Appendix

A Additional Theoretical Results

A.1 Taxes cannot cause Pareto improvements

Although Example 1 and the simulations show that total match value of stable matchings may decrease when the tax rate falls, an arrangement that is stable under a tax rate $\hat{\tau}$ must raise the payoff of at least one agent, relative to an arrangement that is stable under a tax rate $\check{\tau} > \hat{\tau}$.

Proposition 2. *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, and that $[\check{\mu}; \check{t}]$ is stable under tax $\check{\tau}$, with $\check{\tau} > \hat{\tau}$. Then, $[\check{\mu}; \check{t}]$ (under tax $\check{\tau}$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$).*[32]

To see the intuition behind Proposition 2, we consider the case in which $\check{\tau} = 1$ and choose $\check{t} = 0$: If $[\check{\mu}; \check{t}]$ (under tax $\check{\tau} = 1$) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$), then every firm $f \in F$ (weakly) prefers $\check{\mu}(f)$ to $\hat{\mu}(f)$ with the transfer $\hat{t}_{f,\hat{\mu}(f)}$.[33] But then, because $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$,

$$\gamma_{f,\check{\mu}(f)} \overbrace{\geq}^{\text{Pareto}} \gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} \overbrace{\geq}^{\text{Stability}} \gamma_{f,\check{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)},$$

so every f must be offering a weakly positive transfer to $\check{\mu}(f)$ under $\hat{t}$ (that is, $\hat{t}_{f,\check{\mu}(f)} \geq 0$). An analogous argument shows that each worker $w \in W$ must be offering a weakly positive transfer to $\check{\mu}(w)$ under $\hat{t}$ (that is, $\xi_{\hat{\tau}}(\hat{t}_{\check{\mu}(w),w}) \leq 0$). Moreover, Pareto dominance implies that at least one firm or worker must be paying a *strictly* positive transfer. But then, that agent must pay a strictly positive transfer and receive a weakly positive transfer – impossible.

A.2 The Effect of Very Small Taxes

Unlike in non-matching models of taxation, in our setting there is always a nonzero tax that does not generate distortions. To find the minimum tax that generates a distoriton, let $\hat{\mu}$ be an efficient matching. Our results show that if $\check{\mu}$ is stable under $\check{\tau}$, then[34]

$$\check{\tau} \geq \frac{\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\check{\mu})}{\sum_{f \in F} \left(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)}\right)}. \tag{A.1}$$

[32]We say that an arrangement $[\check{\mu}; \check{t}]$ under tax $\check{\tau}$ *Pareto dominates* arrangement $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ if

$$\begin{aligned} \gamma_{f,\check{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} &\geq \gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} && \forall f \in F, \\ \alpha_{\check{\mu}(w),w} + \xi_{\check{\tau}}(\check{t}_{\check{\mu}(w),w}) &\geq \alpha_{\hat{\mu}(w),w} + \xi_{\hat{\tau}}(\hat{t}_{\hat{\mu}(w),w}) && \forall w \in W, \end{aligned}$$

with strict inequality for some $i \in F \cup W$.

[33]Note that under tax $\check{\tau} = 1$, an arrangement with transfers of 0 among match partners Pareto dominates any other arrangement with the same matching. Thus, the transfers between match partners under $[\check{\mu}; \check{t}]$ can be assumed to be 0. Then, the comparison between $[\check{\mu}; \check{t}]$ (under tax $\check{\tau} = 1$) and $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$) amounts to a comparison of agents' match values under $\check{\mu}$ and their total utilities under $[\hat{\mu}; \hat{t}]$.

[34]See Section B.6.

For any inefficient matching $\check{\mu}$, there is a strictly positive minimum tax $\underline{\tau}(\check{\mu})$ at which $\check{\mu}$ could possibly be stable. Since there are finitely many possible matchings, we can just take the minimum of this threshold across inefficient matchings,

$$\underline{\tau}^* = \min_{\{\mu:\mathfrak{M}(\mu)<\mathfrak{M}(\hat{\mu})\}} [\underline{\tau}(\mu)] .$$

For $\tau < \underline{\tau}^*$ only an efficient matching can be stable.[35]

A.3 Multiple matches stable at a given tax rate

Hatfield et al. (2013) shows that in markets with perfect transfers, if two matches are stable, they can be supported by the same transfer vector and all agents are indifferent between the two allocations. These results carryover to the case of taxation as summarized in the following proposition

Proposition 3. *In a wage market if two matches $\check{\mu}$ and $\hat{\mu}$ are both stable at tax rate τ, then*

1. *Any transfer vector that supports one match, also supports the other.*
2. *For any supporting transfer vector, all agents are indifferent between the two allocations.*
3. *The difference in total match value of the two matches equals the difference in revenue for the two stable matches, for a given supporting transfer vector.*

Proof. For any stable outcomes $\hat{\mu}$ and $\check{\mu}$ and a transfer vector t that supports one of the stable matches, we can renormalize worker match utilities $\{\tilde{\alpha}_{f,w}\} = \frac{1}{1-\tau}\{\alpha_{f,w}\}$ and be in a market where the results of Hatfield et al. (2013) apply:

$$\begin{aligned} \gamma_{f,\check{\mu}(f)} - t_{f,\check{\mu}(f)} &= \gamma_{f,\hat{\mu}(f)} - t_{f,\hat{\mu}(f)} && \forall f, \\ \frac{1}{1-\tau}\alpha_{\check{\mu}(w),w} + t_{w,\check{\mu}(w)} &= \frac{1}{1-\tau}\alpha_{\hat{\mu}(w),w} + t_{f,\hat{\mu}(w)} && \forall w. \end{aligned}$$

These equalities hold when we switch back to the original utilities so any supporting transfer vector must support both matches and agents are all indifferent between the two allocations.

Multiplying the second line by $(1 - \tau)$ and summing across agents gives

$$\mathfrak{M}(\check{\mu}) - \mathfrak{M}(\hat{\mu}) = \tau \sum_{f\in F} t_{f,\check{\mu}(f)} - \tau \sum_{f\in F} t_{f,\hat{\mu}(f)}.$$

□

Unfortunately this result about changes in revenue reflecting changes in value is very limited. As the tax rate changes, transfers will change even when the underlying match does not change (so there is no change in total match value). Also, even at the tax rate

[35] One caveat is that if there are multiple efficient matchings (all of which are stable when $\tau = 0$), some of them may *not* be stable in the limit as $\tau \to 0$.

where multiple matches are stable, there may be multiple supporting transfer vectors and the revenue between $[\hat{\mu}, \hat{t}]$ and $[\check{\mu}, \check{t}]$ does not tell us anything about the difference in total match value between $\hat{\mu}$ and $\check{\mu}$.

Similarly, results of Kelso and Crawford (1982) and Hatfield and Milgrom (2005) imply that for any fixed τ, if there are multiple stable arrangements, then workers' and firms' interests are opposed. If all firms prefer $[\mu; t]$ to $[\hat{\mu}; \hat{t}]$, then all workers prefer $[\hat{\mu}; \hat{t}]$ to $[\mu; t]$. Moreover, there exists a firm-optimal (worker-pessimal) stable arrangement that the firms weakly prefer to all other stable arrangements and a worker-optimal (firm-pessimal) stable arrangement that all workers weakly prefer. More generally,

Proposition 4. *In a wage market with proportional taxation, if two distinct matchings $\check{\mu}$ and $\hat{\mu}$ are both stable under tax τ, then*

$$\sum_{w \in W} \left(\alpha_{\check{\mu}(w),w} - \alpha_{\hat{\mu}(w),w} \right) = (1-\tau) \sum_{f \in F} \left(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)} \right) \tag{A.2}$$

Thus, if the firms are not indifferent in aggregate between $\check{\mu}$ and $\hat{\mu}$, then the only tax rate τ under which both $\check{\mu}$ and $\hat{\mu}$ can be stable is

$$\tau = 1 + \frac{\sum_{w \in W} \left(\alpha_{\check{\mu}(w),w} - \alpha_{\hat{\mu}(w),w} \right)}{\sum_{f \in F} \left(\gamma_{f,\check{\mu}(f)} - \gamma_{f,\hat{\mu}(f)} \right)} \tag{A.3}$$

For τ as defined in Equation (A.3) to be less than 1, the fraction on the righthand side must be negative, so that that firms and workers in aggregate disagree about which matching they prefer. In wage markets with proportional taxation, where there is generically a unique stable matching, this opposition of interests carries over to the set of supporting transfer vectors.

In order for there to be multiple values of τ at which two given matchings are both stable, both firms and (following (A.2)) workers must be indifferent between those two matchings.

Corollary 1. *In a wage market with proportional taxation, if there is more than one tax under which two distinct matchings $\check{\mu}$ and $\hat{\mu}$ both are stable, then $\mathfrak{M}(\check{\mu}) = \mathfrak{M}(\hat{\mu})$.*

Corollary 1 implies that for generic match values, there is at most one value of τ at which two matchings $\check{\mu}$ and $\hat{\mu}$ are both stable; since there are finitely many matchings, there is a unique stable matching under almost every tax τ.

A.4 Nonlinear taxes

The wage schedule is in fact progressive, and the worker's income is a concave function of the nominal transfer. We do not have general theoretical results for non-linear taxes, but we can adapt the econometric framework we use to calculate the market equilibrium to the case of piecewise linear taxes.

Assume that the tax rates on workers are $\tau_x^{W,1} < \tau_x^{W,2} < ... < \tau_x^{W,K}$, where $\tau_x^{W,1}$ and $\tau_x^{W,K}$ are respectively the lowest and the top tax rate. Tax rate $\tau_x^{W,k}$ applies to the income above t_x^k, where $t_x^1 = 0$, so that if $\xi_{xy}(t_x^k)$ is the post-tax income of a worker of type x earning t_x^k

and working for a firm of type y, then

$$\xi_{xy}(t_x^{k+1}) = \xi_{xy}(t_x^k) + \left(1-\tau_y^W\right)\left(1-\tau_x^{W,k}\right)\left(t_x^{k+1}-t_x^k\right).$$

More generally, the post-tax income of a worker of type x working for a firm of type y and earning t_x^k is

$$\xi_{xy}(t) = \min_{k\in\{1,\dots,K\}} \left\{\xi_{xy}(t_x^k) + \left(1-\tau_y^W\right)\left(1-\tau_x^{W,k}\right)\left(t-t_x^k\right)\right\}.$$

Hence, the systematic utility of a worker x working for a firm of type y with pre-tax income t is

$$\mathrm{u}_{xy} = \min_{k\in\{1,\dots,K\}} \left\{\alpha_{x,y}^k + \left(1-\tau_y^W\right)\left(1-\tau_x^{W,k}\right)t\right\} \tag{A.4}$$

where $\alpha_{xy}^1 = \alpha_{xy}$, and $\alpha_{x,y}^{k+1} = \alpha_{x,y}^k + \left(1-\tau_y^W\right)\left(\tau_x^{W,k+1}-\tau_x^{W,k}\right)t_x^{k+1}$. On the firm's side, the systematic utility is still

$$\mathrm{v}_{yx} = \gamma_{y,x} - \left(1+\tau_y^F\right)t. \tag{A.5}$$

Substituting out t from equations (A.4) and (A.5), one gets

$$\mathrm{u}_{xy} = \min_{k\in\{1,\dots,K\}} \left\{\alpha_{x,y}^k + \left(1-\tau_y^W\right)\left(1-\tau_x^{W,k}\right)\left(\frac{\gamma_{y,x}-v_{yx}}{1+\tau_y^F}\right)\right\}$$

which can be rewritten as

$$\min_{k\in\{1,\dots,K\}} \left\{\alpha_{x,y}^k - \mathrm{u}_{xy} + \left(1-\tau_y^W\right)\left(1-\tau_x^{W,k}\right)\left(\frac{\gamma_{y,x}-\mathrm{v}_{yx}}{1+\tau_y^F}\right)\right\} = 0.$$

So, letting $\lambda_x^{W,k} = \frac{1}{1-\tau_x^{W,k}}$ (and $\lambda_y^F = \frac{1-\tau_y^W}{1+\tau_y^F}$, as before), we have

$$\min_{k\in\{1,\dots,K\}} \left\{\alpha_{x,y}^k - \mathrm{u}_{xy} + \frac{\lambda_y^F}{\lambda_x^{W,k}}\left(\gamma_{y,x}-\mathrm{v}_{yx}\right)\right\} = 0,$$

which is equivalent to

$$\min_{k\in\{1,\dots,K\}} \left\{\frac{\lambda_x^{W,k}(\alpha_{x,y}^k - \mathrm{u}_{xy}) + \lambda_y^F\left(\gamma_{y,x}-\mathrm{v}_{yx}\right)}{\sigma^W\lambda_x^{W,k} + \sigma^F\lambda_y^F}\right\} = 0. \tag{A.6}$$

By the log-odds formula, we have

$$\sigma^W \ln\frac{\mu_{xy}}{\mu_{x0}} = \mathrm{u}_{xy} \qquad \text{and} \qquad \sigma^F \ln\frac{\mu_{xy}}{\mu_{0y}} = \mathrm{v}_{xy}$$

where μ_{x0} and μ_{0y} adjust so that each agent's match probabilities sum to 1. After plugging in into expression (A.6), we get

$$\min_{k\in\{1,\dots,K\}}\left\{\frac{\lambda_x^{W,k}(\alpha_{x,y}^k+\sigma^W\ln\frac{\mu_{x0}}{\mu_{xy}})+\lambda_y^F\left(\gamma_{y,x}+\sigma^F\ln\frac{\mu_{0y}}{\mu_{xy}}\right)}{\sigma^W\lambda_x^{W,k}+\sigma^F\lambda_y^F}\right\}=0.$$

This implies that

$$\ln\mu_{xy}=\min_{k\in\{1,\dots,K\}}\left\{\frac{\lambda_x^{W,k}(\alpha_{x,y}^k+\sigma^W\ln\mu_{x0})+\lambda_y^F\left(\gamma_{y,x}+\sigma^F\ln\mu_{0y}\right)}{\sigma^W\lambda_x^{W,k}+\sigma^F\lambda_y^F}\right\},$$

which yields

$$\mu_{xy}=\min_{k\in\{1,\dots,K\}}M_{xy}^k\left(\mu_{x0},\mu_{0y}\right),$$

where

$$M_{xy}^k\left(\mu_{x0},\mu_{0y}\right)=\mu_{x0}^{\frac{\sigma^W\lambda_x^{W,k}}{\sigma^W\lambda_x^{W,k}+\sigma^F\lambda_y^F}}\mu_{0y}^{\frac{\sigma^F\lambda_y^F}{\sigma^W\lambda_x^{W,k}+\sigma^F\lambda_y^F}}e^{\frac{\lambda_x^{W,k}\alpha_{x,y}^k+\lambda_y^F\gamma_{y,x}}{\sigma^W\lambda_x^{W,k}+\sigma^F\lambda_y^F}}.$$

We use this to solve for the equilibrium μ_{x0},μ_{0y} such that the probabilities for each x and each y sum to 1.

B Proofs for Section 2 and Appendix A

B.1 Existence of Stable Arrangements & Equivalence with the Core and Competitive Equilibria

In this section, we use results from the literature on matching with contracts to show the existence of stable arrangements in our framework. For a given transfer vector t, the *demand of firm $f\in F$*, denoted $\mathcal{D}^f(t)$, is

$$\mathcal{D}^f(t)\equiv\arg\max_{D\subseteq W}\left\{\gamma_{f,D}-t_{f,D}\right\}.$$

Definition 3 (Kelso and Crawford (1982))**.** *The preferences of firm $f\in F$ are* substitutable *if for any transfer vectors t and $\check{t}$ with $\check{t}\geq t$, there exists, for each $D\in\mathcal{D}^f(t)$, some $\check{D}\in\mathcal{D}^f(\check{t})$ such that*

$$\check{D}\supseteq\{w\in D:t_{f,w}=\check{t}_{f,w}\}.$$

That is, the preferences of $f\in F$ are substitutable if an increase in the "prices" of some workers cannot decrease demand for the workers whose prices remain unchanged.[36]

Theorem 2 of Kelso and Crawford (1982) shows that under the assumption that all firms' preferences are substitutable, there is an arrangement $[\mu;t]$ that is *strict core*, in the sense

[36] Theorem A.1 of Hatfield et al. (2013) shows that in our setting the Kelso and Crawford (1982) substitutability condition is equivalent to the choice-based substitutability condition of Hatfield and Milgrom (2005), that we describe in the main text: *the availability of new workers cannot make a firm want to hire a worker it would otherwise reject.*

that:[37]

- Each agent (weakly) prefers his assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u_i([\mu; t]) \geq 0 \qquad\qquad \forall i \in F \cup W.$$

- There does not exist a firm $f \in F$, a set of workers $D \subseteq W$, and a transfer vector $\check{t}$ such that

$$\begin{aligned} \gamma_{f,D} - \check{t}_{f,D} &\geq \gamma_{f,\mu(f)} - t_{f,\mu(f)}, && \text{and} \\ \alpha_{f,w} + \xi(\check{t}_{f,w}) &\geq \alpha_{\mu(w),w} + \xi(t_{\mu(w),w}) && \forall w \in D, \end{aligned}$$

  with strict inequality for at least one $i \in (\{f\} \cup D)$.

The Kelso and Crawford (1982) (p. 1487) construction of competitive equilibria from strict core allocations then implies that there is some transfer vector $\hat{t}$, having $\hat{t}_{\mu(w),w} = t_{\mu(w),w}$ (for each $w \in W$), such that $[\mu; \hat{t}]$ is stable in our sense.

B.2 Total transfers

We use the following Lemma in proving many of the results.

Lemma 1. *For a given matching μ and transfer vector t, the sum of transfers firms pay to their match partners equals the sum of the transfers paid by workers' match partners,*

$$\sum_{f \in F} t_{f,\mu(f)} = \sum_{f \in F} \sum_{w \in \mu(f)} t_{f,w} = \sum_{w \in W} t_{\mu(w),w} \leq \sum_{w \in W} \xi(t_{\mu(w),w}).$$

Proof. Letting $\mathfrak{B}$ be the set of firms who are matched at μ and $\mathscr{B}$ be the set of workers who are matched at μ, we have

$$\begin{aligned} \mu(f) &\subseteq \mathscr{B} && \forall f \in \mathfrak{B}, \\ \mu(w) &\in \mathfrak{B} && \forall w \in \mathscr{B}. \end{aligned}$$

[37] Strictly speaking, Kelso and Crawford (1982) have one technical assumption not present in our framework: they assume that $\gamma_{f,w} + \alpha_{f,w} \geq 0$, in order to ensure that all workers are matched. However, examining the Kelso and Crawford (1982) arguments reveals that this extra assumption is not necessary to ensure that a strict core arrangement exists – the Kelso and Crawford (1982) salary adjustment processes can be started at some arbitrarily low (negative) salary offer and all of the steps and results of Kelso and Crawford (1982) remain valid, with the caveat that some workers may be unmatched at core outcomes.

We then use the fact that $t_{f,f} = t_{w,w} = 0$, to show that

$$
\begin{aligned}
\sum_{f \in F} t_{f,\mu(f)} &= \sum_{f \in \mathfrak{B}} t_{f,\mu(f)} + \sum_{f \in F \setminus \mathfrak{B}} t_{f,\mu(f)} = \sum_{f \in \mathfrak{B}} t_{f,\mu(f)}, \\
&= \sum_{f \in \mathfrak{B}} \sum_{w \in \mu(f)} t_{f,w} = \sum_{w \in \mathscr{B}} t_{\mu(w),w}, \\
&= \sum_{w \in \mathscr{B}} t_{\mu(w),w} + \sum_{w \in W \setminus \mathscr{B}} t_{\mu(w),w} = \sum_{w \in W} t_{\mu(w),w}.
\end{aligned}
$$

□

B.3 Proof of Proposition 2

First, we show that the arrangements stable under full taxation ($\check{\tau} = 1$) cannot Pareto dominate those stable under tax $\hat{\tau} < 1$.

Claim 1. *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau} < 1$, and that $[\check{\mu}; \check{t}]$ is stable under tax $\check{\tau} = 1$. Then, $[\check{\mu}; \check{t}]$ (under tax $\check{\tau} = 1$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau} < 1$).*

Proof. As no transfers get through under full taxation, an arrangement stable under full taxation is most likely to Pareto dominate some other arrangement when all transfers between match partners are 0. Thus, we assume that $\check{t}_{\check{\mu}(w),w} = 0$ for each $w \in W$. If $[\check{\mu}; \check{t}]$ (under full taxation) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$), then

$$\gamma_{f,\check{\mu}(f)} = \gamma_{f,\check{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \geq \gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)}, \tag{B.1}$$

$$\alpha_{\check{\mu}(w),w} = \alpha_{\check{\mu}(w),w} + \xi_{\check{\tau}}(\check{t}_{\check{\mu}(w),w}) \geq \alpha_{\hat{\mu}(w),w} + \xi_{\hat{\tau}}(\hat{t}_{\hat{\mu}(w),w}), \tag{B.2}$$

with strict inequality for some f or w. However, stability of $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ implies that

$$\gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} \geq \gamma_{f,\check{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)}, \tag{B.3}$$

$$\alpha_{\hat{\mu}(w),w} + \xi_{\hat{\tau}}(\hat{t}_{\hat{\mu}(w),w}) \geq \alpha_{\check{\mu}(w),w} + \xi_{\hat{\tau}}(\hat{t}_{\check{\mu}(w),w}). \tag{B.4}$$

Combining (B.1) and (B.3) gives

$$0 \geq -\hat{t}_{f,\check{\mu}(f)}, \tag{B.5}$$

for each $f \in F$, while combining (B.2) and (B.4) gives

$$0 \geq \xi_{\hat{\tau}}(\hat{t}_{\check{\mu}(w),w}), \tag{B.6}$$

for each $w \in W$. Strict inequality must hold in (B.5) or (B.6) for some f or w.

In the first of these cases, we have

$$\hat{t}_{f',\check{\mu}(f')} > 0$$

for some $f' \in F$; hence, there exists at least one $w \in \hat{\mu}(f')$ for whom

$$\hat{t}_{\check{\mu}(w),w} > 0. \tag{B.7}$$

But (B.7) contradicts (B.6).

In the second case, we have

$$0 > \xi_{\check{\tau}}(\hat{t}_{\check{\mu}(w'),w'}), \tag{B.8}$$

for some $w' \in W$. If we take $f = \check{\mu}(w')$, then (B.8) and (B.6) together imply that

$$0 > \sum_{w \in \check{\mu}(f)} \hat{t}_{\check{\mu}(w),w} = \hat{t}_{f,\check{\mu}(f)},$$

contradicting (B.5). □

For $\check{\tau} < 1$, $\xi_{\check{\tau}}(\cdot)$ is strictly increasing and the conclusion of the proposition follows from the following more general result.

Proposition 2'. *Suppose that $\check{\xi}(\cdot)$ is strictly increasing, that $[\hat{\mu}; \hat{t}]$ is stable under $\hat{\xi}(\cdot)$, and that $[\check{\mu}; \check{t}]$ is stable under $\check{\xi}(\cdot)$, with $\check{\xi}(\cdot) \leq \hat{\xi}(\cdot)$. Then, $[\check{\mu}; \check{t}]$ (under $\check{\xi}(\cdot)$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$).*[38]

Proof. Pareto dominance of $[\check{\mu}; \check{t}]$ (under $\check{\xi}(\cdot)$) over $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$) would imply that

$$\gamma_{f,\check{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \geq \gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)}, \tag{B.9}$$

$$\alpha_{\check{\mu}(w),w} + \check{\xi}(\check{t}_{\check{\mu}(w),w}) \geq \alpha_{\hat{\mu}(w),w} + \hat{\xi}(\hat{t}_{\hat{\mu}(w),w}), \tag{B.10}$$

with strict inequality for some f or w. However, stability of $[\hat{\mu}; \hat{t}]$ under $\hat{\xi}(\cdot)$ implies that

$$\gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} \geq \gamma_{f,\check{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)}, \tag{B.11}$$

$$\alpha_{\hat{\mu}(w),w} + \hat{\xi}(\hat{t}_{\hat{\mu}(w),w}) \geq \alpha_{\check{\mu}(w),w} + \hat{\xi}(\hat{t}_{\check{\mu}(w),w}) \geq \alpha_{\check{\mu}(w),w} + \check{\xi}(\hat{t}_{\check{\mu}(w),w}), \tag{B.12}$$

where the second inequality in (B.12) follows from the fact that $\hat{\xi}(\cdot) \geq \check{\xi}(\cdot)$.

Combining (B.9) and (B.11) gives

$$\hat{t}_{f,\check{\mu}(f)} \geq \check{t}_{f,\check{\mu}(f)}, \tag{B.13}$$

[38] We say that an arrangement $[\check{\mu}; \check{t}]$ (under $\check{\xi}(\cdot)$) *Pareto dominates* arrangement $[\hat{\mu}; \hat{t}]$ under (under $\hat{\xi}(\cdot)$) if

$$\gamma_{f,\check{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \geq \gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} \qquad \forall f \in F,$$

$$\alpha_{\check{\mu}(w),w} + \check{\xi}(\check{t}_{\check{\mu}(w),w}) \geq \alpha_{\hat{\mu}(w),w} + \hat{\xi}(\hat{t}_{\hat{\mu}(w),w}) \qquad \forall w \in W,$$

with strict inequality for some $i \in F \cup W$.

for each $f \in F$, while combining (B.10) and (B.12) gives

$$\begin{aligned} \check{\xi}(\check{t}_{\check{\mu}(w),w}) &\geq \check{\xi}(\hat{t}_{\check{\mu}(w),w}) \\ \check{t}_{\check{\mu}(w),w} &\geq \hat{t}_{\check{\mu}(w),w} \end{aligned} \tag{B.14}$$

for each $w \in W$, where the second line of (B.14) follows from the fact that $\check{\xi}(\cdot)$ is strictly increasing. Strict inequality must hold in (B.13) or (B.14) for some f or w.

In the first of these cases, we have

$$\hat{t}_{f',\check{\mu}(f')} > \check{t}_{f',\check{\mu}(f')}$$

for some $f' \in F$; hence, there exists at least one $w \in \hat{\mu}(f')$ for whom

$$\hat{t}_{\check{\mu}(w),w} > \check{t}_{\check{\mu}(w),w}. \tag{B.15}$$

But (B.15) contradicts (B.14).

In the second case, we have

$$\check{t}_{\check{\mu}(w'),w'} > \hat{t}_{\check{\mu}(w'),w'} \tag{B.16}$$

for some $w' \in W$. If we take $f = \check{\mu}(w')$, then (B.16) and (B.14) together imply that

$$\sum_{w \in \check{\mu}(f)} \check{t}_{\check{\mu}(w),w} > \sum_{w \in \check{\mu}(f)} \hat{t}_{\check{\mu}(w),w};$$

hence, we find that

$$\check{t}_{f,\check{\mu}(f)} > \hat{t}_{f,\check{\mu}(f)},$$

contradicting (B.13). □

B.4 Proof of Theorem 1

If $\hat{\mu} = \check{\mu}$, then the theorem is trivially true. Thus, we consider a wage market in which $[\check{\mu}; \check{t}]$ is stable under tax $\check{\tau}$, $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, $\check{\tau} > \hat{\tau}$, and $\check{\mu} \neq \hat{\mu}$.

The stability conditions for the firms imply that

$$\gamma_{f,\check{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \geq \gamma_{f,\hat{\mu}(f)} - \check{t}_{f,\hat{\mu}(f)}, \tag{B.17}$$

$$\gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} \geq \gamma_{f,\check{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)}; \tag{B.18}$$

these inequalities together imply that

$$\sum_{f \in F} \left(\check{t}_{f,\hat{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \right) \geq \sum_{f \in F} \left(\hat{t}_{f,\hat{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)} \right) \tag{B.19}$$

As the market is a wage market, we have

$$\xi_\tau(\check{t}_{\check{\mu}(w),w}) = (1-\check{\tau})\check{t}_{\check{\mu}(w),w} \quad \text{and} \quad \xi_\tau(\hat{t}_{\check{\mu}(w),w}) = (1-\hat{\tau})\hat{t}_{\check{\mu}(w),w};$$

hence, the stability conditions for the workers imply that

$$\alpha_{\check{\mu}(w),w} + (1-\check{\tau})\check{t}_{\check{\mu}(w),w} \geq \alpha_{\hat{\mu}(w),w} + (1-\check{\tau})\check{t}_{\hat{\mu}(w),w}, \tag{B.20}$$

$$\alpha_{\hat{\mu}(w),w} + (1-\hat{\tau})\hat{t}_{\hat{\mu}(w),w} \geq \alpha_{\check{\mu}(w),w} + (1-\hat{\tau})\hat{t}_{\check{\mu}(w),w}. \tag{B.21}$$

Summing these inequalities and applying Lemma 1, we obtain

$$(1-\hat{\tau})\sum_{f\in F}\left(\hat{t}_{f,\hat{\mu}(w)} - \hat{t}_{f,\check{\mu}(f)}\right) \geq (1-\check{\tau})\sum_{f\in F}\left(\check{t}_{f,\hat{\mu}(f)} - \check{t}_{f,\check{\mu}(f)}\right) \tag{B.22}$$

Combining (B.19) and (B.22), we find that

$$(1-\hat{\tau})\sum_{f\in F}\left(\hat{t}_{f,\hat{\mu}(w)} - \hat{t}_{f,\check{\mu}(f)}\right) \geq (1-\check{\tau})\sum_{f\in F}\left(\hat{t}_{f,\hat{\mu}(w)} - \hat{t}_{f,\check{\mu}(f)}\right) \tag{B.23}$$

Since $\hat{\tau} < \check{\tau}$, (B.23) implies that

$$\sum_{f\in F}\left(\hat{t}_{f,\hat{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)}\right) \geq 0. \tag{B.24}$$

Next, using (B.18) and (B.21), we find that

$$\begin{aligned}
\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\check{\mu}) &= \sum_{f\in F}(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)}) + \sum_{w\in W}(\alpha_{\hat{\mu}(w),w} - \alpha_{\check{\mu}(w),w}) \\
&\geq \sum_{f\in F}\left(\hat{t}_{f,\hat{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)}\right) - (1-\hat{\tau})\sum_{w\in W}\left(\hat{t}_{\hat{\mu}(w),w} - \hat{t}_{\check{\mu}(w),w}\right) \\
&= \hat{\tau}\sum_{f\in F}\left(\hat{t}_{f,\hat{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)}\right) \geq 0,
\end{aligned}$$

where the final inequality follows from (B.24).

B.5 Proof of Proposition 1

Assume a matching $\check{\mu}$ is stable under tax $\check{\tau}$. In a wage market, if we re-normalize the workers utilities by dividing by $(1-\check{\tau})$, then a match that is stable with the renormalized utilities and no taxation is also stable with the original utilities and tax $\check{\tau}$

$$\begin{aligned}
&\alpha_{\check{\mu}(w),w} + (1-\check{\tau})\check{t}_{\check{\mu}(w),w} \geq \alpha_{f,w} + (1-\check{\tau})\check{t}_{f,w}, \\
\Longleftrightarrow\ &\frac{1}{1-\check{\tau}}\alpha_{\check{\mu}(w),w} + \check{t}_{\hat{\mu}(w),w} \geq \frac{1}{1-\check{\tau}}\alpha_{f,w} + \check{t}_{f,w}.
\end{aligned} \tag{B.25}$$

Combining (B.25) with the standard firm stability conditions,

$$\gamma_{f,\hat{\mu}(f)} - \hat{t}_{f,\hat{\mu}(f)} \geq \gamma_{f,D} - \hat{t}_{f,D},$$

gives a matching market with quasilinear utility. It is known (e.g., Kelso and Crawford (1982); Hatfield et al. (2013)) that in such markets, only an efficient matching can stable. So $\check{\mu}$ must maximize the total of the re-normalized match values,

$$\check{\mu} \in \arg\max_{\{\mu\}} \left[\sum_{f \in F} \gamma_{f,\mu(f)} + \sum_{w \in W} \left(\frac{1}{(1-\check{\tau})} \alpha_{\mu(w),w} \right) \right]$$

For the second and third results, define a function

$$W(\lambda^{\alpha}, \lambda^{\gamma}) = \max_{\{\mu\}} \sum_{f \in F} \left(\lambda^{\alpha} \alpha_{f,\mu(f)} + \lambda^{\gamma} \gamma_{f,\mu(f)} \right)$$

which is convex in both λ^{α} and λ^{γ} because it is the maximixation of a linear function. By the envelope theorem, the derivatives of W are

$$\frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\gamma}} = \sum_{f \in F} \gamma_{f,\mu(f)},$$
$$\frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\alpha}} = \sum_{f \in F} \alpha_{f,\mu(f)} = \sum_{w \in W} \alpha_{w,\mu(w)},$$

which are the firms' and workers' match values respectively.

It is sufficient to show that, whenever they exist, (a) $\partial W(\lambda^{\alpha}, \lambda^{\gamma}) / \partial \lambda^{\gamma}$ is nondecreasing in λ^{γ}, and (b) $\partial W(\lambda^{\alpha}, \lambda^{\gamma}) / \partial \lambda^{\alpha}$ is nonincreasing in λ^{γ}. The first point follows directly from the convexity of W.

To see that the cross-derivative is non-positive, note that W is positive homogenous of degree one, so, whenever the derivatives exist, we get by Euler's homogenous function theorem that

$$W(\lambda^{\alpha}, \lambda^{\gamma}) = \lambda^{\alpha} \frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\alpha}} + \lambda^{\gamma} \frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\gamma}}.$$

Hence

$$\begin{aligned} \frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\alpha}} &= \frac{1}{\lambda^{\alpha}} \left(W(\lambda^{\alpha}, \lambda^{\gamma}) - \lambda^{\gamma} \frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\gamma}} \right) \\ &= W(1, \lambda^{\gamma}/\lambda^{\alpha}) - \frac{\lambda^{\gamma}}{\lambda^{\alpha}} \frac{\partial W(1, \lambda^{\gamma}/\lambda^{\alpha})}{\partial \lambda^{\gamma}}, \end{aligned}$$

which means

$$\frac{\partial}{\partial \lambda^{\gamma}} \left(\frac{\partial W(\lambda^{\alpha}, \lambda^{\gamma})}{\partial \lambda^{\alpha}} \right) = \frac{1}{\lambda^{\alpha}} \frac{\partial W(1, \lambda^{\gamma}/\lambda^{\alpha})}{\partial \lambda^{\gamma}} - \frac{1}{\lambda^{\alpha}} \frac{\partial W(1, \lambda^{\gamma}/\lambda^{\alpha})}{\partial \lambda^{\gamma}} - \frac{\lambda^{\gamma}}{\lambda^{\alpha^2}} \frac{\partial^2 W(1, \lambda^{\gamma}/\lambda^{\alpha})}{\partial (\lambda^{\gamma})^2} < 0,$$

again using the convexity of W. Taking $\lambda^{\alpha} = 1$ and $\lambda^{\gamma} = 1 - \check{\tau}$ gives the proposition.

B.6 Derivation of Equation (A.1)

Summing (B.20) across women and (B.17) across men, we find that

$$\sum_{w\in W}\left(\alpha_{\check\mu(w),w}-\alpha_{\hat\mu(w),w}\right)\geq(1-\check\tau)\sum_{w\in W}\left(\check t_{\hat\mu(w),w}-\check t_{\check\mu(w),w}\right) \tag{B.26}$$

$$\sum_{f\in F}\left(\check t_{\hat\mu(w),w}-\check t_{\check\mu(w),w}\right)\geq\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right). \tag{B.27}$$

Since Proposition 1 tells us that

$$\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right)\geq 0,$$

we can combine (B.26) and (B.27) to get

$$\frac{\sum_{w\in W}\left(\alpha_{\check\mu(w),w}-\alpha_{\hat\mu(w),w}\right)}{\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right)}\geq(1-\check\tau),$$

so that we find

$$\begin{aligned}\check\tau&\geq\frac{\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right)}{\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right)}+\frac{\sum_{w\in W}\left(\alpha_{\hat\mu(w),w}-\alpha_{\check\mu(w),w}\right)}{\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right)}\\&=\frac{\mathfrak{M}(\hat\mu)-\mathfrak{M}(\check\mu)}{\sum_{f\in F}\left(\gamma_{f,\hat\mu(f)}-\gamma_{f,\check\mu(f)}\right)}.\end{aligned} \tag{B.28}$$

B.7 Proofs of Proposition 4 and Corollary 1

Suppose that in a wage market, both $[\check\mu;\check t]$ and $[\hat\mu;\hat t]$ are stable under tax τ. The stability conditions for the firms imply that

$$\gamma_{f,\check\mu(f)}-\check t_{f,\check\mu(f)}\geq\gamma_{f,\hat\mu(f)}-\check t_{f,\hat\mu(f)}, \tag{B.29}$$

$$\gamma_{f,\check\mu(f)}-\hat t_{f,\check\mu(f)}\leq\gamma_{f,\hat\mu(f)}-\hat t_{f,\hat\mu(f)}, \tag{B.30}$$

so that

$$\check t_{f,\hat\mu(f)}-\check t_{f,\check\mu(f)}\geq\hat t_{f,\hat\mu(f)}-\hat t_{f,\check\mu(f)}. \tag{B.31}$$

Meanwhile, the stability conditions for the workers imply that

$$\alpha_{\check\mu(w),w}+(1-\tau)\check t_{\check\mu(w),w}\geq\alpha_{\hat\mu(w),w}+(1-\tau)\check t_{\hat\mu(w),w}, \tag{B.32}$$

$$\alpha_{\check\mu(w),w}+(1-\tau)\hat t_{\check\mu(w),w}\leq\alpha_{\hat\mu(w),f}+(1-\tau)\hat t_{\hat\mu(w),w}, \tag{B.33}$$

so that

$$(1-\tau)(\check t_{\hat\mu(w),w}-\check t_{\check\mu(w),w})\leq(1-\tau)(\hat t_{\hat\mu(f),w}-\hat t_{\check\mu(f),w}). \tag{B.34}$$

Summing (B.31) and (B.34) across agents and using Lemma 1, we find that

$$\sum_{f \in F} \left(\check{t}_{f,\hat{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \right) = \sum_{f \in F} \left(\hat{t}_{f,\hat{\mu}(f)} - \hat{t}_{f,\check{\mu}(f)} \right). \tag{B.35}$$

For this equality to hold, we must have equality in (B.31) for each $f \in F$, implying equality in (B.29) and (B.30), for each $f \in F$. Similarly, (B.35) requires that (B.34) hold with equality for each $w \in W$, which implies equality in (B.32) and (B.33), for each $w \in W$. Combining these equalities, and summing across workers $w \in W$, shows that

$$\begin{aligned} \sum_{w \in W} \left(\alpha_{\check{\mu}(w),w} - \alpha_{\hat{\mu}(w),w} \right) &= (1-\tau) \sum_{f \in F} \left(\check{t}_{f,\hat{\mu}(f)} - \check{t}_{f,\check{\mu}(f)} \right) \\ &= (1-\tau) \sum_{f \in F} \left(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)} \right). \end{aligned} \tag{B.36}$$

If the firms are not indifferent in aggregate between $\check{\mu}$ and $\hat{\mu}$, so that

$$\sum_{f \in F} \left(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)} \right) \neq 0, \tag{B.37}$$

we have,

$$\tau = 1 + \frac{\sum_{w \in W} \left(\alpha_{\check{\mu}(w),w} - \alpha_{\hat{\mu}(w),w} \right)}{\sum_{f \in F} \left(\gamma_{f,\check{\mu}(f)} - \gamma_{f,\hat{\mu}(f)} \right)}, \tag{B.38}$$

thereby showing Proposition 4.

To see Corollary 1, observe that (B.38) pins down a unique tax rate in the case that (B.37) holds. Thus, if there are two tax rates under which matchings $\check{\mu}$ and $\hat{\mu}$ are both stable, then we must have

$$\sum_{f \in F} \left(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)} \right) = 0. \tag{B.39}$$

But then, we also have

$$\sum_{w \in W} \left(\alpha_{\hat{\mu}(w),w} - \alpha_{\check{\mu}(w),w} \right) = 0, \tag{B.40}$$

by (B.36). Together (B.39) and (B.40) imply $\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\check{\mu}) = 0$, as desired.

C Simulations of non-wage markets

We run 500 simulations of a one-to-one market with twenty agents on each side of the market and match values independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We vary the tax rate, τ, from 0 to .99 in increments of .01. For each tax rate, we find the manager-optimal stable arrangement and calculate the total match value – if there are multiple stable arrangements, the manager-optimal arrangement is the one preferred by all managers. Non-monotonicities in the total match value of stable matchings appear in over half of the markets (55%). There may be additional non-monotonicities that we do not observe because we cannot vary τ continuously. However, the non-monotonicities we fail to observe necessarily occur over very small ranges of τ, as we observe all non-monotonicities that persist over values of τ for a range of .01 or more.

Table 3: Summary of the non-monotonicities arising in simulated markets.

	Range of τ				
	$[0, .25)$	$[.25, .5)$	$[.5, .75)$	$[.75, 1)$	All τ
Fraction of markets with non-monotonicity	0.006	0.088	0.190	0.394	0.548
Avg size of non-monotonicity, as fraction of range	0.021	0.066	0.111	0.140	0.120
Fraction of deadweight loss from taxation due to non-monotonicity	0.076	0.070	0.051	0.027	0.037

Note: The table summarizes 500 simulations of one-to-one matching markets with 20 agents on each side of the market. All agents' match values are independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We vary the tax rate, τ, from 0 to .99 in increments of .01. For each tax rate, we find the manager-optimal stable arrangement and calculate the total match value. Row 1 presents the fraction of markets that have non-monotonicities in a given tax rate range. Row 2 presents the average size of non-monotonicities within each range, normalized as a fraction of the (within-market) gap between the highest and lowest total stable match values calculated for any tax rate. Row 3 presents the average fraction of taxation deadweight loss that is due to non-monotonicity, across all markets. The deadweight loss from non-monotonicity is computed for each tax rate τ as the difference between the highest total match value for a tax rate $\check{\tau} \geq \tau$ and the total match value under tax rate τ; it is divided by the total deadweight loss from taxation at tax rate τ, which is computed as the difference in total match value between the efficient matching and the matching stable under tax rate τ.

Most markets have relatively small losses from non-monotonicity, mostly occurring at high tax rates, but some have dramatic non-monotonicities. Table 3 summarizes the non-monotonocities arising in our simulations. Row 1 shows the fraction of markets that have non-monotonicities in a given tax rate range. While the majority of non-monotonicities occur at very high tax rates, 10% of our simulation markets have non-monotonicities at tax rates below 50%. Row 2 gives the (normalized) average size of the non-monotonicities in each tax rate range. Again, we see that non-monotonicities are most significant for high tax rates. Row 3 incorporates information on the persistence of non-monotonicities by computing the fraction of the deadweight loss from taxation that is due to a non-monotonicity. The fraction his is relatively high for lower tax rates because there is less total deadweight loss at those tax rates.

Overall, our simulations suggest non-monotonicities in the tax rate are not just artifacts of example selection. However, they also suggest that non-monotonicities are relatively rare at more realistic tax rates ($\tau \in [0, .5)$) and tend not to persist over large ranges of τ.[39]

[39] Increasing the sample size does not appear to decrease the frequency or importance of non-monotonicities.

D Lump Sum Taxation

While not typically phrased in the exact language of taxation, lump sum taxes are present throughout labor markets. They might take the form of costs for hiring (e.g., employee health care costs) or for entering employment (e.g., licensing requirements). In the marriage market context, lump sum taxes can take the form of marriage license fees or tax penalties for marriage.

D.1 Lump Sum Taxation of Transfers

We first consider a lump sum tax that is levied only on (nonzero) transfers between match partners.[40] Such a *lump sum tax on transfers*, ℓ, corresponds to the transfer function

$$\xi_{\ell}^{\mathsf{lump}}(t_{f,w}) \equiv \begin{cases} t_{f,w} - \ell & t_{f,w} \neq 0, \\ t_{f,w} & t_{f,w} = 0. \end{cases}$$

shown in Figure 10. Under this tax structure, the case $\ell = 0$ corresponds to the standard (Shapley and Shubik (1971)) model of matching with transfers and the case $\ell = \infty$ corresponds to (Gale and Shapley (1962)) matching without transfers.

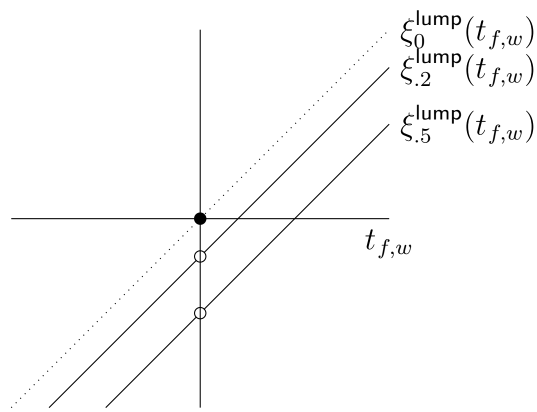

Figure 10: Transfer function $\xi_{\ell}^{\mathsf{lump}}(\cdot)$.

We say that an arrangement or matching is *stable under lump sum tax* ℓ if it is stable given transfer function $\xi_{\ell}^{\mathsf{lump}}(\cdot)$.

A lump sum tax on transfers has an extensive margin effect that makes being unmatched more attractive relative to matching with a transfer. In non-wage markets,[41] a lump sum tax on transfers can also encourage matchings in which transfers are unnecessary.[42] As our

[40] An alternative approach to lump sum taxation, which we discuss in the next section, imposes a flat fee on all matches.

[41] Since it is difficult to observe transfers in non-wage markets, such as marriage markets, it is somewhat hard to imagine taxing them. Nevertheless, lump-sum taxes on transfers could correspond to instituting a lump sum tax on gifts between spouses, and flat fees for matching could correspond to requiring marriage license fees.

[42] Consider the case of balanced one-to-one matching markets. In such markets, lump sum taxes on

next example illustrates, this second distortion can cause the total match value of stable matchings to be non-monotonic in the size of the lump sum tax.

Example 3 (Non-monotonicity)**.** *Consider a one-to-one market with two firms, $F = \{f_1, f_2\}$, two workers, $W = \{w_1, w_2\}$, and match values as pictured in Figure 11a. Worker w_1 likes f_1 – who has a strong preference for w_2 – but w_2 prefers f_2. When transfers are not allowed (or when there is a high lump sum tax on transfers, $\ell \geq 18$), the only stable matching is the matching μ_1 in which $\mu_1(f_1) = w_1$ and $\mu_1(f_2) = w_2$, as shown in Figure 11b. This matching yields total match value of $\mathfrak{M}(\mu_1) = 22$.*

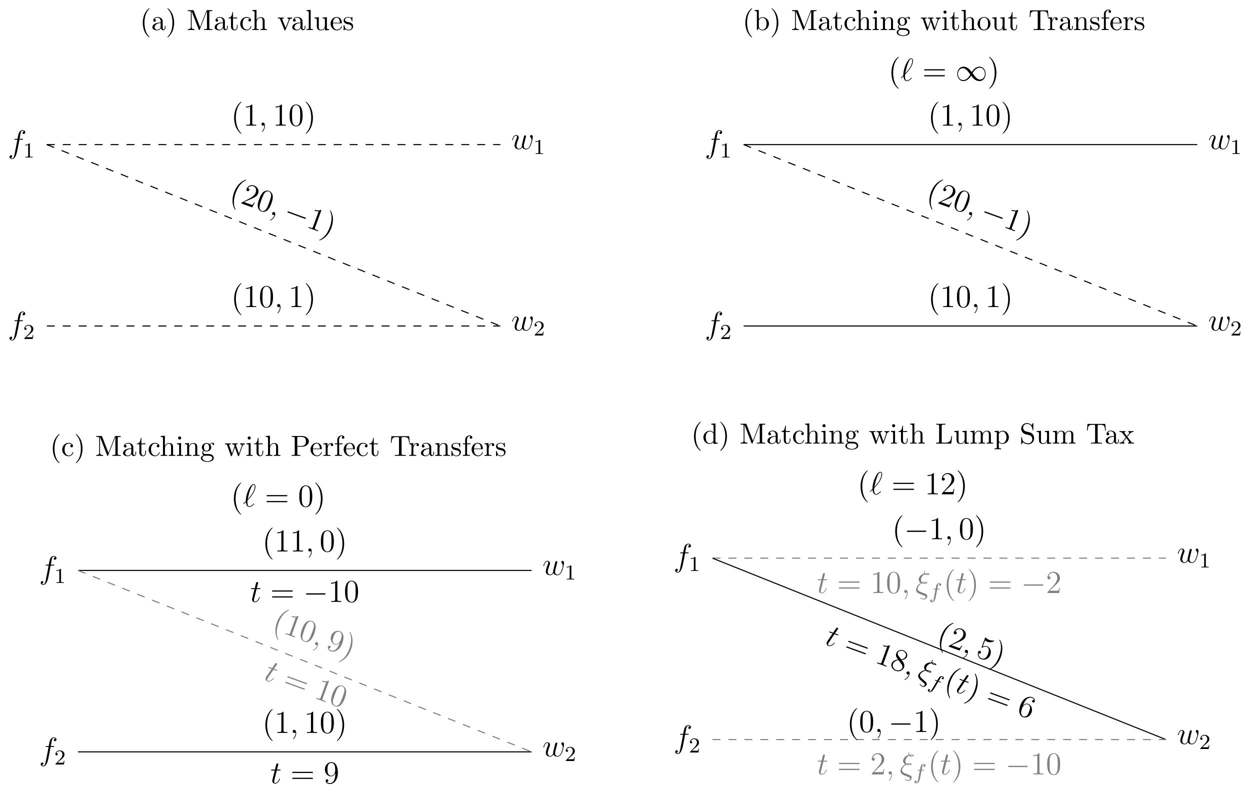

Figure 11: Example 3 – Non-monotonicity under a lump sum tax on transfers.

Note: Utilities, net of transfers, are above the lines (firm's, worker's). Possible supporting transfers (when applicable) are below the lines. Solid lines indicate the stable matching.

When the lump sum tax is lowered to $\ell = 12$, only the matching μ_2 is stable, where $\mu_2(f_1) = w_2$ and w_1 and f_2 are unmatched; this matching gives a total match value $\mathfrak{M}(\mu_2) = 19$, as shown in Figure 11d. When $\ell = 12$, the tax is low enough that f_1 can convince w_2 to match with him, but not low enough for w_1 to hold onto f_1 when it has the option of matching with w_2 (or f_2 to hold onto w_2). Lowering the lump sum tax from 20 to 12 decreases the total match value of the stable matching and decreases the number of agents matched.

transfers promotes pairing (f, w) in which the match value $\gamma_{f,w} + \alpha_{f,w}$ is evenly distributed between the two partners ($\gamma_{f,w} \approx \alpha_{f,w}$), so that transfers are unnecessary.

We use simulations to confirm that Example 3 is not an exceptional case. In the same small markets described in Appendix C, with utilities uniformly distribution on $[-.5, .5]$, we find that match value is non-monotonic in the lump sum tax in 61% of our simulated markets.

In a *strictly positive wage market* worker match values are strictly negative (instead of just non-positive). In strictly positive wage markets, all matchings require a transfer, so a lump sum tax on transfers does not distort agents' preferences among match partners – for a given transfer vector, if a worker prefers firm f_1 to f_2 without a tax, then that worker also prefers f_1 to f_2 under a lump sum tax. Thus, in strictly positive wage markets, the matching distortion of the lump sum tax is only on the extensive margin – the decision of *whether* to match – under a higher lump sum tax, fewer agents find matching desirable. This intuition that lump sum taxes work on the extensive margin is captured in the following lemma, where we use $\#(\mu)$ to denote the number of workers matched in matching μ

Lemma 2. *In strictly positive wage markets, reduction in a lump sum tax on transfers (weakly) increases the number of workers matched in stable matchings. That is, if matching $\check{\mu}$ is stable under lump sum tax $\check{\ell}$, matching $\hat{\mu}$ is stable under lump sum tax $\hat{\ell}$, and $\hat{\ell} < \check{\ell}$, then*

$$\#(\hat{\mu}) \geq \#(\check{\mu}).$$

In non-wage markets, the conclusion of Lemma 2 is not true, in general, because distortion among match partners can dominate the extensive margin effect, as in Example 3.

As lump sum taxes do not distort among match partners in strictly positive wage markets, they can only reduce the efficiency of stable matchings in such markets by reducing the number of workers matched. This idea that the distortion must be on the extensive margin, combined with Lemma 2, gives the following result.

Theorem 2. *In strictly positive wage markets, a reduction in a lump sum tax on transfers (weakly) increases the total match value of stable matchings. That is, if $\check{\mu}$ is stable under lump sum tax $\check{\ell}$, $\hat{\mu}$ is stable under lump sum tax $\hat{\ell}$, and $\hat{\ell} < \check{\ell}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\check{\mu}).$$

Theorem 2 indicates that in strictly positive wage markets, match value increases monotonically as lump sum taxation decreases.

In strictly positive wage markets, we can also bound the total match value loss from a given lump sum tax.

Proposition 5. *In a strictly positive wage market, let $\hat{\mu}$ be an efficient matching, and let $\check{\mu}$ be stable under lump sum tax on transfers $\check{\ell}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\check{\mu}) \leq \check{\ell} \cdot (\#(\hat{\mu}) - \#(\check{\mu})).$$

The intuition for Proposition 5 is that since the workers unmatched under a lump sum tax of $\check{\ell}$ have negative surplus from matching under that lump sum tax, their surplus from matching could not be more than $\check{\ell}$. So the change in total value is less than the change in the number of unmatched workers times a maximum surplus of $\check{\ell}$ per worker.

Finally, we can show that, for a *fixed* limit on the number of workers matched in the presence of a lump sum tax, stable matchings in strictly positive wage markets must generate the maximal match value possible.

Proposition 6. *In a strictly positive wage market, a matching $\breve{\mu}$ can be stable under a lump sum tax on transfers only if*

$$\breve{\mu} \in \underset{\{\mu : \#(\mu) \leq \#(\breve{\mu})\}}{\arg\max} \left[\mathfrak{M}(\mu)\right].$$

Proposition 6 shows that a lump sum tax is an efficient way for a market designer to limit the number of matches (in strictly positive wage markets): the matchings stable under lump sum taxation have maximal value, given the tax's implied limit on the number of agents matched. Analogously, if a market designer wants to encourage matches, a lump-sum subsidy will maximize total match value for a given (subsidy-induced) lower bound on the number of agents matched. For example, Proposition 6 suggests that if a government wants to use tuition subsidies to encourage people to go to school, then uniform tuition subsidies are more efficient than subsidies proportional to the cost of tuition.

D.2 Lump Sum Taxation of Matches

Some fee structures tax *all* pairings, rather than just those that include nonzero transfers. Such *flat fees for matching* can also be interpreted in the language of taxation: they correspond to the transfer function

$$\xi_{\ell}^{\mathsf{fee}}(t_{f,w}) \equiv t_{f,w} - \ell.$$

Figure 12 shows this transfer function for different levels of ℓ.

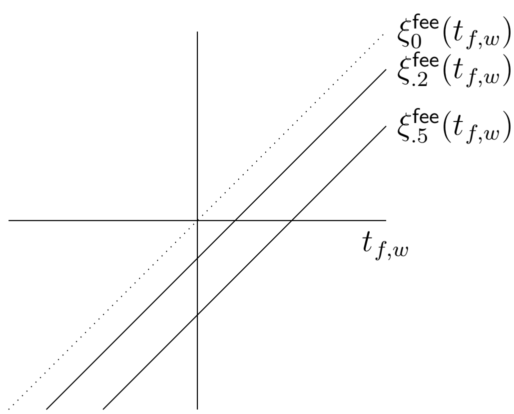

Figure 12: Transfer function $\xi_{\ell}^{\mathsf{fee}}(\cdot)$.

Unlike lump sum taxes on transfers, flat fees for matching never distort among match partners – even in non-wage markets. Flat fees for matching only have extensive margin effects, and thus markets with such fees are similar to strictly positive wage markets with lump sum taxes on transfers.[43] As shown below, the conclusions of Lemma 2, Theorem 2,

[43]Indeed, in strictly positive wage markets, lump sum taxation of transfers is equivalent to lump sum taxation of matchings because workers never match without receiving a strictly positive transfer.

Proof of Theorem 2

As in the proof of Lemma 2, Theorem 2 follows from the following slightly more general result.

Theorem 2'. *A reduction in a flat fee for matching (weakly) increases the total match value of stable matchings. That is, if $\check{\mu}$ is stable under flat fee $\check{\ell}$, $\hat{\mu}$ is stable under flat fee $\hat{\ell}$, and $\hat{\ell} < \check{\ell}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\check{\mu}).$$

Proof. Using (D.2) and Lemma 2', we find that

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\check{\mu}) = \sum_{f \in F} \left(\gamma_{f,\hat{\mu}(f)} - \gamma_{f,\check{\mu}(f)} \right) + \sum_{w \in W} \left(\alpha_{\hat{\mu}(w),w} - \alpha_{\check{\mu}(w),w} \right) \geq \hat{\ell} \cdot (\#(\hat{\mu}) - \#(\check{\mu})) \geq 0;$$

this proves Theorem 2'. □

Proof of Proposition 5

As in the proof of Lemma 2, Proposition 5 follows from the following slightly more general result.

Proposition 5'. *Let $\hat{\mu}$ be an efficient matching, and let $\check{\mu}$ be stable under flat fee $\check{\ell}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\check{\mu}) \leq \check{\ell} \cdot (\#(\hat{\mu}) - \#(\check{\mu})).$$

Proof. The Proposition is immediate from (D.1). □

Proof of Proposition 6

As in the proof of Lemma 2, Proposition 6 follows from the following slightly more general result.

Proposition 6'. *A matching $\check{\mu}$ can be stable under a flat fee only if*

$$\check{\mu} \in \underset{\{\mu : \#(\mu) \leq \#(\check{\mu})\}}{\arg\max} \left\{ \mathfrak{M}(\mu) \right\}.$$

Proof. From (D.1), we see that if $[\check{\mu}; \check{t}]$ is stable under flat fee $\check{\ell}$, then for any matching $\hat{\mu} \neq \check{\mu}$,

$$\mathfrak{M}(\check{\mu}) - \mathfrak{M}(\hat{\mu}) + \check{\ell} \cdot (\#(\hat{\mu}) - \#(\check{\mu})) \geq 0. \tag{D.3}$$

If fewer workers are matched in $\hat{\mu}$ than in $\check{\mu}$ (i.e. $\#(\check{\mu}) \geq \#(\hat{\mu})$), (D.3) implies that

$$\mathfrak{M}(\check{\mu}) - \mathfrak{M}(\hat{\mu}) \geq \hat{\ell} \cdot (\#(\check{\mu}) - \#(\hat{\mu})) \geq 0,$$

so that $\check{\mu}$ must have higher total match value than $\hat{\mu}$. □

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