Demand Estimation with Machine Learning and Model Combination

Patrick Bajari[1], Denis Nekipelov[2], Stephen P. Ryan[3], and Miaoyu Yang[4]

[1]University of Washington and NBER
[2]University of Virginia
[3]University of Texas at Austin and NBER
[4]University of Washington

October 15, 2015

Abstract

We survey and apply several techniques from the statistical and computer science literature to the problem of demand estimation. We derive novel asymptotic properties for several of these models. To improve out-of-sample prediction accuracy and obtain parametric rates of convergence, we propose a method of combining the underlying models via linear regression. We illustrate our method using a standard scanner panel data set to estimate promotional lift and find that our estimates are considerably more accurate in out-of-sample predictions of demand than some commonly-used alternatives. While demand estimation is our motivating application, these methods are widely applicable to other microeconometric problems.

1 Introduction

Over the past decade, there has been a high level of interest in modeling consumer behavior in the fields of computer science and statistics. These applications are motivated in part by the availability of large data sets where the demand for SKU's or individual consumers can be observed. These methods are commonly used in industry in retail, health care or on the internet by firms to use data at large scale to make more rational business decisions. In this paper, we compare these methods to standard econometric models that are used by practitioners to study demand. We are motivated by the problem of finding practical tools that would be of use to applied econometricians in estimating demand with large numbers of observations and covariates, such as in a scanner panel data set.

Many economists are unfamiliar with these methods, so we begin by expositing some commonly used techniques from the machine learning literature. We consider 8 different models that can be used for estimating demand for an SKU. The first two models are well-known to applied econometricians—the conditional logit and a panel data regression model. We then turn to machine learning methods, all of which differ from standard approaches by combining an element of model selection into the estimation procedure. Several of these models can be seen as variants on regularization schemes, which reduce the number of covariates in a regression which receive non-zero coefficients, such as stepwise regression, forward stagewise regression, LASSO, and support vector machines. We also consider two models based on regression trees, which are flexible methods for approximating arbitrary functions: bagging and random forests. While these models may be unfamiliar to many economists, they are surprisingly simple and are based on underlying methods that will be quite familiar. Also, all of the methods that we use are supported in statistical packages. We perform our computations in the open source software package R. Therefore, application of these methods will not require writing complex code from scratch. However, applied econometricians may have to familiarize themselves with alternative software.

We derive novel results for the asymptotic theory for several of the models above. We show, somewhat unsurprisingly, that many of these models do not have standard asymptotics and converge more slowly than the standard square root rate. Since these models do not have standard normal asymptotics, common methods such as the bootstrap cannot be applied for inference. We also propose using an idea dating back at least to Bates and Granger (1969). In a first step, we estimate all of the models on a training data set. In a second step, we then estimate a linear regression, where the regressors are predictions from each submodel. This process has two benefits: first, the linear combination has better predictive accuracy than any of its component models; and second, the linear combination also exhibits parametric rates of convergence under weak conditions.

To illustrate the usefulness of our approach, we apply our method to a canonical demand estimation problem. We use data from IRI Marketing Research via an academic license at the University of Chicago. It contains scanner panel data from grocery stores within one grocery store chain for six years. We used sales data on salty snacks, which is one of the categories provided in the IRI data. We find that the 6 models from the machine learning literature predict demand out of sample in standard metrics much more accurately than a panel data or logistic model. The combined model does even better, increasing predictive fit by two percent against the best single model. We do not claim that these models dominate all methods proposed in the voluminous demand estimation literature. Rather, we claim that as compared to common methods an applied econometrician might use in off the shelf statistical software, these methods are considerably more accurate. Also, the methods that we propose are all available in the well documented, open software package R as well as commercially-available software.

Applied econometricians have sometimes voiced skepticism about machine learning models because they do not have a clear interpretation and it is not obvious how to apply them to estimate causal effects. In this paper, we use an idea proposed by Varian (2014) to estimate the marketing lift attributable to promotions in our scanner panel. The idea is similar to the idea of synthetic controls used in Abadie and Gardeazabal (2003). We begin by training our model on the data where there is no promotion. We then hold the parameters of our model fixed and predict for the observations where there is a promotion. We then take the difference between the observed demand and the predicted demand for every observation in our data. By averaging over all observations in our sample, we construct an estimate of the average treatment effect on the treated.

We believe that this approach might be preferable to instrumental variable methods. In practice, it can be difficult to find instruments that are a priori plausible. When they exist, they may be subject to standard critiques such as the instruments may be weak or the identification may only be local. The logic behind our approach is simply to use lots of data rather than rely on quasi-randomness. As mentioned above, in applied econometrics, there are many forms of data about products that are simply not exploited in empirical studies such as unstructured text or pictures. In some applications, simply using more data and more scalable computations may be a superior strategy to reducing bias in causal lift estimates.

We find quite interesting that a standard panel data model with fixed effects has the "wrong sign" on promotional lift, i.e. promotions decrease demand. By contrast, our models from the statistics and computer science literature have the anticipated sign. We conjecture that this is because they simply use more data and have less bias as suggested by standard omitted variable formulas.

Finally, we can use our model to search for heterogeneity in the treatment effects in an unstructured way. Our model generates a residual for each observation that is treated. We can regress this residual on covariates of interest such as store indicators, brand dummies, hedonic attributes or seasonal factors. Once again, this is a high dimensional regression problem and the estimates would be poorly estimated in a regression framework. We instead propose a method suggested by Belloni et al. (2012) and use a LASSO to select variable and then use standard methods for inference. We believe that this is attractive for applied econometricians since it allows us to learn about heterogeneity in the treatment effect. Also, it could be useful to applied marketers since these variable could be useful in marketing mix models because it allows us to identify a smaller set of variables that predict marketing return.

The paper is organized as follows: Section 2 introduces the underlying statistical model of demand we study; Section 3 discusses the rates of convergence of our estimators; Section 4 covers model combination; Section 5 discusses four additional machine learning models that we use in our application; Section 6 applies the techniques to a scanner data set; and Section 7 concludes.

2 Model

In this section we will base our analysis on the idea that the point of inference of individual models (that will be further used in averaging) is to estimate the conditional expectation $\theta(z) = E[Y \mid Z = z]$. In other words, the concrete machine learning methods will be considered new versions of nonparametric methods whose main goal is *prediction*. When we use the traditional L_2 norm the task of prediction reduces to finding the function that best approximates the conditional expectation of the outcome variable of interest. To fix ideas, we consider a model with a scalar outcome variable Y, a vector of inputs Z and a random disturbance ε, such that for each of n observations i the model for the data generating process is

$$y_i = f(z_i) + \epsilon_i,$$

where $E[\epsilon_i] = 0$ and $E[\epsilon_i^2] = \sigma^2 < \infty$. Our goal will be the inference for $f(\cdot)$.

Before turning to a discussion of the machine learning techniques we will use in this paper, we first consider two common empirical approaches to estimating conditional expectations. To fix ideas, suppose our goal is to estimate the following model of demand. Let there be J products, each endowed with observable characteristics X_j. Let product j have demand in market m at time t equal to:

$$\ln Q_{jhmt} = f(p_{mt}, a_{mt}, X_{mt}, D_{mt}, \epsilon_{jmt}; \theta), \tag{1}$$

where a is a matrix of advertising and promotional measures, D is a vector of demographics, p is a vector of prices, ϵ is an idiosyncratic shock, and θ is a vector of unknown parameters. The above specification is very general. It allows for nesting through the stratification of the error term. Suppose that there are H nests of products; continuing the automobile example, two nests might be entry-level sub-compacts (Ford Fiesta, Toyota Yaris, Mazda 2, Chevrolet Sonic) and luxury performance sedans (BMW M3, Mercedes-Benz AMG C63, Cadillac CTS-V). Nests allow the substitution patterns to vary in a reduced-form way across those different classes of products. One can also extend the model to allow for non-trivial intertemporal shocks, such as seasonality in demand due to environmental conditions or holidays. The specification in Equation 1 is also consistent with models of discrete choice. For example, if the choice problem is discrete, then one obtains quantities by integrating over both the population of consumers and the distribution of errors.

The goal of our exercise is to estimate the relationships between the right-hand side variables and quantities demanded. We discuss several approaches to approximating the model in Equation 1. We first consider more familiar approaches like linear regression and logit models before describing several machine learning approaches which are less known in the econometrics literature. Here, we explain these models in enough detail to provide some intuition.

2.1 Linear Regression

A typical approach to estimating demand would be to approximate Equation 1 using a flexible functional form of the following type:

$$\ln Q_{jhmt} = \alpha' p_{mt} + \beta_1' X_{mt} + \beta_2' D_{mt} + \gamma' a_{mt} + \lambda' \mathcal{I}(X_{mt}, D_{mt}, p_{mt}, a_{mt}) + \zeta_{hm} + \eta_{mt} + \epsilon_{jmt}, \tag{2}$$

where $\mathcal{I}$ is an operator which generates interactions between the observables (e.g. interactions of X, p, and D, e.g. high income neighborhoods have higher demand for expensive imported beer). Dummy variables on nests are captured by ζ_{hm}, while seasonality is captured by the term η_{mt}, which varies by time across markets. In principle, such a model may have thousands of right-hand side variables; for example, an online retailer such as eBay may offer hundreds of competing products in a category, such as men's dress shirts. The demand for one particular good, say white Brooks Brothers dress shirts, may depend on the prices of the full set of competing products offered. In a more extreme example, as offered in Rajaraman and Ullman (2011), Google estimates the demand for a given webpage by using a model of the network structure of literally billions of other webpages on the right-hand side. In practice, however, such models usually only consider a very small subset of all possible right-hand side variables, and those variables are typically chosen in an ad hoc manner by the researcher.

In ordinary least squares (OLS), the parameters of Equation 2, jointly denoted by β, are typically estimated using the closed-form formula:

$$\beta = (X'X)^{-1}(X'Y), \tag{3}$$

where X is the matrix of right-hand side variables and Y is the vector of outcomes.

We note that the formula requires an inversion of $(X'X)$. This imposes a rank and order condition on the matrix X. We highlight this because in many settings, the number of right-hand side variables can easily exceed the number of observations. Even in the simplest univariate model, one can saturate the right-hand side by using a series of basis functions of X. This restriction requires the econometrician to make choices about which variables to include in the regression. We will return to this below, as some of machine learning methods we discuss below allow the econometrician to skirt the order condition by combining model selection and estimation simultaneously.

2.2 Logit Models

A large literature on differentiated products has focused on providing a theoretical foundation for Equation 1 based on an underlying random utility model of discrete choice. This structure gives rise to predictions of choice probabilities that can be used to compute market shares when integrated over the market population; quantities are then computed by multiplying market shares with market size.

A typical discrete choice model is the logit model, where utilities are modeled as functions similar to the right-hand side of Equation 1 and the idiosyncratic error is a Type I Extreme Value. This gives rise to a particularly nice analytical form for the market share:

$$s_j = \frac{\exp(\theta' X_{jhmt})}{\sum_{k \in J} \exp(\theta' X_{khmt})}. \tag{4}$$

Since Berry et al. (1995), many empirical models of differentiated product demand concentrate on estimating a distribution of unobserved heterogeneity, $F(\theta)$, over a typically low-dimensional θ:

$$s_{jhmt} = \int \frac{\exp(\theta' X_{jhmt})}{\sum_{k \in J} \exp(\theta' X_{khmt})} dF(\theta). \tag{5}$$

An attractive feature of this approach is that the method is robust to the inclusion of unobserved heterogeneity and vertical characteristics that are observed to both the firm and consumers. However, this estimator places a significant amount of structure on the demand curve and is computationally burdensome.

3 Implementation and Properties of Selected Machine Learning Methods

3.1 Square-root LASSO

The general class of LASSO methods is designed to perform the computationally efficient and consistent approach to model selection and inference based on penalization. The idea of such a penalization if the following. Suppose that $X \in \mathcal{X}$ is a vector of the p-dimensional space (which is a nonlinear transformation of the space spanned by Z, possibly enlarging the dimension) where function $f(\cdot)$ can be well approximated by some linear function of X. The estimators are considered for the "self-normalized" regressors, such that $\frac{1}{n}\sum_{i=1}^{n} x_{ij}^2 = 1$ for each j. Under appropriate smoothness of $f(\cdot)$ and the appropriately chosen $\mathcal{X}$, the residual from the best linear approximation using s elements of $\mathcal{X}$, denoted $r_i = f(z_i) - x_i'\beta_0$ will be small as measured by the sample sum of squares $c_s^2 = \frac{1}{n}\sum_{i=1}^{n} r_i^2$. For instance, if $f(\cdot)$ is a smooth function and $\mathcal{X}$ is the space of polynomials of $\mathcal{Z}$ then the corresponding sum of squares for the approximation will be determined by order of the residual in the corresponding Taylor expansion of $f(\cdot)$. Provided that the goal of interest is the minimization of the prediction error, Belloni et al. (2014) introduce the norm for estimators $\widehat{\beta}$ of β_0 such that

$$\|\widehat{\beta} - \beta_0\|_{2,n} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i'\widehat{\beta} - x_i'\beta_0)^2}.$$

Note that if $f(\cdot)$ belongs to the linear span of $\mathcal{X}$, then norm $\|\cdot\|_{2,n}$ measures the prediction error for $f(\cdot)$. Otherwise, the error will also reflect the approximation error c_s.

The procedure for finding the tradeoff between the approximation error and estimation error is by choosing the appropriate penalization for the sample least squares objective

$$\widehat{Q}(\beta) = \frac{1}{n}\sum_{i=1}^{n}(y_i - x_i'\beta)^2.$$

Then the sample version of the problem of finding the parsimonious (sparse) linear function can be written as

$$\min_{\beta\in\mathbb{R}^p} \widehat{Q}(\beta) + \frac{\sigma^2\|\beta\|_0}{n}, \tag{6}$$

where $\|\beta\|_0 = \sum_{k=1}^{p} \mathbf{1}\{\beta_k \neq 0\}$ is the ℓ_0 norm counting the number of non-zero components of β. This estimator yields the minimum risk over sparse linear representations of $f(\cdot)$ when the errors are homoskedastic and Gaussian. In this respect this estimator parallels the standard OLS estimator in terms of efficiency. However, the empirical objective function that needs to be minimized is discontinuous with respect to the minimand. Morover it is not convex meaning that known optimization procedures will converge slowly. In fact, the computation problem corresponding to this minimization is known to be NP-hard.

The idea in the Machine Learning literature that allows one to transform a computationally hard problem (6) into a computationally efficient problem that also yields the solution that approximates the solution of (6) well is the following. Since the biggest problem in the objective of (6) is the presence of the ℓ_0 penalty, it gets replaced with the normalized ℓ_1 penalty $\lambda\,\|\beta\|_1$, where $\|\beta\|_1 = \sum_{k=1}^{p}|\beta|_1$. That instantly leads to a convex objective function that can be minimized efficiently. There are three concerns with this approach. First, the penalty constant λ is not known a priori and needs to be estimated as well. Second, in the presense of heteroskedastic errors the choice of unweighted norm $\|\beta\|_1$ does not lead to consistent estimates for parameters. Third, the presense of ℓ_1 penalty leads to the small sample bias in the estimated parameters. Belloni et al. (2014) address all three problems in the following way. First of all, they suggest to replace the objective function $\widehat{Q}(\beta)$ with $\sqrt{\widehat{Q}(\beta)}$. In this case the gradient of $\sqrt{\widehat{Q}(\beta)}$ with respect to β becomes "self-normalized." As a result, the optimal penalty λ no longer depends on σ and is only the function of dimensionality of $\mathcal{X}$ and the sample size. Second, they suggest a linear transformation of β in the ℓ_1 penalty that leads to consistency of the estimator. Finally, they suggest to use the penalized minimization procedure as the tool for selecting the dimensions of X that will be used the approximation and then run the post-estimation procedure that minimizes $\widehat{Q}(\cdot)$ only with respect to the selected coefficients.

Formally, the minimization problem can be characterized as

$$\hat{\beta} \in \arg\min_{\beta \in \mathbb{R}^p} \frac{1}{n} \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda \|\widehat{\Psi}\beta\|_1, \quad \lambda = 2\sqrt{2\log(pn)/n} \tag{7}$$

The optimal choice for the penalty constant in this case is $\lambda = 2\sqrt{\frac{2\log(pn)}{n}}$, which means that no additional steps are required to obtain its value. The transformation matrix Ψ resembles the weights in the WLS procedure such that

$$\widehat{\Psi} = \text{diag}\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_{ij}^2 \epsilon_i^2} + o_p(1),\ j = 1, \ldots, p\right).$$

Provided that the residuals ϵ_i are not immediately available, they are replaced with the estimates $\widehat{\epsilon}_i$ that are the outcome of the iterative procedure. The procedure initilizes at $\widehat{\epsilon}_i = y_i - \bar{y}$ leading to $\widehat{\Psi} = \text{diag}\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_{ij}^2 \widehat{\epsilon}_i^2} + o_p(1),\ j = 1, \ldots, p\right)$. Then the we solve (7) using $\widehat{\Psi}$ as the transformation matrix yielding $\hat{\beta}$. Then at the next step we use the updated errors $\widehat{\epsilon}_i = y_i - x_i'\hat{\beta}$ to construct the new weighting matrix $\widehat{\Psi}$. The procedure iterates till convergence. For the non-zero components of $\widehat{\beta}$ we then run the regular OLS to find the post-selection estimates.

3.2 SVM regression

Support vector machines (SVMs) are a class of estimators designed to solve the problem of prediction of a discrete outcome using a vector of regressors X. We can easily illustrate the idea behind the SVM approach for the settings where the outcome is determined by a linear index of regressors. In other words the statistical model takes the form

$$Y = f(X) + \zeta,$$

where $f(X) = \sum_{k=1}^{p} w_k X_k + b = \langle w,\, X\rangle + b$ and $|\zeta| \leq \epsilon$. The problem as in LASSO is to find a parsimonious set of relevant regressors in terms of their weights w. Allowing for the error in the magnitude of at most ϵ, we can formulate the mathematical problem for finding w from the sample $\{(y_i, x_i)\}_{i=1}^{n}$ as $\min \|w\|^2$ subject to

$$y_i - \langle w, x_i\rangle - b \leq \epsilon, \quad \text{and} \quad \langle w, x_i\rangle + b - y_i \leq \epsilon.$$

Provided that the linear approximation may not exist for a given ϵ, Cortes and Vapnik (1995) suggest introducing "slack" variables ξ and ξ^* that transform the original problem into a feasible problem:

$$\min \|w\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*)$$

subject to

$$y_i - \langle w, x_i \rangle - b \leq \epsilon + \xi_i, \quad \text{and} \quad \langle w, x_i \rangle + b - y_i \leq \epsilon + \xi_i^*$$

for some constant $C > 0$ and non-negative slack variables ξ and ξ^*. The key idea in the SVM is to construct the Lagrangian for the introduced constrained optimization problem and replace the original optimization problem with the corresponding dual problem. Introducing Lagrange multipliers α, α^*, η and η^* we can write the Lagrangian as

$$L = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) + \sum_{i=1}^{n}(\eta_i\xi_i + \eta_i^*\xi_i^*) + \sum_{i=1}^{n}\alpha_i(\epsilon + \xi_i - y_i + \langle w, x_i \rangle + b)$$
$$+ \sum_{i=1}^{n}\alpha_i(\epsilon + \xi_i^* + y_i - \langle w, x_i \rangle - b).$$

The problem dual to the minimization of the Lagrangian with respect to the state variables w, ξ, ξ^* is the problem of maximization with respect to the co-state variables (Lagrange multipliers) α, α^*, η, η^*. After simple re-arrangement this problem can be written as maximization of

$$-\frac{1}{2}\sum_{i,j=1}^{n}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*)\langle x_i,\, x_j \rangle - \epsilon\sum_{i=1}^{n}(\alpha_i + \alpha_i^*) + \sum_{i=1}^{n}y_i(\alpha_i - \alpha_i^*)$$

subject to $\sum_{i=1}^{n}(\alpha_i - \alpha_i^*) = 0$ and $0 \leq \alpha_i,\, \alpha_i^* \leq C$. This is the problem that can be efficiently solved using quadratic programming. The observations i for which α_i or α_i^* are not equal to zero are called the support vectors. The main insight from this formulation is that now the estimated function of interest can be written as

$$\widehat{f}(x) = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)\langle x_i,\, x \rangle + b.$$

This means that the impact of regressors x on the predicted outcome is summarized by the inner product $\langle \cdot, \cdot \rangle$. In the linear case the inner product is the standard inner product in the Euclidean space.

Now suppose that X corresponds to some nonlinear transformation of a smaller set of regressors Z. For instance, X can be different order polynomials of Z. In that case, the inner product $\langle x_i, x\rangle$ will also summarize the mapping $Z \mapsto X$. This means that such an inner product can be explicitly defined as a function of the original inputs z called the kernel function. In this case the estimated regression function can be explicitly written in terms of the kernel function

$$\widehat{f}(z) = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)k(z_i, z) + b.$$

For a given function of two arguments to be a kernel it needs to be shown that it can be represented as an inner product in some transformed space $\mathcal{X}$ (corresponding to $Z \mapsto X$). Several functions has been shown to be kernels and have been applied in practice. For instance, the Gaussian function has been shown to be a kernel:

$$k(z, z') = \exp\left(-\frac{\|z - z'\|^2}{2\sigma^2}\right).$$

Another example of the kernel is

$$k(z, z') = B_{2r+1}(\|z - z'\|),$$

where B_{2r+1} is the B-spline of order $2r + 1$. The difference between the linear case (where the function is estimated as a linear combination of inputs) and the case where the kernel function is used is that the weights w are no longer defined explicitly. To give more intuition, we can consider the case where the kernel can be written explicitly as $k(z, z') = \langle \Phi(z), \Phi(z')\rangle$. Then $\Phi(\cdot)$ is the mapping that creates "regressors" X from Z. This will also allow to write the weights as

$$\widehat{w} = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)\Phi(x_i).$$

We note that unlike LASSO, the regression SVM approach allows the space $\mathcal{X}$ to be explicitly infinite-dimensional via the use of kernels.

3.3 L_2 Boosting

Boosting algorithms are computationally fast methods that have been shown to have similar properties to the LASSO methods in application to high-dimensional linear models. Although initial applications of boosting were for the binary outcome variable, L_2 boosting with the associated quadratic loss function applies to the models with continuous outcome variables. Unlike LASSO L_2 boosting imposes an implicit penalty that is incorporated in the structure of the algorithm itself. The algorithm is designed via the iterative verification of the fit of the model over all of the dimensions of regressors. The algorithm then advances along the dimension of the "best improvement," i.e. the dimension where a given regressor provides the biggest contribution to the least squares residual. The componentwise structure eliminates the need to fit a high-dimensional least squares objective function replacing the estimation with a series of one dimensional least squares procedures. Provided that at each step one needs to verify the fit associated with each regressor in the model, it will be necessary to compute p least squares minima. Due to the structure of the least squares objective function the minimum can be written explicitly via the ratio of two sums further reducing the computational burden.

The algorithm proceeds in four steps. First, the algorithm initializes at step $m = 0$ at some $\widehat{f}^{(0)}(\cdot)$ with the default value $\widehat{f}^{(0)}(\cdot) = \bar{Y}$. Second, at step $m+1$ compute the residuals

$$\widehat{\epsilon}_i^{(m)} = y_i - \widehat{f}^{(m)}(x_i).$$

The model is then fitted using a componentwise linear least squares. (i) Compute the coefficients

$$\widehat{\gamma}_j^{(m)} = \frac{\sum_{i=1}^n x_{ij}\widehat{\epsilon}_i^{(m)}}{\sum_{i=1}^n (x_{ij})^2}.$$

(ii) Find the dimension to be updated $\widehat{j}^{(m)} = \arg\min_{j \leq p} \sum_{i=1}^n \left(\widehat{\epsilon}_i^{(m)} - \widehat{\gamma}_j^{(m)} x_{ij}\right)^2$. (iii) Update the estimated coefficients as $\widehat{f}^{(m+1)}(x) = \widehat{f}^{(m)}(x) + \nu\, \widehat{\gamma}_{\widehat{j}^{(m)}}^{(m)}\, x_{\widehat{j}^{(m)}}$. The choice of the step-length factor ν does not impact the asymptotic behavior of the algorithm and is usually selected to be "sufficiently small" (e.g. $\nu = 0.1$). The componentwise linear least squares thus updates only one dimension of the model at a time. Finally, the second step is iterated until the stopping iteration M has been reached. The recommended choice is $M = K\left(\sqrt{n/\log(p)}\right)^{(3-\delta)/(4-2\delta)}$ for $0 < K < \infty$ and $0 < \delta < 5/8$.

4 Econometric properties of averaged models

Above we discussed three Machine Learning methods. We note that the main difference between the SVM regression, L_2 boosting and square-root LASSO is that the SVM regression does not explicitly use the "generated features" $x = \Phi(z)$. Instead, the (large) vector of generated features is embedded in the kernel function. Our goal is to ensure that the three discussed methods have adequate performance when evaluated in the average model. That would in general require that each of the estimators attains the convergence rate of $o(n^{-1/4})$. That would allow us to use existing results regarding the performance of average predictions and establish asymptotic normality and $O(n^{-1/2})$ convergence rate for the corresponding average predictor.

There are three sets of conditions that need to be imposed on the econometric model to ensure the adequate performance of the resulting estimator. First, we need to ensure that the distributions of regressors and the unobserved disturbances a sufficiently regular so that the concentration inequalities leading to limit theorems can be applied. Second, we need to verify that the sequence of functional spaces that we use to approximate the non-parametric model provides sufficiently good approximation. The properties of approximating functional spaces used for semi- and non-parametric inference have been extensively studied in econometrics, e.g. including Andrews (1991), Newey (1997) and Chen (2007). Finally, we need to impose the constraint on the complexity of the approximating model such that it appropriately trades off the approximation and the estimation errors.

We start with the imposition of properties on the statistical components of the model.

ASSUMPTION 1 *Suppose that the higher order maximum and minimum moments of regressors and disturbances satisfy the following conditions.*

(i) *For* $q > 4$, $\frac{1}{n}\sum_{i=1}^{n} E[|\epsilon_i|^q] < \infty$

(ii) $\inf_{n\geq 1} \min_{1\leq j\leq p} \frac{1}{n}\sum_{i=1}^{n} P^{pj}(z_i)^2 E[\epsilon_i^2] > 0$ *and* $\sup_{n\geq 1} \max_{1\leq j\leq p} \frac{1}{n}\sum_{i=1}^{n} |P^{pj}(z_i)|^3 E[|\epsilon_i|^3] > 0$, *where* $P^{pj}(\cdot)$ *are basis functions of the approximating space described in the next assumption.*

Next we impose a high-level condition on the approximating functional space and the corresponding estimator. The estimator itself will need to be appropriately selected, e.g. using the guidelines in Chen (2007) and will depend on the underlying properties of the function that is approximated (such as smoothness, monotonicity, etc.).

ASSUMPTION 2 *Suppose that*

(i) $f(\cdot) \in \mathcal{F}$ *is an element of a compact subset of separable Hilbert space* $\mathcal{H}$ *and there is a sequence of finite-dimensional vector spaces* $\mathcal{H}^p$ *such that* $\mathcal{H}^p \subset \mathcal{H}^{p+1} \subset \mathcal{H}$ *and there is a sequence of orthogonal bases* $\{P^{pk}(\cdot)\}_{k=0}^{p}$ *such that* $\mathcal{H}^p$ *is a completion of the linear span of* $\{P^{pk}(\cdot)\}_{k=0}^{p}$.

(ii) *For each n and for any $f \in \mathcal{F}$ and any $p \gg n$ there exists a subsequence $\{P^{pk_j}(\cdot)\}_{j=1}^{s}$ with $s \ll p$ such that the class of functions $\mathcal{R}^p = \left\{r^p(\cdot) = f(\cdot) - proj\left(f \,|\, \{P^{pk_j}(\cdot)\}_{j=1}^{s}\right),\, f \in \mathcal{F}\right\}$ has a finite envelope R^p with*

$$E[(R^p)^2] \leq C\, s \log(p)/n$$

(iii) *The inner product in each $\mathcal{H}^p$ and and an element $\Phi^p(\cdot)$ of $\mathcal{H}^p$ generate a kernel $K^p(\cdot, \cdot) = \langle \Phi^p,\, \Phi^p \rangle_{\mathcal{H}^p} \leq \mathcal{K}^p$ such that*

$$proj\left(f \,|\, \{P^{pk_j}(\cdot)\}_{j=1}^{s}\right) = \langle w,\, \Phi^p \rangle_{\mathcal{H}^p},$$

for $w \in \mathcal{H}^p$ and $\|w\|_{\mathcal{H}^p} < \Lambda < \infty$

Assumption 2 essentially specifies the properties of the feature map that approximates the nonparametric regression. Associated with this feature map is the corresponding kernel defined as an inner product in $\mathcal{H}^p$.

Finally, we provide the set of assumptions that ensure the appropriate tradeoff between the sample size and the complexity of the approximating functional space.

ASSUMPTION 3 *Let Δ^p be the subspace of $\mathbb{R}^p$ such that $0 \notin \Delta^p$ and for each $\delta \in \Delta^p$ the component $\delta_{k_j} > 0$ where $0 < k_j \leq p$ is the subsequence defined in Assumption 2 and $\delta_k = 0$ otherwise. Let $l_n \to \infty$. Define $\underline{\lambda}_n = \min\limits_{\delta \in \Delta^p, \|\delta\|_0 \leq s l_n} \frac{\delta'\left(\frac{1}{n}\sum_{i=1}^{n} P^p(z_i)_i P^p(z_i)'\right)\delta}{(\delta'\delta)^{1/2}}$ and $\overline{\lambda}_n = \max\limits_{\delta \in \Delta^p, \|\delta\|_0 \leq s l_n} \frac{\delta'\left(\frac{1}{n}\sum_{i=1}^{n} P^p(z_i)_i P^p(z_i)'\right)\delta}{(\delta'\delta)^{1/2}}$. Suppose that*

(i) $\sup\limits_{n \geq 1} \frac{\overline{\lambda}_n}{\underline{\lambda}_n} < \infty.$

(ii) $\max_{i \leq n, j \leq p} |P^{pj}(z_i)|^2 / l_n = o(1)$, $\log p \leq C(n/\log^2 n)^{1/3}$ *and* $l_n^2 s \log(p) \leq Cn/log\, n.$

(iii) $\mathcal{K}^p \to 0$ *as* $p \to \infty$ *and* $\mathcal{K}^p \sqrt{n} \to \infty$

Based on Assumption 2, we can integrate the results in Belloni et al. (2014), Cortes and Vapnik (1995) and Meinshausen and Bühlmann (2006) to provide the following theorem.

THEOREM 1 *Suppose that statistical model satisfies Assumptions 1, 2, and 3. Consider the $\sqrt{LASSO}$ algorithm with $\lambda = 2\sqrt{2\log(pn)/n}$, the SVM regression algorithm with $\epsilon =$*

$o\left(\mathcal{K}^p\sqrt{n}\right)$ and $C \geq E[(R^p)^2]^{1/2}$, and the L_2 boosting algorithm with $M = O\left(\left(\sqrt{n/\log(p)}\right)^{(3-\delta)/(4-2\delta)}\right)$ with $0 < \delta < 5/8$. Then for each resulting estimator

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(\widehat{f}(z_i) - f(z_i))^2 = o_p(1).$$

The property of the constructed estimator allows us to apply it in the settings of model averaging. The object of interest is the average prediction with the weighing function $w(\cdot)$ such that the final parameter of interest can be expressed as

$$\theta = E[w(Z)f(Z)]. \tag{8}$$

Our next idea is to apply the result in Newey (1994) which assures that as long as model (8) is written in the "orthogonal form" with respect to the nonparametrically estimated function $f(\cdot)$ then the particular estimator of $f(\cdot)$ will not have an impact on the asymptotic distribution and thus the implementation details of the machine learning methods that we used to estimate $f(\cdot)$ will not affect the resulting average prediction.

THEOREM 2 *Suppose that $\widehat{f}(\cdot)$ is a machine learning method satisfying conditions of Theorem 1 and (8) along with the data generating process satisfy Assumptions 5.1-5.3 in Newey (1994). Then the resulting average prediction $\widehat{\theta}$ is asymptotically normal with*

$$\sqrt{n}(\widehat{\theta} - \theta) \xrightarrow{d} N(0, V)$$

with

$$V = Var(\Psi(Y, Z)) = E[w(Z)^2\sigma^2(Y|Z)] + Var(w(Z)f(Z))$$

Proof:

Suppose as before that $z \in \mathcal{Z} \subset \mathbb{R}^d$ with $d \ll n$ and finite. Consider the likelihood of the model $\ell(y, z)$ and consider its parametrization $\ell_t(y, z)$ with some scalar parameter t. Denote $s_t(y|z)$ the score of the conditional distribution $\ell_t(y|z)$ and $s_t(z)$ the score of the unconditional distribution $\ell_t(z)$. The full score $s_t(y, z) = s_t(y|z) + s_t(z)$. Then we can write the tangent space of the model $\mathcal{T}_t = \{a(y, z) + t(z)\}$ where $E[a(Y, Z)^2] < \infty$ and $E[a(Y, Z)|Z = z] = 0$ and $E[t(Z)^2] < \infty$ and $E[t(Z)] = 0$. Now we consider the sequence of parameters θ_t corresponding to the sequence of distributions $\ell_t(y, z)$ and consider the directional derivative

$$\frac{\partial\theta_t}{\partial t} = \frac{\partial}{\partial t}E_t[w(Z)f_t(Z)].$$

We can express this derivative using the fact that $f_t(z) = E_t[Y \mid Z = z]$:

$$\frac{\partial \theta_t}{\partial t} = E[w(Z)f(Z)s_t(Z) + w(Z)Y\, s_t(Y|Z)].$$

Now the idea to find the "orthogonal" form of the estimator will be to find an element of $\mathcal{T}_t$ such that the directional derivative of the parameter of interest can be represented as a projection of the score of the parametrized model on the tangent space. A straightforward verification suggests that

$$\frac{\partial \theta_t}{\partial t} = E[\Psi(Y, Z)s_t(Y, Z)],$$

where

$$\Psi(y, z) = w(z)(y - f(z)) + (w(z)f(z) - E[w(Z)f(Z)]).$$

Next we apply Lemma 5.3 in Newey (1994) to the estimator defined by

$$\widehat{\theta} - E[w(z)f(Z)] = \frac{1}{n}\sum_{i=1}^{n} \Psi(y_i, z_i)$$

with $f(\cdot)$ replaced with $\widehat{f}(\cdot)$. Provided that the estimator $\widehat{f}(\cdot)$ satisfies assumptions of Theorem 1 is satisfies conditions of Lemma 5.3 in Newey (1994). This leads to the result that

$$\sqrt{n}(\widehat{\theta} - \theta) \xrightarrow{d} N(0, V)$$

with

$$V = \mathrm{Var}(\Psi(Y, Z)) = E[w(Z)^2\sigma^2(Y|Z)] + \mathrm{Var}(w(Z)f(Z))$$

Q.E.D.

5 Additional empirical procedures

In addition to the three inference procedures described above we consider four additional machine learning procedures that we also interpret as methods for the estimation of the conditional expectation $E[Y \mid Z = z]$.

5.1 Forward stepwise and forward stagewise models

As in the square-root LASSO procedure and L_2 boosting we consider an explicit mapping $\mathcal{Z} \mapsto \mathcal{X}$. As in standard nonparametric models this mapping can be represented via a collection of basis functions $\{P^{pj}(\cdot)\}_{j=1}^{p}$ where the size of the set of functions p is allowed to slowly grow with the sample size n. Unlike LASSO, where the model is selected by fitting the penalized least squares objective function that leads to corner solutions making multiple coefficients to be equal to zero, the stepwise and stagewise models lead to sparse coefficient sets via the construction of the variable selection procedure.

The incremental version of the stagewise regression model uses the explicitly set step size ϵ that can be calibrated based on the convergence performance of the estimation procedure. The procedure itself is based on the iterative addition of the "explanatory variables" $P^{pj}(\cdot)$. The procedure is initialized from the baseline model $\widehat{f}^{(0)}(z) = \bar{y}$ with the corresponding coefficients of the nonparametric representation $\beta_j^{(0)} \equiv 0$. The corresponding residuals are computed in the standard way as $r_i^{(0)} = y_i - f(z_i)$. Then at iteration k we find the predictor $P^{jp}(\cdot)$ maximizing the correlation between $r_i^{(k)}$ and $P^{jp}(z_i)$. Then the corresponding coefficient in the nonparametric representation is updated as $\beta^{(k+1)} = \beta^{(k)} + \epsilon \,\mathrm{sign}\left(\widehat{\mathrm{corr}}(r^{(k)}, P^{pj}(\cdot))\right)$. The residual then is updated to offset the change in the coefficient β_j. The procedure then stops when the maximum correlation between $r_i^{(k)}$ and $P^{jp}(z_i)$ does not exceed the pre-set tolerance (or the maximum allowed number of steps has been achieved). In our analysis we choose the maximum number of steps at 16,000 to ensure that the procedure is stopped via the tolerance criterion.

In the stepwise algorithm, the set of regressors used for estimation is iteratively updated based on the minimization of the Akaike Information Criterion. The model is initialized at $\widehat{f}^{(0)}(z) = \bar{y}$ same as in the stagewise method. Then at step k we select one of the components $P^{jp}(\cdot)$ for which $\beta_j^{(k-1)} = 0$ that yields the highest contribution to the AIC. This re-computation can be done efficiently by invoking the QR factorization in the corresponding first order condition and thus does not require a repetitive multi-dimensional minimization. The procedure stops once the improvement in the AIC falls below the selected tolerance or the maximum set number of steps is exceed.

5.2 Regression Trees and the Random Forest

The random forest model is different from most of the models that we considered previously in the sense that it is explicitly not using the spectral properties of the functional space that contains $f(z) = E[Y \mid Z = z]$. Instead, the method is based on selecting the best iterative factorization of the state space $\mathcal{Z}$ such that within each element of the partition of the state space function $f(\cdot)$ can be approximated by a constant.

The basic element of the random forest algorithm is a regression tree. The regression tree is defined by two basic parameters: the maximum depth (also called the number of splits) and the size of the terminal nodes (also called leaves) determining the minimum number of points within each partition of the state space. Denote the tree parameters θ. Then given the sample $Z_n = \{z\}_{i=1}^n$ the regression tree is defined by the following algorithm. The algorithm is initialized at $\widehat{f}^{(0)}(z) = \bar{y}$. At step k the state space is split into rectangular areas $R_j^{(k)}$ and the regression tree predicts as

$$\widehat{f}^{(k)}(z) = \sum_j \frac{\sum_{i=1}^n y_i \mathbf{1}\{z_i \in R_j^{(k)}\}}{\sum_{i=1}^n \mathbf{1}\{z_i \in R_j^{(k)}\}} \mathbf{1}\{z \in R_j^{(k)}\}.$$

Then the algorithm is updated by searching for the partition element $R_j^{(k)}$ and the dimension l of z such that if $R_j^{(k)} = [z_{jL}^1, z_{jU}^1] \times \ldots \times [z_{jL}^p, z_{jU}^p]$, then the split of this partitions into two $R_{j_1}^{(k+1)} = [z_{jL}^1, z_{jU}^1] \times \ldots \times [z_{jL}^l, 0.5(z_{jU}^l + z_{jU}^l)] \times \ldots \times [z_{jL}^p, z_{jU}^p]$ and $R_{j_2}^{(k+1)} = [z_{jL}^1, z_{jU}^1] \times \ldots \times [0.5(z_{jU}^l + z_{jU}^l), z_{jU}^l] \times \ldots \times [z_{jL}^p, z_{jU}^p]$ provides the greatest contribution to the sum of squared residuals corresponding to the new partition $R_j^{k+1)}$:

$$\min_{l=1,\ldots,p,\, R_j^{(k)}} \sum_{i=1}^n (y_i - \widehat{f}^{(k+1)}(z_i))$$

over all j and dimensions $l = 1, \ldots, p$. The iterations continue until one of the convergence criteria (the size of the terminal node or the depth of the tree) is met. It can be shown that when the outcome variable is binary, the minimization problem used to construct the tree is analogous to Manski's maximum score problem over the binary splits of the state space.

The random forest model is generated as an ensemble of trees via bootstrap. One draws B bootstrap samples from the original sample X_n. Each such a bootstrap sample X_b and the corresponding parameters θ_b yield the estimated regression tree $\widehat{f}_b(\cdot)$. Then the random forest model provides the value which is the average over the bootstrap samples:

$$\widehat{f}(z) = \frac{1}{B} \sum_{b=1}^B \widehat{f}_b(z).$$

6 Empirical Application

This section compares econometric models with machine learning ones using a typical demand estimation scenario—grocery store sales. The machine learning models in general produce better out-of-sample fits than linear models without loss of in-sample goodness of fit. If we combine all the models linearly with non-negative weights, the resulting combination of models produces better out-of-sample fit than any model in the combination. This section also illustrates how machine learning models could work with unstructured data or sparse data. Unstructured data is not organized to feed into models directly like structured data. For instance, the text description of a bag of chips and the image of the bag are unstructured data. Sparse data is a type of data where most of the elements are zeros. Both of unstructured and sparse data would be hard to handle in econometric models. The last part of this section uses the same data and model structures to estimate the promotional lift of sales.

The data we use is provided by IRI Marketing Research via an academic license at the University of Chicago. It contains scanner panel data from grocery stores within one grocery store chain for six years. We used sales data on salty snacks, which is one of the categories in the IRI data. A unit of observation is product j, uniquely defined by a UPC (Universal Product Code), in store m at week t. There are 3,045,513 observations, which includes 3,337 unique products. Let q_{jmt} be the number of bags of salty snack j sold in store m at week t. If $q_{jmt} = 0$, we do not know if it is due to no sale or out-of-stock and the observation is not filled in. The price p_{jmt} is defined as the quantity weighted average of prices for product j in store m at week t. Therefore if $q_{jmt} = 0$, the weight is also 0. In addition to price and quantity, the data contains attributes of the products (such as brand, volume, flavor, cut type, cooking method, package size, fat and salt levels) and promotional variables (promotion, display and feature). Table 1 shows the summary statistics of quantity, price and dollar spent per transaction. Table 2 shows the number of unique values for the category variables as well as the three most common values of product attributes: brand, volume, flavor, cut type, cooking method, package size, fat and salt levels.

In our application, we compare eight alternative models of demand to predict q_{jmt}. The eight models are linear model, stepwise, forward stagewise, LASSO, random forest, support vector machine, gradient boosting trees, and Logit. Linear model and Logit are traditional econometric models where the others are popular machine learning algorithms. We also perform a linear combination of all the models in the effort to increase prediction accuracy. We use R to compute all the results and a list of related R packages is provided in this

Table 1: Summary Statistics

Variable	Mean	Median
Price	2.12	1.99
Quantity	15.80	6.00
Dollars	28.11	12.19
#Stores	1,560	
#Weeks	313	
#UPC	3,337	
#Obs	3,045,513	

Table 2: Category Variables

Variable	#Levels	Three Most Frequent Values		
Brand	237	Pringles	Utz	Lays
Product Type	4	Potato Chip	Potato Crisp	Potato Chip and Dip
Packaging	20	Bag	Canister	Plastic Wrapped Cardboard Canister
Flavor	207	Original	Sour Cream & Onion	BBQ
Fat Content	16	Missing	Low Fat	Fat Free
Cooking Method	47	Missing	Kettle Cooked	Old Fashion Kettle Cooked
Salt Content	14	Missing	Lightly Salted	Sea Salt
Cutting Type	32	Flat	Missing	Ripple

section.

6.1 Linear Regression Models

The linear regression is a typical approach to estimate demand by approximating the demand using a function form of the following:

$$\ln q_{jmt} = \beta' X_{jmt} + \zeta_{mt} + \eta_{jm} + \epsilon_{jmt}$$

where X_{jmt} is the matrix of attributes including log of own prices, product attributes, advertising and promotion indicators, ζ_{mt} is the market specific seasonal factor, η_{jm} is the product specific market effect and ϵ_{jmt} is an idiosyncratic shock to each product, market and time.

Table 3 shows the output of the linear model where only the significant coefficients are displayed.

Table 3: Linear Regression

Log Quantity	Estimate	Std. Error	t value	Pr($>|t|$)	
Log Price	-0.639	0.055	-11.708	$<$ 2e-16	***
Promotion	0.466	0.039	11.926	$<$ 2e-16	***
Feature: None	-0.630	0.067	-9.334	$<$ 2e-16	***
Display:					
Minor	0.708	0.049	14.341	$<$ 2e-16	***
Major	0.637	0.049	13.119	$<$ 2e-16	***
Brand:					
Herrs	-0.351	0.156	-2.253	0.024	*
Jays	-1.101	0.244	-4.516	0.000	***
Kettle Chips	-0.995	0.236	-4.217	0.000	***
Lays	-0.337	0.159	-2.124	0.034	*
Lays Bistro Gourmet	-0.656	0.188	-3.480	0.001	***
Lays Natural	-1.662	0.327	-5.079	0.000	***
Lays Stax	-1.481	0.183	-8.104	0.000	***
Lays Wow	-0.485	0.204	-2.379	0.017	*
Michael Seasons	-1.655	0.239	-6.921	0.000	***
Pringles	-0.794	0.156	-5.090	0.000	***
Pringles Cheezums	-0.644	0.211	-3.055	0.002	**
Pringles Fat Free	-0.624	0.189	-3.308	0.001	***
Pringles Prints	-1.876	0.314	-5.982	0.000	***
Pringles Right Crisps	-0.881	0.128	-6.892	0.000	***
Ruffles Natural	-1.379	0.389	-3.549	0.000	***
Ruffles Snack Kit	-1.555	0.307	-5.061	0.000	***
Utz	-0.543	0.149	-3.635	0.000	***
Wise	-0.505	0.165	-3.062	0.002	**
Wise Ridgies	-0.984	0.167	-5.888	0.000	***
Volume	0.469	0.113	4.142	0.000	***
Package:					
Canister	0.437	0.091	4.800	0.000	***
Canister In Box	0.453	0.130	3.494	0.000	***
Flavor:					
BBQ	0.167	0.066	2.534	0.011	*
Cheddar	0.241	0.080	3.026	0.002	**
Cheese	-0.443	0.205	-2.164	0.031	*
Ketchup	-0.680	0.244	-2.787	0.005	**
Onion	0.339	0.066	5.107	0.000	***
Original	0.704	0.061	11.588	$<$ 2e-16	***
Spicy	-0.211	0.105	-2.005	0.045	*
Salt: No Salt	-0.446	0.212	-2.099	0.036	*
Type of Cut: Flat	0.308	0.070	4.411	0.000	***
Store Fixed Effects	Yes				
Week Fixed Effects	Yes				
Adjusted R-squared	0.884				
Significance	0 ***	0.001 **	0.01 *	0.05 .	

Table 4: Logit with Regression Selection

Log Share	Estimate	Std. Error	t value	$\Pr(>\lvert t\rvert)$	
Log Price	0.296	0.113	2.624	0.009	**
Promotion	-0.441	5.192	-0.085	0.932	
Feature: None	0.263	0.151	1.745	0.081	.
Display:					
Minor	-0.215	0.104	-2.080	0.038	*
Major	-0.338	0.113	-3.000	0.003	**
Store Fixed Effects	No				
Week Fixed Effects	No				
AIC	6884.4				
Significance	0 ***	0.001 **	0.01 *	0.05 .	

6.2 Logit

We followed Berry et al. (1995) to do a logit-type model of market shares. We project estimated $\hat{q}_{jmt}$ on product attributes dummies, store fixed effects and week fixed effects. Then we sum $\hat{q}_{jmt}$ over stores and weeks. Assuming that market sizes are fixed, we calculate market share by dividing $\hat{q}_{jmt}$ by market size. The log of market share is taken as the dependent variable in our Logit model. Table 4 shows the output of Logit model with traditional regression where only the significant coefficients are displayed.

6.3 Stepwise, Forward Stagewise and Sqrt LASSO

In practice, stepwise and forward stagewise can be realized in R package **lars**. We take the default parameter t and λ in the package. These three model converge in similar ways where in each step, the most important variable is added to the model. We limit the maximum number of steps to 100 in our practice for demonstration purposes because it takes significantly longer to converge if the number of steps is larger.

Square Root Lasso is from the *slim* function of R package **flare**, where *slim* stands for Sparse Linear Regression using Non-smooth Loss Functions and L1 Regularization. In function *slim*, we set type=lq and q=2 (loss function) to use the Square Root Lasso. Default number of lambdas in the sequence is 5. We set it to be 40. Default min/max value of the sequence of lambdas is 0.3*max and $\pi\sqrt{(log(d)/n)}$. The authors of the **flare** package stated that SQRT Lasso is tuning insensitive based on Belloni et al. (2011).

Table 5: Random Forest Variable Importance

Log Quantity	%Increase in Mean Squared Error	Increase in Node Purity
Log Price	74.83	1196.68
Volume	56.81	855.79
Display: Minor	49.79	455.98
Promotion	43.76	519.72
Display: Major	43.29	267.43
Feature: None	42.05	592.37
Brand: Lays	39.82	367.29
Brand: Ruffles	33.21	76.97
Brand: Wavy Lays	32.95	143.46
Flavor: Classic	32.31	219.00
Flavor: Sour Cream & Onion	30.26	62.28
R-Squared	0.42	

6.4 Random Forest

Random forecast is implemented in R package **randomForest**. Random forest has a few parameters about the tree structure: *mtry* as the number of variables to randomly select at each tree, *ntree* as the number of trees in the model, *nodesize* as the minimum size of the terminal nodes of each tree, and *maxnodes* as the maximum number of terminal nodes each tree can have.

We first run a cross-validated version of random forest (function *rfcv*) to determine the best number of variables to sample at each tree. This function uses K-fold cross-validation. We set scale=log, step=0.5, which means we will drop 50% of the variables in every step.After we determine the best number of variables to try at each tree, a random forest model (function *randomForest*) was executed with the optimal *mtry* and the other parameters at default values. The default value for *nodesize* is 5, *ntree* 500 and *maxnodes* NULL. The cross-validation function *rfcv* does not decide the optimal values for these parameters.

Table 5 displays the variable importance of the twelve most important variables in determining Log Quantity using two metrics. The percentage *increase in mean squared error* is the increased percentage of mean squared error if a variable is excluded from the model. Node purity measures how much the additional variable or tree split reduces the residual sum of squares. Thus, the *increase in node purity* measures the size change in node purity if a variable is excluded from the model.

6.5 Support Vector Machine

We use functions *svm* and *tune.svm* in R package **e1071** to implement the support vector machine model. We specify type=eps-regression as our dependent variable continuous.We specify a 10-fold cross-validation in the support vector machine.

We use a tuning function *tune.svm* to do the 10-fold cross-validation of SVM to determine the best parameters of *epsilon*, *cost* and *gamma*. *Epsilon* is ϵ in the loss function with default 0.1. We test *epsilon* from 0 to 1 with interval 0.2. *Cost* is the constant of the regularization term in the Lagrange formulation. We test *cost* from 0.5 to 2 with interval 0.5. *Gamma* is the parameter in the kernel function. We test *gamma* from 0.01 to 0.1 with length of five. The kernel used in the training is *radial basis* - Gaussian. This is tested in our data to out perform the other kernels. The other kernel options are polynomial, linear and sigmoid. Here are the kernel functions:

- linear: $u'v$
- polynomial: $(\gamma u'v + coef0)^{degree}$
- radial basis: $exp(-\gamma|u - v|^2)$
- sigmoid: $tanh(\gamma u'v + coef0)$

6.6 L_2 Boost

L_2 Boost is from function *glmoost* of R package **mboost** for boosting with component-wise least squares. We define the loss function as the squares of errors and define the negative gradient and stopping parameter for the *glmoost* function.

Table 6 shows the coefficients from L_2 Boost where only 36 out of 2243 variables have non-zero coefficients (we show only a portion of the non-zero coefficients).

6.7 Combined Model

In order to compare models, we want to split the data into training and testing set, where we train the model using the training set and pretend the dependent variable in the test set is unknown and predict on the test set. Because we have eight models, we want to assign weight to each model when building a linear combination. However, the weight based on the training set is biased. Some models like linear model tend to get a good in-sample fit but a

Table 6: L_2 Boost Coefficients

Log Quantity	Coefficient
Log Price	-19.57
Promotion	18.24
Feature: Medium Ad	4.79
Feature: None	-19.85
Display: Minor	12.78
Display: Major	18.88
Brand: Kettle Chips	-3.41
Brand: Lance Thunder	-0.48
Brand: Lays	16.50
Brand: Lays Stax	-2.30
Brand: Ruffles	6.26
Brand: Wavy Lays	10.06
Flavor: Classic	11.30
Flavor: Sea Salt & Vinegar	-0.45
Type: Potato Chip and Dip	-0.49
Type: Potato Crisp	-1.10
Package: Canister in Box	-4.08
...	

bad out-of-sample fit, and this grants these model a very large in-sample weight which could be misleading.

Therefore, we randomly split the data into three pieces to do cross validation, where the model weights are determined on the validating set. This three way data partitioning mitigates the possible large out-of-sample error for some models when over fitting happens. 25% of the data is used as the test set, 15% is used as the validate set, and the remaining 60% is used as the training set. Table 8 shows how the data is sliced into three pieces.

6.7.1 Combining Models with Asymptotic Properties

We combine six models that has asymptotic properties using a constrained linear regression, in Table 7. The six models are: linear regression, Square-root Lasso, Support Vector Machine, L_2 Boosting, Logit with market share predicted by a regression model. The response variable is log of quantity sold per week. The covariates are log of price, product attributes variables, promotional variables, store fixed effects, and week fixed effects. We provide the same covariate matrix to all of the models expect for the Logit model, where all the fixed effects are excluded.

We combine them using the following algorithm:

1. Fit the actual dependent variable in the training set on each of the five models.

Table 7: Linear Model Combination: Models with Asymptotics

	Train		Validation		Test		
	RMSE	Std. Err.	RMSE	Std. Err.	RMSE	Std. Err.	Weight
Linear	0.766	0.010	0.994	0.017	1.010	0.015	10.41%
Sqrt Lasso	0.977	0.007	0.984	0.013	0.995	0.009	1.71%
Support Vector Machine	0.543	0.007	0.889	0.018	0.900	0.012	87.57%
L2 Boosting	1.053	0.004	1.016	0.013	1.028	0.012	0.00%
Logit	3.282	0.170	3.509	0.340	3.915	0.263	0.32%
Linearly Combined			0.887		0.898		100.00%
# of Obs	1,827,308		456,827		761,379		
Total Obs	3,045,513						
% of Total	60%		15%		25%		

2. Make out-of-sample prediction using the model coefficients from the first step on the validate set.

3. Take only the validation set. Treat the predicted values of the dependent variable from the five models as regressors, treat the actual value of the dependent variable as the response variable, then do a constrained linear regression. The sum of the coefficients are constrained to be one. Get all the coefficients and regard them as weights in the combined model.

4. Use the fitted models to predict in the test set, and apply the weights from validate set to each model, sum them up and form the linearly combined prediction.

In Table 7, support vector machine gets the most weight out of the five models due to its small prediction error (measured by RMSE) in the validation set. Its prediction error in the test set is also the smallest among all models.

6.7.2 Combining All Models

In Table 8, we compare eight models: two of them are traditional econometrics models and seven of them are more in the context of machine learning as introduced before. Our purpose is to run a horse race of models by comparing out-of-sample prediction errors.

Table 8 shows the comparison of the models. In the scenario of out-of-sample prediction error, the best two models are random forest and support vector machine. The combined model, where we regress the actual value of the response variable on a constrained linear model of the predictions from eight models, outperforms all the eight models, which follows

Table 8: Model Comparison: Prediction Error

	Train		Validation		Test		
	RMSE	Std. Err.	RMSE	Std. Err.	RMSE	Std. Err.	Weight
Linear	0.766	0.010	0.994	0.017	1.010	0.015	17.73%
Stepwise	0.930	0.008	0.969	0.017	0.980	0.014	0.00%
Forward Stagewise	0.977	0.007	0.985	0.015	0.995	0.013	0.00%
Sqrt Lasso	0.977	0.007	0.984	0.013	0.995	0.009	0.00%
Random Forest	0.927	0.007	0.914	0.017	0.916	0.013	37.46%
Support Vector Machine	0.543	0.007	0.889	0.018	0.900	0.012	44.79%
L2 Boosting	1.053	0.004	1.016	0.013	1.028	0.012	0.00%
Logit	3.282	0.170	3.509	0.340	3.915	0.263	0.02%
Linearly Combined			0.879		0.887		100.00%
# of Obs	1,827,308		456,827		761,378		
Total Obs	3,045,513						
% of Total	60%		15%		25%		

the optimal combination of forecasts in Bates and Granger (1969). Random forest and support vector machine get more weights in the combined model due to their good performance out-of-sample.

Based on Section 3, the combination of models converges to asymptotic normal distribution at $\sqrt{n}$ rate, regardless of the what individual models there are. Therefore, we could bootstrap the combined model to get the confidence interval, knowing that it is asymptotically normal.

Table 9 provides the summary statistics of the residual between predicted and actual values of quantity in both validation and testing set.

6.8 Asymptotics

We dedicate this section to show empirically the asymptotics of the selected machine learning models mentioned in Section 3. We use our dataset but limit the number of observations to a sequence of smaller numbers. We run the same model specification as the main application for these datasets of different sizes and compute standard errors of the root mean square error in prediction using subsampling. As Table 10 Figure 1 shows, in both validate and test set, the standard errors are going to zero when the number of observation grows.

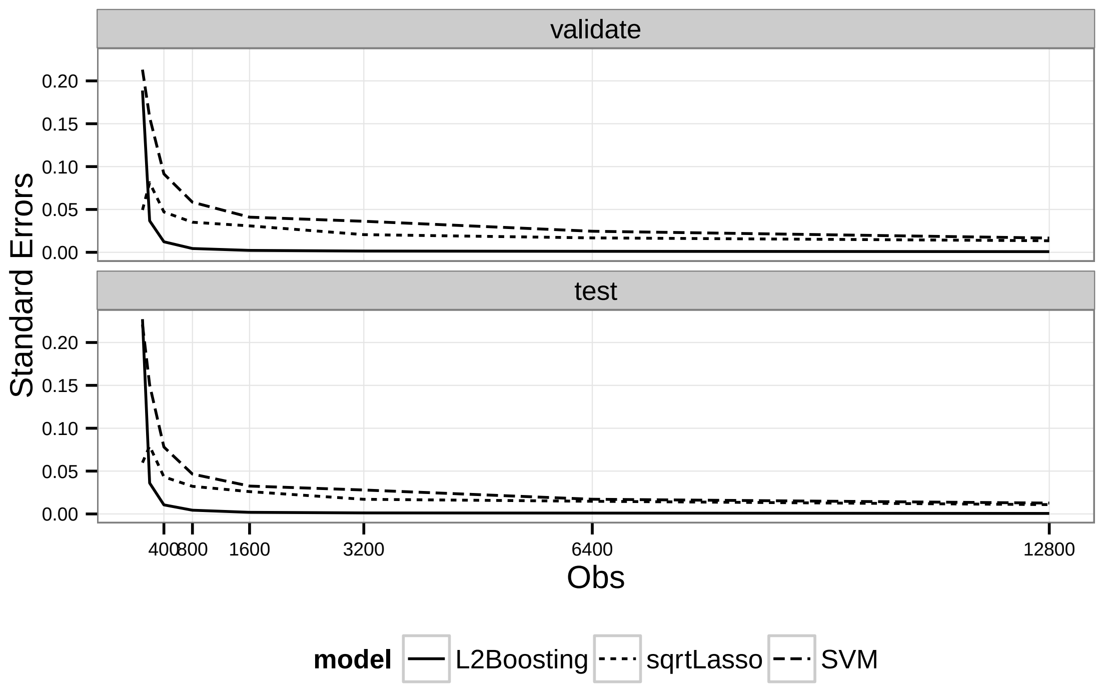
Table 9: Summary Statistics of Residual in Prediction

Validation Set	Mean	Std. Dev.	Median	Min	Max
Linear	-0.019	1.549	-0.015	-5.402	4.862
Stepwise	-0.023	1.440	0.018	-4.942	4.365
Stagewise	-0.023	1.353	0.040	-4.549	3.975
Sqrt Lasso	-0.023	1.347	0.011	-5.501	4.035
Random Forest	0.010	1.424	0.073	-4.591	4.618
Support Vector Machine	-0.010	1.462	0.008	-4.387	5.167
L2 Boost	-0.024	1.399	0.025	-4.952	4.661
Logit	-0.244	3.566	0.154	-39.141	3.135
Combined	-0.004	1.466	0.028	-4.811	5.053
Test Set	**Mean**	**Std. Dev.**	**Median**	**Min**	**Max**
Linear	-0.012	1.533	0.013	-5.385	5.387
Stepwise	-0.029	1.429	-0.026	-5.568	4.726
Stagewise	-0.072	1.402	-0.073	-4.521	3.869
sqrt Lasso	-0.056	1.375	-0.066	-6.262	4.069
Random Forest	0.088	1.489	0.168	-5.912	4.802
Support Vector Machine	0.004	1.515	0.012	-5.771	5.430
L2 Boost	-0.080	1.462	-0.031	-5.888	4.379
Logit	-0.322	3.910	0.154	-38.241	2.833
Combined	-0.033	1.435	-0.038	-5.126	4.698

Table 10: Convergence Rate for Three Machine Learning Models

Sample Size	100	200	400	800	1600	3200	6400	12800
Validate								
Sqrt Lasso	0.2025	0.1380	0.0909	0.0566	0.0382	0.0320	0.0225	0.0162
Support Vector Machine	0.1613	0.1180	0.0767	0.0446	0.0375	0.0222	0.0162	0.0137
L2 Boosting	0.0487	0.0788	0.0467	0.0325	0.0318	0.0223	0.0177	0.0133
Combined	0.1944	0.1468	0.0828	0.0574	0.0417	0.0316	0.0258	0.0159
Test								
Sqrt Lasso	0.2125	0.1364	0.0790	0.0492	0.0292	0.0259	0.0170	0.0130
Support Vector Machine	0.1635	0.1245	0.0765	0.0435	0.0299	0.0186	0.0142	0.0099
L2 Boosting	0.0621	0.0786	0.0466	0.0313	0.0261	0.0185	0.0138	0.0099
Combined	0.2251	0.1476	0.0813	0.0519	0.0367	0.0258	0.0199	0.0136

Figure 1: Standard Errors of RMSE



6.9 Discussion

6.9.1 Variable Selection

When the number of independent variables is large, it is common to have some degree of multicollinearity. The shrinkage models could help us reduce the multicollinearity intelligently. A usual statistics to determine multicollinearity is Variance Inflation Factors (VIF). The VIF for covariate X_j is defined as:

$$VIF_j = \frac{1}{1 - R^2_{-j}} \tag{9}$$

where R^2_{-j} is the R-squared by regressing covariate X_j on the rest of covariates. Table 11 shows some VIFs for the independent variables used in the linear regression model. To compare, after LASSO selects variable, we also have the VIFs for the independent variables in the linear regression using only selected variables. The VIFs in the non-selecting case are much higher than the LASSO selection for many variables. Using 5 as a threshold for multicollinearity, LASSO successfully reduces multicollinearity in the independent variables.

Table 11: Variance Inflation Factors

	VIFs after Selection	
Variable	**Linear**	**LASSO**
Product Type - Potato Chip And Dip	$+\infty$	3.5084
Brand - Ruffles Snack Kit	$+\infty$	3.4729
Logprice	4.1750	3.2319
Volumn	3.9775	3.1541
Cooking - Missing	$+\infty$	3.1100
Cooking - Kettle	$+\infty$	2.6495
Package - Canister	$+\infty$	1.8047
Fat - Regular	76.6610	1.5930
Brand - Lays	104.5904	1.5187
Promotion	1.4806	1.4388
Feature - None	2.3398	1.3369
Brand - Kettle Chips	27.3608	1.3222
Flavor - Original	2.8610	1.2875
Brand - Ruffles	50.1427	1.2802
Salt - Regular	3.0660	1.2732
...		

We believe it's more attractive than ad hoc variable selection procedures commonly used in practice.

6.9.2 Random forest on the individual model pieces

We use the predicted values of all the eight models from the validation set as input variables and use the actual log quantity as the output variable to train the random forest model. We do a 10-fold cross-validated version of random forest to determine the best parameters (same approach as described in Section 6.4) and then predict the output variable log quantity in the test set. Then we evaluate the prediction by the combined model with RMSE.

Table 12 shows combining the eight models using a random forest model instead of linear combination in Table 8. The importance of each individual model is shown in the right column, which is similar to the Weight column of Table 8.

6.9.3 Bag of Words (XXX: move up to main text)

The mere feat of encoding a vector of hedonic attributes commonly used in demand estimation for this data could be cumbersome. In applied econometrics, researchers commonly restrict attention to the products with the largest demand and encode regressors for brands and a small vector of product attributes. In our application, we propose using unsupervised

Table 12: Combining Models in Random Forest

	Train		Validation		Test		
	RMSE	Std. Err.	RMSE	Std. Err.	RMSE	Std. Err.	Var. Imp.
Linear	0.766	0.010	0.994	0.017	1.010	0.015	32.435
Stepwise	0.930	0.008	0.969	0.017	0.980	0.014	32.647
Forward Stagewise	0.977	0.007	0.985	0.015	0.995	0.013	24.607
Sqrt Lasso	0.977	0.007	0.984	0.013	0.995	0.009	23.135
Random Forest	0.927	0.007	0.914	0.017	0.916	0.013	34.845
Support Vector Machine	0.543	0.007	0.889	0.018	0.900	0.012	52.972
L2 Boosting	1.053	0.004	1.016	0.013	1.028	0.012	23.977
Logit	3.282	0.170	3.509	0.340	3.915	0.263	0.932
Combined by Random Forest			0.920		0.902		
# of Obs	1,827,308		456,827		761,378		
Total Obs	3,045,513						
% of Total	60%		15%		25%		

Note: In random forest, variable importance is defined as the average difference in out-of-bag mean squared error with and without permuting the variable, divided by the standard deviation of the difference. The bigger the variable importance value, the more important the variable is.

learning to construct product level regressors. In particular, we use the unstructured text that describes the product in the raw data and apply the bag of words model. This has the advantage of being a simpler computation and allows us to avoid the arduous task of encoding attributes for thousands of products. Since it is more scalable, it allows us to model the demand for all of the products rather than restricting attention to the products with the largest demand. Also, it could be viewed as less ad hoc than hand coding product attributes since it imposes fewer a priori restrictions on the hedonic attributes that we should include as regressors in our model. We note that there is a large literature on this form of unsupervised learning which sometimes goes by the name of feature extraction in computer science. With the exception of Gentzkow and Shapiro (2010), there has been relatively little use of this method in economics. We believe that this is promising for demand estimation because it allows a new source of data to construct covariates such as the raw text in product descriptions or product reviews.

Aside from structured features like package type, cut type of potato chips, the hedonic attributes of the product can also be defined by unstructured texts that describe the product. A bag of words model could easily turn the unstructured texts into large amounts of features. Unsupervised learning (clustering) on these features could be a more scalable approach than hand coding them. The procedures following the unsupervised learning are the same as the way we deal with structured features. Yet this straightforward approach has better prediction power than structured models.

The bag of words model analyzes a corpus of K documents, comprising a dictionary of M words, and finds the relations of words and documents. In our case, the K documents are the n unique potato chips descriptions. We cluster the descriptions, via manipulation of the document-term matrix. A document-term matrix is a mathematical matrix that describes the frequency of terms that occur in a collection of descriptions. In a document-term matrix, rows correspond to each unique product in the collection and columns correspond to terms. Where it is tedious to encode all the features of a product, bag of words provides a simple way to exploit the rich features of products.

In a high dimensional learning problem, only some parts of an observation are important to prediction. For example, the information to correctly categorizing a product may lie in a handful of its words. The extraneous words could prove computational burdensome, so word regularization may be helpful. Possible methods for regularization include LASSO and other shrinkage models. Therefore it's natural to combine the technique of bag of words with our models.

6.9.4 Top/Tail Products

In marketing literature, people usually prune out the tail products (for example Nevo (2001)). But, with the new methods, we want to show that a training data set with all possible products predicts better than a data set with only the top products. To show that, we take only the top twenty products to train the model and predict the sales of the tail products and compare the prediction fit to those in Table 8.

We use the same eight models as modeling examples to demonstrate the difference. We rank all the products by total units sold and only mark the top twenty as the training set. Using exactly the same methodology in our main application, we get the root mean squared error as a measure of fit for eight plus combine model. The prediction error and weights is presented in Table 13.

As Table 13 shows, when we only train on top twenty products, the in-sample predictions (top 20 products) are very good, but the out-of-sample predictions (other products) are not so good, in contrast to Table 8, where the in-sample and out-of-sample error are very close when we randomly split the data for training. As the top twenty products may not necessarily explain all the features of the rest of the products, the predicting power for the other products is therefore weakened.

Table 13: Top 20 Products vs. the Other Products

	Top 20 Products		Other Products		
	RMSE	Std. Err.	RMSE	Std. Err.	Weight
Linear	0.397	0.034	2.037	0.037	35.37%
Stepwise	0.768	0.023	1.437	0.024	0.00%
Forward Stagewise	0.882	0.017	1.371	0.018	0.00%
Sqrt Lasso	0.935	0.015	1.374	0.017	0.00%
Random Forest	0.759	0.018	1.530	0.017	0.00%
Support Vector Machine	0.318	0.042	1.537	0.020	64.63%
L2 Boosting	0.920	0.021	1.378	0.019	0.00%
Logit	1.331	0.124	2.685	0.134	0.00%
Linearly Combined	0.277		1.433		100.00%
# of Obs	504,337		2,541,176		
Total Obs	3,045,513				
% of Total	16.56%		83.44%		

6.9.5 Practical Advantages

There are some other practical advantages to the machine learning models or their generic approaches.

In random forest, the missing values could be imputed, for example, as the median of its column. This imputation will not effect accuracy much since the randomness of subsampling and the trees grown. Another approach for categorical variables is to simply create a new category called "missing." This new category might capture the behavioral differences in observations with missing values and the ones not missing.

If we have a large dataset and we want to do model selection or model assessment, the best approach is to randomly divide the dataset into three parts: a training set, a validation set, and a test set. The training set is used to fit the models; the validation set is used to estimate prediction error for model selection; the test set is used for assessment of the generalization error of the final chosen model. The test set should be kept separately and be brought out only at the end of the analysis.

If, however, we do not have enough data to split three ways, we can efficiently re-use the sample data by cross-validation or bootstrap.

In addition to the models we mentioned in our paper, there are some other very popular machine learning approaches that could be helpful to readers in economics. For example, EM (Expectation-Maximization) algorithm for simplifying difficult maximum likelihood problems; Graphical models for complicated conditional expectations; Neural Networks for

Table 14: Promotion Variables

Variable	Value	Frequency(%)
Promotion	Price Reduction < 5%	74.11
	Price Reduction >5%	25.89
Feature	Large Ad	5.35
	Medium Ad	4.98
	Small Ad	0.33
	None	89.33
Display	None	81.16
	Minor	10.66
	Major	8.18
Total Obs	3,045,513	

extracting linear combination of features and modeling response variable as a non-linear function of the features, and so on.

6.10 Lift Estimates

The other interesting problem we want to look at is estimating promotional lift. In our data, a product is tagged as on promotion if the price reduction is greater than 5%. By considering promotion as a randomized treatment, we follow the methodology suggested by Varian (2014) to estimate the promotional lift. Our models are trained on the control group (no promotion) and then used to predict out of sample on the treated group (on promotion). The control/treatment partition is based on whether the product is on price reduction promotion, instead of random assignment in the last example.

The most important variables we want to study are the price promotion as well as how the store display the products. A product is tagged as in promotion if the price reduction is greater than 5%. There are four levels of feature: large, medium, small and no ad. There are three levels of display: major (including lobby and end-aisle), minor and none. Table 14 shows the frequency of promotion in the dataset.

If everything else is the same in the control group and the treatment group, we believe the difference between predicted and actual in treatment group is our treatment effects. Based on model coefficients from the control group, we predict the quantities using the independent variables in the treated group. The difference between the predicted and the actual value of quantity sold in the treated group is therefore our lift estimates.

The lift estimates from eight models mentioned before are in Table 15. We split the

Table 15: Model Comparison:Promotional Lift

	Mean	t	95% Conf. Int.		Weight
Linear	9.646	8.171	7.332	11.960	23.37%
Stepwise	20.124	19.516	18.103	22.145	7.01%
Stagewise	22.458	21.018	20.363	24.552	0.00%
Sqrt Lasso	22.440	21.006	20.346	24.534	0.00%
Random Forest	18.276	17.705	16.253	20.299	68.00%
Support Vector Machine	25.920	23.428	23.752	28.089	0.00%
L2 Boost	22.995	21.386	20.887	25.102	0.00%
Logit	22.671	20.474	20.500	24.841	1.61%
Linear Combination	19.017	18.456	16.998	21.037	100.00%

control group where there is no promotion into training and validation set. We fit the same models as we used in the main application using the training data and combine them linearly with the validation data set. Then we predict the model using the treated group where there is promotion for the product. The weights form the linear combination of models with the validation set is used to construct the combined model in the treated group. All the machine-learning models calculate a similar promotional lift when the linear model produces a smaller lift for promotion.

6.11 Computation Tools

In this application we face constraints on memory and CPU when processing the data with millions of observations with complicated machine learning models. The time it takes to compute a Random Forest object with data of such size using a single work station could be days, even weeks. A solution to solve the computation constraints is to utilize high performance computing tools, such as parallel processing using Revolution R® and MATLAB® Parallel Computing Toolbox™. Both of them are available on Amazon Web Services(AWS) Marketplace at low costs.

Revolution R has two major packages - RevoR and ScaleR. RevoR automatically uses all available cores and processors to reduce computation times without modifying a standard R script. A real time simulation shows it speeds up computation by three to thirty times compared to a single core standard R process. ScaleR scales up data size to 1 to 16 tera-bytes easily by dividing data into pieces and accessing the pieces with different processors.

In these parallel processing frameworks, a common implementation is MapReduce. It is useful when the data is too large to be held or processed in memory. There are two stages in MapReduce: Map and Reduce. In the Map stage the program (Revolution R$^{\circledR}$ or MATLAB$^{\circledR}$) pre-processes each distributed data trunk in parallel and performs preliminary statistical modeling on each data trunk in parallel. In the Reduce stage, the program gathers all the information for each distributed data trunk from Map stage, summarizes the information and returns it to the user. This is much faster and takes less memory than storing and accessing data directly from memory.

7 Conclusion

In this paper, we survey a set of models from statistics and computer science. We select a few machine learning models to compare with traditional econometric models. The models we focus on in this paper include linear regression as the baseline model, logit as the econometric model, stepwise, forward stagewise, LASSO, random forest, support vector machine and bagging as the machine learning models. We derive novel asymptotic properties for the machine learning models.We use Monte Carlo simulation to demonstrate the properties of the models and also show that combining all the underlying models with a linear regression improves out-of-sample prediction accuracy.

We illustrate the properties of these models by using a real world data set with scanner panel sales data of potato chips. First, we compare the prediction accuracy of these models and the machine learning models consistently give better out-of-sample prediction accuracy while holding in-sample prediction error comparable. By combining all the models via weighted linear regression, we are able to improve the out-of-sample prediction accuracy even more. Second, we compare two scenarios where one, as the traditional marketing literature suggests, prunes the market to only the top sold products, and the other, our approach, uses a mix of both top and tail products. Our approach has better prediction accuracy in demand. Third, we estimate the promotion lift using the predictions for treatment effects. Last, we explored the unstructured text of product description from the raw data and apply the bag of words model. This has the advantage of being a simpler computation and allows us to avoid the arduous task of encoding attributes for thousands of products.

Our approach is robust to a large number of potentially collinear regressors; it scales easily to large data sets; the linear combination method selects the best model automatically and produces the best in-sample and out-of-sample prediction accuracy; and the method can flexibly approximate arbitrary non-linear functions, even when the set of regressors is high dimensional and we also allow for fixed effects.While demand estimation is our motivating application, we believe that the approach we have proposed can be useful in many other microeconometric settings.

References

Abadie, A. and J. Gardeazabal (2003). The economic costs of conflict: A case study of the basque country. *American economic review*, 113–132.

Andrews, D. W. (1991). Asymptotic normality of series estimators for nonparametric and semiparametric regression models. *Econometrica: Journal of the Econometric Society*, 307–345.

Bates, J. M. and C. W. Granger (1969). The combination of forecasts. *Or*, 451–468.

Belloni, A., D. Chen, V. Chernozhukov, and C. Hansen (2012). Sparse models and methods for optimal instruments with an application to eminent domain. *Econometrica 80*(6), 2369–2429.

Belloni, A., V. Chernozhukov, and C. Hansen (2014). Inference on treatment effects after selection among high-dimensional controls. *The Review of Economic Studies 81*(2), 608–650.

Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile prices in equilibrium. *Econometrica 63*(4), 841–90.

Chen, X. (2007). Large sample sieve estimation of semi-nonparametric models. *Handbook of econometrics 6*, 5549–5632.

Cortes, C. and V. Vapnik (1995). Support-vector networks. *Machine learning 20*(3), 273–297.

Gentzkow, M. and J. M. Shapiro (2010). What drives media slant? evidence from u.s. daily newspapers. *Econometrica 78*(1).

Meinshausen, N. and P. Bühlmann (2006). High-dimensional graphs and variable selection with the lasso. *The Annals of Statistics*, 1436–1462.

Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica 69*(2), 307–342.

Newey, W. K. (1994). The asymptotic variance of semiparametric estimators. *Econometrica: Journal of the Econometric Society*, 1349–1382.

Newey, W. K. (1997). Convergence rates and asymptotic normality for series estimators. *Journal of Econometrics 79*(1), 147–168.

Rajaraman, A. and J. D. Ullman (2011). Mining of massive datasets. *Lecture Notes for Stanford CS345A Web Mining 67*(3), 328.

Varian, H. R. (2014). Big data: New tricks for econometrics. *The Journal of Economic Perspectives 28*(2), 3–27.