Networks, Phillips Curves, and Monetary Policy

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Abstract

This paper revisits the positive and normative implications of the benchmark New Keynesian model–which assumes only one sector of production–in a disaggregated multi-sector economy with a general input-output structure. In this setting I derive a general Phillips curve that holds for any sectoral weighted-average inflation rate, and an expression for welfare as a function of the output gap and sectoral inflation rates. With multiple sectors productivity fluctuations generate endogenous cost-push shocks in the consumer-price Phillips curve, pointing at a trade-off between inflation and output stabilization. I construct a novel inflation index that yields a "divine coincidence" Phillips curve with no endogenous cost-push shocks. Targeting this index fully stabilizes the output gap, but is not constrained optimal, because it ignores the negative consequence of sector-level inflation for allocative efficiency. I show that the constrained optimal policy can be implemented by stabilizing an alternative inflation index, which incorporates this trade-off. I calibrate the model using U.S. input-output data and sectoral price adjustment frequencies. The baseline strategy of targeting consumer inflation leads to a welfare loss of 1.12% of per-period GDP. Switching to the optimal policy reduces this loss to 0.28%, but cannot fully eliminate it. Targeting the output gap almost replicates the optimal policy. I validate my framework by showing that the divine coincidence inflation rate provides a better fit for Phillips curve regressions than conventional consumer price specifications. The model also predicts a 30% decline in the slope of the consumer price Phillips curve over the past 70 years, arising from increased intermediate input flows in production.

1 Introduction

The New Keynesian framework informs central banks' approach to monetary policy, and constitutes the theoretical foundation underpinning inflation targeting. It has two key implications. From a positive point of view, inflation and the output gap are linearly related through the Phillips curve. From a normative point of view, welfare depends on inflation and the output gap through a quadratic loss function. The baseline model assumes only one sector of production, whereas in reality there are multiple and heterogeneous sectors, that trade in intermediate inputs. How do the Phillips curve and the loss function depend on the disaggregated structure of the economy? This question has crucial implications for monetary policy and inflation targeting.

Inflation targeting is grounded in the "divine coincidence": prices are stabilized if and only if the output gap is closed. Therefore monetary policy can achieve the first-best outcome by stabilizing the price level. Inflation is a good proxy for the output gap, and a good policy target. The "divine coincidence", however, is an artifact of the one-sector model.[1] Whenever it fails, there is a tradeoff between stabilizing inflation and closing the output gap. In addition, with multiple sectors there are many possible ways to measure aggregate inflation–that is, to map sector-level inflation rates into an average index. The one-sector model cannot tell us which one is correct. In practice consumer price inflation has been taken as the real-world counterpart of inflation in the one-sector model,[2] but this choice has no theoretical backing.[3] Characterizing the optimal response to the inflation-output tradeoff, and the correct indicators to rely on, requires a theoretical framework that relates the Phillips curve and welfare with the production structure.

This paper provides such a framework. I augment the baseline model with a realistic representation of production: there are multiple sectors, which have arbitrary neoclassical production functions and are arranged in an input-output network. Sectors face idiosyncratic productivity shocks and heterogeneous pricing frictions. In this general setting, I analytically characterize the Phillips curve, welfare and optimal policy. I solve the model as a function of three variables, the output gap, sectoral inflation rates and productivity, and a set of steady-state parameters. I then study its quantitative and empirical implications, by constructing time series of these variables and calibrating the parameters to actual US input-output data. The results show that taking into

[1] The "divine coincidence" breaks in simple extensions with two sectors or sticky wages. See Balnchard and Gali (2007), Gali (2008).

[2] Statistical agencies release several different measures of consumer prices: in the US they are the consumer price index (CPI), personal consumption expenditures (PCE), their core versions (excluding sectors with very volatile prices, such as food and energy) and the GDP deflator. Central banks look at all of these measures, and various others (such as wage inflation, commodity prices, import prices, exchange rates...), but they lack a theoretical framework to aggregate them into a proxy for the output gap or into an interest rate target.

[3] For example, for a small open economy producer price inflation is the relevant statistic (see Gali and Monacelli (2005), Gali (2008)).

account the disaggregated structure of the economy is important, both from a theoretical and a quantitative point of view.

The exposition is organized around two sets of results, positive and normative. The positive analysis is concerned with the response of inflation to the output gap and productivity. I first derive it at the sector level, and then aggregate sector-level responses into the Phillips curve. In this way I can characterize the Phillips curve associated with any given inflation index, corresponding to a specific weighting of sectoral inflation rates. The normative analysis focuses on the central bank's problem. I derive the welfare loss as a function of the output gap and sectoral inflation rates, and solve for optimal monetary policy. While consumer prices are not stabilized under zero output gap or when the optimal policy is implemented (they do not preserve the "divine coincidence"), I construct two distinct inflation measures that restore each of these properties. These indicators respectively provide an inflation proxy for the output gap and an optimal policy target.

The Phillips curve relates aggregate inflation (π) with the output gap ($\tilde{y}$):

$$\pi_t = \rho \mathbb{E}\pi_{t+1} + \kappa \tilde{y}_t + u_t \tag{1}$$

where ρ is the discount factor, κ is the slope and u_t is a residual. I focus on two questions. First, for any given aggregate inflation index, I derive the parameters (κ and u) of the corresponding Phillips curve as a function of productivity shocks and model primitives. This provides a framework to interpret Phillips curve regressions. The model predicts how the estimated slope and residuals should vary across specifications and over time. Specifications where the slope is sensitive to the production structure and where productivity shocks generate large residuals are bound to yield noisy and unstable estimates. Second, I derive a "divine coincidence" inflation index that is always stabilized under zero output gap. The corresponding Phillips curve has no endogenous residual, and its slope does not depend on the production struture.

For a given index, the slope of the corresponding Phillips curve is obtained by aggregating the elasticities of sector-level prices to the output gap, according to the given weighting. The output gap acts as a labor demand shifter, and therefore it affects real wages. In turn, real wages impact marginal costs and prices. The wage pass-through is dampened by price rigidities, and more so when there are input-output linkages. In fact, sectors are affected by wage changes directly (if they hire workers) and through intermediate input prices. Suppliers do not fully pass-through wage changes into their prices, so that price rigidities get compounded along the production chain. I derive the pass-through of wages into sectoral prices as a function of input-output linkages and price adjustment frequencies. Aggregate inflation is an average of sector-level effects. Therefore it is also dampened, thereby reducing the slope of the Phillips curve.

The role of input-output linkages is quantitatively significant. I devote special attention to the consumer-price Phillips curve, given its importance in the policy practice. The calibrated network model predicts a slope of around 0.1, which is consistent with empirical estimates (usually between 0.1 and 0.3), while the one-sector model implies a slope of about 1. I also explore the role of the input-output structure for the evolution of the Phillips curve over time, fitting the model to historical input-output tables for each year between 1947 and 2017. The predicted slope is about 30% smaller in 2017 than in 1947, due to larger intermediate input flows between sectors.[4] Mirroring the slope of the Phillips curve, the network model features significantly more monetary non-neutrality than the baseline. The predicted response of inflation to monetary shocks also decreased over time. Intuitively, a smaller slope of the Phillips curve implies that consumer inflation is less sensitive to the output gap. In turn, the output gap is set by monetary policy through interest rates. This is why intermediate input flows reduce the inflation response to monetary (interest rate) shocks.

The residual u_t aggregates the elasticities of sector-level prices with respect to sectoral productivity shocks. Again, productivity shocks affect sectoral marginal costs directly and through input prices. With input-output linkages and/or idiosyncratic shocks, sectors are differentially exposed to productivity changes. Those that are directly hit face a larger change in marginal costs. In addition, under zero output gap real wages adjust to reflect the aggregate productivity change. Wages also have an asymmetric pass-through into sectoral marginal costs. As a consequence, prices cannot be stabilized in every sector, even if the output gap is closed. I derive the response of sectoral prices to productivity and wages as a function of input-output linkages and price rigidities. In general, aggregate inflation cannot be stabilized either, and the Phillips curve has an endogenous residual ($u_t \neq 0$). I use BEA-KLEMS data to construct a time series of the residual in the consumer-price Phillips curve from measured sectoral TFP shocks. The series has a standard deviation of 25 basis points, suggesting that endogenous cost-push shocks are quantitatively significant.

Whenever the Phillips curve has a residual, the "divine coincidence" fails. From equation (1), closing the output gap is equivalent to stabilizing inflation if and only if the residual u is constantly zero. While the consumer-price Phillips curve does not maintain this property in the multi-sector model, I construct a (unique) inflation index that restores it.

This "divine coincidence" index weights sectoral inflation rates according to sales shares, ap-

[4] Other authors attribute the decline in the slope of the Phillips curve to a different channel (see Blanchard (2016)): with better monetary policy inflation is more stable, therefore firms adjust prices less often. This dampens the response of inflation and reduces the slope of the Phillips curve. I mute this channel by assuming constant frequencies of price adjustment. For many sectors it is impossible to track their evolution over time, due to lack of data. For sectors where data are available, Nakamura and Steinnsson (2013) find that the frequency of price adjustment is stable over time.

propriately discounting more flexible sectors. Intuitively, the output gap captures distortions in aggregate demand arising from markups. In our framework sector-level markups change endogenously, due to incomplete price adjustment. Changes in markups, in turn, are reflected in inflation rates. Sectors with more flexible prices respond more to a given underlying markup shock, and therefore need to be discounted. We then want to relate sectoral markups with the aggregate output gap. I show that the correct way to do this is by weighting sectors according to sales shares. Sales shares reflect the full role of each sector in the value added chain, in contrast with consumption shares, which only account for the final stage. I also prove that the slope of the "divine coincidence" Phillips curve is independent of the input-output structure. This is only true for my indicator: for all the others, the slope depends on the production structure.

Interestingly, wage inflation has the highest weight in the "divine coincidence" index. This is because labor has the highest sales share, and wages are quite rigid. Previous contributions (Mankiw and Reis (2006), Blanchard and Gali (2007), Blanchard (2016)) also suggest using wage inflation as an indicator. I provide a formal argument, and characterize the correct weight for wage inflation relative to inflation in other sectors.

I construct a time series of the "divine coincidence" indicator for the US economy from PPI data, input-output tables from the BEA and estimates of price adjustment frequencies from Pasten, Shoenle and Weber (2016). I run Phillips curve regressions over the years 1984-2017, and compare the results with standard measures of consumer inflation. The baseline OLS specification has an R-squared of about 0.2 with the "divine coincidence" index, as opposed to about 0.05 with consumer inflation. Rolling regressions over 20 year windows for the same sample period are always significant, as opposed to about 50% of the time with consumer prices, and the estimated coefficient is stable over time.

Targeting the "divine coincidence" indicator closes the output gap, but does not replicate the efficient equilibrium that prevails under flexible prices. Indeed, in the multi-sector framework it is impossible to achieve the first-best. I derive an expression for the welfare loss as a function of the output gap and sectoral inflation rates. The output gap captures distortions in aggregate demand, while inflation reflects distortions in relative demand across firms and sectors. Within each sector, shocks induce price distortions between the firms that adjust their prices and those that do not. Incomplete price adjustment further results in relative price distortions across sectors. The size of within- and cross-sector price distortions depends on how shocks propagate through the input-output network. Their welfare effect depends on the response of quantities demanded, which is governed by the relevant elasticities of substitution in production and consumption.

Closing the output gap is not constrained optimal: while stabilizing aggregate demand, it does not minimize misallocation. In this sense, the "divine coincidence" does not hold from a normative point of view, unlike in the baseline model. Monetary policy has only one instrument (interest rates or money supply), therefore it needs to trade off aggregate demand against allocative efficiency. The optimal policy can still be implemented via inflation targeting, that is, by stabilizing an appropriate inflation indicator which incorporates the two sides of the tradeoff. The optimal target weights the (inflation proxy for the) output gap against sectoral inflation rates according to the relative marginal benefit and marginal cost of distorting aggregate demand to alleviate the misallocation associated with inflation.

Targeting consumer inflation, as prescribed by the baseline model, leads to a welfare loss of 1.12% of per-period GDP with respect to a world without pricing frictions. This loss decreases to 0.28% under the optimal policy, but it cannot be fully eliminated due to imperfect stabilization. Closing the output gap yields a small additional loss with respect to the optimal policy, therefore the output gap is a good target from a quantitative point of view. Intuitively, monetary policy is a blunt instrument to correct misallocation, because it can only change relative prices proportionately to their elasticity with respect to the output gap. The cost of distorting aggregate demand is larger than the gain, and in practice the optimal output gap is close to zero.

Related literature From a conceptual standpoint, my work is related with the literature on markup distortions, productivity and welfare. In economies with sticky prices, producers who cannot adjust abosorb cost shocks into their markups. My framework can be viewed as an application of Baqaee and Farhi (2017), who consider exogenous markup changes, to an economy with price rigidities, where markups are endogenously determined by productivity shocks.

A large literature relates monetary non-neutrality with input-output linkages and heterogeneous pricing frictions. From a theoretical perspective, Basu (1995) shows that input-output linkages increase monetary non-neutrality. Many subsequent works simulate models which feature intermediate inputs and heterogeneous price adjustment frequencies, finding that both channels are relevant (Carvalho (2006), Carvalho and Nechio (2011), Nakamura and Steinsson (2013) and Pasten, Shoenle and Weber (2016)). With respect to these studies, I provide analytical expressions for the response of inflation and output to monetary shocks as a function of primitives, and derive comparative statics.

The production structure has been related with the presence of an inflation-output trade-off. Blanchard and Gali (2007) and Gali (2008) show that this tradeoff is present in simple extensions of the baseline model, with price and wage rigidities. Models of currency unions feature imperfect stabilization at the country level (Gali and Monacelli (2008)). This papers characterize the Phillips curve and/or optimal policy analytically, but still have a stylized representation of production. Pasten, Shoenle, Weber (2017) instead consider a model with an input-output structure that matches the US network. They simulate the response of real GDP to sector level productivity shocks and find that they produce sizable fluctuations. I combine the two approaches, by providing an analytical characterization of the Phillips curve, welfare and optimal policy in a model that can be calibrated to actual production networks.

Previous works pointed out the limitations of consumer inflation as an indicator for the Phillips curve and monetary policy. From a theoretical perspective, the open economy literature suggests that it is not a good indicator of the output gap (due to the presence of trade linkages across countries). Gali and Monacelli (2005) argue that in a small open economy the relevant statistic for the Phillips curve is the PPI, and Benigno (2004) finds that in a currency area countries should be weighted according to their size, discounting those where prices are more flexible. This closely resembles our "divine coincidence" indicator, in the special case of a two-country model, with one sector per country and no input-output linkages. Mankiw and Reis (2003) instead focus on a closed economy setup, deriving an optimal inflation target under the assumption that the central bank seeks to maximize output stability. They do not consider input-output linkages, but introduce several elements of sectoral heterogeneity, such as size, volatility and price flexibility. They also find that a high weight should be placed on wage inflation, which is indeed what the "divine coincidence" indicator does.

The limitations of consumer prices as an indicator are also well documented empirically, and many studies seek to construct indicators that perform better. Stock and Watson (1999) and Bernanke and Boivin (2003) propose better predictors of future inflation trends, and Stock and Watson (2015) show that using sectoral inflation rates improves the prediction of trend inflation. Orphanides and Van Norden (2002) show that even ex-post measures of the output gap do not predict consumer inflation out of sample. A large empirical literature documents the instability of the Phillips curve: while Gali and Gertler (1999) use real marginal costs as a proxy for the output gap and estimate significant coefficients (and with the correct sign), follow-up studies found that their results do not hold for different time periods or data vintages. Mavroeidis, Plagborg-Muller and Stock (2014) summarize this literature, and document that estimates of the Phillips curve coefficients are very sensitive to the way inflation expectations are accounted for, to the specific measure of output gap used and to the sample period. I connect to this literature by showing that regressions with the "divine coincidence" specification yield more stable and significant estimates over time and across specifications.

The rest of the paper is organized as follows: Section 2 introduces the setup; Section 3 contains some key definitions; Section 4 presents the positive analysis; Section 5 derives the normative results; Section 6 calibrates the model to the US economy; Section 7 discusses results from Phillips curve regressions. Section 8 concludes. Two extensions of the basic model, that introduce exogenous cost-push shocks and dynamics, are presented in Appendix D.

2 Setup

This section introduces the key elements of the network model and lays out the assumptions about preferences, timing and policy instruments. Following the approach in Baqaee and Farhi (2017), we define competitive equilibrium for a given change in sectoral markups. In our setup markups change endogenously because some firms cannot adjust their price, and therefore absorb changes in marginal costs into their markup. Those that can adjust instead keep their markups constant. We assume that fiscal policy eliminates markup distortions in steady-state, and log-linearize the model around the efficient equilibrium. We introduce the variables and steady-state parameters that characterize the log-linearized system.

2.1 Timing and policy instruments

In the main text we consider a one-period model. The economy starts in steady state. Then sectors are hit by unanticipated productivity shocks, and monetary policy is implemented; some firms can adjust their price, while others cannot. The world ends after production and consumption take place. The dynamic version of the model is presented in Appendix D. In the static setup money supply is the only policy instrument (to be replaced with interest rates in the dynamic version), and works through a cash-in-advance constraint.

2.2 Preferences

Consumers derive utility from consumption and leisure, and have homothetic preferences over a bundle of all goods produced in the economy. Their utility function is

$$U = \frac{C^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi}$$

where C is a constant returns to scale aggregator of all goods produced in the economy and L is labor supply.

Consumers maximize utility subject to the cash in advance constraint

$$PC \leq M$$

and the budget constraint

$$PC \leq wL + \Pi - T$$

where P is the price of the consumption bundle, M is total money supply, w is the wage, Π are firm profits (rebated lump-sum to households) and T is a lump-sum transfer (that the government uses to finance input subsidies to firms).

2.3 Production

There are N sectors in the economy (indexed by $i \in \{1, ..., N\}$), and within each sector there is a continuum of firms, producing differentiated varieties. Customers buy a CES bundle of these varieties.

All firms in sector i have the same constrant returns to scale production function

$$Y_i = A_i F_i(L_i, \{x_{ij}\})$$

where L_i is labor, x_{ij} is the quantity of good j used as input, and A_i is a Hicks-neutral, sector-specific productivity shock.[5] We assume that labor is freely mobile across sectors.

Cost minimization and markups All producers in sector i solve the cost-minimization problem

$$mc_i = min_{\{x_{ij}\}, L_i} \ wL_i + \sum_j p_j x_{ij} \ \ s.t. \ A_i F_i \left(L_i, \{x_{ij}\}\right) = \bar{y}$$

Under constant returns to scale marginal costs are the same for all firms, and they use inputs in the same proportions.

In the initial steady-state all firms set their price optimally, solving

$$max_{p_i} \ D_i \left(p_i - (1 - \tau_i) \, mc_i\right) \left(\frac{p_i}{P_i}\right)^{-\epsilon_i}$$

where D_i and P_i are the sector-level demand and price index, and ϵ_i is the elasticity of substitution between varieties within sector i. The government provides input subsidies, in order to eliminate markup distortions (i.e. the markup over pre-subsidy marginal costs is 1). Input subsidies cannot change in response to productivity shocks, and are constrained to be the same for all firms within

[5]Note that this is without loss of generality: factor-biased productivity shocks can be modeled by introducing an additional sector which simply purchases and sells the factor, and letting a Hicks-neutral shock hit this sector.

the same sector. Under the optimal subsidy

$$1 - \tau_i = \frac{\epsilon_i - 1}{\epsilon_i}$$

firms charge price

$$p_i^* = mc_i \tag{2}$$

in steady-state.

With price rigidities, firms within the same sector charge different prices outside of steady-state. Firms who can adjust their price do not change their markup. Firms that cannot adjust must accept a change in their markup, given by

$$d \log \mu_{if}^{NA} = -d \log mc_i$$

2.4 Equilibrium

For given output gap, sectoral probabilities of price adjustment δ_i and sectoral productivity shifters, general equilibrium is given by a vector of firm-level markups, a vector of prices p_i, a nominal wage w, labor supply L, a vector of sectoral outputs y_i, a matrix of intermediate input quantities x_{ij}, and a vector of final demands c_i, such that: a fraction δ_i of firms in each sector i adjust their price; markups are optimally chosen by adjusting firms, while they are such that prices stay constant for the non-adjusting firms; consumers maximize utility subject to the budget and cash-in-advance constraint; producers in each sector i minimize costs and charge the relevant markup; and markets for all goods and labor clear.

3 Definitions

The log-linearized model is fully characterized by three variables (the output gap, the vector of sectoral inflation rates and the vector of sectoral productivity shifters), and a set of steady-state parameters, which capture the input-output structure and sector-level pricing frictions. They are defined below.

3.1 Steady-state parameters

3.1.1 Price rigidity parameters

We model price rigidities by assuming that only a fraction δ_i of the firms in each sector i can adjust their price after observing the shocks. We denote by Δ the diagonal matrix whose i-th element is δ_i.

The Calvo assumption together with firms' optimal pricing equation (2) yield the following mappings between inflation, marginal costs and markups:

$$\pi = \Delta d \log mc = -\Delta \left(I - \Delta\right)^{-1} d \log \mu \tag{3}$$

Remark 1. In this setup wage rigidities can be modeled by adding a labor sector, which sells labor services to all the other sectors and has sticky prices.

3.1.2 Input-output definitions

The input-output structure is characterized by steady-state consumption, labor and input-output shares. We introduce two additional objects, the Leontief inverse and the vector of sales shares, that are derived from the input-output matrix and consumption shares.

Consumption shares The $N \times 1$ vector β denotes expenditure shares in total consumption, and has components

$$\beta_i = \frac{p_i c_i}{PC}$$

Labor shares Sector-level labor shares in total sales are encoded in the $N \times 1$ vector α with components

$$\alpha_i = \frac{wL_i}{p_i y_i}$$

Input-output matrix The input-output matrix Ω is an $N \times N$ matrix, with element i, j given by the expenditure share on input j in i's sales:

$$\omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}$$

Leontief inverse The Leontief inverse of the input-output matrix Ω is the matrix $(I - \Omega)^{-1}$.

It holds that, while ω_{ij} is the fraction of sector i revenues spent on goods from sector j "directly", the Leontief inverse captures the total (direct and indirect) expenditure of sector i on goods from sector j (again as a share of i's revenues). The "indirect" component comes from i's purchases of j's product through intermediate inputs from its suppliers, its suppliers' suppliers, etc. Formally, the Leontief inverse is given by the geometric sum:

$$(I-\Omega)^{-1} = I + \Omega + \Omega^2 + ...$$

where the k-th power captures the expenditure of i on j through paths of lenght k.

In our setup labor is the only factor of production. Therefore labor and intermediate input shares must sum to one in every sector:

$$\alpha_i + \sum_j \omega_{ij} = 1$$

Or equivalently, denoting by $\mathbf{1}$ the column vector with all entries equal to 1:

$$\alpha + \Omega\mathbf{1} = \mathbf{1}$$

This yields the following relation between labor shares and the Leontief inverse:

$$(I-\Omega)^{-1}\alpha = \mathbf{1} \tag{4}$$

The right hand side in equation (4) captures "total" (direct and indirect) sector-level labor shares. Intuitively, producers hire labor directly, or buy it indirectly through intermediate inputs. Given that labor is the only factor of production, equation (4) tells us that the total labor share must be equal to one in every sector.

"Adjusted" Leontief inverse We refer to the matrix $(I-\Omega\Delta)^{-1}$ as the "adjusted" Leontief inverse.

We established that the Leontief inverse captures the direct and indirect expenditure of sector i on goods from sector j, as a share of i's revenue. With flexible prices this is also the elasticity of i's marginal cost to j's marginal cost, because firms charge constant markups. With price rigidities this is no longer true, as marginal cost changes do not get fully passed-through into input prices. In this case the "direct" elasticity of i's marginal cost with respect to j's is $\omega_{ij}\delta_j$, which discounts ω_{ij} by the fraction δ_j of producers in j that adjust their price. Correspondingly, the total (direct and indirect) elasticity of i's marginal cost with respect to j's is given by the (i,j) element of the "adjusted" Leontief inverse.

Sales shares The vector λ of sectoral sales shares in total GDP has components

$$\lambda_i = \frac{P_i Y_i}{PC}$$

Section 4.3 argues that sales shares, as opposed to consumption shares, capture the full role of each sector in the value added chain. Therefore the inflation index in our "divine coincidence" specification of the Phillips is based on sales shares. It holds that sales shares are related with consumption shares and the Leontief inverse as follows:

$$\lambda^T = \beta^T (I - \Omega)^{-1}$$

Elasticities of substitution While the log-linearized model only depends on the input-output concepts defined above, elasticities of substitution matter for the welfare loss derived in Section 5. We denote the elasticity of substitution between varieties from sector i by ϵ_i, the elasticity of substitution between goods i and j in the production of good k by θ_{ij}^k, and their elasticity of substitution in consumption by σ_{ij}^C; the elasticity of substitution between good i and labor in the production of good k is denoted by θ_{iL}^k.

3.2 Variables

3.2.1 Aggregate output gap

The output gap captures distortions in aggregate demand, which we express in terms of final output. Our definition is standard.

Definition 1. The aggregate output gap $\tilde{y}$ is the log-difference between realized output y and efficient output y_{nat}:

$$\tilde{y} = y - y_{nat}$$

Lemma 1 derives the change in efficient output induced by a given productivity shock, and we can solve for the change in realized output as a function of productivity shocks and labor supply using Lemma 2. A consequence of this Lemma is that the output gap can be equivalently expressed in terms of labor demand, as in equation (6).

Lemma 1. *The change in efficient output after a productivity shock $d \log A$ is given by*

$$y^{nat} = \frac{1+\varphi}{\gamma+\varphi} \lambda^T d \log A \tag{5}$$

Proof. See Appendix A1. □

The intuition for this result is simple. From Hulten's theorem, under flexible prices the first-order change in aggregate productivity is an average of sector-level productivity shocks, with weights given by sales shares λ:

$$d\log A_{AGG} = \lambda^T d\log A$$

From the consumption-leisure trade-off, in the flex-prices equilibrium labor supply changes by

$$d\log L^{nat} = \frac{1-\gamma}{\gamma+\varphi}\lambda^T d\log A$$

Equation (5) follows immediately from these two relations, together with the definition of aggregate productivity:

$$y^{nat} = d\log L^{nat} + d\log A_{AGG}$$

Lemma 2. *Around the undistorted steady-state, to a first order the change in aggregate productivity in the economy with price rigidities is the same as in the flex-price equilibrium.*[6]

Proof. See appendix A1. □

Lemma 2 allows to write the output gap as a function of productivity shocks and labor demand. We have:

$$\widetilde{y} = d\log L - d\log L^{nat} = d\log L - \frac{1-\gamma}{\gamma+\varphi}\lambda^T d\log A \tag{6}$$

3.2.2 Sectoral inflation rates

The $N \times 1$ vector of inflation rates is denoted by

$$\pi = \begin{pmatrix} \pi_1 \\ ... \\ \pi_N \end{pmatrix}$$

Remark 2. While the output gap captures distortions in aggregate demand, Section 5.1 shows that the welfare cost of "relative sectoral output gaps" can be written as a function of sectoral inflation rates (see Proposition 4).

[6]There is a second order productivity loss due to incomplete price adjustment. See Section 4.2

4 The Phillips curve

The Phillips curve is a linear relation between aggregate inflation π^{AGG} and the output gap $\tilde{y}$. In a one-period model it is given by

$$\pi^{AGG} = \kappa \tilde{y} + u \tag{7}$$

where κ is the slope and u is the residual. A key result in the one-sector model is that the Phillips curve has no endogenous residual ($u \equiv 0$, the "divine coincidence"). With multiple sectors instead the slope and residual of the Phillips curve depend on our choice of an aggregate inflation index π^{AGG}. Different indicators are characterized by the weight that they assign to sectoral inflation rates.

In this section we address two questions. First, for a given index we derive the parameters of the corresponding Phillips curve (slope and residual). Second, we look for an index that yields a "well-specified" Phillips curve, with no endogenous residual and whose slope does not change with the production structure.

We proceed as follows. Section 4.1 derives the elasticity of inflation rates sector-by-sector with respect to productivity shocks and the output gap, as a function of network primitives. Given an inflation index we then aggregate these elasticities into the slope and residual of the corresponding Phillips curve. We especially focus on consumer inflation, given its importance in the policy practice. We show that the slope of the consumer-price Phillips curve decreases with intermediate input flows, and productivity shocks generate an endogenous residual. In this sense the Phillips curve as usually measured is misspecified. The examples in Section 4.2 illustrate why this is the case. Section 4.3 then derives an inflation index that yields a correct specification, and shows that it is unique.

4.1 Parameters for a given indicator

4.1.1 Notation

We denote by $\mathcal{B}$ the $N \times 1$ vector whose components $\mathcal{B}_i$ are the elasticities of sector i's price with respect to the output gap, and by $\mathcal{V}$ the $N \times N$ matrix whose elements $\mathcal{V}_{ij}$ are the elasticities of sector i's price with respect to a productivity shock to sector j.

Formally, we can express inflation as a function of the output gap and productivity as follows:

$$\underbrace{\pi}_{N\times 1} = \underbrace{\mathcal{B}}_{N\times 1} \widetilde{y} + \underbrace{\mathcal{V}}_{N\times N} \underbrace{d\log A}_{N\times 1} \tag{8}$$

An aggregate inflation index π^{AGG} is characterized by the vector of weights ϕ that it assigns to sectoral inflation rates:

$$\pi^{AGG} \equiv \phi^T \pi = \sum_i \phi_i \pi_i$$

For a given indicator, the corresponding Phillips curve is derived by aggregating both sides of Equation (8):

$$\pi^{AGG} = \underbrace{\phi^T \mathcal{B}}_{\text{slope}} \tilde{y} + \underbrace{\phi^T \mathcal{V} d \log A}_{\text{residual}} \tag{9}$$

Correspondingly, the slope is the aggregate elasticity with respect to the output gap, while the residual is the aggregate elasticity with respect to productivity (as in equation (9)).

Consumer inflation π^C is a special case, obtained by weighting sectoral inflation rates according to consumption shares ($\phi = \beta$):

$$\pi^C = \beta^T \pi$$

The corresponding Phillips curve is

$$\pi^C = \beta^T \mathcal{B} \tilde{y} + \beta^T \mathcal{V} d \log A \tag{10}$$

so that the slope and residual are given by

$$\kappa^C = \beta^T \mathcal{B}$$
$$u^C = \beta^T \mathcal{V} d \log A$$

Sections 4.1.2 and 4.1.3 below characterize the elasticities $\mathcal{B}$ and $\mathcal{V}$, and derive the slope and residual of the consumer-price Phillips curve as a corollary.[7] As we are log-linearizing the model around the initial steady-state, all changes in prices, wages and productivity are relative to this steady-state.

4.1.2 Slope of the Phillips curve

Proposition 1 derives the elasticities of prices with respect to the output gap sector-by-sector. Corollary 1 aggregates them into the slope of the consumer-price Phillips curve.

Proposition 1. *The elasticity of sectoral prices with respect to the output gap is*

$$\mathcal{B} = \Delta \left((I - \Omega \Delta)^{-1} \alpha \right) \frac{\gamma + \varphi}{1 - \bar{\delta}_w} \tag{11}$$

[7] Propositions 1 and 2 can be seen as an application of Proposition 10 in Baqaee and Farhi (2017), recast in terms of sectoral probabilities of price adjustment and in the special case of an efficient initial equilibrium.

where

$$\bar{\delta}_w = \beta^T \Delta \left(I - \Omega\Delta\right)^{-1} \alpha \tag{12}$$

is the pass-through of nominal wages into consumer prices.

Proof. See Appendix A2 □

Corollary 1. *The slope of the consumer-price Phillips curve is given by*

$$\kappa^C = \frac{\bar{\delta}_w}{1 - \bar{\delta}_w} \left(\gamma + \varphi\right) \tag{13}$$

Proof. The result follows immediately from Proposition 1 and Equation 9. □

The elasticity $\mathcal{B}$ and the slope κ^C capture the response of prices to the output gap. Equation (6) shows that the output gap is a labor demand shifter. Therefore it affects real wages, and the effect is governed by the parameters of the labor supply curve. We show (see the proof of Proposition 1) that the elasticity of real wages to the output gap is the sum of the inverse Frish elasticity φ and the wealth effect in labor supply γ, which appears on the right hand side of (11) and (13). Wages in turn affect marginal costs and prices: the price response depends on the pass-through of nominal wages, given by $\Delta\left(\left(I - \Omega\Delta\right)^{-1} \alpha\right)$ for sectoral inflation rates and by $\bar{\delta}_w$ for consumer prices. The denominator in (11) and (13) comes from a general equilibrium effect, and maps changes in real wages into nominal wages (the two differ precisely because of the price response).

In the one-sector benchmark we have $\Omega = \mathbb{O}$ and $\alpha = 1$, so that marginal costs have unit elasticity with respect to wages. Therefore the pass-through is simply given by the price rigidity parameter δ:

$$\bar{\delta}_w = \Delta\left(\left(I - \Omega\Delta\right)^{-1} \alpha\right) = \delta$$

With input-output linkages instead the pass-through gets dampened, as stated in Corollary 2.

Corollary 2. *As long as some sector uses an intermediate input with sticky prices, the pass-through of wages into prices is less than one:*

$$\exists i, j \text{ such that } \omega_{ij}\delta_j < \omega_{ij} \Rightarrow \left(I - \Omega\Delta\right)^{-1} \alpha < \mathbf{1} \tag{14}$$

Correspondingly, the pass-through $\bar{\delta}_w$ is lower than the average price rigidity $\mathbb{E}_\beta(\delta)$:

$$\exists i, j \text{ such that } \omega_{ij}\delta_j < \omega_{ij} \Rightarrow \bar{\delta}_w < \mathbb{E}_\beta(\delta) \tag{15}$$

A reduction in labor shares compensated by a uniform increase in input shares also reduces $\bar{\delta}_w$ *:*

$$d\alpha_i < 0,\ d\omega_{ij} = d\omega_{ik}\ \forall j,k,\ \exists j \text{ such that } \omega_{ij}\delta_j < \omega_{ij} \Rightarrow d\bar{\delta}_w < 0$$

Proof. See Appendix A2. □

The intuition is as follows. Marginal costs and prices are affected by wages directly, or indirectly through input prices. The direct exposure depends on own labor shares, while the indirect one is given by the suppliers' labor share, the suppliers' suppliers labor share, etc., discounted by the fraction of firms that update their prices in each sector. The overall effect is captured by the adjusted Leontief inverse, multiplied by the vector α of steady-state labor shares:

$$\frac{d\log mc}{d\log w} = \left(I - \Omega\Delta\right)^{-1}\alpha \tag{16}$$

We can then use the pricing equation (3) to translate changes in marginal costs into inflation:

$$\frac{d\log p}{d\log w} = \Delta\frac{d\log mc}{d\log w} = \Delta\left(I - \Omega\Delta\right)^{-1}\alpha \tag{17}$$

This gives the pass-through of nominal wages into sectoral inflation rates in equation (11). To obtain the pass-through into consumer prices $\bar{\delta}_w$ we aggregate the sectoral responses in (17) according to consumption shares:

$$\bar{\delta}_w = \beta^T\frac{d\log p}{d\log w} = \beta^T\Delta\left(I - \Omega\Delta\right)^{-1}\alpha \tag{18}$$

Lemma 2 states that with input-output linkages at least one component of the vector $\left(I - \Omega\Delta\right)^{-1}\alpha$ is strictly smaller than one, reflecting the incomplete wage pass-through from price rigidities. Moreover, different sectors have different pass-through (it is higher in sectors with a large direct labor share and flexible prices, and whose suppliers have a large direct labor share and flexible prices...). The inequality in (14) also implies that the pass-through $\bar{\delta}_w$ is smaller than the average price rigidity:

$$\bar{\delta}_w < \mathbb{E}_\beta\left(\delta\right)$$

Section 6.3 calibrates the model to the US economy, and finds that the predicted slope is one order of magnitude smaller when accounting for input-output linkages and wage rigidities.

4.1.3 Endogenous cost-push shocks

The elasticity of sectoral prices with respect to productivity shocks are derived in Proposition 2. Corollary 3 aggregates them into the endogenous residual of the consumer-price Phillips curve. This is in contrast with the "divine coincidence" from the one-sector model: in our framework closing the output gap does not stabilize consumer prices.

Proposition 2. *The elasticity of sectoral prices with respect to productivity shocks is given by*

$$\mathcal{V} = \Delta \left(I - \Omega\Delta\right)^{-1} \left[\frac{1 - \overline{\delta}_A}{1 - \overline{\delta}_w} \alpha \lambda^T - I\right] \tag{19}$$

where

$$\bar{\delta}_A \left(d \log A\right) \equiv \frac{\beta^T \Delta \left(I - \Omega\Delta\right)^{-1} d \log A}{\lambda^T d \log A} \tag{20}$$

is the pass-through of the productivity shock into consumer prices, relative to the aggregate shock.

Proof. See Appendix A2 □

Corollary 3. *The residual in the consumer-price Phillips curve is*

$$u^C = \frac{\overline{\delta}_w - \overline{\delta}_A}{1 - \overline{\delta}_w} \lambda^T d \log A \tag{21}$$

Proof. See Appendix A2 □

The response of sectoral inflation rates under zero output gap depends on the competing effects of wages and productivity on marginal costs. From Equation (6), under zero output gap labor supply is at its efficient level. This can happen if and only if the change in real wages is equal to the change in aggregate productivity, because aggregate productivity coincides with the marginal product of labor.

If aggregate productivity falls ($\lambda^T d \log A < 0$), then wages also fall, and the two have an opposite effect on marginal costs. In the one sector model marginal costs are symmetrically exposed to wages and productivity (they have unit elasticity with respect to both), so that these two forces exactly counteract each other and we have the "divine coincidence". With multiple sectors, instead, marginal costs are not equally exposed to wages and productivity. Therefore they are not necessarily stabilized under zero output gap, and they can move in opposite directions in different sectors.

The change in real wages only depends on aggregate productivity $\lambda^T d \log A$, and we can map it into the change in nominal wages through the general equilibrium multiplier $\frac{1-\bar{\delta}_A}{1-\bar{\delta}_w}$. Following the same reasoning as in Section 4.1.2, wages have heterogeneous pass-through into sectoral marginal costs, based on labor shares and price rigidities. The overal inflation response is given by

$$\underbrace{\Delta\left(I-\Omega\Delta\right)^{-1}\alpha}_{\text{pass-through}}\underbrace{\frac{1-\bar{\delta}_A}{1-\bar{\delta}_w}}_{\text{multiplier}}\underbrace{\lambda^T d\log A}_{\text{real wages}}$$

which yields the first term of equation (19). The effect of productivity on inflation rates instead depends on the specific distribution of sectoral shocks, not just on the aggregate. Productivity affects marginal costs directly, and indirectly through input prices. Again, this effect is asymmetric across sectors: a productivity shock to sector i has a larger impact on i's customers, its customers customers, etc., and the propagation along the chain is larger if prices are more flexible. The overall effect is given by the adjusted Leontief inverse:

$$\frac{\partial \log mc}{\partial \log A} = -\left(I-\Omega\Delta\right)^{-1} \tag{22}$$

Together with the pricing equation (17), equation (22) yields the second term of (19). Whenever the two components of (19) differ across sectors, even under zero output gap it is impossible to stabilize marginal costs and inflation everywhere.

It turns out that consumer inflation is not stabilized either. Overall, its response depends on the relative pass-through of wages and productivity, given by the difference $\bar{\delta}_w - \bar{\delta}_A$. The productivity pass-through $\bar{\delta}_A$ is defined in equation (20), and we introduced the wage pass-through $\bar{\delta}_w$ in Section 4.1.2.[8] From equation (21) we see that, for a negative shock, consumer inflation is positive if and only if $\bar{\delta}_A > \bar{\delta}_w$. That is, iff the pass-through of the productivity shock is larger than the pass-through of nominal wages. The examples in Section 4.2 illustrate that this is the case whenever downstream or flexible sectors are hit by a "worse" shock than the average.

A natural question to ask at this point is whether there are shocks after which prices are stabilized in every sector under zero output gap. Lemma 4 below states that the only shock with this property is an aggregate labor-augmenting shock, which in our setup is equivalent to a productivity shock proportional to sectoral labor shares α.

Corollary 4. *It holds that $\mathcal{V}\alpha = \mathbf{0}$, and α is the only vector with this property.*

Proof. See Appendix A2 □

A consequence of Lemma 4 is that perfect stabilization is impossible not only in the presence

[8]Note that, different from $\bar{\delta}_w$, $\bar{\delta}_A$ is not a constant, but it depends on the specific realization of sectoral productivity shocks.

of asymmetric sector-level shocks, but also after an aggregate TFP shock - except in a horizontal economy,[9] where aggregate TFP shocks and labor augmenting shocks are the same. This is not just a theoretical curiosity: in the calibrated model we find that a 1% negative aggregate TFP shock increases consumer inflation by 0.26%.

4.2 Examples

This section illustrates with three examples the main mechanisms through which the "divine coincidence" fails. The vertical chain isolates the effect of input-output linkages, while heterogeneous adjustment frequencies are the only force at play in the horizontal economy. The oil economy combines the two.

In the vertical chain consumer inflation is dominated by the downstream sector, whose relative consumption share is larger than the relative sales share. In the horizontal economy instead consumer inflation overrepresents more flexible sectors. The two channels interact in the oil economy. This last example rationalizes the common wisdom that oil shocks create an endogenous "cost-push" term in the Phillips curve, highlighting the crucial role of wage rigidities and heterogeneous adjustment probabilities in generating this outcome.

Example 1. Vertical chain

Consider an economy made of two sectors, which we label U (for "upstream") and D (for "downstream"). Both sectors use labor, and D also uses U as an intermediate input. Only D sells to final consumers. As the downstream good is the only consumption good, the residual in the consumer-price Phillips curve is given by inflation in this sector under zero output gap.

[9] That is, an economy without intermediate inputs.

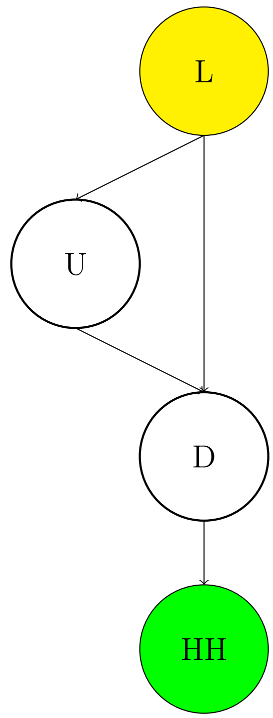

Figure 1: Vertical chain

Let's start by considering a negative productivity shock that hits D. The elasticities $\mathcal{V}_{UD}$ and $\mathcal{V}_{DD}$ of U and D's prices with respect to a productivity shock in D can be derived from Propostion 2, and are given by:

$$\mathcal{V}_{UD} = \overbrace{\underbrace{\delta_U}_{\text{pass-through}} \underbrace{\frac{1-\delta_D}{1-\bar{\delta}_w}}_{\text{multiplier}}}^{\text{nominal wages}} < 0$$

$$\mathcal{V}_{DD} = \left[\overbrace{\underbrace{\bar{\delta}_w}_{\text{pass-through}} \underbrace{\frac{1-\delta_D}{1-\bar{\delta}_w}}_{\text{multiplier}}}^{\text{multiplier}} - \overbrace{\delta_D}^{\text{productivity}} \right] > 0$$

where

$$\bar{\delta}_w = \delta_D \left(\overbrace{\alpha_D}^{\text{direct pass-through}} + \overbrace{(1-\alpha_D)\,\delta_U}^{\text{through } U} \right)$$

The drop in productivity leads to higher marginal costs for D. However the fall in real wages exactly compensates the change in D's productivity. In the one-sector case the two effects cancel out, and marginal costs remain unchanged. This is not true in our example: as long as there is some price stickiness in U, the change in wages is not fully passed-through into D's marginal cost:

$$\alpha_D + (1-\alpha_D)\,\delta_U < 1$$

Thus producers in D want to increase their price, and the residual is positive. Note that inflation has the opposite behavior in U, as nominal wages drop and productivity is unchanged.

The asymmetric exposure of sectoral marginal costs is not merely a result of the asymmetric nature of the shock (it hits only one sector). Indeed, the "divine coincidence" breaks also under an aggregate Hicks-neutral shock. The intuition is as follows: the upstream sector U is more exposed to the change in wages, while the downstream sector is more exposed to the productivity shock. On one side U benefits from the full wage decline, which it doesn't completely pass-through. On the other side, D suffers from both his own negative shock and the component of U's shock that gets passed-through. Therefore under zero output gap marginal costs fall in U and increase in D.

Formally, the pass-through of productivity into D's price is

$$\bar{\delta}_A = \delta_D\left(\overbrace{1}^{\text{direct}} + \overbrace{(1-\alpha_D)\,\delta_U}^{\text{through } U}\right)$$

Therefore we have

$$\bar{\delta}_A > \bar{\delta}_w = \delta_D\left(\alpha_D + (1-\alpha_D)\,\delta_U\right)$$

and consumer inflation is positive.

In this example, consumer inflation is not stabilized because it fails to account for the decrease in upstream prices. This is a general result: Proposition 3 shows that to restore the "divine coincidence" we need to weight sectors by sales shares, which account for their full contribution to total value added.

Example 2. Horizontal economy

Consider the horizontal economy in Figure (2): there are N sectors, $\{1, ..., N\}$, with consumption shares $\beta_1, ..., \beta_N$ and adjustment probabilities $\delta_1, ..., \delta_N$.

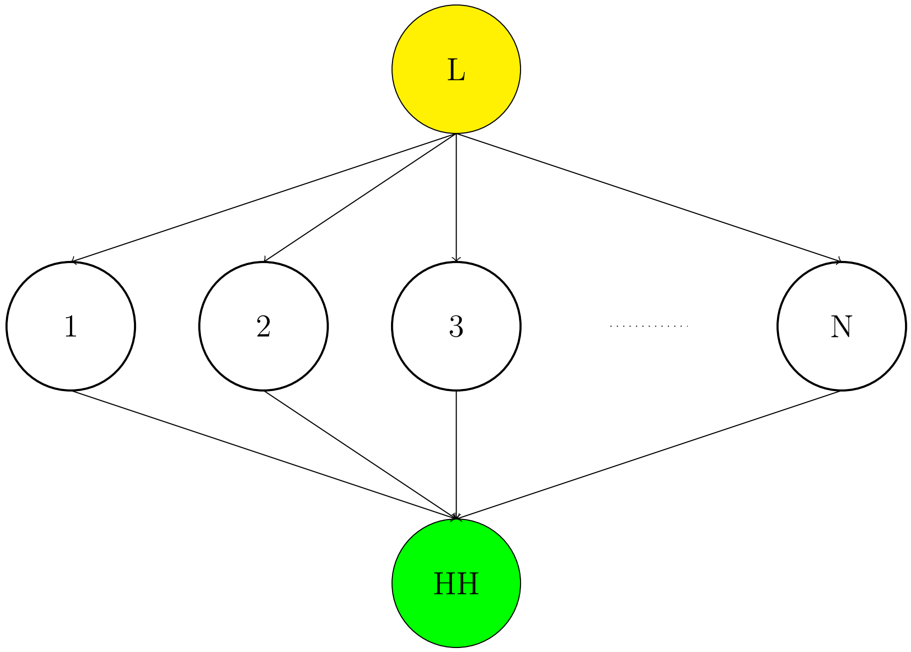

Figure 2: Horizontal economy

Under zero output gap the change in wages reflects the "average" productivity shock $\mathbb{E}_\beta\left(dlogA\right)$. From Proposition 2, inflation in each sector i satisfies

$$\pi_i = \delta_i \left(\underbrace{\frac{1-\bar{\delta}_A}{1-\bar{\delta}_w}\mathbb{E}_\beta\left(dlogA\right)}_{\text{wage}} \quad - \quad \underbrace{dlogA_i}_{\text{productivity}} \right) \tag{23}$$

where

$$\bar{\delta}_w = \mathbb{E}_\beta\left(\delta\right)$$

$$\bar{\delta}_A = \frac{\mathbb{E}_\beta\left(\delta dlogA\right)}{\mathbb{E}_\beta\left(dlogA\right)}$$

We see from (23) that inflation increases in sectors that received a worse shock than the "average" $\frac{1-\bar{\delta}_A}{1-\bar{\delta}_w}\mathbb{E}_\beta\left(dlogA\right)$, and viceversa.

Consumer inflation is obtained by aggregating sectoral inflation rates according to consumption shares:

$$\pi^C = -\frac{Cov_\beta\left(\delta, dlogA\right)}{1-\mathbb{E}_\beta\left(\delta\right)} \tag{24}$$

Therefore consumer inflation is negative if productivity increased more than the average in flexible sectors, and viceversa. Thus the divine coincidence does not hold, because consumer inflation "over-represents" more flexible sectors. This is a general result, and Proposition 3 derives the correct way to discount sectors based on price flexibility.

Example 3. Oil shocks and consumer inflation

This example presents a stylized "oil economy", to explore the channels through which a negative oil shock can increase consumer prices under zero output gap. Section 6 quantifies these channels in the actual US economy, finding that a 10% negative shock raises consumer prices by 0.22% under zero output gap.

The oil sector is among those with the highest sales share ($\simeq .09$, 99th percentile), although it has a relatively small consumption share ($\simeq .0009$, 69th percentile). This is consistent with the fact that oil is an important intermediate input. To capture this feature we study a simplified economy, given by a combination of a vertical chain and a horizontal economy (see Figure 3).

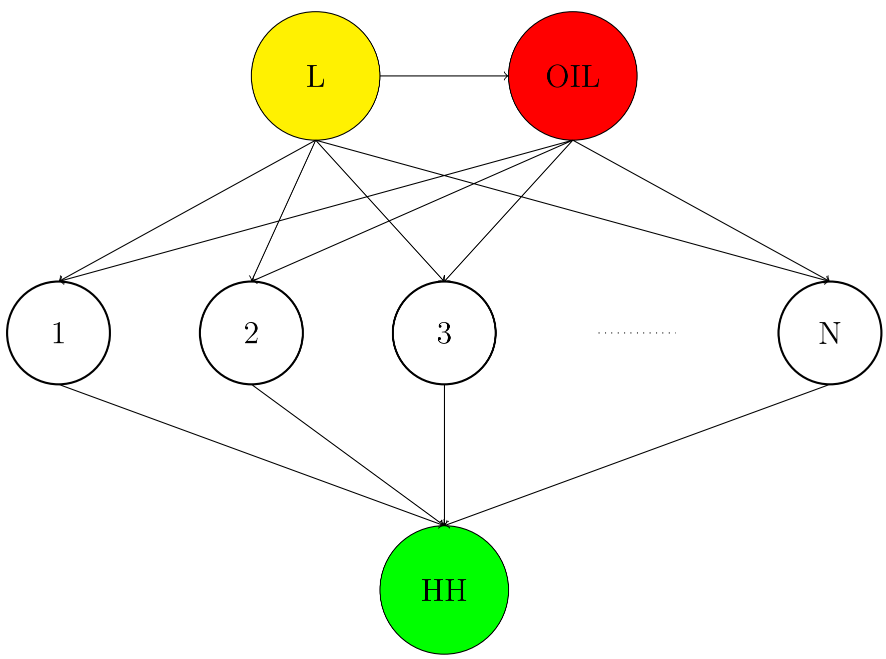

Figure 3: Oil economy

Labor is the first stage, with sticky wages.[10] Then comes oil, and finally the last stage is broken down into multiple sectors. These sectors have heterogeneous consumption shares (β_i), oil input shares ($\omega_{i,oil}$) and adjustment frequencies (δ_i). We see in the data that oil prices are very flexible, therefore oil shocks are (almost) fully passed-through to the final goods sector. On the other hand

[10]See Remark 1 in Section 3.1.1 for a discussion of how we model wage rigidities.

wages are rigid. As a result oil shocks act rather like downstream shocks, in spite of the role of oil as an intermediate input. We know from Example 1 that negative downstream shocks generate positive consumer inflation. Furthermore, we find that oil input shares and adjustment frequencies are positively correlated in the data. Example 2 shows that this further increases the pressure on consumer prices.

Formally, if $\delta_{oil} = 1$ and workers adjust wages with probability δ_L, under zero output gap consumer inflation is given by

$$\pi^C = -\frac{\overbrace{Cov_\beta(\delta, \omega_{oil})}^{\text{horizontal}} + \overbrace{(1-\delta_L)\,\mathbb{E}_\beta(\delta)}^{\text{vertical}}\,\mathbb{E}_\beta(\omega_{oil})}{1-\delta_L \mathbb{E}_\beta(\delta)} d\log A_{oil} \tag{25}$$

It is immediate to see from (25) that, for $d\log A_{oil} < 0$, consumer inflation π_C is decreasing in wage flexibility δ_L and increasing in the covariance between oil shares and adjustment frequencies.

Table 2 in Section 6.4.2 reports the calibrated response of inflation to an oil shock in the US network, under different assumptions about price and wage rigidity. Even if the full netowrk is more complex than the simple economy in this example, we find that our simple model captures well the mechanisms at play. Both wage rigidities and the correlation between oil shares and adjustment frequencies lead negative oil shocks to increase consumer inflation. If instead we assumed wages to be flexible ($\delta_L = 1$) and price adjustment frequencies to be uniform ($\delta_{oil} = \delta_i \equiv \delta_{mean}\ \forall i$), while still accounting for the actual input-output shares, an oil shock would act like an upstream shock, and consumer prices would decrease under zero output gap. The intuition is the same as in Example 1.

4.3 A correct specification for the Phillips curve

Section 4.1.3 shows that the consumer-price Phillips curve has a time-varying slope and an endogenous residual. Examples 1 and 2 suggest that this happens because consumption shares do not capture the full contribution of each sector to value added, and fail to account for the fact that flexible sectors respond more to a given shock. Building on this intuition, Proposition 3 derives an inflation measure that yields a Phillips curve with constant slope and no endogenous residual.

Proposition 3. *Assume that no sector has fully rigid prices ($\delta_i \neq 0\ \forall i$). Then the sales-weighted inflation statistic*

$$SW \equiv \lambda^T (I-\Delta)\,\Delta^{-1}\pi$$

satisfies

$$SW = (\gamma + \varphi)\,\widetilde{y} \tag{26}$$

Moreover, if prices are not fully flexible in all sectors ($\Delta \neq I$), SW is the only aggregate inflation statistic yields a Phillips curve with no endogenous residual.

Proof. Lemma 3 states that the output gap is proportional to a notion of "aggregate" markup, that weights sector level markups according to sales shares.

Lemma 3. *The following relation holds between the output gap and sector-level markups:*

$$(\gamma + \varphi)\,\tilde{y} = -\lambda^T d\log\mu \tag{27}$$

Proof. See Appendix A3. □

Markups can be inferred from inflation rates and price adjustment probabilities from the pricing equation (2):

$$-\,d\log\mu = (I - \Delta)\,\Delta^{-1}\pi \tag{28}$$

Together, Equations (27) and (28) yield the sales-weighted Phillips curve:

$$\lambda^T (I - \Delta)\,\Delta^{-1}\pi = -\lambda^T dlog\mu = (\gamma + \varphi)\,\tilde{y}$$

Lemma 4 implies that $SW = \lambda^T (I - \Delta)\,\Delta^{-1}\pi$ is the only aggregate inflation statistic such that the corresponding Phillips curve has no endogenous residual.

Lemma 4. *If $\Delta \neq I$ then $\lambda^T (I - \Delta)\,\Delta^{-1}$ is the only vector ν that satisfies*

$$\nu^T \mathcal{V} = 0$$

Proof. See appendix A3. □

□

The key result is Lemma 3, which states that the output gap is proportional to a measure of the aggregate markup. Markups can then be inferred from sectoral inflation rates, appropriately corrected for adjustment probabilities. Intuitively, the output gap is proportional to aggregate markups because of "factor suppression": when markups are higher wages correspond to a smaller fraction of workers' marginal product, which reduces labor supply and output. It turns out that the correct way to aggregate sector-level markups into the output gap is to weigh them by sales shares.[11]

[11] This argument is closely related to Proposition 3 in Baqaee and Farhi (2017).

Changes in markups can then be inferred from inflation rates, because both are determined by the same cost shock. Producers who can adjust their price respond to the shock one-for-one, and this is reflected in inflation rates. Those who cannot adjust instead absorb the shock into their markup. For a given cost shock inflation is higher in flexible sectors, and the change in average markup is smaller. Therefore these sectors need to be discounted.

Remark 3. The weights in SW are all positive. Therefore we can have $\lambda^T (I - \Delta) \Delta^{-1} \pi = 0$ only if π_i is positive in some sectors and negative in others, so that under zero output gap there are always sectors where inflation is positive and sectors where it is negative.[12]

5 Welfare function and optimal policy

Section 3 demonstrates that with multiple sectors inflation cannot be stabilized everywhere, even if the output gap is closed. In this Section we derive the welfare loss as a function of the output gap and sectoral inflation rates, and we argue that monetary policy faces a trade-off between closing the output gap and minimizing the distortions associated with inflation. Section 5.1 derives the welfare function and Section 5.2 solves for optimal policy. Section 5.3 derives an inflation target that implements this policy. In our framework, a policy target is a measure of aggregate inflation that is positive if and only if the output gap is above optimal.

Remark 4. We derive optimal policy in terms of the aggregate output gap, although the actual policy instrument is money supply. This is without loss of generality, because there is a one-to-one mapping between the two. From the consumer-price Phillips curve and the cash-in-advance constraint we have

$$dlogM = \pi_C + y = \pi_C + \widetilde{y} + \underbrace{\frac{1+\varphi}{\gamma+\varphi} \lambda^T dlogA}_{y_{nat}} =$$

$$= \left(1 + \kappa^C\right) \widetilde{y} + u^C + \frac{1+\varphi}{\gamma+\varphi} \lambda^T dlogA$$

5.1 Welfare function

We derive a second-order approximation of the welfare loss with respect to the flex-price outcome. In the baseline one-sector model this is given by

$$\mathbb{W} = \frac{1}{2} \left[\ \kappa \tilde{y}^2 + \epsilon \pi^2 \ \right] \tag{29}$$

[12]We know from Lemma 4 that in general π_i cannot be zero in every sector.

We already argued that the output gap corresponds to a distortion in aggregate demand (see Equation (6)). In the baseline model inflation reflects relative price distortions between adjusting and non-adjusting firms: the two face the same marginal cost, but end up charging different prices. Customers buy too much of the varieties whose relative price is lower than in the efficient equilibrium. For a given price distortion, quantities respond more if the elasticity of substitution ϵ is higher. Therefore the welfare cost is increasing in ϵ (see Equation (29)).

In the network model the welfare loss comes from three channels: the output gap (aggregate demand), within-sector misallocation, and cross-sector misallocation. The first two channels are common to the baseline model. In addition, with multiple sectors incomplete price adjustment also results in relative price distortions across sectors. Again, customers buy too much from those whose relative price is lower than in the efficient equilibrium. The welfare loss is increasing in the relevant elasticities of substitution, and depends on how distortions propagate through the input-output network.

Proposition 4 shows that welfare is still a quadratic function of the output gap and inflation, and the two enter separately, as in the baseline model. Interestingly, the loss function does not depend on sectoral productivity shocks directly. Intuitively, misallocation is determined by markup distortions. We derive the welfare function around an efficient steady-state, thererefore there is no interaction between the productivity shock and initial misallocation (the envelope theorem holds). The welfare loss is entirely driven by the change in markups induced by the shock, which we can infer from sectoral inflation rates (see equation (28)).

Proposition 4. *The second-order welfare loss with respect to the flex-price efficient outcome is*

$$\mathbb{W} = \frac{1}{2} \left[\ (\gamma + \varphi)\, \tilde{y}^2 + \pi^T \mathcal{D} \pi \ \right] \tag{30}$$

The matrix $\mathcal{D}$ can be decomposed as $\mathcal{D} = \mathcal{D}_1 + \mathcal{D}_2$, where $\mathcal{D}_1$ captures the productivity loss from within-sector misallocation and $\mathcal{D}_2$ captures the productivity loss from cross-sector misallocation. $\mathcal{D}_1$ is diagonal with elements

$$d^1_{ii} = \lambda_i \epsilon_i \frac{1-\delta_i}{\delta_i} \tag{31}$$

$\mathcal{D}_2$ is positive semidefinite, and it can be written as a function of the substitution operators in production and consumption defined below.[13]

[13] Φ_C and Φ_t are the same as in Baqaee and Farhi (2018). They apply these operators to sector-level price changes and labor shares around a distorted steady-state, to derive the first-order change in allocative efficiency. I work around an efficient steady-state where markup shocks have no first-order effect on allocative efficiency, while the substitution operators applied to sector level price changes characterize the second-order loss.

Definition 2. The substitution operators Φ_t (for sector t) and and Φ_C (for final consumption) are symmetric operators from $\mathbb{R}^N \times \mathbb{R}^{\mathbb{N}}$ to $\mathbb{R}$, defined as

$$\Phi_t(X,Y) = \frac{1}{2}\sum_k \sum_h \omega_{tk}\omega_{th}\theta^t_{kh}(X_k - X_h)(Y_k - Y_h) +$$

$$+\alpha_t \sum_k \omega_{tk}\theta^t_{kL} X_k Y_k$$

and

$$\Phi_C(X,Y) = \frac{1}{2}\sum_k \sum_h \beta_k \beta_h \sigma^C_{kh}(X_k - X_h)(Y_k - Y_h)$$

The elements of $\mathcal{D}_2$ are given by

$$d^2_{ij} = \frac{1-\delta_i}{\delta_i}\frac{1-\delta_j}{\delta_j}\left(\Phi_C\left((I-\Omega)^{-1}_{(i)}, (I-\Omega)^{-1}_{(j)}\right) + \sum_t \lambda_t \Phi_t\left((I-\Omega)^{-1}_{(i)}, (I-\Omega)^{-1}_{(j)}\right)\right) \tag{32}$$

Proof. See appendix B1 □

Although the within- and cross-sector components of the welfare loss are not conceptually different,[14] it is convenient to consider them separately.

From equation (31) we see that the price dispersion loss within each sector is $\epsilon_i \pi_i^2$, the same as in the one-sector model. Proposition **??** tells us that sector-level losses should be aggregated by sales shares, discounting flexible sectors. The intuition is the same as in Section 4.3. Overall, the within-sector component of the total welfare loss is given by

$$\pi^T \mathcal{D}_1 \pi = \sum_i \lambda_i \frac{1-\delta_i}{\delta_i}\epsilon_i \pi_i^2$$

From Equation (32), the total welfare loss from cross-sector misallocation can be written as

$$\pi^T \mathcal{D}_2 \pi = \sum_t \underbrace{\lambda_t}_{\text{aggregation}} \underbrace{\sum_{i,j} \Phi_t(i,j)}_{\text{productivity loss in sector } t} \tag{33}$$

Here we treated final consumption as an additional sector with $\lambda_C = 1$, and with some abuse of

[14] They could be unified into the cross-sector component if we considered a fully disaggregated model, where sectors are identified with individual firms.

notation we defined

$$\Phi_t(i,j) \equiv \Phi_t\left((I-\Omega)^{-1}_{(i)}\frac{1-\delta_i}{\delta_i}\pi_i, (I-\Omega)^{-1}_{(j)}\frac{1-\delta_j}{\delta_j}\pi_j\right)$$

Equation (33) suggests that, for each sector t, misallocation across inputs is equivalent to a (second-order) TFP loss with respect to the flex-price outcome. The aggregate effect is obtained by weighting sectors according to sales shares λ_t, as in Hulten's formula. To derive the productivity loss for each sector t we proceed in two steps. First, we derive the relative price distortions that trigger misallocation across t's inputs. These can be inferred from sectoral inflation rates, as shown in Lemma 5. Second, we translate these relative price distortions into their productivity effect, through the substitution operators.

We define relative prices with respect to nominal wages. Lemma 5 provides the mapping between inflation rates and relative price distortions.

Lemma 5. *Relative price distortions with respect to the flex-price outcome are given by*

$$d\log p - d\log w = (I-\Omega)^{-1}(I-\Delta)\Delta^{-1}\pi \tag{34}$$

Proof. See Appendix B1 □

From Equation (34), relative price distortions can be decomposed into a direct and a propagation effect:

$$d\log p - d\log w = \underbrace{(I-\Omega)^{-1}}_{\text{propagation}}\underbrace{(I-\Delta)\Delta^{-1}\pi}_{\text{markup (direct)}}$$

Here is the intuition. A distortion in the relative price of a sector k can come either directly from a change in k's markup, or indirectly from a change in the markup of some of its inputs. The mapping between markups and inflation rates is given by equation (28):

$$-d\log\mu = (I-\Delta)\Delta^{-1}\pi$$

This corresponds to the direct effect. The Leontief inverse $(I-\Omega)^{-1}$ captures the propagation effect: the price distortion induced in sector k by a change in i 's markup is given by $(I-\Omega)^{-1}_{ik}\,d\log\mu_i$. As a result, the relative price distortion between each input pair (k,h) associated with inflation in sector i is given by

$$\left((I-\Omega)^{-1}_{ki} - (I-\Omega)^{-1}_{hi}\right)\frac{1-\delta_i}{\delta_i}\pi_i$$

The substitution operator Φ_t (see Definition 2) computes the impact on t's productivity. This a second-order effect, therefore it depends on the interaction between distortions induced by all sector pairs (i, j):

$$\text{Loss in } t = \sum_{i,j} \Phi_t(i, j)$$

The distortions induced by π_i and π_j can reinforce or offset each other. $\Phi_t(i, j)$ measures the productivity loss of sector t induced by a 1% increase in i's inflation, given that j's also increased by 1%. It is positive if π_i and π_j produce similar relative price changes across input pairs (k, h), especially those with higher shares or higher elasticity of substitution. In this case π_i and π_j have a reinforcing effect on misallocation. Viceversa, the distortions from π_i and π_j offset each other if they induce opposite relative price changes across input pairs (k, h).

When elasticities of substitution are uniform ($\theta^t_{kh} \equiv \theta^t$) the productivity loss is proportional to the covariance between the price distortions induced by i and j, with probability weights given by t's input shares $\{\omega_{tk}\}_{k=1..N}$:

$$\Phi_t(i, j) = \theta^t Cov_{\Omega_t}\left(\underbrace{(I-\Omega)^{-1}_{(i)} \frac{1-\delta_i}{\delta_i}\pi_i}_{\text{distortion from } i}, \underbrace{(I-\Omega)^{-1}_{(j)} \frac{1-\delta_j}{\delta_j}\pi_j}_{\text{distortion from } j}\right)$$

Otherwise each pair (k, h) needs to be weighted by the relevant elasticity θ^t_{kh}:

$$\Phi_t(i, j) = \underbrace{\omega_{tk}\omega_{th}}_{\text{input shares}} \underbrace{\theta^t_{kh}}_{\text{substitution}} \underbrace{\left((I-\Omega)^{-1}_{ki} - (I-\Omega)^{-1}_{hi}\right) \frac{1-\delta_i}{\delta_i}\pi_i}_{\text{distortion from } i} \underbrace{\left((I-\Omega)^{-1}_{kj} - (I-\Omega)^{-1}_{hj}\right) \frac{1-\delta_j}{\delta_j}\pi_j}_{\text{distortion from } j}$$

5.2 Optimal policy

Optimal monetary policy minimizes the welfare loss, subject to a constraint, given by the response of inflation to the output gap and productivity. In the one-sector model this is captured by the Phillips curve, and the central bank solves

$$\begin{aligned} min_{\pi,\tilde{y}} \quad \mathbb{W} = \tfrac{1}{2}\left[\ \kappa\tilde{y}^2 + \epsilon\pi^2\ \right] \\ s.t.\ \pi = \kappa\tilde{y} \end{aligned} \tag{35}$$

The "divine coincidence" holds: optimal policy achieves the first best by setting $\pi = \tilde{y} = 0$. With multiple sectors the optimal policy problem extends the baseline in two dimentions. First, we replace the within-sector term $\epsilon\pi^2$ with the more complex loss function that we derived in Proposition 4, which correctly aggregates within-sector distortions and accounts for cross-sector distortions. Second, the constraint is not given by the aggregate Phillips curve but it takes into account the full response of inflation rates sector-by-sector. Thus the problem becomes:

$$\begin{gathered} min_{\tilde{y},\pi}\ \frac{1}{2}\left[\ (\gamma+\varphi)\,\tilde{y}^2 + \pi^T \mathcal{D} \pi\ \right] \\ s.t.\ \pi = \mathcal{B}\tilde{y} + \mathcal{V} dlogA \end{gathered} \tag{36}$$

In the multi-sector model there is a trade-off between within- and cross-sector misallocation. Given that some firms cannot change their price, within-sector distortions can be stabilized only if all firms charge the same price as before the shock. However keeping relative sector-level prices fixed is inefficient if relative productivities have changed. Therefore it is impossible to replicate the efficient outcome, even with a full set of sector-level subsidies. In addition, monetary policy is constrained to use a one-dimensional policy instrument (it can only manage the output gap, through interest rates or money supply). This is reflected in the constraint, which shows that monetary policy can only implement relative price changes that are proportional to sectoral elasticities with respect to the output gap $\mathcal{B}$. Proposition 5 characterizes the solution to the optimal policy problem.

Proposition 5. *The value of the output gap that minimizes the welfare loss is*

$$\widetilde{y}^* = -\frac{\mathcal{B}^T \mathcal{D} \mathcal{V} dlogA}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}} \tag{37}$$

Proof. The result follows immediately from solving the minimization problem (36). □

The optimal output gap trades off the marginal cost and benefit of deviating from the efficient aggregate demand level. The denominator in equation (37) reflects the marginal cost, and it is always positive. The cost comes directly from distortions in aggregate demand (whose welfare effect is proportional to the labor supply elasticity $(\gamma + \varphi)$), and indirectly from relative price distortions caused by the output gap (given by $\mathcal{B}^T \mathcal{D} \mathcal{B} \tilde{y}^2$). The numerator in (37) is the marginal benefit. We know from our derivation of the loss matrix $\mathcal{D}$ (see Section 5.1) that

$$\tilde{\pi}^T \mathcal{D} \pi$$

is the welfare effect of inducing inflation $\tilde{\pi}$ for given current inflation π. From the constraint we see that the output gap affects prices proportionately to their elasticity $\mathcal{B}$. Therefore the marginal effect of the output gap on misallocation given that current inflation is π is given by $\mathcal{B}^T\mathcal{D}\pi$. Under zero output gap, the productivity shock induces inflation

$$\pi = \mathcal{V} d \log A$$

so that the overall marginal benefit is given by $\mathcal{B}^T\mathcal{D}\mathcal{V}d\log A$.

5.3 An optimal inflation target

Proposition 6 derives an inflation target that implements the optimal policy. By a policy target we mean an aggregate inflation measure which is positive if and only if the output gap is above optimal.

Proposition 6. *Assume that no sector has fully rigid prices. Then there exists a unique vector of weights ϕ (up to a multiplicative constant) such that the aggregate inflation*

$$\pi_\phi = \phi^T \pi$$

is positive if and only if $\widetilde{y} > \widetilde{y}^*$.

This vector is given by

$$\phi^T = \lambda^T (I - \Delta) \Delta^{-1} + \mathcal{B}^T \mathcal{D} \tag{38}$$

Proof. See Appendix B2 □

Using our correct specification of the Phillips curve, we can re-write the target in terms of inflation and the output gap:

$$\phi^T \pi = (\gamma + \varphi) \tilde{y} + \mathcal{B}^T \mathcal{D} \pi \tag{39}$$

We see from (39) that the output gap is weighted against sectoral inflation rates according to the relative marginal benefit ($\mathcal{B}^T\mathcal{D}\pi$) and marginal cost ($\gamma + \varphi$) of distorting aggregate demand to counteract the misallocation associated with inflation (see Section 5.2).

This result extends with minimal modifications to the dynamic setup (see Appendix D). Here the optimal policy can be implemented via a Taylor rule which targets the inflation statistic in Proposition (6), with an additional correction for inflation expectations.

5.4 Examples

The examples below illustrate optimal monetary policy in the three simple networks introduced in Section 4.2 (vertical chain, horizontal economy and oil economy).

Example 4. Optimal policy in the vertical chain

Consider a negative downstream shock in a two-stage vertical chain, as in Example 1. Can monetary policy do better than implementing a zero output gap? Which sector shall it seek to stabilize?

Recall that marginal costs increase downstream and fall upstream under zero output gap. A positive output gap then increases inflation downstream, while it stabilizes the upstream sector. We argue that this is what the optimal policy should do, because distortions are more costly in the upstream sector, and this sector is also easier to stabilize.

In the vertical chain, the only source of cross-sector misallocation is substitution by D between labor and the intermediate input produced by U. There is no misallocation across consumption goods, because there is only one of them, nor across U's inputs, because it only uses labor. Misallocation between U and labor happens because U's price does not fully adjust to reflect the change in labor costs. Monetary policy can offset this effect by stabilizing wages, and thereby reducing U's desired price adjustment. After a negative productivity shock this entails implementing a positive output gap. This is reflected in the cross-sector component of the optimal output gap:

$$-\frac{(\gamma+\varphi)\,\theta_L^D}{\left(1-\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)\right)^2}\left[\underbrace{(1-\alpha_D)\,\alpha_D}_{\text{input shares}}\ \underbrace{(1-\delta_U)^2\,(1-\delta_D)}_{\propto\frac{1-\delta_U}{\delta_U}\pi_U}\right] dlogA_D > 0$$

The within-sector component of the optimal output gap instead is

$$-\epsilon\frac{(\gamma+\varphi)\,(1-\alpha_D)\,(1-\delta_U)\,(1-\delta_D)}{\left(1-\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)\right)^2}\left[\underbrace{\delta_U}_{\text{benefit for }U} - \underbrace{\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)}_{\text{cost for }D}\right] dlogA_D$$

Here $\frac{\delta_D}{\delta_U}$ is the relative cost of within-sector price dispersion in D and U, and $1-(1-\alpha_D)\,\delta_U$ is the relative marginal effect of monetary policy on inflation in the two sectors. We argued that stabilizing sector U entails implementing a positive output gap. This policy reduces the overall within-sector misallocation if and only if the benefit for U (δ_U) is greater than the loss for D ($\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)$). This is always the case if adjustment frequencies are the same in the two sectors.

Example 5. Optimal policy in the horizontal economy

Consider the horizontal economy in Example 2. The tradeoff between within- and cross-sector misallocation is particularly clear in this setup. On one hand, stabilizing within-sector misallocation would require all firms to charge the same price, which must be the same as in the initial equilibrium, because some firms cannot adjust. On the other, hand under zero output gap sectors that faced a worse shock than the average see an increase in their marginal cost, and viceversa. This should be reflected in relative prices. Therefore any improvement in within-sector misallocation comes at the cost of worse cross-sector misallocation (and viceversa).

Indeed, the relative welfare effect of the output gap on these two channels is given by the ratio of within- and cross-sector elasticities of substitution. The two have opposite sign, reflecting the fact that correcting within- vs cross-sector misallocation requires opposite price adjustments. The marginal gain of increasing the output gap for within-sector misallocation is given by

$$\epsilon \mathbb{E}_{\beta(1-\delta)} \pi$$

and the corresponding cross-sector component is

$$-\sigma \mathbb{E}_{\beta(1-\delta)} \pi$$

Here we denoted by $\mathbb{E}_{\beta(1-\delta)}$ the expectation computed with probability weights

$$\left\{ \frac{\beta_i (1-\delta_i)}{\sum_j \beta_j (1-\delta_j)} \right\}_{i=1,\dots,N}$$

Stabilizing within-sector misallocation is optimal iff the corresponding elasticity is larger than the cross-sector one. Sectors with large consumption share are given higher weight. Flexible sectors are discounted, because only non-adjusting firms face markup distortions.

We can derive the marginal gain of increasing the output gap as a function of productivity by solving for the sector-level inflation induced by the shock under zero output gap. The cross-sector component is

$$-\sigma (\gamma + \varphi) Cov_{\beta(1-\delta)} (\delta, d \log A) \tag{40}$$

The within-sector component can be obtained from (40), multiplying times $-\frac{\epsilon}{\sigma}$. Like the cost-push shock in Example 2, the optimal output gap depends on the correlation between productivity shocks and adjustment frequencies. As discussed in Example 2, this correlation reflects the competing effect of wages and productivity on marginal costs, prices and markups. For the optimal policy, however, the relevant "probability weights" are not just given by plain consumption shares, but they also account for the fraction of non-adjusting firms. Intuitively, fully flexible sectors price at their marginal cost, therefore they do not face any distortion and do not need to be stabilized.

Example 6. Optimal policy in the oil economy

Let's go back to the simple model of an oil economy in Example 3, given by a combination of a vertical chain and a horizontal economy.

Here all the cross-sector misallocation comes from distortions across final goods, and not across the three "stages" (labor, oil and final goods), because oil uses only labor as an input, and we assumed that oil prices are fully flexible. Therefore the cross-sector component of the optimal output gap is the same as the horizontal economy, given by:

$$-\sigma\left(\gamma+\varphi\right)\mathbb{E}_{\beta(1-\delta)}\pi$$

Writing inflation as a function of the productivity shock, we obtain:

$$\sigma\left(\gamma+\varphi\right)\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\left[\mathbb{E}_{\beta(1-\delta)}\left(\delta\omega_{oil}\right)-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\mathbb{E}_{\beta(1-\delta)}\left(\delta\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)\right]dlogA_{oil} \quad (41)$$

With respect to the horizontal economy (see equation (40)), we introduced sticky wages. When wages are fully rigid ($\delta_L = 0$) monetary policy has no effect on marginal costs and therefore on markup distortions, so that the optimal output gap is zero. In addition, in Examples 2 and 5 we argued that the covariance between productivity shocks and adjustment frequencies captures the countervailing effects of wages and productivity on marginal costs and markups. In equation (41) sectoral productivity shocks are replaced by oil input shares, that reflect the pass-through of the oil shock into sectoral marginal costs. When $\delta_L < 1$ the wage channel is muted, and the productivity effect becomes more important. As a result we have:

$$\mathbb{E}_{\beta(1-\delta)}\left(\delta\omega_{oil}\right)-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\mathbb{E}_{\beta(1-\delta)}\left(\delta\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)>Cov_{\beta(1-\delta)}\left(\delta,\omega_{oil}\right)$$

Within-sector misallocation comes from both final goods and labor. Consistent with the intuition from the horizontal economy, within- and cross-sector misallocation have a common component, with opposite sign. From Example (5) we know that, if $\epsilon > \sigma$, optimal policy stabilizes within-sector misallocation. In addition, there is within-sector misallocation in the labor stage as well. From the vertical chain example we know that this can be stabilized with a positive output gap: given that aggregate productivity is lower, wages should decrease. A positive output gap increases labor demand, with an offsetting effect on wages. As a result, the distortion between adjusting and non-adjusting workers becomes smaller, thereby reducing misallocation in the labor sector. The overall effect is in equation (42):

$$\epsilon\left(\gamma+\varphi\right)\frac{\delta_L\mathbb{E}_\beta\left(1-\delta\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\left[\underbrace{\left(1-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)}_{\text{vertical chain}}\right.$$

$$\left.-\left[\underbrace{\mathbb{E}_{\beta(1-\delta)}\left(\delta\omega_{oil}\right)-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\mathbb{E}_{\beta(1-\delta)}\left(\delta\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)}_{\text{horizontal economy}}\right]\right]dlogA_{oil} \tag{42}$$

Quantitatively, the calibration in Section 6.4.2 shows that the optimal output gap is positive.

6 Quantitative analysis

6.1 Data

I calibrate the input-output matrix Ω and the labor and consumption shares α and β based on input-output tables published by the BEA. I use tables for the year 2012, because this is the most recent year for which they are available at a disaggregated level (405 industries). Section 6.3.3 also relies on less disaggregated hisotrical input-output data (46 - 71 industries), always from the BEA input-output accounts, to study the slope of the Phillips curve and monetary non-neutrality over time.

The BEA publishes two direct requirement tables, the Make and Use table, which contain respectively the value of each commodity produced by each industry and the value of each commodity and labor used by each industry and by final consumers. In addition the BEA publishes an Import table that reports the value of commodity imports by industry. The Make and Use matrix (corrected for imports) can be combined, under proportionality assumptions, to compute the matrix Ω of direct input requirements and the labor and consumption shares α and β.

I use data from Pasten, Shoenle and Weber (2017) to calibrate industry-level frequencies of price adjustment. For sectors with missing data I set the adjustment probability equal to the mean. I calibrate the quarterly probability of wage adjustment to 0.25, in line with Barattieri, Basu and Gottschalk (2014) and Beraja, Hurst and Ospina (2016).

I choose values for the elasticities of substitution across inputs and consumption goods based on estimates from the literature. I set the substitution elasticity between consumption goods to $\sigma = 0.9$,[15] the elasticity of substitution between labor and intermediate inputs to $\theta_L = 0.5$,[16] the elasticity of substitution across intermediate inputs to $\theta = 0.001$,[17] and the elasticity of substitution between varieties within each sector to $\epsilon = 8$.[18]

I obtain estimates of annual industry-level TFP changes for the period 1987-2015 from the BEA Integrated Industry-Level Production Account data.

6.2 Welfare loss from business cycles

In a multi-sector economy monetary policy cannot replicate the flex-price efficient outcome. In this section I calibrate the expected welfare loss with respect to this first-best outcome under different policy rules. I assume that productivity shocks are normally distributed, with zero mean and covariance Σ. I calibrate Σ from sector-level TFP shocks as measured in the BEA KLEMS data. I compare the results with two counterfactual scenarios, one where industry-level shocks are iid, and the other where they are perfectly correlated. I keep the variance of aggregate productivity constant across the three specifications. Analytical expressions for the welfare loss under various policy rules are provided in Appendix C.

The results are plotted in Figures 4 and 5. The bars correspond to the percentage of per-period GDP that consumers would be willing to forego in exchange of going from a world with sticky prices to one without pricing frictions, for a given monetary policy rule. Bars of different colors represent different rules (blue for optimal policy, red for output gap targeting, and yellow for consumer inflation targeting).

[15] Atalay (2017), Herrendorf et al. (2013), and Oberfield and Raval (2014)) estimate it to be slightly less than one.

[16] This is consistent with Atalay (2017), who estimates this parameter to be between 0.4 and 0.8.

[17] See Atalay (2017).

[18] This is consistent with estimates of the variety-level elasticity of substitution from the industrial organization and international trade literatures.

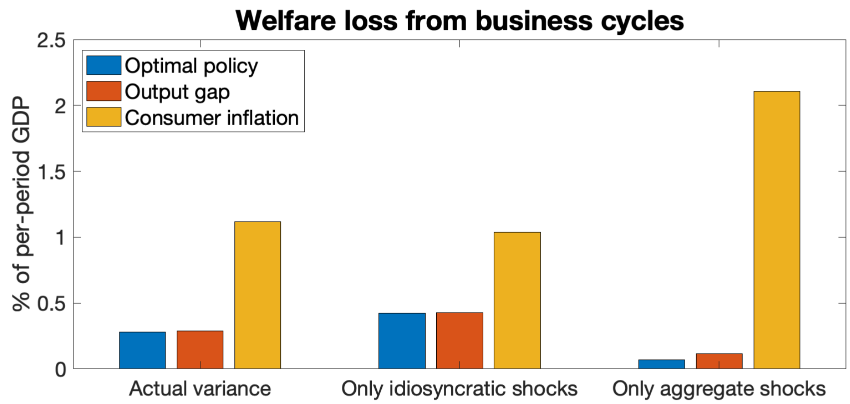

Figure 4: Actual input-output network; different calibrations keep the variance of aggregate output constant

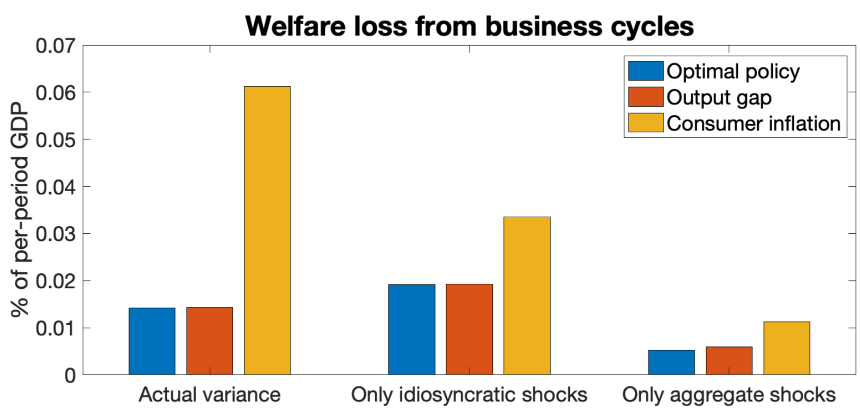

Figure 5: Model with no input-output linkages; different calibrations keep the variance of aggregate output constant

6.2.1 Optimal policy

The blue bars in Figure 4 represent the welfare loss under the optimal policy. In the full calibration business cycles generate a welfare loss of 0.28% of per-period GDP, with respect to an economy without pricing frictions. We see from the figure the idiosyncratic component of the shocks are the main driver. We can compare this estimate to two benchmarks. The first is the one-sector model, where monetary policy can replicate the first best, therefore there is no welfare loss. The second is the well-known Lucas' estimate of the welfare cost of business cycles in an efficient economy with flexible prices.[19] Lucas estimates this cost to be about 0.05% of per-period GDP, one order of magnitude smaller than the additional loss induced by price rigidities in our model.

[19] Here the welfare cost comes from the uncertainty generated by fluctuations in consumption.

Input-output linkages are key for our results: Figure 5 shows that the loss becomes more than one order of magnitude smaller in a horizontal economy with the same wage rigidity and heterogeneous price adjustment frequencies.[20]

6.2.2 Targeting the output gap

The red bars in Figures 4 and 5 show that on average targeting zero output gap yields a small loss with respect to the optimal policy. This result is consistent with our discussion in Section 5.1: the output gap is a very blunt tool to correct misallocation, therefore it is optimal to stabilize aggregate demand.

We get to a similar conclusion when comparing the behavior of SW and the optimal policy target over time (see Figure 6).

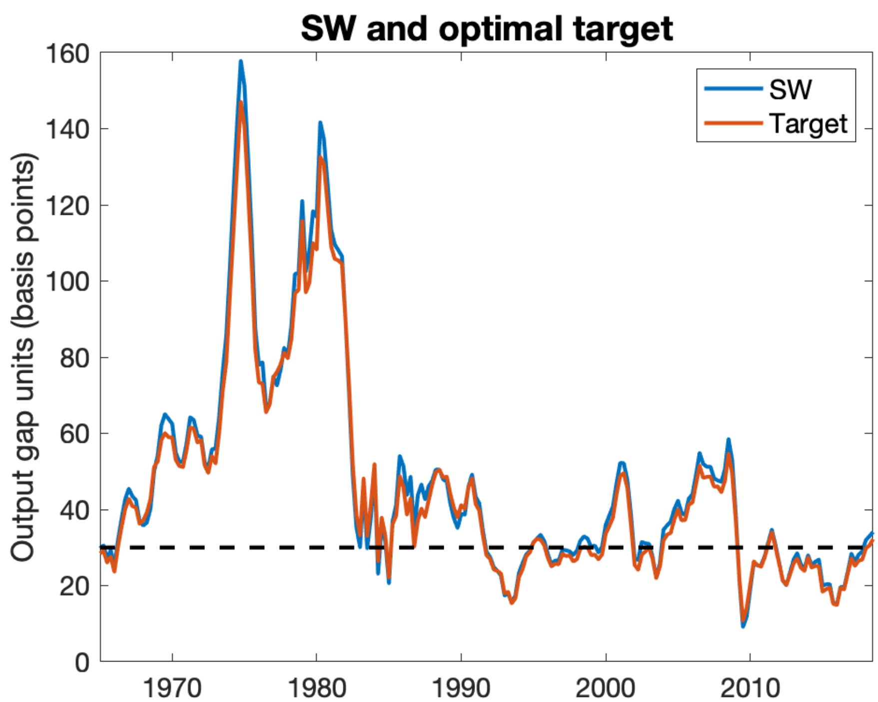

Figure 6: Time series of aggregate inflation (SW) and the optimal policy target

The two series move closely together, although the optimal target is often a few basis points lower than SW, suggesting that the optimal policy should be slightly more expansionary than output gap targeting.

[20]Here consumption shares are calibrated to replicate relative sales shares.

6.2.3 Targeting consumer inflation

The welfare loss under consumer inflation targeting is plotted in the yellow bars in Figures 4 and 5. We see that it is much larger than under the optimal policy, and this result crucially depends on the input-output structure. With the full input-output network (Figure 4), targeting consumer prices is bad even with only aggregate shocks. Figure 5 instead shows that targeting consumer inflation yields a much smaller loss in the calibration without input-output linkages, regardless of the distribution of the shocks.

6.2.4 Within- versus cross-sector misallocation

Figure 7 compares the welfare cost of misallocation between firms in the same sector versus misallocation across different sectors. The within-sector component dominates in the main calibration, which assumes higher substitutability between products from the same sector. The result would be reversed if we assumed uniform substitutability.

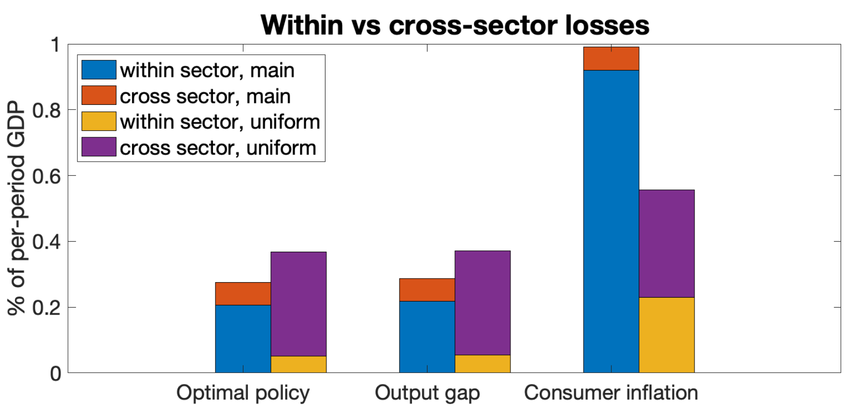

Figure 7: Main calibration: $\epsilon = 8$, $\sigma = 0.9$, $\theta_L = 0.5$, $\theta = 0.001$; uniform elasticities: $\epsilon = \sigma = \theta_L = \theta = 2$

6.3 Slope of the Phillips curve and monetary non-neutrality

This section evaluates quantitatively the role of input-output linkages and heterogeneous pricing frictions for the slope of the Phillips curve and monetary non-neutrality. In the dynamic model we measure monetary non-neutrality from the impulse response of infation to a given real rate shock.[21] We compare the slope and impulse responses in the full calibration and in calibrations

[21]Monetary non-neutrality is closely related with the slope of the Phillips curve. See section 6.3.2 below for a discussion.

that ignore input-output linkages and/or heterogeneous adjustment frequencies. We use historical input-output data to study their evolution over time.

6.3.1 Slope of the Phillips curve

Section 4.1.2 demonstrates that intermediate input flows reduce the slope of the Phillips curve. The calibration suggests that this effect is quantitatively large. Table 1 shows the slope of the Phillips curve in the full calibration relative to alternative calibrations, that assume away input-output linkages, wage rigidities and heterogeneous pricing frictions. The first column compares the full model with the baseline, which ignores both input-output linkages and sticky wages. The slope in the full calibration is more than one order of magnitude smaller. Input-output linkages alone account for a large share of the effect: from the seond column we read that the slope is 60% smaller in the full calibration than in a model without input-output linkages, but with the same price and wage rigidities. The third column shows that heterogeneous pricing frictions instead have little effect. This is not a general result. We find this in our data because adjustment frequencies are not correlated with labor shares, therefore setting them equal to the mean does not affect the wage pass-through $\bar{\delta}_w$ (see Section 4.1.2). By contrast, heterogeneous pricing frictions do matter in the dynamic setting (see Section 6.3.2 below).

full model vs no IO, flex w	full model vs no IO	full model vs δ = mean
0.07	0.38	1.05

Table 1: Slope in the full calibration relative to alternative calibrations

6.3.2 Monetary non-neutrality

In the dynamic setting the impulse response of consumer inflation to real rate shocks is not fully characterized by the slope of the Phillips curve. This represents a departure from both the static version of the network model and the dynamic version of the one-sector model. Using results from Appendix D, the law of motion of inflation is given by

$$\pi_t = \mathcal{B}\tilde{y}_t + \mathcal{V}\log\mu_{t-1} + \rho\mathcal{M}\mathbb{E}\pi_{t+1} \tag{43}$$

Past markups are a state variable, and therefore they appear in Equation (43). The matrix $\mathcal{M}$ (derived in Appendix D) captures the propagation of markups shocks through the network over time. Iterating forward the expression in (43) we obtain

$$\pi_t = \sum_{s \geq 0} (\rho \mathcal{M})^s \left[\mathcal{B} \mathbb{E}_t \tilde{y}_{t+s} + \mathcal{V} \log \mu_{t+s-1} \right] \tag{44}$$

We can aggregate both sides of (44) according to consumption shares, to find

$$\pi_t^C = \sum_{s \geq 0} \beta^T (\rho \mathcal{M})^s \left[\mathcal{B} \mathbb{E}_t \tilde{y}_{t+s} + \mathcal{V} \log \mu_{t+s-1} \right] \tag{45}$$

Equation (45) differs from the one-sector case in two respects. First, in the network model the response of current inflation π_t^C to discounted future output gaps $\rho^s \tilde{y}_{t+s}$ is mediated by the powers of the matrix $\mathcal{M}$. Intuitively, output gaps have a differential effect across sectors. Marginal costs respond more in upstream sectors, and it takes time for the shock to propagate downstream. The response of current inflation π_t^C to future output gaps $\rho^s \tilde{y}_{t+s}$ takes this into account, therefore it is not constant over the time horizon s.Second, current inflation depends on anticipated endogenous markup distortions.

It is through this second channel that heterogeneous adjustment frequencies significantly increase monetary non-neutrality in the dynamic setting (while they have no effect in the static model - see Section 6.3.1). To gain intuition consider two scenarios, one where all sectors have the same adjustment probability, and one where some sectors are more flexible and some are stickier, keeping the average adjustment probability constant. As long as the discount factor is large enough, producers reset their prices to be an "average" of the optimal prices over the period before their next opportunity to adjust.

If all sectors have the same adjustment probability, the producers who can adjust know that many others will also have changed their price by the time they get to adjust again, so they preemptively adjust more. If instead some sectors can adjust very infrequently, producers in the flexible sectors know that they will likely have another opportunity to reset their price before the stickier sectors also get to change theirs. Therefore it is optimal for them to wait. This is captured by the expected markup term in equation (45). The expectations channel gets muted as the discount factor goes to zero, so that heterogeneous adjustment frequencies play no role in the static setting.

Figure 8 plots the impulse response of consumer inflation to a 1% real rate shock. We find that both heterogeneous adjustment frequencies and input-output linkages dampen the response of consumer prices.[22]

[22] Our results are consistent previous works, such as Carvalho (2006), Nakamura and Steinnson (2010).

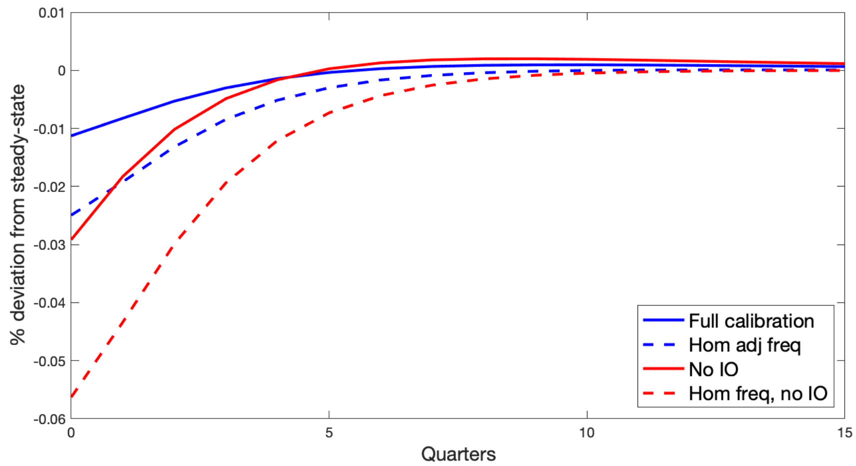

Figure 8: Response to a 1% interest rate shock under a Taylor rule with $\varphi_\pi = 1.24$ and $\varphi_y = .33/12$. Persistence $= 0.5$

6.3.3 Phillips curve and monetary non-neutrality over time

We calibrate the slope of the Phillips curve using historical input-output data for each year between 1947 and 2017. The results are plotted in Figure 9.

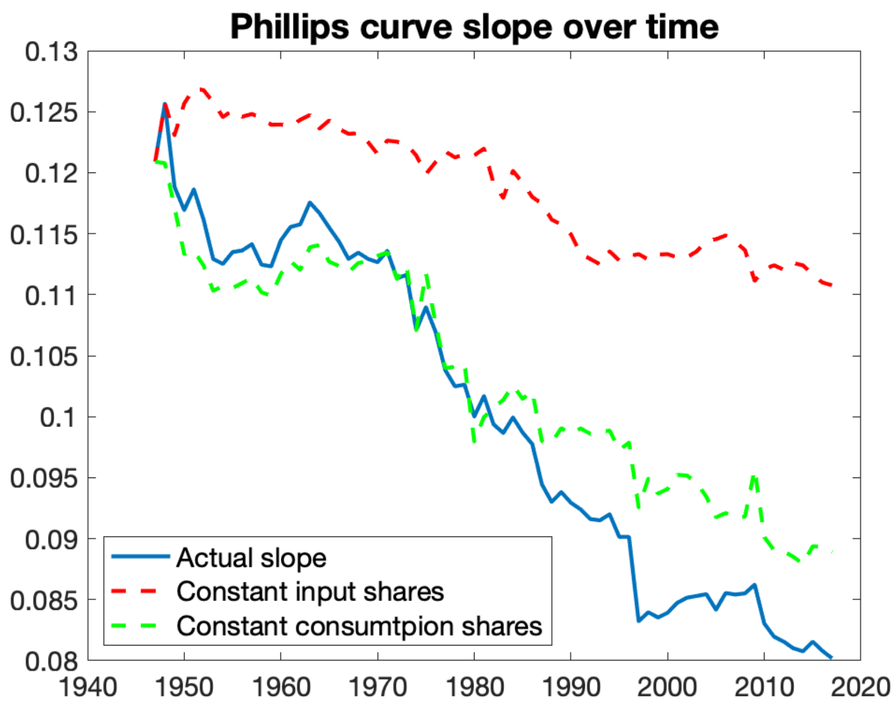

Figure 9: Slope of the Phillips curve over time

The solid line depicts the actual slope, which decreased by about 30% over this time period. Using the results from Section 6.3.1 we can decompose the total change into two effects. The first captures changes in the input-output structure, while the second captures shifts in consumption shares. These two channels are reflected in the expression for the wage pass-through $\bar{\delta}_w$, which in turn determines the slope:

$$\bar{\delta}_w = \underbrace{\beta^T}_{\text{consumption}} \underbrace{\Delta \left(I - \Omega\Delta\right)^{-1} \alpha}_{\text{input-output}}$$

The dashed green line in Figure 9 plots the slope for constant consumption shares (set at their 1947 value), while letting the input-output matrix change over time. The dashed red line instead keeps input flows constant at their 1947 value and lets consumption shares change. We find that 79% of the overall decline in $\bar{\delta}_w$ can be attributed to increasing intermediate input shares, while the remaining effect comes from shifts in the composition of the consumption basket. Figure 6.3.3 further decomposes the changes in consumption and input-output shares across sectors.

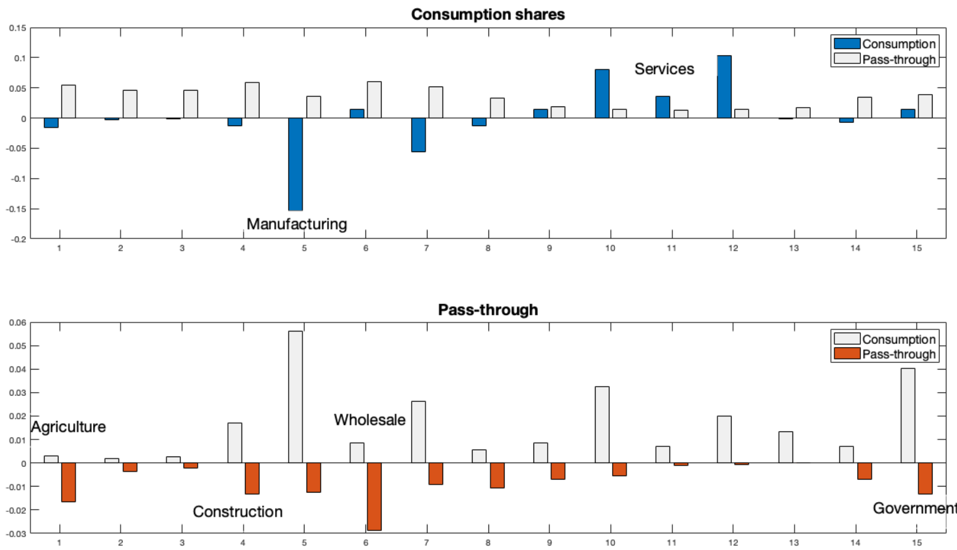

Figure 10: Upper panel: change in consumption shares and average wage pass-through. Lower panel: change in pass-through and average consumption shares.

Formally, we can split the overall change in the pass-through $\bar{\delta}_w$ into a change in sector-level pass-through for constant consumption shares, and a change in consumption shares for constant pass-through:

$$\bar{\delta}_w^{2017} - \bar{\delta}_w^{1947} = \frac{\beta_{1947}^T + \beta_{2017}^T}{2} \left(PT_{2017} - PT_{1947} \right) + $$
$$+ \left(\beta_{2017}^T - \beta_{1947}^T \right) \frac{PT_{1947} + PT_{2017}}{2}$$

where we used the notation

$$PT \equiv \Delta \left(I - \Omega \Delta \right)^{-1} \alpha$$

The grey bars in the two plots respectively represent average pass-through and average consumption shares. The bars in color represent changes in consumption shares and input-output shares. From the upper plot we see that consumption shifted away from manufacturing (where wages have high pass-through) towards services (where they have lower pass-through). The lower plot shows that increasing intermediate input shares (weakly) reduced the pass-through in all sectors, and more so in sectors with high consumption shares (such as construction, manufacturing and government). Both channels reduced the slope, but quantitatively the change in input shares accounts for most of the effect.

We find similar results in the dynamic setting. Figure 11 plots the impact response of inflation to a 1% shock in real rates between 1947 and 2017. Inflation has become less responsive to monetary policy over time, and again the increase in intermediate input shares explains most of the effect.

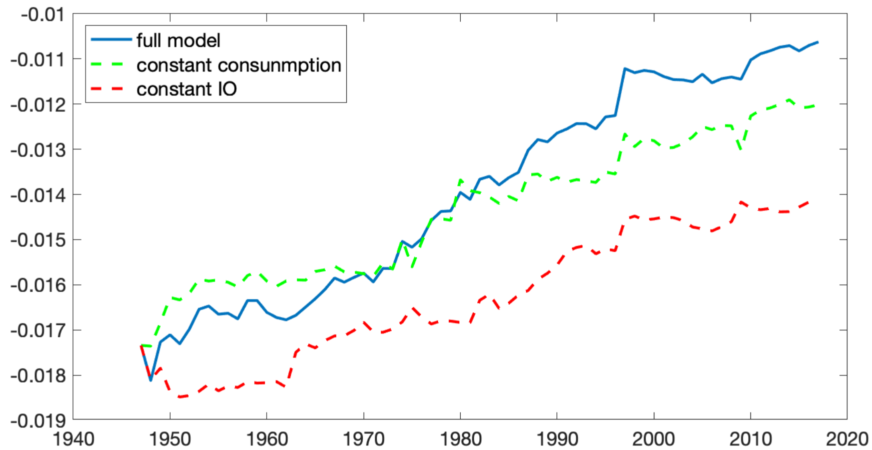

Figure 11: Impact response of consumer inflation and output to a 1% real rate shock

6.4 Endogenous residuals

6.4.1 Time series

We construct a time series for the endogenous residual term in the consumer-price Phillips curve using sector-level masures of yearly TFP shocks from the BEA KLEMS data. Shocks are then aggregated into the residual using the results in Section 4.1.3. Figure 12 plots the results. The residual has a mean of -0.16 and a standard deviation of 0.25, both of which are large relative to the calibrated slope (that is 0.09). We also find that the residual traks oil prices quite closely, as shown in the figure.

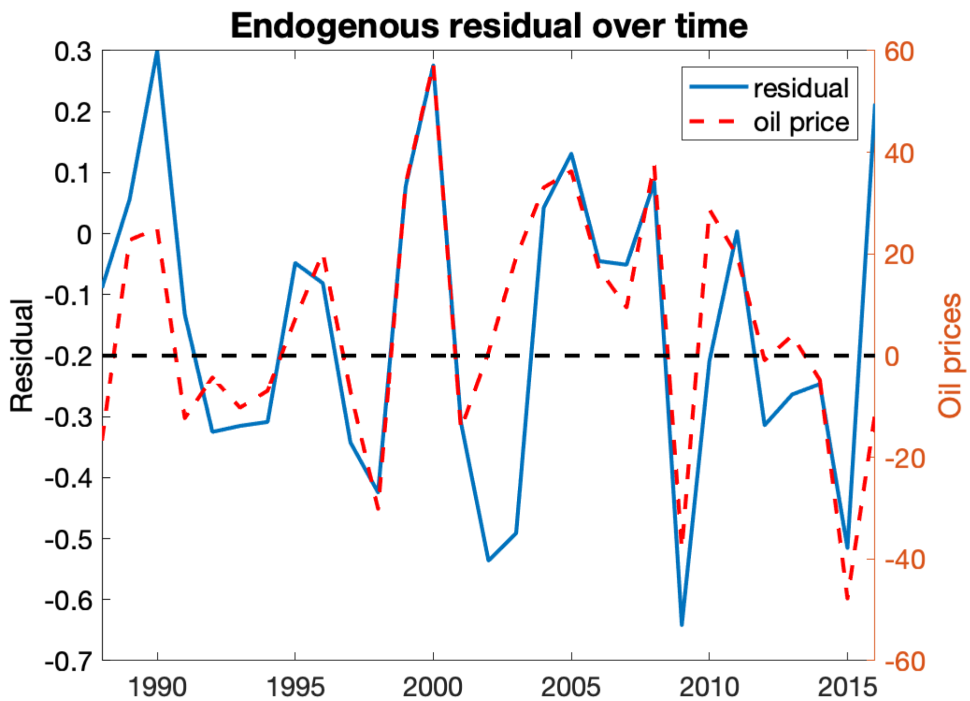

Figure 12: Time series of the endogenous residual and oil prices

6.4.2 Oil shocks

Consistent with the common wisdom in the policy practice, Figure 12 above and Examples 3 and 6 in Sections 4.2 and 5.2 point at the importance of oil shocks in explaining inflation dynamics.

Table 2 reports the calibrated response of inflation to an oil shock in the US network, under different assumptions about price and wage rigidity.

	δ = actual	$\delta \equiv \delta_{mean}$, $\delta_{oil} = 1$	$\delta \equiv \delta_{mean}$
sticky wages	0.22	0.07	-0.00
flexible wages	0.18	0.01	-0.06

Table 2: Consumer inflation after a 10% negative productivity shock to the oil sector, full model

Even if the netowrk structure is more complex than in Example 3, we see that the simple model from the example captures well the mechanisms at play. Both wage rigidities and the correlation between oil shares and adjustment frequencies contribute to the effect of oil shocks on consumer inflation, and the effect goes in the direction predicted by the simple model in the example. If instead we calibrated the model to the real input-output shares, but assumed wages to be flexible ($\delta_L = 1$) and price adjustment frequencies to be uniform ($\delta_{oil} = \delta_i \equiv \delta_{mean}\ \forall i$), an oil shock would act like an upstream shock (see the vertical chain in Example 1), and under zero output gap consumer prices would slightly decline after a negative oil shock.

To complement the discussion in Example 6, Table 3 presents the optimal monetary policy response to a 10% negative oil shock. We express optimal policy in terms of the output gap (in percentage points). In the main calibration we find that the central bank should respond by implementing a positive output gap. The implied percentage change in output is obtained by adding the log change in natural output, $y_{nat} = -0.69$.

	full model	$\delta = \delta_{mean}$, $\delta_{oil} = 1$	$\delta = \delta_{mean}$
sticky wages	0.11	0.16	0.18
flex wages	-0.03	0.06	0.09

Table 3: Optimal output gap (in percentage points) after a 10% negative oil shock

7 Phillips curve regressions

In this section run Phillips curve regressions with the sales-weighted inflation SW implied by the model and and with various measures of consumer prices. We construct a time series of SW for the US economy based on PPI data from the BLS. In the main text we focus on our preferred specification (with no lags and a proxy for inflation expectations, consistent with the model), whereas results for other specifications (including lags and/or excluding expectations) are reported in Appendix E.

7.1 Data

We use sector-level PPI data from the BLS, and aggregate it based on BEA input-output weights and price adjustment frequences from Pasten, Shoenle and Weber (2018). Appendix $E1$ describes more in detail how we construct sector-level price series. We measure inflation as the percentage change from the same quarter of the previous year.

Figure 13 plots SW against two measures of consumer price inflation (CPI and PCE) and against aggregate producer price inflation (PPI), from 1984 to 2018. We refer the reader to Appendix $E1$ for a more detailed comparison between the weighting of sectoral inflation rates in the PCE (or core PCE) and in SW. In the appendix we also provide a breakdown of the weights attributed to the top-15 sectors in SW. A kew difference between the two is that the PCE puts no weight on wage inflation, which instead gets about 18% of the weight in SW. Other important sectors in SW are professional services, financial intermediation and durable goods, whereas the PCE places high weight on health care, real estate and nondurable goods.

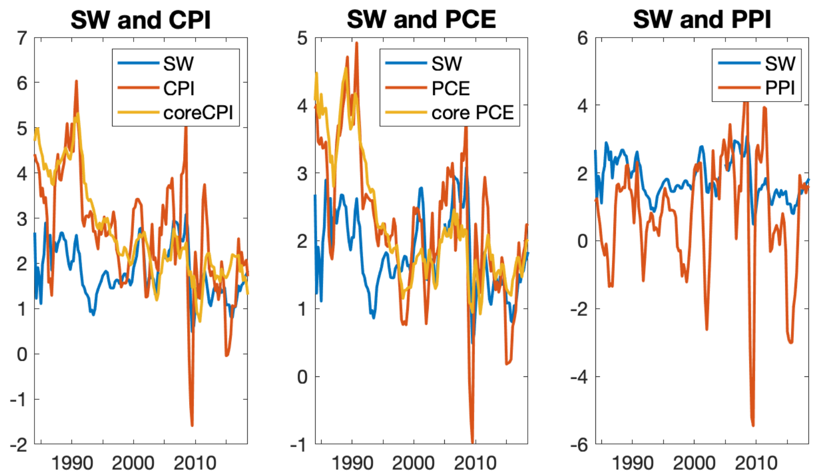

Figure 13: Comparison of SW against consumer and producer prices (1965-2018)

We construct a proxy for inflation expectations by exploiting the statistical properties of the inflation process, whose changes are well approximated by an IMA(1,1) (as discussed in Stock and Watson (2007)). We estimate the IMA(1,1) parameters and use them to construct a forecast series for each of the inflation measures that we use in the regressions. The forecast series are plotted in Appendix $E1$. For consumer inflation it has been shown that survey measures of forcasted inflation (such as the SPF) are well approximated by this IMA(1,1) forecast.

Appendix $E1$ also reports scatterplots of the output gap and SW or the various measures of consumer inflation.

7.2 Regressions over the full sample period

Table 7 reports results for a plain regression with just inflation and the output gap:

$$\pi_t = c + \kappa y_t + u_t \tag{46}$$

Table 8 reports results for our preferred specification with inflation expectations:

$$\pi_t = c + \rho \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t \tag{47}$$

The calibrated model is consistent with empirical estimates for the coefficients on both consumer prices and SW (see Table 6), while the R-squared is much higher for the optimal indicator SW implied by the model. Results for other specifications (with lags and/or other measures of the output gap, and for gap levels versus inflation changes) and residual plots are reported in Appendix $E2$.

	SW	consumer prices
κ	-3.00	-0.09

Table 4: Calibrated slope of the Phillips curve ($\gamma = 1$, $\varphi = 2$)

	SW	CPI	core CPI	PCE	core PCE
gap	-3.8814**	-0.2832**	-0.1839**	-0.1667**	-0.1007*
	(0.6329)	(0.0729)	(0.0642)	(0.0628)	(0.0565)
intercept	1.9842**	2.9052**	2.9021**	2.3978**	2.372**
	(0.0475)	(0.1196)	(0.1052)	(0.103)	(0.0926)
R-squared	0.2154	0.0991	0.0566	0.0489	0.0227

Table 5: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	-1.1054**	-0.1613**	-0.0344	-0.062	0.0047
	(0.3275)	(0.0809)	(0.052)	(0.0487)	(0.0368)
inflation expecations	0.8287**	0.4846**	0.5446**	0.6364**	0.6406**
	(0.0383)	(0.1557)	(0.0559)	(0.0621)	(0.045)
intercept	0.3484**	1.3851**	1.3193**	0.5522**	0.8388**
	(0.0789)	(0.5021)	(0.1818)	(0.196)	(0.1228)
R-squared	0.8234	0.159	0.4425	0.4635	0.6072

Table 6: CBO unemployment gap

7.3 Rolling regressions

We run rolling Phillips curve regressions (with a 20 year window) over the period Januray 1984 - July 2018 for different inflation measures (SW, CPI, PCE, core CPI, core PCE). Here we report results for our preferred specification (47) with inflation expectations. We take the CBO unemployment gap as our measure for the output gap. Appendix $E3$ reports results for different measures of the output gap and for gap levels versus inflation changes.

Figure 14 compares the strength and stability of the estimated relation for different choices of the dependent variable. The left panel reports the average R-squared over the sample period, the middle panel reports the fraction of windows in which the estimated coefficient is significant, and the right panel gives a measure of the stability of our estimated coefficient, as proxied by its standard deviation relative to the mean. We find that SW dominates consumer prices along all these demensions: the R-squareds are consistently higher, the estimated coefficient is always significant and the variance of the estimate is lower. Plots of the estimated coefficients and confidence intervals are reported in Appendix $E3$.

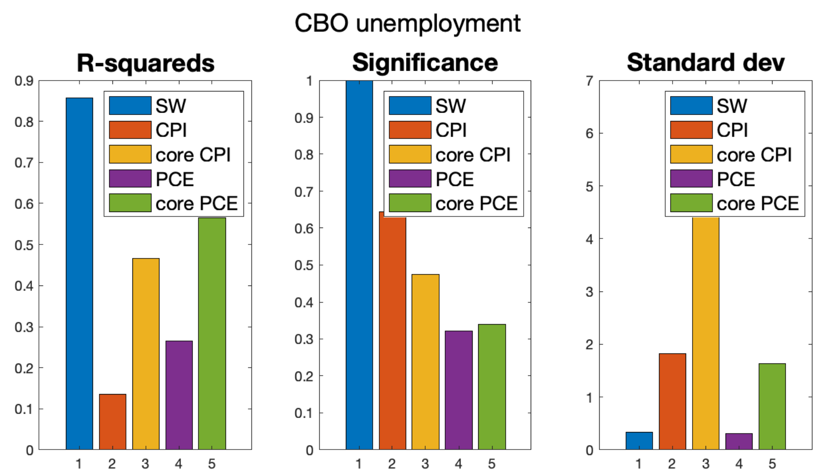

Figure 14: Summary statistics for rolling Phillips curve regressions (20y window, 1984-2018, unemployment gap as dependent variable)

8 Conclusion

This paper revisits the "positive" and "normative" implications of the New Kenynesian model under a realistic production structure, with multiple sectors, input-output linkages and heterogeneous shocks and pricing frictions. We show that the consumer-price Phillips curve is misspecified: the slope changes with the input-output structure and productivity shocks create an endogenous residual. We propose a correct specification based on the model, which aggregates sectoral inflation rates according to sales shares and appropriately discounts flexible sectors. We compare Phillips curve regressions with consumer prices and with our indicator, finding that the estimated coefficients are consistent with the calibrated model and the R-squared is significantly higher with our preferred indicator.

Monetary policy cannot replicate the first best, but the optimal policy can still be implemented by targeting an appropriately defined inflation indicator. We evaluate the performance of the two standard targets in the Taylor rule, output gap and consumer inflation, finding that targeting the output gap is close to optimal, while stabilizing consumer prices generates an expected loss of 0.79% of per-period GDP with respect to the optimal policy.

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Appendix A: Positive analysis

A1: Natural output and output gap

Proof of Lemma 1**:**

Given that the flex-price equilibrium is efficient, it solves the problem

$$max_{C,L,\{L_i,y_i,\{x_{ij}\}\}} \frac{C\left(\{y_i\}\right)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi} \quad s.t. \quad \begin{array}{c} y_i + \sum_j x_{ij} = A_i F_i\left(\{x_{ij}\}, L_i\right) \ \forall i \\ \sum_i L_i = L \end{array} \tag{48}$$

Our objective is to compute $\frac{dlogC^*}{dlogA_i}$.

Observe that the optimization problem in (48) can be solved in two steps: first choose $\{L_i, y_i, \{x_{ij}\}\}$ for given L, and then choose the optimal L.

Formally, solving problem (48) is equivalent to solving

$$\frac{C^*\left(L;A\right)^{1-\gamma}}{1-\gamma} = max_{\{L_i,y_i,\{x_{ij}\}\}} \frac{C\left(\{y_i\}\right)^{1-\gamma}}{1-\gamma} \quad s.t. \quad \begin{array}{c} y_i + \sum_j x_{ij} = A_i F_i\left(\{x_{ij}\}, L_i\right) \ \forall i \\ \sum_i L_i = L \end{array} \tag{49}$$

and

$$max_L \frac{C^*(L;A)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi} \tag{50}$$

The solution of (50) must satisfy

$$C^*(L;A)^{\gamma} L^{\varphi} = \frac{\partial C^*}{\partial L}$$

Using the envelope theorem in problem (49) we have that

$$\frac{\partial C^*}{\partial L} = C^{*\gamma} \nu_L(A)$$

where ν_L is the Lagrange multiplier associated to the constraint $\sum_i L_i = L$. From the equation

$$L^{\varphi} = \nu_L(A)$$

we have

$$\frac{dlogL}{dlogA_i} = \frac{1}{\varphi} \frac{dlog\nu_L}{dlogA_i}$$

Finally, applying again the envelope theorem to problem (49) we have

$$\frac{dlogC^*}{dlogA_i} = C^{*\gamma}\left(\frac{\nu_L L}{\varphi C^*}\frac{dlog\nu_L}{dlogA_i} + \frac{\nu_i F_i\left(\{x_{ij}\}, L_i\right)}{C^*}\right)$$

We now need to prove that $C^{*\gamma}\frac{\nu_i F_i(\{x_{ij}\},L_i)}{C^*} = \lambda_i$ (where λ_i is the share of i's sales in GDP), and that

$$C^{*\gamma}\frac{\nu_L L}{\varphi C^*}\frac{dlog\nu_L}{dlogA_i} = \frac{1}{\varphi}\lambda_i - \frac{\gamma}{\varphi}\frac{dlogC^*}{dlogA_i} \tag{51}$$

This in turn implies that

$$\frac{dlogC^*}{dlogA_i} = \frac{1+\varphi}{\gamma+\varphi}\lambda_i$$

Let's first prove that $C^{*\gamma}\nu_i$ is equal to the price of good i relative to the CPI in the competitive equilibrium, so that $C^{*\gamma}\frac{\nu_i F_i(\{x_{ij}\},L_i)}{C^*} = \lambda_i$.

From the FOCs of (49) we have $C_i = C^{\gamma}\nu_i$ and from consumer optimization in the competitive equilibrium we have $\frac{C_j}{C_i} = \frac{p_j}{p_i}$. Thus

$$\frac{C_j}{C_i} = \frac{\nu_j}{\nu_i} = \frac{p_j}{p_i}$$

Using the fact that C is homogeneous of degree one, and normalizing $\sum_j \frac{p_j y_j}{C} = 1$, we have

$$\frac{\sum C_j y_j}{C_i} = \frac{C}{C_i} = \frac{C}{p_i} \Rightarrow p_i = C_i$$

The FOCs for (49) in turn imply that $p_i = C^{\gamma}\nu_i$.

Let's now derive equation (51). From the FOCs of (49) it holds that $C^{\gamma}\nu_L = C^{\gamma}\nu_i A_i F_{iL} = p_i A_i F_{iL} = w \;\forall i$, where the last equality follows from firm optimization in the competitive equilibrium. Moreover, from the consumers' budget constraint we have that $w = \frac{C^*}{L}$. Thus

$$C^{*\gamma}\frac{\nu_L L}{\varphi C^*}\frac{dlog\nu_L}{dlogA_i} = \frac{1}{\varphi}\left(\frac{dlogw}{dlogA_i} - \gamma\frac{dlogC^*}{dlogA_i}\right)$$

To conclude the proof we need to show that

$$\frac{dlogw}{dlogA_i} = \lambda_i$$

Using again the consumers' budget constraint we have

$$\frac{dlogw}{dlogA_i} = \frac{\partial logC^*}{\partial logA_i} + \left(\frac{\partial logC^*}{\partial logL} - 1\right)\frac{dlogL}{dlogA_i} = \lambda_i$$

Proof of Lemma 2 **:**

As explained in the proof of Lemma **??**, the undistorted steady-state maximizes productivity by optimally allocating labor both within and across sectors. Therefore around this steady-state the productivity loss induced by a change in firm-level markups is zero to a first order.

A2: Sector-level inflation

Proof of Propositions 2 **and** 1

We first solve for the change in marginal costs as a function of the change in prices, wages and productivity:

$$dlogmc_i = \widetilde{\alpha}_i dlogw + \sum_j \widetilde{\omega}_{ij} dlogp_j - dlogA_i$$

We can write the change in sectoral prices as function of the change in marginal costs using the Calvo assumption:

$$dlogmc_i = \widetilde{\alpha}_i dlogw - dlogA_i + \sum_j \widetilde{\omega}_{ij}\delta_j dlogmc_j$$

and solve for the change in marginal cost as a function of the change in wages and productivity:

$$dlogmc = \left(I - \widetilde{\Omega}\Delta\right)^{-1}(\widetilde{\alpha} dlogw - dlogA) \tag{52}$$

The change in consumer prices is

$$dlogP = \beta^T dlogp = \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}(\widetilde{\alpha} dlogw - dlogA) \tag{53}$$

From the consumption-leisure trade-off, we have

$$dlogw = dlogP + (\varphi dlogL + \gamma dlogy) =$$

$$= \left(\varphi dlogL + \gamma\tilde{y} + \gamma y^{nat} + dlogP\right) =$$

$$= \left((\gamma + \varphi)\,\widetilde{y} + \lambda^T dlogA + dlogP\right)$$

We can then use (53) to solve for the change in wages as a function of the output gap and productivity shocks. We have:

$$dlogw = \frac{(\gamma + \varphi)\,\widetilde{y} + \beta^T \left[\left(I - \widetilde{\Omega} \right)^{-1} - \Delta \left(I - \widetilde{\Omega}\Delta \right)^{-1} \right] dlogA}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta \right)^{-1} \widetilde{\alpha}} \tag{54}$$

Lemma 6 shows that the denominator in (**??**) is always well defined.

Lemma 6. $1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta \right)^{-1} \widetilde{\alpha} > 0.$

Proof. First note that, by definition of labor and input shares, it holds that $\tilde{\alpha} = (I - \Omega)\,U$, where U is a Nx1 vector with all entries equal to 1. Thus we have that

$$\beta^T \left(I - \widetilde{\Omega} \right)^{-1} \widetilde{\alpha} = \beta^T \left(I - \widetilde{\Omega} \right)^{-1} (I - \Omega)\,U =$$

$$\beta^T U = \sum_j \beta_j = 1$$

To prove Lemma 6 it is enough to show that

$$\beta^T \Delta \left(I - \widetilde{\Omega}\Delta \right)^{-1} \widetilde{\alpha} < \beta^T \left(I - \widetilde{\Omega} \right)^{-1} \widetilde{\alpha}$$

A sufficient condition for this to hold is that

$$\Delta \left(I - \widetilde{\Omega}\Delta \right)^{-1}_{ij} < (I - \Omega)^{-1}_{ij} \;\; \forall i, j$$

Note that

$$\Delta \left(I - \widetilde{\Omega}\Delta \right)^{-1}_{ij} = \delta_i \left(I - \widetilde{\Omega}\Delta \right)^{-1}_{ij} < \left(I - \widetilde{\Omega}\Delta \right)^{-1}_{ij}$$

therefore it is sufficient to prove that

$$\left(I - \widetilde{\Omega}\Delta \right)^{-1}_{ij} < (I - \Omega)^{-1}_{ij} \;\; \forall i, j$$

We can do so using the relations

$$\left(I - \widetilde{\Omega}\Delta \right)^{-1} = I + \widetilde{\Omega}\Delta + \left(\widetilde{\Omega}\Delta \right)^2 + ...$$

$$\left(I - \widetilde{\Omega} \right)^{-1} = I + \widetilde{\Omega} + \widetilde{\Omega}^2 + ...$$

This yields

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}=\mathbb{I}(i=j)+\omega_{ij}\delta_j+\sum_k\omega_{ik}\omega_{kj}\delta_j\delta_k+...<$$

$$\mathbb{I}(i=j)+\omega_{ij}+\sum_k\omega_{ik}\omega_{kj}+...=\left(I-\widetilde{\Omega}\right)^{-1}_{ij}$$

□

To find marginal costs as function of the output gap and productivity shocks, plug (**??**) into (52):

$$dlogmc=\frac{(\gamma+\varphi)\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}\widetilde{y}+$$

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}\left(\frac{\widetilde{\alpha}\left[\lambda^T-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}-I\right)dlogA \tag{55}$$

The expressions for the elasticities $\mathcal{B}$ and $\mathcal{V}$ in Section 3.2 follow immediately from (55).

Proof of Lemma 2**:**

In our setup labor is the only factor of production. Then labor and input shares must sum to one:

$$\alpha+\Omega\mathbf{1}=\mathbf{1}$$

therefore it holds that $(I-\Omega)^{-1}\alpha=\mathbf{1}$. The result

$$\exists i,j\text{ such that }\omega_{ij}\delta_j<\omega_{ij}\Longrightarrow(I-\Omega\Delta)^{-1}\alpha<\mathbf{1}$$

follows immediately from the fact that each term in the geometric sum

$$(I-\Omega\Delta)^{-1}\alpha=\left(I+\Omega\Delta+(\Omega\Delta)^2+...\right)\alpha$$

has at least one component that is smaller than in the corresponding term of

$$(I-\Omega)^{-1}\alpha=\left(I+\Omega+\Omega^2+...\right)\alpha$$

It then follows that

$$\bar{\delta}_w=\sum_i\beta_i\delta_i\left[(I-\Omega\Delta)^{-1}\alpha\right]_i<\sum_i\beta_i\delta_i\equiv\mathbb{E}_\beta(\delta)$$

Equation **??** is obtained by differentiating (12).

Proof of Lemma 4**:**

We first show that $\mathcal{V}\alpha = 0$, that is, α belongs to $ker(\mathcal{V})$.

Recall the expression for $\mathcal{V}$:

$$\mathcal{V} = \frac{\left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha} \left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}} - \left(I - \widetilde{\Omega}\Delta\right)^{-1}$$

Thus we have

$$\mathcal{V}\alpha = \frac{\left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha} \left[1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \alpha\right]}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}} - \left(I - \widetilde{\Omega}\Delta\right)^{-1} \alpha = 0$$

We then prove that α is the only element of $ker(\mathcal{V})$. Note that for every vector $x \neq 0$ such that $\mathcal{V}x = 0$ it must hold that

$$\left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha} \frac{\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] x}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}} = \left(I - \widetilde{\Omega}\Delta\right)^{-1} x \iff$$

$$\widetilde{\alpha}_j \frac{\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] x}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}} = x_i \ \forall i \tag{56}$$

and

$$\mathcal{C} \equiv \frac{\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] x}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}} \neq 0$$

(otherwise we would have $x = 0$). From (56) we then have that

$$\frac{\alpha_i}{\alpha_j} = \frac{x_i}{x_j} \ \forall i, j$$

so that x is proportional to the vector of labor shares α.

A3: Output gap and aggregate inflation

Proof of Lemma 3:

The output gap is a labor demand shifter. Therefore it impacts real wages, according to the parameters of the labor supply curve. From the consumers' optimal labor supply decision we have:

$$\left(logw - logw^{nat}\right) - \left(logP - logP^{nat}\right) = \gamma\tilde{y} + \varphi\left(logL - logL^{nat}\right) = (\gamma + \varphi)\,\tilde{y} \tag{57}$$

where the last equality follows from Lemma 2.

We next need to compute the left hand side of (57), which corresponds to the change in real wages induced by markup distortions.

Let's first introduce two definitions.

Definition 3. The cost-based input-output matrix $\widetilde{\Omega}$ is an $N \times N$ matrix with element i, j given by the expenditure share on input j in i's cost:

$$\widetilde{\omega}_{ij} = \frac{p_j x_{ij}}{mc_i y_i}$$

Definition 4. The sector-level steady-state labor shares in marginal costs are encoded in the $N \times 1$ vector $\tilde{\alpha}$ with components

$$\tilde{\alpha}_i = \frac{wL_i}{mc_i y_i}$$

In a steady-state with optimal subsidies it holds that $\Omega = \widetilde{\Omega}$ and $\alpha = \tilde{\alpha}$.

To solve for real wages as a function of sector-level markups we first need to consider how they impact marginal costs and prices. We have:

$$dlogmc_i = \widetilde{\alpha}_i dlogw + \sum_j \widetilde{\omega}_{ij} dlogp_j - dlogA_i$$

and

$$dlogp_i = dlogmc_i + dlog\mu_i \tag{58}$$

$$\Rightarrow dlogmc = \left(I - \widetilde{\Omega}\right)^{-1}\left(\widetilde{\alpha} dlogw - dlogA + \widetilde{\Omega} dlog\mu\right) \tag{59}$$

$$\Rightarrow dlogP = \beta^T (dlogmc + dlog\mu) = dlogw + \tilde{\lambda}^T\left(dlog\mu - dlogA\right)$$

It follows that

$$dlogw - dlogP = \tilde{\lambda}^T\left(dlogA - dlog\mu\right) \tag{60}$$

For constant productivity we then have

$$\left(logw - logw^{nat}\right) - \left(logP - logP^{nat}\right) = -\lambda^T dlog\mu \tag{61}$$

Equations (57) and (61) together give the result.

Proof of Lemma 4:

We need to prove that all the vectors $x \neq 0$ satisfying $x^T\mathcal{V} = \mathbf{0}$ are proportional to $(I - \Delta)\,\Delta^{-1}\lambda$. Proposition 3 implies that $\lambda^T\,(I - \Delta)\,\Delta^{-1}\mathcal{V} = \mathbf{0}$.

Consider then all vectors x such that $x^T\mathcal{V} = \mathbf{0}$. Note that

$$x^T\mathcal{V} = \mathbf{0} \Longleftrightarrow x^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\left[\widetilde{\alpha}\left[\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] - \left(1 - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}\right)I\right] = \mathbf{0}$$

$$\Longleftrightarrow \tilde{x}^T\left[\widetilde{\alpha}\left[\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] - \left(1 - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}\right)I\right] = \mathbf{0} \tag{62}$$

where $\tilde{x}^T \equiv x^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}$.

To prove the Lemma we need to show that all vectors $\tilde{x}$ satisfying (62) are proportional to $\lambda^T\,(I - \Delta)\left(I - \widetilde{\Omega}\Delta\right)^{-1}$.

From (62) we have the relation

$$\left(1 - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}\right)\tilde{x}_j = \tilde{x}^T\widetilde{\alpha}\left[\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_j \quad \forall j \tag{63}$$

from which we derive the condition

$$\frac{\tilde{x}_i}{\tilde{x}_j} = \frac{\left[\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_i}{\left[\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_j}$$

Note that we must have $\tilde{x}^T\tilde{\alpha} \neq 0$, otherwise we would get $\tilde{x}^T = \mathbf{0}$, while we want $\tilde{x} \neq 0$. Moreover, the ratio on the RHS is well defined, because $\left[\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_j > 0\ \forall j$ (see Lemma 6).

Thus, $\tilde{x}^T$ must be proportional to the vector

$$\lambda^T - \beta^T\Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1} = \beta^T\left[(I - \Omega)^{-1} - \Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] =$$

$$= \beta^T\left[(I - \Omega)^{-1}(I - \Omega\Delta) - \Delta\right](I - \Omega\Delta)^{-1} =$$

$$= \beta^T (I - \Omega)^{-1} (I - \Delta) (I - \Omega\Delta)^{-1} = \lambda^T (I - \Delta) (I - \Omega\Delta)^{-1}$$

Appendix B: Optimal policy

B1: Welfare function

Proof of Lemma 5**:**

From equation (59), for each sector i we have

$$dlogp_i - dlogw = (I - \Omega)^{-1} dlog\mu$$

We can then use Lemma **??** to substitute for markups as a function of inflation rates.

Proof of Proposition 4**:**

In what follows, I will use the second-order approximation

$$\frac{Z - Z^*}{Z} \simeq log\left(\frac{Z}{Z^*}\right) + \frac{1}{2}log\left(\frac{Z}{Z^*}\right)^2$$

I will prove below that we can write the second-order log change in output with respect to the efficient outcome as

$$log\left(\frac{Y}{Y^*}\right) \equiv \hat{y} = \hat{l} - d$$

Using this result we can approximate the utility function around the efficient outcome as

$$\frac{U - U^*}{U_c C} \simeq \hat{y} + \frac{1}{2}\hat{y}^2 + \frac{1}{2}\frac{U_{cc}C}{U_c}\hat{y}^2 + \frac{U_l L}{U_c C}\left(\hat{l} + \frac{1}{2}\frac{U_{ll}N}{U_l}\hat{l}^2\right) =$$

$$= \hat{y} + \frac{1-\gamma}{2}\hat{y}^2 - \left(\hat{l} + \frac{1+\varphi}{2}\hat{l}^2\right) =$$

$$= \hat{y} + \frac{1-\gamma}{2}\hat{y}^2 - \left(\hat{y} + d + \frac{1+\varphi}{2}\hat{y}^2\right) =$$

$$= -\frac{\gamma+\varphi}{2}\tilde{y}^2 - d$$

where the last equality follows from the fact that, to the second order, $\hat{y}^2 = \tilde{y}^2$.

I will now derive the approximation

$$\hat{y} = \hat{l} - d$$

and the explicit expression for the second order component d.

For given labor supply and Hicks-neutral productivity shifters there is a productivity loss if firm's markups are not 1, which is the case whenever there are price rigidities and the economy is hit by a productivity shock. To a first-order this loss is zero around the efficient steady-state, so that $\hat{l}$ is the only component. Away from the efficient outcome a change in firm-level markups produces two effects. First, sector-level productivities are lower than in the efficient steady state (i.e. more labor is required to produce one unit of sectoral output), due to relative price distortions across firms within the sector. I will denote by a_i the logarithm of the "effective" productivity of sector i relative to the actual TFP A_i.

Second, there is a change in "sector-level markups", defined as

$$\mu_i = \frac{p_i}{mc_i}$$

where p_i is the sectoral price index (and note that the marginal cost is the same for all producers in sector i). This change in markups generates relative price distortions across sectors.

Both of these channels are associated with an aggregate productivity loss. I will refer to the first channel as the "within-sector" component of this loss, and to the second channel as the "cross-sector" component. I will now derive a first-order approximation of both, and then compute the second order approximation around the efficient steady-state.

Note that aggregate productivity can be expressed as a function of real wages and labor shares. Denoting the aggregate labor share by $\Lambda = \frac{wL}{GDP}$, by definition we can write aggregate output as

$$Y = \frac{1}{\Lambda}\frac{w}{P}L$$

Here $\frac{w}{P}$ is the real wage, and $\frac{1}{\Lambda}\frac{w}{P}$ acts as a measure of aggregate labor productivity. In log deviations from steady-state we have:

$$\hat{Y} = \hat{w} - \hat{P} - \hat{\Lambda} + \hat{l} \tag{64}$$

To compute the change in aggregate productivity we need to know how real wages and Λ respond in equilibrium to sectoral productivities and markups.

Real wages are derived in Lemma 3 (see equation (**??**)). Combining (**??**) with (64) we obtain the first-order approximation

$$dlogY - dlogL = \tilde{\lambda}^T (a - dlog\mu) - dlog\Lambda \tag{65}$$

We then need to compute $dlog\Lambda$ as function of the change in sectoral markups and productivities. It holds that

$$\Lambda + \lambda^T \left(1 - \frac{1}{\mu}\right) \equiv 1$$

Therefore we have

$$dlog\Lambda = -\frac{1}{\Lambda}\left(\sum_i d\lambda_i \left(1 - \frac{1}{\mu_i}\right) + \sum_i \lambda_i \frac{dlog\mu_i}{\mu_i}\right)$$

Using (65) we find that, around the efficient steady state,

$$dlogY - dlogL = \underbrace{\tilde{\lambda}^T a}_{\text{within sector}} + \underbrace{\left(\frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T\right) dlog\mu}_{\text{cross-sector}} \tag{66}$$

As $\frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T = 0$ around $\mu \equiv 1$, the first-order productivity loss from cross-sector misallocation is zero. To compute the second-order loss we need to derive again the cross-sector component in equation (66) .

Note that, since the first order effect on both cross-sector misallocation and sector-level productivities is zero, the second-order terms in $(dlogA)(dlog\mu)$ are also going to be zero. Therefore we only need to derive the cross-sector component with respect to sector-level markups. We have:

$$\left(d^2logY - d^2logL\right) - \tilde{\lambda}^T a = \frac{1}{\Lambda}\left(-\left(\sum_i \lambda_i \frac{dlog\mu_i}{\mu_i}\right)^2 + 2\sum_i d\lambda_i \frac{dlog\mu_i}{\mu_i} + \sum_i \frac{\lambda_i}{\mu_i}(dlog\mu_i)^2\right) - \sum_i d\tilde{\lambda}_i dlog\mu_i =$$

$$= -\frac{1}{2}\sum_i \sum_j \widetilde{d}^2_{ij} dlog\mu_i dlog\mu_j \tag{67}$$

where

$$\widetilde{d}^2_{ij} = \sum_h \sum_k \beta_h \beta_k \sigma_{hk} \left[(I-\Omega)^{-1}_{hi} - (I-\Omega)^{-1}_{ki}\right] \left[(I-\Omega)^{-1}_{hj} - (I-\Omega)^{-1}_{kj}\right] +$$

$$+ \sum_t \lambda_t \sum_h \sum_k \omega_{th}\omega_{tk}\theta^t_{hk} \left[(I-\Omega)^{-1}_{hi} - (I-\Omega)^{-1}_{ki}\right] \left[(I-\Omega)^{-1}_{hj} - (I-\Omega)^{-1}_{kj}\right] +$$

$$+ \sum_t \lambda_t \alpha_t \sum_h \omega_{th}\omega_{tk}\theta^t_{hk} (I-\Omega)^{-1}_{hi} (I-\Omega)^{-1}_{hj} =$$

$$= \Phi_C\left((I-\Omega)^{-1}_{(i)}, (I-\Omega)^{-1}_{(j)}\right) + \sum_t \lambda_t \Phi_t\left((I-\Omega)^{-1}_{(i)}, (I-\Omega)^{-1}_{(j)}\right)$$

To derive the welfare loss as a function of sector-level inflation rates we need to solve for the endogenous change in sector-level markups due to price rigidities.

In this setup (where price adjustment is iid across firms and there are CES preferences "within sectors") firm-markups are the same as in the efficient equilibrium for the producers who can adjust their price. For those who cannot adjust, the change in log markup is the opposite as the change in log marginal cost. Thus the change in sector-level markups is $dlog\mu = -(I-\Delta)\, dlogmc = -(I-\Delta)\,\Delta^{-1}\pi$

Therefore we can re-write (67) as

$$d^2logY - d^2logL = \tilde{\lambda}^T a - \frac{1}{2}\pi^T \mathcal{D}_2 \pi$$

with

$$d^2_{ij} = \frac{1-\delta_i}{\delta_i}\frac{1-\delta_j}{\delta_j}\tilde{d}^2_{ij}$$

It remains to compute the "within-sector" component $\lambda^T a$.

Index by t the different varieties of product i and note that, given the CES assumption, sectoral output can be written as

$$Y_i = A_i F\left(\{x_{ij}\}, L_i\right) \frac{p_i^{-\epsilon_i}}{\int p_{it}^{-\epsilon_i} dt} \tag{68}$$

where

$$x_{ij} = \int x_{ij}(t)dt$$

$$L_i = \int L_i(t)dt$$

From (68) we see that the "effective" productivity of sector i is

$$\tilde{A}_i = A_i \frac{p_i^{-\epsilon_i}}{\int p_{it}^{-\epsilon_i} dt}$$

so that

$$a_i = \epsilon_i \left[\frac{\int p_{it}^{-\epsilon_i} dlogp_{it} dt}{\int p_{it}^{-\epsilon_i} dt} - \frac{\int p_{it}^{1-\epsilon_i} dlogp_{it} dt}{\int p_{it}^{1-\epsilon_i} dt}\right] \tag{69}$$

Given the Calvo assumption, around the efficient steady state we have that

$$\frac{\int p_{it}^{-\epsilon_i} dlogp_{it} dt}{\int p_{it}^{-\epsilon_i} dt} = \frac{\int p_{it}^{1-\epsilon_i} dlogp_{it} dt}{\int p_{it}^{1-\epsilon_i} dt} = \delta dlogmc_i$$

so that $a_i = 0$.

Let's now compute the second-order loss. The second derivative of $\tilde{\lambda}^T a$ with respect to sector-level productivities is $\sum_i \tilde{\lambda}_i d^2 log\left(\frac{\tilde{A}_i}{A_i}\right)$.

The expression for $d^2 log\left(\frac{\tilde{A}_i}{A_i}\right)$ can be obtained by deriving (69) with respect to $\{dlogp_{it}\}$, to find[23]

$$d^2 log\left(\frac{\tilde{A}_i}{A_i}\right) = \epsilon_i \left[\int \left(logp_{it} - logp_i\right)^2 dt - \left(\int \left(logp_{it} - logp_i\right) dt\right)^2\right] =$$

$$= \epsilon_i \frac{1-\delta_i}{\delta_i} \pi_i^2$$

We can thus express the second-order welfare loss from within-sector misallocation as

$$\frac{1}{2} \pi \mathcal{D}_1 \pi$$

where

$$d_{ij}^1 = \begin{cases} 0 & if\ i \neq j \\ \lambda_i \epsilon_i \frac{1-\delta_i}{\delta_i} & if\ i = j \end{cases}$$

Lemma 7. *The elasticity of sectoral prices with respect to initial markups $dlog\mu_{-1}$ is given by the matrix $\mathcal{V}$. The optimal output gap is*

$$\widetilde{y}^* = -\frac{\mathcal{B}^T \mathcal{D} \mathcal{V} dlog\mu_{-1}}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}} \tag{70}$$

Proof. If initial sector-level markups are given by the vector $dlog\mu_{-1} \neq 0$, then sectoral inflation rates are given by

$$\pi_i = \delta_i \left(dlogmc_i - dlog\mu_{i-1}\right)$$

and sector-level markups are still given by (28). We can then proceed as in Propositions 2 and 1 to derive

$$\pi = \Delta \left(I - \Omega \Delta\right)^{-1} \left(\alpha dlogw - dlog\mu_{-1}\right)$$

[23]This is the same as in the traditional NK model (Gali (2008) Ch.4)

and

$$dlogw = \frac{\gamma+\varphi}{1-\beta^T\left(I-\Omega\Delta\right)^{-1}\alpha}\left(\tilde{y}-\tilde{y}_{-1}\right) - \frac{\beta^T\Delta\left(I-\Omega\Delta\right)^{-1}}{1-\beta^T\left(I-\Omega\Delta\right)^{-1}\alpha} dlog\mu_{-1}$$

From this we can immediately solve for sectoral inflation rates as a function of $\tilde{y}$,$\tilde{y}_1$ and $dlog\mu_{-1}$.

Welfare is the same function of the output gap and sectoral inflation rates as in (30), because both the variance of firm-level prices within sectors and sector-level markups are the same function of sectoral inflation rates. The optimal output gap then follows from the first order condition. □

B2: Policy target

Proof of Proposition 6**:**

We look for weights ϕ such that

$$\phi^T\left(\mathcal{B}\widetilde{y}+\mathcal{V}dlogA\right) > 0 \Longleftrightarrow \widetilde{y} > \widetilde{y}^* \tag{71}$$

I will first construct a vector ϕ that satisfies the condition

$$\phi^T\left(\mathcal{B}\widetilde{y}+\mathcal{V}dlogA\right) = 0 \Longleftrightarrow \widetilde{y} = \widetilde{y}^* \tag{72}$$

and then argue that this vector also satisfies (71).

Note that, as long as $\phi^T\mathcal{B} \neq 0$, we have

$$\phi^T\left(\mathcal{B}\widetilde{y}+\mathcal{V}dlogA\right) = 0 \Longleftrightarrow \widetilde{y} = -\frac{\phi^T\mathcal{V}dlogA}{\phi^T\mathcal{B}}$$

while the optimal output gap is

$$\widetilde{y}^* = -\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}dlogA}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$$

Thus (72) is satisfied for all realizations of $dlogA$ if and only if ϕ is such that

$$\frac{\phi^T\mathcal{V}dlogA}{\phi^T\mathcal{B}} = \frac{\mathcal{B}^T\mathcal{D}\mathcal{V}dlogA}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}} \quad \forall dlogA$$

In turn, this is true if and only if

$$\phi^T\left[I-\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]\mathcal{V} = 0 \tag{73}$$

that is, iff ϕ is a left eigenvector of the matrix $\left[I-\frac{\mathcal{B}\mathcal{B}^T\Delta\Xi\Delta}{\gamma+\varphi+\mathcal{B}^T\Delta\Xi\Delta\mathcal{B}}\right]\mathcal{V}$, relative to the eigenvalue 0.

We already proved in Lemma **??** that $\lambda^T (I - \Delta) \Delta^{-1}$ is a left eigenvector of the matrix $\mathcal{V}$ relative to the eigenvalue 0 (and it is the only such eigenvector). Therefore, as long as $\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]$ is invertible, $\phi^T = \lambda^T (I - \Delta) \Delta^{-1} \left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1}$ is the (unique) desired eigenvector of the matrix $\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right] \mathcal{V}$.

The matrix $\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]$ is indeed invertible: it is immediate to see that $\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$ has only one non-zero eigenvalue, $\frac{\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}} < 1$, and $\mathcal{B}$ is the unique corresponding eigenvector.

Next, to satisfy condition (71) we need

$$\phi^T \left(\mathcal{B}\widetilde{y} + \mathcal{V} dlogA\right)$$

to be increasing in the output gap $\widetilde{y}$, which is true iff $\phi^T \mathcal{B} > 0$. To prove this we use the fact that $\mathcal{B}$ is an eigenvector of $\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$ relative to the eigenvalue $\frac{\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$. Therefore it is also an eigenvector of $\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1}$, relative to the eigenvalue $\frac{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi} > 1$. Thus we have

$$\phi^T \mathcal{B} = \lambda^T (I - \Delta) \Delta^{-1} \left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1} \mathcal{B} =$$
$$= \gamma + \varphi + \mathcal{B}^T\mathcal{D}\mathcal{B} > 0$$

(see Lemma **??**).

Finally, to obtain the formulation in (38) we observe that

$$\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1} = I + \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}} + \left(\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right)^2 + ...$$

and

$$\left(\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right)^n = \left(\frac{\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right)^{n-1} \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$$

so that

$$\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1} = I + \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi}$$

Moreover, we have that

$$\frac{\lambda^T (I - \Delta) \Delta^{-1} \mathcal{B}}{\gamma + \varphi} = \frac{\lambda^T (I - \Delta) (I - \Omega\Delta)^{-1} \alpha}{1 - \beta^T \Delta (I - \Omega\Delta)^{-1} \alpha} = 1$$

so that

$$\lambda^T (I-\Delta)\Delta^{-1}\left[I-\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1}=\lambda^T (I-\Delta)\Delta^{-1}+\mathcal{B}^T\mathcal{D}$$

Appendix C: Welfare loss from business cycles

The expressions for the expected welfare loss under different policy rules are as follows (recall: Σ is the covariance matrix of sectoral shocks).

- Optimal policy

$$\frac{1}{2}\left[\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}-\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}\right]$$

loss from non-zero output gap:

$$\frac{1}{2}(\gamma+\varphi)\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})^2}$$

gain in allocative efficiency from non-zero output gap:

$$\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}-\frac{1}{2}\mathcal{B}^T\mathcal{D}\mathcal{B}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})^2}$$

net misallocation loss:

$$\frac{1}{2}\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}-\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}+\frac{1}{2}\mathcal{B}^T\mathcal{D}\mathcal{B}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})^2}$$

- zero consumer inflation vs optimal policy:

$$\frac{1}{2}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}+$$

$$+\frac{1}{2}\left[\frac{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}{(\beta^T\mathcal{B})^2}\beta^T\mathcal{V}-2\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}}{\beta^T\mathcal{B}}\right]\Sigma\mathcal{V}^T\beta$$

loss from non-zero output gap:

$$\frac{1}{2}(\gamma+\varphi)\frac{\beta^T\mathcal{V}\Sigma\mathcal{V}^T\beta}{(\beta^T\mathcal{B})^2}$$

total loss from misallocation:

$$\frac{1}{2}\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}+\left[\frac{1}{2}\mathcal{B}^T\mathcal{D}\mathcal{B}\frac{\beta^T\mathcal{V}}{\left(\beta^T\mathcal{B}\right)^2}-\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}}{\beta^T\mathcal{B}}\right]\Sigma\mathcal{V}^T\beta$$

- zero output gap vs optimal policy

$$\frac{1}{2}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{\left(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}\right)}$$

total loss from misallocation:

$$\frac{1}{2}\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}$$

Appendix D: Cost-push shocks and Dynamics

Appendix D1: Exogenous cost-push shocks

As discussed in Section 4.1.3, productivity shocks generate an endogenous "cost-push" term in the Phillips curve. In this section I extend the analysis to allow for sector-level "exogenous" cost-push shocks, which I model as a change in producers' desired markup $d\log\mu^D$. Lemma (8) derives sectoral inflation rates and the Phillips curve.

Lemma 8. *The elasticity of sectoral prices with respect to cost-push shocks is given by*

$$\left(\frac{\mathcal{B}\lambda^T}{\gamma+\varphi}-\mathcal{V}\right) \tag{74}$$

The sales-based Phillips curve is

$$SW=\left(\gamma+\varphi\right)\tilde{y}+\lambda^T dlog\mu^D \tag{75}$$

while the CPI-based Phillips curve is

$$\pi^C=\kappa\tilde{y}+u+v \tag{76}$$

where

$$u=\frac{\bar{\delta}_w-\bar{\delta}_A}{1-\bar{\delta}_w}\lambda^T dlogA$$

$$v = \frac{\bar{\delta}_\mu}{1 - \bar{\delta}_w} \lambda^T dlog\mu^D$$

$$\delta_\mu = \frac{\beta^T \Delta \left(I - \Omega\Delta\right)^{-1} dlog\mu^D}{\lambda^T dlog\mu^D}$$

The two Phillips curves expressed in terms of deviations from steady-state output are

$$SW = \left(\gamma + \varphi\right) y + \lambda^T \left(dlog\mu^D - dlogA\right) \tag{77}$$

$$\pi^C = \kappa y + \frac{\bar{\delta}_\mu \lambda^T dlog\mu^D - \bar{\delta}_A \lambda^T dlogA}{1 - \bar{\delta}_w} \tag{78}$$

Note that for $dlogA = -dlog\mu^D < 0$ we have

$$\frac{u}{\lambda^T dlogA} < \frac{v}{\lambda^T dlog\mu^D}$$

Similar to the baseline model, the central bank faces a worse trade-off after a cost-push shock than after a negative productivity shock of the same size (i.e. $dlogA = -dlog\mu^D$). This is because the change in firms' desired price is the same for the two shocks, but in the cost-push case natural output hasn't changed. In other words, inflation is the same after the two shocks for a given deviation of output from steady-state, while for a given output gap the cost-push shock generates higher inflation. Correspondingly, an additive "aggregate" cost-push term appears in the Phillips-curve.

Lemma (9) solves for the optimal policy response.

Lemma 9. *The optimal monetary policy response to a cost-push shock $dlog\mu^D$ implements output gap*

$$\tilde{y}^*_{CP} = \frac{\mathcal{B}^T \Delta \left(\mathcal{D} \left(\overbrace{\frac{\mathcal{B}\lambda^T}{\gamma + \varphi}}^{\text{inflation-output trade-off}} - \overbrace{\mathcal{V}}^{\text{propagation}} \right) - \overbrace{\mathcal{D}_2 \Delta \left(I - \Delta\right)^{-1}}^{\text{direct effect}} \right)}{\left(\gamma + \varphi\right) + \mathcal{B}^T \mathcal{D} \mathcal{B}} dlog\mu^D \tag{79}$$

Under the optimal policy the inflation target derived in Proposition 6 takes value

$$\pi_\phi = \left(\lambda^T - \mathcal{B}^T \mathcal{D}_2 \Delta \left(I - \Delta\right)^{-1}\right) dlog\mu^D \tag{80}$$

Comparing (79) with (37) above, we see that the optimal response to productivity and cost-push shocks has a common component (the "propagation" term in (79)). Here, monetary policy seeks to address the relative price distortions induced by the shock through input-output linkages. These are the same regardless of whether inflation responded to fluctuations in productivity or desired markups. In the case of a productivity shock this is the only effect.

Shocks in desired markups instead by definition have also a "direct" effect on relative prices. Monetary policy can partly offset this effect, because by moving the output gap it can decrease the relative markup of the sectors hit by the shock (third term in (79)). Whether this entails increasing or decreasing the output gap depends on how exposed these sectors are to monetary shocks relative to the others. Increasing the output gap is optimal when marginal costs are relatively more exposed to monetary shocks in the sectors hit by the cost-push shock than in the rest of the economy. Whenever this is the case the policy target is positive, as reflected in the second term of (80).

Finally, the first term in equation (79) comes from the fact that monetary policy faces a "worse" trade-off under the cost-push shock than under the productivity shock, because natural output has not fallen. This is also captured by the first term of the policy target (80), which has the same intuition as in the one-sector model: in the face of a cost-push shock the central bank trades off the output loss with the increase in inflation. Therefore after a cost-push shock the output gap should be lower than after an equally-sized productivity shock, while the output level and inflation should be higher. With multiple sectors however this channel is potentially counteracted by the response to the "direct" effect.

Appendix D2: Dynamics - Main results

Consumers

Consumers' intertemporal preferences are given by

$$U = \sum_{t=0}^{\infty} \rho^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right)$$

where C_t is a CRS bundle of all goods produced in the economy and L_t is labor supply.

In each period consumers are subject to the budget constraint

$$P_t C_t + B_{t+1} \leq w_t L_t + \Pi_t - T_t + (1 + i_t) B_t$$

where $w_t L_t$ is labor income, Π_t are firm profits (rebated lump-sum to households), T_t is a lump-sum transfer (that the government uses to finance input subsidies to firms), B_t is the quantity of risk-free bonds paying off in period t owned by the household and i_t are nominal interest rates.

Consumer optimization yields the Euler equation

$$U_{ct} = \rho(1 + i_{t+1})\mathbb{E}\left[U_{ct+1}\frac{P_{ct}}{P_{ct+1}}\right] \tag{81}$$

where P_{ct} are consumer prices at time t. We can log-linearize equation (81) and impose market clearing for final goods to find

$$y_t = \mathbb{E}\left[y_{t+1}\right] - \frac{1}{\gamma}\left(i_{t+1} - \mathbb{E}\left[\pi^c_{t+1}\right] - \log\rho\right)$$

In gaps, this becomes

$$\tilde{y}_t = \mathbb{E}\left[\tilde{y}_{t+1}\right] - \frac{1}{\gamma}\left(i_{t+1} - \mathbb{E}\left[\pi^c_{t+1}\right] - r^n_{t+1}\right) \tag{82}$$

where r^n_{t+1} is the natural interest rate, satisfying

$$r^n_{t+1} = \log\rho + \gamma\lambda^T\mathbb{E}\left[logA_{t+1} - logA_t\right]$$

Policy instruments

I consider a cashless economy, in which interest rates are the only policy instrument. At each period t the central bank sets the risk-free rate i_{t+1}.

Production

Within each period the production technology is as described in Section 2.3. Sectoral productivity shifters A_{it} vary across periods.

As in the one-period model, I assume that the government sets input subsidies to eliminate markup distortions. That is, in steady-state all firms optimally set their markup over pre-subsidy marginal costs at 1.

The government cannot change input subsidies in response to productivity shocks, and it cannot give different subsidies to different firms within the same sector.

All producers minimize costs given wages and input prices: producers in sector i solve

$$mc_{it} = min_{\{x_{ijt}\},L_{it}}\ w_tL_{it} + \sum_j p_{jt}x_{ijt} \quad s.t.\ A_{it}F_i\left(L_{it}, \{x_{ijt}\}\right) = \bar{y}$$

Under constant returns to scale marginal costs are the same for all firms, and they use inputs in the same proportions. However not all firms can adjust prices. Therefore within the same sector firms charge different markups outside of steady-state.

Sector-level inflation dynamics

The firms who can update their price solve

$$p_{it}^{*} = max_{p_i}\ \mathbb{E}\left[\sum_t SDF_t\,(1-\delta_i)^t\, Y_{it}(p_i)\,(p_i - (1-\tau_i)\, mc_{it})\right] \tag{83}$$

The subsidies

$$1-\tau_i = \frac{\epsilon_{it}^{*}-1}{\epsilon_{it}^{*}}$$

eliminate markup distortions in steady-state. With these subsidies in place the optimal reset price is

$$p_{it}^{*} = \frac{\mathbb{E}\sum_t \left[\frac{\epsilon_{it}}{\epsilon_i^{*}} SDF_t\,(1-\delta_i)^t\, Y_{it}(p_i) mc_{it}\right]}{\mathbb{E}\sum_t \frac{\epsilon_{it}-1}{\epsilon_i^{*}-1}\left[SDF_t\,(1-\delta_i)^t\, Y_{it}(p_i)\right]} \tag{84}$$

Log-linearizing equation (84) yields the following expression for sector-level inflation rates:

$$\pi_{it} = \frac{\delta_i\,(1-\rho(1-\delta_i))}{1-\delta_i}\left(d\log\mu_{it}^{D} - \log\mu_{it}\right) + \rho\mathbb{E}_t\left[\pi_{it+1}\right] \tag{85}$$

where μ_{it} is the "sector-level" markup:

$$log\mu_{it} = logp_{it} - logmc_{it}$$

and $\mu_t^{D} = \frac{\epsilon_t}{\epsilon_t - 1}$ is the firms' desired markup.

Equilibrium

Equilibrium is defined in a similar way as in section 2.5.

For given sectoral probabilities of price adjustment δ_i, sectoral productivity shifters A_{it} and interest rates i_t for each period t, general equilibrium is given by a vector of firm-level markups μ_{fit}, a vector of prices p_{it}, a nominal wage w_t, labor supply L_t, a vector of sectoral outputs y_{it}, a matrix of intermediate input quantities x_{ijt}, and a vector of final demands c_{it} for each period t such that: a fraction δ_i of firms in each sector i can adjust their price in every period; markups are chosen optimally by adjusting firms (see problem (83)), while they are such that prices stay constant for the non-adjusting firms; consumers maximize intertemporal utility subject to the budget constraints; producers in each sector i minimize costs and charge the relevant markup; and markets for all goods and labor clear.

The sales-based Phillips curve

Similar to the one-period case, the Phillips curve constructed using SW as our measure of aggregate inflation inherits the properties of the Phillips curve in the one-sector model.[24]

Proposition 7. *It holds that*

$$SW_t \equiv \lambda^T \left(I - \hat{\Delta}\right) \hat{\Delta}^{-1} \pi_t = \rho \mathbb{E}\left(SW_{t+1}\right) + \kappa \tilde{y}_t + \lambda^T d \log \mu_t^D \tag{86}$$

where

$$\kappa = \gamma + \varphi$$

and $\hat{\Delta}$ is a diagonal matrix with elements

$$\hat{\Delta}_{ii} = \frac{\delta_i \left(1 - \rho(1 - \delta_i)\right)}{1 - \rho \delta_i (1 - \delta_i)}$$

Response of inflation rates and markups to productivity and monetary shocks

Proposition (8) characterizes the evolution of sector-level inflation, inflation expectations and markups for given productivity shocks $logA_t - logA_{t-1}$, monetary policy $\tilde{y}_t$, and past markups. Note that, different from the one-sector model and the sales-based Phillips curve in Section 8, past markups are state variables.[25]

Proposition 8. *Denote by*

$$\mathcal{M} \equiv \left(\frac{\hat{\mathcal{B}} \lambda^T}{\gamma + \varphi} - \hat{\mathcal{V}} \right) \left(I - \hat{\Delta}\right) \hat{\Delta}^{-1}$$

The evolution of sectoral markups and inflation rates is given by the following system of difference equations:

$$\begin{pmatrix} \rho \mathbb{E} \pi_{t+1} \\ log \mu_t \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{M}^{-1} \hat{\mathcal{V}} \\ -\left(I - \hat{\Delta}\right) \hat{\Delta}^{-1} \left(I - \mathcal{M}^{-1}\right) & -\left(I - \hat{\Delta}\right) \hat{\Delta}^{-1} \mathcal{M}^{-1} \hat{\mathcal{V}} \end{pmatrix} \begin{pmatrix} \pi_t \\ log \mu_{t-1} \end{pmatrix} +$$

[24] Note that the sales-based Phillips curve in (26) does not depend on past markups. This is a consequence of Lemma 7 in Appendix A2.

[25] The SW-Phillips curve is independent of past markups, therefore the actual state variables are "relative" past markups $\mathcal{V} d \log \mu_{t-1}$. Given these, the system is invariant to the "aggregate" past markup $\lambda^T d \log \mu_{t-1}$.

$$+\begin{pmatrix} -\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)-\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1}d\log\mu_t^D \\ -\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right) \end{pmatrix} \tag{87}$$

Proof. See Appendix D3 □

The first equation in (87) extends the results from Sections 4.1.2 and 4.1.3 to the dynamic setup. We can re-write it as

$$\pi_t=\underbrace{\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}+log\mu_{t-1}\right)}_{\text{productivity and past markups}}+\underbrace{\left(\frac{\hat{\mathcal{B}}\lambda^T}{\gamma+\varphi}-\hat{\mathcal{V}}\right)\left(d\log\mu_t^D+\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\rho\mathbb{E}\pi_{t+1}\right)}_{\text{cost-push shock}} \tag{88}$$

The first term contains the elasticities of sectoral inflation rates with respect to productivity and monetary shocks, which are the same as in the static setup. In addition, we now have to account for pre-existing markups, due to the fact that some producers could not adjust their price in past periods. To gain intuition, Lemma 7 in Appendix B2 shows that in the static setting the response of inflation to productivity shocks and initial markups is the same, because both induce the same desired price changes under zero output gap. Finally, expected future inflation acts as a "cost-push" shock, and indeed it has the same effect on sectoral inflation rates. To map inflation expectations into the corresponding "cost-push" shock by setting:

$$dlog\mu_{\mathbb{E}}^D=\left(I-\Delta\right)\Delta^{-1}\rho\mathbb{E}\pi_{t+1} \tag{89}$$

Consumer price Phillips curve

We can aggregate sectoral inflation rates into the CPI-based Phillips curve from equation (88):

$$\pi_t^C=\kappa\tilde{y}_t+\rho\mathbb{E}\pi_{t+1}^C+u_t+v_t \tag{90}$$

where

$$u_t=\frac{\bar{\delta}_w-\bar{\delta}_A}{1-\bar{\delta}_w}\lambda^T\left(dlogA_t-dlogA_{t-1}\right)+\frac{\bar{\delta}_w-\bar{\delta}_{\mu_{-1}}}{1-\bar{\delta}_w}\lambda^T dlog\mu_{t-1}$$

$$v_t=\frac{\bar{\delta}_w-\bar{\delta}_{\pi^C}}{1-\bar{\delta}_w}\rho\mathbb{E}\pi_{t+1}^C+\frac{\bar{\delta}_{\mu^D}}{1-\bar{\delta}_w}\lambda^T d\log\mu_t^D$$

$$\bar{\delta}_{\mu_{-1}}=\frac{\beta^T\Delta\left(I-\Omega\Delta\right)^{-1}dlog\mu_{t-1}}{\lambda^T dlog\mu_{t-1}}$$

$$\bar{\delta}_{\pi^C}=\frac{\beta^T\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Omega\right)\mathbb{E}\pi_{t+1}}{\mathbb{E}\pi_{t+1}^C}$$

$$\bar{\delta}_{\mu^D} = \frac{\beta^T \Delta \left(I - \Omega\Delta\right)^{-1} d\log\mu^D}{\lambda^T d\log\mu^D}$$

Equation (90) highlights that past markups and inflation expectations also create an endogenous cost-push term in the Phillips curve, exactly in the same way as productivity shocks. This is not surprising, given that their propagation across the input-output network is the same.

Optimal policy

Proposition 9 characterizes the dynamic welfare loss and the central bank's problem without commitment.

Proposition 9. *Given a path $\{y_t, \pi_t, z_{t+1}\}_{t=0}^{\infty}$ for the output gap, sectoral inflation rates and markups, the expected second-order welfare loss is given by*

$$\sum_{t=0}^{\infty} \rho^s \mathbb{E}\left[\left(\gamma+\varphi\right)\tilde{y}_t^2 + \pi_t^T \mathcal{D}_1 \pi_t + z_{t+1}^T \mathcal{D}_2 z_{t+1}\right]$$

subject to (87), where

$$z_t \equiv -\left(I - \hat{\Delta}\right)^{-1} \hat{\Delta} log\mu_{t-1}$$

Without commitment the central bank takes future output gaps and inflation rates as given, and chooses $\{\tilde{y}_t, \pi_t, z_{t+1}\}$. Thus the central bank solves

$$min_{\{\tilde{y}_t, \pi_t\}} \left(\gamma+\varphi\right)\tilde{y}_t^2 + \pi_t^T \mathcal{D}_1 \pi_t + z_{t+1}^T \mathcal{D}_2 z_{t+1}$$

subject to (87). The FOCs yield

$$\tilde{y}_t^* = -\underbrace{\frac{\mathcal{B}^T \mathcal{D}\mathcal{V}\left(dlogA_t - dlog\mu_{t-1}\right)}{\left(\gamma+\varphi\right) + \mathcal{B}^T\mathcal{D}\mathcal{B}}}_{\text{productivity and past markups}} - \underbrace{\frac{\mathcal{B}^T\left(\mathcal{D}\mathcal{M} - \mathcal{D}_2\right)}{\left(\gamma+\varphi\right) + \mathcal{B}^T\mathcal{D}\mathcal{B}}\rho\mathbb{E}\pi_{t+1}}_{\text{cost-push shock}} \tag{91}$$

Proof. See Appendix D3 □

In the same spirit as Proposition 8, Proposition 9 decomposes the optimal output gap into the response to productivity shocks and past markups, which is the same as in the static setup (see Proposition 5), and the response to inflation expectations, which act as a "cost-push" shock (see Section 5.1). Lemma 10 below shows that the optimal policy can be implemented with a targeting rule, in the same way as in the static setup (see Proposition 6).

Lemma 10. *Assume that the productivity shocks follow an AR1 process:*

$$logA_{t+1} - logA_t = \eta \left(logA_t - logA_{t-1}\right) + u_{t+1}$$

with $\mathbb{E}u_{t+1} = 0$ *and* $\eta < 1$.

Then there is a unique path of inflation rates such that the optimal output gap is implemented in each period.

Denote by

$$\begin{cases} \phi_t^T \equiv \frac{\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}+\mathcal{B}^T\mathcal{D}}{\gamma+\varphi} \\ \phi_{t+1}^T \equiv \rho\frac{\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}-\mathcal{B}^T\mathcal{D}_2}{\gamma+\varphi} \end{cases} \tag{92}$$

For $\zeta > \gamma$ *the interest rate rule*

$$i_t = \underbrace{\overbrace{r_t^n + \gamma\left[\mathbb{E}\widetilde{y}_{t+1}^* - \widetilde{y}_t^*\right]}^{\text{real rate under opt policy}} + \beta^T\mathbb{E}\pi_{t+1}^*}_{\text{nominal rate under optimal policy}} + \zeta\underbrace{\left(\phi_t\pi_t + \phi_{t+1}\mathbb{E}\pi_{t+1}\right)}_{\text{inflation target}} \tag{93}$$

implements the optimal policy.

The nominal rate can be expressed as a function of productivity shocks and current and expected inflation using the relation

$$\gamma\left[\mathbb{E}\tilde{y}_{t+1}^* - \tilde{y}_t^*\right] + \beta^T\mathbb{E}\pi_{t+1}^* = \kappa_t\pi_t^* + \kappa_{t+1}\mathbb{E}\pi_{t+1}^* + \kappa_A\left(dlogA_t - dlogA_{t-1}\right)$$

where

$$\begin{cases} \kappa_t^T \equiv -\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\left(I + \left[\mathcal{M} - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}_2}{\gamma+\varphi}\right]^{-1}\hat{\mathcal{V}}\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\right) \\ \kappa_{t+1}^T \equiv -\left(\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\left(\left(1+\frac{\gamma}{(\gamma+\varphi)\rho}\right)I - \left[\mathcal{M} - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}_2}{\gamma+\varphi}\right]^{-1}\left(\frac{\gamma\left(I+\mathcal{B}\mathcal{B}^T\mathcal{D}\right)}{(\gamma+\varphi)\rho} - \hat{\mathcal{V}}\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\right)\right) + \beta^T\right) \\ \kappa_A \equiv \eta\frac{\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}}{(\gamma+\varphi)}\left[\mathcal{M} - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}_2}{\gamma+\varphi}\right]^{-1}\hat{\mathcal{V}} \end{cases}$$

Proof. See Appendix D3 □

Comparing equation (42) from the static setup with equation (92) we see that they yield a very similar inflation target. The key difference is that in the dynamic setup the central bank should not just target current inflation, but also inflation expectations. The intuition is that inflation expectations act as a negative cost-push shock, and we know that with cost-push shocks the "static" inflation target is not zero under the optimal policy. Specifically, from Lemma 9 and equation (80) we know that, after a shock $dlog\mu^D$ to desired markups, the inflation target takes value

$$\pi_T = \left(\lambda^T - \mathcal{B}^T \mathcal{D}_2 \left(I - \Delta\right)^{-1}\right) dlog\mu^D$$

under the optimal policy. Correspondingly, from equation (89) we see that the time-t inflation target should be adjusted by

$$\left(\lambda^T \left(I - \Delta\right) \Delta^{-1} - \mathcal{B}^T \mathcal{D}_2\right) \rho \mathbb{E} \pi_{t+1}$$

Lemma 11 shows that the central bank can implement zero output gap in all periods by targeting it directly in the nominal rate, as long as the targeting rule is "reactive enough".

Lemma 11. *There is a unique path of inflation rates such that the output gap is constantly zero. This can be implemented with the interest rate rule*

$$i_t = \underbrace{r_t^n + \beta^T \mathbb{E} \pi_{t+1}^{zg}}_{\text{nominal rate under zero output gap}} + \zeta \tilde{y}_t \tag{94}$$

with $\zeta > 0$.

Proof. See Appendix D3. □

Appendix D3: Dynamics - Proofs

Proof of Proposition 8

We characterize the response of sectoral inflation rates and markups as a function of initial markups (the only state variable), productivity shocks and monetary policy.

Denote by $\hat{\Delta}$ the diagonal matrix with

$$\hat{\delta}_i = \frac{\delta_i \left(1 - \rho(1 - \delta_i)\right)}{1 - \rho \delta_i \left(1 - \delta_i\right)}$$

The first step is to solve for the growth rate of sector-level markups, remembering that it is given by the log-difference between the growth rates of prices and marginal costs:

$$- \left(log\mu_t - log\mu_{t-1}\right) = \alpha \left(logw_t - logw_{t-1}\right) - \left(I - \Omega\right) \pi_t - \left(logA_t - logA_{t-1}\right) \Rightarrow$$

$$- \left(log\mu_t - log\mu_{t-1}\right) = - \left(I - \Omega\right) \hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} \left(-log\mu_t\right) + \alpha \left(logw_t - logw_{t-1}\right) +$$

$$-\left[(logA_t - logA_{t-1}) + (I-\Omega)\left[\rho\mathbb{E}\pi_{t+1} + \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D\right]\right] \Rightarrow$$

$$\left(\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1} + (I-\Omega)\right)\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1}(-log\mu_t) =$$

$$= (-log\mu_{t-1}) + \alpha\left(logw_t - logw_{t-1}\right) - \left[(logA_t - logA_{t-1}) + (I-\Omega)\left[\rho\mathbb{E}\pi_{t+1} + \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D\right]\right]$$

Denote by

$$x_t \equiv -\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} log\mu_t$$

$$x_t^D \equiv \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D$$

and re-write

$$\left(\hat{\Delta}^{-1} - \Omega\right)x_t = \left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}x_{t-1} + \alpha\left(logw_t - logw_{t-1}\right) - \left[(logA_t - logA_{t-1}) + (I-\Omega)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right] \Rightarrow$$

$$x_t = \hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\left[\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}x_{t-1} + \alpha\left(logw_t - logw_{t-1}\right) - \left[(logA_t - logA_{t-1}) + (I-\Omega)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right]\right]$$

From the consumers' labor-leisure trade-off, wages evolve according to

$$logw_t - logw_{t-1} = (\gamma+\varphi)\left(\tilde{y}_t - \tilde{y}_{t-1}\right) + \lambda^T\left(logA_t - logA_{t-1}\right) +$$

$$+\beta^T\left(x_t + x_t^D + \rho\mathbb{E}\left(\pi_{t+1}\right)\right) \tag{95}$$

so that

$$logw_t - logw_{t-1} = \frac{\gamma+\varphi}{1-\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\alpha}\left(\tilde{y}_t - \tilde{y}_{t-1}\right) +$$

$$+\frac{\lambda^T - \beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}}{1-\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\alpha}\left[(logA_t - logA_{t-1}) + (I-\Omega)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right] +$$

$$+\frac{\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}}{1-\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\alpha}\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}x_{t-1} \tag{96}$$

Combining (95) and (96) we obtain

$$x_t = \hat{\mathcal{B}}\left(\tilde{y}_t - \tilde{y}_{t-1}\right) + \hat{\mathcal{V}}\left[(logA_t - logA_{t-1}) + (I-\Omega)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right] + \mathcal{M}x_{t-1} \tag{97}$$

Lemma 12. *As long as no sector is fully flexible ($\delta_i < 1\ \forall i$), the matrix $\mathcal{M}$ is invertible. Moreover, all of its eigenvalues have modulus smaller than one.*

Proof. It holds that

$$\mathcal{M} = \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Delta\right)\Delta^{-1}$$

The matrix $\left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)$ has eigenvalues 1 (and all vectors orthogonal to β are corresponding eigenvectors) and $\frac{1}{1-\beta^T\Delta(I-\Omega\Delta)^{-1}\alpha}$, with corresponding eigenvector $\mathcal{B}$. Therefore it is invertible. The matrix $\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Delta\right)\Delta^{-1}$ is invertible because we assumed that no sector has fully rigid or fully flexible prices. Thus $\mathcal{M}$ is invertible.

To prove that all eigenvectors are smaller than one in modulus, note that $\mathcal{M}\mathbf{1} = \mathbf{1}$:

$$\mathcal{M}\mathbf{1} = \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\Delta\left(I-\Omega\Delta\right)^{-1}\left(\Delta^{-1}-\Omega-(I-\Omega)\right)\left(I-\Omega\right)^{-1}\alpha =$$

$$= \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\Delta\left(I-\Omega\Delta\right)^{-1}\left(\left(I-\Omega\Delta\right)\Delta^{-1}\left(I-\Omega\right)^{-1}-I\right)\alpha =$$

$$= \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\left(\left(I-\Omega\right)^{-1}-\Delta\left(I-\Omega\Delta\right)^{-1}\right)\alpha =$$

$$= \mathbf{1} - \Delta\left(I-\Omega\Delta\right)^{-1}\alpha + \frac{\mathcal{B}}{\gamma+\varphi}\left(1-\beta^T\Delta\left(I-\Omega\Delta\right)^{-1}\alpha\right) = \mathbf{1}$$

In addition $\mathcal{M}$ has all positive elements, because both $\left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)$ and $\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Delta\right)\Delta^{-1}$ have positive elements. These two properties imply that all of its eigenvalues must be smaller than one in modulus. □

Denoting by $z_t \equiv x_{t-1}$, equations (85) and (97) can then be combined to obtain the following system of difference equations in π_t and z_t:

$$\begin{pmatrix} \rho\mathbb{E}\pi_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -I \\ I-\mathcal{M}^{-1} & I \end{pmatrix}\begin{pmatrix} \pi_t \\ z_t \end{pmatrix} + \begin{pmatrix} -\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\left(\tilde{y}_t-\tilde{y}_{t-1}\right)+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)-x_t^D \\ \mathcal{M}^{-1}\left(\hat{\mathcal{B}}\left(\tilde{y}_t-\tilde{y}_{t-1}\right)+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right) \end{pmatrix} \tag{98}$$

Finally, it is useful to re-write (98) substituting out for the past output gap, using Lemma 3 (see equation (**??**)):

$$\begin{pmatrix} \rho\mathbb{E}\pi_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} \\ I-\mathcal{M}^{-1} & \mathcal{Z} \end{pmatrix}\begin{pmatrix} \pi_t \\ z_t \end{pmatrix} + \begin{pmatrix} -\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)-x_t^D \\ \mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right) \end{pmatrix}$$

where

$$\mathcal{Z} \equiv \mathcal{M}^{-1}\hat{\mathcal{V}}\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}$$

To obtain the system in (87) just use the relation

$$z_t \equiv -\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} log\mu_{t-1}$$

Proof of Lemma 11

To prove that there is a unique path of inflation rates and markups that implements zero output gap we start from the system

$$\begin{pmatrix} \mathbb{E}\pi_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho}\mathcal{M}^{-1} & -\frac{1}{\rho}\mathcal{Z} \\ I-\mathcal{M}^{-1} & \mathcal{Z} \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \end{pmatrix} + \begin{pmatrix} -\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_t - logA_{t-1}\right) - \frac{1}{\rho}x_t^D \\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_t - logA_{t-1}\right) \end{pmatrix} \tag{99}$$

and want to show that the matrix

$$\mathcal{A} = \begin{pmatrix} \frac{1}{\rho}\mathcal{M}^{-1} & -\frac{1}{\rho}\mathcal{Z} \\ I-\mathcal{M}^{-1} & \mathcal{Z} \end{pmatrix}$$

has N eigenvectors greater than 1, and N smaller than 1.

This is enough to guarantee that the system has a unique bounded solution for any given past markups z_t and productivity/markup shocks $logA_t - logA_{t+1}$ and x_t^D. That is, given an initial condition for z_t, imposing that $|lim_{t\to\infty}\pi_{it}^*| < \infty \ \forall i$ and $|lim_{t\to\infty}z_{it}^*| < \infty \ \forall i$ pins down a unique initial value for π_t^*. To see this observe that, given our assumption about the productivity process,

$$\mathbb{E}lim_{t\to\infty}\begin{pmatrix} \pi_t^* \\ z_t^* \end{pmatrix} = lim_{t\to\infty}\mathcal{A}^t\begin{pmatrix} \pi_0^* \\ z_0 \end{pmatrix} + lim_{t\to\infty}\left(\sum_{s\le t}\eta^s\mathcal{A}^{t-s}\right)\begin{pmatrix} -\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) - \frac{1}{\rho}x_0^D \\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) \end{pmatrix}$$

In turn, we can write

$$\begin{pmatrix} -\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) - \frac{1}{\rho}x_0^D \\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) \end{pmatrix} = a_1w_1 + ... + a_{2N}w_{2N}$$

where $\{w_1, ..., w_{2N}\}$ are eigenvectors of $\mathcal{A}$. Denote by $\{\nu_1, ..., \nu_{2N}\}$ the corresponding eigenvalues. Then we have

$$lim_{t\to\infty}\left(\sum_{s\le t}\eta^s\mathcal{A}^{t-s}\right)\begin{pmatrix} -\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) - \frac{1}{\rho}x_0^D \\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) \end{pmatrix} =$$

$$= \sum_{i/\nu_i<1} \frac{a_i w_i}{1-\nu_i} + lim_{t\to\infty}\mathcal{A}^t \sum_{i/\nu_i>1} \frac{\nu_i}{\nu_i-\eta} a_i w_i$$

To have a unique bounded solution we need the condition

$$lim_{t\to\infty}\mathcal{A}^t \begin{pmatrix} \pi_0^* \\ z_0 \end{pmatrix} = -lim_{t\to\infty}\mathcal{A}^t \sum_{i/\nu_i>1} \frac{\nu_i}{\nu_i-\eta} a_i w_i \tag{100}$$

to yield a unique solution π_0^*, which happens if and only if the matrix $\mathcal{A}$ has N eigenvalues with absolute value greater or equal to 1 and N eigenvalues with absolute value smaller than 1.

Note that (for $x_t^D \equiv 0$) the two equations in (99) yield the optimal reset price equation

$$\rho\mathbb{E}\pi_{t+1} = \pi_t - z_{t+1}$$

It is convenient to substitute this to the first equation and use it together with the second to look for eigenvectors. Assume then that $\begin{pmatrix} \pi \\ z \end{pmatrix}$ is an eigenvector relative to the eigenvalue ν. From the optimal reset price equation we find

$$\nu z = (1-\rho\nu)\,\pi$$

The second equation yields

$$\nu z = \left(I - \mathcal{M}^{-1}\right)\pi + \mathcal{Z}z$$

For $\nu = 0$ we have $\pi = 0$ and $z = \mathcal{M}^{-1}\hat{\Delta}\hat{\mathcal{B}}$.

For $\nu = \frac{1}{\rho}$ we have $z = 0$ and $\pi = \begin{pmatrix} 1 \\ ... \\ 1 \end{pmatrix}$.

Otherwise, we can substitute into the second equation to solve as a function of π:

$$\rho\nu\pi = \mathcal{M}^{-1}\pi - \frac{1-\rho\nu}{\nu}\mathcal{Z}\pi$$

Also note that all eigenvectors of $\mathcal{M}$ except $\begin{pmatrix} 1 \\ ... \\ 1 \end{pmatrix}$ are orthogonal to $\lambda^T (I-\Delta)\,\Delta^{-1}$. Therefore if $\pi$is an eigenvector of $\mathcal{M}$, with corresponding eigenvalue $\xi \neq 0$, then $\mathcal{Z}\pi = \pi$. Thus the equation becomes

$$\frac{\rho\nu^2 - \rho\nu + 1}{\nu}\pi = \frac{1}{\xi}\pi$$

Since $\pi \neq 0$ we must have

$$\frac{\rho\nu^2 - \rho\nu + 1}{\nu} = \frac{1}{\xi}$$

From Lemma 12, it holds that this equation has two solutions, ν^+ and ν^-, with $0 < \nu^- < 1$ and $\nu^+ > 1$. Thus we have $N-1$ couples of solutions (one smaller than 1 and one greater than 1), plus 0 and $\frac{1}{\rho}$. Therefore the matrix $\mathcal{A}$ has N eigenvalues greater than 1 and N smaller than 1 in absolute value, as we wanted to show.

It remains to prove that the interest rate rule

$$i_t = \underbrace{r_t^n + \beta^T \mathbb{E}\pi_{t+1}^{zg}}_{\text{nominal rate under zero output gap}} + \zeta\tilde{y}_t$$

with $\zeta > 0$ implements zero output gap in every period.

Under this rule the system becomes

$$\begin{pmatrix} \rho\mathbb{E}\pi_{t+1} \\ z_{t+1} \\ \mathbb{E}\tilde{y}_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1}\hat{\mathcal{B}} \\ I - \mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1}\hat{\mathcal{B}} \\ 0 & 0 & \zeta + 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \\ \tilde{y}_t \end{pmatrix} + \begin{pmatrix} -\mathcal{M}^{-1}\hat{\mathcal{V}} \\ \mathcal{M}^{-1}\hat{\mathcal{V}} \\ 0 \end{pmatrix} (logA_t - logA_{t-1}) + \begin{pmatrix} -I \\ 0 \\ 0 \end{pmatrix} x_t^D$$

Note that the solution to the previous system is still a solution of the new system. To prove that there are no additional solutions we will show that the matrix

$$\tilde{\mathcal{A}} \equiv \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1}\hat{\mathcal{B}} \\ I - \mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1}\hat{\mathcal{B}} \\ 0 & 0 & \zeta + 1 \end{pmatrix}$$

has the same eigenvalues and eigenvectors as $\mathcal{A}$ above, plus the eigenvalue $\nu = \zeta + 1$, with associated eigenvector $\begin{pmatrix} \pi \\ z \\ \tilde{y} \end{pmatrix}$ such that

$$\pi = \left(I - \frac{1 - \rho\nu + \rho\nu^2}{\nu} \mathcal{G} \right)^{-1} \hat{\mathcal{B}}$$

$$z = \frac{1 - \rho\nu}{\nu} \pi$$

$$\tilde{y} = (1 - \rho\nu) \frac{\lambda^T (I - \Delta) \Delta^{-1}}{\gamma + \varphi} \pi$$

with

$$\mathcal{G} \equiv \Delta \left(I - \Omega\Delta\right)^{-1} \left(I - \Delta\right) \Delta^{-1}$$

This implies that for $\zeta > 0$ the new system has a unique bounded solution, which proves the result.

Let's then study the eigenvalues and eigenvectors of $\tilde{\mathcal{A}}$. Denote the eigenvalues by ν, and the first N components of the corresponding eigenvector by π. From the first two rows and the definition of eigenvector we derive the conditions

$$z = \frac{1 - \rho\nu}{\nu} \pi$$

$$\tilde{y} = \left(1 - \rho\nu\right) \frac{\lambda^T \left(I - \Delta\right) \Delta^{-1}}{\gamma + \varphi} \pi$$

$$\left(I - \frac{1 - \rho\nu + \rho\nu^2}{\nu} \mathcal{G}\right) \pi = \hat{\mathcal{B}} \frac{\lambda^T \left(I - \Delta\right) \Delta^{-1}}{\gamma + \varphi} \left(I - \frac{1 - \rho\nu + \rho\nu^2}{\nu} \mathcal{G}\right) \pi$$

The last condition implies

$$\left(I - \frac{1 - \rho\nu + \rho\nu^2}{\nu} \mathcal{G}\right) \pi = \hat{\mathcal{B}}$$

From the last row of $\tilde{\mathcal{A}}$ we derive the relation

$$\left(1 + \zeta - \nu\right) \left(1 - \rho\nu\right) \frac{\lambda^T \left(I - \Delta\right) \Delta^{-1}}{\gamma + \varphi} \pi = 0$$

which we know is satisfied by the eigenvalues/eigenvectors of $\mathcal{A}$. In addition, it is also satisfied for $\nu = 1 + \zeta$ and $\pi = \left(I - \frac{1 - \rho\nu + \rho\nu^2}{\nu} \mathcal{G}\right)^{-1} \hat{\mathcal{B}}$. This proves the result.

Proof of Proposition 9

Within each period, the cross-sector misallocation loss is the same function of sector-level markups derived in Section 4. Therefore it can be written as

$$x_t^T \mathcal{D}_2 x_t$$

where now $\mathcal{D}_2$ is defined as

$$\Xi_2 = \left(I - \hat{\Delta}\right) \hat{\Delta}^{-1} \tilde{\mathcal{D}}_2 \hat{\Delta}^{-1} \left(I - \hat{\Delta}\right)$$

and $\tilde{\mathcal{D}}_2$ is as in Section 4.

The within-sector productivity loss is given by

$$\sum_{i=1}^{N} \lambda_i \epsilon_i \left[\int (logp_{ift} - logp_{it})^2 \, df - \left(\int (logp_{ift} - logp_{it}) \, df \right)^2 \right]$$

as derived in Proposition 4.

The following lemma shows how this discounted sum can be written as a function of sectoral inflation rates.

Lemma 13. *It holds that*

$$\sum_{s \geq 0} \rho^s \left(\sum_{i=1}^{N} \lambda_i \epsilon_i \left[\int (logp_{ift+s} - logp_{it+s})^2 \, df - \left(\int (logp_{ift+s} - logp_{it+s}) \, df \right)^2 \right] \right) = \sum_{s \geq 0} \rho^s \pi_{t+s}^T \mathcal{D}_1 \pi_{t+s}$$

where Ξ_1 *is a diagonal matrix with elements*

$$\xi_{1ii} = \lambda_i \epsilon_i \frac{1 - \hat{\delta}_i}{\hat{\delta}_i}$$

Proof. To prove the lemma it is enough to show that

$$\sum_{s \geq 0} \rho^s \left[\int (logp_{ift+s} - logp_{it+s})^2 \, df - \left(\int (logp_{ift+s} - logp_{it+s}) \, df \right)^2 \right] = \frac{1 - \hat{\delta}_i}{\hat{\delta}_i} \sum_{s \geq 0} \rho^s \pi_{t+s}^2$$

Given the Calvo assumption, in each sector i for the fraction δ_i of firms who adjust prices we have

$$logp_{ift} - logp_{it} = (1 - \delta_i)(logp_{it}^* - logp_{it-1}) = \frac{1 - \delta_i}{\delta_i} \pi_{it}$$

For the remaining fraction $(1 - \delta_i)$ we have

$$logp_{ift} - logp_{it} = (-\delta_i)(logp_{it}^* - logp_{it-1}) + (logp_{ift-1} - logo_{it-1}) = (logp_{ift-1} - logp_{it-1}) - \pi_{it}$$

Hence we find that, defining

$$\mathcal{D}_{it} \equiv \int (logp_{ift+s} - logp_{it+s})^2 \, df - \left(\int (logp_{ift+s} - logp_{it+s}) \, df \right)^2$$

around a steady-state where $logp_{ift} - logp_{it} = 0 \; \forall f$, we have

$$D_{it} = (1-\delta_i)\left(\frac{1-\delta_i}{\delta_i}\pi_{it}^2 + D_{it-1}\right)$$

It follows that

$$\sum_s \rho^s D_{it+s} = \sum_s \rho^s \frac{1-\delta_i}{\delta_i}\pi_{is}^2 \left(\sum_{\tau \geq s} \left(\rho\left(1-\delta_i\right)\right)^{\tau - s}\right) =$$

$$= \frac{1-\delta_i}{\tilde{\delta}_i}\sum_s \rho^s \pi_{is}^2$$

$\square$

Therefore at each time t the central bank solves the following problem:

$$min_{\{y_{t+s}, \mathbb{E}\pi_{t+s}, \mathbb{E}z_{t+s+1}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \rho^s \mathbb{E}\left[\left(\gamma+\varphi\right)\tilde{y}_{t+s}^2 + \pi_{t+s}^T \mathcal{D}_1 \pi_{t+s} + z_{t+s+1}^T \mathcal{D}_2 z_{t+s+1}\right]$$

subject to the constraint given by (**??**).

Since we assumed no commitment, the central bank takes $\{y_{t+s}, \mathbb{E}\pi_{t+s}, \mathbb{E}z_{t+s+1}\}_{s=1}^{\infty}$ as given and the problem reduces to

$$min_{\{\tilde{y}_t, \pi_t\}} \left(\gamma+\varphi\right)\tilde{y}_t^2 + \pi_t^T \mathcal{D}_1 \pi_t + z_{t+1}^T \mathcal{D}_2 z_{t+1}$$

again subject to (**??**).

The FOCs yield

$$\left(\gamma+\varphi\right)\tilde{y}_t^* = -\mathcal{B}^T\left[\mathcal{D}_1 \pi_t^* + \mathcal{D}_2 z_{t+1}^*\right] \tag{101}$$

The expression in (91) can be derived by solving for π_t^* and z_{t+1}^* as a function of productivity shocks, sector-level initial markups and inflation expectations from (99).

Proof of Lemma 10

Under the optimal policy (87) can be re-written as

$$\begin{pmatrix} \mathbb{E}\pi_{t+1}^* \\ z_{t+1}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho}\mathcal{X}^{-1} & -\frac{1}{\rho}\mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z} \\ I - \mathcal{X}^{-1} & \mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z} \end{pmatrix} \begin{pmatrix} \pi_t^* \\ z_t \end{pmatrix} + \begin{pmatrix} -\frac{1}{\rho}\mathcal{X}^{-1}\mathcal{W}\hat{\mathcal{V}}\left(logA_t - logA_{t-1}\right) \\ \mathcal{X}^{-1}\mathcal{W}\hat{\mathcal{V}}\left(logA_t - logA_{t-1}\right) \end{pmatrix} \tag{102}$$

where

$$\mathcal{W} \equiv I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma + \varphi + \mathcal{B}^T\mathcal{D}\mathcal{B}}$$

$$\mathcal{X} \equiv \mathcal{W}\mathcal{M} + \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}_2}{\gamma + \varphi + \mathcal{B}^T\mathcal{D}\mathcal{B}}$$

Let's first show that this system has a unique bounded solution for any given past markups z_t and productivity shocks $logA_t - logA_{t+1}$. This can be done in the same way as in the proof of Lemma 11, by showing that the matrix

$$\mathcal{A} \equiv \begin{pmatrix} \frac{1}{\rho}\mathcal{X}^{-1} & -\frac{1}{\rho}\mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z} \\ I - \mathcal{X}^{-1} & \mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z} \end{pmatrix}$$

has N eigenvalues with absolute value greater or equal to 1 and N eigenvalues with absolute value smaller than 1. Lemma 14 proves this result.

Lemma 14. *The matrix $\mathcal{A}$ has N eigenvalues with absolute value greater or equal to* 1 *and N eigenvalues with absolute value smaller than* 1.

Proof. The eigenvectors $\begin{pmatrix} \pi \\ z \\ \tilde{y} \end{pmatrix}$ and eigenvalues ν of $\mathcal{A}$ solve

$$\left(I - \frac{1 - \rho\nu + \rho\nu^2}{\nu}\mathcal{G}\right)\pi = \hat{\mathcal{B}}$$

$$z = \frac{1 - \rho\nu}{\nu}\pi$$

and

$$\left[\frac{\lambda^T (I - \Delta)\Delta^{-1} + \mathcal{B}^T\mathcal{D}}{\gamma + \varphi}\right][I - \rho\nu\mathcal{X}]\,\pi = 0$$

It holds that there are exactly N values of νgreater than 1 and N values smaller than 1 that satisfy both conditions. □

To construct our "policy target" note that (**??**), together with the observation that $\lambda^T\left(I - \hat{\Delta}\right)\hat{\Delta}^{-1}\mathcal{M} = \lambda^T\left(I - \hat{\Delta}\right)\hat{\Delta}^{-1}$ and $\lambda^T\left(I - \hat{\Delta}\right)\hat{\Delta}^{-1}\mathcal{Z} = \mathbf{0}$, imply that the linear combination

$$\left[\frac{\lambda^T (I - \Delta)\Delta^{-1} + \mathcal{B}^T\mathcal{D}}{\gamma + \varphi}\right]\left[(I - \mathcal{X})\,\pi_t^* + \mathcal{X}z_{t+1}^*\right] \equiv$$

$$\equiv \phi_t^T\pi_t^* + \phi_{t+1}^T\mathbb{E}\pi_{t+1}^*$$

is constantly zero. The target as expressed in (92) can be obtained straightforwardly from the definition of $\mathcal{X}$.

Finally, to express the real interest rate as a function of $logA_t - logA_{t-1}$, π_t^* and z_{t+1}^* we need to compute the value of

$$\gamma\left[\mathbb{E}\tilde{y}_{t+1}^* - \tilde{y}_t^*\right] + \beta^T \mathbb{E}\pi_{t+1}^*$$

as a function of these variables under the optimal policy. This can be done using (102):

$$\gamma\left[\mathbb{E}\tilde{y}_{t+1}^* - \tilde{y}_t^*\right] + \beta^T \mathbb{E}\pi_{t+1}^* =$$

$$= \left(\frac{\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}}{(\gamma+\varphi)\rho}\left(I-\mathcal{X}^{-1}\right) + \frac{\beta^T}{\gamma\rho}\right)\pi_t^* +$$

$$-\left(\frac{\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}}{(\gamma+\varphi)}\left(\frac{\left(I-\mathcal{X}^{-1}\right)}{\rho} + \left(I-\mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z}\right)\right) + \frac{\beta^T}{\gamma\rho}\right)z_{t+1}^* +$$

$$+\eta\frac{\lambda^T\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}}{(\gamma+\varphi)}\mathcal{X}^{-1}\mathcal{W}\hat{\Delta}\hat{\mathcal{V}}\left(dlogA_t - dlogA_{t-1}\right)$$

Thus, if the optimal policy is implemented, it holds that

$$\gamma\left[\mathbb{E}\tilde{y}_{t+1}^* - \tilde{y}_t^*\right] + \beta^T \mathbb{E}\pi_{t+1}^* = \kappa_t\pi_t^* + \kappa_{t+1}\mathbb{E}\pi_{t+1}^* + \kappa_A\left(dlogA_t - dlogA_{t-1}\right) + \zeta\left(\phi_t^T\pi_t^* + \phi_{t+1}^T\mathbb{E}\pi_{t+1}^*\right)$$

for all $\zeta \in \mathbb{R}$.

This proves that if the optimal policy is implemented (93) must hold.

Let's then check that the reverse is also true.

Lemma (15) shows that there is a unique bounded solution to (87) when adding (93), and it is equal to the optimal one.

Lemma 15. *The interest rate rule*

$$i_t = \underbrace{\overbrace{r_t^n + \gamma\left[\mathbb{E}\widetilde{y}_{t+1}^* - \widetilde{y}_t^*\right]}^{\text{real rate under opt policy}} + \beta^T\mathbb{E}\pi_{t+1}^*}_{\text{nominal rate under optimal policy}} + \zeta\underbrace{\left(\phi_t^T\pi_t + \phi_{t+1}^T\mathbb{E}\pi_{t+1}\right)}_{\text{inflation target}}$$

with $\zeta > \gamma$ implements the optimal policy in every period.

Proof. Under this rule the system becomes

$$\begin{pmatrix} \rho\mathbb{E}\pi_{t+1} \\ z_{t+1} \\ \mathbb{E}\tilde{y}_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1}\hat{\mathcal{B}} \\ I-\mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1}\hat{\mathcal{B}} \\ \phi_\pi^T+\phi_z^T-\left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{M}^{-1} & \left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{Z} & \left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{M}^{-1}\hat{\mathcal{B}}+1 \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \\ \tilde{y}_t \end{pmatrix} +$$

$$+\begin{pmatrix} -\mathcal{M}^{-1}\hat{\mathcal{V}} \\ \mathcal{M}^{-1}\hat{\mathcal{V}} \\ \phi_A^T+\left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{M}^{-1}\hat{\mathcal{V}} \end{pmatrix}(logA_t-logA_{t-1})$$

The solution to the previous system is still a solution of the new system. To prove that there are no additional solutions we will show that the matrix

$$\tilde{\mathcal{A}} \equiv \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1}\hat{\mathcal{B}} \\ I-\mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1}\hat{\mathcal{B}} \\ \phi_\pi^T+\phi_z^T-\left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{M}^{-1} & \left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{Z} & \left(\phi_z^T+\frac{\beta^T}{\gamma\rho}\right)\mathcal{M}^{-1}\hat{\mathcal{B}}+1 \end{pmatrix}$$

has the same eigenvalues and eigenvectors as $\mathcal{A}$ above, plus the eigenvalue $\nu=\frac{\zeta}{\gamma}$, with associated eigenvector

$$\pi=\left(I-\frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)^{-1}\hat{\mathcal{B}}$$

$$z=\frac{1-\rho\nu}{\nu}\pi$$

$$\tilde{y}=(1-\rho\nu)\frac{\lambda^T(I-\Delta)\Delta^{-1}}{\gamma+\varphi}\pi$$

This implies that for $\zeta>\gamma$ the new system has a unique bounded solution, which proves the result.

Let's then study the eigenvalues and eigenvectors of $\tilde{\mathcal{A}}$. Denote the eigenvalues by ν, and the first N components of the corresponding eigenvector by π. As in the proof of Lemma 11, from the first two rows and the definition of eigenvector we derive the conditions

$$z=\frac{1-\rho\nu}{\nu}\pi$$

$$\tilde{y}=(1-\rho\nu)\frac{\lambda^T(I-\Delta)\Delta^{-1}}{\gamma+\varphi}\pi$$

$$\left(I-\frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)\pi=\hat{\mathcal{B}}$$

From the last row of $\tilde{\mathcal{A}}$ we now have the relation

$$\frac{\lambda^T (I-\Delta)\,\Delta^{-1}}{\gamma+\varphi}\left[\rho\nu I-\mathcal{X}^{-1}+\frac{1-\rho\nu}{\nu}\mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z}\right]\pi =$$

$$= -\frac{\zeta}{\gamma\nu}\left[\frac{\lambda^T (I-\Delta)\,\Delta^{-1}+\mathcal{B}^T\mathcal{D}}{\gamma+\varphi}\right][I-\rho\nu\mathcal{X}]\,\pi \tag{103}$$

It holds that, for every $\nu \in \mathbb{C}$,

$$\frac{\lambda^T (I-\Delta)\,\Delta^{-1}}{\gamma+\varphi}\left[\rho\nu I-\mathcal{X}^{-1}+\frac{1-\rho\nu}{\nu}\mathcal{X}^{-1}\mathcal{W}\mathcal{M}\mathcal{Z}\right]\pi =$$

$$= -\left[\frac{\lambda^T (I-\Delta)\,\Delta^{-1}+\mathcal{B}^T\mathcal{D}}{\gamma+\varphi}\right][I-\rho\nu\mathcal{X}]\,\pi \tag{104}$$

Since the eigenvalues/eigenvectors of $\mathcal{A}$ satisfy

$$\left[\frac{\lambda^T (I-\Delta)\,\Delta^{-1}+\mathcal{B}^T\mathcal{D}}{\gamma+\varphi}\right][I-\rho\nu\mathcal{X}]\,\pi = 0$$

(104) implies that they also satisfy (103). In addition it is immediate to see from (104) that (103) is satisfied for $\nu = \frac{\zeta}{\gamma}$ and $\pi = \left(I - \frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)^{-1}\Delta\mathcal{B}$. □

Appendix E: Empirics

Appendix E1: Data

Sector-level PPI series

This section describes the construction of SW. The main data source for sector-level price series is PPI data from the BLS. In this dataset the sample period varies across sectors: most manufacturing series are available by the mid-1980s, while most serivice series are available by 2006. To make up for the incomplete series we run a Lasso regression of each incomplete series on disaggregated (338 sectors) PCE components for the period in which both are available. We then use the Lasso approximation to extend the series as far back as 1984. Summary statistics for the Lasso regressions are reported in Table 7. There are 172 incomplete series in total, and those with the highest weight in SW are reported in Table 8. There are also 67 missing price series with respect to the BEA classification. We make up for 40 of them by using the concordance between NAICS sectors and PCE series provided by the BEA. Information about the missing series with highest weight is reported in Table 9.

Mean	**Max**	**Min**
88	127	19

Table 7: Number of series in Lasso approximation

	Weight in SW	Initial date
Employment services	0.85	1994-09-01
Management consulting services	0.55	2006-09-01
Insurance agencies, brokerages, and related activities	0.47	2003-03-01
Architectural, engineering, and related services	0.45	1997-03-01
Automotive equipment rental and leasing	0.45	1992-03-01
Custom computer programming services	0.41	2006-09-01
Specialized design services	0.37	19970301
Nursing and community care facilities	0.36	2004-03-01
Services to buildings and dwellings	0.36	1995-03-01
Environmental and other technical consulting services	0.36	2006-09-01
Wireless telecommunications carriers (except satellite)	0.31	1993-09-01
Office administrative services	0.27	1994-09-01
Satellite, telecommunications resellers, and all other telecommunications	0.23	1993-09-01
Other computer related services, including facilities management	0.22	2006-09-01
Internet publishing and broadcasting and Web search portals	0.21	2010-03-01

Table 8: Weights of top-15 incomplete series in SW (in %)

	Weight in SW	Added?
Oilseed farming	4.00	0
Funds, trusts, and other financial vehicles	2.02	1
Management of companies and enterprises	0.29	0
Sound recording industries	0.22	1
Elementary and secondary schools	0.20	1
Monetary authorities and depository credit intermediation	0.18	1
State and local government hospitals and health services	0.13	0
State and local government passenger transit	0.12	0
Other educational services	0.12	1
Motion picture and video industries	0.12	1
Transit and ground passenger transportation	0.10	1
Limited-service restaurants	0.10	1
Federal general government (nondefense)	0.09	0
Full-service restaurants	0.08	1
Promoters of performing arts and sports and agents for public figures	0.07	1

Table 9: Weights of top-15 missing series in SW (in %)

Comparison between PCE and SW

Figure 15 compares the weights assigned to different sectors by SW and PCE, at an aggregated 21-sector level.

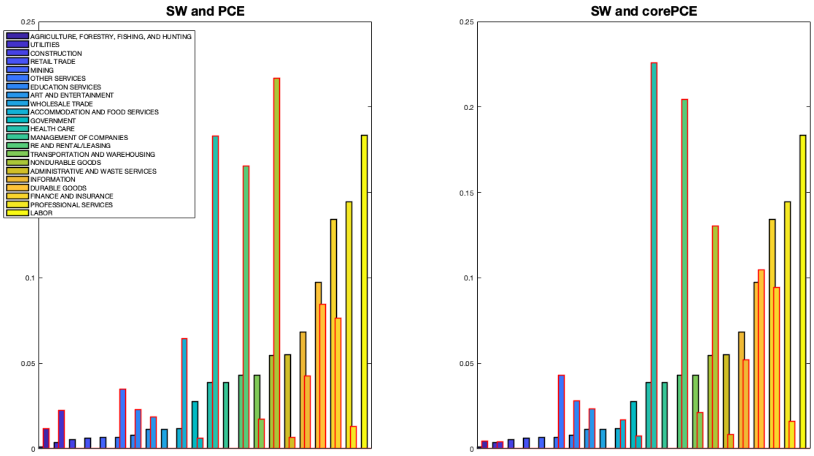


Figure 15: SW and PCE weights (The bars are ordered so that sectoral weights in SW are increasing. Those with red borders correspond to the PCE)

Table 10 reports the weights of the top-15 sectors in SW in percentage of the total (at disaggregated 405 sector level).

Industry name	Weight (SW)	Weight (Domar)	Weight (PCE)
Labor	18.3221	27.8648	0
Insurance agencies, brokerages, and related activities	9.23917	1.39786	0
Management of companies and enterprises	3.887	1.68309	0
Architectural, engineering, and related services	2.51957	0.812411	0
Insurance carriers, except direct life	2.13001	1.04094	2.5369
Warehousing and storage	2.12367	0.344483	0.0019132
Accounting, tax preparation, bookkeeping, and payroll services	2.05855	0.53267	0.17815
Other real estate	2.05001	2.87134	0.057851
Legal services	1.87954	0.893466	1.0623
Advertising, public relations, and related services	1.68975	0.415808	0.017779
Hospitals	1.65114	1.17451	9.6864
Employment services	1.63912	0.913483	0.012342
Management consulting services	1.63082	0.569068	0
Wired telecommunications carriers	1.44281	0.78146	2.0335
All other miscellaneous professional, scientific, and technical services	1.31821	0.312412	0

Table 10: Weights of top-15 series in SW (in %)

Proxy for inflation expectations

Figure 16 plots the actual inflation series and the inflation expectation series constructed with our IMA(1,1) model for all measures used in the regressions.

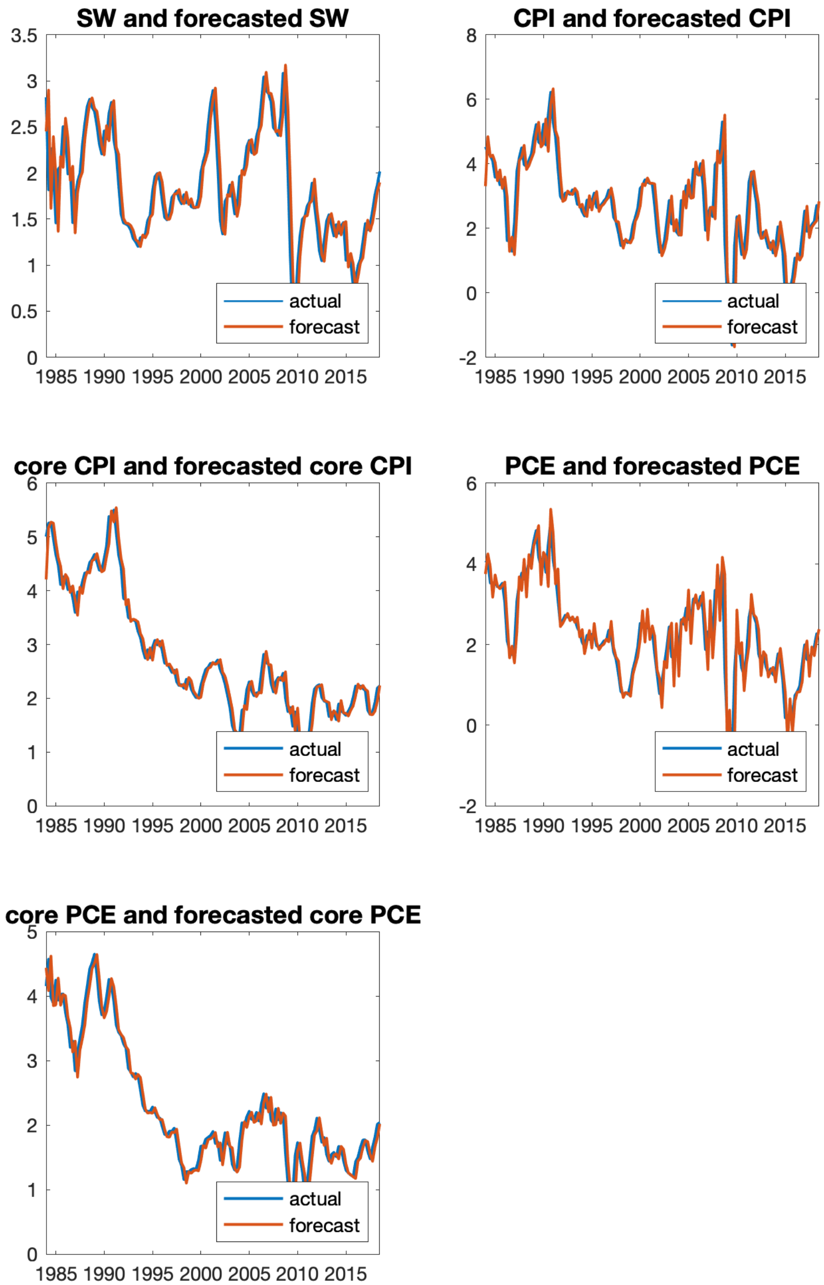


Figure 16: Actual and forecasted inflation

Summary statistics

Scatterplots in levels

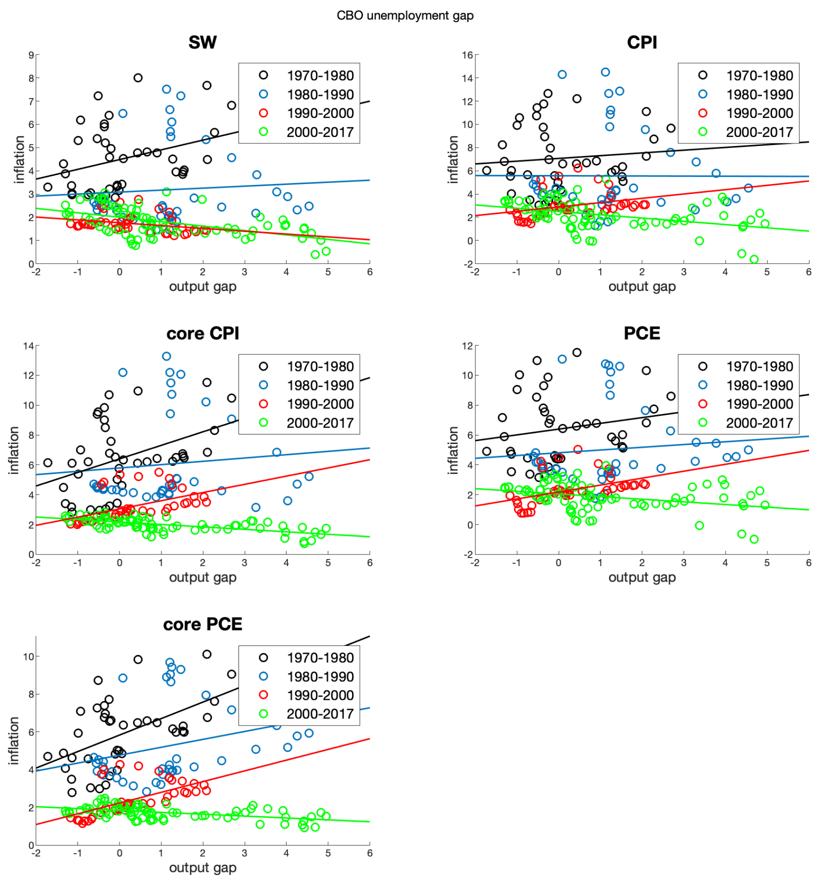


Figure 17: Inflation and unemployment gap

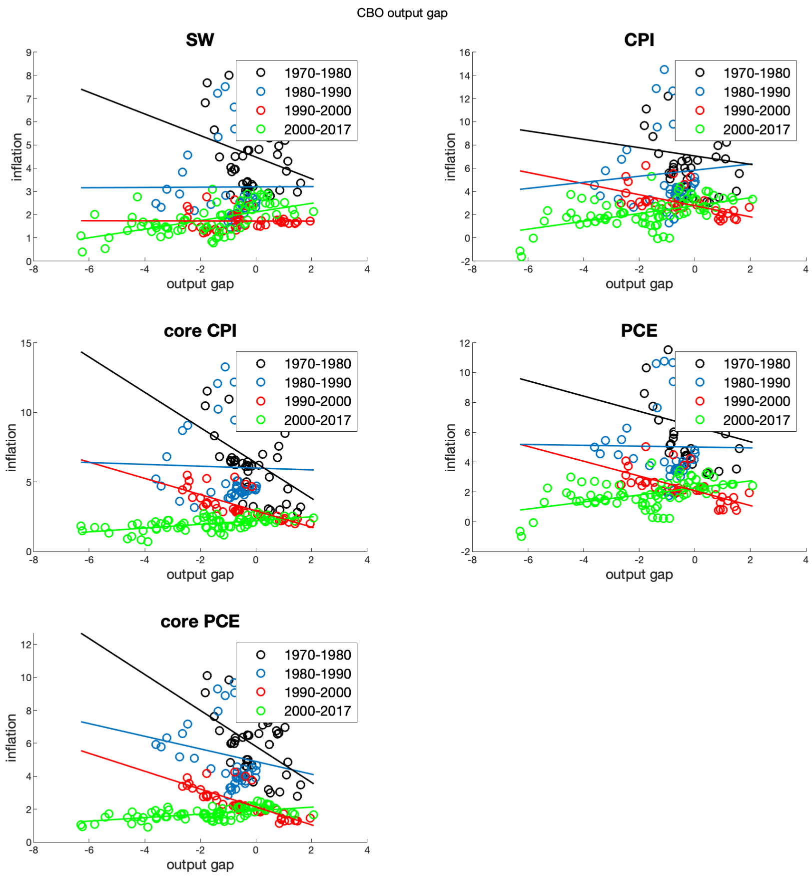


Figure 18: Inflation and output gap

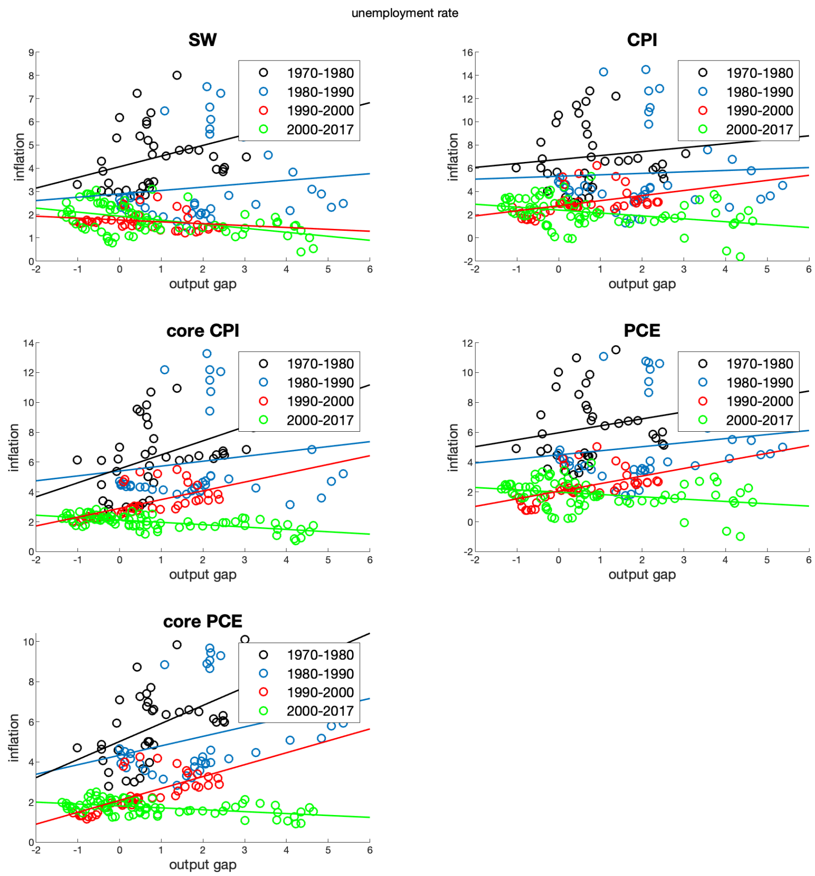


Figure 19: Inflation and unemployment rate

Scatterplots in changes (inflation change vs gap level)

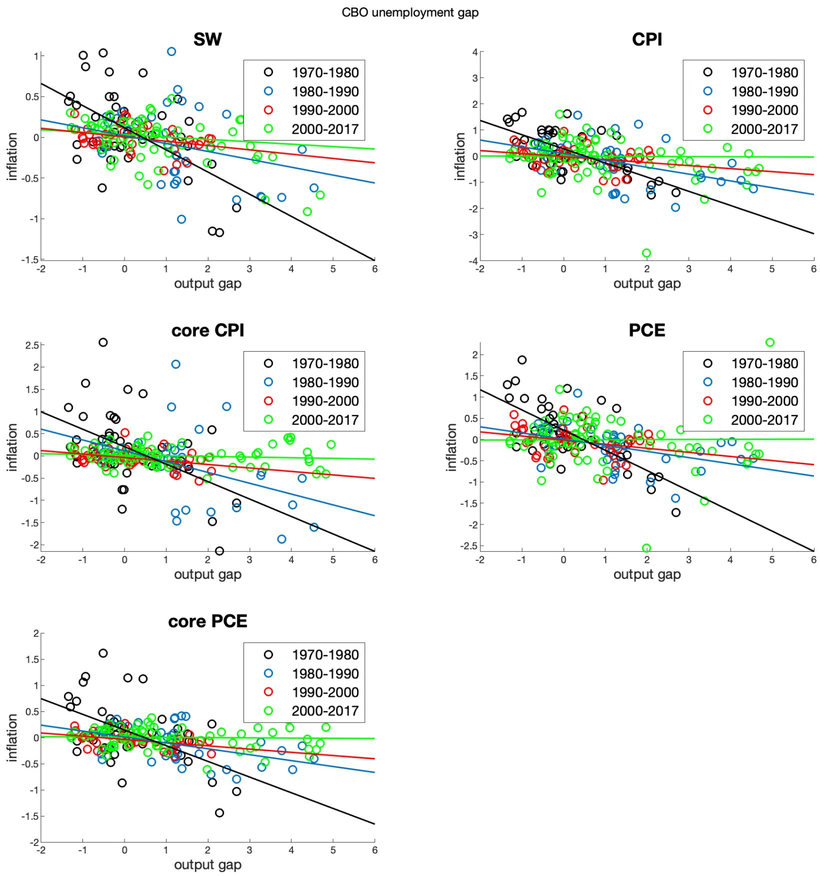


Figure 20: Inflation changes and unemployment gap

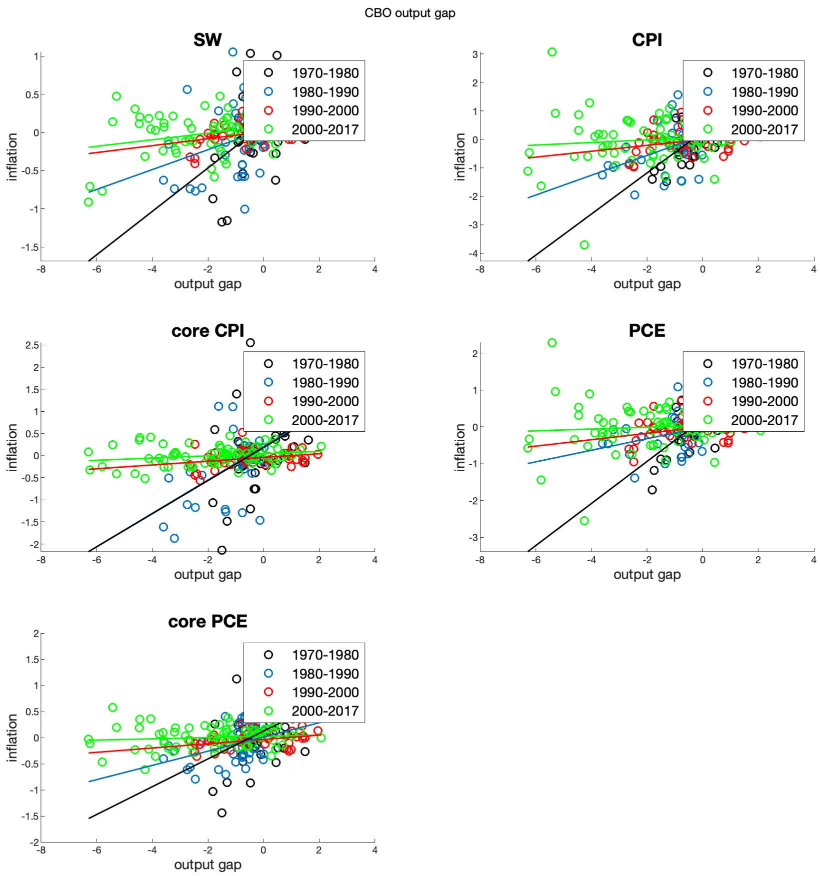


Figure 21: Inflation changes and output gap

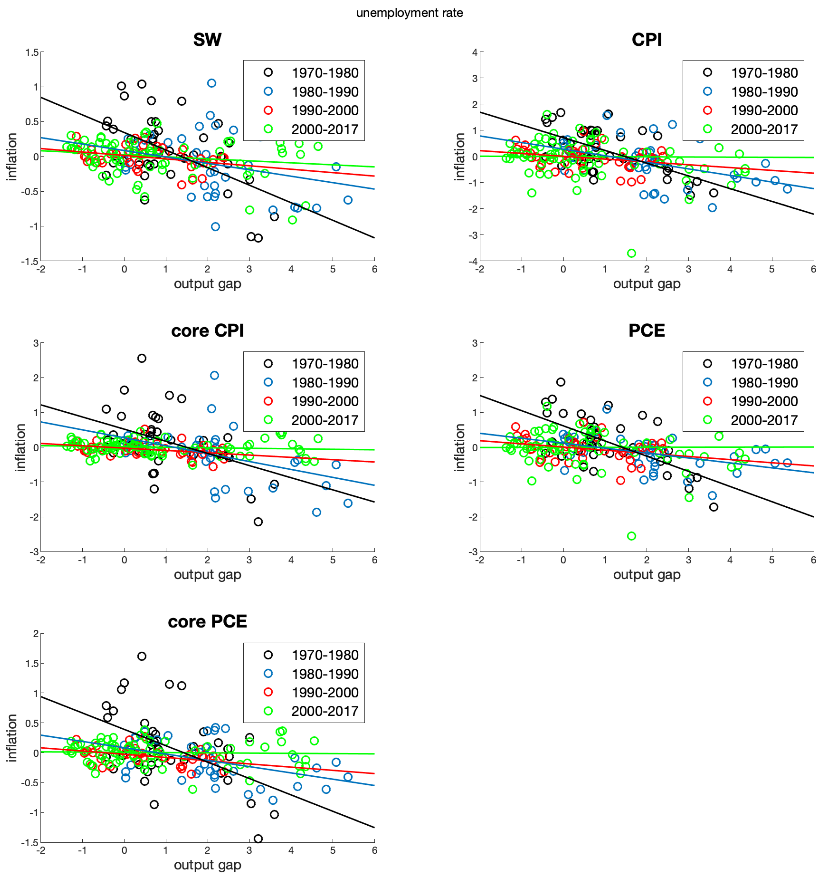


Figure 22: Inflation changes and unemployment rate

Appendix E2: Regressions over the full sample

Baseline specification:

$$\pi_t = c + \kappa \tilde{y}_t + u_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-3.8814**	-0.2832**	-0.1839**	-0.1667**	-0.1007*
	(0.6329)	(0.0729)	(0.0642)	(0.0628)	(0.0565)
intercept	1.9842**	2.9052**	2.9021**	2.3978**	2.372**
	(0.0475)	(0.1196)	(0.1052)	(0.103)	(0.0926)
R-squared	0.2154	0.0991	0.0566	0.0489	0.0227

Table 11: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	3.144**	0.2791**	0.1728**	0.1837**	0.1162**
	(0.5538)	(0.0618)	(0.055)	(0.0532)	(0.0482)
intercept	2.0189**	3.0193**	2.9661**	2.4878**	2.4325**
	(0.0522)	(0.1271)	(0.1131)	(0.1095)	(0.0992)
R-squared	0.1905	0.1297	0.0673	0.08	0.0407

Table 12: CBO output gap

	SW	CPI	core CPI	PCE	core PCE
gap	-3.084**	-0.1405*	0.0028	-0.036	0.0545
	(0.6645)	(0.0759)	(0.0661)	(0.0644)	(0.057)
intercept	1.9621**	2.8021**	2.7595**	2.2996**	2.2514**
	(0.0505)	(0.1259)	(0.1096)	(0.1067)	(0.0945)
R-squared	0.1359	0.0244	0	0.0023	0.0066

Table 13: unemployment rate

Residual plots:

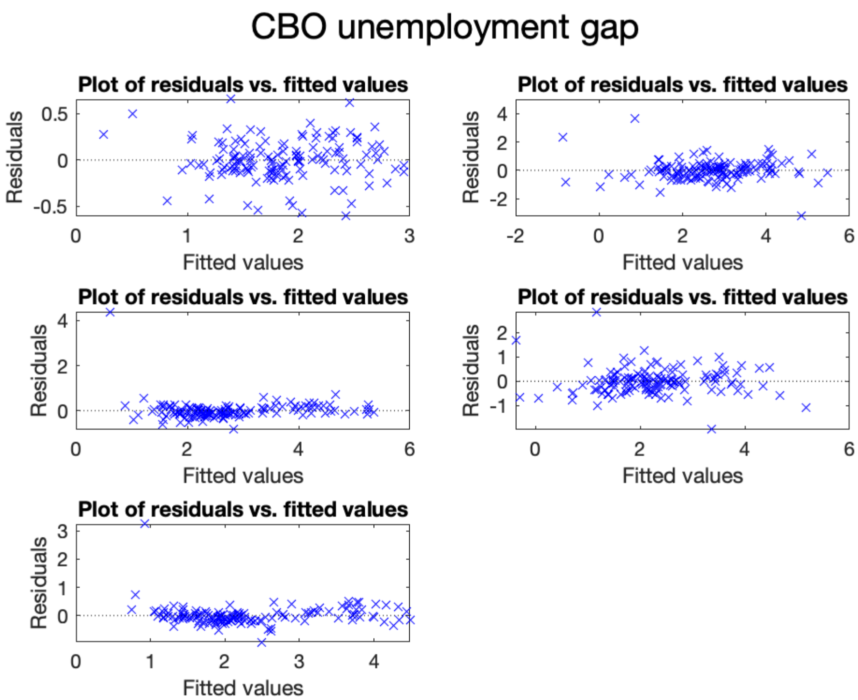


Figure 23:

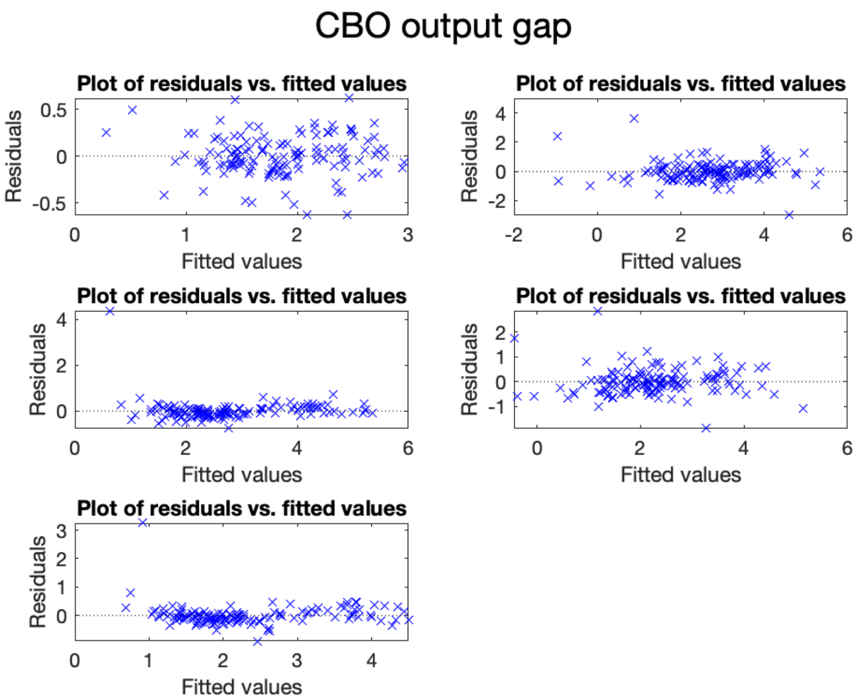
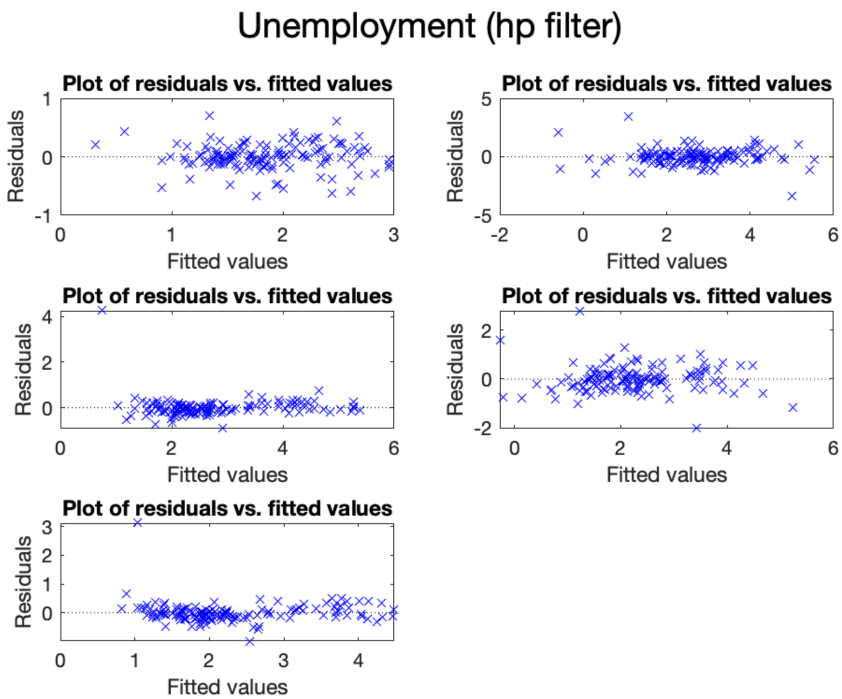

CBO output gap
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values

Figure 24:



Unemployment (hp filter)
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values
Plot of residuals vs. fitted values
Residuals
Fitted values

Figure 25:

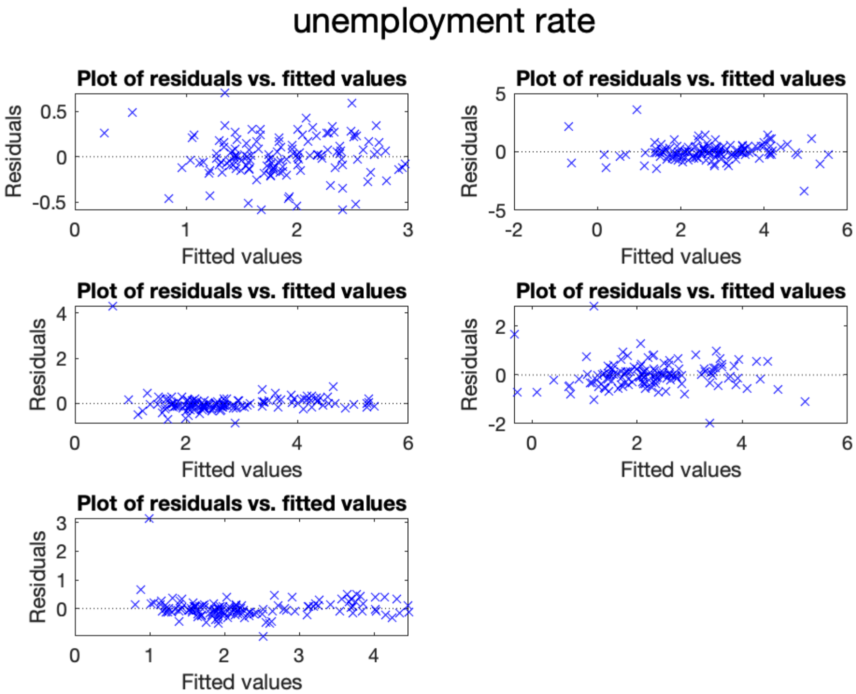


Figure 26:

With oil prices:

$$\pi_t = c + \kappa \tilde{y}_t + \pi_{oil} + u_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-3.6385**	-0.2198**	-0.2038**	-0.1194**	-0.1066*
	(0.6294)	(0.0655)	(0.0643)	(0.0584)	(0.0573)
intercept	1.9532**	2.7286**	2.9576**	2.266**	2.3883**
	(0.0483)	(0.1099)	(0.1078)	(0.0979)	(0.0961)
oil prices	0.0032**	0.0185**	-0.0058*	0.0138**	-0.0017
	(0.0013)	(0.003)	(0.0029)	(0.0027)	(0.0026)
R-squared	0.2488	0.2959	0.0829	0.2049	0.0257

Table 14: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	2.8985**	0.2137**	0.1961**	0.1351**	0.1243**
	(0.5562)	(0.0562)	(0.0553)	(0.0501)	(0.0492)
intercept	1.9843**	2.8179**	3.038**	2.3383**	2.4576**
	(0.0536)	(0.1184)	(0.1164)	(0.1055)	(0.1036)
oil prices	0.0031**	0.0179**	-0.0064**	0.0133**	-0.0022
	(0.0014)	(0.003)	(0.0029)	(0.0027)	(0.0026)
R-squared	0.2199	0.3108	0.0985	0.2221	0.0458

Table 15: CBO output gap

	SW	CPI	core CPI	PCE	core PCE
gap	-2.7813**	-0.0591	-0.0158	0.0259	0.0524
	(0.6655)	(0.0684)	(0.067)	(0.0596)	(0.0582)
intercept	1.9278**	2.601**	2.8056**	2.1465**	2.2564**
	(0.0516)	(0.116)	(0.1136)	(0.101)	(0.0987)
oil prices	0.0033**	0.0196**	-0.0045	0.0149**	-0.0005
	(0.0014)	(0.0031)	(0.0031)	(0.0027)	(0.0027)
R-squared	0.1707	0.2418	0.0155	0.1816	0.0069

Table 16: unemployment rate

With lags:

$$\pi_t = c + \kappa \tilde{y}_t + \sum_{s=1}^{4} \gamma_s \pi_{t-s} + u_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-1.3803**	-0.0696	-0.0232	-0.0285	-0.0069
	(0.4361)	(0.0444)	(0.027)	(0.0339)	(0.0235)
intercept	0.6573**	0.7961**	0.367**	0.515**	0.2801**
	(0.1085)	(0.1717)	(0.1109)	(0.128)	(0.0928)
lag 1	0.7443**	0.9835**	0.9983**	1.0655**	1.068**
	(0.0858)	(0.0869)	(0.0867)	(0.0867)	(0.0866)
lag 2	0.1224	-0.2051*	-0.0391	-0.3083**	-0.2537**
	(0.1064)	(0.1218)	(0.1225)	(0.1265)	(0.1264)
lag 3	-0.1573	-0.0051	-0.068	0.0912	0.1143
	(0.1065)	(0.1219)	(0.1225)	(0.1265)	(0.1264)
lag 4	-0.0356	-0.0469	-0.0138	-0.0624	-0.0438
	(0.0799)	(0.0843)	(0.0848)	(0.0848)	(0.0849)
R-squared	0.6954	0.705	0.8465	0.7436	0.8396

Table 17: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	1.3534**	0.0931**	0.0359	0.0524*	0.0212
	(0.3627)	(0.0378)	(0.0231)	(0.0293)	(0.0203)
intercept	0.6586**	0.8709**	0.4035**	0.5697**	0.3093**
	(0.1043)	(0.1717)	(0.1122)	(0.1294)	(0.0947)
lag 1	0.7359**	0.9662**	0.9875**	1.0502**	1.0595**
	(0.0845)	(0.0861)	(0.0866)	(0.0864)	(0.0867)
lag 2	0.1261	-0.2035*	-0.0362	-0.3064**	-0.2521**
	(0.105)	(0.1202)	(0.1217)	(0.1253)	(0.1259)
lag 3	-0.1512	-0.006	-0.066	0.0883	0.1131
	(0.1051)	(0.1203)	(0.1217)	(0.1253)	(0.1259)
lag 4	-0.0237	-0.0367	-0.012	-0.0522	-0.0397
	(0.0789)	(0.0833)	(0.0842)	(0.0843)	(0.0847)
R-squared	0.7035	0.7126	0.8484	0.7484	0.8408

Table 18: CBO output gap

	SW	CPI	core CPI	PCE	core PCE
gap	-1.0051**	-0.0248	0.0067	0.0035	0.0172
	(0.4293)	(0.0433)	(0.0263)	(0.0334)	(0.0232)
intercept	0.6012**	0.7115**	0.3251**	0.4737**	0.2612**
	(0.1071)	(0.1648)	(0.105)	(0.1232)	(0.0887)
lag 1	0.773**	1.002**	1.0058**	1.0747**	1.068**
	(0.0861)	(0.0867)	(0.0865)	(0.0864)	(0.0863)
lag 2	0.1231	-0.2067*	-0.04	-0.3094**	-0.2536**
	(0.1081)	(0.1228)	(0.1228)	(0.1268)	(0.1262)
lag 3	-0.1607	-0.0036	-0.0688	0.0937	0.1147
	(0.1082)	(0.1228)	(0.1228)	(0.1268)	(0.1262)
lag 4	-0.0373	-0.0461	-0.0127	-0.0655	-0.0445
	(0.0812)	(0.085)	(0.085)	(0.0851)	(0.0848)
R-squared	0.6854	0.7003	0.8457	0.7423	0.8402

Table 19: unemployment rate

With inflation expectations:

$$\pi_t = c + \kappa \tilde{y}_t + \rho \mathbb{E}\pi_{t+1} + \epsilon_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-1.1054**	-0.1613**	-0.0344	-0.062	0.0047
	(0.3275)	(0.0809)	(0.052)	(0.0487)	(0.0368)
inflation expecations	0.8287**	0.4846**	0.5446**	0.6364**	0.6406**
	(0.0383)	(0.1557)	(0.0559)	(0.0621)	(0.045)
intercept	0.3484**	1.3851**	1.3193**	0.5522**	0.8388**
	(0.0789)	(0.5021)	(0.1818)	(0.196)	(0.1228)
R-squared	0.8234	0.159	0.4425	0.4635	0.6072

Table 20: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	1.0861**	0.1881**	0.0412	0.0881**	0.0084
	(0.2714)	(0.0678)	(0.0449)	(0.0417)	(0.032)
inflation expecations	0.8297**	0.4412**	0.5398**	0.6231**	0.6365**
	(0.0368)	(0.1515)	(0.0561)	(0.0617)	(0.0455)
intercept	0.3668**	1.6124**	1.3548**	0.6459**	0.8614**
	(0.0772)	(0.4987)	(0.1892)	(0.2005)	(0.1291)
R-squared	0.8288	0.1808	0.4442	0.4744	0.6073

Table 21: CBO output gap

	SW	CPI	core CPI	PCE	core PCE
gap	-0.9404**	-0.0049	0.0781	-0.0505	0.0757**
	(0.3185)	(0.0788)	(0.0499)	(0.0477)	(0.0355)
inflation expecations	0.8468**	0.6312**	0.5668**	0.6549**	0.6432**
	(0.0372)	(0.1518)	(0.0537)	(0.0608)	(0.0434)
intercept	0.3108**	0.8344*	1.1705**	0.4941**	0.7757**
	(0.0762)	(0.4879)	(0.1711)	(0.1851)	(0.1155)
R-squared	0.8202	0.1344	0.4507	0.4616	0.6198

Table 22: unemployment rate

With lags and inflation expectations:

$$\pi_t = c + \kappa\tilde{y}_t + \rho\mathbb{E}\pi_{t+1} + \sum_{s=1}^{4}\gamma_s\pi_{t-s} + \epsilon_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-1.1389**	-0.0488	-0.0081	-0.016	0.0073
	(0.3111)	(0.049)	(0.0274)	(0.0327)	(0.022)
inflation expecations	1.0886**	0.0987	0.1086**	0.215**	0.1927**
	(0.0952)	(0.0984)	(0.0378)	(0.0591)	(0.04)
intercept	0.3695**	0.5385*	0.2741**	0.2951**	0.1857**
	(0.0812)	(0.3089)	(0.1166)	(0.1355)	(0.0876)
lag1	-0.3659**	0.9746**	0.8858**	0.9401**	0.8238**
	(0.1147)	(0.0873)	(0.0936)	(0.09)	(0.0948)
lag2	0.2617**	-0.2079*	-0.0063	-0.2927**	-0.1762
	(0.0767)	(0.1219)	(0.1199)	(0.1212)	(0.1181)
lag3	-0.2055**	-0.0078	-0.0721	0.0827	0.0875
	(0.0759)	(0.1219)	(0.1194)	(0.1212)	(0.1171)
lag4	0.0353	-0.0509	-0.0068	-0.1163	-0.0043
	(0.0572)	(0.0844)	(0.0826)	(0.0825)	(0.079)
R-squared	0.8469	0.7072	0.8506	0.7688	0.8645

Table 23: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	1.0634**	0.0822*	0.0226	0.0383	0.0076
	(0.2599)	(0.0416)	(0.0236)	(0.0284)	(0.0191)
inflation expecations	1.0744**	0.0618	0.1033**	0.2066**	0.1884**
	(0.0944)	(0.0967)	(0.0378)	(0.0591)	(0.0401)
intercept	0.3678**	0.7088**	0.3114**	0.3494**	0.2103**
	(0.0787)	(0.3064)	(0.1187)	(0.1382)	(0.09)
lag1	-0.3551**	0.961**	0.8831**	0.9324**	0.824**
	(0.1132)	(0.0866)	(0.0933)	(0.0896)	(0.0948)
lag2	0.2629**	-0.2053*	-0.0057	-0.2917**	-0.1771
	(0.0758)	(0.1205)	(0.1195)	(0.1205)	(0.118)
lag3	-0.2004**	-0.0078	-0.0709	0.0806	0.0872
	(0.0751)	(0.1206)	(0.119)	(0.1205)	(0.1171)
lag4	0.0436	-0.0404	-0.0068	-0.1063	-0.0034
	(0.0566)	(0.0837)	(0.0823)	(0.0824)	(0.0791)
R-squared	0.8503	0.7135	0.8515	0.7715	0.8646

Table 24: CBO output gap

	SW	CPI	core CPI	PCE	core PCE
gap	-1.0106**	0.0026	0.0214	-0.0048	0.0303
	(0.303)	(0.047)	(0.0265)	(0.032)	(0.0216)
inflation expecations	1.1128**	0.1424	0.1163**	0.2187**	0.198**
	(0.0958)	(0.0976)	(0.0376)	(0.059)	(0.0396)
intercept	0.3354**	0.3572	0.2385**	0.2744**	0.1753**
	(0.079)	(0.2931)	(0.1091)	(0.1283)	(0.083)
lag1	-0.3745**	0.9843**	0.8803**	0.9422**	0.8131**
	(0.1159)	(0.0871)	(0.0936)	(0.0901)	(0.0945)
lag2	0.2653**	-0.2102*	-0.0047	-0.293**	-0.1736
	(0.0773)	(0.1223)	(0.1196)	(0.1213)	(0.1173)
lag3	-0.2082**	-0.0076	-0.0724	0.0835	0.0867
	(0.0765)	(0.1224)	(0.1191)	(0.1213)	(0.1163)
lag4	0.0359	-0.0535	-0.0056	-0.118	-0.0042
	(0.0576)	(0.0848)	(0.0824)	(0.0826)	(0.0785)
R-squared	0.8445	0.705	0.8513	0.7684	0.8664

Table 25: unemployment rate

In changes (inflation change vs gap level):

$$\pi_t - \pi_{t-1} = c + \kappa \tilde{y}_t + u_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-0.6945**	-0.0287	-0.0212*	-0.0169	-0.0105
	(0.3258)	(0.0404)	(0.0119)	(0.0296)	(0.0123)
intercept	0.0182	0.008	-0.003	-0.0006	-0.0078
	(0.0244)	(0.0661)	(0.0195)	(0.0485)	(0.0201)
R-squared	0.0323	0.0037	0.0227	0.0024	0.0054

Table 26: CBO unemployment gap

	SW	CPI	core CPI	PCE	core PCE
gap	0.7805**	0.0472	0.02*	0.0299	0.0103
	(0.2768)	(0.0345)	(0.0102)	(0.0254)	(0.0106)
intercept	0.0365	0.0422	0.0046	0.022	-0.0035
	(0.0262)	(0.0713)	(0.0211)	(0.0524)	(0.0218)
R-squared	0.0552	0.0136	0.0273	0.0101	0.007

Table 27: CBO output gap

	SW	CPI	core CPI	PCE	core PCE
gap	-0.7464**	-0.0317	-0.0238**	-0.0199	-0.0133
	(0.3264)	(0.0405)	(0.0119)	(0.0297)	(0.0123)
intercept	0.0212	0.0113	-0.0002	0.0024	-0.0052
	(0.0247)	(0.0668)	(0.0197)	(0.049)	(0.0203)
R-squared	0.037	0.0045	0.0284	0.0033	0.0085

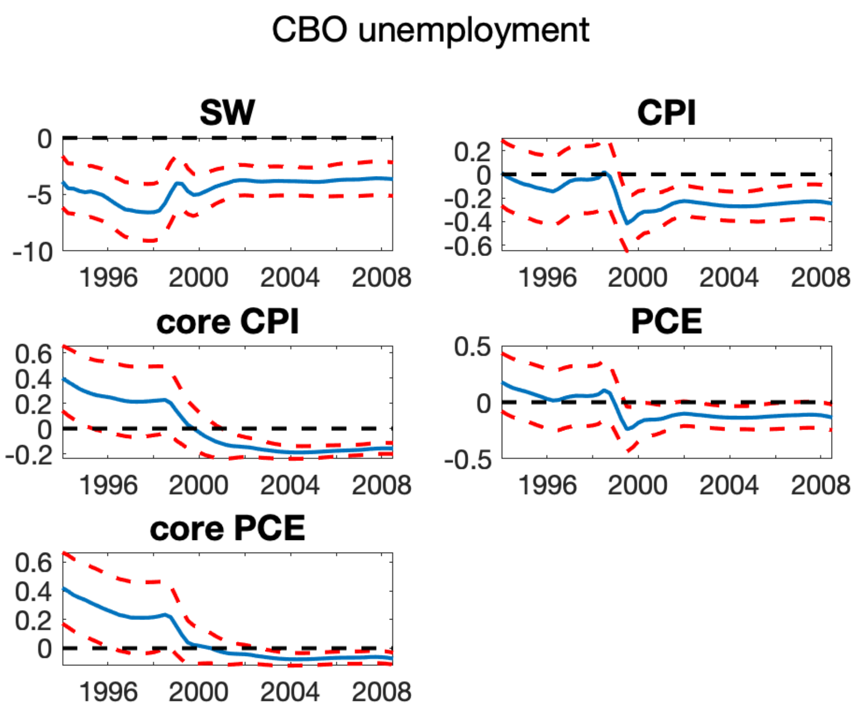
Table 28: unemployment rate

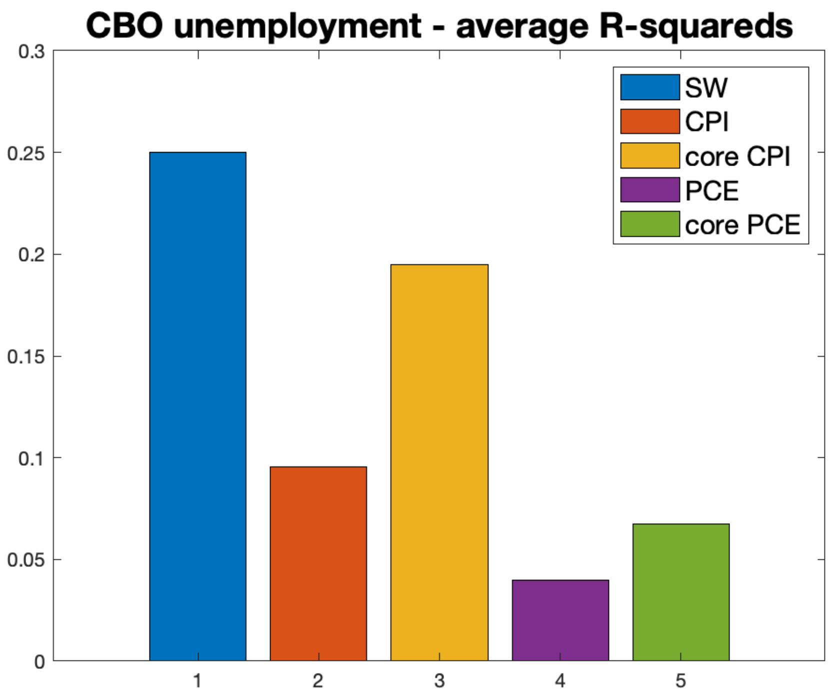
Appendix E3: Rolling regressions

The figures below plot estimated coefficients (with confidence intervals) and R-squareds for several different specifications.

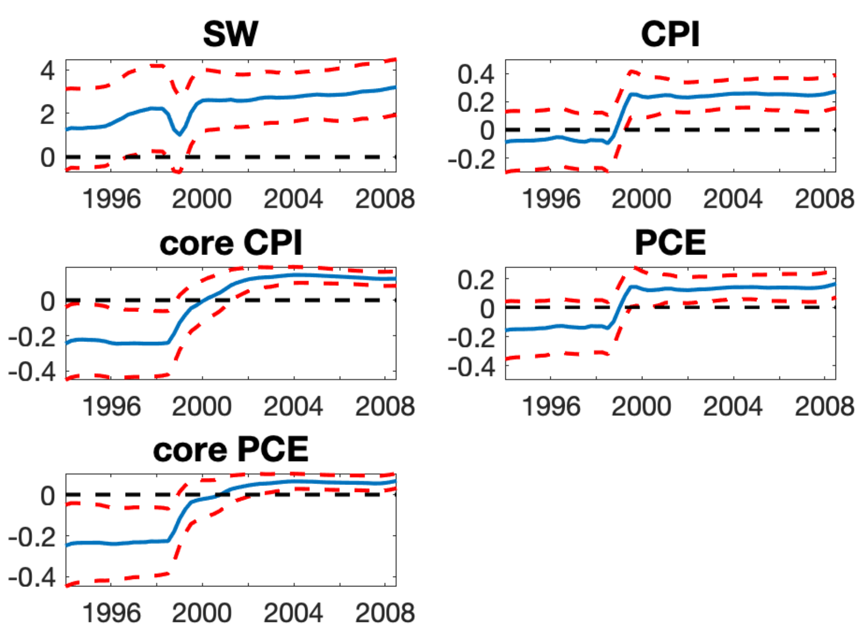
Baseline:

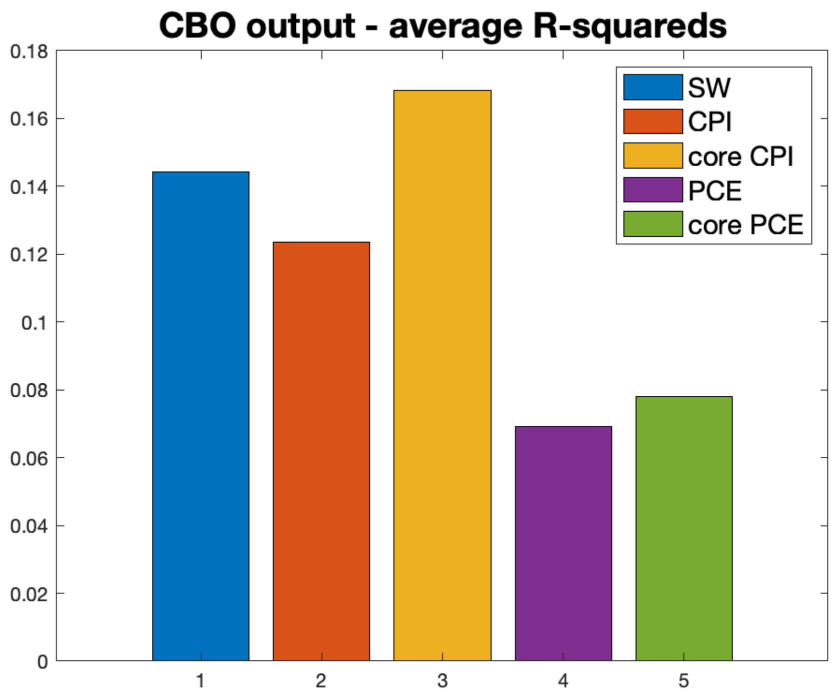
$$\pi_t = \kappa \tilde{y}_t + \epsilon_t$$

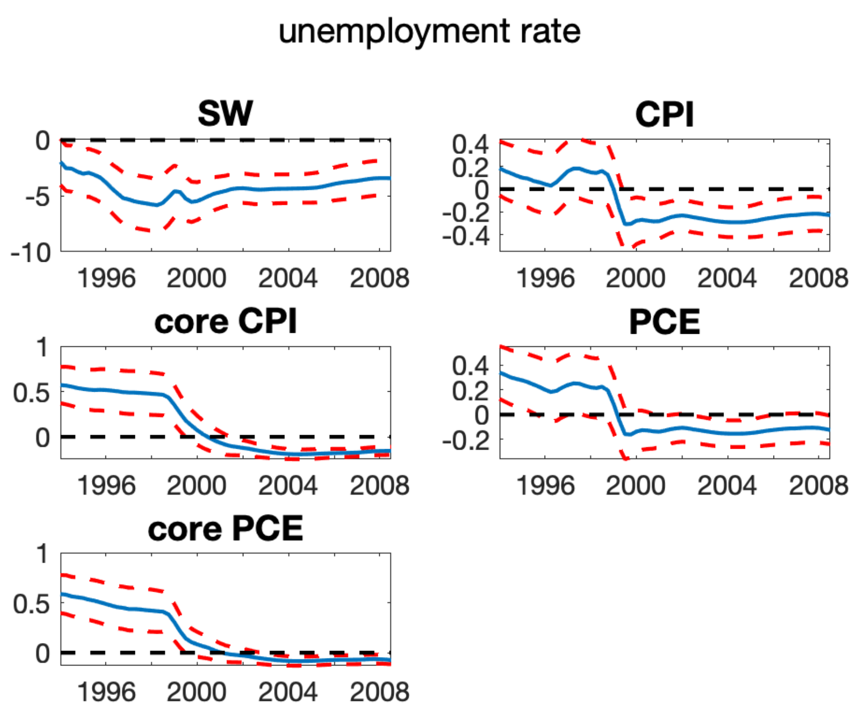


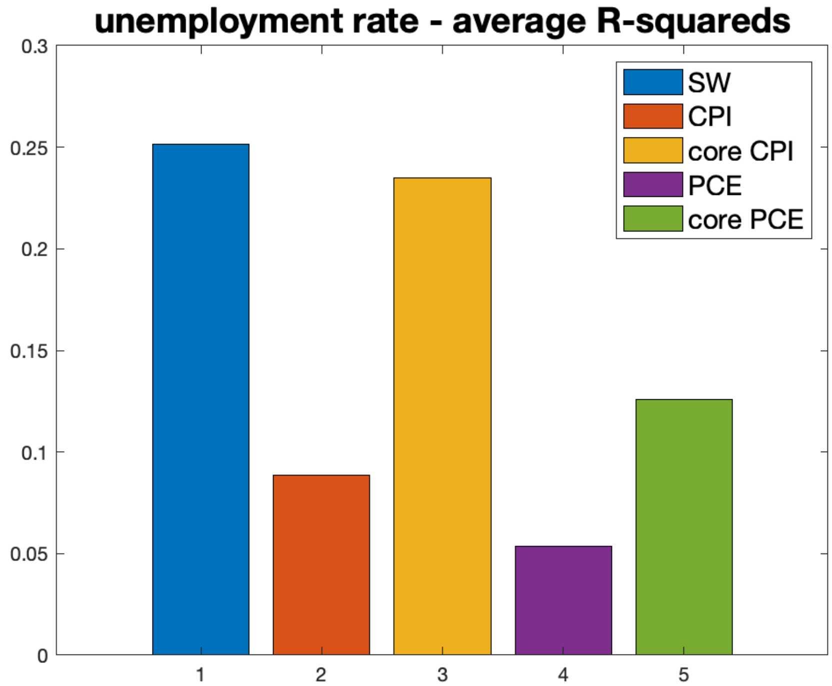

CBO unemployment - average R-squareds
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
SW
CPI
core CPI
PCE
core PCE

CBO output


SW
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008

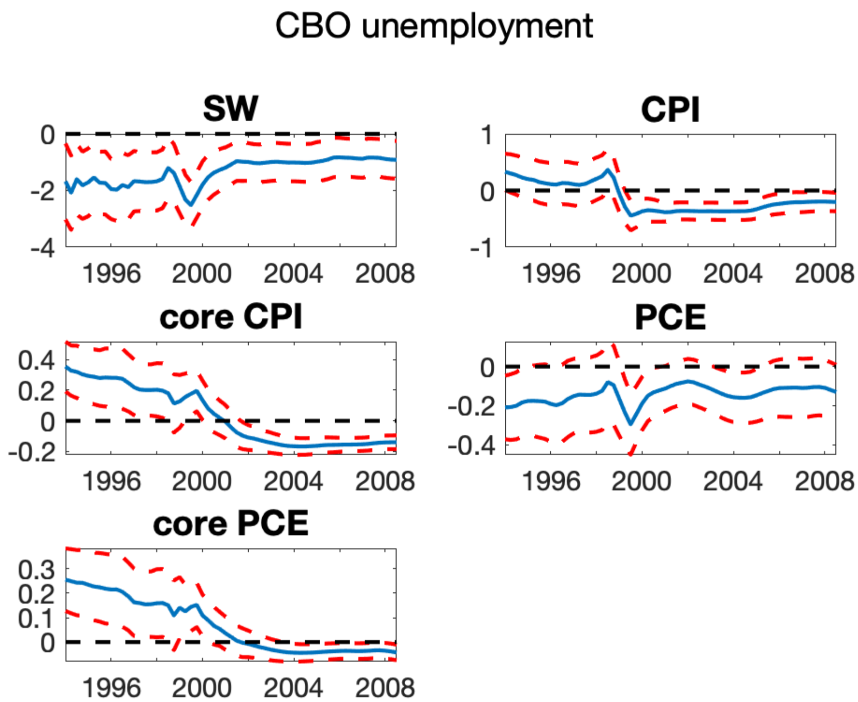

CBO output - average R-squareds
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
SW
CPI
core CPI
PCE
core PCE

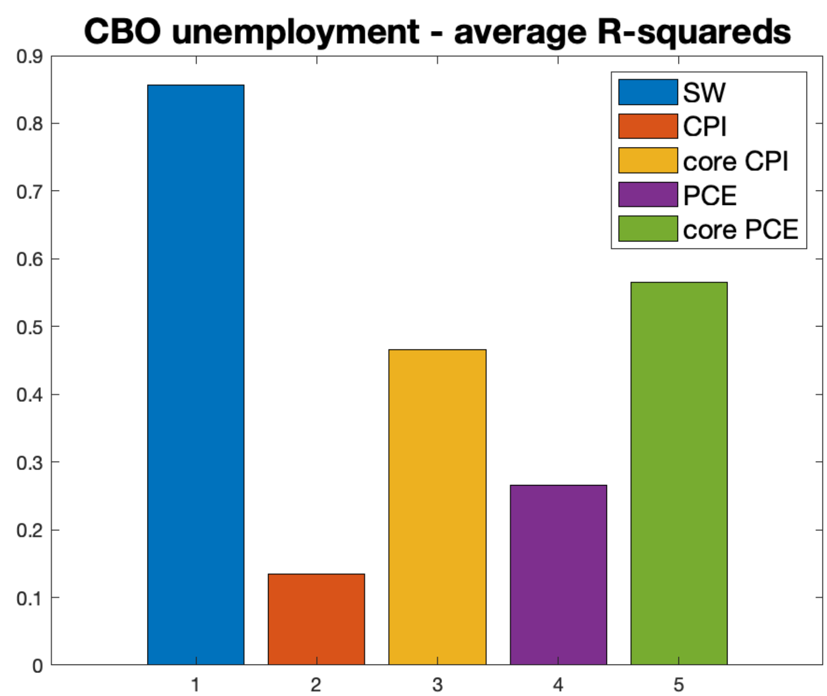

unemployment rate
SW
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008

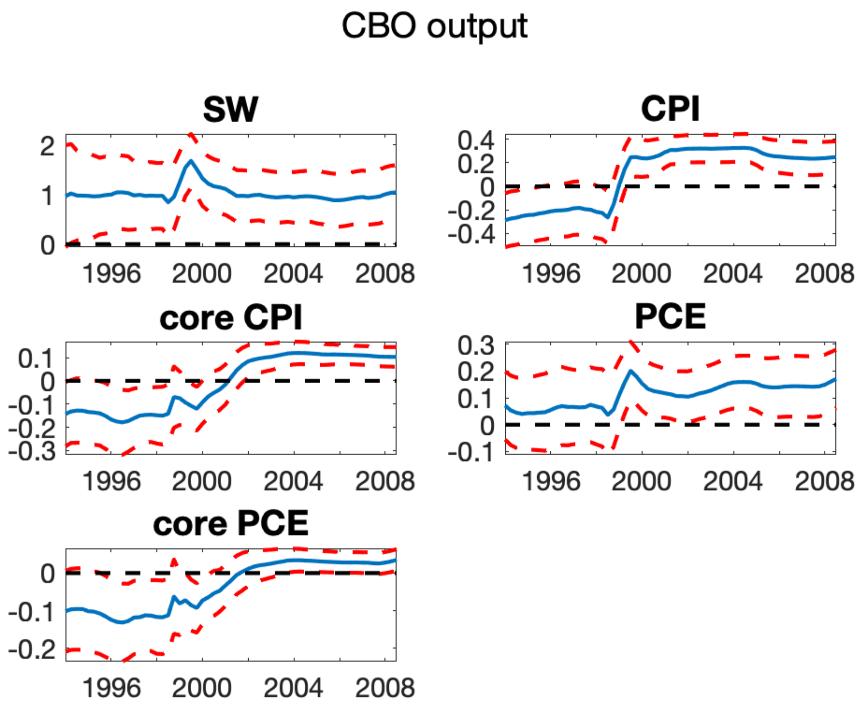


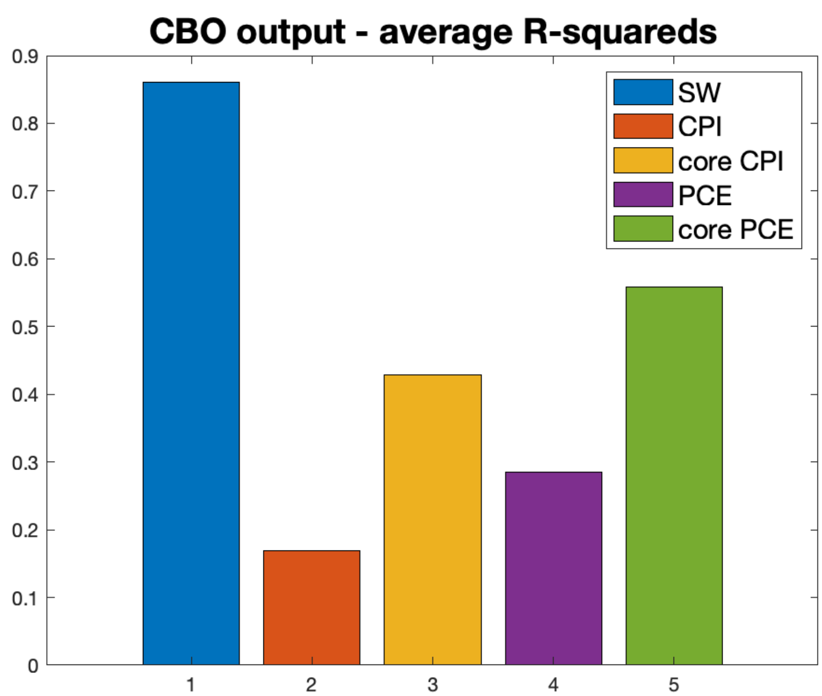
With expectations:

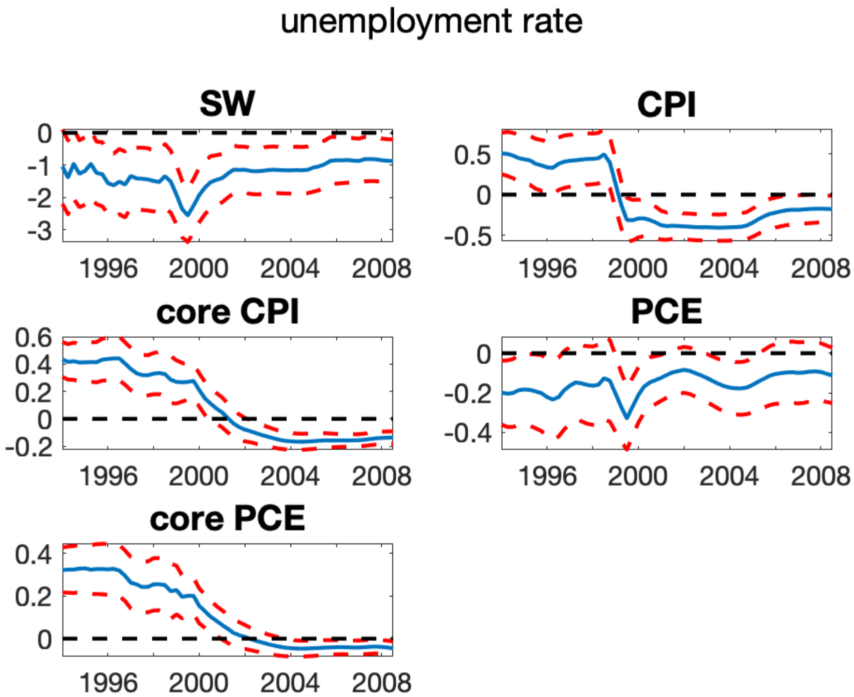
$$\pi_t = \kappa \tilde{y}_t + \rho \mathbb{E} \pi_{t+1} + \epsilon_t$$

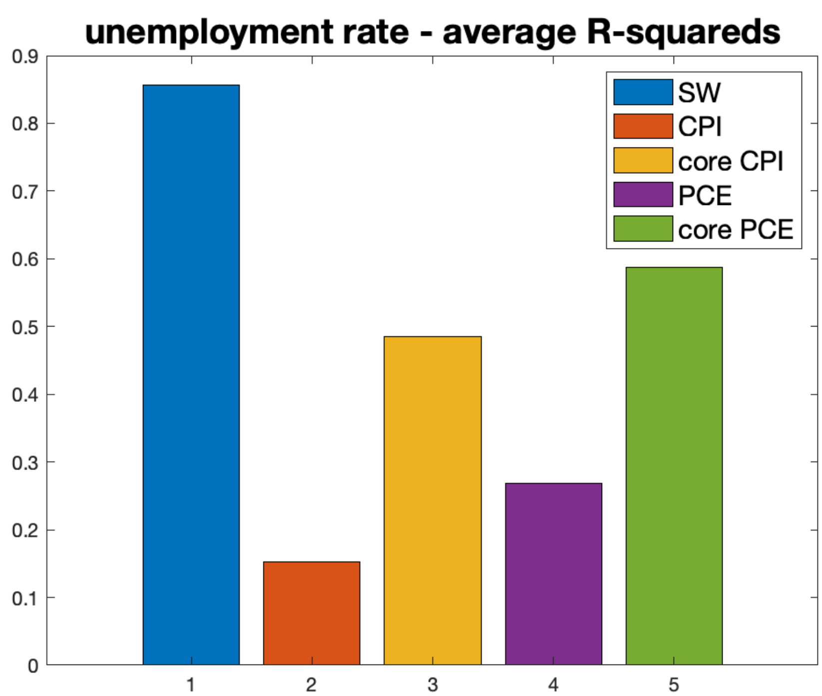

CBO unemployment
SW
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008


CBO unemployment - average R-squareds
SW
CPI
core CPI
PCE
core PCE


CBO output
SW
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008


CBO output - average R-squareds
SW
CPI
core CPI
PCE
core PCE
1
2
3
4
5

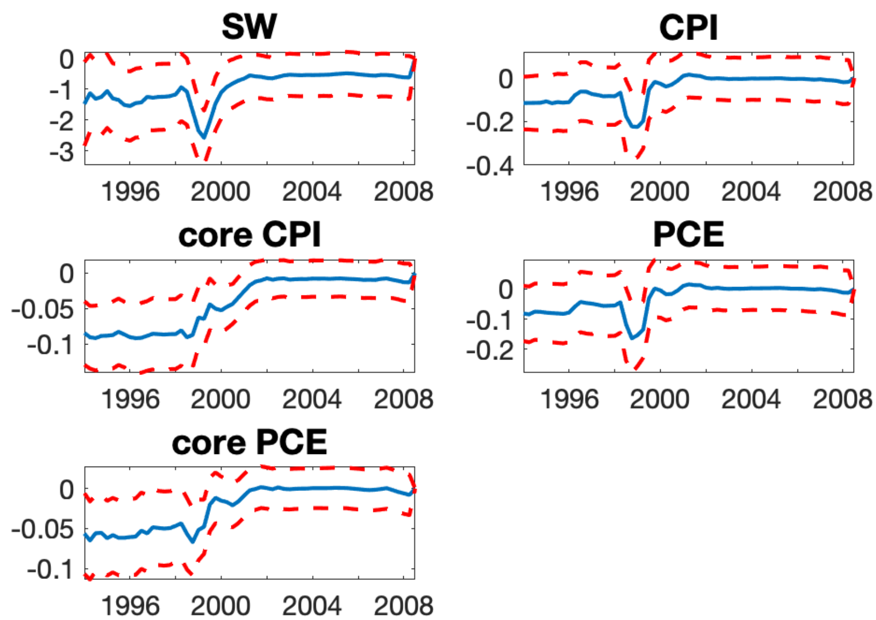


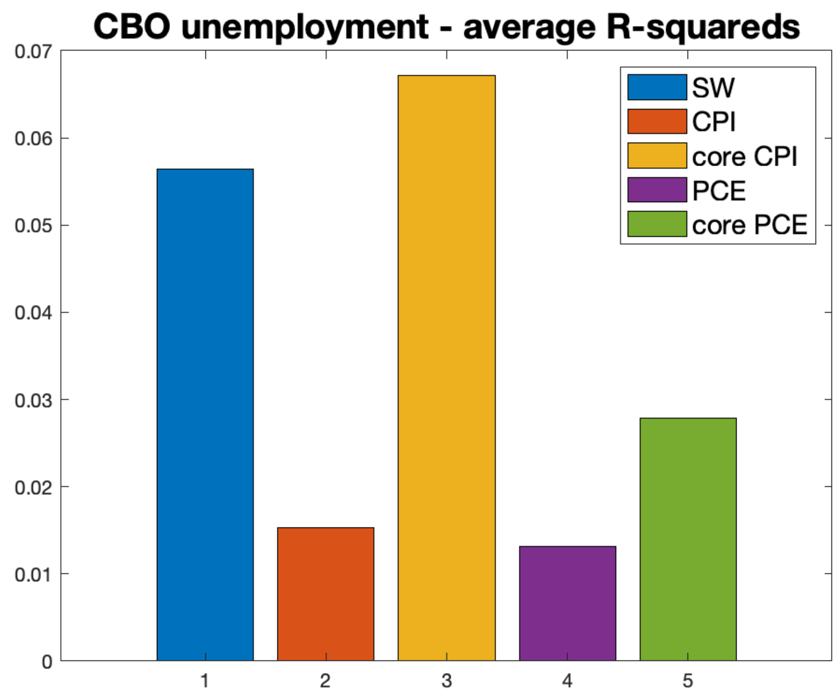


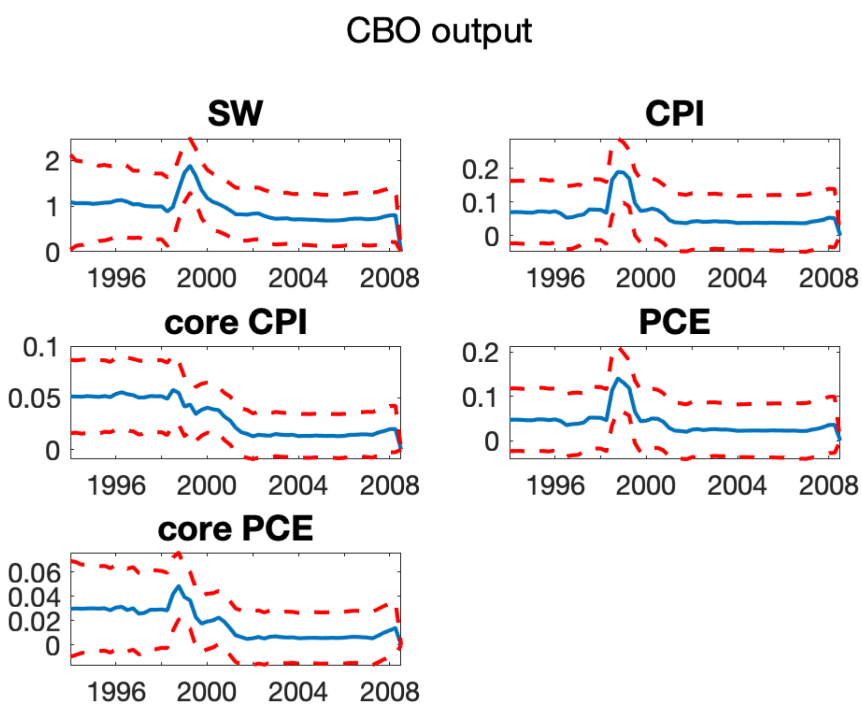
In changes (inflation change vs gap level):

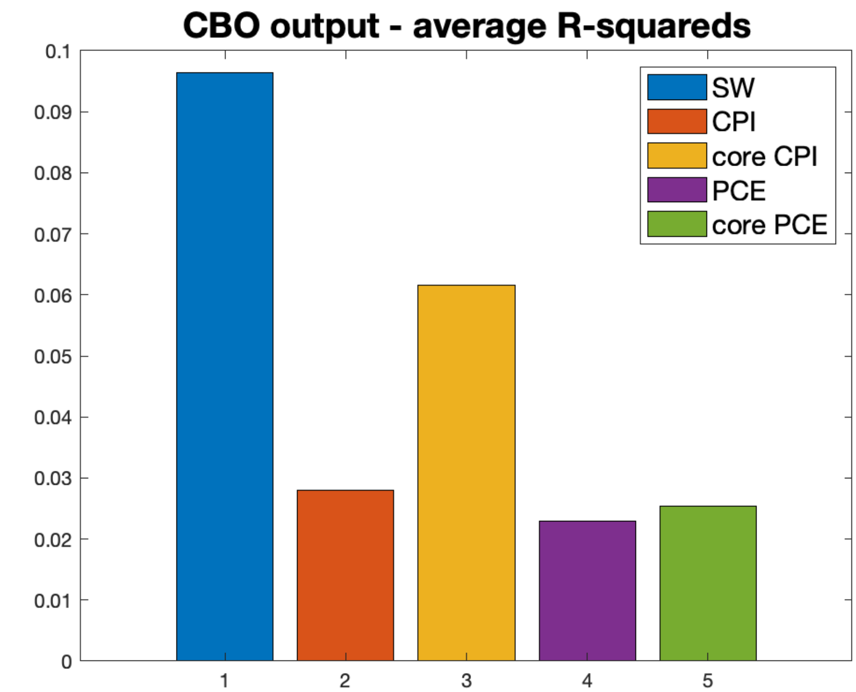
$$\pi_t - \pi_{t-1} = \kappa \tilde{y}_t + \epsilon_t$$

CBO unemployment

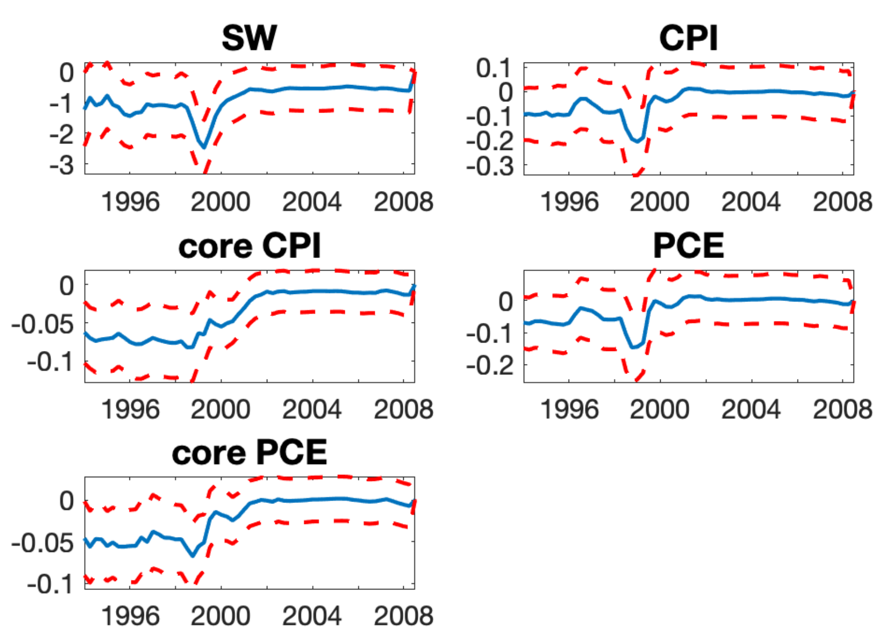


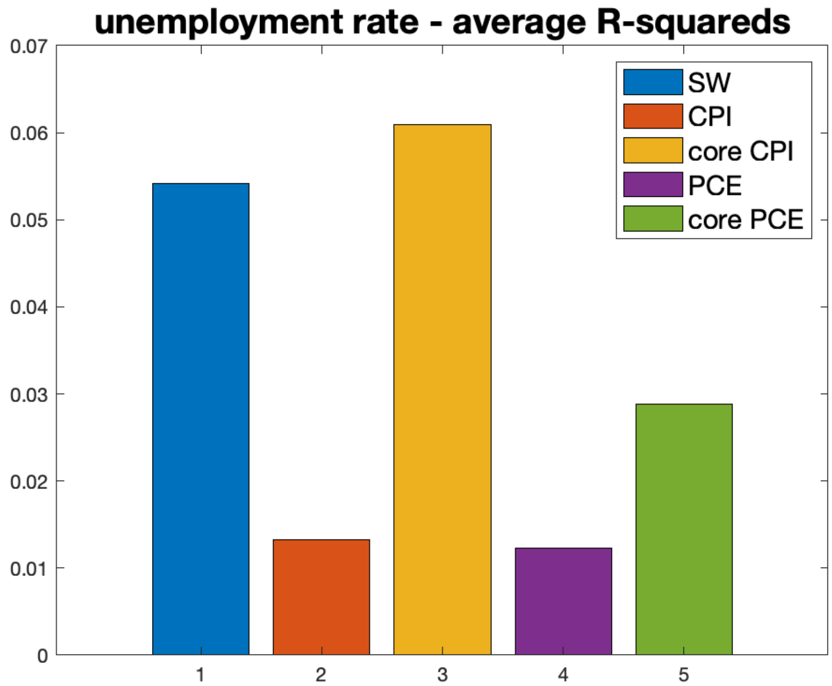

CBO unemployment - average R-squareds
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
SW
CPI
core CPI
PCE
core PCE


CBO output
SW
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008



unemployment rate




unemployment rate - average R-squareds
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
SW
CPI
core CPI
PCE
core PCE