Organization of Funding across Fields: Budget Apportionment and Application Incentives

Marco Ottaviani*

January 26, 2020

Abstract

Some of the world's largest research funding agencies allocate funds to different fields in proportion to the share of applications received in each field, thus equalizing the success rate across fields. Casting the problem in a simple supply and demand framework, we characterize the equilibrium acceptance standard and the resulting amount of applications when submissions are costly. We show that in all stable equilibria an increase in the accuracy of evaluation in a field reduces applications in that field. Multiple equilibria can result when the distribution of types does not have increasing hazard rate. Fields have perverse incentives to reduce the accuracy of evaluation in order to increase the number of successful applications in their field. Benchmarking current merit scores with respect to previous rounds—a practice introduced at the National Institutes of Health in 1988—generates virtuous incentives to step up the accuracy of the evaluation.

Keywords: Evaluation across fields, proportional budget allocation, payline, percentile, benchmarking, signal dispersion, information accuracy, unraveling, mean residual quantile function.

*Bocconi University, BIDSA, CEPR, and IGIER, marco.ottaviani@unibocconi.it. This work is partly inspired by my experience as grant applicant, panel reviewer, journal editor, and university administrator. I thank without implication Estelle Cantillon, Nenad Kos, Danielle Li, Meg Meyer, and Reinhilde Veugelers for helpful discussion, Aldo Lucia for unfettering research assistance, and seminar participants at Brown, Duke, Naples, Toronto, and NBER Science of Science Funding.

Some of the world's largest research funding organizations assign grants to different fields of research by allocating funds in proportion to the applications received in each field. The EU research funding agency, the European Research Council (ERC), explained the working of this scheme in the 2008 Work Programme for its second year of operation:[1]

> ". . . an indicative budget will be allocated to each panel, in proportion to the budgetary demand of its assigned proposals. This indicative budget is calculated as the cumulative grant request of all proposals to the panel divided by the cumulative grant request of all proposals to the domain of the call, multiplied by the total indicative budget of the domain."

Proportional allocation of budget across fields works as follows: A total budget T is assigned to all fields $i = 1, 2, ..., N$. If applications received in the different fields are $A_1, A_2, ..., A_N$, the budget allocated to field i is

$$\frac{A_i}{\sum_{j=1}^{N} A_j} T, \tag{1}$$

in proportion to the applications received in field i relatively to the applications received in all fields. This proportional allocation formula implies that the *success rate* in field i, defined as the fraction of funded projects over applications received in field i

$$p := \frac{\frac{A_i}{\sum_{j=1}^{N} A_j} T}{A_i} = \frac{T}{\sum_{j=1}^{N} A_j}, \tag{2}$$

is automatically equalized across all fields. In the context of research funding, grant applications in each field are assigned to a different panel (or study section) of evaluators with expertise in the field. Expert evaluators in each panel are then asked to select the most fundworthy applications so as to exhaust $100 \times p$ per cent of the budget requested by the applications in the field.

Canadian public research funding agencies, such as the Canada Institutes of Health Research and the Social Sciences and Humanities Research Council of Canada, also allocate their budget proportionally to different fields.[2] A number of institutes and centers of the US National Institutes of Health (NIH), the largest research funding organization in the world, award research grants across different fields in the life sciences through a similar scheme.[3] NIH institutes and centers apportion their budget—in turn

[1] See European Research Council (2007). The total budget allocated to the ERC for the period 2014-2020 is 13.1 billion euros.

[2] The US National Science Foundation and the major UK research councils, such as the Medical Research Council and the Economic and Social Research Council, do not allocate funds proportionally to different fields. However, they publish success rates for different programs, resulting in an implicit pressure to equalize success rates across disciplines.

[3] Out of the NIH overall annual budget of \$39.2 billion for 2018, roughly \$30.2 billion was apportioned to finance extramural research in various forms (grants, contracts, etc.).

determined by the congressional appropriations process—on the basis of the evaluation by specialized expert panels, known as study sections. After ranking and percentiling the scores given by each study section, grants are financed when they meet a target percentile, known as "payline", equalized across study sections so as to exhaust the budget of the individual institute/center according to (2).[4]

The proportional formula (1) by construction allocates a larger fraction of the overall budget to a field that attracts more applications. By automatically equalizing the fraction of successful projects over applications across different fields, proportional allocation appears to be fair in treating all fields in the same way. Proportional allocation also eliminates administrative discretion and political meddling in funding allocation, given that the budget allocation is determined automatically only on the basis of relative demand from applications across fields. As another important virtue, the proportional allocation scheme has the merit of flexibly responding to demand-side signals.

The simplicity of this scheme, however, can be deceptive when it is costly to submit applications and fields are heterogeneous, as it is typically the case. Casting the problem in a simple supply and demand framework, we characterize the equilibrium acceptance standard and the resulting amount of applications with costly submissions. Our first main result is that proportional allocation is biased against fields in which the evaluation of quality is more accurate, or, equivalently, more consensual. We show that *in all stable equilibria* an increase in dispersion (or, equivalently, a decrease in the accuracy) of the evaluation signal in a field unambiguously increases applications in that field. The model predicts that fields with more agreement about the ranking of applications in equilibrium attract fewer applications compared to less consensual fields in which there is wider dispersion of opinions.

The analysis also uncovers a second critical drawback of proportional allocation by characterizing when it leads to *multiple equilibria*. We show that if either the density of types is increasing or the type distribution has increasing hazard rate, the equilibrium is unique. Multiple equilibria allocations arise when the density of types features segments that decrease sufficiently fast. Intuitively, an increase in demand by low types then generates such a large increase in awards that the panel, so as to keep the success rate constant at p, ends up reducing the acceptance standard by more than it is needed to encourage additional demand.

Third, we turn to the perverse incentives that proportional allocation creates for fields. By increasing

[4]Percentile paylines differs across NIH institutes/centers, reflecting the budget available for distribution as well as the amount of applications received at each institute/center. In addition, more favorable percentile paylines are typically adopted for some special categories of applicants, such as Early Stage Investigators. See https://www.einstein.yu.edu/administration/grant-support/nih-paylines.aspx for a list of NIH Institutes that publish paylines and equalize percentile scores across study groups. See also Azoulay, Graff Zivin, Li, and Sampat's (2019) Appendix A for a more detailed description of the mechanism used by the NIH to rank proposals

applications, each field obtains a larger fraction of the overall budget, at the expense of other fields. Indeed, many scientific associations coordinate field-level activities by advertising the availability of grants and by supporting the submission of applications through information sessions, seminars on grant-writing, or even seed grants or matching funds for applicants. We also show that with proportional allocation fields have a perverse incentive to decrease the accuracy of evaluation in their study section, so as to increase the number of successful applications in their field. However, we find that the NIH practice of computing percentiles by benchmarking current scores in each evaluation cycle against the scores from the same study section given in recent cycles introduces virtuous incentives to step up the accuracy of the evaluation.

More generally, elements of proportionality are present in a wide variety of allocation schemes. For examples, editorial boards at academic journals exert pressure to equalize the success rate across editors who deal with different subfields. Similarly, admission boards at universities might be tempted to equalize admission rates across different majors or degree programs. Our analysis stresses the danger of giving in to the temptation of equalizing success rates across fields.

Literature. There is no previous literature on proportional allocation mechanisms and a dearth of work on budget allocation across fields. Peirce (1867) pioneers the normative theory of the allocation of resources across research fields. As stressed at least since Arrow (1962), market forces tend to underprovide research, mostly because invention is non-rival. Governments, however, have limited information about the benefits of research in different fields. For an early attempt to quantify the social benefits of medical research across diseases see Weisbrod (1963).[5] Weinstein and Zeckhauser (1973) link the problem of the optimal allocation of budget to fields to the decision theoretic approach underlying hypothesis testing.

At a positive level, the description of the actual process for determining NIH funding by the federal government in the early days inspires Wildavsky's (1964) formulation of the incremental nature of budget apportionment; see also Davis, Dempster, and Wildavsky (1964).[6] Zuckerman and Merton (1971) notice that acceptance rates at leading scholarly journals vary across academic disciplines, with higher rejection rates in social sciences and humanities compared to physical sciences.[7] Rejection rates also vary along similar lines across directorates at the National Science Foundation, which does not

[5] In a review of the NIH, Zeckhauser (1967) also argues that disease burden should guide funding choices.

[6] Savage (1999) gives a historical account of the influence process behind university earmarks in comparison to merit-based public funding of research.

[7] Zukerman and Merton (1971, page 77) write: ". . . the more humanistically oriented the journal, the higher the rate of rejecting manuscripts for publication; the more experimentally and observationally oriented, with an emphasis on rigour of observation and analysis, the lower the rate of rejection."

allocate automatically funds with the proportional formula.

In his broad overview of research funding, Lazear (1997) touches upon allocation across fields, but mostly looks at how research funding agencies should optimally trade off mean returns with riskiness. Building on a setting with continuous types and scale-location signal similar to ours, Leslie (2005) sketches a model of the demand for submissions to academic journals—key to our contribution is consideration of the supply side.[8] Scotchmer (2004, Chapter 8) formulates a simple dynamic model of demand for funding where high quality researchers sort into applying and are disciplined to deliver because of the expectation of repeated funding. See also Stephan (2012, Chapter 6) for a broad discussion and references on science funding.[9]

The contest literature focuses mostly on elicitation of contestants' effort incentives, see Moldovanu and Sela (2001), Che and Gale (2003), and Siegel (2009); our model, instead, zooms in on the noisy evaluation process of contestants' types. Closer to our setting, Morgan, Sisak, and Várdy (2018) analyze the incentives of applicants to select different fields in a setting with exogenous supply; instead, we focus on endogenously determining the supply through proportional budget allocation when applicants cannot pick field but can only choose whether or not to apply. The agency literature that analyzes how to optimally constrain biased evaluators is more tangentially related; see, e.g., Che, Dessein, and Kartik (2013), Alonso (2018), and Frankel (2018).

1 Model

Each field i is populated by a continuum of n_i risk-neutral agents representing the pool of potential applicants. If a fraction a_i of the total number n_i of potential applicants in the field apply, each requesting the allowed budget of q_i in that field, the total funds requested in field i are $A_i = n_i q_i a_i$.[10] The overall budget determines the success rate, a.k.a. payline, according to (2).

Consider field i. Each agent in the field is characterized by a type θ, representing the agent's quality or merit, which the agent observes. Agent types in field i are distributed according to G_i, with

[8] See also Cotton (2013). Models of publication selection, such as Taylor and Yildirim (2011), mostly focus on discrimination issues, which we skirt.

[9] Gans and Murray (2012) overview the main funding sources available for scientists (government, private firms' internal R&D, and foundations) and compare their different disclosure and openness requirements. Similar allocation problem arises for arts funding; Cowen (2002) argues that the vitality of the U.S. arts scene results from an ingenious combination of direct public subsidies and indirect schemes that encourage private charitable giving.

[10] In practice, grant calls typically set maximum budgets for applications, sometimes depending on the career stage of the applicant; the ERC sets the maximum allowed budgets at the same level in all fields. Given that almost all applicants request the maximum funding allowed, we do not model the individual choice of budget by the applicant. In the more general case in which grant applicants request different budgets, panel i selects the projects with the highest score so as to distribute the fraction $100 \times p$ of the total funds applied for in field i.

continuous and differentiable density g_i and support equal to the interval between $\underline{\theta}_i$ and $\overline{\theta}_i$, which can be either bounded or unbounded.

To apply agents in field i must spend $c_i > 0$, capturing the application cost in terms of resources, time, and inconvenience. If the application is successfully funded, the agent obtains a payoff equal to v_i, the value of the benefit from being awarded a research grant, scholarship, or admission. Focus on the interesting case with $v_i > c_i$ and define $\gamma_i := c_i/v_i$ as the cost-benefit ratio.[11] Agents are atomistic and thus do not take into account the impact of their application on the success rate.[12]

For every agent who decides to apply, the evaluation panel in field i observes a signal drawn from the location-scale family

$$x|\theta \sim F_i\left(\frac{x-\theta}{\sigma_i}\right), \tag{3}$$

with location parameter θ, the agent type, and scale parameter σ, the signal dispersion.[13] We assume that F_i has a logconcave density f_i.[14] Given the location structure (3), the signal distributions for different types θ are horizontally parallel. This property is illustrated in Figure 1 for types $\theta = \underline{\theta}$ (light gray), $\hat{\theta}$ (gray), and $\overline{\theta}$ (black), where we drop the subscript i. Denoting $F_{\theta,\sigma}(x) = F\left(\frac{x-\theta}{\sigma}\right)$, we have $F_{\theta,\sigma}(x) = F_{\overline{\theta},\sigma}\left(x+\overline{\theta}-\theta\right)$: the signal distribution for any type θ can be obtained from the signal distribution for type $\overline{\theta}$ by sliding horizontally to the right by $\overline{\theta}-\theta$.[15]

Section 2 analyzes the model with a single field facing a given payline, fixed at p. This analysis is directly relevant for a field that is so small, relative to the amount of applications submitted in the other fields, that it does affect the payline. More importantly, the partial equilibrium model with fixed payline is a fundamental building block for Section 3 where the characterize the full equilibrium with endogenous payline (2).

[11] The model can also easily accommodate the addition of an embarrassment or psychological cost d borne by the agent when the application is turned down; the cost benefit ratio then becomes $\gamma = (c+d)/(v+d)$.

[12] With a finite population of agents, an additional application in a field would lead to an increase in funding available—and thus in the success rate—for that field, even holding constant the behavior of other agents. This effect, however, vanishes as the number of agents and applicants increase.

[13] A more dispersed signal is less valuable in any monotone decision problem; see Lehmann (1988) and Persico (2000). Li and Agha (2015) present recent evidence on the accuracy of grant evaluation at the NIH.

[14] Given the restriction to location experiments, the assumption of logconcavity is equivalent to the Monotone Likelihood Ratio Property; see, for example, Lehmann and Romano (2005, Example 8.2.1, page 323) for a proof. This implies that updating is monotonic, so that the evaluator selects the best $100 \times p$ per cent of the applications by accepting whenever $x \geq \hat{x}$.

[15] The normal signal displayed in Figure 1 has a symmetric density f, with $f(x) = f(-x)$, so that $F(0) = 1/2$. We derive additional results for symmetric signals, but otherwise do not impose this restriction.

2 Single Field Facing Fixed Payline

This section characterizes the partial equilibrium in a single field for a fixed payline. The properties of the partial equilibrium we derive here are robust to the endogenization of the payline, as we will see in Section 3. Given the focus on a single field facing a fixed payline, in this section we drop the subscript i for the field. Under proportional allocation the evaluator accepts the top $100 \times p$ per cent of the applications a received, based on the signal x. Thus, the evaluator selects the applications most worthy of funding under the constraint that the success rate is no more than p. This allocation rule defines a game; we assume that players have common knowledge of the game and its parameters. The game proceeds as follows:

1. Each agent privately observes her type θ and decides whether to apply;
2. The evaluator observes a signal realization x for every applicant and accepts the top p applications.

Section 2.1 characterizes the application demand a by the agents if they expect the evaluator to assign grants to applications with signal realization $x \geq \hat{x}$; we show that agents with type above a threshold level $\hat{\theta}$ apply. Given that $a = 1 - G\left(\hat{\theta}\right)$ agents apply, according to the proportional rule the evaluator can accept at most pa applications. As explained in Section 2.2, the evaluator assigns grants to the $100 \times p$ per cent applications with the highest signal realizations, $x \geq \hat{x}$.

Knowing the model parameters and the information structure, agents share the same expectation about the acceptance behavior of the evaluator; this expectation is correct in equilibrium. Section 2.3 characterizes the Bayes-Nash equilibria with proportional allocation. Given that agents have measure zero and are atomistic, any deviation by an individual agent has no impact on the acceptance standard set by the evaluator $\hat{x}$; thus, the equilibrium outcome is identical to the one that would result if agents and evaluator were to act simultaneously. Section 2.4 performs comparative statics of the equilibrium with respect to the dispersion σ of the evaluator's signal, our key parameter of interest.

2.1 Incentives for Application Demand

Suppose that agents expect the evaluator to adopt the acceptance standard $\hat{x}$, whereby all applications with signal $x \geq \hat{x}$ are accepted. Agents apply provided that the expected payoff from applying is positive,

$$v\left[1 - F_{\theta,\sigma}\left(\hat{x}\right)\right] - c \geq 0.$$

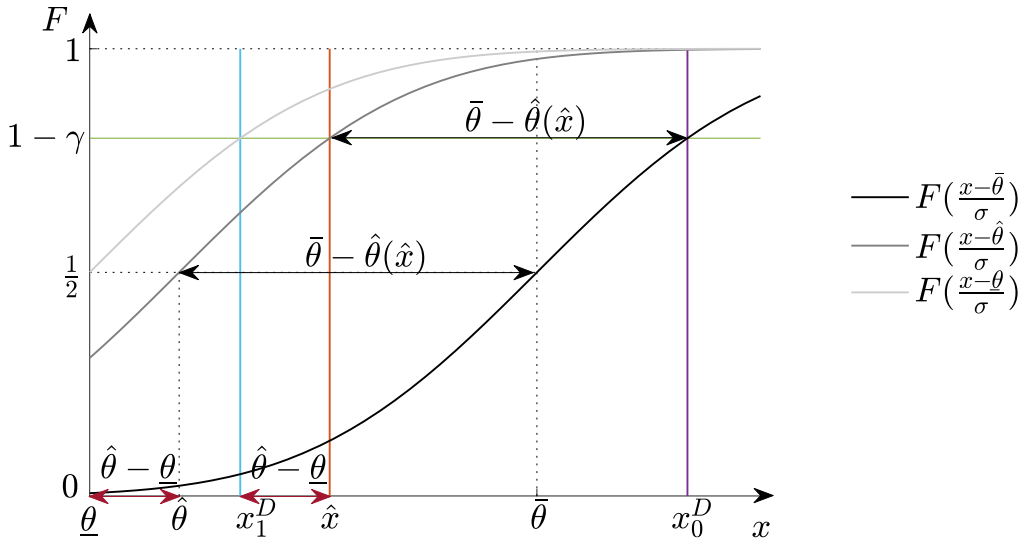

Figure 1: Derivation of demand from signal distribution for type $\theta = \bar{\theta}$, for F normal with $\sigma = 1$. The light blue and violet vertical bars correspond to the acceptance standards for which all ($a = 1$) and zero ($a = 0$) researchers apply. For intermediate acceptance standard at $\hat{x}$ (orange bar), the marginal type is $\hat{\theta}$.

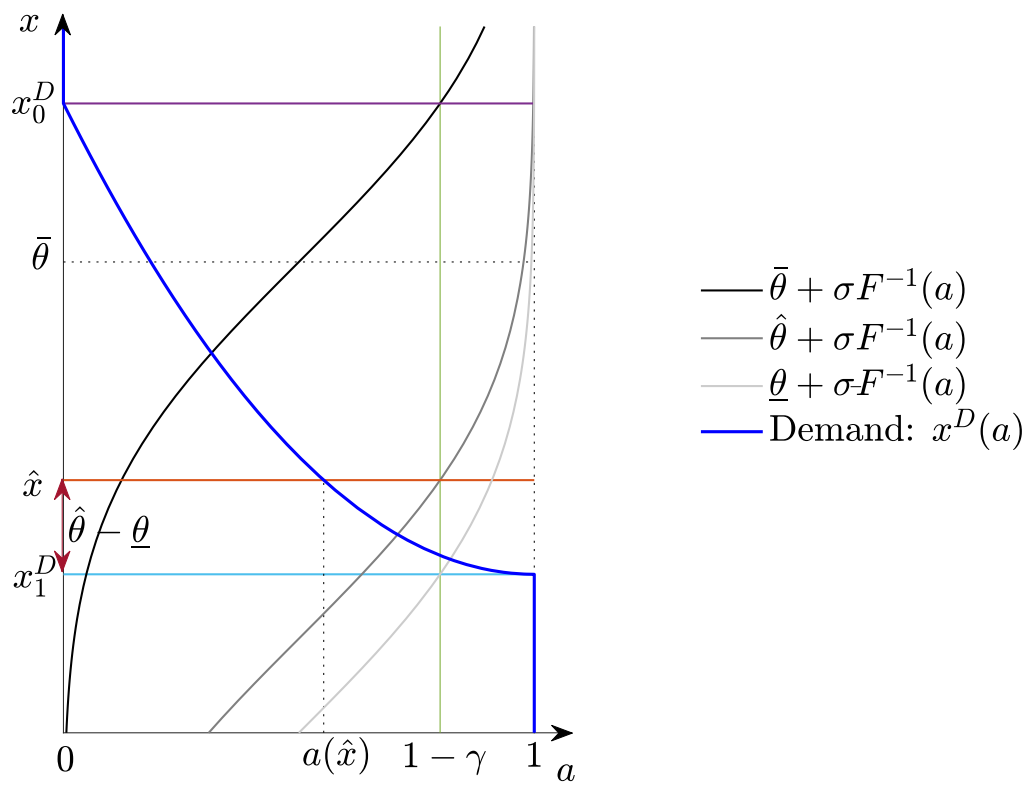

Figure 2: Construction of demand function from signal distribution and type distribution, for F normal and $G(\theta) = \sqrt{\theta}$.

By (3), the type of the marginal applicant who is exactly indifferent between applying and not applying is then

$$\hat{\theta}(\hat{x}) = \hat{x} - \sigma F^{-1}(1-\gamma), \tag{4}$$

where $\gamma = c/v$. As illustrated in Figure 1, for a given acceptance standard $\hat{x}$ (orange vertical line) the marginal type, $\hat{\theta}$, expects to be accepted with probability $1 - F\left(\frac{\hat{x}-\hat{\theta}}{\sigma}\right) = \gamma$. Given that all agents with $\theta \geq \hat{\theta}$ apply, application demand at acceptance standard $\hat{x}$ is

$$a^D(\hat{x}) = 1 - G(\hat{\theta}(\hat{x})) = 1 - G\left(\hat{x} - \sigma F^{-1}(1-\gamma)\right). \tag{5}$$

Figure 2 represents application demand a, on the horizontal axis, as a function of the acceptance standard $\hat{x}$, on the vertical axis, which plays a role similar to the price. The vertical intercept x_0^D is the standard at which the highest type $\overline{\theta}$ is exactly indifferent between applying and not. As illustrated in Figure 1, at $\hat{x} = x_0^D$ the highest type, $\overline{\theta}$, expects to be accepted with probability $1 - F\left(\frac{x_0^D - \overline{\theta}}{\sigma}\right) = \gamma$. Equivalently, the vertical demand intercept is $x_0^D = F_{\overline{\theta},\sigma}^{-1}(1-\gamma) = \overline{\theta} + \sigma F^{-1}(1-\gamma)$.

Proposition 1 (Demand) *Application demand (5)*
(a) *decreases in the standard, $da^D/d\hat{x} \leq 0$;*
(b) *decreases in the cost-benefit ratio, $da^D/d\gamma \leq 0$;*
(c) *increases/decreases in signal dispersion $\partial a^D/\partial\sigma \gtreqless 0$ whenever the acceptance standard on the demand curve is above/below the marginal type, $\hat{x} - \hat{\theta} = \sigma F^{-1}(1-\gamma) \gtreqless 0$. If the signal distribution is symmetric $F(0) = 1/2$, demand **(i)** increases in signal dispersion $da^D/d\sigma \geq 0$ in a* tight contest *with $\gamma \leq 1/2$ and **(ii)** decreases $da^D/d\sigma \leq 0$ in a* loose contest *with $\gamma \geq 1/2$*

Inequalities (a) and (b) hold strictly when demand is interior. According to part (a), demand slopes down; by (4), the marginal type $\hat{\theta}$ increases in $\hat{x}$. Figure 2 also represents the inverse of the signal distributions from Figure 1—with the axes reversed—as signal quantile functions, the increasing curves for types $\theta = \underline{\theta}$ (light gray), $\hat{\theta}$ (gray), and $\overline{\theta}$ (black). Inverting (5), the inverse demand function is then

$$\hat{x}^D(a) = \sigma F^{-1}(1-\gamma) + G^{-1}(1-a), \tag{6}$$

corresponding to the blue curve displayed in bold in Figure 2. By construction, the acceptance standard at which demand reaches the upper corner $a = 1$ is $x_1^D = \underline{\theta} + \sigma F^{-1}(1-\gamma)$, so that for $\hat{x} \leq x_1^D$ the acceptance probability for all types is more than γ.

For part (b), an increase in γ shifts demand (5) down to the left; from (6) the vertical intercept of demand is lowered. For part (c), the impact of signal dispersion σ on the demand curve is determined

by the location of the marginal type. For a given $\tilde{\sigma}$, at standard $\hat{x}(\tilde{\sigma})$ consider the marginal type $\hat{\theta}(\tilde{\sigma})$, who is exactly indifferent between applying and not. Demand increases with signal dispersion whenever the acceptance probability for $\hat{\theta}(\tilde{\sigma})$ increases in σ,

$$\frac{\partial}{\partial\sigma}\left[1-F\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right)\right]=\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma^2}f\left(\frac{\hat{x}(\tilde{\sigma})-\hat{\theta}(\tilde{\sigma})}{\sigma}\right)>0,$$

i.e., whenever $\hat{x}(\tilde{\sigma})>\hat{\theta}(\tilde{\sigma})$. Otherwise, demand decreases in dispersion. To illustrate, note that with perfect information, $\sigma=0$, type $\hat{\theta}$ is accepted for sure at standard $\hat{x}<\hat{\theta}$, expecting always a signal $x=\hat{\theta}$. As signal dispersion increases, the signal distribution rotates clockwise around $\hat{\theta}$. Given that $\hat{x}<\hat{\theta}$, the acceptance probability is reduced, and thus demand decreases in dispersion.

Symmetric Signal. When the signal is symmetric, $F(0)=1/2$, the comparative statics depends on whether the marginal type expects to be accepted less or more than 50% of the times. From (4) and symmetry of the distribution F, the marginal applicant is above or below the acceptance standard, $\hat{\theta}\lesseqqgtr\hat{x}$, whenever $\gamma\lesseqqgtr 1/2$. Therefore, $da^D/d\sigma\gtreqqless 0\Leftrightarrow\gamma\lesseqqgtr 1/2$:[16]

- In a tight contest (i), the application cost is less than half the value of the award, $c<v/2$. The cost-to-value ratio is sufficiently low that the acceptance probability is below $1/2$ for the marginal applicant. By $F^{-1}(1-\gamma)>0$, the marginal applicant type is below the acceptance standard. An increase in dispersion σ increases the acceptance probability for the marginal applicant and thus increases incentives to apply. This means that the marginal applicant decreases—and thus demand increases—in signal dispersion σ. In practice this result is relevant for common calls that tend to be are highly competitive because application costs are typically small relatively to awards, resulting in a tight contest.

- The logic is flipped in a loose contest (ii), in which the application cost is more than half the value of the award, $c>v/2$. Incentives to apply are then so limited that the marginal type is above the acceptance standard, $\hat{\theta}>\hat{x}$; in a loose contest, the marginal applicant increases—and thus demand decreases—in σ. While less typical, the loose contest case with high cost benefit ratio $\gamma>1/2$ could arise when applicants face a high stigma from being turned down.[17]

[16] It is worth drawing a parallel between our tight vs. loose contests and Johnson and Myatt's (2006) niche vs. mass markets. Like in their setting, the flipping of the comparative statics is based on whether the marginal type is to the left or to the right of the rotation point of the distribution. In our setting, however, demand by the privately informed agents depends on their type as well as the noisy signal possessed by the evaluator about the type; comparative statics is with respect to the private information of the evaluator.

[17] This amounts to setting the parameter d introduced in footnote 11 at a high level, as can be the case in internal grant competitions for university faculty members.

Next we consider an example in which demand always decreases as dispersion increases.

Example: Positive Exponential Signal. For example, if the signal distribution is $F(x) = \exp(x)$ for $x \in (-\infty, 0)$, we have $F^{-1}(1-\gamma) = \ln(1-\gamma) < 0$ so that demand always decreases in dispersion, $\partial a^D / \partial \sigma < 0$. Intuitively, a^D decreases in dispersion given that the marginal applicant is a relatively high type. With this signal structure, the marginal applicant is always discouraged—and thus demand decreases—as the signal becomes noisier.

In spite of the ambiguous comparative statics of demand with respect of dispersion, we will see that equilibrium applications—in all sensible cases in which the equilibrium is stable—always increase in dispersion.

2.2 Proportional Supply for Given Payline: When Demand Creates Supply

The panel funds applications for which the signal is above the acceptance standard $\hat{x}$.[18] Recall that the amount of applications received by applicants with $\theta \geq \hat{\theta}$ is $a = 1 - G(\hat{\theta})$ and that an applicant of type θ clears the bar with probability $1 - F\left(\frac{\hat{x}-\theta}{\sigma}\right)$. The acceptance standard $\hat{x}^S$ on the proportional supply, such that exactly a fraction p of the a applicants are successful, solves

$$\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1 - F\left(\frac{\hat{x}^S-\theta}{\sigma}\right)\right] g(\theta) d\theta}{a} = p. \tag{8}$$

This supply equation determines the acceptance standard $\hat{x}$ that guarantees that the fraction of projects selected for funding among the applicants—themselves the top a agents in the population—is equal to p, the required success rate.

Figure 3 illustrates the supply function in the $(a, \hat{x})$ space.[19] The construction relies on the fact that applicants with higher types enjoy a higher probability of acceptance than weaker candidates. Given a,

[18] By logconcavity of f, $E[\theta|a,x]$ is increasing in x; see footnote 14. The behavior of the evaluator is natural and can be rationalized if the evaluator aims at maximizing the expected quality of accepted applications subject to the constraint that exactly pa applications are accepted. Equivalently, accepting the $100 \times p$ per cent applications with the highest signal realizations is optimal for an evaluator who maximizes the total expected worthiness of accepted applications over the acceptance set X

$$\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[\int_X E[\theta|x,a] \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) dx\right] g(\theta) d\theta \tag{7}$$

subject to the success rate being lower than p, $\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[\int_X \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) dx\right] \frac{g(\theta)}{a} d\theta \leq p$.

[19] The vertical intercept for $a = 0$ is at $x_0^S = \bar{\theta} + \sigma F^{-1}(1-p)$, the standard for which the acceptance probability for the highest type is p. The vertical intercept for $a = 1$ is at x_1^S solving $\int_{\underline{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{x_1^S-\theta}{\sigma}\right)\right] g(\theta) d\theta = p$.

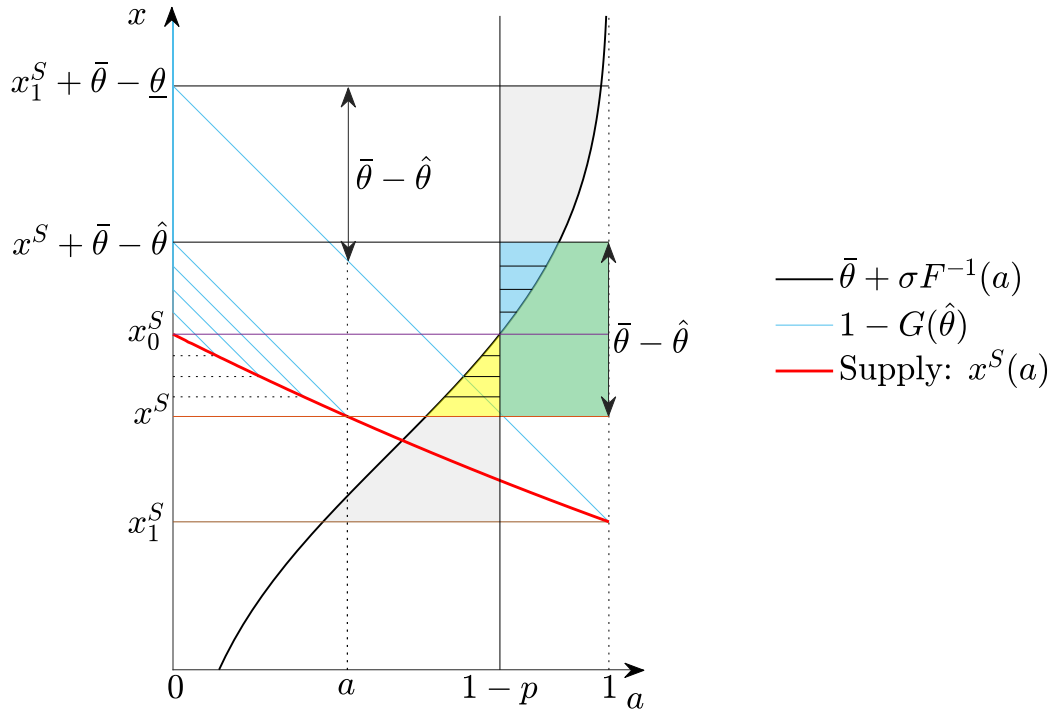

Figure 3: Proportional supply function for normal example with uniform $G(\theta) = \theta$.

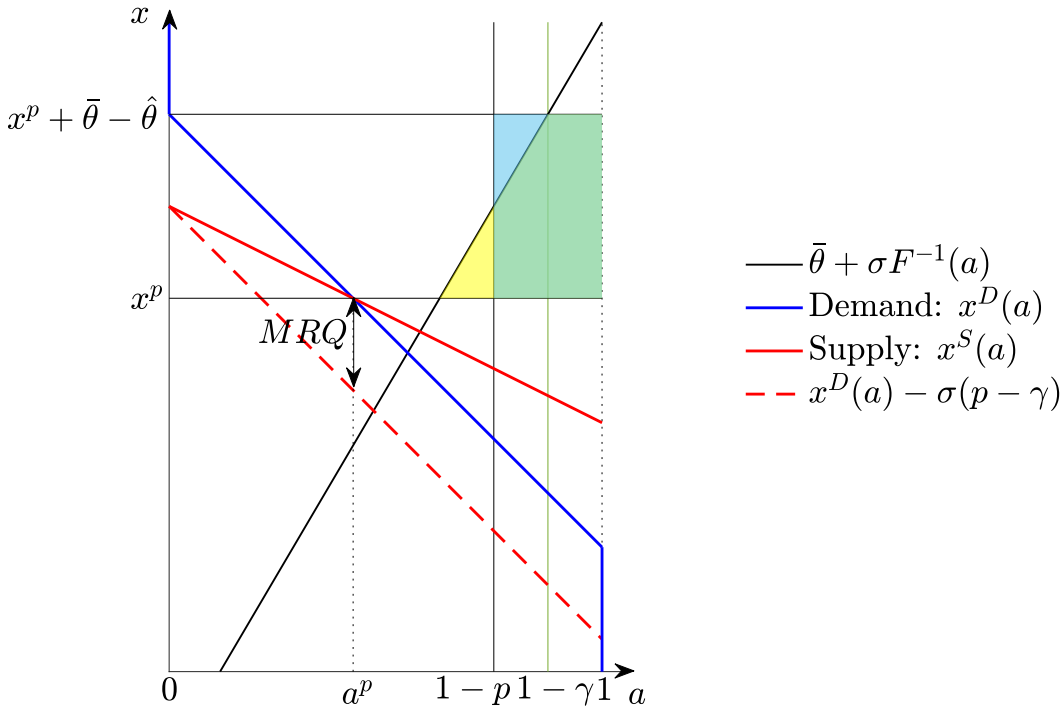

Figure 4: Construction of proportional equilibrium in example with uniformly distributed types and uniform signal.

with proportional funding the acceptance standard $\hat{x}^S$ is set so that the average probability of winning is p across all applicants, or

$$\int_{\hat{x}^S-\sigma F^{-1}(1-p)}^{\bar{\theta}} \left[\left(1-F\left(\frac{\hat{x}^S-\theta}{\sigma}\right)\right)-p\right] g(\theta)d\theta = \int_{G^{-1}(1-a)}^{\hat{x}^S-\sigma F^{-1}(1-p)} \left[p-\left(1-F\left(\frac{\hat{x}^S-\theta}{\sigma}\right)\right)\right] g(\theta)d\theta. \tag{9}$$

The argument of the integral on left-hand side of (9) is the difference between the acceptance probability for stronger applicants with types $\theta \in \left[\hat{x}^S-\sigma F^{-1}(1-p), \bar{\theta}\right]$ and the average acceptance probability p.[20] The proportional supply $\hat{x}^S(a)$ is such that the excess acceptance probability (weighed by the corresponding density of agent types) for stronger applicants on the left-hand side—the yellow area in Figure 3 when agent types are uniformly distributed, $g(\theta)=1$—is equal to the integral of the difference between p and the acceptance probability for weaker applicants with types $\theta \in \left[G^{-1}(1-a), \hat{x}^S-\sigma F^{-1}(1-p)\right]$ on the right-hand side of (9)—the blue area in Figure 3.

Proposition 2 (Proportional Supply with Fixed Payline) *Proportional supply solving (8)*
(a) *decreases in applications, $d\hat{x}^S/da \leq 0$;*
(b) *decreases in the success rate, $d\hat{x}^S/dp \leq 0$.*

According to part (a), the acceptance standard on the proportional supply $\hat{x}^S$ is a downward sloping function of applications, a. As applications increase, the average quality of applicants is reduced. To keep the success rate at the same level for the resulting worse pool of applicants, the acceptance standard must be reduced. Thus, the proportional supply curve slopes down, unlike classic supply curves, which always slope up.

For part (b), when the success rate is increased, the acceptance standard for any a must be reduced. This second property of supply will play an important role in the construction of the full equilibrium, where the success rate is endogenously determined on the basis of applications in all fields.

Uniform Signal. With signal $F_{\theta,\sigma}(x) = 1/2+(x-\theta)/\sigma$, a uniform distribution of length σ centered around θ, the supply is

$$\hat{x}^S(a) = \sigma\left(\frac{1}{2}-p\right)+G^{-1}(1-a)+\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1-G(\theta)}{a}d\theta.$$

Comparing this expression with (6), the supply has a similar structure to the inverse demand with two key differences:[21] (i) the cost-benefit ratio γ, determining incentives to apply on the demand

[20] Note that the acceptance probability of type $\hat{x}^S-\sigma F^{-1}(1-p)$ is exactly p.
[21] See the Supplementary appendix for more details.

side, is replaced by the success rate p for average applicants on the supply and (ii) the marginal type $G^{-1}(1-a) = \hat{\theta}$ is replaced by the average inframarginal type

$$E\left[\theta|\theta \geq G^{-1}(1-a)\right] = \int_{\hat{\theta}(a)}^{\bar{\theta}} \theta \frac{g(\theta)}{1-G\left(\hat{\theta}(a)\right)} d\theta.$$

Example: Uniform Types and Signal. What is the impact of an increase in signal dispersion σ on the standard $\hat{x}^S$ that induces a success rate of p given that the top $100 \times a$ per cent of agents apply? The forces at play are nicely illustrated by the example with uniformly distributed types. According to Lemma 1 in Appendix B, an increase in dispersion shifts up the proportional supply $\hat{x}^S$ for any a if and only if $p < 1/2$. Consider the realistic scenario (i) with less than fifty-fifty success rate, $p < 1/2$. Then, an increase in dispersion raises the average acceptance probability of applicants. To bring down the average success rate to p the acceptance standard must be raised: $d\hat{x}^S/d\sigma > 0$. In the knife-edge case with $p = 1/2$, the proportional supply is constant in signal dispersion: an increase in dispersion pulls down the acceptance probability of each applicant stronger than the median by exactly the same amount as it pushes up the acceptance probability of a corresponding applicant weaker than the median. In soft contests with $p > 1/2$, the logic in (i) is flipped: $d\hat{x}^S/d\sigma < 0$.

2.3 Partial Equilibrium for Fixed Payline

Figure 4 illustrates the equilibrium construction. At any given acceptance bar $\hat{x}$ the upward sloping curve in the figure represents the distribution function $F\left(\frac{\hat{x}-\bar{\theta}}{\sigma}\right)$ corresponding to the highest type, $\theta = \bar{\theta}$. For any given standard $\hat{x}$, the acceptance probability is equal to $1 - F\left(\frac{\hat{x}-\bar{\theta}}{\sigma}\right)$. Thanks to the location structure of the experiment, the acceptance probability for an agent of type $\theta < 1$ can be read on this same curve by sliding to the right by $\bar{\theta} - \theta$, thus obtaining $1 - F\left(\frac{\hat{x}-\theta}{\sigma}\right)$.

The demand condition (5) requires that the acceptance probability for the marginal type $\hat{\theta}$ which generates demand $a = 1 - G\left(\hat{\theta}\right)$ is exactly equal to γ, or

$$1 - F\left(\frac{\hat{x} - G^{-1}(1-a)}{\sigma}\right) = \gamma,$$

represented by the crossing of the distribution function with the vertical green line. The supply condition (8) requires that the average acceptance probability satisfies (8), so that the total amount of prizes assigned, equal to the acceptance probability for types from the highest (corresponding to $x = \hat{x}$) to the marginal (corresponding to $x = \hat{x} + \bar{\theta} - G^{-1}(1-a)$), weighted by their corresponding density, is equal to pa, the amount of prizes available for distribution. Notice that the acceptance probability for

the marginal type $\hat{\theta} = G^{-1}(1-a)$ is $1 - F\left(\frac{\hat{x}-G^{-1}(1-a)}{\sigma}\right) = \gamma < p$; by the location property of the distribution, this probability can be read off the distribution function $F\left(\frac{x-\bar{\theta}}{\sigma}\right)$ in the graph by setting $x = \hat{x} + \bar{\theta} - G^{-1}(1-a)$. At the other end, the acceptance probability for the top type $\theta = \bar{\theta}$ when the acceptance bar is at $\hat{x}$, $1 - F\left(\frac{\hat{x}-\bar{\theta}}{\sigma}\right)$, must necessarily be higher than p.

Proposition 3 (Uniqueness and Stability of Partial Equilibrium with Fixed Payline) ***(a)*** *If the average mean residual life of the type distribution weighted by the signal distribution is lower than the Mills ratio of the type distribution*

$$\frac{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right)[1-G(t)]\,dt}{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right) g(t)\,dt} < \frac{1-G(\theta)}{g(\theta)}, \tag{KEY}$$

there is a unique equilibrium and this equilibrium is stable.

(b) *When the support of the type distribution is bounded, $\bar{\theta} < \infty$, if $p \in (\gamma, \hat{p})$, with $p = \hat{p}$ such that the vertical intercept of supply for $a = 1$ satisfies $x_1^S = \sigma F^{-1}(1-\gamma)$, there is an interior equilibrium with $a^E \in (0,1)$.*

(c) *Each of the following conditions is sufficient for (KEY) and thus for uniqueness and stability of equilibrium: (1) the distribution of types has increasing density*

$$\frac{d}{d\theta} g(\theta) \geq 0 \tag{S1:ID}$$

or (2) the distribution of types has increasing hazard rate

$$\frac{d}{d\theta} \frac{g(\theta)}{1-G(\theta)} \geq 0 \tag{S2:IHR}$$

Condition (KEY) in part (a) characterizes the condition for the supply to cross demand from below, resulting in an interior equilibrium that is unique and stable. Our stability notion is classic: starting from any non-equilbrium allocation $\left(a^0, x^0\right)$ any tâtonnement supply and demand adjustment, $a^{t+1} = a^D\left(\hat{x}^S(a^t)\right)$ and $x^{t+1} = \hat{x}^S\left(a^D(x^t)\right)$, leads to the equilibrium, $\lim_{t\to\infty}(a^t, x^t) \to (a^p, x^p)$.

Condition (KEY) is satisfied by a wide set of distributions. Part (c) gives two alternative sufficient conditions for (KEY). First, when the density of types is (weakly) increasing, sufficient condition (S1:ID) guarantees that the interior equilibrium is unique and stable, for all signal distributions. Second, when the type distribution has increasing hazard rate sufficient condition (S2:IHR) guarantees uniqueness and stability, again regardless of the signal distribution. Condition (S2:IHR) covers a broad set of distributions, given that all logconcave densities have increasing hazard rates; thus, if the density of types is strongly unimodal the equilibrium is unique, interior and stable.

Example: Exponential Types. In an important boundary case the type distribution is negative exponential $g(\theta) = \alpha \exp(-\alpha\theta)$, with constant mean residual life. Condition (KEY) is then verified with equality for all signal distributions, so that depending on the parameters there is either a unique stable equilibrium at $a = 0$ or a unique stable equilibrium at $a = 1$; for a non-generic boundary region of parameters any $a \in [0,1]$ is an equilibrium. To illustrate, if the signal distribution is also exponential, $F(x) = 1 - \exp(-x)$, inverse demand is $\hat{x}^D(a) = -(\ln a)/\alpha - \sigma \ln \gamma$ and supply is $\hat{x}^S(a) = -(\ln a)/\alpha - \sigma \ln[p(\alpha\sigma - 1)/(\alpha\sigma)]$. Then, $a^p = 0$ if $(p-\gamma)/p < 1/(\alpha\sigma)$, $a^p = 1$ when the inequality is reversed, and any $a^p \in [0,1]$ when equality holds.

Uniform Signal. When restricting attention to uniform signal, $f(x) = 1$, condition (KEY) boils down to the property that the type distribution has Decreasing Mean Residual Life

$$\frac{\int_\theta^{\bar{\theta}} [1 - G(t)]\, dt}{1 - G(\theta)} < \frac{1 - G(\theta)}{g(\theta)} \Leftrightarrow \frac{\partial}{\partial \theta} \frac{\int_\theta^{\bar{\theta}} [1 - G(t)]\, dt}{1 - G(\theta)} < 0, \tag{DMRL}$$

a condition weaker than (S2:IHR).[22]

Example: Beta (with $\alpha = 1$) Types. For a particularly tractable DMRL example, when the type distribution is $G(\theta) = 1 - (1-\theta)^\beta$ with $\bar{\theta} = 1$, corresponding to a Beta with parameters $\alpha = 1$ and general β, for $p \in (\gamma, p = 1/(1+\beta) + \gamma)$ and $\sigma \in (0, \bar{\sigma} = 1/[(1+\beta)(p-\gamma)])$ the unique equilibrium is interior at $a^p = [(1+\beta)\sigma(p-\gamma)]^\beta$ and $\hat{x}^p = 1 + \sigma[1/2 + \beta\gamma - (1+\beta)p]$.[23]

To understand the conditions for the equilibrium to be interior in part (b), note that when the types have a bounded support if $p < \gamma$ proportional supply starts off above demand; thus, there is a stable equilibrium at the corner $a^p = 0$ with no demand. If $p \geq \hat{p}$, at the boundary $a = 1$ the vertical intercept of supply is below the vertical intercept of demand, $x_1^S \leq \sigma F^{-1}(1-\gamma)$, so that there is a corner equilibrium in which all agents apply, $a^p = 1$, and $\hat{x} = x_1^S$.

When the type distribution has unbounded support ($\bar{\theta} = \infty$), both demand and supply start off (as $a \to 0$) at infinity. The equilibria depend on the features of the Mean Residual Quantile (MRQ) function

[22] DMRL is equivalent to logconcavity of the right-hand integral of the survival function of the type distribution

$$\frac{\partial^2 \log \int_\theta^1 [1 - G(t)]\, dt}{\partial \theta^2} < 0.$$

A sufficient condition for DMRL is that the survival function $1 - G$ is logconcave or, equivalently, that the hazard rate $g/(1-G)$ is increasing (see, e.g., Bagnoli and Bergstrom (1998), Corollary 3). In turn, a sufficient condition for logconcavity of $1 - G$ is that the density g is logconcave (see Bagnoli and Bergstrom (1998), Theorem 6).

[23] For $p \in (\gamma, \hat{p})$ and $\sigma > \bar{\sigma}$, as well as $p > \hat{p}$, there is a unique and stable equilibrium at the corner $a = 1$. For $p < \gamma$ (as well as for $p \in (\gamma, \hat{p})$ and $\sigma = 0$) there is a unique and stable equilibrium at $a = 0$.

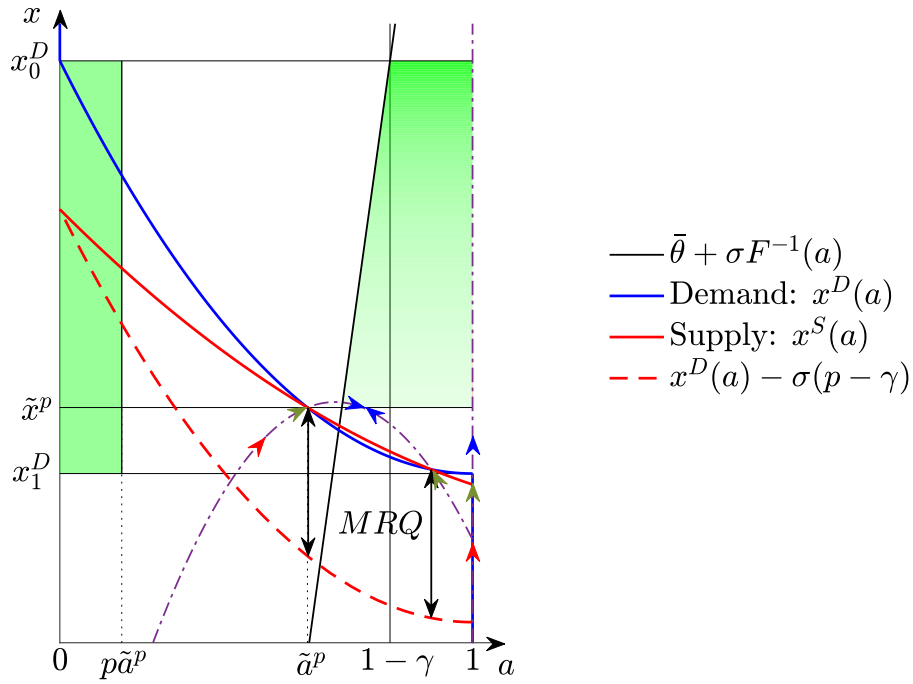

Figure 5: Example of multiple equilibria when DMRL is violated.

computed at $1-a$, defined as

$$MRQ(1-a) = \frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} [1-G(t)]\,dt}{a} = E\left[\theta - G^{-1}(1-a) \,|\, \theta \geq G^{-1}(1-a)\right],$$

representing the average excess type beyond the marginal type when the marginal type is at the $100a\%$ point (from the top) of the type distribution G.[24] If $\lim_{a\to 0} MRQ(1-a) < \sigma(p-\gamma)$, then supply starts off below demand, so that the first equilibrium (i.e., the equilibrium with lower a) is stable; this condition is satisfied for all distributions with light top tail for which $\lim_{a\to 0} MRQ(1-a) = 0$.[25] If, in addition, $\lim_{a\to 1} MRQ(1-a) = E[\theta] > \sigma(p-\gamma)$, the first equilibrium is necessarily interior and $a=1$ is not an equilibrium. If, instead, $\lim_{a\to 0} MRQ(1-a) = \infty$, then there is always a stable equilibrium with unraveling $a=0$.

Equilibrium Multiplicity. Turning to multiple equilibria, part (c) implies that for multiplicity of equilibria to result it is necessary that the hazard rate be decreasing *and* that the density be decreasing in some interval of types. When the signal is uniform, the equilibrium condition can be rewritten as

[24] On the definition of MRQ see also Nair, Sankaran, and Balakrishnan (2010, Chapter 2.4)

[25] When the type distribution is exponential, $G(\theta) = 1-\exp(-\alpha\theta)$, with constant $MRQ = 1/\alpha$, there is a unique and stable equilibrium with unraveling $a=0$ if $\sigma < 1/(\alpha p - \alpha\gamma)$ and with full coverage $a=1$ if $\sigma > 1/(\alpha p - \alpha\gamma)$. In the non-generic case $\sigma = 1/(\alpha p - \alpha\gamma)$, there is a continuum of equilibria with any $a \in [0,1]$.

$MRQ(1-a) = \sigma(p-\gamma)$. Violation of DMRL is sufficient for the existence of a region of parameters for which there are multiple equilibria—and all equilibria for which the condition is reversed are unstable. Intuitively, the density of types must decrease so steeply in θ that an increase in demand by low types generates such a large increase in the supply of awards that the acceptance standard (so as to keep the success rate constant at p) must be reduced by more than it is needed to encourage additional demand.

Example: Haupt and Schäbe Types. Figure 5 displays a tractable example with non-monotonic *MRQ* where we can obtain a closed-form solution of the full set of multiple equilibria. Suppose types follow the square root distribution $G(\theta) = \sqrt{\theta}$ (corresponding to a Beta with parameters $\alpha = 1/2, \beta = 1$) and the signal is uniform.[26] Given that the Mean Residual Quantile is not monotonic in a (here, it initially increases and then decreases), there is a set of parameters for which multiple equilibria result. As illustrated by Figure 5, for $\sigma \in (\underline{\sigma} = 1/[3(p-\gamma)], \overline{\sigma} = 3/[8(p-\gamma)])$ there are two internal crossings of supply and demand, the first corresponding to a stable equilibrium and the second to an unstable equilibrium; there is also a corner equilibrium at $a = 1$.[27]

Example: Pareto-Lomax Types. We conclude by discussing the pattern of equilibria resulting when the type distribution has a thick top tail. For example, if the type distribution is Pareto type II, a.k.a. Lomax, $G(\theta) = 1-(1+\beta\theta)^{-\alpha}$ on the support $[0,\infty)$ for $\alpha > 1$, with $MRQ(1-a) = a^{-1/\alpha}/[\beta(\alpha-1)]$ decreasing in a, there is always a stable equilibrium with unraveling at $a = 0$, which is unique for $\sigma < 1/[\beta(\alpha-1)(p-\gamma)]$. For $\sigma > 1/[\beta(\alpha-1)(p-\gamma)]$, there is also an unstable interior equilibrium at $a = [\sigma\beta(\alpha-1)(p-\gamma)]^{-\alpha}$ as well as a stable equilibrium with full coverage at $a = 1$. The general pattern established in Proposition 3 is again confirmed. Given that it is realistic for the distribution of ability types to feature a thick tail, multiplicity of equilibria is a serious practical concern.

2.4 Impact of Signal Dispersion and Unraveling

Now, our headline result. For all parameter values, applications in all stable equilibria increase in signal dispersion:

[26] The Supplementary Appendix reports closed-form expressions for the all equilibria resulting with the more general Haupt and Schäbe distribution $G(\theta) = -\eta + \sqrt{\eta^2 + (1+2\eta)\theta}$, with $\eta \in [0,\infty)$.

[27] The stable equilibrium is at $\tilde{a}^p = \left\{3 - \sqrt{3[3-8\sigma(p-\gamma)]}\right\}/4$ and the unstable equilibrium at $\tilde{\tilde{a}}^p = \left\{3 + \sqrt{3[3-8\sigma(p-\gamma)]}\right\}/4$.

Proposition 4 (Impact of Dispersion in Partial Equilibrium) *Under proportional allocation, in every stable equilibrium the equilibrium level of applications (strictly) increases (when interior) in signal dispersion σ, $da^E/d\sigma \geq 0$. In every unstable equilibrium the equilibrium level of applications decreases in signal dispersion σ.*

If the evaluator signal is *completely uninformative* ($\sigma \to \infty$), the scheme becomes a lottery. Given that the signal contains no information, the evaluator selects winners randomly. All agents apply, expecting to win with probability $p > \gamma$. As σ is decreased, at some point some agents at the bottom of the distribution expect that their acceptance probability is too low to justify spending the application cost. By the monotone structure of the equilibrium, only top researchers self select into applying. Within this self-selected pool, only the top p applications are successful.

As σ is reduced further, better and better low-end applicants withdraw, and the bar is continuously raised. An increase in signal dispersion (i.e., a reduction in signal accuracy) induces contrasting effects on demand and supply, but in the end unambiguously increases applications in all stable equilibria.

Example: Uniform Types and Signal. To further understand the logic of this general result, it is useful to initially focus on the example with uniformly distributed types, where the equilibrium is always unique and stable. Consider our headline case with a tight and tough contest, $\gamma < p < 1/2$. As illustrated in Figure 6, a reduction in signal dispersion, corresponding to an increase in accuracy of the evaluator signal (I) shifts down demand (given that $\gamma < 1/2$) pushing a down and (II) shifts down supply (given that $p < 1/2$) pushing a up—however, given that supply is flatter, effect (I) dominates and a goes down. Next, in a tight and soft contest, $\gamma < 1/2 < p$, (II') supply now also shifts up—so both demand and supply push a down. Finally, in a loose and soft contest, $1/2 < \gamma < p$, (I') demand now shifts up, but now (II') supply shifts up more than demand—again a goes down.[28]

Unraveling. Consider the limit as the signal becomes perfectly informative, $\sigma \to 0$. At the limit with $\sigma = 0$ when the evaluator has a *perfectly informative* signal about each applicant, agents with type below the acceptance standard are sure they will not be approved. Applicants only spend $c > 0$ if they are sure they will be successful. However, only a fraction $p < 1$ of these applicants must win, according to the fixed payline. Given that the evaluator selects the top $100 \times p$ per cent of the applications received, agents not in the top $100 \times p$ per cent are rejected and so are better off saving the application cost. So, the only equilibrium has zero applications.[29] The equilibrium completely unravels when $\sigma = 0$.

[28] Clearly, the equilibrium is always at the corner $a = 0$ in a loose and tough contest, $p < 1/2 < \gamma$.

[29] Or, equivalently, only the highest type $\overline{\theta}$ (measure-zero) applies and is awarded a fraction p of the grant.

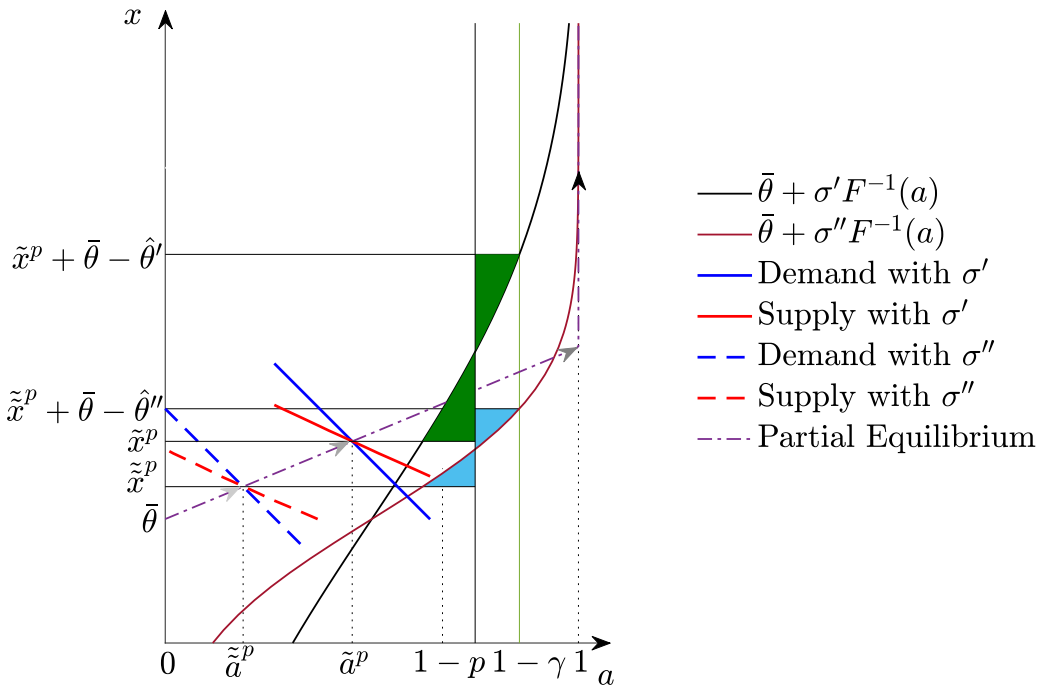

Figure 6: Partial equilibrium path as signal dispersion σ increases in example with G uniform and F normal.

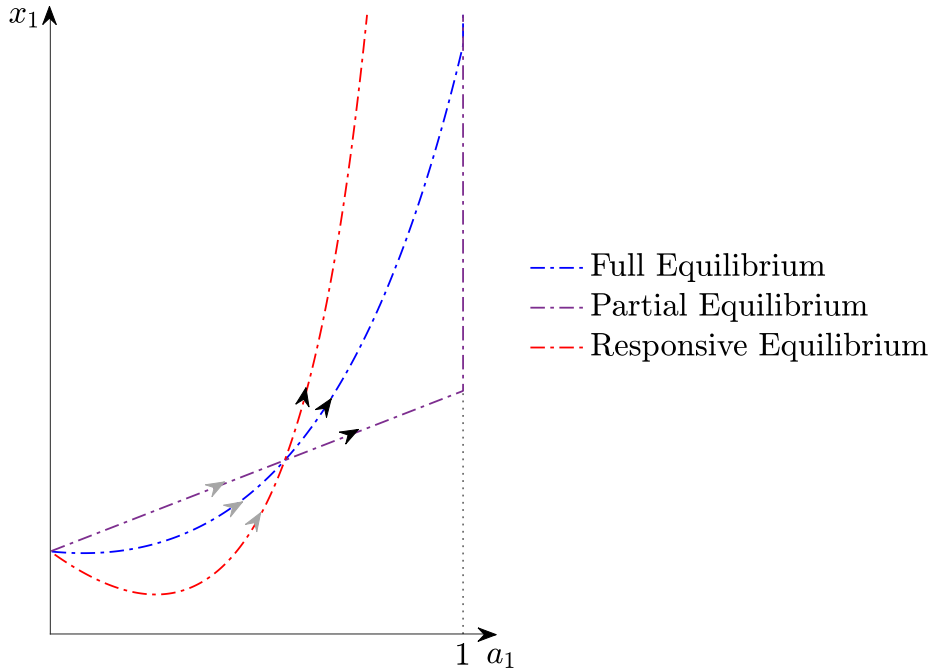

Figure 7: Comparison of path for partial equilibrium, responsive equilibrium, and full equilibrium in example with G uniform and F uniform.

The unraveling logic underlying the fact that the equilibrium amount of applications decreases in the accuracy of evaluation is a major shortcoming of proportional allocation. More generally, according to Proposition 4, an increase in signal accuracy results in a reduction in applications in all stable equilibria.

Multiple Equilibria Paths. When DMRL is violated and multiple equilibria arise, equilibria are generically odd in number and follows an alternating stability pattern. To illustrate the pattern, Figure 5 displays the path of multiple equilibria resulting with types following the square root distribution $G(\theta) = \sqrt{\theta}$ and a uniform signal. Whenever the mean residual quality is non-monotonic, as in this example, there exists a critical level $\underline{\sigma}$ for the variability such that for $\sigma < \underline{\sigma}$ there is a unique stable equilibrium which increases in a, at $\sigma = \underline{\sigma}$ a second equilibrium with $a = 1$ appears, while for $\sigma \in (\underline{\sigma}, \bar{\sigma})$ there are three equilibria (an interior stable equilibrium increasing in σ, an interior unstable equilibrium decreasing in σ, and a stable corner equilibrium at $a = 1$), at a second critical level $\sigma = \bar{\sigma} > \underline{\sigma}$ there are two equilibria (an interior equilibrium stable at the left and unstable at the right and $a = 1$), for $\sigma > \bar{\sigma}$ a single equilibrium at $a = 1$. Applications in all unstable equilibria decrease in signal dispersion.

Revisiting the Pareto example presented in the last paragraph of the previous section, also note that when there are two stable equilibria the basin of attraction of the larger among the stable equilibria increases in σ, again in the spirit of Proposition 4. Given that unstable equilibria are not robust to perturbations, our robust conclusion is that proportional allocation has the perverse property that in all stable equilibria applications decrease as signal accuracy improves.

3 Full Equilibrium

Recall that the total funds requested in field i are $A_i = n_i q_i a_i$ when a fraction a_i of the n_i potential applicants in the field apply, each requesting the allowed budget of q_i. So far we determined the field-level partial equilibrium application rate a_i, and thus $A_i = n_i q_i a_i$, in each field i for given success rate p, at the crossing between the field-level demand and the field-level supply induced by that field's demand. Even though each atomistic agent has a negligible impact on the success rate, applicants collectively determine $a_1, a_2, ..., a_N$. Given these field-level demands and the overall budget T, the equilibrium success rate solves the proportional allocation formula (2).

This section characterizes the full equilibrium with proportional allocation by simultaneously solving with respect to $a_1, a_2, ..., a_N, x_1, x_2, ..., x_N, p$ the $2N+1$ equations representing demand and proportional supply for each of the N fields plus the balanced budget equation (2). We proceed in two steps.

First, we extend Section 1's partial equilibrium analysis with fixed payline to the case with a single field facing a responsive payline determined by (2). Second, we characterize the full equilibrium with $N \geq 2$ fields, where the general equilibrium effects generated by the adjustments in the fields are taken into account.

3.1 Incorporating Payline Response: Single Field

The analysis in Section 2 covers the case with a small field that does not affect the payline—similar to the small country analysis in international trade. We now characterize the partial equilibrium for a large field that impacts the payline, while still disregarding the general equilibrium effects generated by the fact that the allocation in the other fields also depends on the payline. This case can also be interpreted as the full equilibrium resulting with a single field, $N = 1$.

Proposition 5 (Full Equilibrium with Single Field) ***(a)*** *In every stable interior partial equilibrium, applications increase in the success rate p, $da_i^p/dp > 0$.* ***(b)*** *If (KEY) holds, the full equilibrium with a single field, $N = 1$ is unique and stable.* ***(c)*** *In every stable interior equilibrium with responsive payline the impact of dispersion on equilibrium applications is still positive but dampened compared to the case with fixed payline: $da_i^p/d\sigma_i > da_i^i/d\sigma_i > 0$.*

According to part (a), all stable partial equilibria a_i^p—and thus the right-hand side of (2)—are decreasing in the success rate p. This is a key step in the characterization of the fixed-point problem once the payline is endogenized. Condition (KEY) guarantees that the equilibrium with responsive payline is unique and stable, part (b). According to part (c), the impact of dispersion on applications is dampened when the payline is endogenized; intuitively, the payline adjusts adversely as applications increase, thus discouraging additional applications. Figure 7 displays in blue the path of the full equilibrium for a single field $i = N = 1$ as σ_i increases; full equilibrium applications increase less fast than in the path for the partial equilibrium in purple.

3.2 Equilibrium with Multiple Fields

The logic of the previous proposition is general and allows us to characterize the full equilibrium with multiple fields. If the partial equilibrium with fixed payline in every field $i = 1, ..., N$ is unique and stable (i.e., by Proposition 3, if (KEY) holds for every field), the full equilibrium is unique and stable:

Proposition 6 (Characterization of Full Equilibrium) *Under condition (KEY), when interior **(a)** the full equilibrium is unique and stable; **(b)** full equilibrium applications in any field i (when interior) **(i)** increase in the dispersion of evaluation in the same field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_i} > 0,$$

*and **(ii)** decrease in the dispersion of the evaluation in any other field*

$$\frac{da_i^{1,\ldots,N}}{d\sigma_j} < 0;$$

***(c)** full equilibrium applications (when interior) are less responsive to own dispersion than under partial equilibrium with fixed payline, but more responsive than under partial equilibrium with endogenous payline*

$$\frac{da_i^p}{d\sigma_i} > \frac{da_i^{1,\ldots,N}}{d\sigma_i} > \frac{da_i^r}{d\sigma_i} > 0.$$

When the partial equilibrium impact of own dispersion σ_i on applications in field i is positive (as it is the case for stable partial equilibria), holding fixed the applications in the other fields $j \neq i$, the endogenous adjustment of the payline (2) dampens the partial equilibrium effect, but does not change its sign, $da_i^p/d\sigma_i \geq da_i/d\sigma_i \geq 0$, consistent with the result for a single field in Proposition 5.c. The full equilibrium impact also takes into account the adjustment of applications in the other fields $j \neq i$ as the payline deteriorates. Given that partial equilibrium demand in each of the other fields decreases in the payline by Proposition 5.a, the general equilibrium adjustment in turn dampens the reduction in the payline, but without overturning the sign of the impact.

Figure 7 displays in red the equilibrium path for field $i = 1$ with $N = 2$ when the adverse impact of the payline is incorporated, but applications in the other field $j = 2$ are held constant. Consistent with part (c), when σ_i increases from the baseline, full equilibrium applications (red) increase less fast than in partial equilibrium (purple) but faster when only the adverse response of the payline is taken into account but the applications in the other fields different from i are held constant. The comparative statics for unstable equilibria is clearly reversed.

Unraveling in Full Equilibrium. We argue in a number of steps that applications unravel necessarily in all fields i with perfect evaluation $\sigma_i = 0$, provided that there is at least one field j with noisy evaluation $\sigma_j > 0$.

First, note that with perfect evaluation in field i, demand is $a_i^D(\hat{x}) = 1 - G(\hat{x})$ and supply is $\hat{x}_i^S(a_i, a_{-i}) = G^{-1}\left(1 - \frac{Ta_i}{a_i + a_{-i}}\right)$ for given $a_{-i} = \sum_{j \neq i} a_j$. The partial equilibrium correspondence for a field with $\sigma_i = 0$ is $a_i^r(a_{-i}) = \max\langle T - a_{-i}, 0\rangle$.

Second, if evaluation is perfect, $\sigma_i = 0$, in *all* fields $i = 1, ..., N$, there is a large set of multiple equilibria with $p = 1$. Any $\mathbf{a}$ such that $\sum_{i=1}^{N} a_i = T$, so that all agents who apply are sure to win, is an equilibrium, given that from $a_j^r\left(a_{-j}\right) = T - a_{-j}$ we have $a_{-i}^r(a_i) = \sum_{j \neq i} a_j^r\left(a_{-j}\right) = T - a_i$. The winning applicants can be from any of the fields. In particular, there is a symmetric equilibrium in which $a_i = T/N$ for all i. There are also very extreme equilibria in which applications in a field are zero, provided the size of the other fields is sufficiently large to scoop up all the available funds, T.

Finally, suppose that there is at least one field j with $\sigma_j > 0$. Given that this field can sustain a payline $p < 1$, applications necessarily unravel in all the fields with $\sigma_i = 0$, as claimed.

Example: Exponential Types. When types are exponentially distributed $G_i(\theta) = 1 - \exp(-\alpha_i \theta)$ and the signal is also exponential in every field with σ_i and γ_i, the full equilibrium takes a particularly simple form. Order fields $i = 1, ..., N$ by the index

$$\frac{\sigma_i - \frac{1}{\alpha_i}}{\sigma_i \gamma_i}, \tag{10}$$

from lowest to highest. The index increases in dispersion σ_i and decreases in the cost-benefit ratio γ_i as well as in the expectation of the prior type distribution $1/\alpha_i$ in the field. In the generic case in which there are no ties in the index across fields, there is a unique full equilibrium with applications

$$a_i^{1,...,N} = \max\left\langle \min\left\langle \frac{T - \sum_{j=i+1}^{N} n_j q_j}{n_i q_i}, 1 \right\rangle, 0 \right\rangle. \tag{11}$$

Building on the characterization of the partial equilibrium in Section 2.3, full equilibrium applications in field N with the highest index (10) are $a_N^{1,...,N} = \min\left\langle \frac{T}{n_N q_N}, 1 \right\rangle$. Proceeding recursively, we have $a_{N-1} = \max\left\langle \min\left\langle \frac{T - n_N q_N}{n_{N-1} q_{N-1}}, 1 \right\rangle, 0 \right\rangle$ and so on, thus obtaining (11). Equilibrium multiplicity arises if there are ties in the indices in some fields. To illustrate the construction with ties, suppose that k fields, $i = N-k+1, ..., N-1, N$, share the same highest index and that the total size of these fields is more than T

$$\sum_{i=N-k+1}^{N} n_i q_i > T,$$

so that these fields are constrained. Then, any allocation $a_i, ..., a_N$ that satisfies the budget with equality

$$\sum_{i=N-k+1}^{N} a_i n_i q_i = T$$

is part of a full equilibrium. To summarize, the exponential example confirms the general pattern whereby the amount of equilibrium applications in a field increases in dispersion and decreases in the

cost-benefit ratio relative to other fields. A striking feature of this example is that fields with *worse* pools (i.e., agents with lower average types, $1/\alpha_i$) generate *more* applications in equilibrium! This property is due to the positive skewness of the exponential distribution. Intuitively, as prior expected quality deteriorates, the type distribution becomes more positively skewed, with density more steeply decreasing. This pathological result holds more generally with other similarly skewed distributions, such as gamma.

Equilibrium Multiplicity: From Partial to Full Equilibria. To illustrate the possibility of multiple full equilibria when condition (KEY) does not hold, let us endogenize the payline in the example with type distribution $G_i(\theta) = \sqrt{\theta}$ for field i. The bending curve in Figure 8 represents the partial equilibrium correspondence for applications in field $a = a_i$ depending on applications in all other fields $b = \sum_{j \neq i} a_j$. Not to compound the equilibrium multiplicity across fields, suppose that in the other fields $j \neq i$ the distribution satisfies (KEY), so that in those fields the partial equilibrium with fixed payline is unique, and thus the partial equilibrium correspondence in each of those fields is monotonic. For concreteness, we take twelve other fields with uniform types and uniform signals. The downward sloping curve in Figure 8 represents the sum of the partial equilibrium correspondences for the sum of applications in all other fields as a function of a. The crossings of the two partial equilibrium correspondences represent the equilibria. Figure 9 displays the path of the impact of dispersion. In line with Proposition 6, the impact of dispersion on applications in full equilibrium is dampened compared to the partial equilibrium.

Example: Uniform Types and Signal. Suppose types are uniformly distributed, $G_i(\theta) = \theta$, and the signal is uniform, $F_i(x) = 1/2 + x$, in every field. Recall that field i is characterized by size n_i, grant size q_i, cost-benefit ratio γ_i, and dispersion σ_i. Crossing of demand $a^D(\hat{x}) = 1 - \hat{x} + \sigma(1/2 - \gamma)$ and supply $\hat{x}^S(a) = 1 + \sigma(1/2 - p) - a/2$ gives the partial equilibrium level of applications $a_i^p = 2\sigma_i(p - \gamma_i)$. Substituting a_i^p into (2), the equilibrium success rate solves

$$p = \frac{T}{\sum_{i=1}^N n_i q_i a_i^p} = \frac{T}{\sum_{i=1}^N n_i q_i \min\langle 2\sigma_i(p - \gamma_i), 1\rangle}.$$

Focusing on the case with interior equilibrium applications in each field, $\sigma_i < (p - \gamma_i)/2$ for all i, solution of this equation gives the equilibrium success rate

$$p = \frac{1}{2}\frac{\sum_{i=1}^N n_i q_i \sigma_i \gamma_i}{\sum_{i=1}^N n_i q_i \sigma_i} + \sqrt{\left(\frac{1}{2}\frac{\sum_{i=1}^N n_i q_i \sigma_i \gamma_i}{\sum_{i=1}^N n_i q_i \sigma_i}\right)^2 + \frac{1}{2}\frac{T}{\sum_{i=1}^N n_i q_i \sigma_i}},$$

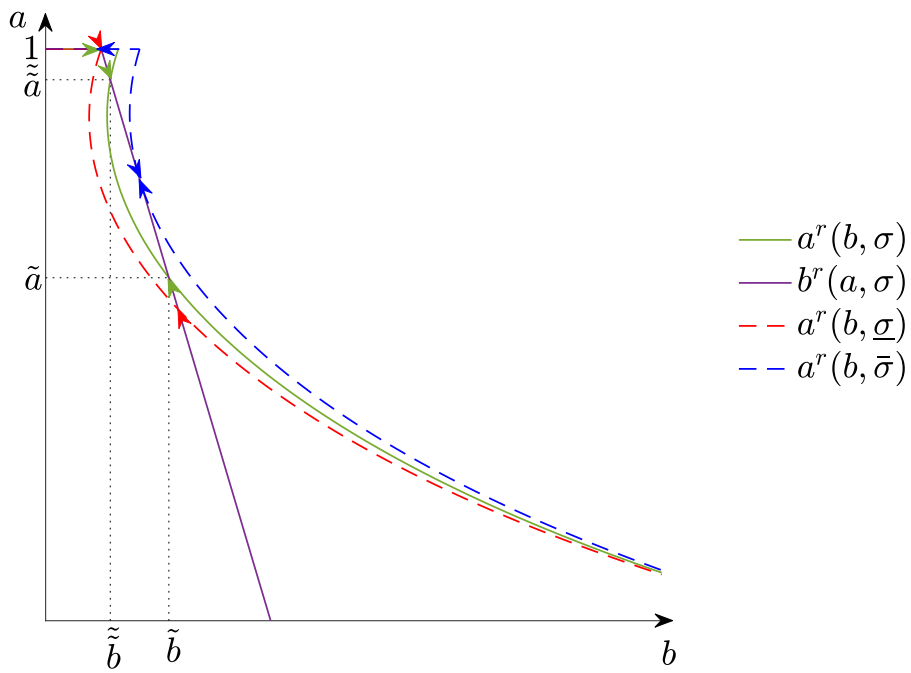

Figure 8: Construction of full equilibria by crossing partial equilibrium correspondences in example with 13 panels, 12 with types uniformly distributed and 1 with types following the square root distribution.

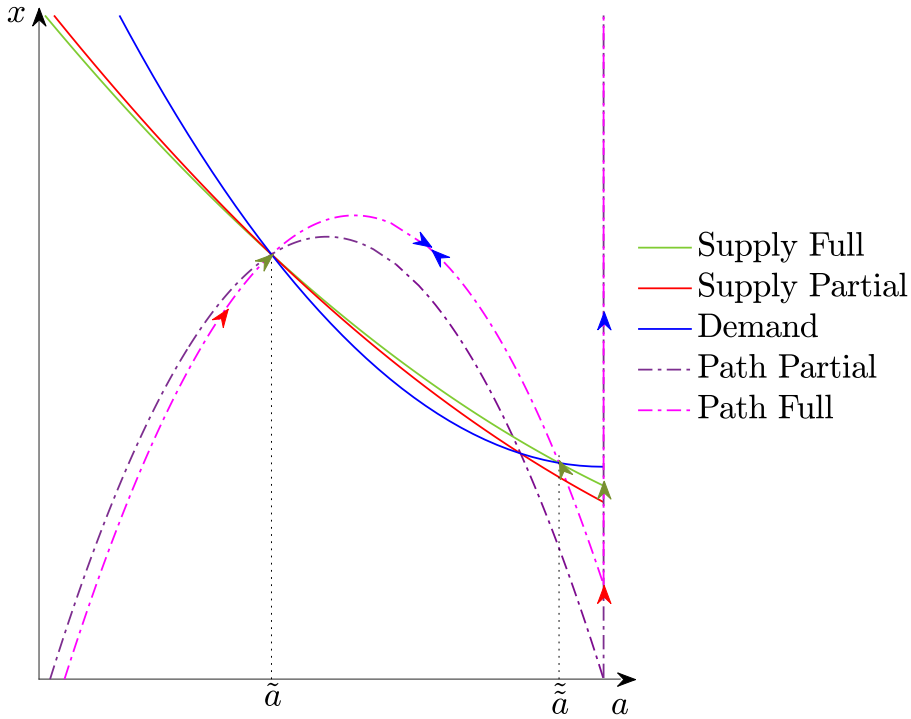

Figure 9: Multiple equilibrium paths: from partial equilibria with fixed payline to full equlibria.

so that equilibrium applications in field i are

$$A_i = n_i q_i \sigma_i \left(\frac{\sum_{j=1}^N n_j q_j \sigma_j \gamma_j}{\sum_{l=1}^N n_l q_l \sigma_l} - \gamma_i \right) + 2 n_i q_i \sigma_i \left(\sqrt{\left(\frac{1}{2} \frac{\sum_{j=1}^N n_j q_j \sigma_j \gamma_j}{\sum_{l=1}^N n_l q_l \sigma_l} \right)^2 + \frac{1}{2} \frac{T}{\sum_{l=1}^N n_l q_l \sigma_l}} - \frac{\gamma_i}{2} \right)$$

and overall applications are

$$\sum_{i=1}^N A_i = \sqrt{\left(\sum_{j=1}^N n_j q_j \sigma_j \gamma_j \right)^2 + 2T \sum_{l=1}^N n_l q_l \sigma_l} - \sum_{j=1}^N n_j q_j \sigma_j \gamma_j.$$

3.3 Welfare Performance and Design Tweaks

To evaluate the welfare performance of proportional allocation define social welfare as the sum of the welfare of the evaluator and the welfare of agents in all the fields

$$W = \sum_{i=1}^N \int_{\hat{\theta}_i}^{\overline{\theta}} \left\{ \int_{\hat{x}_i}^{\overline{x}} \left[\underbrace{E\left(\theta | x; \theta \geq \hat{\theta}_i\right) - k}_{\text{evaluator expected net merit}} + \underbrace{v_i}_{\text{agent benefit}} \right] f(x|\theta)\, dx - \underbrace{c_i}_{\text{agent cost}} \right\} g(\theta)\, d\theta,$$

where $E\left(\theta | x; \theta \geq \hat{\theta}_i\right)$ is computed by Bayes' rule taking into account the information contained in the fact that agents of higher types self select into applying and k is the evaluator's opportunity cost of funds.

To illustrate how inefficient proportional allocation can be, consider two fields 1 and 2, identical ($\gamma_1 = \gamma_2$ and $f_1 = f_2$) other than for the fact that evaluation is perfect in field 1, $\sigma_1 = 0$, but completely uninformative in field 2, $\sigma_2 = \infty$. The evaluator's value from awarding a grant is equal to the agent's type θ. Suppose the total budget is equal to $T = 1$ and that the opportunity cost for the evaluator is $f \in (1/2 = E[\theta], 1)$ in either field, so that accepting a random applicant gives a negative value, justifying the evaluation process. The optimal policy for the evaluator is: in field 1 accept all applicants with $\theta \geq f$ by setting $\hat{x} = \hat{\theta} = f$ so that the top $1 - f$ agents apply, yielding evaluator surplus $(1-f)^2/2 > 0$; reject all applicants in field 2, which then attracts no application. With budget $T = 1$, the proportional allocation equilibrium in the two fields is $a_1 = 0$ and $a_2 = 1$, yielding evaluator surplus of $(1/2 - f) < 0$. The loss of evaluator surplus from proportional allocation relative to the optimal allocation is $f^2/2$.[30] In this admittedly extreme scenario, proportional allocation is actually the worst possible allocation system.

[30] In the more general case with budget T, the optimal policy yields evaluator surplus $(1-f)^2/2 > 0$ if $T > f$ and $T(1-f-T) + T^2/2$ if $T < f$ in field 1. With proportional allocation, the equilibrium is as in the text unless $T \in [0, \gamma]$ (in which case $a_2 = 0$, yielding evaluator surplus of 0) or $T \in [\gamma, 1]$ (when $a_2 = 1$ and $\alpha = T$, with evaluator surplus $(1/2 - f)T < 0$). The loss of evaluator surplus from proportional allocation is then 0 if $T \in [0, \gamma]$, $T(1-T)/2$ for $T \in [\gamma, f]$, $(1-f)^2/2 - (1/2 - f)T$ for $T \in [f, 1]$, and $f^2/2$ for $T \geq 1$.

More generally, elements of proportionality are welfare improving when fields have different cost-benefit ratios $\gamma_i \neq \gamma_j$. It is natural to wonder whether there are simple modifications of the proportional allocation formula (1) that improve the welfare performance of the resulting equilibrium outcome. While we leave a thorough investigation of this design problem to future work, here we sketch how our model can be easily adapted to attack this key policy question.

Consider the following quasi-proportional generalization of the proportional allocation rule (1)

$$\frac{A_i^\rho}{\sum_{j=1}^N A_j^\rho} T, \tag{12}$$

with $\rho \geq 0$. We verified that when the allocation rule is sub-subproportional decreasing (e.g., for $\rho < 1$) there is no unraveling, but our main comparative statics result holds, so that stable equilibrium allocation increases in dispersion σ. When the allocation is super-proportional ($\rho \geq 1$), unraveling can take place for $\sigma < \underline{\sigma}$, with $\underline{\sigma}$ bounded away from zero, also when (DMRL) holds.

Consider the design problem of choosing the proportionality coefficient ρ to maximize overall social welfare W. For a specification of the model with normally distributed types and signals (see Supplementary Appendix B for details), we verified numerically that when all fields have the same dispersion $\sigma_i = \sigma$ but vary in terms of the cost-benefit ratios $\gamma_i \neq \gamma_j$, it is socially optimal to introduce some element of proportionality $\rho^* > 0$. Intuitively, proportionality allocates more resources to fields with more favorable cost-benefit ratios; this responsiveness is socially beneficial. If, instead, fields have identical $\gamma_i = \gamma$ but differ in terms of $\sigma_i \neq \sigma_j$, fair and unresponsive allocation with $\rho^* = 0$ is optimal. More generally, the optimal level of proportionality increases in the variation of cost-benefit ratio γ but decreases in the variation in dispersion σ across fields.

4 Field Game

We now turn to the perverse incentives that the proportional allocation formula creates for fields. Researchers in a given field face a collective action problem, which they can solve by forming a scientific association that represents their interests. Scientific associations can naturally coordinate field-level outcomes through a number of activities, such as advertising the availability of grants and supporting the submission of applications through seed grant schemes, information sessions, and seminars on grant writing.

Consider the following game between field associations. First, each field association $i = 1, \ldots, N$ simultaneously sets its application rate a_i, thus determining applications $A_i = n_i q_i a_i$. This stage captures the ability of associations to incentivize applications, to advise and assist applicants, as well as to affect

the accuracy of the evaluation process in their panel. Second, according to (1) field association i obtains the fraction

$$p(A_i, A_{-i}) = \min\left\langle \frac{T}{A_i + A_{-i}}, 1 \right\rangle$$

of the overall budget T, where $A_{-i} = \sum_{j \neq i} n_j q_j a_j$ denotes the sum of the applications submitted by the competing fields; thus, this is an aggregative game. By increasing applications, each field association obtains a larger fraction of the overall budget. The incentive to increase applications, however, is curbed by the fact that field associations also take into account the cost of applications. Given total applications A_{-i} in the other fields, field association i's maximizes the overall payoff obtained by all researchers within the field

$$\max_{a_i} v_i p(A_i, A_{-i}) A_i - c_i A_i. \tag{13}$$

We are looking for the Nash equilibrium solution resulting in the first stage, where each field best replies to the equilibrium applications in the other fields.

Given the payoff function induced by the proportional allocation formula, the solution turns out to have a very simple structure. Each field's problem is equivalent to a monopoly problem with inverse (residual) demand function $P_i(A_i, A_{-i}) := v_i p(A_i, A_{-i})$ and with constant marginal cost c_i. The proportional allocation rule actually induces a hyperbolic demand with elasticity

$$\varepsilon_i = \frac{dA_i}{dP_i} \frac{P_i}{A_i} = -1 - \frac{A_i}{A_{-i}}.$$

Given that demand is *always* more than unit elastic, $\varepsilon < -1$, each field has an incentive to increase applications so as to attract more funding to their field out of the fixed available budget for all fields, at the expense of other fields. Thus, the revenue maximizing level of applications is at the corner, $A_i = 1$. Intuitively, if applications were costless, taking as given the sum of applications A_{-i} from the other fields, each field has an incentive to flood the system so as to maximize the funding obtained.

Marshall's second law of demand holds: demand becomes less elastic ($|\varepsilon_i|$ decreases) as demand A_i increases, $\partial \varepsilon_i / \partial A_i > 0$, guaranteeing concavity and uniqueness of the solution. The first-order condition for firm i can be rewritten according to the familiar Lerner formula equating the markup to the inverse of the elasticity

$$\frac{P_i - c_i}{P_i} = \frac{p - \gamma_i}{\gamma_i} = \frac{1}{-\varepsilon},$$

from which we obtain the expression for the best reply

$$A_i = \min\left\langle \sqrt{\frac{A_{-i} T}{\gamma_i}} - A_{-i}, 1 \right\rangle. \tag{14}$$

The best reply is downward sloping. To see why, note that demand becomes more elastic as total demand by competing fields increases, $\partial \varepsilon_i / \partial A_{-i} < 0$, so that an increase in applications in the other fields A_{-i} reduces the optimal A_i chosen by a field. Thus, competition is in strategic substitutes. The best reply is interior $A_i < 1$ for $A_{-i} > \left(T + \sqrt{T^2 - 4T\gamma_i}\right) / (2\gamma_i) - 1$; otherwise, the best reply is $A_i = 1$.

Proposition 7 (Equilibrium in Field Game) *In the interior equilibrium of the field game, applications in field i are*

$$A_i^{(N)} = (N-1)\, T \frac{\sum_{j=1}^{N} \gamma_j - (N-1)\, \gamma_i}{\left(\sum_{j=1}^{N} \gamma_j\right)^2} \tag{15}$$

and the equilibrium success rate (as well as payline) is $p = \left(\sum_{j=1}^{N} \gamma_j\right) / (N-1)$. *If the N fields have identical* $\gamma_i = \gamma$, *the equilibrium surplus in each field is* vT/N^2 *and the total surplus is* vT/N. *In the limit as* $N \to \infty$, *the success rate p converges to* γ *and the surplus of each field as well as the total surplus of all fields converges to zero.*

Because field associations internalize the cost of applications, they do not completely flood the market with applications. However, as the number of fields increases, competition also increases—the entire rent from inframarginal applications is fully dissipated in the limit as the number of fields goes to infinity.

5 Percentiles and Benchmarking: The Organization of Evaluation

We now turn to a variation of the proportional allocation system that is in place at the NIH. Recall that NIH grants are awarded by the Councils of one of 27 NIH institute/centers (such as the National Cancer Institute and the National Institute for Allergies and Infectious Diseases) according to a system of dual review. For the first-level review, applications from different institutes/centers are assigned to one of the some 180 specialized study sections. Study sections evaluate and score applications based on their field-specific scientific expertise. The second-level review is made by the Council of the institute/center based on the merit scores assigned by the study section. A number of the largest institutes/centers fund applications with percentiled score above their payline, set so as to exhaust the budget obtained from Congress through the appropriations process.[31]

However, there is a small tweak in the NIH system compared to the baseline proportional allocation used by the ERC and other research funding organizations. The NIH computes percentiles by pooling

[31]For a list of NIH paylines for the last three years see https://www.einstein.yu.edu/administration/grant-support/nih-paylines.aspx.

scores from the three most recent evaluation cycles. This system was introduced after various attempts to normalize scores so as to make them more easily comparable across study sections.[32] Starting in October 1988, normalization was eventually replaced with percentiling: "all of the NIH funding components will be utilizing percentile values. This action will emphasize the importance of relative rank and provide compensation for widely differing scoring practices that have occurred among IRGs in recent years", as announced by National Institutes of Health (1988).[33] The announcement also explains that percentiles for applications in each evaluation cycle are calculated by pooling the current scores with the scores given by the same study sections to the applications evaluated in the preceding two cycles, a system that is still in operation today.[34] As we argue next, what might look like an inessential detail actually turns out to dampen the force leading to unraveling. Also, we show that benchmarking induces virtuous incentives for accuracy, a countervailing force to the vicious incentive to reduce accuracy we highlighted so far.

Benchmarking Dampens Unraveling. To illustrate the impact of benchmarking on unraveling, consider a field and assume that the distribution of agents is the same across two consecutive evaluation cycles. With benchmarking, the supply equation (8) is replaced by

$$\frac{a}{a+b}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{x-\theta}{\sigma}\right)\right]\frac{g(\theta)}{a}d\theta+\frac{b}{a+b}\int_{G^{-1}(1-b)}^{\bar{\theta}}\left[1-F\left(\frac{x-\theta}{\sigma}\right)\right]\frac{g(\theta)}{b}d\theta=p$$

Once applicants from the current and the past cycle are pooled, the mixture distribution of the signal becomes relevant. The left-hand side is the average of the distribution of signals for applicants from the current cycle (with conditional density $g(\theta)/\left[1-G\left(G^{-1}(1-a)\right)\right]=g(\theta)/a$) and past cycle weighted by their relative sizes.

We now argue that unraveling breaks down once current applications are benchmarked against previous applications: some applications are submitted in the second cycle provided that some applications $b>0$ were submitted in the previous cycle. To see this, note that the intercept of the supply with benchmarking, x_0^{Sb}, is equal to the approval standard that would have resulted if only applications b were submitted, i.e., $\hat{x}^S(b)$. In turn, $\hat{x}^S(b)<x_0^S$ given that the supply is downward sloping; thus, even

[32] See Mandel (1996) for a historical account of the long process that led to the introduction of the payline, percentiling, and benchmarking of applications across study sections at the NIH. As reported by Mandel (1996, pages 164-165), the NIH started normalizing scores in 1971 given that "variations in scoring and success rates among study sections could not be explained in terms of scientific merit criteria alone. . . . To minimize skewing effects when applications from high-scoring and low-scoring study sections were interdigitated . . . transforming raw priority scores on a Gaussian curve . . .".

[33] IRGs stands for Integrated Review Groups, clusters of study sections around a general scientific area.

[34] See https://www.niaid.nih.gov/grants-contracts/understand-paylines-percentiles for a detailed account.

as the dispersion $\sigma \to 0$, the highest type $\overline{\theta}$ has an incentive to apply.[35] More generally, benchmarking dampens the negative impact of information accuracy on the equilibrium incentives to apply.

Benchmarking Rewards Accuracy. Next, we argue that benchmarking can actually reverse the perverse comparative statics of proportional allocation with respect to information accuracy. To this end, consider the impact of benchmarking on the distribution of scores that represent posterior expectations $E[\theta|a,x]$, rather on the distribution of signals that we analyzed so far.[36] The black curve in Figure 10 displays the distribution of the posterior expectation generated by a normal signal with σ_b, corresponding to scores from the previous cycle. Now, pool those scores with the scores resulting in the current cycle generated from a normal signal with lower dispersion $\sigma_a < \sigma_b$ (green distribution). Note that the distribution of posterior expectations $E[\theta|a,x]$ about application quality θ, given the evaluator's noisy signal x, becomes *more* dispersed as the signal becomes *less* dispersed (or more accurate).[37]

The blue distribution is the resulting mixture distribution that is used to determine which applications are above the payline. For a given payline p, represented by the horizontal line, if the current distribution (in green) is more dispersed than the past distribution (in black), the proposals above the payline are disproportionately originating from the current cycle—in addition to a fraction p of applicants, the applications displayed in violet are also accepted. Through this mechanism, by improving its accuracy in this cycle compared to the previous cycle, a panel is able to increase the fraction of successful applications above the payline. Under the reasonable assumption that panel reviewers aim at assigning as many grants as possible to applicants in their study section (possibly at the expense of panels in other fields), they now have an incentive to be more accurate than in the previous cycle, so as to increase dispersion in the posterior expectation and thus increase the number of funded applications in their panel. Through this channel, the NIH method of computing percentiles relative to the applications previously evaluated by the same panel incentivizes accurate evaluation, triggering a virtuous

[35] In the uniform-uniform example, partial equilibrium demand in this round is

$$a^{Pb} = \sigma(p-\gamma) - b + \sqrt{[\sigma(p-\gamma)-b]^2 + b[b+2\sigma(p-\gamma)]}.$$

When evaluation becomes perfectly accurate, the equilibrium application rate converges to $\left(\sqrt{2}-1\right)b > 0$. Note that $\partial^2 a^{Pb}/\partial\sigma\partial b < 0$ so that benchmarking dampens the positive impact of dispersion equilibrium demand.

[36] So far we represented the acceptance standard in term of the signal x, rather than the posterior expectation $E[\theta|a,x]$ about the application merit θ. The two approaches are equivalent when all applicants are evaluated with a common signal structure F and homogeneous dispersion σ, given that $E[\theta|a,x]$ is increasing in x by logconcavity of f (equivalent to the monotone likelihood ratio property in our setting with location signal; see footnotes 14 and 18).

[37] To confirm the intuition, in the limit as signal dispersion $\sigma \to \infty$, the distribution of the posterior expectation becomes a step function at the prior $E[\theta]$.

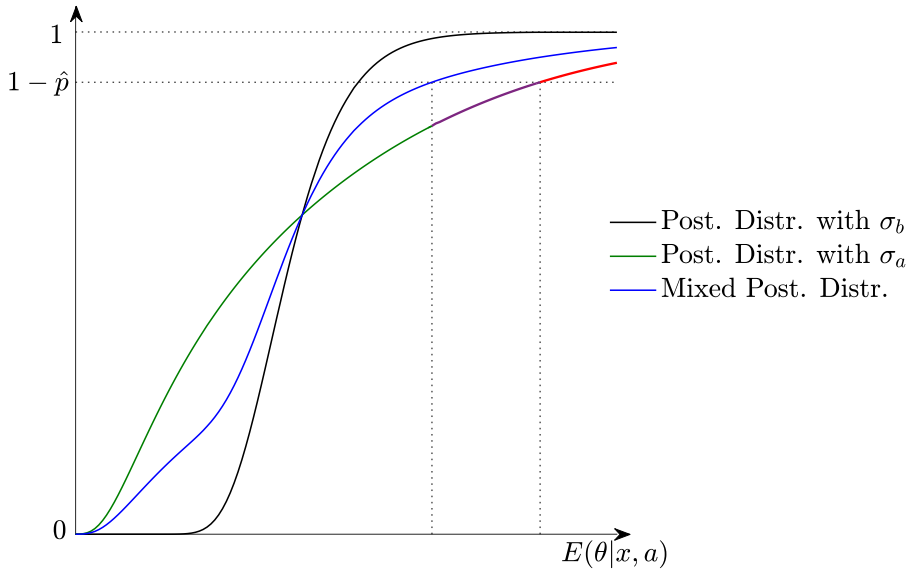

Figure 10: Impact of improvement in accuracy relative to previous round.

cycle.

Panel Organizational Design and Within-Field Heterogeneity. The same logic we highlighted in the previous paragraph also explains the impact of a merger between heterogeneous fields. Consider two fields characterized by signal dispersions $\sigma_1 > \sigma_2$, but otherwise identical. If each field had its own panel, the same fraction of applications would be funded in the two fields, though field 2 would attract less applications than field 1. What would result if the two fields were pooled into a single panel, while retaining the evaluation by reviewers specialized in each field? Fix for now the application levels in the two fields. Given that the distribution of scores of applications from field 1 (with more dispersed signal or less consensual evaluation, corresponding to the black curve in Figure 10) is less dispersed than field-2 applications (green curve), field 1 suffers relative to field 2. This pattern is compatible with Martin, Lindquist, and Kotchen's (2008) finding that clinical research has lower success rate relative to basic research at the NIH—when clinical research is pooled with basic research it suffers from being less consensual. Less consensual fields thus have a strong incentive to separate and lobby to have their own panel—once separated, not only the fraction of accepted applications will increase, but also application incentives will improve, setting in place a spiral with an increase in applications and in acceptances. When, instead, a more consensual field remains isolated with its own panel, it will experience dwindling applications.

6 Conclusion

The mechanism that leads to unraveling in our model—with no applications being submitted in equilibrium for fields with perfect evaluation—is somewhat reminiscent of Akerlof's (1970) market for lemons. However, in our setting unraveling leads to breakdown of applications in fields where information is symmetric, rather than asymmetric as in Akerlof. Agents who are able to predict how they will be evaluated prefer to hold out and save the application cost, unless they are confident of being accepted. Fields with accurate evaluation are driven out by fields with noisier evaluation. Proportional allocation creates perverse incentives for fields with asymmetric information to thrive.

Even at research funding agencies that do adopt proportional allocation, success rates across fields are closely monitored. Even when differences in success rates across fields in non-proportional systems persist over time, there is an implicit pressure to reduce the budget for fields with higher success rates in favor of fields with lower success rates.

Beyond research funding, our analysis of proportional allocation is relevant for large research fellowships programs, such as the EU-wide Marie Skłodowska-Curie Action (MSCA) scheme (with a total budget of 6.16 billion euros for the period 2014-2020, assigned in proportion to applications across all university fields) and doctoral fellowship programs in Canada.[38] The drawbacks we highlighted are particularly severe for mechanisms that equalize the success rate among very heterogeneous fields—as it is the case for the ERC and MSCA—but we expect it to be somewhat less problematic at the NIH, which focuses on medical research, even though life sciences are far from homogeneous. It is only understandable that some institutes/centers at the NIH prefer not to publish paylines, thus retaining the flexibility of setting different paylines for proposals from different fields.

General-interest academic journals are often subject to a similar pressure to allocate space to different subfields in proportion to submissions. When co-editors are given a common target acceptance rate, fields with less accurate evaluation will attract more submissions. Similarly, university admission boards are tempted to admit students to different programs in proportion to applications—or to increase slots available in areas that attract more applications. Giving in to this temptation leads to a race to the bottom in terms of quality of admitted students.

[38] Such as the SSHRC Doctoral Fellowships program covering all humanities and social sciences.

References

Akerlof, George A. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics*, 84(3): 488–500.

Alonso, Ricardo. 2018. "Recruiting and Selecting for Fit." Working Paper, London School of Economics.

Arrow, Kenneth J. 1962. "Economic Welfare and the Allocation of Resources for Invention." In: *The Rate and Direction of Inventive Activity: Economic and Social Factors* (Richard R. Nelson, ed.), Princeton: Princeton University Press.

Azoulay, Pierre, Joshua S. Graff Zivin, Danielle Li, Bhaven N. Sampat. 2019. "Public R&D Investments and Private-sector Patenting: Evidence from NIH Funding Rules." *Review of Economic Studies*, 86(1): 117–152.

Azzalini, Adelchi, and Antonella Capitanio. 2014. *The Skew Normal and Related Distributions*. Cambridge: Cambridge University Press.

Bagnoli, Mark, and Ted Bergstrom. 2005. "Log-concave Probability and its Applications." *Economic Theory*, 26(2), 445–469.

Bradley, David M., and Ramesh C. Gupta. 2003. "Limiting Behaviour of the Mean Residual Life." *Annals of the Institute of Statistical Mathematics*, 55(1): 217–226.

Calabria, Raffaela, and Gianpaolo Pulcini. 1987. "On the Asymptotic Behaviour of the Mean Residual Life Function." *Reliability Engineering*, 19(3): 165–170.

Che, Yeon-Koo, and Ian Gale. 2003. "Optimal Design of Research Contests." *American Economic Review*, 93(3): 646–671.

Che, Yeon-Koo, Wouter Dessein, and Navin Kartik. 2013. "Pandering to Persuade." *American Economic Review*, 103(1): 47–79.

Cole, Jonathan, and Stephen Cole. 1981. *Peer Review in the National Science Foundation: Phase Two of a Study*. National Academy Press.

Cotton, Christopher. 2013. "Submission Fees and Response Times in Academic Publishing." *American Economic Review*, 103(1): 501–509.

Cowen, Tyler. 2009. *Good and Plenty: The Creative Successes of American Arts Funding*. Princeton University Press.

Davis, Otto A., Michael A. H. Dempster, and Aaron Wildavsky. 1966. "A Theory of the Budgetary Process." *American Political Science Review*, 60(3): 529–547.

European Commission. 2007. *ERC Work Programme 2008*. European Commission C (2007) 5746, 29 November 2007.

Frankel, Alexander. 2018. "Selecting Applicants." Working Paper, Chicago Booth School of Business.

Gans, Joshua S., and Fiona E. Murray. 2012. "Funding Scientific Knowledge: Selection Disclosure and the Public-Private Portfolio." Chapter 1 in: *The Rate and Direction of Inventive Activity Revisited* (Joshua Lerner and Scott Stern, eds.), National Bureau of Economic Research, Cambridge, MA.

Johnson, Justin P. and David P. Myatt. 2006. "On the Simple Economics of Advertising, Marketing, and Product Design." *American Economic Review*, 96(3): 756–784.

Lazear, Edward P. 1997. "Incentives in Basic Research." *Journal of Labor Economics*, 15(1), S167-S197.

Lehmann, Erich L. 1988. "Comparing Location Experiments." *Annals of Statistics*, 16(2): 521–533.

Lehmann, Erich L., and Joseph P. Romano. 2005. *Testing Statistical Hypotheses*. Third Edition, Springer.

Leslie, Derek. 2005. "Are Delays in Academic Publishing Necessary?" *American Economic Review*, 95(1): 407-413.

Li, Danielle and Leila Agha. 2015. "Big Names or Big Ideas: Do Peer-Review Panels Select the Best Science Proposals?" *Science*, 348(6233): 434–438.

Mandel, Richard. 1996. *Half a Century of Peer Review, 1946-1996*. Division of Research Grants, National Institutes of Health.

Martin, Michael R., Teresa Lindquist, and Theodore A. Kotchen. 2008. "Why Are Peer Review Outcomes Less Favorable for Clinical Science than for Basic Science Grant Applications?" *American Journal of Medicine*, 121(7): 637–641.

Moldovanu, Benny and Aner Sela. 2001. "The Optimal Allocation of Prizes in Contests" *American Economic Review*, 91(3), 542–558.

Morgan, John, Dana Sisak, and Felix Várdy. 2018. "The Ponds Dilemma." *Economic Journal*, 128(611): 1634–1682.

Nair, N. Unnikrishnan, P.G. Sankaran, and N. Balakrishnan. 2010. *Quantile-Based Reliability Analysis*. Birkhäuser Springer.

National Institutes of Health. 1988. *Guide for Grants and Contracts. Notice: New Basis for Priority Score Percentiles at NIH*, http://grants.nih.gov/grants/guide/historical/1988_08_12_Vol_17_No_26.pdf.

Peirce, Charles S. 1879. "Note on the Theory of the Economy of Research," *United States Coast Survey*, reprinted in 1967, *Operations Research*, 15(4): 643–648.

Persico, Nicola. 2000. "Information Acquisition in Auctions," *Econometrica*, 68(1), 135–148.

Savage, James D. 1999. *Funding Science in America: Congress, Universities, and the Politics of the Academic Pork Barrel*. Cambridge University Press.

Scotchmer, Suzanne. 2004. *Innovation and Incentives*. MIT Press.

Siegel, Ron. 2009. "All-Pay Contests." *Econometrica*, 77(1), 71–92.

Stephan, Paula. 2012. *How Economics Shapes Science*. Harvard University Press.

Taylor, Curtis R. and Huseyin Yildirim. 2011. "Subjective Performance and the Value of Blind Evaluation." *Review of Economic Studies*, 78(2): 762–794.

Weinstein, Milton and Richard Zeckhauser. 1973. "Critical Ratios and Efficient Allocation." *Journal of Public Economics*, 2(2): 147–157.

Weisbrod, Burton. 1961. *Economics of Public Health*. University of Pennsylvania Press.

Wildavsky, Aaron. 1964. *The Politics of the Budgetary Process*. Little, Brown and Company.

Zeckhauser, Richard. 1967. *Some Thoughts on the Allocation of Resources to BioMedical Research*. Occasional Paper No. 4. U.S. Department of Health, Education and Welfare: Office of Planning and Evaluation.

Zuckerman, Harriet, and Robert K. Merton. 1971. "Patterns of Evaluation in Science: Institutionalisation, Structure and Functions of the Referee System." *Minerva*, 9(1): 66-100.

7 Appendix: Proofs

Proof of Proposition 1. Differentiating the demand equation

$$D(x,a;\sigma,\gamma) = 1 - F\left(\frac{x - G^{-1}(1-a)}{\sigma}\right) - \gamma = 0 \tag{16}$$

gives

$$\begin{aligned} D_x &= -\tfrac{1}{\sigma} f\left(\tfrac{x-G^{-1}(1-a)}{\sigma}\right) \quad & D_a &= -\tfrac{1}{\sigma} f\left(\tfrac{x-G^{-1}(1-a)}{\sigma}\right) \tfrac{1}{g\left(G^{-1}(1-a)\right)} \\ D_\gamma &= -1 & D_\sigma &= \tfrac{\hat{x}-G^{-1}(1-a)}{\sigma^2} f\left(\tfrac{\hat{x}-G^{-1}(1-a)}{\sigma}\right). \end{aligned} \tag{17}$$

Part (a) follows from $da^D/dx = -D_x/D_a = -g\left(G^{-1}(1-a)\right) < 0$; similarly, part (b) from $da^D/d\gamma = 1/D_a < 0$. For part (c), $da^D/dx = -D_\sigma/D_a \gtreqless 0$ if and only if the marginal applicant is below or above the acceptance standard on the demand curve, $G^{-1}(1-a) = \hat{\theta} \lesseqgtr \hat{x}$. From (4) this holds whenever $F^{-1}(1-\gamma) \gtreqless 0 \Leftrightarrow 1-\gamma \gtreqless F(0)$. If the distribution F is symmetric, $F(0) = 1/2$, we conclude that $da^D/d\sigma \gtreqless 0 \Leftrightarrow \gamma \lesseqgtr 1/2$.

Proof of Proposition 2. Differentiating the supply equation

$$S(x,a;\sigma,p) = \frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1 - F\left(\frac{x-\theta}{\sigma}\right)\right] g(\theta)\, d\theta}{a} - p = 0, \tag{18}$$

we have

$$\begin{aligned} S_x &= -\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta) d\theta}{a} \quad & S_a &= \tfrac{1}{a}\left[1 - F\left(\tfrac{x-G^{-1}(1-a)}{\sigma}\right) - \frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1-F\left(\frac{x-\theta}{\sigma}\right)\right] g(\theta) d\theta}{a}\right] \\ S_p &= -1 & S_\sigma &= \frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \frac{x-\theta}{\sigma^2} g(\theta) d\theta}{a} \end{aligned} \tag{19}$$

where $S_x < 0$, $S_p < 0$, and $S_a < 0$ because along the supply the acceptance probability of the marginal type $G^{-1}(1-a)$ must be below the average success rate p. Part (a) and (b) follow.

Proof of Proposition 3. (a) The supply curve is flatter than the inverse demand

$$\frac{d\hat{x}^S}{da} = -\frac{S_a}{S_x} > -\frac{D_a}{D_x} = \frac{1}{da^D/d\hat{x}} \Leftrightarrow \begin{vmatrix} D_x & D_a \\ S_x & S_a \end{vmatrix} < 0 \tag{20}$$

Substituting from (17) and (19), we have

$$\begin{aligned} &\frac{\begin{vmatrix} D_x & D_a \\ S_x & S_a \end{vmatrix}}{-\frac{1}{a}\frac{1}{\sigma} f\left(\frac{x-G^{-1}(1-a)}{\sigma}\right)} \\ = \;& 1 - F\left(\frac{x - G^{-1}(1-a)}{\sigma}\right) - \frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1 - F\left(\frac{x-\theta}{\sigma}\right)\right] g(\theta)\, d\theta}{a} + \frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta)\, d\theta}{g\left(G^{-1}(1-a)\right)} \end{aligned} \tag{21}$$

Integration by parts gives

$$\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta)\, d\theta}{g(G^{-1}(1-a))} = \frac{-F\left(\frac{x-\theta}{\sigma}\right) g(\theta)\Big|_{G^{-1}(1-a)}^{\bar{\theta}} + \int_{G^{-1}(1-a)}^{\bar{\theta}} F\left(\frac{x-\theta}{\sigma}\right) g'(\theta)\, d\theta}{g(G^{-1}(1-a))}$$

$$= -\frac{g\left(\bar{\theta}\right) F\left(\frac{x-1}{\sigma}\right)}{g(G^{-1}(1-a))} + F\left(\frac{x-G^{-1}(1-a)}{\sigma}\right) + \int_{G^{-1}(1-a)}^{\bar{\theta}} F\left(\frac{x-\theta}{\sigma}\right) \frac{g'(\theta)}{g(G^{-1}(1-a))} d\theta$$

$$= F\left(\frac{x-G^{-1}(1-a)}{\sigma}\right) - F\left(\frac{x-\bar{\theta}}{\sigma}\right) - \int_{G^{-1}(1-a)}^{\bar{\theta}} \left[F\left(\frac{x-\bar{\theta}}{\sigma}\right) - F\left(\frac{x-\theta}{\sigma}\right)\right] \frac{g'(\theta)}{g(G^{-1}(1-a))} d\theta,$$

where the last line used $g\left(\bar{\theta}\right) = \int_{G^{-1}(1-a)}^{\bar{\theta}} g'(\theta)\, d\theta + g\left(G^{-1}(1-a)\right)$. Substituting this last equation into (21) and using $\int_{G^{-1}(1-a)}^{\bar{\theta}} g(\theta)\, d\theta = a$, we conclude that

$$\begin{vmatrix} D_x & D_a \\ S_x & S_a \end{vmatrix} \lesseqgtr 0 \Leftrightarrow \int_{G^{-1}(1-a)}^{\bar{\theta}} \left[F\left(\frac{x-\theta}{\sigma}\right) - F\left(\frac{x-\bar{\theta}}{\sigma}\right)\right] \left(\frac{g(\theta)}{a} + \frac{g'(\theta)}{g(G^{-1}(1-a))}\right) d\theta \gtreqless 0. \quad (22)$$

Using the definition $a = 1 - G\left(\hat{\theta}\right)$, we conclude that the equilibrium is unique and stable if and only if

$$\int_{\theta}^{\bar{\theta}} \left[F\left(\frac{x-t}{\sigma}\right) - F\left(\frac{x-\bar{\theta}}{\sigma}\right)\right] \left(\frac{g(t)}{1-G(\theta)} + \frac{g'(t)}{g(\theta)}\right) dt > 0. \quad (23)$$

Integrating by parts and simplifying, this is equivalent to

$$-\left[F\left(\frac{x-\theta}{\sigma}\right) - F\left(\frac{x-\bar{\theta}}{\sigma}\right)\right] \left(\frac{G(\theta)}{1-G(\theta)} + \frac{g(\theta)}{g(\theta)}\right) + \int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right) \left(\frac{G(t)}{1-G(\theta)} + \frac{g(t)}{g(\theta)}\right) dt > 0$$

Collecting terms, we can rewrite characterization (23) for uniqueness and stability of equilibrium as

$$\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-t}{\sigma}\right) \left(\frac{g(t)}{g(\theta)} - \frac{1-G(t)}{1-G(\theta)}\right) dt > 0 \quad (24)$$

or equivalently as (KEY).

(b) The conditions for the equilibrium to be interior are explained in the text below the statement of the proposition.

(c) Sufficient condition (S1:ID) follows from characterization (23), given that the term in brackets is positive—if the density g is increasing, (23) holds. Sufficient condition (S2:IHR) follows from characterization (24)—if the hazard rate $g/(1-G)$ is increasing, (24) holds.

Proof of Proposition 4. (i) Applying the implicit function theorem to the system (16) and (18) gives

$$\frac{da}{d\sigma} = -\frac{\begin{vmatrix} D_x & D_\sigma \\ S_x & S_\sigma \end{vmatrix}}{\begin{vmatrix} D_x & D_a \\ S_x & S_a \end{vmatrix}} \quad (25)$$

We already characterized the sign of the determinant at the denominator in the proof of Proposition 4. From (17) and (19), the determinant at the numerator of (25) is equal to

$$\begin{aligned} J_{[a]\sigma} &:= \begin{vmatrix} D_x & D_\sigma \\ S_x & S_\sigma \end{vmatrix} = -\frac{1}{a\sigma} f\left(\frac{x-G^{-1}(1-a)}{\sigma}\right) \\ &\left[\int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \frac{x-\theta}{\sigma} g(\theta)\, d\theta - \frac{x-G^{-1}(1-a)}{\sigma} \int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta)\, d\theta\right] \\ = \; & -\frac{1}{a\sigma} f\left(\frac{x-G^{-1}(1-a)}{\sigma}\right) \int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{G^{-1}(1-a)-\theta}{\sigma} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta)\, d\theta > 0. \end{aligned}$$

Combining this inequality with (22), from (25) we conclude that equilibrium applications increase (decrease) in dispersion for all stable (unstable) equilibria, i.e., depending on (22).

Proof of Proposition 5. (a) Following the same steps as in the proof of Proposition 4, from

$$\begin{vmatrix} D_x & D_p \\ S_x & S_p \end{vmatrix} = \frac{1}{\sigma} f\left(\frac{x-G^{-1}(1-a)}{\sigma}\right) > 0$$

we conclude that in any stable (or unstable) partial equilibrium $da_i^p/dp \geq 0$, i.e., that applications increase (or decrease) in the success rate p, strictly so when the equilibrium is interior.

(b) Note that

$$J^i = \begin{vmatrix} \frac{\partial D_i}{\partial x_i} & \frac{\partial D_i}{\partial a_i} \\ \frac{\partial S_i}{\partial x_i} & \frac{\partial S_i}{\partial a_i} - \frac{\partial p}{\partial a} \end{vmatrix} = J^{ip} - \frac{\partial D_i}{\partial x_i}\frac{\partial p}{\partial a} < J^{ip}, \tag{26}$$

where the inequality follows from $\partial D_i/\partial x_i < 0$ and $\partial p/\partial a < 0$. If (KEY) holds, we have $J^{ip} < 0$, which in combination with (26), implies that $J^i < 0$. Thus, the partial equilibrium with responsive payline (or, equivalently, the full equilibrium with a single field) is unique and stable.

(c) As shown in part (b), every partial equilibrium that is stable (i.e., with $J^{ip} < 0$) is also stable when the adverse response of the payline is taken into account (given that $J^i < J^{ip} < 0$). Applying the implicit function theorem to the demand and supply systems with fixed and responsive payline, we conclude that responsiveness of the payline dampens the positive impact of dispersion on the level of equilibrium applications,

$$-\frac{J_{[a_i]\sigma_i}}{J^{ip}} = \frac{da_i^p}{d\sigma_i} > \frac{da_i^i}{d\sigma_i} = -\frac{J_{[a_i]\sigma_i}}{J^i} > 0.$$

Proof of Proposition 6. (a) If $T = 0$ there is unique equilibrium at the corner $p = a_i = 0$ for all i. For $T > 0$, there is an equilibrium with $a_i > 0$ for some i. Condition (KEY) guarantees that any given p determines a unique vector of field-level application rates $a_1, a_2, ..., a_N$; given that the right hand side of (2) is decreasing in p, the overall equilibrium is unique. Turning to stability, recall that the full

equilibrium solves the system of $2N$ demand and supply equations obtained by replacing the budget (2) into the supply equations. The determinant of the Jacobian of this system

$$J^{1,\ldots,N} := \begin{vmatrix} \frac{\partial D_1}{\partial x_1} & \frac{\partial D_1}{\partial a_1} & 0 & 0 & 0 & 0 \\ \frac{\partial S_1}{\partial x_1} & \frac{\partial S_1^p}{\partial a_1} - \frac{\partial p}{\partial a} & 0 & -\frac{\partial p}{\partial a} & 0 & -\frac{\partial p}{\partial a} \\ & & \ddots & & & \\ 0 & 0 & \frac{\partial D_i}{\partial x_i} & \frac{\partial D_i}{\partial a_i} & 0 & 0 \\ 0 & -\frac{\partial p}{\partial a} & \frac{\partial S_i}{\partial x_i} & \frac{\partial S_i^p}{\partial a_i} - \frac{\partial p}{\partial a} & 0 & -\frac{\partial p}{\partial a} \\ & & & & \ddots & \\ 0 & 0 & 0 & 0 & \frac{\partial D_N}{\partial x_N} & \frac{\partial D_N}{\partial a_N} \\ 0 & -\frac{\partial p}{\partial a} & 0 & -\frac{\partial p}{\partial a} & \frac{\partial S_N}{\partial x_N} & \frac{\partial S_N^p}{\partial a_N} - \frac{\partial p}{\partial a} \end{vmatrix}$$

is equal to

$$J^{1,\ldots,N} = \prod_{i=1}^{N} \underbrace{J^{ip}}_{-} - \sum_{i=1}^{N} \underbrace{\overbrace{\frac{\partial p}{\partial a}}^{-} \overbrace{\frac{\partial D_i}{\partial x_i}}^{-}}_{+} \prod_{j \neq i} \underbrace{J^{jp}}_{-}. \tag{27}$$

Note that if the partial equilibrium with fixed payline in each field i is stable, $J^{ip} < 0$, as guaranteed by (KEY), for all $i = 1, \ldots, N$, this determinant $J^{1,\ldots,N}$ has a negative sign when the number N of markets is odd and a positive sign for N even. Thus, the full equilibrium is stable.

(b) Turn to the comparative statics of the full equilibrium. (i) Given that $J^{1,\ldots,N}$ and $J^{1,\ldots,N \setminus i}$ have opposite sign, we obtain that for any locally stable selection of the partial equilibrium, full equilibrium demand in any field i increases in the dispersion of the evaluation in that field

$$\frac{da_i^{1,\ldots,N}}{d\sigma_i} = -\frac{\overbrace{J_{[a_i]\sigma_i}}^{+} J^{1,\ldots,N \setminus i}}{J^{1,\ldots,N}} > 0,$$

given that $J_{[a_i]\sigma_i} > 0$, as shown in Proposition 4. (ii) For any locally stable selection of the partial equilibrium, full equilibrium demand in any field i decreases in the dispersion of the evaluation in any other field j

$$\frac{da_i^{1,\ldots,N}}{d\sigma_j} = -\frac{\overbrace{J_{[a_j]\sigma_j}}^{+} \overbrace{\frac{\partial p}{\partial a} \frac{\partial D_i}{\partial x_i}}^{+} \prod_{k \neq i,j} J^{ip}}{J^{1,\ldots,N}} < 0,$$

given that the sign of $\prod_{k \neq i,j} J^{ip}$ is the same as the sign of $J^{1,\ldots,N}$. The comparative statics for unstable equilibria is reversed.

(c) Having established the last inequality in part (b) we now obtain inequalities (i) and (ii) in

$$\frac{da_i^p}{d\sigma_i} = -\frac{J_{[a_i]\sigma_i}}{J^{ip}} \underset{(i)}{>} \frac{da_i^{1,\ldots,N}}{d\sigma_i} = -\frac{J_{[a_i]\sigma_i} J^{1,\ldots,N\setminus i}}{J^{1,\ldots,N}} \underset{(ii)}{>} \frac{da_i^r}{d\sigma_i} = -\frac{J_{[a_i]\sigma_i}}{J^i} > 0.$$

To prove (i), note that from (27) for fields $1,\ldots,N\setminus i$ we have

$$J^{ip} J^{1,\ldots,N\setminus i} = \prod_{i=1}^N J^{ip} - \frac{\partial p}{\partial a} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j} J^{kp}.$$

Combining this equation with (27) for fields $1,\ldots,N$ we obtain

$$J^{1,\ldots,N} = J^{ip} J^{1,\ldots,N\setminus i} - \frac{\partial p}{\partial a} \frac{\partial D_i}{\partial x_i} \prod_{j\neq i} J^{jp},$$

so that condition (i) is equivalent to

$$\frac{J^{ip} J^{1,\ldots,N\setminus i}}{J^{1,\ldots,N}} = \frac{J^{1,\ldots,N} + \frac{\partial p}{\partial a} \frac{\partial D_i}{\partial x_i} \prod_{j\neq i} J^{jp}}{J^{1,\ldots,N}} < 1,$$

which clearly holds given that $\prod_{j\neq i} J^{jp}$ and $J^{1,\ldots,N}$ have opposite sign for every N.

To establish (ii), note that

$$\begin{aligned} J^i J^{1,\ldots,N\setminus i} &= \prod_{i=1}^N J^{ip} - \frac{\partial p}{\partial a} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j} J^{kp} \\ &\quad - \frac{\partial p}{\partial a} \frac{\partial D_i}{\partial x_i} \prod_{j\neq i} J^{jp} + \left(\frac{\partial p}{\partial a}\right)^2 \frac{\partial D_i}{\partial x_i} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j,i} J^{kp} \\ &= \prod_{i=1}^N J^{ip} - \frac{\partial p}{\partial a} \sum_{i=1}^N \frac{\partial D_i}{\partial x_i} \prod_{k\neq i} J^{kp} + \left(\frac{\partial p}{\partial a}\right)^2 \frac{\partial D_i}{\partial x_i} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j,i} J^{kp}, \end{aligned}$$

so that

$$J^i J^{1,\ldots,N\setminus i} = J^{1,\ldots,N} + \left(\frac{\partial p}{\partial a}\right)^2 \frac{\partial D_i}{\partial x_i} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j,i} J^{kp}.$$

Substituting into condition (ii), this is equivalent to

$$\frac{J^i J^{1,\ldots,N\setminus i}}{J^{1,\ldots,N}} = \frac{J^{1,\ldots,N} + \left(\frac{\partial p}{\partial a}\right)^2 \frac{\partial D_i}{\partial x_i} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j,i} J^{kp}}{J^{1,\ldots,N}} > 1,$$

which always holds given that $\left(\frac{\partial p}{\partial a}\right)^2 \frac{\partial D_i}{\partial x_i} \sum_{j\neq i} \frac{\partial D_j}{\partial x_j} \prod_{k\neq j,i} J^{kp}$ and $J^{1,\ldots,N}$ have opposite sign for every N.

Proof of Proposition 7. The equilibrium (15) is found by solving simultaneously the N best replies (14). To verify, from (15) we have

$$A_{-i} = \sum_{j \neq i} A_j = \frac{(N-1)^2 T \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2},$$

which substituted into the best reply (14) gives

$$A_i = \sqrt{\frac{A_{-i} T}{\gamma}} - A_{-i} = \frac{(N-1) T}{\sum_{j=1}^N \gamma_j} - \frac{(N-1)^2 T \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2} = (N-1) T \frac{\sum_{j=1}^N \gamma_j - (N-1) \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2},$$

as desired. Total applications are

$$\sum_{j=1}^N a_j^{(N)} = (N-1) T \frac{\sum_{j=1}^N \gamma_j - (N-1) \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2} + \frac{(N-1)^2 T \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2} = \frac{(N-1) T}{\sum_{j=1}^N \gamma_j},$$

so that the equilibrium success rate is

$$p = \frac{\sum_{j=1}^N \gamma_j}{N-1}.$$

Substituting into (13), the equilibrium surplus in field i is

$$\begin{aligned} & v_i \left(\frac{\sum_{j=1}^N \gamma_j}{N-1} \right) \left((N-1) T \frac{\sum_{j=1}^N \gamma_j - (N-1) \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2} \right) - c_i \left((N-1) T \frac{\sum_{j=1}^N \gamma_j - (N-1) \gamma_i}{\left(\sum_{j=1}^N \gamma_j\right)^2} \right) \\ = & \left(v_i - \frac{N-1}{\sum_{j=1}^N \gamma_j} c_i \right) T \frac{\sum_{j=1}^N \gamma_j - (N-1) \gamma_i}{\sum_{j=1}^N \gamma_j}. \end{aligned}$$

Under symmetry ($\gamma_i = \gamma$), we have demand at each field $a^{(N)} = T(N-1) / \left(\gamma N^2\right)$, total demand $N a^{(N)} = T(N-1) / (\gamma N)$, and success rate $p = N\gamma / (N-1)$. The equilibrium surplus in each field is then vT/N^2 and total surplus vT/N, both converging to zero as $N \to \infty$.

PARAMETERS	
Figures 1 and 2	$\gamma = 1/5$, F normal, $\sigma = 0.4$, $G(\theta) = \sqrt{\theta}$ $\theta = 1$ (blue), $\theta = 1 - a^D(\hat{x}) \tilde{=} 0.2$ (gray), $\theta = 1 - a^D\left(x_1^D\right) = 0$ (light gray)
Figure 3	$p = 1/4$, F uniform, $\sigma = 1$, $G(\theta) = \theta$
Figure 4	$\gamma = 1/8$, $p = 1/4$, F, G uniform, $\sigma = 1$
Figure 5	$\gamma = 1/5$, $p = 1/4$, F uniform, $\sigma = 7.2$, $\overline{\sigma} = 7.5$, $\underline{\sigma} \approx 6.67$, $G(\theta) = \sqrt{\theta}$
Figure 6	$\gamma = 1/7$, $p = 1/4$, F normal, G uniform θ, $\sigma = 0.6$ (black), $\sigma = 0.25$ (purple)
Figure 7	2 markets, $\gamma_1 = 1/5$, $\gamma_2 = 1/4$, $\lambda = 1/4$, F_1, F_2, G_1, G_2 uniform, $T = .3$
Figure 8 and 9	12 markets, $\sigma_1 = 8, \gamma_1 = 1/5$, $\gamma_j = 1/5, \underline{\sigma}_j \tilde{=} 6.9$, $\sigma_j = 7.2, \sigma_j = 7.5$, for $j = 2, .., 12$, F_i, G_i uniform for $i = 1, .., 12$, $T = 2.55$
Figure 10	$\mu = 2$, $\kappa = 1, a = .3$, F, G normal, $\sigma_a = .3$, $\sigma_b = 2$

8 Supplementary Material: Analytical Examples in Figures

This appendix presents some background results for the analysis of the examples of the information structure used to illustrate the working of the model in the figures.

Uniform Signal Example. Suppose that the signal $x \sim U[\theta - \sigma/2, \theta + \sigma/2]$ follows a uniform distribution centered around θ of length σ, so that $F_{\theta,\sigma}(x) = 1/2 + (x-\theta)/\sigma$. This example is particularly tractable and allows us to obtain closed-form expressions for demand, supply, and equilibria when the type distribution is uniform (a special case of the beta distribution considered below) and square root (a special case of the Haupt and Schäbe distribution considered below). Substituting $F(x) = 1/2 + x$ into the supply equation (8) we obtain

$$\frac{\left(\frac{1}{2} - \frac{\hat{x}}{\sigma}\right)\int_{G^{-1}(1-a)}^{\bar{\theta}} g(\theta)\,d\theta + \frac{1}{\sigma}\int_{G^{-1}(1-a)}^{\bar{\theta}} \theta g(\theta)\,d\theta}{a} = p.$$

Integrating by parts

$$\int_{G^{-1}(1-a)}^{\bar{\theta}} \theta g(\theta)\,d\theta = aG^{-1}(1-a) - \int_{G^{-1}(1-a)}^{\bar{\theta}} [1 - G(\theta)]\,d\theta,$$

and solving we find the following explicit expression for the supply curve

$$\hat{x}^S(a) = \sigma\left(\frac{1}{2} - p\right) + G^{-1}(1-a) + \int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1 - G(\theta)}{a} d\theta,$$

where the last addend is equal to $E\left[\theta - G^{-1}(1-a) \,|\, \theta \geq G^{-1}(1-a)\right]$. Comparing with (6), with uniform signal the supply has a similar structure to the inverse demand

$$\hat{x}^D(a) = \sigma\left(\frac{1}{2} - \gamma\right) + G^{-1}(1-a),$$

with two key differences: (i) instead of the cost-benefit ratio γ the supply features the success rate p in the intercept and (ii) the marginal type $G^{-1}(1-a) = \hat{\theta}$ on the demand side is replaced by the average inframarginal type on the supply

$$E\left[\theta | \theta \geq G^{-1}(1-a)\right] = G^{-1}(1-a) + \int_{G^{-1}(1-a)}^{\bar{\theta}} \frac{1 - G(\theta)}{a} d\theta = \int_{\hat{\theta}(a)}^{\bar{\theta}} \theta \frac{g(\theta)}{1 - G\left(\hat{\theta}(a)\right)} d\theta.$$

Beta (with $\alpha = 1$) and Uniform Types Example. We now derive the closed-form expressions for the unique interior equilibrium resulting with uniform signal and type distribution $G(\theta) = 1 - (1-\theta)^{\beta}$,

corresponding with a Beta with parameters $\alpha = 1$ and general β; the special case with $\beta = 1$ corresponds to uniformly distributed types, illustrated in Figure 4. This example satisfies the DMRL condition. Setting the average winning probability among applicants equal to p, the winning probability of the highest type $\theta = 1$ is $2p - \gamma$. The equilibrium conditions for demand (5) and supply (8) boil down to

$$\begin{aligned} a^D(\hat{x}) &= \left[1 - \hat{x} + \sigma\left(\frac{1}{2} - \gamma\right)\right]^{\beta} \\ \hat{x}^S(a) &= 1 + \sigma\left(\frac{1}{2} - p\right) - \frac{\beta}{\beta+1} a^{\frac{1}{\beta}}, \end{aligned}$$

resulting in the unique interior equilibrium

$$\begin{aligned} \hat{x} &= 1 + \sigma\left[\frac{1}{2} + \beta\gamma - (1+\beta)\, p\right] \\ a &= [(1+\beta)\,\sigma\,(p-\gamma)]^{\beta}. \end{aligned}$$

for $p \in (\gamma, \overline{p} = 1/(1+\beta) + \gamma)$. For $p \geq \overline{p}$, the unique equilibrium is at the corner $a = 1$ with $\hat{x} = 1/(1+\beta) + \sigma(1/2 - p)$. Equilibrium demand a always increases in σ, with $\lim_{\sigma \to 0} a = 0$ and $\lim_{\sigma \to \infty} a = 1$, where we reach the corner solution with no demand $x = 1, a = 0$ at the boundary $\sigma = 0$ and the corner solution with demand by all types for a bounded level of dispersion, $\overline{\sigma} = 1/[(1+\beta)(p-\gamma)]$.

Lemma 1 *If types are uniformly distributed, the acceptance standard on the supply curve*
(i) *increases in evaluator signal dispersion* $d\hat{x}^S/d\sigma \geq 0$ *in a* tough contest *with* $p \leq 1/2$*;*
(ii) *decreases* $d\hat{x}^S/d\sigma \leq 0$ *in a* soft contest *with* $p \geq 1/2$.

Proof of Lemma 1. For part (i), if $p < 1/2$ the acceptance probability for weaker candidates, which is below 50%, clearly increases in dispersion σ, as the right tail of the distribution gets larger. If, in addition, p is sufficiently small, $\hat{x}^S > 1$ so that the acceptance probability of the strongest applicant is also less than 50% and thus also decreases in σ. Then, given that all applicants a are more likely to be accepted at the initial acceptance standard, the acceptance standard must increase in order to keep the success rate equal to p.

When, instead, p is large so that $\hat{x}^S < 1$, so that the acceptance probability of stronger applicants is above 50% and thus decreases in σ, the acceptance probability for most applicants increases in dispersion given that $p < 1/2$. To see this, consider first the uniform example case where f is loglinear; in this case, the acceptance probability of the median applicant is equal to p by construction; given that

the overall success rate must be kept fixed at $p < 1/2$ more that half of the applicants enjoy a higher probability of acceptance as dispersion is raised. If f is strictly logconcave, the acceptance probability of the median applicant is even lower than p, so that this median applicant always benefits from an increase in dispersion—actually, the acceptance probability for more than 50% of applicants increases with dispersion. Overall, in case (i) to bring down the average success rate to p the acceptance standard must be raised: $d\hat{x}^S/d\sigma > 0$.

In the knife-edge case with $p = 1/2$. By symmetry of the signal distribution, $F^{-1}(1/2) = 0$, the vertical intercept is $x_0^S = 1$. More generally, in (and only in) this case the acceptance probability of the median applicant is exactly equal to the average success rate p. As dispersion increases, the acceptance probabilities of applicants is spread in a completely symmetric way, so that the acceptance standard is constant in σ. Regardless of F, the proportional supply is then $\hat{x}^S(a) = 1 - a/2$, which is invariant in signal dispersion. If, instead, $p > 1/2$ as in case (ii), the logic in (i) is flipped so that $d\hat{x}^S/d\sigma < 0$.

Lemma 2 *If types are uniformly distributed, the equilibrium acceptance standard* $d\hat{x}^E/d\sigma \gtreqless 0 \Leftrightarrow p \lesseqgtr \tilde{p}$ *with* $\tilde{p} \leq 1/2(1/2+\gamma)$.

Proof of Lemma 2. With uniform signal when the success rate is $\tilde{p}_U = 1/2(\gamma + 1/2)$, half way between the cost-to-value ratio γ and the rotation point of the signal distribution $1/2 = F_{\bar{\theta},\sigma}(\bar{\theta}) = F(0)$, the approval standard is necessarily constant at $\hat{x} = \hat{\theta}$ for all σ. For distributions with f strictly logconcave, we now show that the success rate at which the equilibrium acceptance standard is constant in σ is strictly below the one we found for the case with loglinear density, $p < \tilde{p}_U$. If the density is strictly logconcave, the density is strictly decreasing above the symmetry point of the distribution; thus for $\theta = \hat{\theta}$, $F_{\hat{\theta},\sigma}(x)$ is strictly concave for $x \geq \hat{\theta}$, as illustrated for the normal example in Figure 2. Thus, at $\alpha = \hat{\alpha}_U$ we have

$$\int_{F^{-1}_{\hat{\theta},\sigma}(1-\tilde{p}_U)}^{\bar{\theta}+a} \left[F\left(\frac{\hat{\theta}-(1-x)}{\sigma}\right) - (1-\tilde{p}_U) \right] dx > \int_{\bar{\theta}}^{F^{-1}_{\hat{\theta},\sigma}(1-\tilde{p}_U)} \left[(1-\tilde{p}_U) - F\left(\frac{\hat{\theta}-(1-x)}{\sigma}\right) \right] dx.$$

This means that the equilibrium acceptance standard must be $\hat{x}^P < \bar{\theta}$ at $p = p_U$. In order to raise the acceptance standard to $\hat{\theta}$, the level such that $d\hat{x}^E/d\sigma = 0$, it is necessary to reduce $\tilde{p}$ below $\tilde{p}_U$. By continuity, there exists $\tilde{p} < \tilde{p}_U$ such that $\hat{x}^P = \hat{\theta}$ for all σ. We conclude that in general $d\hat{x}^E/d\sigma \gtreqless 0$ for $p \lesseqgtr \tilde{p}$ with $\tilde{p} \leq \tilde{p}_U$.

Haupt and Schäbe Types Example. Next, we report closed-form expressions for all the equilibria resulting with Haupt and Schäbe distribution $G(\theta) = -\eta + \sqrt{\eta^2 + (1+2\eta)\theta}$, where $\eta \in [0,\infty)$; see,

e.g., Nair, Sankaran, and Balakrishnan (2010, page 243). As $\eta \to \infty$ the distribution converges to the uniform; for $\eta = 0$ we recover the square root distribution used for Figure 5. Even though the hazard rate is non-monotonic for $\eta \in [1/3, 1)$, for $\eta \in [1/3, \infty)$ the DMRL condition is satisfied—the unique, interior, and stable equilibrium is equal to $\tilde{a}^p = \frac{3(1+\eta)-\sqrt{9\eta^2+3(2\eta+1)(3-8\sigma(p-\gamma))}}{4}$. For $\eta \in [0, 1/3)$ DMRL is violated, so that for $\sigma \in ((1+3\eta)/[3(1+2\eta)(p-\gamma)], 3(1+\eta)^2/[8(1+2\eta)(p-\gamma)])$ in addition to the interior stable equilibrium at $\tilde{a}^p$, there is second interior but unstable equilibrium at $\tilde{\tilde{a}}^p = \frac{3(1+\eta)+\sqrt{9\eta^2+3(2\eta+1)(3-8\sigma(p-\gamma))}}{4}$, as well as a stable equilibrium at the corner $\tilde{\tilde{\tilde{a}}}^p = 1$.

Normal Example. Suppose that types are normally distributed with $\theta \sim N(\mu, \kappa^2)$ with mean μ and standard deviation κ and that the signal $x \sim N(\theta, \sigma^2)$ follows a normal distribution with mean θ and standard deviation σ. Given that $F\left(\frac{x-\theta}{\sigma}\right) = \Phi\left(\frac{x-\theta}{\sigma}\right)$ and $G(\theta) = \Phi\left(\frac{\theta-\mu}{\kappa}\right)$, the demand is

$$a^D(\hat{x}) = 1 - \Phi\left(\frac{x - \sigma\Phi^{-1}(1-\gamma) - \mu}{\kappa}\right)$$

and the supply solves

$$\frac{\int_{\mu+\kappa\Phi^{-1}(1-a)}^{\infty}\left[1-\Phi\left(\frac{x-\theta}{\sigma}\right)\right]\frac{1}{\kappa}\varphi\left(\frac{\theta-\mu}{\kappa}\right)d\theta}{a} = p.$$

Using the well-known formula for conditioning on a variable being greater than a certain value we obtain

$$X\big|\theta > \mu + \kappa\Phi^{-1}(1-a) \sim ESN\left(\mu, \sqrt{\kappa^2+\sigma^2}, \frac{\kappa}{\sigma}, -\Phi^{-1}(1-a)\right) \tag{28}$$

where $ESN(\mu, \sigma, \alpha, \tau)$ is an extended skew normal distribution with the density

$$h(x;a) = \frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right)\frac{\Phi\left(\frac{\tau\sqrt{1+\alpha^2}+\alpha x-\mu}{\sigma}\right)}{\Phi\left(\frac{\tau-\mu}{\sigma}\right)};$$

see Azzalini and Capitanio's (2014, page 36, equation 2.39). Thus, the supply is

$$1 - H(x;a) = \int_{\hat{x}}^{\infty}\frac{1}{\sqrt{\kappa^2+\sigma^2}}\varphi\left(\frac{x-\mu}{\sqrt{\kappa^2+\sigma^2}}\right)\frac{\Phi\left(\frac{-\Phi^{-1}(1-a)\sqrt{1+\left(\frac{\kappa}{\sigma}\right)^2}+\frac{\kappa}{\sigma}x-\mu}{\sqrt{\kappa^2+\sigma^2}}\right)}{\Phi\left(\frac{-\Phi^{-1}(1-a)-\mu}{\sqrt{\kappa^2+\sigma^2}}\right)}dx = p.$$

The distribution of θ when demand is a (or equivalently when $\theta > \hat{\theta} = \Phi_{\mu,\kappa}^{-1}(1-a) = \mu + \kappa\Phi^{-1}(1-a)$) is a truncated normal with density

$$g\left(\theta|\theta \geq \Phi_{\mu,\kappa}^{-1}(1-a)\right) = \frac{\frac{1}{\kappa}\varphi\left(\frac{\theta-\mu}{\kappa}\right)}{1-\Phi\left(\frac{\Phi_{\mu,\kappa}^{-1}(1-a)-\mu}{\kappa}\right)}I_{\theta\geq\mu+\kappa\Phi^{-1}(1-a)}.$$

The posterior about θ conditional on (x,a) is a truncated normal with parameters $\tilde{\mu}(x)$ and $\tilde{\sigma}^2$ and truncation at $\Phi_{\mu,\kappa}^{-1}(1-a) = \mu + \kappa\Phi^{-1}(1-a)$; the density of this posterior is

$$g(\theta|x,a) = \frac{\frac{1}{\tilde{\sigma}}\varphi\left(\frac{\theta-\tilde{\mu}(x)}{\tilde{\sigma}}\right)}{1-\Phi\left(\frac{\Phi_{\mu,\kappa}^{-1}(1-a)-\tilde{\mu}(x)}{\tilde{\sigma}}\right)} I_{\theta\geq\mu+\kappa\Phi^{-1}(1-a)}.$$

The posterior expectation conditional on (x,a) is then

$$E(\theta|x,a) = \tilde{\mu}(x) + \frac{\varphi\left(\frac{\mu+\kappa\Phi^{-1}(1-a)-\tilde{\mu}(x)}{\tilde{\sigma}}\right)}{1-\Phi\left(\frac{\mu+\kappa\Phi^{-1}(1-a)-\tilde{\mu}(x)}{\tilde{\sigma}}\right)}\tilde{\sigma}. \tag{29}$$

Conditional on a, signal x is the sum of a truncated normal with location μ and scale κ^2 truncated at $\Phi_{\mu,\kappa}^{-1}(1-a) = \mu + \kappa\Phi^{-1}(1-a)$ and an independent normal $N(0,\sigma)$. Note that a normal with parameters (μ,κ) truncated at $\mu + \kappa\Phi^{-1}(1-a)$ is an extended skew normal with parameters $\left(\mu,\kappa,\alpha=\infty,-\Phi^{-1}(1-a)\right)$.[39] Given that $\lim_{\alpha\to\infty}\alpha\left(1+\left(1+\alpha^2\right)\sigma^2/\kappa^2\right)^{-1/2} = \kappa/\sigma$, it follows from Azzalini and Capitanio's (2014, page 37) Proposition 2.9 that the sum of an ESN with parameters $\left(\mu,\kappa,\alpha=\infty,-\Phi^{-1}(1-a)\right)$ and an independent Normal with parameters $(0,\sigma)$ is an ESN with parameters $\left(\mu,\sqrt{\kappa^2+\sigma^2},\kappa/\sigma,-\Phi^{-1}(1-a)\right)$.

[39] This observation follows from equation (2.41) on page 37 in Azzalini and Capitanio (2014), once we take X_1 to be perfectly correlated with X_0 by setting $\delta = 1$ in their notation and then using equation (2.15) on page 29.