

Nonparametric Welfare Analysis

Jerry A. Hausman and Whitney K. Newey

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Unobserved individual heterogeneity is important in demand analysis.

R-squared often low in applications, implying that the great majority of demand variation due to unobserved heterogeneity.

Important to account correctly for it.

Demand functions could vary in general ways across individuals.

Price and income elasticities unlikely to be confined to a curve.

Demand might arise from combined discrete/continuous choice.

A priori reason to think heterogeneity is multidimensional or general.

Exact consumer surplus quantifies welfare effects of price changes.

Average surplus is common welfare measure.

We show that for continuous demand average welfare is not identified from distribution of demand for fixed price and income.

Use bounds on income effects to derive bounds on average surplus.

Bounds based on the average of quantity demanded (and e.g. not log of quantity).

For two good case also give general bounds based only on utility maximization.

Empirical application is based on independence of preferences and budget sets.

Possibly conditioned on covariates and control functions.

Under independence average demand is conditional expectation of quantity.

Can be estimated by nonparametric, semiparametric, or parametric regression in cross section data and used to construct bounds based on income effects.

Distribution of demand can be similarly estimated and used to construct general bounds.

Apply results to gasoline demand.

Use data from 2001 U.S. National Household Transportation Survey.

Find tight bounds for average surplus based on income effect bounds.

Deadweight loss bounds are substantially wider.

General bounds for average surplus are wider but still informative.

Model here is a special case of McFadden and Richter (1991) and McFadden (2004).

Like Lewbel (2001) in assuming single valued, smooth demand as often do for empirical demand analysis.

Other recent work on demand models with general heterogeneity includes Hoderlein and Stoye (2009), Dette, Hoderlein, and Neumeyer (2016).

Kitamura and Stoye (2016) consider empirical implications of stochastic revealed preference.

Focus here is on estimating objects of interest, namely exact consumer surplus and deadweight loss.

Other recent work considers identification and estimation of consumer surplus when heterogeneity is a scalar that enters nonseparably.

Blundell, Horowitz, and Parey (2016), Hoderlein and Vanhems (2013), Blundell, Kristensen, Matzkin (2014).

Recently Lewbel and Pendakur (2016) have estimated surplus with restricted multivariate heterogeneity.

Here go beyond these specifications in allowing for general heterogeneity.

Inference results here based on Hausman and Newey (1995) results for nonparametric consumer surplus estimates.

Demand Functions with General Heterogeneity.

Denote price vector by p and income by y , relative to price of a numeraire good a .
Let $x = (p^T, y)^T$.

Denote vector heterogeneity by η ; could be infinite dimensional.

Think of a value of η corresponding to a consumer.

Demand function $q(x, \eta)$.

$$q(x, \eta) = \arg \max_{q \geq 0, a \geq 0} U(q, a, \eta) \quad s.t. \quad p^T q + a \leq y.$$

Here we are assuming that demand is single valued.

Essentially the same as strictly quasi-concave utility.

Utility maximization imposes restrictions on demand functions.

Slutzky condition: *For each η the demand function $q(x, \eta)$ is continuously differentiable in $x \gg 0$ and $\partial q(x, \eta)/\partial p + q(x, \eta)[\partial q(x, \eta)/\partial y]^T$ is negative semi-definite and symmetric.*

Implied by solution to utility maximization. By Hurwicz and Uzawa (1971), this condition is sufficient for existence of a utility function $U(q, a, \eta)$ where $q(x, \eta)$ is solution to the above optimization.

Also assume sufficient conditions for making probability statements.

Let r denote a possible value of quantity demanded and G a distribution of η .

The CDF of quantity for a fixed price and income x is

$$F(r|x, q, G) = \int \mathbf{1}(q(x, \eta) \leq r)G(d\eta).$$

Model we consider is one with CDF of this form for $q(x, \eta)$ satisfying Slutzky condition and a distribution of η .

$$F(r|x, q, G) = \int \mathbf{1}(q(x, \eta) \leq r) G(d\eta).$$

A random utility model (RUM) like McFadden (2005) and McFadden and Richter (1991).

We specialize to single valued demands that are smooth in price and income.

Smoothness should be useful for nonparametric estimation.

Much of the literature has focused on restrictions on F implied by RUM.

McFadden (2005) gives inequalities that are necessary and sufficient.

With two goods there is a simple characterization in terms of quantile $Q(\tau|x) = \inf\{r : F(r|x, q, G) \geq \tau\}$.

The RUM holds if and only if $Q(\tau|x)$ is a demand function for each $0 < \tau < 1$.

Useful in identification analysis to follow.

Exact Consumer Surplus

We consider equivalent variation; could do a similar analysis for compensating variation.

Expenditure function is

$$e(p, u, \eta) = \min_{q \geq 0, a \geq 0} \{p^T q + a \text{ s.t. } U(q, a, \eta) \geq u\}.$$

Equivalent variation for price change from p^0 to p^1 with income \bar{y} and the utility at price p^1 equal to u^1 is

$$S(\eta) = \bar{y} - e(p^0, u^1, \eta).$$

For $\Delta p = p^1 - p^0$ the corresponding deadweight loss is

$$D(\eta) = S(\eta) - q(p^1, \bar{y}, \eta)^T \Delta p.$$

Helpful to derive surplus and deadweight loss from the demand function.

Let t be a scalar and $p(t)$ a continuously differentiable price path with $p(0) = p^0$, $p(1) = p^1$.

Shephard's Lemma implies that $S(\eta)$ is the solution $s(0, \eta)$ at $t = 0$ to the ordinary differential equation

$$\frac{ds(t, \eta)}{dt} = -q(p(t), \bar{y} - s(t, \eta), \eta)^T \frac{dp(t)}{dt}, \quad s(1, \eta) = 0.$$

$S(\eta)$ does not depend on price path as long as Slutsky (symmetry) condition is satisfied.

Intuitively we get exact surplus by integrating the demand function while compensating income so that utility remains constant at u^1 .

Change in price of one good, say the first, is a common example.

Here $p^0 = (p_1^0, \bar{p}_2^T)^T$ and $p^1 = (p_1^1, \bar{p}_2^T)^T$ for some fixed \bar{p}_2 .

Natural choice of price path is $p(t) = tp^1 + (1-t)p^0 = (p_1^0 + t\Delta p_1, \bar{p}_2^T)^T$, where $\Delta p_1 = p_1^1 - p_1^0$.

Here $S(\eta)$ will be the solution at $t = 0$ to the equation

$$\frac{ds(t, \eta)}{dt} = -q_1(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y} - s(t, \eta), \eta)\Delta p_1, \quad s(1, \eta) = 0.$$

We see that with multiple goods the exact consumer surplus for a price change for a single good can be computed from the demand function for that good, Hausman (1981).

Objects of interest are the average surplus \bar{S} and deadweight loss \bar{D} across individuals

$$\bar{S} = \int S(\eta)G(d\eta), \bar{D} = \int D(\eta)G(d\eta).$$

Hicks (1939) showed that $\bar{S} > 0$ means that possible to make everyone better off under p^0 .

Also average surplus used as practical measure of social welfare.

Average demand $\bar{q}(x) = \int q(x, \eta)G(d\eta)$ is important.

Average surplus depends only on average demand when income effect is constant.

For $\partial q_1(p_1, \bar{p}_2, y, \eta)/\partial y = b$ the surplus for a change in p_1 solves

$$\frac{ds(t, \eta)}{dt} = -[q_1(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y}, \eta) - bs(t, \eta)]\Delta p_1, \quad s(1, \eta) = 0.$$

This is a linear differential equation with explicit solution

$$S(\eta) = \Delta p_1 \int_0^1 q_1(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y}, \eta) \exp(-tb\Delta p_1) dt.$$

Taking expectations under the integral gives

$$\bar{S} = \Delta p_1 \int_0^1 \bar{q}_1(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y}) \exp(-tb\Delta p_1) dt.$$

With one price changing and income effect constant for that good, average surplus is integral of average demand.

Related to classic Gorman (1961) aggregation result, but only requires constant income effect for goods with changing prices.

Identification

We consider identification of objects of interest when we know the CDF $F(r|x, q, G)$ of demand over a set $\bar{\chi}$ of prices and income.

Corresponds to knowing the distribution of demand in cross-section data, where we only observe one price and income for each individual.

If more than one value of x were observed for each individual, as in panel data, then one could identify some joint distributions of demand at different values of x .

We leave consideration of this topic to future research.

We adapt a standard framework to our setting, as in Hsiao (1983).

Structure is demand function and heterogeneity distribution pair (q, G) .

Definition 1: (q, G) and (\tilde{q}, \tilde{G}) are observationally equivalent if and only if for all r and $x \in \bar{\chi}$,

$$F(r|x, q, G) = F(r|x, \tilde{q}, \tilde{G}).$$

We consider identification of object $\delta(q, G)$ that is a function of the structure (q, G) , for example average surplus.

Definition 2: The identified set for δ corresponding to (q_0, G_0) is $\Lambda(q_0, G_0) = \{\delta(\tilde{q}, \tilde{G}) : (q_0, G_0) \text{ and } (\tilde{q}, \tilde{G}) \text{ are observationally equivalent}\}$.

Here (q_0, G_0) are the true values of the demand function and heterogeneity distribution.

The set $\Lambda(q_0, G_0)$ is nonempty since $\delta(q_0, G_0) \in \Lambda(q_0, G_0)$.

Sharpness of $\Lambda(q_0, G_0)$ holds automatically here because we are explicitly formulating the identified set in terms of all the restrictions on F , and we are assuming that F is all we know.

Viewing demand as a stochastic process indexed by x helps explain identification.

Here $q(x, \eta)$ is a function of x for each η , that varies stochastically with η , i.e. $q(x, \eta)$ is a stochastic process.

A structure (q, G) can be thought of as a *demand process*.

The distribution of $q(x, \eta)$ for fixed x is the *marginal distribution*.

The distribution of $(q(x^1, \eta), \dots, q(x^K, \eta))^T$ is a *joint distribution*.

$F(r|x, q, G)$ is the marginal distribution of the demand process.

Here two demand processes are observationally equivalent if and only if they have the same marginal distribution.

Cross-section data only identifies marginal distribution, because only observe one price and income for each individual.

Objects $\delta(q, G)$ that depend only on the marginal distribution of the demand process are point identified.

They are the same for all observationally equivalent structures.

Example of point identified object is average demand

$$\bar{q}(x) = \int q(x, \eta)G(d\eta) = \int rF(dr|x, q, G)$$

Functionals of average demand are also identified.

More generally, functionals of the (marginal) distribution of demand F are identified.

Joint distributions of the demand process are not identified.

For example, the joint distribution of $(q(\tilde{x}, \eta), q(\bar{x}, \eta))^T$ for two different values of x , is not identified.

Intuitively, joint distributions are not identified because we only know the marginal distribution.

Similarly, objects that depend on varying x for fixed η are not identified.

For example, average surplus will not be identified, as we show.

Another unidentified feature of the demand process is whether $q(x, \eta)$ depends monotonically on a scalar function of η .

Assumed by Blundell, Kristensen, and Matzkin (2011).

Can be thought of as $q(x, \eta)$ being perfectly predictable for all x if we know $q(\bar{x}, \eta)$ for some \bar{x} .

That is,

$$\text{Var}(q(x, \eta) | q(\bar{x}, \eta)) = 0.$$

This is characteristic of the joint distribution of the demand process.

Is not identified from the marginal distribution.

Interesting example is a two good set up with linear varying coefficients demand, where.

$$q_0(x, \eta) = \eta_1 + \eta_2 p + \eta_3 y, \quad \eta \text{ has distribution } G_0.$$

A familiar specification that satisfies Slutsky condition for restrictions on G_0 .

By earlier result the conditional quantile $Q(\tau|x)$ is a demand function for all $0 < \tau < 1$.

Consider the quantile demand process

$$\tilde{q}(x, \tilde{\eta}) = Q(\tilde{\eta}|x), \quad \tilde{\eta} \sim U(0, 1).$$

By standard arguments the CDF of $Q(\tilde{\eta}|x)$ is $F(r|x, q_0, G_0)$, so quantile demand is observationally equivalent to linear random coefficients demand.

Cannot nonparametrically distinguish linear random coefficients from demand process with scalar heterogeneity.

Also, if η_3 varies then true average surplus different than quantile average surplus so *average surplus is not identified*.

Income Effect Bounds

Since average surplus is not point identified we must content ourselves with bounds, i.e. with finding the identified set.

We start with sets based on bounds on income effects. These are computationally simple.

Also can get bounds based just on utility maximization. These are more complicated to compute.

Income effects idea: With bounds on income effects the true demand is bounded above and below by a function with constant income effect.

With constant income effect average surplus depends just on average demand.

True average surplus lies between upper and lower bounds that can be computed from average demand.

Recall that $\bar{q}(x) = \int q(x, \eta)G(d\eta)$ is average demand.

For any constant B let

$$\bar{S}_B = \int_0^1 [\bar{q}(p(t), \bar{y})^T \frac{dp(t)}{dt}] e^{-Bt} dt.$$

This is the solution at $t = 0$ to the linear differential equation

$$\frac{d\bar{s}_B(t)}{dt} = -\bar{q}(p(t), \bar{y})^T \frac{dp(t)}{dt} + B\bar{s}_B(t), \quad \bar{s}_B(1) = 0.$$

\bar{S}_B would be the average surplus if just the price of the first good were changing and the income effect for the first good was constant across individuals and income, given by

$$\partial q_1(p(t), y, \eta) / \partial y = B / \Delta p_1.$$

If i) $q(p(t), \bar{y} - s, \eta)^T dp(t)/dt \geq 0$ for $s \in [0, S(\eta)]$, ii) there are constants \underline{B} and \overline{B} such that $\underline{B} \leq [\partial q(x, \eta)/\partial y]^T dp(t)/dt \leq \overline{B}$ for all $x \in \chi$; iii) all prices in $p(t)$ are bounded away from zero then

$$\begin{aligned} \bar{S}_{\overline{B}} &\leq \bar{S} \leq \bar{S}_{\underline{B}}, \\ \bar{S}_{\overline{B}} - \bar{q}(p^1, \bar{y})^T \Delta p &\leq \bar{D} \leq \bar{S}_{\underline{B}} - \bar{q}(p^1, \bar{y})^T \Delta p. \end{aligned}$$

Condition i) automatically satisfied when only the price of the first good is changing and $p_1^1 > p_1^0$.

Surplus bounds hold under weaker conditions than bounded income effects.

The key ingredient for these average surplus bounds are bounds on the income effect $[\partial q(x, \eta)/\partial y]^T dp(t)/dt$.

Economics can deliver income effect bounds.

Focus on a price change in the first good, where \underline{B} and \overline{B} are bounds on $\Delta p_1 \partial q_1(x, \eta) / \partial y$ and $\Delta p_1 > 0$.

If q_1 is a normal good then the income effect is nonnegative, so we can take $\underline{B} = 0$.

Then an upper bound for average equivalent variation and deadweight loss can be obtained from Marshallian surplus for average demand as

$$\bar{S} \leq \bar{S}_M = \int_0^1 [\bar{q}(p(t), \bar{y})^T dp(t) / dt] dt, \bar{D} \leq \bar{S}_M - \bar{q}(p^1, \bar{y})^T \Delta p.$$

Upper bound on average deadweight loss could be useful for policy purposes.

Proceed with a tax if average public benefits (e.g. environmental benefits) exceed average deadweight loss and have separability conditions.

Economics can also deliver upper bounds on income effects.

If no more than a fraction π of additional income is spent on q_1 then

$$\partial q_1(x, \eta) / \partial y \leq \pi / p_1 \leq \pi / p_1^0.$$

Then $\bar{B} = \Delta p_1 \pi / p_1^0 = \pi (p_1^1 / p_1^0 - 1)$ is upper bound on $[\partial q(x, \eta) / \partial y]^T dp(t) / dt$.

See gasoline demand application below for example.

Quantiles also provide information about income effects.

By Hoderlein and Mammen (2007),

$$\frac{\partial Q_1(\tau|x)}{\partial y} = E\left[\frac{\partial q_1(x, \eta_i)}{\partial y} \mid q_1(x, \eta_i) = Q_1(\tau|x)\right] \leq \bar{B}/\Delta p_1$$

Upper bound must be larger than quantile income derivative.

Does not identify the bounds, because there is variation in income effect "left over" from quantile.

The conditional quantile is also informative about the surplus bounds.

Let S^τ be the exact surplus obtained by treating $Q_1(\tau|x)$ as if it were a demand function

S^τ is the solution $s^\tau(0)$ at $t = 0$ to the differential equation

$$\frac{ds^\tau(t)}{dt} = -Q_1(\tau|p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y} - s^\tau(t))\Delta p_1, \quad s^\tau(1) = 0.$$

With two goods and scalar heterogeneity, the average surplus would be $\int_0^1 S^\tau d\tau$.

In general $\int_0^1 S^\tau d\tau$ is between the surplus bounds,

$$\bar{S}_{\underline{B}} \leq \int_0^1 S^\tau d\tau \leq \bar{S}_{\underline{B}}.$$

Can be used to compare surplus bounds to quantile average surplus.

Surplus bounds are relatively insensitive to income effect bounds when a small proportion of income is spent on the good.

Related to Hotelling (1938), that when expenditure is small approximate consumers surplus is typically close to actual consumers surplus.

Differentiating \bar{S}_B with respect to B to obtain

$$\begin{aligned}\bar{y}^{-1} \frac{\partial \bar{S}_B}{\partial B} &= -\bar{y}^{-1} \int_0^1 [\bar{q}(p(t), \bar{y})^T dp(t)/dt] t e^{-Bt} dt \\ &= - \int_{p_1^0}^{p_1^1} [\bar{q}_1(p_1, \bar{p}_2, \bar{y}) p_1 / \bar{y}] \left(\frac{1 - p_{10}/p_1}{\Delta p_1} \right) \exp\left(-B \frac{p_1 - p_1^0}{\Delta p_1}\right) dp_1.\end{aligned}$$

In this way the bounds are less sensitive to B when share $\bar{q}_1(p_1, \bar{p}_2, \bar{y}) p_1 / \bar{y}$ of income spent on the first good is smaller.

General Bounds with Two Goods

Average surplus bounds based just on utility maximization

Find supremum and infimum of average surplus over approximation to the set of demand processes that are consistent with the distribution of demand.

We focus here on the two good case.

The approximation is based on a series expansion around the quantile demand.

Coefficients of the series terms have a discrete distribution.

Consider a demand process of the form

$$\tilde{q}(x, \eta) = Q(\tilde{\eta}|x) + \sum_{j=1}^J \check{\eta}_j m_j(x)$$

where $m_j(x)$, $j = 1, \dots, J$ are approximating functions, $\tilde{\eta} \sim U(0, 1)$, and $\check{\eta} = (\check{\eta}_1, \dots, \check{\eta}_J)^T$ has a discrete distribution with L points of support $\{\check{\eta}^1, \dots, \check{\eta}^L\}$ that is independent of $\tilde{\eta}$.

Draw $\check{\eta}^\ell$ at random, keeping only those where $\tilde{q}^\ell(x, \tilde{\eta}) = Q(\tilde{\eta}|x) + \sum_{j=1}^J \check{\eta}_j^\ell m_j(x)$ satisfies the Slutsky condition over a grid of values for $\tilde{\eta}$ and x .

Let $\rho_\ell = \Pr(\check{\eta}_i = \check{\eta}^\ell)$ and $F(r|x) = Q^{-1}(r|x)$ be the CDF corresponding the quantile $Q(\tau|x)$. Integrating over $\tilde{\eta}$ gives

$$F(r|x, \tilde{q}, \tilde{G}) = \sum_{\ell=1}^L \rho_\ell F(r - \sum_{j=1}^J \check{\eta}_j^\ell m_j(x)|x).$$

Integrate over $\tilde{\eta}$ to smooth out the CDF.

Allow the mixture probabilities ρ_ℓ to vary to find max and min of average surplus.

Look for the max and min average surplus subject to restrictions imposed by the data.

Let $\tilde{S}^\ell(\tilde{\eta})$ be the surplus for $\tilde{q}^\ell(x, \tilde{\eta}) = Q(\tilde{\eta}|x) + \sum_{j=1}^J \tilde{\eta}_j^\ell m_j(x)$.

Let $\bar{S}^\ell = \int_0^1 \tilde{S}^\ell(\tilde{\eta}) d\tilde{\eta}$, where this integral is approximated using a grid of $\tilde{\eta}$ values.

Here \bar{S}^ℓ is the average surplus for the demand model with $\check{\eta} = \check{\eta}^\ell$.

The average surplus for the demand model where $\Pr(\check{\eta} = \check{\eta}^\ell) = \rho_\ell$ and $\tilde{\eta}$ is distributed independently of $\check{\eta}$ is

$$\sum_{\ell=1}^L \rho_\ell \bar{S}_\ell$$

Approximate upper bound for surplus solves the linear program

$$\begin{aligned} & \max_{\rho_1, \dots, \rho_L} \sum_{\ell=1}^L \rho_\ell \bar{S}_\ell \\ \text{s.t. } & F(r_m | x_m) = \sum_{\ell=1}^L \rho_\ell F(r_m - \sum_{j=1}^J \eta_j^\ell m_j(x_m) | x_m), \\ & (r_m, x_m) \in \Gamma, \rho_\ell \geq 0, \sum_{\ell=1}^L \rho_\ell = 1, \end{aligned}$$

where Γ is a grid where the distribution constraints are imposed.

Use the same grid for x as when checking to make sure that model with $\check{\eta}^\ell$ satisfies the Slutsky conditions.

This is a linear program so computation is straightforward.

As for other estimators of partially identified objects (e.g. Manski and Tamer, 2002), it may be important to include some slackness in the constraints.

Do this by requiring that constraints be approximately satisfied.

Solve instead

$$\max_{\rho_1, \dots, \rho_L} \sum_{\ell=1}^L \rho_{\ell} \bar{S}_{\ell} \quad \text{s.t.}$$

$$\sum_{(r_m, x_m) \in \Gamma} [F(r_m | x_m) - \sum_{\ell=1}^L \rho_{\ell} F(r_m - \sum_{j=1}^J \eta_j^{\ell} m_j(x_m) | x_m)]^2 \leq \varepsilon,$$

$$\rho_{\ell} \geq 0, \quad \sum_{\ell=1}^L \rho_{\ell} = 1.$$

for some $\varepsilon > 0$. ε is a slackness variable.

This is a quadratic program that is also relatively easy to compute.

Approximation to true bounds depends on large J and L .

The choice of J and L and inference are beyond scope of paper.

Bounds of interest even for fixed J .

As long as $\check{\eta}^\ell = 0$ for some ℓ the average quantile surplus will be between the bounds, so that the bounds give a measure of how much average surplus can vary away from the quantile surplus.

Can compare the general bounds to those based on income effects.

Width of bounds increase with J , so if they are wider than bounds based on income effects then we can be assured that using income effects produces narrower bounds.

This series approximation empirically implements the random utility model.

Idea is that

$$\tilde{q}(x, \eta) = Q(\tilde{\eta}|x) + \sum_{j=1}^J \check{\eta}_j m_j(x),$$

can approximate any demand process for large enough J and sufficiently many support points for $\check{\eta}$.

Based on approximation that is uniform over sets of functions with bounded derivatives for fixed J , as $\check{\eta}$ varies.

Differs from Kitamura and Stoye (2012) where revealed stochastic preference inequalities are imposed.

Here we impose the Slutsky conditions on a grid and then interpolate between points using a series approximation.

This approach relies on and exploits smoothness in underlying demand functions.

Empirical Application

Previous results use average and distribution of demand for fixed price and income.

These are identified when prices and income in the data are independent of preferences.

That is, when the data are (q_i, x_i) , $(i = 1, \dots, n)$ with $q_i = q_0(x_i, \eta_i)$ and x_i and η_i are statistically independent.

Then

$$E[q_i | x_i = x] = \bar{q}_0(x), \Pr(q_i \leq r | x_i = x) = F(r | x, q_0, G_0).$$

Average demand is the conditional expectation of quantity q_i conditional on $x_i = x$.

Similarly the distribution of demand is conditional CDF of q_i given $x_i = x$.

The conditional expectation of quantity, and not some other function of quantity like the log, is special.

$E[q_i | x_i = x]$ equals the average demand, which is used for bounds.

$E[q_i|x_i = x]$ could be estimated by nonparametric regression; see gasoline demand application.

Or semiparametric or parametric when there are many prices (goods).

Independence of η_i and x_i encompasses statistical version of preferences not varying with prices.

Also individual is small relative to the market and observations come from different markets.

Independence of income and preferences can be a concern with dynamic consumption.

Is an important starting point; commonly imposed in gasoline demand applications.

Can allow for covariates.

Index specification where w are covariates, $v(w, \delta)$ affects utility, with η_i and $(x_i^T, w_i^T)^T$ are independent.

For example w could be demographic variables representing observed components of the utility.

The demand function $q_0(x, v(w, \delta_0), \eta)$ depends on the index $v(w, \delta_0)$.

Average demand is then

$$\bar{q}_0(x, v(w, \delta_0)) = \int q_0(x, v(w, \delta_0), \eta) G(d\eta) = E[q_i | x_i = x, v(w_i, \delta_0) = v]$$

This is a partial index regression of quantity q_i on x_i and $v(w_i, \delta)$.

Similar approaches are common in demand analysis.

Endogeneity can be allowed with an estimable control variable ξ .

Assume x_i and η_i are independent conditional on ξ_i and the conditional support of ξ_i given x_i equals the marginal support of ξ_i .

In that case it follows as in Blundell and Powell (2003) and Imbens and Newey (2009) that

$$\int E[q_i | x_i = x, \xi_i = \xi] F_\xi(d\xi) = \bar{q}_0(x),$$
$$\int \Pr(q_i \leq r | x_i = x, \xi_i = \xi) F_\xi(d\xi) = F(r|x, q_0, G_0),$$

Conditions for existence of a control variable are quite strong, see Blundell and Matzkin (2014).

This approach does allow some forms of endogeneity.

Gasoline Demand

Data similar to Blundell, Horowitz, and Parey (2013).

From 2001 U.S. National Household Transportation Survey (NHTS).

Conducted every 5-8 years by Federal Highway Administration.

Nationally representative repeated cross-section.

24-hour travel behavior of randomly-selected households

Includes detailed trip data and household characteristics (income, age, number of drivers, etc.).

restrict data to households with 1-2 gasoline-powered cars, vans, SUVs, pickup trucks.

Only households with positive miles driven, use daily consumption, monthly state prices, annual income, drop Alaska and Hawaii.

Use up to a 4th degree polynomial in logs with covariates

$$\widehat{\bar{q}}(x, w) = \sum_{j,k,\ell=1}^4 \hat{\beta}_{j,k,\ell} (\ln p)^j (\ln y)^k (v(w, \hat{\delta}))^\ell$$

Assume preference independent of p, y, w .

Also use control function for gasoline price using state tax rates as instruments and also distance of the state from the Gulf of Mexico, as in Blundell, Horowitz and Parey (2012).

Use the estimated residuals from this first stage $\hat{\xi}_i$ to include in fourth order multivariate series regression,

$$E[\widehat{q_i} | x, w, \xi] = \sum_{j,k,\ell,m=1}^4 \tilde{\beta}_{j,k,\ell} (\ln p)^j (\ln y)^k (w' \delta)^\ell (\hat{\xi})^m$$

Plug in $\hat{\xi}_i$ and average over them to get $\widehat{\bar{q}}(x, w)$.

To set bounds on income effects, assume gasoline is a normal good, so $\underline{B} = 0$.

To set upper bound on income effect we estimate $Q(\tau|x)$ using local linear quantile regression evaluated at median price and income.

We find that $\partial\hat{Q}(\tau|x_{med})/\partial y$ is increasing in τ .

Take $\bar{B} = .0197 = 20 \times \partial\hat{Q}(.9|x_{med})/\partial y$.

This income effect is very large, corresponding to more than two cents of every additional dollar of income being spent on gasoline.

Confident all have smaller income effect.

Estimate linear, varying coefficients demand get average income effect of .000726

Get standard deviation of the income effect, is .00241.

Here .0197 would be extremely large relative to the distribution of income effects.

Estimated conditional quantile of log quantity conditional on log price and income.

Used cubic approximation in log price and log quantity for all quantiles, following preliminary fit of median.

Estimated .01, .02, ..., .99 quantiles subject to Slutsky inequality (in logs).

Imposed Slutsky on grid of 81 price and income points, a randomly selected point in a grid of empirical support.

J corresponds to cubic in logs.

Set $L = 1000$.

This is the number of support points for the coefficient vector in the deviation from the quantile, drawn randomly, each satisfying Slutsky conditions.

Also try redrawing these to see if the η values effects results.

Grid of r values for constraints is 5 values at unconditional quantiles.

Base CDF on estimated conditional quantiles $\hat{Q}_q(.01|x), \dots, \hat{Q}_q(.99|x)$.

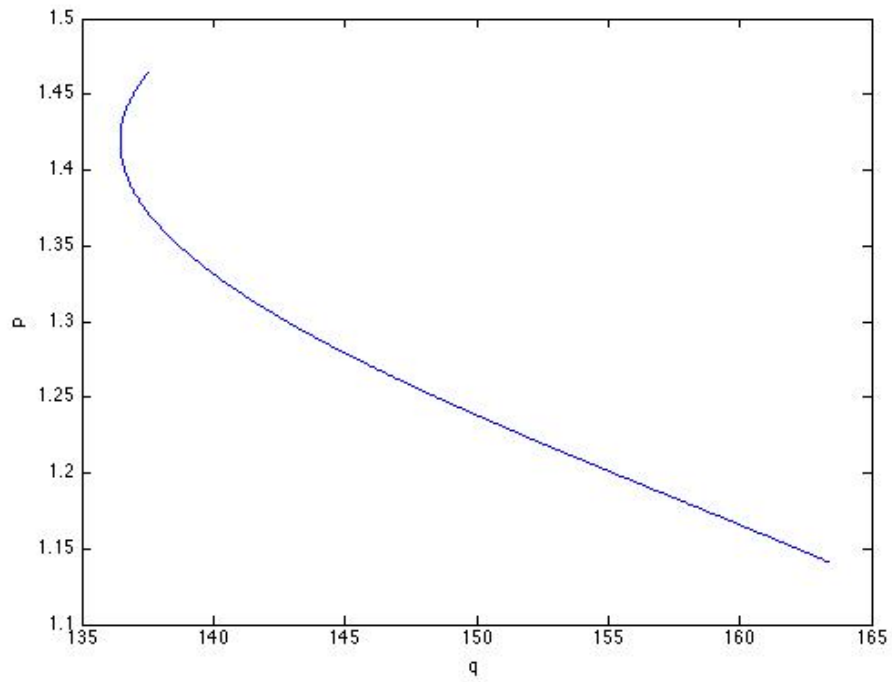
Use as our estimated CDF

$$\hat{F}(r|x) = 99^{-1} \sum_{k=1} \Phi\left(\frac{r - \hat{Q}_q(\tau_k|x)}{.01}\right)$$

Here ε is the squared deviation of constraint from zero, across all constraints.

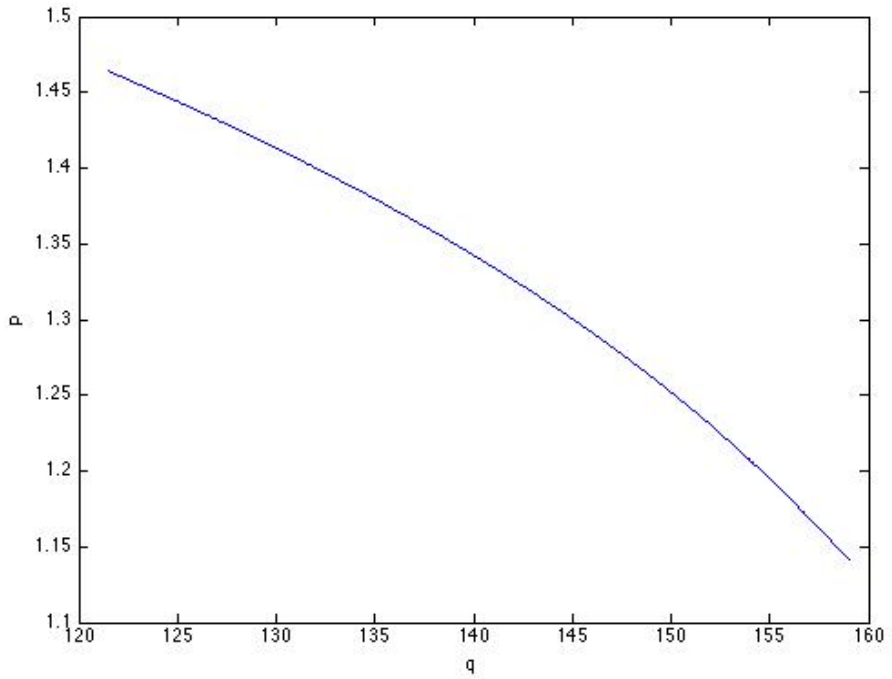
We maximize and minimize the average surplus, subject to the sum the squared deviation of the left hand and right side of the constraints being less than ε .

Figure 1. Estimated Demand: OLS



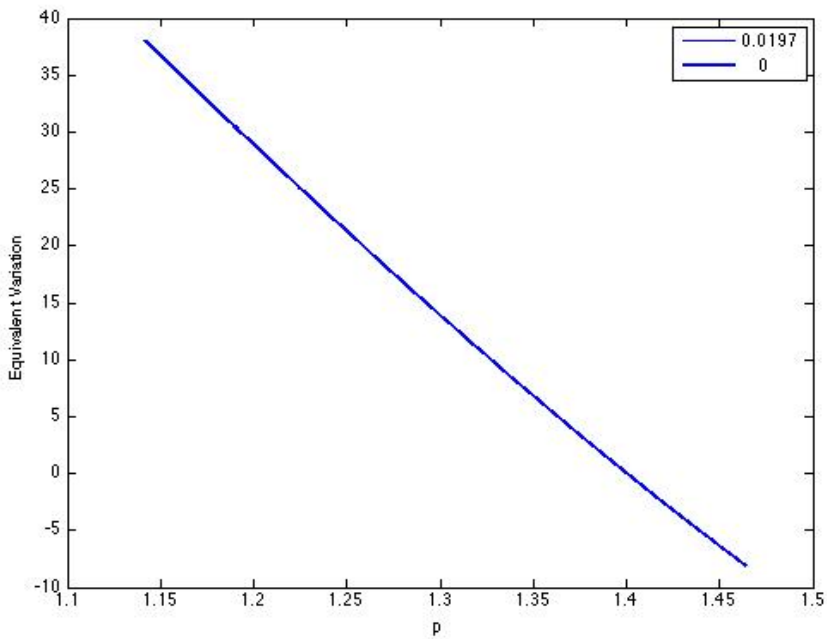
Notes: Demand estimated from 3rd order series regression evaluated at median income.

Figure 2. Estimated Demand: Control Function



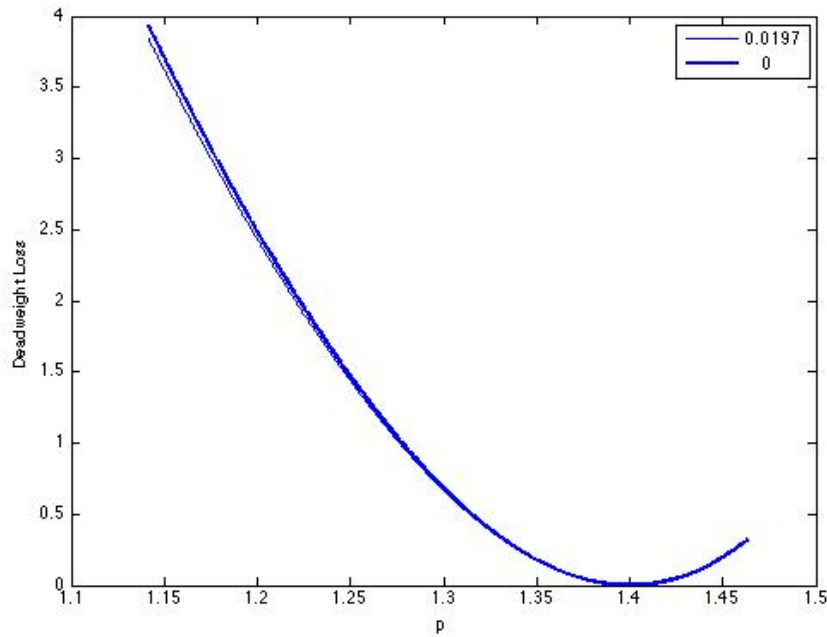
Notes: Demand estimated from 3rd order power series control function regression evaluated at median income.

Figure 3. Equivalent Variation Bounds



Notes: Graph shows change in equivalent variation for a price increase from p to \$1.40, evaluated at upper and lower bounds of income derivative and at median income and estimated from 3rd order power series control function estimated demand.

Figure 4. Deadweight Loss Bounds



Notes: Graph shows change in deadweight loss for a price increase from p to \$1.40, evaluated at upper and lower bounds of income derivative and at median income and estimated from 3rd order power series control function estimated demand.

Table 1. Summary Statistics

Variable	Mean	Median	Std Dev	Min	Max
price (\$)	1.33	1.32	0.08	1.14	1.46
quantity (gallons)	4.90	2.65	7.53	0.01	195.52
income (1,000 \$)	62.19	47.5	47.47	2.08	170.72
number of drivers	2.04	2	0.78	1	7
public transit availability	0.24	0	0.42	0	1
Observations	8,908				

Table 2. Estimated Price Elasticities

Quantiles	OLS Estimates			Control Function Estimates		
	0.25	0.5	0.75	0.25	0.5	0.75
Order 1	-0.698 (0.254)	-0.656 (0.244)	-0.631 (0.243)	-1.111 (0.282)	-1.060 (0.280)	-1.043 (0.291)
Order 2	-1.597 (0.469)	-0.798 (0.283)	0.069 (0.509)	-1.675 (0.476)	-1.350 (0.320)	-1.111 (0.657)
Order 3	-1.214 (0.753)	-0.798 (0.570)	0.271 (0.721)	-1.037 (0.860)	-1.102 (0.670)	-0.872 (0.952)
Order 4	-0.713 (0.877)	-0.583 (0.623)	0.140 (0.801)	-0.389 (1.158)	-0.588 (0.707)	-0.853 (1.032)

Table 3. Estimated Income Elasticities

Quantiles	OLS Estimates			Control Function Estimates		
	0.25	0.5	0.75	0.25	0.5	0.75
Order 1	0.168 (0.022)	0.157 (0.019)	0.151 (0.017)	0.167 (0.022)	0.159 (0.019)	0.157 (0.018)
Order 2	0.221 (0.032)	0.244 (0.025)	0.261 (0.031)	0.210 (0.032)	0.233 (0.025)	0.262 (0.034)
Order 3	0.217 (0.057)	0.221 (0.040)	0.236 (0.039)	0.220 (0.059)	0.221 (0.041)	0.256 (0.043)
Order 4	0.266 (0.067)	0.305 (0.054)	0.275 (0.047)	0.267 (0.074)	0.294 (0.058)	0.277 (0.052)

Table 4. Bounds on Equivalent Variation Estimates

	From \$1.20 to 1.30		From \$1.20 to 1.40	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Order 1	16.777	16.794	32.281	32.343
	0.349	0.349	0.502	0.501
	[16.104, 17.468]		[31.355, 33.270]	
Order 2	15.147		28.829	28.884
	0.443	0.443	0.753	0.751
	[14.275, 16.005]		[27.405, 30.309]	
Order 3	14.972	14.987	28.845	28.900
	0.647	0.646	0.884	0.882
	[13.715, 16.244]		[27.163, 30.583]	
Order 4	14.625	14.639	28.546	28.601
	0.660	0.659	0.924	0.922
	[13.340, 15.925]		[26.788, 30.360]	

Table 5. Bounds on Deadweight Loss Estimates

	From \$1.20 to 1.30		From \$1.20 to 1.40	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Order 1	0.646	0.663	2.467	2.529
	0.175	0.175	0.669	0.668
	[0.319, 0.990]		[1.219, 3.776]	
Order 2	0.735		2.821	2.876
	0.233	0.233	0.728	0.727
	[0.277, 1.178]		[1.452, 4.245]	
Order 3	0.485	0.500	2.434	2.489
	0.393	0.393	1.04	1.039
	[-0.272, 1.257]		[0.447, 4.475]	
Order 4	0.321	0.335	1.827	1.882
	0.498	0.497	1.137	1.135
	[-0.641, 1.298]		[-0.343, 4.052]	

Table 6: Average Surplus General Bounds

ε	Data 1			Data 2		
	Max CS	Min CS	Dif CS	Max CS	Min CS	Dif CS
.001	56.283	45.771	10.512	56.451	45.774	10.677
.0001	53.073	48.927	4.146	53.070	48.924	4.146
.00001	51.603	50.088	1.515	51.606	50.130	1.476

