STRATEGIC EXPERIMENTATION WITH ASYMMETRIC INFORMATION

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I study strategic experimentation, with one player initially being better informed about the state of nature than the other. Players are otherwise symmetric, and observe past experimentation decisions and outcomes. I construct an equilibrium in which a mutual encouragement effect arises: as the public information becomes discouraging, the informed player's high effort continuously brings in good news, encouraging the uninformed player to experiment; in return, the uninformed player's experimentation pattern yields an increasing reward, encouraging the informed player to experiment. Due to this effect, players' total effort can increase over time, and the uninformed player may grow increasingly optimistic, despite the discouraging public information. Moreover, creating information asymmetry improves total welfare if the informed player's initial signal is sufficiently precise.

Keywords: Strategic experimentation, signaling, learning, asymmetric information.

1. INTRODUCTION

Experimentation, including learning from one's own and others' experiments, is an important mechanism through which agents discover and explore new ideas, thereby promoting technological change, and driving economic growth.[1] In many relevant applications, some agent initially is *better informed* about the value of experimentation, either due to agents' heterogeneous background or companies' marketing effort. For

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[1]Endogenous technological change is a key driver of economic growth, as argued by endogenous growth theory (Romer, 1990; Aghion and Howitt, 1992). On the role of experimentation in the discovery and selection of new ideas, see Romer (1994, page 12), Nelson and Winter (1994, Chapter 11).

example, in a strategic alliance that is marketing a new product, an incumbent firm knows more about the potential demand than an entrant; in a village where farmers experiment with a new fertilizer, an experienced farmer knows more about the fertilizer's profitability than a novice farmer; when physicians learn through each others' prescriptions the efficacy of a new drug, a specialist physician knows more about the pros and cons of the drug (due to education or pharmaceutical detailing[2]).

In such environments, on the one hand, the information gathered from experimentation is a public good, creating a free-riding problem: agents experiment less than if they acted cooperatively. This free-riding problem has been studied under symmetric information by Bolton and Harris (1999), Keller et al. (2005), and formed the basis of a large literature.[3] On the other hand, exactly because information is a public good, with information asymmetry, the better informed "leader" then has an incentive to use his private information to motivate the less-informed "follower" to acquire more information, which may counterbalance free-riding.

How does the initial asymmetric information affect agents' experimentation behavior? Does it mitigate or exacerbate free-riding? For policy makers who aim to improve technology diffusion or for companies that market new products, would it be desirable to induce information asymmetry initially (for instance, by hiding information from some agent, or by targeting a certain agent)?

The central contribution of this paper is to show that initial information asymmetry can qualitatively change agents' experimentation behavior — agents can *increase* experimentation even after a history of unsuccessful experiments (that is, experiments that do not lead to any breakthrough), unlike in the symmetric information setting. The key mechanism is a novel *mutual encouragement effect*: a better-informed player — the leader — signals good news through persistent high (experimentation) effort, encouraging an uninformed player — the follower — to experiment; the follower follows his lead, and increases her effort over time to encourage the leader to persevere when the leader becomes too pessimistic. Thanks to this effect, inducing information asymmetry mitigates free-riding, leads to more learning and can improve total welfare.

This paper builds on the two-player version of the exponential-bandit model (Keller et al., 2005). At each point in time, each player must divide a unit of resource between

[2] Detailing refers to the activity of pharmaceutical sales representatives providing physicians with "details" related to a drug—approved scientific information, benefits, side effects, or adverse events.

[3] The free-riding effect has also been well documented empirically, for instance, by Foster and Rosenzweig (1995).

a safe project with known payoffs and a risky project of unknown quality. Learning is conclusive: only good risky projects deliver payoffs (breakthroughs), governed by a Poisson process. Players observe past experimentation decisions and payoffs. I add one source of information asymmetry: at date 0, one player, called the informed player (he), privately observes a binary noisy signal. He thus becomes either an *optimistic* type whose posterior belief is higher than the uninformed player's (she), or a *pessimistic* type.

With asymmetric information, a public history carries two components of information. The first component is the information generated from the experimentation technology, represented by the informed player's beliefs. This component is "passive," as it depends only on the public history. The second component is the private information that the informed player reveals to the uninformed player, represented by the uninformed player's belief about the informed player being the optimistic type, called his reputation. This component is "strategic," as it depends also on the informed player's strategies.

I construct a Markov perfect equilibrium (MPE) using these two components of information as state variables. During a gradual revelation phase of this equilibrium, the pessimistic type mixes between mimicking the optimistic type's high effort and revealing himself such that, as long as he keeps mimicking, his reputation gradually increases. This rising reputation induced by the informed player's high effort (the strategic component) counterbalances the pessimism induced by the absence of a breakthrough (the passive component), and encourages the uninformed player to increase her effort over time.[4]

This rising effort dynamics of the uninformed player occurs when the pessimistic type's belief is such that neither player would experiment if his signal were public. Intuitively, during the gradual revelation phase, the pessimistic type has to be indifferent between mimicking the optimistic type so as to be willing to convince the uninformed player to exert effort, and revealing himself, thereby inducing both players to stop experimentation. The marginal value of both players' efforts to the pessimistic type is dropping over time due to the absence of a breakthrough; for him to be indifferent, the uninformed player has to accelerate her information production to reward the pessimistic type's persistence.

The joint behavior pattern during the gradual revelation phase — the informed

[4]The uninformed player may even become increasingly optimistic about the risky project before a breakthrough occurs, which is another novel qualitative impact of information asymmetry.

player maintaining high effort and the uninformed player increasing her effort despite the absence of a breakthrough — admits the following intuitive interpretation. Leaders motivate followers through role modeling: a leader articulates an appealing vision, which may or may not be reachable; however, as the leader sees further and more accurately than his follower,[5] his putting in long hours during setbacks gradually convinces the follower of his optimism about the vision, and hence motivates the follower to work harder. That leaders enhance followers' commitment to their visions through role modeling is a recurring theme in both modern leadership theories[6] and leadership guidelines in popular management books.[7]

The constructed MPE exists if the initial signal of the informed player is precise enough, and the fraction of the pessimistic type is not too low. If the prior belief is not too low, then the distribution of the equilibrium paths of the constructed MPE is unique among the MPEs that satisfy (1) that players play the symmetric MPE after the informed player reveals his type (on path); (2) a criterion in the spirit of D1.

The mutual encouragement effect leads to interesting welfare implications. Suppose a social planner (she), who cares about total welfare, observes the informed player's private signal. Would she hide it from the uninformed player? I find that, if the signal is precise enough, she would hide it. Intuitively, in such cases, the optimistic type, being still optimistic before revealing his type, has little to learn, and thus suffers little from asymmetric information. The pessimistic type and the uninformed player, on the other hand, benefit significantly from the mutual encouragement effect. As a result, asymmetric information improves total welfare.

Drawing from this welfare implication, a policymaker aiming at promoting new technology adoption may find it desirable to target certain individuals first by giving them relevant information or training. Companies promoting new experience goods might find it profitable to target some consumers, say early adopters, or experts; indeed, pharmaceutical companies spend huge amounts of money targeting marketing activities at "opinion leaders," for instance, by giving them detailed information about

[5] See for instance, page 2 of March and Weil (2009).

[6] Examples are charismatic leadership theory, transformational leadership theory (Bass and Bass, 2009; Yukl, 2010), and authentic leadership theory (Gardner et al., 2005; Avolio and Gardner, 2005).

[7] For instance, Yukl (2010) gives the following guidelines "for leaders seeking to inspire followers and enhance their self-confidence and commitments to the mission": articulate a clear and appealing vision; explain how the vision can be attained; act confident and optimistic; express confidence in followers; use dramatic, symbolic actions to emphasize key values; and lead by example (role modeling). See page 290–293.

their new drugs, a process called detailing (Nair et al., 2010).

2. LITERATURE REVIEW

Strategic experimentation in a non-competitive team environment was first introduced by Bolton and Harris (1999), in a two-armed Brownian bandit model. They analyze how an encouragement effect — a player's future effort encourages another player to experiment now — interacts with the free-riding effect. Keller et al. (2005) propose the exponential bandit model to analyze the experimentation problem. Notably, they find that no encouragement effect arises in (Markov) equilibria: players do not acquire more information than a single player does.[8] In both papers, players are symmetrically informed. This paper introduces initial asymmetric information and shows how it generates a new encouragement effect.

This paper is closely related to the recent literature on private learning. Bonatti and Hörner (2011) study moral hazard in teams (with hidden effort) in an exponential bandit framework. Building on this model, Guo and Roesler (2016) analyze a collaboration problem with public and irreversible exit decisions, in which players may privately learn the quality of their project over time if it is bad. Their paper is the closest to my paper in that both papers study signaling in experimentation problems. But in their paper, signaling is through exit decisions rather than through effort, which leads to different behavior predictions and welfare implication. There, information asymmetry creates inefficiency.

Private learning is also examined by Heidhues et al. (2015) and Das (2017), with public effort but private payoffs or signals, with or without competition. In both papers, equilibrium effort strategies are cutoff strategies. Therefore, the joint behavior pattern in this paper does not occur in theirs. Halac et al. (2017) study mechanism design in contests for experimentation, in which both experimentation decisions and experimentation outcomes are private. They find that a "hidden equal-sharing" contest can outperform a "public winner-takes-all contest." Their mechanism differs from mine in that, signaling does not play a role in theirs.

More broadly, that asymmetric information may improve welfare also relates to

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[8] This encouragement effect may occur in MPEs that are not limit equilibria of discrete-time games as the length of a period shrinks to 0.

Note that the encouragement effect defined by Keller et al. (2005) differs from that defined by Bolton and Harris (1999): the former is an equilibrium property and the latter concerns best responses. This paper follows the former definition.

the leadership literature. Hermalin (1998) and Komai et al. (2007) analyze a static model of moral hazard in team, in which the leader who knows the state of the world signals to the followers the value of their joint project by working hard, thereby partially overcoming the free-riding problem. Different from them, this paper focuses on dynamics. Also, their environments have payoff externalities, which lead to different welfare implications from my paper.

3. THE MODEL

Time is continuous, indexed by $t \in [0, \infty)$. There are two players. Each player is endowed with one unit of a divisible resource per unit of time, and must divide it between a safe project and a risky project. A safe project delivers a known return; the return of a risky project depends on its quality θ, unknown and common to both players, with $\theta = g$ referring to a good project, and $\theta = b$ to a bad one. If a player allocates a fraction $a_t \in [0, 1]$ of resource to the risky project over a time interval $[t, t + dt]$, and hence $(1 - a_t)$ to the safe project, then the player receives $(1 - a_t)sdt$ from the safe project, and a lump-sum payoff h with probability $a_t\lambda\mathbb{1}_{\{\theta=g\}}dt$ from the risky project, where $\lambda > 0$. That is, a bad risky project delivers zero payoffs whereas a good risky project delivers lump-sum payoffs, called breakthroughs, that arrive at a Poisson rate. Learning is thus conclusive: a single breakthrough perfectly reveals good quality. At any time t, players observe past experimentation decisions and payoffs. Both players prefer a good risky project to a safe project, and a safe project to a bad risky project: $\lambda h > s > 0$. They discount future payoffs with a common discount rate $r > 0$.

Initially, players assign a common prior probability q_0 to the risky projects being good. At time 0, one player, called the informed player (player I, he), receives a favorable signal s_+ with probability ρ_θ, and an unfavorable signal s_- with probability $1-\rho_\theta$. The favorable signal s_+ is more likely to occur to a good risky project than to a bad risky project: $0 \leq \rho_b < \rho_g < 1$. By Bayes' rule, after receiving signal s_+, I adjusts his belief upward to some q_0^+, strictly higher than the uninformed player's (player U, she) posterior q_0, thereby becoming an *optimistic* type; otherwise, he adjusts his belief downward to some $q_0^- < q_0$, thereby becoming a *pessimistic* type. The parameters q_0, ρ_g, and ρ_b are common knowledge.[9] The information asymmetry is the only divergence from the canonical exponential-bandit model (Keller et al., 2005).

[9]Such information asymmetry would arise, for instance, if I is an incumbent and U is a new entrant who is uncertain of how long I has been experimenting before time 0.

Remark 3.1 *[A joint project interpretation.] Because a breakthrough publicly reveals good quality and there is no payoff externality, the game essentially ends once a breakthrough occurs. It then becomes a dominant strategy for a player to use the risky project forever, bringing a discounted payoff $\lambda h/r$. Note also that the player who receives the first breakthrough enjoys an additional payoff h relative to the other player. Therefore, the model admits the following joint project interpretation: instead of working on two risky projects of the same quality, the two players work on one joint risky project; a breakthrough occurs to the risky project with the same probability as in our model, bringing a lump-sum payoff $\lambda h/r$ to each player, and an additional intrinsic satisfaction h to the first player who experiences the breakthrough; the project is completed once a breakthrough arrives.*

3.1. *The cooperative solution*

If players act cooperatively to maximize their joint surplus, the informed player would reveal his signal truthfully to the uninformed player.[10] Therefore, from time 0 on, both players would share a common posterior belief q_t, which decreases over time as long as players experiment and no breakthrough arrives. Both players adopt a cutoff strategy: experimenting if q_t is higher than a cutoff $q_2^* \in (0,1)$, defined by

$$r(\lambda q_2^* h - s) + 2\lambda q_2^*(\lambda h - s) = 0, \tag{3.1}$$

and stopping otherwise. On the left-hand side, the first term is the flow marginal benefit of experimentation at the cooperative cutoff q_2^* (relative to the safe return; in the sequel, all return is relative to the safe return if not mentioned), and the second term the marginal option value of information to both players. Equation (3.1) says that at the optimal cutoff, the total marginal benefit of experimentation is 0 (a smooth pasting condition).

For future use, we also introduce the single-player solution, a similar cutoff strategy with cutoff q_1^*, where q_1^* is determined by equation (3.1) with the number 2 being replaced by 1. Since the option value of information to two players is twice as much as that to a single player (at the same belief) whereas the flow benefit is the same, a two-player team in the cooperative solution acquire more information than does a single player: $q_2^* < q_1^*$. Intuitively, the more valuable the information, the more information player(s) should acquire.

[10] He can do so by playing some action for an infinitesimal amount of time.

4. BELIEFS AND THE EQUILIBRIUM CONCEPT

Following the experimentation literature with symmetric information (Bolton and Harris, 1999; Keller et al., 2005), I focus on Markov perfect equilibrium (MPE). Different from them, there is no single state variable for the solution concept, because players do not share a common posterior belief. Now a public history carries two components of information. The first is the information obtained from the experimentation technology, depending only on the public history, independent of players' equilibrium strategies, hence is called "passive." This component of information can be represented by how the informed player updates his beliefs. The other component is the informed player's private information revealed through his actions, depending also on his equilibrium strategies and hence is called "strategic." This component of information can be represented by how the uninformed player updates her belief about the informed player being the optimistic type. Based on this observation, we define state variables. Strategies, belief systems, and equilibria are defined afterward.

4.1. *The state variables*

The passive component—the background belief. Consider an outsider who knows the model except that he mistakenly believes that neither player has observed the initial signal of the informed player's. Assume that he starts with the same prior belief $p_0 \equiv q_0$ and observes the same public histories as our players do. Denote his posterior belief at time t by p_t, and call it the *background belief*.[11]

Of course, this background belief differs from I's posterior belief. But if after a public history, the outsider is told of I's private signal, he would then adjust his belief to exactly I's. That is, after a public history, if the background belief is p, type s_+'s posterior belief must be $\boldsymbol{q}^+(p)$ determined by Bayes' rule,

$$\boldsymbol{q}^+(p) = \frac{p\rho_g}{p\rho_g + (1-p)\rho_b}, \tag{4.1}$$

and type s_-'s must be $\boldsymbol{q}^-(p)$ given by

$$\boldsymbol{q}^-(p) = \frac{p(1-\rho_g)}{p(1-\rho_g) + (1-p)(1-\rho_b)}. \tag{4.2}$$

Equations (4.1) and (4.2) imply that the background belief p and the signals of the informed player, s_- and s_+, are sufficient to track the informed player's posterior beliefs. To track U's belief, we need the strategic component of information.

The strategic component—I's reputation: the probability U assigns to I being

[11] Appendix D gives a formal definition of the background belief.

type s_+, denoted by μ. Together with the background belief, I's reputation determines U's posterior belief about the risky project by

$$\boldsymbol{q}^U(p,\mu) \equiv \mu\boldsymbol{q}^+(p) + (1-\mu)\boldsymbol{q}^-(p), \tag{4.3}$$

and hence directly affects U's instantaneous payoff. As U's incentives are affected by I's strategies, which are type dependent, U's belief about I's types is necessary to compute U's continuation payoffs.

In sum, the background belief, p, and I's reputation, μ, are sufficient to represent the two components of information and are thus used as state variables.

To reduce the burden of notation, denote the expected arrival rate of breakthroughs per unit of effort for type s_+, type s_-, and U at state (p,μ), by $\lambda^{I+}(p)$, $\lambda^{I-}(p)$, and $\lambda^U(p,\mu)$, respectively, which are equal to the corresponding posterior beliefs about the risky project multiplied by the arrival rate of a good risky project λ.

Remark 4.1 *Singling out this passive background belief from a public history is not only technically convenient, but also empirically relevant. We interpret the outsider that we introduce to define the background belief as an econometrician who mistakenly believes that players are symmetrically informed. He, therefore, misspecifies the true asymmetric information model as a symmetric information model. We will discuss the empirical consequences of such a misspecification later.*

4.2. *Strategies and systems of beliefs*

Players' strategies are Markov. A pure strategy for U is a mapping from the state space into the effort space, $a^U : [0,1]^2 \to [0,1]$, with $a^U(p,\mu)$ denoting U's effort level at state (p,μ). Type s_+'s and type s_-'s pure strategies are similarly defined and denoted by a^{I+} and a^{I-} respectively. We are interested in equilibria in which both U and type s_+ play pure strategies, and type s_- plays a pure strategy after his type is revealed (on path). In such equilibria, a mixed strategy for type s_- is a mixture over his pure strategies, and can be defined based on Aumann (1964).[12] Abusing notation, we still use a^{I-} to denote the pessimistic type's strategy.

[12]Type s_-'s pure strategy can be taken as choosing a time (or, a belief level) at which to stop mimicking type s_+; his mixed strategy can then be taken as a distribution over such stopping times. Specifically, let a^{I+} denote a pure strategy for the optimistic type, and $a^{I-}(\cdot,0)$ a pure strategy for the pessimistic type after he reveals his type. A mixed strategy for the type s_- is implemented as follows: at the start of the game, type s_- draws a number from the uniform distributed on $[0,1]$; if r is realized, then type s_- plays a^{I+} as long as the background belief is strictly higher than $\hat{p}(r)$,

A belief system is denoted by $\mu(s_+|\cdot)$, which associates to each public history, a probability that U assigns to I being type s_+.

4.3. *Equilibrium*

Given a Markov strategy profile (a^{I-}, a^{I+}, a^U) and a system of beliefs $\mu(s_+|\cdot)$, the expected average payoff to type s_l, $l \in \{+,-\}$, at time 0, is

$$E^l_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}\left[\int_0^\infty re^{-rt}\left(\left(1-a^{Il}_t\right)s + a^{Il}_t \lambda h \mathbb{1}_{\{\theta=g\}}\right)dt\right],$$

which is equal to

$$E^l_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}\left[\int_0^\infty re^{-rt}\left(\left(1-a^{Il}_t\right)s + a^{Il}_t \lambda^{Il}(p_t)h\right)dt\right], \tag{4.4}$$

by the law of iterated expectations, where $E^l_{(a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}$ denotes type s_-'s expectation under the probability distribution induced by the tuple $(a^{I-}, a^{I+}, a^U, \mu(s_+|\cdot))$.

Similarly, the expected average payoff of player U at time 0 is

$$E^U_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}\left[\int_0^\infty re^{-rt}\left(\left(1-a^{U}_t\right)s + a^{U}_t \lambda^{U}(p_t,\mu_t)h\right)dt\right],$$

where $E^U_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}$ is defined similarly.

The tuple $(a^{I-}, a^{I+}, a^U, \mu(s_+|\cdot))$ is an MPE if given the other player's strategy and the belief system, a player finds it optimal to play his or her equilibrium strategy, and if the belief system satisfies Bayes' rule whenever possible.

4.4. *The evolution of the state variables*

Let a^I_t and a^U_t denote I's and U's efforts at time t respectively. By Bayes' rule, given an action path $(a^I_t, a^U_t)_{t\geq 0}$ (on or off the equilibrium path), before a breakthrough occurs, the background belief process $(p_t)_{t\geq 0}$ evolves according to[13]

$$dp_t = -p_t(1-p_t)(a^I_t + a^U_t)\lambda dt. \tag{4.5}$$

The evolution of a reputation process $(\mu_t)_{t\geq 0}$ depends on the equilibrium prescription. Fix a candidate equilibrium $(a^{I+}, a^{I-}, a^U; \mu(s_+|\cdot))$. We focus on how μ_t evolves along the path such that *no breakthrough has occurred and I has been taking type s_+'s*

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and plays $a^{I-}(\cdot,0)$ otherwise, where $\hat{p} : [0,1] \to [0,1]$ is a decreasing function that assigns to each realization from the uniform distribution a cutoff background belief at which type s_- reveals himself.

[13]To see this, suppose at background belief p_t, players take efforts (a^I_t, a^U_t) during a dt duration of time. In the absence of a breakthrough, the background belief at $t + dt$, p_{t+dt}, satisfies $p_{t+dt} = \frac{p_t(1-(a^I_t+a^U_t)\lambda dt)}{p_t(1-(a^I_t+a^U_t)\lambda dt)+(1-p_t)}$ by Bayes' rule. Therefore, the belief change in this time interval, $dp_t \equiv p_{t+dt} - p_t$, is given by equation (4.5).

(prescribed) effort.[14]

If the equilibrium involves pooling from time 0 to some time T, then U's belief about the risky project coincides with the background belief over the time interval $[0, T]$. As a result, I's reputation μ_t at background belief p_t for $t \in [0, T]$ is equal to

$$(4.6) \qquad \boldsymbol{\mu}^o(p_t) \equiv p_t\rho_g + (1 - p_t)\rho_b,$$

where the mapping $\boldsymbol{\mu}^o : [0, 1] \to [0, 1]$ is called a *pooling path*. Since $p_0 = q_0$, $\boldsymbol{\mu}^o(q_0)$ is I's reputation at time 0. Note that during pooling, I's reputation μ_t decreases as p_t decreases over time. Intuitively, since signal s_+ is more likely to occur to a good risky project, as U becomes more and more pessimistic about the risky project being good, she becomes more and more pessimistic about s_+ having occurred.

Once the equilibrium diverges from pooling, I's reputation μ_t would differ from $\boldsymbol{\mu}^o(p_t)$. Conditional on I being type s_-, a candidate equilibrium $(a^{I+}, a^{I-}, a^U; \mu(s_+|\cdot))$ induces a distribution over the set of equilibrium paths, which, for each t, gives a distribution over the set of time-t histories such that no breakthrough has occurred before type s_- reveals himself (by stopping taking type s_+'s equilibrium effort). This set of time-t histories consists of histories such that either "type s_- has revealed himself before or at t" or "type s_- has not revealed himself and no breakthrough has arrived up to time t." Let Y_t denote the probability that type s_- has revealed himself before or at t conditional on this set. The function $Y : [0, \infty) \to [0, 1]$ is then a cumulative distribution function (CDF) over the time at which type s_- reveals himself conditional on no breakthrough having occurred before type s_-'s revelation.[15]

[14]Once I's action differs from type s_+'s (prescribed) action on the equilibrium path, I's reputation will stay at 0; once a breakthrough occurs, his reputation ceases to matter.

[15] Specifically, there are three types of time-t public histories (on path): (1) histories such that a breakthrough has occurred and I has been taking type s_+'s (prescribed) action, (2) the history such that no breakthrough has occurred and I has been taking type s_+'s action, and (3) histories such that I has stopped taking type s_+'s action. Fix the candidate equilibrium. Conditional on a history being either of the second or of the third type, if I is of type s_+ then the second-type history occurs with probability 1, whereas if I is of type s_- then the second-type history occurs with probability $1 - Y_t$ and the third type occurs with probability Y_t.

We now show how Y_t is determined by the candidate equilibrium. According to type s_-'s mixed strategy induced by the randomization device in Footnote 12, if the random number r is such that p_t is below the threshold $\hat{p}(r)$, then type s_- must stop mimicking type s_+ before or at t, resulting in a time-t history of the third type; otherwise, if r is such that p_t is strictly above $\hat{p}(r)$, then type s_- must have been mimicking type s_+ at least until t (because the background beliefs before time t is higher than p_t, which is strictly higher than $\hat{p}(r)$), resulting in a time-t history is of the second

By Bayes' rule, I's reputation μ_t satisfies

$$\mu_t = \frac{\boldsymbol{\mu}^o(p_t)}{\boldsymbol{\mu}^o(p_t) + [1-\boldsymbol{\mu}^o(p_t)]\,(1-Y_t)}. \tag{4.7}$$

Written in its differential form, the reputation process $(\mu_t)_{t\geq 0}$ evolves according to

$$\frac{d\mu_t}{\mu_t(1-\mu_t)} = \frac{d\boldsymbol{\mu}^o(p_t)}{\boldsymbol{\mu}^o(p_t)\,(1-\boldsymbol{\mu}^o\,(p_t))} + \frac{dY_t}{1-Y_t}, \tag{4.8}$$

where $\frac{dY_t}{1-Y_t}$ denotes the probability that type s_- reveals himself during the time interval $[t, t+dt)$, conditional on *no breakthrough having occurred and I having been taking type s_+'s effort*. We call $y_t \equiv \frac{dY_t/dt}{1-Y_t}$ type s_-'s *revealing rate* at time t.

4.5. *The continuation game after revelation (on path)*

I focus on the equilibria in which after I's type is revealed (on path), players play the unique symmetric MPE under symmetric information as the continuation equilibrium. Keller et al. (2005) characterize this equilibrium: players exert effort 1 when they are sufficiently optimistic (when their *posterior belief* is above a threshold q^S); they exert effort 0 when they are sufficiently pessimistic (when their posterior belief is below the single-player cutoff q_1^*); and they exert interior effort when in between. Denote this MPE by $a^S : [0,1] \to [0,1]$, whose argument is players' true *posterior belief*.

Two features of this equilibrium will change qualitatively under asymmetric information. First, there is no encouragement effect: players acquire the same amount of information as a single player does, as they stop experimenting at the single-player cutoff belief q_1^*. Second, effort decreases over time in the absence of a breakthrough.

Figure 1 translates this equilibrium in the language of the background belief. The dashed curve corresponds to the continuation equilibrium following type s_-'s revelation, where the cutoff p_1^{S-} is defined to be the background belief at which players are willing to switch from effort 1 to interior efforts: $\boldsymbol{q}^-(p_1^{S-}) = q^S$, and p_1^{*-} the background belief at which players' posterior belief is at the single-player cutoff: $\boldsymbol{q}^-(p_1^{*-}) = q_1^*$. The solid curve corresponds to the continuation equilibrium following type s_+'s revelation, with the two cutoffs similarly defined. To avoid redundancy, whenever no confusion arises, we call p_1^{*-} type s_-'s single-player cutoff (background belief), $p_2^{*-} \equiv (\boldsymbol{q}^-)^{-1}(q_2^*)$ his cooperative cutoff, and p^{S+} type s_+'s switching cutoff.

type. Therefore, Y_t by definition is equal to the probability that r is below $\hat{p}^{-1}(p_t)$, which is equal to $\hat{p}^{-1}(p_t)$ (as r is uniformly distributed).

4.6. *The odds ratio*

A crucial factor driving the momentum of the mutual encouragement effect is the belief difference between I's two types, measured by $\frac{\boldsymbol{q}^+(p)}{1-\boldsymbol{q}^+(p)} / \frac{\boldsymbol{q}^-(p)}{1-\boldsymbol{q}^-(p)}$. By Bayes' rule (equations (4.1) and (4.2)), this ratio is also equal to an *odds ratio* O defined by

$$O \equiv \frac{\rho_g/(1-\rho_g)}{\rho_b/(1-\rho_b)}, \tag{4.9}$$

the ratio of the odds of signal s_+ occurring to a good project to the odds of it occurring to a bad project. Therefore, the odds ratio O measures both the belief difference between I's two types and the informativeness of I's private signal.

The following assumption greatly eases the exposition of the mutual encouragement effect. Section 6 discusses what happens if this assumption does not hold.

Assumption 1 *The odds ratio O is greater than or equal to $O^S \equiv \frac{q^S}{1-q^S} / \frac{q_2^*}{1-q_2^*}$.*

Under Assumption 1, when type s_-'s posterior belief is at the cooperative cutoff q_2^*, type s_+'s would be weakly greater than the switching cutoff q^S (after the same public history). It means the belief difference between I's two types is large, so that after the players with public information s_- find it optimal to stop experimenting when playing cooperatively, the players with public information s_+ still experiment with the full resource for at least some time (when playing the symmetric MPE). The parameters in Figure 1 satisfy this assumption, because at the background belief p_2^{*-} (type s_-'s cooperative cutoff), players with public information s_+ are still willing to exert effort 1.

5. MPE WITH GRADUAL REVELATION

This section constructs the MPE of interest (and shows its existence). We first highlight its main structure and elaborate the equilibrium behavior dynamics and belief dynamics. Detailed equilibrium construction is postponed to the last subsection and equilibrium uniqueness to Section 6.

Recall that we focus on equilibria such that, after I's type is revealed (on path), the players play the symmetric MPE a^S (defined in Section §4.5). In the sequel, by type s_- revealing himself, we mean that he plays this equilibrium strategy, and immediately after this, U follows suit. Type s_+'s strategy before U assigns probability one on him being type s_+ is simple: he exerts effort 1. All equilibrium descriptions are conditioned on no breakthrough having occurred.

The equilibrium has three phases.

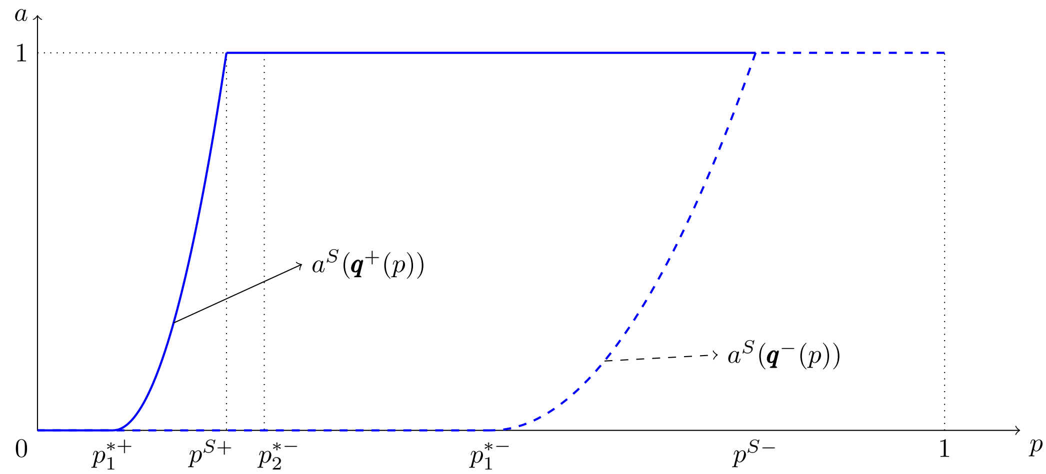

Figure 1: The continuation equilibrium after I's type is revealed (on path)

1. When type s_- is sufficiently optimistic — over background beliefs $(p_{gr}, 1)$, p_{gr} to be determined — the equilibrium involves *pooling*, during which, both players exert effort 1. As a result, I's reputation gradually decreases over time (along the pooling path $\boldsymbol{\mu}^o$). The eroding reputation path is illustrated by the dash-dot line over the interval $(p_{gr}, 1]$ in Figure 2: as time passes by, I's reputation decreases along this line from right to left until p reaches p_{gr}.
2. When type s_-'s belief is intermediate — over background beliefs $(p_2^{*-}, p_{gr}]$ — the equilibrium involves *gradual revelation*, during which, type s_+ still exerts effort 1, whereas type s_- mixes between mimicking type s_+ and revealing himself, such that as long as he keeps mimicking, his reputation gradually increases, along a *gradual revelation path* $\hat{\boldsymbol{\mu}} : [p_2^{*-}, p_{gr}] \to [0, 1]$. This rising reputation path is illustrated by the solid curve in Figure 2: as time passes by, I's reputation increases along this line from right to left.
3. When type s_- is sufficiently pessimistic — over background beliefs $(0, p_2^{*-}]$ — the equilibrium involves *separation*, during which, type s_+ plays the symmetric MPE strategy under symmetric information s_+, whereas type s_- stops experimenting immediately. Referring to Figure 2, if I stops experimentation, the state variables jump on the line $\mu = 0$ and then cease to move; otherwise, the state variables jump on the line $\mu = 1$ and move along it from right to left until experimentation ends.[16]

[16] Of course, the state variables stop moving when the background belief reaches p_1^{*+}, below which, even type s_+ stops experimenting.

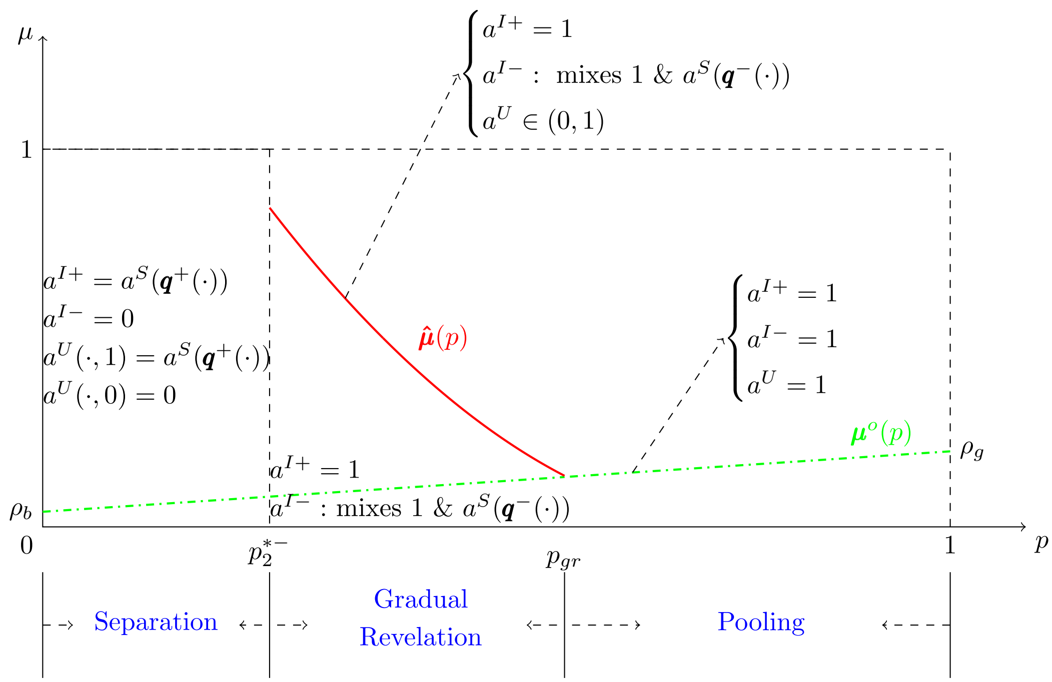

Figure 2: An MPE with gradual revelation

Call this equilibrium an *MPE with gradual revelation*. Figure 3 illustrates how phase transitions occur. If the prior belief q_0 (which, recall, is equal to p_0) lies in the pooling region $(p_{gr}, 1)$, say at the closed circle on the solid part of $\boldsymbol{\mu}^o$, then the equilibrium begins with the pooling phase, during which, the state variables move from right to left along the pooling path $\boldsymbol{\mu}^o$ until the background belief reaches p_{gr}. After this, the gradual revelation begins, during which, the state variable move along the gradual revelation path $\hat{\boldsymbol{\mu}}$ until the background belief reaches p_2^{*-}. After this follows the separation phase. The solid arrowed curve illustrates how the state variables evolve over time, conditional on no breakthrough and I having been playing type s_+'s effort.

If the prior belief q_0 lies in the gradual revelation region $(p_2^{*-}, p_{gr}]$, say, at the open circle on the dashed part of $\boldsymbol{\mu}^o$, then type s_- reveals with some probability such that upon non-revealing, the state variables immediately jump up on the curve $\hat{\boldsymbol{\mu}}$. The gradual revelation phase then begins and the equilibrium dynamics are the same as in the previous case. The dashed arrowed curve illustrates how the state variables evolve over time conditional on no breakthrough and I having been playing type s_+'s effort.

We are ready to present the first main result of the paper — the qualitative features

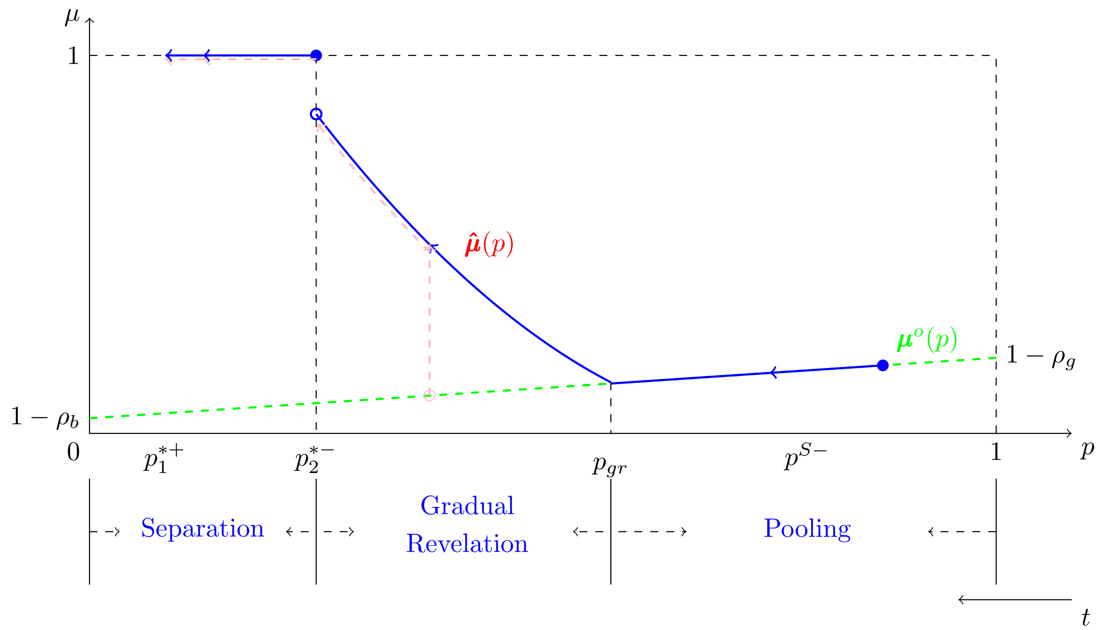

Figure 3: An MPE with gradual revelation: two paths of the state variables

of the behavior dynamics and belief dynamics.

Proposition 1 *During the gradual revelation phase of the MPE with gradual revelation, conditional on no breakthrough having occurred and I having been playing type s_+'s effort,*

1. *U's effort gradually increases over time, when the background belief is between type s_-'s cooperative cutoff p_2^{*-} and his single-player cutoff p_1^{*-};*
2. *I's reputation gradually rises over time;*
3. *U's belief about the risky project is either increasing or U-shaped over time, if the informativeness of I's initial signal is intermediate (that is, if the odds ratio O is not too high but still satisfies Assumption 1).*

We have illustrated the rising reputation in Figure 3. In Figure 4, the arrowed curve displays the uninformed player's effort path conditional on her facing an optimistic type: U increases her effort over time when the background belief is between p_1^{*-} and p_2^{*-}. We postpone discussing U's decreasing effort (over time) during the gradual revelation phase until Section 5.4.1.

Figure 5 and Figure 6 contrast two distinct paths of U's belief about the risky

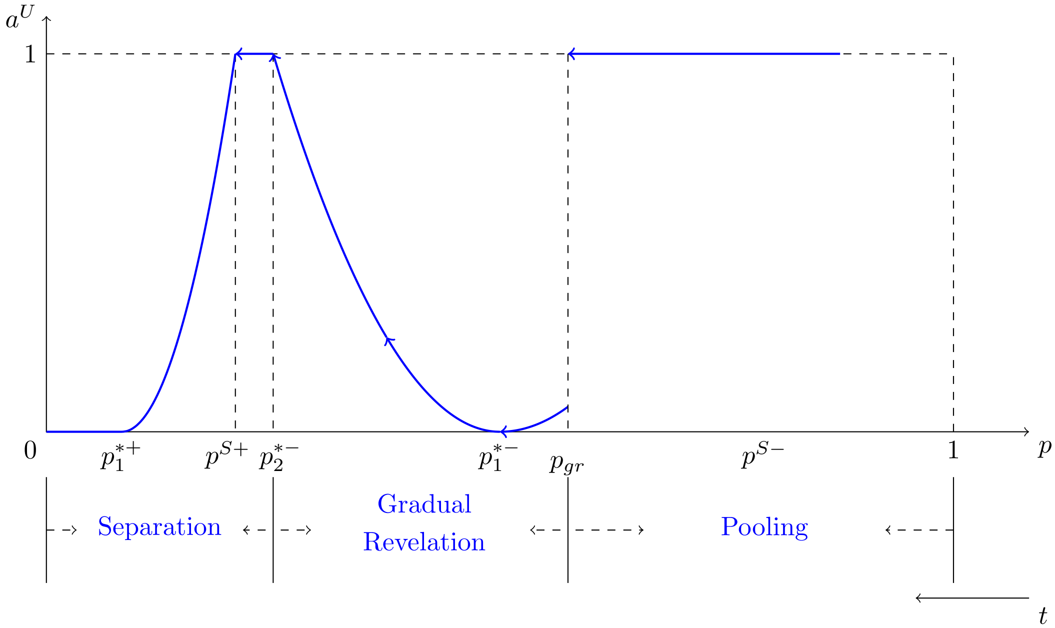

Figure 4: The uninformed player's effort

project's quality. In Figure 5, U's belief decreases over time before the separation phase occurs; this occurs in an environment with a high odds ratio. In Figure 6, U's belief is U-shaped before the separation phase occurs; this typically occurs in an environment with an intermediate odds ratio.

Compared with the symmetric MPE under symmetric information, two features of the current equilibrium stand in sharp contrast.

First, the uninformed player can *increase effort*, and *become more optimistic* about the risky project over time, despite the absence of a breakthrough. She does so because I's high effort continually brings in good news, compensating for the absence of a breakthrough, and encouraging her to experiment.

Second, the pessimistic type *experiments beyond the single-player cutoff*, until the cooperative cutoff, with positive probability. He does so because the uninformed player responds to his hard work by also working hard, thereby producing more information over time, encouraging him to experiment at beliefs he would not were he alone or were his signal public.

We, therefore, have identified a *mutual encouragement effect*: I's rising reputation compensates the dropping background belief, encouraging U to experiment; U's increasing effort compensates type s_-'s growing pessimism, encouraging him to per-

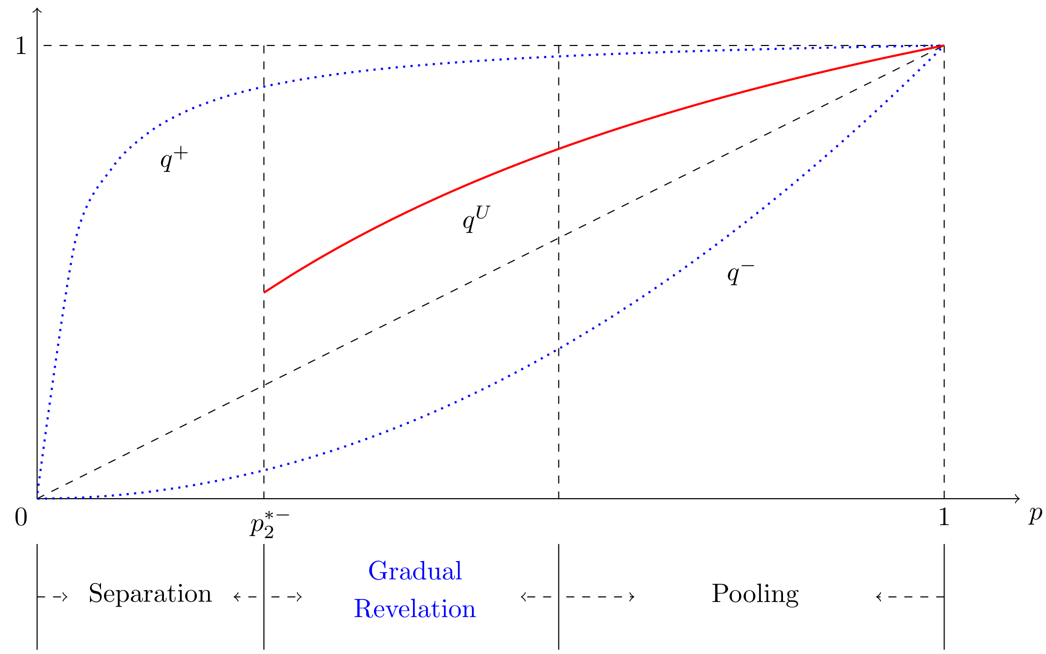

Figure 5: U's growing pessimism before separation (a large odds ratio)

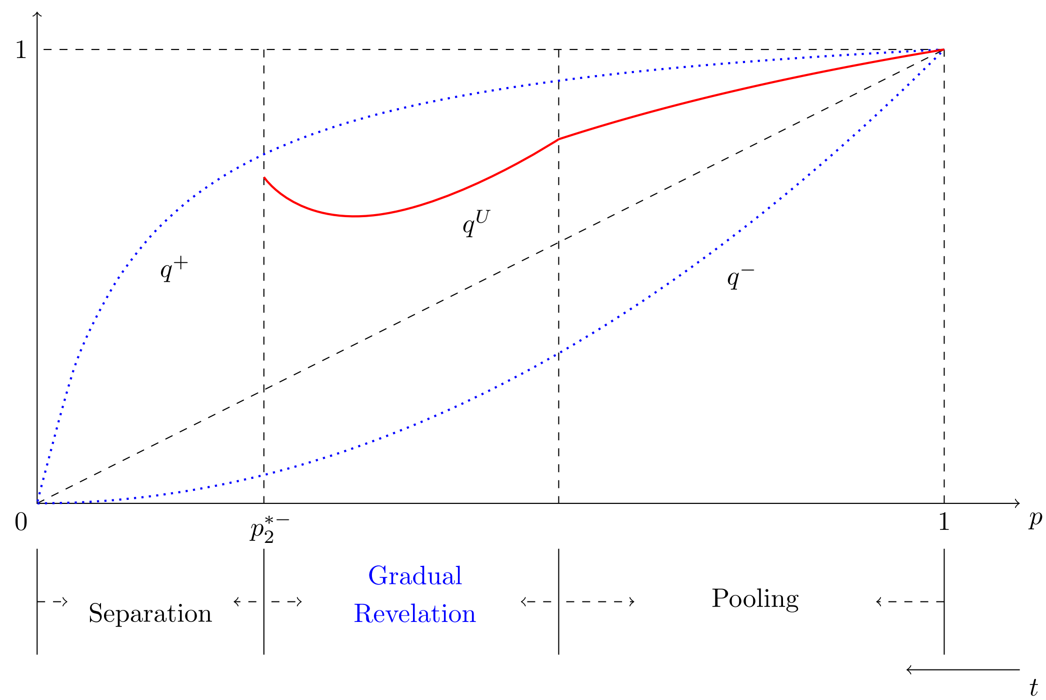

Figure 6: U's growing optimism before separation (an intermediate odds ratio)

severe. Driven by this effect, the joint behavior pattern — the informed player keeps exerting high effort and the uninformed player increases effort, despite the absence of a breakthrough — does not occur in any MPE of the symmetric information game.[17] This pattern leads to qualitatively different empirical predictions, which we will discuss in Section 8.

The above discussion highlights two requirements for the mutual encouragement effect to arise. First, it is able to counterbalance the deterioration of the background belief. This is guaranteed by Assumption 1, which ensures that a perfect reputation brings in sufficiently good news to encourage U to experiment (given the continuation equilibrium). Second, it is needed to counterbalance the deterioration of the background belief, which is guaranteed by the fraction of type s_+ being not too high:

Assumption 2 *Conditional on $\theta = g$, signal s_+ is not too likely: $\rho_g \leq \frac{s}{(r+\lambda)h+\lambda h-s}$.*

Proposition 2 *Under Assumption 1 and 2, an MPE with gradual revelation exists.*

If Assumption 2 is not satisfied, then the gradual revelation phase can be empty: the pooling phase lasts until the background belief reaches p_2^{*-}.

We now elaborate on the intuitions behind Proposition 1.

5.1. *The informed player's rising reputation*

During the gradual revelation phase, I's rising reputation counterbalances the declining background belief, maintaining U's indifference about experimentation, thereby incentivizing her to take interior effort. To see this, consider the following two main elements that drive U's experimentation incentives.[18]

1. U's instantaneous marginal benefit of experimentation, which depends only on her belief about the risky project. The higher her belief, the higher her willingness to experiment.

[17] To be specific, it does not occur in any MPE that is a limit MPE of the discrete-time experimentation games as the length of periods goes to zero. Hörner et al. (2014) (in Lemma 1) show that in any perfect Bayesian equilibrium of such discrete time game, players do not experiment when their posterior is below the single-player cutoff. Using this result, we can show that in any limit MPE, total effort cannot strictly decrease in players' posterior.

[18] I's current effort also affects U's experimentation incentive, due to the strategic substitutability of players' current effort decisions, as in the symmetric information game. This element is absent here because I's current effort is 1 with probability 1.

2. U's continuation marginal benefit of experimentation, which depends on her expected continuation value. The higher her expected continuation value, the higher her incentive to speed up experimentation so as to enjoy it earlier.

Suppose instead I's reputation does not increase. Then as time passes by, U becomes more pessimistic and hence her instantaneous marginal benefit decreases. Moreover, with both of her ex post continuation values decreasing, together with I's dropping reputation, so is her expected continuation value. Consequently, if at some point in time she is indifferent about experimentation, she would strictly prefer not to experiment afterward.

Therefore, for U to be indifferent, I's reputation must rise over time. Indeed, during this phase, the incentive-enhancing effect of I's rising reputation (driven by the strategic component) exactly balances out the incentive-dampening effect of the deteriorating background belief (driven by the passive component), maintaining U's willingness to take the effort in Figure 4 — in particular, to increase her effort when the background belief is between (type s_-'s single-player cutoff) p_1^{*-} and (type s_-'s cooperative cutoff) p_2^{*-}.

5.2. *The uninformed player's increasing effort*

When the background belief is between the pessimistic type's single-player cutoff p_1^{*-} and his cooperative cutoff p_2^{*-}, U's increasing effort compensates his growing pessimism, keeping him indifferent between mimicking type s_+ (by continuing experimenting) and revealing himself (by stopping experimenting). As a result, he is willing both to experiment beyond his single-player cutoff, and to stop so that mimicking type s_+ indeed continually carries encouraging news.

Specifically, by revealing himself, he induces both players to stop experimenting as the background belief is below his single-player cutoff; he thereby receives zero relative to the safe return. By continuing mimicking type s_+ for a dt duration of time, he receives an instantaneous benefit, $r\left(\lambda^{I-}(p)\,h - s\right)dt$, which is decreasing with the absence of a breakthrough, and a continuation benefit, an upward jump of his continuation value in case a breakthrough arrives, $\lambda h - s$, with probability $\left(1 + a^U\right)\lambda^{I-}(p)\,dt$.[19] For him to be indifferent, the two options must give him the

[19] In case no breakthrough arrives, type s_-'s continuation value stays at the safe return s and hence he receives no continuation benefit after this event.

same payoff. That is, U's effort must satisfy

$$a^U(p, \hat{\boldsymbol{\mu}}(p)) = \frac{r(s - \lambda^{I-}(p)\, h)}{\lambda^{I-}(p)\, (\lambda h - s)} - 1, \text{ for } p \in [p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]. \tag{5.1}$$

which increases over time, as p deteriorates. Intuitively, since effort becomes less and less valuable to type s_- due to the absence of a breakthrough, U must produce more information — that is, to increase her effort — to reward type s_-'s perseverance.

We thus call this sub-phase of the gradual revelation phase the "rewarding sub-phase." Note that in Figure 4, $a^U = 0$ at $p = p_1^{*-}$. This is because, p_1^{*-} being type s_-'s single-player cutoff, U does not need to provide any extra reward for him to experiment. Note also that $a^U = 1$ at $p = p_2^{*-}$. This is because, p_2^{*-} being his cooperative cutoff, U needs to respond one for one to type s_-'s effort, so that type s_- indirectly internalizes the social benefit of his effort.[20] The gradual revelation phase ends at p_2^{*-} because U reaches the budget limit that she can reward I's hard working. U's effort in the other sub-phase of the gradual revelation phase, that is, when p is in (p_1^{*-}, p_{gr}), is left to the final subsection.

5.3. *U's growing optimism about the risky project*

How much encouraging information should I reveal to U to maintain U's experimentation incentive? It depends on how informative I's private signal is:

Lemma 1 *There exists $\tilde{O} \in (O^S, \infty)$ such that, during the pooling phase and the gradual revelation phase of the MPE with gradual revelation, the uninformed player's belief about the risky project's quality q^U*

1. *strictly decreases over time, if the odds ratio is sufficiently high—if $O \in [\tilde{O}, \infty)$;*
2. *is U-shaped—it first decreases over time, and then after reaching some point in the gradual revelation region, it begins to increase—if odds ratio is intermediate, that is, if $O \in [O^S, \tilde{O})$.*

U's growing optimism in Proposition 1 follows from the second case. We here give an intuition for why near the end of the gradual revelation phase, U becomes increasingly pessimistic over time if I's private signal is sufficiently informative ($O \in [\tilde{O}, \infty)$), and increasingly optimistic if it is intermediately informative ($O \in [O^S, \tilde{O})$).

[20]Conditional on signal s_- being realized, since players are symmetric, I's benefit from (the information produced by) U's effort is exactly equal to U's benefit from I's effort. Therefore, by rewarding I's effort with (the same amount of) U's effort, it is as if adding to I's incentive U's benefit from I's effort, thereby making I internalize the social benefit of his effort.

For ease of illustration, we decompose U's marginal benefit of experimentation (see Section §5.1) into the following three parts: (1) her instantaneous marginal benefit, (2) her option value of the information generated from the experimentation technology — the passive component, and (3) her option value of the information revealed by I's action — the strategic component. The first two parts, combined together, depend only on U's belief about the risky project's quality q^U and changes in the same direction as q^U changes. The third part depends on the spread of the informed player's private information, measured by the spread of the informed player's beliefs q^- and q^+; the bigger the spread, the more useful I's private information to U, and hence the higher her experimentation incentive.

A drop in q^- widens the spread between q^- and q^+. This means that if μ were to change in such a way that the uninformed player's belief q^U increases, then the uninformed player's continuation marginal benefit of experimentation must also increase. On the contrary, a drop in q^+ shrinks the spread between q^- and q^+. This means that if μ were to change in such a way that the uninformed player's belief decreases, then her continuation marginal benefit of experimentation must also decrease.

When the odds ratio is sufficiently large, during the gradual revelation phase, the optimistic type's belief q^+ is close to 1 and hence barely decreases over time (by Bayes' rule). As a result, the effect of the dropping q^- (due to the lack of a breakthrough) dominates. From the above analysis, if U's belief q^U does not decrease over time, then her total marginal benefit of experimentation would strictly increase, and consequently she would not be indifferent about experimentation, which could not occur in an MPE with gradual revelation.

When the odds ratio is intermediate (greater than and close to a^S), U is willing to experiment only if I is sufficiently likely to be type s_+, that is, only if I's reputation μ is close to 1.[21] As a result, the dropping q^- ceases to matter as its impact is weighted by $1-\mu$, and the effect of the dropping q^+ dominates. If U's belief q^U does not increase over time, then her total marginal benefit of experimentation would be strictly decreasing, which could not occur in an MPE with gradual revelation.

We have completed the (sketch of) proof of Proposition 1.

[21] See the discussion of Lemma 3 in the last subsection for further explanation.

5.4. *Detailed equilibrium construction*

This subsection studies the following questions. First, if the MPE with gradual revelation is an equilibrium, what conditions do U's effort a^U and the gradual revelation path $\hat{\boldsymbol{\mu}}$ necessarily satisfy? Second, if a^U and $\hat{\boldsymbol{\mu}}$ indeed satisfy these conditions, is the MPE with gradual revelation indeed an equilibrium? To reduce the burden of notations, we omit the arguments $(p, \hat{\boldsymbol{\mu}}(p))$ of the continuation value functions W^{I+}, W^{I-}, and W^U, and of the pure effort strategies a^{I+} and a^U, whenever no confusion arises.

5.4.1. *Necessary conditions for the equilibrium construction*

1. U's effort function a^U for $p \in (p_1^{*-}, p_{gr})$. At any state $(p, \hat{\boldsymbol{\mu}}(p))$ during gradual revelation, type s_- faces two options: mimicking type s_+ (by exerting effort a^{I+}) and revealing himself.

If he mimics, he will receive continuation value $W^{I-}(p, \hat{\boldsymbol{\mu}}(p))$, which satisfies the Hamilton–Jacobi–Bellman (HJB) equation:

$$\begin{aligned} r\left(W^{I-} - s\right) &= a^{I+}\left[r\left(\lambda^{I-}(p)\,h - s\right) - \lambda p\,(1-p)\,\frac{dW^{I-}}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}\right)\right] \\ &\quad + a^U\left[-\lambda p\,(1-p)\,\frac{dW^{I-}}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}\right)\right]. \end{aligned} \tag{5.2}$$

This equation says that type s_-'s flow continuation value — the left-hand side, must be equal to the sum of his instantaneous benefit $r\left[a^{I+}\left(\lambda^{I-}(p)\,h - s\right)\right]$ and the value of information. The latter consists of two parts: in case a breakthrough arrives, occurring at a rate of $\left(a^{I+} + a^U\right)\lambda^{I-}(p)$, his continuation value increases by $(\lambda h - W^{I-})$; in case no breakthrough arrives, his continuation value changes at a rate of $\frac{dW^{I-}}{dp}\frac{dp_t}{dt} = -\frac{dW^{I-}}{dp}\left(a^{I+} + a^U\right)\lambda p\,(1-p)$.

If he does not mimic, then players will play the symmetric MPE (with signal s_- public), whereby type s_- receives a continuation value denoted by $w^S\left(\boldsymbol{q}^-(p)\right)$. Note that in this symmetric MPE, a player is indifferent between experimenting and not experimenting when the background belief is in $(p_1^{*-}, p_{gr}]$ (because $p_{gr} < p^{S-}$), meaning that type s_- would obtain the same continuation value by exerting effort a^{I+} instead of a^S given that U plays a^S. Therefore, $w^S\left(\boldsymbol{q}^-(p)\right)$ satisfies (omitting the argument $\boldsymbol{q}^-(p)$):

$$\begin{aligned} r\left(w^S - s\right) &= a^{I+}\left[r\left(\lambda^{I-}(p)\,h - s\right) - \lambda p\,(1-p)\,\frac{dw^S}{dp} + \lambda^{I-}(p)\left(\lambda h - w^S\right)\right] \\ &\quad + a^S\left[-\lambda p\,(1-p)\,\frac{dw^S}{dp} + \lambda^{I-}(p)\left(\lambda h - w^S\right)\right]. \end{aligned} \tag{5.3}$$

Since type s_- is indifferent between these two options, we have $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = w^S(\boldsymbol{q}^-(p))$. This equality, equations (5.2) and (5.3), and the fact that information is valuable (that is, the terms in the square brackets on the second line of equation (5.2) is positive), imply that

$$a^U(p, \hat{\boldsymbol{\mu}}(p)) = a^S\left(\boldsymbol{q}^-(p)\right), \quad p \in (p_1^{*-}, p_{gr}].$$

That is, U exerts the same level of effort whether the informed player continues exerting high effort a^{I+} or not. Intuitively, since type s_- is willing to take type s_+'s effort a^{I+} even after losing his reputation, mimicking type s_+ is costless and hence he should not be rewarded for doing so. We thus call $(p_1^{*-}, p_{gr}]$ the *non-responding* region of the gradual revelation phase.

The non-responding region is empty if $p_{gr} \leq p_1^{*-}$. For a given odds ratio, if the informed player is likely to be type s_+, that is, if ρ_g is high, then U is willing to experiment even at low background beliefs, implying a short gradual revelation phase and hence a low p_{gr}, and consequently, an empty non-responding region. It can be shown that for each odds ratio satisfying Assumption 1, there is a threshold of ρ_g below which, the non-responding region exists, and above which, it does not.

U's effort is summarized in Lemma 2 and illustrated by Figure 4.

Lemma 2 *If the MPE with gradual revelation is an equilibrium, then along the gradual revelation path* $\hat{\boldsymbol{\mu}}$,

1. *over the rewarding region* $(p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]$, *U's effort satisfies equation* (5.1), *and hence is strictly increasing over time.*
2. *over the non-responding region* $(\min\{p_1^{*-}, p_{gr}\}, p_{gr}]$ *(if nonempty), U's effort equals the symmetric MPE effort under public information* s_-, *and hence is strictly decreasing over time.*

2. The gradual revelation path $\hat{\boldsymbol{\mu}}$. By Lemma 2, U's effort is interior, meaning that she is indifferent about experimentation. This condition pins down $\hat{\boldsymbol{\mu}}$. To show this, we need to analyze the value of information to U, in particular, the rate at which I reveals his private information to U.

Equation (4.8) links to each (differentiable) gradual revelation path $\hat{\boldsymbol{\mu}}$ a CDF Y over the times at which type s_- stops mimicking type s_+'s effort, conditional on no breakthrough having occurred and type s_- having not revealed himself. Specifically, suppose over time $[t, t+dt]$, I's effort is a^{I+} and U's is a. Then the fact that the state variables move along $\hat{\boldsymbol{\mu}}$ implies that type s_- reveals himself at a *rate* of

$$\frac{dY_t/dt}{1-Y_t} = \left(\frac{\hat{\boldsymbol{\mu}}_p(p)}{\hat{\boldsymbol{\mu}}(p)(1-\hat{\boldsymbol{\mu}}(p))} - \frac{\boldsymbol{\mu}_p^o(p)}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))}\right)\frac{dp_t}{dt}$$

$$(5.4) \qquad = \left(\frac{\boldsymbol{\mu}_p^o(p)}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))} - \frac{\hat{\boldsymbol{\mu}}_p(p)}{\hat{\boldsymbol{\mu}}(p)(1-\hat{\boldsymbol{\mu}}(p))}\right)(a^{I+}+a)p(1-p)\lambda,$$

where $\hat{\boldsymbol{\mu}}_p$ denotes the derivative of $\hat{\boldsymbol{\mu}}$. Note that U's effort a affects type s_-'s revealing rate. Intuitively, the higher her effort, the quicker the negative information (i.e., no breakthrough) accumulates, and hence the higher the rate at which type s_- needs to reveal himself so that non-revealing brings encouraging information fast enough.

The value of information to U consists of three parts:

- in case a breakthrough arrives, which occurs at a rate of $(a^{I+}+a)\lambda^U$, U's continuation value jumps by $(\lambda h - W^U)$;
- in case no breakthrough arrives and I continues exerting effort a^{I+}, U's continuation value changes at a rate of $\frac{dW^U}{dp}\frac{dp_t}{dt}$;
- in case no breakthrough arrives and I stops exerting effort a^{I+}, which occurs at a rate of $(1-\hat{\boldsymbol{\mu}}(p))\frac{dY_t/dt}{1-Y_t}$, U's continuation value drops by $|W^U(p,0) - W^U(p,\hat{\boldsymbol{\mu}}(p))|$.

Summing up and applying equation (5.4), the value of information to U is thus $(a^{I+}+a)A(p,\hat{\boldsymbol{\mu}}(p))$, where $A(p,\hat{\boldsymbol{\mu}}(p))$ denotes U's continuation marginal benefit of experimentation (omitting the arguments $(p,\hat{\boldsymbol{\mu}}(p))$ of A, W^U, and λ^U):

$$A \equiv \left(\frac{\boldsymbol{\mu}_p^o(p)(1-\hat{\boldsymbol{\mu}}(p))}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))} - \frac{\hat{\boldsymbol{\mu}}_p(p)}{\hat{\boldsymbol{\mu}}(p)}\right)p(1-p)\lambda\left(W^U(p,0) - W^U\right)$$

$$(5.5) \qquad -\lambda p(1-p)\frac{dW^U}{dp} + \lambda^U\left(\lambda h - W^U\right)$$

Using the fact that type s_+'s effort is 1 during gradual revelation, U's continuation value function W^U satisfies the HJB equation:

$$(5.6) \qquad r\left(W^U - s\right) = \max_{a\in[0,1]} a\left[r\left(\lambda^U h - s\right) + A\right] + A.$$

For U to be indifferent, her marginal benefit must be 0:

$$(5.7) \qquad r\left(\lambda^U h - s\right) + A = 0.$$

Equations (5.6) and (5.7) imply that U's continuation value function is given by

$$(5.8) \qquad W^U - s = s - \lambda^U h.$$

U's indifference condition (5.7) and her continuation value function (5.8) give an ODE that the gradual revelation path $\hat{\boldsymbol{\mu}}$ must satisfy:

$$(5.9) \qquad \hat{\boldsymbol{\mu}}_p(p) = g(p,\hat{\boldsymbol{\mu}}(p)), \qquad p \in (p_2^{*-}, p_{gr}),$$

where the formula of g is given in Appendix B.1.3 due to its complexity.

The following lemma characterizes the gradual revelation path $\hat{\boldsymbol{\mu}}$.

Lemma 3 *If the MPE with gradual revelation is an equilibrium, then the gradual revelation path* $\hat{\boldsymbol{\mu}}$ *is the unique solution to the first order ODE problem defined by equation* (5.9)*, with the initial value condition*

$$\hat{\boldsymbol{\mu}}(p) = \frac{s - \lambda^{I-}(p)\, h}{w^S(\boldsymbol{q}^+(p)) - s + \lambda^{I+}(p)\, h - \lambda^{I-}(p)\, h}, \qquad p = p_2^{*-}, \tag{5.10}$$

and the boundary p_{gr} *being the smallest* p *satisfying*

$$\hat{\boldsymbol{\mu}}(p_{gr}) = \boldsymbol{\mu}^o(p_{gr}). \tag{5.11}$$

The initial value condition comes from the value matching condition of W^U at p_2^{*-}:

$$s - \lambda^U h = \hat{\boldsymbol{\mu}}(p) \left(w^S \left(\boldsymbol{q}^+(p) \right) - s \right), \qquad p = p_2^{*-}, \tag{5.12}$$

where the left-hand side is U's continuation value (5.8) at the end of gradual revelation ($p = p_2^{*-}$) and the right-hand side is U's expected continuation value at the beginning of separation: with probability $\hat{\boldsymbol{\mu}}(p_2^{*-})$, U faces type s_+ and hence achieves a continuation value $w^S\left(\boldsymbol{q}^+\left(p_2^{*-}\right)\right) - s$, and with the complementary probability, U faces type s_- and experimentation ends.

Finally, the gradual revelation path $\hat{\boldsymbol{\mu}}$ must lie above the pooling path $\boldsymbol{\mu}^o$ and intersects at the boundary of gradual revelation p_{gr}, explained by condition (5.11).

5.4.2. *Sufficient conditions for the equilibrium construction and existence*

The previous subsection shows that players' (on-path) behaviors in an MPE with gradual revelation are necessarily characterized by Lemma 2 and Lemma 3. This subsection shows that these are also sufficient for the equilibrium to exist. Proposition 2 follows from Lemma 4.

Lemma 4 *Under Assumptions 1 and 2, an MPE with gradual revelation can be sustained as an equilibrium, if during gradual revelation, the uninformed player's effort is as in Lemma 2, and the gradual revelation path* $\hat{\boldsymbol{\mu}}$ *is the unique solution to the ODE problem defined by* (5.9)*,* (5.10)*, and* (5.11)*.*

To provide an example, the MPE with gradual revelation, with the system of beliefs and type s_+'s strategy specified below, is an equilibrium.

(1) The belief system. Low effort completely depletes reputation: if I takes an effort strictly lower than type s_+'s equilibrium effort, he will be taken as type s_-. The belief updating rule for $a^I = a^{I+}(p, \mu)$ is pinned down by Bayes' rule.

(2) Type s_+'s strategy. Type s_+ plays the symmetric MPE strategy under symmetric information as long as his reputation is strictly positive; otherwise he plays

the single-player solution. With Assumption 1, the strategy implies he always exerts effort 1 before the separation phase occurs, hence is consistent with our equilibrium prescription.

6. MULTIPLICITY OF EQUILIBRIA

6.1. *Equilibrium Uniqueness*

Not surprisingly, the asymmetric information game has multiple MPEs, due to the arbitrariness of assigning off-equilibrium beliefs, and to the multiplicity of (asymmetric) MPEs even under symmetric information (caused by the strategic substitutability of current effort decisions). To select sensible equilibria, I focus on MPEs that survive a criterion in the spirit of the D1 criterion, and that players play the symmetric MPE under symmetric information after the uninformed player assigns probability 1 to the informed player being a certain type. For simplicity, call the former restriction D1, and the latter restriction SMPE.

Proposition 3 *There exists $\bar{O}$, such that if the odds ratio O is greater than $\bar{O}$ and the prior belief q_0 is not low (above p_2^{*-}), then the distribution of the equilibrium path of any MPE satisfying D1 and SMPE coincides with that of the MPE with gradual revelation in Section 5.*

The proof is in the Online Appendix. The intuition behind this result is that, under Assumption 1, during a gradual revelation phase, if there are no reputation concerns, then type s_+ strictly prefers to experiment, whereas type s_- either strictly prefers not to experiment (in the rewarding region) or is indifferent (in the non-responding region). Therefore, the reason that type s_+ might choose effort lower than 1 in some MPE must be that effort 1 leads to a continuation equilibrium in which, U free-rides in the future. However, if the odds ratio is sufficiently high (i.e. $O \geq \bar{O}$) and if the prior belief q_0 is not too low, then before separation, type s_+ would still be sufficiently optimistic, meaning that there is little for him to learn from the information acquired by the uninformed player. As a result, type s_+ has little reputation concerns and would prefer to experiment before separation occurs.

6.2. *MPEs when Assumption 1 does not hold*

If Assumption 1 does not hold, then MPEs satisfying D1 and SMPE still exist. An MPE close to the MPE with gradual revelation has an additional pooling phase between the gradual revelation phase and the separation phase.

I focus on MPEs with gradual revelation because it delivers new insights, and qualitatively different behavior and belief dynamics. Moreover, it is useful to construct other equilibria. It represents one extreme where U's experimentation incentive is maintained through the encouraging private information gradually revealed by the informed player. In another extreme, U's experimentation incentive can be maintained by the informed player gradually reducing his effort (because current effort decisions are strategic substitutes), which can indeed occur in equilibrium if the odds ratio is small. Between these two extremes, it is possible to construct hybrid MPEs, in which, U's experimentation incentive is maintained by the two forces combined together.

7. WELFARE ANALYSIS

Does inducing information asymmetry, by hiding information from one player, improve welfare? To answer this question, I compare players' ex ante total welfare at the common prior belief q_0 (which, recall, is equal to the initial background belief p_0) in the MPE with gradual revelation:

$$W^U(q_0, \boldsymbol{\mu}^o(q_0)) + \boldsymbol{\mu}^o(q_0) W^{I+}(q_0, \boldsymbol{\mu}^o(q_0)) + (1 - \boldsymbol{\mu}^o(q_0)) W^{I-}(q_0, \boldsymbol{\mu}^o(q_0)), \tag{7.1}$$

and that in the symmetric MPE of the symmetric information game in which the informed player's private information is made public:

$$2\boldsymbol{\mu}^o(q_0) w^S(\boldsymbol{q}^+(q_0)) + 2(1 - \boldsymbol{\mu}^o(q_0)) w^S(\boldsymbol{q}^-(q_0)). \tag{7.2}$$

Asymmetric information is said to *improve* welfare if the former is greater than the latter, and *deteriorate* welfare if the former is smaller than the latter.

Thanks to the mutual encouragement effect, asymmetric information creates a benefit: in case I holds signal s_-, players experiment more than in the symmetric information benchmark. Asymmetric information may also incur a cost: in case I holds signal s_+, then during the gradual revelation phase, U experiments less than under symmetric information. However, if the informed player's private signal is informative enough, the benefit outweighs the cost, as is stated in proposition 4.

PROPOSITION 4 *If the odds ratio is high enough, that is,* $O \in [1 + 2\frac{\lambda}{r}, \infty)$*, then asymmetric information improves welfare, and strictly so if the common prior belief* q_0 *is in the gradual revelation region or the pooling region* $(p_2^{*-}, 1)$*.*

Interested readers may refer to Proposition 5 at the end of this section for a detailed welfare characterization when the odds ratio is intermediate. In Proposition 4, asymmetric information can strictly improve welfare only if q_0 is not in the separation region, because it does not affect welfare after separation.

We elaborate on the intuition of Proposition 4. Compared with symmetric information, in the asymmetric information game, type s_+ exerts the same level of effort, type s_- more effort; U exerts less effort than in the symmetric MPE when s_+ is public, and more when s_- is public. At the interim stage (right after I learns his type),

- type s_+ suffers from asymmetric information because he does not learn as much as he learns from U's experimentation under symmetric information due to U's lower effort, except when $\rho_b = 0$. When $\rho_b = 0$ (or, O is infinity), type s_+ knows that the risky project is good and hence does not need to learn from U.
- Type s_- (weakly) benefits from asymmetric information because he always has the option to reveal himself by exerting some low effort, whereby he guarantees himself the same payoff as in the symmetric information benchmark.
- U benefits from asymmetric information. U always has the option of matching her effort to I's. Doing so, in case I holds signal s_+, both players would experiment as under symmetric information with s_+ public, whereby U achieves the same ex post continuation value as under symmetric information. In case I holds signal s_-, both players would experiment more than under symmetric information with s_- public, but still less than in the cooperative solution; as a result, U achieves a strictly greater continuation value than under symmetric information. Therefore, with asymmetric information, by taking this effort-matching option, U can guarantee herself a higher interim value (in the MPE with gradual revelation) than in the symmetric benchmark.

Type s_+'s loss from asymmetric information is decreasing in the odds ratio. Intuitively, the greater the belief difference between I's types, the more optimistic type s_+ is during gradual revelation, and hence the less he needs to learn from U's experimentation, consequently the less he suffers from asymmetric information. Type s_- and U's gain from asymmetric information is increasing in the odds ratio. Intuitively, the greater the belief difference, the less type s_- needs to stop experimenting to compensate U during gradual revelation, and hence the higher probability that players continue experimenting over time, implying a higher welfare gain.

At one extreme when the odds ratio O is infinity (that is, if $\rho_b = 0$), type s_+ does not suffer from asymmetric information; asymmetric information thus results in a

Pareto improvement. At the other extreme when the odds ratio is equal to O^S, U is willing to experiment at the end of the gradual revelation phase only if she believes I is very likely to be type s_+; as a result, type s_+'s loss dominates, and asymmetric information deteriorates welfare (at least for background beliefs close to p_2^{*-}). By continuity and the monotonicity of the ex post gains and losses in the odds ratio, there exists a threshold such that, asymmetric information improves ex ante total welfare universally if the odds ratio is above the threshold, and does not if the odds ratio is below the threshold (and if the fraction of the pessimistic type is not too low so that the gradual revelation phase is not empty).

We are thus done with the main message. We now discuss the welfare impact of asymmetric information when the informativeness of I's private signal is intermediate, that is, when $O \in [O^S, 1 + 2\frac{\lambda}{r})$. If the fraction of type s_+ is high so that the gradual revelation phase is empty, then both players experiment at least until the background belief hits the pessimistic type's cooperative cutoff p_2^{*-}. Asymmetric information in this case leads to a Pareto improvement. Proposition 5 focuses on the parameter region where the gradual revelation phase is not empty, as is guaranteed by Assumption 2:

PROPOSITION 5 *Assume that the odds ratio is intermediate:* $O \in [O^S, 1 + 2\frac{\lambda}{r})$*, and that Assumption 2 is satisfied. Then*

1. *either asymmetric information deteriorates welfare, and strictly so if the common prior* q_0 *lies in the gradual revelation region or the pooling region* $(p_2^{*-}, 1)$*;*
2. *or there exists* $\tilde{p} \in (p_2^{*-}, p^{S-}]$*, such that asymmetric information deteriorates welfare if the common prior* q_0 *is in* $(0, \tilde{p})$*, and improves it if* q_0 *is in* $(\tilde{p}, 1)$*.*

A sufficient and necessary condition for the second case to occur is ρ_g being either sufficiently low or sufficiently high. Intuitively, when ρ_g is sufficiently low, then there is a large fraction of type s_-, implying the players' expected gain (occurring only in case I holds signal s_-) is large, relative to their expected loss (occurring only in case I holds signal s_+). When ρ_g is sufficiently high, then there is a small fraction of type s_-, implying a short gradual revelation phase, and hence a small welfare loss (occurring during gradual revelation), relative to the the gain (occurring during pooling).

8. EMPIRICAL IMPLICATIONS

1. On testing "learning from others' experimentation." Suppose a new seed variety is introduced in a village, and we want to test whether farmers learn from each other's experimentation. Should we test the hypothesis "a farmer's land allocated to the new

seed is positively correlated with his/her neighbor's yield" (for instance, as is done by Munshi (2004))? The result of this paper—players can increase experimentation over time despite the absence of a breakthrough—implies that, if asymmetric information is present, then even if this hypothesis is rejected, we still cannot conclude that farmers do not learn from each other.

One can amend the test by focusing on positive news, that is, by testing whether a farmer allocates a larger land to the new seed in reaction to an increase of his/her neighbors' yield. This amendment improves the test because, with asymmetric information, although players may not react to negative news (the absence of breakthroughs) by experimenting less, they do react to positive news by experimenting more.

2. Divergent learning dynamics. Note that, if the non-responding region of the gradual revelation phase is not empty ($p_{gr} > p_1^{*-}$) and if the prior q_0 is not too low ($q_0 > p_1^{*-}$), then, conditional on the informed player being the pessimistic type and the risky project being bad, with positive probability the pessimistic type reveals himself at a late time (after the background belief reaches p_1^{*-}), after which, experimentation ends immediately; and with positive probability the pessimistic type reveals himself at an early time (before the background belief reaches p_1^{*-}) and players play the symmetric MPE, in which, free-riding is so severe that they never abandon the bad projects in finite time.[22]

Therefore, combining the joint-project interpretation (see page 7), the MPE with gradual revelation predicts that two identical groups of players receiving the same information can exhibit divergent learning dynamics. In one group, the leader leads by example for a long time; as a result, learning is fast, and the joint project is completed or abandoned in finite time. In another group, the leader leads by example for a short period of time; as a result, players free ride, projects are highly inertial with little learning, and bad projects are not abandoned in finite time. That failing projects of strategic alliances are highly inertial are well documented in the management literature (Doz, 1996, for instance).

3. Further empirical predictions. This paper predicts that experienced players experiment more than inexperienced players do and that their experimentation behavior is less sensitive to negative news or other players' experimentation behavior. These predictions are in line with the empirical findings of Bandiera and Rasul (2006), and Conley and Udry (2010).

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[22]This property of the symmetric MPE is proved by **?**.

9. CONCLUSION

This paper has studied the impact of initial information asymmetry on agents' experimentation behavior, using the canonical exponential-bandit model. It has shown that a novel mutual encouragement effect can arise, which drives players to increase experimentation despite the absence of breakthroughs, and to acquire more information than under symmetric information.

The paper has shown that, if a social planner has a piece of private information, then revealing the information to only one player generates higher welfare than revealing it to both players or to neither. Suppose now the social planner has more alternatives: she can send to each player as a signal a garbling of the initial information structure. Is revealing the information to one player and having the other player uninformed *optimal* (in terms of ex-ante welfare)? Interestingly, over a certain parameter region, it is; that is, creating an informed leader and an uninformed follower can be better than any other alternative. This occurs, for instance, when the favorable signal reveals good quality of the risky project and the favorable signal is likely to occur. This paper leaves open the question of how to allocate information over the parameter region where revealing information to exactly one player is not optimal. It would be interesting to analyze this problem in future research.

APPENDIX A: SOME BEST RESPONSES

A.1. *U's best response to I's pooling strategies*

The following lemma analyzes U's best response to I's pooling strategies over some interval of background beliefs $[\underline{p}, \bar{p}]$. Let $\boldsymbol{\mu}(p)$ denote I's reputation at background belief $p \in [\underline{p}, \bar{p}]$, determined by Bayes' rule (equation (4.7) with Y_t being a constant).

LEMMA 5 *Assume over some interval $[\underline{p}, \bar{p}]$ both types of I are prescribed to exert effort 1. Let $\tilde{a}^U : [\underline{p}, \bar{p}] \times [0,1] \to [0,1]$ be U's best response to this strategy profile, and $\tilde{W}^U : [\underline{p}, \bar{p}] \times [0,1] \to \mathbb{R}$ her corresponding continuation value function. Then*

1. *U finds it optimal to use corner solutions, that is, either to exert effort 1, or not to experiment. At any point of p at which U switches actions, U's continuation value satisfies $\tilde{W}^U(p, \boldsymbol{\mu}) - s = s - \lambda^U(p, \boldsymbol{\mu})h$.*
2. *If $\tilde{W}^U(p, \boldsymbol{\mu}) - s > s - \lambda^U(p, \boldsymbol{\mu})h$, then $\tilde{a}^U(p, \boldsymbol{\mu}) = 1$; if $\tilde{W}^U(p, \boldsymbol{\mu}) - s < s - \lambda^U(p, \boldsymbol{\mu})h$, then $\tilde{a}^U(p, \boldsymbol{\mu}) = 0$; otherwise, $\tilde{a}^U(p, \boldsymbol{\mu}) \in [0,1]$.*
3. *If U's value function satisfies the boundary condition $W^U(\underline{p}, \boldsymbol{\mu}) - s = s - \lambda^U(\underline{p}, \boldsymbol{\mu})h$, then she finds it optimal to adopt a cutoff strategy: to exert effort 1 if $p \geq p^*$, and not to experiment otherwise, for some $p^* \in [\underline{p}, \bar{p}]$.*
4. *If on top of the boundary condition in point 3, at $p = \underline{p}$ is also satisfied*
$$r\left(\lambda^U(p, \boldsymbol{\mu})h - s\right) + \lambda p(1-p)\frac{d\lambda^U(p, \boldsymbol{\mu})}{dp}h + \lambda^U(p, \boldsymbol{\mu})\left(\lambda h - s - \left(s - \lambda^U(p, \boldsymbol{\mu})h\right)\right) \geq 0.$$
then U finds it optimal to exert effort 1 over $[\underline{p}, \bar{p}]$.

PROOF: *Point 1.* Given I's strategy profile, U's value function $\tilde{W}^U$ satisfies the following HJB equation, for $p \in (\underline{p}, \bar{p})$,

$$\begin{aligned} r\tilde{W}^U(p, \boldsymbol{\mu}) &= \max_{a \in [0,1]} a\left[r\left(\lambda^U(p, \boldsymbol{\mu})h - s\right) - \lambda p(1-p)\frac{d\tilde{W}^U(p, \boldsymbol{\mu})}{dp} + \lambda^U(p, \boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p, \boldsymbol{\mu})\right)\right] \\ &\quad + \left[-\lambda p(1-p)\frac{d\tilde{W}^U(p, \boldsymbol{\mu})}{dp} + \lambda^U(p, \boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p, \boldsymbol{\mu})\right)\right] + rs. \end{aligned}$$

At any state $(p, \boldsymbol{\mu})$ where U is willing to switch actions, and hence is indifferent between experimenting and not experimenting, we have

$$r\left(s - \lambda^U(p, \boldsymbol{\mu})h\right) = -\lambda p(1-p)\frac{d\tilde{W}^U(p, \boldsymbol{\mu})}{dp} + \lambda^U(p, \boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p, \boldsymbol{\mu})\right). \tag{A.1}$$

Consequently, $\tilde{W}^U$ satisfies the HJB equation

$$r\tilde{W}^U(p,\boldsymbol{\mu}) - rs \;=\; -\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right).$$

The above two equations imply that at any p where U switches actions,

$$\text{(A.2)}\qquad \tilde{W}^U(p,\boldsymbol{\mu}) - s = s - \lambda^U(p,\boldsymbol{\mu}).$$

Since there is no subinterval of $[\underline{p},\bar{p}]$ over which $\tilde{W}^U$ satisfy both equation (A.1) and (A.2), there is no subinterval of $[\underline{p},\bar{p}]$ over which U exerts interior effort.

Point 2 is obvious from the above analysis.

Point 3. We will show that $\tilde{W}^U(p,\boldsymbol{\mu}) - s$ intersects $s - \lambda^U(p,\boldsymbol{\mu})$ at most once over $(\underline{p},\bar{p})$. Together with *Point 2*, this property implies *Point 3*.

To show this property, it is sufficient to show that if there is some $\tilde{p} \in [\underline{p},\bar{p})$ such that $\tilde{W}^U(\tilde{p},\boldsymbol{\mu}) - s = s - \lambda^U(\tilde{p},\boldsymbol{\mu})$, and $\frac{d\tilde{W}^U(\tilde{p}+,\boldsymbol{\mu}(\tilde{p}+))}{dp} > -\frac{d\lambda^U(\tilde{p},\boldsymbol{\mu}(\tilde{p}))}{dp}$, then $\tilde{W}^U(p,\boldsymbol{\mu}) - s > s - \lambda^U(p,\boldsymbol{\mu})$ for all $p \in (\tilde{p},\bar{p})$. Suppose by contradiction that there is some $\check{p} \in (\tilde{p},\bar{p})$ such that $\tilde{W}^U(\check{p},\boldsymbol{\mu}) - s = s - \lambda^U(\check{p},\boldsymbol{\mu})$, then we must have $\frac{d\tilde{W}^U(\check{p}-,\boldsymbol{\mu}(\check{p}))}{dp} < -\frac{d\lambda^U(\check{p},\boldsymbol{\mu}(\check{p}))}{dp}$. Since $s - \lambda^U(p,\boldsymbol{\mu})$ strictly decreases in p, we have $\tilde{W}^U(\check{p},\boldsymbol{\mu}) < \tilde{W}^U(\tilde{p},\boldsymbol{\mu})$. But these inequalities imply that equation (A.1) cannot be satisfied at both $\tilde{p}$ and $\check{p}$. A contradiction.

Point 4. The sufficient condition given in Point 4, together with equation (A.1) and $W^U(\underline{p},\boldsymbol{\mu}) - s = s - \lambda^U(\underline{p},\boldsymbol{\mu})$, implies that $\frac{d\tilde{W}^U\left(\underline{p}+,\boldsymbol{\mu}(\underline{p}+)\right)}{dp} > -\frac{d\lambda^U\left(\underline{p},\boldsymbol{\mu}(\underline{p})\right)}{dp}$. Using the same argument involved in proving *Point 3*, we conclude that $\tilde{a}^U(p,\boldsymbol{\mu}) = 1$ over $[\underline{p},\bar{p}]$. *Q.E.D.*

More generally, if a player exerts a constant effort over some interval of background belief, then the other player finds it optimal to use corner solutions over this interval, with at most two cutoffs. The proof is similar and hence omitted.

APPENDIX B: EQUILIBRIUM CONSTRUCTION AND ANALYSIS

For brevity, all the derivatives of functions at $p = p_2^{*-}$ refer to the right derivatives.

B.1. *Characterization of the gradual revelation phase*

B.1.1. *U's strategy (proof of the second case of Lemma 2)*

We are left to derive U's strategy during the non-responding region of the gradual revelation phase (that is, for $p \in (p_1^{*-}, p_{gr})$, if nonempty), assuming that the equilibrium with gradual revelation is an equilibrium.

Proof of the second case of Lemma 2: First, $W^{I-}(p,0)$ satisfies the same HJB equation as equation (5.2), with the arguments $(p, \hat{\boldsymbol{\mu}}(p))$ of all the functions replaced by $(p,0)$. Since for $p \in (p_1^{*-}, p_{gr})$, $a^{I-}(p,0) = a^S(\boldsymbol{q}^-(p)) \in (0,1)$, the terms in equation (5.2) that are directly affected by type s_-'s effort must be 0:

$$\left[r\left(\lambda^{I-}(p)\,h - s\right) - \lambda p\,(1-p)\,\frac{dW^{I-}(p,0)}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}(p,0)\right)\right] = 0.$$

Since $\lambda^{I-}(p)\,h - s < 0$ for $p \in (p_1^{*-}, p_{gr})$, we have

$$\left[-\lambda p\,(1-p)\,\frac{dW^{I-}(p,0)}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}(p,0)\right)\right] > 0.$$

During the gradual revelation phase, type s_- must be indifferent between revealing and not revealing, that is, $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = W^{I-}(p,0)$. This equality, together with both $W^{I-}(p, \hat{\boldsymbol{\mu}}(p))$ and $W^{I-}(p,0)$ satisfying the HJB equation (5.2), implies that $a^U(p, \hat{\boldsymbol{\mu}}(p)) = a^U(p,0)$ over (p_1^{*-}, p_{gr}). Since by construction $a^U(p,0) = a^S(\boldsymbol{q}^-(p))$, we have $a^U(p, \hat{\boldsymbol{\mu}}(p)) = a^S(\boldsymbol{q}^-(p))$. Therefore, before a breakthrough occurs, the uninformed player's effort is decreasing over time during the non-rewarding region. *Q.E.D.*

B.1.2. *U's HJB equation and experimentation incentive*

A heuristic derivation of U's HJB equation, equation (5.6): Suppose the time-t state is $(p_t, \hat{\boldsymbol{\mu}}(p_t))$, and that U considers exerting effort $\tilde{a} \in [0,1]$ during the time interval $[t, t+dt)$ as long as I exerts effort 1, and playing according to the candidate equilibrium strategy at other states.

The flow continuation value of doing so, $r(W^U(p_t, \hat{\boldsymbol{\mu}}) - s)$, should equal the right-hand side of equation (5.6): the expected instantaneous payoff (per unit of time), $r\tilde{a}\left(\lambda^U(p_t, \hat{\boldsymbol{\mu}})\,h - s\right)$, plus the value of information (per unit of time), $E[W(p_{t+dt}, \hat{\boldsymbol{\mu}}(p_{t+dt})) - W(p_t, \hat{\boldsymbol{\mu}}(p_t))]/dt$. The latter can be decomposed into three parts. The first two parts resemble those in equation (5.2): the rate of change in U's continuation value in case no breakthrough arrives and type s_- does not reveal his type in $[t, t+dt)$, and the change in U's continuation value in case a breakthrough arrives in $[t, t+dt)$, multiplied by the arrival rate of a breakthrough. The third part comes from the possibility that I reveals he is type s_- in $[t, t+dt]$ in the absence of a breakthrough, an event that reduces U's continuation value by an amount $|W^U(p,0) - W^U(p, \hat{\boldsymbol{\mu}})|$. We now calculate the probability of this event.

Given players' effort path, the background belief at time $t+dt$ will be $p_t - (1+\tilde{a})\,\lambda p\,(1-p)\,dt$. According to the equilibrium prescription, type s_- will reveal at a rate such that the action of non-revealing (continuing exerting effort 1) will keep the state variables on the curve $\hat{\boldsymbol{\mu}}$. That is, I's reputation at time $t+dt$ will be $\hat{\boldsymbol{\mu}}\left(p_t-(1+\tilde{a})\,\lambda p_t\left(1-p_t\right)dt\right)$. Therefore, I's reputation during the time interval $[t,t+dt]$ adjusts by an amount

$$d\mu_t \equiv \hat{\boldsymbol{\mu}}\left(p_t-(1+\tilde{a})\,\lambda p_t\left(1-p_t\right)dt\right)-\hat{\boldsymbol{\mu}}\left(p_t\right)=\hat{\boldsymbol{\mu}}_p\left(p_t\right)dp_t.$$

Using Bayes' rule (4.8), we have, the rate at which I reveals that he is type s_- is

$$\left(1-\hat{\boldsymbol{\mu}}\left(p_t\right)\right)\frac{dY_t/dt}{1-Y_t} \quad = \quad \left(\frac{\boldsymbol{\mu}_p^o(p_t)\left(1-\hat{\boldsymbol{\mu}}\left(p_t\right)\right)}{\boldsymbol{\mu}^o(p_t)\left(1-\boldsymbol{\mu}^o\left(p_t\right)\right)}-\frac{\hat{\boldsymbol{\mu}}_p\left(p_t\right)}{\hat{\boldsymbol{\mu}}\left(p_t\right)}\right)(1+\tilde{a})p_t(1-p_t)\lambda.$$

Rearranging terms, we have, at $p=p_t$, U's continuation value function satisfies

$$r(W^U\left(p,\hat{\boldsymbol{\mu}}\right)-s) \quad = \quad \tilde{a}\left[r\left(\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right)+A\left(p,\hat{\boldsymbol{\mu}}\right)\right]+A(p,\hat{\boldsymbol{\mu}}).$$

Using optimality of $\tilde{a}$, we obtain the HJB equation (5.6).

During gradual revelation, U's equilibrium effort is interior (except at $p=p_1^{*-}$). Therefore, her IC condition (5.7) must hold. This condition, together with the HJB equation (5.6), implies that U's value function satisfies equation (5.8). *Q.E.D.*

B.1.3. *The gradual revelation path $\hat{\boldsymbol{\mu}}$ (proof of Lemma 3)*

The Formula of g. Define functions $B:[0,1]^2\to\mathbb{R}$ and $C:[0,1]^2\to\mathbb{R}$ by

$$B\left(p,\mu\right) \quad = \quad \left(s-\lambda^U\left(p,\mu\right)h\right)-\left(s-\lambda^{I-}\left(p\right)h\right)a^{I-}\left(p,0\right),$$

$$C\left(p,\mu\right) \quad = \quad r\left(\lambda^U\left(p,\mu\right)h-s\right)+\lambda p\left(1-p\right)\lambda_p^U\left(p,\mu\right)h+\lambda^U\left(p,\mu\right)\left(\lambda h-s-\left(s-\lambda^U\left(p,\mu\right)h\right)\right),$$

where λ_p^U denotes the partial derivative function of λ^U with respect to its first argument p.[23] The function B is to be interpreted as U's continuation value drop caused by type s_-'s revelation, and C as U's marginal benefit of experimentation excluding the part obtained from the private information revealed by I. Then, using U's indifference condition (5.7) and her value function (5.8), we have

$$\text{(B.1)}\quad \hat{\boldsymbol{\mu}}_p \quad = \quad g\left(p,\hat{\boldsymbol{\mu}}\right)\equiv-\left(\frac{C\left(p,\hat{\boldsymbol{\mu}}\right)}{\lambda p\left(1-p\right)B\left(p,0\right)}-\phi\left(p,\hat{\boldsymbol{\mu}}\right)\frac{B\left(p,\hat{\boldsymbol{\mu}}\right)}{B\left(p,0\right)}\right)\hat{\boldsymbol{\mu}},\quad p\in(p_2^{*-},p_{gr}),$$

where

$$\text{(B.2)}\quad \phi\left(p,\mu\right) \quad \equiv \quad \frac{\left(1-\mu\right)\boldsymbol{\mu}_p^o\left(p\right)}{\boldsymbol{\mu}^o\left(p\right)\left(1-\boldsymbol{\mu}^o\left(p\right)\right)}=\frac{\lambda^{I+}\left(p\right)-\lambda^U\left(p,\mu\right)}{\lambda p\left(1-p\right)}.$$

We prove Lemma 3 in four steps. Step 1 derives some convenient formulas for $\hat{\boldsymbol{\mu}}_p/\hat{\boldsymbol{\mu}}$ and for $d\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)/dp$, which will be used in later steps. Step 2 shows that if $\hat{\boldsymbol{\mu}}$ satisfies ODE (5.9), then it is strictly decreasing, ensuring that type s_-'s revealing rate is strictly positive and is hence feasible (see Lemma 6). Step 3 shows that the

[23] That is, $\lambda_p^U\left(p,\mu\right)\equiv\mu\frac{d\lambda^{I+}(p)}{dp}+(1-\mu)\frac{d\lambda^{I-}(p)}{dp}=\mu\lambda_p^{I+}\left(p\right)+(1-\mu)\lambda_p^{I-}\left(p\right)$.

ODE problem defined by equations (5.9)-(5.11) has a unique solution (see Lemma 7). Step 4 shows that a gradual revelation path is absolutely continuous with p, and hence satisfies equations (5.9)-(5.11). Reading from Step 4 to 2, Lemma 3 follows.

Proof of Lemma 3: *Step 1. Convenient formulas for $\hat{\boldsymbol{\mu}}_p/\hat{\boldsymbol{\mu}}$ and $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp$.*

Define a function $D:[0,1]^2\to\mathbb{R}$ by (omitting the argument (p,μ) of λ^U)

$$\text{(B.3)}\qquad D(p,\mu)\ \equiv\ \frac{r\left(\lambda^U h-s\right)+\lambda^U\left(\lambda h-s-s+\lambda^U h\right)}{B(p,\mu)}+\lambda^U-\lambda.$$

Let λ^U_μ denote the partial derivative function of λ^U with respect to its second argument μ. Combining equations (B.1) and (B.2), at p such that $\hat{\boldsymbol{\mu}}(p)\neq 0$, we have

$$\begin{aligned}
-\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}} &= \frac{C(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}-\phi(p,\hat{\boldsymbol{\mu}})\frac{B(p,\hat{\boldsymbol{\mu}})}{B(p,0)}\\
\text{(B.4)}\qquad &= \frac{\lambda^U_p(p,\hat{\boldsymbol{\mu}})h}{B(p,0)}+\frac{B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[D(p,\hat{\boldsymbol{\mu}})+\lambda-\lambda^{I+}(p)\right]\\
\text{(B.5)}\qquad &= \frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp}{B(p,\hat{\boldsymbol{\mu}})}+\frac{1}{\lambda p(1-p)}\left[D(p,\hat{\boldsymbol{\mu}})+\lambda-\lambda^{I+}(p)\right].
\end{aligned}$$

The last equality uses $d\lambda^U/dp=\lambda^U_p+\lambda^U_\mu\hat{\boldsymbol{\mu}}_p$ and $B(p,0)=B(p,\hat{\boldsymbol{\mu}})+\lambda^U_\mu\hat{\boldsymbol{\mu}}h$.

To derive some formulas for $d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp$, we first need an expression for $\frac{\lambda^U_p}{\lambda^U_\mu\hat{\boldsymbol{\mu}}}$.

$$\begin{aligned}
\frac{\lambda^U_p}{\lambda^U_\mu\hat{\boldsymbol{\mu}}} &= \frac{\lambda^U_p}{\hat{\boldsymbol{\mu}}\left(\lambda^{I+}(p)-\lambda^{I-}(p)\right)}\\
&= \frac{\rho_g(1-\rho_g)}{p(1-p)(\rho_g-\rho_b)}\left[\frac{p+(1-p)\frac{1-\rho_b}{1-\rho_g}}{p+(1-p)\frac{\rho_b}{\rho_g}}\frac{\rho_b}{\rho_g}+\left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\frac{p+(1-p)\frac{\rho_b}{\rho_g}}{p+(1-p)\frac{1-\rho_b}{1-\rho_g}}\frac{1-\rho_b}{1-\rho_g}\right]\\
\text{(B.6)}\quad &= \frac{\lambda}{\lambda p(1-p)}\left[\left(1-\frac{\lambda^{I+}(p)}{\lambda}\right)+\frac{1}{\hat{\boldsymbol{\mu}}}\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b}+\left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\frac{\lambda^{I-}(p)}{\lambda}\right].
\end{aligned}$$

Subtracting $\frac{\lambda^U_p}{\lambda^U_\mu\hat{\boldsymbol{\mu}}}+\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp}{B(p,\hat{\boldsymbol{\mu}})}$ from both sides of equation (B.5), using equation (B.6), and rearranging terms, we have

$$
\begin{aligned}
&-\frac{d\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)}{dp} \\
\text{(B.7)}\qquad &= \frac{\lambda_{\mu}^{U}\hat{\boldsymbol{\mu}}B\left(p,\hat{\boldsymbol{\mu}}\right)}{\lambda p\left(1-p\right)B\left(p,0\right)}\left[D\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I-}\left(p\right)-\frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_{b}\left(1-\rho_{g}\right)}{\rho_{g}-\rho_{b}}\lambda+\lambda^{I-}\left(p\right)\right)\right] \\
\text{(B.8)}\qquad &= \frac{\lambda_{\mu}^{U}\hat{\boldsymbol{\mu}}B\left(p,\hat{\boldsymbol{\mu}}\right)}{\lambda p\left(1-p\right)B\left(p,0\right)}\left[D\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I-}\left(p\right)-\frac{\left(\lambda-\lambda^{I-}\left(p\right)\right)\lambda^{I-}\left(p\right)}{\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)-\lambda^{I-}\left(p\right)}\right] \\
\text{(B.9)}\qquad &= \frac{\lambda_{\mu}^{U}\hat{\boldsymbol{\mu}}B\left(p,\hat{\boldsymbol{\mu}}\right)}{\lambda p\left(1-p\right)B\left(p,0\right)}\left[D\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda-\lambda^{I+}\left(p\right)-\frac{\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\left(\lambda-\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)}{\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)-\lambda^{I-}\left(p\right)}\right].
\end{aligned}
$$

Step 2. Monotonicity of $\hat{\boldsymbol{\mu}}$ (or, feasibility of type s_{-}'s revealing strategy).

We will treat the point p^{S-} with care because $B(p,0)>0$ for $p\in[p_{2}^{*-},p^{S-})$ and $B(p^{S-},0)=0$. The latter equality follows from $a^{I-}(p^{S-},0)=a^{S}\circ\boldsymbol{q}^{-}(p^{S-})=1$, where $\boldsymbol{q}^{-}(p^{S-})$ is the posterior belief at which players switch from 1 to interior effort in the symmetric MPE.

Lemma 6 *Let $\alpha\in(p_{2}^{*-},p^{S-})$ and $\hat{\boldsymbol{\mu}}|_{[p_{2}^{*-},\alpha]}$ be a solution to the ODE problem defined by (5.9) restricted over $[p_{2}^{*-},\alpha]$ and the initial condition (5.10). If $\hat{\boldsymbol{\mu}}\in(0,1)$ and $B\left(p,\hat{\boldsymbol{\mu}}\right)>0$ over (p_{2}^{*-},α), then $\hat{\boldsymbol{\mu}}_{p}<0$ over (p_{2}^{*-},α).*

Proof of Lemma 6: Take p in the gradual revelation region. By equation (B.5), if $d\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)/dp\leq0$, then by the definition of λ^{U}, we have $\hat{\boldsymbol{\mu}}_{p}<0$; if $d\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)/dp>0$ and $D\left(p,\hat{\boldsymbol{\mu}}\right)>0$, we also have $\hat{\boldsymbol{\mu}}_{p}<0$. We now show $D\left(p,\hat{\boldsymbol{\mu}}\right)>0$ for any p in the gradual revelation region. Lemma 6 then follows.

We first show, if $D\left(p,\hat{\boldsymbol{\mu}}\right)>0$ at $p=p_{2}^{*-}$ (the left boundary of the gradual revelation phase), then $D\left(p,\hat{\boldsymbol{\mu}}\right)>0$ for any p in the gradual revelation region. Suppose $D\left(p,\hat{\boldsymbol{\mu}}\right)>0$ at $p=p_{2}^{*-}$, and there exists some p such that $D\left(p,\hat{\boldsymbol{\mu}}\right)$ is negative. Then there exists some $\tilde{p}$ such that $D\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)=0$, and that $d\lambda^{U}\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)/dp\leq0$. Equation (B.7) then implies that at $D\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)>0$. A contradiction.

we now show $D\left(p,\hat{\boldsymbol{\mu}}\right)>0$ at $p=p_{2}^{*-}$. From here until the end of this proof, if not mentioned, p is fixed at p_{2}^{*-}. Using the initial condition (5.10), at $p=p_{2}^{*-}$, we have

$$
\frac{\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\left(\lambda h-s\right)}{s-\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)h}=\frac{s\left(\lambda^{I+}\left(p\right)-\lambda^{I-}\left(p\right)\right)+\lambda^{I-}\left(p\right)\left(w^{S}\left(\boldsymbol{q}^{+}\left(p\right)\right)-s\right)}{\left(s-\lambda^{I-}\left(p\right)h\right)\left(w^{S}\left(\boldsymbol{q}^{+}\left(p\right)\right)-s\right)}\left(\lambda h-s\right).
$$

Applying the definition of D, and the fact that $a^S \circ \boldsymbol{q}^-(p) = 0$ at $p = p_2^{*-}$, we have

$$D\left(p, \hat{\boldsymbol{\mu}}\right) + \lambda - \lambda^{I+}(p) = \frac{s\left(\lambda^{I+}(p) - \lambda^{I-}(p)\right)\frac{\lambda h - w^S\left(\boldsymbol{q}^+(p)\right)}{w^S\left(\boldsymbol{q}^+(p)\right) - s} + \lambda^{I-}(p)\left(\lambda^{I+}(p) - \lambda h\right)}{s - \lambda^{I-}(p)\,h}.$$

Using the definition of p_2^{*-} (in equation (3.1)) and that of λ^{I-}, we have

$$D\left(p, \hat{\boldsymbol{\mu}}\right) + \lambda - \lambda^{I+}(p) = \frac{-w^S\left(\boldsymbol{q}^+(p)\right)\left(r + 2\lambda^{I+}(p)\right) + (2\lambda + r)\,\lambda^{I+}(p)\,h}{2\left(w^S\left(\boldsymbol{q}^+(p)\right) - s\right)}. \tag{B.10}$$

By Assumption 1, $p_2^{*-} \geq p^{S+}$, we have $a^S\left(\boldsymbol{q}^+(p)\right) = 1$ at $p = p_2^{*-}$. Recall that w^S is the continuation value function corresponding to the symmetric MPE under symmetric information a^S, whose argument is the *true posterior*.

Since in MPE a^S, players exert effort 1 if their common posterior is above q^S, w^S satisfies the following HJB equation at background beliefs p such that $\boldsymbol{q}^+(p) > q^S$,

$$rw^S\left(\boldsymbol{q}^+\right) - rs = r\left(\lambda \boldsymbol{q}^+ h - s\right) - 2\lambda \boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right) w_q^S\left(\boldsymbol{q}^+\right) + 2\lambda \boldsymbol{q}^+\left(\lambda h - w^S\left(\boldsymbol{q}^+\right)\right),$$

where the argument of $\boldsymbol{q}^+$ is omitted. Rearranging terms, we have

$$-w^S\left(\boldsymbol{q}^+\right)\left(r + 2\lambda \boldsymbol{q}^+\right) + (2\lambda + r)\,\lambda \boldsymbol{q}^+ h = 2\lambda \boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right) w_q^S\left(\boldsymbol{q}^+\right). \tag{B.11}$$

With this equation, equation (B.10) becomes

$$D\left(p, \hat{\boldsymbol{\mu}}\right) + \lambda - \lambda^{I+}(p) = \frac{\lambda \boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right) w_q^S\left(\boldsymbol{q}^+\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)}. \tag{B.12}$$

Therefore,

$$\begin{aligned}
D\left(p, \hat{\boldsymbol{\mu}}\right) &= \frac{\lambda \boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right) w_q^S\left(\boldsymbol{q}^+\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)} - \left(1 - \boldsymbol{q}^+\right)\lambda \\
&= \left(1 - \boldsymbol{q}^+\right)\lambda \frac{\boldsymbol{q}^+ w_q^S\left(\boldsymbol{q}^+\right) - \left(w^S\left(\boldsymbol{q}^+\right) - s\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)} && \text{(B.13)} \\
&> \left(1 - \boldsymbol{q}^+\right)\lambda \frac{\left(\boldsymbol{q}^+ - q_1^*\right) w_q^S\left(\boldsymbol{q}^+\right) - \left(w^S\left(\boldsymbol{q}^+\right) - s\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)} \\
&> 0 && \text{(B.14)}
\end{aligned}$$

The second-to-last inequality is due to $w_q^S(\boldsymbol{q}^+) > 0$ at $p = p_2^{*-}$; the last inequality is due to the convexity of w^S over $[q_1^*, 1]$, and that $w^S\left(q_1^*\right) = s$. *Q.E.D.*

Step 3. Existence and uniqueness of a solution to the ODE problem (5.9)-(5.11).

LEMMA 7 *If $\hat{\boldsymbol{\mu}}(p_2^{*-})$ defined by (5.10) is such that $\hat{\boldsymbol{\mu}}(p_2^{*-}) < \boldsymbol{\mu}^o(p_2^{*-})$, then the ODE problem defined by (5.9)-(5.11) has a unique solution $\hat{\boldsymbol{\mu}}$. Moreover, the right boundary p_{gr} is in (p_2^{*-}, p^{S-}).*

Proof of Lemma 7: Let $\epsilon \in (0, p^{S-})$ be the unique solution to the equation $s - \lambda^U \left(p^{S-} - \epsilon, \boldsymbol{\mu}^o \left(p^{S-} - \epsilon\right)\right) h = W^{S-}\left(p^{S-} - \epsilon\right) - s$. Uniqueness follows from the fact that both sides are continuous in ϵ, the left-hand side is increasing in ϵ, the right-hand side is decreasing in ϵ, and that the left-hand side is strictly greater than the right-hand side at $\epsilon = p^{S-}$ and strictly smaller than the latter at $\epsilon = 0$. As $\epsilon > 0$, we have $a^{I-}(p, 0) < 1$ over $[p_2^{*-}, p^{S-} - \epsilon]$,[24] and hence $B(p, 0)$ is bounded away from 0 for $p \in [p_2^{*-}, p^{S-} - \epsilon]$. $C(p, \hat{\boldsymbol{\mu}})$, $\phi(p, \hat{\boldsymbol{\mu}})$, and $B(p, \hat{\boldsymbol{\mu}})$ are also bounded for $p \in [p_2^{*-}, p^{S-} - \epsilon]$ and $\hat{\boldsymbol{\mu}} \in [0, 1]$.

Consider ODE (5.9) but restricted over $[p_2^{*-}, p^{S-} - \epsilon]$. That is,

$$\text{(B.15)} \quad \hat{\boldsymbol{\mu}}_p = -\left(\frac{C(p, \hat{\boldsymbol{\mu}})}{\lambda p (1-p) B(p, 0)} - \phi(p, \hat{\boldsymbol{\mu}}) \frac{B(p, \hat{\boldsymbol{\mu}})}{B(p, 0)}\right) \hat{\boldsymbol{\mu}}, \qquad p \in (p_2^{*-}, p^{S-} - \epsilon),$$

with the initial condition (5.10). $\hat{\boldsymbol{\mu}}_p$ as a function of $(p, \hat{\boldsymbol{\mu}})$, defined by equation (B.15), is bounded, and Lipschitz continuous. Applying standard uniqueness theorems (for example, Picard–Lindelöf theorem), this initial value problem has a unique solution. Let $\hat{\boldsymbol{\mu}}$ denote this solution (abusing notations). Moreover, if $\hat{\boldsymbol{\mu}}$ reaches 0 at some $\hat{p}$, then the function defined by $\hat{\boldsymbol{\mu}}(p) = 0$ for all $p \in [\hat{p}, p^{S-} - \epsilon]$ solves ODE (B.15) when it is restricted over $[\hat{p}, p^{S-} - \epsilon]$. Therefore, by uniqueness, the solution to the ODE problem defined by (B.15) and (5.10) must be such that, once it reaches 0, it stays at 0 for larger p's. We will use Claims 1 to 3 to establish Lemma 7.

Claim 1 *Over the interval* $[p_2^{*-}, p^{S-} - \epsilon]$, *we have* $\hat{\boldsymbol{\mu}} \in (0, 1)$, *and* $B(p, \hat{\boldsymbol{\mu}}) > 0$.

Claim 2 $\hat{\boldsymbol{\mu}}$ *is decreasing over the interval* $[p_2^{*-}, p^{S-} - \epsilon]$.

Claim 3 $\hat{\boldsymbol{\mu}}\left(p^{S-} - \epsilon\right) < \boldsymbol{\mu}^o\left(p^{S-} - \epsilon\right)$.

Claim 2 and 3, together with $\hat{\boldsymbol{\mu}}\left(p_2^{*-}\right) > \boldsymbol{\mu}^o\left(p_2^{*-}\right)$ and the continuity of $\hat{\boldsymbol{\mu}}$ and $\boldsymbol{\mu}^o$, imply that there exists a unique $p_{gr} \in (p_2^{*-}, p^{S-} - \epsilon)$ such that $\hat{\boldsymbol{\mu}}(p_{gr}) = \boldsymbol{\mu}^o(p_{gr})$, and that $\hat{\boldsymbol{\mu}}(p) > \boldsymbol{\mu}^o(p)$ for $p \in [p_2^{*-}, p_{gr})$. Therefore, $\hat{\boldsymbol{\mu}}$ restricted over $[p_2^{*-}, p_{gr}]$ is the unique solution to the ODE problem (5.9)-(5.11), as desired. *Q.E.D.*

We are left to prove the Claims 1 to 3.

Proof of Claim 1: Suppose by negation that there is some $p \in [p_2^{*-}, p^{S-} - \epsilon]$ such that $B(p, \hat{\boldsymbol{\mu}}) \leq 0$; denote the smallest p satisfying this inequality as $\tilde{p}$. By the definition of the function B and that $a^{I-}(p, \hat{\boldsymbol{\mu}}) < 1$ for $p \in [p_2^{*-}, p^{S-} - \epsilon]$, we have

[24] Recall that over $[p_2^{*-}, p^{S-}]$, $a^{I-}(p, 0) \equiv a^S \circ \boldsymbol{q}^-(p)$, and is strictly increasing from 0 to 1.

$\hat{\boldsymbol{\mu}}(\tilde{p}) > 0$. Since $B(p, \hat{\boldsymbol{\mu}}) > 0$ at $p = p_2^{*-}$ and B is continuous, we have $\tilde{p} > p_2^{*-}$, $B(p, \hat{\boldsymbol{\mu}}) > 0$ for $p \in [p_2^{*-}, \tilde{p})$, and $B(\tilde{p}, \hat{\boldsymbol{\mu}}) = 0$. The latter two inequalities imply that $\frac{dB(\tilde{p},\hat{\boldsymbol{\mu}})}{dp} \leq 0$. Moreover, by definition, $B(p, \hat{\boldsymbol{\mu}}) = \left(s - \lambda^U(p,\mu)\,h\right) - (w^S(\boldsymbol{q}^-(p)) - s)$, and hence $\tilde{p}$ must be in the region where players exert interior effort in the symmetric MPE when s_- is public. $\frac{dB(\tilde{p},\hat{\boldsymbol{\mu}})}{dp} \leq 0$ implies that $\frac{d\left(s-\lambda^U(\tilde{p},\mu)h\right)}{dp} \leq \frac{dw^S(\boldsymbol{q}^-(\tilde{p}))}{dp}$.

But then, ODE (B.15), which implies U's indifference condition (5.7), cannot be satisfied at $p = \tilde{p}$. This is because, as $\lambda^U(\tilde{p},\mu) > \lambda^{I-}(\tilde{p})$, the instantaneous marginal benefit of experimenting is strictly higher for U than for type s_-; the value of a breakthrough weighted by the arrival rate is higher higher for U as well:

$$\lambda^U(\tilde{p},\mu)\left(\lambda h - s - (s - \lambda^U(\tilde{p},\mu)\,h)\right) > \lambda^{I-}(\tilde{p})\left(\lambda h - s - (w^S(\boldsymbol{q}^-(p)) - s)\right);$$

the rate of change in continuation values in case of no breakthrough is also higher for U: $\frac{d\left(s-\lambda^U(\tilde{p},\mu)h\right)}{dp}\frac{dp_t}{dt} \geq \frac{w^S(\boldsymbol{q}^-(\tilde{p}))}{dp}\frac{dp_t}{dt}$, as $\frac{dp_t}{dt} < 0$. Therefore, if at some $\tilde{p}$ type s_- is indifferent about experimentation, then using $B(\tilde{p}, \hat{\boldsymbol{\mu}}) = 0$, U's indifference condition (5.7) cannot be satisfied. A contradiction to $\hat{\boldsymbol{\mu}}$ being a solution to ODE (B.15). *Q.E.D.*

Proof of Claim 2: Using Claim 1, Lemma 6, and the fact that $\hat{\boldsymbol{\mu}}(p_2^{*-}) \in (0,1)$, we have that $\hat{\boldsymbol{\mu}}$ is decreasing until it reaches 0 (if ever), and then because of our selection of $\hat{\boldsymbol{\mu}}$, it stays at 0 until p reaches $p^{S-} - \epsilon$. Therefore, $\hat{\boldsymbol{\mu}}$ is decreasing over $[p_2^{*-}, p^{S-} - \epsilon]$. *Q.E.D.*

Proof of Claim 3: Suppose by contradiction that $\hat{\boldsymbol{\mu}}\left(p^{S-} - \epsilon\right) \geq \boldsymbol{\mu}^o\left(p^{S-} - \epsilon\right)$. Then $W^U(p^{S-} - \epsilon, \hat{\boldsymbol{\mu}}(p^{S-} - \epsilon))$ defined by equation (5.8) is smaller than $w^S(\boldsymbol{q}^-(p^{S-} - \epsilon))$ by the choice of ϵ. Using $B(p, \hat{\boldsymbol{\mu}}) = W^U(p, \hat{\boldsymbol{\mu}}) - w^S(\boldsymbol{q}^-(p))$, we have that $B(p^{S-} - \epsilon, \hat{\boldsymbol{\mu}}(p^{S-} - \epsilon)) \leq 0$, which contradicts Claim 1. *Q.E.D.*

Step 4. A gradual revelation path $\hat{\boldsymbol{\mu}}$ is absolutely continuous. First, $\hat{\boldsymbol{\mu}}$ is continuous over (p_2^{*-}, p_{gr}). Suppose by negation that there is some $\tilde{p} \in (p_2^{*-}, p_{gr})$ at which $\hat{\boldsymbol{\mu}}$ is discontinuous, that is, $\hat{\boldsymbol{\mu}}(\tilde{p}+) < \hat{\boldsymbol{\mu}}(\tilde{p}-)$.[25] Since over a small right neighborhood of $\tilde{p}$, player U is indifferent between experimenting and not experimenting, we must have $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) = s + s - \lambda^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))h$, by equation (5.8). Similarly, we have $W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-)) = s + s - \lambda^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))h$. Since λ^U strictly increases in its second argument, these two inequalities imply that $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) > W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))$. But this contradicts the fact that $W^U \geq s$, as $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))$ is the average between $W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))$ and s.

[25]Note in our candidate equilibrium, $\hat{\boldsymbol{\mu}}$ is discontinuous if and only if type s_- reveals with a lump-sum probability, hence at any p, $\hat{\boldsymbol{\mu}}$ can only jump downward.

Similarly, $\hat{\boldsymbol{\mu}}$ is continuous at p_{gr}. The difference between this case and the previous case is that, over a small right neighborhood of p_{gr}, player U strictly prefers to experiment, and type s_- strictly prefers not to reveal. Therefore, by Point 2 of Lemma 5, we have $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) \geq s + s - \lambda^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))h$. Continuity of $\hat{\boldsymbol{\mu}}$ at p_{gr} follows the same logic as in the previous case. We show in the online appendix that $\hat{\boldsymbol{\mu}}$ does not have singular continuous part.

Since $\hat{\boldsymbol{\mu}}$ is monotone (see Lemma 6), continuous, and does not have a singular continuous part, it must be absolutely continuous, and hence satisfies ODE defined by (5.9), (5.10), and (5.11). *Q.E.D.*

B.2. *Verification (proof of Lemma 4)*

We first check type s_+'s incentive, and then check type s_-'s and U's.

In the separation phase ($p \leq p_2^{*-}$), if the state is $(p, 0)$, then U does not experiment, and hence it is optimal for type s_+ to adopt the single-player solution. If the state is $(p, 1)$, type s_+ has no incentive to deviate to lower efforts because doing so would reduce his reputation to 0 and hence he would obtain the continuation value in the single-player solution, which is lower than following the equilibrium prescription, which gives him the continuation value associated with the symmetric MPE. Deviating to higher efforts cannot improve his continuation value either.

Before separation ($p > p_2^{*-}$), according to the equilibrium prescription, I's reputation can only be 0 or $\hat{\boldsymbol{\mu}}$, on and off path. If the state is $(p, 0)$, then U will play the symmetric MPE effort with public information s_-. Type s_+'s optimal strategy is to adopt the single-player solution, because if I's reputation is fixed at 0, then for $p \in [p^{S-}, 1]$, type s_- being willing to experiment implies that type s_+ is willing to experiment, and for $p \in [p_2^{*-}, p^{S-})$, U's effort being 0 implies the optimality of type s_+'s single-player solution. If the state is on the gradual revelation path, $(p, \hat{\boldsymbol{\mu}})$, type s_+ is willing to exert effort 1. This is because if he deviates, he will still find effort 1 optimal before separation, but since the deviation reduces U's effort, it reduces what he obtains before separation; Moreover, type s_+'s continuation value at the beginning of the separation phase is lower if he deviates today (see the previous paragraph). In total, type s_+ has no profitable deviation.

We now check type s_-'s and U's incentive to deviate phase by phase.

1. Separation ($p \leq p_2^{*-}$). The nontrivial case is when $p_1^{*+} < p \leq p_2^{*-}$, and $\mu = \boldsymbol{\mu}^o(p)$. Type s_- has no incentive to deviate because, given the updating rule and that U will choose the same action as he does (i.e. the symmetric MPE effort cor-

responding to the information conveyed by I's action), the tradeoff of experimenting or not he faces is exactly the same as is faced by a two-player team with public information s_-: in both situations, experimenting yields the same instantaneous payoff and twice the value of information of a single player. Since a two-player team finds it optimal to stop if $p < p_2^{*-}$, so does type s_-.
U has no incentive to deviate given that I has revealed his type and plays the corresponding symmetric MPE strategy.

2. Gradual revelation ($p_2^{*-} < p < p_{gr}$).
(2.1) The state is along $\hat{\boldsymbol{\mu}}$. Since we construct U's effort function to make type s_- indifferent between revealing him self and mimicking type s_+, type s_- has no incentive to deviate. Similarly, we construct the gradual revelation path $\hat{\boldsymbol{\mu}}$ to make U indifferent between experimenting and not experimenting, U has no incentive to deviate either.
(2.2) The state is along $\boldsymbol{\mu}^o$. If type s_- deviates to an effort level different from his equilibrium efforts, he would reveal himself, thereby obtaining a continuation value at most $w^S(\boldsymbol{q}^-(\cdot))$, which is his value of following the equilibrium strategy. Therefore, he has no incentive to deviate.

3. Pooling ($p > p_{gr}$). Whether type s_- deviates or not, he obtains the same continuation value at p_{gr}: his symmetric MPE payoff. If he does not deviate, then both players exert effort 1 until p reaches p_{gr}, which maximizes the players' joint surplus before p reaches p_{gr}, and hence gives type s_- a higher payoff than if he deviates (which gives him his symmetric MPE payoff).
Applying Point 4 of Lemma 5 for $\underline{p} = p_{gr}$ and $\bar{p} = 1$, U has no incentive to deviate, if we show that the left-hand side of the inequality in Point 4 is positive at $p = p_{gr}$ with $\boldsymbol{\mu}$ replaced by I's pooling reputation $\boldsymbol{\mu}^o$. This condition is indeed satisfied, because $\frac{d\lambda^U(p,\boldsymbol{\mu}^o)}{dp} > 0$, and at $p = p_{gr}$,
$$r\left(\lambda^U\left(p, \boldsymbol{\mu}^o\right) h - s\right) + \lambda^U\left(p, \boldsymbol{\mu}^o\right)\left(\lambda h - s - \left(s - \lambda^U\left(p, \boldsymbol{\mu}^o\right) h\right)\right) > 0.$$
The latter follows from $D(p, \hat{\boldsymbol{\mu}}) > 0$ at $p = p_{gr}$ (see the proof of Lemma 6), and by the definition of p_{gr}, $\hat{\boldsymbol{\mu}}(p_{gr}) = \boldsymbol{\mu}^o(p_{gr})$.

APPENDIX C: WELFARE ANALYSIS

This section proves Proposition 4 and Proposition 5. We first analyze welfare when the state variables are in the gradual revelation phase, and then extend the result to the pooling region. The two propositions follow after these.

C.1. *Welfare analysis in the gradual revelation phase*

Assume that the prior q_0 is in the gradual revelation region. In the MPE with gradual revelation, type s_- would reveal himself with a positive probability, such that mimicking type s_+'s effort immediately pushes I's reputation from $\boldsymbol{\mu}^o(q_0)$ to $\hat{\boldsymbol{\mu}}(q_0)$. We now calculate the welfare gain of inducing asymmetric information from this point on. To this aim, let $\Delta W(p, \hat{\boldsymbol{\mu}})$ be the difference between the total welfare under asymmetric information and that under symmetric information when the state variables in the MPE with gradual revelation are $(p, \hat{\boldsymbol{\mu}})$: expression (7.1) minus expression (7.2), with $\boldsymbol{\mu}^o$ replaced by $\hat{\boldsymbol{\mu}}$, and q_0 by p.

$$\Delta W(p, \hat{\boldsymbol{\mu}}) = W^U(p, \hat{\boldsymbol{\mu}}) + \hat{\boldsymbol{\mu}} W^{I+}(p, \hat{\boldsymbol{\mu}}) - 2\hat{\boldsymbol{\mu}} w^S(\boldsymbol{q}^+(p)) - (1 - \hat{\boldsymbol{\mu}}) w^S(\boldsymbol{q}^-(p)). \tag{C.1}$$

Whether inducing asymmetric information improves welfare or not at the prior q_0 (in the gradual revelation region) is equivalent with whether $\Delta W(q_0, \hat{\boldsymbol{\mu}})$ is positive or not. Therefore, in the sequel, we look at the sign of $\Delta W(p, \hat{\boldsymbol{\mu}})$ at $p = q_0$.

LEMMA 8 *During the gradual revelation phase,*

1. *If* $O \geq 1 + 2\frac{\lambda}{r}$, *then* $\Delta W(p, \hat{\boldsymbol{\mu}}) > 0$ *over* $(p_2^{*-}, p_{gr}]$.
2. *If* $O^S \leq O < 1 + 2\frac{\lambda}{r}$, *and*
   (a) *if there exists* $\tilde{p}_O \in (p_2^{*-}, p_{gr})$ *such that* $\Delta W(\tilde{p}_O, \hat{\boldsymbol{\mu}}) = 0$, *then* $\Delta W(p, \hat{\boldsymbol{\mu}}) < 0$ *over* $(p_2^{*-}, \tilde{p}_O)$, *and* $\Delta W(p, \hat{\boldsymbol{\mu}}) > 0$ *over* $(\tilde{p}_O, p_{gr}]$;
   (b) *otherwise,* $\Delta W(p, \hat{\boldsymbol{\mu}}) < 0$ *over* (p_2^{*-}, p_{gr}). *(*p_{gr} *is not included in the interval because we might have* $\Delta W(p_{gr}, \hat{\boldsymbol{\mu}}) = 0$.*)*

We will use the following two Claims. In Claim 5, the derivatives refer to right derivatives.

CLAIM 4 *During the gradual revelation phase, if there is some* $\tilde{p} \in (p_2^{*-}, p_{gr})$ *at which* $\Delta W(\tilde{p}, \hat{\boldsymbol{\mu}}) = 0$, *then* $\frac{d\Delta W(\tilde{p}, \hat{\boldsymbol{\mu}})}{dp} > 0$.

CLAIM 5 *At* $p = p_2^{*-}$, $\Delta W(p, \hat{\boldsymbol{\mu}}) = 0$, $\frac{d\Delta W(p, \hat{\boldsymbol{\mu}})}{dp} = 0$, *and*

1. $\frac{d^2\Delta W(p, \hat{\boldsymbol{\mu}})}{dp^2} > 0$, *if* $O > 1 + 2\frac{\lambda}{r}$;
2. $\frac{d^2\Delta W(p, \hat{\boldsymbol{\mu}})}{dp^2} < 0$, *if* $O \in [O^S, 1 + 2\frac{\lambda}{r})$;
3. $\frac{d^2\Delta W(p, \hat{\boldsymbol{\mu}})}{dp^2} = 0$ *and* $\frac{d^3\Delta W(p, \hat{\boldsymbol{\mu}})}{dp^3} > 0$, *if* $O = 1 + 2\frac{\lambda}{r}$.

Proof of Lemma 8: The statement in the first case of Lemma 8 is implied by Claim 4, the first and the third case of Claim 5. The statement in the second case is implied by Claim 4 and the second case of Claim 5. *Q.E.D.*

We are left to prove Claims 4 and 5. We will use the following equation, which is derived in the Online Appendix.

Claim 6 *During the gradual revelation phase, ΔW satisfies*

$$\begin{aligned} & \left(1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda p\left(1-p\right)\frac{d\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)}{dp} \\ = & -\left[r+\left(1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)+\left(1-\hat{\boldsymbol{\mu}}\right)y\left(p,\hat{\boldsymbol{\mu}}\right)\right]\Delta W\left(p,\hat{\boldsymbol{\mu}}\right) \\ & -\left(1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)r\left[\hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}^+\left(p\right)\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)+\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right]. \end{aligned} \tag{C.2}$$

Proof of Claim 4: From equation (C.2), at $\tilde{p}$ such that $\Delta W\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)=0$, the sign of $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp}$ is the same with that of

$$-\left[\hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}^+\left(p\right)\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)+\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right] \tag{C.3}$$

at $p=\tilde{p}$. Using the expression $W^U\left(p,\hat{\boldsymbol{\mu}}\right)-s=s-\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h$ and $\Delta W\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)=0$ to replace $\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h-s$, we have, the sign of $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp}$ is the same with that of

$$\hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}^+\left(p\right)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)+\left(1-\hat{\boldsymbol{\mu}}\right)\left(w^S\left(\boldsymbol{q}^-\left(p\right)\right)-s\right) \tag{C.4}$$

at $p=\tilde{p}$. Note that a necessary condition for $\Delta W\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)=0$ for some $\tilde{p}$ is that the odds ratio O is finite, implying that $\frac{d\boldsymbol{q}^+(p)}{dp}>0$.

Step 1. $d\left(w^S\left(\boldsymbol{q}^+\left(p\right)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)/dp>0$ and $\frac{d\left(w^S\left(\boldsymbol{q}^-(p)\right)-s\right)}{dp}>0$ during gradual revelation. Since both $w^S\left(\boldsymbol{q}^+\left(\cdot\right)\right)$ and $w^S\left(\boldsymbol{q}^-\left(\cdot\right)\right)$ increase in p, the two inequalities hold if we show $d\left(-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h\right)/dp>0$. To show this, we take $W^{I+}\left(\cdot,\hat{\boldsymbol{\mu}}\right)$ and λ^{I+} as functions of type s_+'s posterior belief q^+; we can do so because $\frac{d\boldsymbol{q}^+(p)}{dp}>0$. Using the HJB equation of W^{I+}, we have, (omitting the arguments $\left(p,\hat{\boldsymbol{\mu}}\right)$ of W^{I+} and a^U)

$$\begin{aligned} & r\left(W^{I+}-\lambda^{I+}\left(p\right)h\right) \\ = & \left(1+a^U\right)\left[-\lambda\boldsymbol{q}^+\left(p\right)\left(1-\boldsymbol{q}^+\left(p\right)\right)\frac{dW^{I+}}{dq^+}+\lambda^{I+}\left(p\right)\left(\lambda h-s-\left(W^{I+}-s\right)\right)\right]. \end{aligned} \tag{C.5}$$

Suppose by negation $\frac{dW^{I+}(p,\hat{\boldsymbol{\mu}})}{dq^+}\geq\lambda h$. Then the right-hand side of equation (C.5) would be smaller than

$$\left(1+a^U\right)\left[-\lambda\boldsymbol{q}^+\left(p\right)\left(1-\boldsymbol{q}^+\left(p\right)\right)\lambda h+\lambda^{I+}\left(p\right)\left(\lambda h-s-\left(W^{I+}-s\right)\right)\right].$$

The terms in the square brackets are equal to $\lambda^{I+}(p)\,h - W^{I+}(p,\hat{\boldsymbol{\mu}})$, which is strictly negative at during the gradual revelation phase, contradicted with the fact that the left-hand side of equation (C.5) is strictly positive (because the odds ratio is finite and $\tilde{p} < 1$).

Step 2. $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp} > 0$. Suppose by negation that $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp} \leq 0$, implying that expression (C.4) is negative. Since $\hat{\boldsymbol{\mu}} > 0$, $w^S(\boldsymbol{q}^+(p)) - W^{I+}(p,\hat{\boldsymbol{\mu}}) + \lambda^{I+}(p)\,h - s$ is also negative. This property, together with $\hat{\boldsymbol{\mu}}$ being decreasing in p (Proposition 1), and the inequalities in Step 1, we have, expression (C.4) is strictly increasing in p at $\tilde{p}$. This contradicts $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp} \leq 0$. *Q.E.D.*

Proof of Claim 5: Obviously, inducing asymmetric information does not affect welfare at the beginning of the separation phase: $\Delta W\left(p_2^{*-},\hat{\boldsymbol{\mu}}\right) = 0$. Applying this equality and $a^U\left(p_2^{*-},\hat{\boldsymbol{\mu}}\right) = 1$ to equation (C.2), we have $\frac{d\Delta W\left(p_2^{*-},\hat{\boldsymbol{\mu}}\right)}{dp} = 0$.

Taking right derivatives on both sides of equation (C.2), and using $\Delta W = 0$, $d\Delta W/dp = 0$, and $\hat{\boldsymbol{\mu}}\left(w^S(\boldsymbol{q}^+(p)) - s\right) = s - \lambda^U(p,\hat{\boldsymbol{\mu}})\,h$ at $p = p_2^{*-}$ (the initial value condition of $\hat{\boldsymbol{\mu}}$), we have, at $p = p_2^{*-}$,

$$\left(1 + a^U(p,\hat{\boldsymbol{\mu}})\right)\lambda p(1-p)\frac{d^2\Delta W(p,\hat{\boldsymbol{\mu}})}{dp^2} = -\frac{da^U(p,\hat{\boldsymbol{\mu}})}{dp} r\hat{\boldsymbol{\mu}}\left(\lambda^{I+}(p)\,h - s\right). \tag{C.6}$$

Since $\frac{da^U(p,\hat{\boldsymbol{\mu}})}{dp} < 0$ at $p = p_2^{*-}$, we have, at $p = p_2^{*-}$,

- $\frac{d^2\Delta W(p,\hat{\boldsymbol{\mu}})}{dp^2} > 0$ if $\lambda^{I+}(p)\,h - s > 0$;
- $\frac{d^2\Delta W(p,\hat{\boldsymbol{\mu}})}{dp^2} < 0$ if $\lambda^{I+}(p)\,h - s < 0$.

This statement implies the statements in the first two cases in Claim 5, because at $p = p_2^{*-}$, $O > 1 + 2\frac{\lambda}{r}$ implies $\lambda^{I+}(p)\,h - s > 0$; and $O < 1 + 2\frac{\lambda}{r}$ implies $\lambda^{I+}(p)\,h - s < 0$.

If $O = 1 + 2\frac{\lambda}{r}$, then $\lambda^{I+}(p)\,h - s = 0$ at $p = p_2^{*-}$, and hence $\frac{d^2\Delta W(p,\hat{\boldsymbol{\mu}})}{dp^2} = 0$ from equation (C.6). Taking right derivative with respect to p on both sides of equation (C.6), we have $\left(1 + a^U(p,\hat{\boldsymbol{\mu}})\right)\lambda p(1-p)\frac{d^3\Delta W(p,\hat{\boldsymbol{\mu}})}{dp^3} = -\frac{da^U(p,\hat{\boldsymbol{\mu}})}{dp} r\hat{\boldsymbol{\mu}}\frac{\lambda^{I+}(p)}{dp}h > 0$. The third case follows. *Q.E.D.*

Proof of Claim 6: Rewrite the HJB equations of W^U, W^{I+}, $w^S(\boldsymbol{q}^+(\cdot))$, and $w^S(\boldsymbol{q}^-(\cdot))$ in the following way. In the sequel, for brevity, we omit the arguments $(p,\hat{\boldsymbol{\mu}})$ of W^U, W^{I+}, ΔW, λ^U, and a^U.

$$\begin{aligned} r\left(W^U - s\right) &= a^U r\left[\lambda^U h - s\right] - (1-\hat{\boldsymbol{\mu}})\,y\left(W^U - W^U(p,0)\right) \\ &\quad + \left(1 + a^U\right)\left[-\lambda p(1-p)\frac{dW^U}{dp} + \lambda^U\left(\lambda h - s - \left(W^U - s\right)\right)\right]. \end{aligned} \tag{C.7}$$

This equation is the same with the HJB equation (5.6), except that we collect the two terms representing the usual value of information together, corresponding to the event that no revealing and no breakthrough arrives and to the event that a breakthrough arrives.

$$
\begin{aligned}
r\left(W^{I+}-s\right) &= r\left[\lambda^{I+}(p)\,h-s\right] \\
&\quad +\left(1+a^{U}\right)\left[-\lambda p(1-p)\frac{dW^{I+}}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(W^{I+}-s\right)\right)\right].
\end{aligned} \tag{C.8}
$$

$$
\begin{aligned}
& r\left(w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s\right) \\
= {} & r\left[\lambda^{I+}(p)\,h-s\right] \\
& +\left(1+a^{U}\right)\left[-\lambda p(1-p)\frac{dw^{S}\left(\boldsymbol{q}^{+}(p)\right)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s\right)\right)\right] \\
& +\left(1-a^{U}\right)\left[-\lambda p(1-p)\frac{dw^{S}\left(\boldsymbol{q}^{+}(p)\right)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s\right)\right)\right] \\
= {} & r\left[\lambda^{I+}(p)\,h-s\right] \\
& +\left(1+a^{U}\right)\left[-\lambda p(1-p)\frac{dw^{S}\left(\boldsymbol{q}^{+}(p)\right)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s\right)\right)\right] \\
& +\frac{\left(1-a^{U}\right)}{2}\left[w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right].
\end{aligned} \tag{C.9}
$$

The first equality is due to the fact that $p>p^{S+}$, and hence both players exert effort 1 in the symmetric MPE when s_{+} is public. In the second equality, we replace the value of information by $\left[w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]/2$, which is obtained from the first equality. The term

$$
\left(1-a^{U}\right)\left[w^{S}\left(\boldsymbol{q}^{+}(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]
$$

can be interpreted as the welfare loss to both players in case player I is type s_{+}, caused by the lack of effort of player U (in the equilibrium of asymmetric information game, compared with the symmetric MPE in the symmetric information game).

$$
\begin{aligned}
& r\left(w^{S}\left(\boldsymbol{q}^{-}(p)\right)-s\right) \\
= {} & r\left[\lambda^{I-}(p)\,h-s\right] \\
& +\left(1+a^{U}\right)\left[-\lambda p(1-p)\frac{dw^{S}\left(\boldsymbol{q}^{-}(p)\right)}{dp}+\lambda^{I-}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}^{-}(p)\right)-s\right)\right)\right].
\end{aligned} \tag{C.10}
$$

Written in this way, this equation says, in the symmetric information game with public signal s_{-}, U's payoff can be obtained by her exerting effort 1 for $p\in\left(p_{2}^{*-},p_{gr}\right)$ and type s_{-} exerting effort $a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)$. This is true because type s_{-} obtains the symmetric information payoff $w^S\left(\boldsymbol{q}^-(\cdot)\right)$ in the asymmetric information setup and we swap the effort strategies of the two players. We write $w^S\left(\boldsymbol{q}^-(\cdot)\right)$ in this way so that the total effort is $\left(1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)$, the same under symmetric information, which makes comparison easy. Loosely speaking, in the asymmetric information game, player U enjoys an additional flow payoff $r\left(1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left(s-\lambda^{I-}(p)\,h\right)$ due to effort saving in case I has signal s_-, compared with the symmetric information case.

We now combine these four HJB equations to derive an HJB equation of ΔW.

By definition of ΔW (equation (C.1)), we have

$$\begin{aligned}\frac{d\Delta W}{dp} &= \left(\frac{dW^U}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}}{dp}-2\hat{\boldsymbol{\mu}}\frac{dw^S\left(\boldsymbol{q}^+(p)\right)}{dp}-(1-\hat{\boldsymbol{\mu}})\frac{dw^S\left(\boldsymbol{q}^-(p)\right)}{dp}\right)\\ &\quad+\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\hat{\boldsymbol{\mu}}\left(W^{I+}-2w^S\left(\boldsymbol{q}^+(p)\right)+w^S\left(\boldsymbol{q}^-(p)\right)\right) \qquad \text{(C.11)}\\ &= \left(\frac{dW^U}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}}{dp}-2\hat{\boldsymbol{\mu}}\frac{dw^S\left(\boldsymbol{q}^+(p)\right)}{dp}-(1-\hat{\boldsymbol{\mu}})\frac{dw^S\left(\boldsymbol{q}^-(p)\right)}{dp}\right)\\ &\quad+\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\left(\Delta W-W^U+w^S\left(\boldsymbol{q}^-(p)\right)\right). \qquad \text{(C.12)}\end{aligned}$$

Using again the definition of ΔW and the four HJB equations (C.7) to (C.10), we obtain an HJB equation for $\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)$, with a term

$$\left(\frac{dW^U}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}}{dp}-2\hat{\boldsymbol{\mu}}\frac{dw^S\left(\boldsymbol{q}^+(p)\right)}{dp}-(1-\hat{\boldsymbol{\mu}})\frac{dw^S\left(\boldsymbol{q}^-(p)\right)}{dp}\right).$$

We then apply equalities (C.11) and (C.12) to replace this term, and apply equations (5.4) and (B.2) to replace $\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}$. Rearranging terms, we obtain the HJB equation

$$\begin{aligned}r\Delta W &= -r\left(1-a^U\right)\left[\hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}^+(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right)+\lambda^U h-s\right]\\ &\quad+\left(1+a^U\right)\left[-\lambda p\,(1-p)\,\frac{d\Delta W}{dp}-\lambda^U\Delta W\right]-(1-\hat{\boldsymbol{\mu}})\,y\Delta W, \qquad \text{(C.13)}\end{aligned}$$

which reduces to equation (C.2) in the claim. *Q.E.D.*

C.2. *Welfare analysis in the pooling phase*

During the pooling phase, both players exert effort 1, hence the total effort level is the same as in the symmetric MPE when s_+ is public, and the same as in the symmetric MPE with s_- being public for $p\in[p^{S-},1]$ but strictly higher for $p\in(p_{gr},p^{S-})$. Therefore, the same property in Claim 4 holds over the interval $[p_{gr},p^{S-})$; The sign of ΔW over the interval $(p^{S-},1)$ is the same with that at $p=p^{S-}$.

Therefore, the result in Lemma 8 can be extended to both the gradual revelation phase and the pooling phase, with p_{gr} being replaced by 1 (except that at $p = 1$, $\Delta W = 0$).

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Online Appendix: Strategic Experimentation with Asymmetric Information

Miaomiao Dong

D. BACKGROUND BELIEF

This section defines the background belief formally.

Let Ω_N be the set of standard Poisson process paths and $\Omega \equiv \{g, b\} \times \{s_+, s_-\} \times \Omega_N$. Let $\mathcal{P}(\{g, b\} \times \{s_+, s_-\})$ be the power set of $\{0, 1\} \times \{s_+, s_-\}$, $(\mathcal{F}_t^N)$ the filtration generated by the standard Poisson process N, and $(\mathcal{F}_t) \equiv \mathcal{P}(\{g, b\} \times \{s_+, s_-\}) \otimes (\mathcal{F}_t^N)$ and $\mathcal{F} \equiv \mathcal{F}_\infty$. For a given prior p_0, an effort path $a \equiv (a_t^I, a_t^U)_{t \geq 0}$ induces a distribution P_{a,p_0} over the filtered space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0})$. For each $\theta \in \{g, b\}$, each $s_l \in \{s_+, s_-\}$, each t, let $\{\theta, s_l, N^t = 0^t\} \subset \Omega$ denote the event that the risky project's quality is θ, the signal s_l has occurred to I, and no breakthrough has arrived until time t. Let the events $\{\theta, N^t = 0^t\}$,$\{s_l, N^t = 0^t\}$, $\{N^t = 0^t\} \subset \Omega$ be similarly defined. P_{a,p_0} satisfies,

$$\begin{aligned} P_{a,p_0}(\theta, s_l, N^t = 0^t) &= P_{a,p_0}(s_l, N^t = 0^t|\theta)P_{a,p_0}(\theta) \\ &= P_{a,p_0}(s_l|\theta)P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta), \end{aligned}$$

where the second inequality is follows from the fact that given a and conditional on θ, the random variable s_l and the process N are independently distributed. Note that $P_{a,p_0}(\theta) = p_0$, $P_{a,p_0}(s_l|\theta)$ does not depend on a and p_0, and $P_{a,p_0}(N^t = 0^t|\theta)$ does not depend on p_0.

Given P_{a,p_0}, if $\sum_\theta P_{a,p_0}(s_l|\theta)P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta) > 0$, the distribution of θ conditional on $\{s_l, N^t = 0^t\}$, is given by

$$P_{a,p_0}(\theta|s_l, N^t = 0^t) = \frac{P_{a,p_0}(s_l|\theta)P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta)}{\sum_\theta P_{a,p_0}(s_l|\theta)P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta)} \tag{D.1}$$

In the asymmetric information game, $P_{a,p_0}(\theta|s_l, N^t = 0^t)$ is the conditional probability that type s_l assigns on the risky project's quality being θ, after he observes the effort history and and that no breakthrough has occurred up to time t. We now show the following two statements (in italic).

(i) For a given effort path a, the informed player's posterior does not depend on whether he observes s_l before the event $\{N^t = 0^t\}$ or afterward.

Dividing both the numerator and denominator of the right-hand side of equation (D.1) by $\sum_{\tilde{\theta}} P_{a,p_0}(s_l|\tilde{\theta})P_{a,p_0}(\tilde{\theta})$, we have

$$\begin{aligned}
P_{a,p_0}(\theta|s_l, N^t = 0^t) &= \frac{P_{a,p_0}(N^t = 0^t|\theta)[P_{a,p_0}(s_l|\theta)P_{a,p_0}(\theta)/\sum_{\tilde{\theta}} P_{a,p_0}(s_l|\tilde{\theta})P_{a,p_0}(\tilde{\theta})]}{\sum_\theta P_{a,p_0}(N^t = 0^t|\theta)[P_{a,p_0}(s_l|\theta)P_{a,p_0}(\theta)/\sum_{\tilde{\theta}} P_{a,p_0}(s_l|\tilde{\theta})P_{a,p_0}(\tilde{\theta})]} \\
&= \frac{P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta|s_l)}{\sum_\theta P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta|s_l)}
\end{aligned}$$

The second equality is by Bayes' rule. This equality can be interpreted as follows: after observing the signal s_l, a player (or an outsider) with prior p_0 updates his or her prior to $P_{a,p_0}(\theta|s_l)$ (which is independent of a); then the player observes a path of effort up to time t, an experimentation result history up to time t, $\{N^t = 0^t\}$, and he or she updates belief according to Bayes' rule, using $P_{a,p_0}(\theta|s_l)$ as the new "prior." This is how the informed player in the asymmetric information game updates his belief.

Similarly, dividing both the numerator and denominator of the right-hand side of equation (D.1) by $\sum_{\tilde{\theta}} P_{a,p_0}(N^t = 0^t|\tilde{\theta})P_{a,p_0}(\tilde{\theta})$, we have

$$\begin{aligned}
P_{a,p_0}(\theta|s_l, N^t = 0^t) &= \frac{P_{a,p_0}(s_l|\theta)[P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta)/\sum_{\tilde{\theta}} P_{a,p_0}(N^t = 0^t|\tilde{\theta})P_{a,p_0}(\tilde{\theta})]}{\sum_\theta P_{a,p_0}(s_l|\theta)[P_{a,p_0}(N^t = 0^t|\theta)P_{a,p_0}(\theta)/\sum_{\tilde{\theta}} P_{a,p_0}(N^t = 0^t|\tilde{\theta})P_{a,p_0}(\tilde{\theta})]} \\
&= \frac{P_{a,p_0}(s_l|\theta)P_{a,p_0}(\theta|N^t = 0^t)}{\sum_\theta P_{a,p_0}(s_l|\theta)P_{a,p_0}(\theta|N^t = 0^t)}. \qquad \text{(D.2)}
\end{aligned}$$

This right-hand side of equation (D.2) can be interpreted as the posterior belief of a player (or an outsider) in the following counterfactual world: after observing a path of effort up to time t, an experimentation result history up to time t, $\{N^t = 0^t\}$, the player (or the outsider) who starts with a prior belief p_0 updates his or her belief to $P_{a,p_0}(\theta|N^t = 0^t)$; then the player observes the noisy signal s_l, and he or she updates belief according to Bayes' rule, using $P_{a,p_0}(\theta|N^t = 0^t)$ as the new "prior."

(ii) In the asymmetric information game, any two public histories that lead to the same posterior of type s_- must also lead to the same posterior of type s_+. This is because on the right-hand side of equation (D.2), $P_{a,p_0}(s_l|\theta)$ is independent of a and p_0, if the left-hand side when s_l replaced by s_- (which represents type s_-'s posterior) is equal to some q^-, then there is a unique value of $P_{a,p_0}(\theta|N^t = 0^t)$ satisfying equation (D.2), denoted as p, and hence a unique value of the left-hand side of equation (D.2) when s_l replaced by s_+ (which represents type s_+'s posterior), denoted as q^+. That is, one variable, be it q^- or p, is sufficient to represent the posteriors of the two types of the informed player. This paper uses p, that is, $P_{a,p_0}(\theta|N^t = 0^t)$, and call it the "background belief."

E. DYNAMICS OF U'S BELIEF ABOUT THE RISKY PROJECT

This section proves Lemma 1, which follows from Lemma 9 and Lemma 10.

LEMMA 9 *There exists $\tilde{O} \in (1,\infty)$, such that, if $O > \tilde{O}$, then $\frac{\lambda^U(p,\hat{\boldsymbol{\mu}}(p))}{dp}|_{p=p_2^{*-}} > 0$; if $O \in [O^S, \tilde{O})$, then $\frac{\lambda^U(p,\hat{\boldsymbol{\mu}}(p))}{dp}|_{p=p_2^{*-}} < 0$.*

PROOF: Let $\boldsymbol{q}_2^{*+} : [1,\infty) \to [0,1]$ be defined by $\boldsymbol{q}_2^{*+}(O) = \frac{1}{1+(\frac{1}{q_2^*}-1)\frac{1}{O}}$, for $O \in [1,\infty)$, where $\boldsymbol{q}_2^{*+}(O)$ refers to type s_+'s posterior belief about the risky project, when type s_-'s posterior is q_2^* and the odd ratio is O. Let $\hat{\boldsymbol{\mu}}^*(O)$ denote the initial value of $\hat{\boldsymbol{\mu}}$ at p_2^{*-}, implied by the initial value condition (5.10), when the odd ratio is O. (The notations $\boldsymbol{q}_2^{*+}$ and $\hat{\boldsymbol{\mu}}^*$ are only used in this proof.)

Using equations (B.3) (the definition of function D), (B.8), and (B.13), we have, $d\lambda^U(p,\hat{\boldsymbol{\mu}}(p))/dp|_{p=p_2^{*-}} < 0$ if and only if at $p = p_2^{*-}$,

$$\lambda\left(1-\boldsymbol{q}_2^{*+}(O)\right)\frac{\left[w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)\boldsymbol{q}_2^{*+}(O)-\left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s\right)\right]}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s}$$
$$+q_2^*\lambda-\frac{(\lambda-q_2^*\lambda)\,q_2^*\lambda}{\lambda^U(p,\hat{\boldsymbol{\mu}})-q_2^*\lambda} \;>\; 0. \tag{E.1}$$

Using again the initial value condition (5.10) and the value matching condition (5.12), we have, at $p = p_2^{*-}$, $\lambda^U(p,\hat{\boldsymbol{\mu}}) = z(\boldsymbol{q}_2^{*+}(O)) \equiv s - \frac{(s-q_2^*h)\left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s+\boldsymbol{q}_2^{*+}(O)\lambda h-q_2^*\lambda h}$. Define the following functions:

$$\hat{D}_1(q) \;\equiv\; \lambda(1-q)\frac{\left(w_q^S(q)\,q-\left(w^S(q)-s\right)\right)}{w^S(q)-s}, \tag{E.2}$$

$$\hat{D}_2(q) \;\equiv\; q_2^*\lambda-\frac{(\lambda-q_2^*\lambda)\,q_2^*\lambda}{z(q)-q_2^*\lambda}. \tag{E.3}$$

Then the left-hand side of inequality (E.1) equals to

$$\hat{D}(O) \;\equiv\; \hat{D}_1\left(\boldsymbol{q}_2^{*+}(O)\right)+\hat{D}_2\left(\boldsymbol{q}_2^{*+}(O)\right). \tag{E.4}$$

We will show in the following two claims that $\hat{D}$ is strictly decreasing in O, $\hat{D} > 0$ at $O = O^S$, and that $\hat{D} < 0$ as $O \to \infty$. By continuity of $\hat{D}$, there is a unique $\tilde{O} \in (O^S,\infty)$, such that $\hat{D}(O) > 0$ for $O \in [O^S,\tilde{O})$, and $\hat{D}(O) < 0$ for $O > \tilde{O}$. We are left to prove these claims. Recall that O^S is the odds ratio such that $\boldsymbol{q}_2^{*+}\left(O^S\right) = q^S$, that is, when type s_-'s belief is at q_2^*, type s_+'s is at q^S.

CLAIM 7 *$\hat{D}(O)$ strictly decreases in O.*

Claim 8 $\hat{D}\left(O^S\right) > 0$ *and* $\lim_{O\to\infty} \hat{D}\left(O\right) < 0$.

Proof of Claim 7: If we show that $\hat{D}_1\left(q\right)$ and $\hat{D}_2\left(q\right)$ strictly decreases in q, then, since $\boldsymbol{q}_2^{*+}\left(O\right)$ strictly increases in O, we have that $\hat{D}\left(O\right)$ strictly decreases in O.

(i) $\hat{D}_1\left(q\right)$ strictly decreases in q.

Replacing w_q^S in the expression of $\hat{D}_1$ by equation (B.11), we have

$$\hat{D}_1\left(q\right) = \lambda\left(\frac{q\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right) - \frac{r}{2\lambda}s}{w^S\left(q\right)-s} - \left(\frac{r}{2\lambda}+1\right)\right).$$

Taking derivative with respect to q and rearranging terms, we have

$$\begin{aligned}\frac{d\hat{D}_1\left(q\right)}{dq} &= \lambda\frac{\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)\left(w^S\left(q\right) - s - qw_q^S\left(q\right)\right) + w_q^S\left(q\right)\frac{r}{2\lambda}s}{\left(w^S\left(q\right)-s\right)^2} \\ &= \lambda\frac{\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)\left(w^S\left(q\right) - s - \left(q-q_2^*\right)w_q^S\left(q\right)\right)}{\left(w^S\left(q\right)-s\right)^2} \\ &< 0,\end{aligned}$$

where the second inequality follows from $\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)q_2^* = \frac{r}{2\lambda}s$, and the third from the convexity of w^S and that $w^S(q_2^*) = s$.

(ii) $\hat{D}_2\left(q\right)$ strictly decreases in q.

$\hat{D}_2\left(q\right)$ being strictly decreasing in q is equivalent with the function z being strictly decreasing in q, which is equivalent with $\frac{q\lambda h - q_2^*\lambda h}{w^S(q)-s}$ being strictly decreasing in q. The derivative of $\frac{q\lambda h - q_2^*\lambda h}{w^S(q)-s}$ with respect to q, $\frac{w^S(q)-s-\left(q-q_2^*\right)w_q^S(q)}{(w^S(q)-s)^2}\lambda h$, is indeed strictly negative, because the numerator is strictly decreasing in q (as $-w_{qq}^S < 0$ for $q \in [q^S, 1]$) and hence is smaller than the value of the numerator at $q = q_1^*$: *Q.E.D.*

Proof of Claim 8: *For brevity, we use q to indicate $\boldsymbol{q}_2^{*+}\left(O\right)$ in the proof of this claim, whenever no confusion arises.* As $O \to \infty$, we have $\boldsymbol{q}_2^{*+}\left(O\right) \to 1$, and $\lim_{O\to\infty}\hat{\boldsymbol{\mu}}^*\left(O\right) \in (0,1)$ by the initial value condition. Therefore, as $O \to \infty$, we have $\hat{D}_1 \to 0$, $\hat{D}_2 \to -\left(\lim_{O\to\infty}\frac{1}{\hat{\boldsymbol{\mu}}^*(O)} - 1\right)\lambda q_2^* < 0$. Therefore, $\hat{D}(O) < 0$ if O is large enough.

If $O = O^S$, then $\hat{\boldsymbol{\mu}}^*\left(O\right) = 1$ by the initial value condition. Define $\hat{D}_3(q) \equiv \left(q - q_2^*\right)w_q^S\left(q\right) - \left(w^S\left(q\right) - s\right)$, which is strictly increasing in q over $[q^S, 1]$ from the proof of the previous Claim. Using $\hat{D}\left(q\right) = \hat{D}_1\left(q\right) - \frac{\lambda}{O-1}$, We have,

$$\text{(E.5)} \qquad \frac{1}{\lambda}\hat{D}\left(q\right) = \left(1-q\right)\frac{\hat{D}_3}{w^S\left(q\right)-s} + \frac{qw_q^S\left(q\right)}{w^S\left(q\right)-s}\frac{1-q}{q}q_2^* - \frac{1}{O-1}.$$

If we show that

$$\text{(E.6)}\qquad \frac{qw_q^S(q)}{w^S(q)-s}\frac{1-q}{q}q_2^* - \frac{1}{O-1} > \frac{1}{O}\left[\frac{1}{O-1} - \frac{qw_q^S(q)}{w^S(q)-s}\frac{1-q}{q}q_2^*\right],$$

then we have $\frac{qw_q^S(q)}{w^S(q)-s}\frac{1-q}{q}q_2^* - \frac{1}{O-1} > 0$. This inequality, together with $\hat{D}_3 > 0$ and equation (E.5), implies $\hat{D} > 0$. We are left to show inequality (E.6). First, by the definition of $\boldsymbol{q}_2^{*+}$, we have $\frac{1-q}{q}q_2^* = \frac{1}{O}(1-q_2^*)$. Then,

$$\begin{aligned}
\frac{qw_q^S(q)}{w^S(q)-s}\frac{1-q}{q}q_2^* - \frac{1}{O-1} &= \left(\frac{(q-q_2^*)\,w_q^S(q)}{w^S(q)-s} + \frac{qw_q^S(q)}{w^S(q)-s}\frac{q_2^*}{q}\right)\frac{1}{O}(1-q_2^*) - \frac{1}{O-1}\\
&= \left(\frac{(q-q_2^*)\,w_q^S(q)}{w^S(q)-s} + \frac{qw_q^S(q)}{w^S(q)-s}\frac{1-q}{q}q_2^*\right)\frac{1}{O} - \frac{1}{O-1}\\
&> \left[\frac{1}{O-1} - \frac{qw_q^S(q)}{w^S(q)-s}\frac{1-q}{q}q_2^*\right]\frac{1}{O}.
\end{aligned}$$

The last inequality uses $\frac{(q-q_2^*)w_q^S(q)}{w^S(q)-s} > 1$, and $\frac{1}{O} - \frac{1}{O-1} = \frac{1}{O(O-1)}$. *Q.E.D.*

Q.E.D.

Lemma 10 *For any* $p \in (p_2^{*-}, p_{gr})$ *such that* $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp} = 0$, *we have* $\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp^2} > 0$.

Proof: Suppose there exists some $p' \in (p_2^{*-}, p_{gr})$ such that $\frac{d\lambda^U(p',\hat{\boldsymbol{\mu}})h}{dp} = 0$.
Since $\hat{\boldsymbol{\mu}}\lambda_\mu^U(p,\hat{\boldsymbol{\mu}}) = \lambda^U(p,\hat{\boldsymbol{\mu}}) - \lambda^{I-}(p)$, and $B(p,\hat{\boldsymbol{\mu}}) = s + s - \lambda^U(p,\hat{\boldsymbol{\mu}}) - w^S(\boldsymbol{q}^-)$, we have, at $p = p'$, $\frac{d\left(\hat{\boldsymbol{\mu}}\lambda_\mu^U(p,\hat{\boldsymbol{\mu}})\right)}{dp} = -\lambda\frac{d\boldsymbol{q}^-(p)}{dp}$, and $\frac{dB(p,\hat{\boldsymbol{\mu}})}{dp} = -w_q^S(\boldsymbol{q}^-(p))\frac{d\boldsymbol{q}^-(p)}{dp}$. Using these two equations, and taking derivative on both sides of equality (B.9) with respect to p at $p = p'$, and applying $\frac{d\lambda^U(p',\hat{\boldsymbol{\mu}})h}{dp} = 0$, we have, at $p = p'$,

$$\begin{aligned}
&-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0)\\
= \;&\frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\Bigg[-\lambda\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h - s\right) + \lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - w^S\left(\boldsymbol{q}^-\right) - B(p,\hat{\boldsymbol{\mu}})\right)\right)\\
&+\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\lambda\boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)w_q^S\left(\boldsymbol{q}^-\right)\Bigg].
\end{aligned}$$

If $p' \in [p_2^{*-}, p_1^{*-}]$, then $w^S(\boldsymbol{q}^-(p')) = s$ and hence $w_q^S(\boldsymbol{q}^-(p')) = 0$. Apply inequality (B.14) and the definition of function D, we have $\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}|_{p=p'} > 0$.

If $p' \in (p_1^{*-}, p_{gr}]$, (which is possible only if $(p_1^{*-}, p_{gr}]$ is nonempty,) then by the fact that $a^S\left(\boldsymbol{q}^-(p')\right) \in (0,1)$, we have

$$\lambda \boldsymbol{q}^- \left(1-\boldsymbol{q}^-\right) w_q^S\left(\boldsymbol{q}^-\right) = r\left(\lambda \boldsymbol{q}^- h - s\right) + \lambda \boldsymbol{q}^- \left(\lambda h - w^S\left(\boldsymbol{q}^-\right)\right).$$

Using this equation to replace w^S and w_q^S, we have

$$\begin{aligned} -\frac{d^2\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)}{dp^2} B\left(p,0\right) &= \frac{d\boldsymbol{q}^-\left(p\right)/dp}{\lambda p\left(1-p\right)} \Bigg[-\left(\frac{\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)}{\lambda \boldsymbol{q}^-} - 1\right)\lambda r s + \lambda\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right) B\left(p,\hat{\boldsymbol{\mu}}\right) \\ &\quad + \frac{\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)\left(\lambda \boldsymbol{q}^- - \lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)}{\lambda \boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)}\left(r\left(\lambda \boldsymbol{q}^- h - s\right) + \lambda \boldsymbol{q}^-\left(\lambda h - w^S\left(\boldsymbol{q}^-\right)\right)\right)\Bigg]. \end{aligned}$$

After some algebra, the terms on the second line of the previous equation can be reduced to

$$\frac{\left(\lambda \boldsymbol{q}^- - \lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)}{\left(1-\boldsymbol{q}^-\right)}\left(r\left(\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h - s\right) + \lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)\left(\lambda h - w^S\left(\boldsymbol{q}^-\right)\right)\right) + \frac{\left(\lambda \boldsymbol{q}^- - \lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)^2}{\lambda \boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)} r s,$$

which can be further reduced to

$$-\lambda\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right) B\left(p,\hat{\boldsymbol{\mu}}\right) + \frac{\left(\lambda \boldsymbol{q}^- - \lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)^2}{\lambda \boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)} r s,$$

if we use the fact that the right-hand side of equation (B.9) equals to 0 at $p = p'$ (that is, when $d\lambda^U/dp = 0$). Therefore, at $p = p'$, we have,

$$-\frac{d^2\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)}{dp^2} B\left(p,0\right) = \frac{d\boldsymbol{q}^-\left(p\right)/dp}{\lambda p\left(1-p\right)}\left[-\frac{\lambda - \lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)}{1-\boldsymbol{q}^-}\left(\frac{\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)}{\lambda \boldsymbol{q}^-} - 1\right) r s\right] < 0,$$

implying that $\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}|_{p=p'} > 0$. *Q.E.D.*

F. EQUILIBRIUM UNIQUENESS

This section proves Proposition 3.

PROOF: We will prove the proposition in two steps.

Step 1. There exists some $\bar{O} \in (1,\infty)$ such that if the odds ratio $O > \bar{O}$, and if the initial prior belief p_0 is higher than p_2^{*-}, then in any continuation equilibrium, separation much occur once the background belief reaches p_2^{*-}, and that type s_+'s effort is 1 before separation.

Note that due to the one-to-one relationship between the background belief p and type s_-'s posterior belief q^-, any MPE with state variables (p,μ) can be equivalently expressed as an MPE with state variables (q^-, μ), and vice versa. For convenience, we use (q^-, μ) as the state variables in this proof.

Assume that type s_-'s prior belief q_0^- is higher than the cooperative cutoff q_2^*. Fix an odds ratio level O and consider an MPE of the game corresponding to the odds ratio O. Let $\tilde{q}_O^-$ be the highest background belief at which, separation occurs *at all reputation levels (that is, there exists no path of play, on or off the equilibrium path, along which mimicking type s_+ from belief q_0^- to $\tilde{q}^-$ is not worse than switching to the symmetric MPE when his type is public, for type s_-)*. If separation does not occur over when type s_+'s posterior belief is higher than q_2^*, we simply set $\tilde{q}_O^-$ as the corresponding posterior belief of type s_-'s when type s_+'s posterior belief is exactly q_2^* (because after this, no experimentation occurs and without loss the equilibrium phase can be taken as separation). Let $\boldsymbol{q}_O : [0, 1] \to [0, 1]$ be the function that maps type s_-'s posterior belief to type s_+'s corresponding posterior belief.

We now show that there exists some $\bar{O} \in (1, \infty)$ such that if $O > \bar{O}$, then when type s_-'s posterior belief reaches $\tilde{q}_O^-$, type s_+'s posterior belief will be higher than q^s: $\boldsymbol{q}_O(\tilde{q}_O^-) > q^s$, meaning that after separation, if I is believed to be type s_+, players will still exert effort 1 for some time. Suppose by contradiction that there exists a sequence (O_n) that converges to infinity and that $\boldsymbol{q}_{O_n}(\tilde{q}_{O_n}^-) \le q^s$ for all n. For simplicity, denote $\tilde{q}_{O_n}^-$ as $\tilde{q}_n^-$, and $\boldsymbol{q}_{O_n}(\cdot)$ as $\boldsymbol{q}_n(\cdot)$. Take some $\Delta \in (0, 1 - q^s)$, and let n be so large that there exists $\hat{q}_n^-$ such that $\hat{q}_n^-$ is close to 0 and $\boldsymbol{q}_n(\hat{q}_n^-) \ge q^s + \Delta$. By assumption, we have $\hat{q}_n^- > \tilde{q}_n^-$. By the definition of $\tilde{q}_n^-$, there is a continuation equilibrium in which type s_- is willing to mimic type s_+ until his belief reaches $\tilde{q}_n^-$. For n sufficiently large, over $[\tilde{q}_n^-, \hat{q}_n^-]$, type s_- is willing to mimic type s_+ only if type s_+'s effort is high for a short period of time; but then, by D1, type s_+ by sticking to effort 1 for a short period at the beginning (starting from $\hat{q}_n^-$ to $\hat{q}_n^- - \eta$), can prove that he indeed is type s_+, because type s_- would never have benefited from such a deviation, whatever the reputation he is going to get (that is, whatever the continuation equilibrium is after the deviation). This deviation is indeed profitable for type s_+ if the odds ratio is high, because the benefit — both players exert effort 1 over $[\tilde{q}_n^-, \hat{q}_n^- - \eta]$, outweighs the cost — U might exert low effort over $[\hat{q}_n^- - \eta, \hat{q}_n^-]$. A contradiction.

Let $O > \bar{O}$. Obviously, we must have $\tilde{q}_O^- \ge q_2^*$; otherwise type s_- would strictly prefer to mimic type s_+ until his belief reaches q_2^*. We now show that $\tilde{q}_O^- = q_2^*$. Suppose by contradiction that $\tilde{q}_O^- < q_2^*$. Then there must exist some reputation level (which induces a continuation equilibrium) such that type s_+'s effort is lower than 1 over $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$ for some ϵ small; otherwise, type s_- would not have been willing to pool. Let α denote U's average effort over $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$. Now consider the deviation of exerting effort 1 over $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$. Since separation would occur once type s_-'s belief reaches $\tilde{q}_O^-$, whether such a deviation is profitable or not depends only on the payoff that is collected during $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$. We now show that if the equilibrium satisfies D1 then type s_+ strictly benefits from the deviation. First, for the equilibrium to satisfy D1, U's average effort during $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$ induced by the deviation cannot be strictly lower than α. Suppose this is not true, then there must exist some $\hat{\epsilon} \in (0, \epsilon)$ such that in this continuation equilibrium, U's average effort is strictly lower than 1 over $(\tilde{q}_O^-, \tilde{q}_O^- + \hat{\epsilon})$, and that if U's effort over $(\tilde{q}_O^-, \tilde{q}_O^- + \hat{\epsilon})$ is replaced by 1 then her average effort over $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$ would be α. Then, when type s_-'s belief reaches $\tilde{q}_O^- + \hat{\epsilon}$, U should have believed that I is of type s_+, as type s_- would never have deviated had type s_- expected that U would play the continuation equilibrium; that is, U's effort over $(\tilde{q}_O^-, \tilde{q}_O^- + \hat{\epsilon})$ should be 1, meaning that her average effort over $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$ should be at least α (whatever continuation equilibrium strategy she is willing to play), a contradiction. Second, since U's average effort during $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$ is (weakly) higher than α, type s_+ strictly benefits from deviating to effort 1 over $(\tilde{q}_O^-, \tilde{q}_O^- + \epsilon)$.

Using the same argument, one can show that for an MPE to satisfy D1, we must have type s_+'s effort to be 1 over for $q^- \geq q_2^*$, if O is higher than $\bar{O}$.

Step 2. If the odds ratio O is higher than $\bar{O}$ and if the initial prior belief p_0 is higher than p_2^{*-}, then the distribution of the equilibrium paths of any MPE that satisfies D1 and SMPE coincides with the MPE with gradual revelation.

In Step 1, we have shown that under the above conditions, type s_+'s equilibrium effort (on and off path) must be 1 if the background belief is in $[p_2^{*-}, 1]$. Take an odds ratio $O > \bar{O}$ and an MPE of the game with the corresponding odds ratio O. We now show that the equilibrium structure must be pooling and followed by semi-separation, that is, pooling at high background beliefs and semi-separation at lower background beliefs. Suppose by contradiction that there is an interval $[\hat{p}_1, \hat{p}_2]$ in $[p_2^{*-}, 1]$ during which the equilibrium involves pooling, and there exists some $\tilde{p} > \hat{p}_2$ at which the equilibrium involves semi-separation (That is, either at $\tilde{p}$ type s_- reveals with positive probability or around $\tilde{p}$ there is gradual revelation). Let $\hat{p}_1$ be the infimum of background beliefs at which pooling occurs. During pooling, when the background belief p is in some right neighborhood of $\hat{p}_1$, we must have $W^U(p, \mu) - s \geq s - \lambda^U(p, \mu)$, so that U is willing to exert some effort; otherwise, type s_- would strictly prefer not to mimic type s_+ at some p close to $\hat{p}_1$, which contradicts with the assumption that separation does not occur before $\hat{p}_1$. The fact that U is willing to exert effort at any

p close to $\hat{p}_1$, together with Point 3 of Lemma 5, implies that U will exert effort 1 during pooling. But then type s_- would strictly prefer to mimic type s_+ right before a pooling phase (that is, when p is sufficiently close and larger than the upper bound background belief of a pooling phase). That is, semi-separation cannot occur at (or around) any background belief higher than the upper bound background belief of a pooling phase. A contradiction.

The rest of the proof of Step 2 follows from the proof of Lemma 3. *Q.E.D.*