\text{Decomposition: } & X \perp\!\!\!\perp (W,Y)|Z \Rightarrow X \perp\!\!\!\perp Y|Z.\\
\text{Weak Union: } & X \perp\!\!\!\perp (W,Y)|Z \Rightarrow X \perp\!\!\!\perp W|(Y,Z).\\
\text{Contraction: } & X \perp\!\!\!\perp Y|Z \text{ and } X \perp\!\!\!\perp W|(Y,Z) \Rightarrow X \perp\!\!\!\perp (W,Y)|Z.\\
\text{Intersection: } & X \perp\!\!\!\perp W|(Y,Z) \text{ and } X \perp\!\!\!\perp Y|(W,Z) \Rightarrow X \perp\!\!\!\perp (W,Y)|Z.\\
\text{Redundancy: } & X \perp\!\!\!\perp Y|X.
\end{aligned}$$

The intersection relation is only valid for strictly positive probability distribution.

Proof of Theorem T-4:

Proof. 1. Our goal is to identify $\boldsymbol{P}_S$, but according to Equation (22), that is, $\boldsymbol{P}_Z = \boldsymbol{A_S}\boldsymbol{P}_S$. By Theorem **T-1**, we have that:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 3 & 1 \\ 1 & 2 & 3 & 2 & 2 & 2 & 1 \\ 1 & 2 & 3 & 3 & 2 & 3 & 3 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{A_S} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \quad \therefore \quad \text{rank}(\boldsymbol{A_S}) = 7.$$

Thus, according to Corollary **T-3**, $\boldsymbol{P}_S$ is identified through $\boldsymbol{P}_S = \boldsymbol{A}_{\boldsymbol{S}}^{+}\boldsymbol{P}_Z$ as $\left(\boldsymbol{I}_9 - \boldsymbol{A}_{\boldsymbol{S}}^{+}\boldsymbol{A_S}\right) = \boldsymbol{0}$.

2. The identified causal parameters is a direct consequence of facta that response-types probabilities are identified according to item (1) and that according to Corollary **T-3**, if $\left(\boldsymbol{I}_\kappa - \boldsymbol{A}_{\boldsymbol{D}}^{+}\boldsymbol{A_D}\right)'\lambda = \boldsymbol{0}$, for a λ be a real-valued vector of dimension $\kappa = N_S \cdot |\,\text{supp}(T)|$, then $\lambda'\boldsymbol{Q}_S$ is identified.

3. According to Theorem **T-12**, we have that:

$$E(X_\omega \cdot \mathbf{1}[T_\omega = t]|Z_\omega) = \sum_{s\in\text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega]\, E(X_\omega|T_\omega = t, S_\omega = s)\, P(S_\omega = s).$$

But if $X_\omega \perp\!\!\!\perp T_\omega|S_\omega$, the above equation is simplified by:

$$E(X_\omega \cdot \mathbf{1}[T_\omega = t]|Z_\omega) = \sum_{s\in\text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega]\, E(X_\omega \cdot \mathbf{1}[S_\omega = s]),$$

which is identical to the equation for propensity scores (18) when $\mathbf{1}[T_\omega = t]$ is replaced by $X_\omega \cdot \mathbf{1}[T_\omega = t]$ and $\mathbf{1}[S_\omega = s]$ is replaced by $X_\omega \cdot \mathbf{1}[S_\omega = s]$. Thereby $\boldsymbol{Q}_Z = \mathbf{A_S}\boldsymbol{Q}_S$ (instead of $\boldsymbol{Q}_Z = \mathbf{A_D}\boldsymbol{Q}_S$) when X_ω is the targeted variable of $\boldsymbol{Q}_Z$ and $\boldsymbol{Q}_S$. Thus, by the rationale of item (1), $E(X_\omega \cdot \mathbf{1}[S_\omega = s])$ is identified for all $s \in \text{supp}(S)$. The proof is completed by the fact that probabilities $P(S_\omega = s)$ are identified for all $s \in \text{supp}(S)$. □

Proof of Theorem T-5:

Proof. According to the definition of S_ω :

$$\text{for each } \; v \in \text{supp}(V), \;\; \text{exists a unique } \; s \in \text{supp}(S) \text{ such that } s = f_S(v)$$
$$\therefore (T_\omega|Z_\omega = z, V_\omega = v) = (T_\omega|Z_\omega = z, S_\omega = s) \text{ such that } s = f_S(v).$$

But T_ω is deterministic conditioned on S_ω and T_ω. And, in particular:

$$(T_\omega|Z_\omega = z_i, S_\omega = s_j) = \boldsymbol{A}[i,j] = \sum_{t\in\text{supp}(T)} t \cdot \boldsymbol{A}_t[i,j].$$

The ordering of the values that the instrumental variable Z_ω takes (i.e. $\{z_1, z_2, z_3\}$) is arbitrary. Thus, without loss of generality, let $\phi_t : [1, \ldots, 3] \to [1, \ldots, 3]$ be the permutation function such that $P(T_\omega = t|Z_\omega = z_{\phi_t(1)}) < P(T_\omega = t|Z_\omega = z_{\phi_t(2)}) < P(T_\omega = t|Z_\omega = z_{\phi_t(3)})$. Also the ordering of the values that the Response variable S_ω takes (i.e. $\{s_1, \ldots, s_7\}$) is also arbitrary. In the same token, let $\psi_t : [1, \ldots, 7] \to [1, \ldots, 7]$ be a permutation function such that $\sum_{i=1}^{3} \boldsymbol{A}[i, \psi_t(1)] \geq \ldots \geq \sum_{i=1}^{3} \boldsymbol{A}[i, \psi_t(7)]$. Let $\widetilde{\boldsymbol{A}}_t$ be the matrix generated by the permutated lines of matrix $\boldsymbol{A}_t$ according to ϕ_t and the permuted columns according to ψ_t. Specifically, $\widetilde{\boldsymbol{A}}_t[i, j] = \boldsymbol{A}_t[\phi_t(i), \psi_t(j)]; i \in \{1, 2, 3\}$ and $j \in \{1, \ldots, 7\}$. Also we can generate a one-to-one correspondence between support of Z into the set $\{1, 2, 3\}$ and between support of S_ω into the set $\{1, \ldots, 7\}$. Thus, in order to proof the theorem, it suffices to show that there exist functions $\varphi_t : \{1, \ldots, 7\} \to \mathbb{R}$ and $\zeta_t : \{1, 2, 3\} \to \mathbb{R}$ such that

$$\widetilde{\boldsymbol{A}}_t[i, j] = \mathbf{1}[\varphi_t(j) \leq \zeta_t(i)]; i \in \{1, 2, 3\} \text{ and } j \in \{1, \ldots, 7\}.$$

Now, due to our particular permutation functions ϕ_t and ψ_t, matrix $\widetilde{\boldsymbol{A}}_t$ takes the following composition regardless of the value t takes in $\{1, 2, 3\}$:

$$\widetilde{\boldsymbol{A}}_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Now let $\widetilde{\boldsymbol{P}}_Z(t)$ be the propensity score vector generated by the line permutations of $\boldsymbol{P}_Z(t)$ according to permutation function ϕ_t and let $\widetilde{\boldsymbol{P}}_S$ be the response-types probabilities vector generated by the line permutations of $\boldsymbol{P}_S$ according to permutation function ψ_t. But $\boldsymbol{P}_Z(t) = \boldsymbol{A}_t \boldsymbol{P}_S$ holds. Thereby $\widetilde{\boldsymbol{P}}_Z(t) = \widetilde{\boldsymbol{A}}_t \widetilde{\boldsymbol{P}}_S$ also holds and we can express the propensity scores in $\widetilde{\boldsymbol{P}}_Z(t)$ as:

$$P(T_\omega = t|Z_\omega = z_{\phi_t(i)}) = \sum_{j=1}^{7} \widetilde{\boldsymbol{A}}_t[i, j] \cdot P(S_\omega = s_{\psi_t(j)}). \tag{41}$$

Now note that each line if $\widetilde{\boldsymbol{A}}_t$ is a sequence of elements 1 followed by elements zero. Upon this fact, we can express $\widetilde{\boldsymbol{A}}_t[i, j]$ as:

$$\begin{aligned} \widetilde{\boldsymbol{A}}_t[i, j] &= \mathbf{1}[\varphi_t(j) \leq \zeta_t(i)], \\ \text{where } \varphi_t(j) &= \sum_{j'=1}^{7} \mathbf{1}[j' \leq j] \, P(S_\omega = s_{\psi_t(j')}), \\ \text{and } \zeta_t(i) &= P(T_\omega = t|Z_\omega = z_{\phi_t(i)}). \end{aligned}$$

□

Proof of Theorem T-6

Proof. It is useful to prove the equivalence result by adopting an alternative ordering of statements:

$$\text{(v)} \;\Rightarrow\; \text{(iii)} \;\Rightarrow\; \text{(i)} \;\Rightarrow\; \text{(ii)} \;\Rightarrow\; \text{(iv)} \;\Rightarrow\; \text{(v)}.$$

Moreover I restate the Generalized Monotonicity (iii) in a more convenient form. Item (iii) states that as Z shifts from z to z' then the choice indicator, i.e. $\mathbf{1}[(T_\omega = t]|Z_\omega = z, V_\omega = v)\big)$ can change to either direction, that is,

$$\begin{aligned} &\big(\mathbf{1}[T_\omega = t]|Z_\omega = z, V_\omega = v\big) - \big(\mathbf{1}[(T_\omega = t]|Z_\omega = z', V_\omega = v)\big) \geq 0 \\ \text{or } &\big(\mathbf{1}[T_\omega = t]|Z_\omega = z, V_\omega = v\big) - \big(\mathbf{1}[(T_\omega = t]|Z_\omega = z', V_\omega = v)\big) \leq 0, \end{aligned}$$

however the direction of change must be the same *for all* $v \in \text{supp}(V)$.

This choice rule imposes a restriction on the treatment choice across agents (i.e. across $v \in \textit{supp}(V)$). Suppose two agents associated with unobserved variables $v, v' \in \text{supp}(V)$ such that

$$\big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v)\big) = 1 \text{ and } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v')\big) = 1$$

then, it *cannot* be the case that :

$$\big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v)\big) = 0 \text{ and } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v')\big) = 0.$$

Otherwise we would have that:

$$\begin{aligned} &\big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v)\big) - \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v)\big) > 0 \\ &\big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v')\big) - \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v')\big) < 0. \end{aligned}$$

which contradicts (iii). Thus an alternative way to express the Generalized Monotonicity of Item (iii) is:

$$\begin{aligned} \text{If } &\big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v)\big) = 1 \text{ and } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v')\big) = 1, \\ \text{then } &\big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v)\big) = 1 \text{ or } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v')\big) = 1. \end{aligned}$$

1. (v) $\Rightarrow$ (iii): In order to prove (iii), it suffices to use (v) to show that for some $v, v' \in \text{supp}(V)$ and for some $z, z' \in \text{supp}(Z)$:

$$\text{if } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v)\big) = 1 \text{ and } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v')\big) = 1, \tag{42}$$

$$\text{then } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z', V_\omega = v)\big) = 1 \text{ or } \big(\mathbf{1}[T_\omega = t]|(Z_\omega = z, V_\omega = v')\big) = 1. \tag{43}$$

Let an agent ω whose unobserved characteristics are denoted by $V_\omega = v; v \in \text{supp}(V)$, associated with preferences $\succsim_\omega$. Also let $t, t' \in \text{supp}(T)$ and $z, z' \in \text{supp}(Z)$. Then, according to item (v),

$$(t, z) \succsim_\omega (t', z') \text{ iff } \varphi(v, t) + \zeta(z, t) + \psi(v, z) \geq \varphi(v, t') + \zeta(z', t') + \psi(v, z').$$

Thus choices (42) can be expressed as :

$$
\begin{aligned}
\varphi(v,t)+\zeta(z,t)+\psi(v,z) &\geq \varphi(v,\tilde{t})+\zeta(z,\tilde{t})+\psi(v,z)\;\forall\,\tilde{t}\in \text{supp}(T)\setminus\{t\}\\
\text{and } \varphi(t,v')+\zeta(t,z')+\psi(v',z') &\geq \varphi(v',\tilde{t})+\zeta(z',\tilde{t})+\psi(v',z')\;\forall\,\tilde{t}\in \text{supp}(T)\setminus\{t\}\\
\therefore \big(\varphi(v,t)+\zeta(z',t)\big)+\big(\varphi(v',t)+\zeta(z,t)\big) &\geq \big(\varphi(v,\tilde{t})+\zeta(z',\tilde{t})\big)+\big(\varphi(v',\tilde{t})+\zeta(z,\tilde{t})\big)\\
\Rightarrow \big(\varphi(v,t)+\zeta(z',t)\big) &\geq \big(\varphi(v,\tilde{t})+\zeta(z',\tilde{t})\big)\\
\text{or } \big(\varphi(v',t)+\zeta(z,t)\big) &\geq \big(\varphi(v',\tilde{t})+\zeta(z,\tilde{t})\big)\\
\Rightarrow \big(\varphi(v,t)+\zeta(z',t)+\psi(v,z')\big) &\geq \big(\varphi(v,\tilde{t})+\zeta(z',\tilde{t})+\psi(v,z')\big)\\
\text{or } \big(\varphi(v',t)+\zeta(z,t)+\psi(v',z)\big) &\geq \big(\varphi(v',\tilde{t})+\zeta(z,\tilde{t})+\psi(v',z)\big)\\
\Rightarrow \big(\mathbf{1}[T_\omega=t]|(Z_\omega=z',V_\omega=v)=1\big) &\text{ or } \big(\mathbf{1}[T_\omega=t]|(Z_\omega=z,V_\omega=v')=1\big)
\end{aligned}
$$

which implies the choice restrictions of Equations (43) as desired.

2. (iii) $\Rightarrow$ (i): According to the definition of S_ω, for every $v \in \text{supp}(V)$, there is a unique $s \in \text{supp}(S)$ such that $f_S(v) = s$ where f_S is given by (13). Thus we can restate the Generalized Monotonicity (iii) as:

$$
\begin{aligned}
&\text{If } \big(\mathbf{1}[T_\omega=t]|(Z_\omega=z,S_\omega=s_j)\big)=1 \text{ and } \big(\mathbf{1}[T_\omega=t]|(Z_\omega=z',S_\omega=s_{j'})\big)=1,\\
&\qquad\text{then it cannot be that}\\
&\big(\mathbf{1}[T_\omega=t]|(Z_\omega=z',S_\omega=s_j)\big)=0 \text{ and } \big(\mathbf{1}[T_\omega=t]|(Z_\omega=z,S_\omega=s_{j'})\big)=0.
\end{aligned}
\tag{44}
$$

Now, according to the definition of $\mathbf{A}_t$,

$$
\big(\mathbf{1}[T_\omega=t]|(V_\omega=v,Z_\omega=z_i)\big)=\big(\mathbf{1}[T_\omega=t]|(S_\omega=s_j,Z_\omega=z_i)\big)=\mathbf{A}_t[i,j];\,f_S(v)=s_j
$$

Thus, Equation (44) implies that none of the 2×2 sub-matrices of every matrix $\mathbf{A}_t$; $t \in \text{supp}(T)$ can be of the type:

$$
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ nor } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
$$

Ryser (1957) show that this is the necessary and sufficient condition for a binary matrix, e.g. $\mathbf{A}_t$, to be lonesum.

3. (i) $\Rightarrow$ (ii): Let the permutation function $\sigma_t : [1,\ldots,N_Z] \to [1,\ldots,N_Z]$ such that we obtain

$$
r_{\sigma(1),t} \;\geq\; \ldots \;\geq\; r_{\sigma(N_Z),t}. \tag{45}
$$

Let $\mathbf{B}$ be the matrix generated by line permutations of $\mathbf{A}_t$ according to σ, that is $\mathbf{B}[i,\cdot] = \mathbf{A}_t[\sigma_t(i),\cdot]$; $i = 1,\ldots,N_Z$. We can permute columns of $\mathbf{B}$ such that its first row is $(\,\mathbf{1}^{r_{\sigma_t(1),t}}\,,\,\mathbf{0}^{N_S-r_{\sigma_t(1),t}}\,)$. Let $\mathbf{B}_1$ be the matrix generated by this column permutation. By Equation (45) we know that the number of elements one in the second line must be $r_{\sigma(1),t}$ or less. Now, by the lonesum property of the matrix, we know that no 2×2

sub-matrices of $\mathbf{B}$ can be of the type:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Thus, the last $N_S - r_{\sigma_t(1),t}$ entries of the second row of $\mathbf{B}_1$ must be all zeros. In the same fashion, we can further permute columns of $\mathbf{B}_1$ into $\mathbf{B}_2$ such that the second row is of type $(\ \mathbf{1}^{r_{\sigma_t(2),t}}\ ,\ \mathbf{0}^{N_S - r_{\sigma_t(2),t}}\ )$. Iterating this process N_Z times generates the maximal of $\mathbf{A}_t$. Moreover, let $\kappa_t : [1,\ldots,N_S] \to [1,\ldots,N_S]$ be generated by these iterations, that is, a permutation function $\kappa_t : [1,\ldots,N_S] \to [1,\ldots,N_S]$ such that we obtain $c_{\kappa_t(1),t} \geq \ \ldots \ \geq c_{\kappa_t(N_S),t}$. Intuitivelly, κ_t represents the permutation function that swaps columns of matrix $\mathbf{A}_t$ such that every line of $\mathbf{A}_t$ is a sequence of elements 1 followed by elements 0. Under this notation, each element of the Maximal of $\mathbf{A}_t$ is defined by $\bar{\mathbf{A}}_t[i,j] = \mathbf{A}_t[\sigma_t(i), \kappa_t(j)]$ for $i \in \{1,\ldots N_Z\}$, $j \in \{1,\ldots,N_S\}$.

4. (ii) $\Rightarrow$ (iv): We use the fact that T is deterministic conditional on Z and V. Thus, to prove item (iv), it suffices to show that there exist functions $\varphi : \text{supp}(V) \times \text{supp}(T) \to \mathbb{R}$ and $\zeta : \text{supp}(Z) \times \text{supp}(T) \to \mathbb{R}$ such that:

$$(T_\omega | Z_\omega = z, V_\omega = v) = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}[\varphi(v,t) + \zeta(z,t) \geq 0], \tag{46}$$

Now, according to the definition of S_ω, for each value $v \in \text{supp}(V)$ that V_ω takes, there is a unique $s_j \in \mathit{supp}(S)$ such that $s_j = f_S(v)$. Thus, for any $z_i \in \text{supp}(Z)$, we can write:

$$(T_\omega | Z_\omega = z_i, V_\omega = v) = (T_\omega | Z_\omega = z_i, S_\omega = s_j) \text{ such that } s_j = f_S(v).$$

Thus, our proof reduces to show that there exist functions $\tilde{\varphi}$ and $\tilde{\zeta}$ such that

$$(T_\omega | Z_\omega = z_i, S_\omega = s_j) = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}[\tilde{\varphi}(j,t) + \tilde{\zeta}(z_i,t) \geq 0], \tag{47}$$

where $j \in \{1,\ldots,N_S\}$ and $z_i \in \{z_1,\ldots,z_{N_Z}\}$. We follow the notation of the previous item of this proof and Equation (22), i.e. $P_Z(t) = \mathbf{A}_t\, P_S$, to write the following relations:

$$P(T_\omega = t | Z_\omega = z_i) = \sum_{j=1}^{N_S} \mathbf{A}_t[i,j]\, P(S_\omega = s_j) = \sum_{j=1}^{N_S} \bar{\mathbf{A}}_t[\sigma_t^{-1}(i), j]\, P(S_\omega = s_{\kappa_t(j)}), \tag{48}$$

but, according to (ii), $\bar{\mathbf{A}}_t[\sigma_t^{-1}(i), \cdot] = (\ \mathbf{1}^{r_{i,t}}\ ,\ \mathbf{0}^{N_S - r_{i,t}}\ )$. In other words:

$$\bar{\mathbf{A}}_t[\sigma_t^{-1}(i), j] = 0 \text{ if } j > r_{i,t} \text{ and } \bar{\mathbf{A}}_t[\sigma_t^{-1}(i), j] = 1 \text{ if } j \leq r_{i,t}. \tag{49}$$

Moreover, we can write:

$$P(T_\omega = t | Z_\omega = z_i) = \sum_{j=1}^{r_{i,t}} P(S_\omega = s_{\kappa_t(j)}). \tag{50}$$

We can use Equation (50) to write $\bar{\mathbf{A}}_t[\sigma_t^{-1}(i), j]$ of Equation (49) as following:

$$\bar{\mathbf{A}}_t[\sigma_t^{-1}(i), j] = \mathbf{1}\Big[P(T_\omega = t | Z_\omega = z_i) \geq \sum_{j'=1}^{j} P(S_\omega = s_{\kappa_t(j')})\Big]. \tag{51}$$

We now draw of the relation $\bar{\mathbf{A}}_t[i, j] = \mathbf{A}_t[\sigma_t(i), \kappa_t(j)]$ of item (iv) and Equation (51) to write:

$$\mathbf{A}_t[i, j] = \mathbf{1}\Big[P(T_\omega = t | Z_\omega = z_i) \geq \sum_{j'=1}^{\kappa_t^{-1}(j)} P(S_\omega = s_{\kappa_t(j')})\Big]. \tag{52}$$

Note that $P(T_\omega = t | Z_\omega)$ only depends on Z and t while and $\sum_{j'=1}^{\kappa_t^{-1}(j)} P(S_\omega = s_{\kappa_t(j')})$ only depends on S_ω (and thereby V_ω as $S_\omega = f_S(V_\omega)$) and t. Now, by the definition of $\mathbf{A}_t[i, j]$ we have that $\mathbf{A}[i, j] = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{A}_t[i, j]$. But $(T_\omega | Z_\omega = z_i, S_\omega = s_j) = \mathbf{A}[i, j]$, thus:

$$\begin{aligned}(T_\omega | Z_\omega = z_i, S_\omega = s_j) &= \sum_{t \in \text{supp}(T)} t \cdot \mathbf{A}_t[i, j] \\ &= \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}\Big[P(T_\omega = t | Z_\omega = z_i) \geq \sum_{j'=1}^{\kappa_t^{-1}(j)} P(S_\omega = s_{\kappa_t(j')})\Big]\end{aligned} \tag{53}$$

where the last equation comes from (52). We can map Equation (53) into Equation (47) as:

$$\tilde{\varphi}(j, t) = \sum_{j'=1}^{\kappa_t^{-1}(j)} P(S_\omega = s_{\kappa_t(j')}) \tag{54}$$

$$\text{and } \tilde{\zeta}(z_i, t) = P(T_\omega = t | Z_\omega = z_i),$$

where Equation (54) denotes the cumulative probability S_ω up to element s_j where the ordering of its support is defined according to the permutation function κ_t. Moreover, each strata probability is strictly bigger than zero, that is, $P(S_\omega = s) > 0 \,\forall\, s \in \text{supp}(S)$. This generates a strict inequality in Equation (55) below:

$$(T_\omega | Z_\omega = z_i, S_\omega = s_j) = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}\Big[P(T_\omega = t | Z_\omega = z_i) > \sum_{j'=1}^{\kappa_t^{-1}(j)} P(S_\omega = s_{\kappa_t(j')})\Big]. \tag{55}$$

5. (vi) ⇒ (v): It suffices to prove that the treatment choice defined Equation (46), that is,

$$(T_\omega | Z_\omega = z, V_\omega = v) = \sum_{t \in \text{supp}(T)} t \cdot \mathbf{1}[\varphi(v, t) + \zeta(z, t) \geq 0], \tag{56}$$

can be expressed as an additive function $u(t, z, v) = \varphi(v, t) + \zeta(z, t) + \psi(v, z)$, such that

$$\Big(\mathbf{1}[T_\omega = t] | Z_\omega = z, V_\omega = v\Big) = 1 \quad \Rightarrow \quad \arg\max_{t' \in \text{supp}(T)} \Big(\varphi(v, t) + \zeta(z, t) + \psi(v, z)\Big) = t.$$

The treatment choice of Model (**??**) under Assumptions **??**–**??** takes a single value $T_\omega = t \in \text{supp}(T)$ when conditioned on $Z_\omega = z \in \text{supp}(Z), V_\omega = v \in \text{supp}(V)$. Thus, according to Equation (56) there is a unique

$t \in \text{supp}(T)$ such that:

$$\varphi(v,t)+\zeta(z,t)\geq 0 \text{ and } \varphi(v,t')+\zeta(z,t')<0\,\forall\, t'\in \text{supp}(T)\setminus\{t\} \tag{57}$$

thereby $t = \arg\max_{t'\in\text{supp}(T)}\big(\varphi(v,t')+\zeta(z,t')\big)$. Let a function $\psi(v,z):\text{supp}(V)\times\text{supp}(Z)\to\mathbb{R}$, then we also have that $t=\arg\max_{t'\in\text{supp}(T)}\big(\varphi(v,t')+\zeta(z,t')+\psi(v,z)\big)$ as $\psi(v,z)$ is constant as t' varies in $\text{supp}(T)$.

□

Proof of Theorem T-7:

Proof. The **TOT** parameter as defined in Equation 28 is the ratio between the outcome expectation conditioned on different values of the instrumental variables divided by the difference in propensity scores associated of choices $\tau\in\text{supp}(T)$ that consist of choices *induced* by the change in instrumental variables. Notationally, let $z_i, z_{i'}\in\text{supp}(Z)$ and $t\in\tau\subset\text{supp}(T)$. If τ consists of all choices induced by the change in instrumental variables from $z_{i'}$ to z_i. Therefore it must be the case that:

$$\boldsymbol{A}[i,j]\neq\boldsymbol{A}[i,j']\Rightarrow\boldsymbol{A}[i,j]\in\tau \text{ and } \boldsymbol{A}[i',j]\notin\tau \tag{58}$$

In other words, for any $t\in\tau$, it must be the case that:

$$\boldsymbol{A}_t[i,j]-\boldsymbol{A}_t[i',j]\in\{0,1\} \tag{59}$$

Now $\boldsymbol{A}_t[i,j]$ only takes value 1 for a single element $t\in\text{supp}(T)$, therefore it is also true that:

$$\sum_{t\in\tau}\left(\boldsymbol{A}_t[i,j]-\boldsymbol{A}_t[i',j]\right)\in\{0,1\}. \tag{60}$$

Now by Equation (58) and Equation (60) we have that:

$$\boldsymbol{A}[i,j]\neq\boldsymbol{A}[i,j']\Rightarrow\sum_{t\in\tau}\left(\boldsymbol{A}_t[i,j]-\boldsymbol{A}_t[i',j]\right)=1.$$

Now the numerator of Equation (28) for $t \in \text{supp}(T)$ and $z_i, z_{i'} \in \text{supp}(Z)$ can be expressed as:

$$
\begin{aligned}
& E(Y_\omega|Z_\omega = z_i) - E(Y_\omega|Z_\omega = z_{i'}) = \\
= & \sum_{j=1}^{N_S} \big(E(Y_\omega|Z_\omega = z_i, S_\omega = s_j) - E(Y_\omega|Z_\omega = z_{i'}, S_\omega = s_j)\big) \, P(S_\omega = s_j) \\
= & \sum_{j=1}^{N_S} \big(E(Y_\omega|Z_\omega = z_i, S_\omega = s_j, T_\omega = \boldsymbol{A}[i,j]) - E(Y_\omega|Z_\omega = z_{i'}, S_\omega = s_j, T_\omega = \boldsymbol{A}[i',j])\big) \, P(S_\omega = s_j) \\
= & \sum_{j=1}^{N_S} \big(E(Y_\omega|S_\omega = s_j, T_\omega = \boldsymbol{A}[i,j]) - E(Y_\omega|S_\omega = s_j, T_\omega = \boldsymbol{A}[i',j])\big) \, P(S_\omega = s_j) \\
= & \sum_{j=1}^{N_S} \big(RATE_{s_j}(\boldsymbol{A}[i,j], \boldsymbol{A}[i',j])\big) \, P(S_\omega = s_j),
\end{aligned}
\tag{61}
$$

where the first equality comes from the law of iterated expectations. The second comes from the fact that T_ω is deterministic conditioned on S_ω and Z_ω, thus $Y_\omega \perp\!\!\!\perp T_\omega|(S_\omega, Z_\omega)$ in the empirical model. The third equality comes from Lemma **L-2**. The last equality comes from the definition of $RATE$. Following the same rationale, let $z_i, z_{i'} \in \text{supp}(Z)$ and $t, t' \in \text{supp}(T)$, then the denominator of Equation (28) can be expressed as:

$$
\begin{aligned}
& P(T_\omega = t|Z_\omega = z_i) - P(T_\omega = t'|Z_\omega = z_{i'}) = \\
= & \sum_{j=1}^{N_S} \big(P(T_\omega = t|Z_\omega = z_i, S_\omega = s_j) - P(T_\omega = t'|Z_\omega = z_{i'}, S_\omega = s_j)\big) \, P(S_\omega = s_j) \\
= & \sum_{j=1}^{N_S} \big(\mathbf{1}[\boldsymbol{A}[i,j] = t] - \mathbf{1}[\boldsymbol{A}[i',j] = t']\big) \, P(S_\omega = s_j) \\
= & \sum_{j=1}^{N_S} \big(\boldsymbol{A}_t[i,j] - \boldsymbol{A}_t[i',j]\big) \, P(S_\omega = s_j)
\end{aligned}
\tag{62}
$$

□

Proof of Lemma L-3:

Proof. The independence relation $S_\omega \perp\!\!\!\perp Z_\omega$ comes from the fact that $V_\omega \perp\!\!\!\perp Z_\omega$ and that S_ω is a function of only V_ω. Let $t \in \text{supp}(T)$, then $Y(t) \perp\!\!\!\perp T|S$ is a consequence of the independence of error term and the Structural Equations of the general IV model:

$$
\Big((V_\omega, \epsilon_\omega) \perp\!\!\!\perp Z_\omega \Big) \Rightarrow \Big(f_Y(t, V_\omega, \epsilon_\omega) \perp\!\!\!\perp g_T(Z_\omega, f_S(V_\omega)) | f_S(V_\omega) \Big) \Rightarrow \Big(Y_\omega(t) \perp\!\!\!\perp T_\omega | S_\omega \Big)
$$

Also,

$$
\Big((V_\omega, \epsilon_\omega) \perp\!\!\!\perp Z_\omega \Big) \Rightarrow \Big((f_Y(t, V_\omega, \epsilon_\omega), f_S(V_\omega)) \perp\!\!\!\perp Z_\omega \Big) \Rightarrow \Big((Y_\omega(t), S_\omega) \perp\!\!\!\perp Z_\omega \Big).
$$

We now apply the Weak Union Property of conditional independence relations of Dawid (1976) [28] to obtain $Y_\omega(t) \perp\!\!\!\perp Z_\omega|S_\omega$. But T_ω is a linear function of Z_ω when conditioned on S_ω (see (80)), thus we have that $Y_\omega(t) \perp\!\!\!\perp (T_\omega, Z_\omega)|S_\omega$. Again, by Weak Decomposition we have that $Y_\omega(t) \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)$. We according to Representation 11:

$$\Big(Y_\omega(t) \perp\!\!\!\perp Z_\omega|(T_\omega, S_\omega)\Big) \Rightarrow \Big(\sum_{t\in\text{supp}(T)} Y_\omega(t)\cdot \mathbf{1}[T_\omega = t] \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)\Big) \Rightarrow \Big(Y_\omega \perp\!\!\!\perp Z_\omega|(S_\omega, T_\omega)\Big).$$

□

Proof of Theorem T-8:

Proof.

$$\begin{aligned}
&E(Y_\omega|G_\omega = g, Z_\omega = z_j, T_\omega = t) = \\
&\quad= \sum_{s_i\in\text{supp}(S)} E(Y_\omega|S_\omega = s_i, G_\omega = g, Z_\omega = z_j, T_\omega = t)\, P(S_\omega = s_i|G_\omega = g, T_\omega = t, Z_\omega = z_j) \\
&\quad= \sum_{s_i\in\text{supp}(S)} E(Y|G_\omega = g, S_\omega = s_i)\, P(S_\omega = s_i|T_\omega = t, Z_\omega = z_j) \\
&\quad= \sum_{s_i\in\text{supp}(S)} E(Y|G_\omega = g, S_\omega = s_i)\frac{P(T_\omega = t|S_\omega = s_i, Z_\omega = z_j)\, P(S_\omega = s_i|Z_\omega = z_j)}{P(T_\omega = t|Z_\omega = z_j)} \\
&\therefore E(Y_\omega|G_\omega = g, Z_\omega = z_j, T_\omega = t)\, P(T_\omega = t|Z_\omega = z_j) = \sum_{s\in\text{supp}(S)} \boldsymbol{A}_t[j, s]\, E(Y_\omega|G_\omega = g, S_\omega = s_i)\, P(S_\omega = s_i)
\end{aligned}$$

The first equality comes from the law of iterated expectations. The first term of the second equality comes from $Y_\omega \perp\!\!\!\perp (Z_\omega, T_\omega)|(S_\omega, G_\omega)$ of Lemma **L-3** and the second terms comes from (S_ω, G_ω) and $G_\omega \perp\!\!\!\perp (S_\omega, Z_\omega)|T_\omega$ of Lemma **L-3**. The second equality comes from Bayes Rule. The third equation comes from $S_\omega \perp\!\!\!\perp Z_\omega$ of Lemma **L-3**, the fact that T_ω is deterministic when conditioned on S_ω and Z_ω and the definition $\boldsymbol{A}_t[j, i] = (T_\omega = t|Z = z_j, S = s_i)$. □

Proof of Theorem T-9:

Proof. Consider the following notation:

$$\zeta_Z(g, t, z) = E(Y_\omega|G_\omega = g, Z_\omega = z, T_\omega = t)\, P(T_\omega = t|Z_\omega = z)$$

and

$$\zeta_Z(g, t) = [\zeta_Z(g, t, z_1), \zeta_Z(g, t, z_2), \zeta_Z(g, t, z_3)].$$

[28] The Graphoid axioms are a set of conditional independence relations first presented by Dawid (1976):

$$\begin{aligned}
\text{Symmetry: }& X \perp\!\!\!\perp Y|Z \Rightarrow Y \perp\!\!\!\perp X|Z. \\
\text{Decomposition: }& X \perp\!\!\!\perp (W, Y)|Z \Rightarrow X \perp\!\!\!\perp Y|Z. \\
\text{Weak Union: }& X \perp\!\!\!\perp (W, Y)|Z \Rightarrow X \perp\!\!\!\perp W|(Y, Z). \\
\text{Contraction: }& X \perp\!\!\!\perp Y|Z \text{ and } X \perp\!\!\!\perp W|(Y, Z) \Rightarrow X \perp\!\!\!\perp (W, Y)|Z. \\
\text{Intersection: }& X \perp\!\!\!\perp W|(Y, Z) \text{ and } X \perp\!\!\!\perp Y|(W, Z) \Rightarrow X \perp\!\!\!\perp (W, Y)|Z. \\
\text{Redundancy: }& X \perp\!\!\!\perp Y|X.
\end{aligned}$$

The intersection relation is only valid for strictly positive probability distribution.

Let the vector of outcome expectations conditioned on G, Z and T be denoted by $\boldsymbol{Q}_Z(g)$ and defined as:

$$\boldsymbol{Q}_Z(g) = [\zeta_Z(g,1), \zeta_Z(g,2), \zeta_Z(g,3)]'.$$

Also let the vector of outcome expectations conditioned on $G_\omega = g$ and $S_\omega = s$ be $\zeta_S(g,s)$ and defined as

$$\zeta_S(g,s) = E(Y_\omega | G_\omega = g, S_\omega = s)\, P(S_\omega = s).$$

Also let $\boldsymbol{Q}_S(g)$ de defined as:

$$\boldsymbol{Q}_S(g) = [\zeta_S(g,s_1), \ldots, \zeta_S(g,s_7)]'.$$

In this notation, the first equation of **T-8** can be expressed by

$$\boldsymbol{Q}_Z(g) = \boldsymbol{A_S} \boldsymbol{Q}_S(g).$$

But the rank of $\boldsymbol{A_S}$ is equal to 7 for the economically justified response-types. Namely, $\text{rank}(\boldsymbol{A_S}) = 7$, and therefore we can write $\boldsymbol{Q}_S(g) = \boldsymbol{A}_{\boldsymbol{S}}^{+} \boldsymbol{Q}_Z(g)$. Thereby $E(Y|G_\omega = g, S_\omega = s)$ is identified for all $s \in \text{supp}(S)$. Theorem **T-8** states that $E(Y_\omega | T_\omega = t, S_\omega = s)$ is a function of $E(Y|G_\omega = g, S_\omega = s)$ and the observed probabilities. Therefore also identified. □

Web Appendix

B Binary Choice Model with Binary Instrumental Variable

The parsimonious binary choice model with binary instrumental variable consist of the following variables:

1. Instrumental variable $Z_\omega \in \{0,1\}$ denotes a voucher assignment for family ω such that $Z_\omega = 1$ if family ω is a voucher recipient and $Z_\omega = 0$ if family ω receives no voucher.

2. The relocation decision T_ω for family ω such that $T_\omega = 0$ if family ω does not relocate and $T_\omega = 1$ if family relocates.

3. Counterfactual relocation decision $T_\omega(z)$ stands for the relocation decision that family ω would choose if it had been assigned to voucher $z \in \{0,1\}$.

4. Counterfactual outcomes $(Y_\omega(0), Y_\omega(1))$ denote the potential outcomes when relocation choice T_ω is *fixed* at values 0 and 1.

5. The observed outcome for family ω is given by $Y_\omega = Y_\omega(0)(1-T_\omega) + Y_\omega(1)T_\omega$.

6. The response-type variable S_ω that is defined by the unobserved vector of potential relocation decisions that a family ω would choose if voucher assignment were set to zero and one, i.e., $S_\omega = [T_\omega(0), T_\omega(1)]'$.

Table A.1 describes the four vectors of potential response-types that S_ω can take. The model is completed by the standard assumption that the instrumental variable Z_ω is independent of counterfactual variables:

$$(Y_\omega(0), Y_\omega(1), T_\omega(0), T_\omega(1)) \perp\!\!\!\perp Z_\omega. \tag{63}$$

The following equation comes as a direct consequence of Equation (63) and the definition of S_ω :

$$(Y_\omega(0), Y_\omega(1)) \perp\!\!\!\perp Z_\omega | S_\omega. \tag{64}$$

Table A.1: Possible Response-types for the Binary Relocation Choice with Binary Voucher

Voucher Types	Voucher Assignment	Relocation Countefactuals	**Response-types** Never Takers	Compliers	Always Takers	Defiers
No Voucher	$Z_\omega = 0$	$T_\omega(0)$	0	0	1	1
Voucher Recipient	$Z_\omega = 1$	$T_\omega(1)$	0	1	1	0

In this notation, the relocation decision T_ω can be expressed in terms of response-type S_ω as:

$$T_\omega = (1 - Z_\omega)T_\omega(0) + Z_\omega T_\omega(0) \tag{65}$$

$$= [\mathbf{1}(Z_\omega = 0), \mathbf{1}(Z_\omega = 1)] \cdot [T_\omega(0), T_\omega(1)]' \tag{66}$$

$$= [\mathbf{1}(Z_\omega = 0), \mathbf{1}(Z_\omega = 1)] \cdot S_\omega, \tag{67}$$

where Equation (65) comes from the definition of $T_\omega(z); z \in \{0, 1\}$, and Equation (67) comes from the definition of S_ω. A consequence of Equation (67) is that T_ω is deterministic conditioned on Z_ω and S_ω.

The expected value of observed outcomes conditioned on voucher assignment in this model is given by:

$$\begin{aligned} E(Y_\omega|Z_\omega = 1) &= E(Y_\omega|Z_\omega = 1, S_\omega = [0,0]')\, P(S_\omega = [0,0]') + E(Y_\omega|Z_\omega = 1, S_\omega = [0,1]')\, P(S_\omega = [0,1]') \\ &+ E(Y_\omega|Z_\omega = 1, S_\omega = [1,1]')\, P(S_\omega = [1,1]') + E(Y_\omega|Z_\omega = 1, S_\omega = [1,0]')\, P(S_\omega = [1,0]') \quad (68) \\ &= E(Y_\omega(0)|S_\omega = [0,0]')\, P(S_\omega = [0,0]') + E(Y_\omega(1)|S_\omega = [0,1]')\, P(S_\omega = [0,1]') \\ &+ E(Y_\omega(1)|S_\omega = [1,1]')\, P(S_\omega = [1,1]') + E(Y_\omega(0)|S_\omega = [1,0]')\, P(S_\omega = [1,0]'), \quad (69) \end{aligned}$$

where Equation (68) comes from the law of iterated expectations. Equation (69) comes the equation for observed outcome $Y_\omega = Y_\omega(0)(1 - T_\omega) + Y_\omega(1)T_\omega$, the fact that T_{omega} is deterministic conditioned on S_ω and Z_ω and the independence relation $(Y_\omega(0), Y_\omega(1)) \perp\!\!\!\perp Z_\omega|S_\omega$ of Equations 64. In the same fashion, we can express $E(Y_\omega|Z_\omega = 1)$ by:

$$\begin{aligned} E(Y_\omega|Z_\omega = 0) &= E(Y_\omega(0)|S_\omega = [0,0]')\, P(S_\omega = [0,0]') + E(Y_\omega(0)|S_\omega = [0,1]')\, P(S_\omega = [0,1]') \\ &+ E(Y_\omega(1)|S_\omega = [1,1]')\, P(S_\omega = [1,1]') + E(Y_\omega(1)|S_\omega = [1,0]')\, P(S_\omega = [1,0]'). \quad (70) \end{aligned}$$

The Intention-to-treat effect ITT is defined by $E(Y_\omega|Z_\omega = 1) - E(Y_\omega|Z_\omega = 0)$ and refers to the causal effect of the vouchers Z_ω on outcome Y_ω. According to Equations (69)–(70), the ITT can be expressed in terms of response-types as:

$$\begin{aligned} ITT &= E(Y_\omega|Z_\omega = 1) - E(Y_\omega|Z_\omega = 0) \\ &= E(Y_\omega(1) - Y_\omega(0)|S_\omega = [0,1]')\, P(S_\omega = [0,1]') + E(Y_\omega(0) - Y_\omega(1)|S_\omega = [1,0]')\, P(S_\omega = [1,0]'). \end{aligned} \tag{71}$$

Equation (71) states that the ITT is a mixture between the contradicting effects. By contradicting I mean the causal effect relocating compared to not relocation for the compliers ($S_\omega = [0,1]'$) and the causal effect of not relocatong compared to relocating for the definers.

The probability of relocation conditioned on receiving the voucher is expressed in terms of response-types by:

$$\begin{aligned} P(T_\omega|Z_\omega = 1) &= E(\mathbf{1}[T_\omega = 1]|Z_\omega = 1, S_\omega = [0,0]')\, P(S_\omega = [0,0]') + E(\mathbf{1}[T_\omega = 1]|Z_\omega = 1, S_\omega = [0,1]')\, P(S_\omega = [0,1]') \\ &\quad + E(\mathbf{1}[T_\omega = 1]|Z_\omega = 1, S_\omega = [1,1]')\, P(S_\omega = [1,1]') + E(\mathbf{1}[T_\omega = 1]|Z_\omega = 1, S_\omega = [1,0]')\, P(S_\omega = [1,0]') \\ &= P(S_\omega = [0,1]') + P(S_\omega = [1,1]'), \end{aligned} \tag{72}$$

where Equation (72) comes from the fact that T_ω is deterministic conditioned on S_ω and Z_ω. Using the same reasoning, the probability of relocation conditioned on not receiving the voucher is expressed in terms of response-types by:

$$P(T_\omega|Z_\omega = 0) = P(S_\omega = [1,0]') + P(S_\omega = [1,1]'). \tag{73}$$

Thus the difference in propensity of relocation across voucher assignments is given by:

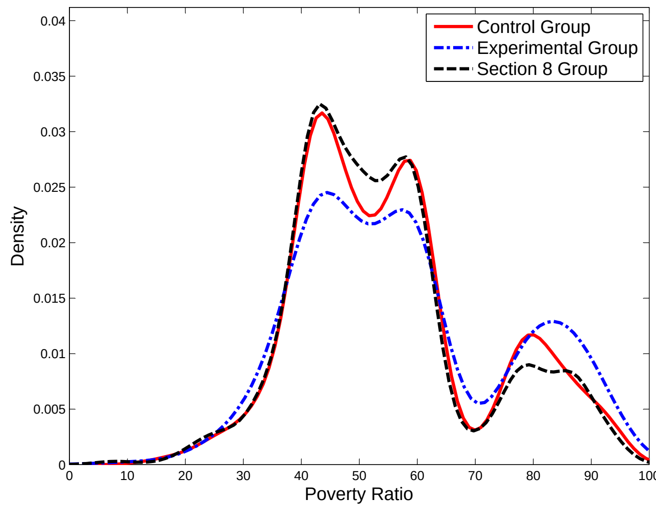
$$P(T_\omega|Z_\omega = 1) - P(T_\omega|Z_\omega = 0) = P(S_\omega = [0,1]') - P(S_\omega = [1,0]'); \tag{74}$$

C Additional Information on Neighborhood Poverty

This section described the distribution of neighborhood poverty of MTO participating families by voucher assignment and relocation decision. Figure 5 shows the probability density estimation of baseline neighborhood poverty by voucher assignment. As expected, poverty distributions conditional on voucher assignments are very similar due to the randomized assignment of vouchers.

Figure 6 presents baseline neighborhood poverty for the Experimental group by neighborhood relocation, i.e., moving with voucher, moving without voucher and not moving. Families that

Figure 5: **Density Estimation of Baseline Neighborhood Poverty (1990 Census) by Voucher Assignment**



This figure presents the density estimation of baseline neighborhood poverty levels by voucher assignment, i.e., Control, Experimental and Section 8 groups. Poverty levels are computed according to the US 1990 Census data as the fraction of households whose income falls below the national poverty threshold for each 1990 census tract. Estimates are based on the normal kernel with optimal normal bandwidth. See columns 2–6 of Table 6 for inference on the average level of neighborhood poverty by voucher assignment.

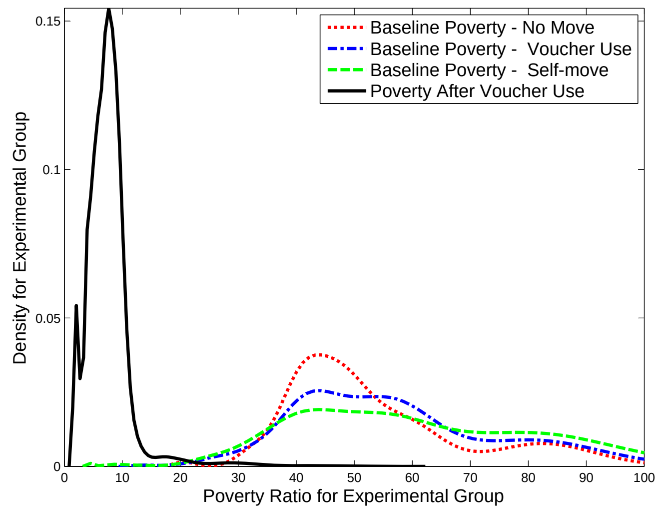
did not move had lived in slightly lower poverty level neighborhoods when compared to families that moved. Figure 6 also shows the poverty density of the neighborhood chosen by families that relocated using the Experimental voucher. The poverty levels of relocation neighborhoods are substantially lower than those of baseline neighborhoods as expected.

Figure 7 examines neighborhood poverty of families assigned to the Section 8 voucher. It shows a similar pattern as that observed in Figure 6. The poverty levels of Section 8 relocation neighborhoods are lower than those of baseline neighborhoods. However poverty levels of Section 8 relocation neighborhoods are higher than those faced by the families that relocated using the Experimental voucher in Figure 6.

D Examples of Response Matrices that Comply with the Separability Condition

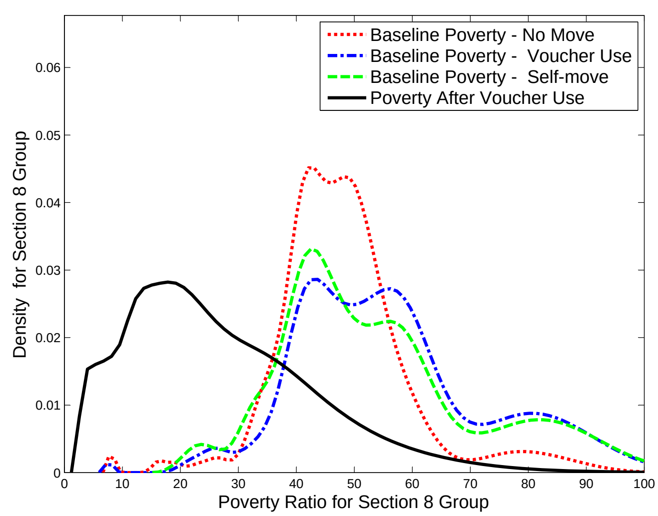
Section 1.1 explains that the MTO project generates 27 possible response-types. These response types consist of all possible counterfactual relocations that generated by a three-valued instrumental

Figure 6: **Density Estimation of Baseline Neighborhood Poverty (1990 Census) of the Experimental Group by Voucher Compliance**



This figure presents the density estimation of baseline neighborhood poverty for the Experimental group conditional on relocation choice, i.e., (1) do not relocate, (2) relocate using the voucher and (3) relocate without using the Experimental voucher. See columns 7–11 of Table 6 for inference on the average level of neighborhood poverty by voucher assignment and compliance. The graph also presents the neighborhood poverty density of the families that use the Experimental voucher after relocation. Estimates are based on the normal kernel with optimal normal bandwidth.

Figure 7: **Density Estimation of Baseline Neighborhood Poverty of the Section 8 Group by Voucher Compliance**



This figure presents the density estimation of baseline neighborhood poverty for the Section 8 group conditional on relocation choice, i.e., (1) do not relocate, (2) relocate using the voucher and (3) relocate without using the Experimental voucher. See columns 12–16 of Table 6 for inference on the average level of neighborhood poverty by voucher assignment and compliance. The graph also presents the neighborhood poverty density of the families that use the Section 8 voucher after relocation. Estimates are based on the normal kernel with optimal normal bandwidth.

Table A.2: All Response Matrices that Comply with Separability Condition **T-5**

Response-types	Response Matrices	
Number	Total Number	Satisfy Separability
1	27	27
2	351	183
3	2,925	553
4	17,550	882
5	80,730	774
6	296,010	354
7	888,030	**66**
8	2,220,075	0
⋮	⋮	⋮
27	27	0

The first column of the table gives the number of response-types in a response matrix. The second column gives the total number of distinct response matrix that can be generated by combination of the number of response-types given in the first column out of the 27 possible response-types. The last column gives the total number of response matrices that satisfy the separability condition of Theorem **T-5**.

variable in a three-choice model. My goal in this section is to examine the number of response matrices that satisfy the Separability Condition stated in Theorem **T-5**.

The number of response matrices generated by the combination of 7 response-types taken from these 27 possible ones totals 888.030. I rely on Heckman and Pinto (2014a) to assert that 66 out of these 888.030 response matrices satisfy the separability condition Theorem **T-5** – including the economically justified response matrix of **T-1**. These 66 response matrices are presented in Table A.3.

The first column of Table A.2 gives the number of response-types in a response matrix. The second column gives the total number of distinct response matrix that can be generated by combination of the number of response-types given in the first column out of the 27 possible response-types. The last column gives the total number of response matrices that satisfy the separability condition of Theorem **T-5**. The table shows that the maximum number of response-types in a response matrix for which the separability condition of Theorem **T-5** holds is 7.

Table A.3: All Response Matrices that Comply with Separability Condition **T-5**

1	2	3	1	1	1	1	1	2	3	1	3	3	3	1	2	3	1	2	2	2	1	2	3	1	2	3	3	1	2	3	1	2	2	2	1	2	3	2	2	3	3
1	2	3	1	1	2	3	1	2	3	1	1	2	3	1	2	3	1	1	2	3	1	2	3	2	2	2	2	1	2	3	3	2	3	3	1	2	3	1	2	1	3
1	2	3	2	3	2	3	1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	1	1	3	1	2	3	1	1	1	3	1	2	3	1	1	1	1
1	2	3	1	1	2	3	1	2	3	1	3	3	3	1	2	3	1	2	3	3	1	2	3	1	2	2	3	1	2	3	1	2	3	3	1	2	3	2	2	2	3
1	2	3	1	1	1	1	1	2	3	1	1	1	2	1	2	3	1	2	1	2	1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	2	3	3
1	2	3	2	3	2	3	1	2	3	2	2	3	2	1	2	3	3	3	3	3	1	2	3	1	1	3	3	1	2	3	1	2	1	2	1	2	3	1	1	1	1
1	2	3	1	1	1	1	1	2	3	1	1	1	1	1	2	3	1	1	1	1	1	2	3	1	3	3	3	1	2	3	1	2	2	3	1	2	3	2	2	2	2
1	2	3	1	2	3	3	1	2	3	1	2	2	3	1	2	3	2	2	2	3	1	2	3	2	2	2	3	1	2	3	3	3	3	3	1	2	3	1	2	3	3
1	2	3	2	2	2	3	1	2	3	3	2	3	3	1	2	3	1	2	3	3	1	2	3	1	1	2	1	1	2	3	1	1	2	1	1	2	3	1	1	1	3
1	2	3	1	1	1	3	1	2	3	1	1	1	3	1	2	3	1	1	1	3	1	2	3	1	3	3	3	1	2	3	1	2	2	2	1	2	3	2	2	2	2
1	2	3	1	2	3	3	1	2	3	1	2	2	2	1	2	3	2	2	2	2	1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	2	2	3
1	2	3	2	2	2	2	1	2	3	3	2	3	3	1	2	3	1	2	3	3	1	2	3	1	1	2	3	1	2	3	1	1	2	3	1	2	3	1	1	3	3
1	2	3	1	1	3	3	1	2	3	1	1	1	2	1	2	3	1	1	1	1	1	2	3	1	1	1	1	1	2	3	2	2	3	3	1	2	3	2	2	2	3
1	2	3	1	2	2	3	1	2	3	1	2	3	2	1	2	3	2	2	3	3	1	2	3	2	3	3	3	1	2	3	1	1	1	1	1	2	3	1	3	3	3
1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	2	1	3	1	2	3	2	1	2	3	1	2	3	1	2	1	3	1	2	3	1	1	2	1
1	2	3	1	1	1	2	1	2	3	1	1	2	3	1	2	3	1	1	3	3	1	2	3	1	1	1	3	1	2	3	2	2	2	2	1	2	3	2	2	2	2
1	2	3	1	3	3	3	1	2	3	1	2	2	2	1	2	3	2	2	2	2	1	2	3	2	3	3	3	1	2	3	1	1	3	3	1	2	3	1	3	3	3
1	2	3	2	2	3	2	1	2	3	3	3	3	3	1	2	3	1	2	2	3	1	2	3	2	1	2	2	1	2	3	1	2	2	3	1	2	3	1	1	2	3
1	2	3	1	1	2	3	1	2	3	1	1	2	2	1	2	3	1	1	1	2	1	2	3	1	1	1	2	1	2	3	2	2	2	3	1	2	3	2	3	3	3
1	2	3	1	3	3	3	1	2	3	1	3	2	3	1	2	3	2	2	3	2	1	2	3	3	3	3	3	1	2	3	1	1	1	1	1	2	3	1	1	1	3
1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	3	3	3	1	2	3	1	2	3	2	1	2	3	1	2	3	3	1	2	3	2	1	2	2
1	2	3	1	2	3	3	1	2	3	1	2	2	3	1	2	3	1	1	2	3	1	2	3	1	1	2	3	1	2	3	2	2	2	2	1	2	3	2	3	3	3
1	2	3	1	1	1	3	1	2	3	1	1	1	1	1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	1	1	3	1	2	3	1	1	1	1
1	2	3	2	2	2	2	1	2	3	3	2	3	3	1	2	3	1	3	3	3	1	2	3	1	2	2	2	1	2	3	1	2	3	3	1	2	3	2	1	2	3
1	2	3	1	2	3	3	1	2	3	1	2	2	2	1	2	3	1	1	2	3	1	2	3	1	1	2	2	1	2	3	2	2	2	3	1	2	3	2	3	3	3
1	2	3	1	1	1	1	1	2	3	1	1	1	3	1	2	3	2	3	2	3	1	2	3	3	3	3	3	1	2	3	1	1	2	1	1	2	3	2	1	2	3
1	2	3	2	2	2	3	1	2	3	3	2	3	3	1	2	3	1	1	1	1	1	2	3	1	3	2	3	1	2	3	1	3	3	3	1	2	3	1	1	1	1
1	2	3	1	2	2	3	1	2	3	1	2	2	3	1	2	3	1	2	3	3	1	2	3	1	2	2	3	1	2	3	2	2	2	2	1	2	3	2	3	3	3
1	2	3	1	1	3	3	1	2	3	1	1	2	1	1	2	3	2	2	2	3	1	2	3	3	2	3	3	1	2	3	1	1	2	3	1	2	3	2	1	2	2
1	2	3	2	2	2	2	1	2	3	3	3	3	3	1	2	3	1	1	1	1	1	2	3	1	1	1	1	1	2	3	1	3	3	3	1	2	3	1	1	1	3
1	2	3	3	3	3	3	1	2	3	3	3	3	3	1	2	3	3	3	3	3	1	2	3	3	3	3	3	1	2	3	3	3	3	3	1	2	3	3	3	3	3
1	2	3	1	2	3	3	1	2	3	1	2	2	3	1	2	3	1	2	2	2	1	2	3	1	1	2	2	1	2	3	1	1	2	3	1	2	3	1	1	1	2
1	2	3	1	2	1	2	1	2	3	1	1	2	1	1	2	3	1	1	2	3	1	2	3	1	3	2	3	1	2	3	1	2	2	2	1	2	3	1	2	3	2

This table presents all response matrices consisting of seven response-types that comply with the separability condition of Theorem **T-5**.

E SARP and the Random Utility Model

The identification analysis of Section 3.1 is the result of the combination of three strategies. The first strategy is to use of MTO vouchers as instrumental variables for neighborhood relocation. The second strategy is o use a causal framework that allows to summarizes the identification problem of neighborhood effects into binary properties of the response matrix. The third one is to rely on economics, i.e. the Strong Axiom of Reveled Preferences (SARP), to reduce the column-dimension of the response matrix and thereby rendering identification results.

A related literature in economics studies the effect of individual rationality on aggregate data. A substantial economic literature uses Random Utility Models (RUM) to examine if observed empirical data on prices and consumed goods is consistent an underlying framework where agents maximize utility representing rational preferences (McFadden, 2005). The term random in RUM refers to unobserved heterogeneity across agents. This literature does not uses SARP to identify causal effects, but rather explore how SARP impacts statistical quantities of observed data. McFadden and Richter (1991) coined the term Axiom of Revealed Stochastic Preference (ARSP) for the collection of inequalities that must hold on aggregate data of prices and consumption when heterogeneous individuals are rational. Blundell et al. (2003, 2008) examines the consequences of revealed preferences on the quantiles of Engel curves. They develop a nonparametric estimation of the demand function for consumption goods. Blundell et al. (2014) uses inequality restrictions generated by revealed preferences to investigate the estimation of consumer demand.

A recent paper of Kitamura et al. (2014) implements a nonparametric test that verifies if empirical data comply with the inequalities generated by ARSP. Kitamura et al. (2014) major insight is to form a coarse partition of each budget set W_i such that no other budget set, say W_j, intersect the interior of the partition subsets associated with W_i. This insight allows to transform a continuous utility maximization problem into a discrete problem were the agent selects a consumption bundle that belongs to a finite list of possible choices. They generate a test that explore the choice restrictions generated by SARP. It is useful to clarify Kitamura et al. (2014) approach using a setup that features the MTO experiment. Our goal is to show that the example also generates the same response matrix of **T-1**.

Let $u_\omega : \text{supp}(K_E) \times \text{supp}(K_S) \times \text{supp}(K_X) \to \mathbb{R}^+$ represent a non-satiable rational preferences for agent ω over the consumption bundle consisting of three goods K_L, K_H and K_X. Let K_X denotes a divisible good in $\mathbb{R}^+$ and K_L, K_H denote indivisible goods whose support is the natural numbers. Let $K = [K_L\,,\, K_H\,,\, K_X]$ to represent a vector of consumption goods associate with the price vector $\boldsymbol{p} = [p_H\,,\, p_L\,,\, p_X] > 0$, such that $\text{supp}(K) = \mathbb{N} \times \mathbb{N} \times \mathbb{R}^+$. Also let the wealth of each agent ω be standardized to 1. Under this setup, the budget plane of any agent ω only depends on price p and is given by $W(\boldsymbol{p}) = \{K \in \mathbb{N} \times \mathbb{N} \times \mathbb{R}^+; pK = 1\}$. The consumption choice for agent ω facing prices p_ω is given by:

$$K_\omega(\boldsymbol{p}_\omega) = \underset{k \in W(\boldsymbol{p}_\omega)}{\arg\max}\, u_\omega(k).$$

The econometrician only observes the random sample of $(K_\omega(\boldsymbol{p}_\omega), \boldsymbol{p}_\omega)$.

Now suppose prices can only take three values $\boldsymbol{p}^C = [1, 1, 1]$, $\mathbf{p}^E = [0.6, 1, 1]$ and $\boldsymbol{p}^S = [0.6, 0.6, 1]$. Under discrete goods and non-satiable preferences, the possible consumption bundles are given by: $K_\omega(\boldsymbol{p}^C) \in \{[1, 0, 0], [0, 1, 0], [0, 0, 1]\}$, $K_\omega(\boldsymbol{p}^E) \in \{[1, 0, 0.4], [0, 1, 0], [0, 0, 1]\}$, and $K_\omega(\boldsymbol{p}^S) \in \{[1, 0, 0.4], [0, 1, 0.4], [0, 0, 1]\}$. We are now able to link this consumer model to the MTO experiment. Good K_L indicates the choice of relocating to a low-poverty neighborhood, K_H indicates the choice of relocating to a high-poverty neighborhood and $[K_L, K_H] = [0, 0]$ denotes no relocation. The price values represent MTO voucher assignments. Baseline price $\boldsymbol{p}^C$ stands for no voucher. Price $\boldsymbol{p}^E$ stands for the experimental voucher, which subsidizes the relocation to low-poverty neighborhood and price $\boldsymbol{p}^S$ stands for Section 8 voucher, which subsidizes the relocation to either low or high-neighborhood relocation.

For each price there are 3 possible consumption bundles. There are also 3 price vector, which totals 27 possible combinations of consumption bundles across price vectors. The same number of possible response-types. We test which of those combinations satisfy SARP, i.e., if the transitive closure of directly revealed preferences is acyclical. The combinations that do not violate SARP are described in Table A.4. There are a total of nine response-type. The two last response-types of Table A.4 are purged due to Assumption **A-3**.

Section 3.1 models neighborhood choice using a more natural approach than the one described above. It does not defines relocation decisions as a goods nor assigns prices to neighborhood choices. Instead, the model of Section 3.1 explores the relation of budget sets generated by voucher assignments and relocation choices. This is a simpler approach as no budget set hyperplane intersects. In

Table A.4: Consumption Bundles

Voucher	Prices	**Possible Consumption Bundles**								
Control	$\boldsymbol{p}^C$	$[0,0,1]$	$[1,0,0]$	$[0,1,0]$	$[0,0,1]$	$[0,0,1]$	$[0,1,0]$	$[0,0,1]$	$[1,0,0]$	$[0,1,0]$
Experimental	$\boldsymbol{p}^E$	$[0,0,1]$	$[1,0,.4]$	$[0,1,0]$	$[1,0,.4]$	$[1,0,.4]$	$[1,0,.4]$	$[0,0,1]$	$[1,0,.4]$	$[1,0,.4]$
Section 8	$\boldsymbol{p}^S$	$[0,0,1]$	$[1,0,.4]$	$[0,1,0.4]$	$[0,1,.4]$	$[1,0,.4]$	$[0,1,.4]$	$[0,1,.4]$	$[0,1,.4]$	$[1,0,.4]$
Voucher	Z Assignment	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
Control	$Z = z_1$	1	2	3	1	1	3	1	2	3
Experimental	$Z = z_2$	1	2	3	2	2	2	1	2	2
Section 8	$Z = z_3$	1	2	3	3	2	3	3	3	2

This tables presents the combinations of possible consumption bundles that survive SARP according to prices $\boldsymbol{p}^C$, $\boldsymbol{p}^E$ and $\boldsymbol{p}^S$. The table also maps these bundles into the neighborhood choice and voucher assignments. This generates nine response-types.

other words, symmetric difference of any two budget sets is empty. Budget sets are either identical, disjoint or proper subsets. SARP restrictions are applied directly to choice rules based on the budget sets relations. Kitamura et al. (2014) tests if there is a distribution across potential rational agent types that would generate the observed distribution of prices and consumption goods. They explain that the agent types distribution is commonly non-identified. In the case of MTO, I was able to identify this distribution, that is, the response-types probabilities of **T-4**.

F Model Specification Tests

This section presents model specification tests for response-types outcome expectations estimated using two methods. The first method does use neighborhood poverty data in the estimation. It refers to the identified parameters described in Section 3.3 of the main paper. Namely, the response-type counterfactual expectations described below:

$E(Y_\omega(1)\|S_\omega = s_1)$	$E(Y_\omega(2)\|S_\omega = s_2)$	$E(Y_\omega(3)\|S_\omega = s_3)$
$E(Y_\omega(1)\|S_\omega = s_7)$	$E(Y_\omega(2)\|S_\omega = s_5)$	$E(Y_\omega(3)\|S_\omega = s_6)$
$E(Y_\omega(1)\|S_\omega \in \{4,5\})$	$E(Y_\omega(2)\|S_\omega \in \{4,6\})$	$E(Y_\omega(3)\|S_\omega \in \{4,7\})$

The second estimation method uses data on neighborhood poverty as proxy for unobserved neighborhood characteristics. It refers to the assumption stated in Section 3.7 of the main paper.

The model specification test compares the outcome expectations using both methods. If the model assumption that neighborhood poverty is a good proxy for unobserved neighborhood characteristics holds (Section 3.7), then the difference between parameters estimated according to these different methods should not be statistically significant.

I present three tables of inference in this section. Table A.5 presents inference on the counterfactual parameters associated with no relocation, that is, $E(Y_\omega(1)|S_\omega = s_1)$, $E(Y_\omega(1)|S_\omega = s_7)$ and $E(Y_\omega(1)|S_\omega \in \{4,5\})$. Table A.6 presents inference on the counterfactual parameters associated with the choice of relocating to a low-poverty neighborhood. Namely $E(Y_\omega(2)|S_\omega = s_2)$, $E(Y_\omega(2)|S_\omega = s_5)$ and $E(Y_\omega(2)|S_\omega \in \{4,6\})$. Finally, Table A.7 presents inference on the counterfactual parameters associated with the choice of relocating to a high-poverty neighborhood. Namely $E(Y_\omega(3)|S_\omega = s_3)$, $E(Y_\omega(3)|S_\omega = s_6)$ and $E(Y_\omega(3)|S_\omega \in \{4,7\})$. The model specification tests presented here focus on MTO income outcomes interim evaluation.

The MTO outcomes of each table are grouped in blocks separated by horizontal lines. The last line of each block of outcomes examines the average of the participant rank across the outcomes within block. First column states the variable name. Second column indicates if the variable is reversed, i.e., multiplied by -1. The remaining columns refer to three sections of empirical estimates.

Each section investigates the estimates conditional on the response-types designated in the header of the table. Each block of analysis examines the difference between outcome estimates conditioned on the response-type. The first column presents the outcome estimate that does rely on the available data on neighborhood characteristics. The second column shows the difference in the outcome expectations of previous column and the one that uses available data on neighborhood poverty. The third column shows double-sided single hypothesis p-value of no difference in outcome expectations. The fourth column presents double-sided multiple hypothesis stepdown p-value associated with joint-hypothesis of no difference in outcome expectations.

Table A.5: Model Specification Tests for No Relocation1 out of 1)

		Response-type s_1 for No move				Response-type s_7 for No move				Response-types s_4 and s_5 for No move			
		Base	**Diff.**	**Inference**		**Base**	**Diff.**	**Inference**		**Base**	**Diff.**	**Inference**	
Variable Name	**Rev.**	**Mean**	**Means**	**Single**	**SD**	**Mean**	**Means**	**Single**	**SD**	**Mean**	**Means**	**Single**	**SD**
Adult Earnings													
Earnings (2001)	No	7363.9	421.7	0.39	0.46	10115.6	3877.8	0.81	0.81	8896.2	-335.4	0.66	0.85
Current Weekly earnings	No	154.2	19.680	**0.10**	0.17	241.9	-186.0	0.38	0.62	193.4	2.709	0.87	0.87
Earnings Range (1 to 6)	No	2.208	0.012	0.89	0.89	2.452	1.179	0.76	0.82	2.388	-0.077	0.57	0.81
Rank Average	No	0.466	0.012	0.40	–	0.531	-0.036	0.88	–	0.502	-0.002	0.92	–
Total Income													
Total income (head)	No	11460.9	456.5	0.30	0.30	12274.4	3073.1	0.84	0.90	11132.0	-284.5	0.66	0.66
Total household income	No	12545.8	1032.5	**0.07**	0.12	13213.4	3001.4	0.88	0.88	13275.3	-781.9	0.35	0.48
Sum of all income	No	15565.5	1114.9	**0.09**	0.14	15513.5	-3556.0	0.67	0.94	14875.6	-737.0	0.42	0.54
Rank Average	No	0.489	0.016	0.28	–	0.488	0.066	0.90	–	0.472	-0.010	0.62	–
Poverty Line													
Income $<$ 50% poverty line	Yes	-0.302	0.034	0.12	0.29	-0.441	-0.083	0.75	0.89	-0.349	0.003	0.91	0.91
Income $\geq$ 150 % poverty line	No	0.179	0.006	0.73	0.73	0.046	0.037	0.89	0.89	0.136	-0.005	0.85	0.97
Income $>$ poverty line	No	0.253	0.026	0.22	0.35	0.277	0.128	0.87	0.96	0.229	-0.016	0.64	0.92
Rank Average	No	0.505	0.010	0.38	–	0.462	0.014	0.96	–	0.486	-0.004	0.75	–
Self-suficiency													
Economic self-sufficiency	No	0.179	-0.012	0.54	0.79	0.073	0.217	0.63	0.63	0.160	-0.014	0.58	0.58
Employed (no welfare)	No	0.468	0.039	0.20	0.42	0.503	-0.404	0.32	0.46	0.434	0.031	0.39	0.61
Not in the labor force	Yes	-0.430	0.005	0.83	0.83	-0.175	-0.241	0.56	0.59	-0.355	0.050	0.11	0.28
Rank Average	No	0.487	0.004	0.71	–	0.518	-0.054	0.72	–	0.493	0.010	0.49	–
Self-reported Employment													
Employed	No	0.489	0.017	0.52	0.52	0.625	-0.311	0.41	0.75	0.554	0.020	0.58	0.84
Employed with health insurance	No	0.240	0.039	0.12	0.28	0.412	-0.017	0.97	0.97	0.260	0.044	0.20	0.44
Employed full-time	No	0.357	0.026	0.34	0.58	0.478	-0.360	0.35	0.61	0.411	0.012	0.73	0.89
Employed above poverty	No	0.288	0.022	0.41	0.57	0.319	-0.164	0.59	0.86	0.338	0.006	0.87	0.87
Job for more than a year	No	0.373	0.022	0.42	0.65	0.405	-0.170	0.55	0.91	0.332	0.056	0.12	0.33
Rank Average	No	0.487	0.011	0.44	–	0.539	-0.109	0.48	–	0.497	0.015	0.41	–

This table presents model specification tests for counterfactuals expectations conditioned on response-types . MTO Outcomes on this table are grouped in blocks separated by horizontal lines. The last line of each block of outcomes examines the average of the participant rank across the outcomes within block. First column states the variable name. Second column indicates if the variable is reversed, i.e., multiplied by -1. The remaining columns refer to three blocks of counterfactual comparison analysis used as a model specification test. Each block of analysis examines the difference between outcome estimates conditioned on response-types. The first column presents the outcome estimate that does rely on the available data on neighborhood characteristics. The second column shows the difference in the outcome expectations of previous column and the one that uses available data on neighborhood quality. The third column shows double-sided single hypothesis p-value of no difference in outcome expectations. The fourth column presents double-sided multiple hypothesis stepdown p-value associated with joint-hypothesis of no difference in outcome expectations.

Table A.6: Model Specification Tests for Low-poverty Relocation1 out of 1)

		Response-type s_2 for Low-poverty Relocation				Response-type s_5 for Low-poverty Relocation				Response-type s_4 and s_6 for Low-poverty Relocation			
		Base	Diff.	Inference		Base	Diff.	Inference		Base	Diff.	Inference	
Variable Name	Rev.	Mean	Means	Single	SD	Mean	Means	Single	SD	Mean	Means	Single	SD
Adult Earnings													
Earnings (2001)	No	11811.7	-2771.3	0.71	0.78	9013.3	1087.7	0.90	0.90	9533.2	-202.4	0.71	0.79
Current Weekly earnings	No	177.0	-78.139	0.57	0.81	236.9	74.185	0.67	0.67	185.2	-5.665	0.65	0.87
Earnings Range (1 to 6)	No	2.939	-0.272	0.83	0.83	2.412	-0.292	0.84	0.94	2.544	-0.030	0.78	0.78
Rank Average	No	0.539	-0.027	0.86	–	0.516	0.016	0.93	–	0.520	-0.005	0.78	–
Total Income													
Total income (head)	No	15894.9	182.1	0.98	0.98	10216.1	-453.0	0.94	0.95	12513.1	-93.182	0.86	0.86
Total household income	No	19961.8	-6235.4	0.59	0.67	8551.0	8036.2	0.53	0.54	14439.7	-534.7	0.41	0.50
Sum of all income	No	21298.8	-7365.6	0.51	0.64	11800.3	10246.2	0.45	0.45	16653.8	-821.3	0.24	0.33
Rank Average	No	0.641	0.013	0.94	–	0.413	-0.003	0.99	–	0.526	-0.011	0.56	–
Poverty Line													
Income < 50% poverty line	Yes	-0.264	-0.171	0.51	0.69	-0.588	0.204	0.61	0.81	-0.293	-0.024	0.33	0.66
Income ≥ 150 % poverty line	No	0.173	-0.201	0.42	0.47	0.209	0.208	0.46	0.51	0.168	-0.005	0.78	0.78
Income > poverty line	No	0.595	0.001	1.00	1.00	0.080	0.103	0.74	0.74	0.327	-0.016	0.48	0.70
Rank Average	No	0.567	-0.063	0.54	–	0.434	0.085	0.54	–	0.517	-0.009	0.57	–
Self-suficiency													
Economic self-sufficiency	No	0.288	0.095	0.72	0.72	0.129	-0.141	0.63	0.76	0.204	0.010	0.59	0.82
Employed (no welfare)	No	0.377	0.159	0.56	0.81	0.758	-0.212	0.62	0.62	0.484	-0.008	0.76	0.76
Not in the labor force	Yes	-0.390	0.127	0.63	0.89	-0.336	-0.063	0.85	0.85	-0.289	-0.018	0.44	0.80
Rank Average	No	0.500	0.060	0.59	–	0.526	-0.059	0.67	–	0.521	-0.004	0.82	–
Self-reported Employment													
Employed	No	0.482	0.037	0.89	0.98	0.758	-0.025	0.95	0.95	0.567	-0.012	0.66	0.83
Employed with health insurance	No	0.281	0.014	0.96	0.96	0.424	0.023	0.94	0.99	0.327	-0.026	0.25	0.58
Employed full-time	No	0.269	-0.050	0.86	0.99	0.564	-0.011	0.98	0.98	0.407	-0.011	0.67	0.92
Employed above poverty	No	0.259	0.038	0.89	0.99	0.493	-0.140	0.71	0.97	0.325	0.013	0.60	0.92
Job for more than a year	No	0.219	0.091	0.76	0.99	0.545	-0.177	0.65	0.94	0.392	-0.008	0.75	0.75
Rank Average	No	0.460	0.027	0.84	–	0.593	-0.043	0.79	–	0.511	-0.005	0.76	–

This table presents model specification tests for counterfactuals expectations conditional on response-types. MTO Outcomes on this table are grouped in blocks separated by horizontal lines. The last line of each block of outcomes examines the average of the participant rank across the outcomes within block. First column states the variable name. Second column indicates if the variable is reversed, i.e., multiplied by -1. The remaining columns refer to three blocks of counterfactual comparison analysis used as a model specification test. Each block of analysis examines the difference between outcome estimates conditioned on response-types. The first column presents the outcome estimate that does rely on the available data on neighborhood characteristics. The second column shows the difference in the outcome expectations of previous column and the one that uses available data on neighborhood quality. The third column shows double-sided single hypothesis p-value of no difference in outcome expectations. The fourth column presents double-sided multiple hypothesis stepdown p-value associated with joint-hypothesis of no difference in outcome expectations.

Table A.7: Model Specification Tests for High-poverty Relocation1 out of 1)

		Response-type s_3 for High-poverty Relocation				Response-type s_6 for High-poverty Relocation				Response-type s_4 and s_7 for High-poverty Relocation			
		Base	**Diff.**	**Inference**		**Base**	**Diff.**	**Inference**		**Base**	**Diff.**	**Inference**	
Variable Name	**Rev.**	**Mean**	**Means**	**Single**	**SD**	**Mean**	**Means**	**Single**	**SD**	**Mean**	**Means**	**Single**	**SD**
Adult Earnings													
Earnings (2001)	No	9361.4	-1749.2	0.47	0.52	8527.9	1575.3	0.63	0.71	9160.5	122.7	0.87	0.87
Current Weekly earnings	No	206.2	-58.413	0.22	0.22	146.3	51.842	0.46	0.46	173.5	3.727	0.81	0.94
Earnings Range (1 to 6)	No	2.517	-0.262	0.51	0.51	2.488	0.243	0.66	0.67	2.443	0.035	0.81	0.97
Rank Average	No	0.526	-0.075	0.22	–	0.489	0.083	0.36	–	0.510	-0.000	1.00	–
Total Income													
Total income (head)	No	12903.4	-2479.9	0.28	0.43	12128.8	2782.5	0.39	0.88	11738.1	-203.7	0.77	0.77
Total household income	No	14441.0	-1260.0	0.69	0.69	14460.6	-724.9	0.89	0.95	12247.9	529.3	0.60	0.75
Sum of all income	No	17057.2	-1565.1	0.62	0.70	16510.6	-494.3	0.93	0.93	13325.9	650.9	0.54	0.74
Rank Average	No	0.553	-0.094	0.18	–	0.523	0.092	0.36	–	0.473	0.004	0.86	–
Poverty Line													
Income $<$ 50% poverty line	Yes	-0.348	-0.142	0.19	0.40	-0.376	0.072	0.57	0.78	-0.395	0.013	0.65	0.65
Income $\geq$ 150 % poverty line	No	0.110	-0.017	0.81	0.81	0.225	-0.002	0.98	0.98	0.112	0.013	0.55	0.80
Income $>$ poverty line	No	0.315	-0.138	0.21	0.35	0.456	0.106	0.45	0.84	0.223	0.019	0.50	0.83
Rank Average	No	0.496	-0.051	0.23	–	0.534	0.028	0.61	–	0.469	0.010	0.62	–
Self-suficiency													
Economic self-sufficiency	No	0.196	-0.012	0.89	0.89	0.186	0.048	0.69	0.69	0.205	-0.004	0.85	0.85
Employed (no welfare)	No	0.510	-0.110	0.29	0.46	0.484	0.198	0.28	0.31	0.429	-0.011	0.74	0.92
Not in the labor force	Yes	-0.287	-0.164	0.12	0.28	-0.571	0.247	0.19	0.34	-0.271	-0.016	0.58	0.92
Rank Average	No	0.520	-0.050	0.22	–	0.469	0.077	0.24	–	0.511	-0.002	0.91	–
Self-reported Employment													
Employed	No	0.594	-0.130	0.22	0.38	0.449	0.177	0.29	0.40	0.555	-0.006	0.87	0.87
Employed with health insurance	No	0.353	-0.145	0.15	0.35	0.236	0.155	0.27	0.33	0.322	-0.005	0.87	0.98
Employed full-time	No	0.433	-0.103	0.33	0.46	0.307	0.105	0.47	0.47	0.391	0.009	0.79	0.97
Employed above poverty	No	0.343	-0.100	0.34	0.34	0.320	0.119	0.43	0.43	0.346	-0.023	0.47	0.85
Job for more than a year	No	0.440	-0.137	0.18	0.36	0.259	0.205	0.20	0.22	0.388	-0.010	0.74	0.98
Rank Average	No	0.525	-0.062	0.19	–	0.464	0.078	0.27	–	0.508	-0.002	0.91	–

This table presents model specification tests for response-type counterfactuals expectations. MTO Outcomes on this table are grouped in blocks separated by horizontal lines. The last line of each block of outcomes examines the average of the participant rank across the outcomes within block. First column states the variable name. Second column indicates if the variable is reversed, i.e., multiplied by -1. The remaining columns refer to three blocks of counterfactual comparison analysis used as a model specification test. Each block of analysis examines the difference between outcome estimates conditioned on response-types. The first column presents the outcome estimate that does rely on the available data on neighborhood characteristics. The second column shows the difference in the outcome expectations of previous column and the one that uses available data on neighborhood quality. The third column shows double-sided single hypothesis p-value of no difference in outcome expectations. The fourth column presents double-sided multiple hypothesis stepdown p-value associated with joint-hypothesis of no difference in outcome expectations.

G Another Approach to Achieve Point-identification

This section is motivated by the work of Altonji et al. (2005) to state identifying assumption for counterfactual outcomes conditioned on response-type.

$RATE_{\{s_4,s_5\}}(2,1)$ can be identified if we assume that unobserved family characteristics that affect outcomes are similar for families of response-types s_4 and s_6. Under this assumption, the expectation of an outcome conditioned on the same neighborhood choice is the same across these response-types, i.e. $E(Y_\omega|T_\omega = 2, S_\omega = s_4) = E(Y_\omega|T_\omega = 2, S_\omega \in \{s_4, s_6\})$. Thereby term $E(Y_\omega|T_\omega = 2, S_\omega \in \{s_4, s_5\})$ of (25) is identified by:

$$E(Y_\omega|T_\omega = 2, S_\omega \in \{s_4, s_5\}) = \frac{E(Y_\omega|T_\omega = 2, S_\omega \in \{s_4, s_6\})\, P(S_\omega = s_4) + E(Y_\omega|T_\omega = 2, S_\omega = s_5)\, P(S_\omega = s_5)}{P(S_\omega = s_4) + P(S_\omega = s_5)}.$$

In the same fashion, $RATE_{\{s_4,s_7\}}(3,1)$ is identified by assuming that unobserved family characteristics that affect outcomes are similar for response-type s_4 and s_7. Under this assumption, we can identify $E(Y_\omega|T_\omega = 3, S_\omega \in \{s_4, s_7\})$ of (26) by:

$$E(Y_\omega|T_\omega = 1, S_\omega \in \{s_4, s_7\}) = \frac{E(Y_\omega|T_\omega = 1, S_\omega \in \{s_4, s_5\})\, P(S_\omega = s_4) + E(Y_\omega|T_\omega = 1, S = s_7)\, P(S_\omega = s_7)}{P(S_\omega = s_4) + P(S_\omega = s_7)}.$$

These assumptions cannot be tested directly. However Item 3 of **T-4** allow us to test if the expectation of pre-program variables X_ω conditioned on the response-types s_4, s_6 and s_4, s_7 mentioned above are equal.

Table A.8 focus on the comparison of pre-intervention variables of response-types s_4 and s_5 and of response-type s_4 and s_6. I cannot reject the hypothesis of no difference in means on pre-intervention variables between the response-types I examine, which is inline with the suggested assumption.

H Extension of the Monotonicity Condition for the Case of MTO

The monotonicity condition of Imbens and Angrist (1994) applies to the choice model in which the agent decides among two treatment options. In the case of MTO, families can decide among three relocation alternatives. A natural approach to examine the identification of neighborhood effects

Table A.8: Pre-program Variables Inference Conditional on Response-types

Variable Name	Response-type Mean s_4	Diff. Means $s_6 - s_4$	Inference Single p-value	Inference Stepdown p-value	Diff. Means $s_5 - s_4$	Inference Single p-value	Inference Stepdown p-value
Family							
Disable Household Member	0.097	0.078	0.706	0.706	0.012	0.974	0.974
No teens (ages 13-17) at baseline	0.304	0.432	0.203	0.395	0.187	0.732	0.883
Household size is 2 or smaller	0.103	0.166	0.414	0.541	0.168	0.682	0.938
Neighborhood							
Baseline Neighborhood Poverty	58.808	-1.409	0.872	0.977	-14.167	0.415	0.774
Victim last 6 months (baseline)	0.311	0.162	0.548	0.946	-0.028	0.960	0.960
Living in neighborhood > 5 yrs.	0.741	-0.200	0.503	0.956	-0.284	0.631	0.965
Chat with neighbor	0.326	0.252	0.330	0.778	0.469	0.415	0.799
Watch for neighbor children	0.439	0.122	0.576	0.942	0.093	0.832	0.957
Unsafe at night (baseline)	0.551	-0.037	0.869	0.869	-0.147	0.737	0.962
Moved due to gangs	0.708	0.100	0.639	0.960	-0.202	0.640	0.943
Schooling							
Has a GED (baseline)	-0.033	0.261	0.290	0.779	0.276	0.502	0.938
Completed high school	0.621	-0.306	0.298	0.641	-0.407	0.438	0.861
Enrolled in school (baseline)	0.197	-0.003	0.984	0.984	-0.196	0.598	0.896
Never married (baseline)	0.486	0.211	0.396	0.792	-0.017	0.972	0.972
Teen pregnancy	0.487	-0.188	0.540	0.721	-0.292	0.538	0.933
Missing GED and H.S. diploma	-0.050	0.120	0.395	0.715	0.087	0.696	0.897
Sociability							
No family in the neigborhood	0.475	0.207	0.424	0.638	0.147	0.785	0.931
Respondent reported no friends	0.350	0.101	0.677	0.677	0.119	0.801	0.801
Welfare/economics							
AFDC/TANF Recepient	0.763	0.031	0.885	0.885	-0.169	0.680	0.824
Car Owner	0.259	-0.099	0.530	0.732	-0.182	0.561	0.811
Adult Employed (baseline)	0.497	-0.286	0.440	0.798	-0.195	0.726	0.726

Notes: This table shows pre-program variables average estimates for MTO pre-program variables conditioned on response-types. All estimates and inference are weighted according to the associated weighting index suggested by the MTO intervention and conditional on the site of implementation. All standard deviations are computed using the method of bootstrap.

Table A.9: MTO Response-types Under Monotonicity Restrictions 75

Voucher Assignment	Z	**Possible Response-types**																
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}	s_{17}
Control	$Z = z_1$	1	1	1	1	1	1	1	1	1	2	2	3	3	3	3	3	3
Experimental	$Z = z_2$	1	1	1	2	2	2	3	3	3	2	2	1	1	2	2	3	3
Section 8	$Z = z_3$	1	2	3	1	2	3	1	2	3	2	3	2	3	2	3	2	3

in the MTO design is to extend the monotonicity condition of Imbens and Angrist (1994) to the case of an unordered choice model in which the agent decides among three treatments options.

Imbens and Angrist (1994) assume that a change in the instrumental variable induces a change in the treatment choice towards a single direction. For instance, consider a model that randomly assigns house subsidizing vouchers that incentivizes neighborhood relocation to families. Then a family is more likely to relocate if is offered the voucher than otherwise. This rationale supports the deletion of the defiers from the set of possible response-types of the binary-treatment, binary-instrument model of the introduction.

In the case of MTO, the experimental voucher incentivizes families to relocation to a low poverty neighborhood while the Section 8 voucher incentivizes families to relocate to both low and high poverty neighborhoods. Thus, using the same reasoning of Imbens and Angrist (1994), it is plausible to assume that the family's relocation choice can change only towards low neighborhood relocation as the voucher changes from no voucher to experimental voucher. Also the family's relocation choice can change only towards low or high neighborhoods relocation as the voucher changes from no voucher to Section 8 voucher. Those assumptions can be formally expressed by:

$$P(\mathbf{1}[T_\omega(z_2) = 2] \geq \mathbf{1}[T_\omega(z_1) = 2]) = 1, \text{ and } P(\mathbf{1}[T_\omega(z_3) \neq 1] \geq \mathbf{1}[T_\omega(z_1) \neq 1]) = 1. \quad (75)$$

The response matrix of **T-1** is consistent with the monotonicity restrictions (75). Even though the restrictions (75) are sensible, they are not sufficiently rich to identify the causal effects of neighborhood relocation. The monotonicity restrictions (75) generate the 17 response-types described in Table A.9, which do not render the identification of any causal parameter.

Sobel (2006) also examines the causal interpretation of the Bloom estimator for the MTO intervention under the the monotonicity restrictions (75).

I A General Model for MTO Intervention

This section investigates the MTO model described in the Section 3 using a novel causal framework. I explain that that the causal operations necessary to examine causality are ill-defined in statistics. I then present a causal framework that is more general an intuitive than the methodology commonly used in economics. Most important, the causal framework presented here solves the problem of translating causal concepts into standard statistical language. That is to say that the causal framework is fully developed within standard statistical theory, it does not require additional mathematical tools to assess causal operations.

I.1 MTO Main Variables and Their Causal Relations

All random variables in the MTO model are defined in the common probability space $(\Omega, \mathscr{F}, P)$ in which Y_ω denotes a measurement of a random variable Y for family $\omega \in \Omega$. A general causal model that describes the MTO intervention stems from seven random variables denoted by set $B = \{X_\omega, Z_\omega, V_\omega, T_\omega, G_\omega, U_\omega, Y_\omega\}$ and described by:

1. X_ω stands for the observed pre-intervention variables (e.g. gender, site, demographics) of family ω. X_ω is an external variable, i.e., a variable not caused by any variable of the model.

2. Z_ω denotes the MTO vouchers caused by some variables in X_ω (e.g., MTO site) of family ω. Its support is given by $\text{supp}(Z) = \{z_1, z_2, z_3\}$ where $Z_\omega = z_1$ stands for Control group; $Z_\omega = z_2$ for Experimental group; and $Z_\omega = z_3$ for Section 8 group.

3. V_ω stands for all unobserved pre-intervention variables. V_ω is caused by X_ω and plays the role of a confounding variable generating selection bias on neighborhood relocation choices.[29] It causes the choice of neighborhood relocation and also affects outcomes.

4. T_ω denotes the choice of neighborhood relocation of family ω caused by V_ω, Z_ω and X_ω. Its support is given by $\text{supp}(T) = \{1, 2, 3\}$, where $T_\omega = 1$ stands for not moving; $T_\omega = 2$ for moving to a low-poverty neighborhood; and $T_\omega = 3$ for moving to a high-poverty neighborhood;

[29] Our results also hold if V_ω causes X_ω instead.

5. G_ω represents unobserved *neighborhood* characteristics faced by family ω. It varies according to neighborhood relocation choice T_ω.

6. U_ω stands for all unobserved post-intervention variables of family ω.. It is caused by previous unobserved characteristics V_ω, X_ω and also neighborhood characteristics G_ω.

7. Y_ω denotes the outcome of interest caused by G_ω, U_ω and X_ω.

MTO voucher Z_ω plays the role of an instrumental variable, i.e., variable Z_ω only affects outcome of of family ω, i.e. Y_ω through its impact on the neighborhood relocation choice T_ω.

I do not assume any special *statistical relationships* among variables. Instead, I assume *causal relations* which, in turn, generate statistical ones. I use the seven error terms $(\epsilon^X_\omega, \epsilon^Z_\omega, \epsilon^V_\omega, \epsilon^T_\omega, \epsilon^G_\omega, \epsilon^U_\omega, \epsilon^Y_\omega)$ associated to each variable in B to express the causal relations described above by the following *autonomous* structural equations:

$$X_\omega = f_X(\epsilon^X_\omega),\ Z_\omega = f_Z(X_\omega, \epsilon^Z_\omega),\ V_\omega = f_V(X_\omega, \epsilon^V_\omega),\ T_\omega = f_T(V_\omega, Z_\omega, X_\omega, \epsilon^T_\omega),$$
$$G_\omega = f_G(T_\omega, \epsilon^G_\omega),\ U_\omega = f_U(G_\omega, V_\omega, X_\omega, \epsilon^U_\omega),\ Y_\omega = f_Y(G_\omega, U_\omega, X_\omega, \epsilon^Y_\omega). \tag{76}$$

By autonomy I mean a stable mechanism represented by a deterministic function that remain invariant under the external manipulation of its arguments (Frisch, 1938). Variables that are arguments in a structural equation *directly cause* its output variable. I also invoke the following regularity conditions:

Assumption A-6. Error terms $(\epsilon^X_\omega, \epsilon^Z_\omega, \epsilon^V_\omega, \epsilon^T_\omega, \epsilon^G_\omega, \epsilon^U_\omega, \epsilon^Y_\omega)$ are mutually independent.

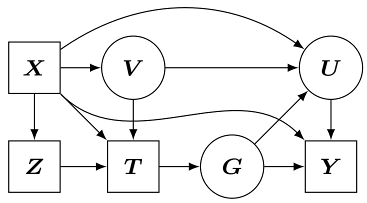
Assumption A-7. The expectation of Y_ω exists, i.e., $E(|Y_\omega|) < \infty$.

Assumption A-8. $P(Z_\omega = z | X_\omega) > 0 \quad \forall \quad z \in \text{supp}(Z)$.

Assumption A-9. $P(T_\omega = t | X_\omega) > 0 \quad \forall \quad t \in \text{supp}(T)$.

Assumption **A-6** imposes no restriction on the statistical dependence among variables in B. Error term ϵ^Z_ω denotes the MTO randomization device and statistical dependencies among variables in B can be modeled through the unobserved variables V_ω, G_ω and U_ω. In other words, Assumption **A-6** is not a binding constraint. Assumption **A-7** assures that the mean treatment parameters

Figure 8: **General Causal Model for MTO Intervention**



This figure represents the MTO Model as a DAG. **[JJH: Define.]** Arrows represent direct causal relations. Circles represent unobserved variables. Squares represent observed variables. Y is the observed outcome. T an observed housing decision that causes outcome Y. V is an unobserved confounding variable generating selection bias. Z is the MTO voucher assignment that plays the role of instrumental variable causing housing decision T. X are pre-intervention variables.

are well-defined. Assumption **A-8** assures that families were randomized to each MTO vouchers with positive probability. Assumption **A-9** assures that each choice of neighborhood relocation is chosen by some families.

Our framework is fully non-parametric. I impose no restrictions on the functional forms of structural equations $(f_X, f_V, f_Z, f_T, f_G, f_U, f_Y)$. Neither do I impose restrictions on the dimension of unobserved variables V, G or U. Thus, without loss of generality, I can assume that V subsumes error term ϵ_T. Thereby I can replace $T_\omega = f_T(V_\omega, Z_\omega, X_\omega, \epsilon_\omega^T)$ of Equation (76) by $T_\omega = f_T(V_\omega, Z_\omega, X_\omega)$. This assures that for all $(v, z, t, x) \in \text{supp}(V) \times \text{supp}(Z) \times \text{supp}(T) \times \text{supp}(X)$,

$$P(T_\omega = t | V_\omega = v, Z_\omega = z, X_\omega = x) = 0 \text{ or } P(T_\omega = t | V_\omega = v, Z_\omega = z, X_\omega = x) = 1 \quad \forall\, t \in \text{supp}(T). \tag{77}$$

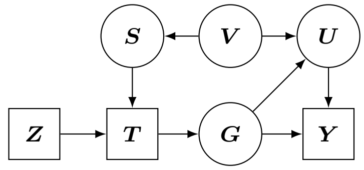
For sake of notational simplicity I also suppress pre-intervention observed variables X_ω. All analysis can be understood as conditioned on X_ω. The modified model is presented below:

$$\begin{aligned} Z_\omega &= f_Z(\epsilon_\omega^Z),\ V_\omega = f_V(\epsilon_\omega^V),\ T_\omega = f_T(V_\omega, Z_\omega), \\ G_\omega &= f_G(T_\omega, \epsilon_\omega^G),\ U_\omega = f_U(G_\omega, V_\omega, \epsilon_\omega^U),\ Y_\omega = f_Y(G_\omega, U_\omega, \epsilon_\omega^Y). \end{aligned} \tag{78}$$

I account for the family choice of neighborhood by adding an unobserved variable S_ω that denotes the unobserved 3-dimensional vector that comprises neighborhood choices across the MTO vouchers $z \in \{1, 2, 3\}$ for family ω :

$$S_\omega = [f_T(V_\omega, z_1)\ ,\ f_T(V_\omega, z_2)\ ,\ f_T(V_\omega, z_3)]' = f_S(V_\omega). \tag{79}$$

Figure 9: **Causal Model for MTO Intervention with Response Variable**



This figure represents the MTO Model as a DAG. Arrows represent direct causal relations. Circles represent unobserved variables. Squares represent observed variables. Y is the observed outcome. T an observed housing decision that causes outcome Y. V is an unobserved confounding variable generating selection bias. Z is the MTO voucher assignment that plays the role of instrumental variable causing housing decision T. X are pre-intervention variables.

Variable S_ω is termed a *Response Variable* and denotes the neighborhood relocation decisions that would have occurred if families had been assigned to Control group $(f_T(V_\omega, z_1))$, Experimental group $(f_T(V_\omega, z_2))$ and Section 8 $(f_T(V_\omega, z_3))$. S is a function of unobserved variables V, therefore it does not add any new information to the model. We can express neighborhood choice T_ω in terms of the response variable S_ω by:

$$T_\omega = [\mathbf{1}[Z_\omega = z_1], \mathbf{1}[Z_\omega = z_2], \mathbf{1}[Z_\omega = z_3]] \cdot S_\omega = g_T(S_\omega, Z_\omega), \tag{80}$$

where $\mathbf{1}[\psi]$ is an indicator function that takes value 1 if ψ is true and 0 otherwise. In summary, Causal Model (78) is recast into:

$$\begin{aligned} &V_\omega = f_V(X_\omega, \epsilon_\omega^V),\ Z_\omega = f_Z(X_\omega, \epsilon_\omega^Z),\ S_\omega = f_S(V_\omega),\ T = g_T(S, Z), \\ &G_\omega = f_G(T_\omega, \epsilon_\omega^G),\ U_\omega = f_U(G_\omega, V_\omega, X_\omega, \epsilon_\omega^U),\ Y_\omega = f_Y(G_\omega, U_\omega, X_\omega, \epsilon_\omega^Y), \end{aligned} \tag{81}$$

where $f_V, f_Z, f_Y, f_T, f_G, f_U, f_Y$ are the same as in (78), $f_S(V_\omega)$ as in (79) and g_T as in (80). Figure 9 represents Model (81) as a DAG.

We use $\text{supp}(S) = \{s_1, \cdots, s_{N_S}\}$ for the finite support of S_ω, i.e., all response-types s such that $P_{\text{E}}(S_\omega = s) > 0$. Recall that neighborhood decisions T_ω and MTO vouchers Z_ω take values in $\{z_1, z_2, z_3\}$. Thus there are a total of $3^3 = 27$ possible values $s \in \text{supp}(S)$ that the Response Variable S can take. A useful property of the Response Variable S_ω in Model (81) is that $Y_\omega \perp\!\!\!\perp T_\omega | (S_\omega, Z_\omega)$ since $T_\omega = g_T(S_\omega, Z_\omega)$, i.e., T_ω is deterministic conditioned on MTO vouchers Z_ω and unobserved Response Variable variable S_ω. Next section describes some statistical relations of the Response Variable S_ω.

I.2 From Causal Relations to Statistical Relations

I draw on the literature of Bayesian Networks to translate causal links into conditional independence relations. Some notation is necessary. Let variables that directly cause a variable $V_\omega \in B$ be called *parents* of V_ω and denoted by $Pa(V_\omega)$. For example, $Pa(Y_\omega) = \{G_\omega, U_\omega\}$ in Model (81). Variables with no parents are termed external variables. Variables directly caused by a variable $V_\omega \in B$ are termed children of V_ω and are defined by $Ch(V_\omega) = \{V'_\omega \in B; V_\omega \in Pa(V'_\omega)\}$, e.g., $Ch(V_\omega) = \{S_\omega, U_\omega\}$ in (81). A causal *chain* is a sequence of variables in B connected by direct cause links, e.g., $Z_\omega \to T_\omega \to G_\omega \to Y_\omega$. $D(V_\omega) \subset B$ denotes the descendants of $V_\omega \in B$, that is, all variables in B connected to V_ω through a chain of causation arising from V_ω, e.g, $D(Z_\omega) = \{T_\omega, G_\omega, U_\omega, Y_\omega\}$ in (81).

The Local Markov Condition (LMC) states that if a model is recursive, that is, no variable is a descendent of itself, then each variable is independent of its non-descendants conditional on its parents (Kiiveri et al., 1984; Lauritzen, 1996). [30] Notationally, LMC can be defined as:

$$V_\omega \perp\!\!\!\perp (B \setminus D(V_\omega)) \mid Pa(V_\omega) \quad \text{for all } V_\omega \in B. \tag{82}$$

Equation (82) uses Dawid's (1979) notation for conditional independence. Let $B_1, B_2, B_3 \subset B$, be a disjoint sets of random variables, then $B_1 \perp\!\!\!\perp B_2 | B_3$ means that variables in B_1 and B_2 are pairwise independent conditioned on all variables in set B_3. In addition to the LMC, I also rely on the Graphoid Axioms (GA) of Dawid (1976) to investigate conditional independence relationships.[31]

Lemma **L-4** uses the LMC (82) and the GA to state two useful properties of Response Variable S of Model (81):

Lemma L-4. (a) $Z_\omega \perp\!\!\!\perp S_\omega$ and (b) $Y_\omega \perp\!\!\!\perp Z_\omega | (S_\omega, T_\omega)$ hold in the Model described by (81).

[30] See Heckman and Pinto (2013b) for a short proof.

[31] The Graphoid Axioms are a set of conditional independence relations first presented by Dawid (1976):

$$\begin{aligned}
\text{Symmetry: } & X \perp\!\!\!\perp Y|Z \Rightarrow Y \perp\!\!\!\perp X|Z. \\
\text{Decomposition: } & X \perp\!\!\!\perp (W,Y)|Z \Rightarrow X \perp\!\!\!\perp Y|Z. \\
\text{Weak Union: } & X \perp\!\!\!\perp (W,Y)|Z \Rightarrow X \perp\!\!\!\perp W|(Y,Z). \\
\text{Contraction: } & X \perp\!\!\!\perp Y|Z \text{ and } X \perp\!\!\!\perp W|(Y,Z) \Rightarrow X \perp\!\!\!\perp (W,Y)|Z. \\
\text{Intersection: } & X \perp\!\!\!\perp W|(Y,Z) \text{ and } X \perp\!\!\!\perp Y|(W,Z) \Rightarrow X \perp\!\!\!\perp (W,Y)|Z. \\
\text{Redundancy: } & X \perp\!\!\!\perp Y|X.
\end{aligned}$$

The intersection relation is only valid for strictly positive probability distribution.

Proof. See Proof in Section I.5 □

The first relationship in **L-2** states that the voucher indicator Z_ω is independent of the potential choices of neighborhood relocations in S_ω. The second relationship states that conditioning on neighborhood decision and the unobserved Response Variable S_ω renders independence between outcomes and assigned vouchers. Next section defines a causal framework used to investigate neighborhood effects in the MTO experiment.

I.3 A Causal framework

Our goal is to evaluate causal effects of neighborhood relocation T_ω on outcome Y_ω. The concept of average causal effect dates back to Haavelmo (1944) and refers to the expected difference between counterfactual outcomes generated by *fixing* T_ω for all participants of the sample space. *Fixing* is a causal operation that sets variable T_ω to a value $t \in \text{supp}(T)$ in the structural equations that have T_ω as input, that is, f_G in Equations (81).

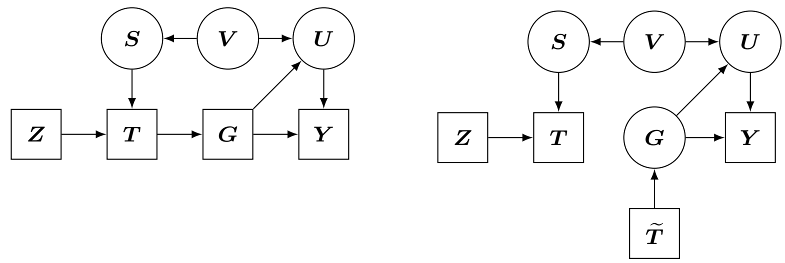
The concept of fixing is an ill-defined concept in statistical theory.[32] This generates some confusion in realms of statistics and economics. For instance, fixing depends on the direction of causal relations while statistical theory lacks directionality. Fixing differs from statistical conditioning in that fixing a variable does not affect the distribution of its non-descendants. Conditioning, on the other hand, affects the dependency structure of all random variables. Most important, fixing does not comply to range of statistical manipulations such as the law of iterated expectation. See Heckman and Pinto (2014b) for a recent discussion.

I use a simple yet intuitive approach that harmonizes causal concepts with standard statistical tools. I sidestep the complexities of the fixing operator by distinguishing the model that generates data, termed *empirical model*, from a *hypothetical model* suitable for analysing causal effects of T_ω on Y_ω. I incorporate the core idea underlying fixing, namely an independent variation of the treatment status T_ω, by introducing an the external *hypothetical variable* that replaces the T_ω in the equations we fix upon. Specifically the hypothetical model stems two properties: (1) it shares the same structural equations and distribution of error terms of the empirical model; (2) it adds the *hypothetical variable* $\widetilde{T}_\omega$ that is not caused by any variable in the system but replaces the T-input

[32] See Heckman and Pinto (2014b) for a recent discussion on fixing.

Figure 10: **Empirical and Hypothetical MTO Causal Models**

A. MTO Empirical Model	**B. MTO Hypothetical Model**



This figure represents two causal frameworks as DAGs: (1) the empirical model for MTO Intervention with Response Variable S; (2) the hypothetical model associated with this empirical model. Arrows represent direct causal relations. Circles represent unobserved variables. Squares represent observed variables. Y is the observed outcome. T an observed housing decision that causes outcome Y. $\widetilde{T}$ then unobserved hypothetical variable. V is an unobserved confounding variable generating selection bias. S is an unobserved Response Variable. Z is the MTO voucher assignment that plays the role of instrumental variable causing housing decision T.

in the structural equations that have T_ω as an argument.

Notationally, our hypothetical model is generated by substituting Equation $G_\omega = f_G(T_\omega, \epsilon_\omega^G)$ in (81) by $G_\omega = f_G(\widetilde{T}_\omega, \epsilon_\omega^G)$, that is:

$$\begin{aligned} &X_\omega = f_X(\epsilon_\omega^X),\ V_\omega = f_V(\epsilon_\omega^V),\ Z_\omega = f_Z(\epsilon_\omega^Z),\ T_\omega = f_T(V_\omega, Z_\omega, \epsilon_\omega^T), \\ &G_\omega = f_G(\widetilde{T}_\omega, \epsilon_\omega^G),\ U_\omega = f_U(G_\omega, V_\omega, \epsilon_\omega^U),\ Y_\omega = f_Y(G_\omega, U_\omega, \epsilon_\omega^Y). \end{aligned} \tag{83}$$

The MTO Empirical (81) and Hypothetical (83) models are represented as DAGs in Panels A and B of Figure 10. I use $B_{\text{E}}, P_{\text{E}}, E_{\text{E}}, Pa_{\text{E}}, Ch_{\text{E}}, D_{\text{E}}$ for the variable set, probability measure, expectation, parents, children and descendent of the Empirical model respectively. I use $B_{\text{H}}, P_{\text{H}}, E_{\text{H}}, Pa_{\text{H}}, Ch_{\text{H}}, D_{\text{H}}$ for the Hypothetical model counterpart.

Heckman and Pinto (2013a,b) use the hypothetical model to avoid the statistical inconsistencies of fixing. They show that the distribution of variables in the empirical model when T_ω is *fixed* at t translates to the distribution of variables in the hypothetical model *conditioned* on $\widetilde{T}_\omega = t$. This allow us to define counterfactual outcomes within the domain of standard statistical theory. For instance, the average treatment effect between counterfactual outcomes Y when neighborhood choice T is fixed at t against t' is given by a conditional expectation in the Hypothetical model: $E_{\text{H}}(Y|\widetilde{T}_\omega = t) - E_{\text{H}}(Y|\widetilde{T}_\omega = t');\ t, t' \in \text{supp}(T)$.

Our approach offers a clear identification concept: treatment effects are identified when causal parameters defined in a hypothetical model can be expressed through observed data generated by the empirical model. Hence, identification depend on rules that bridge the empirical and hypothetical probability measures. The next theorem is useful in this regard.

Theorem T-11. Let $V_\omega, Z_\omega \in \mathrm{B_E}$ be two disjoint sets of variables in the empirical model associated with the hypothetical where $\mathrm{B_H} = \mathrm{B_E} \cup \{\widetilde{T}_\omega\}$. Thus:

1. $P_\mathrm{H}(V_\omega|\mathbf{Pa}_\mathrm{H}(V_\omega)) = P_\mathrm{E}(V_\omega|\mathbf{Pa}_\mathrm{E}(V_\omega)) \,\forall\, V_\omega \in \mathrm{B_E} \setminus \{\mathrm{Ch_H}(\widetilde{T}_\omega)\}$;

2. $P_\mathrm{H}(V_\omega|\mathbf{Pa}_\mathrm{H}(V_\omega) \setminus \{\widetilde{T}_\omega\}, \widetilde{T}_\omega = t) = P_\mathrm{E}(V_\omega|\mathbf{Pa}_\mathrm{E}(V_\omega) \setminus \{T_\omega\}, T_\omega = t) \,\forall\, V_\omega \in \mathrm{Ch_H}(\widetilde{T}_\omega)$;

3. $P_\mathrm{H}(V_\omega|Z_\omega) = P_\mathrm{H}(V_\omega|Z_\omega, \widetilde{T}_\omega) = P_\mathrm{E}(V_\omega|Z_\omega)$ if $V_\omega, Z_\omega \in \mathrm{B_E} \setminus \mathrm{D_H}(\widetilde{T}_\omega)$;

4. $P_\mathrm{H}(V_\omega|Z_\omega, T_\omega = t, \widetilde{T}_\omega = t) = P_\mathrm{E}(V_\omega|Z_\omega, T_\omega = t) \,\forall\, V_\omega, Z_\omega \in \mathrm{B_E}$

Proof. See proof in Section I.5. □

Item (1) of Theorem **T-11** states that the distribution of variables not directly caused by the hypothetical variable remain the same in both the hypothetical and the empirical models when conditioned on their parents. Item (2) states that children of the hypothetical variable have the same distribution in both models when conditioned to the same parents. Item (3) states that the hypothetical variable does not affect the distribution of its non-descendants. Item (4) states that variables in both models share the same conditional distribution when the hypothetical variable $\widetilde{T}_\omega$ and the variable being fixed T_ω take the same value t.

I.4 Applying the Causal Framework to MTO

I now use the causal framework just described to investigate useful properties of Response Variable S regarding the identification of treatment effects. First, I obtain the following relations by applying LMC (82) and GA to the MTO Hypothetical Model (83):

Lemma L-5. $Y_\omega \perp\!\!\!\perp T_\omega|(S_\omega, \widetilde{T}_\omega)$ and $S_\omega \perp\!\!\!\perp \widetilde{T}_\omega$ hold in the Hypothetical model (83).

Proof. See proof in Section I.5. □

Lemma **L-5** states that counterfactual outcomes are independent of neighborhood relocation when conditioned on S. Also, both S_ω and T_ω are non-descendants of the hypothetical variable $\widetilde{T}_\omega$. Thus, by item (3) of Theorem **T-11**, we have that:

$$P_{\mathrm{H}}(S_\omega = s) = P_{\mathrm{E}}(S_\omega = s); s \in \mathrm{supp}(S). \tag{84}$$

$$\text{and } \; P_{\mathrm{H}}(T_\omega = t | S_\omega = s) = P_{\mathrm{E}}(T_\omega = t | S_\omega = s); t \in \mathrm{supp}(T), s \in \mathrm{supp}(S). \tag{85}$$

Equations (84)–(85) state that Hypothetical and Empirical models share the same distribution of S_ω and T_ω. We use the properties of S_ω stated in Lemma **L-5** to express counterfactual outcomes of the Hypothetical model in terms of the conditional expectation in the Empirical model. Suppose $P_{\mathrm{H}}(T_\omega = t | S_\omega = s) > 0$ for all $s \in \mathrm{supp}(S)$, then:

$$\begin{aligned} \mathbf{E}_{\mathrm{H}}(Y | \widetilde{T}_\omega = t) &= \sum_{s \in \mathrm{supp}(S)} \mathbf{E}_{\mathrm{H}}(Y_\omega | \widetilde{T}_\omega = t, S = s) \, P_{\mathrm{H}}(S = s | \widetilde{T}_\omega = t) \\ &= \sum_{s \in \mathrm{supp}(S)} \mathbf{E}_{\mathrm{H}}(Y_\omega | \widetilde{T}_\omega = t, T_\omega = t, S_\omega = s) \, P_{\mathrm{H}}(S = s) \\ &= \sum_{s \in \mathrm{supp}(S)} \mathbf{E}_{\mathrm{E}}(Y_\omega | T_\omega = t, S_\omega = s) \, P_{\mathrm{E}}(S_\omega = s), \end{aligned} \tag{86}$$

where the first equation comes from the law of iterated expectations. The first and second terms of the second equality comes from the first and second relations of Lemma (**L-5**). The first term of Equation (86) comes from items (4) of Theorem **T-11** and the second one comes from (84).

Equation (86) expresses counterfactual $\mathbf{E}_{\mathrm{H}}(Y_\omega | \widetilde{T}_\omega = t)$, defined in the hypothetical model, into quantities defined in the empirical model, i.e., a weighted average of outcome Y_ω conditioned on treatment $T_\omega = t$ over the values that the Response Variable S_ω takes. Conceptually, the Response Variable S_ω solves the problem of confounding effects of unobserved variables V_ω by generating a coarse partition of sample space such that the distribution of V_ω is the same across relocation choices within the response-types generated by S_ω.

Result (86) motivate us to define the Response Average Treatment Effect ($RATE$), that is, the causal effect of T_ω on Y_ω when T_ω is fixed at t against t' conditioned on response-type $S_\omega = s$:

$$RATE_s(t, t') = \mathbf{E}_{\mathrm{H}}(Y_\omega | \widetilde{T}_\omega = t, S_\omega = s) - \mathbf{E}_{\mathrm{H}}(Y_\omega | \widetilde{T}_\omega = t', S_\omega = s).$$

We can evoke Lemma **L-5** and Theorem **T-11** once more to write $RATE$ in terms of the Empirical model:

$$RATE_s(t,t') = \mathbf{E}_{\mathrm{E}}(Y_\omega|T_\omega = t, S_\omega = s) - \mathbf{E}_{\mathrm{E}}(Y_\omega|T = t', S_\omega = s). \tag{87}$$

If $P_{\mathrm{E}}(T_\omega = t|S_\omega = s) > 0$ and $P_{\mathrm{E}}(T_\omega = t'|S_\omega = s) > 0$ for all $s \in \mathrm{supp}(S), t, t' \in \mathrm{supp}(T)$ then the Average Treatment Effect (ATE) can also be expressed as a weighted average of $RATE$s:

$$ATE(t,t') = \mathbf{E}_{\mathrm{H}}(Y_\omega|\widetilde{T}_\omega = t) - \mathbf{E}_{\mathrm{H}}(Y_\omega|\widetilde{T}_\omega = t') = \sum_{s \in \mathrm{supp}(S)} RATE_s(t,t')\, P_{\mathrm{E}}(S_\omega = s). \tag{88}$$

Example I.1 applies the concepts discussed here to the standard binary-treatment RCT with full compliance.

Example I.1. Let a instrumental variable Z_ω represents the treatment assignment of family ω: $Z_\omega = 1$ if family ω is treated and $Z_\omega = 0$ otherwise(control). Let the Response Variable S_ω consists of a two-dimensional vector that describes the realization of the treatment T_ω when Z_ω takes values 0 and 1. Under full compliance, treatment T_ω takes the same value of Z_ω, i.e. $P_{\mathrm{E}}(T_\omega = Z_\omega) = 1$, and the support of Response Variable S_ω consist of a single element that takes value 0 for $Z_\omega = 0$ and 1 for $Z_\omega = 1$, i.e. $P_{\mathrm{E}}(S_\omega = s_1) = 1$, where $s_1 = [0,1]'$. The equations below explain the well-known fact that treatment group comparison beget causal effects in RCT:

$$\begin{aligned} ATE(1,0) &= \sum_{s \in \mathrm{supp}(S)} RATE_s(1,0)\, P_{\mathrm{H}}(S_\omega = s) \\ &= RATE_{s_1}(1,0)\, P_{\mathrm{E}}(S_\omega = s_1) \\ &= RATE_{s_1}(1,0) \\ &= \mathbf{E}_{\mathrm{E}}(Y_\omega|T_\omega = 1, S_\omega = s_1) - \mathbf{E}_{\mathrm{E}}(Y_\omega|T_\omega = 0, S_\omega = s_1) \\ &= \mathbf{E}_{\mathrm{E}}(Y_\omega|T_\omega = 1) - \mathbf{E}_{\mathrm{E}}(Y_\omega|T_\omega = 0), \end{aligned} \tag{89}$$

where the first equality comes from (88). The second equality comes from the fact $\mathrm{supp}(S)$ is a unity set and Equation (84). The third from $P_{\mathrm{E}}(S_\omega = s_1) = 1$. The fourth from Equation (87). The fifth from the conditional independence $Y_\omega \perp\!\!\!\perp S_\omega|T_\omega$ as S_ω is constant.

In contrast to the RCT Example I.1, the Response Variable S_ω is not observed in the MTO Experiment. The problem of identification of neighborhood effects is summarised into the task of assessing unobserved $RATE$s based on observed data. Next section uses the economic behavior of MTO families to further investigate this problem.

According to Equations (87)–(88), the identification of causal effects of T on Y relies on the evaluation of unobserved quantities $\mathbf{E}_{\text{E}}(Y_\omega|T_\omega = t, S_\omega = s),\ P_{\text{E}}(S_\omega = s)\ \forall s \in \text{supp}(S), t \in \text{supp}(T)$ for strata values $s \in \text{supp}(S)$ based on observed quantities $\mathbf{E}_{\text{E}}(Y_\omega|T_\omega = t, Z_\omega = z)$, $P_{\text{E}}(T_\omega = t, Z_\omega = z); t \in \text{supp}(T), z \in \text{supp}(Z)$. The next theorem uses MTO vouchers Z_ω and Lemma **L-2** to express these unobserved quantities in terms of observed ones:

Theorem T-12. The following equations hold for the the empirical model described by (81):

$$E_{\text{E}}(Y_\omega \cdot \mathbf{1}[T_\omega = t]|Z_\omega = z) = \sum_{s \in \text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega = z]\, E_{\text{E}}(Y_\omega|T_\omega = t, S_\omega = s)\, P_{\text{E}}(S_\omega = s). \tag{90}$$

Proof. See proof in Section I.5. □

Equation (90) expresses strata outcome expectations:

$$E_{\text{E}}(Y_\omega|T_\omega = t, S_\omega = s); (s,t) \in \text{supp}(S) \times \{1,2,3\}$$

in terms of outcome expectations conditional on MTO voucher assignments

$$E_{\text{E}}(Y_\omega \cdot \mathbf{1}[T_\omega = t]|Z_\omega = z) = E_{\text{E}}(Y_\omega|T_\omega = t, Z_\omega = z)\, P_{\text{E}}(T_\omega = t|Z_\omega = z); t \in \{1,2,3\}, z \in \{z_1, z_2, z_3\}.$$

If we set outcome Y_ω as a constant, then Equation (90) generates the following equality:

$$P_{\text{E}}(T_\omega = t|Z_\omega = z) = \sum_{s \in \text{supp}(S)} \mathbf{1}[T_\omega = t|S_\omega = s, Z_\omega = z]\, P_{\text{E}}(S_\omega = s). \tag{91}$$

Equation (91) expresses strata probabilities $P_{\text{E}}(S_\omega = s); s \in \text{supp}(S)$ in terms of propensity scores $P_{\text{E}}(T_\omega = t|Z_\omega = z); t \in \{1,2,3\}, z \in \{z_1, z_2, z_3\}$.

The left-hand side of both Equations (91)–(90) is observed, the right-hand side is not. An identification problem exists as there are 9 possible values for the right-hand side for each equation in **T-12** (combination of 3 possible neighborhood decisions in supp(T) and 3 possible MTO vouchers in supp(Z)), while there are up to 27 possible values for the left-hand side (i.e. the total number of possible elements in the support of S_ω). The analysis of Section 3.2 follows.

Kling et al. (2007) postulate that neighborhood poverty is a good proxy for the unobserved neighborhood characteristics that affect outcomes. They assume linearity and evaluate the impact of poverty levels on outcomes through a Two Stage Least Square (TSLS) model using MTO vouchers as instrumental variables. Clampet-Lundquist and Massey (2008) also assumes that poverty levels are among the main driving forces generating neighborhood effects. In this section I draw on their insight to show that the average neighborhood causal effects we can be non-parametrically identified.

Model 81 allows for both family and neighborhood unobserved characteristics to cause the outcome of interest. In it, variable G_ω represents unobserved neighborhood characteristics that are affected by neighborhood relocation choice T_ω and cause family unobserved characteristics U_ω, which, in turn, cause outcome Y_ω.

Kling et al. (2007) insight is to use neighborhood poverty as a good proxy for unobserved neighborhood characteristics G_ω. By neighborhood poverty I mean the neighborhood poverty levels faced by families in each housing spell since the onset of the program. This assumption does not modify the structural equations used to model the MTO experiment but changes the status of variable G_ω from unobserved to observed. In particular, it does not modify the confounding effects of family unobserved characteristics on outcomes and neighborhood choices.

Unnoticed by previous literature is that an observed G_ω allows for the non-parametric identification of the average treatment effect of neighborhood relocation. I rely on the following lemma to show this identification result. It uses LMC (82) and the Graphoid relationships to investigate useful conditional independence relationships in the hypothetical model.

Lemma L-6. In the Hypothetical Model (83), (1) $Y_\omega \perp\!\!\!\perp \widetilde{T}_\omega | G_\omega$, (2) $T_\omega \perp\!\!\!\perp G_\omega$, and (3) $Y_\omega \perp\!\!\!\perp \widetilde{T}_\omega | (G_\omega, T_\omega)$ hold.

Proof. See proof in Section I.5. □

I use Lemma **L-6** and Theorem **T-11** state the following theorem:

Theorem T-13. The expected value of counterfactual outcomes of Hypothetical Model (83) can be identified through observed quantities of the associated Empirical Model (81) if G_ω is observed by:

$$E_{\mathrm{H}}(Y_\omega|\widetilde{T}_\omega = t) = \int\int E_{\mathrm{E}}(Y_\omega|G_\omega = g, T_\omega = t') \cdot \mathbf{PR}_{\mathrm{E}}(t, t'|G_\omega = g)\, d\mathbf{F}_{(T_\omega, G_\omega)}(t', g), \tag{92}$$

$$\text{where } \mathbf{F}_{(T_\omega, G_\omega)}(t', g) = P_{\mathrm{E}}(T_\omega \leq t', G_\omega \leq g) \text{ and}$$

$$\mathbf{PR}_{\mathrm{E}}(t, t'|G_\omega = g) = \left(\frac{P_{\mathrm{E}}(T_\omega = t|G_\omega = g)}{P_{\mathrm{E}}(T_\omega = t)}\right)\left(\frac{P_{\mathrm{E}}(T_\omega = t'|G_\omega = g)}{P_{\mathrm{E}}(T_\omega = t')}\right)^{-1}. \tag{93}$$

Proof. See Mathematical Appendix. □

Theorem **T-13** is proved in three steps. First, the LMC (82) and the Graphoid relationships allow us to show that $Y_\omega \perp\!\!\!\perp \widetilde{T}_\omega|G_\omega$, $T_\omega \perp\!\!\!\perp G_\omega$, and $Y_\omega \perp\!\!\!\perp \widetilde{T}_\omega|(G_\omega, T_\omega)$ hold for the Hypothetical Model 83. Upon these relations, we can express $E_{\mathrm{H}}(Y_\omega|\widetilde{T}_\omega = t)$ into the hypothetical counterpart of the statistical quantities presented in Equation 92. Finally, Theorem **T-11** is used to translated these quantities defined in the hypothetical model into the empirical model counterparts.

Equation (92) expresses the expected counterfactual outcome when neighborhood choice takes value t, i.e. $E_{\mathrm{H}}(Y_\omega|\widetilde{T}_\omega = t)$ as the multiplication of two terms, $E_{\mathrm{E}}(Y_\omega|G_\omega = g, T_\omega = t')$ and $\mathbf{PR}_{\mathrm{E}}(t, t'|G_\omega = g)$, weighted by the values that T_ω and G_ω take. The first term is simply the outcome expectation conditioned on neighborhood choice and poverty levels. The second term, defined in (93), is called *Proportion Ratio.* It stands for the ratio between the propensity of choosing a treatment $t \in \mathrm{supp}(T)$ conditional on poverty level G_ω and measured in terms of its unconditional propensity divided by the same measure for a treatment $t' \in \mathit{supp}(T)$. The counterfactual expectation (92) can be understood as a weighted average of conditional expectations $E_{\mathrm{E}}(Y_\omega|G_\omega = g, T_\omega = t')$ as $\mathbf{PR}_{\mathrm{E}}(t, t'|G_\omega = g)$ are always positive and

$$\int\int \mathbf{PR}_{\mathrm{E}}(t, t'|G_\omega = g)\, d\mathbf{F}_{(T_\omega, G_\omega)}(t', g) = 1.$$

If the neighborhood choice T_ω does not affect the levels of neighborhood poverty, i.e. $T_\omega \perp\!\!\!\perp G_\omega$, then the causal effect of neighborhood relocation must be zero. In this case, the distribution of G_ω conditional on $T_\omega = t$ is the same for all $t \in \mathrm{supp}(T)$. Thereby $\mathbf{PR}_{\mathrm{E}}(t, t'|G_\omega = g)$ always takes value 1 and causal effects are zero as all outcome counterfactual expectations attain the same value.

We now examine the case where neighborhood relocation affects the levels of poverty faced by participating families. By this I mean that the neighborhood choice made at the onset of the intervention affect not only the subsequent neighborhood families relocate, but all the subsequent neighborhoods they lived from onset until the time of survey. Suppose that the poverty distribution for families who chose not to relocate ($T_\omega = 1$) stochastically dominate the poverty distribution of families who chose to move to a low-poverty neighborhood ($T_\omega = 2$). In this case, the Proportion Ratio that assess the counterfactual outcome of moving to a low-poverty neighborhood, i.e. $\mathbf{PR}_{\mathrm{E}}(2,1|G_\omega = g)$ is larger for low-poverty levels. These events are also more likely for $T_\omega = 2$. Otherwise stated, the Proportion Ratio weights poverty levels according to its relative likeliness across neighborhood choices.

I.5 Proofs

This section presents the proofs of lemmas and theorems of Sections I.1, I.3 and I.4. The subscript ω is not shown for sake of notational simplicity.

Proof of Lemma L-4:

Proof. By LMC (82) on Z we obtain $(V,S) \perp\!\!\!\perp Z$, by the Decomposition, we obtain $S \perp\!\!\!\perp Z$. To prove that $Y \perp\!\!\!\perp Z|(S,T)$ holds, we use the fact that $(V,G,U,Y) \perp\!\!\!\perp T|(Z,S)$ holds as T is deterministic conditional on Z and S. By LMC (82) on S we obtain $(Z,S,T) \perp\!\!\!\perp U|(V,G)$. By LMC (82) on V we obtain $(Z,V,S) \perp\!\!\!\perp G|T$. By Contraction on $(Z,S,T) \perp\!\!\!\perp U|(V,G)$ and $(Z,V,S) \perp\!\!\!\perp G|T$ we obtain $(Z,S) \perp\!\!\!\perp (G,U)|(V,T)$. By LMC (82) on T we obtain $(Z,V,S,T) \perp\!\!\!\perp Y|(G,U)$. By Contraction on $(Z,S) \perp\!\!\!\perp (G,U)|(V,T)$ and $(Z,V,S,T) \perp\!\!\!\perp Y|(G,U)$ we obtain $(Z,S) \perp\!\!\!\perp (G,U,Y)|(V,T)$. By LMC (82) on G we obtain $(V,G,U,Y) \perp\!\!\!\perp T|(Z,S)$. By LMC (82) on Z we obtain $(V,S) \perp\!\!\!\perp Z$. By Contraction on $(V,G,U,Y) \perp\!\!\!\perp T|(Z,S)$ and $(V,S) \perp\!\!\!\perp Z$ we obtain $V \perp\!\!\!\perp (Z,T)|S$. By Contraction on $(Z,S) \perp\!\!\!\perp (G,U,Y)|(V,T)$ and $V \perp\!\!\!\perp (Z,T)|S$ we obtain $Z \perp\!\!\!\perp (V,G,U,Y)|(S,T)$, and therefore $Z \perp\!\!\!\perp Y|(S,T)$ holds by Decomposition. $\square$

Proof of Theorem T-11:

Proof. Let Ancestors of V, that is $\mathrm{An}(V)$, refers to all variables in B connected to V through a chain of causation arriving at V, e.g, $\mathrm{An}(T) = \{V, Z\}$ in (81).

1. If $V \in \mathrm{B_E} \setminus \{\mathrm{Ch_H}(\widetilde{T})$, then $(V|\mathbf{Pa}_\mathrm{H}(V))$, in the hypothetical model, and $(V|\mathbf{Pa}_\mathrm{E}(V))$, in the empirical model, are a function of the error term ϵ_V associated with V. Equality in distribution comes from the fact that the empirical and the hypothetical models share the same structural equations and the same distribution of error terms.

2. Likewise item 1, $(V|\mathbf{Pa}_\mathrm{H}(V) \setminus \{\widetilde{T}\}, \widetilde{T} = t)$ and $(V|\mathbf{Pa}_\mathrm{E}(V) \setminus \{T\}, T = t)$ are a function of the error term ϵ_V associated with V.

3. If $V \in \mathrm{B_E} \setminus \mathrm{D_H}(\widetilde{T})$ then $\widetilde{T} \notin \mathrm{An_H}(V)$ and thereby $\widetilde{T} \notin \mathrm{An_H}(V')\ \forall V' \in \mathrm{An_H}(V)$. But V can be expressed as by the same function of error terms associated with its ancestors in both empirical and hypothetical model. Thereby it share the same distribution in either models. In particular the joint distribution of all variables $V \in \mathrm{B_E}; \widetilde{T} \notin \mathrm{An_H}(V)$ is the same in both models. Also, by LMC 82, $\tilde{T} \perp\!\!\!\perp \mathrm{B_E} \setminus \mathrm{D_H}(\widetilde{T})$.

4. The joint distribution of all variables in $\mathrm{B_E} \setminus \{T\}$ of the empirical model when T is conditioned on $T = t$ is defined by the structural equations that replace input T by t. This coincides with the characterization of the joint distribution of these variables in in hypothetical model when both T and $\tilde{T}$ take the value t.

□

Proof of Lemma L-5:

Proof. By LMC (82) on T we obtain $(V, G, U, Y, \widetilde{T}) \perp\!\!\!\perp T|(Z, S)$. By LMC (82) on Z we obtain $(V, S, G, U, Y, \widetilde{T}) \perp\!\!\!\perp Z$. By Contraction on $(V, G, U, Y, \widetilde{T}) \perp\!\!\!\perp T|(Z, S)$ and $(V, S, G, U, Y, \widetilde{T}) \perp\!\!\!\perp Z$ we obtain $(V, G, U, Y, \widetilde{T}) \perp\!\!\!\perp (Z, T)|S$, and therefore $Y \perp\!\!\!\perp T|(S, \widetilde{T})$ holds by Decomposition and Weak Union. □

Proof of Lemma T-12:

Proof.

$$E_{\mathrm{E}}(Y|T=t,Z=z) = \sum_{s\in\mathrm{supp}(S)} E_{\mathrm{E}}(Y|T=t,S=s,Z=z)\, P_{\mathrm{E}}(S=s|T=t,Z=z)$$
$$= \sum_{s\in\mathrm{supp}(S)} E_{\mathrm{E}}(Y|T=t,S=s,Z=z)\frac{P_{\mathrm{E}}(T=t|S=s,Z=z)\, P_{\mathrm{E}}(S=s|Z=z)}{P_{\mathrm{E}}(T=t|Z=z)}$$
$$\therefore E_{\mathrm{E}}(Y|T=t,Z=z)\, P_{\mathrm{E}}(T=t|Z=z) =$$
$$= \sum_{s\in\mathrm{supp}(S)} \mathbf{1}[T=t|S=s,Z=z]\, E_{\mathrm{E}}(Y|T=t,S=s)\, P_{\mathrm{E}}(S=s). \tag{94}$$

The second equality comes from Bayes Rule. The first term of Equation (94) comes from the fact that T is deterministic conditional on Z and S. The second and third terms come from $Y \perp\!\!\!\perp Z|(S,T)$ and $S \perp\!\!\!\perp Z$ of Lemma **L-2**. □

Proof of Lemma L-6:

Proof. By LMC (82) on Y we obtain $(\widetilde{T},V,Z,S,T) \perp\!\!\!\perp Y|(G,U)$. By LMC (82) on U we obtain $(\widetilde{T},Z,S,T) \perp\!\!\!\perp U|(V,G)$. By Contraction on$(\widetilde{T},V,Z,S,T) \perp\!\!\!\perp Y|(G,U)$ and $(\widetilde{T},Z,S,T) \perp\!\!\!\perp U|(V,G)$ we obtain $(\widetilde{T},Z,S,T) \perp\!\!\!\perp (U,Y)|(V,G)$. By LMC (82) on G we obtain $(V,Z,S,T) \perp\!\!\!\perp G|\widetilde{T}$. By LMC (82) on $\widetilde{T}$ we obtain $(V,Z,S,T) \perp\!\!\!\perp \widetilde{T}$. By Contraction on$(V,Z,S,T) \perp\!\!\!\perp G|\widetilde{T}$ and $(V,Z,S,T) \perp\!\!\!\perp \widetilde{T}$ we obtain $(V,Z,S,T) \perp\!\!\!\perp (\widetilde{T},G)$. By Contraction on$(\widetilde{T},Z,S,T) \perp\!\!\!\perp (U,Y)|(V,G)$ and $(V,Z,S,T) \perp\!\!\!\perp (\widetilde{T},G)$ we obtain$\widetilde{T} \perp\!\!\!\perp (V,Z,S,T,U,Y)|G$. Thus, by holds by Decomposition, Weak Union and Symmetry we obtain (1) $Y \perp\!\!\!\perp \widetilde{T}|G$ and (3) $Y \perp\!\!\!\perp \widetilde{T}|(T,G)$. Now by LMC (82) on G we obtain $(V,Z,S,T) \perp\!\!\!\perp G|\widetilde{T}$. By LMC (82) on $\widetilde{T}$ we obtain $(V,Z,S,T) \perp\!\!\!\perp \widetilde{T}$. By Contraction on $(V,Z,S,T) \perp\!\!\!\perp G|\widetilde{T}$ and $(V,Z,S,T) \perp\!\!\!\perp \widetilde{T}$ we obtain $(V,Z,S,T) \perp\!\!\!\perp (\widetilde{T},G)$ and therefore (2) $T \perp\!\!\!\perp G$ holds by Decomposition. □

Proof of Lemma T-13:

Proof. We now use Lemma **L-6** to express the counterfactual expectation $E_{\mathrm{H}}(Y|\tilde{T}=t)$ defined in the hypothetical model in terms of observed quantities of the empirical model:

$$
\begin{aligned}
&E_{\mathrm{H}}(Y|\tilde{T}=t)\\
&\quad=\sum_{g\in\mathrm{supp}(G)} E_{\mathrm{H}}(Y|G=,\tilde{T}=t)Pr_{\mathrm{H}}(G=g|\tilde{T}=t)\\
&\quad=\sum_{g\in\mathrm{supp}(G)} E_{\mathrm{H}}(Y|G=g)Pr_{\mathrm{H}}(G=g|\tilde{T}=t)\\
&\quad=\sum_{g\in\mathrm{supp}(G)}\left(\sum_{t'\in\mathrm{supp}(T)} E_{\mathrm{H}}(Y|T=t',G=g)Pr_{\mathrm{H}}(T=t'|G=g)\right)P_{\mathrm{H}}(G=g|\tilde{T}=t)\\
&\quad=\sum_{g\in\mathrm{supp}(G)}\left(\sum_{t'\in\mathrm{supp}(T)} E_{\mathrm{H}}(Y|T=t',G=g)Pr_{\mathrm{H}}(T=t')\right)P_{\mathrm{H}}(G=g|\tilde{T}=t)\\
&\quad=\sum_{g\in\mathrm{supp}(G)}\left(\sum_{t'\in\mathrm{supp}(T)} E_{\mathrm{H}}(Y|T=t',\tilde{T}=t',G=g)\,P_{\mathrm{H}}(T=t')\right)P_{\mathrm{H}}(G=g|\tilde{T}=t)\\
&\quad=\sum_{g\in\mathrm{supp}(G)}\left(\sum_{t'\in\mathrm{supp}(T)} \underbrace{E_{\mathrm{E}}(Y|G=g,T=t')}_{\text{by Item (4) of T-11}}\ \underbrace{P_{\mathrm{E}}(T=t')}_{\text{by Item (1) of T-11}}\right)\underbrace{P_{\mathrm{E}}(G=g|T=t)}_{\text{by Item (3) of T-11}}.
\end{aligned}
$$

The second equality comes from relationship (1) $Y \perp\!\!\!\perp \tilde{T}|G$ of Lemma **L-6**. The fourth equality comes from relationship (2) $T \perp\!\!\!\perp G$ of Lemma **L-6**. The fifth equality comes from relationship (3) $Y \perp\!\!\!\perp \tilde{T}|(G,T)$ of Lemma **L-6**. The last equality links the distributions of the hypothetical model with the distributions of the empirical model. The first term uses Item (4) of Theorem **T-11** to equate $Pr_{\mathrm{H}}(Y|T=t',\tilde{T}=t',G=g) = Pr_{\mathrm{E}}(Y|G=g,T=t')$. The second term uses the fact that T is not a child of $\tilde{T}$; thus, by Item (1) of Theorem **T-11**, $Pr_{\mathrm{H}}(T=t') = Pr_{\mathrm{E}}(T=t')$. Finally, LMC (82) for G generates $G \perp\!\!\!\perp T|\tilde{T}$ in the hypothetical model. Then, by Item (3) of Theorem **T-11**, $Pr_{\mathrm{H}}(G|\tilde{T}=t) = Pr_{\mathrm{E}}(G|T=t)$. The final equation of the theorem is obtained by applying Bayes Rule and rearranging the terms of the last equation above. □