Monetary Policy and the Redistribution Channel*

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Abstract

This paper evaluates the role of redistribution in the transmission mechanism of monetary policy to consumption. Using consumer theory, I show that redistribution has aggregate effects whenever marginal propensities to consume (MPCs) covary, across households, with balance-sheet exposures to aggregate shocks. Unexpected inflation gives rise to a *Fisher channel* and real interest rate shocks to an *interest rate exposure channel*; both channels are likely to contribute to the expansionary effects of accommodative monetary policy. Indeed, using a sufficient statistic approach, I find that redistribution could be the dominant reason why aggregate consumer spending reacts to transitory changes in the real interest rate, provided households' elasticities of intertemporal substitution are reasonably small (0.3 or less in the United States). I then build and calibrate a general equilibrium model with heterogeneity in MPCs, and I evaluate how the redistribution channel alters the economy's response to shocks. When household assets and liabilities have short effective maturities, the interest rate exposure channel raises the elasticity of aggregate demand to real interest rates, which dampens fluctuations in the natural rate of interest in response to exogenous shocks and amplifies the real effects of monetary policy shocks. The model predicts that if U.S. mortgages all had adjustable rates—as they do in the U.K.—the effect of interest-rate changes on consumer spending would more than double. In addition, this effect would be asymmetric, with rate increases reducing spending by more than cuts would increase it.

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1 Introduction

The role that monetary policy plays in redistributing income and wealth has been at the center stage of discussions between economic commentators and policymakers in the past few years.[1] There is a clear sense that households are not all equally affected by low interest rates, but no consensus on who gains and who loses. Consider the many ways in which accommodative monetary policy can affect an asset holder: he may lose from low returns, or benefit due to capital gains; he may lose from inflation eroding his savings, or benefit if a recession is avoided.

A conventional view is that redistribution is a side effect of monetary policy changes, separate from the issue of aggregate stabilization which these changes aim to achieve. This view is implicit in most models of the monetary policy transmission mechanism, which feature a representative agent. By contrast, in this paper I argue that redistribution is a *channel* through which monetary policy affects macroeconomic aggregates. Specifically, I contend that the redistributive effects of *accommodative* monetary policy contribute to *increasing* aggregate consumption demand, because those who gain have higher marginal propensities to consume (MPCs) than those who lose—and inversely for contractionary monetary policy. The simple argument goes back to Tobin (1982):

> Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and for debtors. But [...] the population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth or from current income.

However, I find that Tobin's distinction between debtors and creditors is not precise enough to evaluate the aggregate effect of redistribution via monetary policy on household spending. Because accommodative monetary policy tends to raise inflation *and* to lower real interest rates, there are at least two dimensions of heterogeneity—and hence two channels of redistribution—to consider. Let me discuss these in turn.

First, current and future inflation revalue nominal balance sheets—the first by redenominating assets and liabilities, and the second by increasing nominal interest rates and hence the rates at which future flows are discounted. Nominal creditors lose and nominal debtors gain: this is the *Fisher channel*, which has a long history in the literature since Fisher (1933). This channel has been explored by Doepke and Schneider (2006a), who measure the balance sheet exposures of various sectors and groups of households in the United States to different inflation scenarios. Net nominal positions (NNPs) quantify these exposures for the case of unexpected increases in the price level.

Real interest rate changes create a second, more subtle form of redistribution. Households' balance sheets do not only consist of their financial assets and liabilities: they also include their

[1] For recent examples see Martin Wolf's contention that "the Federal Reserve's policies have benefited the relatively well off; it is trying to raise the prices of assets which are overwhelmingly owned by the rich." ("Why inequality is such a drag on economies", Financial Times, September 30, 2014) or William D. Cohan, "How Quantitative Easing Contributed to the Nation's Inequality Problem", New York Times Dealbook, October 24, 2014. Among central bankers' speeches on the topic, see Benoît Coeuré, "Savers Aren't Losing Out", November 11, 2013; James Bullard, "Income Inequality and Monetary Policy: A Framework with Answers to Three Questions", June 26, 2014 or Yves Mersch, "Monetary Policy and Economic Inequality", October 17, 2014.

future incomes and consumption plans. Hence, to determine if someone benefits from falls in real interest rates, one should not look at the increase in the prices of the financial assets that this person holds. Instead, one should consider whether his total assets have longer durations than his total liabilities. *Unhedged interest rate exposures* (UREs)—the difference between maturing assets and liabilities at a point in time—are the correct measure of balance-sheet exposures to real interest rate changes, just like net nominal positions are for price level changes. For example, agents whose financial wealth is primarily invested in short-term certificates of deposit tend to have positive UREs, while those with large investments in long-term bonds or adjustable-rate mortgage holders tend to have negative UREs. Real interest rate falls redistribute away from the first group towards the second group: this is what I call the *interest rate exposure channel*.

In this paper I focus my attention on this second channel for two reasons. First, equilibrium real interest rates fluctuate over the business cycle, making the redistribution they induce important at all times. By contrast, the Fisher channel is likely to be more muted in countries with low and stable inflation—though it is very important during large regime shifts in monetary policy such as the ones that motivate Doepke and Schneider (2006a), or during debt deflation episodes (Bernanke, 1983). Second, while Tobin (1982) and many others have long argued that nominal debtors have higher MPCs than nominal creditors, the literature has not asked whether *agents with unhedged borrowing requirements* (negative URE) have higher MPCs than *agents with unhedged savings needs* (positive URE). In this paper I find that they do, using two complementary approaches: one reduced-form and one structural. I first develop a theory to identify a sufficient statistic, the *cross-sectional covariance between MPCs and UREs*, that quantifies the effect of redistribution via real interest rates on aggregate demand. I measure this statistic in the data and find that it is plausibly large. I then calibrate a dynamic general equilibrium model that confirms this finding. I use the model to assess the performance of my sufficient statistic in predicting the aggregate effect of changes in monetary policy, and to run various counterfactual experiments.

In first part of the paper, I develop my sufficient statistic approach. In partial equilibrium, I consider an optimizing agent with a given initial balance sheet, who values nondurable consumption and leisure, and is subject to an arbitrary change in his income path and in the whole term structure of nominal and real interest rates. I show that, irrespective of the form of his utility function, the wealth effect component of his consumption response is given by the product of his marginal propensity to consume—the partial derivative of his consumption with respect to a current increase in his income—with a balance-sheet revaluation term in which NNPs and UREs appear. Importantly, I show that, for the case of transitory shocks, this result is robust to the presence of incomplete markets, idiosyncratic risk, and (certain kinds of) borrowing constraints. In other words, the MPC out of a windfall income transfer is relevant to determine the response of optimizing consumers to *any other* change in their balance sheet—in particular those induced by inflation or real interest rate changes. To the best of my knowledge, this result is new to the incomplete-markets consumption literature.[2]

[2]A precursor to this finding is Kimball (1990), who provides equations that characterize MPC, analyzes how it

I then sum across the individual-level predictions and exploit the fact that financial assets and liabilities net out in general equilibrium to obtain the response of aggregate consumption to simultaneous transitory shocks to the real interest rate, output, and the price level. Independently of agents' utility functions, the specification of production, and the asset market structure, the cross-sectional covariances between MPCs and exposures to the aggregate shocks are sufficient statistics for the part of this response that is due to redistribution.[3] I denote by $\mathcal{E}_r$ the covariance between MPCs and UREs, and call it the *redistribution elasticity of aggregate demand* with respect to real interest rates.

My theory shows that it is possible to decompose the change in aggregate consumption that results from a change in real interest rates into the sum of a redistribution component (which depends on $\mathcal{E}_r$) and an intertemporal substitution component (which depends on consumers' Elasticities of Intertemporal Substitution, or EIS). There is a large literature estimating "the" EIS—a key parameter in dynamic macroeconomic models—using aggregate or household-level data. While there is no agreement on its exact value, most studies point to a number between 0 and 2, with some consensus in macroeconomics for a value below 1 (see for example Hall, 2009 or Havránek, 2013). To assess how large the redistribution channel is without taking a stand on the value of the EIS, I derive a measure σ^* that corresponds to the average level of this elasticity that makes the interest rate exposure channel and the substitution channel equal in magnitude, for the case of a purely transitory change in the real interest rate and exogenous labor supply. σ^* is positive when $\mathcal{E}_r$ is negative; that is, when negative-URE households have higher marginal propensities to consume than positive-URE ones. I define a method for measuring UREs, in an exercise similar to Doepke and Schneider's work on exposures to price changes.

Turning to the data, I estimate $\sigma^* = 0.12$ in Italy using a survey containing a self-reported measure of MPC (Jappelli and Pistaferri, 2014), and $\sigma^* = 0.3$ in the United States using a procedure that exploits the randomized timing of tax rebates as a source of identification for MPC (Johnson, Parker and Souleles, 2006, hereafter JPS). These numbers confirm that there exists a redistribution channel of monetary policy, acting in the same direction as the substitution channel.[4] They also show that this channel is quantitatively significant, especially when the EIS is reasonably small. Therefore, representative-agent analyses that abstract from redistribution can be significantly off.

My reduced-form approach has the virtue of being very robust, but it requires precise measures of MPC and URE, which are challenging to obtain jointly. Moreover, it cannot directly handle persistent changes in real interest rates, nor can it be used to perform policy experiments that affect

depends on income uncertainty and market structure, and notes in passing that "the marginal propensity to consume out of wealth figures into the interest elasticity of consumption, as a factor in the wealth effect term".

[3]In addition to the Fisher and the interest rate exposure channels, my analysis highlights the presence of an *earnings heterogeneity channel* which arises when monetary policy does not change all household incomes proportionally. In section 6.6 I discuss the role of this channel in determining the aggregate effect of monetary policy in the context of my general equilibrium model.

[4]This finding is also consistent with recent survey evidence from the United Kingdom reported in Bank of England (2014). Faced with a hypothetical two-percentage-point increase in interest rates, overwhelmingly more mortgagors reported that they would cut spending, and that they would do so by larger amounts, than savers reported that they would increase spending.

MPCs or UREs themselves. A calibrated general equilibrium model addresses these shortcomings by imposing more structure. I construct a Bewley-Huggett-Aiyagari incomplete markets model with nominal, long-term, circulating private IOUs (as in Huggett, 1993) and endogenous labor supply. The model features rich heterogeneity in MPCs and UREs, and perfect aggregation of labor supply owing to GHH preferences (Greenwood, Hercowitz and Huffman, 1988).

I calibrate the model to the U.S. economy and quantitatively evaluate, in its steady-state, the size of all my sufficient statistics. I find that the interest rate exposure channel has the same sign, and comparable magnitude, as in my reduced-form analysis. I also find that the Fisher channel is consistent with a substantial increase in demand from an unexpected increase in the price level. Hence, in the model, *both* channels should contribute to the expansionary effects of accommodative monetary policy. I confirm this by studying the economy's transitional dynamics after unanticipated shocks.

In the version of the model with flexible prices, redistributive shocks have no output effects due to the absence of wealth effects on labor supply. Thus, I study the effects of these shocks on equilibrium interest rates, and find that these effects depend on the economy's maturity structure. When financial assets are long term,[5] the interest rate exposure channel is smaller, which lowers the total elasticity of aggregate demand to real interest rates, and increases the fluctuations in the natural rate of interest in response to exogenous shocks. Intuitively, under a long maturity structure, debtors—the high-MPC agents in the economy—roll over a smaller fraction of their liabilities each period, and their consumption plans are therefore less sensitive to changes in real interest rates.

Under sticky prices, the transmission mechanism of monetary policy works as follows. A surprise fall in the nominal interest rate creates a drop in the real interest rate, which boosts demand—both through the substitution channel and through the interest rate exposure channel. The latter effect is stronger when the economy's maturity structure is shorter. This increase in demand then gets amplified through four channels. First, the average income increase translates into a spending increase though the economy's average MPC. Second, consumers' incomes are impacted in heterogeneous ways: in my model, the increase in output disproportionately benefits the high earners, who have lower MPCs; hence the earnings heterogeneity channel acts as a mitigating factor. Third, higher hours worked increase spending on their own, due to the complementarity between consumption and labor supply inherent in GHH preferences. Finally, as in standard New Keynesian models, firms' marginal costs are raised, which can create inflation and boost demand further via the Fisher channel.[6] In my calibration, the fixed point of this process of income-spending increases reflects substantial amplification of monetary policy shocks.

I use the model to ask the extent to which monetary policy transmission would differ if the all assets and liabilities in the U.S. economy—calibrated with an EIS of 0.5—were short term. I find

[5] In my model, all financial assets have an offsetting liability within the household sector, so that experiments shortening the maturity of one necessarily shorten the maturity of the other.

[6] In the current version of the model, I shut down the Fisher channel by assuming that prices are fully sticky. In general, the Fisher channel will increase in strength with the degree of price flexibility.

that monetary policy shocks would have more than double their current effect on household non-durable consumption. This is consistent with the cross-country structural vector autoregression study of Calza, Monacelli and Stracca (2013), which finds that consumption reacts much more strongly to identified monetary policy shocks in countries where mortgages predominantly have adjustable rates. It also confirms a widely held-view in policy circles that mortgage structure plays a role in the monetary policy transmission mechanism (Cecchetti, 1999; Miles, 2004). One interpretation is that the substitution effect is stronger in these countries, since agents effectively participate more in financial markets. My paper offers an alternative interpretation that does not rely on limited participation: in adjustable-rate mortgage (ARM) countries, monetary policy affects household spending predominantly by redistributing wealth.[7]

Finally, I derive in the context of the model a first-order approximation for the impulse response to a one-time monetary policy shock—it involves MPC-based sufficient statistics—and I compare this prediction to the full nonlinear impulse response to a shock. While the approximation is excellent for any small shock, as well as for larger increases in interest rates, in my ARM calibration I find that it overpredicts the increase in output that results from a moderate fall in the policy rate. This asymmetry in the effects of monetary policy comes from the differential response of borrowers at their credit limit to rises and falls in income: while these borrowers save an important fraction of the gains they get from low interest rates—which reduce the payments they have to make on the credit limit—they adjust spending one for one with every dollar increase in the payments they have to make when interest rates rise. The prediction that interest rate hikes lower output more than falls increase it has received support in the empirical literature (Cover, 1992; de Long and Summers, 1988; Tenreyro and Thwaites, 2013). An influential interpretation of this fact, which dates back to Keynes, relies on the presence of downward nominal wage rigidities. My explanation is that MPC differences are smaller for falls than for rises in interest rates, so that the redistribution channel is smaller for the former than for the latter.

Literature review. This paper contributes to several strands of the literature.

First, an extensive empirical literature has documented that marginal propensities to consume are large and heterogenous in the population (Parker, 1999; Johnson et al., 2006; Parker, Souleles, Johnson and McClelland, 2013; Broda and Parker, 2014; see Jappelli and Pistaferri, 2010 for a survey). In particular, the literature has found a dependence of MPCs on household balance sheet positions, which motivates my analysis (Mian, Rao and Sufi, 2013; Mian and Sufi, 2014; Baker, 2014; Jappelli and Pistaferri, 2014). Recently, di Maggio, Kermani and Ramcharan (2014) and Keys, Piskorski, Seru and Yao (2014) have measured the consumption response of households to changes in the interest rates they pay on their mortgages. My theory shows that these papers quantify

[7] Aside from limited participation, I am leaving a number of other redistributive channels out of my analysis. First, since I abstract away from aggregate risk, in my framework monetary policy cannot change risk premia. Second, since I assume that all assets are remunerated at the risk-free rate, my analysis does not address the unequal incidence of inflation due to larger cash holdings by the poor (Erosa and Ventura, 2002; Albanesi, 2007). Hence my analysis can best be seen to apply to conventional monetary policy actions in modern developed countries with low and stable inflation targets.

an important leg of the redistribution channel of monetary policy. In appendix B.2, I illustrate how such cross-sectional studies of the link between household consumption and balance sheet structure can be mapped into my theoretical framework.

Two empirical papers have explored two of the channels of monetary policy that I highlight in isolation, focusing on the redistribution itself rather than its aggregate demand effect. As described above, Doepke and Schneider (2006a) quantify the redistribution that the Fisher channel could create under different scenarios of surprise inflation. Coibion, Gorodnichenko, Kueng and Silvia (2012) use the Consumer Expenditure Survey and find that identified monetary policy accommodations lower income inequality. To the extent that lower income agents have higher MPCs, this suggests that the *earnings heterogeneity channel* (a term I borrow from their paper) may increase aggregate demand following falls in interest rates.[8]

On the theoretical front, my work belongs to a long tradition in macroeconomics that uses micro-founded general equilibrium models to explain why unanticipated increases in nominal interest rates tend to lower household spending. The vast majority of the New Keynesian literature—with which I share my focus on household consumption, and my emphasis on sticky prices in the last section—analyzes this question in a framework where households do not have net financial positions (see the textbook expositions of Woodford, 2003 and Galí, 2008). In this context, intertemporal substitution is the dominant reason why consumers respond to changes in real interest rates.[9] My analysis brings wealth effects to the forefront of the analysis of monetary policy shocks, by emphasizing that, when MPCs and balance sheets are heterogenous, there exists a direct or "first-round" effect of redistribution that transmits real interest rate shocks to aggregate consumption demand.

My analysis is also related to a literature that emphasizes the role of firms' or banks' net worth in amplifying the effects of monetary policy on investment (for example, Bernanke and Gertler, 1995; Bernanke, Gertler and Gilchrist, 1999; Adrian and Shin, 2010; Brunnermeier and Sannikov, 2014). While this literature mainly stresses the Fisher channel, both asset-liability duration mismatches and changes in borrowers' interest expenses are understood to create a link between monetary policy and net worth. In this paper, I show that the concept of unhedged interest rate exposures makes these insights applicable to the study of consumption, the largest component of output.

A number of models use redistribution as a mechanism that can generate macroeconomic effects of monetary policy even without nominal rigidities. Among these are models that emphasize limited participation (Grossman and Weiss, 1983; Rotemberg, 1984; Alvarez, Atkeson and Edmond, 2009) or wealth effects on labor supply (Sterk and Tenreyro, 2014 combine them with

[8] My consumer theoretic analysis may help to refine the theoretical framework of Coibion et al. (2012). In particular, their *savings redistribution channel* ("an unexpected increase in interest rates or decrease in inflation [which] benefits savers and hurts borrowers") can usefully be decomposed into my *Fisher channel* and my *interest rate exposure channel*: the two are very different. Typical borrowers with fixed-rate mortgages, for example, have much to lose from decreases in inflation but close to nothing from rises in real interest rates.

[9] Wealth effects do matter in New Keynesian representative-agent models, but only through "second-round", general equilibrium effects (higher interest rates lower aggregate spending which in turn lowers household incomes).

substitution into durable goods purchases). By contrast, I consider agents who participate in financial markets at all times, and I generally abstract away from wealth effects on labor supply, so I introduce nominal rigidities to translate the link between redistribution and demand that I highlight into an effect on final output.

Since the pioneering work of Harberger (1964), sufficient statistics have been used in public finance to evaluate the welfare effect of hypothetical policy changes in a way that is robust to the specifics of the underlying structural model (see Chetty, 2009 for a survey). My sufficient statistics are useful to evaluate the impact on *aggregate demand* of hypothetical changes in macroeconomic aggregates in a similarly robust way. All that is required is information on household balance sheets, income and consumption levels, and their MPCs. Farhi and Werning (2013b) bridge the public finance approach and my positive approach: MPCs enter as sufficient statistics for their optimal macro-prudential interventions under nominal rigidities.

The importance of MPC differences in the determination of aggregate demand is well understood by the theoretical literature on fiscal transfers (for example Galí, López-Salido and Vallés, 2007; Oh and Reis, 2012; Farhi and Werning, 2013a; McKay and Reis, 2013). MPC differences between borrowers and savers, in particular, have been explored a source of aggregate effects from shocks to asset prices (King, 1994), or to borrowing constraints (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2012; Korinek and Simsek, 2014). Guerrieri and Lorenzoni (2012) notably study, as I do, a Bewley-Huggett-Aiyagari model with rich heterogeneity in asset positions and MPCs (together with heterogeneity in marginal propensities to work). None of these studies, however, focus on the role of MPC differences in generating aggregate effects of monetary policy.

My dynamic general equilibrium model belongs to a recent literature studying monetary policy in New Keynesian environments with incomplete markets. Sheedy (2014) focuses on a normative question: when the only available assets are nominal and risk-free, the central bank can exploit its influence on the price level to ameliorate market incompleteness through the Fisher channel. He explores the quantitative importance of this objective, relative to the traditional aggregate stabilization role of monetary policy. Two papers close to my work are Gornemann, Kuester and Nakajima (2012) and McKay, Nakamura and Steinsson (2014). Gornemann et al. (2012) examine quantitatively the population distribution of consumption and welfare gains after monetary policy shocks, in a calibrated model that matches the U.S. business cycle moments, as well as its wealth distribution in steady state. They find that a small fraction of wealthy agents gain, and others lose, from contractionary monetary policy shocks. My framework suggests that this can arise from rich households both earning more in profits *and* gaining from positive unhedged interest rate exposures. It also suggests a simple way to calculate, in their model, the part of the aggregate output effect that is due to redistribution, a question which motivates them but which they do not address. McKay et al. (2014) compare, as I do, the aggregate effects of monetary policy shocks when markets are incomplete relative to the representative-agent case, but they focus on shocks announced well in advance. They show that incomplete markets dampen the effect of this forward guidance. This contrasts to the response to contemporaneous monetary policy shocks that I highlight, which tends to be higher under incomplete markets—especially when debt is short-term debt—because of the negative correlation between marginal propensities to consume and unhedged interest rate exposures that the model generates.

Finally, a few other dynamic general equilibrium models examine the impact of mortgage structure on the monetary transmission mechanism. As in my paper, Calza et al. (2013) and Rubio (2011) find, in the calibrations of their models, significantly larger effects from monetary policy shocks under variable- than under fixed-rate mortgages (FRMs). I highlight the role of unhedged interest rate exposures in accounting for part of these results. Garriga, Kydland and Sustek (2013) study a flexible-price, limited participation model and also find much larger output effects under ARMs than FRMs, mainly acting through investment.

Layout. The remainder of this paper is structured as follows. Section 2 presents a partial equilibrium decomposition of consumption responses to shocks into substitution and wealth effects, starting from a single-agent model under perfect foresight and showing that the result survives under incomplete markets. Section 3 provides my aggregation result and some simple general equilibrium applications. Section 4 assesses the quantitative magnitude of the redistribution channel by measuring σ^* in survey data. Finally, section 5 builds and calibrates a Huggett model, which section 6 uses to assess quantitatively the role of the economy's maturity structure in shaping the cyclical properties of the natural rate of interest and the ability of monetary policy to increase household spending. Section 6 also investigates the asymmetric effects of increases and cuts in interest rates. Section 7 concludes.

2 Household balance sheets and wealth effects

In this section, I consider the role of households' balance sheets in determining their consumption and labor supply adjustments to a macroeconomic shock. I first highlight the forces at play in a life-cycle labor supply model (Modigliani and Brumberg, 1954; Heckman, 1974) featuring perfect foresight and balance sheets with an arbitrary maturity structure. Following an unexpected shock, balance sheet revaluations—as well as marginal propensities to consume and work—play a crucial role in determining both the welfare response and the wealth effects on consumption and labor supply (theorem 1). I isolate in particular the role of unhedged interest rate exposures. Under certain conditions, the result in theorem 1 survives the addition of idiosyncratic income uncertainty (theorem 2) and therefore applies to a large class of microfounded models of consumption behavior.

2.1 Perfect-foresight model

Consider a household with arbitrary non-satiable preferences over nondurable consumption $\{c_t\}$ and hours of work $\{n_t\}$.[10] I assume no uncertainty for simplicity: the same insights obtain when markets are complete. The household is endowed with a stream of real unearned income $\{y_t\}$. He has perfect foresight over the general level of prices $\{P_t\}$ and the path of his nominal wages $\{W_t\}$, and holds long-term nominal and real contracts. Time is discrete, but the horizon may be finite or infinite, so I do not specify it in the summations. The agent solves the following utility maximization problem:

$$\begin{aligned} \max \quad & U\left(\{c_t, n_t\}\right) \\ \text{s.t.} \quad & P_t c_t = P_t y_t + W_t n_t + \left({}_{t-1}B_t\right) + \sum_{s \geq 1} \left({}_t Q_{t+s}\right)\left({}_{t-1}B_{t+s} - {}_t B_{t+s}\right) \\ & \qquad + P_t\left({}_{t-1}b_t\right) + \sum_{s \geq 1} \left({}_t q_{t+s}\right) P_{t+s}\left({}_{t-1}b_{t+s} - {}_t b_{t+s}\right) \end{aligned} \tag{1}$$

In the flow budget constraint (1), ${}_tB_{t+s}$ denotes a nominal payment the household arranges in period t to be paid out to him in period $t+s$, whereas ${}_tb_{t+s}$ denotes a payment in real terms. Correspondingly, ${}_tQ_{t+s}$ is the time-t price of a nominal zero-coupon bond paying at $t+s$, and ${}_tq_{t+s}$ the price of a real zero-coupon bond. This asset structure is the most general one that can be written for this dynamic environment with no uncertainty. Examples of nominal assets include deposits, long-term bonds or most typical mortgages. Examples of real assets include stocks (which here pay a riskless real dividend stream and therefore are priced according to the risk-free discounted value of this stream), inflation-indexed government bonds, or price-level adjusted mortgages.

The only restriction on the environment is an assumption of no arbitrage, which results in a Fisher equation for the nominal term structure:

$${}_tQ_{t+s} = \left({}_tq_{t+s}\right)\frac{P_t}{P_{t+s}} \quad \forall t, s$$

I begin the analysis of the consumer problem at $t = 0$. The environment allows for a very rich description of the household's initial holdings of financial assets, denoted by the consolidated claims, nominal $\{{}_{-1}B_t\}_{t\geq 0}$ and real $\{{}_{-1}b_t\}_{t\geq 0}$, due in each period. I write the initial real term structure as $q_t \equiv \left({}_0q_t\right)$ and real wages at t as $w_t \equiv \frac{W_t}{P_t}$.

Using either a terminal condition if the economy has finite horizon, or a transversality condition if the economy has infinite horizon, the flow budget constraints consolidate as expected into an intertemporal budget constraint:

$$\sum_{t\geq 0} q_t c_t = \sum_{t \geq 0} q_t \left(y_t + w_t n_t + \left({}_{-1}b_t\right) + \left(\frac{{}_{-1}B_t}{P_t}\right)\right) \tag{2}$$

Just as in any consumer theory problem, this one features a degree of indeterminacy in prices. The budget constraint (2) is unchanged when all discount rates q_t are multiplied by a constant.[11] I

[10] The analysis extends in a straightforward way to include additional endowment goods—such as housing—in preferences, and this is omitted for brevity.

[11] Note that there is no *nominal* indeterminacy in the usual sense, since the nominal denomination of assets means that the problem is generally different when the path for the price level is altered.

choose the *present value* normalization, $q_0 = 1$, which discounts future cash flows to date-0 terms.[12] I define present value wealth, W, as the value of the right-hand side of (2) when $q_0 = 1$. W can in turn be decomposed as the sum of human wealth (the present value of all future income) and financial wealth W^F:

$$W^F = \sum_{t \geq 0} q_t \left(({}_{-1}b_t) + \left(\frac{{}_{-1}B_t}{P_t} \right) \right)$$

From (2) it is clear that all financial assets with the same present value—that deliver the same level of financial wealth W^F—are equivalent from the point of view of the solution. This observation is summarized in the following proposition:

Proposition 1. *Financial assets with the same initial present value W^F deliver the same solution to the consumer problem.*

This framework predicts that a given household, holding a mortgage with outstanding nominal principal L (normalizing the price level at $P_0 = 1$), formulates the same plan $\{c_t, n_t\}_{t \geq 0}$ for consumption and labor supply irrespective of whether this liability is in the form of:

a) an adjustable-rate mortgage (ARM): ${}_{-1}B_0 = -L$ (the household has sold a short-term bond which is rolled over at the going nominal market interest rate every period)
b) a fixed-rate mortgage (FRM), where nominal payments are fixed and contracted in advance for T periods: ${}_{-1}B_t = -M$ for $t = 0 \ldots T$ (the outstanding principal is calculated as the outstanding present value of mortgage payments M: $L = \sum_{t=0}^{T} Q_t M$)
c) a price-level adjusted mortgage (PLAM), where the payments are pre-specified in real terms and get revalued with the price level: ${}_{-1}b_t = -m$ for $t = 0 \ldots T$ (the outstanding principal discounts these payments using the *real* term structure: $L = \sum_{t=0}^{T} q_t m$)

2.2 Adjustment after a shock

I now conduct an exercise where, *keeping balance sheets fixed* at $\{{}_{-1}B_t\}_{t \geq 0}$ and $\{{}_{-1}b_t\}_{t \geq 0}$, all of the variables relevant to the consumer choice problem are altered at once:

a) the price level $\{P_0, P_1 \ldots\}$
b) the real term structure[13] $\{q_0 = 1, q_1, q_2 \ldots\}$
c) the agent's unearned income sequence $\{y_0, y_1 \ldots\}$
d) the stream of real wages $\{w_0, w_1 \ldots\}$

I consider the *first-order* change in consumption dc_0, labor supply dn_0, and welfare dU that results from this change in the environment. It is simplest to interpret this exercise as an *unanticipated shock*, to which dc_0, dn_0 and dU are the approximate impulse responses on impact. However, the

[12] For some purposes, in particular when thinking through the impact of changes of the real interest rate at date 0 only, it will be convenient to choose a *future value* normalization, with $q_T = 1$ at some arbitrary date T in the future.

[13] To prevent arbitrage opportunities after the shock, I assume that the nominal term structure adjusts instantaneously to make the Fisher equation hold at the post-shock sequences of interest rates and prices.

analysis in this section is entirely partial equilibrium. In general equilibrium, the changes in a)-d) are linked and the income terms may include insurance payments; I postpone discussing these issues to section 3.

Several important quantities from consumer theory are defined along the initial path and can be evaluated at the initial sequence of prices, wealth and utility level: those include the marginal propensity to consume $MPC = \frac{\partial c_0}{\partial y_0}$, the marginal propensity to supply labor $MPN = \frac{\partial n_0}{\partial y_0}$, and the Hicksian (or compensated) demand elasticities $\epsilon^h_{x_0,y_t} = \frac{\partial x^h_0}{\partial y_t}\frac{y_t}{x_0}$ for $x \in \{c, n\}$ and $y \in \{q, w\}$.[14] Slutsky's equations can then be adapted to this dynamic context (see appendix C.1) to yield:

Theorem 1 (Generalized impulse response.). *To first order, the date-0 responses of consumption, labor supply and welfare to the considered change are given by*

$$dc_0 \simeq MPC d\Omega + c_0 \left(\sum_{t\geq 0} \epsilon^h_{c_0,q_t} \frac{dq_t}{q_t} + \sum_{t\geq 0} \epsilon^h_{c_0,w_t} \frac{dw_t}{w_t} \right)$$

$$dn_0 \simeq MPN d\Omega + n_0 \left(\sum_{t\geq 0} \epsilon^h_{n_0,q_t} \frac{dq_t}{q_t} + \sum_{t\geq 0} \epsilon^h_{n_0,w_t} \frac{dw_t}{w_t} \right)$$

$$dU \simeq U_{c_0} d\Omega$$

where $d\Omega = dW - \sum_{t\geq 0} c_t dq_t$*, the net-of-consumption wealth change, is given by*

$$\begin{aligned} d\Omega = & \underbrace{\sum_{t\geq 0} (q_t y_t) \frac{dy_t}{y_t}}_{\text{Real unearned income change}} + \underbrace{\sum_{t\geq 0} (q_t w_t n_t) \frac{dw_t}{w_t}}_{\text{Real earned income change}} \\ & + \underbrace{\sum_{t\geq 0} q_t \left(y_t + w_t n_t + \left(\frac{{}_{-1}B_t}{P_t} \right) + ({}_{-1}b_t) - c_t \right) \frac{dq_t}{q_t}}_{\text{Revaluation of net savings flows}} - \underbrace{\sum_{t\geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0} \right) \frac{dP_t}{P_t}}_{\text{Revaluation of net nominal position}} \end{aligned} \tag{3}$$

These equations separate the substitution and the wealth effects that result from the shock. In general, consumers substitute intertemporally with respect to current and all future interest rate and wage changes. All wealth effects get aggregated into a net term, $d\Omega$, which affects consumption and labor supply after multiplication by the marginal propensity to consume and work, respectively. Most parametrizations of utility specify consumption and leisure to be *normal*, that is, to increase with exogenous increases in income ($MPC > 0$ and $MPN \leq 0$).

Note that theorem 1 makes no assumption on horizon or the form of the utility function. This is in line with Campbell (2006)'s recommendation for "normative household finance [to] emphasize results that are robust to alternative specifications of household utility". The only assumption is that of a linear budget constraint with fixed, known prices, which Deaton and Muellbauer (1980) call the *neoclassical model*.

Determinants of the net wealth change. The net wealth change $d\Omega$ in (3) is the key expression determining both welfare and wealth effects in theorem 1. Observe first the structure of this term:

[14] Appendix C.2 specifies these elasticities for several standard parametrizations of utility.

it is a sum of products of *balance-sheet exposures* by *changes in aggregates*. For example, the exposure to a price increase at date t is the net nominal payment stream due to be received at that date: $Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)$. An unexpected increase dP_t in the price level at t lowers the present value of this stream and creates a balance-sheet loss for a nominal asset holder which is valued at $-Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)\frac{dP_t}{P_t}$.

Just as an increase in the price level at date t "acts" upon the net nominal payment stream at that date, equation (3) shows that changes in real discount rates act upon the present value of what I call *unhedged interest rate exposures* (UREs)

Definition 1. A household's date-t *unhedged interest rate exposure* measured at date -1 is the difference between *all* his maturing assets (including his income) and liabilities (including his planned consumption) at time t:

$$ {}_{-1}URE_t \equiv y_t + w_t n_t + \left(\frac{{}_{-1}B_t}{P_t}\right) + ({}_{-1}b_t) - c_t $$

Hence ${}_{-1}URE_t$ represents the *net saving requirement* of the household at time t, from the point of view of date -1. Because it includes the *stocks* of financial assets that mature at date t rather than interest *flows*, it might significantly diverge from traditional measures of savings at date t, in particular if investment plans have very short durations. The following examples and observations help clarify the role of UREs.

Example 1 (No wealth effect)**.** Consider a household whose initial financial assets are entirely indexed to inflation, and are arranged so that dividend payments match the difference between his planned consumption path and his other sources of income:

$$ {}_{-1}B_t = 0; \quad {}_{-1}b_t = c_t - (y_t + w_t n_t) \quad \forall t \tag{4} $$

This household has no exposure to price changes or real interest rate changes at any date (${}_{-1}URE_t = 0\ \forall t$). Following a shock that does not change his income ($dy_t = dw_t = 0\ \forall t$), his consumption and labor supply responses are purely driven by substitution effects, and his welfare is unaffected to first order. (The second-order term is positive, reflecting the gain from the ability to reoptimize at the new prices.)

Observation 1. *The composition of a household's balance sheet is important to understand his consumption, labor supply, and welfare response to changes in interest rates.*

The financial balance sheet described by equations (4) is sometimes called the *Arrow trading plan*, where all trades are pre-arranged. In practice, a household investing all wealth in real annuities might achieve a plan close to this one. According to proposition 1, any investment plan for a given level of financial wealth leads to the same life-cycle path for consumption and labor supply before the shock. But theorem 1 makes clear that these plans have different consumption and welfare implications after the shock. With short durations (for example, ${}_{-1}b_t = ({}_{-1}B_t) = 0$ for $t \geq 1$), date-0 unhedged interest rate exposure and net asset position are closely aligned.

Observation 2. *Asset value changes give incomplete information to understand the effects of monetary policy on household welfare.*

In the model just presented and in its extensions in the rest of the paper, monetary policy influences asset values through two channels: risk-free real discount rate effects and expected inflation effects. But these asset value changes do not enter $d\Omega$ directly, so they are not relevant *on their own* to understand who gains and who loses from monetary policy, contrary to what popular discussions sometimes imply. For example, it is sometimes argued that accommodative monetary policy benefits bondholders by increasing bond prices. Theorem 1 shows that such a conclusion cannot be reached without knowledge of the consumption plan a given bondholder is trying to finance. Monetary policy has no effect on bondholders whose dividend streams initially match the difference between their target consumption and other sources of income. Accommodative monetary policy benefits households who hold long-term bonds to finance short-term consumption, through the capital gains it generates. It hurts households who finance a long consumption stream with short-term bonds, by lowering the rates at which they reinvest their wealth.[15] Unhedged interest rate exposures constitute the welfare-relevant metric for the impact of real interest rate changes on households.

Example 2 (Purely transitory change)**.** Suppose one-off unexpected inflation revises all prices by $\frac{dP_t}{P_t} = \frac{dP}{P}$ for $t \geq 0$, the real interest rate changes for one period only ($\frac{dq_t}{q_t} = -dr$ for $t \geq 1$), and income and wages change at $t = 0$ only by $dy_0 = dy$, $dw_0 = dw$. Dropping all date-0 subscripts for ease of notation, I obtain

$$\begin{aligned} d\Omega &= dy + ndw + \sum_{t\geq 1} q_t \left({}_{-1}URE_t\right)(-dr) - \sum_{t\geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0}\right)\frac{dP}{P} \\ &= dy + ndw + URE \cdot dr - NNP \cdot \frac{dP}{P} \end{aligned} \tag{5}$$

In (5), the household's net nominal position is defined as the market value of his nominal liabilities: $NNP \equiv \sum_{t\geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0}\right)$. The term that appears as the relevant balance-sheet exposure to a transitory dr change is the unhedged interest rate exposure for date zero,

$$URE \equiv {}_{-1}URE_0$$

as can be seen by using the intertemporal budget constraint[16]

$$URE + \sum_{t\geq 1} q_t \left({}_{-1}URE_t\right) = 0 \tag{6}$$

A temporary rise in the real interest rate benefits a household with a short-term unhedged saving need—such as a holder of short-term financial assets. The exact present-value balance-sheet gain is given by $URE \cdot dr$.

Example 3 (Permanent change)**.** Consider an unexpected increase in the inflation rate, $\frac{dP_t}{P_t} = td\pi$,

[15] This remark echoes the prescriptions of the literature on long-term portfolio choice, which, following the pioneering work of Merton (1971), argues that long-horizon investors should invest in long-term inflation indexed bonds (Campbell and Viceira, 2002). My analysis focuses on ex-post wealth effects rather than ex-ante portfolio choice.

[16] An alternative way to see the role of the date-0 unhedged interest rate exposure is to use future value normalization for discount rates: $dq_0 = q_0 dr$. After rediscounting to express the wealth change in present value, the relevant term in $d\Omega$ is $\frac{1}{q_0}\left[q_0\left({}_{-1}URE_0\right)dr\right] = URE \cdot dr$

and a permanent change in the real interest rate, $\frac{dq_t}{q_t} = -tdr^\ell$, for $t \geq 1$. I obtain:

$$d\Omega = \underbrace{\sum_{t\geq 1} tq_t \left({}_{-1}URE_t\right)}_{\equiv -URE^\ell} \left(-dr^\ell\right) - \underbrace{\sum_{t\geq 0} tQ_t \left(\frac{{}_{-1}B_t}{P_0}\right)}_{\equiv NNP^\ell} d\pi \tag{7}$$

It can be useful to write the exposure of a balance sheet to a permanent change in inflation as $NNP^\ell = NNP \cdot D^N$, where $D^N \equiv \frac{NNP^\ell}{NNP}$ is the *duration* of the position. On the other hand, the present value of the time path of unhedged interest rate exposures is zero by (6). Appendix B.1 presents an example where URE^ℓ, as defined in (7), has the same sign as URE and is much larger in magnitude, though this need not be true in general.

From the welfare change to the behavioral response. Doepke and Schneider (2006a) calculate NNP and NNP^ℓ for various groups of U.S. households and show that these numbers are large and heterogenous in the population: they are very positive for rich, old households and negative for the young middle class with fixed-rate mortgage debt. Theorem 1 shows that these numbers are not only relevant for welfare, but also *for the predicted behavioral response* to these inflation scenarios. For example, in partial equilibrium, before real interest rates adjust and induce intertemporal substitution, an agent's consumption response to an unexpected increase in the price level is given, to first order, by $MPC \cdot NNP \cdot \frac{dP}{P}$.

Example 4 (Purely transitory change, separable utility)**.** Assume that utility is separable over time and that labor is supplied inelastically, $U(\{c_t\}) = \sum_{t\geq 0} \beta^t u(c_t)$. Then Appendix C.3 shows that the consumption response at date 0, $dc \equiv dc_0$, is given, to first order, by

$$dc \simeq \underbrace{MPC \cdot \left(dy + ndw + URE \cdot dr - NNP\frac{dP}{P}\right)}_{\text{Wealth effect}} - \underbrace{\sigma c\,(1 - MPC)\,dr}_{\text{Substitution effect}} \tag{8}$$

where σ is the local elasticity of intertemporal substitution:

$$\sigma \equiv -\frac{u'(c)}{cu''(c)} \tag{9}$$

A temporary increase in the real interest rate lowers consumption demand via intertemporal substitution, and increases it via a wealth effect for an agent with positive URE (respectively, decreases it for an agent with negative URE). Note that the Hicksian elasticity is scaled down from the Frisch elasticity σ by a $1 - MPC$ term (this is a general result for separable preferences; see Houthakker, 1960).

Example 5 (Mortgage type and consumption wealth effects)**.** Theorem 1 predicts that adjustable-rate mortgage holders lower their consumption by more than fixed-rate mortgage holders in response to increases in interest rates because of a wealth effect. Appendix B develops this argument in detail using a numerical example, shows the accuracy of the approximation implied by theorem 1 for different interest-rate scenarios, and proposes a structural interpretation to regressions that compare households with exogenously different mortgage types.

Even though theorem 1 assumes no uncertainty and perfect foresight, it applies directly to environments with uncertainty but where markets are complete, *except* for the shock that is unexpected (all summations are then over states as well as dates). An important feature of all these environments is that the marginal propensity to consume, MPC, is the same out of all forms of wealth ($\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial W}$): a dollar received today has the same impact on consumption as a hypothetical amount received in the future, provided that amount is worth a dollar in present value terms. However, a key result of the next section is that theorem 1—and in particular equation (8)—extends to environments with incomplete markets and idiosyncratic income uncertainty.

2.3 The consumption response to shocks under incomplete markets

A large empirical literature, cited in the introduction, measures the marginal propensity to consume out of transitory income shocks. In this section, I show that the theoretical MPC out of current income ($MPC = \frac{\partial c}{\partial y}$) remains a key sufficient statistic for predicting behavior with respect to other changes in consumer balance sheets. I first present the general framework under which my results can be derived, and then present theorem 2 under the case of inelastic labor supply. I develop extensions in appendix C.3.

I consider a dynamic, incomplete-market partial equilibrium consumer choice model. The consumer faces an idiosyncratic process for real wages $\{w_t\}$ and unearned income $\{y_t\}$, and has utility function over the sequence $\{c_t, n_t\}$, which has an expected utility form and is separable over time:

$$\mathbb{E}\left[\sum_t \beta^t U\left(c_t, n_t\right)\right] \tag{10}$$

The horizon is still not specified in the summation. As we will see, it will only influence behavior through its impact on the MPC. To model market incompleteness in a general form, I assume that the consumer can trade in N stocks and as well as in a nominal long-term bond. In period t, stocks pay real dividends $\mathbf{d}_t = (d_{1t} \dots d_{Nt})$ and can be purchased at real prices $\mathbf{S}_t = (S_{1t} \dots S_{Nt})$; the consumer's portfolio of shares is denoted by θ_t. The long-term bond is modeled as in Hatchondo and Martinez (2009): it can be bought at time t at price Q_t and is a promise to pay a geometrically declining nominal coupon with pattern $\left(1, \delta_N, \delta_N^2, \dots\right)$ starting at date $t+1$ and until the horizon end-point. The household's budget constraint at date t is therefore

$$P_t c_t + Q_t\left(\Lambda_{t+1} - \delta_N \Lambda_t\right) + \theta_{t+1} \cdot P_t \mathbf{S}_t = P_t y_t + P_t w_t n_t + \Lambda_t + \theta_t \cdot \left(P_t \mathbf{S}_t + P_t \mathbf{d}_t\right)$$

Each period, a borrowing limit may restrict trading. I write this constraint with the generic form

$$\mathbb{S}\left(\Lambda_{t+1}, \theta_{t+1}; P_t, Q_t, \mathbf{S}_t\right) \geq 0 \tag{11}$$

and pay special attention to borrowing limits that have the form

$$\mathbb{S}^*\left(\Lambda_{t+1}, \theta_{t+1}; P_t, Q_t, \mathbf{S}_t\right) \equiv Q_t \Lambda_{t+1} + \theta_{t+1} \cdot P_t \mathbf{S}_t + \frac{\overline{D} P_t}{R_t} \geq 0 \tag{12}$$

for some $\overline{D}$, where R_t is the real interest rate at t. $\mathbb{S}^*$ is a natural specification in that it restricts the real market value of all claims viewed from date $t+1$ to be bounded below by a constant $-\overline{D}$.

After dividing through by the price level at time t, defining real bond positions as $\lambda_t \equiv \frac{\Lambda_t}{P_{t-1}}$ and writing $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ for the inflation rate between $t-1$ and t, the budget constraint becomes

$$c_t + Q_t\left(\lambda_{t+1} - \delta_N \frac{\lambda_t}{\Pi_t}\right) + (\theta_{t+1} - \theta_t)\cdot \mathbf{S}_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot \mathbf{d}_t$$

Define the household's net nominal position at t as the real market value of his nominal assets:

$$NNP_t \equiv (1 + Q_t \delta_N)\frac{\lambda_t}{\Pi_t}$$

and the time-t unhedged interest rate exposure as:

$$URE_t \equiv y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot \mathbf{d}_t - c_t = Q_t\left(\lambda_{t+1} - \delta_N \frac{\lambda_t}{\Pi_t}\right) + (\theta_{t+1} - \theta_t)\cdot \mathbf{S}_t$$

Note that, to the extent that the portfolio choice problem has a unique solution at date $t-1$, URE_t is uniquely defined in each state at time t. Suppose that at a given time 0, the consumer is making an unconstrained choice between consumption, hours worked and investment into all available assets, without hitting the constraint in (11). His optimization problem can be represented using the recursive formulation

$$\max_{c,n,\lambda',\theta'} U(c,n) + \underbrace{\beta \mathbb{E}\left[V\left(\lambda',\theta';y',w',Q',\Pi',\mathbf{d}',\mathbf{S}'\right)\right]}_{\equiv W(\lambda',\theta')}$$

$$\text{s.t.} \qquad c + Q\left(\lambda' - \delta_N \frac{\lambda}{\Pi}\right) + \left(\theta' - \theta\right)\mathbf{S} = y + wn + \frac{\lambda}{\Pi} + \theta \mathbf{d} \tag{13}$$

Note that the function V corresponds to the value from optimizing given a starting real level of bonds λ' and shares θ', and includes the possibility of hitting future borrowing constraints.

Consider the predicted effects on consumption resulting from a simultaneous unexpected change in unearned income dy, the real wage dw, the price level $\frac{dP}{P} = \frac{d\Pi}{\Pi}$ and the real interest rate dr. Since the changes are purely transitory, the only effect on asset prices from the real interest rate change comes from the change in discounting: $\frac{dQ}{Q} = \frac{dS_j}{S_j} = -dr$ for $j = 1\ldots N$. Similarly, transitory changes do not alter the value from future optimization starting at (λ',θ')— that is, the function W is unchanged. One can now apply the implicit function theorem to the set of $N+2$ first-order conditions which, together with the budget constraint, characterize the solution to the problem in (13). Writing $MPC = \frac{\partial c}{\partial y}$, the partial derivatives of consumption with respect to all changes can then be expressed as a function of MPC and a limited set of constants and elasticities that depend on the utility function. One crucial such constant is the local elasticity of intertemporal substitution (9). In the simplest case where n hours are inelastically supplied

$$U(c,n) = u(c) \tag{14}$$

the result from Example 4 carries through:

Theorem 2. *Suppose utility is separable (10) and labor supply is inelastic (14). Assume that either a) the consumer is locally optimizing, or b) the consumer is at a binding borrowing constraint (MPC=1) and* $\mathbb{S} = \mathbb{S}^*$. *Then his total change in consumption dc resulting from a purely transitory, simultaneous small* *change in income dy, wages dw, the price level dP and the real interest rate dr is given by (8):*

$$dc \simeq MPC\left(dy + ndw + UREdr - NNP\frac{dP}{P}\right) - \sigma c\,(1 - MPC)\,dr \tag{8}$$

The full proof is given in appendix C.3. The intuition is simple: when the consumer is locally optimizing, MPC summarizes the way in which he reacts to all balance-sheet revaluations, income being only one such revaluation. When the borrowing limit *is* binding, his consumption adjustment depends on the way the borrowing limit changes when the shock hits. Under a specification that maintains the real value of claims next period as a constant ($\mathbb{S} = \mathbb{S}^*$), the changes in dr and dP free up borrowing capacity exactly in the amount $UREdr - NNP\frac{dP}{P}$.

Appendix C.4 shows that theorem 2 extends to situations with endogenous labor supply, and conjectures an extension to multi-period shocks. I now explore the implications of this result for the general equilibrium aggregation.

3 Aggregation and the redistribution channel

This section shows how the microeconomic demand responses derived in section 2 aggregate in general equilibrium to explain the economy-wide response to shocks in a large class of heterogenous-agent models (theorem 3). The key insight is that aggregate consumption responds to redistribution according to sufficient statistics—covariances between marginal propensities to consume and agents' balance-sheet exposures to macroeconomic aggregates—that are independent of the particular model generating these MPCs and exposures.

3.1 Fixed balance sheets in response to shocks

Taking theorem 1 as a starting point for general equilibrium aggregation following a shock requires assuming that fixed balance sheets are a useful first step for such an exercise. This assumption is restrictive in that it gives balance sheets an active role—contrary to a complete-markets world in which they are pure accounting devices—but it does not rule out the possibility of insurance. First, as noted in Example 1, balance sheets can be *arranged* so as not to generate wealth effects from price-level or real interest rate changes. Second, at the level of generality of the next section, it is possible to count ex-post insurance transfers between agents as part of the individual-level revisions in income after shocks. My assumption is that insurance beyond what is observable in balance sheets is limited enough that fixed balance sheets are indeed a more useful starting point than fixed Pareto weights in the analysis of redistributive effects of monetary policy.

In the later sections, I will progressively make the fixed-balance sheet assumption stronger to fully understand its implications. When I specify the production side of the model I will rule out insurance transfers; and in the dynamic general equilibrium model of sections 5 and 6, I will further assume that balance sheets are chosen without anticipating the shocks.[17]

[17]The assumption of fixed balance sheets is the one adopted by the overwhelming majority of papers in the financial

3.2 Aggregation result

Consider a general-equilibrium closed economy populated by $i = 1 \dots I$ heterogenous agents facing the same prices. There is no government, labor is the only factor of production, and there is no aggregate risk: a net supply of $j = 1 \dots N$ trees pays dividends in terms of consumption goods which are fixed in the aggregate (though they can vary with consumers' idiosyncratic states). Absent a government, there is no net supply of nominal assets, and absent accumulable capital, agents cannot save or borrow in the aggregate. Goods market clearing then requires that aggregate consumption be equal to aggregate income from all sources, including dividends from trees. Writing $d_{t,j}$ for the aggregate dividend of tree j, this condition (omitting $t = 0$ subscripts) is

$$C \equiv \sum_{i=1}^{I} c_i = \sum_{i=1}^{I} y_i + \sum_{i=1}^{I} w_i n_i + \sum_{j=1}^{N} d_j \equiv Y$$

Equivalently, the aggregate date-0 unhedged interest exposure of the economy is zero:

$$\sum_{i=1}^{I} URE_i = 0 \tag{15}$$

while the absence of net supply of financial assets is a statement that the aggregate net nominal position of the economy is zero:

$$\sum_{i=1}^{I} NNP_i = 0 \tag{16}$$

Consider a change that upsets the equilibrium for one period only. Aggregation of consumer responses as described by theorem 2 shows that a change in monetary policy operates on aggregate consumption via five channels, as described by the following theorem.

Theorem 3 (Aggregate demand response to a one-time shock)**.** *In a general equilibrium where heterogenous agents have inelastic labor supply and are all either locally optimizing, or subject to a borrowing constraint of the form $\mathcal{S}^*$, the changes dC, dY_i, dP, dr are linked to first order by*

$$dC \simeq \underbrace{\left(\sum_i \frac{Y_i}{Y} MPC_i\right) dY}_{\text{Aggregate income channel}} + \underbrace{\mathcal{C}\text{ov}_I\left(MPC_i, dY_i - Y_i \frac{dY}{Y}\right)}_{\text{Earnings heterogeneity channel}} - \underbrace{\mathcal{C}\text{ov}_I\left(MPC_i, NNP_i\right)}_{\text{Fisher channel}} \frac{dP}{P}$$
$$+ \left(\underbrace{\mathcal{C}\text{ov}_I\left(MPC_i, URE_i\right)}_{\text{Interest rate exposure channel}} \underbrace{- \sum_i \sigma_i \left(1 - MPC_i\right) c_i}_{\text{Substitution channel}} \right) dr \tag{17}$$

Theorem 3 shows that, in a large class of models with heterogenous agents, a small set of sufficient statistics is enough to understand and predict the first-order response of aggregate demand to a macroeconomic shock.

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frictions literature (Kiyotaki and Moore, 1997; Bernanke et al., 1999). It is well known that complete markets without additional frictions reduce the strength of these effects very significantly (Krishnamurthy, 2003; di Tella, 2013; Carlstrom, Fuerst and Paustian, 2014). Opportunities to hedge against macroeconomic shocks are likely to be more limited for households than they are for firms.

Suppose that[18]

$$\text{Cov}_I\left(MPC_i, URE_i\right) < 0 \quad \text{and} \quad \text{Cov}_I\left(MPC_i, NNP_i\right) < 0 \tag{18}$$

so that agents with unhedged borrowing requirements (respectively net nominal borrowers) have higher marginal propensities to consume than agents with unhedged savings needs (respectively net nominal asset holders). Suppose that, as a result of a transitory monetary accommodation, the real interest rate falls ($dr < 0$) and the price level unexpectedly increases ($\frac{dP}{P} > 0$).[19] Then both the interest-rate exposure channel and the Fisher channel contribute *independently* to increasing aggregate demand—the right-hand side of (17). If, as a result, output increases and this disproportionately benefits high-MPC agents, the earnings heterogeneity channel raises aggregate demand further. The covariance terms in equation (17) constitute the redistribution channel of monetary policy.

The rest of this paper explores the implications of Theorem 3. First, in section 3.3 I show it can be used to solve for endogenous variables as a function of exogenous ones under various specifications of the production side of the economy. Section 4 uses household-level cross-sectional data to measure $\text{Cov}_I\left(MPC_i, URE_i\right)$, confirming that it is negative and plausibly large. Section 5 builds a full general equilibrium model and generates the negative covariances in (18) endogenously.

3.3 Three general equilibrium applications

Theorem 3 holds irrespective of the underlying model generating MPCs and exposures at the micro level, as well as the relationship between dY, dP and dr at the macro level. Here I develop three examples of specifications of the production side of the economy. Under each, one endogenous aggregate can be solved for as a function of one exogenous one. These examples show that the interest-rate exposure channel is always a key component of the elasticity of aggregate demand to a change in the real interest rate, and preview a few of the results from my calibrated model in sections 5 and 6. I find it useful to introduce the following definitions.

Definition 2. The *redistribution elasticities of aggregate demand* to the real interest rate $\mathcal{E}_r$ and the price level $\mathcal{E}_P$ are defined, respectively, as

$$\mathcal{E}_r \equiv \frac{\mathcal{C}\text{ov}_I\left(MPC_i, URE_i\right)}{\sum_i c_i} = \text{Cov}_I\left(MPC_i, \frac{URE_i}{\mathbb{E}_I\left[c_i\right]}\right)$$
$$\mathcal{E}_P \equiv -\text{Cov}_I\left(MPC_i, \frac{NNP_i}{\mathbb{E}_I\left[c_i\right]}\right)$$

while the average elasticity of intertemporal substitution $\overline{\sigma}$ and the Hicksian scaling factor S are

[18] For clarity, I distinguish in my notation the aggregate $\mathcal{C}\text{ov}_I\left(a_i, b_i\right) \equiv \sum_i a_i b_i - \frac{1}{I}\left(\sum_i a_i\right)\left(\sum_i b_i\right)$ from the more conventional $\text{Cov}_I\left(a_i, b_i\right) \equiv \frac{1}{I}\mathcal{C}\text{ov}_I\left(a_i, b_i\right)$ which appear in the elasticity expressions below. I also define $\mathbb{E}_I\left[a_i\right] \equiv \frac{1}{I}\sum_i a_i$.

[19] In practice, monetary accommodations tend to affect the price level with a lag, but Theorem 3 applies irrespective of the time horizon, which can be chosen to be long enough for these effects to operate.

defined as

$$\overline{\sigma} \equiv \mathbb{E}_I\left[\sigma_i \frac{(1-MPC_i)\, c_i}{\mathbb{E}_I\left[(1-MPC_i)\, c_i\right]}\right]$$
$$S \equiv \mathbb{E}_I\left[(1-MPC_i)\,\frac{c_i}{\mathbb{E}_I\left[c_i\right]}\right]$$

Example 6 (Current account response upon opening up to international trade)**.**

Consider an endowment economy with flexible prices, where individual i has a fixed endowment income Y_i. The economy is initially closed, with $C_t = Y_t$ and a current account $CA_t = Y_t - C_t = 0$ at all times. The sequence of real interest rates $\{r_0, r_1, r_2 \ldots\}$ clears the goods and asset markets. Suppose that at date 0 the economy unexpectedly opens up to international goods and asset trade, and that all agents can access international financial markets at the prevailing sequence of world interest rates $\{r_0^*, r_1, r_2, \ldots\}$. For instance, $r_0^* < r_0$ could capture a temporary savings glut in the rest of the world. Using (17), the resulting instantaneous change in the current account as a share of output is

$$\frac{dCA}{Y} = -\frac{dC}{C} \simeq -\left(\mathcal{E}_r - \overline{\sigma} S\right) dr \tag{19}$$

where $dr = \frac{r_0^* - r_0}{1+r_0}$. When $r_0^* < r_0$ and condition (18) holds so that $\mathcal{E}_r < 0$, opening up to trade creates a consumption boom and a deficit on the current account whose magnitude is given by (19) to first order.

Example 7 (Equilibrium real interest rate change in response to exogenous shocks)**.**

Consider again the economy of example 6. Assume that $t = 1$ is the terminal date at which all financial assets must be repaid; initial financial assets are arbitrary. In order to understand how the redistribution channel alters the determination of equilibrium real interest rates, consider the following two unexpected shocks at date 0. The first shock is an endowment change that affects each agent proportionally, $\frac{dY_i}{Y_i} = \frac{dY}{Y}$; it can be thought of as a total factor productivity shock that "lifts all boats equally". In equilibrium, the goods market must clear: $dC = dY$. Using (17), this induces an equilibrium real interest rate change equal to

$$dr \simeq \frac{1 - \mathbb{E}_I\left[\frac{Y_i}{Y} MPC_i\right]}{\mathcal{E}_r - \overline{\sigma} S} \frac{dY}{Y} \tag{20}$$

An increase in the aggregate endowment raises desired savings, and (20) expresses the amount by which this in turn depresses the real interest rate.

The second experiment is an unexpected increase in the price level, $\frac{dP}{P}$, that does not change agents' endowments. The equilibrium interest rate change at date 0 is now

$$dr \simeq -\frac{\mathcal{E}_P}{\mathcal{E}_r - \overline{\sigma} S} \frac{dP}{P} \tag{21}$$

Here, assuming (18) still holds, the rise in the price level creates a positive demand pressure via the Fisher channel, which is counteracted by an increase in the equilibrium real rate of interest, mitigating this pressure via the substitution channel and the interest-rate exposure channel. These two examples are an illustration of the following proposition:

Proposition 2. *A larger interest-rate exposure channel (a more negative $\mathcal{E}_r$) dampens the fluctuations in the equilibrium real rate of interest in response to exogenous shocks.*

Example 8 (Output effects of monetary policy under nominal rigidities)**.**

Consider an economy with perfectly sticky wages. Labor is the only source of income. Agent i initially supplies labor n_i; agents accommodate demand increases by working more at the going wage, with the burden of increased labor supply falling on all agents in proportion to their current level of work.[20] A perfectly competitive firm has production function $Y = \sum_{i=1}^{I} n_i$. Real wages are therefore $w_i = 1$, and individual incomes are $Y_i = n_i$.

Suppose the central bank has a steady-state policy of setting its nominal interest rate equal to the natural rate of interest and is targeting the natural level of output. Consider the consequence of a one-period deviation from this level by $d\imath$, followed by a return to the steady-state policy. Since wages are perfectly sticky and firms have constant returns to production, consumer price inflation is zero, and the nominal interest rate change creates a real interest rate change $dr = d\imath$. Since the central bank stabilizes future incomes, all resulting changes take place over one period, and Theorem 3 applies.

In this case, the Fisher channel is shut down because nominal prices do not change ($dP = 0$), and the earnings heterogeneity channel is shut down given the equiproportionate rule for demand accommodation. Hence (17) allows us to solve for the demand increase as

$$\frac{dC}{C} \simeq \frac{1}{1 - \mathbb{E}_I\left[\frac{Y_i}{Y} MPC_i\right]} \left(\mathcal{E}_r - \overline{\sigma} S\right) dr \tag{22}$$

A given fall in the real interest rate generates a larger increase in demand when $\mathcal{E}_r$ is more negative. This is summarized in the following proposition.

Proposition 3. *A larger interest-rate exposure channel (a more negative $\mathcal{E}_r$) amplifies the real effects of monetary policy shocks.*

In the special case of a representative agent ($I = 1$) with EIS σ, we have $\mathcal{E}_r = 0$, $\overline{\sigma} = \sigma$ and $S = 1 - MPC$; and equation (22) yields the well-known New Keynesian impulse response:

$$\begin{aligned} \frac{dC}{C} &\simeq -\frac{1}{1 - MPC} \sigma \left(1 - MPC\right) dr \\ &= -\sigma dr \end{aligned}$$

One can interpret the cancellation of the MPC terms as the fixed point of an infinite round of income-spending increases, involving the traditional Keynesian multiplier, $\frac{1}{1-MPC}$.

[20] I consider this model of the labor market and rationing rule for simplicity and continuity of exposition with Theorem 3. The same results obtain in a more conventional New Keynesian model with flexible wages, but require the use of a modified version of Theorem 3 that allows for substitution effects on consumption as real wages change (see section 3.4.1).

3.4 Extensions

3.4.1 Elastic labor supply

While theorem 3 applies to cases in which labor supply is inelastic or, by extension, to those in which all agents are off their labor supply curves, it can readily be extended to include elastic labor supply. These extensions are useful, for example, when the model has a standard New Keynesian production side, with sticky prices but flexible wages. Changes in real wages induce substitution effects which are not accounted for in equation (17).

The extension to a case where preferences are separable between consumption and leisure is straightforward. Write ψ_i for agent i's Frisch elasticity of labor supply. Theorem $2'$ in appendix C.4 shows that expression (17) obtains with two simple adjustments: one for the fact that the income term now includes the substitution response of hours, $dY_i = dy_i + (1+\psi_i)\, n_i dw_i$, and another scaling up MPC in the intertemporal substitution term to take into account the fact that part of every increase in income is used on leisure: the new expression for the substitution channel is $\sum_i \sigma_i \left(1 - MPC_i \left(1 + \frac{\psi_i}{\sigma_i}\frac{w_i n_i}{c_i}\right)\right) c_i$.

Under non-separable preferences, increases in hours worked change the marginal utility of consumption. When preferences have the particular form of complementarity embedded in $U(c,n) = u(c - v(n))$, appendix theorem $2''$ shows that a new term appears reflecting the increase in agents' desired consumption following an increase in hours:

$$dC \simeq \underbrace{\left(\sum_i \frac{Y_i}{Y} MPC_i\right) dY}_{\text{Aggregate income channel}} + \underbrace{\mathcal{C}\text{ov}_I\left(MPC_i, dY_i - Y_i\frac{dY}{Y}\right)}_{\text{Earnings heterogeneity channel}} - \underbrace{\mathcal{C}\text{ov}_I\left(MPC_i, NNP_i\right)}_{\text{Fisher channel}}\frac{dP}{P}$$
$$+ \left(\underbrace{\mathcal{C}\text{ov}_I\left(MPC_i, URE_i\right)}_{\text{Interest rate exposure channel}} - \underbrace{\sum_i \sigma_i \xi_i \left(1 - MPC_i\right) c_i}_{\text{Substitution channel}}\right) dr + \underbrace{\sum_i \psi_i \left(1 - MPC_i\right) n_i dw_i}_{\text{Complementarity channel}} \tag{23}$$

where $0 < \xi_i < 1$ is a constant. The complementarity channel that arises in (23) has been argued to be reasonable to explain, for example, the observed hump shapes in the life-cycle profile of earnings and consumption (Heckman, 1974; Aguiar and Hurst, 2005).

3.4.2 Outside assets and trading with other sectors

The market clearing equations (15) and (16), which can be rewritten in terms of cross-sectional means, $\mathbb{E}_I[URE_i] = \mathbb{E}_I[NNP_i] = 0$, respectively state that the unhedged interest rate exposure and net nominal positions of the household sector must be zero. Equivalently, unexpected real interest rate and price-level changes create *pure redistribution* within the household sector. There are reasons to expect these equalities to fail in the data. It is useful to reflect upon why this might be true and discuss how this may alter the aggregate predictions from the model.

Doepke and Schneider (2006a) find that the net nominal position of U.S. households is positive. This means households tend to lose in the aggregate, mostly to the benefit of the government sector, from an unexpected rise in inflation. Similarly, there are reasons to expect to find a positive aggregate URE in the data. The main one is that the household sector tends to be maturity mismatched, holding relatively short-term assets (deposits) and relatively long-term liabilities (fixed-rate mortgages); this is the natural counterpart to the reverse situation in the banking sector. In addition, in periods where the government is increasing its debt and has large flow borrowing requirements, these flows must be financed and households are natural counterparts for them.[21]

Since households are the ultimate claimants on the financial sector and the government, gains to these sectors that occur as a result of lower real interest rates or unexpected inflation must ultimately be rebated back to them. Consider the case of a purely Ricardian model, such as a simple Real Business Cycle model. There, we know that the trading plan between the household and the government is irrelevant. If the government happens to be a flow borrower (have negative URE) when a shock takes place which results in lower real interest rates, this creates a present-value gain to the government, and a lump-sum transfer to the representative household must take place to ensure that both agents' present-value budget constraints are still satisfied. The same holds true for the maturity-mismatched financial sector, which might rebate gains from lower interest rates through lower fees or higher dividends.[22]

When $\mathbb{E}_I [URE_i] > 0$ and strictly no transfer takes place, the redistribution elasticity $\mathcal{E}_r$ needs to be replaced by a term I call the "no rebate" elasticity,

$$\mathcal{E}_r^{NR} = \mathbb{E}_I [MPC_i] \frac{\mathbb{E}_I [URE_i]}{\mathbb{E}_I [c_i]} \tag{24}$$

However, given the logic that gains and losses must be rebated, this term is only a partial-equilibrium upper bound. An agnostic procedure is to assume a uniform rebating rule, in which case the covariance formula applies. Rebate rules might in practice target higher or lower MPC agents, so that the precise number may depart from the covariance expression in either direction. In the measurement part that follows, I use $\mathcal{E}_r$ as my benchmark, and compute the no-rebate $\mathcal{E}_r^{NR}$ as a robustness check.

Open-economy considerations can strengthen this rebating logic further. In the international financial accounts of the United States, the Rest of the World has long liabilities (FDI) and shorter assets (Treasury securities)—that is, its aggregate URE is positive—and therefore it tends to lose when interest rates fall, creating additional gains that must ultimately accrue to households.

Although my results in the next sections suggest otherwise, it is interesting to note the theoretical possibility that the interest-rate exposure term—either $\mathcal{E}_r$ or $\mathcal{E}_r^{NR}$ if none of the gains to

[21] In the United States, households do not hold many government securities directly. The major counterparts to the recent \$8trn increase in federal debt have been the U.S. financial sector, the Federal Reserve system, and the Rest of the World. See Bank of England (2012) and McKinsey Global Institute (2013) for sectoral studies of winners and losers from the current monetary policy environment.

[22] In a world where financial frictions are important, the "stealth recapitalization" of the banking sector (Brunnermeier and Sannikov, 2012, 2014) from lower real interest rates may have large additional effects on aggregate demand, notably via investment, beyond the ones induced by a rebating of gains to the household sector.

other sectors are rebated to households—may not only be positive, but larger than $\overline{\sigma}S$. In this case, real interest rate increases *raise* aggregate consumption demand, altering significantly the conventional understanding of how monetary policy operates.[23]

4 Measuring the redistribution elasticity of aggregate demand

A clear picture that emerges from section 3 is that the overall elasticity of aggregate demand to changes in real interest rates is, in general, the sum of a redistribution component and an intertemporal substitution component. In a representative-agent model, the latter is the only channel of transmission from real interest rates to consumption. The magnitude of this substitution channel depends crucially on the size of the elasticity of intertemporal substitution (EIS). In this section I treat the EIS as a parameter σ and ask how large the redistribution component (which I call the interest rate exposure channel) is, *relative* to the substitution channel at any given σ.

In order to do this, I make several simplifications. I maintain my focus on a purely transitory change in the real interest rate, for which theorem 3 established the existence of a sufficient statistic for the interest rate exposure channel. Longer-run changes in real interest rates tend to increase both the intertemporal substitution term and the redistribution term (see appendix B and section 6), so the relative magnitudes that I obtain here plausibly extrapolate to these changes as well. I assume that all agents have the same EIS σ and supply labor inelastically—cases with endogenous labor supply reduce the size of the substitution channel, so that this assumption provides a lower bound on the relative magnitude of the redistribution component. Manipulating (17), I find that the *partial elasticity* of demand to the real interest rate, $\frac{\partial C}{C}\frac{1}{\partial r}$, is given by[24]

$$\left(\underbrace{\mathrm{Cov}_I\left(MPC_i, \frac{URE_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\mathcal{E}_r} - \sigma \underbrace{\mathbb{E}_I\left[\left(1 - MPC_i\right)\frac{c_i}{\mathbb{E}_I\left[c_i\right]}\right]}_{S}\right) \tag{25}$$

A key finding of this section is that the redistribution elasticity of aggregate demand $\mathcal{E}_r$ is *negative*, so that the redistribution effect and the substitution effect act in the same direction. I further quantify the magnitude of (the absolute value of) $\mathcal{E}_r$ so that it can be compared to $\sigma S < \sigma$. I also measure the "no rebate" (NR) version of this elasticity, $\mathcal{E}_r^{NR}$, defined in (24), which assumes that gains and losses to other domestic agents are not rebated to households and is therefore a lower bound for the effects of redistribution on aggregate demand (see the discussion in section 3.4).

Since all quantities in (25) are measurable at the household or at the group level *except* for the

[23] This theoretical possibility is sometimes mentioned in economic discussions of monetary policy. See Raghuram Rajan ("Interestingly [...] low rates could even hurt overall spending"), "Money Magic", Project Syndicate, November 11, 2013; or Charles Schwab "Raise Interest Rates, Make Grandma Smile", Wall Street Journal, November 20, 2014

[24] This is only a partial elasticity since it holds other macroeconomic aggregates fixed. As illustrated in section 3.3, in general equilibrium, changes in real interest rates are accompanied by other adjustments in macroeconomic quantities which also have effects consumption. For example, in the case of monetary policy shocks in a New Keynesian model, the central-bank-induced change in the real interest rate creates multiplier effects, and the general equilibrium elasticity of demand to dr is larger than the partial elasticity (see Example 8).

EIS σ, a useful way of organizing the results is to determine the value of the EIS that would make the substitution and the redistribution effects equal in magnitude. I call this value σ^*; it is defined by

$$\sigma^* \equiv \frac{-\mathcal{E}_r}{S} = \frac{-\mathrm{Cov}_I\left(MPC_i, \frac{URE_i}{\mathbb{E}_I[c_i]}\right)}{\mathbb{E}_I\left[(1 - MPC_i)\frac{c_i}{\mathbb{E}_I[c_i]}\right]}$$

Knowing σ^* allows us to say how much a representative-agent model should *add* to its assumed EIS of σ to correctly predict the magnitude of the economy's response to one-time shocks. Taking full account of the redistribution channel is more complex than assuming that the representative-agent is "more elastic" with respect to real interest rate changes—both because longer-term changes to interest rates do not simply scale the redistribution and the substitution component, and because the redistribution component is a function of other model primitives such as the market structure. Nevertheless, the value of σ^* is a useful starting point in evaluating the importance of redistribution in the transmission of shocks.

The literature has used different ways to measure the marginal propensity to consume out of transitory income shocks (see Jappelli and Pistaferri, 2010 for a survey). It is first important to determine what qualifies as a "transitory income shock" from the point of view of the theory as outlined above. A simple approach has been to ask households to self-report the part of any hypothetical windfall that they would immediately spend. This has the benefit of circumventing the general equilibrium issue of determining the source of this windfall. The Italian Survey of Household Income and Wealth (SHIW) contains such a question in 2010 (Jappelli and Pistaferri, 2014), and I use data from this survey as my first measure of MPC.

One concern with self-reported answers to hypothetical situations is that they are not informative about how households would actually behave in these situations. For this reason, the literature has looked at cleanly identified settings allowing estimation of MPC from actual behavior. Perhaps the best setting such settings are the large-scale 2001 and 2008 U.S. tax rebates, whose timing of receipt was randomized (Johnson et al., 2006; Parker et al., 2013). Since these studies exploit variation in timing for a policy announced ahead of time, they identify the MPC out of an *expected* increase in income. This is, in general, different from the theoretically-consistent MPC out of an unexpected increase. In a benchmark incomplete market model, unless borrowing constraints are binding and are not adjusting in response to the expected tax rebate, two consumers that differ only in their timing of receipt should adjust their consumption profile by similar amounts when they receive the news (reflecting the net gain from the present value of the transfer as well as Ricardian offsets) and not react differentially when they receive the transfer. However, to the extent that borrowing constraints are rigid and binding, or if households are surprised by the receipt despite its announcement, the estimation gets closer to the MPC that is important for the theory; and in general provides a lower bound for it.[25] I use the data from the 2001 rebates collected in

[25]See Kaplan and Violante (2014) for another discussion of the interpretation of the coefficients estimated by JPS, and a model in which even households with positive total assets can be at a binding limit on their liquid transactions account, and modify their consumption upon receipt but not upon news of the transfer.

the U.S. Consumer Expenditure Survey (CEX) and analyzed in Johnson et al. (2006) as my second measure of MPC.

Section 4.1 explains how one should conceptually measure a theory-consistent URE from household-level survey data. The data requirements are very stringent: one needs consumption, income and detailed information about assets and liabilities at the household level. Surveys such as the SHIW and the CEX do not measure assets and liabilities very precisely, so that there is likely to be some measurement error in URE. For example, more detailed measures of U.S. household wealth, such as those from the Federal Reserve's Survey of Consumer Finances, tend to show that asset positions (and hence UREs) are much more dispersed than what is reported in the CEX. This is likely to bias my estimate of σ^* downwards. In section 4.2 I present the two datasets, and in sections 4.3 and 4.4 I measure my key moments $\mathcal{E}_r^{NR}$, $\mathcal{E}_r$ and σ^* using the two procedures for calculating MPC just outlined.

4.1 Conceptual measurement issues

As defined in section 2.2, URE_i measures the total resource flow that a household i needs to invest over the first period of his consumption plan. From the surveys, I construct URE_i as

$$URE_i = Y_i - C_i + B_i - D_i \tag{26}$$

where Y_i is income from all sources including dividends and interest payments, C_i is consumption including expenditures on durable goods[26] as well as mortgage payments and installments on consumer credit, and B_i and D_i represent, respectively, asset and liability stocks that mature over the period.

Even though $\mathcal{E}_r$ is a unitless number, the choice of time units is important: MPC needs to be measured over a period consistent with the choice of time units for the numerator and denominator of $\frac{URE_i}{\mathbb{E}_I[c_i]}$. Ideally, all measurement would be done over a quarter, which is the frequency at which models analyzing monetary policy are calibrated. This is what I do with the CEX.[27]

Consumption, income and MPC in the SHIW are only available at annual frequency. Because it is not obvious how to translate an annual measure of MPC into a quarterly one, I measure $\mathcal{E}_r$ once at annual frequency, and once at quarterly frequency where I use $MPC^Q = \frac{MPC^A}{3}$ to reflect the tendency of households with precautionary savings motives to spend more of their income in the first quarter of a one-off transfer receipt.[28]

[26] The theory presented in section 2 can be extended to include durable goods. Wealth effects depend on a measure of unhedged interest rate exposures that subtracts durable *expenditures*. This is intuitive: a consumer who has a plan to borrow large amounts to finance a durable good is hurt by a rise in real interest rates that raises the financing cost. However, the presence of durable goods also creates additional substitution effects between nondurable and durable consumption, since a real interest rate increase raises the user cost of durables and makes consumers substitute away from them. This effect only mitigates the substitution effect on nondurable consumption further.

[27] Households are interviewed every quarter. Although they are asked to report a monthly break-down of expenditure by month, most researchers aggregate the data back to quarterly frequency to prevent recall-driven serial correlation in consumption.

[28] This theoretical pattern is also consistent with empirical behavior (see the dynamic specifications of Johnson et al., 2006; Parker et al., 2013 and Broda and Parker, 2014).

Given the limited information regarding asset maturities in both the SHIW and the CEX, maturing asset stocks (B_i) are difficult to determine precisely. In my quarterly calibrations, I treat time and savings deposits as maturing in the quarter. In annual calibrations, I treat them as maturing within the year. These are likely to be good effective lower and upper bounds for deposit durations. Doepke and Schneider (2006a) calculate that, since the beginning of the 2000s, the average duration of U.S. financial assets has been around 4 years. Whenever I have good information on assets beyond deposits, I count one-fourth of the stock towards B_i in the annual calibration (respectively one-sixteenth in the quarterly calibration). I treat adjustable-rate mortgages, just as deposits, as maturing liability stocks within the quarter or within the year depending on the calibration. Fixed-rate mortgages are not counted additionally in B_i since mortgage payments—which include amortization—are already subtracted from URE as part of the consumption measure.

Finally, I assume away timing differences in the reporting of consumption and income in my calculation of URE. To the extent that these create noise in my URE estimate, they may tend to raise the observed cross-sectional dispersion in URE but will also likely reduce its covariance with MPC, so that it is unclear that it will impart a clear directional bias in my estimate of $\mathcal{E}_r$.

4.2 Data

Survey of Household Income and Wealth 2010 Italy's 2010 Survey of Household Income and Wealth (SHIW) contains a question which can be used as an empirical measure of MPC:

> "Imagine you unexpectedly receive a reimbursement equal to the amount your household earns in a month. How much of it would you save and how much would you spend? Please give the percentage you would save and the percentage you would spend."

Jappelli and Pistaferri (2014) present a detailed analysis of the data and of the empirical determinants of MPC.[29]

Consumer Expenditure Survey, 2001-2002 (JPS sample) My data for the Consumer Expenditure Survey comes from the Johnson et al. (2006) (JPS) dataset, which I merged with the main survey data to add information on total consumption expenditures, as well as assets and liabilities separately. The dataset covers households with interviews between February 2001 and March 2002. I restrict my sample to households who have income information. This leaves me with 9,443 interviews of 4,583 households.

I define maturing assets are the sum of total assets in checking, brokerage and other accounts, savings, S&L, credit unions and other accounts as well as one fourth of the amount in U.S. savings

[29]Note that the time frame for MPC is not specified in the question, as issue that is left unresolved in Jappelli and Pistaferri (2014). A follow-up question in the 2012 SHIW separates durable and nondurable consumption, and specifies the time frame as a full year. The equivalent "MPC" out of both durable and nondurable consumption has close to the same distribution as that of MPC in the 2010 SHIW (respective means are 47 in 2010 and 45 in 2010) which suggests that households tended to assume that the question referred to the full year.

	SHIW		CEX	
Variable	mean	n.s.d.	mean	n.s.d.
Income from all sources (Y_i, per year)	36,114	0.90	45,617	1.01
Consumption incl. mortgage payments (C_i, per year)	27,976	0.61	36,253	0.79
Deposits and maturing assets (B_i)	14,200	1.45	7,147	0.77
ARM mortgage liabilities and consumer credit (D_i)	6,228	1.03	2,872	0.22
Unhedged interest rate exposure (URE_i, per year)	16,110	1.92	13,639	1.27
Unhedged interest rate exposure (URE_i, per Q)	10,007	7.07	6,616	3.39
Marginal Propensity to Spend (annual)	0.47	0.35		
Count	7,951		9,443	

"mean" is the sample mean computed using sample weights (in € for SHIW; current USD for CEX)

"n.s.d" is the normalized standard deviation, $sd_I\left(\frac{X_i}{\mathbb{E}_I[C_i]}\right)$ for $X_i = Y_i, C_i, B_i, URE_i$ and $sd_I(MPC_i)$ for MPC

Table 1: Main summary statistics from the two datasets

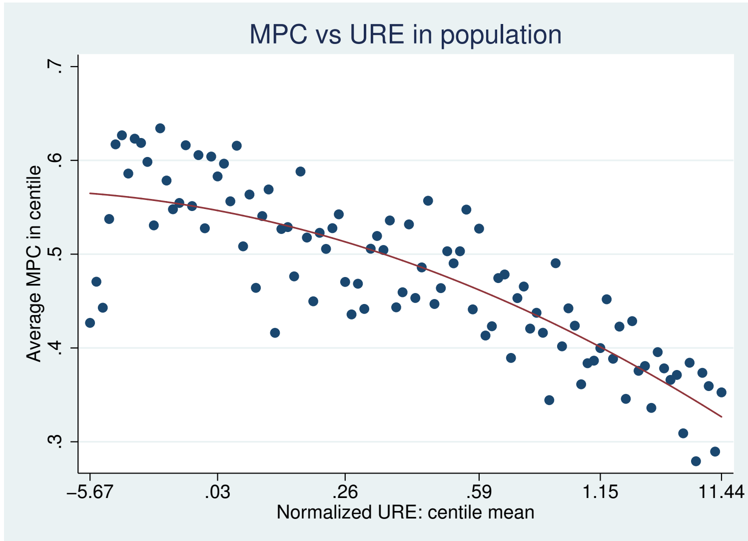
bonds. Because the CEX has poor data on mortgages, I count all of "total amount owed to creditors" from the fifth interview towards D_i is order to calculate my unhedged interest rate exposure measure.

Summary statistics Table 1 reports the main relevant summary statistics from the two datasets, with appendix A.2 providing further details. For each dataset, the first column reports sample means in current euro for the SHIW (respectively current dollars for the CEX). Consumption is always below income at the mean. This is in part because the coverage of the consumption data in the surveys is less than the whole of personal consumption expenditures from national accounts, in part because consumption tends to be underreported due to imperfect recall. The second column reports a normalized population standard deviation measure, where all variables except for MPC are normalized by the sample mean of consumption, which allows comparison of cross-sectional dispersions across surveys and is consistent with the normalization implicit in the definition of $\mathcal{E}_r$ in (25).

4.3 The redistribution elasticity in the SHIW

Figure 1 illustrates that the empirical correlation between MPC and URE is negative in the SHIW. This is reminiscent of the finding from Jappelli and Pistaferri (2014) that MPC covaries with net liquid assets in this survey. A direct implication is that $\mathcal{E}_r < 0$: falls in interest rates increase demand via the redistribution channel.

Table 2 computes the key moments $\mathcal{E}_r^{NR}$, $\mathcal{E}_r$ and σ^* using the household-level information. Sampling uncertainty is taken into account using the survey's sampling weights. The main quantitative result is that, depending on the frequency at which the estimation is done, σ^* is around 0.1. In other words, and using my preferred, annual-frequency estimate, I find that *in Italy, the redistribution channel explains as much of the demand response to changes in real interest rates as the substitution channel if the EIS is equal to 0.12.* This baseline number is regarded by some as a plausible



The figure presents the average reported MPC in each percentile of URE

Figure 1: Correlation between MPC and URE in the population

Time Horizon		Annual		Quarterly	
Parameter		Estimate	95% C.I.	Estimate	95% C.I.
No-rebate elasticity	$\widehat{\mathcal{E}_r^{NR}}$	0.21	[0.17; 0.23]	0.15	[0.12; 0.19]
Redistribution elasticity	$\widehat{\mathcal{E}_r}$	-0.06	[-0.09; -0.04]	-0.07	[-0.10; -0.03]
Scaling factor	$\widehat{S}$	0.55	[0.53; 0.57]	0.85	[0.83; 0.87]
Equivalent EIS	$\widehat{\sigma^*} = -\frac{\widehat{\mathcal{E}_r}}{\widehat{S}}$	0.12	[0.06; 0.17]	0.08	[0.03; 0.12]

All statistics computed using survey weights

Table 2: Measures of $\mathcal{E}_r^{NR}$, $\mathcal{E}_r$ and σ^* using the SHIW

value for the EIS (for example Hall, 1988, or the meta-analysis in Havránek, 2013). Hence this first measure already suggests that the redistribution channel has quantitative legs.

Combining information from tables 1 and 2, we can decompose the annual $\mathcal{E}_r$ measure as

$$\mathcal{E}_r = \underbrace{\mathrm{Corr}_I\left(MPC_i, \frac{URE_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{\simeq -0.09} \underbrace{\mathrm{Sd}_I\left(MPC_i\right)}_{0.35} \underbrace{\mathrm{Sd}_I\left(\frac{URE_i}{\mathbb{E}_I\left[c_i\right]}\right)}_{1.92}$$

The low absolute value for the correlation between MPC and URE suggests that $\mathcal{E}_r$ could plausibly be several times larger in a setting with less measurement error in MPCs and UREs. The next section takes a different approach to measuring MPCs and finds precisely this.[30]

[30] A feature of table 2 is that the no-rebate redistribution elasticity $\mathcal{E}_r^{NR}$ is positive, in other words, the negative correlation between MPC and URE is not enough, in this case, to overwhelm the positive average URE in the data. In addition to all the reasons given in section 3.4 for why $\mathcal{E}_r$ is a more reasonable calculation than $\mathcal{E}_r^{NR}$ in a world where measurement is perfect, the SHIW's total consumption data is sparse. Hence average consumption is underestimated and average URE is overestimated in the survey. This problem is not as severe in the CEX which has a more comprehensive measure of consumption than the SHIW, but it is still present. My benchmark estimate $\mathcal{E}_r$ removes this bias exactly if underreporting in consumption is uncorrelated with MPC.

4.4 The redistribution elasticity from the 2001 tax rebates in the CEX

In this section I take a different route towards calculating the value of my key redistribution elasticities. Instead of relying on a survey-based measure, I compute the MPC out of the 2001 tax rebate using the Johnson et al. (2006) (JPS) procedure, stratifying by URE. I then use the estimates by bin to form a covariance. To be specific, I split the sample into J groups ranked by their URE. I then run the main JPS estimating equation

$$C_{i,m,t+1} - C_{i,m,t} = \alpha_m + \beta X_{i,t} + \sum_{j=1}^{J} MPC_j R_{i,t+1} QURE_{i,j} + u_{i,t+1} \tag{27}$$

where $C_{i,m,t}$ is the level of household i's consumption expenditures in month m and at date t, α_m are month fixed effects absorbing seasonal variation in expenditures, $X_{i,t}$ are the controls used by JPS in their main specification (age and changes in family composition), $R_{i,t+1}$ is the dollar amount of the rebate at $t+1$, and $QURE_{i,j}$ is a dummy indicating that household i's URE is in group $j = 1 \ldots J$. This procedures exploits variation in timing of the rebate across households in the same exposure group to identify the propensity to consume out of the expected one-time transfer that the stimulus payment provides.

In each URE bin, I next calculate the average normalized URE, $NURE_j$, as the average over households in group j of $\frac{URE_j}{\bar{c}}$, where $\bar{c}$ is average consumption expenditure in the sample. I finally compute my estimators as

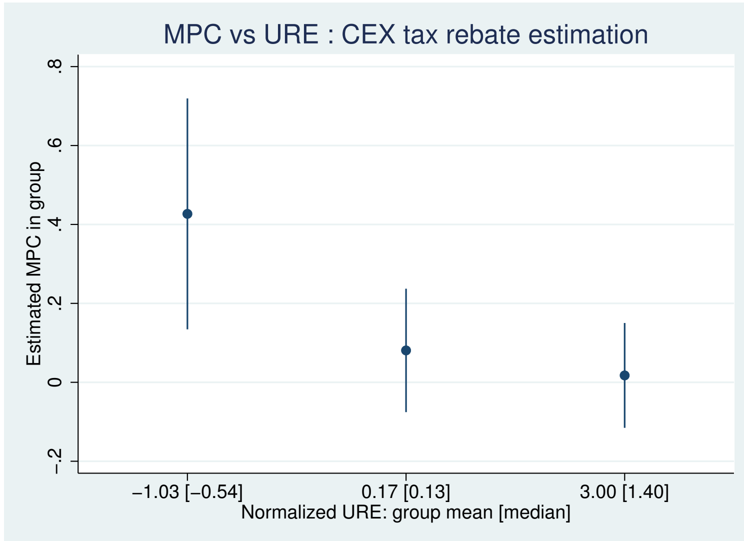
$$\begin{aligned}
\widehat{\mathcal{E}_r^{NR}} &= \frac{1}{J}\sum_{j=1}^{J} MPC_j NURE_j \\
\widehat{\mathcal{E}_r} &= \widehat{\mathcal{E}_r^{NR}} - \left(\frac{1}{J}\sum_{j=1}^{J} MPC_j\right)\left(\frac{1}{J}\sum_{j=1}^{J} NURE_j\right) \\
\widehat{S} &= 1 - \left(\frac{1}{J}\sum_{j=1}^{J} MPC_j\right)
\end{aligned}$$

where MPC_j is the point estimate in group j from (27).[31] In order to take into account sampling uncertainty, I compute the distribution of these estimators using a Monte-Carlo procedure, resampling the panel at the household level with replacement.

Figure 2 illustrates the procedure for $J = 3$, using expenditures on food as the headline consumption estimate. There is a clear gradient in MPC, with households with lower (and on average negative) URE displaying a much higher marginal propensity to consume, confirming my claim that $\mathcal{E}_r < 0$.

Table 3 repeats the exercise of table 2, where this time the moment estimation is done at the group and not the individual level. The quantitative results are large. Using food consumption as the source of MPC estimation in (27), σ^* is estimated to be 0.3, which is well within the range

[31] Note that I simply take $\widehat{S}$ to be the sample counterpart to $1 - \mathbb{E}_I[MPC]$. The procedure cannot simultaneously give an estimate of the covariance between MPC and consumption. In the SHIW data, the difference between average MPC and consumption-weighted MPC is small, so this is unlikely to significantly affect the value of σ^*.



The figure presents the estimated MPC, together with 95% confidence intervals, in each URE bin

Figure 2: MPC estimated in URE bins (JPS procedure, food consumption)

Consumption measure		Food		All nondurable	
Parameter		Estimate	95% C.I.	Estimate	95% C.I.
No-rebate elasticity	$\widehat{\mathcal{E}_r^{NR}}$	-0.12	[-0.27; 0.02]	-0.33	[-0.65; -0.02]
Redistribution elasticity	$\widehat{\mathcal{E}_r}$	-0.24	[-0.42; -0.07]	-0.64	[-0.97; -0.32]
Scaling factor	$\widehat{S}$	0.82	[0.69; 0.95]	0.56	[0.33; 0.78]
Equivalent EIS	$\widehat{\sigma^*} = -\frac{\widehat{\mathcal{E}_r}}{\widehat{S}}$	0.30	[0.05; 0.54]	1.15	[0.24; 2.07]

Confidence intervals are bootstrapped by resampling households 100 times with replacement

Table 3: Moments of the redistribution channel computed using the JPS procedure

of typical values for the EIS. Using all nondurable consumption instead, σ^* becomes as high as 1.15, suggesting that redistribution may even play a dominant role in the transmission of shocks to real interest rates. Even the no-rebate elasticity $\mathcal{E}_r^{NR}$ is negative, so that the negative correlation $\mathcal{E}_r$ is strong enough to overwhelm the effect of a positive aggregate URE. Appendix tables 8 and 9 also shows these results to be robust to using medians instead of means within URE bins, and to using any different number of bins J for 2 to 10. In every case, the correlation is very negative and the standard deviation of MPC is substantial. Even though the standard deviation of URE is only 1.27 in this dataset (table 1), these numbers combine to make σ^* large in magnitude and give the redistribution channel of monetary policy quantitative support.

4.5 Towards a calibrated general equilibrium model

In this section I proposed a quantification of the redistribution channel that operates through real interest rates. The main caveat is that this calculation is subject to a substantial degree of measurement error, as illustrated by the range of values for σ^* obtained across datasets and methods.

Section 4.1 discussed why unhedged interest rate exposures are difficult to measure: in particular, it is known that consumption tends to be measured with error in surveys, and a precise attribution of financial stocks to flow interest rate exposures is difficult without many more details on the composition of wealth (in particular, asset duration) than is available in most surveys. Another issue, given the impossibility of observing individual households' actual plans for income and consumption over the long run, is that no empirical methodology can evaluate the redistribution channel induced by a *long-term* change in real interest rates, which requires knowledge of future URE terms.

Building a fully-specified general equilibrium model gives up the benefits of the sufficient statistic approach, but it addresses these shortcomings. It allows us to evaluate the interest-rate exposure channel based on a model-consistent measure of MPC, and to provide a cross-check on the reduced-form exercise of this section. In addition, it allows us to evaluate the other components of the redistribution channel identified in theorem 3, and it allows all these components to interact in determining macroeconomic responses to shocks. This is the goal of the next sections.

5 A Huggett model with long-term nominal assets

I now build a calibrated, general equilibrium model that features heterogeneity in unhedged interest rate exposures and marginal propensities to consume. Its preferences and market structure are of the Bewley-Huggett-Aiyagari class—a benchmark for both microeconomic and macroeconomic analyses of consumption behavior (see the surveys in Heathcote, Storesletten and Violante, 2009 and Attanasio and Weber, 2010), which can generate average marginal propensities to consume in line with the empirical evidence (Carroll, Slacalek and Tokuoka, 2014). I depart from the standard model by introducing assets that are nominal and have long maturities.

I choose a simple specification for the production side of the economy, just rich enough to illustrate how the forces highlighted in theorem 3 all interact in general equilibrium. Thus, I assume that labor is the only factor of production, with preferences such that there are no wealth effects on labor supply. This assumption has great modeling appeal, because it allows for a simple aggregation result for GDP. It may also be a reasonable description of preferences for the study of cyclical phenomena such as the ones I am interested in.[32] I also assume that all claims in the economy are pure circulating private IOUs, as in Huggett (1993). Two observations motivate this choice. First, in the Flow of Funds data of the United States, total household financial liabilities and

[32]While wealth effects on labor supply are important to understand long-run trends, and even though some papers do point towards diminished labor supply in response to receipts of inheritances (Holtz-Eakin, Joulfaian and Rosen, 1993) or lottery prizes (Imbens, Rubin and Sacerdote, 2001), I am not aware of evidence that these effects act over very short horizons. Most households are plausibly not on their short-run labor supply curve: one way to capture this would be to assume that rigidities prevent labor markets from clearing instantly. In a model where labor markets do clear, such as the one I build, it is then natural to specify the absence of wealth effects as stemming from preferences. This preference specification has received support in structurally estimated macroeconomic models (Schmitt-Grohé and Uribe, 2012). A model without income effects on labor supply in the presence of heterogeneity is also considered a good benchmark in several other contexts: analyses of the Mirrleesian model of income taxation (for example Diamond, 1998), and studies of capital taxation (Aiyagari, 1995) or redistribution with incomplete markets (Heathcote, 2005; Correia, 2010).

interest-paying assets held directly are roughly balanced at any point in time (see Appendix A.1). Second, the rates of return on these assets are directly influenced by changes in monetary policy. In this model, therefore, changes in inflation or in real interest rates do not create an aggregate wealth effect on their own—only purely redistributive effects.

In order to focus on the role of individual-level heterogeneity in determining the aggregate response to shocks, I continue to consider equilibria that feature perfect foresight over macroeconomic aggregates and are perturbed by unexpected shocks. In this section I describe the model and properties of these equilibria for given initial distributions. I calibrate the model to the United States, and compute the moments of the redistribution channel identified in theorem 3. In section 6 I will consider transitional dynamics following shocks, and illustrate the quantitative importance of the redistribution channel for the determination of the natural rate of interest and for the aggregate effects of monetary policy shocks.

5.1 Environment

The model is an infinite-horizon economy with a continuum of ex-ante identical, but ex-post heterogenous households indexed by $i \in [0,1]$. Agents face idiosyncratic uncertainty with respect to their productivity $\left\{e_t^i\right\}$ and their discount factor $\left\{\beta_t^i\right\}$. The process for the idiosyncratic state $\mathbf{s}_t^i = \left(e_t^i, \beta_t^i\right)$ is uncorrelated across agents and follows a Markov chain $\Gamma\left(\mathbf{s}'|\mathbf{s}\right)$ over time. This Markov chain is assumed to have a stationary distribution $\varphi\left(\mathbf{s}\right)$, which is also the initial cross-sectional distribution of idiosyncratic states. There is no aggregate uncertainty: the path $\left\{\mathbf{S}_t\right\}$ for all macroeconomic variables is perfectly anticipated.

Household i has GHH preferences over the sequence $\left\{c_t^i, n_t^i\right\}$:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\left(\beta_t^i\right)^t u\left(c_t^i - v\left(n_t^i\right)\right)\right] \tag{28}$$

where the outside felicity function u is such that the elasticity of intertemporal substitution in net consumption is a constant σ, and the disutility function v over working hours has constant elasticity ψ,

$$u\left(g\right) = \frac{g^{1-\sigma^{-1}}}{1-\sigma^{-1}} \quad v\left(n\right) = b\frac{n^{1+\psi^{-1}}}{1+\psi^{-1}}$$

where b is a constant.

The final good that enters consumers' utility is produced with a technology

$$Y_t = \left[\int_0^1 \left(x_t^j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

where x_t^j is the quantity of intermediate good $j \in [0,1]$ used as input and $\epsilon > 1$ is the constant elasticity of substitution across goods. All intermediate goods are produced with an identical linear technology

$$x_t^j = A_t l_t^j \tag{29}$$

where $l_t^j = \int_i e_t^i n_t^{i,j} di$ is the number of efficiency units of work entering the production of good j (with $n_t^i = \int_j n_t^{i,j} dj$) and A_t is aggregate productivity.

5.2 Markets and government

Households. Households only have access one type of nominal, risk-free, long-term bond with rate of decay δ_N (see section 2.3). They are subject to an affine tax schedule on labor income. Each period, a household with productivity e_t^i seeks to maximize (28) subject to the budget constraint

$$P_t c_t^i + Q_t \left(\Lambda_{t+1}^i - \delta_N \Lambda_t^i\right) = (1-\tau) W_t e_t^i n_t^i + P_t T_t + \Lambda_t^i$$

where P_t is the nominal price of the final good, Q_t the nominal price of a bond paying the sequence of coupons $\left(1, \delta_N, \delta_N^2, \ldots\right)$ starting in period $t+1$, Λ_t^i the nominal coupon payment due at time $t+1$, τ the marginal tax rate on labor income, W_t the nominal market wage per efficient unit of work, and T_t a real lump-sum transfer from the government, common across individuals.

A borrowing constraint further limits the size of bond issuances so that the market value of real end-of-period liabilities is bounded below by a limit $\overline{D}_t$ at time t:

$$Q_t \Lambda_{t+1}^i \geq -\overline{D}_t P_t \tag{30}$$

Maximization with respect to n_t^i yields a static first-order condition for hours supplied:

$$n_t^i = \left[\frac{1}{b} e_t^i (1-\tau) \frac{W_t}{P_t}\right]^{\psi} = \left[\frac{1}{b} e_t^i w_t\right]^{\psi} \tag{31}$$

where $w_t \equiv (1-\tau) \frac{W_t}{P_t}$ is the post-tax real market wage.

Define i's *net consumption* g_t^i and the net income function z as

$$g_t^i \equiv c_t^i - v\left(n_t^i\right) \qquad z(e, w) \equiv \max_{\hat{n}} \left\{w \cdot e \cdot \hat{n} - v(\hat{n})\right\}$$

The consumer's idiosyncratic state is summarized by his real bond position $\lambda_t^i \equiv \frac{\Lambda_t^i}{P_{t-1}}$. From his point of view, the relevant components of the aggregate state are $\left(w_t, T_t, Q_t, \Pi_t, \overline{D}_t\right) \subseteq \mathbf{S}_t$, where $\Pi_t = \frac{P_t}{P_{t-1}}$ denotes the inflation rate at t. His optimization problem is characterized by the Bellman equation:

$$\begin{aligned} V_t(\lambda, \mathbf{s}) = \quad & \max_{g, \lambda'} \quad u(g) + \beta(\mathbf{s}) \mathbb{E}\left[V_{t+1}\left(\lambda', \mathbf{s}'\right) | \mathbf{s}\right] \\ & \text{s.t.} \qquad g + Q_t \left(\lambda' - \delta_N \frac{\lambda}{\Pi_t}\right) = z(e(\mathbf{s}), w_t) + T_t + \frac{\lambda}{\Pi_t} \\ & \qquad\qquad Q_t \lambda' \geq -\overline{D}_t \end{aligned} \tag{32}$$

Proposition 4. *The consumption policy function $c_t(\lambda, \mathbf{s})$ is concave in bond holdings λ, and strictly concave for λ sufficiently high that the borrowing constraint does not bind.*

Proposition 4, which follows from a result of Carroll and Kimball (1996),[33] implies that the marginal propensity to consume is declining in bonds held, generating a natural link between MPCs and asset positions which is key to the redistribution channel.

[33] See Appendix D for the proofs to all propositions of this section.

Firms. The final good is produced by a perfectly competitive firm, which takes as given the prices $\left\{P_t^j\right\}$ of intermediate goods. Profit maximization leads to a final good price of P_t, zero profits (so that it is not necessary to be specific about the firm's ownership), and isoelastic demand for intermediate goods:

$$x_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} Y_t \quad \text{where} \quad P_t \equiv \left[\int_0^1 \left(P_t^j\right)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{33}$$

Labor markets are competitive and wages are fully flexible. Every intermediate goods firm $j \in [0,1]$ produces under monopolistic competition. The firm's nominal profits in period t, when its current price is p, are given by

$$F_t^j(p) = p x_t^j(p) - W_t l_t^j(p) = \left(p - \frac{W_t}{A_t}\right)\left(\frac{p}{P_t}\right)^{-\epsilon} Y_t \tag{34}$$

I consider two assumptions on price setting. When prices are flexible, producers set them in each period and state to maximize (34). This results in an identical price across all firms in each period t, equal to

$$P_t = P_t^j = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{A_t} \tag{35}$$

I also consider the opposite assumption of perfectly sticky prices. This is meant to capture the macroeconomic adjustments that take place under nominal rigidities in the simplest possible way; under the shocks I consider, incentives to change prices vanish in the long run. Under sticky prices, firm j has a price P_t^j at time t and cannot change it. It accommodates the demand x_t^j that is forthcoming at that price by hiring workers at the going real wage, and its profits are determined by $F_t^j(P_t^j)$.

Fiscal policy. To simplify the treatment of firm ownership given the double heterogeneity of consumers and firms, I assume that the government owns all the firms. Each period, it collects their nominal profits and runs the personal income tax system. Moreover, it maintains a strict balanced budget every period, and therefore sets the lump-sum transfer equal to total collections:

$$P_t T_t = \int_j F_t^j(P_t^j) + \tau \int_i W_t e_t^i n_t^i di \tag{36}$$

5.3 Aggregation and analysis

This section present several aggregation results that obtain when consumers choose hours worked optimally and markets for intermediate goods, final goods, and labor clear. The latter two conditions are expressed as

$$C_t \equiv \mathbb{E}_I\left[c_t^i\right] = Y_t \qquad \int_j l_t^j dj = \int_i e_t^i n_t^i di \tag{37}$$

Proposition 5. *When households optimally choose labor supply and intermediate-goods and labor markets* *clear, (per capita) GDP is equal to*

$$Y_t = \frac{1}{\triangle_t} A_t N(w_t) \tag{38}$$

where $N(w_t) \equiv \kappa w_t^{\psi}$ *is the total supply of effective hours,* $\kappa \equiv \frac{1}{b^{\psi}} \mathbb{E}\left[e^{1+\psi}\right]$ *is a constant, and* $\triangle_t \equiv \int_{j=0}^{1} \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} dj \geq 1$ *is a measure of price dispersion.*

Proposition (5) illustrates the simplicity of aggregation in this model, despite the heterogeneity of consumers and firms. GDP is only a function of three parameters: technology A_t, a summary measure of heterogeneity in production outcomes $\triangle_t$, and aggregate labor supply which depends only on the net-of-tax real wage w_t.

Define gross nonfinancial income Y_t^i as i's real earnings inclusive of the lump-sum tax, and net financial income by subtracting the disutility of labor supply from gross income:

$$Y_t^i \equiv w_t e_t^i n_t^i + T_t \qquad Z_t^i \equiv z\left(e_t^i, w_t\right) + T_t$$

Proposition 6. *When households optimally choose labor supply and intermediate-goods and labor markets clear, the tax intercept share of GDP is:*

$$\frac{T_t}{Y_t} = \widetilde{\tau}_t$$

where $\widetilde{\tau}_t$ *is the* labor wedge, *a summary measure of the economy's distortions:*

$$1 - \widetilde{\tau}_t = \frac{w_t}{A_t} \triangle_t = (1-\tau) \frac{W_t}{P_t A_t} \triangle_t \tag{39}$$

Moreover, households i's total nonfinancial gross and net incomes as a share of per capita GDP are:

$$\frac{Y_t^i}{Y_t} = (1-\widetilde{\tau}_t) \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + \widetilde{\tau}_t \qquad \frac{Z_t^i}{Y_t} = \frac{1-\widetilde{\tau}_t}{1+\psi} \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + \widetilde{\tau}_t \tag{40}$$

Note that $\mathbb{E}_I\left[Y_t^i\right] = Y_t$. The balanced-budget and full profit taxation assumptions therefore lead to a very simple link between the labor wedge and the progressivity of the tax system in this model. Fundamental inequality in productivity e_t^i translates into inequality in pre-tax earnings $\left(e_t^i\right)^{1+\psi}$. As $\widetilde{\tau}_t$ varies from zero to one, agents' relative incomes alternate between their fundamental level and the one arising under perfect equality. A higher $\widetilde{\tau}_t$, in turn, indicates a more distorted economy—which may result from high tax rates, high monopoly power or a negative output gap (a recession) under sticky prices.[34]

Definition 3. The *moments of the redistribution channel* in the model are defined as follows. Gross-of-tax income-weighted MPC $\mathcal{M}_t^g$ and (net-of tax) income-weighted MPC $\mathcal{M}_t$ are

$$\mathcal{M}_t^g \equiv \mathbb{E}_I\left[\frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} MPC_t^i\right] \qquad \mathcal{M}_t \equiv \mathbb{E}_I\left[\frac{Y_t^i}{Y_t} MPC_t^i\right]$$

The redistribution elasticities with respect to the labor wedge $\widetilde{\tau}$, the price level P, and the real

[34] In a flexible-price steady-state, $\widetilde{\tau}_t = \tau^*$ is a constant and Y_t is declining τ^* (see proposition 9). This implies a simple tradeoff between efficiency and equity, which one can resolve by setting the steady-state tax rate τ.

interest rate r are

$$\mathcal{E}_{\tau,t} \equiv -\text{Cov}_I\left(MPC_t^i, \frac{Y_t^i}{Y_t}\right) \quad \mathcal{E}_{P,t} \equiv -\text{Cov}_I\left(MPC_t^i, \frac{NNP_t^i}{\mathbb{E}_I\left[c_t^i\right]}\right) \qquad \mathcal{E}_{r,t} \equiv \text{Cov}_I\left(MPC_t^i, \frac{URE_t^i}{\mathbb{E}_I\left[c_t^i\right]}\right)$$

Note that the average MPC is $\mathbb{E}_I\left[MPC_t^i\right] = \mathcal{M}_t + \mathcal{E}_{\tau,t}$. Finally, the share of net in gross consumption and the Hicksian scaling factor are

$$\xi_t^i \equiv 1 - \frac{v\left(n_t^i\right)}{c_t^i} \qquad S_t \equiv \mathbb{E}_I\left[\xi_t^i\left(1 - MPC_t^i\right)\frac{c_t^i}{\mathbb{E}_I\left[c_t^i\right]}\right]$$

All of these cross-sectional moments can be directly computed from the policy functions in an equilibrium. In particular,

$$MPC_t^i = \frac{\Pi_t}{1+\delta_N Q_t}\frac{\partial c_t\left(\lambda, \mathbf{s}^i\right)}{\partial \lambda} \qquad URE_t^i = Y_t^i + \frac{\lambda_t^i}{\Pi_t} - c_t^i \qquad NNP_t^i = \left(1 + Q_t\delta_N\right)\frac{\lambda_t^i}{\Pi_t}$$

The following propositions follow from the concavity of the consumption function:

Proposition 7. *$\mathcal{E}_{P,t} > 0$ and, provided that all assets and liabilities are short term ($\delta_N = 0$), $\mathcal{E}_{r,t} < 0$.*

In the steady-state calibration we will see that $\mathcal{E}_r < 0$ also holds for longer maturities—consistent with the reduced-form analysis presented in section 4. A key result, however, will be that the absolute value of $\mathcal{E}_r$ is declining in δ_N. The cross-sectional moments from definition 3 are useful to predict the response of aggregate consumption to macroeconomic shocks that last for one period, as the following proposition (a version of theorem 3 in the model) indicates.

Proposition 8 (Response to one-time shocks in the model)**.** *Assume that final goods, intermediate goods, and labor markets initially clear. Consider a shock at t that changes dY_t, $d\tilde{\tau}_t$, dR_t and dw_t for one period only, and revises all future prices by dP_t. Then aggregate consumption changes by approximately*

$$\frac{dC_t}{C_t} \simeq \mathcal{M}_t\frac{dY_t}{Y_t} + \mathcal{E}_{\tau,t}\frac{d\tilde{\tau}_t}{1-\tilde{\tau}_t} + \mathcal{E}_{P,t}\frac{dP_t}{P_t} + \left(\mathcal{E}_{r,t} - \sigma S_t\right)\frac{dR_t}{R_t} + \psi\left(1 - \mathcal{M}_t^g\right)\left(1-\tilde{\tau}_t\right)\frac{dw_t}{w_t} \tag{41}$$

Moreover, provided that the tax rate τ is kept constant:

$$\frac{d\tilde{\tau}_t}{1-\tilde{\tau}_t} = -\frac{dw_t}{w_t} + \frac{dA_t}{A_t} - \frac{d\triangle_t}{\triangle_t}$$

and provided that the current account remains closed:

$$\frac{dC_t}{C_t} = \frac{dY_t}{Y_t} = \frac{dA_t}{A_t} + \psi\frac{dw_t}{w_t} - \frac{d\triangle_t}{\triangle_t}$$

Each of the term in (41) corresponds to one of the channels highlighted in (23). In sections 6.4-6.6, I will apply the proposition to evaluate the accuracy of the theorem's prediction relative to the full nonlinear solution to the response of shocks in the calibrated model.

5.4 Equilibrium and steady-state with flexible prices

Define the real interest rate between t and $t+1$ as

$$R_t = \frac{1+\delta_N Q_{t+1}}{Q_t \Pi_{t+1}} \tag{42}$$

When prices are fully flexible, R_t is determined in equilibrium. Through its influence on the nominal bond price Q_t, the monetary authority controls the path of the inflation rate Π_t directly (for example, pinning it down through a Taylor rule). All price level changes are perfectly anticipated by households and firms, and have no effect on real variables. The following defines a flexible-price equilibrium:

Definition 4. Given an initial distribution $\Psi_0(\mathbf{s}, \lambda)$ over idiosyncratic states and bond positions, an initial price level P_0, and paths for productivity $\{A_t\}$, inflation $\{\Pi_t\}$ and borrowing limits $\{\overline{D}_t\}$, a flexible-price equilibrium is a sequence of consumption rules $\{c_t(\mathbf{s}, \lambda)\}$, next-period bond choices $\{\lambda_{t+1}(\mathbf{s}, \lambda)\}$, distributions $\{\Psi_t(\mathbf{s}, \lambda)\}$ and aggregate prices $\{R_t, Q_t, P_t, W_t, w_t, \triangle_t\}$ and quantities $\{N_t, C_t, Y_t, T_t\}$ such that: consumers optimally choose hours worked according to (31) and make a dynamic consumption and bond purchase decision consistent with (32), final and intermediate-goods firms maximize profits, leading to (35), the price level evolves according to $P_t = \Pi_t P_{t-1}$ and bond prices according to (42), the government's budget constraint (36) is satisfied, markets for labor, intermediate goods, final output and bonds clear:

$$C_t \equiv \int c_t(\mathbf{s}, \lambda)\, d\Psi_t(\mathbf{s}, \lambda) = Y_t \quad Q_t \int \lambda_{t+1}(\mathbf{s}, \lambda)\, d\Psi_t(\mathbf{s}, \lambda) = 0 \tag{43}$$

and the evolution of the bond distribution is consistent with $\{\lambda_{t+1}(\mathbf{s}, \lambda)\}$.

In equilibrium, using the results in section 5.3, the following proposition holds:

Proposition 9. *When prices are flexible, the labor wedge is constant*

$$\widetilde{\tau}_t = \tau^* \equiv 1 - (1-\tau)\frac{\epsilon - 1}{\epsilon} \quad \triangle_t = 1 \quad \forall t$$

Moreover, output—and therefore aggregate consumption—is entirely determined by current productivity, the degree of monopoly power, and the tax system:

$$Y_t = \kappa (1-\tau^*)^{\psi} A_t^{1+\psi} \quad \forall t$$

A corollary of proposition 9 is that (unexpected) redistributive policies—such as targeted lump-sum transfers from one group of agents to another, or inflationary shocks that erode the real value of debts and assets—do not affect aggregate output. They do, however, change *relative* consumption and welfare levels, as well as the market-clearing real interest rate, in a way that depends on the strength of the redistribution channel as explored in section 6.4. This result can be viewed as a useful benchmark, highlighting the importance of general equilibrium when thinking through the aggregate effects of redistributive policy.

I define a steady-state as a flexible-price equilibrium with constant productivity A, debt limit $\overline{D}$ and inflation Π, attaining a constant real interest rate R^* and a stationary distribution for bonds $\Psi(\mathbf{s}, \lambda)$. The steady-state has the following important property:

Proposition 10 (Invariance of steady-state to the maturity structure)**.** *Two economies that differ only in their maturity structure of financial assets and liabilities δ_N attain the same steady-state interest rate R^*, with the same joint distribution over bond market values $b = Q\lambda$ and idiosyncratic states* $\mathbf{s}$*.*

The logic behind this proposition is simple. When agents face a constant term structure of interest rates R^*, short and long-term assets span the same set of contingencies, and a constant borrowing limit specification (30) is is also neutral with respect to maturity. Crucially for my argument, unhedged interest rate exposures do vary with δ_N. Changing δ_N allows us to change asset durations $\frac{\Pi R}{\Pi R - \delta_N}$ and the strength of the interest rate exposure channel without changing any of the other steady-state properties of the model.

5.5 Equilibrium with fully sticky prices

When prices are fully sticky, the inflation rate is constant at $\Pi_t = 1$, and the monetary authority, by changing the nominal interest rate, controls the real interest rate R_t directly.

Definition 5. Given an initial distribution $\Psi_0(\mathbf{s}, \lambda)$ over idiosyncratic states and bond positions, an initial distribution for prices $\left\{P_0^j\right\}$, and paths for productivity $\{A_t\}$, the real interest rate $\{R_t\}$ and borrowing limits $\{\overline{D}_t\}$, a sticky-price equilibrium is a sequence of consumption rules $\{c_t(\mathbf{s}, \lambda)\}$, next-period bond choices $\{\lambda_{t+1}(\mathbf{s}, \lambda)\}$, distributions $\{\Psi_t(\mathbf{s}, \lambda)\}$ and aggregate variables $\{\Pi_t, Q_t, P_t, W_t, w_t, N_t, C_t, Y_t, T_t, \triangle_t\}$ such that: consumers optimally choose hours worked according to (31) and make a dynamic consumption and bond purchase decision consistent with (32), final goods firm maximize profits and intermediate-goods firms satisfy demand at prices $\left\{P_0^j\right\}$, the price level is a constant $P_t = P_0$, $\Pi_t = 1$, bond prices evolve according to (42), the government's budget constraint (36) is satisfied, markets for labor, consumption and bonds clear as in (43), and the evolution of the bond distribution is consistent with $\{\lambda_{t+1}(\mathbf{s}, \lambda)\}$.

In a fully sticky-price equilibrium, when the central bank temporarily and unexpectedly sets a real interest rate R_t below its "natural" level that prevails under flexible prices R_t^*, aggregate demand increases due to both a substitution and a redistribution effect, and firms accommodate by producing more. Price dispersion Δ_t is fixed at its initial level Δ_0. According to (38), an increase in production Y_t requires an increase in the real wage w_t. This leads to a fall in the labor wedge (39), and therefore to a fall in the progressivity of the tax system.[35] In section 6.5 I will discuss how all these macroeconomic changes interact with the heterogeneity in determining the aggregate effect of monetary policy shocks.

6 Model-based evaluation of the redistribution channel

6.1 Steady-state calibration and solution method

I perform my calibration at quarterly frequency. I target an annual equilibrium real interest rate of 3% and a household debt/PCE ratio 113%—the U.S. level for 2013, which in that year is virtually

[35] Note that this result is not associated with a traditional automatic stabilizer logic. During booms, firm profits are low since their prices are too low; since the government collects these profits and runs a balanced budget, these low profits diminish lump-sum transfers to households. Hence current booms—low wedges —exacerbate relative income inequality and current recessions mitigate it.

Parameter	Description	Value
β^I	Impatient discount factor	0.95
β^P	Patient discount factor	0.993
ψ	Elasticity of labor supply	1
σ	Elasticity of substitution in net consumption	0.5
δ_N	Asset/liability coupon decay rate	0.95
$\overline{D}$	Borrowing limit (% of annual PCE per capita)	185%

Table 4: Calibration parameters

equal to the stock of interest-paying assets held by the household sector.[36] I also target an average asset duration of 4.5 years. This is an average of the durations of U.S. household assets and liabilities reported by Doepke and Schneider (2006a) at the end of their data sample (see their figure 3). I consider a flexible-price steady-state with no inflation: $\Pi = 1$. These considerations imply a choice of $\delta_N = 0.95$.[37]

In order to discipline the micro-level facts, I use the 2009 wave of the Panel Study of Income Dynamics (PSID). I aim to obtain, in equilibrium, asset dispersions that match that those observed in that survey, and to bring consumers' marginal propensities to consume in the model in line with the empirical evidence of papers such as Johnson et al. (2006), Parker et al. (2013) and Baker (2014). I target an average quarterly marginal propensity to consume of 0.25—which is close to a consensus number from the empirical literature. I achieve these joint objectives through a combination of relatively tight borrowing limits ($\overline{D} = 7.4$, or 185% of per capita annual consumption, when the natural borrowing limit is over 1300%) and a preference process where agents alternate between patience (discount factor β^P) and impatience (discount factor β^I). I specify that the stationary population distribution must contain patient and impatient agents in equal numbers, and that consumers stay in their patience state for 50 years on average. This process is meant to capture slow-moving preference heterogeneity. It is similar to the one which Krusell and Smith (1998) found useful to match the wealth distribution and which Carroll et al. (2014) used to generate high marginal propensities to consume on average in the population.

Table 4 summarizes my benchmark parameters. Since my calibration for the income process targets pre-tax earnings rather than productivity, the elasticity of labor supply ψ does not play a major role in the flexible price version of my model. However, it does play a role in determining the response of real wages to monetary policy shocks in the sticky price version. Since the structural vector autoregression evidence (for example Christiano, Eichenbaum and Evans (2005)) is for a muted response, I calibrate ψ on the high end of what standard estimates from analyses of panel data imply, and set $\psi = 1$. I set the elasticity of intertemporal substitution in net consumption to $\sigma = 0.5$, which is well within the range of typical calibrations. Since I am ultimately interested in comparing the substitution and the redistribution channel, this allows me a priori not to stack the

[36]Sources: NIPA and U.S. Financial Accounts. See Appendix A.1 for details.

[37]The duration of the nominal bond is $\frac{R\Pi}{R\Pi - \delta_N} = \mathcal{D}$, so $\frac{\delta_N}{\Pi} = R\left(1 - \frac{1}{\mathcal{D}}\right)$.

		Steady-state value	
		$\delta_N = 0.95$	$\delta_N = 0$
Redistribution elasticity for r	$\mathcal{E}_r$	-0.09	-1.76
Hicksian scaling factor	S	0.57	
Equivalent EIS	$\sigma^* = -\frac{\mathcal{E}_r}{S}$	0.15	3
Total partial elasticity to r	$\mathcal{E}_r - \sigma S$	-0.38	-2.05
Average MPC	$\mathbb{E}_I\left[MPC^i\right]$	0.25	
Average MPC (net-income weighted)	$\mathcal{M}$	0.16	
Average MPC (gross-income weighted)	$\mathcal{M}^g$	0.09	
Redistribution elasticity for P	$\mathcal{E}_P$	1.77	
Redistribution semi-elasticity for $\widetilde{\tau}$	$\mathcal{E}_\tau/(1-\tau^*)$	0.16	

Table 5: Calibration outcomes for the steady-state moments of Proposition 8

cards against the substitution channel.[38]

The mix of labor income tax τ and the elasticity of substitution between goods ϵ is irrelevant, conditional on the labor wedge τ^*. I calibrate this wedge jointly with the earnings process. I then normalize GDP and average hours to 1 per quarter, which allows me to calibrate A and b given τ^*, ψ and the process for e.[39] Further details on the calibration and the numerical solution technique are provided in appendix E.

6.2 Calibration outcomes

Table 5 displays the model statistics from definition 3, which proposition 8 showed to be important to analyze the redistribution channel. The average MPC is 0.25 per quarter, which is in line with the empirical evidence. 22% of borrowers are at their borrowing limit, so that it is important to understand their behavior when interest rates change. The model generates a large dispersion in MPCs, with some agents with high cash-on-hand only slightly above the permanent-income level (below 0.01), and many constrained agents with much larger MPCs (figure 3).[40]

Given the long maturities, the standard deviation of unhedged interest rate exposures is moderate. The redistribution elasticity $\mathcal{E}_r = -0.09$ has a magnitude which is comparable to the one I obtained in the SHIW, and below that I obtained in the CEX. However, if we change maturities to be much shorter ($\delta_N = 0$, which is the short-term debt typically modeled with a duration of 1 quarter), this elasticity rises dramatically, much above the assumed level of the EIS. This impor-

[38] The size of the redistribution channel, however, depends endogenously on σ in this model since the EIS determines the sizes of asset positions that agents are ready to take; so in equilibrium higher σ is associated with both a higher substitution channel—σS in Definition 3—and a higher redistribution channel ($\mathcal{E}_r$)

[39] This involves backing A and b from $Y = (1-\tau^*)^\psi\, \mathbb{E}\left[e^{1+\psi}\right] \frac{A^{1+\psi}}{b^\psi} = 1$ and $\frac{\mathbb{E}[n]}{Y} = \frac{\mathbb{E}[e^\psi]}{\mathbb{E}[e^{1+\psi}]A} = 1$

[40] Note that while the MPC of agents exactly at the borrowing limit is equal to 1 by definition, the Lagrange multiplier on their borrowing constraint is small enough that even a small positive transfer (lower than the amount it takes to move them to the next point on the asset grid) leads them to start smoothing substantially, as their MPC falls discontinuously to a number below 0.5. In other words, the large persistence in earnings creates an incentive for households to smooth non-infinitesimal positive shocks to income. This has an important implication for the asymmetry of the reaction to positive and negative shocks to interest rates.

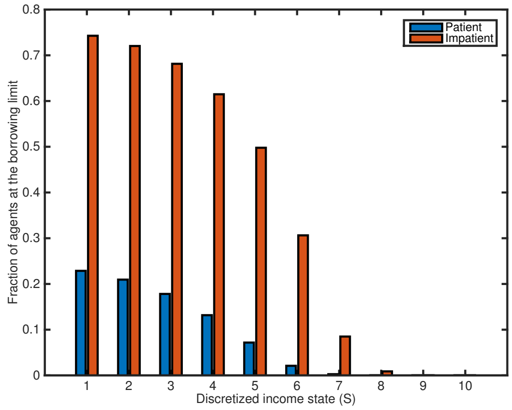


Figure 3: Share of constrained agents

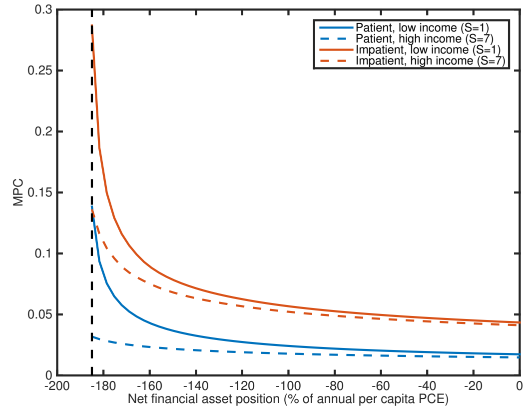


Figure 4: MPCs by asset and state

tant finding has implications for the determination of the natural rate of interest under flexible prices (section 6.4) and for the effects of monetary policy changes (section 6.5).

One unexpected feature of the model comes from the elasticity of aggregate demand with respect to rises in the price level, $\mathcal{E}_P = 1.77$. This number implies a very powerful redistribution through the Fisher channel. There is a strong intuition for this result. Inflation redistributes along the asset dimension, which in this class of models is highly correlated with MPC (see proposition 4). On the other hand, incomes and MPCs are much less correlated (their covariance in the model is -0.09): agents can smooth shocks to income quite well, and MPCs are driven by preference heterogeneity much more than by income heterogeneity. Figure 4 illustrates these statements in the context of the calibration. The gradient is clearly much higher with respect to asset level than it is with respect to income (moving from solid to dashed lines). As I discuss in section 6.4, this implies that targeted income-redistributive policies have less demand impact than price level changes in this world with nominal assets. First, I need to discuss how borrowing limits adjust away from steady-state.

6.3 Transition after shocks and borrowing constrained agents

In the following sections I will compute transitional dynamics starting from steady-state. Following the logic of section 2, I compute the impulse-response, perfect-foresight path for macroeconomic aggregates in deviations from steady-state, following an unexpected shock and holding asset positions fixed. The channels illustrated in theorem 3 all play a role. I illustrate the comparison between economies that vary in terms of the strength of their interest rate exposure channels by comparing the benchmark U.S. calibration to the calibration with short maturities only.

An important question in computing transitional dynamics is how borrowing limits adjust. A natural choice for borrowing limits is one that holds the *real coupon payment in the next period fixed*:

$$\overline{D_t} = Q_t \overline{d} \quad \forall t \tag{44}$$

The borrowing limit can then be written

$$\lambda_{t+1} \geq -\overline{d}$$

and is equivalent to a restriction on *flow payments* in the next period (which we can think of mortgage payments including amortization when durations are long), as opposed to the present value of liabilities, as would be implied if $\overline{D_t}$ was not adjusting following changes in Q_t.

In addition to being a natural one, the specification of the adjustment process in (44) implies that proposition 8 holds exactly, including for agents at a binding borrowing limit. It is crucial to understand how these agents are affected depending on the maturity of the debt in the economy, δ_N. For simplicity, assume that the inflation rate is constant at $\Pi_t = 1$, so that nominal and real interest rates are equal. Consider an agent with income y_t^i who maintains himself at the borrowing limit in an initial steady-state where the real interest rate is R and the bond price is constant at $Q = \frac{1}{R-\delta_N}$. His consumption is equal to his income, minus the interest payment on the value of the borrowing limit $\overline{D} = Q\overline{d}$.

$$c_t^i = Y_t^i - \overline{D}\,(R-1)$$

Across economies with different debt maturities δ_N, $\overline{D}$ is a constant, so that the steady-state payments are the same, but the exposure of these payments to real interest rate changes differ. Indeed we can decompose:

$$\overline{D}\,(R-1) = (R-\delta_N)\,\overline{D} - \overline{D}\,(1-\delta_N) = \overline{d} + \underline{URE}$$

where $\overline{d} = (R-\delta_N)\,\overline{D}$ is the part that is precontracted and $\underline{URE} = -\overline{D}\,(1-\delta_N)$ the part that is subject to interest changes. Hence, economies with different δ_N involve very different levels of unhedged interest rate exposures for borrowing-constrained agents, ranging from the full principal $-\overline{D}$ when $\delta_N = 0$ to none when $\delta_N = 1$. In the benchmark calibration with $\delta_N = 0.95$, the minimum income level of agents is $Y^i = 0.413$, $\overline{d} = 0.413$, and $\underline{URE} = -0.358$. In other words, these highly indebted agents use their full income for interest payments and amortization, and then borrow to maintain their consumption level, so that their effective interest payments are $(R-1)\,\overline{D} = 0.055$. On the other hand, in the ARM calibration, $\overline{d} = 7.455$ and $\underline{URE} = -7.4$.

Suppose that current and future real interest rates increase, so that Q_t falls, and suppose the change is small enough that it does not alter the agent's decision to stay at the borrowing limit. His new consumption level is then

$$\begin{aligned} c_t^{i*} &= Y_t^i - \left(\overline{d} + \frac{Q_t}{Q}\underline{URE}\right) \\ &= Y_t^i - \overline{D}\,(R-1) - \left(\frac{Q_t - Q}{Q}\right)\underline{URE} \end{aligned}$$

For short-term changes, $\frac{Q_t - Q}{Q} \simeq -\frac{dR}{R}$ and we obtain the prediction from theorem 8, $c_t^{i*} - c_t^i \simeq \underline{URE}\frac{dR}{R}$, which is largest in an economy with $\delta_N = 0$.[41]

[41]Note that for changes in real interest rates that last for more than a period, there is a larger change in Q_t in an economy with long-term debt, so less debt is rolled over but the price is more affected, offsetting part of the first effect.

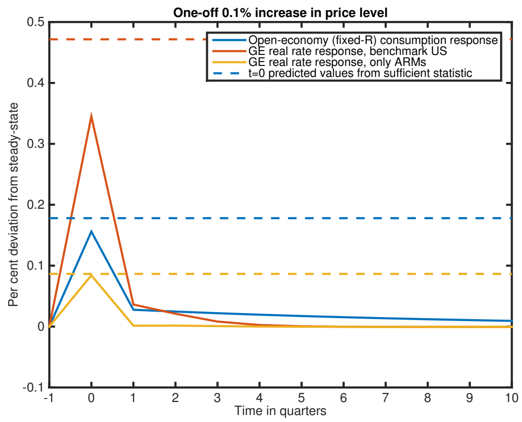


Figure 5: Price level shock $\frac{dP}{P} = 0.1\%$

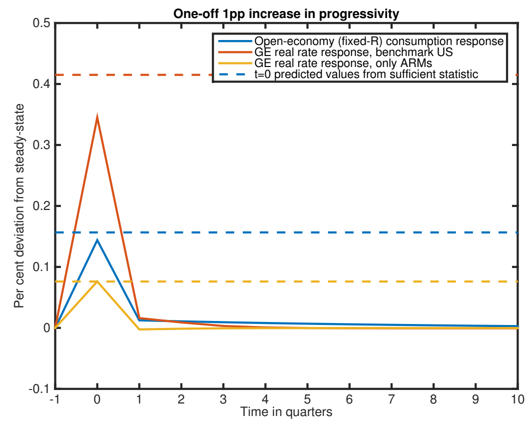


Figure 6: Tax progressivity $d\tau = 1\text{pp}$

6.4 The redistribution channel and the natural rate of interest

Proposition 9 made clear that under flexible prices, aggregate output in the model is only a function of current technological parameters and taxes. The real interest rate provides the adjustment mechanism that brings demand back in line with supply in response to shocks that redistribute income through taxation or real wealth through inflation. This section expands upon the examples provided in example 7 in the context of a full dynamic determination of the natural rate of interest, and explores the quantitative ability of the sufficient statistics provided given in Table 5 to predict the first-order response to small shocks.

I look at two classes of shocks that act purely through redistribution. The first one is a 0.1% one-off increase in the price level that redistributes via the Fisher channel.[42] The second is a 1 percentage point effective rise in the progressivity of the tax system, from $\tau^* = 40\%$ to 41%, which is entirely performed through lump-sum redistribution and therefore does not affect work incentives. The steady-state moments computed in section 6.2 predict that these two shocks should have approximately the same effect on the equilibrium real interest rate, because (to first order) they have the same aggregate demand effect.

Figures 5 and 6 plots the dynamic path of adjustment of the economy to both shocks. As a benchmark, the blue line plots the consumption response in a hypothetical open economy where the real interest rate did not change. Dotted lines correspond to the first-order approximations of these responses using the sufficient statistics approach. While both shocks are fairly small, they nevertheless translate into a fairly substantial 0.17% aggregate consumption effect in partial equilibrium. In general equilibrium, the real interest rate adjusts more in the benchmark calibration than in the US calibration, illustrating again the proposition presented in example 7. Moreover, the first-order approximation gets responses that are close to correct relative to the nonlinear solu-

[42]This shock is similar in spirit to that performed by Doepke and Schneider (2006b), but I consider a special class of models in which I know GDP stays constant. I focus attention on a small shock to evaluate the accuracy of my sufficient statistics; section 6.6 explores the case of large shocks.

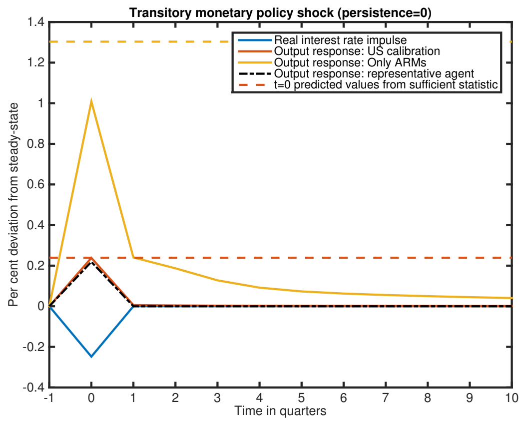


Figure 7: M.P. shock ($\rho_R = 0$)

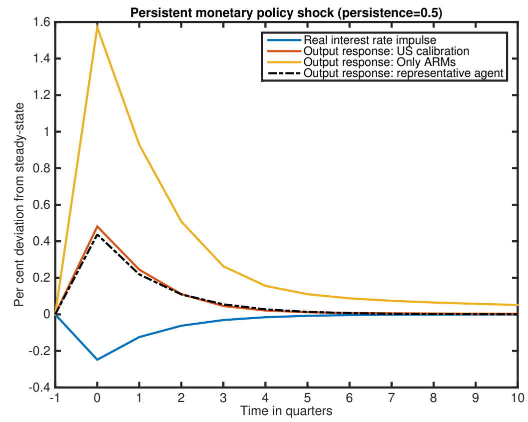


Figure 8: M.P. shock ($\rho_R = 0.5$)

tion, in particular for the demand response in partial equilibrium. Methodologically, these results therefore illustrate the usefulness of the sufficient statistic approach in predicting general equilibrium effects. Theoretically, they stress the importance of the redistribution channel, and of the maturity structure, in determining the natural rate of interest.

6.5 The redistribution channel and monetary policy shocks

I now consider the way in which the redistribution channel alters the effects of monetary policy shocks. Starting from the steady-state, I assume that the central bank unexpectedly lowers the real interest rate, and then lets it gradually return to its steady-state level with the following path:

$$R_t - R^* = \rho_R \left(R_{t-1} - R^* \right)$$

This corresponds to the typical *monetary policy shock* that is analyzed in the New Keynesian literature, for the special case in which prices are fully sticky.

Figures 7 and 8 show the effect on aggregate GDP of a transitory and a persistent monetary policy shock. The initial impulse is a 100 annualized basis points fall in the nominal interest rate, which translates into a fall in the real interest rate (the light blue line) since prices are fully sticky. In the benchmark U.S. calibration (red line), the response of the economy to a monetary policy shock is slightly above that predicted by representative-agent version of the model, consistent with a redistribution channel amplifying the effects of real interest rates changes. However, while $\sigma^* = 0.15$ and $\sigma = 0.5$ may suggest that the total response with heterogeneity should be 30% above that without, the actual magnitude is lower because earnings heterogeneity channel lowers the multiplier from income increases, as explained below. For an equivalent economy with short maturities, the response is more than twice as large as both the benchmark U.S. response and the representative-agent response. There are two equivalent interpretations for this result.[43] The first

[43]The figures also illustrate that the redistribution channel is not simply making the representative-agent "more

interpretation is that unhedged interest rate exposures are smaller for all agents in the economy, and in particular for the high-MPC borrowers, so that their consumption is less affected by a change in the real interest rate. The second interpretation is that when assets have long maturities, as they do in the benchmark, expansionary monetary policy creates capital gains for asset holders and upward revaluation of liabilities for borrowers. These redistribute against the MPC gradient and make monetary policy less potent in affecting output. With short maturities, on the other hand, real interest rate changes create redistribution that is more aligned with the MPC gradient. This explains the larger effects of monetary policy shocks on output in this case. This prediction of the model is consistent with the cross-country structural VAR evidence presented in Calza et al. (2013). It suggests that wealth redistribution is the primary reason why monetary policy affects consumption in a country like the United Kingdom where mortgages have adjustable rates.

Figure 7 also compares the output response in the full solution of the model to the predictions from the first-order approximation, discussed in more detail below. The approximation is excellent, especially for the benchmark calibration, illustrating again the fruitfulness of the sufficient statistic approach in quantifying the aggregate effects of monetary policy. Section 6.6 will explain why the response in the ARM-only calibration is below that from the first-order approximation.

Understanding the role of redistribution Consider the case $\rho_R = 0$: the shock lasts for only one period. With fully rigid prices the Fisher channel is shut down: $dP = d\triangle = 0$. Applying proposition 8 at the steady-state, and dropping time subscripts for ease of notation, the following identities hold:

$$\frac{dC}{C} = \frac{dY}{Y} \qquad -\frac{d\widetilde{\tau}}{1-\widetilde{\tau}} = \frac{dw}{w} = \frac{1}{\psi}\frac{dY}{Y}$$

and the predicted response of consumption and GDP to a monetary policy shock is

$$\frac{dC}{C} = \frac{(\mathcal{E}_r - \sigma S)}{\tau^*(1-\mathcal{M}^g) + \mathcal{M}^g - \mathcal{M} + \frac{\mathcal{E}_\tau}{\psi}}\frac{dR}{R} = -\mu^{het} S\left(\sigma^* + \sigma\right)\frac{dR}{R} \tag{45}$$

Figure 9 helps interpret equation (45). Consider first the representative agent version of this model. For such a case we have, from definition 3, $S = \xi\,(1 - MPC)$, $\mathcal{E}_r = \mathcal{E}_\tau = 0$, and $\mathcal{M}^g = \mathcal{M} = MPC$. Hence (45) yields[44]

$$\frac{dC}{C} = \frac{-\sigma\xi\,(1-MPC)}{\tau^*(1-MPC)}\frac{dR}{R} = -\frac{\xi}{\tau^*}\sigma\frac{dR}{R} \tag{46}$$

In the standard New Keynesian model, a monetary accommodation lowers the real interest rate, which raises aggregate demand through the substitution channel, and gets further amplified through the aggregate income channel. The MPC cancels out of this calculation, leaving the denominator of (46). Relative to the standard model, this model features consumption/labor complementarities from GHH preferences. These provide strong amplification of increases in hours

elastic". The relative magnitudes of the redistribution and the substitution channel is above 4 in the case of a short-lived shock.

[44] Appendix D.8 verifies that this expression also obtains from writing the loglinearized equations characterizing the model.

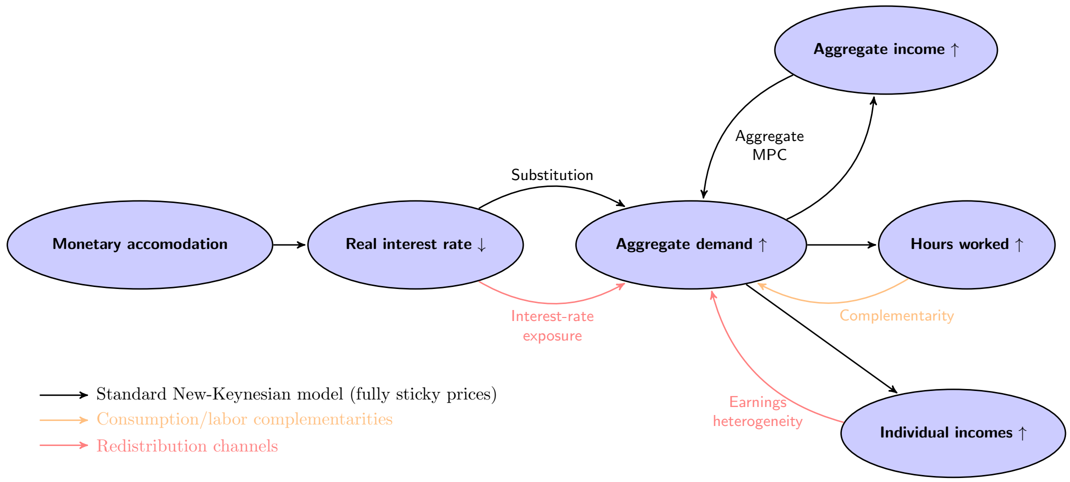


Figure 9: Monetary policy transmission mechanism in the model

worked on consumption. Combining the effect of this multiplier from the scaling of net consumption to gross consumption, we obtain a factor $\xi/\tau^* = 1.75$ multiplying $\sigma\frac{dR}{R}$ in (46).

In the presence of heterogeneity, the redistribution channel enters at two levels. First, lower real interest rates boost aggregate demand though the interest rate exposure channel as well as the substitution channel ($\mathcal{E}_r < 0$), which adds a negative term to the numerator of (46). Second, monetary policy raises incomes heterogeneously: wages rise and profits fall, resulting in a lower tax intercept and less effective redistribution during the boom. Because high income agents have lower MPC, this acts to lower demand. Given the calibrated $\psi = 1$, this results in a factor of $\mu^{het}S = 1.47$ multiplying $(\sigma^* + \sigma)\,\frac{dR}{R}$ in (45). Hence, in this particular calibration, the "impulse" reponse to a monetary policy shock is larger (from the interest-rate exposure channel), but the multiplier is lower (from the earnings heterogeneity channel) than what the representative-agent model predicts; with the first effect dominating slightly.[45]

Adding price adjustment to the picture would only enhance the departures of the impulse response to monetary policy from the representative-agent benchmark. As is clear from the magnitude of $\mathcal{E}_P$ and from section 6.4, the Fisher channel is strong in the calibrated model, since MPCs and nominal asset positions are highly correlated. This suggests that inflation can be a powerful amplification mechanism of monetary policy acting through redistribution.

6.6 Asymmetric effects of increases and cuts in interest rates

This section explores in more detail the quality of the first-order approximation (45), relative to the full nonlinear solution of the model, according to the size of the one-off real interest rate change of $dr = \frac{dR}{R}$.

[45] This multiplier effect is only illustrative of the role of the earnings heterogeneity channel in determining the full response to monetary policy shocks. A model with wage rigidities would alter the sign of this channel (which would be more consistent with the empirical evidence of Coibion et al. (2012)) and increase the multiplier relative to the case without heterogeneity.

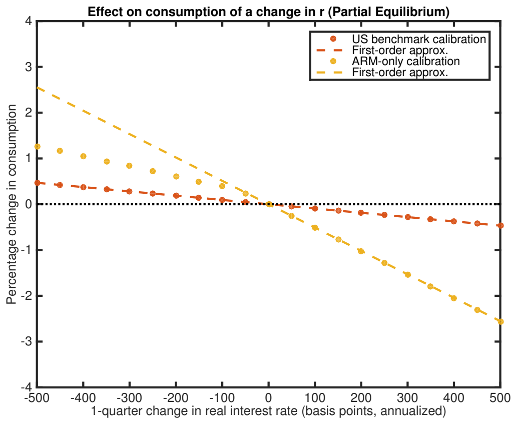


Figure 10: Change in r: partial equilibrium

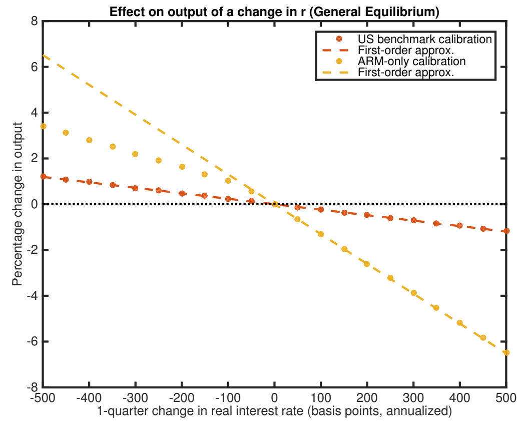


Figure 11: Change in r: general equilibrium

In figure 10 I show the aggregate consumption effects that result from this change in partial equilibrium, that is, before any general equilibrium effects on income—as in the current account experiment of Example 6. The corresponding first-order approximation—the numerator of (45)—is very good, including for relatively large increases in the real interest rate, as well as for small decreases. However, in the ARM calibration, it overpredicts the consumption response for falls in the real interest rate of 100bp or more. This asymmetric effect can be traced back to the asymmetric behavior of the 22% of agents who are at their borrowing limit in this economy in response to increases and falls in income. While they have to cut consumption one for one in response to falls in income, their MPC out of moderate increases in income is below 0.3, as figure 4 illustrated. Because their debt is short term, falls in interest rates have large effects on their interest payments. This reduces effective MPC differences, and therefore the size of the redistribution channel, that results from falls in interest rates. Increases in interest rates do not have the same feature, since the MPC of borrowers out of increases in income payments is exactly one, as captured by the sufficient statistics.

Figure 11 shows that this asymmetric effect remains in general equilibrium. Is is slightly less strong than in partial equilibrium because the incomes of poor agents—which make up a disproportionate fraction of agents at the borrowing limit—do not rise as fast as those of the rest of the population as the tax system becomes less progressive in the boom.

This type of asymmetric effects of monetary policy changes receives support from the empirical evidence (see for example Cover, 1992; de Long and Summers, 1988 and recently Tenreyro and Thwaites, 2013). My explanation, which has to do with asymmetric MPC differences in response to policy rate changes, provides an alternative to the traditional Keynesian interpretation of this fact, which relies on downward nominal wage rigidities.[46]

[46]While my U.S. benchmark calibration does not feature asymmetric effects of interest rates, in practice, the refinancing option embedded in fixed rate mortgages in the United States is likely to create an asymmetric effect in the opposite direction from the one I stress in this section.

7 Conclusion

This paper contributes to our understanding of the role of heterogeneity in the transmission mechanism of monetary policy. I established the precise sense in which a systematic covariance between agents' marginal propensities to consume and exposures to macroeconomic shocks generates a redistribution channel. I showed that the covariance between marginal propensities to consume and unhedged interest rate exposures is a sufficient statistic for the importance of the redistribution channel through real interest rates, and proposed a measure of this statistic in survey data. My results suggest that, if the Elasticity of Intertemporal Substitution is about 0.3—regarded by many as a plausible value—changes in real interest rates could be affecting consumption demand via redistribution as much as they do via the standard substitution channel present in representative-agent models. This is a quantitatively large effect which suggests that, while heterogeneity and inequality are important issues in their own right, they are also key to understanding aggregate consumption dynamics.

An important finding of my paper is that the monetary policy transmission mechanism operates differently in economies with short and long asset maturities, due to the different nature of the redistribution caused by changes in interest rates. This finding expands upon the popular view that lower interest rates benefit holders of long-term assets and are prejudicial to holders of short-term assets. When assets have relatively long maturities, lower real interest rates indeed tend to benefit asset holders and therefore to redistribute against the gradient of MPC in the economy. This makes interest rate cuts less effective at increasing aggregate demand. My calibrated model suggests that these effects—which one can interpret as a quantification of the macroeconomic effects of market incompleteness—are also large. It also suggests that monetary policy and mortgage design policies are intertwined, confirming a widely-held view in policy circles.

My results capture some of the general equilibrium, macroeconomic consequences of the presence of large and heterogeneous marginal propensities to consume, which are a robust feature of household micro data. Beyond the role of wealth redistribution for the macroeconomic effects of fiscal or monetary policy, this raises other questions—such as the role of inequality in the distribution of income in determining aggregate demand—which I leave for future research.

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A Data

A.1 Aggregate data

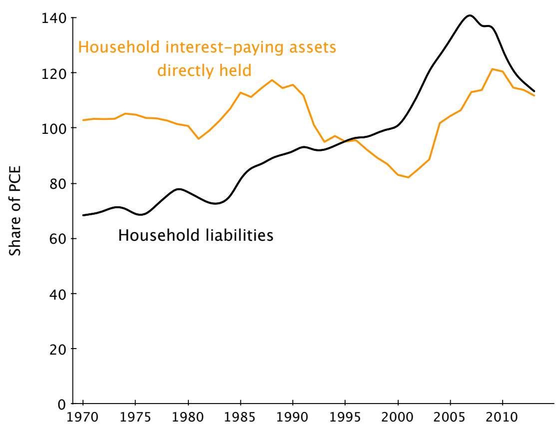


Figure 12: Monetary assets and liabilities

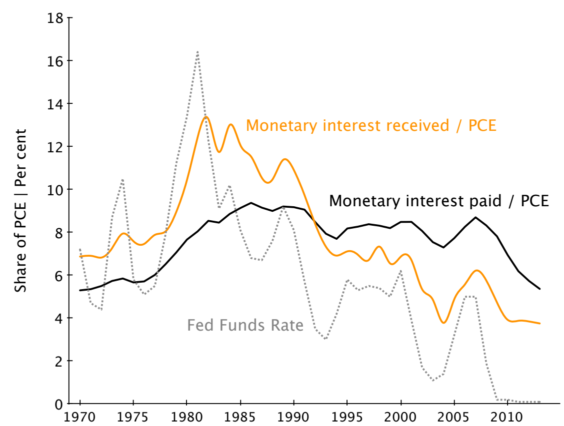


Figure 13: Monetary interest flows

Data sources: monetary interest paid and received by households is from NIPA table 7.11 (lines 15 and 32 respectively). Personal Consumption Expenditure (PCE) is from NIPA table 2.3.5, line 1. Household liabilities and interest-paying assets are from the U.S. Financial Accounts (Board of Governors Z.1 release). Interest-paying liabilities is the sum of mortgages (table L.217, line 7) and consumer credit (table L.222, line 1). Interest-paying assets includes time and savings deposits (L.205[13]) and credit market instruments (the sum of L.208[18], L.209[6], L.210[6], L.211[8], L.212[13], L.216[36], L.217[14] and L.222[3])

A.2 Micro data

Tables 6 and 7 present summary statistics from the surveys used in section 4 with more information on the distributions than those available in table 1.

Survey of Household Income and Wealth 2010

A striking feature of the Italian data is that fewer than 10% of households report to own mortgages. The share of adjustable mortgages is around 50%, which is consistent with official sources (Eurosystem, 2009).

Variable	count	mean	p5	p25	p50	p75	p95
Income from all sources (Y_i, €/year)	7,951	36,114	9,565	19,857	30,719	45,340	81,320
Consumption incl. mortgage payments (C_i, €/year)	7,951	27,976	10,700	17,060	24,000	33,600	57,600
Deposits and maturing assets (B_i, €)	7,951	14,200	0	1,000	5,156	15,054	50,000
ARM mortgage liabilities and consumer credit (D_i, €)	7,951	6,228	0	0	0	0	26,800
Unhedged interest rate exposure (€/yr)	**7,951**	**16,110**	**-21,862**	**1,093**	**10,974**	**26,646**	**71,610**
Unhedged interest rate exposure (€/Q)	**7,951**	**10,007**	**-17,328**	**594**	**6,407**	**16,871**	**52,054**
Total fixed-income financial assets (€)	7,951	15,133	0	0	0	4,285	99,000
Total financial liabilities (€)	7,951	27,481	0	1,359	7,000	24,064	91,104
Marginal Propensity to Spend	7,951	47	0	20	50	80	100

All statistics are computed using survey weights

Table 6: Summary statistics from the Italian SHIW 2010

Consumer Expenditure Survey

Variable	count	mean	p5	p25	p50	p75	p95
Income from all sources ($/year)	9,443	45,617	6,700	18,612	36,000	62,828	115,000
Consumption incl. mortgage payments ($/year)	9,443	36,253	9,544	18,724	28,464	44,114	90,296
Deposits and maturing assets ($/year)	9,443	7,147	0	0	0	2,100	30,100
Consumer credit ($)	9,443	2,872	0	0	0	2,000	14,600
Unhedged interest rate exposure ($/yr)	9,443	13,639	-41,948	-6,000	6,192	27,616	84,126
Unhedged interest rate exposure ($/Q)	9,443	6,616	-16,514	-2,288	1,377	7,813	35,520
Net liquid assets ($)	7,202	7,340	0	5	1,329	6,400	38,000

All statistics are computed using survey weights

Table 7: Summary statistics from the JPS CEX sample

A.3 Additional data tables

Consumption measure	Food		All nondurable	
	Estimate	95% C.I	Estimate	95% C.I
$\widehat{\mathcal{E}_r^{PE}}$	-0.065	[-0.14; 0.01]	-0.17	[-0.33 -0.02]
$\widehat{\mathcal{E}_r}$	-0.123	[-0.21; -0.04]	-0.32	[-0.48; -0.16]
$\widehat{S}$	0.82	[0.69; 0.95]	0.56	[0.33; 0.78]
$\widehat{\sigma^*} = -\frac{\widehat{\mathcal{E}_r}}{\widehat{S}}$	0.149	[0.03; 0.27]	0.58	[0.12; 1.04]

Confidence intervals are bootstrapped by resampling households 100 times with replacement

Table 8: Estimates from table 3 using median instead of mean URE per bin

J	2	3	4	5	7	10
$\widehat{\sigma^*}$	0.25	0.30	0.29	0.28	0.35	0.31
95% C.I.	[0.03; 0.47]	[0.05; 0.55]	[0.05; 0.52]	[0.04; 0.53]	[0.08; 0.62]	[0.00; 0.60]

Confidence intervals are bootstrapped by resampling households 100 times with replacement

Table 9: Estimated $\widehat{\sigma^*}$, using J bins of URE (consumption measure: food expenditures)

B Mortgage type and unhedged interest-rate exposures

B.1 An example

Consider a consumer who, midway through his life-cycle (at a date labelled $t = 0$), has an amount L in debt which must be repaid before his death at date T. The general level of prices is constant and normalized to $P = 1$. I abstract away from intertemporal substitution in labor supply and assume that this consumer faces a certain stream of unearned income $y_t = y$, and has expected utility with constant elasticity of substitution σ over consumption alone:

$$U\left(\{c_t, n_t\}\right) = \sum_{t=0}^{T-1} \beta^t \left(\frac{c_t^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \right) \tag{47}$$

The rate of interest is initially equal to the rate of time preference: $q_t = \beta^t$. It is clear that the solution to this optimization problem is

$$c_t = c = y - R_T L$$

where R_T is the annuitization factor for T periods, which is also the household's marginal propensity to consume, $MPC = \sum_{t=0}^{T-1} q_t = R_T = \frac{1-\beta}{1-\beta^T}$. The market value of financial wealth is projected to evolve as $W_t^F = -\frac{1-\beta^{T-t}}{1-\beta^T} L$, so that the debt is repaid in full by year T.

Consider the following three financial arrangements, each having the same present value $-L$, and so, as per proposition 1, leading to the same lifetime consumption plan under the initial path for real interest rates:

a) an adjustable-rate mortgage: ${}_{-1}B_{ARM,0} = -L$, ${}_{-1}B_{ARM,t} = 0$ for $t > 0$
b) a fixed-rate mortgage: ${}_{-1}B_{FRM,t} = -R_T L$ for $t = 0 \ldots T-1$
c) a bullet loan, due at the end of life, ${}_{-1}B_{bullet,T-1} = -\frac{1}{\beta^{T-1}} L$, ${}_{-1}B_{bullet,T-1} = 0$ for $t < T-1$

Figure 15 illustrates the solution under a simple calibration in which income is constant at $y = \$25\,000$ per year, initial debt is $L = \$100\,000$, the discount factor is $\beta = 0.97$, the elasticity of intertemporal substitution is $\sigma = 0.25$ and the horizon is $T = 30$ years. Here the annual real interest rate is $R = 3.1\%$ and the annual marginal propensity to consume is $MPC = R_T = 5\%$, so that consumption is equal to $c = \$20\,000$ per year. The remaining \$5000 can be interpreted as payment of interest and principal under the ARM and FRM plans, or as a savings build-up towards repayment of the bullet loan. The bottom panel of the figure shows the time path of unhedged interest rate exposures for each of the three mortgage types under consideration. Under an ARM, URE is very negative at date 0[47] and then becomes positive and equal to $R_T L$ throughout the consumer's remaining life. A bullet loan has a symmetric pattern of UREs, positive everywhere and very negative in the last period. A Fixed-Rate Mortgage, in this case, achieves the Arrow trading plan with URE exactly equal to zero throughout the life-cycle.

[47]The figure is truncated at URE=-\$10,000 for readability. Under an ARM, date-0 URE is $-\left(1 - R_T\right) L \simeq -\$95\,000$. Under a bullet loan, (non-discounted) URE in the last period is $-\$236\,000$.

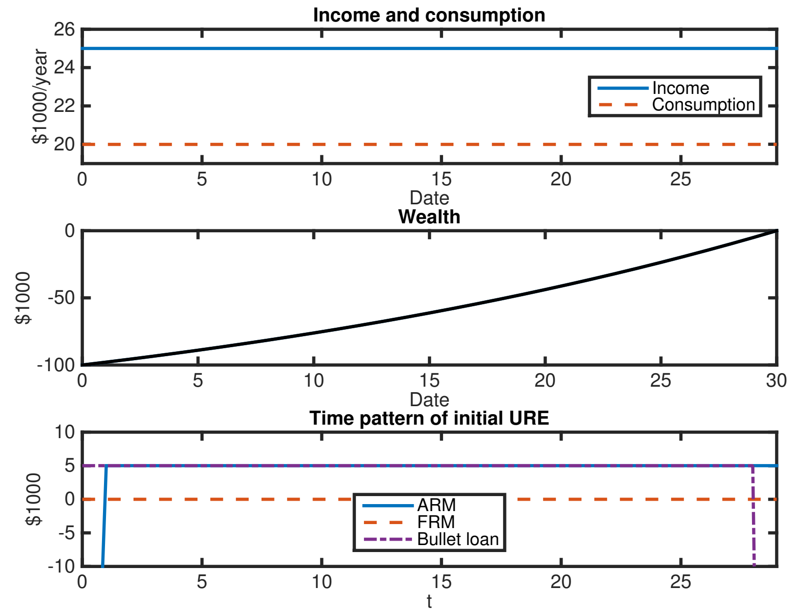


Figure 14: Solution to the life-cycle model and time path of URE under different mortgages

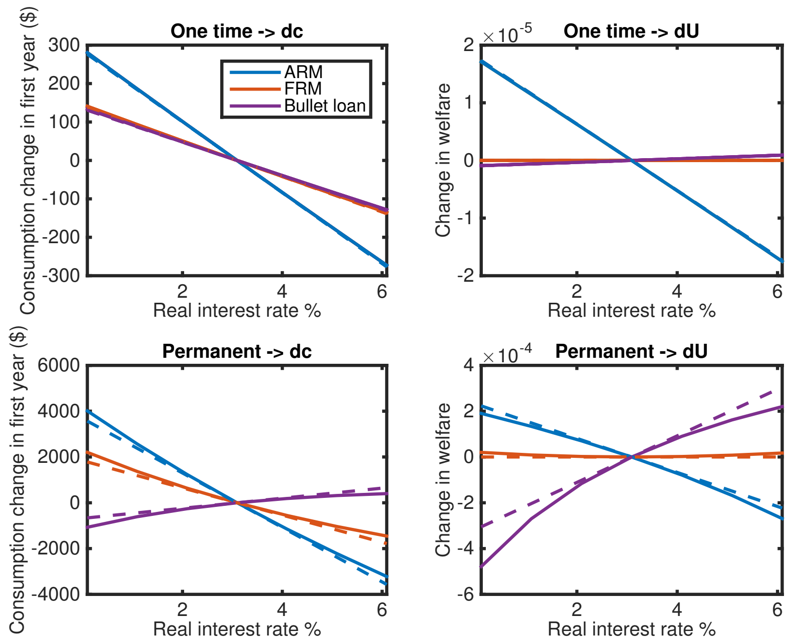
Suppose real interest rates unexpectedly change at date 0, and consider the impact on the consumer's welfare and consumption level in the first year following the change. Appendix C.2.1 shows that if real present-value discount factors change by $\{dq_s\}$, under the utility function (5), theorem 1 specializes to

$$dc_0 = MPC d\Omega + \sigma c \left[MPC \sum_{s=1}^{T-1} dq_s \right]$$
$$dU = u'(c)\, d\Omega$$
$$d\Omega = \sum_{s=1}^{T-1} q_s \left({}_{-1}URE_s \right) \frac{dq_s}{q_s}$$

For example, a *one-time change* dr in the real interest rate between periods 0 and 1 changes discount factors by $dq_s = -q_s dr$ for $s \geq 1$. We saw in (8) that the resulting consumption change is

$$dc_0 \quad = \quad [MPC \cdot URE - \sigma c (1 - MPC)]\, dr \tag{48}$$

The top panels of figure 15 display the first-year consumption change dc_0 and change in welfare dU under various levels for the real interest rate in the first year. A rise in the real interest above its initial level of $R = 3.1\%$ always decreases consumption from intertemporal substitution, but by much more moderate amounts when the consumer holds an FRM or a bullet loan than when he holds an ARM. The consumption change of an FRM holder is entirely due to intertemporal substitution, since he has no interest rate exposure and so never experiences wealth effects. Quantitatively, this substitution effect amounts to $-\sigma c (1 - MPC)$, which is a fall of about \$50 in annual consumption per percentage point rise in the real interest rate. Since the date-0 URE term is positive but small for the bullet loan holder, his wealth effect from the rise in interest rates translates into a small increase in welfare and a level of consumption slightly above that of the FRM holder. On the other hand, ARM holders experience a very large wealth loss from this temporary rise in interest rates since their initial unhedged interest rate exposure is almost as large as their loan balance. Precisely, $URE_{ARM} = -(1 - R_T) L$, implying a \$950 present-value loss—translating into an additional fall in consumption of \$50—per percentage point fall in the short-term real interest rate.



The figure shows the actual consumption change (solid lines) and the predictions from the approximation of theorem 1 (dashed lines) under various types of mortgages. Differences between colored lines are due to differential wealth effects from the rate change.

Figure 15: Effect of a change in real interest rates on dc and dU with different mortgages

This simple calibration—where under ARMs, income and substitution effects are of comparable magnitudes—illustrates the quantitative results in the paper. In the population, marginal propensities to consume tend to be much larger than the 5% annual number in this simple example, and balance sheet positions can be much larger than the 5 times annual consumption assumed here. To the extent that these positions are relatively short term, the income effect can therefore overwhelm the substitution effect for many individuals. In the aggregate, when MPCs and UREs are aligned, this can imply that real interest rate changes affect consumption through redistribution as much as through substitution. Section 4 makes this observation more precise.

The bottom panel of figure 15 considers the effect of a *permanent* change in the real interest rate throughout the consumer's remaining life. Such a change dr modifies present-value discount factors by $dq_s = -q_s s dr$ for $s \geq 1$. From (7),

$$d\Omega^{\ell} = \left[\sum_{s=1}^{T-1} q_s s \left(_{-1}URE_s\right)\right](-dr) \equiv URE^{\ell} \cdot dr$$

Define the duration of a "perpetuity" bond paying 1 in each year between date 1 and date $T-1$ as

$$D^{\ell} = \frac{\sum_{s=1}^{T-1} s q_s}{\sum_{s=1}^{T-1} q_s}$$

Hence theorem 1 implies that the first-order response of consumption in response to this shift in the real yield curve is

$$dc_0^{\ell} = \left[MPC \cdot URE^{\ell} - \sigma c \left(1 - MPC\right) D^{\ell}\right] dr$$

The bottom panel of figure 15 shows that the approximation is excellent for small changes in the real interest rate. Long-term changes create larger substitution effects than short-term changes (in the calibration, the long-term substitution effect is $D^\ell = 12.8$ years times the short-term one). This is visible by comparing the response of fixed-rate mortgage holders for whom wealth effects are zero under all scenarios. But long-term real interest rate changes also create much larger wealth effects. In the calibration, for an ARM holder $URE^\ell = -\$1.2\text{m}$, which is a present-value loss of \$12 000 per percentage point rise in the real interest rate, translating into a \$600 of additional cut in annual consumption. For a bullet loan holder $URE^\ell = \$1.7\text{m}$, enough for the wealth effect of a prolonged change in real interest rates to overwhelm its substitution effect, as is visible in the bottom left panel of figure 15.

Comparing across lines on the left panels of figure 15 at a given interest rate level, we difference out substitution effects on consumption and obtain the difference in wealth effects across mortgage types m and m', $\sum_{t\geq 0} q_t \left({}_{-1}URE_{m,t}\right) - \sum_{t\geq 0} q_t \left({}_{-1}URE_{m't}\right)$. Another way of looking at this difference in difference is to notice that it subtracts out the valuation of future consumption and income, with the remainder being a difference in financial wealth alone, $\sum_{t\geq 0} q_t \left({}_{-1}b_{m,t}\right) - \sum_{t\geq 0} q_t \left({}_{-1}b_{m',t}\right)$. When interest rates rise, we might observe households with ARMs lowering their consumption more than households with FRMs: the theory predicts that this is due to a wealth effect, which can be quantified based on observable balance sheet information; and not—at least to the extent that borrowing constraints do not strictly bind—a "disposable income" effect. The next section builds on this observation and provides a structural interpretation to studies that regress consumption changes on mortgage type.

B.2 Interpretation of consumption/balance-sheet regressions

Consider a regression that compares households with a standard fixed-rate mortgage to those with an adjustable-rate mortgage around a monetary policy change. For a cross-section of households i, we have data on their change in consumption $\triangle c_i$ around the event, and an indicator of whether they held an adjustable-rate mortgage ARM_i in the pre-period. Suppose that we run

$$\triangle c_i = \alpha + \beta ARM_i + \epsilon_i \tag{49}$$

and that we have an ideal instrument for ARM_i, so that the assignment to balance-sheets can be taken to be as good as random. As an example, suppose we use as an instrument a dummy for whether the mortgage is above or below the conforming loan limit, CLL_i. Vickery (2007) and Moench, Vickery and Aragon (2010) show that this is a powerful predictor of mortgage choice, with loans above the conforming loan limit having a much higher chance of being adjustable rates; suppose further that the exclusion restriction is valid so that the only influence of conforming loan limits on consumption changes is through mortgage type. Write $\triangle c_{i,ARM}$ for the change in consumption consumer i if he holds an adjustable-rate mortgage and $\triangle c_{i,FRM}$ for the change in consumption that would result if he held a fixed-rate mortgage with the same principal outstanding. The instrumented regression of $\triangle c_i$ on ARM_i then produces a treatment effect[48] of

$$\begin{aligned} \beta &= \mathbb{E}\left[\triangle c_{i,ARM} - \triangle c_{i,FRM}\right] \\ &= \mathbb{E}\left[c_{i,ARM} - c_{i,FRM}\right] \end{aligned} \tag{50}$$

Write $\overline{c_i}$ for i's consumption plan absent any change in monetary policy. Then according to theorem 1,

$$\begin{aligned} c_{i,ARM} - c_{i,FRM} &= \left(c_{i,ARM} - \overline{c_i}\right) - \left(c_{i,FRM} - \overline{c_i}\right) \\ &\simeq dc_{i,ARM} - dc_{i,FRM} \\ &= MPC_i \left(dW^F_{i,ARM} - dW^F_{i,FRM}\right) \end{aligned}$$

[48] For example, the case of the CLL instrument, this result requires that potential outcomes be independent of the assignment to the CLL cutoff, that the exclusion restriction be valid, and that the first stage be positive. With heterogenous treatment effects, (50) is technically a local average treatment effect, where the average is taken over "compliers" whose decision to purchase an adjustable rate mortgage is changed by the conforming loan limit. See Imbens and Angrist (1994).

In other words, the regression differences out all of the influences on consumption that are induced by the monetary policy change (in particular, intertemporal substitution and general equilibrium effects on income) and captures purely the differential balance-sheet revaluation experienced by the household under the two potential assignments to mortgage structure.[49]

Both the ARM and the FRM are nominal contracts. Since an ARM contract is entirely short term, its present value is isolated from movements in the nominal term structure $\frac{dQ_t}{Q_t} = \frac{dq_t}{q_t} - \frac{dP_t}{P_t}$, so:

$$dW^F_{i,ARM} = 0$$

A consumer with a fixed-rate mortgage that specifies nominal payments M_i, on the other hand, experiences a financial wealth revaluation from a change in the nominal term structure $\{dQ_t\}$ of

$$dW^F_{i,FRM} = -\sum_{t=1}^{T_i} Q_t \left(\frac{M_i}{P_0}\right) \frac{dQ_t}{Q_t}$$

Hence, a fixed-rate borrower experiences a present-value gain from a rise in interest rates that lowers discount rates dQ_t, since the monetary policy change lowers the present value of his liabilities.[50]

For example, a parallel rise in the term structure due to a combination of a permanent rise in inflation and/or a permanent rise in the real interest rate, where all nominal interest rates are altered by $d\iota$, leads to

$$dW_{i,FRM} = -D_i \cdot L_i \cdot d\iota$$

where L_i is the consumer's mortgage principal outstanding, and D_i its duration. In that case

$$\beta \quad = \quad -\mathbb{E}\left[MPC_i \cdot D_i \cdot L_i\right] d\iota \tag{51}$$

that is, the regression coefficient has a structural interpretation as an average product of MPCs by individual characteristics (mortgage durations and balances) that may be part of the dataset.[51] In this case we could use $\frac{\triangle c_i}{D_i L_i}$ as a left-hand side variable instead of $\triangle c_i$ in (49), and infer an average MPC.

This result is a useful benchmark, which runs counter to the intuition that consumption should fall because the rise in interest rates is akin to a "negative income shock" for the adjustable-rate mortgage holder: it shows that it is more accurate to think of it as a negative *wealth* shock, whose relevant denominator is the change in wealth and not the change in mortgage payments. However, it is important to highlight that it derived under the condition that borrowing constraints do not bind. If borrowing constraints bind for many consumers—as they do in the model of section 5—the result may down, depending on how borrowing limits adjust, and the income shock interpretation may be closer to accurate for some households. Another caveat is that in the case of *falls* in interest rates, in countries like the United States, the difference in financial wealth effect between FRM and ARM holders is lower than suggested by equation (51) due to the possibility of refinancing FRMs at a low cost.

The general idea behind equation (51) is that a perfect instrument differences out general equilibrium effects of monetary policy changes and singles out the out the first-round source of differential consumption adjustments across consumers—which here is a relative wealth change. In two recent papers, di Maggio et al. (2014) and Keys et al. (2014) investigate the response of car expenditures across counties with different

[49] As I show in section 2.3, this result is true even if households face incomplete markets, providing borrowing constraints are not binding and the change is transitory. The result appears extends to any change in the term structure, providing borrowing constraints do not bind during the time over which the financial wealth effects operate (see Conjecture 1, work in progress).

[50] In section B.1 we saw that the ARM borrower experienced a negative wealth effect, while the FRM borrower experienced none, following a rise in interest rates. The two results are not contradictory: this section focuses on financial wealth revaluations alone, while the previous section included human wealth in the calculation. Of course both ways of looking at this differential prediction yield the same conclusion.

[51] The bottom left panel of 15 provides a quantitative example: if interest rates rise permanently by 1% per year, the average marginal propensity to consume is 5%, the average mortgage liability is \$100 000, and average mortgage duration is 12 years, then we may expect $\beta = -(.05)\,(12)\,(100\,000)\,(.01) = -\$600$, that is, adjustable-rate-mortgage borrowers consume \$600 less per year than fixed-rate mortgage borrowers as a result of the interest rate change.

shares of adjustable vs fixed rate mortgages after the recent fall in interest rates.[52] Like other county-level regressions such as those of Mian et al. (2013), such a regression may capture general equilibrium effects on local income of wage changes, in addition to the first-round effect of monetary policy on wealth. The models in my paper are a step towards understanding these general equilibrium effects in a world where aggregate demand is affected by the wealth distribution.

C Proofs for section 2

C.1 Proof of theorem 1

Proof. It is convenient to define the expenditure function over the sequences $\{q_t\}$ and $\{w_t\}$:

$$e\left(\{q_t\},\{w_t\},U\right)=\min\left\{\sum_t q_t\left(c_t-w_tn_t\right)\quad \text{s.t.}\quad U\left(\{c_t,n_t,h_t\}\right)\geq U\right\}$$

and let c_t^h, n_t^h be the resulting Hicksian demands. The envelope theorem implies a version of Shephard's lemma:

$$\begin{aligned} e_{q_t} &= c_t-w_tn_t \quad \forall t \\ e_{w_t} &= -q_tn_t \quad \forall t \end{aligned}$$

Define the indirect utility function to attain unearned wealth $\widetilde{W}=\sum_{t\geq 0}q_t\left(y_t+\left({}_{-1}b_t\right)+\left(\frac{{}_{-1}B_t}{P_t}\right)\right)$ (wealth exclusive of earned income whose price is changing) as

$$V\left(\{q_t\},\{w_t\},\widetilde{W}\right)=\max\left\{U\left(\{c_t,n_t\}\right)\quad \text{s.t.}\quad \sum_t q_t\left(c_t-w_tn_t\right)=\widetilde{W}\right\}$$

and let c_t, n_t be the resulting Marshallian demands. Differentiating along the identities

$$\begin{aligned} c_0^h\left(\{q_t\},\{w_t\},U\right) &= c_0\left(\{q_t\},\{w_t\},e\left(\{q_t\},\{w_t\},U\right)\right) \\ n_0^h\left(\{q_t\},\{w_t\},U\right) &= n_0\left(\{q_t\},\{w_t\},e\left(\{q_t\},\{w_t\},U\right)\right) \end{aligned}$$

we find that Marshallian derivatives are

$$\begin{aligned} \frac{\partial c_0^h}{\partial q_t} &= \frac{\partial c_0}{\partial q_t}+\frac{\partial c_0}{\partial \widetilde{W}}e_{q_t} & \frac{\partial c_0^h}{\partial w_t} &= \frac{\partial c_0}{\partial w_t}+\frac{\partial c_0}{\partial \widetilde{W}}e_{w_t} \\ \frac{\partial n_0^h}{\partial q_t} &= \frac{\partial n_0}{\partial q_t}+\frac{\partial n_0}{\partial \widetilde{W}}e_{q_t} & \frac{\partial n_0^h}{\partial w_t} &= \frac{\partial n_0}{\partial w_t}+\frac{\partial n_0}{\partial \widetilde{W}}e_{w_t} \end{aligned}$$

denoting $MPC\equiv\frac{\partial c_0}{\partial \widetilde{W}}=\frac{\partial c_0}{\partial W}$ and $MPN\equiv\frac{\partial n_0}{\partial \widetilde{W}}=\frac{\partial n_0}{\partial W}$,[53] and using Shephard's lemma we obtain

$$\begin{aligned} \frac{\partial c_0^h}{\partial q_t} &= \frac{\partial c_0}{\partial q_t}+MPC\cdot\left(c_t-w_tn_t\right) & \frac{\partial c_0^h}{\partial w_t} &= \frac{\partial c_0}{\partial w_t}-MPC\cdot q_tn_t \\ \frac{\partial n_0^h}{\partial q_t} &= \frac{\partial n_0}{\partial q_t}+MPN\cdot\left(c_t-w_tn_t\right) & \frac{\partial n_0^h}{\partial w_t} &= \frac{\partial n_0}{\partial w_t}-MPN\cdot q_tn_t \end{aligned}$$

Applying a Taylor expansion to the consumption function $c_0\left(\{q_t\},\{w_t\},\{P_t\},\{y_t\}\right)$ and using the above values for derivatives evaluated at the initial sequence $\{q_t\},\{w_t\},\{P_t\},\{y_t\}$, we have, for sufficiently

[52] The assignment of counties to mortgage structure is not quasi-random—in part because higher house prices increase the fraction of loans above the conforming loan limit—but both papers have a way of controlling for this.

[53] Under a present-value normalization with $q_0=1$, $MPC=\frac{\partial c_0}{\partial y_0}$ and $MPN=\frac{\partial n_0}{\partial y_0}$

small changes in aggregates:

$$\begin{aligned}
dc_0 &\simeq \sum_{t\geq 0}\frac{\partial c_0}{\partial q_t}dq_t+\sum_{t\geq 0}\frac{\partial c_0}{\partial w_t}dw_t+\frac{\partial c_0}{\partial \widetilde{W}}d\widetilde{W}\\
&\simeq \sum_{t\geq 0}\left(\frac{\partial c_0^h}{\partial q_t}-MPC\cdot(c_t-w_tn_t)\right)dq_t+\sum_{t\geq 0}\left(\frac{\partial c_0^h}{\partial q_t}+MPC\cdot q_tn_t\right)dw_t\\
&\quad +MPC\left(\sum_{t\geq 0}\left(y_t+({}_{-1}b_t)+\left(\frac{{}_{-1}B_t}{P_t}\right)\right)dq_t-\sum_{t\geq 0}q_t\left(\frac{{}_{-1}B_t}{P_t}\right)\frac{dP_t}{P_t}+\sum_{t\geq 0}q_tdy_t\right)\\
&\simeq c_0\sum_{t\geq 0}\underbrace{\frac{q_t}{c_0}\frac{\partial c_0^h}{\partial q_t}}_{\epsilon^h_{c_0,q_t}}\frac{dq_t}{q_t}+c_0\sum_{t\geq 0}\underbrace{\frac{w_t}{c_0}\frac{\partial c_0^h}{\partial q_t}}_{\epsilon^h_{c_0,w_t}}\frac{dw_t}{w_t}+MPCd\Omega
\end{aligned}$$

where

$$\begin{aligned}
d\Omega &= \sum_{t\geq 0}q_tdy_t+\sum_{t\geq 0}q_tn_tdw_t\\
&\quad +\sum_{t\geq 0}\left(y_t+w_tn_t+({}_{-1}b_t)+\left(\frac{{}_{-1}B_t}{P_t}\right)-c_t\right)dq_t-\sum_{t\geq 0}q_t\left(\frac{{}_{-1}B_t}{P_t}\right)\frac{dP_t}{P_t}\\
&= \sum_{t\geq 0}q_ty_t\frac{dy_t}{y_t}+\sum_{t\geq 0}q_tw_tn_t\frac{dw_t}{w_t}\\
&\quad +\sum_{t\geq 0}q_t\left(y_t+w_tn_t+({}_{-1}b_t)+\left(\frac{{}_{-1}B_t}{P_t}\right)-c_t\right)\frac{dq_t}{q_t}-\sum_{t\geq 0}Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)\frac{dP_t}{P_t}
\end{aligned}$$

where the last line uses the Fisher equation, $\frac{q_t}{P_t}=\frac{Q_t}{P_0}$ to rewrite future real wealth in date-0 terms. Using the same calculation for n_0, we obtain the labor supply response formula in theorem 1. The welfare response follows from application of the envelope theorem to the indirect utility function:

$$\begin{aligned}
\frac{\partial V}{\partial q_t} &= -U_{c_0}\left(\{c_t,n_t,h_t\}\right)\cdot(c_t-w_tn_t)\\
\frac{\partial V}{\partial w_t} &= U_{c_0}\left(\{c_t,n_t,h_t\}\right)\cdot(q_tn_t)\\
\frac{\partial V}{\partial \widetilde{W}} &= U_{c_0}\left(\{c_t,n_t,h_t\}\right)
\end{aligned}$$

therefore a Taylor expansion yields

$$\begin{aligned}
dU &\simeq \sum_{t\geq 0}\frac{\partial V}{\partial q_t}dq_t+\sum_{t\geq 0}\frac{\partial V}{\partial w_t}dw_t+\frac{\partial V}{\partial \widetilde{W}}d\widetilde{W}\\
&\simeq U_{c_0}\left(\{c_t,n_t\}\right)\cdot\left(\sum_{t\geq 0}(w_tn_t-c_t)\,dq_t+\sum_{t\geq 0}q_tn_tdw_t+d\widetilde{W}\right)\\
&\simeq U_{c_0}\left(\{c_t,n_t\}\right)\cdot d\Omega
\end{aligned}$$

as was to be shown.

□

C.2 Value of elasticities for common utility functions

C.2.1 Inelastic labor supply

Suppose that labor supply is inelastic (so that we can consider all income, $Y_t = y_t + nw_t$, as unearned) and utility is time-separable, but not necessarily homothetic

$$U(\{c_t, n\}) = \sum_t \beta^t u(c_t) \tag{52}$$

The budget constraint is

$$\sum_{t\geq 0} \frac{q_t}{q_0} c_t = W$$

Online appendix OA.1.1 derives the following facts: the marginal propensity to consume is

$$MPC = \left(1 + \sum_{t\geq 1} \frac{q_t}{q_0} \frac{\sigma(c_t) c_t}{\sigma(c_0) c_0}\right)^{-1} \tag{53}$$

and the Hicksian elasticities are:

$$\begin{aligned} \epsilon^h_{c_0,q_t} &= MPC \frac{q_t}{q_0} \sigma(c_t) \frac{c_t}{c_0} \quad t \geq 1 \\ \epsilon^h_{c_0,q_0} &= -\sigma(c_0)(1 - MPC) \end{aligned}$$

Hence, under inelastic labor supply, theorem 1 predicts that

$$\begin{aligned} dc_0 &= MPC d\Omega - c_0 \sigma(c_0)(1 - MPC) \frac{dq_0}{q_0} + MPC \sum_{t\geq 1} \frac{q_s}{q_0} c_s \sigma(c_s) \frac{dq_s}{q_s} \\ dU &= u'(c_0) d\Omega \\ d\Omega &= \sum_{t\geq 0} q_t dY_t + \sum_{t\geq 0} q_t \left(Y_t + \left(\frac{{}_{-1}B_t}{P_t}\right) + ({}_{-1}b_t) - c_t\right) \frac{dq_t}{q_t} - \sum_{t\geq 0} Q_t \left(\frac{{}_{-1}B_t}{P_0}\right) \frac{dP_t}{P_t} \end{aligned}$$

Application: one-time change. For the case of a one-time change, $\frac{dP_t}{P_t} = \frac{dP}{P}$ for $t \geq 0$, and $\frac{dq_t}{q_t} = -dr$ for $t \geq 1$, we have

$$\begin{aligned} MPC\left(\sum_{t\geq 1} \frac{q_s}{q_0} c_s \sigma(c_s)\right)(-dr) &= MPC(\sigma(c_0) c_0)\left(MPC^{-1} - 1\right) \\ &= -\sigma(c_0) c_0 (1 - MPC) dr \end{aligned}$$

Combining with $d\Omega$ from (5) we obtain

$$dc_0 = MPC \cdot \left(dy + ndw + URE \cdot dr - NNP \cdot \frac{dP}{P}\right) - c_0 \sigma(c_0)(1 - MPC) dr$$

C.2.2 Separable preferences over consumption and labor

Consider the standard macroeconomic specification of preferences

$$U(\{c_t, n_t\}) = \sum_t \beta^t (u(c_t) - v(n_t)) \quad u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \quad v(n) = b \frac{n^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \tag{54}$$

Online appendix OA.1.2 derives the values of Hicksian elasticities ϵ^h and marginal propensities in the general case. Here I just report their value for an infinite horizon model around a steady-state with no growth, that is, where $\frac{q_s}{q_0} = \beta^s$ and $w_s = w^*, \forall s$. These are elasticities relevant to determine the impulse responses in many RBC models, for example. Writing $\vartheta \equiv \frac{w^* n^*}{c^*}$ for the share of earned income in consumption and $\kappa \equiv \frac{\frac{\psi}{\sigma}\vartheta}{1+\frac{\psi}{\sigma}\vartheta} \in (0,1)$ we have:

ϵ^h	q_0	$q_s, s \geq 1$	w_0	$w_s, s \geq 1$	Marg. propensity	
c_0	$-\sigma\beta$	$\sigma(1-\beta)\beta^s$	$\sigma\kappa(1-\beta)$	$\sigma\kappa(1-\beta)\beta^s$	MPC	$(1-\kappa)(1-\beta)$
n_0	$\psi\beta$	$-\psi(1-\beta)\beta^s$	$\psi(1-\kappa(1-\beta))$	$-\psi\kappa(1-\beta)\beta^s$	MPN	$-\frac{1}{w^*}\kappa(1-\beta)$

Table 10: Steady-state moments, separable preferences

C.2.3 GHH preferences

Consider now a GHH preference specification

$$U(\{c_t, n_t\}) = \sum_t \beta^t u(c_t - v(n_t)) \quad u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \quad v(n) = b\frac{n^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}$$

Define $\xi = \left(1 - \frac{v(n^*)}{c^*}\right) = \left(1 - \frac{w^* n^*}{c^*}\left(\frac{\psi}{1+\psi}\right)\right) \in (0,1)$, online appendix OA.1.3 shows that the relevant elasticities, in a steady-state with no growth, are given by the following table:

ϵ^h	q_0	$q_s, s \geq 1$	w_0	$w_s, s \geq 1$	Marg. propensity	
c_0	$-\sigma\xi\beta$	$\sigma\xi(1-\beta)\beta^s$	$(1+\psi)(1-\xi)$	0	MPC	$1-\beta$
n_0	0	0	ψ	0	MPN	0

Table 11: Steady-state moments, GHH preferences

C.3 Proof of theorem 2

This section proves a main lemma and derives theorem 2 in the case where $N = 0$. Online appendix OA.2 shows that the result extends to the case with an arbitrary number of stocks.

Lemma 1. *Let $c(z,b,q)$ and $q(z,b,q)$ be the solution to the following consumer choice problem under concave preferences over current consumption $u(c)$ and assets $V(a)$*

$$\begin{aligned} \max \quad & u(c) + V(a) \\ s.t. \quad & c + q(a-b) = z \end{aligned}$$

then to first order

$$dc \simeq MPC\,(dz - (a-b)\,dq + q\,db) - \sigma(c)\,c\,(1-MPC)\,\frac{dq}{q}$$

where $\sigma(c) \equiv -\frac{u'(c)}{cu''(c)}$ is the local elasticity of intertemporal substitution and $MPC = \frac{\partial c}{\partial z}$

Proof. The following first-order conditions are necessary and sufficient for optimality:

$$u'(c) = \frac{1}{q} V'(a) \tag{55}$$

I first obtain the expression for MPC by considering an increase in income dz alone. Consider how that increase is divided between consumption and savings. (55) implies

$$u''(c)\, dc = \frac{1}{q} V''(a)\, da \tag{56}$$

where the changes dc and da are related to dz through the budget constraint

$$dc + q da = dz \tag{57}$$

Define $MPC = \frac{\partial c}{\partial z}$ and $MPA = \frac{\partial a}{\partial z}$. Then (56) implies

$$\frac{MPA}{MPC} = \frac{qu''(c)}{V''(a)}$$

Combining with (57), the marginal propensity to consume satisfies

$$MPC = \frac{\partial c}{\partial z} = 1 - qMPA = 1 - \frac{q^2 u''(c)}{V''(a)} MPC \tag{58}$$

and is equal to

$$MPC = \frac{1}{1 + q^2 \frac{u''(c)}{V''(a)}} = \frac{V''(a)}{V''(a) + q^2 u''(c)}$$

Consider now the overall effect on c and a of a change in z, b, and q. Applying the implicit function theorem to the system of equations

$$\begin{cases} qu'(c) - V'(a) = 0 \\ c + q(a - b) - z = 0 \end{cases}$$

results in the following expression for partial derivatives:

$$\begin{bmatrix} \frac{\partial c}{\partial z} & \frac{\partial c}{\partial b} & \frac{\partial c}{\partial q} \\ \frac{\partial a}{\partial z} & \frac{\partial a}{\partial b} & \frac{\partial a}{\partial q} \end{bmatrix} = - \underbrace{\begin{bmatrix} qu''(c) & -V''(a) \\ 1 & q \end{bmatrix}^{-1}}_{\equiv A} \begin{bmatrix} 0 & 0 & u'(c) \\ -1 & -q & (a-b) \end{bmatrix}$$

now

$$\det(A) = q^2 u''(c) + V''(a) = \frac{V''(a)}{MPC}$$

and so

$$A^{-1} = \frac{MPC}{V''(a)} \begin{bmatrix} q & V''(a) \\ -1 & qu''(c) \end{bmatrix}$$

therefore

$$\begin{bmatrix} \frac{\partial c}{\partial z} & \frac{\partial c}{\partial b} & \frac{\partial c}{\partial q} \end{bmatrix} = MPC \begin{bmatrix} \frac{-q}{V''(a)} & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & u'(c) \\ -1 & -q & (a-b) \end{bmatrix}$$

This results directly in

$$\begin{aligned} \frac{\partial c}{\partial z} &= MPC \\ \frac{\partial c}{\partial b} &= qMPC \end{aligned}$$

Moreover, using (58) together with $u'(c) \equiv -c\sigma(c) u''(c)$ we find

$$-q\frac{u'(c)}{V''(a)}MPC = \frac{\sigma(c)c}{q}q^2\frac{u''(c)}{V_{aa}(a,\tilde{q})}MPC = \frac{\sigma(c)c}{q}(1-MPC)$$

so that

$$\frac{\partial c}{\partial q} = \frac{\sigma(c)c}{q}(1-MPC) - (a-b)MPC$$

Using a first-order Taylor expansion, the total differential is then approximately

$$\begin{aligned} dc &\simeq \frac{\partial c}{\partial z}dz + \frac{\partial c}{\partial b}db + \frac{\partial c}{\partial q}dq \\ &= MPC(dz + qdb - (a-b)dq) + \sigma(c)c(1-MPC)\frac{dq}{q} \end{aligned} \tag{59}$$

as claimed. □

Proof of theorem 2. **Case a)**: For simplicity I restrict to the special case where $N = 0$, so that the the long-term bond constitutes the only means of transferring wealth through time. Online appendix OA.2 shows that the result extends to cases with $N > 0$. The notation of theorem 2 can be cast using that of lemma 1 by using the mapping

$$q \equiv Q \quad z \equiv y + wn + \frac{\lambda}{\Pi} \quad a \equiv \lambda' \quad b \equiv \delta_N\frac{\lambda}{\Pi}$$

with $\frac{dP}{P} = \frac{d\Pi}{\Pi}$ and $\frac{dQ}{Q} = -dr$. Hence $dz = dy + ndw - \frac{\lambda}{\Pi}\frac{dP}{P}$, $db = -\delta_N\frac{\lambda}{\Pi}\frac{dP}{P}$ and $\frac{dq}{q} = -dr$; so

$$dz + qdb - (a-b)dq = dy + ndw - \underbrace{(1+Q\delta_N)\frac{\lambda}{\Pi}}_{\text{NNP}}\frac{dP}{P} + \underbrace{\left(\lambda' - \delta_N\frac{\lambda}{\Pi}\right)Q}_{\text{URE}}dr$$

Inserting this equation into (59) yields the desired result.

Case b): The consumption of an agent at the borrowing limit when $\mathbb{S} = \mathbb{S}^*$ is given by

$$c = y + wn + (1+Q\delta_N)\frac{\lambda}{\Pi} + \theta\cdot(\mathbf{d}+\mathbf{S}) + \frac{\overline{D}}{R}$$

under the considered change,

$$dc \simeq dy + ndw - \underbrace{(1+Q\delta_N)\frac{\lambda}{\Pi}}_{\text{NNP}}\frac{dP}{P} + \underbrace{\left(Q\delta_N\frac{\lambda}{\Pi} + \theta\cdot\mathbf{S} + \frac{\overline{D}}{R}\right)}_{-\text{URE}}(-dr)$$

Since this agent has $MPC = 1$, and hence satisfies $\sigma c(1-MPC) = 0$, the result follows. □

C.4 Extensions

Separable preferences Suppose that preferences are separable between consumption and hours worked:

$$U(c,n) = u(c) - v(n) \tag{60}$$

Define the elasticity of intertemporal substitution as in (9) and the Frisch elasticity of labor supply (local to the point of initial choice) as

$$\psi \equiv \frac{v'(n)}{nv''(n)}$$

Also define the total marginal propensity to spend in the present by

$$\widetilde{MPC} = MPC\left(1 + \frac{\psi}{\sigma}\frac{wn}{c}\right)$$

when income is increased by 1, consumption rises by MPC and hours worked fall by $MPN = -\frac{\psi}{\sigma}\frac{n}{c}MPC$, lowering earned income by $\frac{\psi}{\sigma}\frac{wn}{c}MPC$.

Theorem 2′. *When preferences are separable as given by (60), the change in consumption dc resulting from a purely transitory, simultaneous change in dy, dw, dP and dr is given by*

$$dc \simeq MPC\left(dy + (1+\psi)\, ndw + UREdr - NNP\frac{dP}{P}\right) - \sigma c\left(1 - \widetilde{MPC}\right) dr \tag{61}$$

The proof is given in online appendix OA.3, and is similar in spirit to that of theorem 2. The separability of preferences delivers a clean result regarding the way in which increases in wages translate into current consumption. From the point of view of the consumption decision itself, it is sufficient to know how much earnings would change if the marginal utility of consumption was held constant, that is, $(1+\psi)\, ndw$.

Non-separable preferences Suppose now that preferences are not separable. Instead, assume that consumption and work effort complement each other in utility, to the point that wealth effects on labor supply are inexistent: preferences take the form

$$U(c,n) = u(c - v(n)) \tag{62}$$

Also denote by ξ the share of net consumption in gross consumption:

$$\xi = 1 - \frac{v(n)}{c} = 1 - \left(1 + \frac{1}{\psi}\right)\frac{wn}{c}$$

Theorem 2″. *When preferences are non-separable as given by (62), the change in consumption dc resulting from a purely transitory, simultaneous change in dy, dw, dP and dr is given by*

$$dc \quad \simeq \quad MPC\left(dy + n(1+\psi)\, dw + UREdr - NNP\frac{dP}{P}\right) + (1 - MPC)\,\psi ndw - \sigma c\xi\,(1 - MPC)\, dr$$

The proof is given in online appendix OA.4. Instead of contributing equally to the increase in consumption dc as in theorem 2′, the two components of the increase in earnings induced by higher wages no longer have a symmetric role. The non-behavioral part is treated by the consumer as unearned income and contributes to an increase in consumption through the MPC. The behavioral part ($wdn = \psi ndw$), however, now has a one-for-one effect on consumption, due to the particular form of complementarity between hours and consumption assumed here.

Long-term change Consider preferences that are separable over consumption only, and a finite horizon T

$$\mathbb{E}\left[\sum_{t=1}^{T}\beta^t u(c_t)\right]$$

In section C.2.1 I derived the implications of theorem 1 under separable preferences of this kind. The following is a conjecture that the results carry through under incomplete markets, provided that borrowing constraints never bind, as is the case in a Bewley model under the natural borrowing limit. Uncertain future terms need to be appropriately evaluated given the information available at the initial date, which involves the use of a modified probability measure.

Conjecture 1. *When markets are incomplete with respect to idiosyncratic shocks and borrowing constraints never bind, the consumption change to an unexpected shock to the paths for $\{q_t\}$ and $\{P_t\}$ under perfect foresight is given* *by*

$$dc_0 = MPCd\Omega - c_0\sigma(c_0)(1-MPC)\frac{dq_0}{q_0} + MPC\sum_{t\geq 1}\frac{q_t}{q_0}\mathbb{E}_0^{\mathbb{Q}}[c_t\sigma(c_t)]\frac{dq_t}{q_t}$$

$$d\Omega = \sum_{t\geq 0} q_t \mathbb{E}_0^{\mathbb{Q}}[URE_t]\frac{dq_t}{q_t} - \sum_{t\geq 0} Q_t\left(\frac{{}_{-1}B_t}{P_0}\right)\frac{dP_t}{P_t}$$

where $\mathbb{Q}$ is a measure with density

$$\xi_T = \frac{u''(c_T)\,MPC_T}{\mathbb{E}[u''(c_T)\,MPC_T]} \qquad \left[\frac{d\mathbb{Q}}{d\mathbb{P}}\right]_t = \mathbb{E}_t[\xi_T]$$

D Proofs for section 5

D.1 Proof of proposition 4

When the borrowing constraint binds, the consumption function is linear in λ. Otherwise, an Euler equation characterizes consumers' optimal consumption plans. The envelope theorem applied to z, $\frac{dz(e,w)}{de} = w \cdot n$, shows that these consumers are exposed to residual idiosyncratic risk provided that $w(\mathbf{S}_t) > 0$. An adaptation of the Carroll and Kimball (1996) results then shows that the net consumption function g is strictly concave in λ. The result follows because $c = g + v(n)$ where n is independent of λ.

D.2 Proof of proposition 5

Integrating (37) using households' first-order condition for labor supply (31) as well as the existence of a stationary distribution $\varphi(\mathbf{s}_t)$ for idiosyncratic states, I obtain

$$\int_i e_t^i n_t^i di = \kappa w_t^{\psi} \equiv N(w_t) \tag{63}$$

where $\kappa = \frac{1}{b^{\psi}}\int [e(\mathbf{s})]^{1+\psi}\varphi(\mathbf{s})\,d\mathbf{s} = \frac{1}{b^{\psi}}\mathbb{E}\left[e^{1+\psi}\right]$ is a time-invariant cross-sectional moment of idiosyncratic productivity. Hence the supply curve of total effective hours worked, $N(\cdot)$, has the same constant elasticity ψ with respect to the economy's post-tax real wage $w_t \equiv (1-\tau)\frac{W_t}{P_t}$ as individual labor supply (31).

Next, integrating (33) and (29) across firms, and using intermediate-good market clearing, GDP is equal to

$$\begin{aligned} Y_t &= \frac{1}{\triangle_t}\int_{j=0}^{1} x_t^j dj \\ &= \frac{1}{\triangle_t}A_t\int_{j=0}^{1} l_t^j dj \end{aligned}$$

where

$$\triangle_t = \int_{j=0}^{1}\left(\frac{P_t^j}{P_t}\right)^{-\epsilon} dj$$

Using labor market clearing (37), together with (63), we have

$$Y_t = \frac{1}{\triangle_t}A_t\int_i e_t^i n_t^i di = \frac{1}{\triangle_t}A_t N(w_t) \tag{38}$$

D.3 Proof of proposition 6

Using the government budget constraint constraint (36), the definition of firm profits in (34), and labor market clearing (37)

$$\begin{aligned} P_t T_t &= \int_j F_t^j (P_t^j) dj + \tau \int_i W_t e_t^i n_t^i di \\ &= \int_j P_t^j x_t^j dj - W_t \int_j l_t^j dj + \tau \int_i W_t e_t^i n_t^i di \\ &= P_t Y_t - (1-\tau) W_t N(w_t) \end{aligned}$$

Exploiting the relationship between aggregate hours and output in (38)

$$\begin{aligned} T_t &= Y_t - (1-\tau) \frac{W_t}{P_t} \frac{\triangle_t}{A_t} Y_t \\ &= Y_t \underbrace{\left(1 - (1-\tau) \frac{W_t}{P_t} \frac{\triangle_t}{A_t} \right)}_{\equiv \tilde{\tau}_t} \end{aligned}$$

Moreover, noting from (63) that

$$e_t^i n_t^i = \frac{1}{b^\psi} \left(e_t^i \right)^{1+\psi} w_t^\psi = \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} N(w_t) \tag{64}$$

we find that real non-financial income for an individual with productivity e_t^i is

$$\begin{aligned} Y_t^i &\equiv w_t e_t^i n_t^i + T_t \\ &= w_t N(w_t) \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + \tilde{\tau}_t Y_t \end{aligned}$$

using (38) again,

$$\begin{aligned} Y_t^i &= w_t \frac{\triangle_t}{A_t} \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} Y_t + \tilde{\tau}_t Y_t \\ &= Y_t \left((1-\tilde{\tau}_t) \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + \tilde{\tau}_t \right) \end{aligned}$$

Finally, note that from the definition of net income z,

$$\begin{aligned} z(e,w) &= we \left(\frac{1}{b} we \right)^\psi - \frac{b}{1+\psi^{-1}} \left(\frac{1}{b} we \right)^{1+\psi} \\ &= \frac{1}{b^\psi} (we)^{1+\psi} \left(1 - \frac{1}{1+\psi^{-1}} \right) \\ &= \frac{1}{1+\psi} \frac{1}{b^\psi} (we)^{1+\psi} \end{aligned}$$

Hence

$$\begin{aligned} Z_t^i &= z\left(e_t^i, w_t\right) + T_t \\ &= \frac{1}{1+\psi} \frac{1}{b^\psi} (w_t)^{\psi+1} \left(e_t^i\right)^{1+\psi} + T_t \end{aligned}$$

abd using (64) we finally obtain

$$\begin{aligned} Z_t^i &= \frac{1}{1+\psi} w_t N(w_t) \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + T_t \\ &= Y_t \left(\frac{1-\tilde{\tau}_t}{1+\psi} \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + \tilde{\tau}_t \right) \end{aligned}$$

D.4 Proof of proposition 7

We have

$$\mathcal{E}_{P,t} = \mathrm{Cov}_I \left(MPC_t^i, \frac{NNP_t^i}{\mathbb{E}_I\left[c_t^i\right]} \right) = \frac{1}{\mathbb{E}_I\left[c_t^i\right]} \mathrm{Cov}_I \left(\frac{\partial c_t\left(\lambda_t^i, \mathbf{s}^i\right)}{\partial \lambda_t^i}, \lambda_t^i \right) < 0$$

since $\frac{\partial c_t\left(\lambda, \mathbf{s}^i\right)}{\partial \lambda}$ is declining in λ by Proposition 4. Moreover, whenever debt is short-term, the budget constraint implies

$$URE_t^i\left(\lambda, \mathbf{s}^i\right) = Y_t^i + \frac{\lambda}{\Pi_t} - c_t^i = Q_t \lambda_{t+1}\left(\lambda, \mathbf{s}^i\right)$$

where λ_{t+1} is increasing in λ. Hence, using the result on the covariance of monotone functions of random variables in Schmidt (2003), we find

$$\mathcal{E}_{r,t} = \frac{Q_t}{\mathbb{E}_I\left[c_t^i\right]} \mathrm{Cov}_I \left(\frac{\partial c_t\left(\lambda_t^i, \mathbf{s}^i\right)}{\partial \lambda_t^i}, \lambda_{t+1}\left(\lambda_t^i, \mathbf{s}^i\right) \right) < 0$$

D.5 Proof of proposition 8

First rewrite the extension of theorem 3 for GHH preferences in (23) in terms of elasticities, using the fact that all individuals have a common net EIS and Frisch elasticity ($\sigma^i = \sigma$, $\psi^i = \psi$), together with market clearing:

$$C_t = E_I\left[c_t^i\right] = Y_t$$

This yields

$$\begin{aligned} \frac{dC_t}{C_t} \simeq & \underbrace{\mathbb{E}_I\left[\frac{Y_t^i}{Y_t} MPC_t^i\right]}_{\mathcal{M}_t} \frac{dY_t}{Y_t} + \frac{1}{Y_t}\mathrm{Cov}_I\left(MPC_t^i, dY_t^i - Y_t^i \frac{dY_t}{Y_t}\right) - \underbrace{\mathrm{Cov}_I\left(MPC_t^i, \frac{NNP_t^i}{\mathbb{E}_I\left[c_t^i\right]}\right)}_{\mathcal{E}_{P,t}} \frac{dP_t}{P_t} \\ & + \left(\underbrace{\mathrm{Cov}_I\left(MPC_t^i, \frac{URE_t^i}{\mathbb{E}_I\left[c_t^i\right]}\right)}_{\mathcal{E}_{r,t}} - \sigma \underbrace{\mathbb{E}_I\left[\xi_t^i\left(1 - MPC_t^i\right)\frac{c_t^i}{\mathbb{E}_I\left[c_t^i\right]}\right]}_{\mathcal{S}_t} \right) \frac{dR_t}{R_t} + \frac{\psi}{Y_t}\mathbb{E}_I\left[\left(1 - MPC_t^i\right) n_t^i e_t^i\right] dw_t \end{aligned}$$

Next, using (40),

$$Y_t^i = Y_t \left(\frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} + \tilde{\tau}_t \left(1 - \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} \right) \right)$$

so

$$\begin{aligned} dY_t^i - \frac{Y_t^i}{Y_t} dY_t &= Y_t \left(1 - \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]} \right) d\tilde{\tau}_t \\ &= -\frac{Y_t^i - Y_t}{1 - \tilde{\tau}_t} d\tilde{\tau}_t \end{aligned}$$

hence

$$\frac{1}{Y_t}\mathrm{Cov}_I\left(MPC_t^i, dY_t^i - Y_t^i\frac{dY_t}{Y_t}\right) = \underbrace{-\mathrm{Cov}_I\left(MPC_t^i, \frac{Y_t^i}{Y_t}\right)}_{\mathcal{E}_{\tau,t}}\frac{d\widetilde{\tau}_t}{1-\widetilde{\tau}_t}$$

Finally, from (64),

$$\frac{e_t^i n_t^i}{Y_t} = \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]}\frac{N\left(w_t\right)}{Y_t} = \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]}\frac{\triangle_t w_t}{A_t}\frac{1}{w_t} = \frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]}\frac{1-\widetilde{\tau}_t}{w_t}$$

so that

$$\begin{aligned}\frac{\psi}{Y_t}\mathbb{E}_I\left[\left(1-MPC_t^i\right)n_t^i e_t^i\right]dw_t &= \psi\left(1-\widetilde{\tau}_t\right)\mathbb{E}_I\left[\left(1-MPC_t^i\right)\frac{\left(e_t^i\right)^{1+\psi}}{\mathbb{E}\left[e^{1+\psi}\right]}\right]\frac{dw_t}{w_t} \\ &= \psi\left(1-\widetilde{\tau}_t\right)\left(1-\mathcal{M}_t^g\right)\frac{dw_t}{w_t}\end{aligned}$$

Combining all results yields the formula in (41). The other results in proposition 8 obtain by differentiating $1-\widetilde{\tau}_t = \frac{w_t}{A_t}\triangle_t$, from which we find

$$-\frac{d\widetilde{\tau}_t}{1-\widetilde{\tau}_t} = \frac{dw_t}{w_t} - \frac{dA_t}{A_t} + \frac{d\triangle_t}{\triangle_t}$$

as well as by differentiating $Y_t = \frac{\kappa}{\triangle_t}A_t n_t^{\psi}$, which yields

$$\frac{dY_t}{Y_t} = -\frac{d\triangle_t}{\triangle_t} + \frac{dA_t}{A_t} + \psi\frac{dw_t}{w_t}$$

D.6 Proof of proposition 9

When prices are flexible, intermediate good price setting results in (35). Hence all firms set the same nominal price at all times ($\Delta_t = 1$), and the post-tax real wage is

$$w_t = (1-\tau)\frac{W_t}{P_t} = (1-\tau)\frac{\epsilon-1}{\epsilon}A_t = (1-\tau^*)A_t$$

where τ^* is the constant defined as

$$\tau^* \equiv 1-(1-\tau)\frac{\epsilon-1}{\epsilon}$$

then, from the definition of the labor wedge in (39), it follows that

$$\widetilde{\tau}_t = \tau^* \quad \forall t$$

and it follows from proposition 5 that

$$Y_t = A_t N\left(w_t\right) = \kappa\left(1-\tau^*\right)^{\psi}A_t^{1+\psi}$$

D.7 Proof of proposition 10

In flexible price equilibrium, real wages are the constant $(1-\tau^*)A$, the tax intercept is the constant $T = \tau^* Y$, and the bond price is the constant $Q = \frac{1}{\Pi R - \delta_N}$. Hence, the budget constraint and borrowing constraint in (32) rewrite

$$\begin{aligned} g + \frac{1}{\Pi R-\delta_N}\lambda' &= z\left(e\left(\mathbf{s}\right),\left(1-\tau^*\right)A\right) + T + \frac{R}{\Pi R-\delta_N}\lambda' \\ \frac{1}{\Pi R-\delta_N}\lambda' &\geq -\overline{D}\end{aligned}$$

Define gross assets as $a = \frac{1}{\Pi R - \delta_N}\lambda + \overline{D}$ and cash-on-hand as

$$\chi = z\left(e\left(\mathbf{s}\right), \left(1-\tau^*\right)A\right) + T + \frac{R}{\Pi R - \delta_N}\lambda + \overline{D}$$

The consumer's net consumption policy is then the solution to the stationary Bellman equation

$$V\left(\chi;\mathbf{s}\right) = \max_{\hat{a}\geq 0} u\left(\chi - \hat{a}\right) + \beta\mathbb{E}\left[V\left(Z\left(\mathbf{s}\right) + R\hat{a} - \overline{D}\left(R-1\right);\mathbf{s}'\right)|\mathbf{s}\right] \tag{65}$$

The maturity structure parameter δ_N does not enter equation (65). Hence in steady state, aggregate consumption demand is neutral with respect to this parameter. Since aggregate labor supply is also unaffected by it, this proves the proposition.

D.8 Monetary policy shock in the representative-agent model

Following the tradition in the New Keynesian literature, I present the equations in log-linearized form. Letting $r_t = \log\frac{R_t}{R^*}$, the Euler equation is:

$$\mathbb{E}_t\left[\hat{c}_{t+1}\right] - \hat{c}_t = \sigma\xi r_t + \left(1-\tau^*\right)\left(\mathbb{E}_t\left[\hat{n}_{t+1}\right] - \hat{n}_t\right) \tag{66}$$

where $\xi = 1 - \frac{v(N)}{C} = 1 - \left(1-\tau^*\right)\frac{\psi}{1+\psi}$. Equation (66) illustrates the amplification mechanism inherent in the complementarities between hours and consumption. Since productivity is unchanged following a shock to the real interest rate, a change in consumption must be matched by a change in hours worked, $\widehat{c_t} = \widehat{n_t}$. This in turn raises the marginal utility of consumption. In this way, a one percentage point increase in consumption around the steady-state raises net consumption by τ^* per cent. Since consumers substitute intertemporally with respect to *net* consumption, a multiplier of $\frac{1}{\tau^*}$ applies from net to gross consumption. In equilibrium, $\hat{c}_t$ follows the following dynamic equation in response to interest rate shocks:

$$\hat{c}_t = \mathbb{E}_t\left[\hat{c}_{t+1}\right] - \frac{\sigma\xi}{\tau^*}r_t$$

which can be solved for the path of $\widehat{c_t}$ given the path for r_t, assuming that the central bank's commitment to future stabilization ensures a return to the initial steady-state in the long run.

E Calibration of the earnings process and solution technique

The steady-state wedge τ^* plays a crucial role in the analysis. As is clear from proposition 6, it determines the degree of inequality in earnings, and hence the strength of the precautionary savings motive, given a process for idiosyncratic uncertainty.[54] I therefore calibrate it jointly with the productivity process as follows.

Since Lillard and Weiss (1979) and MaCurdy (1982), a large literature has fitted earnings processes to panel data on labor earnings, in particular to PSID data on male earnings. A consensus from the literature is that the earnings process features an important degree of persistence: the data on annual, log pre-tax labor earnings is reasonably described by an $AR(1)$ process with a large autoregressive root, possibly a unit root.

Since my model with infinitely-lived agents exploits the existence of a stationary distribution to define steady-state aggregates, I postulate that individual-level productivity follows an $AR(1)$ process in logs at quarterly frequency

$$\log e_t^i = \rho\log e_{t-1}^i + \sigma_e\sqrt{1-\rho^2}\epsilon_t^i \quad \epsilon_t^i \sim \mathcal{N}\left(0,1\right) \tag{67}$$

with $\rho < 1$. (67) admits the stationary distribution $\log e^{SS} \sim \mathcal{N}\left(0,\sigma_e^2\right)$. In the steady-state of my model, pre-tax earnings $e_t^i n_t^i$ are proportional to $\left(e_t^i\right)^{1+\psi}$. This implies that the steady-state variance of log earnings is $\left(1+\psi\right)^2\sigma_e^2$. Moreover the process, sampled at annual frequency, is an $AR(1)$ with root ρ^4.

[54] In section 6.5 we saw that it is also crucially related to the output multiplier when prices are sticky.

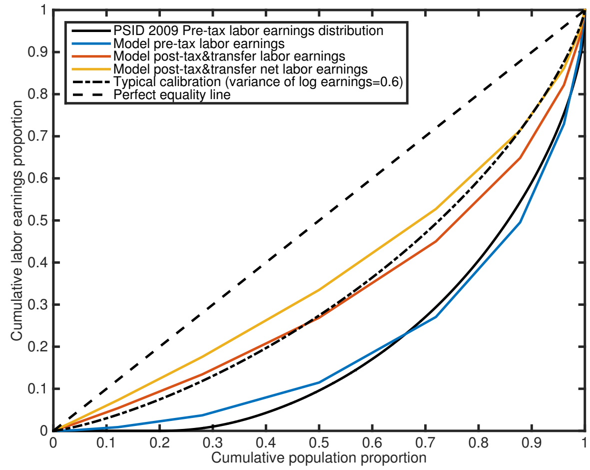


Figure 16: Calibration of pre- and post-tax earnings process

In the PSID, the cross-sectional standard deviation of log head pre-tax earnings in 2009 is 1.04. This number been relatively stable over time since 1968. I therefore set $\sigma_e = \frac{1.04}{1+\psi}$. I then discretize the idiosyncratic productivity process by using a 10-point Markov Chain using the procedure described in Tauchen (1986). Figure 16 plots the PSID Lorenz curve for post-tax earnings against that obtained in the model, illustrating that a lognormal distribution fits the majority of the earnings distribution very well.

Typical calibrations of the earnings process in this class of models (for example, Aiyagari, 1995; Floden and Lindé, 2001 or Guerrieri and Lorenzoni, 2012) assume smaller standard deviations for log earnings than 1.04, because they calibrate residual earnings uncertainty and therefore do not seek to match the earnings distribution as I do here. In my model, the driver of consumption-smoothing behavior is *post*-tax-and-transfer earnings, whose standard deviation is controlled by τ^*. I therefore set $\tau^* = 0.4$ to match the post-tax labor earnings distribution that these studies typically take (with a standard deviation of logs of 0.6).

The value of ρ^4 is more controversial. Two papers that estimate the process in the PSID and use it to calibrate an incomplete-market model are Heaton and Lucas (1996) and Floden and Lindé (2001). The former use a value of 0.53, while the latter use 0.91. I settle for $\rho^4 = 0.8$, implying a quarterly degree of persistence $\rho = 0.9457$.

Numerical solution technique To compute policy functions for steady states and transitional dynamics, I use the method of endogenous gridpoints (Carroll, 2006; Guerrieri and Lorenzoni, 2012). The algorithm for finding the steady-state is standard. For transitional dynamics, I use the following procedure. Starting from a date T at which the economy is assumed to have returned to steady-state, I compute the policy functions backwards using the path for known macroeconomic variables. I then compute the bond distribution forward from the initial distribution Ψ_0. For each date at which there is an excess demand or supply in the goods market, I adjust the real interest rate (for the flexible-price economy) or the real wage (for the sticky-price economy) in the direction of market clearing. Using this new path, I start the procedure again and iterate until convergence. Details are provided in the online appendix, section OB.