Pundits and Quacks*

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Abstract

Do asset prices aggregate investors' private information about financial analysts? We show that in the medium run the market typically gets trapped: investors ignore their private information and blindly follow analyst recommendations. As time goes by and recommendations accumulate then – unless assets are too volatile – arbitrage based on the inferred analyst ability becomes profitable again. Thus while the market gets trapped in the medium run it may, in the long run, sort the pundits from the quacks.

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1 Introduction

The intensified division of labour taking place within the finance industry raises new and important challenges. Sell-side and independent financial analysts now account for a major part of the information used by investors to determine what stocks to trade.[1] According to Ramnath et al. (2008): "Analysts are viewed as sophisticated processors of financial information who are less likely than naive investors to misunderstand the implications of financial information". Yet while independent analysts have the potential to improve markets' efficiency, their existence paves the way for a textbook moral hazard: analysts take risks in making recommendations, but do not fully internalize the costs of those risks.[2] This could be unproblematic, if quack analysts were quickly driven out of the market. But empirical as well as anecdotal evidence suggests otherwise.[3] This paper formally investigates financial markets' performance at sorting the pundits from the quacks.

It is a well-known fact that markets can learn about analysts' ability based on public information regarding underlying assets.[4] This wisdom is important, but incomplete. Public information varies across time, and asset classes. Growth stocks, for instance, may take years before generating revenue. Financial markets must rely then on the private information of investors, in order to learn about analysts' ability.[5] The question is then: Do asset prices aggregate investors' private information about financial analysts? Alas, our paper shows that prices may aggregate investors' private information regarding analysts' ability, but in the long run only, i.e. when that information matters least.

[1]According to R.J. Wayman, Vice President and Portfolio Manager for Sweetwater Asset Management: "Independent research firms are becoming the main source of information on the majority of stocks". `http://www.investopedia.com/articles/analyst/03/031803.asp`.

[2]In short, analysts lack 'skin in the game'. Potential conflicts of interests is, in practice, another important problem. We abstract here from that issue. The interested reader is referred to Ramnath et al. (2008) for an extensive survey of that literature.

[3]The presence of persistent differences in the quality of analyst recommendations has been found in, for instance, Li (2005). Furthermore, pundits in television shows that make stock recommendations are often noted to be poor predictors of stock returns, although they have a significant short-term effect on trading. See for instance `http://en.wikipedia.org/wiki/Wall_Street_Week#The_Rukeyser_Effect`.

[4]See, e.g., Trueman (1994). To the extent that public information is available regarding underlying assets, comparing the predictions of analysts with the realized values of forecasted variables provides an immediate way of evaluating analyst ability.

[5]Investors' private information about analysts is exogenous in the model we consider. Throughout, we remain agnostic about the precise source of this information. Personal contacts is one possibility. The quality of the analyst's reports is another plausible source of information (Hirst et al. (1995) and Asquith et al. (2005) find evidence in this sense). For instance, an investor may discover that an analyst has copied-pasted the arguments he uses, or find flaws in the analyst's underlying reasoning; alternatively, the investor may deem the arguments laid forward by the analyst perceptive and original.

The basic model we analyze has the following features. An asset of unknown value is traded in a market with a single long-run player (the *analyst*) and an infinite sequence of short-run players (the *investors*). The analyst is of one of two types: informed or uninformed.[6] An informed analyst's recommendations are correlated with the asset's true value. A bad analyst, however, knows nothing more about the asset than the public information and makes strategic recommendations aiming to maximize his *reputation*.[7] A new investor arrives each period, possessing two pieces of private information: the latest recommendation from the analyst and, an imperfectly informative signal of the analyst's true ability. The investor then decides whether or not to follow the analyst's recommendation, given competitive bid and ask prices. A *liquidity trader* makes his decision based on factors exogenous to the model; a *speculator*, on the other hand, trades to maximize profits. The recommendation and the trade become known publicly when the period ends, at which point the market updates its belief that the analyst is informed (i.e., the reputation of the analyst). If all information could be centralized then the market would know the true type of the analyst. We explore whether, and how, the private information of investors concerning the analyst is aggregated in the decentralized market setting described above.

Unlike all previous studies, feedback about the analyst's ability is entirely endogenous.[8] Two channels in our model transmit information to the market about the analyst's true type: (i) investors' decision to follow or not the analyst's advice and, (ii) recommendations themselves. How much information each channel conveys is determined endogenously, and varies with time. We say that a *reputational trap* occurs when both channels are mute and no information about the analyst's true type is transmitted to the market in that period.

Our analysis of the basic model revolves around two principal results:

1. We show that in the medium run, the market typically gets trapped. Speculators follow blindly the recommendations made, and prices fail to aggregate investors' information

[6]We do not model the source of a good analyst's superior information directly. A good analyst may distinguish himself purely by his ability to analyze data, but could also retrieve information from his network of connections within monitored firms.

[7]The basic model assumes that the analyst maximizes his expected reputation one period ahead. We show in a later section that all our results extend to much broader settings, where the analyst maximizes the expected discounted sum of future income, and income in a given period is an increasing function of the analyst's reputation.

[8]In our model, feedback about the analyst's ability occurs solely through prices and recommendations made (the public information). In particular, assets' true values are never observed. All the paper's main results remain valid if public information about the asset's true value is available. This extension is considered in an online-appendix.

concerning analyst ability. The uninformed analyst thus maintains a lasting reputation and affect prices durably.

2. As time goes by and recommendations accumulate, arbitrage based on the (inferred) ability of the analyst becomes profitable again. Thus while the market gets trapped in the medium run, it sorts in the long run the pundits from the quacks.

The basic intuition behind the first point is the following. By raising the expected quality of advice, better reputation increases the average foregone profits from ignoring a recommendation. Hence when reputation is high, so too are foregone profits. Above a threshold reputation speculators must then choose, in equilibrium, to follow *all* advice stemming from the analyst.

The occurrence of reputational traps hinges upon three simple conditions: (a) non-zero measure of liquidity traders, (b) imperfectly informed investors, and (c) good analyst reputation. Without condition (a), prices would be 'too' elastic for a trap to occur: a speculator would then be unable to turn a profit from his private information. When the condition holds however, a speculator benefits from prices' relative inelasticity, and the expected profit from following the analyst's advice is increasing in the reputation of the analyst. Next, condition (b) ensures the willingness of speculators to attribute negative ability signals to sheer luck, and more so the better the reputation of the analyst. In view of the previous remarks, condition (c) is the final ingredient leading to a reputational trap: good reputation induces speculators to disregard their information concerning the analyst, and blindly follow the recommendations made.

While conditions (a) and (b) relate to the parameters of our model, analyst reputation evolves endogenously in our analysis. Since no restriction is placed on the initial reputation of the analyst, reputational traps will in general occur *after* the analyst has built a reputation for being informed, i.e., in the medium run only. Whether reputational traps are relevant in the short run too, depends on the analyst's initial reputation.

The occurrence of reputational traps naturally places a question mark over the possibility to learn the analyst's true type based solely on the private information of investors. Yet, reputational traps are, almost always, transient events. The intuition is the following. The recommendations of the informed analyst are correlated with the asset's true value. So if the analyst is informed, prices will eventually converge. This implies in turn that the uninformed analyst must either make prices converge too, or lose his reputation.[9] However, as prices

[9]This mechanism is reminiscent of, e.g., Prendergast and Stole (1996). Effectively, the analyst here makes

converge, speculators tend to gain less from trading based on the analyst's recommendations, all the while standing to gain more from arbitrage based on their private signal of the analyst: the more information contained in the public history, the more critical the knowledge of the true type of the analyst. An overwhelming dominance of 'buy' recommendations, say, will push prices up toward their highest value. On the other hand, a speculator with a negative ability signal will see his valuation of the asset revert toward the unconditional mean. The higher the price, the more the trader believes the asset to be overvalued. As prices get close to their upper bound, this will induce him to trade *against* historical trends (i.e. sell the asset), independently of the recommendation made. Speculators' actions then start reflecting again their private information concerning the analyst, at which point the market exits the trap it was in.

We go on to show that the market almost surely learns the analyst's true type. Among other things, our analysis of the long run provides important new insights relative to the work of, e.g., Benabou and Laroque (1992), or Cripps et al. (2004). In both papers, a long-run player is faced with a sequence of short-run players. The long-run player is either a commitment type – committed to playing action A, say – or a strategic type – with a short-run incentive to play action B, say. The long-run player's action is unobserved, but generates a public signal y distributed according to F^A if he takes action A, and F^B if he takes action B. Both papers assume that $F^A \neq F^B$, and that the distributions are fixed through time. These assumptions guarantee a positive lower bound on the equilibrium amount of information conveyed in any given round about the true type of the long-run player.[10] In our model, information about the analyst depends on the strategy of speculators and thus, in contrast to the aforementioned papers, there is no uniform positive lower bound regarding the information conveyed about the long-run player in any given round: it may be nothing (as in the case of a reputational trap), or close to nothing. It is prices' convergence which ultimately forces investors to release information concerning the true type of the analyst. *Learning about analysts is thus inseparable from the convergence of asset prices.* This key and novel feature of our analysis has important implications. We show for instance that if asset fundamentals are sufficiently volatile then learning about analysts may collapse in the medium *and in the long run.*[11] Markets are then

'persistent' recommendations in an attempt to convince the market that he is informed.

[10] In general, the distribution of y may also depend on the actions of the short-run players, but the assumptions made in both papers are such that there always is a positive lower bound on the information revealed in any given round.

[11] Our long-run learning results thus contrast sharply with those of, e.g., Benabou and Laroque (1992) and Trueman (1994) where learning is entirely unrelated to assets' underlying volatility. See Proposition 5 of the

forced to rely on public information regarding underlying assets, in order to sort the pundits from the quacks.

To derive empirical predictions, we then endogenize the decision of the analyst to make a recommendation at all, in any given period. First, we show that in this case the rate of recommendations evolves with time. The intuition is as follows. In general, the uninformed analyst will prefer to avoid making recommendations, so as to minimize the information released concerning his true ability. In a reputational trap however, the analyst knows that the market will follow his recommendations with probability one. He then needs not fear making recommendations. The rate of recommendations thus rises in a reputational trap, and falls outside it. Since reputational traps occur when reputation is good *and* prices moderate, the recommendation rate will increase under these conditions, rather than being monotonic in either of the variables.[12]

Second, our model predicts large and sudden asset price movements. The reason is simple. When the market is trapped, it stops aggregating the information concerning the ability of the analyst, but goes on accumulating information about the asset in the form of new recommendations (at an increased rate moreover, given previous remarks). Large price movements occur when the market starts learning the analyst's true ability again, and suddenly appreciates/depreciates all the information accumulated *during* the reputational trap.

Finally, the price impact from a recommendation is gradual in our model. A recommendation is first incorporated 'at a discount', and later adapted according to the evolution of the analyst's reputation. Past recommendations thus have a contemporaneous effect on prices, so long as the market goes on learning the true ability of the analyst.[13]

Related literature. This paper examines a previously unexplored aspect of financial markets' (in)efficiency: Do asset prices aggregate investors' private information about analysts? Whereas the amount of empirical work on financial analysts is enormous, theoretical contributions are relatively few. Two important exceptions are Benabou and Laroque (1992), and Trueman (1994).[14]

online appendix, and the discussion related to that result.

[12]To the best of our knowledge, no other paper makes predictions about the *rate* of recommendations of financial analysts.

[13]See, e.g., Michaely and Womack (2005) for empirical evidence of this phenomenon.

[14]Other papers include Admati and Pfleiderer (1986), and Ottaviani and Sørensen (2006). Admati and Pfleiderer (1986) examine a monopolistic analyst who, in order to overcome the dilution in the value of information due to its leakage through informative prices, may prefer to sell noisier versions of the information he actually has. In that paper, no uncertainty remains in equilibrium about the quality of the information

In Benabou and Laroque (1992) all analysts are informed, but engage in insider trading and have therefore strong short-run incentives to deceive the market. The uncertainty is about the 'honesty' of the analyst. Trueman (1994) studies a model of reputational cheap talk.[15] As in our paper good and bad analysts coexist, but trade plays no role (there are no traders). Uninformed analysts slant reports toward the prior, in an effort to appear informed.[16] In both Benabou and Laroque (1992) and Trueman (1994), public information about underlying assets is plentiful.[17] We aim, by contrast, to understand the extent to which markets may dispense with this public information in order to evaluate analysts' true ability.

Our paper is also related to the study of multidimensional uncertainty in financial markets explored in, e.g., Avery and Zemsky (1998) and Park and Sabourian (2011). However, the *structure* of the multidimensional uncertainty we analyze is entirely novel. In our model, the first layer of uncertainty is related to assets' true values. The second layer, on the other hand, pertains to the *quality* of information accumulated concerning assets' true values.

Finally, our model brings together two major strands of the economics literature: the market microstructure literature initiated by Glosten and Milgrom (1985), and the reputation in repeated games literature introduced by the work of Kreps and Wilson (1982) and Milgrom and Roberts (1982). To the best of our knowledge, our paper is the first to combine these separate frameworks into a single model.

The paper is organized as follows. Section 2 presents the basic model. Section 3 analyses the medium run, and Section 4 the long run. Section 5 extends the model to analyze the role played by the volatility of assets, to derive empirical implications, and to demonstrate the robustness of our results to more general specifications. Proofs of all results are presented in an appendix. We discuss further extensions of the model in an online-appendix.

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provided by the analyst. Ottaviani and Sørensen (2006) show that, in a winner-takes-all contest, analysts gain from moving away from the prior mean and that as a result, equilibrium forecasts tend to be overly differentiated. As in Trueman (1994) and Benabou and Laroque (1992), assets' true values are observed at the end of each period.

[15]Our setup is connected more generally to the extensive reputational cheap talk literature, where an agent of unknown ability aiming to maximize his reputation communicates his information to an uninformed principal. In all that literature however, the realization of the state is observed before the reputation updating. The price mechanism – when there is a price at all – plays no role in the feedback about the agent.

[16]See also Ottaviani and Sørensen (2006).

[17]Benabou and Laroque (1992) and Trueman (1994) assume that the underlying assets' true values are observed at the end of each period.

2 Basic model

The basic model has the following broad features. There is one long-run player (the analyst) and an infinite sequence of short-run players (the investors).[18] Each period, a new investor is given the option to trade an asset of unknown value based on the analyst's recommendation. The analyst may be a pundit or a quack. A pundit's recommendations are correlated with the asset's true value, but a quack is uninformed and makes strategic recommendations with a view to maximize his reputation. Each investor possesses a piece of information concerning the true type of the analyst. We now lay out the details and notation of this model.

Asset and Financial Analyst. The traded asset has fundamental value $\theta \in \{-1, 1\}$, and mean zero.[19] All supplementary information about the asset – if any – is provided by the *analyst* in the form of recommendations to sell/buy the asset; $r_t \in \{-1, 1\}$ denotes the recommendation made in period t.

The analyst is either informed or uninformed ('good' or 'bad'); τ denotes the analyst's true type: $\tau = G$ if he is good, $\tau = B$ if he is bad. His type is drawn at the beginning of the game and known only to himself: $\tau = G$ with probability $\lambda_0 \in (0, 1)$, and $\tau = B$ with complementary probability $1 - \lambda_0$. The parameter λ_0 defines the *reputation* of the analyst at the beginning of the game.

The informed analyst makes recommendations correlated with the asset's true value:[20]

$$\mathbb{P}(r_t = \theta | \tau = G) = \phi,$$

$\phi \in (1/2, 1)$. The recommendations of the good analyst are conditionally independent across time periods.

The bad analyst knows the information publicly available, but nothing more than that, and

[18]The market microstructure is a dealer model adapted from Glosten and Milgrom (1985).

[19]As in Glosten and Milgrom (1985), one may think of θ as the expected sum of discounted dividend payments, where the dividend d_t in period t takes a value in $\{-1, 0, 1\}$. The frequency φ of dividend payments, and the relative frequency of high/low dividends then together determine θ. As $\varphi \to 0$, public information about θ vanishes. See also the online appendix for a model with public information about the asset's true value.

[20]In the terminology of the reputation literature, the informed analyst is a behavior type. Implicit in our formulation is the assumption that the good analyst receives a sequence of signals correlated with the asset's true value, and commits to reveal those signals truthfully. We have opted for a simple formulation of the good analyst's behavior. In Rudiger and Vigier (2014) we allow the good analyst to be strategic as well, and show that there is an equilibrium in which he is truthful.

makes strategic recommendations with a view to maximize his reputation next period.[21] The analyst's reputation is updated using all public information (we later describe the updating process).[22]

Investors. A new investor arrives to the market each period.[23] The investor knows the public information (later specified), and two more pieces of information: the recommendation of the analyst, r_t, and a signal $s_t \in \{0, 1\}$ of the analyst's true ability distributed according to[24]

$$\mathbb{P}(s_t = 0|\tau = G) = 1 - \pi$$
$$\mathbb{P}(s_t = 0|\tau = B) = 1.$$

The signals are conditionally independent across investors. The parameter π captures the precision of the signal s_t. If π were zero then investors would always follow the recommendations made. We rule out this trivial and uninteresting scenario, by assuming $\pi > 0$. Observe too that if $\pi = 1$ then each investor knows the true type of the analyst.[25]

An investor is of one of two types, defining his motives for trading the asset. Each investor knows his own type, while other market participants have probabilistic beliefs about an investor's type. A fraction $\mu \in (0, 1)$ of investors maximize expected profits from trade, while remaining investors trade for exogenous motives, unrelated to profits. Call these investors *speculators* and *liquidity traders*, respectively. A liquidity trader buys, abstains or sells the asset with probability 1/3 each. We let $y_t \in \{a, n, b\}$ denote the trade of investor t: $y_t = a$ if he buys the asset, $y_t = n$ if he abstains, and $y_t = b$ if he chooses to sell the asset.[26]

[21] We assume that the expert is myopic, to ease the exposition. We relax the assumption in Section 5.3, and show that the main results and insights are all unaffected.

[22] Chen et al. (2005) find that the market's response to analysts' recommendations is consistent with investors learning about analysts' forecasting ability in a Bayesian fashion as more observations of past recommendations become available.

[23] As in Glosten and Milgrom (1985) investors are short-lived, for tractability.

[24] The specific signal structure chosen here plays no essential role, and merely simplifies the analysis. More general signal structures are considered in the online appendix.

[25] As indicated earlier, we remain largely agnostic in this paper about the source of the information summarized in s_t. Personal contacts is one possibility. The quality of the analyst's reports is another plausible source of information (Hirst et al. (1995) and Asquith et al. (2005) find evidence in this sense), or any combination of these and other sources. At any rate, we make no assumption about the strength of the signal s_t, other than $\pi > 0$.

[26] As in Glosten and Milgrom (1985) the size of trades is irrelevant, since speculators are risk neutral. It is assumed that each trade involves one unit of the asset.

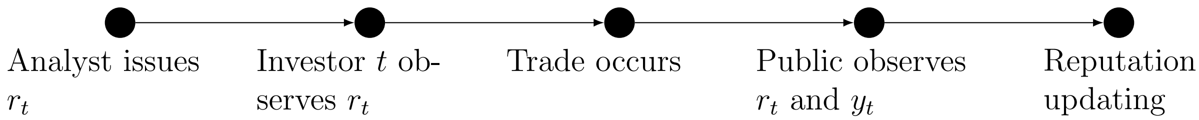

Figure 1: Timing within period t

Timing. The timing of the game within each period is as follows (c.f. Figure 1). Investor t observes the recommendation of the analyst, r_t, and chooses whether or not to follow this recommendation given the ask price and bid price offered. The investor then leaves the market. The recommendation r_t and trade y_t become known publicly when the period ends. This timing assumption is crucial, but reflects a real-life feature: analyst recommendations are first disclosed to client investors before being publicized (see, e.g., Michaely and Womack (2005)).[27] The market then revises its belief about the analyst and updates the prices posted next period.

Some notation is useful. We let $\mathcal{H}_t$ denote the public information at the beginning of period t. Following the timing above:

$$\mathcal{H}_t := \{\mathcal{R}_{t-1}, \mathcal{Y}_{t-1}\},$$

where $\mathcal{R}_{t-1} := (r_0, ..., r_{t-1})$ denotes the sequence of recommendations, and $\mathcal{Y}_{t-1} := (y_0, ..., y_{t-1})$ the sequence of trades, up to but excluding period t. $\mathbb{P}_t(\cdot) := \mathbb{P}(\cdot|\mathcal{H}_t)$ is the probability operator conditional on $\mathcal{H}_t$. $\mathbb{E}_t[\cdot] := \mathbb{E}[\cdot|\mathcal{H}_t]$ is similarly defined. The reputation of the analyst in period t, λ_t, is defined as the probability that the analyst is informed, given all public information at the beginning of period t:

$$\lambda_t := \mathbb{P}_t(\tau = G).$$

The (Bayesian) reputation updating equation is

$$\lambda_{t+1}(r_t, y_t) := \frac{\lambda_t \mathbb{P}_t(r_t, y_t|G)}{\lambda_t \mathbb{P}_t(r_t, y_t|G) + (1-\lambda_t)\mathbb{P}_t(r_t, y_t|B)}. \tag{1}$$

[27]In fact, powerful investors typically become aware of analysts' recommendations before these are even disclosed to clients. See, e.g., `http://www.nytimes.com/2014/01/09/business/blackrock-agrees-to-stop-pursuing-nonpublic-views.html`, or Rudiger and Vigier (2015) for a related theoretical study.

Strategies. The strategic players of our model are (i) the analyst of type $\tau = B$, and (ii) the speculators. The vector $\underline{\sigma}_{\mathcal{H}_t}$ denotes the strategy of the uninformed analyst in period t given history $\mathcal{H}_t$, with $\sigma^r_{\mathcal{H}_t}$ indicating the probability of recommending r. The vector $\underline{\xi}_{\mathcal{H}_t}$ denotes a speculator's strategy in period t given $\mathcal{H}_t$, with $\xi^y_{\mathcal{H}_t}(r_t, s_t)$ denoting the probability that he takes action y having observed recommendation r_t and signal s_t. To shorten notation, and when this is unlikely to create confusion, we will use $\underline{\sigma}_t, \underline{\xi}_t$ instead of $\underline{\sigma}_{\mathcal{H}_t}, \underline{\xi}_{\mathcal{H}_t}$.

Equilibrium. Prices are assumed competitive, reflecting all information available publicly at the time of trade.[28] This includes the trade order itself, and the behavior of all strategic players:

$$p^y_t = \mathbb{E}_t[\theta | y_t = y; \underline{\xi}_t, \underline{\sigma}_t]. \tag{2}$$

The structure of the game described above is common knowledge. A *competitive market equilibrium* is defined by sequences $\left(\underline{\xi}_{\mathcal{H}_t}\right)_{\mathcal{H}_t}$ and $\left(\underline{\sigma}_{\mathcal{H}_t}\right)_{\mathcal{H}_t}$ maximizing expected profits for speculators and expected reputation for the analyst of type $\tau = B$, with prices given by (2).

Definition 1. *The sequences* $\left(\underline{\xi}_{\mathcal{H}_t}\right)_{\mathcal{H}_t}$ *and* $\left(\underline{\sigma}_{\mathcal{H}_t}\right)_{\mathcal{H}_t}$ *constitute a* ***competitive market equilibrium*** *if and only if*

- $\xi^a_t(r_t, s_t) > 0 \Rightarrow \mathbb{E}_t[\theta | r_t, s_t; \underline{\sigma}_t] \geq p^a_t$, *with* p^a_t *given by (2).*
- $\xi^b_t(r_t, s_t) > 0 \Rightarrow \mathbb{E}_t[\theta | r_t, s_t; \underline{\sigma}_t] \leq p^b_t$, *with* p^b_t *given by (2).*
- $\mathbb{E}_t[\theta | r_t, s_t; \underline{\sigma}_t] > p^a_t \Rightarrow \xi^a_t(r_t, s_t) = 1$, *with* p^a_t *given by (2).*
- $\mathbb{E}_t[\theta | r_t, s_t; \underline{\sigma}_t] < p^b_t \Rightarrow \xi^b_t(r_t, s_t) = 1$, *with* p^b_t *given by (2).*
- $\sigma^r_t > 0 \Rightarrow r \in \arg\max_{r_t} \mathbb{E}_t[\lambda_{t+1}(r_t, y_t) | \tau = B; \underline{\sigma}_t, \underline{\xi}_t]$, *with* $\lambda_{t+1}(r_t, y_t)$ *given by (1).*

Part one (resp. part two) says that speculators do not buy the asset (resp. sell the asset) if they expect to lose money by doing so. Part three (resp. part four) says that speculators always buy the asset (resp. sell the asset) if by doing so they expect to make strictly positive profits. Part five says that an analyst of type $\tau = B$ only issues a recommendation if no other recommendation (on average) yields higher reputation next period.

[28] To avoid unnecessary cluttering, we do not model market makers explicitly in this paper.

3 Analysis: the medium run

We explore in this section the basic workings of our model and show that the market gets trapped in the medium run, failing to properly aggregate investors' private information concerning the ability of the analyst.

Denote by θ_t^G the asset's expected value in period t, assuming the analyst is informed. Let $\theta_t^G(r)$ denote the corresponding expected value, where we augment the information set with the recommendation made in period t. Thus

$$\theta_t^G := \mathbb{E}_t[\theta|\tau = G],$$
$$\theta_t^G(r) := \mathbb{E}_t[\theta|r_t = r, \tau = G].$$

Let q_t^r be the probability that the informed analyst recommends r in period t,

$$q_t^r := \mathbb{P}_t(r_t = r|\tau = G).$$

Straightforward calculations yield[29]

$$q_t^+ = \frac{1}{4}\left[1 + (2\phi - 1)\theta_t^G\right]$$
$$q_t^- = \frac{1}{4}\left[1 - (2\phi - 1)\theta_t^G\right].$$

Consider next the speculators. An investor possesses two pieces of information with which to make profits: the latest recommendation, and his private signal of the analyst's ability. Let $v_t(r, s)$ denote the valuation of an investor with recommendation r and ability signal s. An investor with $s_t = 1$ assigns probability 1 to the informed analyst. This yields

$$v_t(1, 1) = \theta_t^G(1),$$
$$v_t(-1, 1) = \theta_t^G(-1).$$

An investor with $s_t = 0$, on the other hand, updates his assessment of the analyst in two ways: first according to the likelihood $(1 - \pi)/1$ of a negative ability signal, and second according

[29] We use the short-hand notation q_t^+ and q_t^- for q_t^{+1} and q_t^{-1}. Similarly for σ_t^r.

to the likelihood $q_t^{r_t}/\sigma_t^{r_t}$ of the recommendation made; this yields

$$v_t(1,0) = \frac{\lambda_t q_t^+(1-\pi)}{\lambda_t q_t^+(1-\pi) + (1-\lambda_t)\sigma_t^+}\theta_t^G(1),$$
$$v_t(-1,0) = \frac{\lambda_t q_t^-(1-\pi)}{\lambda_t q_t^-(1-\pi) + (1-\lambda_t)\sigma_t^-}\theta_t^G(-1).$$

We next examine the ranking of the valuations derived above. Much of our model's interest springs from the fact that rather than being fixed, the ranking of the valuations – and, by way of consequence, the information reflected in prices – typically evolves over time. To help fix ideas we focus in what follows on histories where $\theta_t^G(-1) > 0$. This condition is satisfied whenever $\sum_0^{t-1} r_{t'} \geq 2$, which we will refer to as cases of 'bullish' history.

Claim 4 in the appendix establishes that, if history is bullish, then in any equilibrium:

$$v_t(-1,0) < \min\{v_t(-1,1), v_t(1,0)\}, \tag{3}$$
$$v_t(1,1) > \max\{v_t(-1,1), v_t(1,0)\}. \tag{4}$$

The intuition is straightforward. An investor with $(r_t = -1, s_t = 0)$ discounts the bullish history *and* receives a recommendation to sell the asset. At the other extreme, an investor with $(r_t = 1, s_t = 1)$ endorses the bullish history *and* receives a recommendation to buy the asset. These investors therefore respectively have the lowest and highest valuations.

We deduce from the former inequalities that, in equilibrium, a speculator with $(r_t = 1, s_t = 1)$ (resp. $(r_t = -1, s_t = 0)$) must buy the asset (resp. sell the asset) with probability 1, since he has the highest (resp. lowest) possible valuation. Speculators with $(r_t = 1, s_t = 0)$ or $(r_t = -1, s_t = 1)$, on the other hand, face a dilemma. A speculator with $(r_t = 1, s_t = 0)$, for instance, discounts the bullish history, but is simultaneously advised to buy the asset. In general, his trading decision will thus vary according to the relative weight attached to these two factors. The implications of his decision, however, are crucial for the transmission of information concerning the analyst. If he abstains from buying, the market will be able to distinguish him (statistically) from the investor with $(r_t = 1, s_t = 1)$, and thereby learn something about s_t and hence the analyst's true type. If he always buys the asset, however, the market will learn nothing concerning the analyst.

Our first theorem shows that, in the medium run, the market typically fails in equilibrium to aggregate the information of investors concerning the ability of the analyst, inducing

$$\mathbb{P}_t(\lambda_{t+1} = \lambda_t) = 1. \tag{5}$$

The basic intuition is the following. By raising the expected quality of advice, better reputation increases the average foregone profits from ignoring a recommendation. Hence when reputation is high, so too are foregone profits. Above a threshold reputation speculators must then choose, in equilibrium, to follow *all* advice stemming from the analyst.

Theorem 1. *If $\mu < 1$ and $\pi < 1$ then there exists a (unique) threshold reputation, $\hat{\lambda}(\mathcal{R}_t)$, such that above that threshold, in equilibrium the market fails to learn anything about the true type of the analyst:*

$$\mathbb{P}_t(\lambda_{t+1} = \lambda_t) = 1 \Leftrightarrow \lambda_t \geq \hat{\lambda}(\mathcal{R}_t).$$

The greater the mass of speculators, μ, and/or the greater the precision of speculators' information, π, the higher the threshold $\hat{\lambda}(\mathcal{R}_t)$.

Note a few important things. The threshold $\hat{\lambda}(\mathcal{R}_t)$ is a function of past recommendations only. The analyst's reputation in period t on the other hand, naturally depends on the entire history as well as the equilibrium considered. Theorem 1 says however that provided $\lambda_t \geq \hat{\lambda}(\mathcal{R}_t)$ then (5) must hold, independently of the equilibrium considered.

We refer to the situation captured by (5) as a *reputational trap.* The occurrence of a reputational trap hinges upon three simple conditions: (a) non-zero mass of liquidity traders ($\mu < 1$), (b) imperfectly informed investors ($\pi < 1$), and (c) high analyst reputation ($\lambda_t \geq \hat{\lambda}(\mathcal{R}_t)$). Condition (a) bounds the elasticity of prices. Consider a speculator with a positive recommendation, looking to buy the asset. In the absence of liquidity traders, a 'lemons' situation ensues in which the market unravels: prices adjust, and only the speculator with the highest valuation would ever buy the asset. But a speculator's trade order would then reveal his information of the analyst since, as we saw earlier, a speculator with $(r_t = 1, s_t = 1)$ always has the highest valuation. Condition (a) is thus necessary for the market to get trapped.[30] Conditions (b) and (c) are complementary. When they hold, speculators willingly attribute negative ability signals to sheer luck. This allows them to disregard their information concerning the analyst, and blindly follow the recommendations made.

[30] When $\mu = 1$ there is also a 'no trade' equilibrium in which the market breaks down, and a trap trivially ensues. Condition (a) is necessary for the result to hold in *all* equilibria.

The remaining of this section elaborates the various steps leading to Theorem 1, and derives a closed-form expression of the threshold $\hat{\lambda}(\mathcal{R}_t)$ defined in the statement of the theorem. We first construct an equilibrium in which (5) holds, above a threshold reputation (Lemma 1). The second step shows that the threshold found in step one is in fact 'uniform', i.e. applies across all possible equilibria. This step is hard; the reader who wishes to skip the details of the analysis can go directly to the next section.

Two channels in our model transmit information in equilibrium to the market about the analyst's true type. First, speculators' decision to follow or not the analyst's advice. Second, recommendations themselves. Information about the analyst's ability is transmitted through the recommendations because the extent to which speculators follow the analyst's advice will in general not be uniform across the different possible recommendations. Therefore, as the bad analyst wishes to minimize the information channeled regarding his type, he will favor the recommendations most likely to be followed by the speculators. In turn, in equilibrium the recommendations themselves will generally be informative about the true type of the analyst.

The next definitions formalize the distinction made between the two channels identified above.

Definition 2. *Given trading strategy $\underline{\xi}_t$, say that **screening is efficient** if trades fully reveal private signals of the analyst's ability, in the following sense: $\xi_t^y(r,1) > 0 \Rightarrow \xi_t^y(r,0) = 0$, $\forall\, y, r$.*

Definition 3. *Given trading strategy $\underline{\xi}_t$, say that **screening breaks down** if trades are uninformative about private signals of the analyst's ability, in the following sense: screening breaks down on the positive side (resp., negative side) if there exist $y \in \{a, b, n\}$ such that $\xi_t^y(1,1) = 1 = \xi_t^y(1,0)$ (resp., such that $\xi_t^y(-1,1) = 1 = \xi_t^y(-1,0)$). Say that screening breaks down if it does so on both the positive and the negative side.*

Consider an equilibrium and history $\mathcal{H}_t$, and suppose (5) holds. Clearly, screening must break down in the equilibrium considered. If it did not, then the investor's trade in period t would act as a (noisy) signal of the private information s_t regarding the ability of the analyst, and (5) would fail to hold. The next observation establishes the converse of this result.

Observation 1. *Consider an equilibrium and history $\mathcal{H}_t$ where screening breaks down in period t. Then (5) holds.*

To see why the observation is true, suppose for the sake of contradiction that (5) does not hold. The recommendation made in period t must then be informative about the analyst, i.e.

the equilibrium must be such that $\underline{\sigma}_t \neq \underline{q}_t$. Let r and r' such that $\sigma_t^r > q_t^r$ and $\sigma_t^{r'} < q_t^{r'}$. In that case (1) yields $\lambda_{t+1}(r, y) < \lambda_t$ and $\lambda_{t+1}(r', y) > \lambda_t$, for all y. Recommending r' is thus unambiguously more advantageous for the uninformed analyst. This is inconsistent with the uninformed analyst maximizing his expected reputation, and so (5) must hold, by contradiction.

Observation 1 stresses the strategic implications of our model. Consider for comparison the case of an uninformed analyst mechanically (uniformly) randomizing between buy and sell recommendations, i.e. behaving according to $(\sigma_t^-, \sigma_t^+) = (1/2, 1/2)$ regardless of the history. Suppose the history were bullish: the market would then expect the informed analyst to be bullish too. But then, an analyst who emits buy and sell recommendations with equal probability will (on average) reveal himself by appearing bearish, comparatively. Thus, in that case, in contrast to our model screening may break down and yet some information be conveyed in equilibrium to the market about the true type of the analyst.

We now ask: When might speculators choose to ignore their private information of the analyst's true type? Continue assuming history is bullish. We noted earlier that when history is bullish then, in equilibrium, a speculator with $(r_t = -1, s_t = 0)$ sells the asset with probability 1. Similarly, a speculator with $(r_t = 1, s_t = 1)$ buys the asset with probability 1. Screening thus breaks down if and only if (a) speculators with $(r_t, s_t) = (1, 0)$ buy with probability 1, while (b) speculators with $(r_t, s_t) = (-1, 1)$ sell with probability 1. We next derive the necessary and sufficient conditions for (a) and (b) to hold in equilibrium.

Given competitive prices – and in view of Observation 1 – for (a) to be an equilibrium requires

$$\frac{\lambda_t(1-\pi)q_t^+}{\lambda_t(1-\pi)q_t^+ + (1-\lambda_t)q_t^+} \cdot \theta_t^G(1) \geq \frac{\gamma q_t^+}{\gamma q_t^+ + 1 - \gamma} \cdot \lambda_t \theta_t^G(1) + \frac{1-\gamma}{\gamma q_t^+ + 1 - \gamma} \cdot p_t^0,$$

where $\gamma := \frac{\mu}{\mu + (1-\mu)/3}$ and $p_t^0 := \mathbb{E}_t[\theta]$. The left-hand side of this inequality is $v_t(1, 0)$, when $\sigma_t^+ = q_t^+$. The right-hand side is obtained as follows. With probability μq_t^+ investor t is a speculator observing $r_t = 1$. With probability $(1-\mu)/3$, he is a liquidity trader looking to buy the asset. In the former case the expected asset value is $\lambda_t \theta_t^G(1)$. In the latter case, it is simply the public valuation p_t^0. We find, by substituting $p_t^0 = \lambda_t \theta_t^G$, that the inequality above is satisfied for $\lambda_t \geq \hat{\lambda}^+(\mathcal{R}_t)$, where

$$\hat{\lambda}^+(\mathcal{R}_t) = 1 - \frac{1-\pi}{\pi} \cdot \frac{(1-\gamma)\left(1 - \frac{\theta_t^G}{\theta_t^G(1)}\right)}{\gamma q_t^+ + (1-\gamma)\frac{\theta_t^G}{\theta_t^G(1)}}. \tag{6}$$

Similarly, for (b) to be an equilibrium requires

$$\theta_t^G(-1) \leq \frac{\gamma q_t^-}{\gamma q_t^- + 1 - \gamma} \cdot \lambda_t \theta_t^G(-1) + \frac{1-\gamma}{\gamma q_t^- + 1 - \gamma} \cdot p_t^0.$$

Substituting again $p_t^0 = \lambda_t \theta_t^G$, this inequality is satisfied for $\lambda_t \geq \hat{\lambda}^-(\mathcal{R}_t)$, where

$$\hat{\lambda}^-(\mathcal{R}_t) = 1 - \frac{(1-\gamma)\left(1 - \frac{\theta_t^G(-1)}{\theta_t^G}\right)}{\gamma q_t^- \frac{\theta_t^G(-1)}{\theta_t^G} + 1 - \gamma}. \tag{7}$$

Setting $\hat{\lambda}(\mathcal{R}_t) = \max\{\hat{\lambda}^+(\mathcal{R}_t), \hat{\lambda}^-(\mathcal{R}_t)\}$ yields the next lemma.

Lemma 1. *Consider history $\mathcal{H}_t$ such that $\lambda_t \geq \hat{\lambda}(\mathcal{R}_t)$. Then an equilibrium exists in which (5) holds.*

As is usual in models of strategic communication, multiple equilibria typically exist in the model we explore. This remark puts a question mark on the scope of our first lemma. Our next result addresses the issue, and shows that if an equilibrium exists in which the market gets trapped then this must in fact be the unique equilibrium, given that history.

Lemma 2. *Consider history $\mathcal{H}_t$ and suppose an equilibrium exists in which (5) holds. Then (5) holds in any equilibrium, given that history.*

The proof of the lemma contains various steps. We here sketch the main arguments.[31] Let $E1$ denote the trapped equilibrium, and suppose another equilibrium exists, $E2$ say. We proceed to analyze the second equilibrium.

Denote by $\underline{\sigma}_1$ and $\underline{\sigma}_2$ the strategy played by the uninformed analyst in these equilibria, respectively. First, suppose that $\sigma_2^+ > \sigma_1^+$. The expected reputation from issuing a positive recommendation must then be lower in $E2$ than in $E1$: one, a positive recommendation is more likely to be sent by the bad analyst in $E2$ than in $E1$; two, there can be no less screening in $E2$ than in $E1$, since in $E1$ screening altogether breaks down.

[31] A detailed proof is given in the appendix.

The key step of the proof is then as follows. We show that *the less often a recommendation is issued by the uninformed analyst, the less speculators gain from screening it.* Since no screening occurs in $E1$, and $\sigma_2^- < \sigma_1^-$, then screening must break down on the negative side in $E2$. But then the expected reputation from issuing a sell recommendation is greater in $E2$ than it is in $E1$. Combining this observation with our first remark above finally shows that in $E2$, the expected reputation from recommending $r_t = -1$ must be greater than the expected reputation from recommending $r_t = 1$. But this is inconsistent with equilibrium.

These arguments thus establish $\underline{\sigma}_1 = \underline{\sigma}_2$. The proof is concluded by showing that in $E2$, screening breaks down too, and that prices in the two equilibria are identical. The two equilibria are thus the same.

Combining Lemmas 1 and 2 shows that if $\lambda_t \geq \hat{\lambda}(\mathcal{R}_t)$ (where $\hat{\lambda}(\mathcal{R}_t) = \max\{\hat{\lambda}^+(\mathcal{R}_t), \hat{\lambda}^-(\mathcal{R}_t)\}$, given by (6) and (7)), then (5) holds in equilibrium. That $\hat{\lambda}(\mathcal{R}_t)$ is increasing in μ and in π can be seen immediately upon inspection of (6)-(7). The comparative statics is intuitive. The less the mass of liquidity traders, the more responsive the prices to the trade orders. This in turn discourages speculators to ignore their signal of the analyst's ability, reducing thereby the occurrence of reputational traps. Similarly, an increase in the parameter π raises the precision of investors' information regarding the analyst, thus discouraging the former to ignore that information. These remark conclude the proof of Theorem 1.

4 Analysis: the long run

We examine in this section the long-run properties of our model. We establish that the reputational traps uncovered in Section 3 are transient events, and that the market can in the long run sort the pundits from the quacks based solely on the private information of investors.

The occurrence of reputational traps naturally places a question mark over the possibility to learn the analyst's true type and, in turn, over the possibility for prices to converge to the right value. The following lemma is a key result of our long-run analysis.

Lemma 3. *Reputational traps are transient events, a.s., in any competitive market equilibrium.*

The basic intuition is as follows. When the market is trapped, the uninformed analyst can hide his type insofar as he pretends to be good. If he were good, however, then prices would converge to the asset's true value. This forces the analyst of type $\tau = B$ to make prices converge too, or else lose his reputation. However, as prices converge, speculators tend to gain less from trading based on the recommendations and, all the while, stand to gain more from arbitrage based on their information of the analyst's true ability. To be sure, the signal s_t determines the 'discount factor' of *all* past recommendations. Hence, the more information contained in the public history, the more critical the signal of the analyst's true type. An overwhelming dominance of 'buy' recommendations, say, will push prices up toward their highest value. A speculator with a negative ability signal on the other hand will see his valuation of the asset revert toward the unconditional mean. Hence, the higher the price the more the speculator will view the asset as overvalued. As prices approach their upper bound, this will induce him to trade *against* historical trends (i.e. sell the asset), independently of the recommendation made. By acting as contrarians in the long run, speculators thus eventually release information about the true type of the analyst.

We go on to show that the market almost surely learns the type of the analyst. Learning of the analyst's true type is perhaps surprising in the model we explore and, at any rate, contrasts sharply with the classic analysis of Benabou and Laroque (1992), or Cripps et al. (2004). In both papers, a long-run player is faced with a sequence of short-run players. The long-run player is either a commitment type – committed to playing action A, say – or a strategic type – with a short-run incentive to play action B, say. The long-run player's action is unobserved, but generates a public signal y distributed according to F^A if he takes action A, and F^B if he takes action B. Both the aforementioned papers assume that $F^A \neq F^B$, and that the distributions are fixed through time. These assumptions guarantee a positive lower bound on the equilibrium amount of information conveyed in any given round about the true type of the long-run player.[32] In our model, information about the analyst depends on the strategy of speculators and thus, in contrast to the aforementioned papers, there is no uniform positive lower bound regarding the information conveyed about the analyst in any given round: it may be nothing (as in the case of a reputational trap), or close to nothing. It is prices' convergence which ultimately forces investors to release information concerning the true type of the analyst.[33]

Theorem 2. *The market a.s. learns the analyst's true type in the long run. In particular:*

1. *Conditional on type $\tau = G$, prices converge almost surely to the true asset value θ.*

2. *Conditional on type $\tau = B$, prices converge almost surely to zero.*

[32]In general, the distribution of y may also depend on the actions of the short-run players, but the assumptions made by both papers are such that there always is a positive lower bound on information revealed.

[33]This key feature of our analysis is exploited in Section 5.1 to show a counterpart to Theorem 2.

Markets can learn about analysts' ability if public information regarding underlying assets is available. The necessity of this information to learn about analysts in the medium run was established in Theorem 1. Theorem 2, by contrast, shows that public information may be dispensed with in the long run. The next section further qualifies this result. There, we show that high asset volatility hampers learning about analysts in the long run too. Financial markets performance at sorting the pundits from the quacks based on investors' private information may therefore be poor in the medium, *and in the long run.*

5 Some extensions of the basic model

We extend in this section the basic model along three broad directions, with a view to explore some issues related to the central theme of our paper. First, Section 5.1 examines the impact of assets' underlying volatility. Section 5.2 endogenizes the decision to make new recommendations. Section 5.3, finally, investigates the impact from expanding the time horizon of the uninformed analyst. Other, more trivial, extensions of the basic model are developed in an online-appendix.[34]

5.1 Fundamental volatility and learning analyst ability

We assumed in the basic model that the asset's fundamental value (θ) was fixed through time. We now show that relaxing this assumption worsens considerably the market's ability to learn the true type of the analyst in the long run, expanding thereby the scope of our paper's main insights.[35] The intuition is simple. In the framework we explore above, whenever the market is in a reputational trap, historical information embedded in prices accumulates and eventually takes precedence over the current recommendation of the analyst. When this occurs, investors start turning to their private information with a view to evaluate the credence of this historical information. At that point, the market exits the trap (c.f. Section 4). By contrast, when the fundamental value of the asset is very volatile, current recommendations continue to be highly informative relative to historical information. This allows the analyst to sustain his grip on the market.

[34]This includes: adding public information about the asset's true value (e.g. dividend payments), allowing more general structures for the private information of investors, and adding other assets and/or analysts to the model.

[35]Introducing fundamental volatility effectively *prolongs* the influence of the medium-run effects uncovered in Section 3.

Let θ_t denote the fundamental value in period t. The basic model of Section 2 is retrieved by setting $\theta_t = \theta$, $\forall t$. We consider next the limit case in which the model has i.i.d. fundamental values.[36] The random variables $\{\theta_t\}$ are distributed independently, with mean zero, and take values in $\{-1, 1\}$. Let in what follows $\hat{\lambda} = \max\{\lambda_0, 1 - 2\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\}$.

Proposition 1. *With i.i.d. fundamental values, the market's ability to learn the true type of the analyst collapses, and reputation is (uniformly) bounded above. There exists $\varepsilon > 0$ such that, for all $\mathcal{H}_t$, $\lambda_t < \hat{\lambda} + \varepsilon < 1$ and :*

$$\mathbb{P}(\lim \lambda_t = 1 | \tau = G) = 0$$
$$\mathbb{P}(\lim \lambda_t = 0 | \tau = B) < 1.$$

Proposition 1 underscores the importance of the remarks made on the role which prices play in bringing about learning of the analyst's true type. When the asset's fundamental value is very volatile, prices have no time to accumulate historic information, and the 'value' of speculators' information about analyst ability remains constant. In that case, if and when the analyst gains sufficient reputation, speculators will follow his recommendations always, resulting in an indefinite trap.

5.2 New recommendations and prices' behavior

We endogenize in this subsection the decision of the analyst to make a recommendation or not, and explore the implications of our model regarding trading activity and prices' behavior over time. We show that trading activity peaks when the market is trapped, and that the occurrence of reputational traps can explain sudden and large price movements. Finally, we look at how our model is consistent with the empirical fact that previous recommendations may exert a contemporaneous effect on prices.

We have so far assumed that the good analyst receives each period a signal correlated with the asset's true value. This has forced the bad analyst to make recommendations each period in order to avoid losing his reputation. In practice, naturally, even a good analyst may in some periods fail to possess new information concerning the asset. Enhancing the model in this way opens up important new strategic considerations for the bad analyst, who may

[36] This simple model is studied, e.g., in Benabou and Laroque (1992). It amounts to trading a new asset each period. While admittedly unrealistic, the i.i.d. model allows us to convey an essential insight in a simple way.

now decide in any given period whether or not to make a recommendation at all. Avoiding recommendations is attractive insofar as it minimizes information released about one's true type, but may in equilibrium arise suspicion regarding the ability of the analyst. We next explore the implications of this trade-off.

Formally, the basic model is modified as follows. Each period the analyst now makes a choice $r_t \in \{-1, 0, 1\}$, where $r_t = 0$ is used to indicate the absence of a recommendation in period t. The good analyst is informed (i.e. makes a recommendation) on average half of the time, so that

$$\mathbb{P}(r_t = 0 | \tau = G) = 1/2$$
$$\mathbb{P}(r_t = \theta | \tau = G) = \phi/2,$$

where $\phi \in (1/2, 1)$. The rest of the model is as described in Section 2, except that now if no recommendation is made in period t, then trader t remains uninformed about the true type of the analyst.[37] It is precisely this feature which makes it attractive for the bad analyst to avoid making a recommendation.

We begin by showing that the rate at which the analyst makes recommendations varies with time.[38] The intuition is as follows. So long as speculators screen the recommendations, the uninformed analyst will prefer to avoid making recommendations, so as to minimize the information released concerning his true ability. In a reputational trap however, the analyst knows that the market will follow his recommendations with probability one. He then needs not fear making new recommendations. The rate of recommendations thus rises when the market is trapped.

Observation 2. *The rate of recommendations varies with time, reaching a maximum when the market is trapped.*

We next record an important implication of the former observation. The arrival of new recommendations evidently provides speculators with incentives to trade based on the information provided by the analyst. Thus more recommendations typically spur more trade. The probability that a recommendation induces trade however, is normally less than one, unless

[37]To be sure, the absence of a recommendation may in itself be informative in equilibrium about the true type of the analyst. But s_t is unobserved unless $r_t \neq 0$. This assumption is justified by the fact that the quality of analyst reports is informative about analyst ability, and an analyst who does not make a recommendation needs not write a report either.

[38]Strictly speaking, only the rate at which the *bad* analyst makes new recommendations varies with time.

namely the market finds itself in a reputational trap. We thus obtain:

Observation 3. *Trading activity varies with time, and peaks when the market is trapped.*

We turn next to the implications of our model regarding prices' behavior. Let $d_t = \left|\sum_0^{t-1} r_{t'}\right|$ record the 'net' amount of information concerning the asset accumulated up to period t.

Observation 4. *The greater the information accumulated about the asset, the more sensitive the price to the reputation of the analyst. Formally:*

$$\frac{\partial}{\partial d_t}\left(\left|\frac{\partial p_t^0}{\partial \lambda_t}\right|\right) > 0.$$

Observation 4 allows us to see why the occurrence of reputational traps can explain sudden and large price movements. As the market gets trapped it stops aggregating information about the analyst, but goes on accumulating recommendations stemming from the analyst (at an increased rate moreover, given Observation 2). All information accumulated *during* the reputational trap is then re-evaluated at once the instant the market exits the trap, and learning about the analyst's true type again starts operating. Interestingly, rather than being caused by the release of new information (as in, e.g., Lee (1998)), the 'price crash' depicted here results from the *simultaneous depreciation of all past accumulated information.*

We complete this section with an important remark. Prices take time in the model we explore to fully incorporate the recommendations of the analyst. Here, a recommendation is first incorporated 'at a discount', and later adapted according to the evolution of the analyst's reputation.[39] Past recommendations thus affect prices contemporaneously, so long as the market goes on learning the true ability of the analyst.

Observation 5. *All past recommendations have a contemporaneous effect on the price, unless the market is trapped, in which case only the latest recommendation affects the price.*

5.3 Farsighted analyst

We relax in this subsection the 'myopia' of the strategic analyst, and show that the results obtained in the basic model approximate well those of the more general framework considered here.

[39] See, e.g., Michaely and Womack (2005) for empirical evidence of such behavior.

Consider the basic model of Section 3, modified by the assumption that the uninformed analyst now maximizes the expected discounted flow of his future income, where income in a given period is an increasing function of the analyst's reputation in that period. Let δ denote the discount factor and $f(.)$ an increasing function relating income to reputation. Each period t, the analyst of type $\tau = B$ maximizes $\mathbb{E}_t[\sum_{s=t+1}^{\infty} \delta^{s-(t+1)} f(\lambda_s)|B]$, where $f'(\cdot) > 0$. The basic model of Section 3 is retrieved by setting $\delta = 0$.

The basic forces inducing screening to break down in the framework we explore are largely unaffected by the time horizon of the strategic analyst. As a sketch argument, suppose the uninformed analyst plays $\underline{\sigma}_t$ in period t, and let $m_t(\sigma_t^+) = \lambda_t q_t^+ + (1-\lambda_t)\sigma_t^+$. Assuming as usual a bullish history, screening will break down on the positive side if

$$\frac{\lambda_t(1-\pi)q_t^+}{\lambda_t(1-\pi)q_t^+ + (1-\lambda_t)\sigma_t^+} \cdot \theta_t^G(1) \geq \frac{\gamma\lambda_t q_t^+}{\gamma m_t(\sigma_t^+) + 1 - \gamma} \cdot \theta_t^G(1) + \frac{1-\gamma}{\gamma m_t(\sigma_t^+) + 1 - \gamma} \cdot p_t^0.$$

Recall $q_t^+ \geq 1 - \phi > 0$. So if $\pi < 1$ and $\mu < 1$ (implying $\gamma < 1$) then as λ_t goes to one, the left-hand side goes to $\theta_t^G(1)$, and the right-hand side goes to some strictly convex combination of $\theta_t^G(1)$ and θ_t^G, which is obviously less than $\theta_t^G(1)$. Thus, for any σ_t^+ there exists a threshold for λ_t, above which the highlighted inequality will hold. A uniform threshold is then retrieved by letting σ_t^+ span $[0,1]$.

Consider next the effect of the time horizon on the behavior of the strategic analyst. If screening were constant, i.e. if speculators' strategies were time-invariant, then an increase in today's reputation would induce a first-order stochastic dominance shift of reputation in any future period. The objective of the analyst in that case would reduce to maximizing reputation one period ahead. In the framework we explore, however, the degree of screening carried out by speculators varies with time. The time horizon will therefore in principle affect the behavior of the strategic analyst, who may be willing to sacrifice reputation today in order to reap the benefits from less screening some time in the future.

We next illustrate (informally first, then with a proposition) why these effects are unimportant for the qualitative results of our paper. Suppose that the analyst looks forward n periods, i.e. the analyst cares about his stream of income (and hence, his reputation) starting from period $t+1$ all the way to period $t+n$. Define $\hat{\lambda}_k(\mathcal{R}_t)$, $k \in \{1,...,n\}$, as follows. First, set $\hat{\lambda}_1(\mathcal{R}_t) = \hat{\lambda}(\mathcal{R}_t)$, the threshold elicited in Section 3. Set then, recursively, $\hat{\lambda}_k(\mathcal{R}_t) = \max\{\hat{\lambda}_{k-1}(\mathcal{R}_t, +1), \hat{\lambda}_{k-1}(\mathcal{R}_t, -1)\}$. The idea is the following. If $\lambda_t \geq \hat{\lambda}_n(\mathcal{R}_t)$ then screening will break down in at least the next n periods. But the analyst cares about n periods only. So if today his reputation is above $\hat{\lambda}_n(\mathcal{R}_t)$ then effectively, from his perspective, the strategy of speculators *is* time-invariant. This simple example thus suggests how, by raising appropriately the threshold reputation of the analyst, results and insights obtained in the basic model do carry over as we extend the time horizon of the strategic analyst.

We next summarize the remarks of the previous paragraphs in a formal proposition, and show in addition that the long-run behavior of the basic model is wholly unaffected by the time horizon of the strategic analyst.

Proposition 2. *Let $\delta \in [0, 1)$, $\mu < 1$, and $\pi < 1$. Fix $\epsilon > 0$. There exists a threshold reputation $\hat{\lambda}(\mathcal{R}_t, \delta)$ such that above that threshold:*

$$\mathbb{P}_t\big(|\lambda_{t+1} - \lambda_t| < \epsilon\big) = 1.$$

As δ tends to zero, $\hat{\lambda}(\mathcal{R}_t, \delta) \to \hat{\lambda}(\mathcal{R}_t)$, the threshold reputation of Theorem 1 obtained in the basic model. Furthermore, in the long run, the market almost surely learns the analyst's true type. In particular:

1. *Conditional on type $\tau = G$, prices converge almost surely to the true asset value θ.*

2. *Conditional on type $\tau = B$, prices converge almost surely to zero.*

Results obtained in the basic model thus approximate well those of the more general framework considered here.

6 Conclusion

Do asset prices aggregate investors' private information concerning financial analysts? We uncovered in this paper a market failure previously ignored, by showing that investors' private information concerning financial analysts is typically not aggregated into prices in the medium run. This phenomenon – which we coin *reputational trap* – could allow incompetent analysts to sustain a good reputation longer than they should, and implies that bad analysts will tend to cause substantial short-term price fluctuations: in a bullish market, a bad analyst drives prices up when the market is trapped, inducing sudden price reversals as (and if) the market eventually learns the true type of the analyst.

We also present several results with regards to the long-run convergence of prices. The difficulty lies in the manner in which the market provides feedback about analyst ability. Effectively, due to the existence of reputational traps, the market's feedback about analysts is not 'bounded away from zero'. It turns out that the driver of convergence in our model is the price mechanism itself, which ensures that the more information becomes released about an asset, the more profitable for speculators to screen analysts. However, when assets' fundamentals are very volatile then historic information rapidly depreciates. The 'value' of speculators' information about analysts remains roughly constant, in that case. Such conditions may induce perpetual, or very long, reputational traps.

7 Appendix

In order to avoid repeating very similar proofs twice, we will work in this appendix with the slightly more general model of Section 5.2, i.e. we allow the analyst not to make a recommendation in a given period.

To shorten notation, we will make use throughout the appendix of

$$\gamma := \frac{\mu}{\mu + (1-\mu)/3},$$

$$p_t^0 = \mathbb{E}_t[\theta],$$

$$\lambda_{t+1}^e(r, \underline{\sigma}_t, \underline{\xi}_t) := \mathbb{E}_t[\lambda_{t+1}(r_t = r, y_t) | \tau = B; \underline{\sigma}_t, \underline{\xi}_t].$$

The parameter γ records the weight of speculators relative to liquidity traders taking any given action (buy/sell/abstain). The variable p_t^0 is the public valuation at the beginning of period t. The variable $\lambda_{t+1}^e(r, \underline{\sigma}_t, \underline{\xi}_t)$ finally, denotes the uninformed analyst's expected reputation next period from recommending r today, when his strategy is $\underline{\sigma}_t$ and speculators' strategy is $\underline{\xi}_t$.

We will, wherever possible, drop time subscripts in order to unclutter notation. The following equilibrium conditions, stating that the uninformed analyst is indifferent between all recommendations, are easily established using standard arguments:

$$\lambda^e(-1, \underline{\sigma}, \underline{\xi}) = \lambda^e(0, \underline{\sigma}, \underline{\xi}) = \lambda^e(1, \underline{\sigma}, \underline{\xi}). \tag{8}$$

We begin with a series of preliminary results. Claims 1-3 establish the intuitive results that more screening from speculators and more aggressive behavior from the uninformed analyst induce worse expected reputation for the analyst. Claim 4 orders speculators' valuations, in equilibrium. Claims 5 and 6 are instrumental in proving equilibrium uniqueness.

Claim 1. *Consider $\underline{\sigma}$ such that $\sigma^r > 0$, $\forall r$.*

1. *If $\underline{\xi}_1$ entails a break-down of screening on the positive side then $\lambda^e(1, \underline{\sigma}, \underline{\xi}_1) \geq \lambda^e(1, \underline{\sigma}, \underline{\xi}_2)$, with strict inequality unless $\underline{\xi}_2$ entails a break-down of screening on the positive side too.*

2. *If $\underline{\xi}_1$ entails a break-down of screening on the negative side then $\lambda^e(-1, \underline{\sigma}, \underline{\xi}_1) \geq \lambda^e(-1, \underline{\sigma}, \underline{\xi}_2)$, with strict inequality unless $\underline{\xi}_2$ entails a break-down of screening on the negative side too.*

Proof of Claim 1: Let, for $\mathbb{P}(y|r, \underline{\xi}, B) > 0$:

$$L(y|r, \underline{\xi}) = \frac{\mathbb{P}(y|r, \underline{\xi}, G)}{\mathbb{P}(y|r, \underline{\xi}, B)}.$$

Note that

$$\mathbb{E}[L(y|r, \underline{\xi})|r, \underline{\xi}, B] = \sum_{y:\mathbb{P}(y|r,\underline{\xi},B)>0} \mathbb{P}(y|r, \underline{\xi}, B) \frac{\mathbb{P}(y|r, \underline{\xi}, G)}{\mathbb{P}(y|r, \underline{\xi}, B)} = 1.$$

Using Bayes' rule:

$$\lambda^e(r, \underline{\sigma}, \underline{\xi}) = \mathbb{E}[M_{\sigma^r}(L(y|r, \underline{\xi}))|r, \underline{\xi}, B], \tag{9}$$

where $M_{\sigma^r}(x) = \frac{\lambda q^r x}{\lambda q^r x + (1-\lambda)\sigma^r}$ is concave.

Observe next that if $\underline{\xi}_1$ entails a break-down of screening on the positive side then $L(y|r = 1, \underline{\xi}_1) = 1$ for all y. So either $\underline{\xi}_2$ entails a break-down of screening on the positive side too or, conditional on the expert being bad, the distribution of $L(y|r = 1, \underline{\xi}_2)$ is a mean-preserving spread of the distribution of $L(y|r = 1, \underline{\xi}_1)$. In the latter case we obtain, by (9) and concavity of M_{σ^+}, $\lambda^e(1, \underline{\sigma}, \underline{\xi}_2) < \lambda^e(1, \underline{\sigma}, \underline{\xi}_1)$.

The proof of Part 2 of the Claim is similar, and omitted.

■

Claim 2. *If $\underline{\xi}_1$ entails a break-down of screening then for any $\underline{\xi}_2$:*

1. $\sigma_2^+ > \sigma_1^+ \Rightarrow \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1) > \lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2)$.

2. $\sigma_2^- > \sigma_1^- \Rightarrow \lambda^e(-1, \underline{\sigma}_1, \underline{\xi}_1) > \lambda^e(-1, \underline{\sigma}_2, \underline{\xi}_2)$.

Proof of Claim 2: Using the same notation as in the proof of Claim 1:

$$\begin{aligned}\lambda^e(1,\underline{\sigma}_2,\underline{\xi}_2) &= \mathbb{E}[M_{\sigma_2^+}\big(L(y|r=1,\underline{\xi}_2)\big)|r=1,\underline{\xi}_2,B] \\ &< \mathbb{E}[M_{\sigma_1^+}\big(L(y|r=1,\underline{\xi}_2)\big)|r=1,\underline{\xi}_2,B] \\ &= \lambda^e(1,\underline{\sigma}_1,\underline{\xi}_2) \\ &\leq \lambda^e(1,\underline{\sigma}_1,\underline{\xi}_1).\end{aligned}$$

The first inequality follows from the fact that $M_{\sigma_2^+}(x) < M_{\sigma_1^+}(x)$ for all x. The last inequality is an application of Claim 1.

The proof of Part 2 of the Claim is similar, and omitted.

■

Claim 3. *Let $r, r' \in \{-1, 1\}$. If for $y \neq \hat{y}$ we have $\xi^y(r,0) = \xi^{\hat{y}}(r,1) = 1$ and moreover $\frac{\sigma^r}{q^r} > \frac{\sigma^{r'}}{q^{r'}}$, then $\lambda^e(r',\underline{\sigma},\underline{\xi}) > \lambda^e(r,\underline{\sigma},\underline{\xi})$.*

Proof of Claim 3: The proof is similar to that of Claim 2 and is therefore omitted.

■

We next prove equations (3) and (4) of Section 3. Recall that $v(r,s)$ denotes the valuation of an investor with recommendation r and ability signal s. By extension, $v(0)$ is used to denote the valuation of an investor without a recommendation.[40]

Claim 4. *Let $\theta^G(-1) > 0$. Then, in any equilibrium:*

1. *$v(1,1)$ is (strictly) the highest of all valuations.*
2. *$v(-1,0)$ is (strictly) the lowest of all valuations.*
3. *$v(-1,0) < p^0 < v_t(1,1)$.*

In particular, in any equilibrium: $\xi^a(1,1) = 1$ and $\xi^b(-1,0) = 1$.

Proof of Claim 4: Part 1 is immediate. We prove the second part. Let $(\underline{\sigma},\underline{\xi})$ denote an arbitrary equilibrium strategy-pair.

[40] Recall in particular that under the assumptions of Section 5.2, s_t is unobserved unless $r_t \neq 0$.

Step 1: $v(-1,0) < v(0)$. Let $\beta(r,s)$ denote an investor's updated belief of the analyst's type after observing (r,s). We have:

$$\lambda^e(0,\underline{\sigma},\underline{\xi}) = \lambda^e(-1,\underline{\sigma},\underline{\xi}) > \beta(-1,0).$$

Step 1 now follows, since:

$$v(-1,0) = \beta(-1,0)\theta^G(-1) < \lambda^e(0,\underline{\sigma},\underline{\xi})\theta^G(-1) = \beta(0)\theta^G(-1) < \beta(0)\theta^G(0) = v(0).$$

Step 2: $v(-1,0) < v(1,0)$. Suppose, for the sake of contradiction, that $v(-1,0) \geq v(1,0)$. Then $\beta(-1,0) > \beta(1,0)$, and so $\sigma^-/q^- < \sigma^+/q^+$. But $v(-1,0) \geq v(1,0)$ also implies (using Step 1) that $v(1,0)$ is the lowest valuation, in which case $\xi^b(1,0) = 1$. Since $\xi^a(1,1) = 1$ (by Part 1 of the Claim), then Claim 3 yields $\lambda^e(1,\underline{\sigma},\underline{\xi}) < \lambda^e(-1,\underline{\sigma},\underline{\xi})$. But this is impossible, in equilibrium.

To prove part 3 of the claim, note that $v(-1,0) \geq p^0$ implies $\beta(-1,0) > \lambda$. But then $\lambda^e(-1,\underline{\sigma},\underline{\xi}) > \lambda$, which is impossible by (8). That $p^0 < v(1,1)$ is immediate.

■

Our next result establishes a key step in the proof of Lemma 2.

Claim 5. *Let $\theta^G(-1) > 0$, and E_i, $i = 1,2$, two equilibria. Let $\Delta\sigma^r := \sigma_2^r - \sigma_1^r$. If $\Delta\sigma^- < 0$, and $\Delta\sigma^0, \Delta\sigma^+ \geq 0$, then $p_1^b \leq p_2^b$.*

Proof of Claim 5: We give here a very general proof of the result. A less abstract (but longer) proof can be found in Rudiger and Vigier (2014). The proof we give requires some notation and terminology, which we now define. Let $I := \{0,...,n\}$. Consider a discrete random variable θ, with realizations $\{\theta_i\}_{i\in I}$, $\theta_i < \theta_j$ for $i < j$, and probability distribution $G_i := \mathbb{P}(\theta = \theta_i)$, $i \in I$. Define a signal s by the non-negative weights $\{W_i(s)\}_{i\in I}$, where $W_i(s) \leq G_i$, $\forall i$. Let moreover $W(s) := \sum_I W_i(s)$, and $E(s) := \sum_I \frac{W_i(s)}{W(s)}\theta_i$. Given two signals s and s' with $W_i(s) + W_i(s') \leq G_i$, $i \in I$, define a new signal $s + s' := \{W_i(s) + W_i(s')\}_{i\in I}$. Similarly, given two signals s and s' with $W_i(s') \leq W_i(s)$, $i \in I$, define a new signal $s - s' := \{W_i(s) - W_i(s')\}_{i\in I}$. The following property follows directly: $E(s+s') = \frac{W(s)}{W(s)+W(s')}E(s) + \frac{W(s')}{W(s)+W(s')}E(s')$.

The interpretation should be clear. A signal s is obtained by taking probability mass from the different realizations of the random variable θ; $W(s)$ is the total probability that s obtains given the prior on θ, and $E(s)$ is the posterior mean of θ when s is observed. We now make the link with the model of this paper. At the beginning of an arbitrary period where $\theta^G(-1) > 0$, the asset's true value θ can be viewed as a discrete random variable with (ordered) realizations $\{0, \theta^G(-1), \theta^G(0), \theta^G(-1)\}$, and probability distribution $G_0 = 1 - \lambda$, $G_1 = \lambda q^-$, $G_2 = \lambda q^0$, and $G_3 = \lambda q^+$. The probability mass is then split into 6 different signals: s_a, corresponding to the information of speculators with $(r, s) = (-1, 0)$, s_b corresponding to the information of speculators with $(r, s) = (-1, 1)$, s_c corresponding to $r = 0$, s_d corresponding to $(r, s) = (1, 0)$, s_e corresponding to $(r, s) = (1, 1)$, and $\tilde{s}$ corresponding to the information of liquidity traders. The signals are easily computed as functions of $\underline{\sigma}$ (and of the primitives of the model). The signal s_a, e.g., is given by $W_0(s_a) = \mu\sigma^-$, $W_1(s_a) = \mu(1 - \pi)q^-$, and $W_2(s_a) = W_3(s_a) = 0$. The signal $\tilde{s}$ is obtained by retrieving probability mass uniformly from the prior distribution. Hence: $W_i(\tilde{s}) = (1 - \mu)G_i$, $\forall i$.

Now fix a candidate bid price $z \in [-1, 1]$. For any z, we can define a new signal $\hat{s}_z$ as

$$\hat{s}_z := \frac{1}{3}\tilde{s} + \sum_{E(s_k)<z} s_k.$$

Intuitively, the signal $\hat{s}_z$ corresponds to the information available given that a sell order was passed at bid price $p^b = z$. Note in particular that the unique equilibrium bid price – for $\underline{\sigma}$ fixed – is given by $\inf\{z : E(\hat{s}_z) \leq z\}$.

We can now conclude the proof of the claim. Suppose there exist two equilibria, E and E' say, where E entails $\underline{\sigma}$ and E' entails $\underline{\sigma}'$, and which satisfy the conditions laid out in the statement of the claim, i.e. such that $\underline{\sigma}'$ may be obtained from $\underline{\sigma}$ by shifting weight away from $r = -1$ and onto other recommendations. Observe that, for any $z \in [-1, 1]$, we can decompose

$$\hat{s}'_z = \left(\hat{s}_z - \underline{s}\right) + \overline{s}_z,$$

where $W_0(\underline{s}) = W(\underline{s}) = \sigma^- - \sigma'^-$, and $E(\overline{s}_z) \geq z$. The intuition is the following. As we move from $\underline{\sigma}$ to $\underline{\sigma}'$ we shift 'bad' probability mass from the signal with the worse posterior, s_a, to signals with better posteriors, s_c and s_d.[41] Either $E(s'_c)$ and/or $E(s'_d)$ fall below z, or they don't. If they don't then $\hat{s}'_z$ is obtained from $\hat{s}_z$ by simply removing some probability mass originating from G_0. If they do then $\hat{s}'_z$ is obtained from $\hat{s}_z$ by adding s_c and s_d; but in that case, by construction: $E(s_c)$, $E(s_d) \geq z$.

[41] C.f. Claim 4.

Observe next that

$$E(\hat{s}'_z) \geq E(\hat{s}_z + \overline{s}_z) \geq \frac{W(\hat{s}_z)}{W(\hat{s}_z) + W(\overline{s}_z)} E(\hat{s}_z) + \frac{W(\overline{s}_z)}{W(\hat{s}_z) + W(\overline{s}_z)} z,$$

and hence:

$$E(\hat{s}_z) \geq z \Rightarrow E(\hat{s}'_z) \geq z.$$

We obtain finally, using an earlier remark, that $p^{b'} \geq p^b$, and the claim is established.

■

The next claim was proven within the proof of Claim 5. We state it here for the record. In what follows, for a given equilibrium, let $\underline{p}$ denote the vector of bid and ask prices given by (2).

Claim 6. *Prices are uniquely determined by the strategy of the uninformed analyst. Let E_i, $i = 1, 2$ denote two equilibria. Then:*

$$\underline{\sigma}_1 = \underline{\sigma}_2 \Rightarrow \underline{p}_1 = \underline{p}_2.$$

Proof of Lemma 2: As usual we work the proof for the case where $\theta_t^G(-1) > 0$. Other cases can be treated similarly. Let E_i, $i = 1, 2$ denote two equilibria, and assume that in E_1 screening breaks down. The proof has four steps.

Step 1: $\sigma_2^+ \geq \sigma_1^+$. Step 1 is proved by contradiction. Suppose $\sigma_2^+ < \sigma_1^+$. Then $\xi_2^a(1,0) = \xi_1^a(1,0) = 1$. Hence, from Claim 2: $\lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1) = \lambda$. Now either $\sigma_2^0 > \sigma_1^0$ (call this Case 1), or $\sigma_2^- > \sigma_1^-$ (call this Case 2). Case 1 immediately gives $\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1) = \lambda$. Case 2 gives $\lambda^e(-1, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(-1, \underline{\sigma}_1, \underline{\xi}_1) = \lambda$, where we have made use again of Claim 2. Either way, we obtain a contradiction with (8) in the second equilibrium.

Step 2: $\sigma_2^- \geq \sigma_1^-$. The proof of Step 2 is again by contradiction. Suppose $\sigma_2^- < \sigma_1^-$. Three cases must be considered: $\Delta\sigma^0 \geq 0$ and $\Delta\sigma^+ \geq 0$ (Case 1), $\Delta\sigma^0 > 0$ and $\Delta\sigma^+ < 0$ (Case 2), $\Delta\sigma^0 < 0$ and $\Delta\sigma^+ > 0$ (Case 3). In Case 1, Claim 5 yields $p_2^b \geq p_1^b$. Hence (generically) $\xi_2^b(-1,1) = \xi_1^b(-1,1) = 1$, from which we obtain using Claim 2 that $\lambda^e(-1, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(-1, \underline{\sigma}_1, \underline{\xi}_1) = \lambda$. But then we have a contradiction with (8) in the second equilibrium since

$\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) \leq \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1) = \lambda$. Consider Case 2 next. Reproducing the arguments of Step 1 yields $\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1) = \lambda = \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1) < \lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2)$. Again, we obtain a contradiction with (8) in the second equilibrium. Consider finally Case 3. Using Claim 2 yields $\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1) = \lambda = \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1) > \lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2)$. Yet again, we obtain a contradiction with (8) in the second equilibrium.

Step 3: $\underline{\sigma}_2 = \underline{\sigma}_1$. If not, then combining Steps 1 and 2 yields $\sigma_2^0 < \sigma_1^0$ and hence $\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1) = \lambda$. This necessarily contradicts (8) in the second equilibrium, since the bad analyst is unable to improve his reputation on average.

Step 4: $E_2 = E_1$. Combining Step 3 and Claim 6 yields $\underline{p}_1 = \underline{p}_2$. So $\underline{\xi}_1 = \underline{\xi}_2$ (generically).

■

Before proving Theorem 2, we establish a series of claims. Claim 7 establishes a lower bound on the rate at which the uninformed analyst issues recommendations each period. Claim 8 lower-bounds the probability that a significant jump in reputation occurs, whenever screening is efficient. Claim 9 shows that unless reputation converges to zero or one then screening will eventually become efficient. Claim 10 concludes that reputation converges to either zero or one, almost surely. Finally, Claim 11 shows that reputation converges to the 'correct' value. With these results in hand, we then prove the theorem.

Claim 7. *There exists $\delta > 0$ such that in any equilibrium, for any history $\mathcal{H}_t$ and recommendation r: $\sigma_t^r > \delta$.*

Proof of Claim 7: Notice that due to liquidity traders we can find $\ell > 0$ such that for any r_t, y_t and $\underline{\xi}_t$ then (using notation from Claim 1):

$$\ell^{-1} < L_t(y_t | r_t, \underline{\xi}_t) < \ell.$$

Note also that for any history $\mathcal{H}_t$ and any recommendation r then $q_t^r \in [\frac{1-\phi}{2}, \frac{1}{2}]$. Thus for any $\mathcal{H}_t$, r_t, y_t and $\underline{\xi}_t$:

$$\frac{\mathbb{P}_t(y_t, r_t | \underline{\sigma}_t, \underline{\xi}_t, G)}{\mathbb{P}_t(y_t, r_t | \underline{\sigma}_t, \underline{\xi}_t, B)} = \frac{q_t^{r_t} \mathbb{P}_t(y_t | r_t, \underline{\xi}_t, G)}{\sigma_t^{r_t} \mathbb{P}_t(y_t | r_t, \underline{\xi}_t, B)} > \frac{(1-\phi)\ell^{-1}}{2\sigma_t^{r_t}}.$$

Hence:

$$\sigma_t^{r_t} < \frac{1-\phi}{2}\ell^{-1} \Rightarrow \frac{\mathbb{P}_t(y_t, r_t|\underline{\sigma}_t, \underline{\xi}_t, G)}{\mathbb{P}_t(y_t, r_t|\underline{\sigma}_t, \underline{\xi}_t, B)} > 1.$$

The claim now follows by setting $\delta = \frac{1-\phi}{2}\ell^{-1}$, since if the bad analyst were ever to issue recommendation r with less than probability δ, then he could, by recommending r, improve his reputation with certainty. But this is impossible in equilibrium.

■

Claim 8. *For all $\tilde{\lambda} \in (0,1)$, there exists $\varepsilon_{\tilde{\lambda}} > 0$ and $\delta_{\tilde{\lambda}} > 0$ with the following property: if $|\lambda_t - \tilde{\lambda}| < \delta_{\tilde{\lambda}}$ then, in any equilibrium where screening is efficient:*

$$\mathbb{P}_t(|\lambda_{t+1} - \lambda_t| \geq \varepsilon_{\tilde{\lambda}}) \geq b,$$

where $b > 0$ is independent of $\tilde{\lambda}$.

Proof of Claim 8: Let $\tilde{\lambda} \in (0,1)$, $\lambda_t = \tilde{\lambda}$, and consider an equilibrium where screening is efficient. We will show that we can find $\varepsilon_{\tilde{\lambda}}$, and b independent of $\tilde{\lambda}$ such that the highlighted equation of the claim holds.

One recommendation $r^* \in \{-1, 1\}$ must have $\sigma_t^{r^*} \leq q_t^{r^*}$ (this follows from Observation 2). Let y^* denote the action of the speculator with $(r_t = r^*, s_t = 1)$. Since screening is efficient: $\xi_t^{y^*}(r^*, 0) = 0$. Using (1) yields

$$\begin{aligned}
\lambda_{t+1}(r_t = r^*, y_t = y^*) &= \frac{\lambda_t q_t^{r^*}(\gamma\pi + (1-\gamma))}{\lambda_t q_t^{r^*}(\gamma\pi + (1-\gamma)) + (1-\lambda_t)\sigma_t^{r^*}(1-\gamma)} \\
&\geq \frac{\lambda_t(\gamma\pi + (1-\gamma))}{\lambda_t(\gamma\pi + (1-\gamma)) + (1-\lambda_t)(1-\gamma)} \\
&> \lambda_t.
\end{aligned}$$

Let $\varepsilon_{\tilde{\lambda}}$ be defined by the difference between the final two terms of the right-hand side:

$$\varepsilon_{\tilde{\lambda}} = \frac{\tilde{\lambda}(\gamma\pi + (1-\gamma))}{\tilde{\lambda}(\gamma\pi + (1-\gamma)) + (1-\tilde{\lambda})(1-\gamma)} - \tilde{\lambda}.$$

Then $\varepsilon_{\tilde{\lambda}}$ and $b = \frac{1-\mu}{3} \cdot \min\{\delta, \frac{1-\phi}{2}\}$ together satisfy the conditions we were looking for, with δ defined by Claim 7.

The claim follows by continuity of $\varepsilon_{\tilde{\lambda}}$ (defined above) as a function of $\tilde{\lambda}$.

■

The sequence $\{\lambda_t\}$ is a bounded martingales with respect to the filtration $\{\mathcal{H}_t\}$. The Martingale Convergence Theorem thus applies. We let in what follows the random variable λ denote the (a.s.) limit of the sequence $\{\lambda_t\}$.

Claim 9. *Let W denote the event $\lambda \notin \{0,1\}$. Then, conditional on W, screening eventually becomes efficient, a.s..*

Proof of Claim 9: The process $\{\theta_t^G(0)\}$ is a bounded martingale with respect to the filtration $\{\mathcal{H}_t\}$, under $\tau = G$. The Martingale Convergence Theorem thus applies. Moreover, clearly, $\lim_{t\to\infty} \theta_t^G(0) \in \{-1,1\}$. This implies, in turn, that either $\lambda = 0$, or $\lim_{t\to\infty} \theta_t^G(0)$ exists and takes values in $\{-1,1\}$. The random variable $\lim_{t\to\infty} \theta_t^G(0)$ is thus well-defined under W. Denote $\theta^G := \lim_{t\to\infty} \theta_t^G(0)$, where the random variable is now defined over the entire W.

We condition henceforth on W. Suppose to fix ideas that $\theta^G = 1$ (the case $\theta^G = -1$ is similar and omitted). Then, for any $\epsilon > 0$, we can find a time from which point onwards $|\theta_t^G(r) - 1| < \epsilon$, for all r. Choosing ϵ small enough yields from some time onwards $v_t(-1,1) = \theta_t^G(-1) > \lambda_t \theta_t^G = p_t^0$, since by assumption the limit $\lambda < 1$. Next, applying Claim 4 gives

$$v_t(-1,0) < p_t^b < p_t^0 < v_t(-1,1),$$

and shows that screening eventually becomes efficient (a.s.) on the negative side.

We proceed to show that screening eventually becomes efficient on the positive side too. Applying again Claim 4, observe that

$$p_t^a \geq \frac{1-\gamma}{(1-\gamma)+\gamma\lambda_t\pi} p_t^0 + \frac{\gamma\lambda_t\pi}{(1-\gamma)+\gamma\lambda_t\pi} \theta_t^G(1).$$

As $\lambda > 0$, from some time onwards p_t^a is thus bounded away from p_t^0. But then $v_t(1,0) \geq p_t^a$ must imply $\beta_t(1,0) > \lambda_t$ (recall that $\beta(r,s)$ denotes an investor's updated belief of the analyst's type after observing (r,s)), which in turn implies $\lambda^e(1, \underline{\sigma}_t, \underline{\xi}_t) > \lambda_t$, contradicting (8). Hence $v_t(1,0) < p_t^a$, and screening eventually becomes efficient on the positive side too.

■

Claim 10. *Let $\lambda = \lim\limits_{t\to\infty} \lambda_t$. Then $\lambda \in \{0,1\}$, a.s..*

Proof of Claim 10: Let $W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$ denote the event $|\lambda - \tilde{\lambda}| < \delta_{\tilde{\lambda}}/2$, where $\tilde{\lambda} \in (0,1)$ and $\delta_{\tilde{\lambda}}$ as defined in Claim 8. Clearly, we can choose $\delta_{\tilde{\lambda}}$ such that $W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2) \subset W$, where as before W denotes the event $\lambda \in (0,1)$. Using Claim 9, for any $\omega \in W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$ we can (almost surely) define a smallest time $T(\omega)$ such that for all $s \geq T(\omega)$: (a) screening is efficient and (b) $|\lambda_s - \lambda| < \delta_{\tilde{\lambda}}/2$. Let $V_k = \{\omega \in W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2) : T(\omega) = k\}$. Then $W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2) = \bigcup V_k$. Furthermore, applying Claim 8:

$$\mathbb{P}\big(|\lambda_{s+1} - \lambda_s| \geq \varepsilon_{\tilde{\lambda}} | V_k\big) \geq b \ , \quad s \geq k.$$

But $\{\lambda_t\}$ converges a.s., and hence also in probability. Hence, $\mathbb{P}\big(V_k\big) = 0$, for all k, and ultimately $\mathbb{P}\big(W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)\big) = 0$.

Let $B(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$ denote the open ball with center $\tilde{\lambda}$ and radius $\delta_{\tilde{\lambda}}/2$. For any n, the interval $[1/n, 1 - 1/n]$ has an open cover consisting of open balls $B(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$, $\tilde{\lambda} \in [1/n, 1-1/n]$. By compactness, we can extract a finite sub-cover $\Big(B(\tilde{\lambda}_s, \delta_{\tilde{\lambda}_s}/2)\Big)_{s=1}^{S}$. Then

$$\mathbb{P}(\lambda \in [1/n, 1-1/n]) \leq \mathbb{P}\Big(\lambda \in \bigcup B(\tilde{\lambda}_s, \delta_{\tilde{\lambda}_s}/2)\Big) \leq \sum \mathbb{P}\Big(\lambda \in B(\tilde{\lambda}_s, \delta_{\tilde{\lambda}_s}/2)\Big) = 0.$$

This being true for all n, we finally obtain $\mathbb{P}(\lambda \in \{0,1\}) = 1$.

■

Claim 11. *The market learns the expert's type, almost surely. Let $\lambda = \lim_{t\to\infty} \lambda_t$: if $\tau = G$ then $\lambda = 1$ a.s.; if $\tau = B$ then $\lambda = 0$ a.s..*

Proof of Claim 11: Consider first $\tau = B$. In any equilibrium:

$$\begin{aligned}
\mathbb{E}_t\Big[\frac{\lambda_{t+1}}{1-\lambda_{t+1}}\Big|B\Big] &= \sum_{r_{t+1}, y_{t+1}} \mathbb{P}_t(r_{t+1}, y_{t+1}|B) \cdot \frac{\lambda_t \mathbb{P}_t(r_{t+1}, y_{t+1}|G)}{(1-\lambda_t)\mathbb{P}_t(r_{t+1}, y_{t+1}|B)} \\
&= \frac{\lambda_t}{1-\lambda_t} \sum_{r_{t+1}, y_{t+1}} \mathbb{P}_t(r_{t+1}, y_{t+1}|G) \\
&= \frac{\lambda_t}{1-\lambda_t}.
\end{aligned}$$

Hence $\mathbb{E}\Big[\frac{\lambda_t}{1-\lambda_t}\Big|B\Big] = \frac{\lambda_0}{1-\lambda_0}$, for all t. Fatou's Lemma then gives $\mathbb{E}\Big[\frac{\lambda}{1-\lambda}\Big|B\Big] \leq \frac{\lambda_0}{1-\lambda_0}$. Hence $\mathbb{P}(\lambda = 1|B) = 0$.

Similar derivations give $\mathbb{E}\left[\frac{1-\lambda_t}{\lambda_t}\middle|G\right] = \frac{1-\lambda_0}{\lambda_0}$, for all t, and so $\mathbb{E}\left[\frac{1-\lambda}{\lambda}\middle|G\right] \leq \frac{1-\lambda_0}{\lambda_0}$. Hence $\mathbb{P}(\lambda = 0|G) = 0$.

Claim 11 now follows immediately, by application of Claim 10.

■

Proof of Theorem 2: If the expert is a good expert then the Law of Large Numbers yields $\lim_{t\to\infty} \theta_t^G(0) = \theta$ a.s.. Both parts of the theorem now follow from Claim 11.

■

Proof of Proposition 1: The threshold value $\hat{\lambda}$ is retrieved from (6)-(7), setting $\theta_t^G = 0$. Any value ε no less than the maximum reputational jump of an analyst with current reputation $\hat{\lambda}$ will satisfy the condition stated in the proposition.

■

Proof of Observations 2-4: If $\sigma_t^0 < q_t^0$ then $\lambda_t^e(0, \underline{\sigma}_t, \underline{\xi}_t) > \lambda_t$, contradicting (8). Hence $\sigma_t^0 \geq q_t^0 = 1/2$, with strict inequality unless the market is trapped. This gives Observation 2.

Next, note that for any history $\mathcal{H}_t$ and in any equilibrium

$$\mathbb{P}_t(y_t \in \{a,b\}) = \frac{2(1-\mu)}{3} + \mu \sum_{(r,s)} \mathbb{P}_t(r,s)\left[\xi_t^a(r,s) + \xi_t^b(r,s)\right],$$

where by definition: $\xi_t^a(r,s) + \xi_t^b(r,s) \leq 1$, for all (r,s). When the market is trapped this last inequality becomes an equality. This gives Observation 3.

Observation 4 is immediate, since $p_t^0 = \mathbb{E}_t[\theta] = \lambda_t \theta_t^G$.

■

Proof of Proposition 2: The arguments of the proof for break-down of screening follow along the lines of Theorem 1. Let $\beta_t(r,s)$ denote an investor's updated belief of τ from observing $(r_t, s_t) = (r,s)$. Observe that for all $r \in \{-1,1\}$ and all t, $q_t^r \geq 1-\phi > 0$. Thus, if $\pi < 1$, we can find $\epsilon > 0$ such that $|1-\lambda_t| < \epsilon/2 \Rightarrow |1-\beta_t(r,s)| < \epsilon$, for all $(r,s) \in \{-1,1\} \times \{0,1\}$. Letting ϵ tend to zero, $v_t(r,s)$ then approaches $\theta_t^G(r)$, for all $(r,s) \in \{-1,1\} \times \{0,1\}$. p_t^a (resp. p_t^b), by contrast, is bounded away from $\theta_t^G(1)$ (resp. $\theta_t^G(-1)$), so long as $\mu < 1$. These arguments establish that if $\pi < 1$ and $\mu < 1$ then screening necessarily breaks down, above a threshold level of reputation.

We next establish an analogue of Observation 1 from Section 3 and show that we can find $\epsilon(.) : [0, 1) \to \mathbb{R}_+$, $\lim_{\delta\to 0} \epsilon(\delta) = 0$, such that if in equilibrium screening breaks down then $|\lambda_{t+1} - \lambda_t| \leq \epsilon(\delta)$. Fix δ, and consider an equilibrium in which screening breaks down. λ_{t+1} is then a deterministic function of r_t, which we may write $\lambda_{t+1}(r)$ for $r_t = r$. Moreover, $|\lambda_{t+1}(r) - \lambda_{t+1}(-r)|$ is strictly increasing in $\|\underline{q}_t - \underline{\sigma}_t\|$. It is now immediate to see from the form of the analyst's payoffs that fixing $\|\underline{q}_t - \underline{\sigma}_t\| > 0$ and letting δ tend to zero must either make payoffs from recommending $r_t = 1$ strictly greater than payoffs from recommending $r_t = -1$, or the converse. This is impossible, in equilibrium. Hence, for each $\delta \in [0, 1)$ we can find $\eta(\delta) > 0$, such that $\|\underline{q}_t - \underline{\sigma}_t\| < \eta(\delta)$, in any equilibrium. This in turn delivers, for each $\delta \in [0, 1)$, an $\epsilon(\delta) > 0$ such that $|\lambda_{t+1} - \lambda_t| \leq \epsilon(\delta)$, in any equilibrium. Furthermore $\lim_{\delta\to 0} \eta(\delta) = 0$ and so, evidently, $\lim_{\delta\to 0} \epsilon(\delta) = 0$.

■

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