Microeconomic Origins of Macroeconomic Tail Risks[*]

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Abstract

We document that even though the normal distribution provides a good approximation to GDP fluctuations, it severely underpredicts "macroeconomic tail risks," that is, the frequency of large economic downturns. Using a multi-sector general equilibrium model, we show that the interplay of idiosyncratic microeconomic shocks and sectoral heterogeneity results in systematic departures in the likelihood of large economic downturns relative to what is implied by the normal distribution. Notably, we also show that such departures can happen while GDP is approximately normally distributed away from the tails, highlighting the qualitatively different behavior of large economic downturns from small or moderate fluctuations. We further demonstrate the special role that input-output linkages play in generating "tail comovements," whereby large recessions involve not only significant GDP contractions, but also large simultaneous declines across a wide range of sectors.

Keywords: Business cycles, Domar weights, large economic downturns, network heterogeneity, input-output linkages, tail risks.

1 Introduction

Most empirical studies in macroeconomics approximate the deviations of aggregate economic variables from their trends with a normal distribution. Besides its relative success in capturing salient features of the behavior of aggregate variables in the U.S. and other OECD countries, this approach has a natural justification: since most macro variables, such as GDP, are obtained from combining more disaggregated ones, it is reasonable to expect that a central limit theorem-type result should imply that they are normally distributed. As an implicit corollary to this observation, most of the literature treats the standard deviations of aggregate variables as sufficient statistics for measuring aggregate economic fluctuations.

That the normal distribution is, for the most part, a good approximation to aggregate fluctuations can be seen from panels (a) and (b) of Figure 1. These panels depict the quantile-quantile (Q-Q) plots of the U.S. postwar quarterly output growth and the HP-detrended GDP against the standard normal distribution, while excluding tail risks or large deviations from the sample — defined as quarters in which GDP growth or detrended output are above the top 5% or below the bottom 5% of their empirical distributions. The close correspondence between the normal distribution, shown as the dashed red line, and the two truncated data series shows that, once large deviations are excluded, the normal distribution provides a good approximation to GDP fluctuations.[1]

This picture changes dramatically, however, once large deviations are also taken into account. Panels (c) and (d) of Figure 1 show the same quantile-quantile plots for the entire U.S. postwar sample. Both graphs now exhibit sizable and systematic departures from the normal line at the ends.[2] This observation highlights that even though the normal distribution provides a good approximation to GDP fluctuations in most of the sample, it severely underestimates what is perhaps the most consequential aspect of economic fluctuations: the likelihood of large economic downturns.

In this paper, we argue that such macroeconomic tail risks can have their origins in idiosyncratic microeconomic shocks to disaggregated sectors, and demonstrate that sufficiently high levels of sectoral heterogeneity can lead to systematic departures in the frequency of large economic downturns from what is implied by the normal distribution. Crucially, we also prove that macroeconomic tail risks can coexist with approximately normally distributed fluctuations away from the tails, consistent with the pattern of U.S. GDP fluctuations documented in Figure 1. Consequently, our results show that the microeconomic nature of macroeconomic tail risks can be

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[1]To test for this claim formally, we first excluded the top and bottom 5% of observations from U.S. GDP growth and detrended log GDP between 1947:Q1 and 2015:Q1, and then performed the Anderson-Darling and Jarque-Bera tests of normality. The resulting test statistics for GDP growth are 0.64 and 1.07, respectively, and the test statistics for detrended (with HP filter 1600) log GDP are, respectively, 0.40 and 0.57. In none of these cases we can reject normality at the 10% significance level.

[2]Repeating the tests in footnote 1 for the full sample of log GDP growth and detrended quarterly GDP leads to a strong rejection of normality in both cases: the Anderson-Darling and Jarque-Bera tests yield, respectively, test statistics of 1.91 and 21.65 for GDP growth, hence rejecting normality at the 1% level. Similarly for the detrended quarterly GDP, the two tests lead to test statistics of 2.14 and 27.38, again rejecting normality at the 1% level.
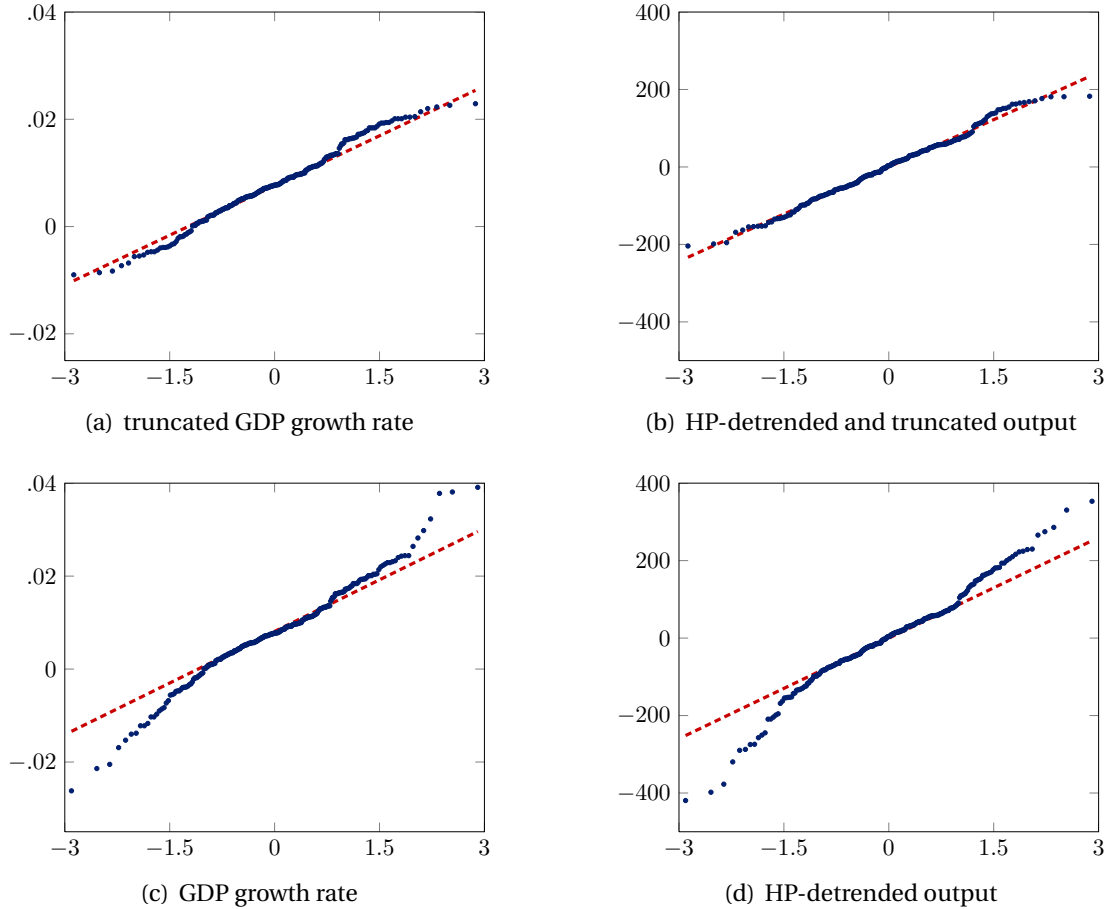
Figure 1. The quantile-quantile (Q-Q) plots of the postwar U.S. GDP fluctuations (1947:Q1 to 2015:Q1) versus the standard normal distribution, shown by the dashed red line. The horizontal and vertical axes correspond, respectively, to the quantiles of the standard normal distribution and the sample data. Panels (a) and (b) depict the Q-Q plots for the GDP growth rate and HP-detrended output. The linearity of the points suggests that both truncated datasets are approximately normally distributed. Panels (c) and (d) depict the Q-Q plots of the two datasets after removing the top and bottom 5% of data points. The deviation from the dashed red line suggests that both datasets exhibit heavier tails compared to the normal distribution.

quite distinct from the determinants of small or moderate fluctuations, underscoring the importance of separately focusing on large downturns.

We develop these ideas in the context of a model economy comprising of n competitive sectors that are linked to one another via input-output linkages and are subject to idiosyncratic productivity shocks. Using an argument similar to those of Hulten (1978) and Gabaix (2011), we first show that aggregate output depends on the distribution of microeconomic shocks as well as the empirical distribution of (sectoral) *Domar weights,* defined as sectoral sales divided by GDP. We also establish that the empirical distribution of Domar weights is in turn determined by the extent of heterogeneity in (i) the weights households place on the consumption of each sector's output (which we refer to as *primitive heterogeneity*); and (ii) the sectors' role as input-suppliers to one another (which we refer to as *network heterogeneity*).

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Using this characterization, we investigate whether microeconomic shocks can translate into significant macroeconomic tail risks, defined as systematic departures in the frequency of large economic downturns from what is predicted by the normal distribution.[3] Our main result establishes that macroeconomic tail risks can emerge if two conditions are satisfied. *First*, microeconomic shocks themselves need to exhibit some minimal degree of tail risk relative to the normal distribution (e.g., by having exponential tails), as aggregating normally distributed shocks can only result in normally distributed GDP fluctuations. *Second*, the economy needs to exhibit sufficient levels of *sectoral dominance*, in the sense that the most dominant disaggregated sectors (i.e., those with the largest Domar weights) ought to be sufficiently large relative to the variation in the importance of all sectors. This condition guarantees that the tail risks present at the micro level do not wash out after aggregation. We then demonstrate that macroeconomic tail risks can emerge even if the central limit theorem holds so that, in a pattern consistent with Figure 1, fluctuations are normally distributed away from the tails.

Our result that high levels of sectoral dominance transform microeconomic shocks into sizable macroeconomic tail risks is related to the findings of Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), who show that microeconomic shocks can lead to aggregate volatility (measured by the standard deviation of GDP) if some sectors are much larger than others or play much more important roles as input-suppliers in the economy. However, the role played by the heterogeneity in Domar weights in generating aggregate volatility is quite distinct from its role in generating tail risks. Indeed, we show that structural changes in an economy can simultaneously reduce aggregate volatility while increasing macroeconomic tail risks, in a manner reminiscent of the experience of the U.S. economy over the last several decades, where the likelihood of large economic downturns may have increase behind the façade of the "Great Moderation".

Our main results show that the distribution of microeconomic shocks and the Domar weights in the economy serve as sufficient statistics for the likelihood of large economic downturns. Hence, two economies with identical Domar weights exhibit equal levels of macroeconomic tail risks, regardless of the extent of network and primitive heterogeneity. Nevertheless, we also establish that economic downturns that arise due to the presence of each type of heterogeneity are meaningfully different in nature. In an economy with no network heterogeneity — where Domar weights simply reflect the differential importance of the disaggregated sectors in household preferences — large economic downturns are a consequence of contractions in sectors with high Domar weights, while other sectors are, on average, in a normal state. In contrast, large economic downturns that arise from the interplay of microeconomic shocks and network heterogeneity display *tail comovements*: they involve not only very large drops in GDP, but also significant simultaneous contractions across a wide

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[3]Formally, we measure the extent of macroeconomic tail risks by the likelihood of a τ standard deviation decline in log GDP relative to the likelihood of a similar decline under the normal distribution in a sequence of economies with both the number of sectors (n) and the size of the deviation (τ) growing to infinity. The justification for these choices is provided in Section 3.

range of sectors within the economy.[4] This observation motivates our next result, where we show that an economy with high levels of sectoral interconnectivity (such as an economy with substantial network heterogeneity) exhibits more tail comovements relative to another economy with identical Domar weights, but with only primitive heterogeneity.

As our final theoretical result, we characterize the extent of macroeconomic tail risk in the presence of heavy-tailed microeconomic shocks (e.g., shocks with Pareto tails). Using this characterization, we demonstrate that sufficient levels of sectoral dominance can translate light-tailed (such as exponential) shocks into macroeconomic tail risks that would have only emerged in the absence of such heterogeneity with heavy-tailed shocks.[5]

We conclude the paper by undertaking a simple quantitative exercise to further illustrate our main results. Assuming that microeconomic shocks have exponential tails — chosen in a way that is consistent with GDP volatility observed in the U.S. data — we find that the empirical distribution of Domar weights in the U.S. economy is capable of generating departures from the normal distribution similar to the patterns documented in Figure 1. We then demonstrate that the extent of network heterogeneity in the U.S. economy plays an important role in creating macroeconomic tail risks. Finally, we show that input-output linkages in the U.S. data can lead to tail comovements, highlighting the importance of intersectoral linkages in translating microeconomic shocks into macroeconomic tail risks.

Related Literature Our paper belongs to the small literature that focuses on large economic downturns. A number of papers, including Cole and Ohanian (1999, 2002) and Kehoe and Prescott (2002), have used the neoclassical growth framework to study Great Depression-type events in the United States and other countries. More recently, there has been a growing emphasis on deep Keynesian recessions due to liquidity traps and the zero lower bound on nominal interest rates (such as Christiano, Eichenbaum, and Rebelo (2011), Eggertsson and Krugman (2012), Eggertsson and Mehrotra (2014)). Relatedly, Christiano, Eichenbaum, and Trabandt (2015) argue that financial frictions can account for the key features of the recent economic crisis. Though our paper shares with this literature the emphasis on large economic downturns, both the focus and the underlying economic mechanisms are substantially different.

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[4] In fact, using shipments data for 459 manufacturing industries from the NBER productivity database between 1958–2009 suggests that there are significant levels of tail comovements. In particular, we find that a two standard deviation decline in GDP, which takes place on annual data in 1973 and 2008, is associated with a two standard deviation decline in 10.68% and 13.73% of manufacturing industries, respectively. These numbers are much greater than the average in the rest of the sample (3.17%) or more relevantly, the average in the rest of the sample once we also exclude 1974 and 2009 (2.06%), the years immediately following 1973 and 2008, where the fraction of manufacturing industries experiencing more than two standard deviation declines is also very high. Clearly, they are also much greater than what we should observe if shipments in these industries were independently distributed.

[5] Fama (1963) and Ibragimov and Walden (2007) observe that the presence of extremely heavy-tailed shocks with infinite variances leads to the break down of the central limit theorem, and hence, to systematic deviations from the normal distribution. Our results, in contrast, are about the (arguably more subtle and interesting phenomenon of) emergence of macroeconomic tail risks in the absence of heavy-tailed micro or macro shocks.

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Our paper is also related to the literature on "rare disasters", such as Rietz (1988), Barro (2006), Gabaix (2012), Nakamura, Steinsson, Barro, and Ursúa (2013) and Farhi and Gabaix (2015), which argues that the possibility of rare but extreme disasters is an important determinant of risk premia in asset markets. Gourio (2012) studies a real business cycle model with a small risk of economic disaster. This literature, however, treats the frequency and the severity of such rare disasters as exogenous. In contrast, we provide a possible explanation for the endogenous emergence of such macroeconomic tail risks due to the propagation and amplification of microeconomic shocks. Furthermore, we characterize how the distributional properties of micro shocks coupled with the input-output linkages of the economy shape the likelihood and depth of large economic downturns.

Our paper is most closely related to and builds on the literature that studies the microeconomic origins of economic fluctuations. Gabaix (2011) argues that if the firm size distribution is sufficiently heavy-tailed (in the sense that the largest firms contribute disproportionally to GDP), firm-level idiosyncratic shocks may translate into aggregate fluctuations. Relatedly, Acemoglu et al. (2012) show that the propagation of microeconomic shocks over input-output linkages can result in aggregate volatility. On the empirical side, Carvalho and Gabaix (2013) explore whether changes in the sectoral composition of the postwar U.S. economy can account for the Great Moderation and its unwinding, while Foerster, Sarte, and Watson (2011) and Atalay (2014) study the relative importance of aggregate and sectoral shocks in aggregate economic fluctuations. Complementing these studies, di Giovanni, Levchenko, and Méjean (2014) use a database covering the universe of French firms and document that firm-level shocks contribute significantly to aggregate volatility, while Carvalho, Nirei, Saito, and Tahbaz-Salehi (2015) and Acemoglu, Akcigit, and Kerr (2015a) provide firm and sectoral-level evidence for the transmission of shocks over input-output linkages.[6]

Even though the current paper has much in common with the above mentioned studies, it also features major differences from the rest of the literature. First, rather than the focusing on the standard deviation of GDP as a notion of aggregate fluctuations, we study the determinants of macroeconomic tail risks, which, to the best of our knowledge, is new. Second and more importantly, this shift in focus leads to a novel set of economic insights: our results establish more than the limitations of the standard deviation of GDP as a measure of the frequency and depth of large economic downturns. They also show that the extent of such macroeconomic tail risks is determined by the interplay between the shape of the distribution of microeconomic shocks and the heterogeneity in Domar weights (as captured by our notion of sectoral dominance), a result with no counterpart in the previous literature.

Our paper is also related to Fagiolo, Napoletano, and Roventini (2008), Cúrdia, Del Negro, and Greenwald (2014) and Ascari, Fagiolo, and Roventini (2015), who document that the normal distribution does not provide a good approximation to many macroeconomic variables in OECD

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[6]Other studies in this literature include Jovanovic (1987), Durlauf (1993), Horvath (1998, 2000), Dupor (1999), Carvalho (2010) and Burlon (2012). For a survey of this literature, see Carvalho (2014). Kozeniauskas, Orlik, and Veldkamp (2014) study how uncertainty in microeconomic and macroeconomic variables are linked.

countries. Similarly, Atalay and Drautzburg (2015) find substantial differences in the extent to which different industries' employment growth rates depart from the normal distribution and compute the contribution of the independent component of industry-specific productivity shocks to the skewness and kurtosis of aggregate variables. In contrast, we provide a theoretical framework for how such departures from normality may arise.

Finally, our paper is linked to the growing literature that focuses on the role of power laws and large deviations in various contexts. For example, Gabaix, Gopikrishnan, Plerou, and Stanley (2003, 2006) provide a theory of excess stock market volatility in which market movements are due to trades by very large institutional investors, whereas Kelly and Jiang (2015) investigate the effects of time-varying extreme events in asset markets.

Outline of the Paper The rest of the paper is organized as follows. We present the model in Section 2. In Section 3, we formally define our notion of tail risks. Our main results are presented in Section 4, where we show that the severity of macroeconomic tail risks is determined by the interaction between the nature of heterogeneity in the economy's Domar weights and the distribution of microeconomic shocks. We present our results on tail comovements and the extent of tail risks in the presence of heavy-tailed microeconomic shocks in Sections 5 and 6, respectively. Section 7 contains our quantitative exercises and Section 8 concludes. All proofs and some additional mathematical details are provided in the Appendix.

2   Microeconomic Model

In this section, we present a simple multi-sector model that forms the basis of our analysis. The model is a static variant of the model of Long and Plosser (1983), which is also analyzed by Acemoglu et al. (2012).

Consider a static economy consisting of n competitive sectors denoted by $\{1, 2, \ldots, n\}$, each producing a distinct product. Each product can be either consumed or used as input for production of other goods. Firms in each sector employ Cobb-Douglas production technologies with constant returns to scale that transform labor and intermediate goods into final products. In particular,

$$x_i = \Xi_i \zeta_i l_i^{1-\mu} \left(\prod_{j=1}^n x_{ij}^{a_{ij}} \right)^\mu, \tag{1}$$

where x_i is the output of sector i, Ξ_i is a Hicks-neutral productivity shock, l_i is the amount of labor hired by the firms in sector i, x_{ij} is the amount of good j used for production of good i, $\mu \in [0, 1)$ is the share of material goods in production, and $\zeta_i > 0$ is some normalization constant.[7] The exponent $a_{ij} \geq 0$ in (1) represents the share of good j in the production technology of good i. A larger a_{ij}

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[7]In what follows, we set $\zeta_i = (1 - \mu)^{-(1-\mu)} \prod_{j=1}^n (\mu a_{ij})^{-\mu a_{ij}}$, which simplifies the key expressions without any bearing on our results.

means that good j is more important in producing i, whereas $a_{ij} = 0$ implies that good j is not a required input for i's production technology.[8] We summarize the intersectoral input-output linkages with matrix $A = [a_{ij}]$, which we refer to as the economy's *input-output matrix*.

We assume that productivity shocks Ξ_i are independent and identically distributed across sectors and denote the common cumulative distribution function (CDF) of $\epsilon_i = \log(\Xi_i)$ by F. We assume that the microeconomic shock to sector i, ϵ_i, has a symmetric distribution around the origin with full support over $\mathbb{R}$ and a finite standard deviation, which, without much loss of generality, we normalize to one. Following Foss, Korshunov, and Zachary (2011), we say ϵ_i has *light tails* if $\mathbb{E}[\exp(b\epsilon_i)] < \infty$ for some $b > 0$. This assumption ensures that all moments of ϵ_i are finite. In contrast, we say microeconomic shocks have *heavy tails* if $\mathbb{E}[\exp(b\epsilon_i)] = \infty$ for all $b > 0$.

The economy is also populated by a representative household, who supplies one unit of labor inelastically. We assume that the representative household has logarithmic preferences over the n goods given by

$$u(c_1, \ldots, c_n) = \sum_{i=1}^{n} \beta_i \log(c_i),$$

where c_i is the amount of good i consumed and $\beta_i > 0$ is i's share in the household's utility function. Without loss of generality, we assume that $\sum_{i=1}^{n} \beta_i = 1$.

The competitive equilibrium of this economy is defined in the usual way: it consists of a collection of prices and quantities such that (i) the representative household maximizes her utility; (ii) the representative firm in each sector maximizes its profits while taking the prices and the wage as given; and (iii) all markets clear.

Throughout the paper, we refer to the logarithm of real value added in the economy as *aggregate output* and denote it by y. Our first result provides a convenient characterization of aggregate output as a function of microeconomic shocks and the technology and preference parameters.

Proposition 1. *The aggregate output of the economy is given by*

$$y = \log(\text{GDP}) = \sum_{i=1}^{n} v_i \epsilon_i, \tag{2}$$

where

$$v_i = \frac{p_i x_i}{\text{GDP}} = \sum_{j=1}^{n} \beta_j \ell_{ji} \tag{3}$$

and ℓ_{ji} is the (j, i) element of the economy's Leontief inverse $L = (I - \mu A)^{-1}$.

This result is related to Hulten (1978), who shows that in a competitive economy with constant returns to scale technologies, aggregate output is a linear combination of sectoral-level productivity shocks, with coefficients v_i given by the *Domar weights* (Domar, 1961; Hulten, 1978), defined as

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[8]The assumption that firms employ constant returns to scale technologies implies that $\sum_{j=1}^{n} a_{ij} = 1$ for all i.

the sectoral sales divided by GDP. Proposition 1, however, also establishes that with Cobb-Douglas preferences and technologies, these weights take a particularly simple form: the Domar weight of each sector depends only on the preference shares, $(\beta_1, \ldots, \beta_n)$, and the corresponding column of the economy's Leontief inverse, which measures that sector's importance as an input-supplier to other sectors in the economy.

The heterogeneity in Domar weights plays a central role in our analysis. Equation (3) provides a clear decomposition of this heterogeneity in terms of the structural parameters of the economy. At one extreme, corresponding to an economy with no input-output linkages (i.e., $\mu = 0$), the heterogeneity in Domar weights simply reflects differences in preference shares: $v_i = \beta_i$ for all sectors i. We refer to this source of heterogeneity in Domar weights as *primitive heterogeneity*.[9] At the other extreme, corresponding to an economy with identical β_i's, the heterogeneity in v_i's reflects differences in the roles of different sectors as input-suppliers to the rest of the economy (as in Acemoglu et al. (2012)), a source of heterogeneity which we refer to as *network heterogeneity*. In general, the empirical distribution of Domar weights is determined by the combination of primitive and network heterogeneity.

Finally, we define a *simple economy* as an economy with symmetric preferences (i.e., $\beta_i = 1/n$) and no input-output linkages (i.e., $\mu = 0$). As such, a simple economy exhibits neither primitive nor network heterogeneity. Hence, all sectors have identical Domar weights and the economy's aggregate output is a simple average of microeconomic shocks: $y = \frac{1}{n} \sum_{i=1}^{n} \epsilon_i$.

3 Defining Tail Risks

The central question this paper focuses on is whether idiosyncratic, microeconomic shocks to disaggregated sectors can lead to the emergence of macroeconomic tail risks. In this section, we first provide a formal definition of this notion and then explain the motivation for our choice. We then argue that to formally capture whether macroeconomic tail risks can originate from microeconomic shocks, one needs to focus on the extent of tail risks in a sequence of economies in which the number of sectors grows.

3.1 Macroeconomic Tail Risks

As explained in the Introduction, our purpose is to measure the likelihood of (very) large deviations of macroeconomic variables from their trends. That is, the "tail risks" we are interested in do not correspond to small or regular variations in output, but rather represent the frequency and likelihood of events such as the Great Depression or the severe recession following the recent financial crisis.

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[9]We use the term primitive heterogeneity as opposed to "preference heterogeneity" since, in general (with non-Cobb-Douglas technologies), differences in the other "primitives" (such as average sectoral productivities) play a similar role to the β_i's in the determination of the Domar weights.

We therefore define an economy's τ-*tail ratio* as the likelihood that aggregate output deviates τ standard deviations from its mean relative to the likelihood of an identical deviation under the normal distribution:

$$R(\tau) = \frac{\log \mathbb{P}(y < -\tau\sigma)}{\log \Phi(-\tau)},$$

where τ is a positive constant, $\sigma = \text{stdev}(y)$ is the standard deviation of the economy's aggregate output (which we refer to as *aggregate volatility*), and Φ is the CDF of the standard normal distribution.[10] This ratio, which is always a positive number, has a natural interpretation: $R(\tau) < 1$ if and only if the likelihood of a τ standard deviation decline in aggregate output is greater than the corresponding likelihood under the normal distribution. Moreover, the further an economy's τ-tail ratio is below unity, the larger the likelihood of observing a $\tau\sigma$ deviation relative to the normal distribution.

Definition 1. The economy *exhibits macroeconomic tail risks* (relative to the normal distribution) if

$$\lim_{\tau \to \infty} R(\tau) = 0.$$

In other words, if an economy exhibits macroeconomic tail risks, then for any arbitrary $r > 1$ there exists a large enough T such that for all $\tau > T$, the likelihood that aggregate output exhibits a $\tau\sigma$ deviation from the mean is at least r times larger than the corresponding likelihood under the normal distribution. This definition therefore provides a natural notion for deviations from the normal distribution at the tails. In addition, the limiting behavior of $R(\tau)$ provides an attractive measure for the extent of macroeconomic tail risks in a given economy: a more rapid rate of decay of $R(\tau)$ corresponds to a greater risk that aggregate output exhibits large deviations from its mean.

Observe that, by construction, our notion of tail risk does not reflect differences in the magnitude of aggregate volatility, as it compares the likelihood of large deviations relative to a normally distributed random variable of the same standard deviation. Hence, even though increasing the standard deviation of sectoral shocks impacts the economy's aggregate volatility, it does not have an impact on the extent of macroeconomic tail risks.

We also remark that even though measures such as kurtosis — frequently invoked to measure deviations from normality (Fagiolo, Napoletano, and Roventini, 2008; Atalay and Drautzburg, 2015) — are informative about the likelihood of large deviations, their major shortcoming as measures of tail risk is that they are also affected by regular fluctuations in aggregate output. In contrast, the notion of tail risk introduced in Definition 1 depends only on the distribution of aggregate output far away from the mean. The following example highlights the distinction between our notion of tail risk and kurtosis.

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[10]The assumption that microeconomic shocks have a symmetric distribution around the origin guarantees that mean aggregate output is equal to zero; that is, $\mathbb{E}y = 0$.

Example 1. Consider an economy in which aggregate output y has the following distribution: with probability $p > 0$, it has a symmetric exponential distribution with mean zero and variance σ^2, whereas with probability $1 - p$, it is uniformly distributed with the same mean and variance. It is easy to verify that the excess kurtosis of aggregate output, defined as $\kappa_y = \mathbb{E}[y^4]/\mathbb{E}^2[y^2] - 3$, satisfies

$$\kappa_y = (1 - p)\kappa_{\text{uni}} + p\kappa_{\text{exp}}, \tag{4}$$

where $\kappa_{\text{uni}} < 0$ and $\kappa_{\text{exp}} > 0$ are, respectively, the excess kurtoses of the uniform and exponential distributions.[11] Therefore, for small enough values of p, aggregate output exhibits a smaller kurtosis relative to that of the normal distribution. This is despite the fact that, for all values of $p > 0$, the likelihood that y exhibits a large enough deviation is greater than what is predicted by the normal distribution. In contrast, our notion adequately captures this type of tail risk: for any $p > 0$, there exists a τ large enough such that $R(\tau) < 1$, and the economy exhibits macroeconomic tail risks in the sense of Definition 1.

A similar argument to that in Example 1 readily shows that any normalized moment of aggregate output satisfies a relationship identical to (4), and is similarly inadequate as a measure of tail risk.

3.2 Micro-Originated Tail Risks

Definition 1 formally defines macroeconomic tail risk in a given economy, regardless of its origins. However, what we are interested in is whether such tail risks can emerge as a consequence of idiosyncratic shocks to disaggregated sectors. In what follows, we argue that to meaningfully represent whether macroeconomic tail risks can originate from microeconomic shocks, one needs to focus on the extent of tail risks in "large economies", formally represented as a sequence of economies where the number of sectors grows, i.e., where $n \to \infty$. This increase in the number of sectors can be interpreted as focusing on finer and finer levels of disaggregation.[12]

The key observation is that in any economy that consists of finitely many sectors, idiosyncratic microeconomic shocks do not fully wash out, and as a result, would have some macroeconomic impact. Put differently, even in the presence of independent, sectoral-level shocks $(\epsilon_1, \ldots, \epsilon_n)$, the economy as a whole is subject to some residual level of aggregate uncertainty, irrespective of how large n is. The following result formalizes this idea:

Proposition 2. *If microeconomic shocks exhibit tail risks, any economy consisting of finitely many sectors exhibits macroeconomic tail risks.*

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[11]Excess kurtosis is defined as the difference between the kurtosis of a random variable and that of a normally distributed random variable. Therefore, the excess kurtosis of the normal distribution is normalized to zero, whereas the excess kurtoses of uniform and symmetric exponential distributions are equal to $-5/6$ and 3, respectively.

[12]In Appendix B, we show that, as far as Domar weights are concerned, the only restriction necessary for a sequence to correspond to different levels of disaggregations is for the Domar weight of an aggregated sector to be equal to the sum of the Domar weights of the subindustries that belong to that sector.

This observation underscores that to asses whether microeconomic shocks lead to the emergence of macroeconomic tail risks in a meaningful fashion, one has to focus on a sequence of economies with $n \to \infty$ and measure how the extent of tail risks decreases along this sequence. This is indeed the strategy adopted by Gabaix (2011) and Acemoglu et al. (2012) who study whether microeconomic shocks can lead to non-trivial levels of aggregate volatility (and is implicit in Lucas'(1977) famous argument that micro shocks should be irrelevant at the aggregate level).

Yet, this strategy raises another technical issue. As highlighted by Definition 1, our notion of tail risks entails studying the deviations of aggregate output from its mean as $\tau \to \infty$. This means that the order in which τ and n are taken to infinity becomes crucial. To highlight the dependence of the rates at which these two limits are taken, we index τ by the level of disaggregation of the economy, n, and study the limiting behavior of the sequence of tail ratios,

$$R_n(\tau_n) = \frac{\log \mathbb{P}(y_n < -\tau_n \sigma_n)}{\log \Phi(-\tau_n)},$$

as $n \to \infty$, where y_n is the aggregate output of the economy consisting of n sectors, $\sigma_n = \mathrm{stdev}(y_n)$ is the corresponding aggregate volatility, and $\{\tau_n\}$ is an increasing sequence of positive real numbers such that $\lim_{n \to \infty} \tau_n = \infty$.

To determine how the sequence $\{\tau_n\}$ should depend on the level of disaggregation, n, we rely on Lucas'(1977) irrelevance argument, which maintains that idiosyncratic, sectoral-level shocks in a simple economy with no network or primitive heterogeneity should have no aggregate impact as the size of the economy grows. Our next result uses this argument to pin down the rate of dependence of τ on n.

Proposition 3. *Consider a sequence of simple economies; that is, $\mu = 0$ and $\beta_i = 1/n$ for all i.*

(a) If $\lim_{n \to \infty} \tau_n/\sqrt{n} = 0$, then $\lim_{n \to \infty} R_n(\tau_n) = 1$ for all light-tailed microeconomic shocks.

(b) If $\lim_{n \to \infty} \tau_n/\sqrt{n} = \infty$, then there exist light-tailed micro shocks such that $\lim_{n \to \infty} R_n(\tau_n) = 0$.

In other words, as long as $\lim_{n \to \infty} \tau_n/\sqrt{n} = 0$, the rate at which we take the two limits is consistent with the idea that in simple economies (with no primitive or network heterogeneity across firms/sectors), microeconomic shocks have no major macroeconomic impact. In contrast, if the rate of growth of τ_n is so fast that $\lim_{n \to \infty} \tau_n/\sqrt{n} = \infty$, then Lucas'(1977) argument for the irrelevance of microeconomic shocks in a simple economy would break down. Motivated by these observations, in the remainder of the paper, we set $\tau_n = \sqrt{n}$ and obtain the following definition:

Definition 2. A *sequence* of economies *exhibits macroeconomic tail risks* if

$$\lim_{n \to \infty} R_n(\sqrt{n}) = 0.$$

As a final remark, note that as in the case of a single economy, this definition also suggests that the rate at which $R_n(\sqrt{n})$ converges to zero provides a natural measure for the extent of macroeconomic

tail risks in a given sequence of economies: a more rapid rate of decay of $R_n(\sqrt{n})$ corresponds to a greater likelihood of a large deviation of aggregate output from its mean.[13]

4   Micro Shocks, Macro Tail Risks

In this section, we study whether idiosyncratic microeconomic shocks can translate into sizable macroeconomic tail risks and present our main results. Taken together, our results illustrate that the severity of macroeconomic tail risks is determined by the interaction between the extent of heterogeneity in Domar weights and the distribution of microeconomic shocks.

4.1   Normal Shocks

We first focus on the case in which microeconomic shocks are normally distributed. Besides providing us with an analytically tractable example of a distribution with extremely light tails, the normal distribution serves as a natural benchmark for the rest of our results.

Proposition 4. *Suppose microeconomic shocks are normally distributed. Then, no sequence of economies exhibits macroeconomic tail risks.*

Thus, in the presence of normally distributed microeconomic shocks, GDP fluctuations can be well-approximated by a normal distribution, even at the tails. More importantly, this result holds irrespective of the nature of input-output linkages or firm size distribution.

4.2   Exponential-Tailed Shocks

Next, we focus on economies in which microeconomic shocks belong to the subclass of light-tailed distributions with exponential tails. Formally, we say that microeconomic shocks have *exponential tails* if there exists a constant $\gamma > 0$ such that

$$\lim_{z \to \infty} \frac{1}{z} \log(1 - F(z)) = -\gamma.$$

For example, any microeconomic shock with a CDF given by $1 - F(z) = Q(z)e^{-\gamma z}$ for $z \geq 0$ and some polynomial function $Q(z)$ belongs to the class of shocks with exponential tails, with the case where $Q(z)$ is constant corresponding to the (symmetric) exponential distribution.

Note that in our terminology, even though exponential-tailed shocks belong to the class of light-tailed distributions, they exhibit microeconomic tail risks, as they have tails that are heavier than that of the normal distribution. Our main result in this section provides a characterization of when such microeconomic tail risks translate into macroeconomic tail risks.

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[13]We provide a more formal definition for comparing the extent of tail risks across different sequences of economies in Section 6.2.

To present our results, we introduce the measure of *sectoral dominance* of a given economy as

$$\delta = \frac{v_{\max}}{\|v\|/\sqrt{n}},$$

where n is the number of sectors in the economy, $v_{\max} = \max\{v_1, \ldots, v_n\}$ and $\|v\| = \left(\sum_{i=1}^{n} v_i^2\right)^{1/2}$ is the second (uncentered) moment of the economy's Domar weights. Intuitively, δ measures how important the most dominant sector in the economy is compared to the variation in the importance of all sectors as measured by $\|v\|$. The normalization factor $\sqrt{n}$ reflects the fact that δ captures the extent of this dominance *relative to a simple economy*, for which $v_{\max} = 1/n$ and $\|v\| = 1/\sqrt{n}$. This of course implies that the sectoral dominance of a simple economy is equal to 1. We also remark that even though, formally, sectoral dominance depends on the largest Domar weight, a high value of δ does not necessarily imply that a single sector is overwhelmingly important relative to the rest of the economy. Rather, the presence of a group of sectors that are large relative to the amount of dispersion in Domar weights would also translate into a high level of sectoral dominance. The next theorem contains our main results.[14]

Theorem 1. *Suppose that microeconomic shocks have exponential tails.*

(a) A sequence of economies exhibits macroeconomic tail risks if and only if $\lim_{n\to\infty} \delta = \infty$.

(b) Consider two sequences of economies with sectoral dominances δ and δ' that exhibit macroeconomic tail risks. The first sequence exhibits greater macroeconomic tail risks if $\lim_{n\to\infty} \delta/\delta' = \infty$.

(c) A sequence of economies for which $\lim_{n\to\infty} \delta/\sqrt{n} = 0$ and $\lim_{n\to\infty} \delta = \infty$ exhibits macroeconomic tail risks, even though aggregate output is asymptotically normally distributed, in the sense that $y/\sigma \to \mathcal{N}(0,1)$ in distribution.

Statement (a) of the theorem states that in the presence of exponentially-tailed shocks, economies with *limited sectoral dominance*, defined as those for which $\liminf_{n\to\infty} \delta < \infty$, exhibit no macroeconomic tail risks. This is due to the fact that in the absence of a dominant sector or a group of sectors, microeconomic tail risks wash out in the aggregate with no sizable macroeconomic effects. In contrast, in economies with non-trivial sectoral dominance (where $\delta \to \infty$), microeconomic tail risks do not entirely cancel each other out, even in a very large economy, leading to the emergence of aggregate tail risks. This result thus complements those of Gabaix (2011) and Acemoglu et al. (2012) by establishing that heterogeneity in Domar weights is key not only in generating aggregate volatility, but also in translating microeconomic tail risks into macroeconomic tail risks. However, as we show in Subsection 4.4, the role played by the heterogeneity in Domar weights in creating aggregate volatility is fundamentally distinct from its role in generating tail risks.

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[14]Note that when we work with a sequence of economies, all our key objects, including δ, $v_{\max}$ and $\|v\|$ depend on the level of disaggregation n. However, in what follows, we simplify the notation by suppressing their dependence on n. We make the dependence on n clear in the proofs in the Appendix.

Statement (b) of Theorem 1 establishes that the limiting behavior of the economies' sectoral dominance determines not just the presence but also the extent of macroeconomic tail risk.

Finally, the last part of the theorem shows that significant macroeconomic tail risks can coexist with a normally distributed aggregate output, as predicted by the central limit theorem. Though it may appear contradictory at first, this coexistence is quite intuitive: the notion of asymptotic normality implied by the central limit theorem considers the likelihood of a $\tau\sigma$ deviation from the mean as the number of sectors grows, while keeping the size of the deviations τ fixed. In contrast, per our discussion in Section 3, tail risks correspond to the likelihood of large deviations, formally captured by taking the limit $\tau \to \infty$. Statement (c) of Theorem 1 thus underscores that the determinants of large deviations can be fundamentally distinct from the origins of small or moderate deviations. This result also explains how, consistent with the patterns documented for the U.S. in Figure 1, aggregate output can be well-approximated by a normal distribution away from the tails, even though it may exhibit significantly greater likelihood of tail events.

The juxtaposition of Theorem 1 and Proposition 4 also highlights the important role that the nature of microeconomic shocks play in shaping aggregate tail risks. In particular, replacing normally distributed microeconomic shocks with exponential shocks — which have only slightly heavier tails — may dramatically increase the likelihood of large economic downturns. This is despite the fact that the distribution of microeconomic shocks has no impact on the standard deviation of GDP or the shape of its distribution away from the tails (as shown by part (c) of the Theorem 1).

Example 2. Consider the sequence of economies depicted in Figure 2 in which sector 1 is the sole supplier to k sectors, whereas the output of the rest of the sectors are not used as intermediate goods for production by other sectors. Furthermore, suppose that the economies in this sequence exhibit no primitive heterogeneity, in the sense that households assign an equal weight to all goods produced in the economy; that is, $\beta_i = 1/n$ for all i. It is easy to verify that Domar weights satisfy

$$v_{\max} = v_1 = \frac{\mu k}{n(1-\mu)} + \frac{1}{n}$$

and

$$\|v\| = \frac{1}{n(1-\mu)}\sqrt{(\mu k + 1 - \mu)^2 + (k-1)(1-\mu)^2 + n - k}.$$

As a result, as we increase the level of disaggregation, $\delta \to \infty$ if and only if $k \to \infty$. Thus, by Theorem 1, exponentially distributed microeconomic shocks in such a sequence of economies lead to macroeconomic tail risks provided that $k \to \infty$. Note that macroeconomic tail risks can be present even if sector 1 is an input-supplier to a diminishing fraction of sectors. For example, if $k = \log n$, the fraction of sectors that rely on sector 1 satisfies $\lim_{n\to\infty} k/n = 0$, and the central limit theorem applies.

The next example shows that macroeconomic tail risks can arise in the absence of network heterogeneity as long as the economy exhibits sufficient levels of primitive heterogeneity.
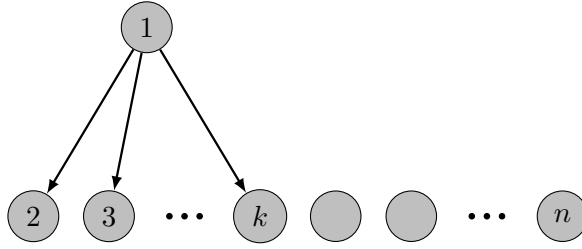
Figure 2. An economy in which sector 1 is the input-supplier to k sectors.

Example 3. Consider a sequence of economies with no input-output linkages (i.e., $\mu = 0$) and suppose that the weights assigned by the representative household to different goods are given by $\beta_1 = s/n$ and $\beta_i = (1 - \beta_1)/(n - 1)$ for all $i \neq 1$. Thus, the representative household values good 1 more than all other goods as long as $s > 1$. Then,

$$v_{\max} = s/n,$$

whereas

$$\|v\| = \frac{1}{n}\sqrt{s^2 + (n - s)^2/(n - 1)}$$

for the economy consisting of n sectors. Therefore, as long $s \to \infty$, this sequence of economies exhibits non-trivial sectoral dominance and sizable levels of macroeconomic tail risks.

Contrasting this observation with Example 2 shows that either network or primitive heterogeneity would be sufficient for the emergence of macroeconomic tail risks.

Though Theorem 1 provides a complete characterization of the conditions under which macroeconomic tail risks emerge from the aggregation of microeconomic shocks, its conditions are in terms of the limiting behavior of our measure of sectoral dominance, δ, which in turn depends on the entire distribution of Domar weights. Our next result focuses on a subclass of economies for which we can directly compute the extent of sectoral dominance.

Definition 3. An economy has *Pareto Domar weights* with exponent $\eta > 0$ if $v_i = ci^{-1/\eta}$ for all i and some constant $c > 0$.

In an economy with Pareto Domar weights, the fraction of sectors with Domar weights greater than or equal to any given k is proportional to $k^{-\eta}$. Consequently, a smaller η corresponds to more heterogeneity in Domar weights and hence a (weakly) larger measure of sectoral dominance. It is easy to verify that if $\eta < 2$ the measure of sectoral dominance of such a sequence of economies grows at rate $\sqrt{n}$, whereas for $\eta > 2$, it grows at rate $n^{1/\eta}$.[15] Nevertheless, in either case, $\lim_{n\to\infty} \delta = \infty$, thus leading to the following corollary to Theorem 1:

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[15]In the knife edge case where $\eta = 2$, sectoral dominance grows at the rate $\sqrt{n/\log n}$. See the proof of Corollary 1 for the exact derivations.

Corollary 1. *Consider a sequence of economies with Pareto Domar weights with common exponent η and suppose that microeconomic shocks have exponential tails.*

(a) The sequence exhibits macroeconomic tail risks for all $\eta > 0$.

(b) $y/\sigma \to \mathcal{N}(0, 1)$ in distribution if $\eta \geq 2$.

Consequently, if $\eta \geq 2$, exponential-tailed microeconomic shocks lead to macroeconomic tail risks, even though aggregate output is asymptotically normally distributed.

4.3   Generalization: Super-Exponential Shocks

In the previous subsection, for the sake of tractability, we focused on economies in which microeconomic shocks have exponential tails. In this subsection, we show that our main results presented in Theorem 1 generalize to a larger subclass of light-tailed microeconomic shocks that are not necessarily exponential. More specifically, we focus on economies in which microeconomic shocks belong to the subclass of *super-exponential* distributions with shape parameter $\alpha \in (1, 2)$, in the sense that

$$\lim_{z \to \infty} \frac{1}{z^{-\alpha}} \log[1 - F(z)] = -c, \tag{5}$$

where F is the common CDF of microeconomic shocks and $c > 0$ is some constant.[16] For example, any shock with a CDF satisfying $1 - F(z) = Q(z) \exp(-cz^{\alpha})$ for some polynomial function $Q(z)$ belongs to this family. Note that we are ruling our the cases of $\alpha = 1$ and $\alpha = 2$ as in such cases microeconomic shocks would have exponential and normal tails, respectively. This observation also highlights that shocks belonging to this subclass of super-exponential distributions have tails that are heavier than that of the normal and lighter than that of the exponential distribution.

Proposition 5. *Suppose that microeconomic shocks have super-exponential tails with shape parameter $\alpha \in (1, 2)$.*

(a) If $\liminf_{n \to \infty} \delta < \infty$, then the sequence of economies exhibits no macroeconomic tail risks.

(b) If $\lim_{n \to \infty} \delta/n^{(\alpha-1)/\alpha} = \infty$, then the sequence of economies exhibits macroeconomic tail risks.

This result thus indicates that the insights of Theorem 1 generalize to economies that are subject to super-exponential shocks. As in the case of exponential-tailed shocks, Proposition 5 shows that the economy's sectoral dominance δ plays a central role in translating microeconomic shocks into macroeconomic tail risks.

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[16]We provide a characterization for a broader class of super-exponential distributions in the proof of Proposition 5.

4.4 Macroeconomic Tail Risks and Aggregate Volatility

We end this section with a discussion clarifying the distinction between the role of sectoral heterogeneity in creating macroeconomic tail risks on the one hand and aggregate volatility on the other.

Recall from Theorem 1 that in the presence of exponential-tailed microeconomic shocks, a sequence of economies with sufficiently high levels of sectoral dominance exhibits macroeconomic tail risks. In particular, microeconomic shocks translate into sizable aggregate tail risks if and only if

$$\lim_{n\to\infty} \delta = \lim_{n\to\infty} \frac{v_{\max}}{\|v\|/\sqrt{n}} = \infty. \tag{6}$$

As already mentioned in Subsection 4.2, this condition holds whenever a sector or a group of sectors play a significant role in determining macroeconomic outcomes relative to the variation in the Domar weights of all sectors.

On the other hand, the characterization in equation (2) implies that aggregate volatility is equal to $\sigma = \text{stdev}(y) = \|v\|$.[17] Therefore, as argued by Gabaix (2011) and Acemoglu et al. (2012), micro shocks generate aggregate volatility if $\|v\|$ decays to zero at a rate slower than $1/\sqrt{n}$; that is, if

$$\lim_{n\to\infty} \frac{\|v\|}{1/\sqrt{n}} = \infty. \tag{7}$$

Contrasting (7) with (6) highlights that even though the intensity of both micro-originated aggregate volatility and tail risks are determined by the extent of heterogeneity in Domar weights, this heterogeneity manifests itself differently in each case: whereas the level of microeconomic tail risks is highly sensitive to the largest Domar weight in the economy, aggregate volatility is determined by the second moment of the distribution of Domar weights. Furthermore, recall from the discussion in Subsections 4.1 and 4.2 that the nature of microeconomic shocks plays a critical role in shaping the extent of macroeconomic tail risks. In contrast, as far as aggregate volatility is concerned, the shape and distributions of micro-shocks (beyond their variance) are immaterial.

Taken together, these observations imply that a sequence of economies may exhibit macroeconomic tail risks even if it does not display non-trivial levels of aggregate volatility, and vice versa. The following examples illustrate these possibilities.

Example 4. Consider a sequence of economies with Pareto Domar weights with common exponent η, that is, $v_i = ci^{-1/\eta}$ for all sectors i. It is easy to verify that as long as $\eta > 2$, aggregate volatility in this sequence of economies decays at the rate $1/\sqrt{n}$ (or more precisely, $\limsup_{n\to\infty} \frac{\|v\|}{1/\sqrt{n}} < \infty$), regardless of the distribution of microeconomic shocks. Hence, microeconomic shocks in such a sequence of economies have no meaningful impact on aggregate volatility. This is despite the fact that, as established in Corollary 1, exponentially-tailed microeconomic shocks lead to macroeconomic tail risks for all positive values of η.

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[17] This object coincides with what Carvalho and Gabaix (2013) refer to as the economy's "fundamental volatility".

Example 5. Next, consider any sequence of economies for which (7) is satisfied. As already argued, in this case, microeconomic shocks lead to non-trivial aggregate volatility, regardless of how microeconomic shocks are distributed. Yet, from Proposition 4, when microeconomic shocks are normally distributed, this sequence of economies exhibits no macroeconomic tail risks.

We end this discussion by showing that the distinct natures of aggregate volatility and macroeconomic tail risks mean that structural changes in an economy can lead to a reduction in the former while simultaneously increasing the latter.

Example 6. Suppose that a structural change in the economy results in a reduction in $\|v\|$ while $v_{\max}$ remains constant. This will reduce the economy's aggregate volatility but increase its sectoral dominance, leading to aggregate fluctuations that are generally more stable but also exhibit greater tail risks. Clearly, the same can be true even if $v_{\max}$ declines, provided that this is less than the reduction in $\|v\|$. The possibility of simultaneous declines in aggregate volatility (during "regular times") and increases in the likelihood of large economic downturns suggests a different perspective on the well-known episode of Great Moderation (referring to the decline in the standard deviation of GDP in the U.S. economy since the 1970s), which came to an end in 2007 with the most severe recession the U.S. economy had experienced since the Great Depression.

5   Tail Comovements

Our results in the previous section show that sufficient levels of sectoral dominance can translate microeconomic tail risks into macroeconomic tail risks. Very deep recessions such as the Great Depression, however, involve not only very large drops in aggregate output, but also significant simultaneous contractions across a range of sectors within the economy. In this section, we investigate this issue and argue that intersectoral input-output linkages play a key role in translating idiosyncratic microeconomic shocks into such simultaneous sectoral contractions.

We start our analysis by formally defining *tail comovements* as the likelihood that all sectors experience a simultaneous τ standard deviation decline in their respective outputs conditional on a $\tau\sigma$ drop in aggregate output.[18] More specifically, for an economy consisting of n sectors, we define

$$C(\tau) = \mathbb{P}\left(\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i \big| y < -\tau\sigma\right), \tag{8}$$

where $\hat{x}_i = \log(x_i)$ is the log output of sector i, $\hat{\sigma}_i = \mathrm{stdev}(\hat{x}_i)$ is output volatility of sector i, and $\sigma = \mathrm{stdev}(y)$ is the economy's aggregate volatility. This statistic measures whether a large contraction in aggregate output would necessarily imply that all sectoral outputs also experience

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[18]As discussed in footnote 4, two standard deviation contractions in the U.S. economy are associated with about 10% of four-digit sectors experiencing similarly large declines. Though it is possible to define tail comovements as the conditional likelihood that 10% (or any other fraction) of sectors experience large declines, it is conceptually simpler and mathematically more convenient to focus on the likelihood that all sectors experience such a decline.

a large decline with high probability. Therefore, in an economy with high levels of tail comovements, micro-originated recessions are very similar to recessions that are consequences of economy-wide aggregate shocks.

In the remainder of this section, we show that the extent of tail comovements, as measured by (8), is determined by the nature of input-output linkages across different sectors. We establish that keeping the distribution of microeconomic shocks and the heterogeneity in Domar weights constant, increasing the extent of sectoral interconnectivity leads to higher levels of tail comovements.

5.1 Sectoral Interconnectivity

Before presenting our main results, we provide a formal notion to compare the extent of sectoral interconnectivity across two economies.

Recall from equation (2) that Domar weights serve as a sufficient statistic for the role of microeconomic shocks in shaping the behavior of aggregate output. On the other hand, equation (3) establishes that Domar weights are in turn determined by the preference parameters, $(\beta_1, \ldots, \beta_n)$, and the economy's input-output linkages as summarized by its Leontief inverse matrix L. As a result, two economies may exhibit different levels of primitive and network heterogeneity, even though their Domar weights are identical. To provide a comparison across such economies, we define the following concept:

Definition 4. Consider two economies with identical Domar weights; i.e., $v_i = v_i'$ for all i. The latter economy exhibits *more sectoral interconnectivity* relative to the former if there exists a stochastic matrix B such that

$$L' = BL, \tag{9}$$

where L and L' are the corresponding Leontief inverse matrices of the two economies, respectively.[19]

Intuitively, pre-multiplication of the Leontief inverse matrix L by the stochastic matrix B ensures that the entries of the resulting Leontief matrix L' are more evenly distributed while at the same time its diagonal elements are smaller than the corresponding elements of L. Therefore, the resulting economy not only exhibits more intersectoral linkages, but also the intensity of such linkages are more equally distributed across pairs of sectors. The following examples clarify these properties.

Example 7. Consider two economies with identical Domar weights and input-output matrices $A = [a_{ij}]$ and $A' = [a_{ij}']$. Furthermore, suppose that input-output linkages in the two economies are related via

$$a_{ij}' = \frac{1}{(1-\rho)} \left(a_{ij} + \frac{\rho(1-\mu)}{n\mu} - \frac{\rho}{\mu} \mathbb{I}_{\{i=j\}} \right)$$

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[19]A square matrix is *stochastic* if it is element-wise nonnegative and each of whose rows add up to 1. The assumption that matrix B is stochastic guarantees that the share of material inputs, μ, in the two economies are equal.

for some constant $0 \leq \rho \leq \mu \min_i a_{ii}$, with $\mathbb{I}_{\{\cdot\}}$ denoting the indicator function (where the restriction on ρ ensures that $a'_{ij} \geq 0$ for all i and j). This transformation reduces the value of a_{ii} for all sectors i (i.e., $a'_{ii} < a_{ii}$) and redistributes it evenly across pairs of sectors $j \neq i$. As a result, input-output linkages in the latter economy are more uniformly distributed.

Indeed, it is easy to show that this intuitive argument is consistent with our formal notion of sectoral interconnectivity in Definition 4: the Leontief inverse matrices of the two economies are related to one another via equation (9) for the stochastic matrix $B = [b_{ij}]$ whose elements are given by $b_{ij} = \rho/n + (1 - \rho)\mathbb{I}_{\{i=j\}}$, thus guaranteeing that the latter economy exhibits greater sectoral interconnectivity.

Example 8. Consider an economy with no input-output linkages, that is, $a_{ij} = 0$ for all pairs of sectors $i \neq j$, so that the economy's Leontief inverse matrix is given by $L = I/(1 - \mu)$, where μ is the share of material goods in the firms' production technology and I is the identity matrix. Domar weights in this economy are proportional to the corresponding preference parameters, i.e., $v_i = \beta_i/(1 - \mu)$ for all i.

This economy exhibits less sectoral interconnectivity, in the sense of Definition 4, relative to all other economies with identical Domar weights. To see this, consider an economy with Leontief inverse matrix L' with a pair of sectors $i \neq j$ such that $a'_{ij} > 0$ and $v'_k = v_k$ for all k. Since the Leontief inverse matrices of the two economies satisfy equation (9) for $B = (1 - \mu)L'$, it is then immediate that the latter economy exhibits higher levels of sectoral interconnectivity.

We end this discussion with a remark on the relationship between primitive and network heterogeneity. Recall that Definition 4 provides a comparison for the extent of interconnectivity across two economies with identical Domar weights. Consequently, in order for all Domar weights to remain unchanged, the transformation in (9) not only impacts the nature of input-output linkages (as summarized by the Leontief inverse matrices), but would also necessarily alter the economy's primitive heterogeneity as well. More specifically, if an economy exhibits more sectoral interconnectivity than another in the sense of (9), the preference shares of the two economies has to be related via $\beta_i = \sum_{j=1}^{n} b_{ji}\beta'_j$, so that $v_i = v'_i$ for all i. This in turn suggests that preference shares in the economy with less sectoral interconnectivity are more evenly distributed across the n goods.

5.2 Input-Output Linkages and Tail Comovements

We now present the main result of this section:

Proposition 6. *If an economy exhibits more sectoral interconnectivity relative to another economy with identical Domar weights, then it also exhibits more tail comovements.*

This result thus highlights the importance of input-output linkages in creating tail comovements across different sectors: given two economies with identical Domar weights, the one with a higher

level of sectoral interconnectivity exhibits more tail comovements, in spite of the fact that the two economies are indistinguishable at the aggregate level.

Proposition 6 also clarifies a key distinction between the nature of economic fluctuations in (i) economies with no input-output linkages but a significant level of primitive heterogeneity (such as the baseline model in Gabaix (2011)) on the one hand, and (ii) economies with a high level of network heterogeneity (such as the ones studied by Acemoglu et al. (2012)) on the other. Whereas large economic downturns of the first type mostly arise as a consequence of negative shocks to sectors with high β_i, fluctuations in the latter category are due to the propagation of shocks over the economy's input-output linkages. Even though the two mechanisms may not be distinguishable in the aggregate, they lead to significantly different levels of tail comovements.

To further clarify this point, consider the economy with no input-output linkages studied in Example 8, which is reminiscent of the islands economies of Gabaix (2011). As our arguments in Section 4 highlight, so long as preference shares are heterogenous enough, the economy exhibits non-trivial levels of macroeconomic tail risks. Nevertheless, Example 8 and Proposition 6 together imply that such an economy exhibits the least amount of tail comovements relative to all other economies with the same Domar weights. In other words, the latter economy experiences large economic downturns at the same frequency of the former, but these downturns are associated with severe contractions across a larger collection of sectors.

We end this section by remarking that even though we presented Proposition 6 for a pair of economies with a given number of sectors n, an identical result holds for two sequences of economies as n grows:

Corollary 2. *Consider two sequences of economies with identical Domar weights. If all economies in the first sequence exhibit more sectoral interconnectivity relative to the corresponding economy in the second sequence, then* $\liminf_{n\to\infty} C_n(\tau_n)/C'_n(\tau_n) \geq 1$ *for all sequences* $\{\tau_n\}$.

6 Heavy-Tailed Shocks: An Equivalence Result

In this section, we strengthen our previous results by showing that the presence of primitive or network heterogeneity can translate light-tailed (e.g., exponential-tailed) idiosyncratic shocks into aggregate effects that can only arise, in the absence of such heterogeneity, with heavy-tailed shocks. In other words, we show that sufficient levels of heterogeneity in the economy's Domar weights have the same effect on the size of macroeconomic tail risks as subjecting firms to shocks with significantly heavier tails. This equivalence result is interesting in part because it clarifies how sectoral heterogeneity magnifies the effects of relatively unlikely shocks by concentrating the risk at the tails.

6.1 Pareto-Tailed Shocks

To present the main result of this section, we focus on an important subclass of heavy-tailed microeconomic shocks, namely shocks with Pareto tails. Formally, we say microeconomic shocks have *Pareto tails* if

$$\lim_{z \to \infty} \frac{1}{\log z} \log[1 - F(z)] = -\lambda,$$

where $\lambda > 2$ is the corresponding Pareto index. The smaller the index parameter λ, the heavier the tail of the distribution. The condition that $\lambda > 2$ is meant to guarantee that the standard deviation of microeconomic shocks is well-defined and finite.

Proposition 7. *Suppose that microeconomic shocks are Pareto-tailed. Then, any sequence of economies exhibits macroeconomic tail risks.*

The intuition for this result is instructive: when microeconomic shocks have Pareto tails, the likelihood that at leasts one sector is hit with a large shock is high. As a result, regardless of the extent of heterogeneity in Domar weights, aggregate output experiences large declines with a relatively high probability, resulting in macroeconomic tail risks. This contrasts with the the case of exponentially-tailed shocks, where macroeconomic tail risks can emerge only if the underlying economy exhibits sizable sectoral dominance.

6.2 Tail Risks Equivalence

We now show that the presence of sufficient level of heterogeneity in Domar weights has the same effect on the size of macroeconomic tail risks as subjecting firms to shocks with Pareto tails.

Definition 5. Consider two sequences of economies with tail risk ratios $R_n(\tau)$ and $R'_n(\tau)$. The former *exhibits more macroeconomic tail risks* than the latter, if there exists a sequence $\tau_n \to \infty$ such that $\lim_{n \to \infty} R_n(\tau_n) = 0$ and $\limsup_{n \to \infty} R'_n(\tau_n) > 0$.

In other words, a sequence of economies exhibits more macroeconomic tail risks than another if the likelihood of a $\tau\sigma$ deviation in the former can be arbitrarily higher for large enough values of τ. Note that this definition does not require the two sequences to be subject to shocks drawn from the same distribution, and hence, can be used to compare the extent of tail risks across economies subject to different types of shocks. However, in the special case where shocks to the two sequences are drawn from a common distribution, it is sufficient to use the rate at which the corresponding τ-tail ratio decays to zero as the measure of macroeconomic tail risks. In this case, Definition 5 provides a natural generalization for the extent of tail risks in a single economy discussed in Subsection 3.1.

Proposition 8. *For a sequence of simple economies subject to Pareto-tailed microeconomic shocks, there exists a sequence of economies subject to exponential-tailed shocks that exhibits at least as much macroeconomic tail risk.*

This result underscores that macroeconomic tail risks can emerge not necessarily due to (aggregate or idiosyncratic) shocks that are drawn from heavy-tailed distributions, but rather as a consequence of the interplay between relatively light-tailed distributions and heterogeneity in Domar weights. Put differently, sufficient levels of sectoral dominance can fundamentally reshape the distribution of aggregate output by concentrating risk at the tails and increasing the likelihood of large economic downturns from infinitesimal to substantial.[20] This observational equivalence result thus provides a novel solution to what Bernanke, Gertler, and Gilchrist (1996) refer to as the "small shocks, large cycles puzzle" by showing that substantial levels of primitive or network heterogeneity can mimic large aggregate shocks.

Proposition 8 also highlights the distinction between our main results and those of Fama (1963) and Ibragimov and Walden (2007) who observe that the presence Pareto-tailed shocks with extremely heavy tails and infinite variances (that is, when the Pareto index satisfies $\lambda < 2$) leads to departures from normality. In contrast to these papers, our results show that sufficient heterogeneity in Domar weights translates light-tailed microeconomic shocks into aggregate effects that are observationally equivalent to those that arise due to heavy-tailed shocks.

We end this discussion with the following corollary to Proposition 8:

Corollary 3. *For a sequence of simple economies subject to Pareto-tailed microeconomic shocks, there exists a sequence of economies with Pareto Domar weights and subject to exponential-tailed microeconomic shocks that exhibits identical levels of macroeconomic tail risks.*

In other words, Pareto distributed Domar weights have the same impact on the level of macroeconomic tail risks as that of Pareto-tailed shocks in a simple economy.

7   A Simple Quantitative Illustration

In this section, we provide a simple quantitative exercise to highlight whether and how microeconomic shocks can lead to macroeconomic tail risks and show that the extent of heterogeneity in Domar weights in the U.S. data is capable of generating departures from the normal distribution similar to the patterns documented in Figure 1. We then provide an illustration of the extent of tail comovements implied by the input-output linkages in the U.S. data.

Throughout this section, we use the 2007 commodity-by-commodity direct requirements table and the corresponding sectoral sales data compiled by the Bureau of Economic Analysis. Using the sales data, we compute each sector's Domar weight as the ratio of its sales over GDP. The direct requirements table gives us the equivalent of our input-output matrix A, with the typical (i, j) entry corresponding — under the Cobb-Douglas technology assumption — to the value of spending on

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[20]In particular, as we show in the proof of Proposition 8, a sequence of economies that are subject to exponential-tailed shocks and whose sectoral dominance satisfy $\lim_{n \to \infty} \delta \sqrt{(\log n)/n} = \infty$ exhibits at least as much macroeconomic tail risks as a sequence of simple economies that are subject to Pareto-tailed shocks.
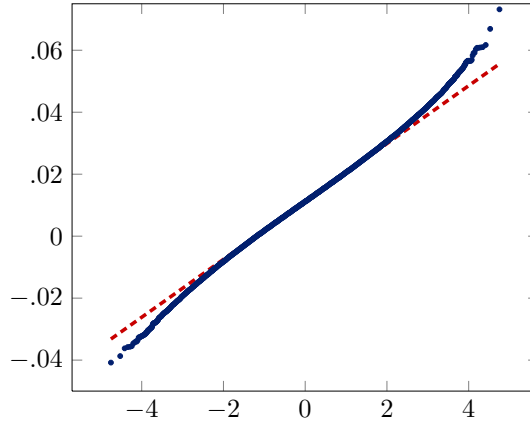
Figure 3. Q-Q plot for aggregate output versus the normal distribution in the presence of exponential shocks, with Domar weights matched to the corresponding values in the U.S. data.

commodity j per dollar of production of commodity i.[21] Though for the sake of simplicity we have thus far assumed that the row sums of A are equal to one (i.e., $\sum_{j=1}^{n} a_{ij} = 1$), we drop this restriction in this section and instead work with the matrix implied by the direct requirements table.[22]

We first study the distribution of aggregate output in our model economy when microeconomic (sectoral) shocks are drawn from a symmetric exponential distribution. We chose the mean and variance of these shocks such that the first two moments of the economy's aggregate output match the first two moments of the U.S. postwar GDP growth rate. The resulting Q-Q plot is depicted in Figure 3.[23] Confirming our theoretical results, the distribution of aggregate output exhibits systematic departures from the normal line at the tails, starting from around two standard deviations away from the mean. The Anderson-Darling and Jarque-Bera tests of normality yield test statistics of 61.22 and 1901.10, respectively, rejecting normality at the 1% level in both cases.

We next investigate the contribution of network heterogeneity in the U.S. data to the extent of macroeconomic tail risks. We focus on the distribution of aggregate output in a counterfactual economy with no primitive heterogeneity, where Domar weights are given by the column sums of the economy's Leontief inverse (divided by $1/n$). The resulting Q-Q plot is depicted in panel (a) of Figure 4. As the figure suggests, the distribution of aggregate output exhibits non-trivial departures from normality at both ends, highlighting the role of network heterogeneity in the emergence of macroeconomic tail risks. The corresponding Anderson-Darling and Jarque-Bera test statistics are, respectively, 35.82 and 1021.35, once again rejecting normality at the 1% level. Finally, Panel (b) of Figure 4 depicts the Q-Q plot of aggregate output when both sources of heterogeneity are shut down, with all Domar weights set equal to $1/n$. Consistent with our theoretical results, aggregate output in this case does not exhibit any meaningful departures from normality (with the Anderson-Darling

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[21]To better approximate the private sector of the economy, in this analysis we exclude 13 sectors corresponding to housing, residential structures, and federal and local government activities. See Acemoglu et al. (2012) for some basic descriptive statistics about the U.S. economy's input-output matrices.

[22]This choice is without any consequence for any of the main points we emphasize in this section, which remain essentially unchanged if we transform A by normalizing its row sums to 1 and then impose $\mu = 0.4$.

[23]We take 500,000 draws from the implied distributions to construct these figures and test statistics.

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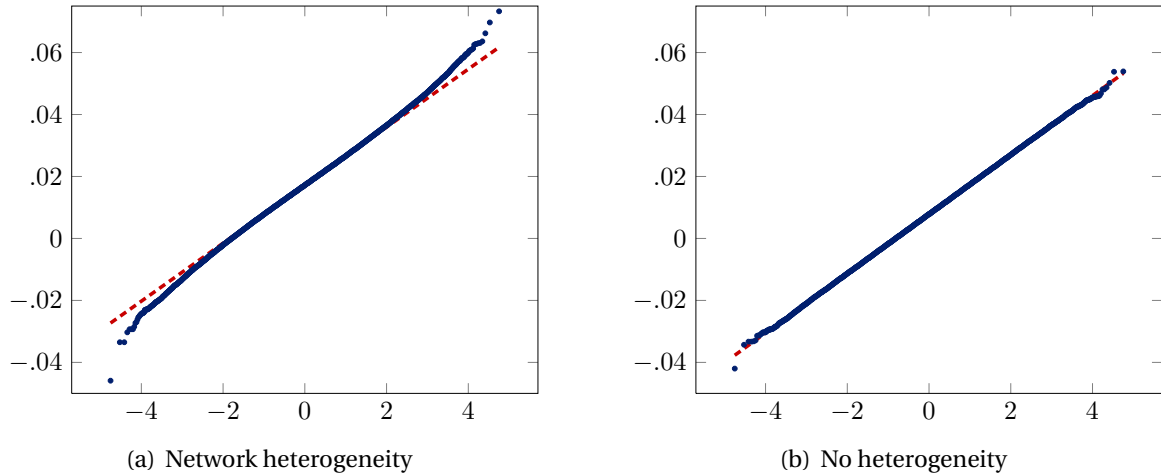(a) Network heterogeneity (b) No heterogeneity

Figure 4. The quantile-quantile plots for aggregate output versus the normal distribution in the presence of exponential shocks. Panel (a) depicts the Q-Q plot for the counterfactual economy with no primitive heterogeneity, with Domar weights set equal to the corresponding column sums of the Leontief inverse matrix divided by n. Panel (b) depicts the same plot for the counterfactual economy with all Domar weights set equal to $1/n$.

and Jarque-Bera tests failing to reject normality at the 15% level) despite the fact that microeconomic shocks have an exponential distribution. Taken together, this exercise confirms that the extent of network heterogeneity in the U.S. data is consistent with the proposition that modest levels of tail risk at the sectoral level can lead to macroeconomic tail risks.[24]

The characterization in Corollary 1 provides an alternative way to asses the role of microeconomic interactions in the emergence of large economic downturns. Recall that, according to this result, Pareto distributed Domar weights can translate exponential-tailed microeconomic shocks into macroeconomic tail risks. Motivated by this observation, Figure 5 plots the empirical counter-cumulative distribution (defined as one minus the empirical cumulative distribution function) of the Domar weights in U.S. data on the log-log scale. It also includes the non-parametric estimates for the empirical counter-cumulative distribution using the Nadaraya-Watson kernel regression (Nadaraya, 1964; Watson, 1964) with a bandwidth selected using least squares cross-validation. The tail of the distribution of Domar weights appears to be approximately linear, corresponding to a Pareto distribution. Taking the tail to correspond to 20% of the sample, we estimate the Pareto index, η, using an ordinary least squares regression with the Gabaix and Ibragimov (2011) correction. We obtain an estimate of $\widehat{\eta}_{\mathrm{OLS}} = 1.45$ with a standard error of 0.24. This is very close to the average slope implied by the non-parametric Nadaraya-Watson regression for the same part of the sample, which is equal to $\widehat{\eta}_{\mathrm{NW}} = 1.36$. This exercise thus suggests that U.S. Domar weights have a distribution

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[24]Using the manufacturing industry data from the NBER productivity database, we also verified that the exponential distribution is a reasonable approximation to the tails of the shocks' distribution at the four-digit level (459 industries). For this exercise, we used the five-factor TFP, which best approximates industry-level shocks, and followed Fagiolo et al. (2008) to estimate the shape parameter in equation (5) above, using maximum likelihood. The mean and the median of this parameter across the 459 manufacturing industries are, respectively, 1.42 and 1.25 (with the 25 and 75 percentiles equal to 0.97 and 1.60, respectively). These results suggest that the exponential distribution provides a better approximation to the tail of the distribution than normal, which would have implied an estimate of α equal to 2.
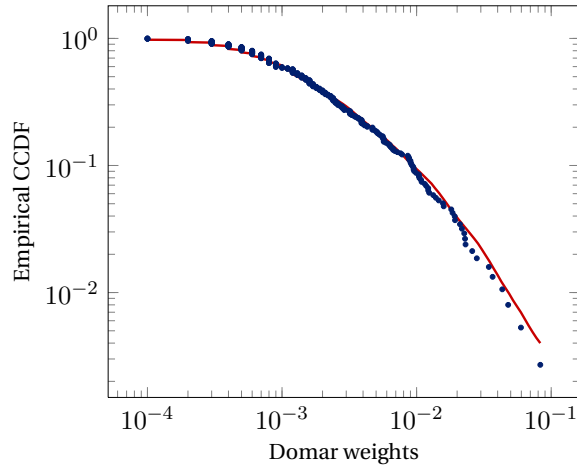
Figure 5. The empirical counter-cumulative distribution of sectoral Domar weights

that is consistent with exponentially-tailed sectoral shocks generating macroeconomic tail risks.[25]

As a final exercise, we assessed the implications of input-output linkages observed in the U.S. data for the extent of tail comovements. Assuming exponentially distributed sectoral shocks, we computed the probability that 10% or more of sectors experience a two standard deviation decline when aggregate output itself declines by two standard deviations or more (recall from footnote 4 that 10% is approximately the fraction of manufacturing sectors experiencing such a decline in the two sharpest U.S. recessions). Given the Domar weights and input-output matrix of the U.S. economy, we find that this number to be equal to 0.17%. We then computed the same number for the counterfactual economy in which Domar weights are identical to that of the U.S. economy but are entirely driven by primitive heterogeneity. Given that, by construction, this counterfactual economy exhibits lower sectoral interconnectivity than the U.S. economy, our theoretical results imply that it should also display less tail comovements. Indeed, in this case the conditional probability that 10% or more of sectors experience a two standard deviation decline is effectively equal to zero, up to seven digits after the decimal point.

8 Conclusions

A noteworthy feature of modern economic fluctuations is the presence of significant "macroeconomic tail risks," defined as a much higher likelihood of large economic downturns relative to what is predicted by the normal distribution. In this paper, we argued that such tail risks may result from the interplay of microeconomic shocks and sectoral heterogeneity, reflecting either network heterogeneity (the differential roles of sectors as input-suppliers) or primitive heterogeneity (their differences in terms of other primitives such as preferences or technology). Our results show that macroeconomic tail risks emerge under two intuitive conditions. First, microeconomic shocks

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[25]Based on the results of Acemoglu et al. (2012), our estimates for the Pareto index of the distribution of Domar weights also imply the presence of significant levels of micro-originated aggregate volatility (measured as standard deviation of aggregate output).

themselves need to exhibit some tail risks — as combinations of normal shocks will lead to normally distributed aggregates. Second, there needs to be sufficient sectoral heterogeneity of a certain type, captured in terms of high levels of "sectoral dominance," which ensures that the largest disaggregated sector or sectors are sufficiently important relative to the variation in the importance of all sectors. We further show that macroeconomic tail risks can emerge even if the central limit theorem applies, so that aggregate output (log GDP) is approximately normally distributed away from the tails.

Another major aspect of large economic downturns is their simultaneous manifestation in many of the economy's disaggregated sectors, what we refer to as tail comovements. We show that keeping the empirical distribution of Domar weights constant, an increase in the extent of sectoral interconnectivity leads to higher levels of tail comovements. This means that when sectoral heterogeneity, at least in part, reflects network heterogeneity, large recessions involve not only significant GDP contractions, but also large simultaneous declines across a wide range of sectors within the economy.

Finally, our quantitative results show that, despite its stylized nature, with the values of sectoral and network heterogeneity as observed in U.S. data, our model generates significant macroeconomic tail risk and tail comovements.

We see our paper as a first step in a systematic investigation of macroeconomic tail risks. Though many commentators view large economic downturns as more consequential than a series of small or moderate recessions, there is relatively little work in understanding whether and how these large economic downturns emerge and whether they are different from regular fluctuations. Our results suggest both ways in which they are similar (in that they are generated by the same interplay of microeconomic shocks and sectoral heterogeneity) and aspects in which they are rather different (the significant departure from the normal distribution at the tail coupled with the approximately normal behavior of aggregate output away from the tails). This perspective naturally lends itself to an investigation of whether certain structural changes can simultaneously stabilize the economy during regular times, while also increasing tail risks.

Several important issues remain open to future research. First, the tractability of our model permits the introduction of various market imperfections into this general framework. This would not only enable an investigation of whether, in the presence of realistic market structures, network and primitive heterogeneities play richer (and more distinct) roles, but also whether large economic downturns necessitate different microeconomic and macroeconomic responses. Second, our analysis was simplified by the log-linear nature of our model economy. An interesting question is whether reasonable nonlinear interactions could exacerbate macroeconomic tail risks. One possibility is to generalize the Cobb-Douglas production technologies, in which case even though versions of equations (2) and (3) would continue to apply, the Domar weights would change endogenously in response to microeconomic shocks. Consequently, depending on elasticities, the shares of sectors hit by negative shocks, and thus the likelihood of large economic downturns would

increase. Third, while we have focused on input-output linkages, other aspects of interactions between microeconomic units may also have major implications for aggregate tail risks. Two natural candidates are the linkages between financial institutions, and between the financial sector and the rest of the economy. The nonlinear contagion mechanisms proposed in recent papers such as Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015b) and Elliott, Golub, and Jackson (2014), as well as other nonlinearities inherent in financial models, such as those emphasized in Brunnermeier, Eisenbach, and Sannikov (2013) may lead to even more macroeconomic tail risks. Finally, even though we have motivated our work with a stylized look at U.S. economic fluctuations, a more systematic empirical investigation to measure and describe the nature of macroeconomic tail risks and to link them to quantitative models would be a natural next step. Particularly important would be to distinguish the economic mechanisms proposed here from alternatives emphasizing the effects of large aggregate shocks (as in the "rare disasters" literature), the implications of time-varying shock variances or model parameters (Engle, 1982; Cogley and Sargent, 2005), the role of financial collapses (Kindleberger, 1978; Reinhart and Rogoff, 2009), and large recessions resulting from nonlinear financial interactions.

Appendix

A   Proofs

We first state and prove two simple lemmas which will be invoked repeatedly.

Lemma A.1. *Let Φ denote the CDF of the standard normal distribution. Then,*

$$\lim_{z \to \infty} \frac{1}{z^2} \log \Phi(-z) = -1/2.$$

Proof. It is well-known that $\lim_{z \to \infty} z\Phi(-z)/\phi(z) = 1$, where ϕ denotes the standard normal density function (e.g., Grimmett and Stirzaker (2001, p. 98)). Consequently,

$$\lim_{z \to \infty} \frac{\log z + \log \Phi(-z)}{\log \phi(z)} = 1,$$

which in turn implies that

$$\lim_{z \to \infty} \frac{\log z + \log \Phi(-z)}{\log \sqrt{2\pi} + z^2/2} = -1.$$

The statement of the lemma follows immediately. □

Lemma A.2. *In any economy, $\sum_{j=1}^{n} \ell_{ij} = 1/(1-\mu)$ for all i, where $L = [\ell_{ij}]$ is the corresponding Leontief inverse matrix.*

Proof. The Leontief inverse matrix, $L = (I - \mu A)^{-1}$, can be written as a power series

$$L = \sum_{k=0}^{\infty} (\mu A)^k.$$

Multiplying both sides of the above equality by the vector of all ones, $\mathbf{1}$, implies that $L\mathbf{1} = \left(\sum_{k=0}^{\infty} \mu^k\right)\mathbf{1}$, where we are using the fact that $A^k\mathbf{1} = \mathbf{1}$ for all $k \geq 0$. Therefore, $L\mathbf{1} = (1-\mu)^{-1}\mathbf{1}$, which means that the row sums of L are equal to $1/(1-\mu)$. $\qquad\square$

Proof of Proposition 1

The first-order conditions of firms in sector i imply that

$$x_{ij} = \mu a_{ij} p_i x_i / p_j \tag{10}$$

$$l_i = (1 - \mu) p_i x_i / w, \tag{11}$$

where w denotes the market wage. Plugging the above into firm i's production function and taking logarithms yields

$$\log p_i + \epsilon_i = (1 - \mu) \log w + \mu \sum_{j=1}^{n} a_{ij} \log p_j.$$

Solving for the equilibrium prices in the above system of equations implies that

$$\log p_i = (1 - \mu) \log w \sum_{j=1}^{n} \ell_{ij} - \sum_{j=1}^{n} \ell_{ij} \epsilon_j,$$

where ℓ_{ij} is the (i, j) element of the economy's Leontief inverse matrix $L = (I - \mu A)^{-1}$. Consequently, by Lemma A.2,

$$\log p_i = \log w - \sum_{j=1}^{n} \ell_{ij} \epsilon_j. \tag{12}$$

Multiplying both sides of the above equation by β_i and summing over all sectors i lead to

$$\sum_{i=1}^{n} \beta_i \log p_i = \log w - \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_i \ell_{ij} \epsilon_j.$$

Moreover, because there is no capital, all firms make zero profits, and total labor supply is normalized to 1, GDP is equal to the market wage, w. Now setting the ideal price index as the numeraire (i.e., $\sum_{i=1}^{n} \beta_i \log p_i = 0$), we obtain $\log w = \sum_{j=1}^{n} v_j \epsilon_j$, where $v_j = \sum_{i=1}^{n} \beta_i \ell_{ij}$. Thus $\log(\text{GDP}) = \sum_{i=1}^{n} v_i \epsilon_i$.

To see that v_i also coincides with the Domar weight corresponding to sector i, note that the market clearing condition for good i is given by

$$x_i = \beta_i w / p_i + \mu \sum_{j=1}^{n} a_{ji} p_j x_j / p_i,$$

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where we are using the fact that $x_{ji} = \mu a_{ji} p_j x_j / p_i$ and that $c_i = \beta_i w / p_i$. Multiplying both sides of the above equality by p_i and solving for the sales of firms in sector i implies

$$p_i x_i = w \sum_{j=1}^{n} \beta_j \ell_{ji},$$

thus establishing that $v_i = p_i x_i / w = p_i x_i / \text{GDP}$. $\qquad\square$

Proof of Proposition 2

Given that microeconomic shocks are independent with a common distribution that is symmetric around the origin, it is immediate that

$$\mathbb{P}(y < -\tau\sigma) \geq \frac{1}{2}\mathbb{P}(v_i\epsilon_i < -\tau\sigma),$$

for some arbitrarily chosen sector i. Since $v_i > 0$ for all i (guaranteed by the fact that $\beta_i > 0$ for all i), we have

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sigma/v_i)}{\log \Phi(-\tau)}.$$

Given that the above inequality has to hold for all i,

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sigma/v_{\max})}{\log \Phi(-\tau)}. \tag{13}$$

Taking limits from both sides of the above inequality, we have

$$\limsup_{\tau\to\infty} R(\tau) \leq \limsup_{\tau\to\infty} \frac{2\log 2}{\tau^2} + \limsup_{\tau\to\infty} \frac{\log F(-\tau\sigma/v_{\max})}{\log \Phi(-\tau)},$$

where we are using Lemma A.1. The first term on the right-hand side of this inequality is equal to zero. On the other hand, the assumption that microeconomic shocks exhibit tail risks implies that the second term is also equal to zero, thus guaranteeing that $\limsup_{\tau\to\infty} R(\tau) = 0$. $\qquad\square$

Proof of Proposition 3

We first state a key result, known as Cramér's theorem, the proof of which can be found in Petrov (1975, p. 218).

Theorem A.1 (Cramér's Theorem)**.** *Let $\epsilon_1, \ldots, \epsilon_n$ be a sequence of i.i.d. light-tailed random variables with zero mean and unit variance. Let $G_n(z) = \mathbb{P}(S_n < z\sqrt{n})$, where $S_n = \epsilon_1 + \cdots + \epsilon_n$. Then, for any sequence $z_n \geq 0$ such that $z_n/\sqrt{n} \to 0$,*

$$\frac{G_n(-z_n)}{\Phi(-z_n)} = \exp\left(-\frac{z_n^3}{\sqrt{n}}\Lambda\left(-\frac{z_n}{\sqrt{n}}\right)\right)\left[1 + O\left(\frac{z_n+1}{\sqrt{n}}\right)\right],$$

as $n \to \infty$, where $\Lambda(z)$ is a power series with coefficients depending on the cumulants of the random variable ϵ_i which converges for sufficiently small values of $|z|$.

Proof of part (a) Turning to the proof of Proposition 3, recall that in a simple economy, all Domar weights are equal to $v_i = 1/n$. Therefore, by Proposition 1, aggregate output is a simple unweighted average of sectoral shocks; that is, $y_n = (1/n) \sum_{i=1}^{n} \epsilon_i$. Furthermore, it is immediate that $\sigma_n = 1/\sqrt{n}$.

In view of the above observations, as long as $\lim_{n \to \infty} \tau_n / \sqrt{n} = 0$, Theorem A.1 implies that

$$\mathbb{P}(y_n < -\tau_n \sigma_n) = \Phi(-\tau_n) \exp\left(-\frac{\tau_n^3}{\sqrt{n}} \Lambda\left(-\frac{\tau_n}{\sqrt{n}}\right)\right) \left[1 + O\left(\frac{\tau_n + 1}{\sqrt{n}}\right)\right],$$

and consequently,

$$\log \mathbb{P}(y_n < -\tau_n \sigma_n) = \log \Phi(-\tau_n) - \frac{\tau_n^3}{\sqrt{n}} \Lambda\left(-\tau_n / \sqrt{n}\right) + O\left(\frac{\tau_n + 1}{\sqrt{n}}\right)$$

$$= \log \Phi(-\tau_n) - \frac{\tau_n^3}{\sqrt{n}} \Lambda\left(-\tau_n / \sqrt{n}\right) + o(1).$$

Dividing both sides of above equality by $\log \Phi(-\tau_n)$ and using Lemma A.1 imply that

$$R_n(\tau_n) = 1 + \frac{2\tau_n}{\sqrt{n}} \Lambda\left(\tau_n / \sqrt{n}\right) + o(1).$$

Now the fact that the power series $\Lambda(z)$ is convergent for small enough values of z guarantees that the second term on the right-hand side above converges to zero as long as $\lim_{n \to \infty} \tau_n / \sqrt{n} = 0$. This implies that $\lim_{n \to \infty} R_n(\tau_n) = 1$. □

Proof of part (b) Recall from the proof of Proposition 2 that inequality (13) is satisfied for any economy and all τ. Therefore, for any sequence $\{\tau_n\}$ such that $\lim_{n \to \infty} \tau_n = \infty$,

$$\limsup_{n \to \infty} R_n(\tau_n) \leq \limsup_{n \to \infty} \frac{2 \log 2}{\tau_n^2} + \limsup_{n \to \infty} \frac{\log F(-\tau_n \sqrt{n})}{\log \Phi(-\tau_n)} = \limsup_{n \to \infty} \frac{\log F(-\tau_n \sqrt{n})}{\log \Phi(-\tau_n)},$$

where we are now using the fact that $v_{\max} = 1/n$ and that $\sigma_n = 1/\sqrt{n}$. Now, suppose that microeconomic shocks have exponential tails with exponent $\gamma > 0$, which clearly belongs to the family of light-tailed distributions. Given that for such a shock distribution $\lim_{z \to \infty}(1/z) \log F(-z) = -\gamma$, we have

$$\limsup_{n \to \infty} R_n(\tau_n) \leq \limsup_{n \to \infty} \frac{2\gamma \sqrt{n}}{\tau_n},$$

where we are once again invoking Lemma A.1. Consequently, for any sequence $\{\tau_n\}$ that satisfies $\lim_{n \to \infty} \tau_n / \sqrt{n} = \infty$, it is immediate that $\limsup_{n \to \infty} R_n(\tau_n) = 0$. □

Proof of Proposition 4

Recall from Proposition 1, that an economy's aggregate output is equal to $y = \sum_{i=1}^{n} v_i \epsilon_i$, where v_i is the Domar weight of sector i. Therefore, when microeconomic shocks are normally distributed, it is immediate that y is also normally distributed, and thus $R(\tau) = 1$, regardless of the value of τ and the number of sectors n. Hence, for any given sequence of economies, it must be the case that $\lim_{n \to \infty} R_n(\sqrt{n}) = 1$. This implies that the sequence of economies in question does not exhibit macroeconomic tail risks. □

Proof of Theorem 1

Proof of part (a)   We start with proving sufficiency. Recall from inequality (13) in the proof of Proposition 2 that for any given economy and all values of τ,

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sigma/v_{\max})}{\log \Phi(-\tau)}.$$

The fact that $\sigma = \|v\|$ implies that for any given economy consisting of n sectors, we have

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sqrt{n}/\delta)}{\log \Phi(-\tau)}, \tag{14}$$

where $\delta = v_{\max}\sqrt{n}/\|v\|$ is the economy's measure of sectoral dominance.

Now, consider a sequence of economies as $n \to \infty$ and suppose that $\tau_n = \sqrt{n}$. For such a sequence, inequality (14) implies that

$$\limsup_{n\to\infty} R_n(\sqrt{n}) \leq \limsup_{n\to\infty} \frac{2\gamma}{\delta_n},$$

where δ_n is the measure of sectoral dominance of the n-sector economy in the sequence. In deriving this inequality we are using Lemma A.1 and the assumption that microeconomic shocks have exponential tails with exponent $\gamma > 0$. Thus, for any sequence of economies for which $\lim_{n\to\infty} \delta_n = \infty$, the right-hand side of the above inequality is equal to zero. Consequently, $\lim_{n\to\infty} R_n(\sqrt{n}) = 0$, guaranteeing that the sequence of economies exhibits macroeconomic tail risks.

To prove the reverse implication, consider an arbitrary economy consisting of n sectors.[26] The assumption that microeconomic shocks have exponential tails with exponent $\gamma > 0$ guarantees that there exists a constant $\hat{\gamma} \leq \gamma$ such that

$$1 - F(z) < e^{-\hat{\gamma}z} \tag{15}$$

for all $z > 0$. On the other hand, given that microeconomic shocks have a symmetric distribution around the origin, we have

$$\frac{1}{2}\mathbb{E}\,|\epsilon_i|^k = \int_0^\infty z^k dF(z) = \int_0^\infty kz^{k-1}\left(1 - F(z)\right)dz$$

for $k \geq 2$, where we have used integration by parts and the fact that

$$0 \leq \lim_{z\to\infty} z^k\left(1 - F(z)\right) \leq \lim_{z\to\infty} z^k e^{-\hat{\gamma}z} = 0.$$

Thus, by (15), there exists a positive constant $r = 1/\hat{\gamma}$ such that

$$\frac{1}{2}\mathbb{E}|\epsilon_i|^k \leq \int_0^\infty kz^{k-1}e^{-z/r}dz = r^k k! \tag{16}$$

---

[26]This part of the argument follows steps similar to those of Teicher (1984).

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for all $k \geq 2$. Consequently, for any positive constant d,

$$
\begin{aligned}
\mathbb{E}\left(e^{dv_i\epsilon_i}\right) &= \sum_{k=0}^{\infty} \frac{(dv_i)^k}{k!} \mathbb{E}\epsilon_i^k \\
&\leq 1 + \sum_{k=2}^{\infty} \frac{(dv_i)^k}{k!} \mathbb{E}|\epsilon_i|^k \\
&\leq 1 + 2\sum_{k=2}^{\infty} (drv_i)^k,
\end{aligned}
$$

where the last inequality is a consequence of (16). Therefore, if $drv_{\max} < 1$, using the fact that $1 + z \leq e^z$ for all z, we have that for all sectors i,

$$
\mathbb{E}\left(e^{dv_i\epsilon_i}\right) \leq 1 + \frac{2(drv_i)^2}{1 - drv_i} \leq \exp\left(\frac{2(drv_i)^2}{1 - drv_{\max}}\right). \tag{17}
$$

On the other hand, from Proposition 1 and Chernoff's inequality, we have

$$
\mathbb{P}(y < -\tau\sigma) \leq e^{-d\tau\sigma}\mathbb{E}\left(e^{dy}\right) = e^{-d\tau\sigma}\prod_{i=1}^{n}\mathbb{E}\left(e^{dv_i\epsilon_i}\right).
$$

Combining this inequality with (17) yields

$$
\log\mathbb{P}(y < -\tau\sigma) \leq -d\tau\sigma + \sum_{i=1}^{n}\frac{2(drv_i)^2}{1 - drv_{\max}} = -d\tau\sigma + \frac{2(dr\|v\|)^2}{1 - drv_{\max}}.
$$

Letting $d = \tau\sigma(4r^2\|v\|^2 + r\tau\sigma v_{\max})^{-1}$ (which satisfies the condition required for deriving (17), $drv_{\max} < 1$) leads to

$$
\log\mathbb{P}(y < -\tau\sigma) \leq \frac{-\tau^2\|v\|}{8r^2\|v\| + 2r\tau v_{\max}},
$$

where we are using the fact that $\|v\| = \sigma$. Therefore, the τ-tail ratio of any economy consisting of n sectors satisfies

$$
R(\tau) \geq \frac{\tau^2(4r^2 + r\tau\delta/\sqrt{n})^{-1}}{-2\log\Phi(-\tau)} \tag{18}
$$

for all $\tau > 0$, where $\delta = v_{\max}\sqrt{n}/\|v\|$ is the economy's measure of sectoral dominance.

Now, consider an arbitrary sequence of economies as $n \to \infty$ and let $\tau_n = \sqrt{n}$. Inequality (18) then implies

$$
\limsup_{n\to\infty} R_n(\sqrt{n}) \geq \limsup_{n\to\infty} \frac{1}{4r^2 + r\delta_n},
$$

where we have once again used Lemma A.1. Consequently, if $\liminf_{n\to\infty}\delta_n < \infty$, the right-hand side of the above inequality would be strictly positive, establishing that $\limsup_{n\to\infty} R_n(\sqrt{n}) > 0$; that is, the sequence of economies does not exhibit macroeconomic tail risks. $\qquad\square$

Proof of part (b) Recall from (14) that for any given economy and all $\tau > 0$,

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sqrt{n}/\delta)}{\log \Phi(-\tau)},$$

where n is the number of sectors in the economy. Therefore, for any sequence of economies as $n \to \infty$ and for any sequence $\tau_n \to \infty$, we have

$$\limsup_{n\to\infty} R_n(\tau_n) \leq \limsup_{n\to\infty} \frac{2\gamma\sqrt{n}}{\tau_n \delta_n}, \tag{19}$$

where $\gamma > 0$ is the exponent of the microeconomic shocks and δ_n is the measure of sectoral dominance of the n-sector economy in the sequence.

On the other hand, inequality (18) implies that for any sequence of economies with measures of sectoral dominance δ'_n, we have

$$\limsup_{n\to\infty} R'_n(\tau_n) \geq \limsup_{n\to\infty} \frac{1}{4r^2 + r\tau_n\delta'_n/\sqrt{n}}. \tag{20}$$

Set $\tau_n = \sqrt{n}/\delta'_n$. The assumption that $\lim_{n\to\infty} \delta_n/\delta'_n = \infty$ means that $\lim_{n\to\infty} \delta_n\tau_n/\sqrt{n} = \infty$. This, coupled with the observation that $\delta_n \leq \sqrt{n}$, implies that $\lim_{n\to\infty} \tau_n = \infty$, guaranteeing that we can use inequalities (19) and (20). Consequently,

$$\limsup_{n\to\infty} R'_n(\tau_n) \geq \limsup_{n\to\infty} \frac{1}{4r^2 + r} > 0$$
$$\limsup_{n\to\infty} R_n(\tau_n) \leq \limsup_{n\to\infty} \frac{2\gamma\delta'_n}{\delta_n} = 0.$$

Thus, by Definition 5, the first economy exhibits greater macroeconomic tail risks. □

Proof of part (c) Since $\lim_{n\to\infty} \delta_n = \infty$, part (a) of the theorem implies that the sequence of economies exhibits macroeconomic tail risks. On the other hand, recall that for any given economy consisting of n sectors, $\delta/\sqrt{n} = v_{\max}/\|v\|$. Therefore, by Theorem 1 of Acemoglu et al. (2012), in any sequence of economies that satisfies $\lim_{n\to\infty} \delta_n/\sqrt{n} = 0$, the random variable y_n/σ_n converges in distribution to the standard normal distribution as $n \to \infty$. □

Proof of Corollary 1

Consider an economy consisting of n sectors. By definition, the Domar weight of sector i is given by $v_i = ci^{-1/\eta}$, where c is a properly chosen normalization constant.[27] It is then immediate that $v_{\max} = c$ and $\|v\| = c\sqrt{\sum_{i=1}^n i^{-2/\eta}}$. Consequently, the economy's measure of sectoral dominance is given by

$$\delta = \left(\frac{1}{n}\sum_{i=1}^n i^{-2/\eta}\right)^{-1/2}.$$

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[27] Recall that, by Lemma A.2, $\sum_{j=1}^n \ell_{ji} = 1/(1-\mu)$ for all sectors j, where μ is the share of intermediate inputs in the firms' production technologies. Therefore, in any economy, $\sum_{i=1}^n v_i = \sum_{i=1}^n \sum_{j=1}^n \beta_j \ell_{ji} = 1/(1-\mu)$.

Now consider a sequence of economies with Pareto Domar weights with common Pareto index η as $n \to \infty$. We analyze the behavior of the measure of sectoral dominance in such a sequence of economies under three separate cases.

First, suppose that $\eta \in (0, 2)$. In this case, the summation $\sum_{i=1}^{n} i^{-2/\eta}$ is convergent and hence bounded from above. It is thus immediate that $\lim_{n\to\infty} \delta_n = \infty$, where δ_n is the measure of sectoral dominance of the n-sector economy in the sequence.

Next, suppose that $\eta = 2$. In this case, $\sum_{i=1}^{n} i^{-2/\eta}$ is nothing but the harmonic series and is therefore upper bounded by $1 + \log n$ for all n. Consequently,

$$\delta_n \geq \left(\frac{1}{n}(1 + \log n) \right)^{-1/2}$$

for all n, which implies that $\lim_{n\to\infty} \delta_n = \infty$.

Finally, if $\eta > 2$, then there exists a constant $\bar{c} > 0$, independent of n, such that

$$\sum_{i=1}^{n} i^{-2/\eta} \leq \bar{c} n^{1-2/\eta}$$

for all n. Hence, $\delta_n \geq n^{1/\eta}/\sqrt{\bar{c}}$, thus once again implying that $\lim_{n\to\infty} \delta_n = \infty$. $\qquad\square$

Proof of Proposition 5

We prove the result for a more general class of super-exponential shocks than the ones considered in Subsection 4.3. In particular, we assume that the CDF of microeconomic shocks satisfies

$$\lim_{z\to\infty} \frac{1}{\rho(z)} \log[1 - F(z)] = -1$$

for some non-negative, increasing function $\rho(z)$ such that

$$\lim_{z\to\infty} \rho(z)/z = \infty$$
$$\lim_{z\to\infty} \rho(z)/z^2 = 0.$$

These conditions guarantee that the tail of the distribution is lighter than that of the exponential distribution, but heavier than that of the normal distribution. It is immediate that the class of distributions that satisfy (5) correspond to the special case in which $\rho(z) = cz^{\alpha}$ for some constant $\alpha \in (1, 2)$.

Proof of part (a) Since super-exponential microeconomic shocks exhibit tails that are lighter than that of the exponential distribution, any given deviation from the mean is more unlikely compared to an identical deviation under the assumption that microeconomic shocks have exponential tails. Hence, in the presence of shocks with super-exponential tails, there exists a constant $r > 0$ such that inequality (18) is satisfied for any arbitrary economy and all values of $\tau > 0$.

Consequently, given a sequence of economies as $n \to \infty$, we have

$$\limsup_{n \to \infty} R_n(\sqrt{n}) \geq \limsup_{n \to \infty} \frac{1}{4r^2 + r\delta_n}.$$

As a result, if $\liminf_{n \to \infty} \delta_n < \infty$, then $\limsup_{n \to \infty} R_n(\sqrt{n}) > 0$, guaranteeing that the sequence of economies does not exhibit macroeconomic tail risks. □

Proof of part (b) Once again, recall from inequality (14) in the Proof of Proposition 2 that the τ-tail ratio of any arbitrary economy of size n satisfies

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sqrt{n}/\delta)}{\log \Phi(-\tau)},$$

for all values of $\tau > 0$, where δ is the economy's measure of sectoral dominance.

Therefore, for a sequence of economies as $n \to \infty$ and for $\tau_n = \sqrt{n}$, the above inequality implies that

$$\limsup_{n \to \infty} R_n(\sqrt{n}) \leq 2 \limsup_{n \to \infty} \frac{1}{n} \log F\left(-n\delta_n^{-1}\right),$$

where δ_n is the measure of sectoral dominance of the n-sector economy in the sequence. Given that shocks have a super-exponential tails, we have

$$\limsup_{n \to \infty} R_n(\sqrt{n}) \leq 2 \limsup_{n \to \infty} \frac{\rho(n\delta_n^{-1})}{n}.$$

Therefore, if $\lim_{n \to \infty} \rho(n\delta_n^{-1})/n = 0$, the right-hand side of the above expression is equal to zero, implying that the sequence of economies exhibits macroeconomic tail risks. Setting $\rho(z) = cz^\alpha$ proves the result for the subclass of super-exponential shocks that satisfy (5). □

Proof of Proposition 6

We start by proving three lemmas. The first lemma determines the equilibrium output of each sector, the second lemma establishes a simple inequality for a collection of non-negative numbers, and our last lemma provides an expression for an economy's measure of tail comovement in terms of the unconditional likelihood that all sectors exhibit a joint deviation from their respective means.

Lemma A.3. *The log output of sector i is equal to $\hat{x}_i = \log(x_i) = \log v_i + \sum_{j=1}^n \ell_{ij}\epsilon_j$.*

Proof. Recall from Proposition 1 that the equilibrium sales of sector i satisfies $p_i x_i = v_i w$, where w is the market wage and v_i is sector i's Domar weight. On the other hand, by equation (12), the equilibrium price of good i is given by

$$\log p_i = \log w - \sum_{j=1}^n \ell_{ij}\epsilon_j,$$

Therefore, the log output of sector i satisfies $\log x_i = \log v_i + \sum_{j=1}^n \ell_{ij}\epsilon_j$. □

Lemma A.4. *Let z_i and q_{ij} be non-negative numbers for all i and j. Then,*

$$\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{n} z_i q_{ij}\right)^2} \leq \sum_{i=1}^{n} z_i \left(\sum_{j=1}^{n} q_{ij}^2\right)^{1/2}.$$

Proof. A simple application of the Cauchy-Schwarz inequality guarantees that

$$\sum_{j=1}^{n} q_{ij} q_{kj} \leq \left(\sum_{j=1}^{n} q_{ij}^2\right)^{1/2} \left(\sum_{j=1}^{n} q_{kj}^2\right)^{1/2}$$

for all i and k. Multiplying both sides of the above inequality by $z_i z_k$ and summing over all i and k thus implies

$$\sum_{j=1}^{n}\left(\sum_{i=1}^{n} z_i q_{ij}\right)\left(\sum_{k=1}^{n} z_k q_{kj}\right) \leq \left(\sum_{i=1}^{n} z_i \left(\sum_{j=1}^{n} q_{ij}^2\right)^{1/2}\right)\left(\sum_{k=1}^{n} z_k \left(\sum_{j=1}^{n} q_{kj}^2\right)^{1/2}\right).$$

Taking square roots from both sides of the above inequality proves the result. $\qquad\square$

Lemma A.5. *For any given economy, the measure of tail comovement is equal to*

$$C(\tau) = \frac{\mathbb{P}\left(\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i\right)}{\mathbb{P}(y < -\tau\sigma)},$$

where $\hat{\sigma}_i$ is output volatility of sector i and $\sigma = \text{stdev}(y)$ is the economy's aggregate volatility.

Proof. From the definition of conditional probability, it is immediate that the statement of the lemma follows once we show that whenever $\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\hat{\sigma}_i$ for all sectors i, then $y < -\tau\sigma$. To this end, recall from Lemma A.3 that $\hat{x}_i = \log v_i + \sum_{j=1}^{n} \ell_{ij}\epsilon_j$, which implies that $\hat{\sigma}_i = \sqrt{\sum_{j=1}^{n} \ell_{ij}^2}$. Therefore, the fact that $\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\sigma_i$ is equivalent to

$$\sum_{j=1}^{n} \ell_{ij}\epsilon_j < -\tau \left(\sum_{j=1}^{n} \ell_{ij}^2\right)^{1/2}.$$

Multiplying both sides of the above inequality by β_i and summing over all sectors implies

$$y < -\tau \sum_{i=1}^{n} \beta_i \left(\sum_{j=1}^{n} \ell_{ij}^2\right)^{1/2},$$

where we are using the fact that $y = \sum_{i,j} \beta_i \ell_{ij}\epsilon_j$, established in Proposition 1. Therefore, by Lemma A.4,

$$y < -\tau \sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{n} \beta_i \ell_{ij}\right)^2}.$$

Finally, the observation that $v_j = \sum_{i=1}^{n} \beta_i \ell_{ij}$ means that the right-hand side of the above inequality is simply equal to $\|v\|$ which is the volatility of aggregate output, $\sigma = \text{stdev}(y)$, thus completing the proof. $\qquad\square$

We now present the proof of Proposition 6. Consider two economies with identical Domar weights (i.e., $v_i = v_i'$ for all i) and denote their corresponding measures of tail comovement with C and C', respectively. Furthermore, assume that the latter economy exhibits more sectoral interconnectivity relative to the former in the sense of Definition 4. In particular, there exists a stochastic matrix $B = [b_{ij}]$ such that

$$\ell_{ij}' = \sum_{k=1}^{n} b_{ik} \ell_{kj}$$

for all pairs of sectors and i and j, where $L = [\ell_{ij}]$ and $L' = [\ell_{ij}']$ are the corresponding Leontief inverse matrices of the two economies, respectively.

To compare the extent of tail comovements in the two economies, recall from Lemma A.5 that

$$C(\tau) = \frac{\mathbb{P}\left(\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i\right)}{\mathbb{P}(y < -\tau\sigma)},$$

where $\hat{\sigma}_i = \text{stdev}(\hat{x}_i)$ is the output volatility of sector i. On the other hand, by Lemma A.3, the log output of sector i is given by $\hat{x}_i = \log v_i + \sum_{j=1}^{n} \ell_{ij}\epsilon_j$, and as a result,

$$\mathbb{P}\left(\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i\right) = \mathbb{P}\left(\sum_{j=1}^{n} \ell_{ij}\epsilon_j < -\tau\hat{\sigma}_i \text{ for all } i\right).$$

Consider the event that

$$\sum_{j=1}^{n} \ell_{ij}\epsilon_j < -\tau\hat{\sigma}_i \qquad \text{for all } i,$$

and pick some arbitrary sector k. Multiplying both sides of the above inequality by b_{ki} and summing over all sectors i implies

$$\sum_{j=1}^{n} \ell_{kj}'\epsilon_j < -\tau \sum_{i=1}^{n} b_{ki}\left(\sum_{j=1}^{n} \ell_{ij}^2\right)^{1/2},$$

where we are using the fact that $\hat{\sigma}_i = (\sum_j \ell_{ij}^2)^{1/2}$. Hence, by Lemma A.4,

$$\sum_{j=1}^{n} \ell_{kj}'\epsilon_j < -\tau\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{n} b_{ki}\ell_{ij}\right)^2} = -\tau\left(\sum_{j=1}^{n} \ell_{kj}'^2\right)^{1/2}.$$

Since the right-hand side of the above inequality is simply equal to $-\tau\hat{\sigma}_k'$, we have

$$\mathbb{P}\left(\hat{x}_i < \mathbb{E}\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i\right) \leq \mathbb{P}\left(\hat{x}_i' < \mathbb{E}\hat{x}_i' - \tau\hat{\sigma}_i' \text{ for all } i\right),$$

and hence,

$$C(\tau) \leq \frac{\mathbb{P}\left(\hat{x}_i' < \mathbb{E}\hat{x}_i' - \tau\hat{\sigma}_i' \text{ for all } i\right)}{\mathbb{P}(y < -\tau\sigma)}.$$

Finally, since the two economies have identical Domar weights, it is immediate that the distribution of aggregate output is also identical in the two economies, and in particular, $\mathbb{P}(y < -\tau\sigma) = \mathbb{P}(y' < -\tau\sigma')$. Using Lemma A.5 one more time then implies that $C(\tau) \leq C'(\tau)$. $\qquad\square$

Proof of Proposition 7

Recall from inequality (14) that

$$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sqrt{n}/\delta)}{\log \Phi(-\tau)}$$

for any given economy and any $\tau > 0$. Therefore, when microeconomic shocks have Pareto tails with Pareto index λ, for a given sequence of economies as $n \to \infty$ and for $\tau_n = \sqrt{n}$, we have

$$\limsup_{n\to\infty} R_n(\sqrt{n}) \leq 2\lambda \limsup_{n\to\infty} \frac{1}{n} \log\left(n/\delta_n\right).$$

The observation that $\delta_n \geq 1$ for any given economy of size n leads to

$$\limsup_{n\to\infty} R_n(\sqrt{n}) \leq 2\lambda \limsup_{n\to\infty} \frac{1}{n} \log\left(n\right).$$

It is then immediate that $\lim_{n\to\infty} R_n(\sqrt{n}) = 0$. $\qquad\square$

Proof of Proposition 8

We first state a theorem, the proof of which can be found in Nagaev (1979, Theorem 1.9).

Theorem A.2. *Let $\epsilon_1, \ldots, \epsilon_n$ be zero-mean, unit variance i.i.d. Pareto-tailed random variables with Pareto index $\lambda > 2$. Furthermore, suppose that $\mathbb{E}|\epsilon_i|^{2+\xi} < \infty$ for some $\xi > 0$. Then,*

$$\mathbb{P}(S_n \geq z_n) = \left(1 - \Phi(z_n/\sqrt{n})\right)(1 + o(1)) + n(1 - F(z_n))(1 + o(1)),$$

as $n \to \infty$ for $z_n \geq \sqrt{n}$, where $S_n = \epsilon_1 + \cdots + \epsilon_n$.

Next, we state and prove a lemma.

Lemma A.6. *Consider a sequence of simple economies and suppose that microeconomic shocks are Pareto-tailed.*

(a) If $\lim_{n\to\infty} \tau_n/\sqrt{\log n} < \infty$, then $\lim_{n\to\infty} R_n(\tau_n) > 0$.

(b) If $\lim_{n\to\infty} \tau_n/\sqrt{\log n} = \infty$, then $\lim_{n\to\infty} R_n(\tau_n) = 0$.

Proof. By Proposition 1, all sectoral Domar weights in a simple economy (that is, an economy with symmetric preferences and no input-output linkages) are identical and are given by $v_i = 1/n$. Consequently, $y_n = (1/n)\sum_{i=1}^{n} \epsilon_i$ and $\sigma_n = 1/\sqrt{n}$, which means that $\mathbb{P}(y_n < -\tau_n\sigma_n) = \mathbb{P}(S_n > \tau_n\sqrt{n})$. Therefore, for any sequence $\{\tau_n\}$ such that $\tau_n \geq 1$ and $\lim_{n\to\infty} \tau_n = \infty$, Theorem A.2 implies that

$$\mathbb{P}(y_n < -\tau_n\sigma_n) = \Phi(-\tau_n)(1 + o(1)) + nF(-\tau_n\sqrt{n})(1 + o(1)). \tag{21}$$

Now, to prove part (a), suppose that $\lim_{n\to\infty} \tau_n/\sqrt{\log n} < \infty$. Under this assumption,

$$\lim_{n\to\infty} \frac{\log \Phi(-\tau_n)}{\log[nF(-\tau_n\sqrt{n})]} = \lim_{n\to\infty} \frac{\tau_n^2/2}{(\lambda/2 - 1)\log n + \lambda \log \tau_n} < \infty,$$

implying that the second term on the right-hand side of (21) never dominates the first term as $n \to \infty$. As a result,

$$\lim_{n \to \infty} \frac{\mathbb{P}(y_n < -\tau_n \sigma_n)}{\Phi(-\tau_n)} < \infty,$$

implying that $\lim_{n \to \infty} R_n(\tau_n) > 0$.

Next, to prove part (b), suppose that $\lim_{n \to \infty} \tau_n / \sqrt{\log n} = \infty$. For such a sequence, we have

$$\lim_{n \to \infty} \frac{\log \Phi(-\tau_n)}{\log[nF(-\tau_n \sqrt{n})]} = \lim_{n \to \infty} \frac{\tau_n^2 / 2}{(\lambda/2 - 1) \log n + \lambda \log \tau_n} = \infty,$$

implying that the second term on the right-hand side of (21) dominates the first term as $n \to \infty$, that is,

$$\mathbb{P}(y_n < -\tau_n \sigma_n) = nF(-\tau_n \sqrt{n})(1 + o(1)).$$

Consequently,

$$\begin{aligned} \lim_{n \to \infty} R_n(\tau_n) &= \lim_{n \to \infty} \frac{\log[nF(-\tau_n \sqrt{n})]}{\log \Phi(-\tau_n)} \\ &= 2\lambda \lim_{n \to \infty} \frac{\log \tau_n}{\tau_n^2} + (\lambda - 2) \lim_{n \to \infty} \frac{\log n}{\tau_n^2}, \end{aligned}$$

where the second equality is due to the assumption that microeconomic shocks have Pareto tails with index λ. Given that both terms on the right-hand side of the above equality converges to zero, it is then immediate that $\lim_{n \to \infty} R_n(\tau_n) = 0$. $\qquad \square$

We now present the proof of Proposition 8. First, consider a sequence of simple economies that are subject to Pareto-tailed microeconomic shocks and denote the corresponding τ-tail ratios with R_n^{par}. Choose the sequence $\{\tau_n^*\}$ such that $\lim_{n \to \infty} \tau_n^* / \sqrt{\log n} = 1$. From part (a) of Lemma A.6, it is immediate that

$$\lim_{n \to \infty} R_n^{\text{par}}(\tau_n^*) > 0. \tag{22}$$

Next, consider a sequence of economies subject to exponential-tailed shocks, whose measure of sectoral dominance satisfy $\lim_{n \to \infty} \delta_n \sqrt{(\log n)/n} = \infty$. Denote the τ-tail ratio of the n-sector economy within such a sequence with R_n^{exp}. Inequality (14) implies that

$$\begin{aligned} \limsup_{n \to \infty} R_n^{\text{exp}}(\tau_n^*) &\le 2\gamma \limsup_{n \to \infty} \frac{\sqrt{n}}{\tau_n^* \delta_n} \tag{23} \\ &= 2\gamma \limsup_{n \to \infty} \frac{\sqrt{n}}{\delta_n \sqrt{\log n}}, \end{aligned}$$

thus leading to

$$\lim_{n \to \infty} R_n^{\text{exp}}(\tau_n^*) = 0. \tag{24}$$

Contrasting (24) with (22) completes the proof. $\qquad \square$

Proof of Corollary 3

Consider a sequence of economies with Pareto Domar weights of common exponent $\eta = 2$; that is, $v_i = ci^{-1/2}$ for all sectors i. For such an economy, it is easy to verify that

$$0 < \liminf_{n \to \infty} \delta_n \sqrt{(\log n)/n} \leq \limsup_{n \to \infty} \delta_n \sqrt{(\log n)/n} < \infty. \tag{25}$$

Suppose that microeconomic shocks have exponential tails and denote the tail ratio of the n-th economy in the sequence with $R_n^{\exp}$. On the other hand, consider a sequence of simple economies subject to Pareto-tailed shocks and denote the corresponding tail ratio with R_n^{Pareto}. We show that the two sequence of economies exhibit identical macroeconomic tail risks. To this end, it is sufficient to show that for any given sequence $\{\tau_n\}$, $\lim_{n \to \infty} R_n^{\exp}(\tau_n) = 0$ if and only if $\lim_{n \to \infty} R_n^{\mathrm{Pareto}}(\tau_n) = 0$.

First, suppose that $\{\tau_n\}$ is such that $\lim_{n \to \infty} R_n^{\mathrm{Pareto}}(\tau_n) = 0$. From part (a) of Lemma A.6 it is immediate that $\lim_{n \to \infty} \tau_n / \sqrt{\log n} = \infty$. On the other hand, recall from (23) that when microeconomic shocks have exponential tails,

$$\limsup_{n \to \infty} R_n^{\exp}(\tau_n) \leq 2\gamma \limsup_{n \to \infty} \frac{\sqrt{n}}{\tau_n \delta_n}.$$

Thus, by (25), it is immediate that $\lim_{n \to \infty} R_n^{\exp}(\tau_n) = 0$.

To prove the converse, suppose that $\{\tau_n\}$ is such that $\lim_{n \to \infty} R_n^{\exp}(\tau_n) = 0$. Inequality (18) implies that for such a sequence,

$$\lim_{n \to \infty} \frac{1}{4r^2 + r\tau_n \delta_n / \sqrt{n}} \leq \lim_{n \to \infty} R_n^{\exp}(\tau_n) = 0.$$

Therefore, $\lim_{n \to \infty} \tau_n \delta_n / \sqrt{n} = \infty$. Combining this with (25) thus implies that $\lim_{n \to \infty} \tau_n / \sqrt{\log n} = \infty$. Consequently, by part (b) of Lemma A.6, $\lim_{n \to \infty} R_n^{\mathrm{Pareto}}(\tau_n) = 0$, completing the proof. $\qquad\square$

B   Aggregation and Disaggregation

In this appendix, we illustrate the relationship between input-output matrices at different levels of disaggregation of an economy with the same input-output structure.

Suppose we observe an economy at two levels of disaggregations n and $m < n$. This means that each sector/industry in the latter representation is a disjoint collection of subindustries in the former. We use the notation $i_m \hookleftarrow i_n$ to denote that sector i_n is one of the subindustries at the level of disaggregation n that has been assigned to the more aggregated sector indexed i_m.

Given that both representations capture the same economy, it must be the case that

$$l_{i_m} = \sum_{i_n : i_m \hookleftarrow i_n} l_{i_n},$$

where l_{i_n} denotes the amount of labor hired by sector i_n at the level of disaggregation n. On the other hand, recall from (11) that

$$l_{i_n} = (1 - \mu)p_{i_n} x_{i_n}/w,$$

where $p_{i_n}x_{i_n}$ is the total sales of industry i_n and w represents the market wage. Therefore, for any sector i_m at the level of disaggregation m,

$$p_{i_m}x_{i_m} = \sum_{i_n:i_m \hookleftarrow i_n} p_{i_n}x_{i_n}.$$

Furthermore, by Proposition 1, sectoral sales are related to the corresponding Domar weights via $v_{i_n} = p_{i_n}x_{i_n}/\text{GDP}$. Consequently, Domar weights at the two levels of disaggregation are related to one another via

$$v_{i_m} = \sum_{i_n:i_m \hookleftarrow i_n} v_{i_n}.$$

That is, the Domar weight of an aggregated sector is equal to the sum of the Domar weights of the subindustries that belong to that sector. The above expression is the only restriction imposed on the Domar weights following disaggregation.

Next, we obtain the relationship between input-output matrices of the economy at the levels of disaggregation m and n. Given that both representations capture the same economy, the total dollar amount of intersectoral trade between two aggregated sectors should be the same regardless of the level of disaggregation. In particular,

$$p_{j_m}x_{i_mj_m} = \sum_{i_n:i_m \hookleftarrow i_n}\sum_{j_n:j_m \hookleftarrow j_n} p_{j_n}x_{i_nj_n},$$

where $p_{j_n}x_{i_nj_n}$ denotes the value of trade between sectors i_n and j_n at the level of disaggregation n. On the other hand, recall form (10) that

$$p_{j_n}x_{i_nj_n} = \mu a_{i_nj_n}p_{i_n}x_{i_n},$$

where $a_{i_nj_n}$ captures the intensity of the input-output linkages between sectors i_n and j_n at the level of disaggregation n. Consequently,

$$a_{i_mj_m}p_{i_m}x_{i_m} = \sum_{i_n:i_m \hookleftarrow i_n}\sum_{j_n:j_m \hookleftarrow j_n} a_{i_nj_n}p_{i_n}x_{i_n}.$$

Replacing for the sectoral sales in terms of the corresponding Domar weights then implies that

$$a_{i_mj_m} = \frac{\sum_{i_n:i_m \hookleftarrow i_n}\sum_{j_n:j_m \hookleftarrow j_n} v_{i_n}a_{i_nj_n}}{\sum_{i_n:i_m \hookleftarrow i_n} v_{i_n}},$$

providing the relationship between the intersectoral input-output matrices of the economy at the two disaggregation levels.

Finally, note also that throughout our analysis, we are imposing that microeconomic shocks have the same distribution at different levels of disaggregation. Therefore, as n changes, the distribution of aggregate output changes as well.

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