Trade, Technology, and the Great Divergence*

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Abstract

Why did per capita income divergence occur so dramatically during the 19th Century, rather than at the outset of the Industrial Revolution? How were some countries able to reverse this trend during the globalization of the late 20th Century? To answer these questions, this paper develops a trade-and-growth model that captures the key features of the Industrial Revolution and Great Divergence between a core industrializing region and a peripheral and potentially lagging region. The model includes both endogenous biased technological change and intercontinental trade. An Industrial Revolution begins as a sequence of more unskilled-labor-intensive innovations in both regions. We show that the subsequent co-evolution of trade and directed technologies can create a delayed but inevitable divergence in demographics and living standards—the peripheral region increasingly specializes in production that worsens its terms of trade and spurs even greater fertility increases and educational declines. Allowing for technological diffusion between regions can mitigate and even reverse divergence, spurring a reversal of fortune for peripheral regions.

Keywords: Industrial Revolution, unified growth theory, endogenous growth, demography, fertility, education, skill premium, North-South model, West-East model

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The huge asymmetries between advanced and developing countries have not disappeared, but they are declining, and the pattern for the first time in 250 years is convergence rather than divergence.

— Michael Spence, *The Next Convergence* (2011)

1. Introduction

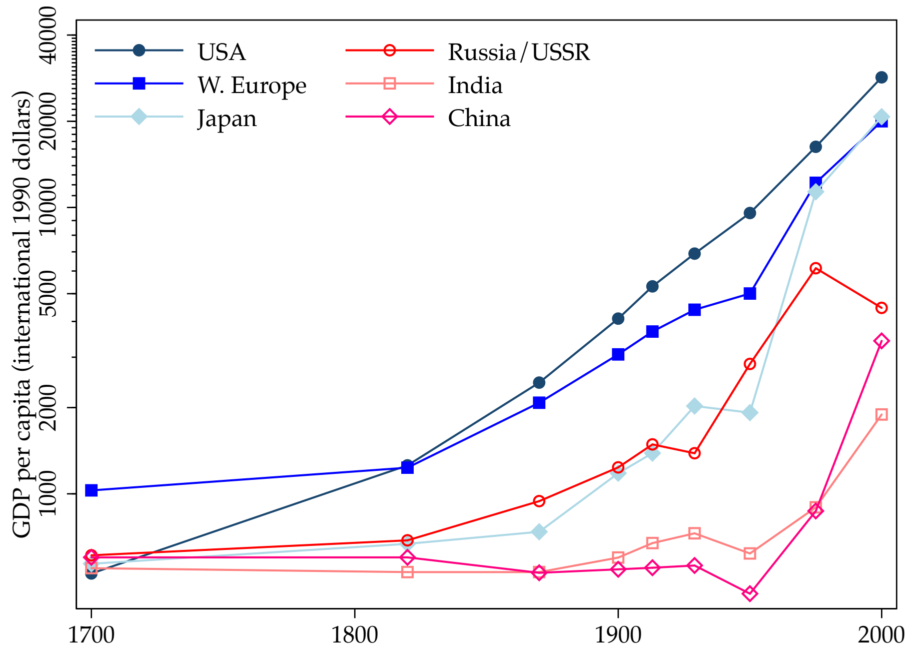
The last two centuries have witnessed dramatic changes in the global distribution of income and it remains as one of the great tasks of economics to explain why. Within that broad debate, there lingers the unresolved question as to whether globalization is a force for convergence or divergence.

As Figure 1 shows, at the dawn of the Industrial Revolution, circa 1750–1800, gaps in living standards between the richest and poorest economies of the world were roughly in the range 2 to 1. With industrialization came both income and population growth in a few core countries. But massive divergence in living standards across the globe did not emerge right away. It only got decisively underway a century later, in the latter half of the 19th century, at the time when the first great era of globalization started to take shape. Today the gap in material living standards between the richest and poorest economies of the world is of the order of 30 or 40 to 1, in large part due to the Great Divergence of the 1850–1950 era and its aftermath. And yet today, as the world experiences a second era of globalization, a few formerly developing countries in the "South" are now on an outward-oriented path towards convergence with the "North" and global inequality is starting to abate.

It seems to us an interesting coincidence that the unprecedented growth in 19th century inter-continental commerce (conceivably creating a powerful force for convergence by inducing countries to exploit their comparative advantages) coincided so precisely with an unprecedented divergence in living standards across the world. Why did incomes diverge just as the world became flatter? And yet, in another interesting coincidence, why are some poor countries today able to replace divergence with convergence, even in the midst of a second era of globalization? We want to confront these questions.

Globalization is a multidimensional phenomenon. In this paper, we focus on two dimensions that seem particularly relevant to the international distribution of income: rising levels of inter-continental trade and the faster diffusion of knowledge between countries. Economic theory is ambiguous about whether the former, in particular, pro-

Figure 1: *Real GDP per capita in six economies since 1700*



Source: Maddison (2010).

motes convergence or divergence between rich and poor countries. Standard neoclassical models may lean toward the former, but models with increasing returns, externalities, or path dependence, can favor the latter. Can a unified model be found which helps explain the very different experiences of the first and second eras of globalization?

The main goal of this paper is to present a unified growth model where both pro- and anti-convergence forces are potentially present, but where their relative strengths are generally state- and history-dependent. We argue that historical trade and technological growth patterns *jointly* sowed the seeds of divergence, contributing enormously to today's great wealth disparities, while they are now operating so as to mitigate these disparities.

Some stylized facts from economic history motivate our search for a new theory. One concerns the nature of industrialization itself—technological change in the Western world was unskilled-labor-intensive during the early Industrial Revolution but became relatively skill-intensive in the late 19th century. For example, the cotton textile industry, which along with metallurgy was at the heart of the early Industrial Revolution, could employ large numbers of unskilled and uneducated workers, thus diminishing the relative demand for skilled labor and education (Galor 2005; Clark 2007; de Pleijt and Weisdorf 2017). By the 1850s, however, two major changes had occurred—technological growth became much more widespread, and it became far more skill-using (Mokyr 2002).

Another factor of great importance was the rising role of international trade in the world economy. Precisely when inter-continental trade of goods became a major factor influencing wages and incomes continues to be a source of much debate; yet the academic consensus suggests that by the 19th century ocean going trade mattered greatly (O'Rourke and Williamson 1994; O'Rourke, Taylor, and Williamson 1996; O'Rourke and Williamson 1999, Chapter 4). Certainly by 1870 a number of innovations had dramatically reduced the cost of steam ocean transport, and real ocean freight rates fell by nearly 35% between 1870 and 1910 (Clark and Feenstra 2003). By 1900 the economic centers of non-Western regions such as Alexandria, Bombay, and Shanghai were fully integrated into the British economy, both in terms of transport costs and capital markets (Clark 2007). If one wishes to understand the Great Divergence over the last two centuries, one is well advised to analyze the role of trade.

Modeling Trade and Divergence We develop a two-region model with several key features mimicking these historical realities. The first key feature of the model is that we endogenize the extent and direction of bias of technological change in both regions. Technologies are sector specific, and sectors have different degrees of skill intensity. Following the endogenous growth literature, we will allow potential innovators in each region to observe factor use in different sectors and tailor their research efforts towards particular sectors, via directed technical change (Acemoglu 2002). Thus, the scope and direction of innovation will depend on each region's factor endowments and hence on its demography.

We also formulate the model in a way that allows for technological diffusion, where technologies can be employed by producers from regions other than where the technologies were originally developed. In our framework innovators from one region cannot prevent or profit from the use of their inventions in other regions, an assumption perhaps suitable for core-peripheral economies (see Eaton and Kortum 1999 for alternative diffusion processes more aligned with relationships among advanced economies).

The second key feature is that we endogenize demography itself. More specifically, we allow households to make education and fertility decisions based on market wages for skilled and unskilled labor, as in other endogenous demography and unified growth theories in which households face a quality-quantity tradeoff with respect to their children (Galor and Mountford 2006, 2008; Klemp and Weisdorf 2018). When the premium for skilled labor rises, families will tend to have fewer but better educated children.

The final feature is that we allow for trade between the two regions. Trade can occur due to differences both in sector-specific technologies (Ricardian) and in factor endowments (Heckscher-Ohlin).[1] Indeed we assume that initially each region has only one exogenous difference—when the model starts, the peripheral region is endowed with more unskilled workers relative to skilled workers, compared with the core region. This will have implications for both the pattern of trade flows and the direction of technical change.

In this manner, given our model, we can ask if such an initial endowment difference between two regions will result in dramatic per capita income divergence over time.[2]

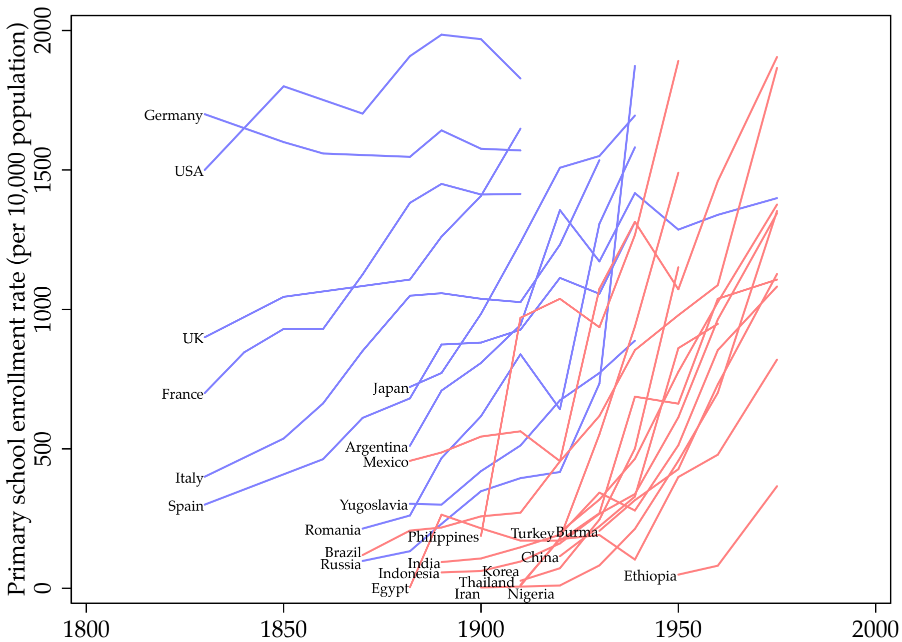
The answer? As is so often the case in economics, we find that "it depends." Why? We simulate the model in two basic ways:

- **North-South model** The first is what we will label the *North-South* model, where technologies are strictly locally developed and employed. We present the results for a baseline case which we suggest roughly captures the dynamics for the period from the 1700s to the mid-1900s. Because of the great abundance of unskilled labor in the world, innovators everywhere first develop unskilled-labor intensive technologies. Early industrialization is thus characterized by a fair amount of unskilled-labor-intensive technological growth and population growth *both* in the North and the South. At first, living standards even *slightly* converge during this period. Once trade becomes more specialized, however, the North becomes more focused on skill-intensive innovation and production. This induces a demographic transition of falling fertility and rising education rates in the North, while the South specializes in unskilled-labor-intensive production, inducing both unskilled-labor-intensive technological growth and further population growth (Figures 2 and 3). This feedback-driven population divergence fosters a deterioration in the South's terms of trade, forcing the South to produce more and more unskilled-labor-intensive production and generating even more fertility increases (Figure 4). Thus, although North and South have static gains from trade, these over time become a dynamic impediment to prosperity in the South, and living standards between the two regions diverge dramatically in the end.

[1] We know that Heckscher-Ohlin trade was important during the 19th century since commodity price convergence induced factor price convergence during this period (O'Rourke and Williamson 1999). And Mitchener and Yan (2014) suggest that unskilled-labor abundant China exported more unskilled-labor-intensive goods and imported more skill-intensive goods from 1903 to 1928, consistent with such a trade model.

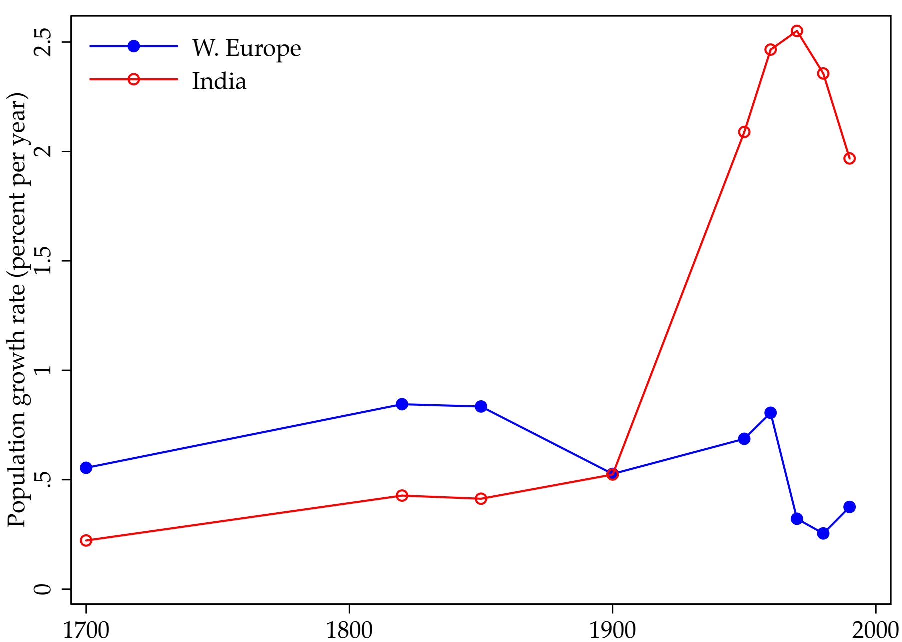
[2] While we can fully acknowledge the great importance of deep factors shaping historic divergence between core and periphery, such as the six "killer aps" in Ferguson (2012) or institutions in Acemoglu and Robinson (2012), here we abstract from all these to highlight the extent to which one proximate difference can shape divergences in growth paths.

Figure 2: *Primary school enrollment rates in many economies since 1800*



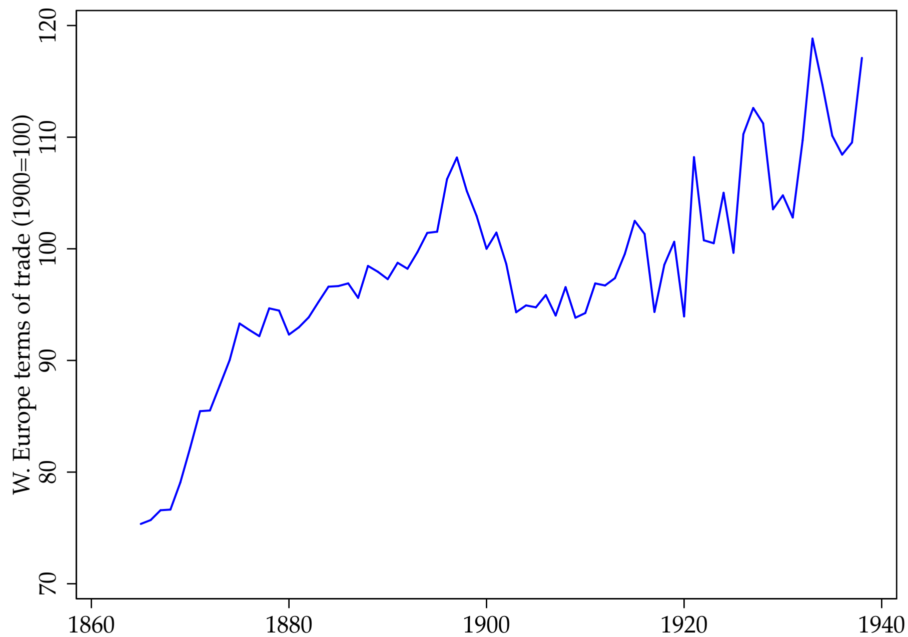
Source: Easterlin (1981).

Figure 3: *Population growth rates in the U.K. and India since 1700*



Source: Maddison (2010).

Figure 4: *Western European terms of trade 1870–1938 (1900 = 100)*



Source: Author's calculations based on Hadass and Williamson (2003).

- **West-East model** We also simulate an alternative scenario more reminiscent of the mid- and later- 20th century, and perhaps the 21st also. We label this case the *West-East* model, where now the skill-abundant region is the West and the unskilled labor abundant region is the East. The set-up is the same as before, except in this case we allow for the possibility of technological diffusion from one region to the other. Each region can either develop their own technologies for local use, or adopt from the pool of available technologies from the world, whichever is larger. We show that in this 'West-East' case trade and technological change can still interact so as to generate some divergence early on. This occurs because skill-oriented technologies developed in the West are somewhat "inappropriate" in the East, given endowment differences. But we also show that this divergence must eventually give way to convergence. This is because trade-induced skilled specialization in the West generates a deep pool of skill-intensive technologies from which even the East, with its relatively low endowment of skilled workers, can benefit in the end. That is, what is in the short run "inappropriate" (for a low-skill, high-fertility East) may in the long run benefit living standards by eventually leading the East down a high-skill, low-fertility path (see the rapid convergence in education for many 20th century economies in Figure 2).

Alternative Stories of Divergence We argue that analyzing the interactions between more and less developed regions—and between trade, innovation, and technological transfers—is critical to understanding the Industrial Revolution, the Great Divergence, and the subsequent convergence paths of some peripheral economies.

Many explanations of divergence rely on institutional differences between various regions of the world (North and Thomas 1973; Acemoglu et al. 2001, 2005). From that perspective economic growth is a matter of establishing the right "rules of the game," and underdevelopment is simply a function of some form of institutional pathology. Our model implicitly assumes that the institutional prerequisites for technological progress are in place, in all times and places. But divergence can still occur, and our model makes the important point that interactions between regions are an independent source of potential divergence and convergence. If so, it may be a mistake to think of differential growth patterns as having been solely generated by institutional differences in economies, which might as well have been operating in isolation from each other.

Another potential explanation for the divergence is that peripheral countries were specializing in *inherently* less-productive industries (Galor and Mountford 2006, 2008). But this is not very convincing—so called low-technology sectors such as agriculture enjoyed large productivity advances during the early stages of the Industrial Revolution (Lipsey and Bekar 1995; Clark 2007). And in the twentieth century, developing countries specialized in textile production which had experienced massive technological improvements more than a century before.

A related puzzle has to do with the size of the developing world. If fully one third of the world had become either Indian or Chinese by the twentieth century (Galor and Mountford 2002), why were Indians and Chinese not wealthier? After all, most semi-endogenous and endogenous growth theories have some form of scale effect, whereby large populations can spur innovation (Acemoglu 2010; Jones 2003).[3] Any divergence story that focuses on the explosive population expansion in peripheral economies faces this awkward implication from the canonical growth literature.

Relation to Galor and Mountford's "Trade and the Great Divergence" The paper presented here relates most closely and obviously to Oded Galor and Andrew Mountford's theoretical works on the Great Divergence (Galor and Mountford 2006, 2008) (henceforward 'GM'). These papers similarly suggest that the developing region's specialization

[3] More specifically, in such seminal endogenous growth models as Romer (1986, 1990), Segerstrom, Anant and Dinopolous (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991), a larger labor force implies faster growth of technology. In "semi-endogenous" growth models such as Jones (1995), Young (1998), and Howitt (1999), a larger labor force implies a higher level of technology.

in unskilled-intensive production stimulated fertility increases which lowered per capita living standards. However, our narrative of the North's launching into modernity and the South's vicious cycle of underdevelopment is distinct in a number of ways.

The first involves the nature of trade. GM establish two basic regimes—an autarkic one where each region evolves independently from the other, and a free trade one where the North specializes in the technologically dynamic sector while the South focuses on the stagnant sector. The implication they suggest is that divergence was an inevitable outcome of 19th Century globalization. In our framework free trade is possible throughout. Here we show that, even as local technologies diverge North and South, trade need not *immediately* foster monotonic divergence between core and periphery. In particular, our model indicates that so long as trade remains in a "diversified" configuration (more on this below) convergence in per capita incomes can occur, even if eventually, in the very long run, divergence will result.

However, this is not the only possibility and our model allows other configurations to appear. This arises because we endogenize both the scope and the direction of technological progress in both regions. GM make assumptions about the timing and speed of a purely exogenous technological growth process which they claim are "consistent with historical evidence." Specifically, they assume that: (1) modernization in either agriculture or in industry is not feasible initially; (2) modernization occurs first in agriculture; and (3) growth in industrial-sector productivity is faster than growth in agricultural-sector productivity. Compelling as these assumptions may seem, they are not universally shared. Rather than consigning the South to the inherently slower-growing industry, we endogenize the direction and speed of technologies in both regions.

These key differences allow our model to address two issues on which the GM approach is silent. The first has to do with the terms of trade between core and periphery. In our North-South setup, the South's specialization in unskilled-intensive goods allows for plenty of technological advance, but this does not promote per capita growth for two reasons. One is that it fosters fertility increases similar to the process outlined in GM. The other is that the South's terms of trade deteriorate over time. As the South's share of the world population grew, it floods world markets with its products. North's skill-intensive products become relatively scarcer, and thus fetch higher prices. The South has to provide more and more low-end exports to buy the same amount of high-end imports from North; through the impact of the terms of trade on factor prices, this raises fertility rates even more. This mechanism, absent in GM's work, suggests that productivity growth (and the scale of the Southern economy that generated this growth) could not save the South—in fact, it contributed to its relative decline.

Also absent from GM is the possibility of technological transfer. In our West-East setup we have this, and it would seem to be of great relevance in the 20th and 21st centuries, as peripheral economies have become increasingly capable of adopting ideas from the world technological frontier as a result of better education, better communications, multinational enterprise, value-chain participation, and so on (Baldwin 2016). Using data for both OECD and non-OECD countries for the latter 20th century, Klenow and Rodriguez-Clare (2005) suggest that during this period most gains in income levels above subsistence have been due to the international diffusion of knowledge. For more recent times technologies can be transferred in ways scarcely possible during the 19th century. For example, to facilitate knowledge transfers, multinationals often use local inventors working in affiliate inventor teams in developing countries, helping poor countries escape the knowledge trap (Branstetter et al. 2018). And the presence of foreign multinational enterprises during the 1990s and 2000s typically promoted the technological catch-up of local firms (Peri and Urban 2006, Bilir and Morales 2016). Our West-East model demonstrates that under the assumption of perfect technological diffusion the trade-technology interactions we emphasize can now work in the opposite direction: they may hasten divergence initially, but they will promote *convergence* in the longer term. Thus we provide the novel insight that the kind of technological diffusion regime in place may play a crucial role in determining whether or not trade generates per capita income convergence or divergence.

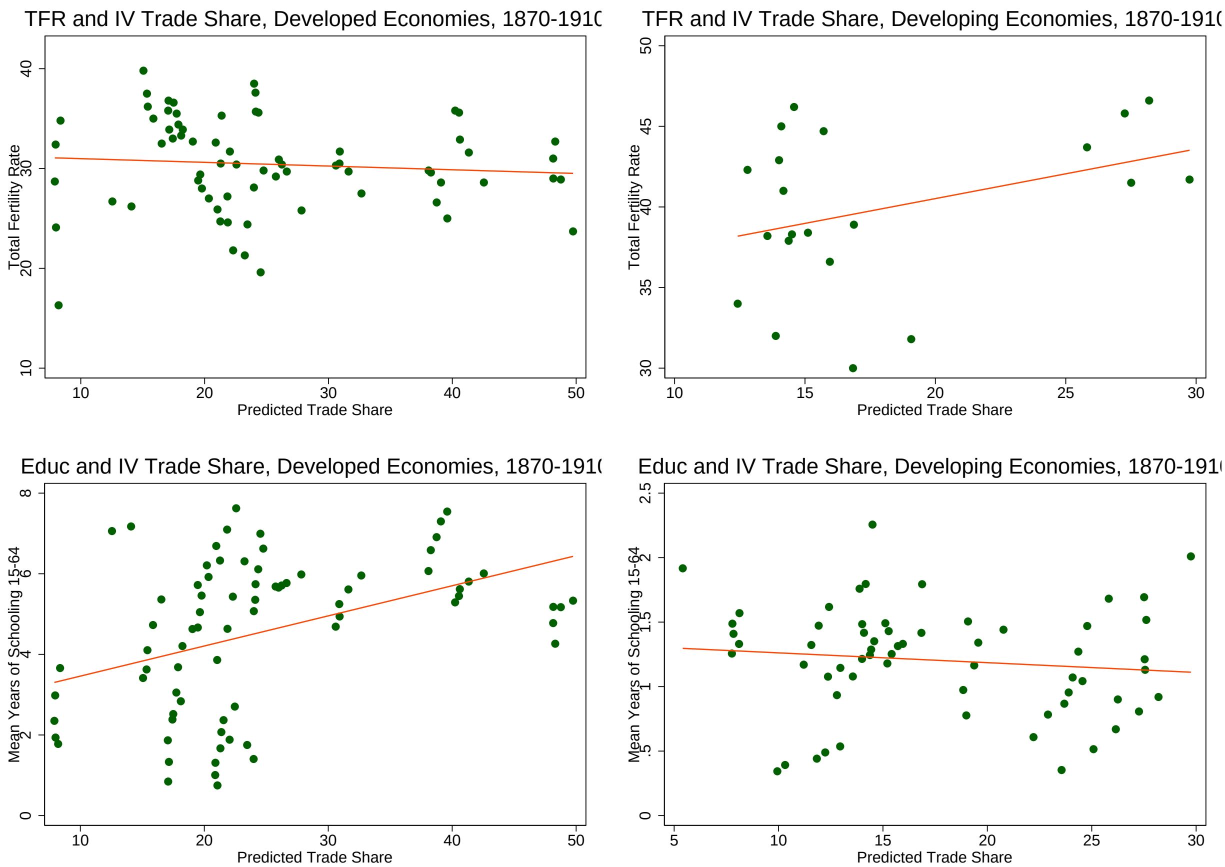
Some Correlations Between Trade and Demographics To briefly explore the empirical relationship between trade and demographic factors suggested by our model we pause to observe some correlations both for the late 19th Century economies and for the contemporaneous period.

First, in Figure 5 we show scatterplots for a decadal panel dataset spanning the years 1870–1910 for both developed regions and developing regions.[4] Here we use Lopez-Cordova and Meissner (2008)'s first-stage estimates for predicted trade shares, and correlate these with fertility and schooling data from Murtin (2013). Here we find that trade is associated with falling fertility and rising education in the "North." In the "South" on the other hand it is the opposite, at least in the cross-sectional relationship.

Next, in Figure 6 we show the same relationships for a cross-section of countries

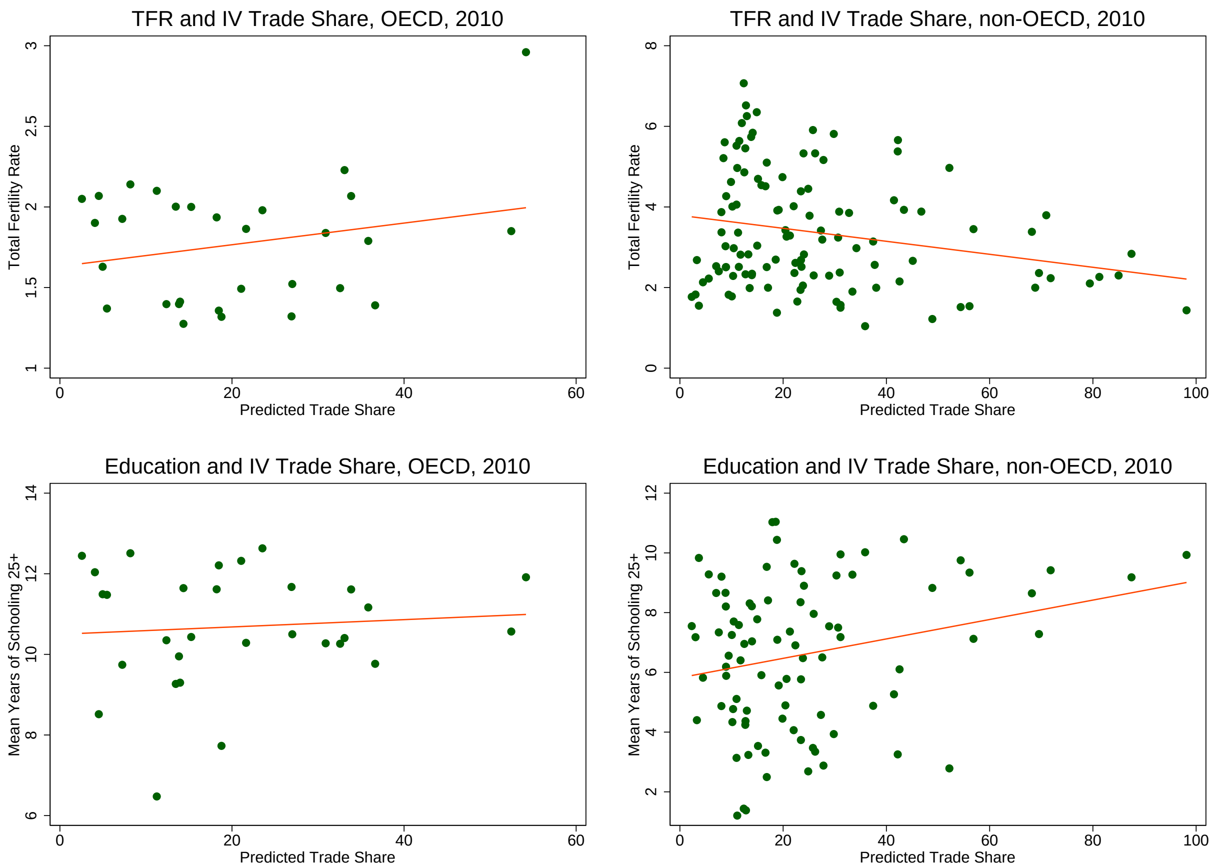
[4]Developed regions are considered to be Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland. Developing regions are considered to be Algeria, Argentina, Brazil, Chile, Costa Rica, Cuba, Egypt, El Salvador, Ghana, Guatemala, Honduras, India, Indonesia, Malyasia, Mexico, Nicagagua, Nigeria, Peru, Philippines, South Africa, Turkey, and Venezuela. Categorization from Murtin (2013). Not all data available for all regions and years.

Figure 5: *Demographics and Trade in the "North-South" World*



Sources: Lopez-Cordova and Meissner (2004); Murtin (2013).
Note: The scatter-plots show that greater trade during the late 19th century is associated with lower fertility and higher education in the developed world (left side diagrams), but *higher* fertility and *lower* education in the developing world (right side diagrams).

Figure 6: *Demographics and Trade in the "West-East" World*



Sources: Frankel et al. (1999); Weil (2013).
Note: The scatter-plots show that greater trade during the early 21st century is very weakly associated with higher fertility and education in OECD countries (left side diagrams), but *lower* fertility and *higher* education in non-OECD countries (right side diagrams).

during more recent times. Now we split up the world between the "West" (OECD countries) and the "East" (non-OECD countries). Now we use predicted trade shares estimated by Frankel and Romer (1999), and correlate these with fertility and education rates taken from Weil (2013). Now we observe the opposite tendency—greater trade is associated with lower fertility and higher education in the developing world as well. Indeed, in the developed world it now appears that the historic relation between trade and demographics has broken down, which would be consistent with regions having undergone, and essentially completed, their demographic transitions.

The basic point we make here is that the effects of globalization in the late 19th Century "North-South" world may well have been quite different from those in the late 20th Century and 21st Century "West-East" world.

2. Production with Given Technologies and Factors

We now present our model which describes a world consisting of a core economy and a peripheral economy and which we will use in both the ''North-South" and "West-East" scenarios in due course.

Total production for a region is given by

$$Y^i = \left(\frac{\alpha}{2} \left(Q_1^i + Z_1 \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(Q_2^i \right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} \left(Q_3^i - Z_3 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

$$Y^j = \left(\frac{\alpha}{2} \left(Q_1^j - Z_1 \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(Q_2^j \right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} \left(Q_3^j + Z_3 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where $\alpha \in [0,1]$ and $\sigma \geq 0$ is the elasticity of substitution among the three intermediate goods Q_1, Q_2, and Q_3. Note that $i \in \{N, W\}$ and $j \in \{S, E\}$; that is, we will refer to country i as either the North or the West, and country j as either the South or the East, depending on the technological regime. Z_1 is j's export of good 1 to country i, while Z_3 is i's export of good 3 to country j. Given initial factor endowments (which we explain below), i always exports sector 3 goods in exchange for sector 1 goods.

We will suppress country superscripts for now, re-introducing them in section 5. The production of intermediate goods is given by:

$$Q_1 = A_1 L_1, \quad (3)$$

$$Q_2 = A_2 L_2^{\gamma} H_2^{1-\gamma}, \quad (4)$$

$$Q_3 = A_3 H_3, \quad (5)$$

where A_1, A_2 and A_3 are the technological levels of sectors 1, 2, and 3, respectively.[5]

In turn, the technological levels of each sector are represented by an aggregation of *sector-specific* machines per worker. Specifically,

$$A_1 = \int_0^{N_1} \left(\frac{x_1(j)}{L_1} \right)^{\alpha} dj, \tag{6}$$

$$A_2 = \int_0^{N_2} \left(\frac{x_2(j)}{L_2^{\gamma} H_2^{1-\gamma}} \right)^{\alpha} dj, \tag{7}$$

$$A_3 = \int_0^{N_3} \left(\frac{x_3(j)}{H_3} \right)^{\alpha} dj, \tag{8}$$

where $x_i(j)$ is the number of machines of type j that can be employed only in sector i. Intermediate producers choose the amounts of these machines to employ, but the number of *types* of machines in each sector is exogenous to producers. Technological progress in sector i can then be represented by growth in this number of machine-types for the sector, which we denote by N_i (we endogenize the growth of these in the next sections by introducing researchers).

Treating technological coefficients as exogenous for the time being, we assume that markets for both the final good and the intermediate goods are perfectly competitive. Thus, prices are equal to unit costs. Solving the cost minimization problems for producers, and normalizing the price of final output to one, yields the unit cost functions

$$1 = \left[\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{9}$$

$$p_1 = \frac{w_l}{A_1}, \tag{10}$$

$$p_2 = \left(\frac{1}{A_2}\right) w_l^{\gamma} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{-\gamma}, \tag{11}$$

$$p_3 = \frac{w_h}{A_3}, \tag{12}$$

where p_i denotes the price for intermediate good Q_i, w_l is the wage paid to L and w_h is the wage paid to H.

[5] Thus sectors vary by *skill*-intensity. While our interest is mainly in the "extreme" sectors (1 and 3), we require an intermediate sector so that production of intermediate goods are determined both by relative prices and endowments, and not pre-determined solely by endowments of L and H. This will be important when we introduce trade to the model.

Full employment of unskilled and skilled labor implies factor-market clearing, with

$$L = \frac{Q_1}{A_1} + \frac{w_l^{\gamma-1} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{1-\gamma} Q_2}{A_2}, \tag{13}$$

$$H = \frac{w_l^{\gamma} w_h^{-\gamma} (1-\gamma)^{\gamma} \gamma^{-\gamma} Q_2}{A_2} + \frac{Q_3}{A_3}. \tag{14}$$

Finally, the demands for intermediate goods from final producers can be derived from a standard CES objective function.[6] Specifically, as shown in the appendix, intermediate goods market clearing requires

$$Q_i = \frac{\xi_i^{\sigma} p_i^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma}} \Bigg) \Bigg(Y, \tag{15}$$

for $i = 1, 2, 3$, and for convenience we define constants $\xi_1 = \xi_3 = \frac{1}{2}\alpha$, and $\xi_2 = 1 - \alpha$.

Provided that we have values for L, H, A_1, A_2, and A_3, along with parameter values, we have thirteen equations [(1)–(5), (9)–(14), and three instances of (15)] with thirteen unknowns [Y, p_1, p_2, p_3, Q_1, Q_2, Q_3, w_l, w_h, L_1, L_2, H_2, H_3]. The solution for all of these variables constitutes the solution for the *static* model in the case of exogenously determined technological and demographic variables.

3. Endogenizing Technologies in Both Regions

In this section we describe how innovators endogenously develop new technologies. In general, modeling purposive research and development effort is challenging when prices and factors change over time. This is because it is typically assumed that the gains from innovation will flow to the innovator over time, and this flow will depend on the price of the product being produced and the factors required for production at each moment in time.[7] If prices and factors are constantly changing (as they will in any economy where trade barriers fall gradually or factors evolve endogenously), a calculation of the true discounted profits from an invention may be impossibly complicated.

To avoid such needless complication but still gain from the insights of endogenous growth theory, we will assume that the gains from innovation last *one time period only.*

[6] Here demands will be negatively related to own price, will be a function of a price index, and will be proportional to total product.

[7] For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is total workforce and L_R is the number of researchers. Calculating this value is fairly straightforward if supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

More specifically, technological progress is sector-specific, and comes about through increases in the numbers of varieties of machines employed in each sector. The new varieties of machines are developed by profit-maximizing inventors, who for one period can *monopolistically* produce and sell the machines to competitive producers of the intermediate goods Q_1, Q_2, or Q_3. However, we assume that the blueprints to these machines become public knowledge in the period after the machine is invented, at which point these machines become old and are *competitively* produced and sold.[8] Thus while we need to distinguish between old and new sector-specific machines, we avoid complicated dynamic programming problems inherent in multiple-period profit streams.[9]

With these assumptions, we can re-define sector-specific technological levels in equations (6)–(8) as a sum over old and new machines at time t (once again suppressing region superscripts) with

$$A_{1,t} = \left(\int_0^{N_{1,t-1}} x_{1,old}(j)^\alpha dj + \int_{N_{1,t-1}}^{N_{1,t}} x_{1,new}(j)^\alpha dj \right) \left(\frac{1}{L_1} \right)^\alpha , \quad (16)$$

$$A_{2,t} = \left(\int_0^{N_{2,t-1}} x_{2,old}(j)^\alpha dj + \int_{N_{2,t-1}}^{N_{2,t}} x_{2,new}(j)^\alpha dj \right) \left(\frac{1}{L_2^\gamma H_2^{1-\gamma}} \right)^\alpha , \quad (17)$$

$$A_{3,t} = \left(\int_0^{N_{3,t-1}} x_{3,old}(j)^\alpha dj + \int_{N_{3,t-1}}^{N_{3,t}} x_{3,new}(j)^\alpha dj \right) \left(\frac{1}{H_3} \right)^\alpha , \quad (18)$$

where $x_{i,old}$ are machines invented before t, and $x_{i,new}$ are machines invented at t. Thus in each sector i there are $N_{i,t-1}$ varieties of old machines that are competitively produced, and there are $N_{i,t} - N_{i,t-1}$ varieties of new machines that are monopolistically produced (again, suppressing country subscripts).

Next, we must describe the producers of the intermediate goods. In each region, these three different groups of producers each separately solve the profit maximization problems

$$\text{Sector 1 producers:} \quad \max_{[L_1, x_1(j)]} \left\{ p_1 Q_1 - w_l L_1 - \int_0^{N_1} \chi_1(j) x_1(j) dj \right\},$$

$$\text{Sector 2 producers:} \quad \max_{[L_2, H_2, x_2(j)]} \left\{ p_2 Q_2 - w_l L_2 - w_h H_2 - \int_0^{N_2} \chi_2(j) x_2(j) dj \right\},$$

$$\text{Sector 3 producers:} \quad \max_{[H_3, x_3(j)]} \left\{ p_3 Q_3 - w_h H_3 - \int_0^{N_3} \chi_3(j) x_3(j) dj \right\},$$

[8] Here one can assume either that patent protection for intellectual property lasts one time period, or that it takes one time period for potential competitors to reverse-engineer the blueprints for new machines.

[9] See Rahman (2013) for more discussion of this simplifying (but arguably realistic) assumption.

where $\chi_i(j)$ is the price of machine j employed in sector i. For each type of producer, their maximization problem with respect to machine j yields machine demands

$$x_1(j) = \chi_1(j)^{\frac{1}{\alpha-1}} (\alpha p_1)^{\frac{1}{1-\alpha}} L_1 , \tag{19}$$
$$x_2(j) = \chi_2(j)^{\frac{1}{\alpha-1}} (\alpha p_2)^{\frac{1}{1-\alpha}} L_2^{\gamma} H_2^{1-\gamma} , \tag{20}$$
$$x_3(j) = \chi_3(j)^{\frac{1}{\alpha-1}} (\alpha p_3)^{\frac{1}{1-\alpha}} H_3 . \tag{21}$$

New machine producers, having the sole right to produce the machine, will set the price of their machines to maximize the instantaneous one-period profit. This price will be a constant markup over the marginal cost of producing a machine. Assuming that the cost of making a machine is unitary implies that their prices will be set at $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1/\alpha$ for new machines. Thus, substituting in this mark-up price, and realizing that instantaneous profits are $(1/\alpha) - 1$ multiplied by the number of new machines sold, instantaneous revenues for new machine producers are given by

$$r_1 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_1)^{\frac{1}{1-\alpha}} L_1 , \tag{22}$$
$$r_2 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_2)^{\frac{1}{1-\alpha}} L_2^{\gamma} H_2^{1-\gamma} , \tag{23}$$
$$r_3 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_3)^{\frac{1}{1-\alpha}} H_3 . \tag{24}$$

Old machines, on the other hand, are competitively produced, and this drives the price down to marginal cost, so prices will be set at $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1$ for all old machines. Sectoral productivities can then be expressed simply as a combination of old and new machines demanded by producers. Plugging in the appropriate machine prices into our machine demand expressions (19)–(21), and plugging these machine demands into our sectoral productivities (16)–(18), we can solve for the productivities

$$A_1 = \left(N_{1,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{1,t} - N_{1,t-1})\right) (\alpha p_1)^{\frac{\alpha}{1-\alpha}} , \tag{25}$$
$$A_2 = \left(N_{2,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{2,t} - N_{2,t-1})\right) (\alpha p_2)^{\frac{\alpha}{1-\alpha}} , \tag{26}$$
$$A_3 = \left(N_{3,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{3,t} - N_{3,t-1})\right) (\alpha p_3)^{\frac{\alpha}{1-\alpha}} . \tag{27}$$

Thus, given the number of types of old and new machines that can be used in each sector (where the evolution of these will be described below in section 5.1), we can then simultaneously solve equations (9)–(15) and (25)–(27) to solve for prices, wages, intermediate goods, and technological levels for a hypothetical economy.

4. Endogenizing Population and Labor-Types in Both Regions

Our next goal is to endogenize the levels of skilled and unskilled labor in this hypothetical economy. We utilize an overlapping generations framework where individuals in each region live for two periods. In the first period, representing youth, individuals work as unskilled workers to earn income for their parents; this income is consumed by their parents. In the second period, representing adulthood, individuals decide whether or not to expend a fixed resource cost to become a skilled worker. Adults also decide how many children to have, and these children earn unskilled income for the adults. Adults, however, forgo some income for child-rearing.

Specifically, we assume that each adult (later indexed by i) has the objective to maximize current-period income. If an adult chooses to remain an unskilled worker (L), she aims to maximize income I_l with respect to her number of children, where

$$I_l = w_l + n_l\, w_l - w_l\, \lambda\, (n_l - 1)^{\phi}\,, \tag{28}$$

where w_l is the unskilled labor wage, n_l is the number of children that the unskilled adult has, and $\lambda > 0$ and $\phi > 1$ are constant parameters that affect the opportunity costs to child-rearing. Note that the costs here include a term in the form $(n_l - 1)$ to ensure that at least replacement fertility is maintained.

If an adult chooses to spend resources to become a skilled worker, she instead maximizes income I_h with respect to her number of children, where

$$I_h = w_h + n_h\, w_l - w_h\, \lambda\, (n_h - 1)^{\phi} - \tau_i\,, \tag{29}$$

where w_h is the skilled labor wage, n_h is the number of children that the skilled adult has, and τ_i is the resources she must spend to become skilled.

We solve the resulting first order conditions to obtain optimal fertility. For convenience we solve for fertility in excess of replacement, denoted n_l^* and n_h^*, to obtain

$$n_l^* \equiv n_l - 1 = (\phi\lambda)^{\frac{1}{1-\phi}}\,, \tag{30}$$

$$n_h^* \equiv n_h - 1 = (\phi\lambda w_h / w_l)^{\frac{1}{1-\phi}}\,. \tag{31}$$

Note that with $w_h > w_l$, and given $\phi > 1$, optimal fertility for a skilled worker is always smaller than that for an unskilled worker (simply because the opportunity costs of child-rearing are larger for skilled workers). Also note that the fertility for unskilled workers is constant, while the fertility for skilled workers is decreasing in the skill premium w_h/w_l.

Finally, we assume that τ varies across each adult i. The resource costs necessary to acquire an education can vary across individuals for many reasons, including differing incomes, access to schooling, or innate abilities. For tractability we assume that over all adults i, the level of τ_i is uniformly distributed across $[0, b]$, where $b > 0$.

An individual adult i who draws a particular τ_i will choose to become a skilled worker only if her optimized income as a skilled worker will be larger than her optimized income as an unskilled worker. Let us call τ^* the *threshold* cost to education; this is the education cost where the adult is indifferent between becoming a skilled worker or remaining an unskilled worker. Solving for this, we get

$$\tau^* = w_h + n_h^* \, w_l - w_h \, \lambda \, n_h^{*\phi} - w_l - w_l \, n_l^* + w_l \, \lambda \, n_l^{*\phi} \, . \tag{32}$$

Only individuals whose τ_i falls below this level will opt to become skilled.

Figure 5 shows the optimal fertility rates for two hypothetical individuals—one with a relatively high τ and one with a relatively low τ. As for earnings, the straight lines show how income increases as adults have more children; their slope is the unskilled wage (w_l) received by children; the own-wage intercept for a skilled worker (w_h) is higher than for an unskilled adult (w_l). As for costs, the cost curves get steeper with more children since $\phi > 1$. For skilled individuals, the cost curve is both higher (to illustrate the resource costs τ necessary to become skilled) and steeper (to illustrate the higher opportunity cost of having children). Notice then that the only difference between the high-τ individual and the low-τ individual is that the latter has a lower cost curve. Given these differences in the fixed costs of education, we can see that the high-τ individual will opt to remain an unskilled worker (and so have a fertility rate of n_l^*), while the low-τ individual will choose to become skilled (and have a fertility rate of n_h^*).

With all the above household machinery in place, we can now describe aggregate supplies of skilled and unskilled labor (the demands for labor are described by full employment conditions (13) and (14)), fertility, and education. Given a total adult population equal to pop, we obtain

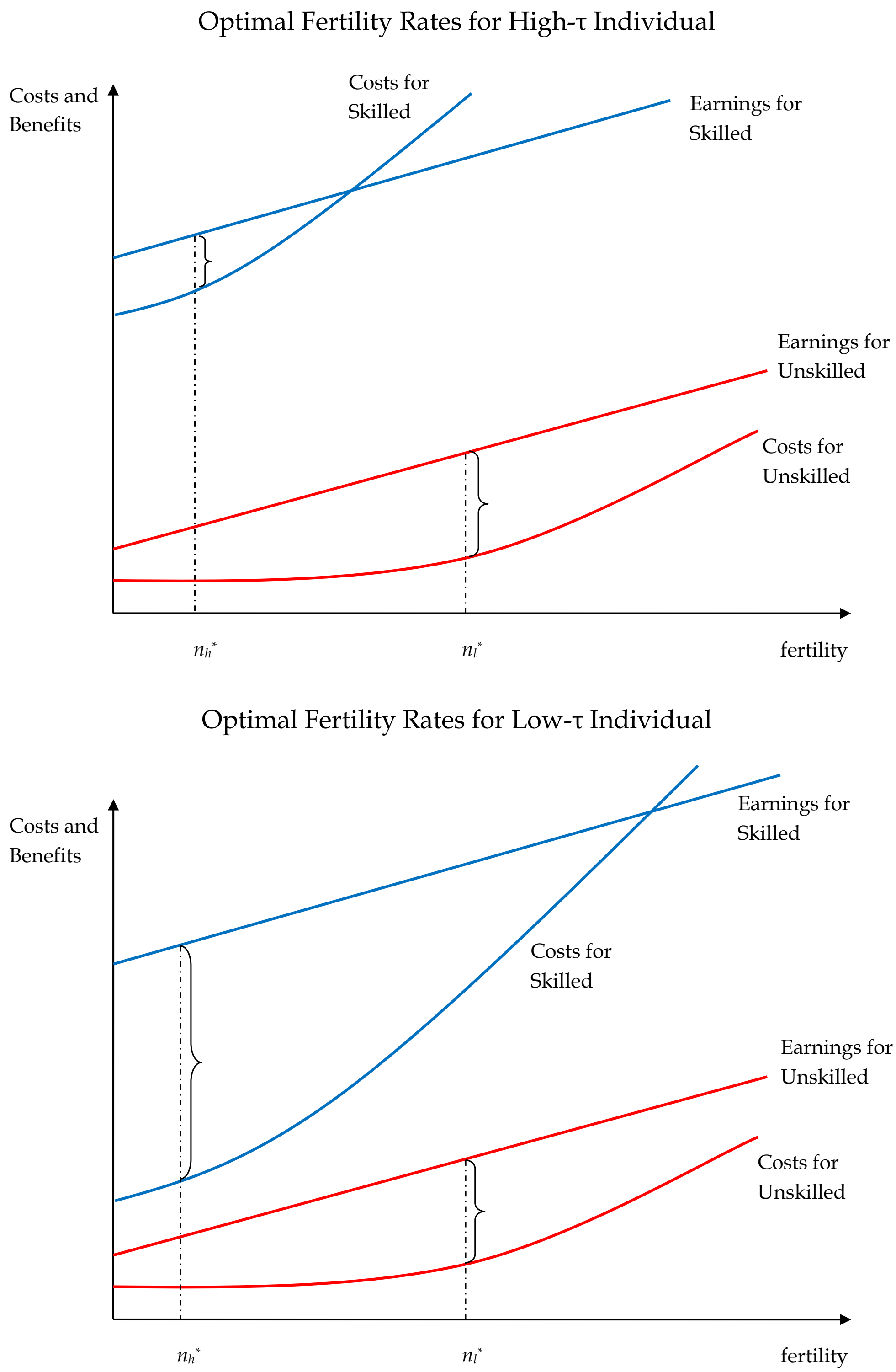
$$H = \left(\frac{\tau^*}{b}\right) pop \, , \tag{33}$$

$$L = \left(1 - \frac{\tau^*}{b}\right) pop + n \cdot pop \, , \tag{34}$$

$$n = \left(1 - \frac{\tau^*}{b}\right) n_l^* + \left(\frac{\tau^*}{b}\right) n_h^* + 1 \, , \tag{35}$$

$$e = \frac{\tau^*}{b} \, , \tag{36}$$

Figure 7: *Optimal Fertility Rates for High and Low τ Individuals (for given w_l and w_h)*



where H is the number of skilled workers (comprised strictly of old workers), L is the number of unskilled workers (comprised of both old workers and all the young), n is aggregate fertility including replacement (the term 1 is added in), e is the fraction of the workforce that gets an education, and n_l^*, n_h^*, and τ^* are the optimized fertility rates and threshold education cost given respectively by (30), (31), and (32).

This completes the description of the static one-country model. The next section uses this model to describe *two* economies that endogenously develop technologies and trade with each other to motivate a story of world economic history.

5. The Roles Played by the Evolution of Trade and Technologies in Historical Divergence/Convergence

In this section we show how interactions between the growth of trade and evolving factor-biased technologies could have contributed to the Great Divergence of the late 19th and early 20th centuries. We go on to show how such interactions could also have induced per capita income convergence in the later 20th century. Our approach is to simulate two economies, now indexed by c for each country or region. The above model describes one hypothetical country—but we now use it to describe either a Northern and a Southern economy, in a setup where applied technologies are strictly locally used; as well as a Western and an Eastern economy, in a setup where technologies developed in one region may be diffused and utilized in the other region. In each case one region will begin with more unskilled labor than the other.

A key issue here is the nature of technological progress and diffusion. We argue that early industrialization was characterized by locally-grown technologies, whereby regions developed their own production processes appropriate for local conditions, and where global technological diffusion was of minimal importance. On the other hand, we later conjecture that 20th century growth saw developing economies move to adopting technologies from the world knowledge frontier (Pack and Westphal 1986; Romer 1992).

These two simulations of our model serve to demonstrate a number of things. Early industrialization in both regions is relatively unskilled labor intensive (O'Rourke, Rahman, and Taylor 2013). Trade between the two regions generates some income convergence early on—specialization induces the North to devote R&D resources to the skill-intensive sector and the South to devote resources to the unskilled-intensive sector. Because the skilled sector is so much smaller than the unskilled sector, the South is able to grow relatively faster at first. But the dynamic effects of these growth paths (notably fertility and education changes) ultimately limit and then sharply reverse income convergence.

The mutually reinforcing interactions between technological growth and intercontinental commerce then kick in and drive a dramatic divergence between the incomes of northern and southern economies.

However, our second set of simulations show that, when industrialization is characterized by diffusion of technologies from the world frontier (generated by both the West and the East), the dynamic of West-East income differentials takes a dramatically different turn as compared with the North-South case. This case still produces some divergence early on, due to the technology-skill mismatch of "inappropriate" imported technologies flowing to but not being used by the East (Basu and Weil 1998). But the West goes on to develop such highly advanced skill-biased technologies that these are, eventually, implemented by the East regardless. The East is then induced to proceed through its own demographic and education transitions, and starts to catch up on the West. Indeed, we demonstrate that if the transition is dramatic enough, the East can even start developing its own skill-intensive technologies and overtake the West as technological leader.

Our simulations reveal how trade and technological change feed off each other to generate growth paths that broadly mirror historic trends. Distinct from Galor and Mountford (2008), we find that the technological environment determines the qualitative impact of trade on convergence-versus-divergence dynamics in the global economy.

5.1. A Dynamic Model—The Evolution of Technology and Trade

How do technologies evolve in each region? We will assume that a region will either develop its own blueprints N, or else adopt blueprints from the world frontier. The upcoming discussion relates to the former case. We turn to the latter case in a moment.

Recall that equations (22)–(24) describe one-period revenues r for research and innovation. Offsetting these are some resource costs $C(.)$ to research and innovation, which we now specify. We assume that these costs are rising in N ("applied" knowledge, the already-known blueprints or machine-types specific to each sector and to each country), and falling in some measure of "general" knowledge, given by B (basic knowledge, which is global, and thus common across all sectors and countries). The former means that it gets harder to make innovations when more innovations have already been made, but the latter means that it gets easier the more basic knowledge you have.

Specifically, we assume that the no-arbitrage (free entry) condition for potential researchers in each region can be written

$$r_i^c \leq C(N_i^c, B)\,, \tag{37}$$

for country $c = n, s$ and sector $i = 1, 3$.[10] Specifically, we assume the following functional form for these research costs,

$$C(N_i^c, B) = \left(\frac{N_{i,t+1}^c}{B_t} \right)^{\nu} , \tag{38}$$

for some $\nu > 0$. Now, given a level of basic knowledge B_t, which we now assume grows at an exogenous rate, and the number of existing machines $N_{i,t+1}^c$, we can determine the resource costs of research and innovation. When basic knowledge is low relative to the number of available machine-types in sector i, the cost of inventing a new machine in sector i is high (see O'Rourke et al. 2013 for a fuller discussion). Thus from (37) and (22)–(24) we see that innovation in sector i becomes more attractive when basic knowledge is large, when the number of machine-types in sector i is low, when the price of good i is high, and when employment in sector i is high.

Note that if $r_i^c > C(N_i^c/B)$ there are profits from research in sector i in region c. However, this will induce local research activity, increasing the number of new machines, and hence the costs of research. In equilibrium, we assume that free entry ensures that N_i adjusts up or down such that costs of research and innovation equal the revenues of new machine production. Thus, increases in global B are matched by increases in local levels of N_i^c such that the no-arbitrage condition holds with equality whenever technological growth in the sector occurs.

Note that there is a nontraded good: we assume that there is no trade in Q_2. Because this good is produced using both L and H, differences in p_2 are very small between the North and the South, and thus the assumption is not very restrictive or important.[11,12]

5.2. Evolution of the World Economy

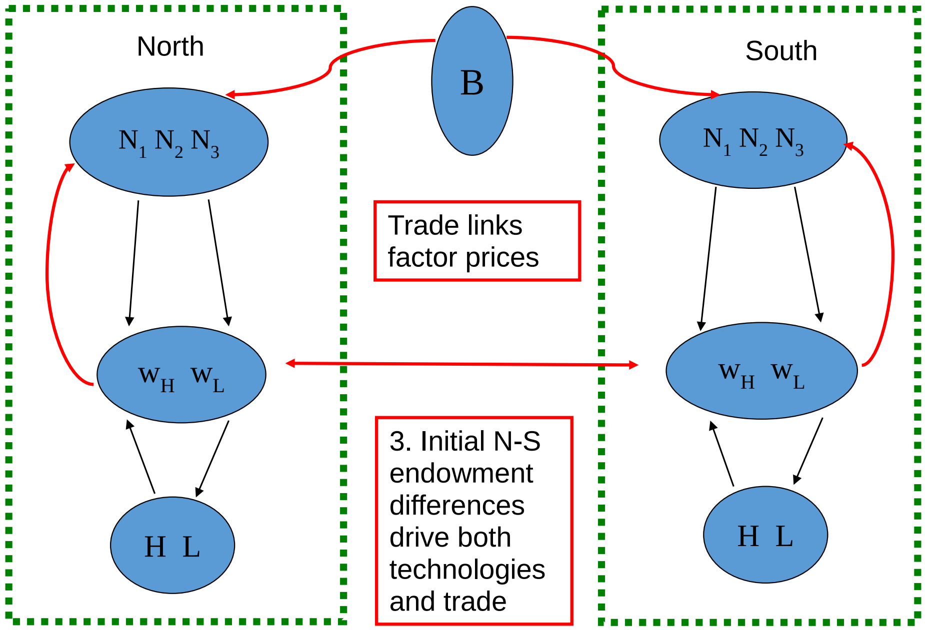
World general equilibrium in any period is a 36-equation system that, given changes in the number of machine blueprints, solves for prices, wages, fertility, education, labor-types, intermediate goods, employment, trade, and sectoral productivity levels for both the North and the South.

[10]For analytical convenience we assume no research occurs in sector 2, so that technological growth is unambiguously factor-biased. Cases where all three sectors grow technologically complicates the model but do not change the evolutions of either economy.

[11]Indeed, trade in all three goods would produce an analytical problem. It is well known among trade economists that when there are more traded goods than factors of production, country-specific production levels, and hence trade volumes, are indeterminate. See Melvin (1968) for a thorough discussion.

[12]One can conceive of Q_2 as the technologically-stagnant and non-tradeable "service" sector. Thus each labor-type can work either in manufacturing or in services.

Figure 8: *Elements of an Open Economy "North-South" Model*

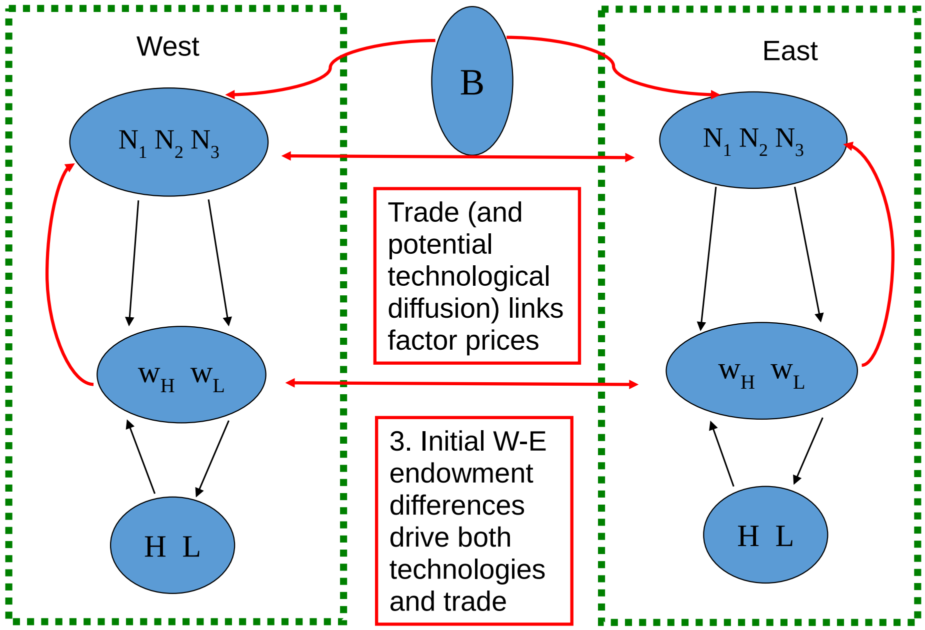


We impose only one parameter difference between the two regions: we will assume that $b^i < b^j$. This is a way to create an initial asymmetry between the regions to match the historical record: it means that there is a bigger range of resource costs for education in the South and East, and this allows us to match the stylized fact that these regions begin with relatively more unskilled labor than skilled labor. All other parameters are *exactly* the same in both regions. The equilibrium is described in more detail in the appendix.

Due to considerable model complexity, we solve for general equilibrium numerically. Specifically, we assume that basic technology (B in equation 38) starts low enough so that technological progress is not possible in any region. Our world is thus at first in stasis. We allow, however, for exogenous growth in basic knowledge, and then solve for the endogenous variables in each period.

Figure 8 provides a diagram summarizing the North-South simulations. The one exogenous growth variable, B, is accessible to both regions. This will affect sector-level productivities in different ways however, as these interact with wages and factors unique to each region. Demography evolves endogenously as well, influencing sectoral technological developments. Figure 9 provides a similar diagram summarizing the West-East simulations. This involves the same setup, except here there is the potential to share applied knowledge (and not just basic knowledge).

Figure 9: *Elements of an Open Economy "North-South" Model*



As a start, we can summarize the evolution of our two economies with a few propositions, starting with the nature of early industrialization in the world.

Proposition 1. *If* $N_1 = N_3$, $L > H$, *and* $\sigma > 1$, *then initial technological growth will be unskilled-labor biased.*

Remark. From (22)–(24) we can see that revenues from innovation rise both in the price of the intermediate good (the "price effect") and in the scale of sectoral employment (the "market-size effect"). If intermediate goods are grossly substitutable, market-size effects will outweigh price effects (see Acemoglu 2002 for more discussion of this).

Recall that GM had to exogenously order the sequence of modernization events by sector. However, in our model, the sequencing now materializes endogenously due to directed technological change. Here we see that, as basic knowledge exogenously grows, sector 1 will be the first to modernize if unskilled labor is relatively abundant. The logical implication of this is that early industrialization around the world (provided there are intellectual property rights in these countries) will be relatively unskilled-labor-intensive, as was indeed the case (O'Rourke et al. 2013). □

Proposition 2. *For certain ranges of factors and technologies, the trade equilibrium implies that* $Q_3^s = 0$*. For other ranges of technologies and factors, the trade equilibrium implies that* $Q_1^n = 0$*.*

Remark. As trade technologies improve, trade costs fall, and economies specialize more and more. And divergent technological growth paths can help reinforce this specialization. There is indeed a point where region i no longer needs to produce any Q_1 (they just import it from region j), and region j no longer needs to produce any Q_3 (they just import it from region i). These cases will be called "specialized trade equilibria" (either partial or full). These cases are described in detail in the appendix. □

Of course, both trade and technological changes will change factor payments. The final proposition states how these changes can affect the supplies of the factors of production themselves.

Proposition 3. *If* $\phi > 1$*, any increase in* w_l *(keeping* w_h *constant) will induce a decrease in* e *and an increase in* n*; furthermore, so long as* ϕ *is "big enough," any increase in* w_h *(keeping* w_l *constant) will induce an increase in* e *and a decrease in* n*.*

Proof. Substituting our expressions for n_l^* and n_h^*, given by (30) and (31), into our expression for τ^*, given by (32), and rearranging terms a bit, we obtain

$$\tau^* = (w_h - w_l) \; - \; w_l \lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right) + \; w_l^{\frac{\phi}{\phi-1}} w_h^{\frac{1}{1-\phi}} \lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right) .$$

What must be true to have the condition $\partial\tau^*/\partial w_l < 0$ hold? Solving for this and rearranging yields

$$\left(\frac{w_l}{w_h}\right)^{\frac{1}{\phi-1}} \; < \; 1 \; + \; \left[\frac{1}{\lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right)} \right]$$

Since the inverse of the skill-premium is always less than one, this expression always holds for any $\phi > 1$. What must be true to have the condition $\partial\tau^*/\partial w_h > 0$ hold? Solving and rearranging yields

$$\lambda^{\frac{1}{\phi}} \phi > \frac{w_l}{w_h} .$$

Thus, for a given value of λ, ϕ needs to be large enough for this condition to hold. Finally, our expression for total fertility, (35), can be slightly rearranged as

$$n = n_l^* \; + \; (n_h^* - n_l^*) \left(\frac{\tau^*}{b} \right) + \; 1 \, .$$

From (30) and (31) we know that the second term is always negative, and that n_l^* is constant. So any increase in education from wage changes will lower aggregate fertility, and any decrease in education from wage changes will increase aggregate fertility. □

5.3. Simulations

Here we simulate the model described above to analyze the potential sources of North-South and West-East divergence in history. Initial basic knowledge and iceberg trade costs are set such that neither technological growth nor trade is possible at first. However, exogenously over time basic knowledge rises and iceberg trade costs fall. We run the simulation for 75 time periods to roughly capture major economic trends during two distinct economic epochs.

The parameter values in the simulations are as follows: $\sigma = 3$, $\alpha = 0.5$, $\gamma = 0.5$, $\lambda = 0.5$, $\phi = 10$, $\nu = 2$. These values ensure that Propositions 1 and 3 hold; beyond that, our qualitative findings are not sensitive to specific parameter values. We set $b^i = 2$, $b^j = 6$, and $pop = 2$; this gives us initial factor endowments of $L_i = 3.14$, $L_j = 3.48$, $H_i = 0.86$, $H_j = 0.52$. Initial machine blueprints for both countries are set to be $N_1 = 10$, $N_2 = 15$, $N_3 = 10$. Initial B is set high enough in all scenarios so that growth in at least one sector is possible early in the simulation; B grows 2 percent each time period.

Divergence Implications Preview Before presenting our three main sets of simulation results in full detail, we begin by previewing our main findings on long-run divergence, since matching these stylized facts was our core empirical goal in constructing this model. Recall that we can solve the model using two different basic sets of assumptions, localized technological development and perfect diffusion of technologies.

There are essentially one of three possible trade equilibria in each period. These differences essentially have to do with different "cones of diversification" (Feenstra 2004). *Diversified trade* is the case where all three sectors are active in both regions. *Partially specialized trade* is where one region has abandoned the production of one sector. In this case the region imports all of that product from the other region. Finally, *fully diversified trade* is where both regions have abandoned production of one sector. As we will see, these differing diversification cones will play a major role in divergence patterns.

The implications that these different equilibria will have on per capita income gaps is shown in the following 2×2 matrix. The no-trade examples are less relevant, so we then go on to a full analysis of three illustrative free-trade simulations (the no diffusion case, the perfect diffusion case, and an intermediate gradual diffusion case) in more detail.

	North-South Model (Localized Technologies)	**West-East Model** (Diffusion of Technologies)
Free Trade	Convergence then Divergence	Divergence then Convergence
No Trade	Stronger Convergence	More Gradual Convergence

The North-South Model of Localized Technological Progress: Analysis In this case the summary of the trade regimes is as follows:

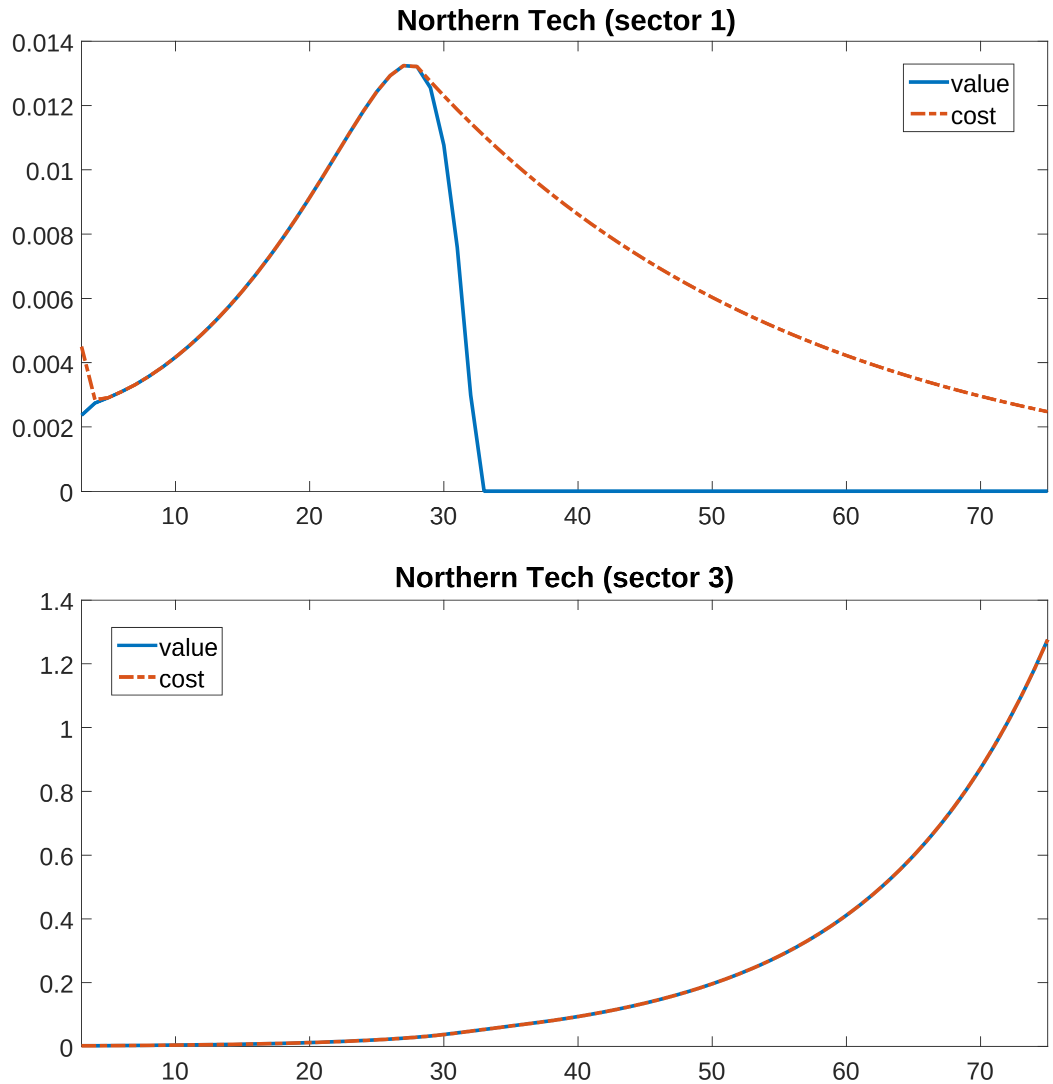
- t = 1–32, partial specialization (no production of Q_3^s)
- t = 33–75, full specialization (no production of Q_1^n or Q_3^s)

Figures 10–14 summarize the evolution of technologies in both regions. In the beginning, the costs of research are prohibitively high everywhere, so technologies are stagnant. But growth in basic knowledge allows us to see the implications of Proposition 1 in Figures 10 and 11. Because there is a greater abundance of unskilled labor relative to skilled labor in both the North and the South, the costs of research first dip below revenues in sector 1 in both regions.

We also see in Figure 10 that once diversified trade occurs, the North abandons unskilled goods production and unskilled technological developments. It then redoubles its energies into skill-intensive technological progress. On the other hand in Figure 11 we observe that the South never produces Q_3^s, and so it never has a chance to innovate for this sector. But it does innovate in sector 1. And over time, we see that Southern innovation in this sector ratchets up due to large scale effects from fertility increases (see bottom of Figure 12). Yet despite the South specializing in a technologically advancing sector, we can see that this does not save it from diverging into relative poverty compared to the North. Why?

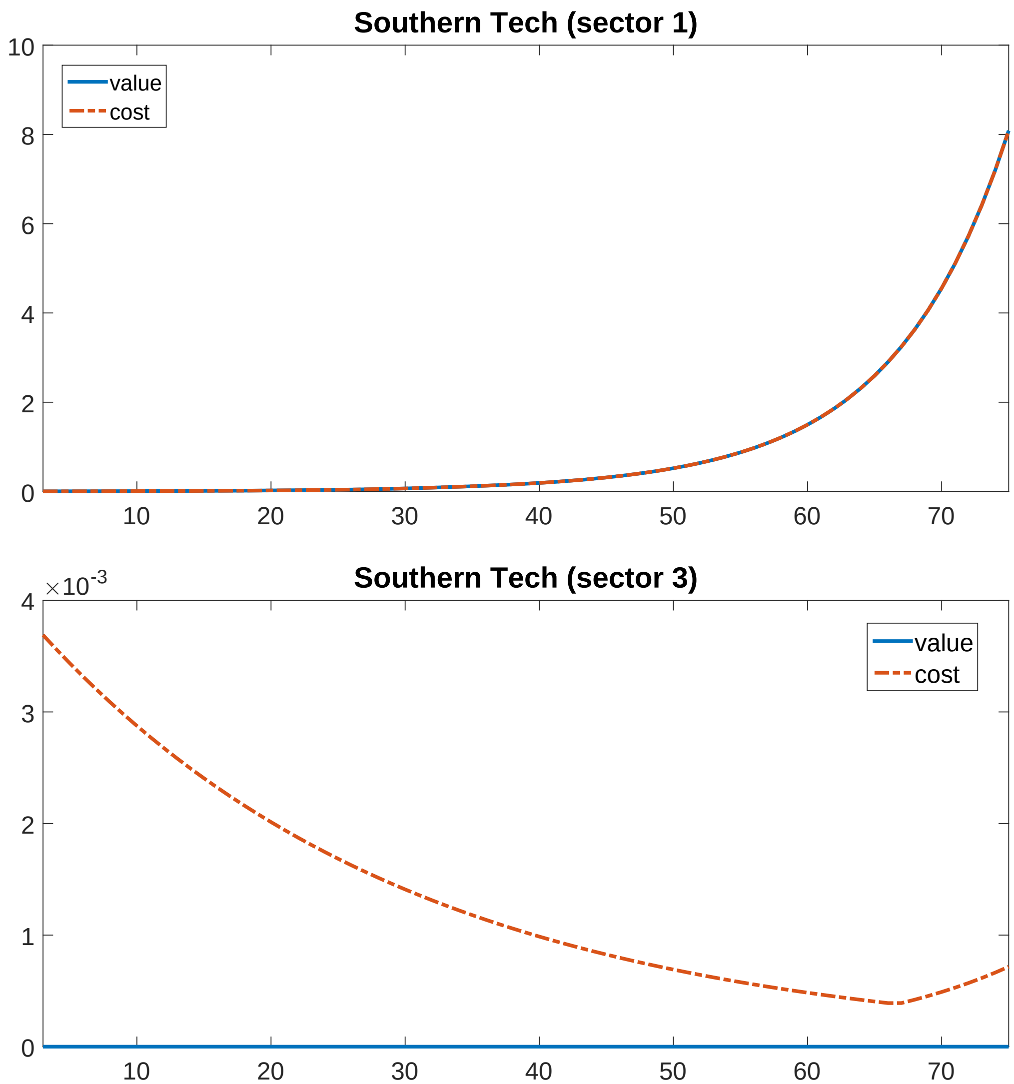
In Figure 13 we see that Northern fertility falls gradually at first due to more balanced technological growth, then falls more precipitously due to greater focus on skill-intensive growth. This demographic transition gets Northern fertility close to replacement fertility. At the same time education rates quickly reach their maximum level of universality. Demographics in the South however diverge dramatically from the North—Southern fertility remains high and education rates actually fall. Southern growth in unskilled-labor-intensive technologies increases the relative returns to unskilled labor, inciting fertility increases and educational decreases, as suggested in Proposition 3.

Figure 10: *Market for Technologies in North—Localized Technologies*



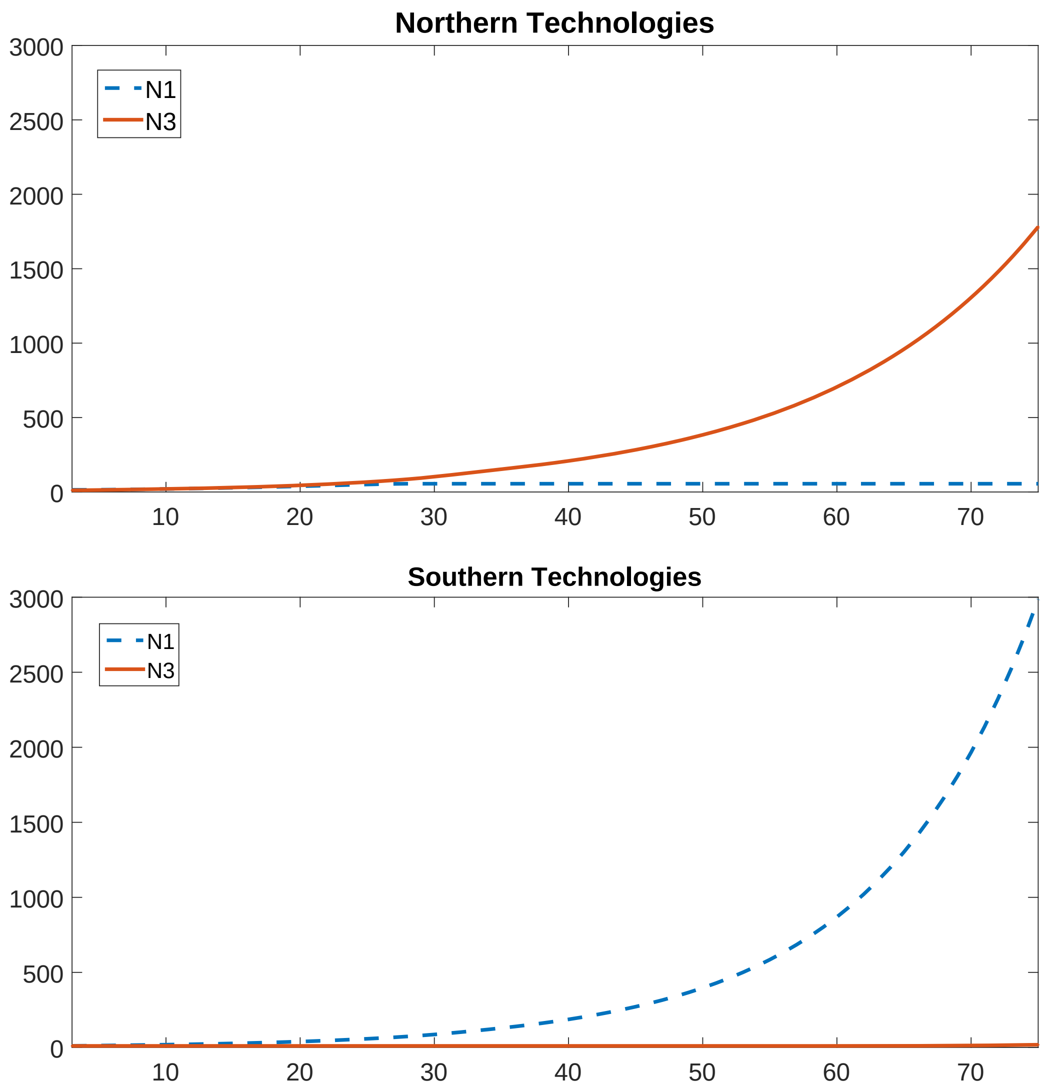
Note: Here we see that technological progress occurs first in the unskilled sector 1 in the North, but that such developments are abandoned around $t = 30$ (top figure). Skilled technologies begin to be developed around $t = 10$ (obscured by scale of bottom figure) and are developed throughout.

Figure 11: *Market for Technologies in South—Localized Technologies*



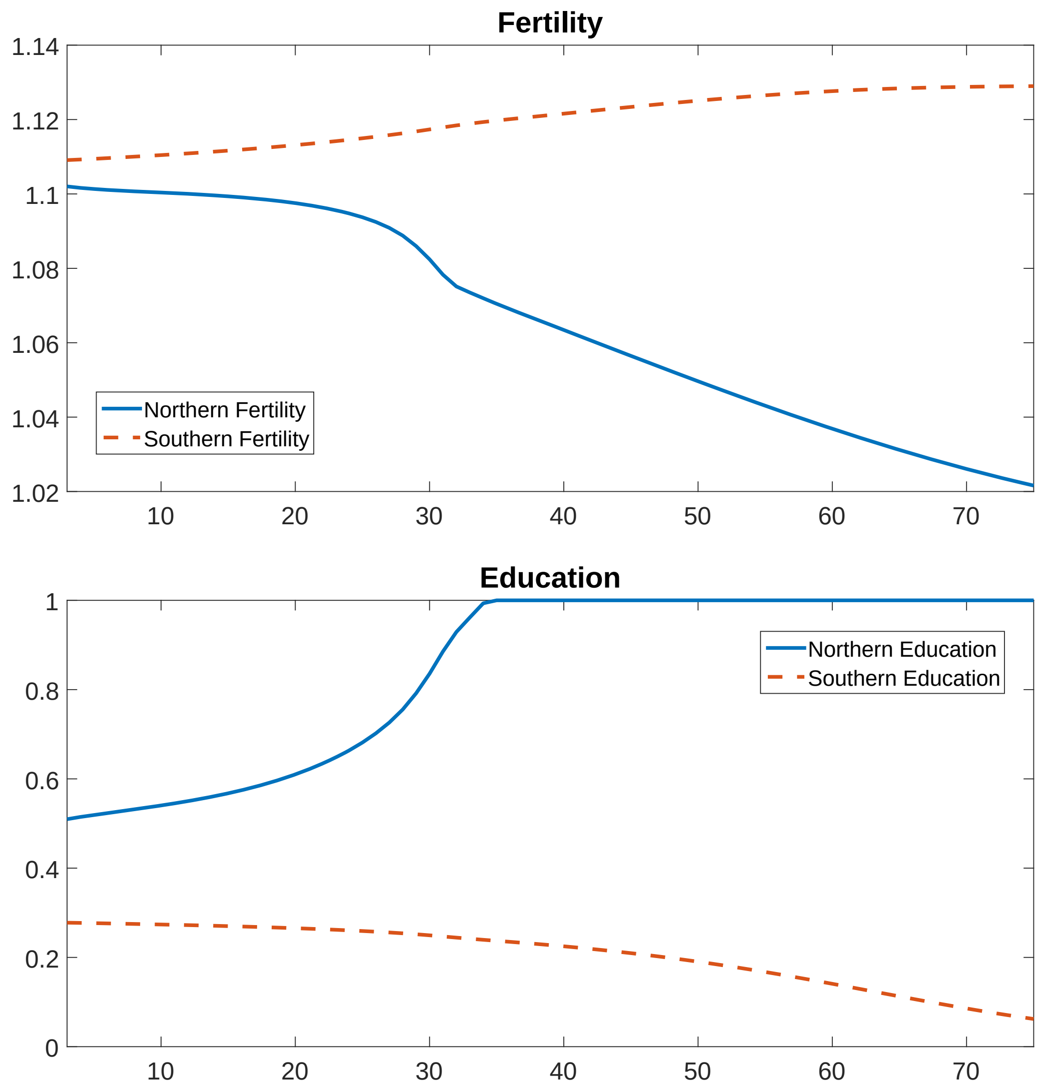
Note: Here we see that technological progress occurs early on in the unskilled sector 1 in the South and are developed throughout (top figure), but skilled technologies in sector 3 are never developed (bottom figure).

Figure 12: *Factor Productivities—Localized Technologies*



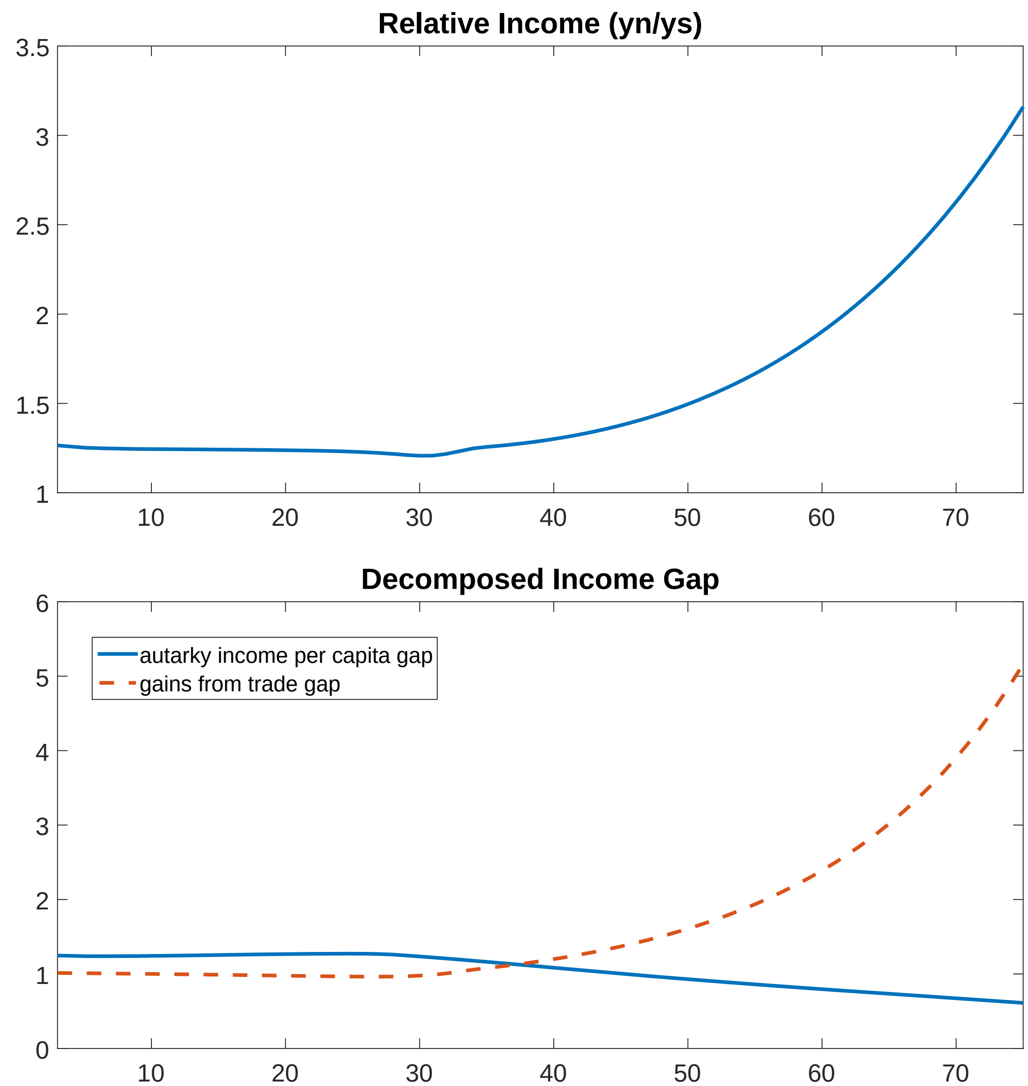
Note: Here we observe technological specialization—the North develops skill-using technologies while the South develops unskilled-using technologies.

Figure 13: *Fertility and Education—Localized Technologies*



Note: Here we observe demographic divergence—the North experiences demographic transition and obtains universal education, while the South experiences higher fertility and deteriorating rates of education.

Figure 14: *Relative Income and Divergence—Localized Technologies*



Note: Here we observe slight income convergence at first, followed by dramatic divergence due to full specialization. As the bottom diagram shows, despite robust technological growth in the South creating a force for convergence (blue line), deteriorating gains from trade in the South more than offsets this effect (red dotted line).

To help us better understand the forces of convergence or divergence that result from these changes, we can decompose the income per capita gap between country i (in this case the North) and country j (in this case the South) into two components. We define Y^i_{aut} as the GDP for country i in a given year, allowing it to have the technologies from the previous, regular equilibrium, but pretending that country i does not trade with country j. GDP per capita can be decomposed as

$$y^i = \left(\frac{Y^i_{aut}}{pop^i}\right)\left(\frac{Y^i}{Y^i_{aut}}\right) \tag{39}$$

So relative per capita incomes of country i versus country j can be decomposed into two multiplicative parts,

$$\frac{y^i}{y^j} = \left(\frac{Y^i_{aut}}{pop^i}\Big/\frac{Y^j_{aut}}{pop^j}\right)\left(\frac{Y^i}{Y^i_{aut}}\Big/\frac{Y^j}{Y^j_{aut}}\right) \tag{40}$$

We will call the first term on the right the "autarkic income per capita gap." We will call the second term the "gains from trade gap."

Figure 14 demonstrates both the decomposed and total income gap between the North and the South. At first, we have a period of convergence. Both countries have lots of unskilled workers and not as many skilled workers, and the South gains in relative terms by developing technologies for this abundant workforce. Indeed, we see the autarky income gap fall throughout. However, as trade continues to grow the South loses in relative terms. Its gains from trade deteriorate because its terms of trade deteriorate—the South must sell more and more of its unskilled good (Q_1) in exchange for its imports of the skilled good (Q_3).

The reason has to do with relative size. The North is small and prosperous; the South is innovative but enormous (its population grows to become ten times that of the North), flooding the world with its output. Divergence here comes about because each region leaves its "cone of diversification." A purely specialized world develops, and the South specializes in a good which generates population growth and deteriorating terms of trade with the North. Technological growth will not save it!

Thus we observe economic divergence that arises from an entirely different source than in GM, but that nevertheless has to do with interactions between trade and technological growth.

The West-East Model of Perfect Technological Diffusion: Analysis In this case the summary of the trade regimes is as follows:

- t = 1–30, partial specialization (no production of Q_3^e)

- t = 31–75, diversified trade

This case is shown in Figures 15–19 and shows a marked contrast to the North-South case. In this model, our two countries (which we now call West and East) can either develop their own technologies for local use, or adopt the sector-specific technologies of the other region, whichever is larger.

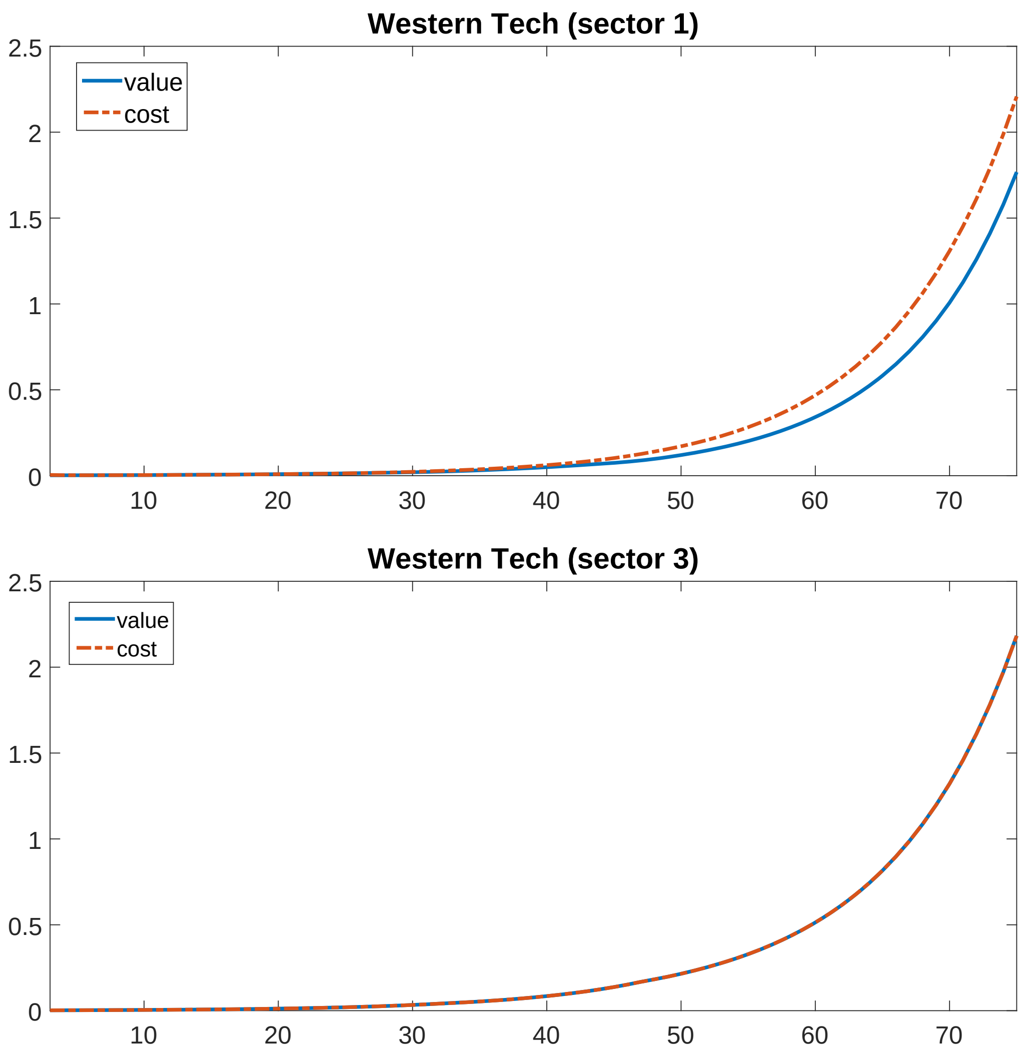
In Figure 15 we see that the West does not eventually abandon production of unskilled labor intensive good Q_1 (as the North did in the previous case). However, it does abandon the development of technologies in this sector. The reason is that it can now just adopt for free the unskilled technologies developed in the East. Similarly, the East eventually produces Q_3 (although we observe that early in the simulation it does not because of partial specialization), but it does not develop its own skill-intensive technologies. Due to the technological sharing, we see in Figure 17 that technologies in both sectors rise for both regions.

We also see, in Figure 18, that there is a demographic divergence between the two regions early on, similar to the North-South case. The East begins to catch up educationally, while the West experiences a fertility boom. Why? It is due to the transition from partial specialization to diversified trade between the two regions. The East can return to its cone of diversification due to its access to improving skill-intensive technologies emanating from the West. Once its skill-intensive sector is thus revived, the East sees the demand for educated workers start to rise: the South's fertility-education poverty trap is avoided here!

Figure 19 shows the evolutions of total and decomposed income gaps. In this case we first observe some divergence in incomes per capita between the two regions, again emanating from the gains from trade gap. This time, divergence occurs because the West does not really need much Q_1 from the East—its adoption of unskilled technologies from the East keeps its production of unskilled goods going, crowding out demand for Eastern trade. But the East does need the West's Q_3, as there is no Eastern production of this good. And technologies flowing from the West are "inappropriate" for the East for this reason. As a result Eastern terms of trade deteriorate.

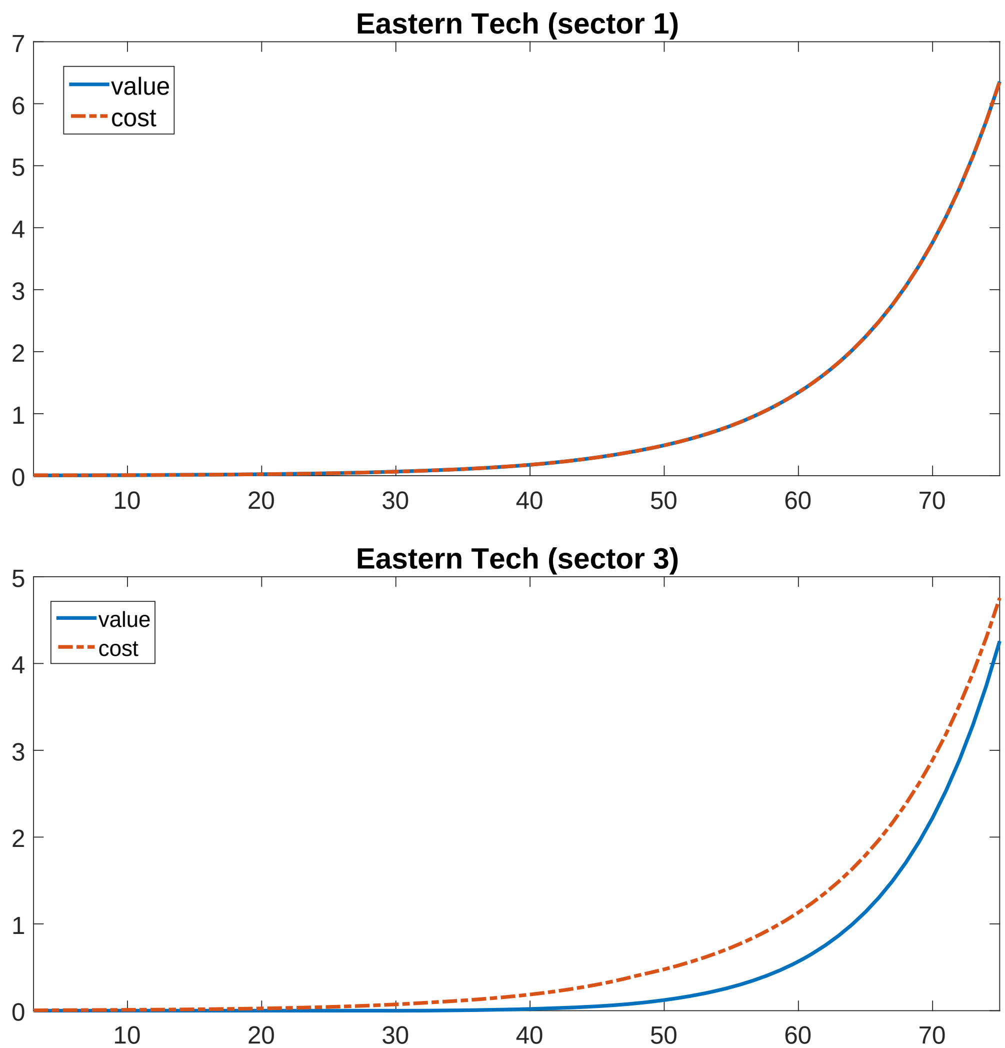
Eventually, however, the pool of Western technologies in sector 3 becomes large enough that the East can adopt them and produce some Q_3 of their own, even with such low education levels. Once both regions are in their cones of diversification, West and East start converging on all fronts (fertility, education, and incomes per capita).

Figure 15: *Market for Technologies in West—Perfect Technological Diffusion*



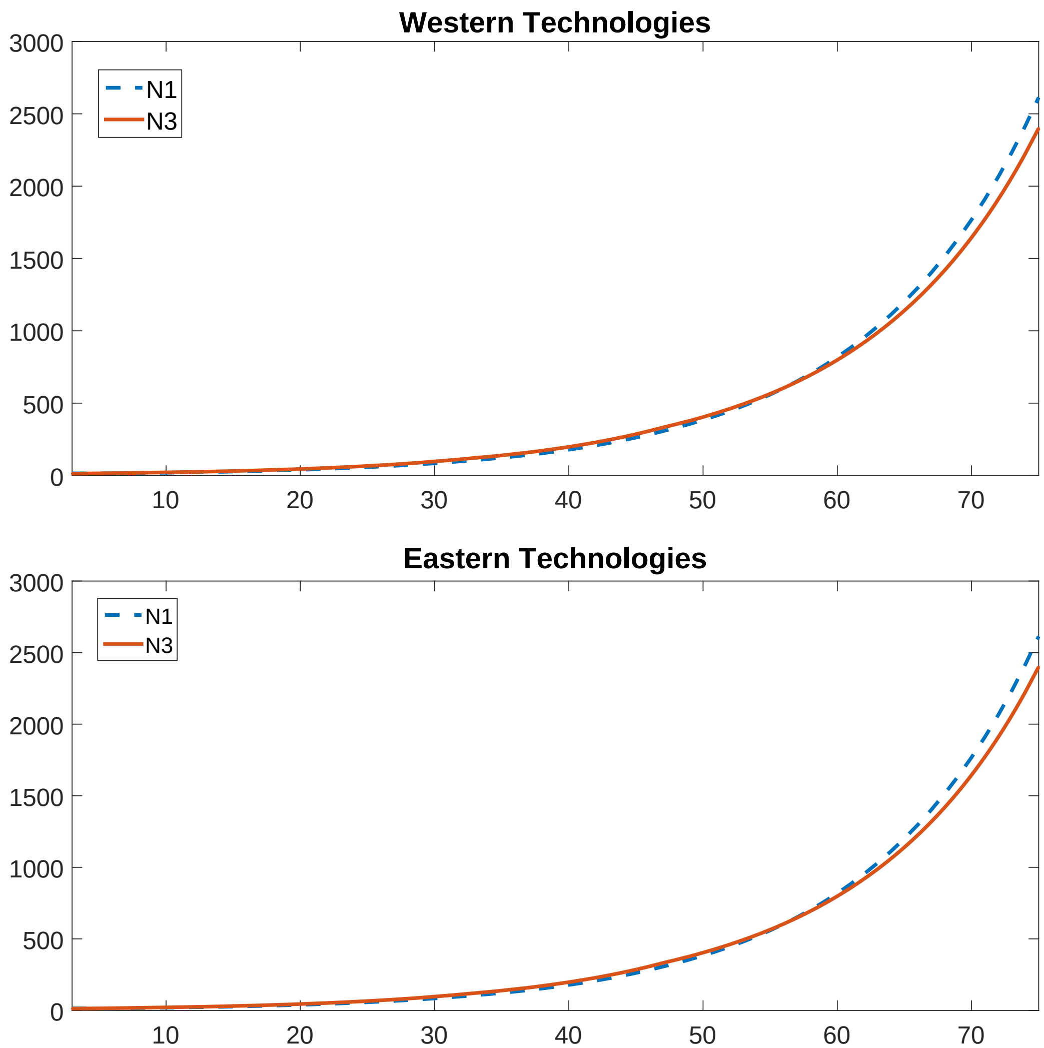
Note: Here we see that the West eventually abandons unskilled technologies (top diagram), while it early on develops, and continues to develop, skilled technologies (bottom diagram).

Figure 16: *Market for Technologies in East—Perfect Technological Diffusion*



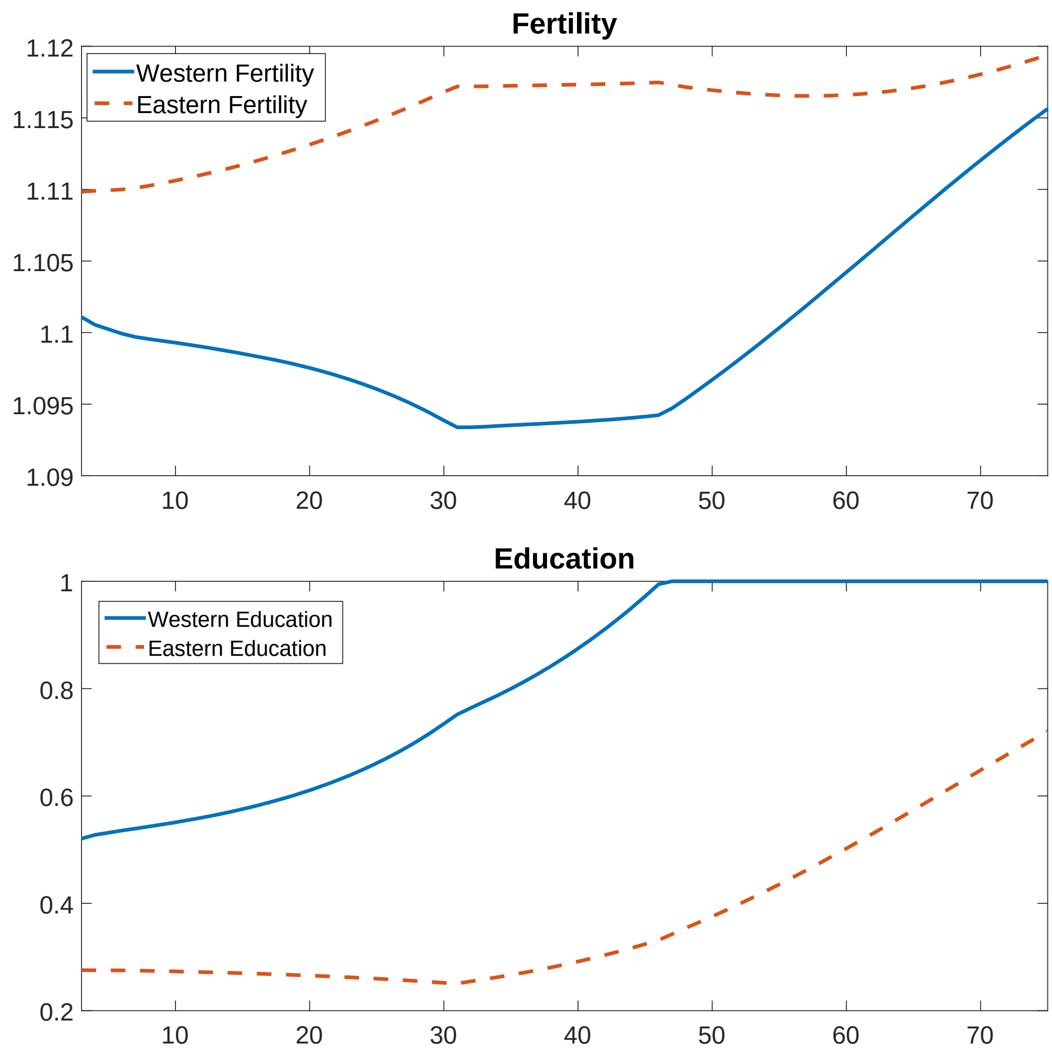
Note: Here we observe the East developing unskilled-intensive technologies throughout (top diagram), while it never develops skill-intensive technologies on its own (bottom diagram).

Figure 17: *Factor Productivities—Perfect Technological Diffusion*



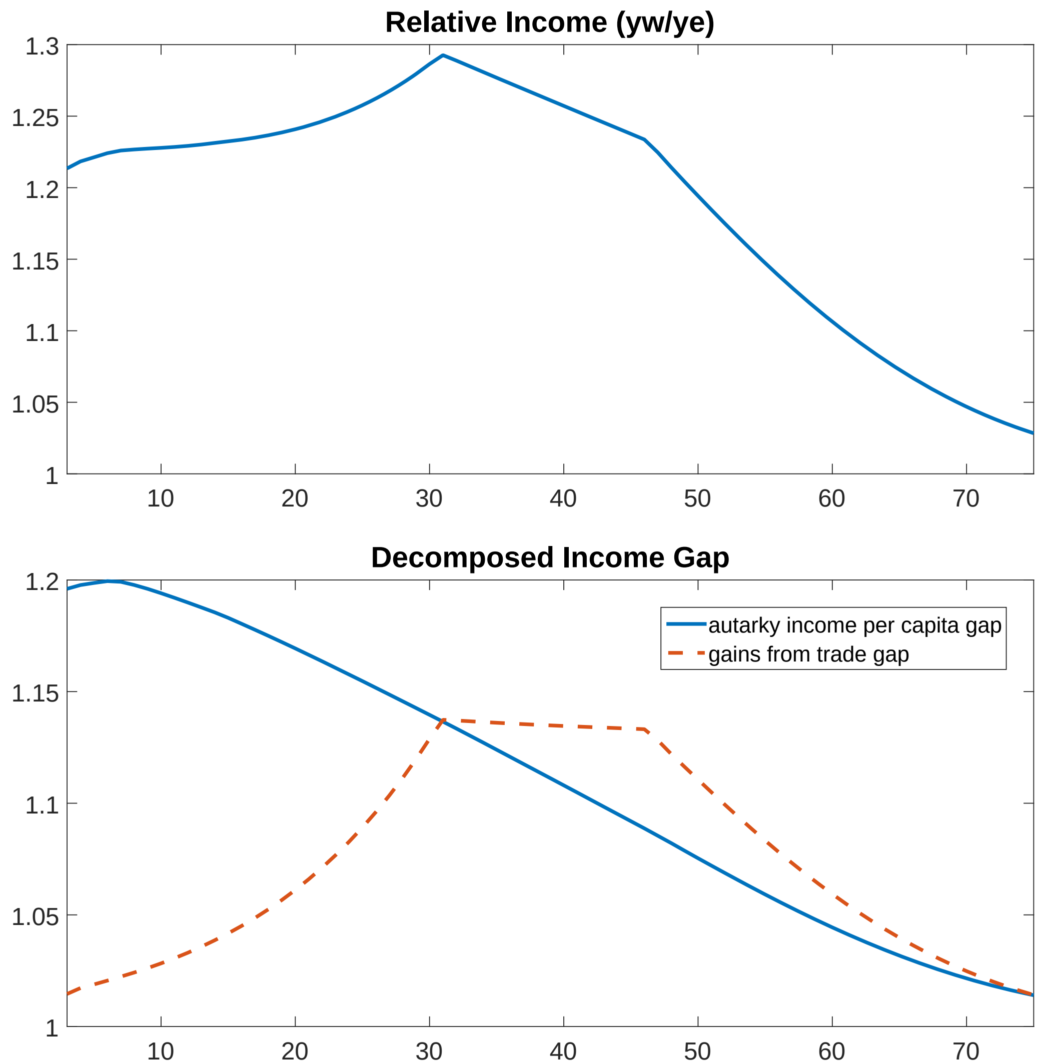
Note: Here we observe that due to perfect technological diffusion, factor productivities are identical in the two regions.

Figure 18: *Fertility and Education—Perfect Technological Diffusion*



Note: Here we observe the West experiencing an early demographic transition, followed by rising fertility later on. The East on the other hand goes through a much delayed increase in rates of education, while fertility remains relatively high throughout.

Figure 19: *Relative Income and Divergence—Perfect Technological Diffusion*



Note: After some income divergence due to a rising gains-from-trade gap (through the "inappropriate technology" channel), the income gap narrows as trade becomes fully diversified.

Note that, in this case, the East goes through its own education growth spurt, but this happens with quite a long delay (see Figure 18). We suggest that this case may better represent certain peripheral economies during the 20th century, which were eventually successful in adopting and implementing skill-intensive technologies developed abroad.

The West-East Model with Gradual Technological Diffusion: Analysis In this case the summary of the trade regimes is as follows:

- t = 1–32, partial specialization (no production of Q_3^e)
- t = 33–75, full specialization (no production of Q_1^w or Q_3^e)
- t = 54–56, partial specialization (no production of Q_1^w)
- t = 57–75, diversified trade

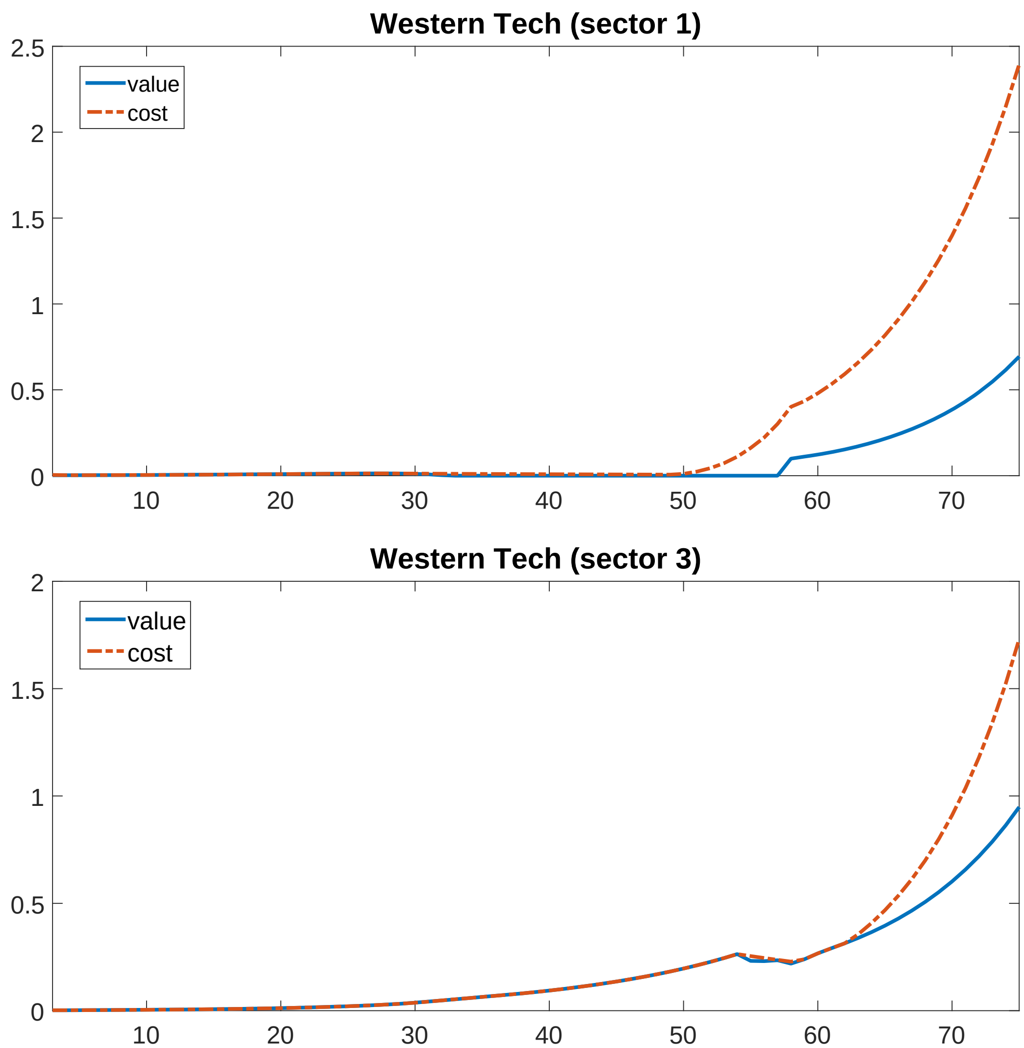
In this final case, we develop a variant of the West-East framework where each region either develops its own technologies or adopts a fraction ρ of the technologies developed in the other region, whichever is larger, where $0 \leq \rho \leq 1$. This may be seen as more realistic than the no diffusion and perfect diffusion cases studied above.

For greater realism, we will have ρ evolve exogenously over time. This is to represent a hybrid case between the earlier two simulations that may help us connect 19th Century divergence with cases of 20th-21st Century convergence. Specifically, here we will have $\rho = 0$ from $t = 1$ to $t = 45$ (and so this case mirrors the North-South framework discussed earlier), then ρ rises by 0.1 each period, reaching to a level of one by $t = 55$. After this point there is perfect technological diffusion, same as the previous West-East case.

Results from this final case are presented in Figures 20–24. Figure 20 shows the striking situation where the West not only abandons technological developments in the unskilled sector; it eventually abandons innovation in the skill-intensive sector.

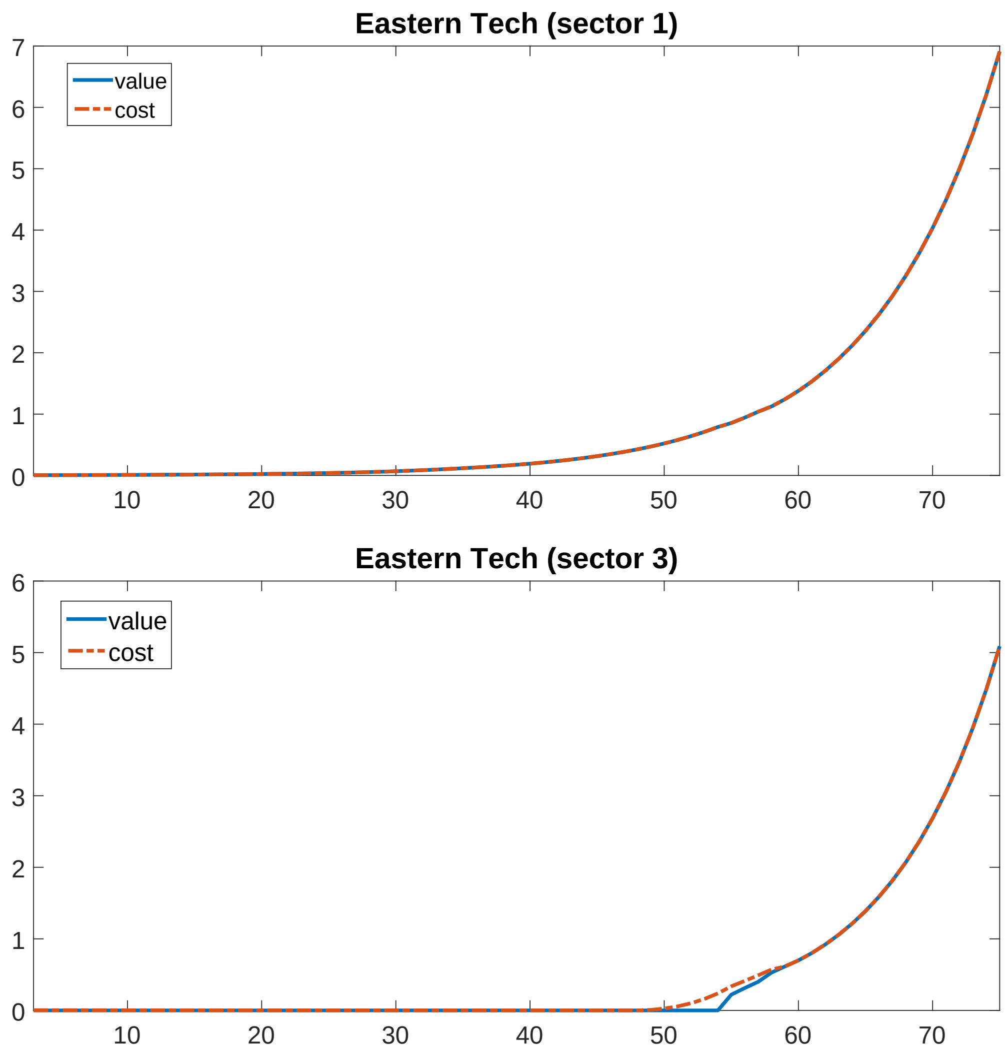
Figure 21 demonstrates why this happens. The East develops unskilled technologies, the same as always. What is remarkable is its development in sector 3. It is hard to see from the diagram, but the East does not innovate in this sector early on (it cannot since Q_3 is not produced). Once East can adopt the skill-intensive technologies from the West, it revives its skill intensive sector as above. At this point there rises a great demand for Eastern educated workers, so education here booms. Soon thereafter ($t = 59$), it can develop its own skilled technologies, as the sector becomes large enough for innovations to be profitable. It then takes over as the innovation leader!

Figure 20: *Market for Technologies in West—Gradual Technological Diffusion*



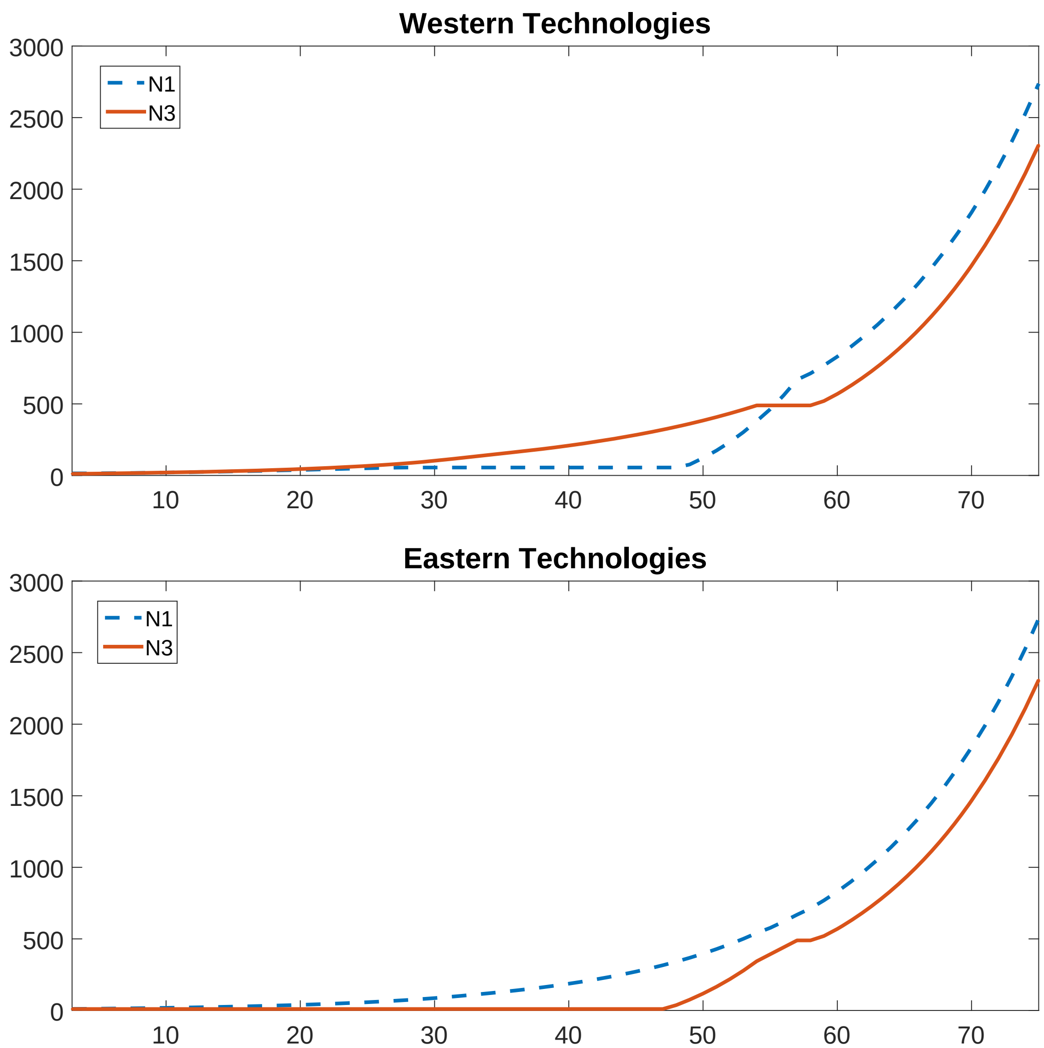
Note: Here the West very early abandons development of unskilled technologies (top diagram). It develops skilled technologies up until the point where it instead adopts these technologies from the East (bottom diagram).

Figure 21: *Market for Technologies in East—Gradual Technological Diffusion*



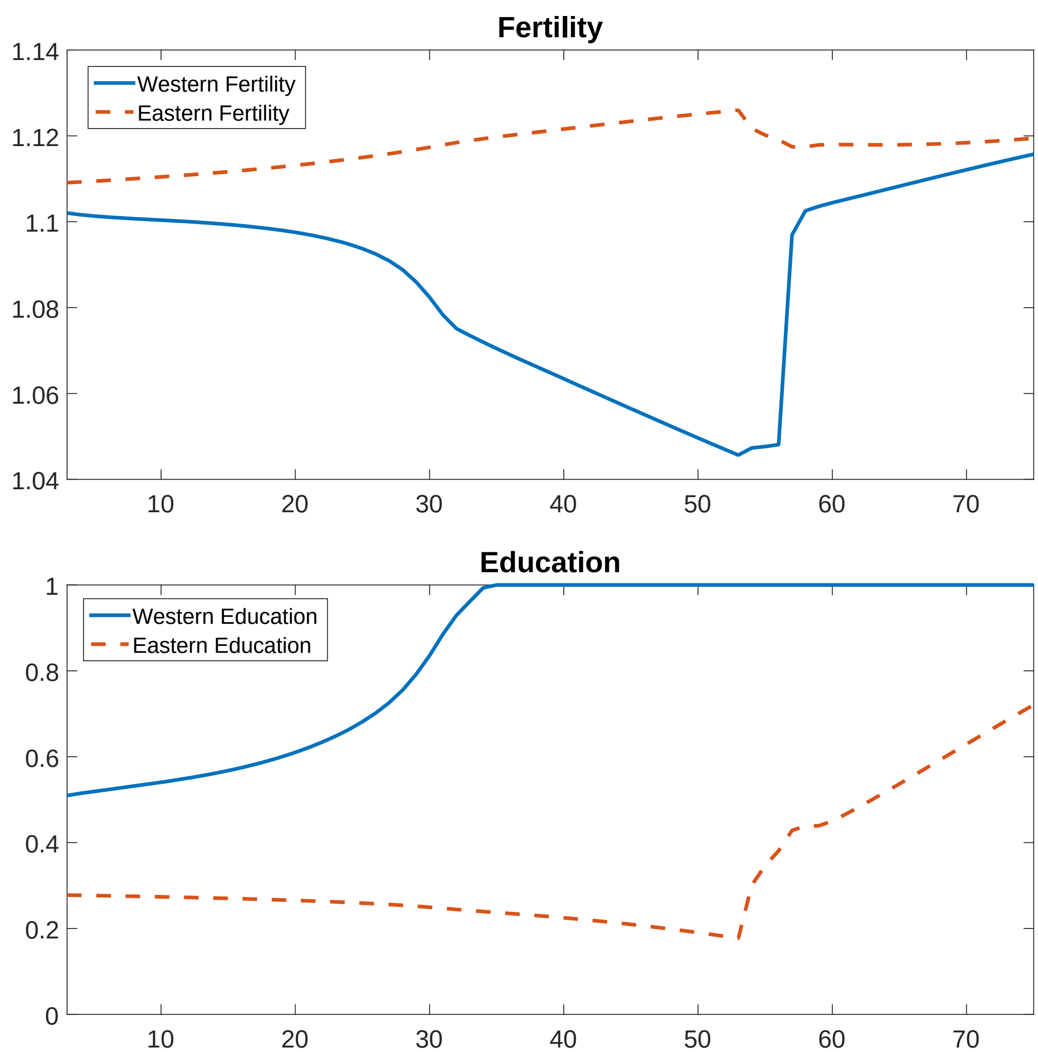
Note: The East develops unskilled technologies throughout (top diagram). It does not develop skilled technologies early on as it has no active sector 3 (obscured in bottom diagram due to scale); but, after $t = 54$ this sector comes active due to knowledge diffusion from the West. Soon after it develops its *own* skilled technologies.

Figure 22: *Factor Productivities—Gradual Technological Diffusion*



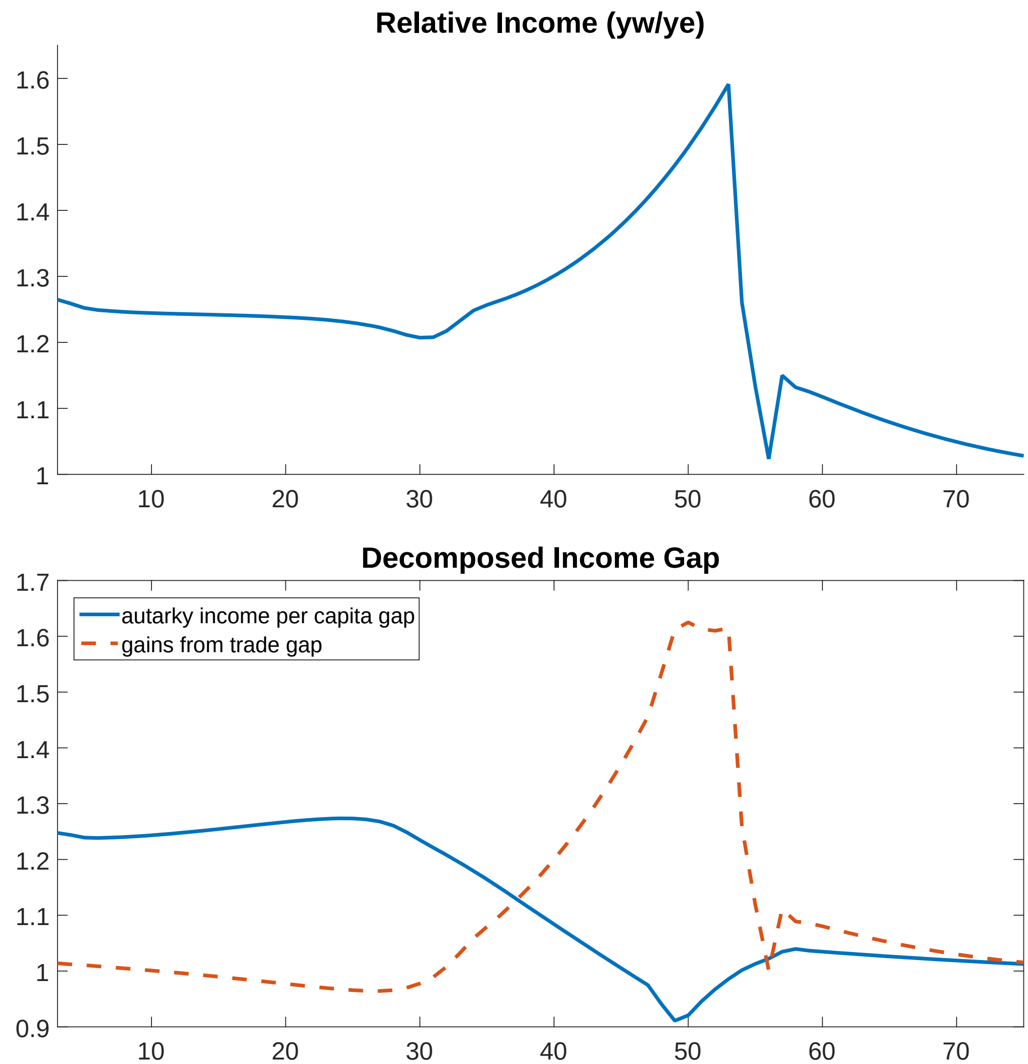
Note: Here we observe technological divergence early on, followed by convergence once knowledge flows become possible.

Figure 23: *Fertility and Education—Gradual Technological Diffusion*



Note: As in the first case of localized technologies, the West goes though a demographic transition, while in the East fertility rises and education falls. The West's sudden adoption of Eastern unskilled technologies generates a large Western fertility boom, while the East's sudden adoption and eventual development of skilled technologies produces an education boom.

Figure 24: *Relative Income and Divergence—Gradual Technological Diffusion*



Note: After a period of divergence due to deteriorating Eastern gains from trade, the East experiences a dramatic reversal of fortune due to its adoption and subsequent development of skilled technologies.

We consider that this final case loosely corresponds to Acemoglu et al. (2006), where relatively backward countries may initially have a technological adoption approach, and then over time switch to an innovation-based strategy. The implied leapfrogging in technological leadership that occurs can only happen, of course, if the East can first adopt the Western pool of skilled technologies. And this pool came about in large part due to trade-induced specialization. Thus we see a case here where globalization can produce divergence in incomes per capita at first, but eventually sows the seeds for eventual technological leapfrogging by the erstwhile laggards.

6. Conclusion

We have shown how different dimensions of globalization can have different implications for convergence. We provide two important innovations from the Galor and Mountford framework. One is that we endogenize the terms of trade. This can generate an entirely different source for divergence than in GM. We demonstrate that even when the South can innovate on its own, specialization patterns can still drive dramatic divergence in incomes. Through this terms of trade effect, we see that larger and in particular growing populations can be economically costly, even in the context where innovation is endogenous.

The other big difference is that we analyze the case, arguably more relevant for contemporary economies, of technology transfer. Our paper suggests that technological adoption fosters convergence, and that continued divergence among economies in the 20th century must be due to other barriers—i.e., outside our model—preventing such adoption. Barriers may be due to blocking efforts by special interests (Parente and Prescott 1999), or financial constraints (Aghion et al. 2005), or institutions (Acemoglu et al. 2001). On the other hand barriers due to the "inappropriateness" of technologies developed by frontier countries (Basu and Weil 1998; Acemoglu and Zilibotti 2001) should not last in the longer term, as we emphasize in our second simulation. It is striking that here, in contrast to GM, globalization will eventually yield income convergence, even as it fosters a more volatile path towards that convergence.

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Appendix

A. Diversified Trade Equilibrium

With trade of goods Q_1 and Q_3 between the North and the South, productions in each region are given by (1) and (2).

For each region $c \in i, j$, the following conditions characterize the diversified trade equilibrium:

$$p_1^j = \frac{w_l^j}{A_1^j}, \tag{41}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma}\left(w_h^c\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma}, \tag{42}$$

$$p_3^i = \frac{w_h^i}{A_3^i}, \tag{43}$$

$$\left(\frac{1}{A_1^c}\right)Q_1^c + \left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma-1}\left(w_h^c\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}Q_2^c = L^c, \tag{44}$$

$$\left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma}\left(w_h^c\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}Q_2^c + \left(\frac{1}{A_3^c}\right)Q_3^c = H^c, \tag{45}$$

$$Q_1^i + Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^i\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^i\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^i\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^i\right)^{1-\sigma}}\right)\cdot Y^i, \tag{46}$$

$$Q_1^j - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^j\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^j\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^j\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^j\right)^{1-\sigma}}\right)\cdot Y^j, \tag{47}$$

$$Q_2^c = \left(\frac{(1-\alpha)^{\sigma}\left(p_2^c\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^c\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^c\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^c\right)^{1-\sigma}}\right)\cdot Y^c, \tag{48}$$

$$Q_3^i - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^i\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^i\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^i\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^i\right)^{1-\sigma}}\right)\cdot Y^i, \tag{49}$$

$$Q_3^j + Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^j\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^j\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^j\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^j\right)^{1-\sigma}}\right)\cdot Y^j, \tag{50}$$

$$A_1^i\left(A_1^i L_1^i + Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{i\,\frac{\sigma-1}{\sigma}}\left(L^i - L_1^i\right)^{-\gamma-\sigma+\sigma\gamma}\left(H^i - H_3^i\right)^{\gamma+\sigma-\sigma\gamma-1}, \tag{51}$$

$$A_3^i\left(A_3^i H_3^i - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{i\,\frac{\sigma-1}{\sigma}}\left(L^i - L_1^i\right)^{-\gamma+\sigma\gamma}\left(H^i - H_3^i\right)^{\gamma-\sigma\gamma-1}, \tag{52}$$

$$A_1^j\left(A_1^jL_1^j-Z_1\right)^{-\frac{1}{\sigma}}=\left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{j\,\frac{\sigma-1}{\sigma}}\left(L^j-L_1^j\right)^{-\gamma-\sigma+\sigma\gamma}\left(H^j-H_3^j\right)^{\gamma+\sigma-\sigma\gamma-1},\tag{53}$$

$$A_3^j\left(A_3^jH_3^j+Z_3\right)^{-\frac{1}{\sigma}}=\left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{j\,\frac{\sigma-1}{\sigma}}\left(L^j-L_1^j\right)^{-\gamma+\sigma\gamma}\left(H^j-H_3^j\right)^{\gamma-\sigma\gamma-1},\tag{54}$$

$$A_1^c=\left(N_{1,t-1}^c+\alpha^{\frac{\alpha}{1-\alpha}}\left(N_{1,t}^c-N_{1,t-1}^c\right)\right)\left(\alpha p_1^c\right)^{\frac{\alpha}{1-\alpha}},\tag{55}$$

$$A_2^c=\left(N_{2,t-1}^c+\alpha^{\frac{\alpha}{1-\alpha}}\left(N_{2,t}^c-N_{2,t-1}^c\right)\right)\left(\alpha p_2^c\right)^{\frac{\alpha}{1-\alpha}},\tag{56}$$

$$A_3^c=\left(N_{3,t-1}^c+\alpha^{\frac{\alpha}{1-\alpha}}\left(N_{3,t}^c-N_{3,t-1}^c\right)\right)\left(\alpha p_3^c\right)^{\frac{\alpha}{1-\alpha}},\tag{57}$$

$$H^c=\left(\frac{\tau^{*c}}{b^c}\right)pop^c,\tag{58}$$

$$L^c=\left(1-\frac{\tau^{*c}}{b^c}\right)pop^c+n^c\,pop^c,\tag{59}$$

$$n^c=\left(1-\frac{\tau^{*c}}{b^c}\right)n_l^{*c}+\left(\frac{\tau^{*c}}{b^c}\right)n_h^{*c},\tag{60}$$

$$e^c=\frac{\tau^{*c}}{b^c},\tag{61}$$

$$\frac{p_1^i}{p_3^i}=\frac{Z_3}{Z_1},\tag{62}$$

$$\frac{p_1^j}{p_3^j}=\frac{Z_3}{Z_1}.\tag{63}$$

Equations (41)–(43) are unit cost functions, (44) and (45) are full employment conditions, (46)–(50) denote regional goods clearance conditions, (51)–(54) equate the marginal products of raw factors, (55)–(57) describe sector-specific technologies, , (58)–(67) describe fertility, education and labor-types for each region, and (68) and (69) describe the balance of payments for each region. Solving this system for the unknowns p_1^i, p_1^j, p_2^i, p_2^j, p_3^i, p_3^j, Q_1^i, Q_1^j, Q_2^i, Q_2^j, Q_3^i, Q_3^j, w_l^i, w_l^j, w_h^i, w_h^j, L_1^i, L_1^j, H_3^i, H_3^j, A_1^i, A_2^i, A_3^i, A_1^j, A_2^j, A_3^j, L^i, L^j, H^i, H^j, n^i, n^j, e^i, e^j, Z_1 and Z_3 constitutes the static partial trade equilibrium. Population growth for each region is given simply by $pop_t^c=n_{t-1}^c\,pop_{t-1}^c$. Each region will produce all three goods so long as factors and technologies are "similar enough." If factors of production or technological levels sufficiently differ, i produces only goods 2 and 3, while j produces only goods 1 and 2. No other specialization scenario is possible for the following reasons: first, given that both regions have positive levels of L and H, full employment of resources implies that they cannot specialize completely in good 1 or good 3. Second, specialization solely in good 2 is not possible either, since a region with a comparative advantage in this good would also have a comparative advantage in either of the other goods. This implies that each country must produce at least two goods. Further, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these goods, regardless of the technological differences between the two regions. See Cuñat and Maffezzoli (2004) for a fuller discussion.

B. Specialized Trade Equilibrium

The specialized equilibrium is one where country i does not produce any good 1 and country j does not produce any good 3. Productions in each region are then given by

$$Y^i = \left(\frac{\alpha}{2} \left(Z_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(Q_2^i\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(Q_3^i - Z_3\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{64}$$

$$Y^j = \left(\frac{\alpha}{2} \left(Q_1^j - Z_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(Q_2^j\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(Z_3\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{65}$$

Once again, we do not permit any trade of good 2. For each region $c \in n, s$, the following conditions characterize this equilibrium.

$$p_1^j = \frac{w_l^j}{A_1^j}, \tag{66}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma}\left(w_h^c\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma}, \tag{67}$$

$$p_3^i = \frac{w_h^i}{A_3^i}, \tag{68}$$

$$\left(\frac{1}{A_2^i}\right)\left(w_l^i\right)^{\gamma-1}\left(w_h^i\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}Q_2^i = L^i, \tag{69}$$

$$\left(\frac{1}{A_2^i}\right)\left(w_l^i\right)^{\gamma}\left(w_h^i\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}Q_2^i + \left(\frac{1}{A_3^i}\right)Q_3^i = H^i, \tag{70}$$

$$\left(\frac{1}{A_1^j}\right)Q_1^j + \left(\frac{1}{A_2^j}\right)\left(w_l^j\right)^{\gamma-1}\left(w_h^j\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}Q_2^j = L^j, \tag{71}$$

$$\left(\frac{1}{A_2^j}\right)\left(w_l^j\right)^{\gamma}\left(w_h^j\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}Q_2^j = H^j, \tag{72}$$

$$Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^i\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^i\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^i\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^i\right)^{1-\sigma}} \right) \cdot Y^i, \tag{73}$$

$$Q_1^j - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^j\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^j\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^j\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^j\right)^{1-\sigma}} \right) \cdot Y^j, \tag{74}$$

$$Q_2^c = \left(\frac{(1-\alpha)^\sigma \left(p_2^c\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^\sigma \left(p_1^c\right)^{1-\sigma} + (1-\alpha)^\sigma \left(p_2^c\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^\sigma \left(p_3^c\right)^{1-\sigma}} \right) \cdot Y^c, \tag{75}$$

$$Q_3^i - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^\sigma \left(p_3^i\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^\sigma \left(p_1^i\right)^{1-\sigma} + (1-\alpha)^\sigma \left(p_2^i\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^\sigma \left(p_3^i\right)^{1-\sigma}} \right) \cdot Y^i, \tag{76}$$

$$Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^\sigma \left(p_3^j\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^\sigma \left(p_1^j\right)^{1-\sigma} + (1-\alpha)^\sigma \left(p_2^j\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^\sigma \left(p_3^j\right)^{1-\sigma}} \right) \cdot Y^j, \tag{77}$$

$$A_3^i \left(A_3^i H_3^i - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right) A_2^{i\,\frac{\sigma-1}{\sigma}} \left(L^i\right)^{-\gamma+\sigma\gamma} \left(H^i - H_3^i\right)^{\gamma-\sigma\gamma-1}, \tag{78}$$

$$A_1^j \left(A_1^j L_1^j - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right) A_2^{j\,\frac{\sigma-1}{\sigma}} \left(L^j - L_1^j\right)^{-\gamma-\sigma+\sigma\gamma} \left(H^j\right)^{\gamma+\sigma-\sigma\gamma-1}, \tag{79}$$

$$A_1^j = \left(N_{1,t-1}^j + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{1,t}^j - N_{1,t-1}^j\right)\right) \left(\alpha p_1^j\right)^{\frac{\alpha}{1-\alpha}}, \tag{80}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{2,t}^c - N_{2,t-1}^c\right)\right) \left(\alpha p_2^c\right)^{\frac{\alpha}{1-\alpha}}, \tag{81}$$

$$A_3^i = \left(N_{3,t-1}^i + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{3,t}^i - N_{3,t-1}^i\right)\right) \left(\alpha p_c^i\right)^{\frac{\alpha}{1-\alpha}}, \tag{82}$$

$$H^c = \left(\frac{\tau^{*^c}}{b^c}\right) pop^c, \tag{83}$$

$$L^c = \left(1 - \frac{\tau^{*^c}}{b^c}\right) pop^c + n^c\, pop^c, \tag{84}$$

$$n^c = \left(1 - \frac{\tau^{*^c}}{b^c}\right) n_l^{*^c} + \left(\frac{\tau^{*^c}}{b^c}\right) n_h^{*^c}, \tag{85}$$

$$e^c = \frac{\tau^{*^c}}{b^c}, \tag{86}$$

$$\frac{p_1^i}{p_3^i} = \frac{Z_3}{Z_1}, \tag{87}$$

$$\frac{p_1^j}{p_3^j} = \frac{Z_3}{Z_1}. \tag{88}$$