Multiproduct Pricing Made Simple

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Overview

- Multiproduct pricing important for:
  - unregulated monopoly
  - oligopoly
  - most efficient prices which cover fixed costs or generate tax revenue (Ramsey prices)
  - optimal regulation when costs are private information
- Key feature is that firm(s)/regulator must decide about price structure as well as overall price level
- This paper:
  - derives simple formulas using notion of consumer surplus as function of *quantities*
  - demonstrates equivalence between symmetric Cournot equilibria and Ramsey prices
  - describes generalized form of homothetic preferences so that pricing decisions can be decomposed into "relative" and "average" decisions
  - firms then have good incentives to choose *relative* quantities

Some (old) literature

- Baumol & Bradford (1970): principles of Ramsey pricing
  - "plausible that damage to welfare minimized if quantities are proportional to the efficient quantities"
- Gorman (1961): conditions on preferences to get linear Engel curves
- Bergstrom & Varian (1985), Slade (1994), Moderer & Shapley (1996): what does an oligopoly maximize?
  - we show it maximizes a Ramsey objective (and *vice versa*)
- Bliss (1988) and Armstrong & Vickers (2001): multiproduct competition with one-stop shopping
  - firms first decide how much surplus to offer customers, then solve Ramsey problem of maximizing profit subject to this constraint
- Marketing literature: patterns of cost passthrough in retailing
  - own-cost passthrough is positive, cross-cost passthrough ambiguous
- Baron & Myerson (1982): optimal regulation of single-product firm with unobserved costs
  - we can sometimes extend this to the multiproduct case

General framework

- There are n products
  - quantity of product i is x_i
  - vector of quantities is $x = (x_1, ..., x_n)$
- Consumers have quasi-linear preferences
  - there is representative consumer with concave gross utility $u(x)$, who maximizes $u(x) - p \cdot x$ when price vector is p
  - inverse demand function is $p_i(x) \equiv \partial u(x)/\partial x_i$ or in vector notation $p(x) \equiv \nabla u(x)$
  - total revenue with quantities x is $r(x) = x \cdot \nabla u(x)$
  - so consumer surplus with quantities x is

$$s(x) \equiv u(x) - x \cdot \nabla u(x)$$

Ramsey monopoly problem

- Products supplied by monopolist with convex cost function $c(x)$
- Ramsey objective with weight $0 \leq \alpha \leq 1$ is

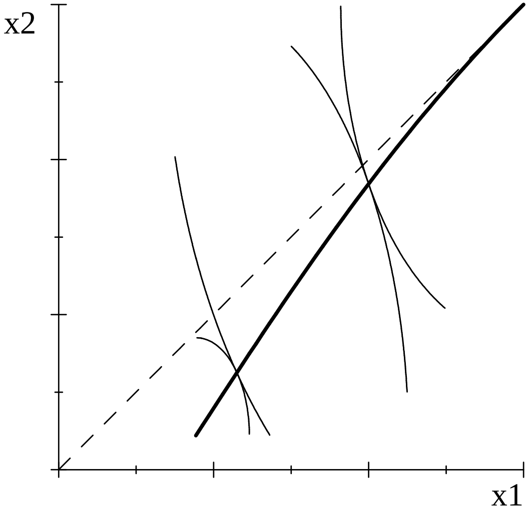
$$[r(x) - c(x)] + \alpha s(x) = [u(x) - c(x)] - (1 - \alpha)s(x)$$

  - $\alpha = 0$ corresponds to profit maximization
  - $\alpha = 1$ corresponds to total surplus maximization
- First-order condition for maximizing Ramsey objective is

$$p(x) = \nabla c(x) + (1 - \alpha)\nabla s(x)$$

  - price above [below] cost for product i if s is increasing [decreasing] in x_i
  - when $c(x)$ is homogeneous degree 1 and $\alpha \approx 1$ Ramsey problem is solved by $x \approx \alpha x^w$, where x^w is efficient quantity vector (with $p = \nabla c$)
  - so equiproportionate quantity reductions a good rule of thumb for small deviations

Ramsey quantities as weight on consumers varies



Cournot competition

- Consider symmetric Cournot market where each multiproduct firm has cost function $c(x)$
  - then symmetric equilibrium (if it exists) has first-order condition for total quantities x

$$p(x) = \nabla c(\tfrac{1}{m}x) + \tfrac{1}{m}\nabla s(x)$$

  - this coincides with optimal quantities in the Ramsey problem of maximizing

$$u(x) - mc(\tfrac{1}{m}x) - (1-\alpha)s(x)$$

when $\alpha = \frac{m-1}{m}$

Theorem

If m firms have the same convex cost function, there exists a symmetric Cournot equilibrium in which quantities maximize the Ramsey objective with $\alpha = \frac{m-1}{m}$. There are no asymmetric equilibria.

- Comparative statics for m straightforward when $c(x)$ is CRS.

Sketch proof of existence of Cournot equilibrium

- When $\alpha = \frac{m-1}{m}$, the Ramsey objective when firm j chooses quantity vector x^j is
$$\frac{1}{m} r(\Sigma_j x^j) + \frac{m-1}{m} u(\Sigma_j x^j) - \Sigma_j c(x^j)$$
which has symmetric solution $x^j \equiv x$, say
    - In particular, choosing $y = x$ maximizes the function
$$\rho(y) \equiv \frac{1}{m} r([m-1]x + y) + \frac{m-1}{m} u([m-1]x + y) - c(y)$$
- A Cournot firm's best response when its rivals each supply x is to choose quantity vector y to maximize
$$\pi(y) \equiv y \cdot p([m-1]x + y) - c(y) \leq \rho(y) - \frac{m-1}{m} u(mx)$$
(inequality follows from concavity of u)
- Hence
$$\pi(x) - \pi(y) \geq \rho(x) - \rho(y) \geq 0$$
and it is an equilibrium for each firm to supply x

Homothetic consumer surplus

Theorem

Consumer surplus $s(x)$ is homothetic in x if and only if

$$u(x) = h(x) + g(q(x))$$

where $h(x)$ and $q(x)$ are both homogeneous degree 1

- "If": We have

$$p(x) = \nabla h(x) + g'(q(x))\nabla q(x)$$

so

$$r(x) = h(x) + g'(q(x))q(x)$$

and hence consumer surplus is

$$s(x) = g(q(x)) - g'(q(x))q(x)$$

which depends only on $q(x)$

Homothetic consumer surplus

- We can write quantities in "polar coordinates" form

$$x = q(x) \cdot \frac{x}{q(x)}$$

  - $x/q(x)$ is homogeneous degree 0, depends only on the ray from origin
  - $q(x)$ measures how far along that ray x lies
  - refer to $q(x)$ as "composite quantity" and $x/q(x)$ as "relative quantities"
  - we know consumer surplus $s(x)$ depends only on the $q(x)$ coordinate
- Three degrees of freedom in the family: $q(x)$, $h(x)$ and $g(q)$
  - this is a much wider class than those where consumer surplus is homothetic in *prices*
  - such preferences must have homothetic $u(x)$, so $h \equiv 0$

Examples

- **Linear demand**: An example with linear $h(x)$ is

$$u(x) = a \cdot x - \tfrac{1}{2} x^T M x \; ; \; p(x) = a - Mx$$

where $a > 0$ and M is a positive definite matrix, so that

$$h(x) = a \cdot x \; , \; q(x) = \sqrt{x^T M x} \; , \; g(q) = -\tfrac{1}{2} q^2$$

- **Logit demand**: An example with linear $q(x)$ has demand function

$$x_i(p) = \frac{e^{a_i - p_i}}{1 + \sum_j e^{a_j - p_j}}$$

which corresponds to the utility function

$$u(x) = \underbrace{a \cdot x + \sum_i x_i \log \frac{q(x)}{x_i}}_{h(x)} + g(q(x))$$

where $q(x) \equiv \sum_j x_j$ and $g(q) = -q \log q - (1-q) \log(1-q)$

Examples

- **Strictly complementary products**:
  - In some natural settings consumption requires prior purchase of a base product ("access")
  - Suppose all who buy access (e.g. to theme park) get gross utility $U(y)$ from complementary services (rides) y.
  - If x_1 consumers acquire access, and each then buys complementary services $y = x_2/x_1$, where x_2 is the total supply of complementary services, gross utility has the form

$$\begin{aligned} u(x_1, x_2) &= x_1 U(y) + g(x_1) \\ &= x_1 U(x_2/x_1) + g(x_1) \end{aligned}$$

  - This is an example with $q(x) = x_1$ so consumer surplus is simply a function of the number of consumers that buy access
  - So services are priced at marginal cost

Consumer optimization

- Consumer surplus with price vector p and quantities x is

$$u(x) - p \cdot x = g(q(x)) - q(x)\frac{p \cdot x - h(x)}{q(x)}$$

  - expressed in terms of the two coordinates $q(x)$ and $x/q(x)$
  - for all $q(x)$ surplus is maximized by minimizing $[p \cdot x - h(x)]/q(x)$
  - this determines the optimal relative quantities given price vector p, say $x^*(p)$
  - let

$$\phi(p) \equiv \min_{x \geq 0} : \frac{p \cdot x - h(x)}{q(x)}$$

    which is concave in p and $x^*(p) \equiv \nabla\phi(p)$

Consumer optimization

- The consumer then optimizes her composite quantity, say Q

$$Q(\phi) \text{ maximizes } g(Q) - Q\phi(p)$$

  and so consumer demand as function of p is

$$x(p) = Q(\phi(p))x^*(p)$$

- Thus $\phi(p)$ is the "composite price"
  - all prices with the same $\phi(p)$ induce the same composite quantity $q(x)$
  - inverse demand for composite quantity is $\phi = g'(Q)$

Market analysis

- Suppose a monopoly or oligopoly supplies the n products
  - with constant-returns-to-scale cost function $c(x)$
- We know Cournot equilibrium coincides with Ramsey optimum, so study the latter objective which is

$$\alpha g(q(x)) + (1-\alpha)q(x)g'(q(x)) - q(x)\frac{c(x)-h(x)}{q(x)}$$

  - again, a function of the two coordinates $q(x)$ and $x/q(x)$
  - regardless of composite quantity, choose relative quantities x^* to minimize $[c(x)-h(x)]/q(x)$
  - so relative quantities the same for all α in Ramsey problem
  - this implies relative price-cost margins also the same for all α
- Monopolist has good incentives to choose its relative quantities
  - sole inefficiency stems from it supplying too little composite quantity

Market analysis

- Let

$$\kappa = \min_{x \geq 0} : \frac{c(x) - h(x)}{q(x)}$$

  - then optimal composite quantity Q maximizes

$$\alpha g(Q) + (1-\alpha) Q g'(Q) - \kappa Q$$

which satisfies the Lerner formula

$$\frac{g'(Q) - \kappa}{g'(Q)} = (1-\alpha)\eta(Q), \text{ where } \eta(Q) \equiv -\frac{Qg''(Q)}{g'(Q)}$$

Theorem

As α increases (or number of Cournot competitors increases), composite quantity increases, composite price decreases, each individual quantity increases equiproportionately, and each price-cost margin contracts equiproportionately

Cost passthrough

- Suppose $c(x) \equiv c \cdot x$, so that $\kappa = \phi(c)$ and $x^* = x^*(c)$
  - all vectors c with the same "composite cost" $\phi(c)$ induce seller to supply same composite quantity and same composite price
- If c_i increases, $\phi(c)$ increases, and so composite quantity decreases along with consumer surplus
  - so our class not rich enough to permit the "Edgeworth paradox", where a higher cost for a product induces firm to reduce all prices
- When $h(x) = a \cdot x$ the Ramsey prices are

$$p_i = \frac{c_i - (1-\alpha)\eta(Q)a_i}{1-(1-\alpha)\eta(Q)}$$

- So with constant η there is zero "cross-cost" passthrough in prices (though quantities are affected unless demands are independent)
- For instance, profit-maximizing ($\alpha = 0$) prices and quantities with linear demand ($\eta = -1$) are

$$p = \tfrac{1}{2}(a + c)$$

Optimal monopoly regulation

- Suppose monopolist has private information about its vector of constant marginal costs, c
  - regulator puts weight $0 \leq \beta \leq 1$ on profit relative to consumer surplus
  - can make transfer to firm to encourage higher quantity
- Look for situations where firm is given discretion over choice of relative quantities
  - cf. Armstrong (1996) and Armstrong & Vickers (2001)
- Consider hypothetical case:
  - regulator knows the efficient relative quantities x^* corresponding to the firm's costs, but not the (scalar) average level of costs, $\kappa = \phi(c)$
  - set of c with the same $x^* = x^*(c)$ is a straight line

Optimal monopoly regulation

- This scalar screening problem can be solved as Baron & Myerson
  - suppose regulator's prior for κ on this iso-x^* line has CDF $F(\kappa \mid x^*)$ and density $f(\kappa \mid x^*)$
  - then optimal quantities for type-κ firm are

$$\underbrace{Q\left(\kappa + (1-\beta)\frac{F(\kappa \mid x^*)}{f(\kappa \mid x^*)}\right)}_{\text{composite price}} \times \; x^*$$

  - each firm supplies the efficient relative quantities x^*
- If this regulatory scheme does not depend on x^* it is valid even when regulator cannot observe x^*
  - i.e., if distribution for cost vector c is such that $\phi(c)$ and $x^*(c)$ are stochastically independent the regulation problem can be solved
  - incentive scheme depends only on the firm's composite quantity
  - firm has freedom to choose its relative quantities

Optimal monopoly regulation

- To illustrate, suppose $u(x) = \sqrt{x_1} + \sqrt{x_2}$, so $q(x) = (\sqrt{x_1} + \sqrt{x_2})^2$, $h(x) \equiv 0$, $g(Q) = \sqrt{Q}$ and

$$\kappa = \phi(c) = \frac{1}{\frac{1}{c_1} + \frac{1}{c_2}}$$

  - method works if $\frac{1}{c_1} + \frac{1}{c_2}$ and $\frac{c_2}{c_1}$ are stochastically independent
  - e.g., if each $\frac{1}{c_i}$ independently comes from exponential distribution (so c_i comes from inverse-χ^2 distribution) optimal prices are

$$p_i = c_i \times [1 + (1 - \beta)\kappa(1 + \kappa)]$$

  - even very high-cost firms produce (so no "exclusion"), though regulated prices can be above unregulated monopoly prices
  - price for one product increases with other product's cost, even though costs are i.i.d.

Conclusions

- Insightful to consider multiproduct pricing problems by way of consumer surplus as a function of quantities
- Ramsey-Cournot equivalence result
- (Surprisingly?) broad class of demand systems with consumer surplus as a homothetic function of quantities is quite tractable
- Relative quantities are efficient
- Multiproduct cost passthrough results
- Conditions identified for optimal monopoly regulation to focus on the level but not the pattern of prices